Errata

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The following errata contains corrections to my article "Polyatomic Logic and Generalized Blok-Esakia" published by the Journal of Logic and Computation. I thank Antonio Cleani for pointing out the typo in the proof of Proposition 5.4, and to Simon Lemal for raising attention to the mistake in Proposition 5.9.

- (*Pp.33 - Proof of Proposition 5.4, 2*): The argument as presented is warped. What should have been written was that we want to prove the following: $Log(\rho(Var(M)) \subseteq \rho(M)$. Indeed assume that $\varphi \in Log(\rho(Var(M))$. Suppose that $B \models M$. Then $\rho(B) \models \varphi$, so $B \models T(\varphi)$. Thus $\varphi \in \rho(M)$. Then by algebraic completeness $Var(\rho(M)) \subseteq Var(Log(\rho(Var(M))) = \rho(Var(M))$.

- *(Pp.31, Proposition 5.9)* - The proof given that τ is a complete homomorphism contains a gap which I have not been able to fix. Indeed, the proof of (3) given does not seem to follow, since the argument I have been able to find for it requires the full power of GMT Blok-Esakia. As such, Proposition 5.9 should be replaced by the following:

  Proposition 5.9. Let $\vdash_s \in \Lambda(\vdash_{\mathbf{X}})$ and $\mathbf{K} \in \Xi(\mathbf{X})$. Assume that ζ is a strongly selective translation.

  1. $\tau(\mathbf{K})$ is a quasivariety.
  2. $\tau(\mathsf{QVar}(\vdash_s)) = \mathsf{QVar}(\tau(\vdash_s))$. Hence $\tau(\mathsf{Log}(\mathbf{K})) = \mathsf{Log}(\tau(\mathbf{K}))$.
  3. τ is a complete meet-semilattice homomorphism.
  4. $\tau(\vdash_s)$ is the least ζ-companion of $\vdash_s$.

  The proof of Proposition 5.10 alludes to the previous proof mentioning that the case for ρ is similar. However, note that in fact ρ appears to be a complete homomorphism. First see that

  $$\rho(\bigcap_{i\in I} K_i) = \bigcap_{i\in I} \rho(K_i)$$

  To see it note that if $A \in \rho(\bigcap_{i\in I} K_i)$ then there is some $B \in \bigcap_{i\in I} K_i$ such that $\theta(B) = A$ and so $B \in K_i$ so $A \in \bigcap_{i\in I} \rho(K_i)$. On the other hand if $A \in \bigcap_{i\in I} \rho(K_i)$, then $A = \theta(C_i)$ where each $C_i \in K_i$. Then $\mathcal{F}(A) \leqslant C_i$ by sobriety, so $\mathcal{F}(A) \in K_i$ for each i. Since $\theta(\mathcal{F}(A)) \cong A$, then $A \in \rho(\bigcap_{i\in I} K_i)$.

  On the other hand we also have that:

  $$\rho(\bigvee_{i\in I} K_i) = \bigvee_{i\in I} \rho(K_i)$$

  Indeed, if $A \in \rho(K_i)$, then $B \in K_i$ so $B \in \bigvee_{i\in I} K_i$, so $A \in \rho(\bigvee_{i\in I} K_i)$, which shows the right to left inclusion. On the other hand if $A \in \rho(\bigvee_{i\in I} K_i)$, then for some $B \in \bigvee_{i\in I} K_i$, $A = \theta(B)$. Now B is a homomorphic image of C and C a subalgebra of D, and $D = \prod_{i\in I} C_i$. Passing θ through everything you obtain that $\theta(A)$ will be the result of the constructions done in $\bigvee_{i\in I} \rho(K_i)$. This shows equality.

  Proposition 5.9 is invoked in the key results of Section 6, namely the Polyatomic Blok-Esakia isomorphism. But in light of the previous fact, these results continue to work: we have that τ and ρ are each others' inverses, and ρ is a complete homomorphism, hence, so is τ.