EECE 4353 Image Processing

Lecture Notes: Reduction of Uncorrelated Noise

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Fall Semester 2016

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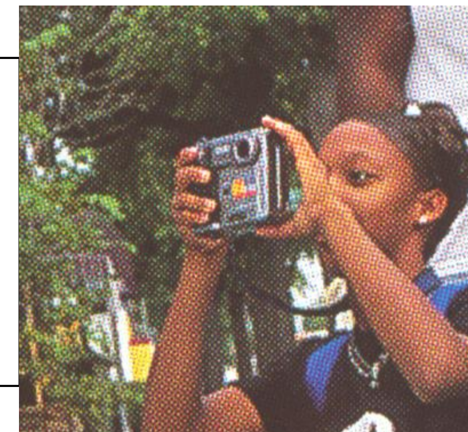
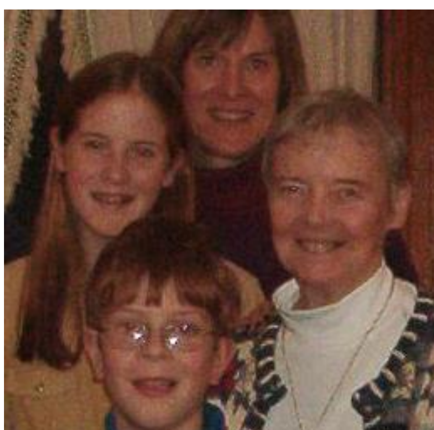
Noise in Images

All images created through optical projection onto a sensor array are noisy.



Correlated noise

- Due to electrical interference
- Due to source / sensor interference
- Halftone distortion / moiré patterns



Uncorrelated noise

- Quantum noise in CCD arrays
- Silver halide grains in film photography
- Neuronal noise in a retina
- Quantization noise in digital photographs

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Image with Additive Noise

noisy image

additive noise

$$\mathbf{J}(r,c) = \mathbf{I}(r,c) + \mathbf{N}(r,c).$$

undegraded image

spatial domain

noisy image

additive noise

$$\mathcal{J}(v,u) = \mathcal{I}(v,u) + \mathcal{N}(v,u).$$

undegraded image

frequency domain

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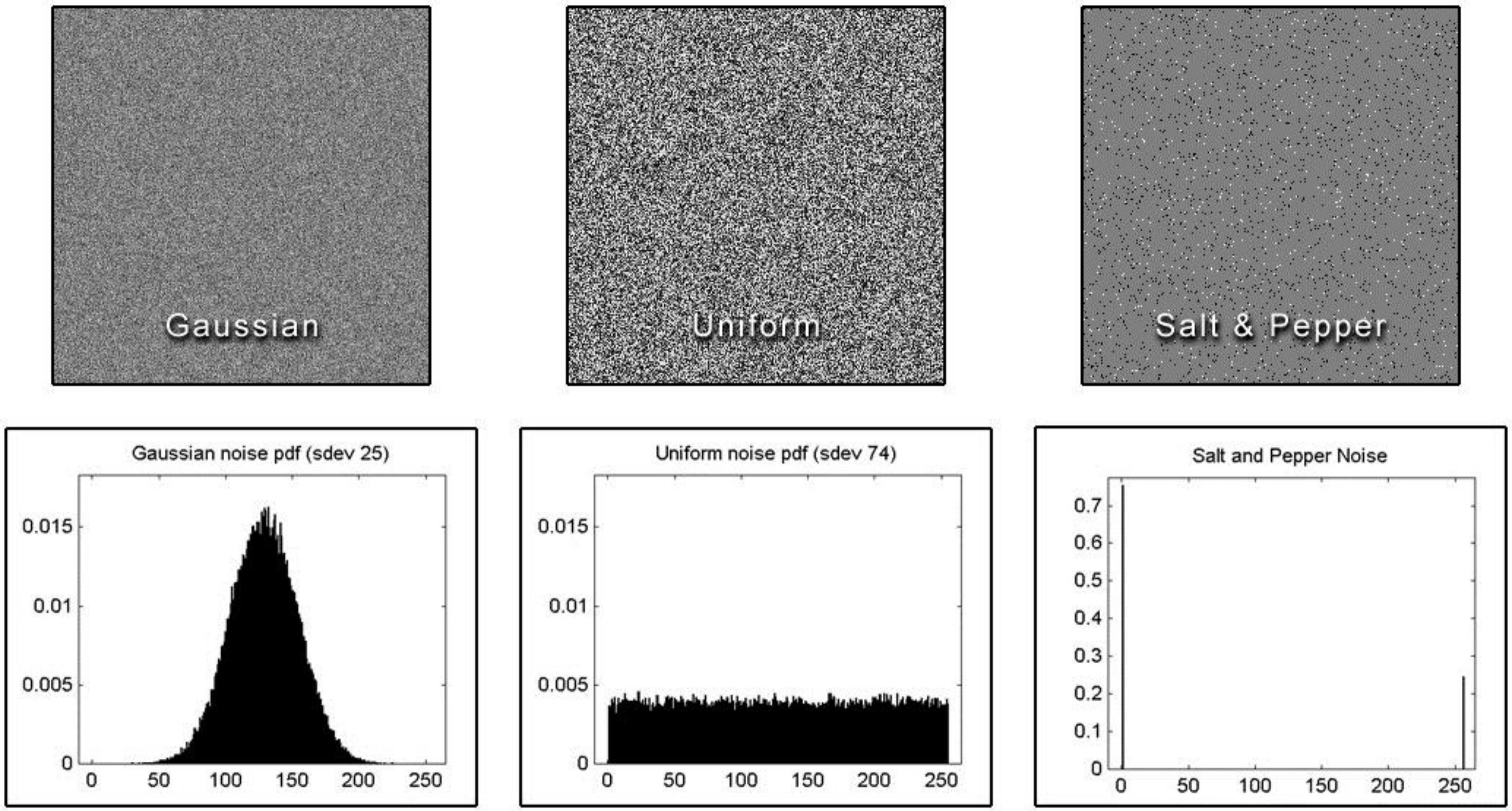
Uncorrelated Noise



Each pixel's value has probability of occurrence given by the associated distribution.

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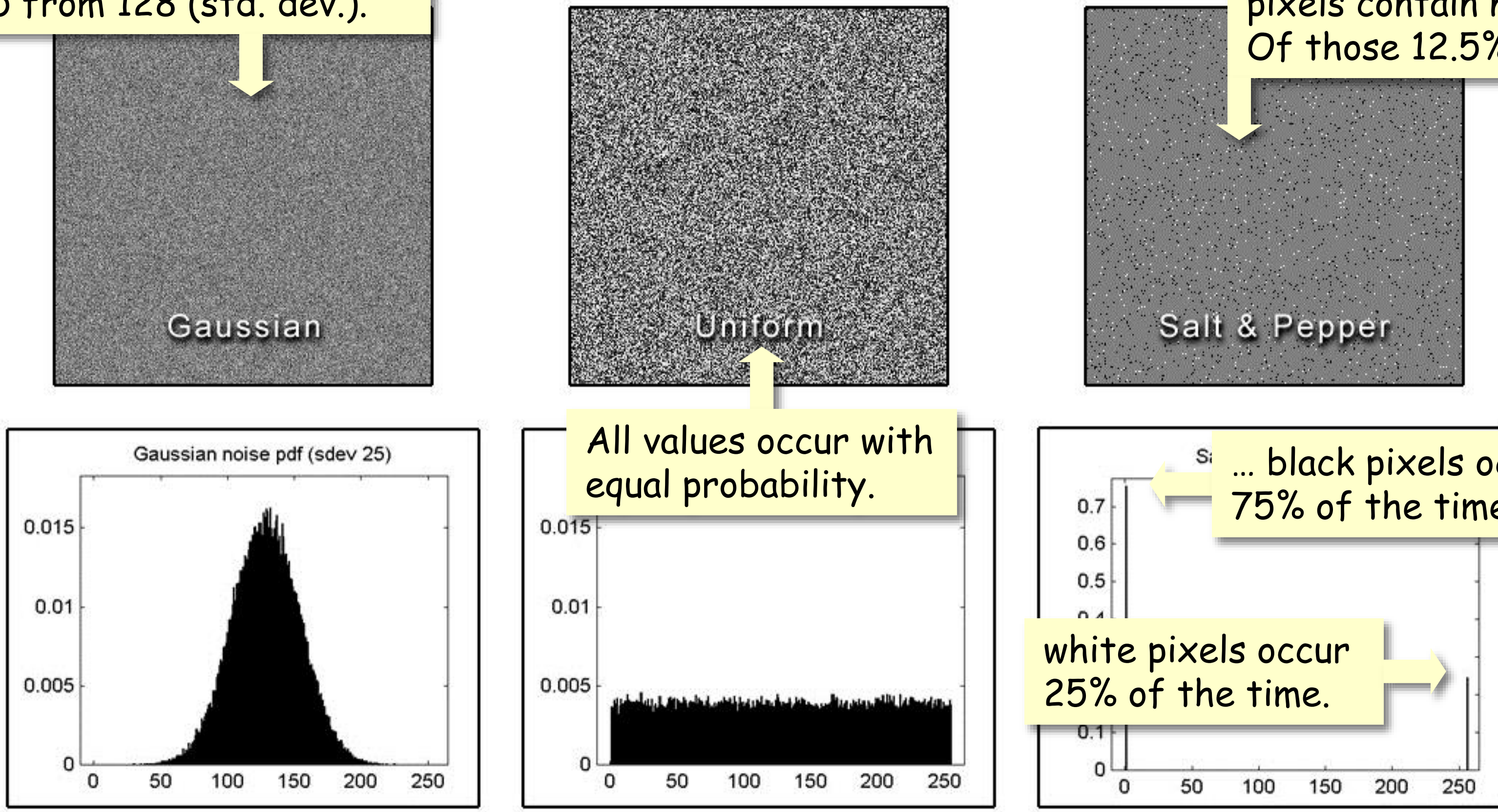
Uncorrelated Noise

The most likely value is 128 with an average difference of 25 from 128 (std. dev.).

This is sparse noise: Only 12.5% of the pixels contain noise. Of those 12.5% …



All values occur with equal probability.

… black pixels occur 75% of the time and

white pixels occur 25% of the time.

Each pixel’s value has probability of occurrence given by the associated distribution.

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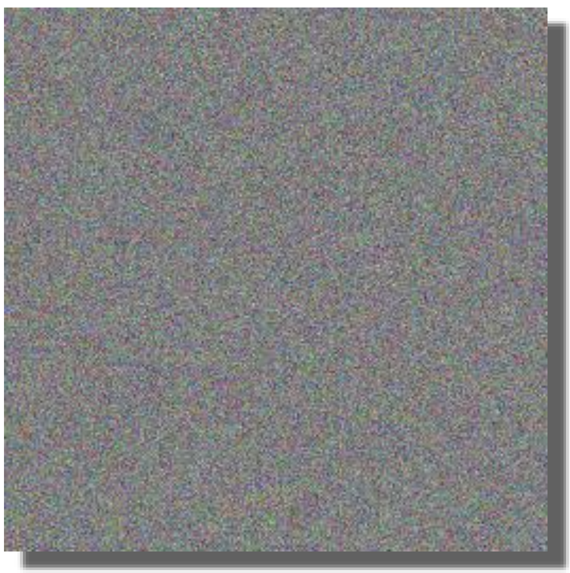
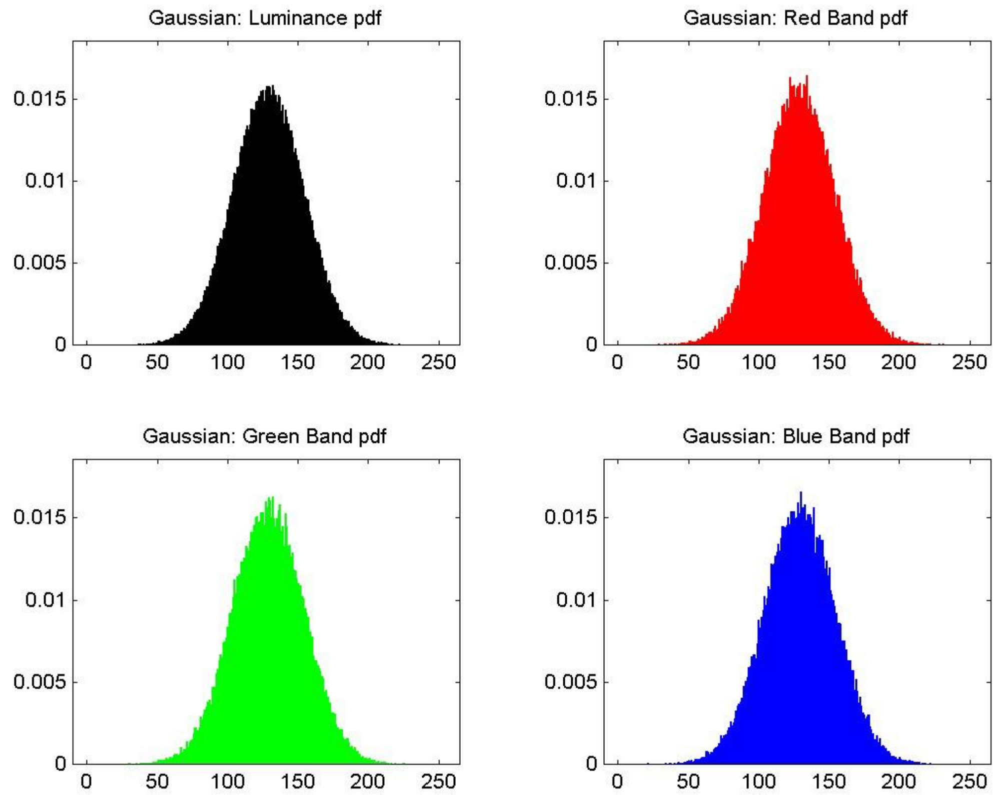
Uncorrelated Color Noise: Gaussian





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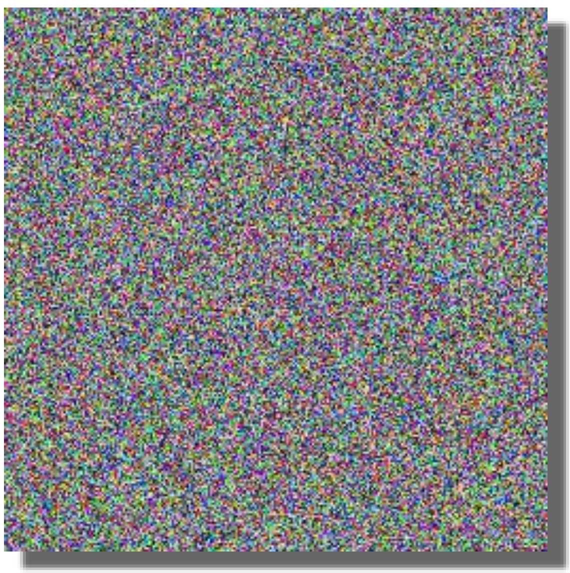
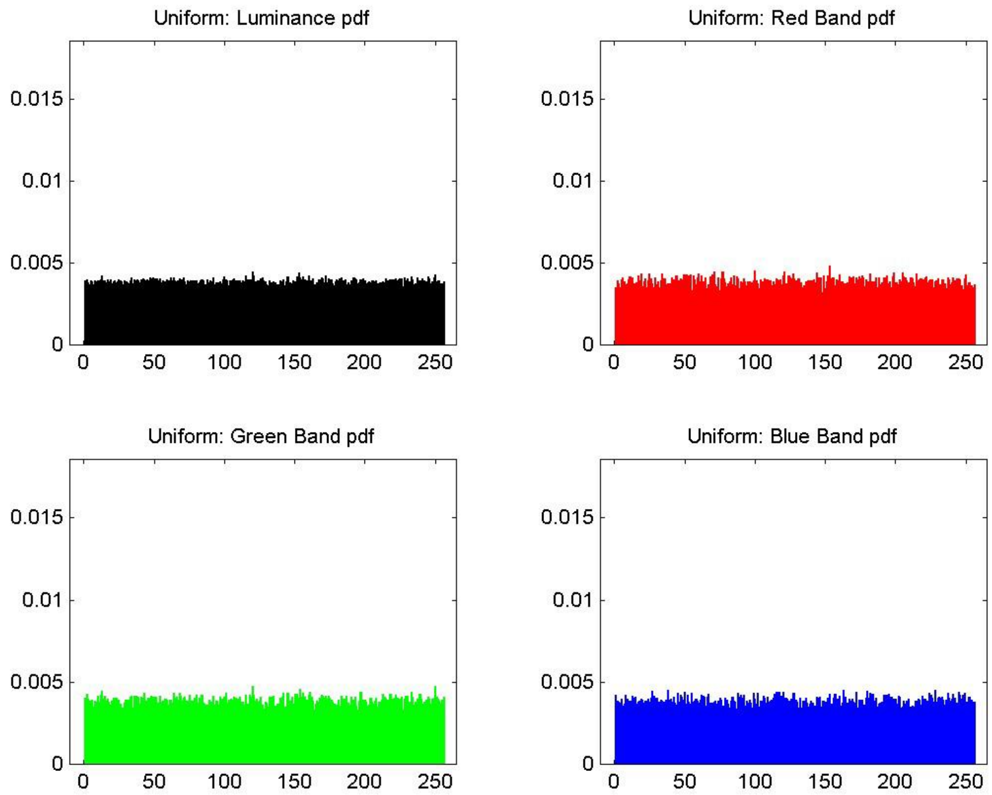
Uncorrelated Color Noise: Uniform





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Noise Enhancement: Problem with Sharpening

- Noise occurs in every natural imaging device
  - Quantum effects in CCD arrays
  - Random distribution of silver halide grains in film
  - Neuronal noise in the retina
- Spatially independent noise
  - The noise in one sensor has no effect on that in its neighbors
  - $\Rightarrow$ the autocorrelation of the signal is an impulse at the origin
  - The chances of getting repeated patterns of any frequency are virtually nil
  - $\Rightarrow$ the frequency spectrum of the noise is flat

Recall: Autocorrelation = inverse Fourier transform of power spectrum; Fourier transform of an impulse at (0,0) is a constant.

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Noise Enhancement: Problem with Sharpening

- The spectra of most natural images fall-off toward the high frequencies.
- IID noise has a flat spectrum.
- Therefore, at some relatively high frequency (HF) the energy in the noise is greater than that in the uncorrupted image.
- Sharpening multiplies the FT of the image by *u* and *v* (or by linear combinations of them) which, at HF, increases the noise more than the uncorrupted image.

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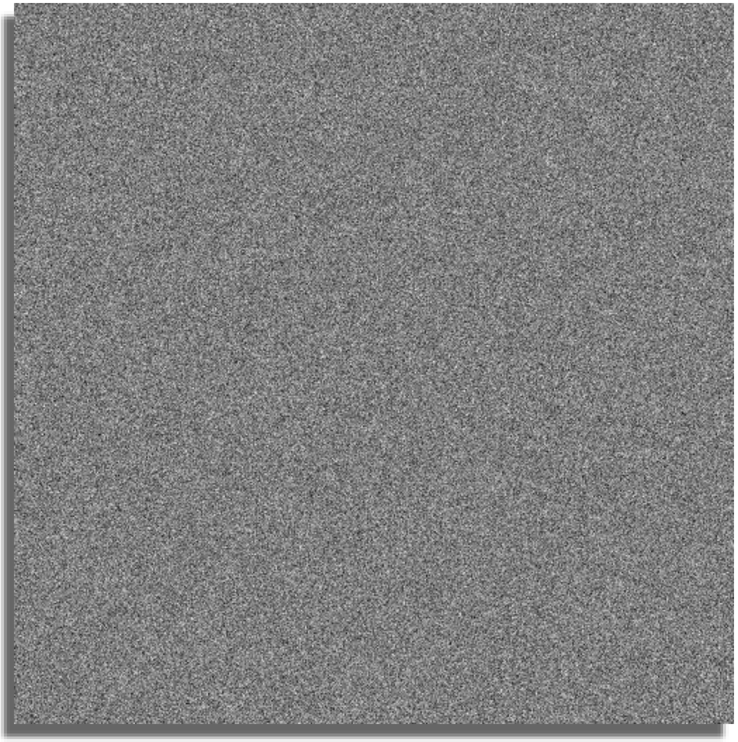
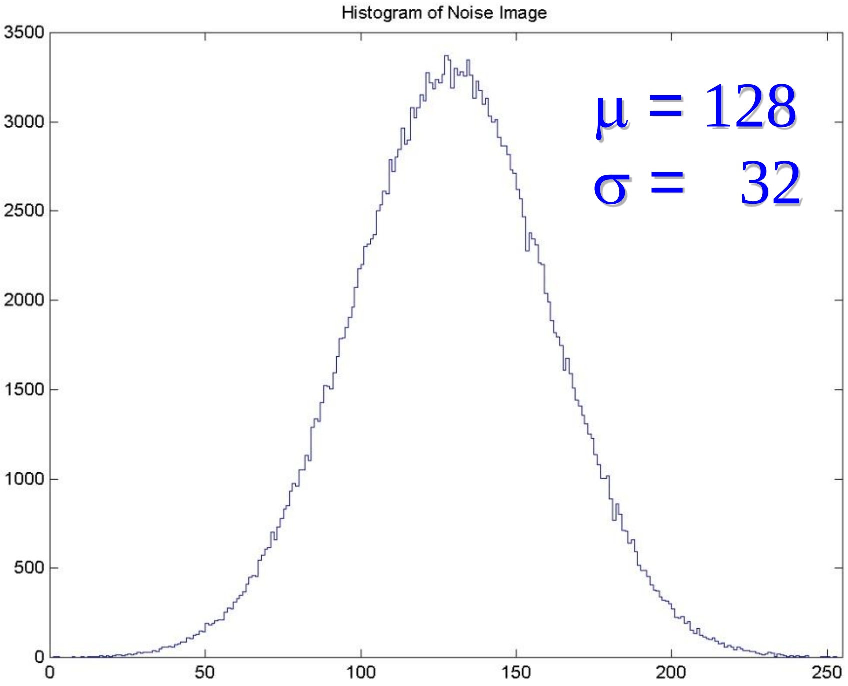
Gaussian IID Noise Field

IID ⇒ no spatial correlation





IID: Independent, Identically Distributed

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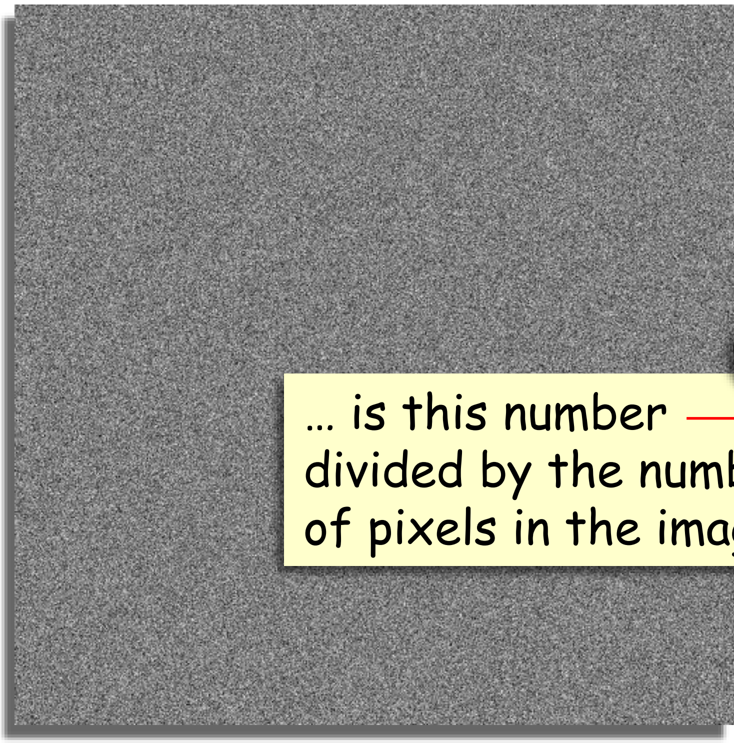
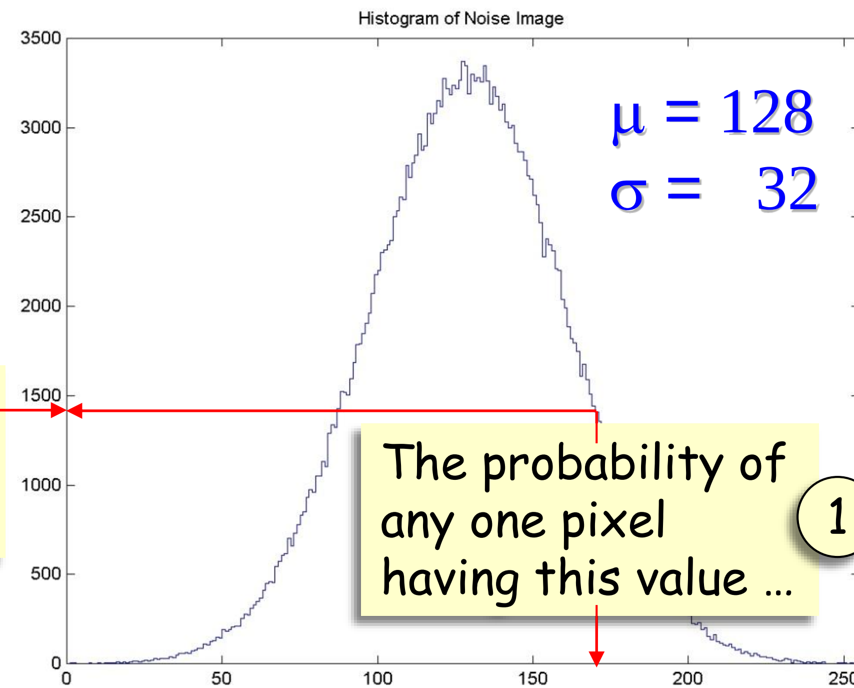
Gaussian IID Noise Field

IID ⇒ no spatial correlation





The probability of any one pixel having this value … ①

② … is this number divided by the number of pixels in the image.

IID: Independent, Identically Distributed

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Autocorrelation of an Image

Let the support of **I** be a torus. (**I** is defined on a torus *a la* the Fourier transform.) Let $\tilde{\mathbf{I}}$ be **I** minus the mean value of **I**. Make a copy of $\tilde{\mathbf{I}}$. Shift the copy by (ρ,χ) on the torus. Pixel-wise multiply the shifted version by the original and sum the products.

$$\mathbf{A}_{\mathbf{I}}(\rho,\chi) = \tfrac{1}{RC}\sum_{r=1}^{R}\sum_{c=1}^{C} \tilde{\mathbf{I}}(r,c)\,\tilde{\mathbf{I}}\big(\psi(r+\rho;R),\psi(c+\chi;C)\big)$$

where

$$\tilde{\mathbf{I}}(\rho,\chi) = \mathbf{I}(\rho,\chi) - \tfrac{1}{RC}\sum_{r=1}^{R}\sum_{c=1}^{C} \mathbf{I}(r,c)$$

and

$$\psi(x;N) = \begin{cases} \mathrm{mod}(x,N) & \text{if} \quad x \geq 0 \\ \mathrm{mod}(x+N,N) & \text{if} \quad x < 0 \end{cases}$$

$\mathbf{A}_{\mathbf{I}}(\rho,\chi)$, the autocorrelation of **I** at offset (ρ,χ), is a measure of the similarity of **I** to itself when shifted by (ρ,χ).

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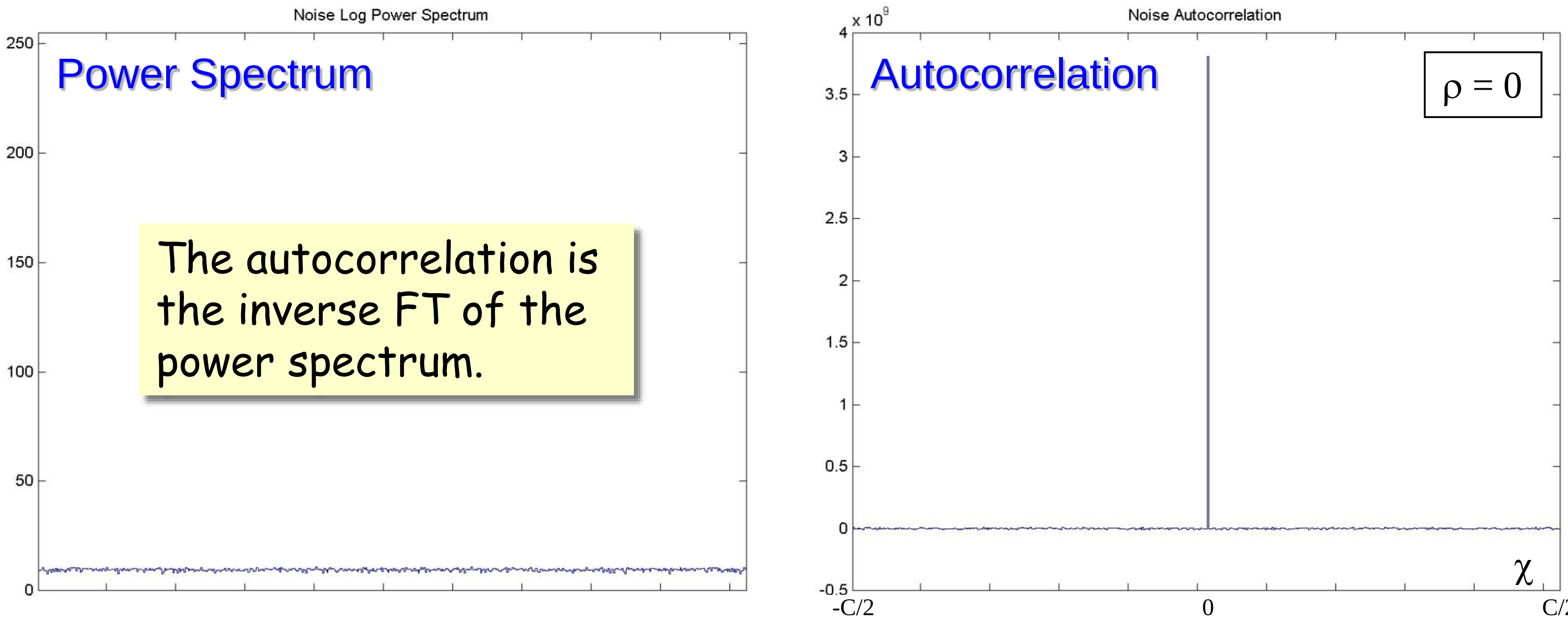
Power Spectrum & Autocorrelation of IID Noise



$$\mathrm{PS}(\mathbf{I}) = \left| \mathcal{F}(\mathbf{I}) \right|^2$$

$$\mathbf{A}_{\mathbf{I}}(\rho, \chi) = \mathrm{Re}\left[\mathcal{F}^{-1}\left\{ \left| \mathcal{F}(\mathbf{I}) \right|^2 \right\} \right]$$

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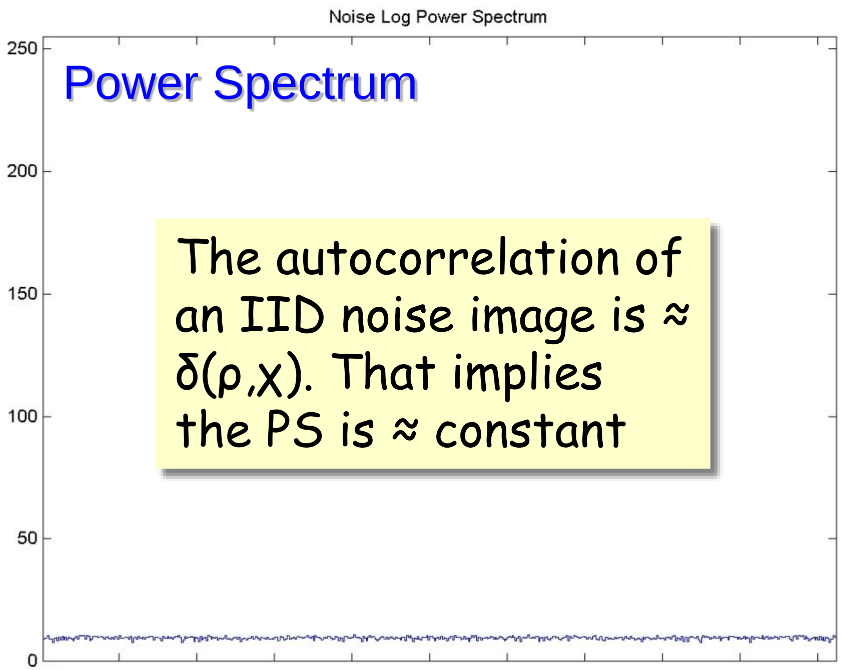
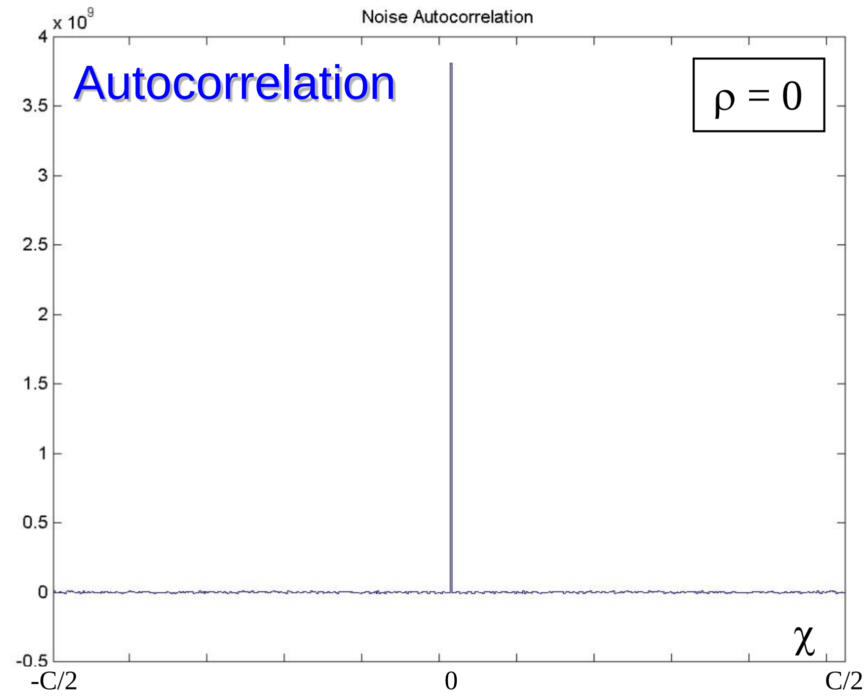
Power Spectrum & Autocorrelation of IID Noise





$$\mathrm{PS}\left(\mathbf{I}\right)=\left|\mathcal{F}\left(\mathbf{I}\right)\right|^2$$

$$\mathbf{A}_{\mathbf{I}}\left(\rho,\chi\right)=\mathrm{Re}\left[\mathcal{F}^{-1}\left\{\left|\mathcal{F}\left(\mathbf{I}\right)\right|^2\right\}\right]$$

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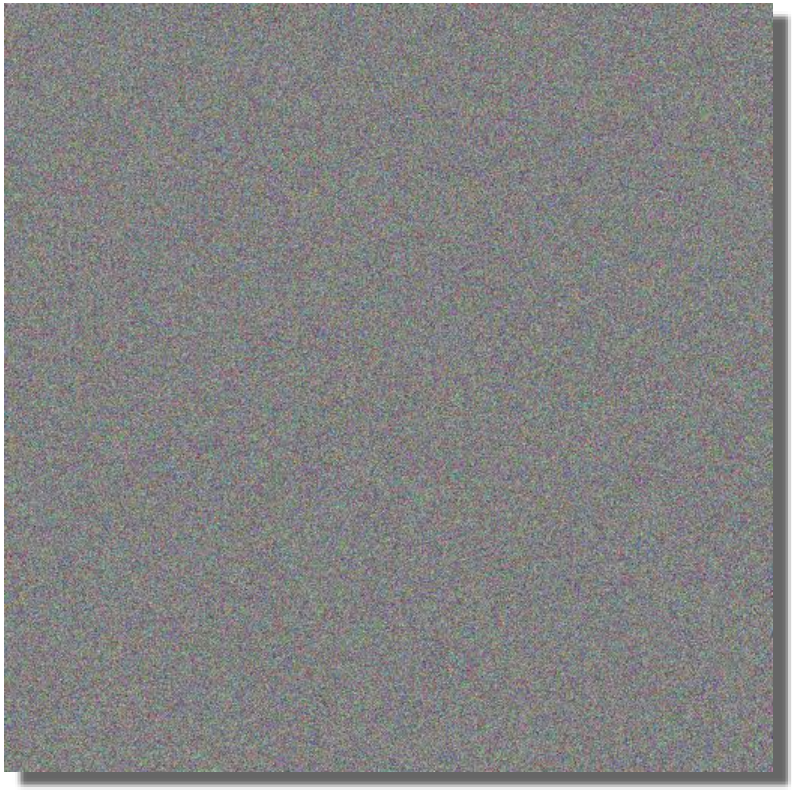
Noise-Free Image and Uncorrelated Noise Field

image



Gaussian noise field

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Spectra of Noise-Free Image and Uncorr. Noise Field

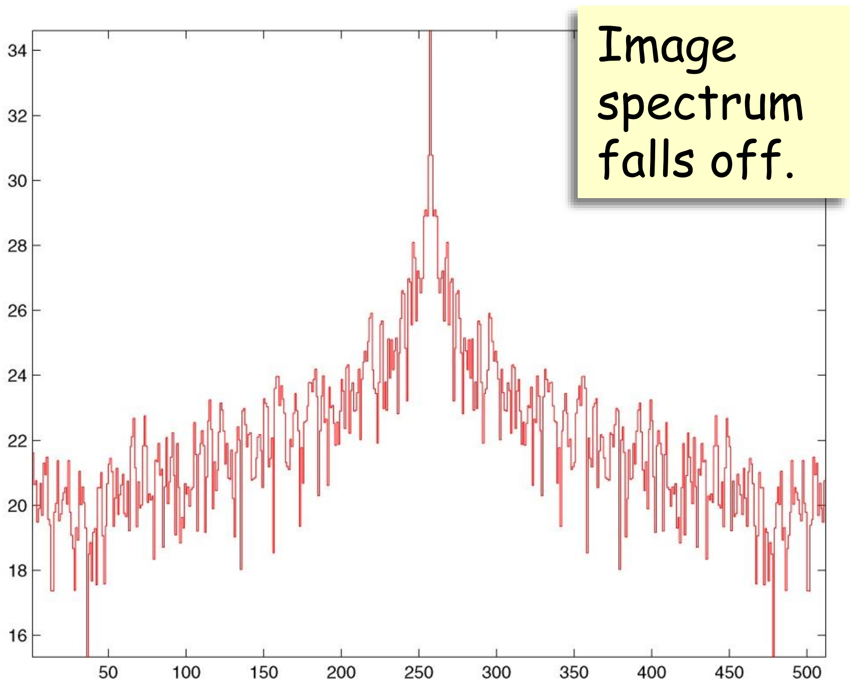
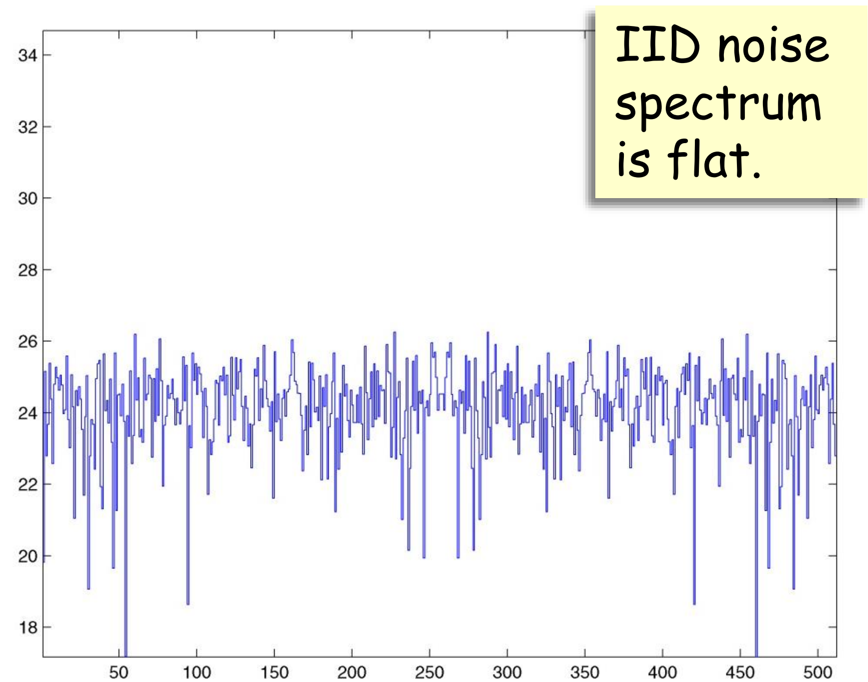


image center row log power spectrum



noise field center row log power spectrum

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Sum of Noise-Free Image and Uncorrelated Noise Field

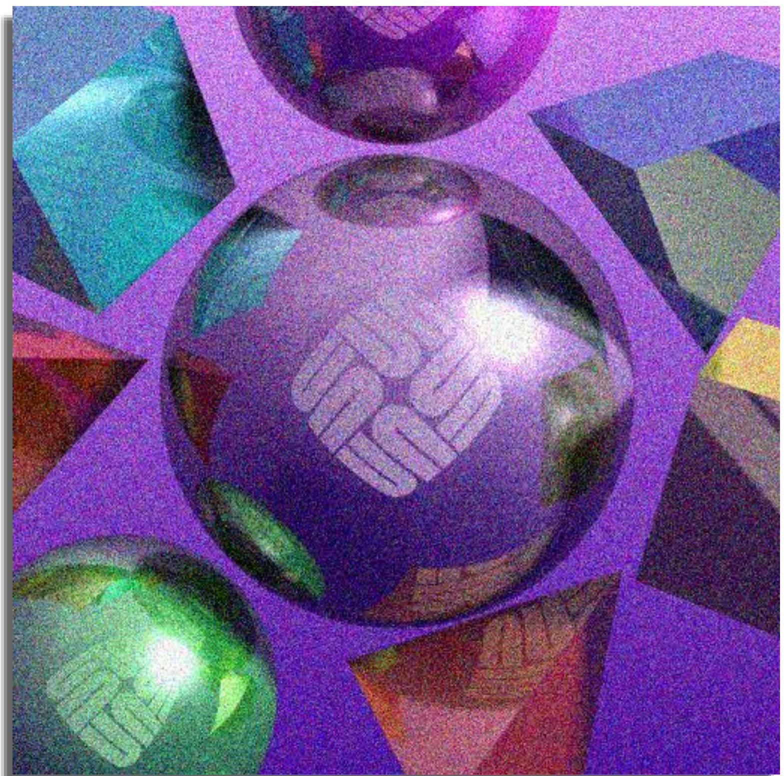
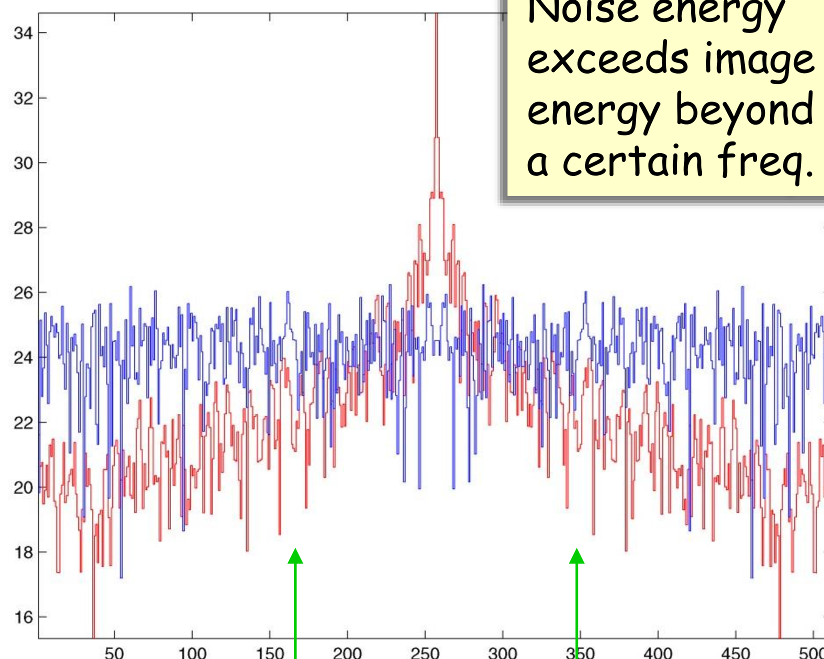


image + noise field

Noise energy exceeds image energy beyond a certain freq.



$|\mathbf{I}(v,u)|^2 < |\mathbf{N}(v,u)|^2$ $|\mathbf{I}(v,u)|^2 > |\mathbf{N}(v,u)|^2$ $|\mathbf{I}(v,u)|^2 < |\mathbf{N}(v,u)|^2$

image + noise field center row log PS

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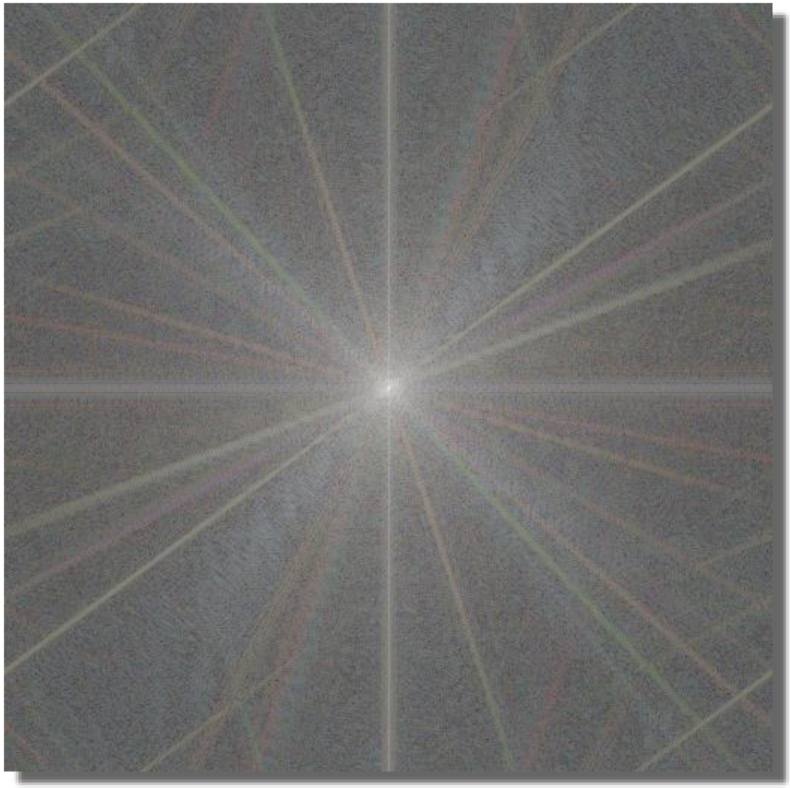
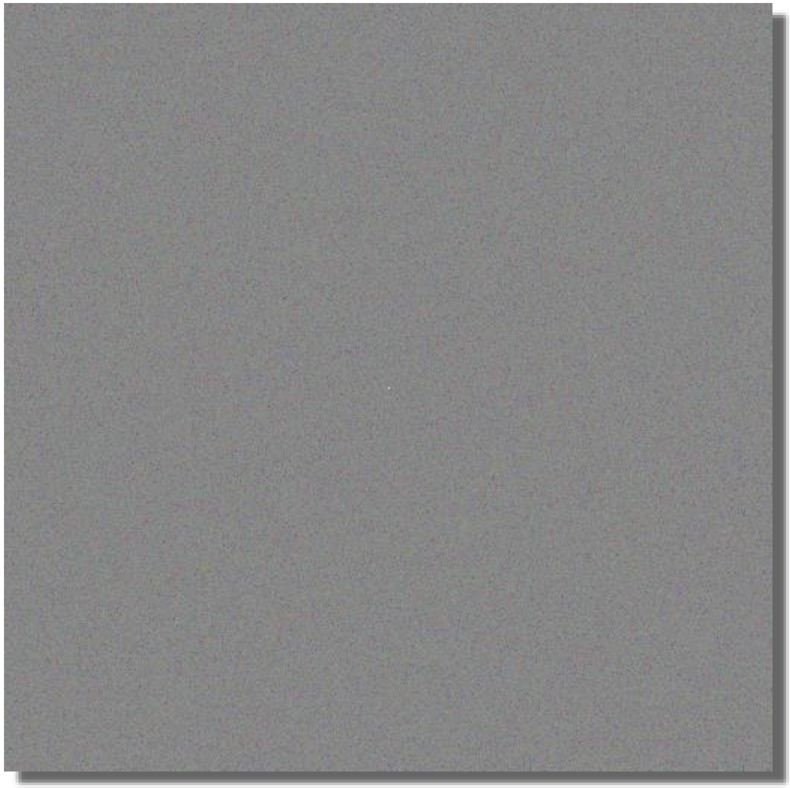
Power Spectra of Noise-Free Image and Noise Field



original image



noise image

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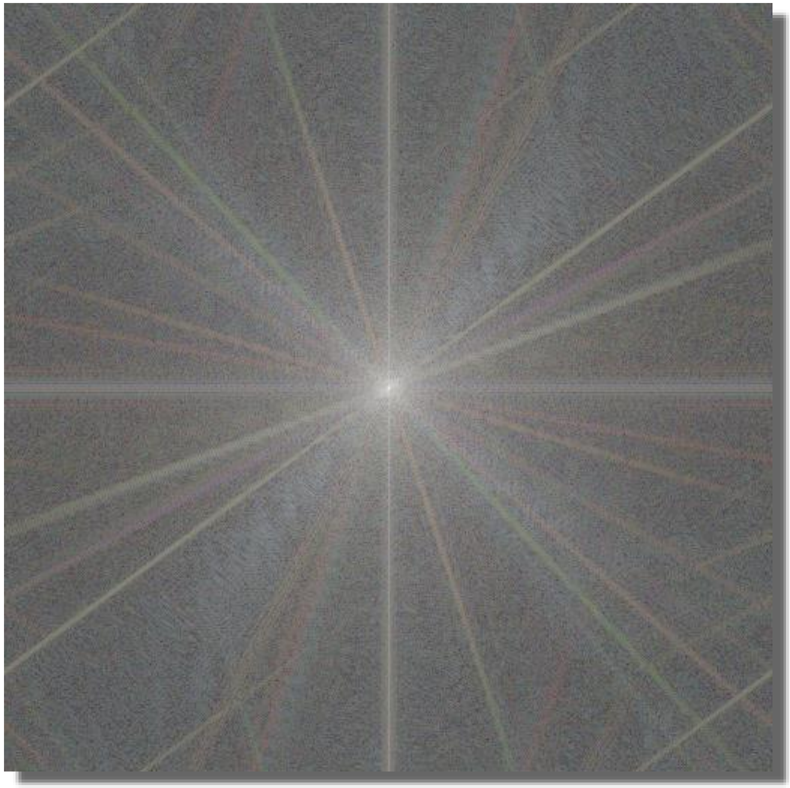
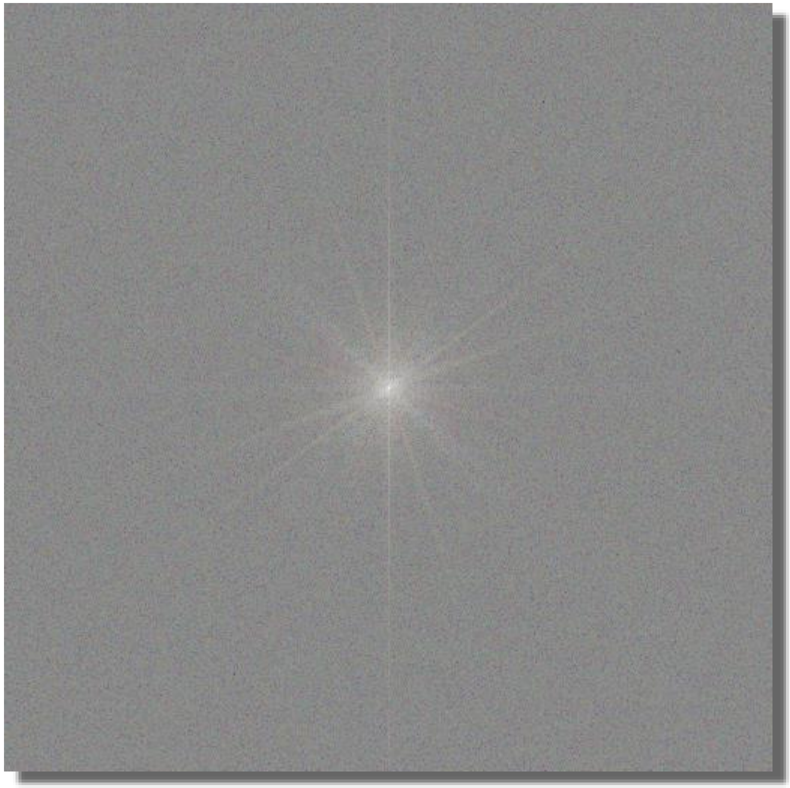
Power Spectra of Sum of Image and Noise Field



original image



noisy image

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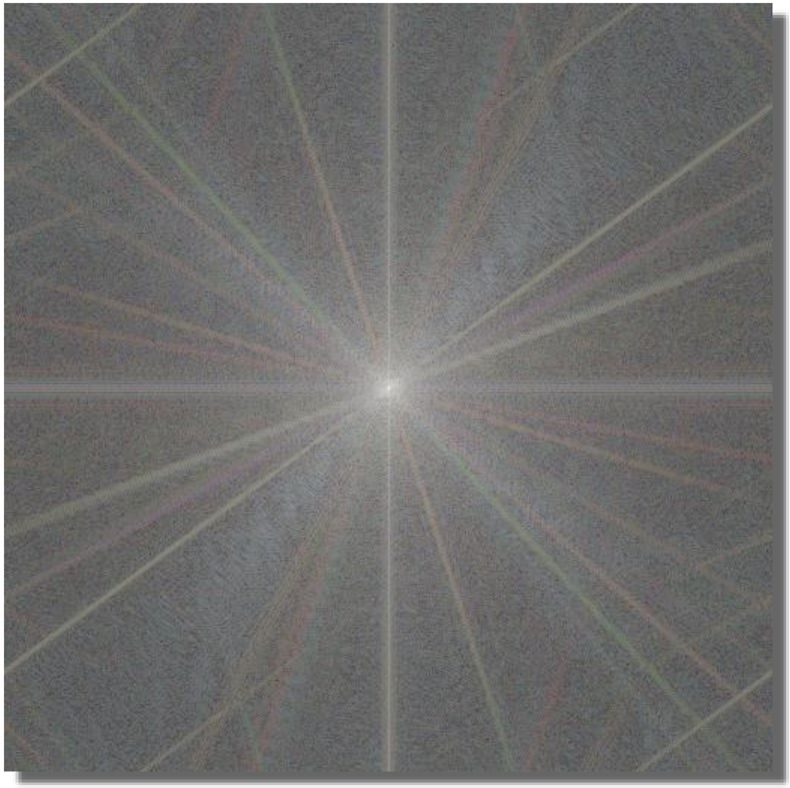
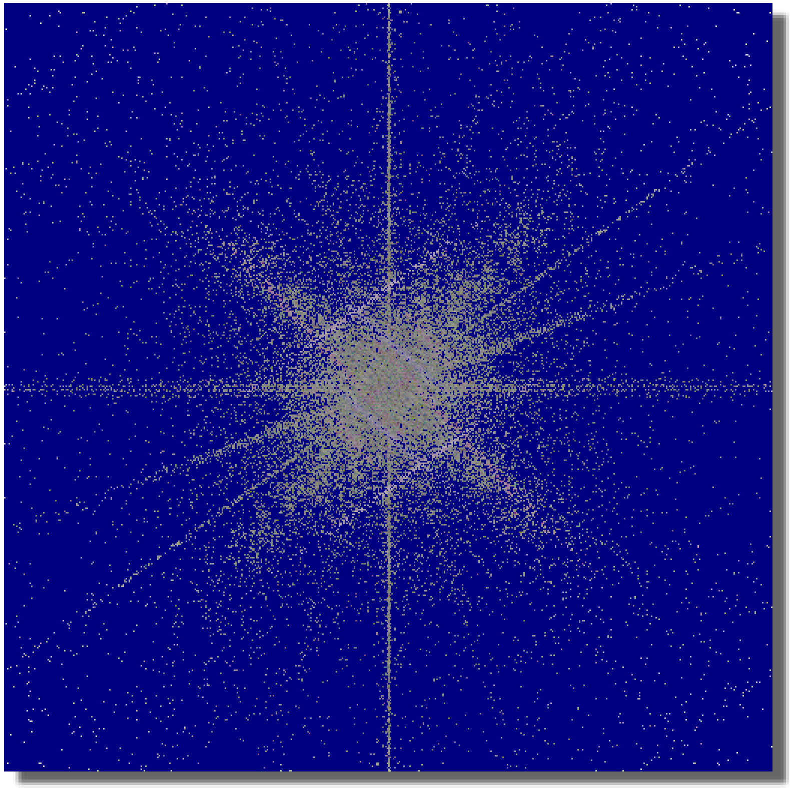
Power Spectra of Sum of Image and Noise Field



original image



blue indicates noise > image

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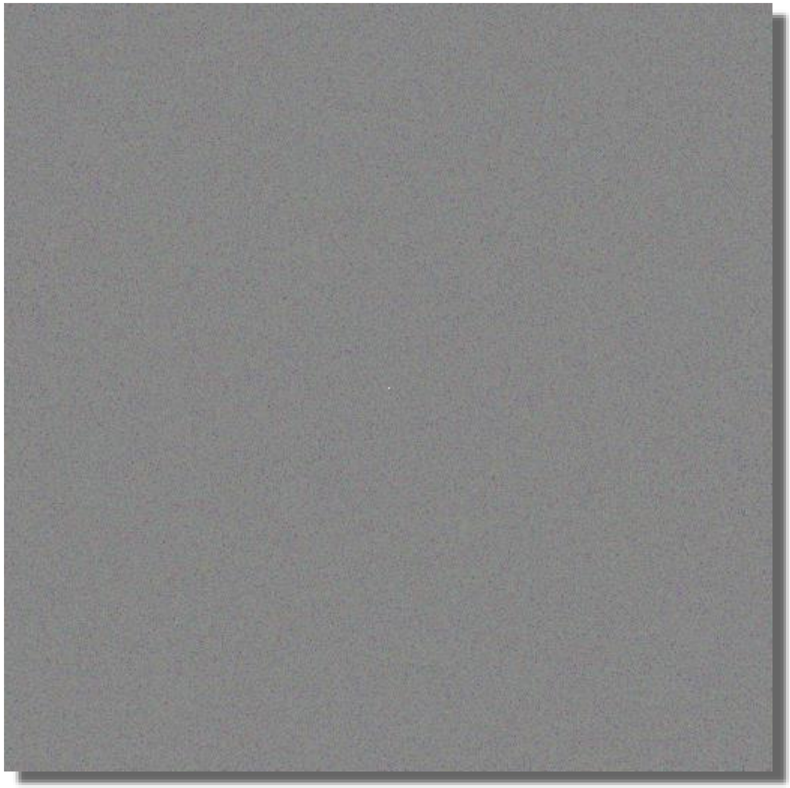
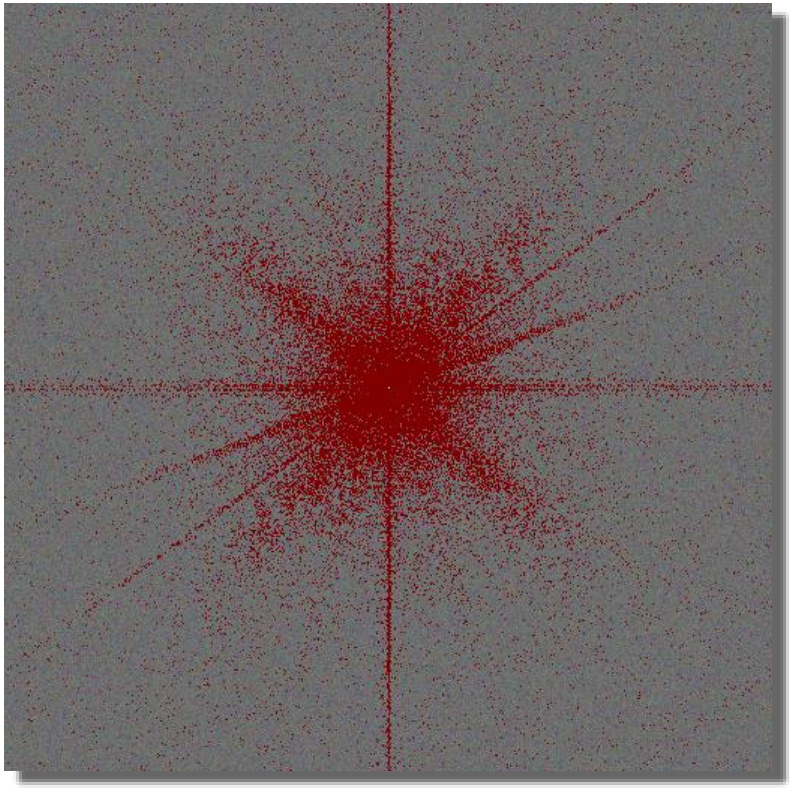
Power Spectra of Sum of Image and Noise Field



noise image



red indicates image > noise

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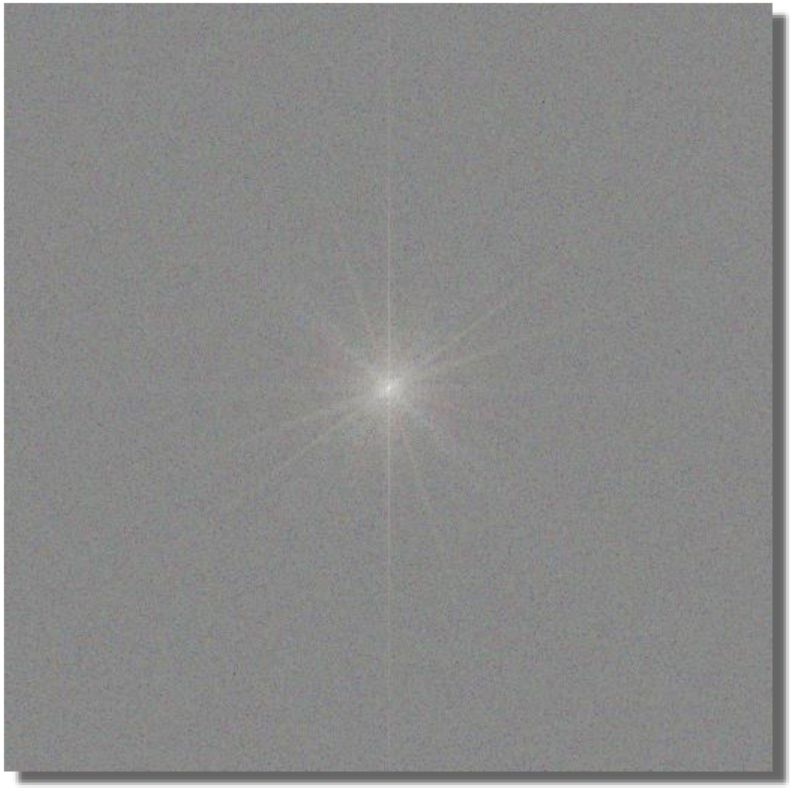
Power Spectra of Sum of Image and Noise Field



noisy image

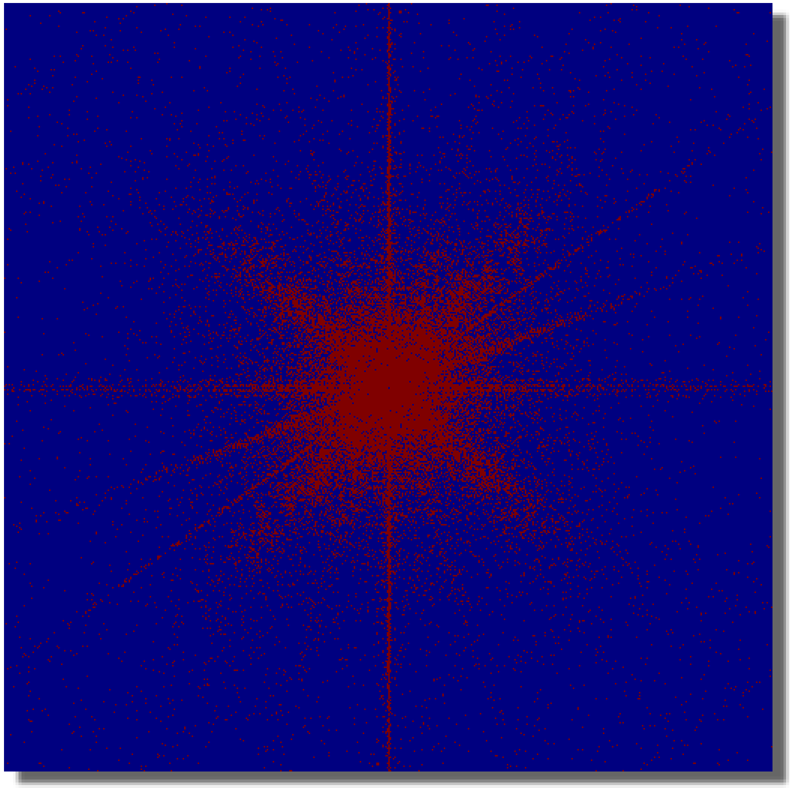


image & noise

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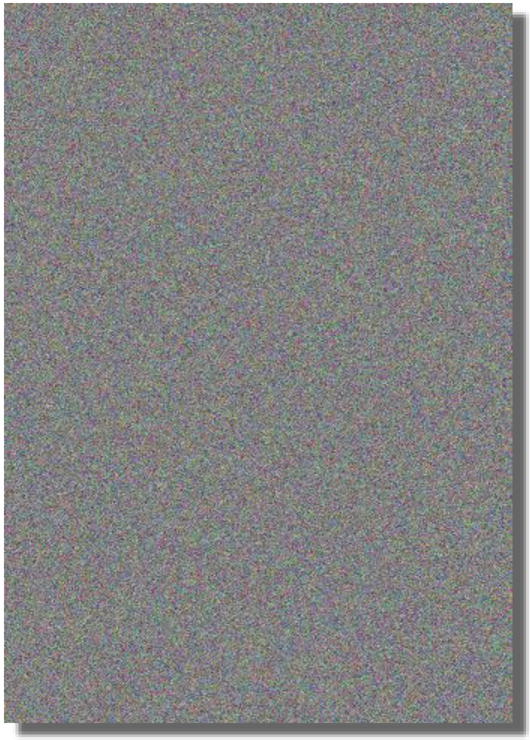
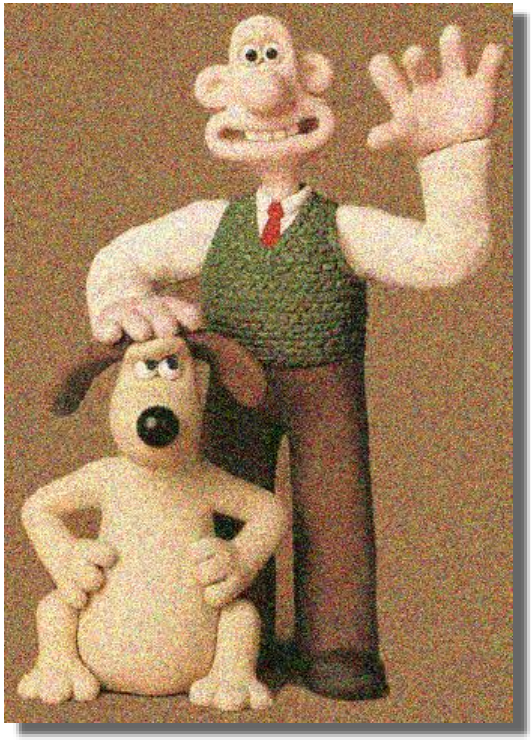
Additive Noise: Another Example

original image



noise image



image+noise

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Additive Noise: Another Example

displayed:
$$\log\left\{\left|\mathcal{F}(\mathbf{I})\right|^2+1\right\}$$

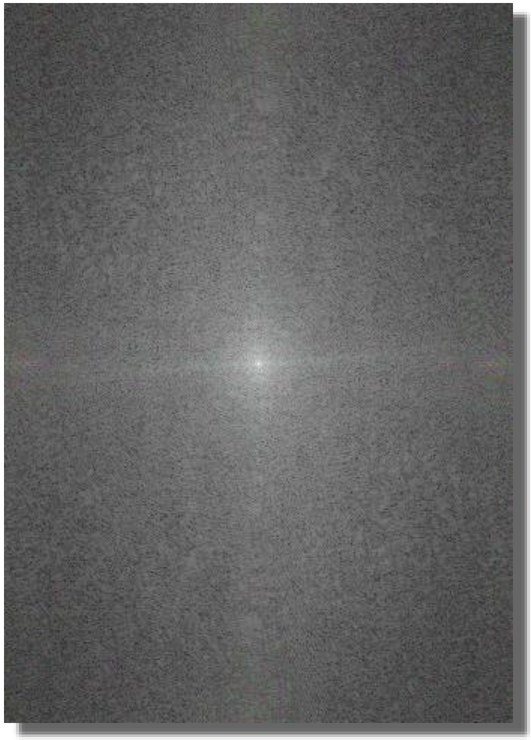
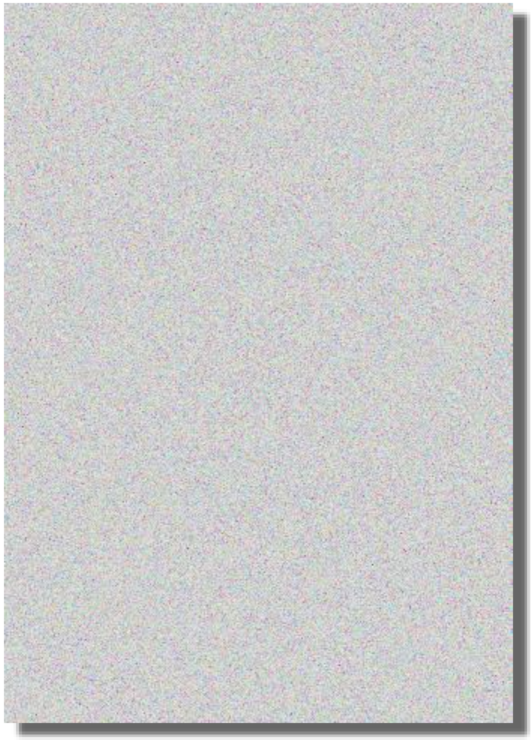
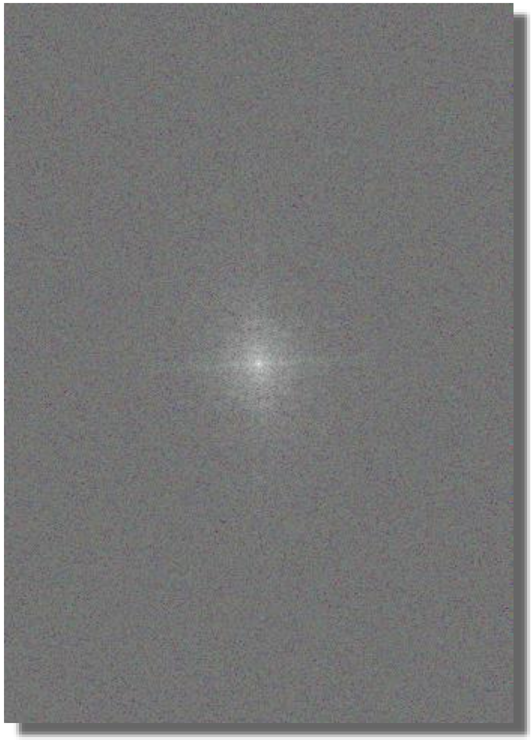


image PS



noise PS



image+noise PS

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Additive Noise: Another Example

displayed:
$\log\left\{ \left| \mathcal{F}\left(\mathbf{I}\right) \right|^2 + 1 \right\}$

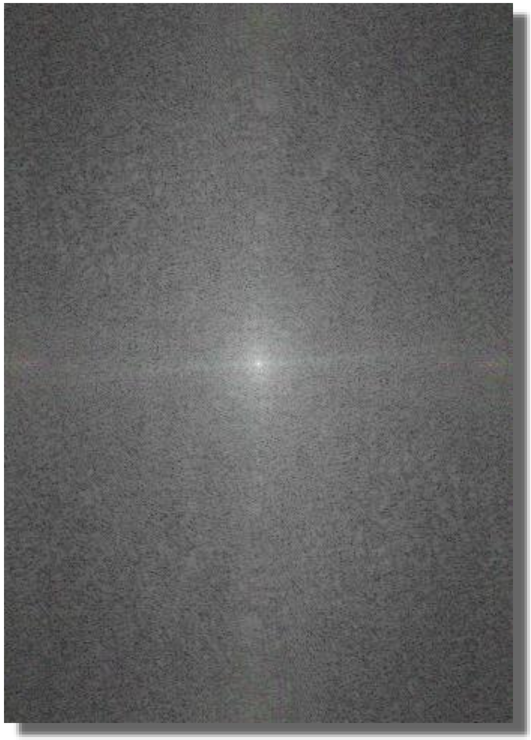
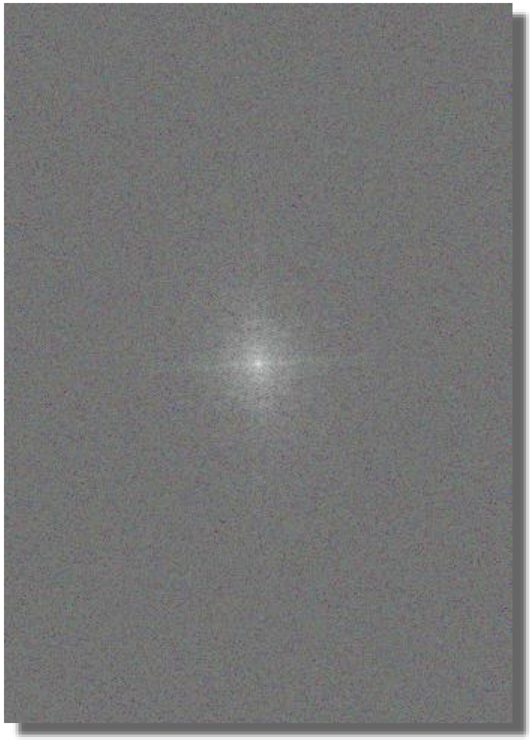


image PS



image+noise PS

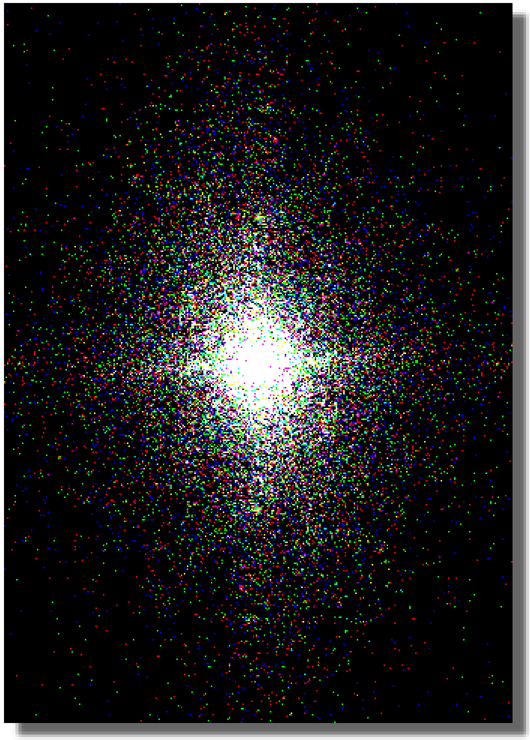


image PS > noise PS

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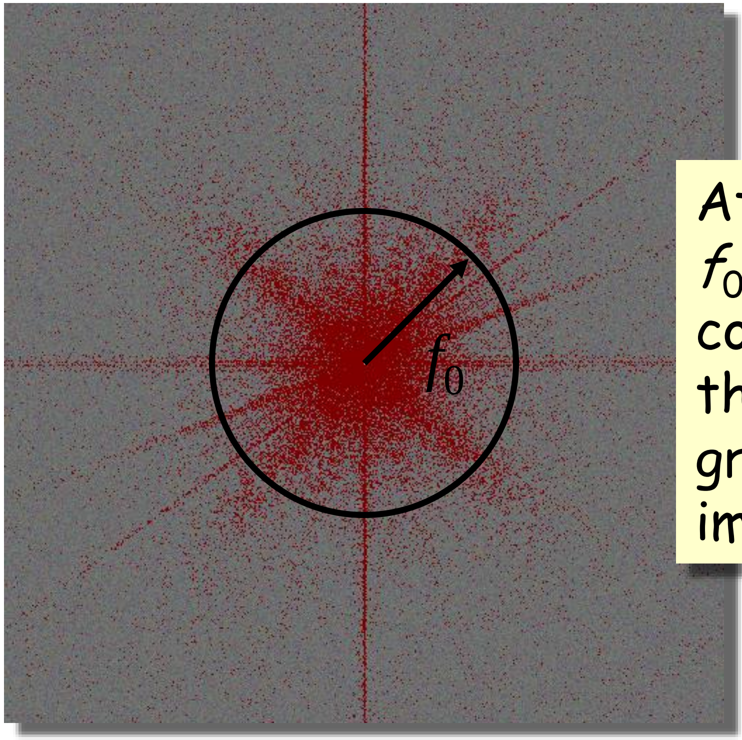
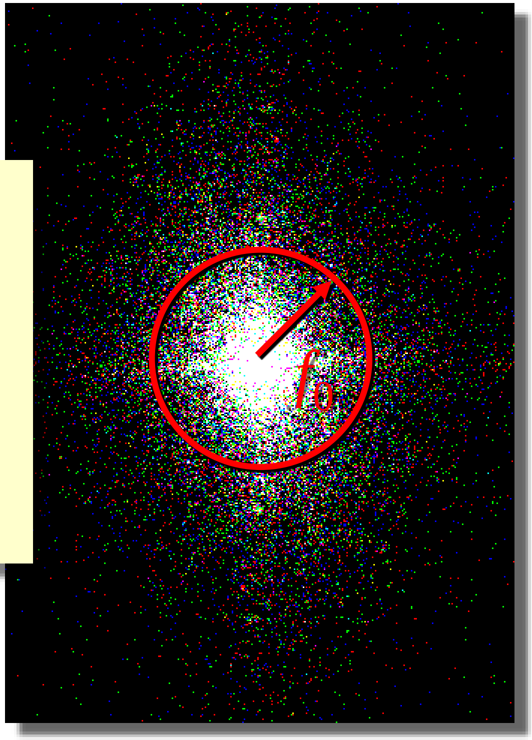
Additive Noise: Reduce Through Blurring?



At some frequency, f_0, there are more components where the noise power is greater than the image power.



red indicates image > noise

image PS > noise PS

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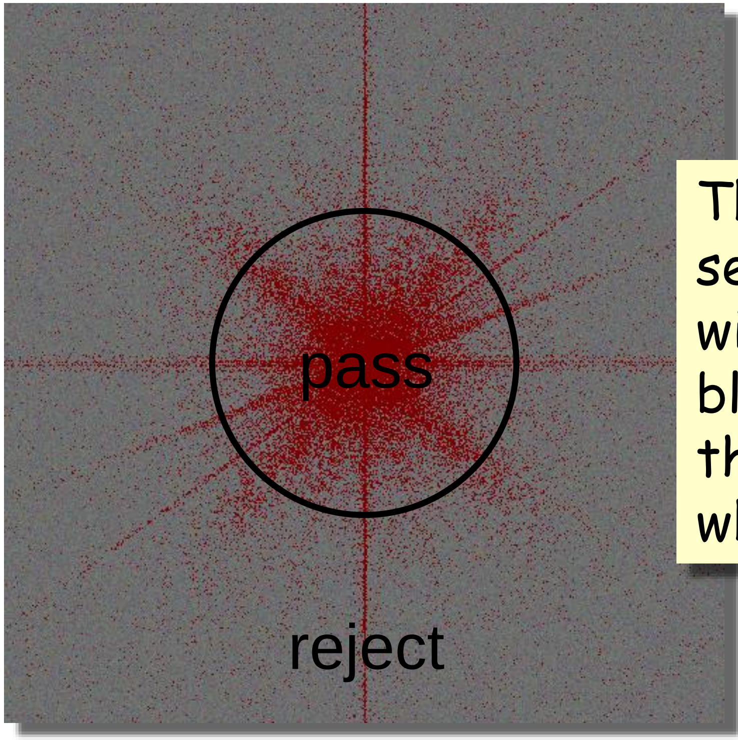
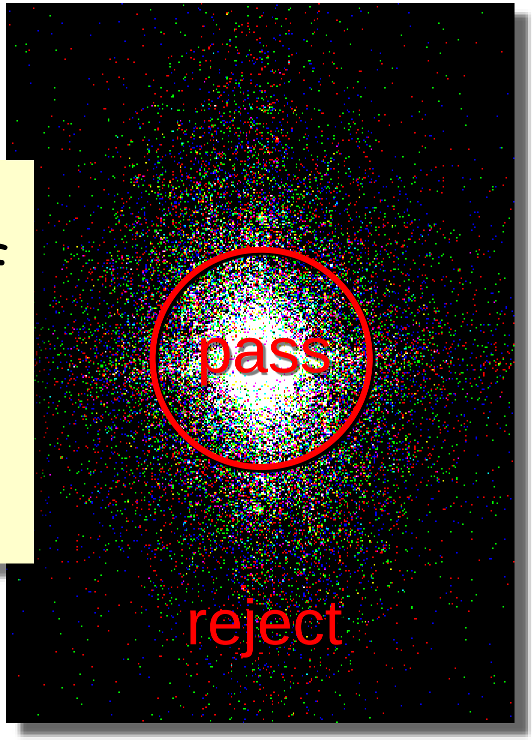
Additive Noise: Reduce Through Blurring?



Thus, it makes sense to apply a LPF with cutoff f0, (a blurring filter) to the images and see what happens.



red indicates image > noise

image PS > noise PS

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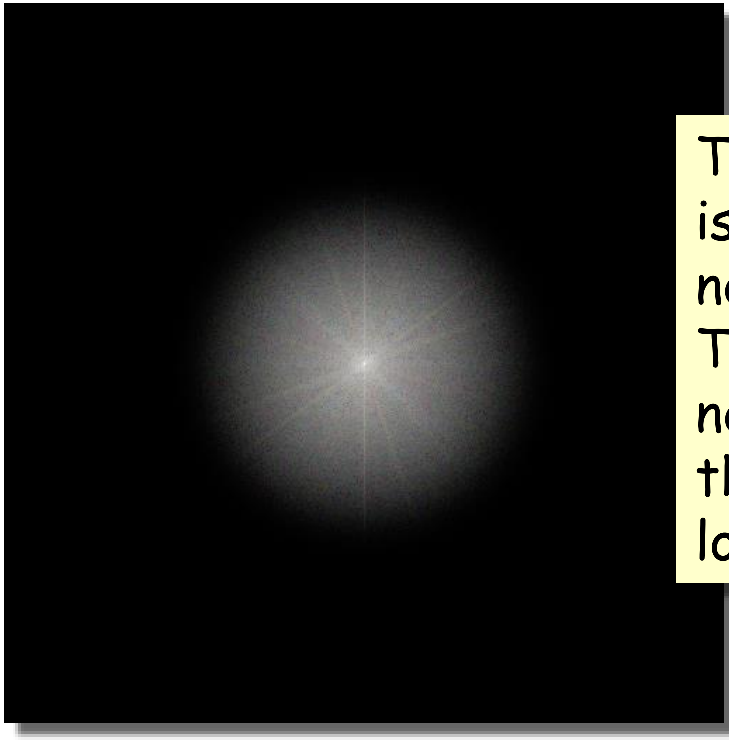
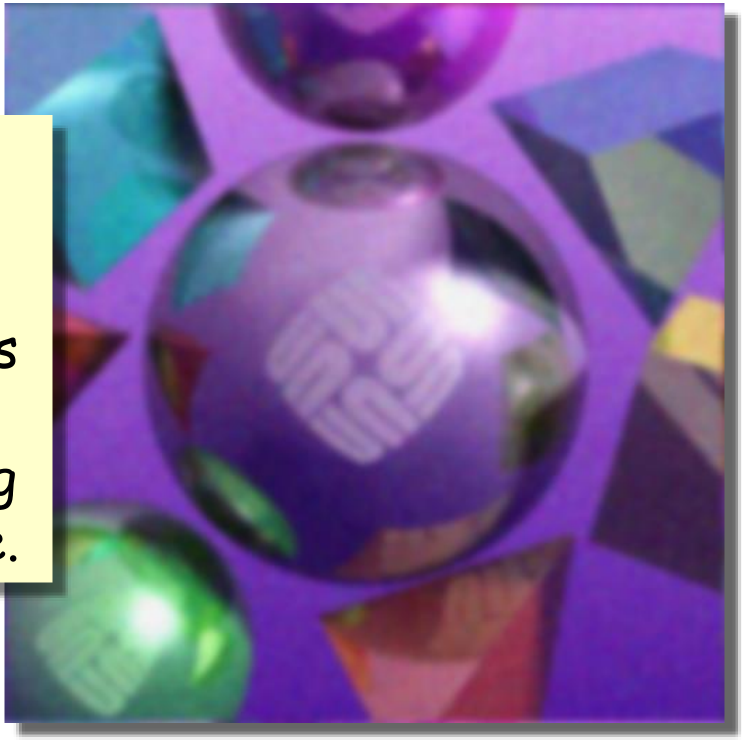
Additive Noise: Reduction Through Blurring.



The result is actually no better. There's less noise but the blurring looks worse.



PS of Gaussian blurred image

Gaussian Blurred Image

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Additive Noise: Reduction Through Blurring.

PS of Gaussian blurred image

The result is actually no better. There's less noise but the blurring looks worse.

Gaussian Blurred Image

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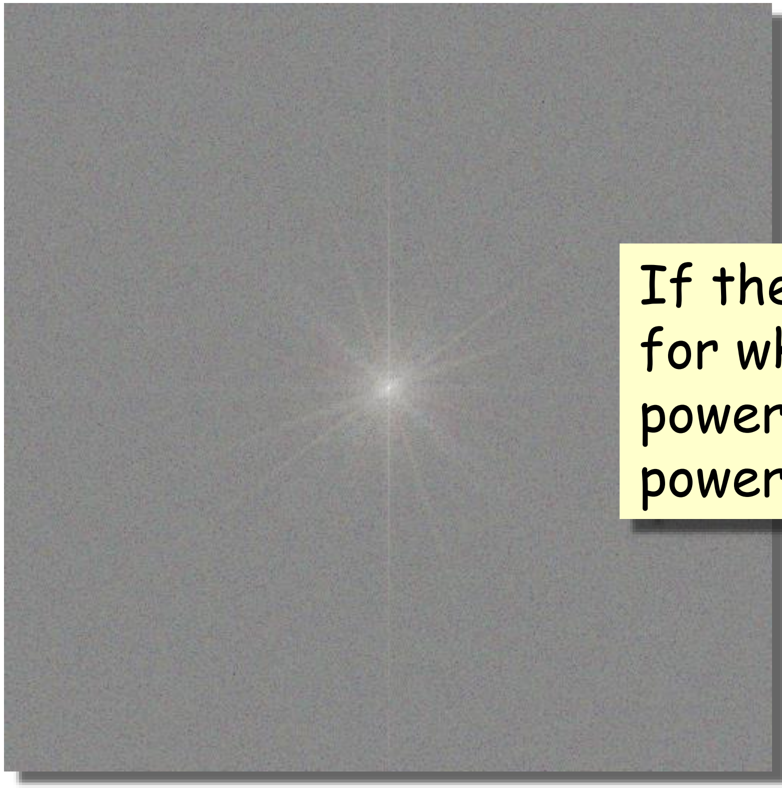
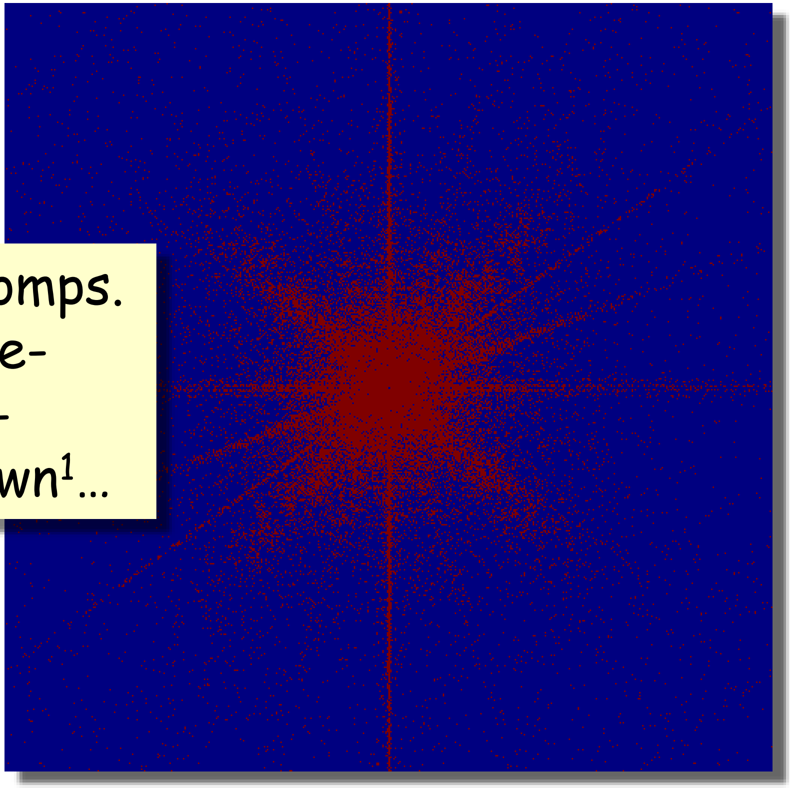
Noise Masking



power spec. of noisy image

If the freq. comps. for which noise-power > image-power are known[1]...



red: image > noise
blue: image < noise

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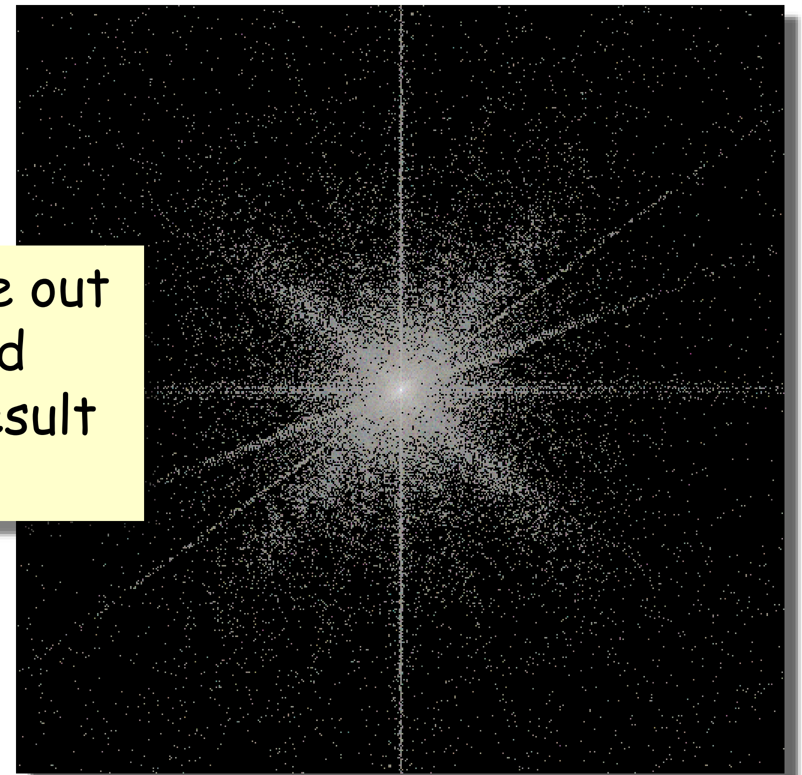
[1]of course they almost never are.

Noise Masking

… mask those out of the FT and invert the result to get…



power spec. of noisy image

image < noise masked out

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Noise Masking

noisy image

... this:

noise-masked mage

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Noise Masking

Although the noise-masked image looks better than the blurred one, it is still noisy. Moreover, this example is unrealistic because we know the exact noise power spectrum. In any real case we will at most know its statistics.

blurred noisy image

noise-masked mage

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Image Degradation Model

So far, we have considered only additive noise. Before going further it will be useful to consider a more general model of image degradation, one that includes convolution with a pointspread[1] function, H, as well as additive noise.

degraded image

pointspread function

$$\mathbf{J}(r,c) = \mathbf{I}(r,c) * \mathbf{H}(r,c) + \mathbf{N}(r,c).$$

undegraded image

additive noise

[1]H is also referred to as the optical transfer function.

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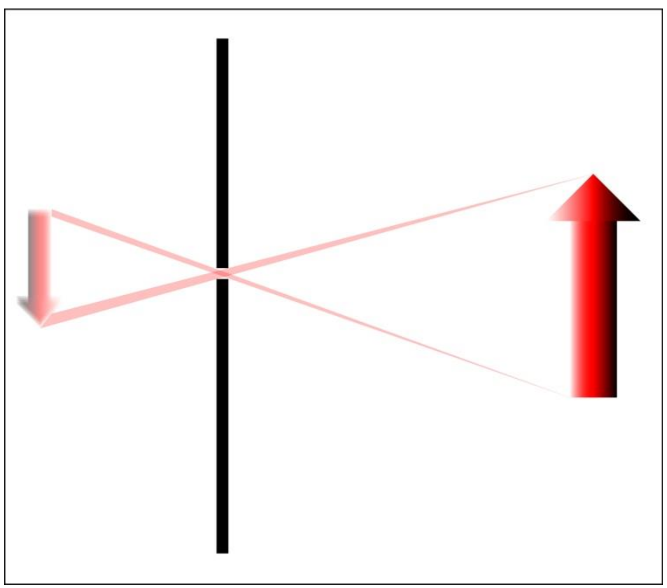
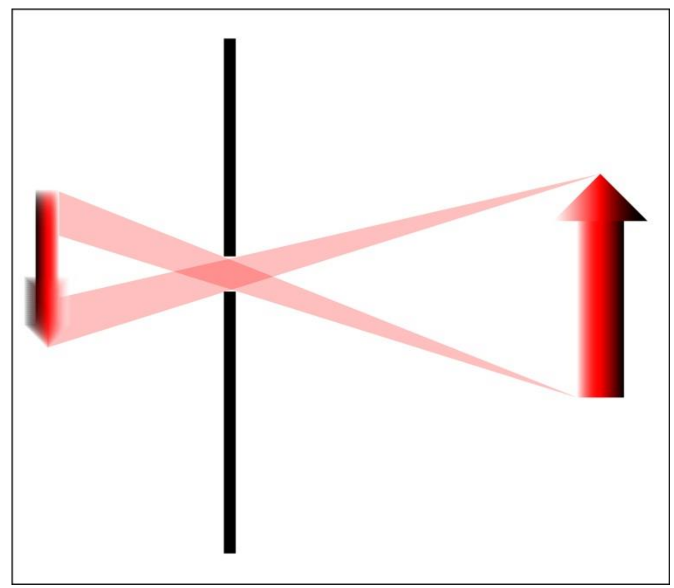
Pointspread Operators

A pointspread operator is a linear model of the distortion acquired during the imaging process. Since it is a linear model, it is a convolution operator. One example of this is aperture distortion, an unavoidable consequence of making an image with a camera that has an opening larger than a point.





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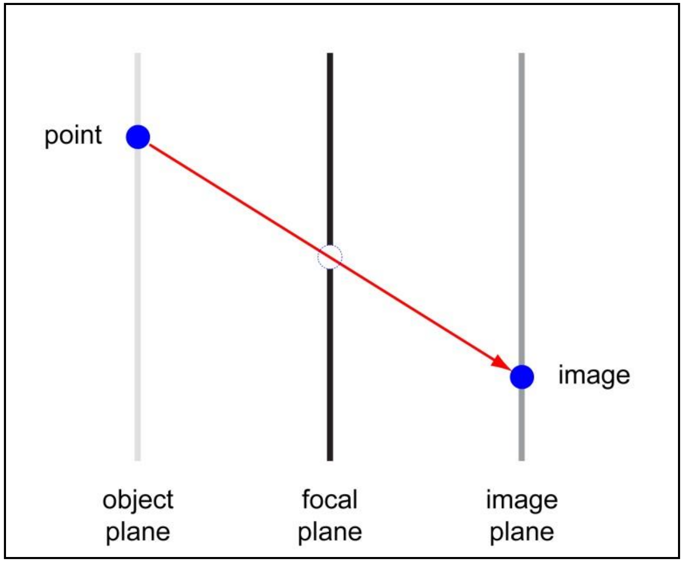
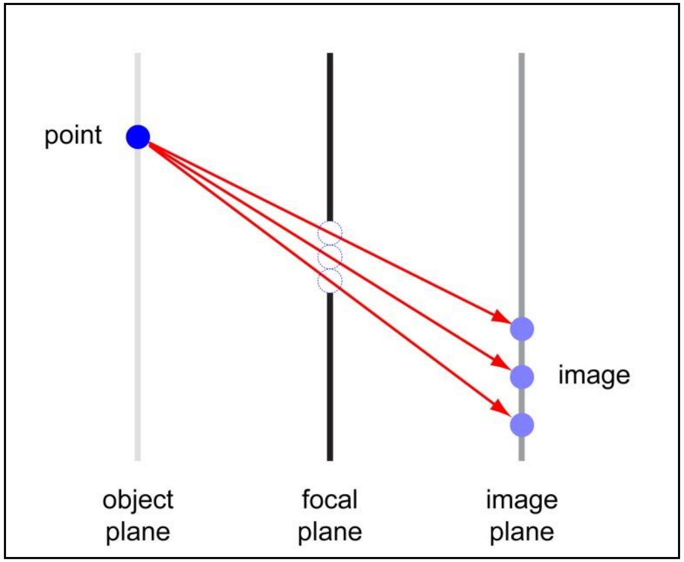
Pointspread Operators

pinhole camera



A pinhole camera maps one object point to one image point; it is one-to-one.

aperture camera



An aperture camera maps one object point to many image points; it spreads the points.

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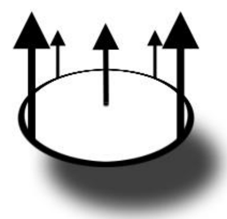
Pointspread Operators and Convolution

$$\mathbf{I}(r,c)$$

$$\mathbf{H}(r,c)$$



$$\mathbf{J}(r,c)=\mathbf{I}(r,c)*\mathbf{H}(r,c)$$

Recall how a convolution works through multiply, shift, and add (See Lect. 7 p. 25ff). That is precisely the effect of imaging through an aperture. It results in a blurry image.

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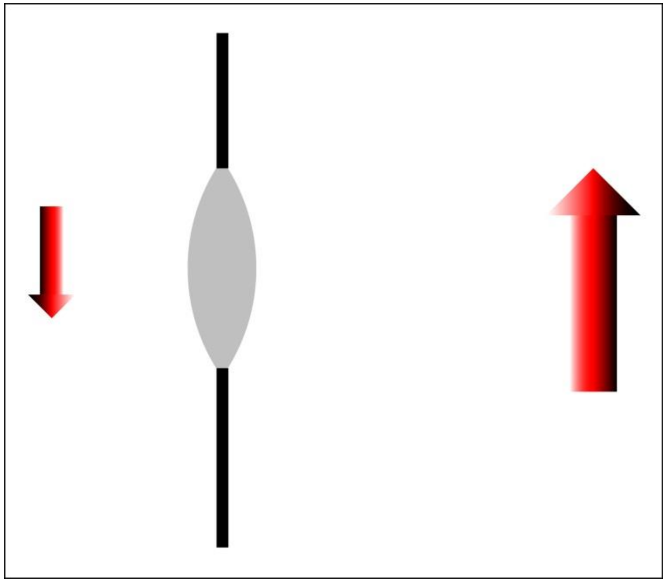
Lenses

A properly designed lens will focus the light emanating from a point and thereby reduce the blurring. But no lens can do this perfectly. In fact, the lens adds its own distortion. The result is an optical transfer function, $\mathbf{H}(r,c)$, that is convolved with the image.



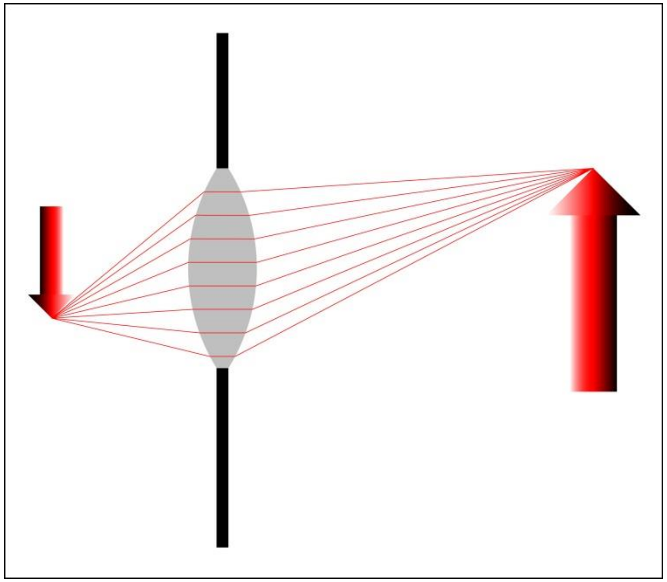


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Image Degradation Model

degraded image

pointspread function

$$\mathrm{J}(r,c) = \mathrm{I}(r,c) * \mathrm{H}(r,c) + \mathrm{N}(r,c).$$

undegraded image

additive noise

Note: The term *pointspread operator* refers to convolution by the pointspread function.

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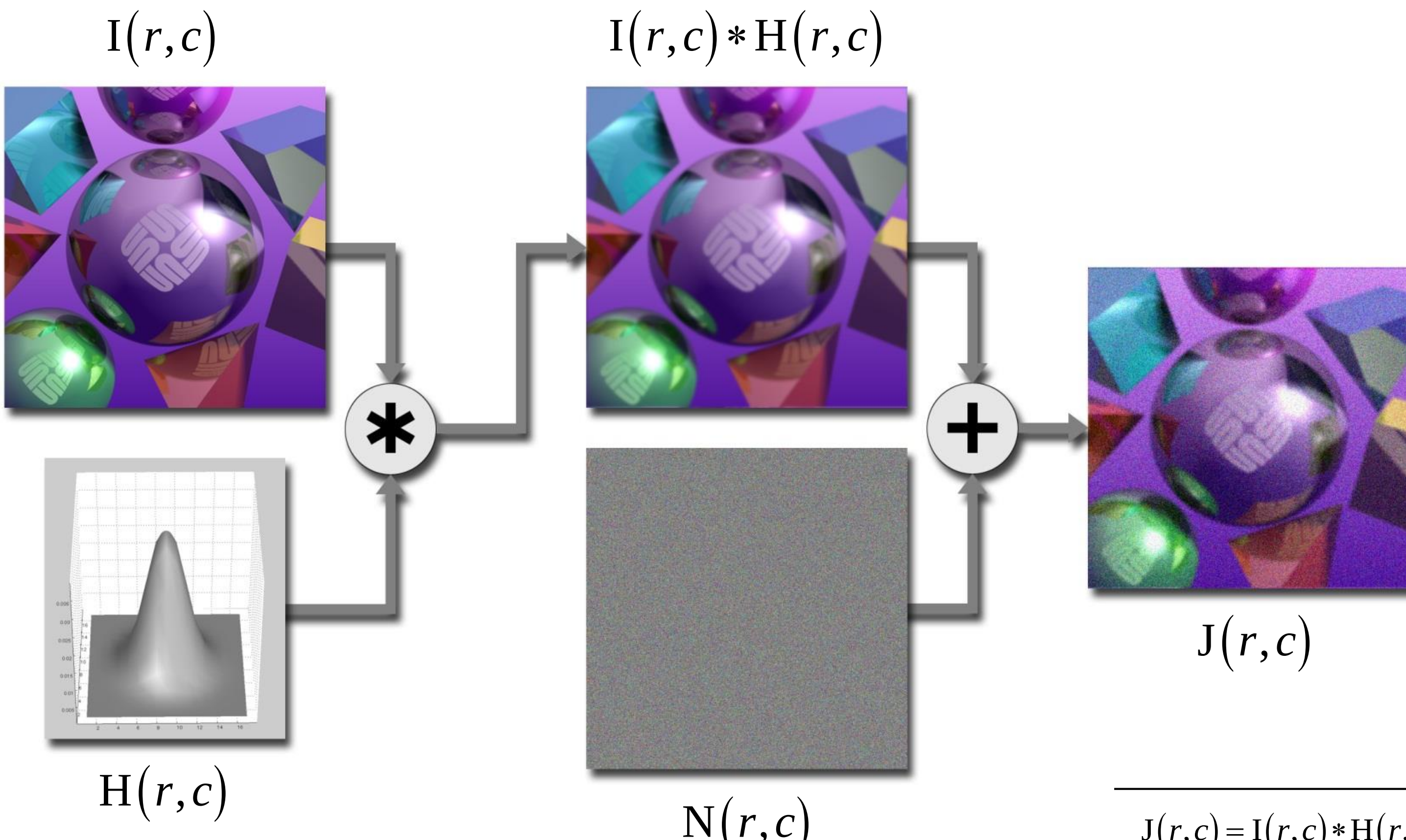
Image Degradation Model



$$J(r,c) = I(r,c)*H(r,c) + N(r,c)$$

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Image Degradation Model (Frequency Domain)

degraded image

pointspread operator

$$\mathcal{J}(v,u) = \mathcal{I}(v,u) \cdot \mathcal{H}(v,u) + \mathcal{N}(v,u).$$

undegraded image

additive noise

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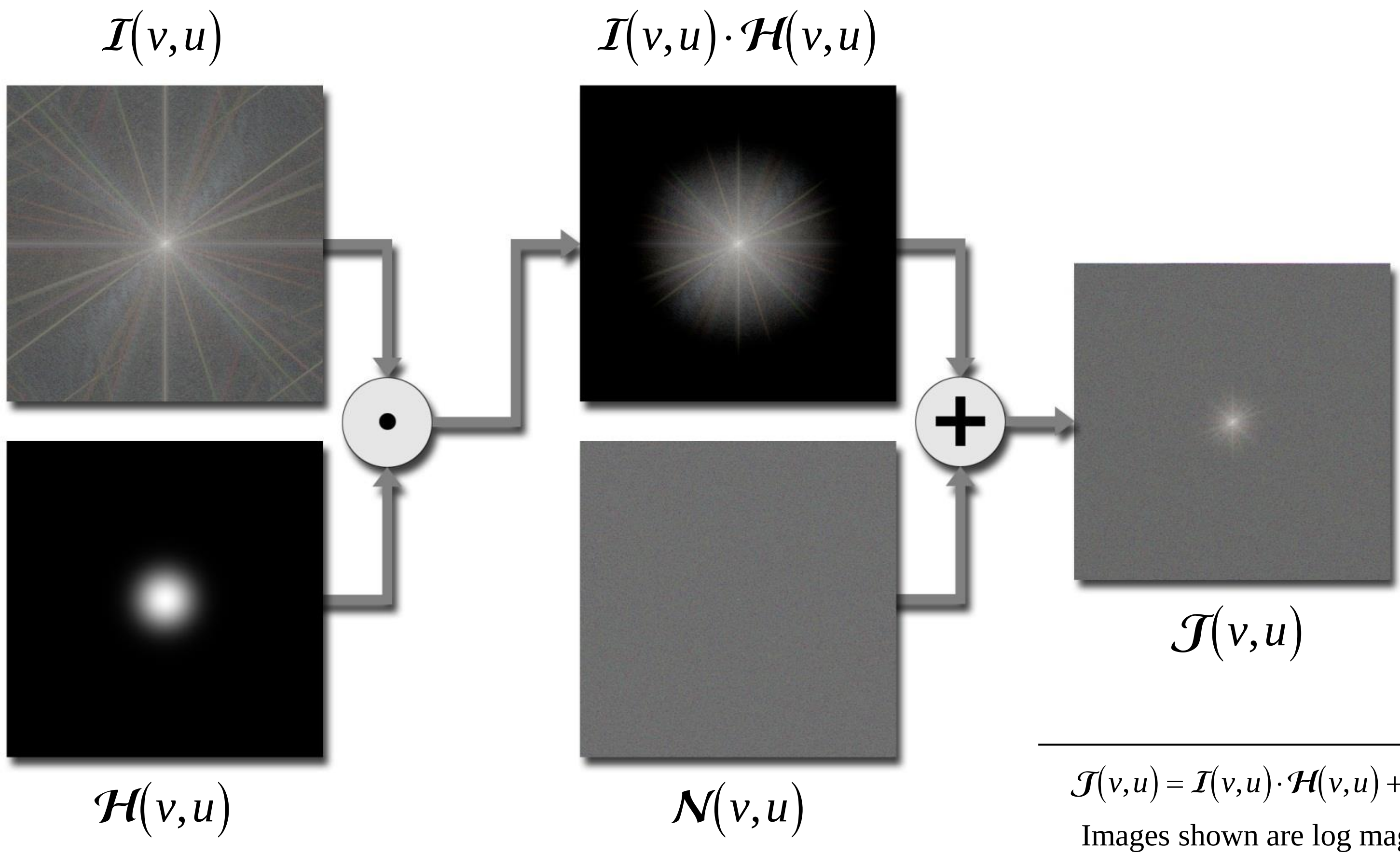
Image Degradation Model (Frequency Domain)



$$\mathcal{J}(v,u) = \mathcal{I}(v,u)\cdot\mathcal{H}(v,u) + \mathcal{N}(v,u).$$

Images shown are log magnitude.

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Image Restoration

Let **I** be a perfect image and let **K** be the image convolved with a pointspread function, **H**. Then in the frequency domain:

$$\mathcal{K}(u,v) = \mathcal{I}(u,v)\,\mathcal{H}(u,v).$$

If the process of imaging adds noise then we get **J** = **K** + **N**, or in freq.:

$$\mathcal{J}(u,v) = \mathcal{K}(u,v) + \mathcal{N}(u,v).$$

We want a filter, **W**, to remove as much of the noise from **J** as possible:

$$\tilde{\mathcal{K}}(v,u) = \mathcal{W}(v,u)\,\mathcal{J}(v,u).$$

Then an estimate of **I** would be the inverse Fourier transform of

$$\tilde{\mathcal{I}}(u,v) = \frac{\tilde{\mathcal{K}}(u,v)}{\mathcal{H}(u,v)} = \frac{\mathcal{W}(u,v)\,\mathcal{J}(u,v)}{\mathcal{H}(u,v)}.$$

We want to find the filter, **W**, that results in the closest possible estimate of **I** *i.e.* the **W** that minimizes the energy of the difference between the estimate and **I**. That is we want to find **W** such that

$$\varepsilon^2 = \iint \left| \mathcal{I} - \tilde{\mathcal{I}} \right|^2 du\,dv$$

is as small as possible. This is called least mean squared (LMS) minimization.

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Image Restoration

There are a number of ways to solve for the minimum squared error. All make use of the assumption that the image and the noise are uncorrelated. Depending on how that fact is used, slightly different solutions are found. The most common one used in image processing is the Wiener filter:

$$\mathcal{W} = \frac{\mathcal{H}^* |\mathcal{I}|^2}{|\mathcal{H}|^2 |\mathcal{I}|^2 + |\mathcal{N}|^2}.$$

Then, with a little bit of algebra, we get

$$\mathcal{W}\mathcal{J} = \frac{|\mathcal{H}|^2 \mathcal{I} + \mathcal{H}^* \mathcal{N}}{|\mathcal{H}|^2 + \dfrac{|\mathcal{N}|^2}{|\mathcal{I}|^2}}.$$

For frequencies (u,v) where noise power is smaller than the image power $\mathcal{W}$ acts like an inverse filter since $\mathcal{N}(u,v)/\mathcal{I}(u,v) < 1$ and

$$\mathcal{W}\mathcal{J}(u,v) \approx \frac{|\mathcal{H}|^2}{|\mathcal{H}|^2} \mathcal{I}(u,v) = \mathcal{I}(u,v),$$

and at frequencies where the noise power dominates, $\mathcal{N}(u,v)/\mathcal{I}(u,v) > 1$ and

$$\mathcal{W}\mathcal{J}(u,v) = \frac{|\mathcal{I}|^2 \mathcal{H}^*}{|\mathcal{I}|^2 |\mathcal{H}|^2 + |\mathcal{N}|^2} \mathcal{N}(u,v),$$

the fraction is small so the noise power is diminished.

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Image Restoration

$$
\begin{aligned}
\varepsilon^2 &= \iint \left| \mathcal{I} - \tilde{\mathcal{I}} \right|^2 du\,dv \\
&= \iint \left| \frac{\mathcal{K}}{\mathcal{H}} - \frac{\mathcal{W}\mathcal{J}}{\mathcal{H}} \right|^2 du\,dv \\
&= \iint |\mathcal{H}|^{-2} \left| \mathcal{K} - \mathcal{W}(\mathcal{K} + \mathcal{N}) \right|^2 du\,dv \\
&= \iint |\mathcal{H}|^{-2} \left| \mathcal{K}(1 - \mathcal{W}) + \mathcal{W}\mathcal{N} \right|^2 du\,dv \\
&= \iint |\mathcal{H}|^{-2} \left[\mathcal{K}(1 - \mathcal{W}) + \mathcal{W}\mathcal{N} \right] \overline{\left[\mathcal{K}(1 - \mathcal{W}) + \mathcal{W}\mathcal{N} \right]} du\,dv \\
&= \iint |\mathcal{H}|^{-2} \left[\left| \mathcal{K}(1 - \mathcal{W}) \right|^2 + \mathcal{K}(1 - \mathcal{W}) \overline{\mathcal{W}\mathcal{N}} + \mathcal{W}\mathcal{N} \overline{\mathcal{K}(1 - \mathcal{W})} + \left| \mathcal{W}\mathcal{N} \right|^2 \right] du\,dv \\
&= \iint |\mathcal{H}|^{-2} \left\{ \left| \mathcal{K}(1 - \mathcal{W}) \right|^2 + 2\,\mathrm{Re}\left[\mathcal{K}(1 - \mathcal{W}) \overline{\mathcal{W}\mathcal{N}} \right] + \left| \mathcal{W}\mathcal{N} \right|^2 \right\} du\,dv \\
&= \iint |\mathcal{H}|^{-2} \left[\left| \mathcal{K}(1 - \mathcal{W}) \right|^2 + \left| \mathcal{W}\mathcal{N} \right|^2 \right] du\,dv + 2\,\mathrm{Re} \iint |\mathcal{H}|^{-2} (1 - \mathcal{W}) \overline{\mathcal{W}} \mathcal{K} \overline{\mathcal{N}}\, du\,dv
\end{aligned}
$$

This is one of the possible derivations of the Wiener filter

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Image Restoration

From the previous page, the squared error is

$$\varepsilon^2 = \iint |\mathcal{H}|^{-2}\left[|\mathcal{K}(1-\mathcal{W})|^2 + |\mathcal{W}\mathcal{N}|^2\right]du\,dv + 2\,\mathrm{Re}\iint |\mathcal{H}|^{-2}(1-\mathcal{W})\,\overline{\mathcal{W}}\mathcal{K}\overline{\mathcal{N}}\,du\,dv.$$

The second term should be small compared to the first since it can be written

$$2\,\mathrm{Re}\iint |\mathcal{H}|^{-1}(1-\mathcal{W})\,\overline{\mathcal{W}}\,\mathcal{I}\,\overline{\mathcal{N}}\,du\,dv,$$

and the image and the noise are assumed to be uncorrelated[1]. Thus the error can be approximated by

$$\varepsilon^2 = \iint |\mathcal{H}|^{-2}\left[|\mathcal{K}(1-\mathcal{W})|^2 + |\mathcal{W}\mathcal{N}|^2\right]du\,dv.$$

The mean squared error, ε^2, is minimized when $\mathcal{W}$ is given by,

$$\mathcal{W} = \frac{\mathcal{H}^*|\mathcal{I}|^2}{|\mathcal{H}|^2|\mathcal{I}|^2 + |\mathcal{N}|^2}.$$

[1] $\iint \mathcal{I}\,\overline{\mathcal{N}}\,du\,dv = 0.$

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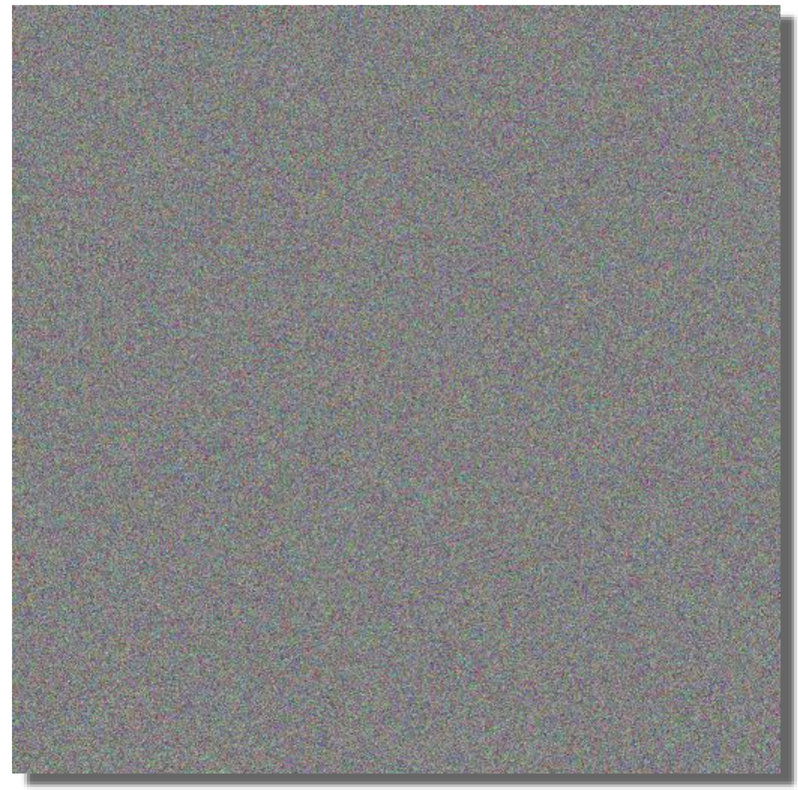
Noise Reduction Through LMS Filtering[1]

image



Gaussian noise field

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[1]Here the PSF is the identity.

Noise Reduction Through LMS Filtering[1]

image

noisy image

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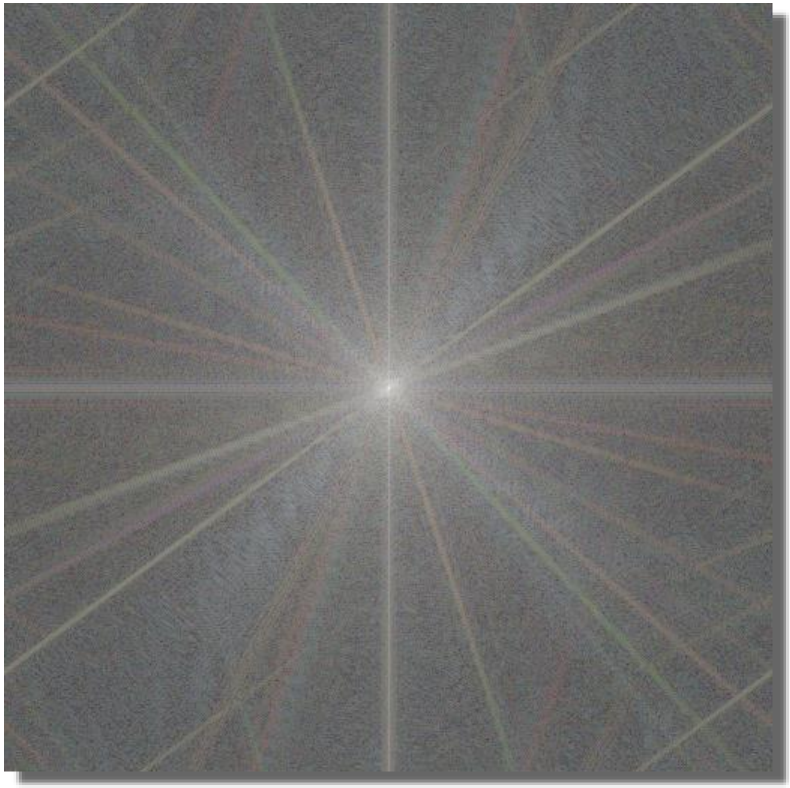
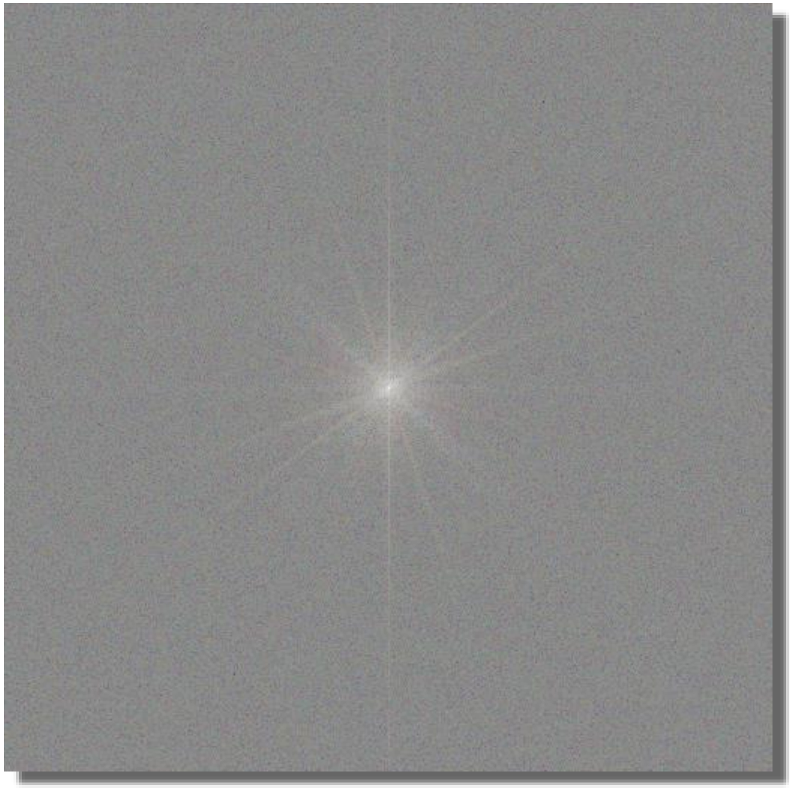
[1]Here the PSF is the identity.

Additive Noise (Power Spectra)



original image



noisy image

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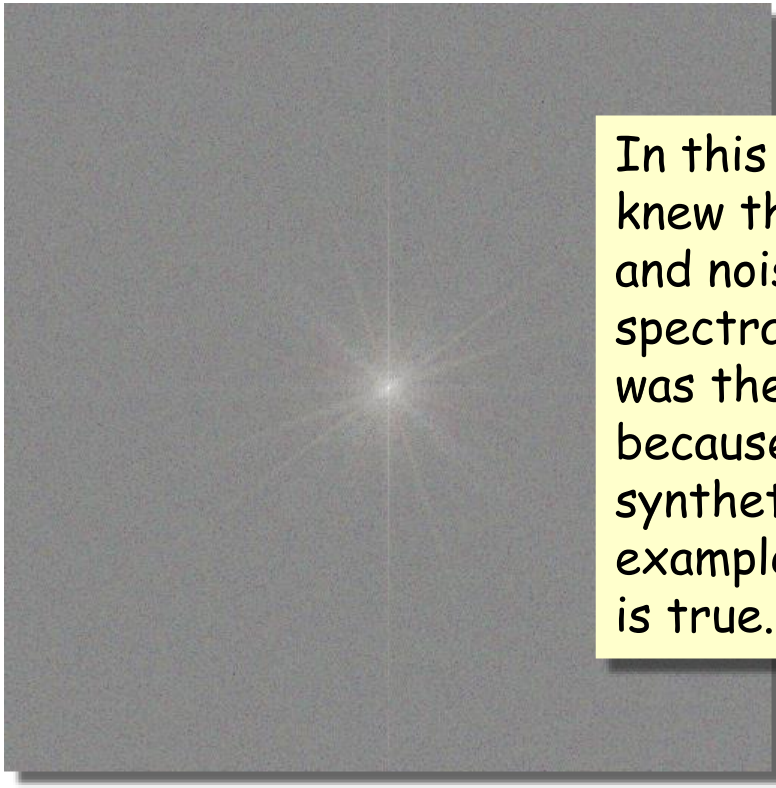
Additive Noise (Power Spectra)



noisy image

In this example we knew the exact image and noise power spectra and the PSF was the identity because the image is synthetic. In a real example, none of that is true.

Wiener filter

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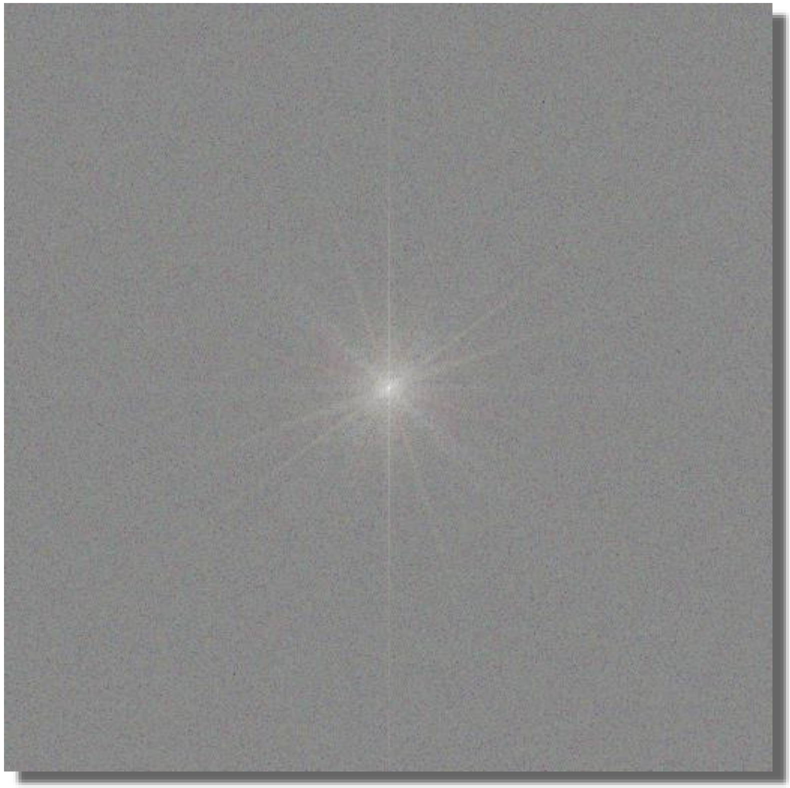
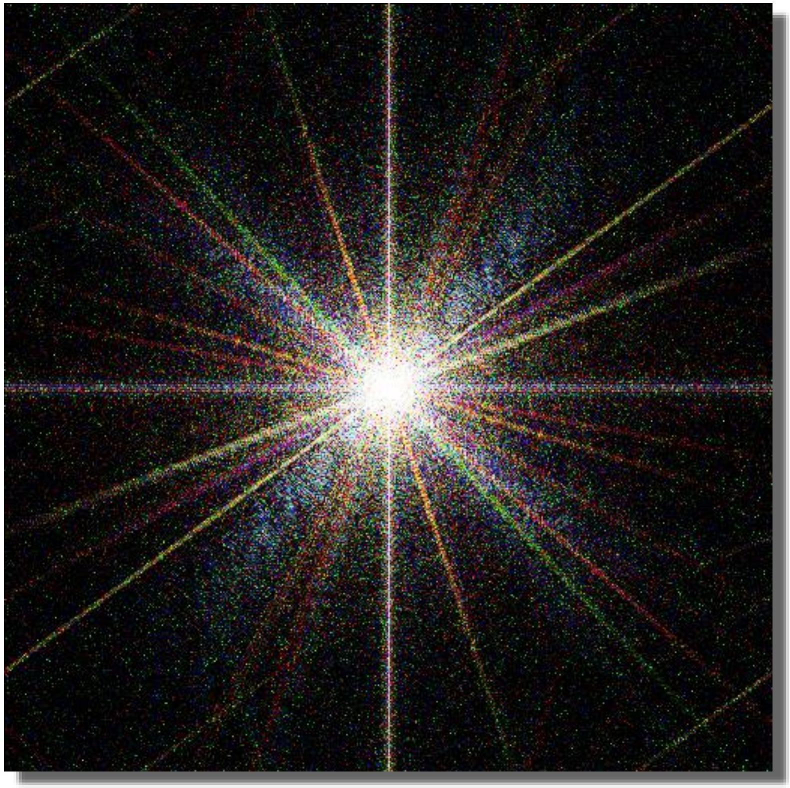
Additive Noise (Power Spectra)



noisy image



Wiener filter

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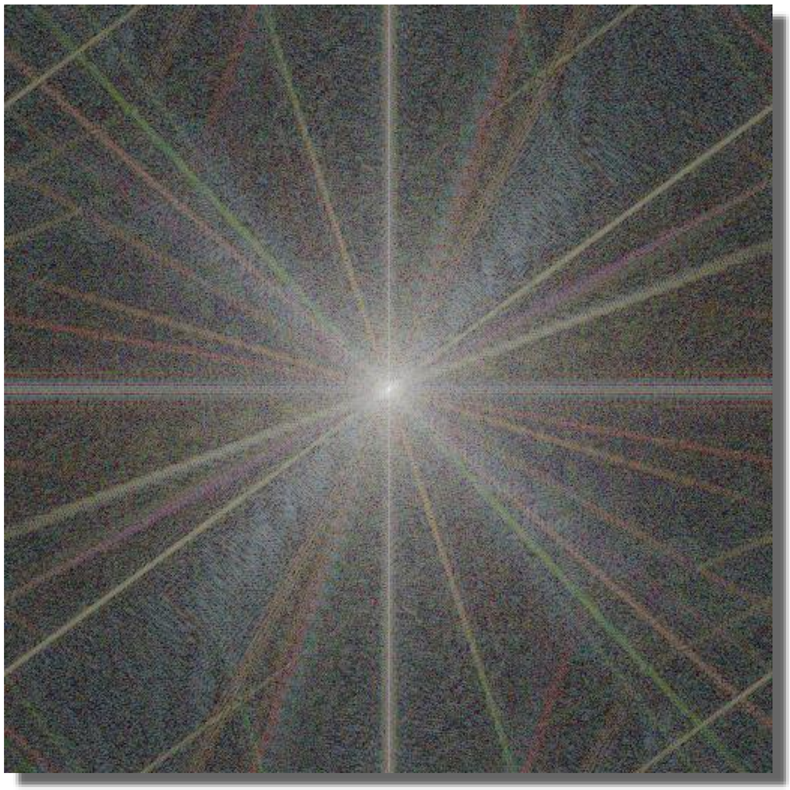
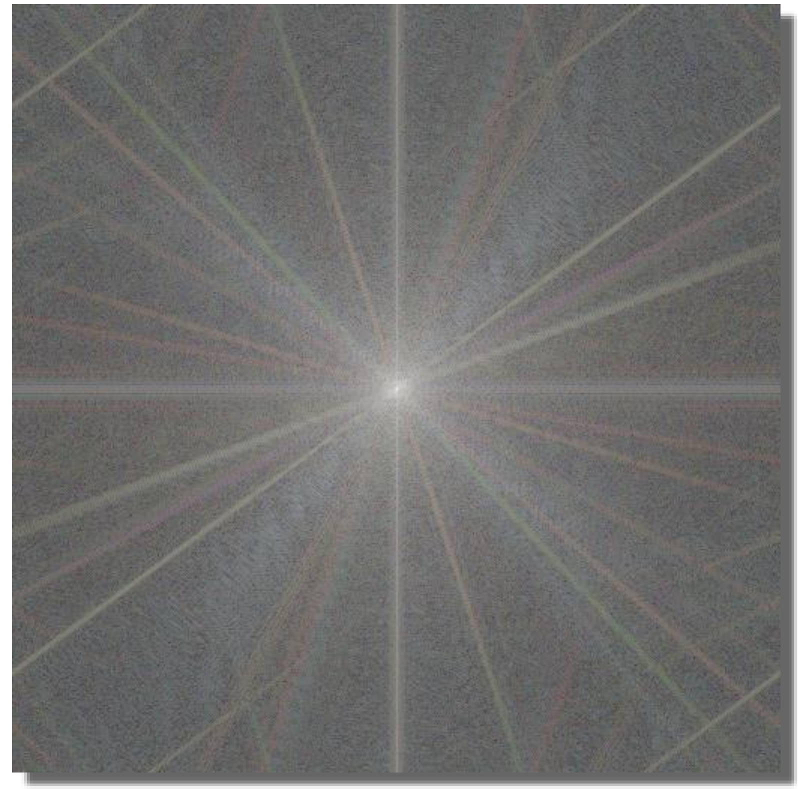
Additive Noise (Power Spectra)



Wiener filtered image



original image

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Additive Noise

noisy image

Wiener filtered image

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Additive Noise

Wiener filtered image

original image

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Additive Noise

Gaussian blurred image

original image

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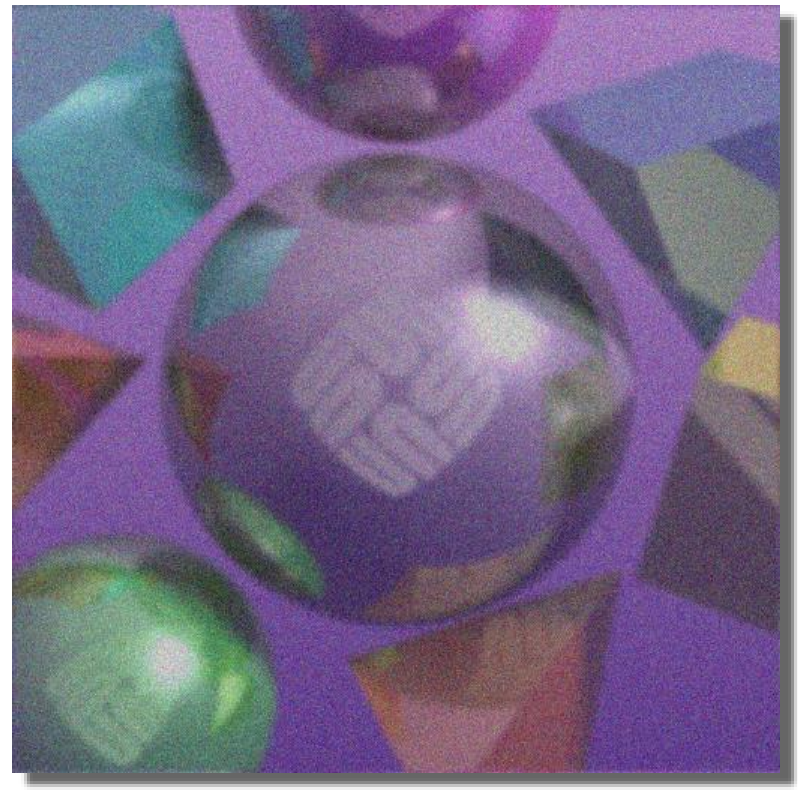
Noise Reduction Through LMS Filtering[1]

image



noisy image $\mathbf{J} = \mathbf{I} * \mathbf{h} + \mathbf{N}$

[1]Least Mean Squared. PSF, **h**, is Gaussian μ=0, σ=2.

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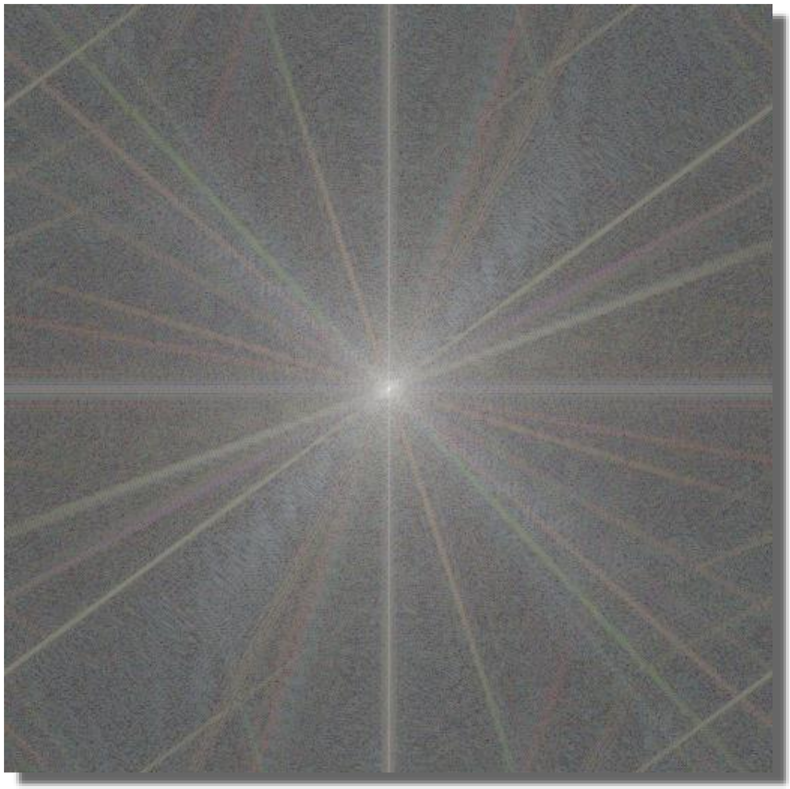
Image*PSF + Noise (Power Spectra)



original image

noisy image **J** = **I*h** + **N**

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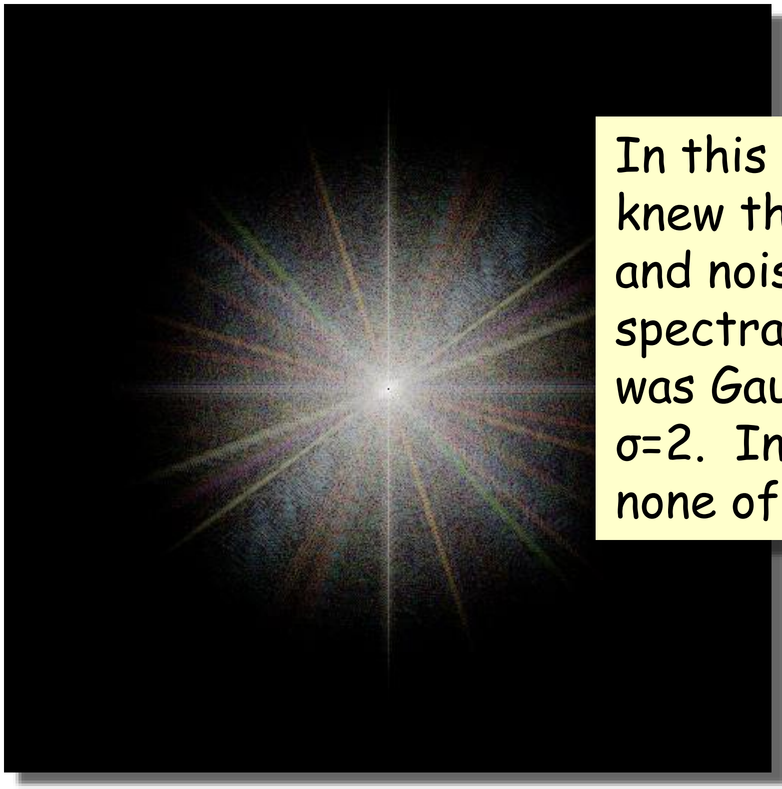
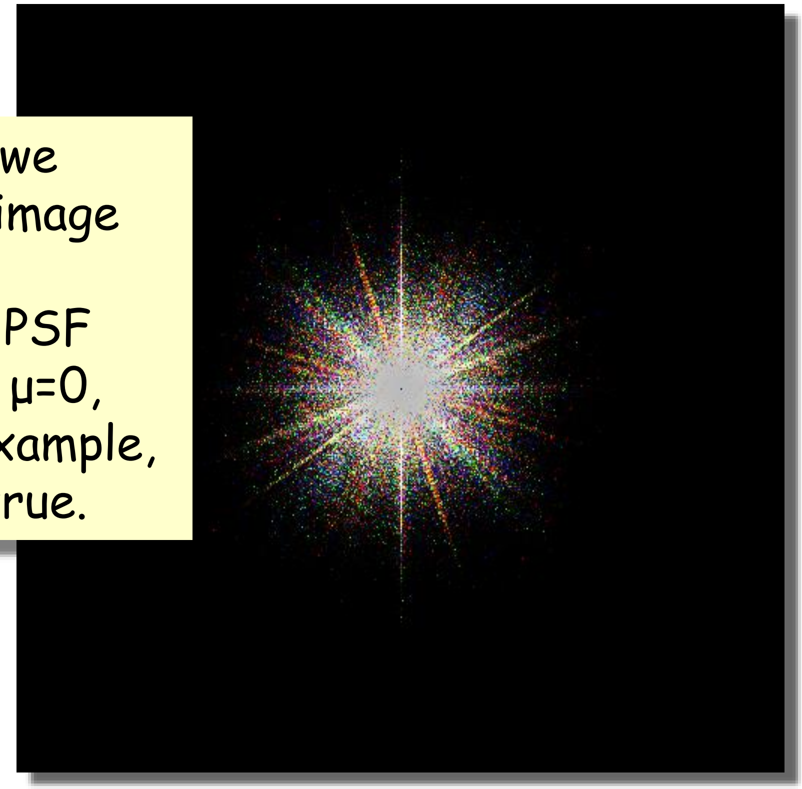
Image*PSF + Noise (Power Spectra)



In this example we knew the exact image and noise power spectra and the PSF was Gaussian w/ μ=0, σ=2. In a real example, none of that is true.



Wiener filtered image

Wiener filter

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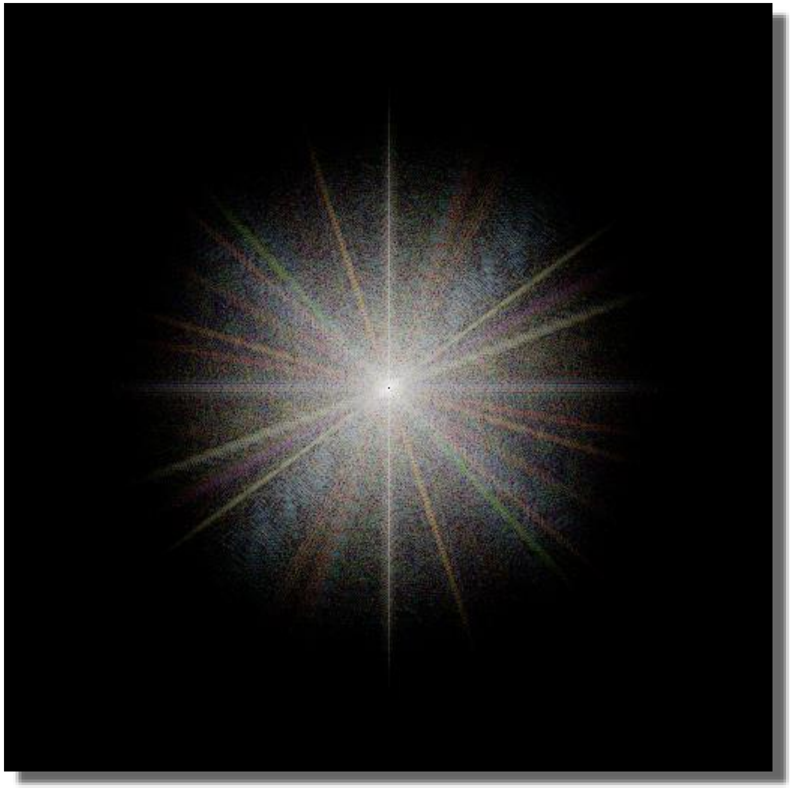
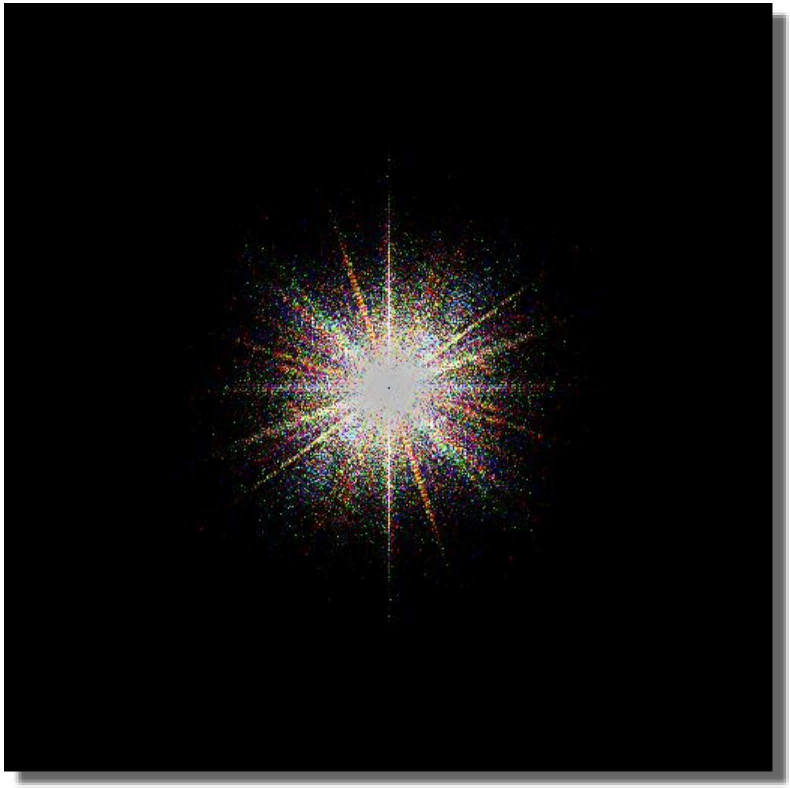
Image*PSF + Noise (Power Spectra)



Wiener filtered image



Wiener filter

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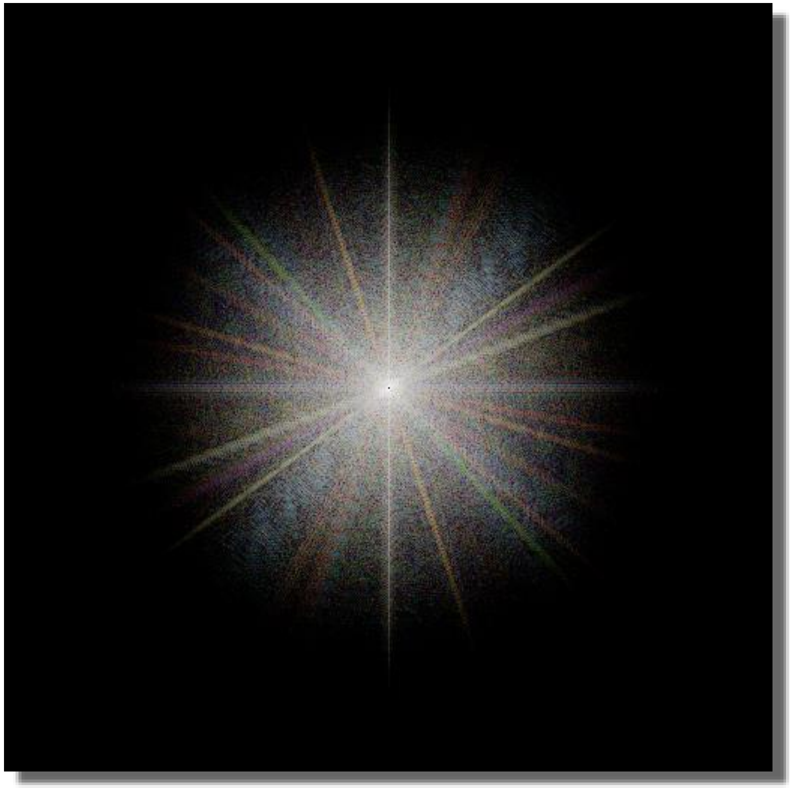
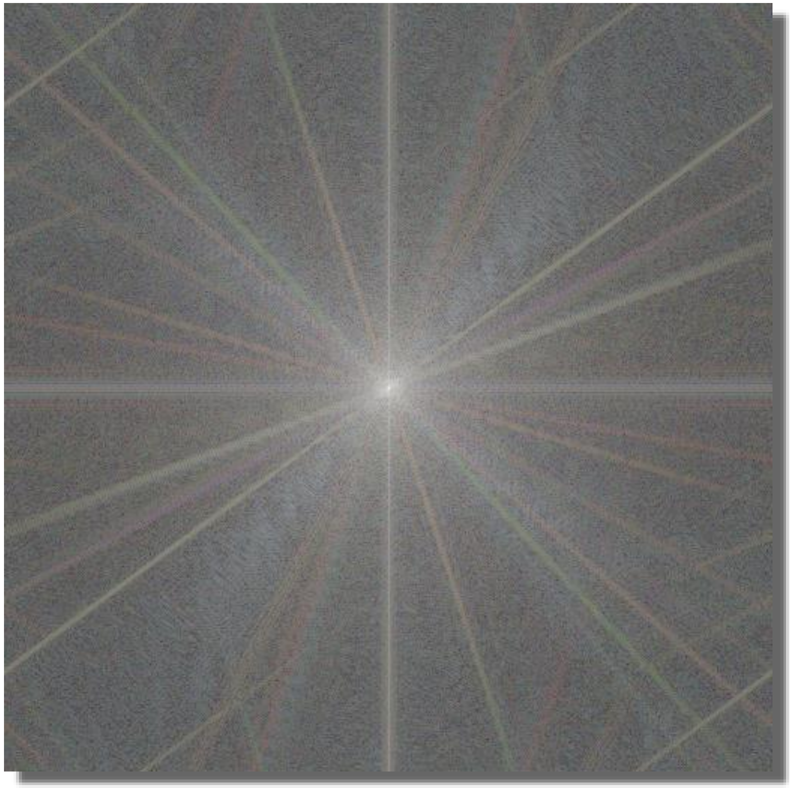
Image*PSF + Noise (Power Spectra)



Wiener filtered image



original image

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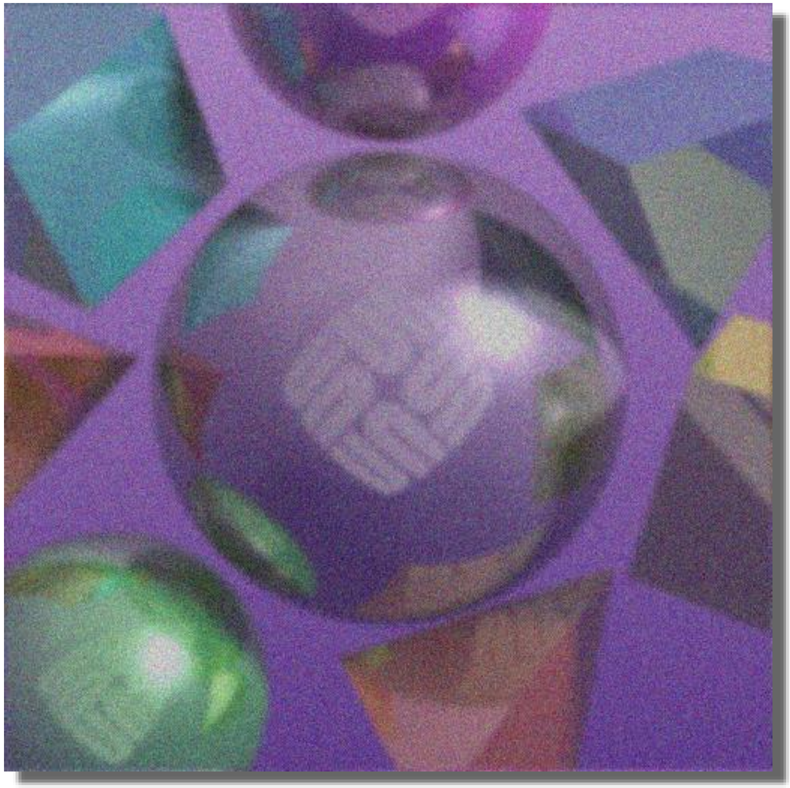
Image*PSF + Noise



noisy image **J** = **I*h** + **N**

Wiener filtered image

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Image*PSF + Noise

Wiener filtered image

original image

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LMS Image Restoration (Real Example)

For this real example we need to estimate the image power spectrum, the pointspread function and the noise power spectrum.

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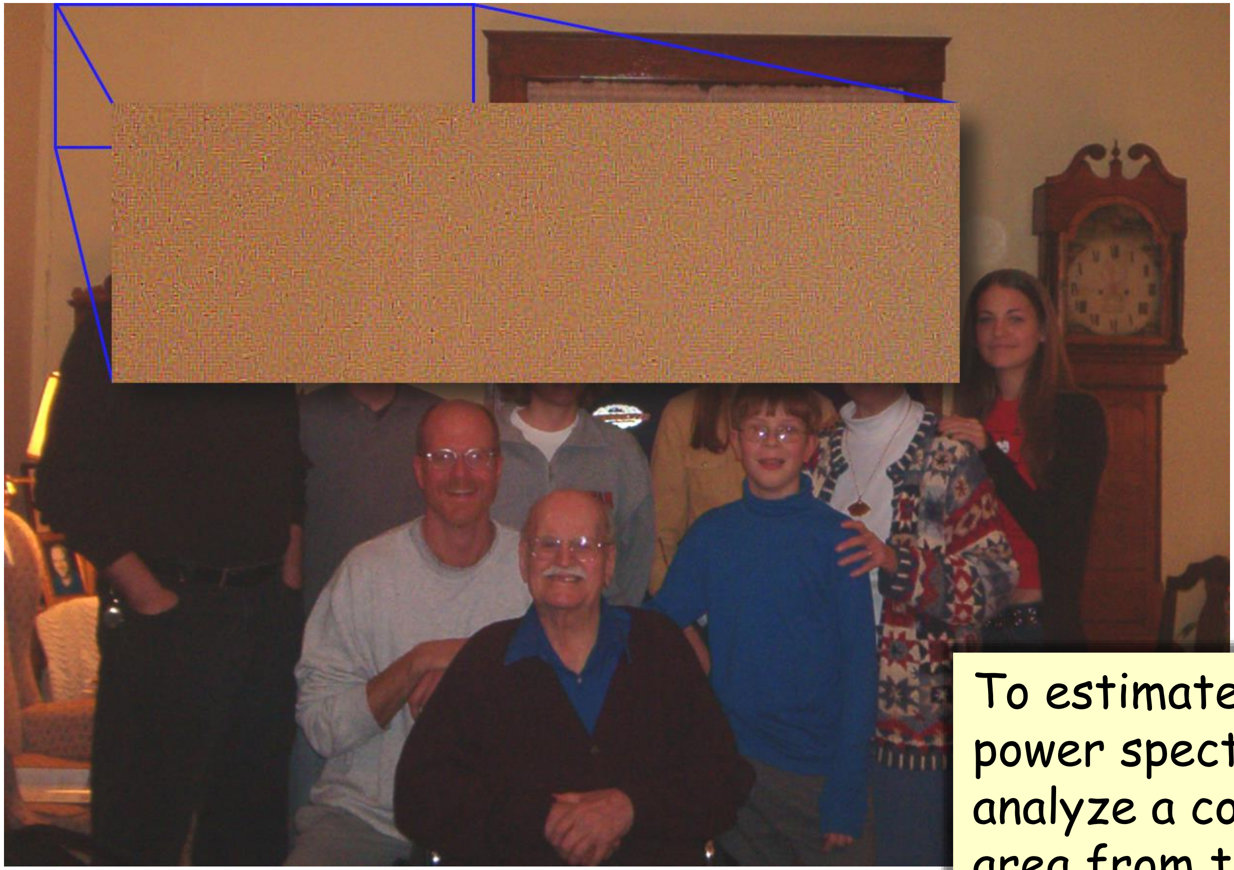
LMS Image Restoration (Real Example)



To estimate the noise power spectrum, analyze a constant area from the image.

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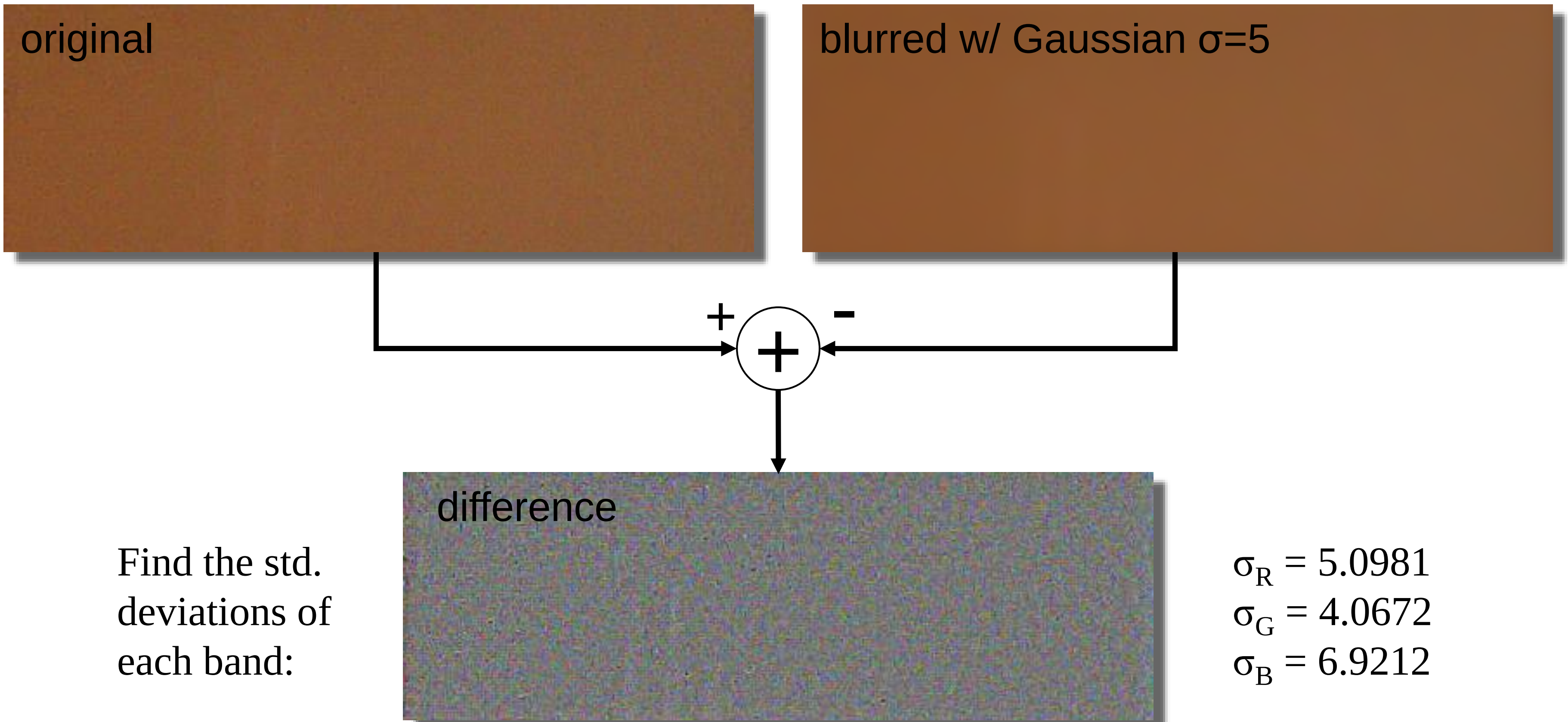
Noise Estimation



Find the std. deviations of each band:

σ_R = 5.0981
σ_G = 4.0672
σ_B = 6.9212

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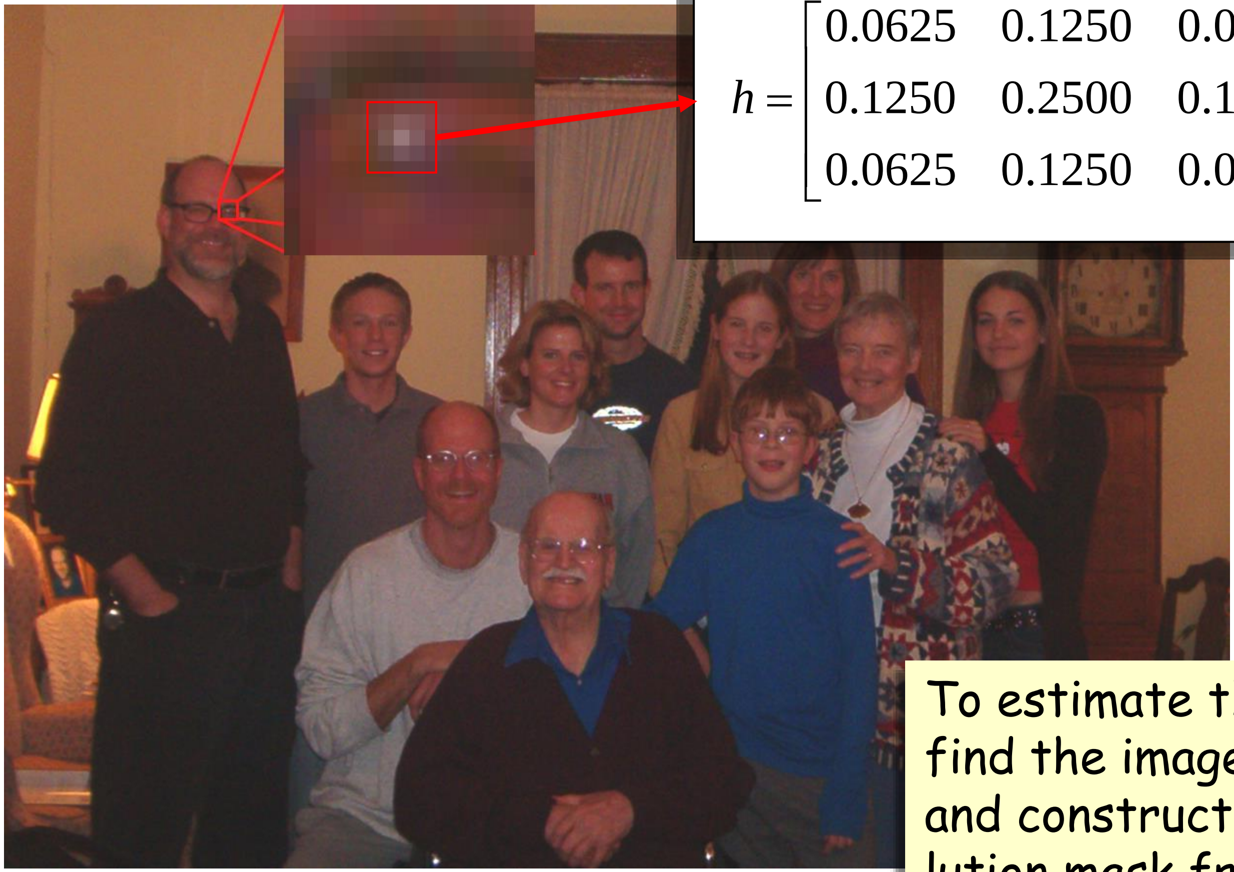
Pointspread Function Estimation



$$h = \begin{bmatrix} 0.0625 & 0.1250 & 0.0625 \\ 0.1250 & 0.2500 & 0.1250 \\ 0.0625 & 0.1250 & 0.0625 \end{bmatrix}$$

To estimate the PSF, find the image of a point and construct a convo-lution mask from it.

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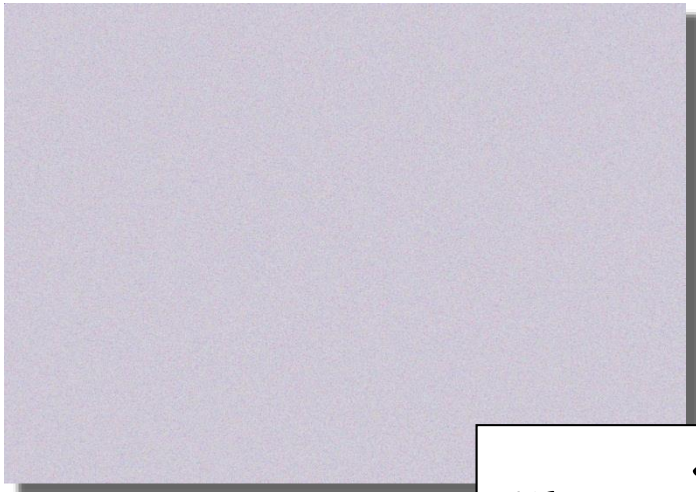
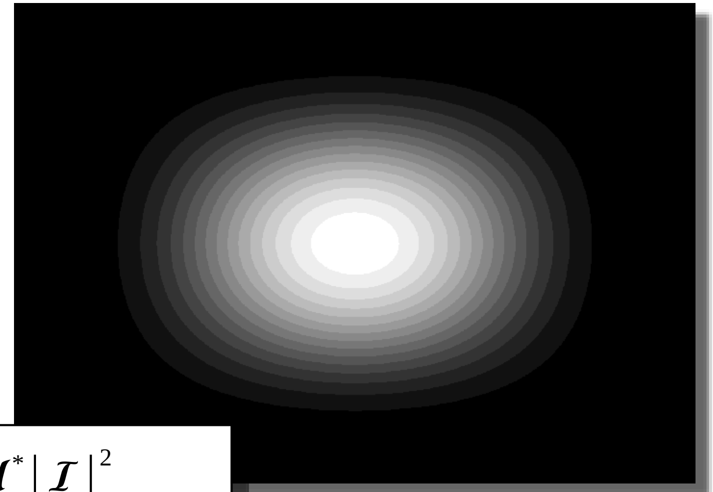
Wiener Filter Estimation

$|\mathcal{N}|^2$

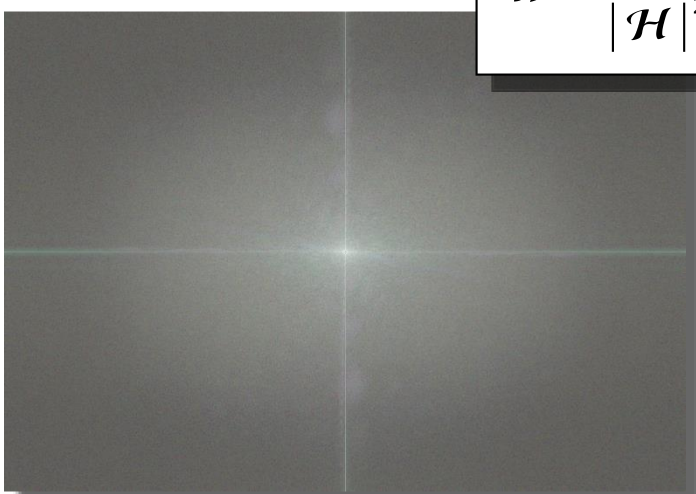


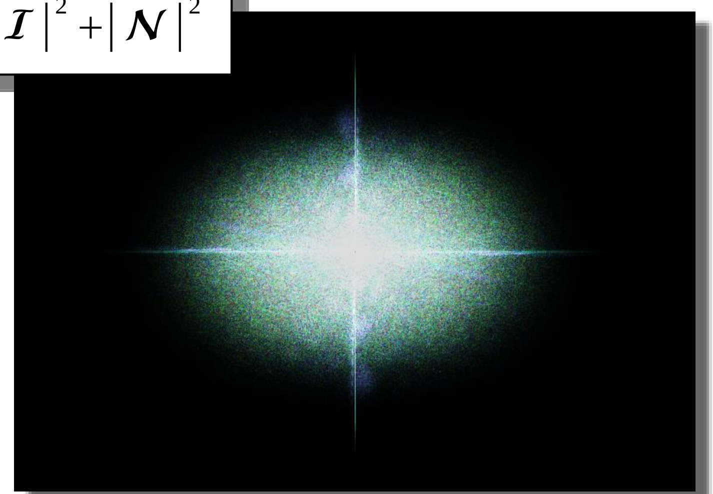


$|\mathcal{H}|^2$

$$\mathcal{W} = \frac{\mathcal{H}^* |\mathcal{I}|^2}{|\mathcal{H}|^2 |\mathcal{I}|^2 + |\mathcal{N}|^2}$$

$|\mathcal{I}|^2$





$\mathcal{W}$

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LMS Image Restoration (original)

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LMS Image Restoration (filtered)

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Detail of Results

The contrast of these has been increased to make the differences more visible.

original image

filtered image

matlab's wiener2

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