Clase10

07 September 2023 09:07

Ley de De Morgan

$\overline{X+Y} = \overline{X}\cdot\overline{Y}$ $\qquad$ $\overline{X\cdot Y} = \overline{X} + \overline{Y}$

Completo: AND, OR, NOT

Completo: AND, NOT

OR, NOT

Ingeniería de Sistemas.
Arquitectura de Sistemas.
Práctico 4. Reducir expresiones

UNIVERSIDAD ORT Uruguay

Ejercicios de Álgebra de Boole.

1. Demostrar las siguientes igualdades utilizando los teoremas y enunciados.
   1. **(a + /b + ab) (a + /b) (/ab) = 0**
   2. **(a + /b + a/b) (a.b + /ac + bc) = ab+/a/bc**
   3. **(ab + c + d) (/c + d) (/c + d + e) = ab/c + d**
   4. **/a (/b/c+ /[(b + /c)(/b + /c)]) = /a(/b+c)**

1) $(a + \bar{b} + ab)\,(a + \bar{b})\,(\bar{a}b) = 0$

Cobertura: a

$(a + ab + \bar{b})\cdot(a + \bar{b})\,(\bar{a}b)$

$= (a + \bar{b})\cdot(a + \bar{b})\quad(\bar{a}b)$

idempotencia

$= (a+\bar{b})\,(\bar{a}b)$

$= \underbrace{a\bar{a}b}_{0} + \underbrace{\bar{b}\bar{a}b}_{0} = 0$

Teoremas y Propiedades

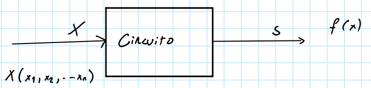
• 1 Variable

- $1 + x = 1$ • $1\cdot x = x$ identidad
- $0 + x = x$ • $0\cdot x = 0$ elementos nulos
- $x + x = x$ • $x\cdot x = x$ idempotencia
- $\overline{\overline{x}} = x$ doble negación
- $x + \bar{x} = 1$ • $x\cdot\bar{x} = 0$ Complemento

• 2 y 3 Variables

- $x + y = y + x$ • $x\cdot y = y\cdot x$ Conmutativa
- $(x+y)+z = x + (y+z)$ • $(x\cdot y)\cdot z = x\cdot(y\cdot z)$ Asociativa
- $x\cdot y + x\cdot z = x\cdot(y+z)$ • $(x+y)\cdot(x+z) = x + y\cdot z$ Distributiva
- $x + x\cdot y = x$ • $x\cdot(x+y) = x$ Cobertura
- $x\cdot y + x\cdot\bar{y} = x$ • $(x+y)\cdot(x+\bar{y}) = x$ Combinación
- $x\cdot y + \bar{x}\cdot z + y\cdot z = x\cdot y + \bar{x}\cdot z$ • $(x+y)\cdot(\bar{x}+z)\cdot(y+z) = (x+y)\cdot(\bar{x}+z)$ Consenso.

Sistemas Combinacionales



Función Lógica

$\underbrace{f_i(x_{r-1}, x_{r-2}, \dots, x_0)}_{r \text{ bits}} = f_i(x) \Big| \begin{matrix} 0 \\ 1 \end{matrix}$ función Lógica

$\Rightarrow 2^r$ bits

Ejemplo 3 bits:

x_0	x_1	x_2	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

→ Las funciones Logicas se pueden dar como una Tabla de Verdad o como una Expresión Algebraica

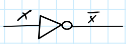
→ Funciones Logicas basicas y sus compuertas

$f(x) = 0$
$f(x) = 1$
} son las funciones más basicas "Las constantes".

función NOT → $f(x) = \bar{x}$

x	$\bar{x}$
0	1
1	0

$(\bar{x}, /x, !x)$

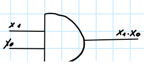


Funciones de dos Bits

AND

x_1	x_0	AND
0	0	0
0	1	0
1	0	0
1	1	1

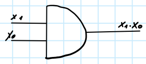
$f(x_1, x_0) = x_1\cdot x_0$



AND

X_1	X_0	AND
0	0	0
0	1	0
1	0	0
1	1	1

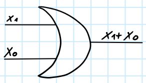
$f(X_1, X_0) = X_1 \cdot X_0$



OR

$f(X_1, X_0) = X_1 + X_0$

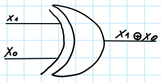
X_1	X_0	OR
0	0	0
0	1	1
1	0	1
1	1	1



XOR (OR Exclusivo)

$f(X_1, X_0) = X_1 \oplus X_0$

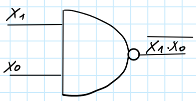
X_1	X_0	XOR
0	0	0
0	1	1
1	0	1
1	1	0



NAND (NOT AND)

$f(X_1, X_0) = \overline{X_1 \cdot X_0}$

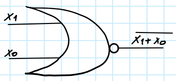
X_1	X_0	NAND
0	0	1
0	1	1
1	0	1
1	1	0



NOR (NOT OR)

$f(X_1, X_0) = \overline{X_1 + X_0}$

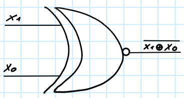
X_1	X_0	NOR
0	0	1
0	1	0
1	0	0
1	1	0
		0



XNOR EQU (NOT XOR)

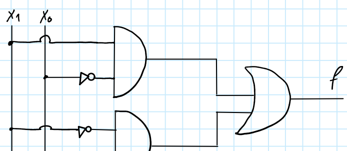
$f(X_1, X_0) = \overline{X_1 \oplus X_0}$

X_1	X_0	$\overline{X_1 \oplus X_0}$
0	0	1
0	1	0
1	0	0
1	1	1

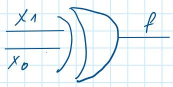


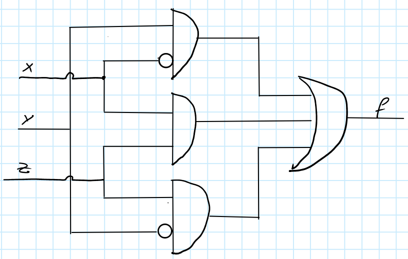
Esempio

$f(X_1, X_0) = X_1 \cdot \overline{X_0} + \overline{X_1} \cdot X_0 = X_1 \oplus X_2$

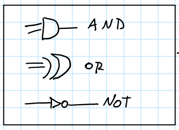


X_1	X_0	f
0	0	0
0	1	1
1	0	1
1	1	0





$f = Y \cdot \overline{X} + X \cdot Z + Z \cdot \overline{Y}$



→ Completo

AND NOT → completo

OR NOT → Completo

NAND → Completo

NOR → Completo

→ Vamos A Demostrar Que NAND y NOR Son Completas.

"Sale en Parcial"

Dem NAND.

$\text{Not } A = A \text{ NAND } A = \text{NOT } \underbrace{(A \text{ AND } A)}_{A} = \text{Not } A$

$A + B = \overline{\bar{A} \cdot \bar{B}} = \bar{A} \text{ NAND } \bar{B} = (A \text{ NAND } A) \text{ NAND } (B \text{ NAND } B)$

De Morgan

$\text{NOT}((\text{NOT}(A)) \text{ AND } (\text{NOT}(B))) = (\text{NOT}(A)) \text{ NAND } (\text{NOT}(B))$

$A \cdot B = \overline{\overline{A \cdot B}} = \text{Not } (A \text{ NAND } B) =$

doble negación

$(A \text{ NAND } B) \text{ NAND } (A \text{ NAND } B)$