Mereotopology

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Mereology

Mereolgy is the branch of ontology in charge of studying the parthood relation. It has three fundamental principles:

- Objects have hierarchically structured and spatially articulated parts.
- Objects are made of matter, which is located where they are. Put otherwise, objects occupy space.
- Objects can be classified into whole and disconnected.

Wholeness & connectedness

How is a intact cup different from a shattered one?

- They have the same parts, so mereologically they are equal.
- Connection between parts

Three strategies

- Whitehead: only admit connected wholes
- Topology (A): Use topology as a ground theory and define mereology over it.
- Topology (B): Use mereology for parthood and topology for connectedness.

Whitehead's approach

x is connected to y $\iff$ there is z overlapping both x and y and there is no part of z that doesn't overlap either x or y. (see blackboard)

Circularity problem

Top (A)

Clarke(1981): x part of y $\iff$ everything connected to x is also connected to y

- P1) Insistence of regions $\rightarrow$ neglect of contact points, boundaries...
- P2) Either of these must hold
  - All spatial entities can be mapped onto their regions
  - Topological regions apply to events and objects alike

Basic axioms

- Pxy or 'x is a part of y' works as primitive
- M=(P1-3): P is a partial order

  We can now define:

  - $Oxy \iff \exists z(Pzx \wedge Pzy)$
  - $Uxy \iff \exists z(Pxz \wedge Pyz)$
  - $PPxy \iff Pxy \wedge \neg Pyx$

Extensions of M

MM=M+(P4) and EM=M+(P5)

- (P4): $PPxy \rightarrow \exists z(Pzy \wedge \neg Ozx)$
- (P5): $\neg Pyx \rightarrow \exists z(Pzy \wedge \neg Ozx)$

Given the definition of PPxy we can see that (P5) implies (P4)

EM implies that objects with the same proper parts are identical

Closure principles

X+(P6-7)=CX

- (P6): $Uxy \to \exists z \forall w (Owz \iff (Owx \vee Owy))$ $[z = x + y]$
- (P7): $Oxy \to \exists z \forall w (Pwz \iff (Pwx \wedge Pwy))$ $[z = x \times y]$

(P7)+(P4)$\to$ (P5), so CMM=CEM

Infinitary fusions

X+(P8)= GX

- (P8): $\exists x \varphi(x) \rightarrow \exists z \forall y(Oyz \iff \exists x(\varphi(x) \wedge Oyx))$
  Every specifiable set of objects corresponds to a whole
- $[z = \sigma x \varphi]$
- $[\pi x \varphi = \sigma z \forall x(\varphi(x) \rightarrow Pxz)]$
- $[\sim x = \sigma z \neg Ozx]$

Atoms

- Atomicity (P9): $\forall x \exists (Pyx \wedge \forall z \neg PPzy)$
- Atomlessness (P10): $\forall x \exists y PPyx$

New relations

- Cxy, 'x is connected to y' as a new primitive
- T= (C1-2): C is reflexive and symmetric
    - $Exy \iff \forall z(Czx \to Czy)$
    - $ECxy \iff Cxy \land \neg Oxy$
- (C3): $Pxy \to Exy$
    - Corollary: $Oxy \to Cxy$

MT

- $IPxy \iff Pxy \wedge \forall z(Czx \rightarrow Ozy)$
- $TPxy \iff Pxy \wedge \neg IPxy$
- $IOxy \iff \exists z(IPzx \wedge IPzy)$
- $TOxy \iff Oxy \wedge \neg IOxy$
- $IUxy \iff \exists z(IPxz \wedge IPyz)$
- $TUxy \iff Uxy \wedge \neg IUxy$

Problem, how are TOxy and ECxy different?

Self-Connectedness 1

$$SCx \iff \forall yz(\forall w(Owx \iff Owy \vee Owz) \to Cyz)$$

CMT,CEMT

CMT=CM+T+(C4)

- (C4): $Cxy \rightarrow Uxy$
- $SCx \wedge SCy \wedge Cxy \rightarrow \exists z(SCz \wedge \forall w(Owz \iff Owx \vee Owy))$

CEMT=CMT+(P5)=CEM+T+(C4)

- $SCx \wedge SCy \wedge Cxy \rightarrow \exists z(SCz \wedge z = x + y)$
- $SCx \iff \forall yz((x = y + z) \rightarrow Czy)$

GEMT

GEMT=GEM+T. Now we can define topological operators:

- $ix = \sigma z(IPzx)$
- $ex = i(\sim x)$
- $cx = \sim (ex)$
- $bx = \sim (ex + ix)$ $\Rightarrow$ Difference between TOxy and ECxy

Topological axioms

GEMTC=GEMT+(C5-7)

- Inclusion(C5): $Px(cx)$
- Idempotence(C6): $c(cx) = cx$
- Additivity(C7): $c(x + y) = cx + cy$

Here you can prove that

- $Cxy \iff Oxy \vee Oxc(y) \vee Oc(x)y$

Self-Connectedness 2

- $SSCx \iff SCx \wedge SCix$
- $MSSCx \iff SSCx \wedge \forall y(SScy \wedge Oyx \rightarrow Pyx)$

  Our wholes will be the $\varphi - MSSC$

Atomicity problem

- Boundarylessness(C9): $\forall x \exists y IPPyx$
- $z \neq U \rightarrow \exists y(y = bz)$
- $y = bz \rightarrow \neg\exists x(IPxy)$

Thank you! Questions?