TOPICS IN ALGEBRAIC LOGIC AND DUALITY THEORY
SEMINAR SHEET 3

- The contents of this seminar sheet will be discussed on **June 10**;
- Pick one or more examples to work out in full detail; you do not need to work on all of the examples.
- Examples are roughly ordered by conceptual difficulty, relative to the material covered in lectures.

(1) **(Craig + Uniform definability = Uniform Craig)** Show that Craig interpolation together with the uniform definability property implies the Uniform Craig property.

(2) **(Characterising formulas in intuitionistic logic)**
Let $(\mathfrak{M}, x)$ and $(\mathfrak{N}, y)$ be two finite intuitionistic models. Show the following: $(\mathfrak{M}, x)$ and $(\mathfrak{N}, y)$ are n-bisimilar if and only if they satisfy the same formulas of implication rank n.
Hint: Use the proof idea from the modal case. Moreover, if x is a point, consider the formulas
$$\Big(Th_{n-1}(x) \to \bigvee_{x \leq z} Th_{n-1}(z)\Big) \text{ and for } x \leq z, \text{ the formula } \neg Th_{n-1}(z).$$

(3) **(Combinatorial lemma)** Give a proof that the combinatorial lemma holds for the following two cases:
(a) **KD**;
(b) **KB**.

(4) Show that every Boolean algebra embeds into an existentially closed Boolean algebra.
Hint: If you know model theory, prove this directly. If you do not, prove the following easier statement: every finite Boolean algebra embeds into an atomless Boolean algebra.