EECE 4353 Image Processing

Lecture Notes: Sharpening and Edge Enhancement

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Fall Semester 2016

Sharpening

- Results from high frequency enhancement since small features correspond to short wavelength sinusoids.
- Relative amplification of high frequencies in the Fourier domain corresponds to differentiation in the spatial domain.
- On a discrete image, differentiation corresponds to pixel differencing.

The Derivative Property of the Fourier Transform

The FT of the partial derivative w.r.t. r (in the row direction) of an image, I …

$$\mathscr{F}\left\{\frac{\partial \mathbf{I}}{\partial r}\right\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tfrac{\partial}{\partial r}\mathbf{I}(r,c)\, e^{-i2\pi(uc+vr)}\, dc\, dr$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{I}(r,c)\cdot \tfrac{\partial}{\partial r} e^{-i2\pi(uc+vr)}\, dc\, dr$$

Integration by parts

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{I}(r,c)\cdot(-i2\pi v)\, e^{-i2\pi(uc+vr)}\, dc\, dr$$

$$= -i2\pi v \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{I}(r,c)\, e^{-i2\pi(uc+vr)}\, dc\, dr$$

$$= -i2\pi v\, \mathscr{F}\{\,\mathbf{I}\,\} = -i2\pi v\, \mathbf{F}(u,v).$$

… is equal to the product of the FT of the image and the corresponding frequency variable, v.

This results in horizontal HF enhancement

Differentiation is Highpass Filtering

Directional derivative in *c*.

Vertical HF Enhancement

$$\mathcal{F}\left\{\frac{\partial \mathbf{I}}{\partial c}\right\}(u,v) \propto u\mathcal{F}\{\mathbf{I}\}(u,v)$$

$$\mathcal{F}\left\{\frac{\partial \mathbf{I}}{\partial r}\right\}(u,v) \propto v\mathcal{F}\{\mathbf{I}\}(u,v)$$

Directional derivative in r.

Horizontal HF Enhancement

Fourier Transforms of Sums of Derivatives

$$\mathfrak{F}\left\{\left[\frac{\partial}{\partial r}+\frac{\partial}{\partial c}\right]\mathbf{I}\right\}=-i2\pi\left(u+v\right)\mathfrak{F}\left\{\mathbf{I}\right\}=-i2\pi\left(u+v\right)\mathbf{F}\left(u,v\right).$$

Sum of first-order partial derivatives…

…linear amplification of high frequencies

$$\mathfrak{F}\left\{\left[\frac{\partial^2}{\partial r^2}+\frac{\partial^2}{\partial c^2}\right]\mathbf{I}\right\}=-4\pi^2\left(u^2+v^2\right)\mathfrak{F}\left\{\mathbf{I}\right\}=-4\pi^2\left(u^2+v^2\right)\mathbf{F}\left(u,v\right).$$

Sum of second-order partial derivatives…

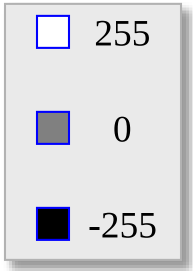
…quadratic amplification of high frequencies

Sharpening via Differencing or Highpass Filtering

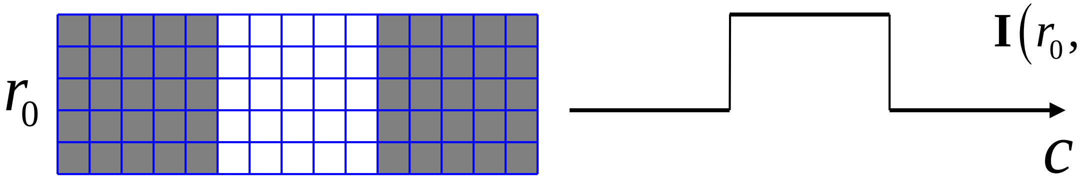
Sharpening results from **adding to the image a copy of itself** that has been:

- Pixel-differenced in the spatial domain:
  - Each pixel in the output is a difference between itself and a weighted average of its neighbors.
  - Is a convolution whose weight matrix sums to 0.
- Highpass filtered in the frequency domain:
  - High frequencies are enhanced or amplified.
  - Individual frequency components are multiplied by an increasing function of ω such as $\alpha\omega = \alpha\sqrt{(u^2+v^2)}$, where α is a constant.

Horizontal Differences

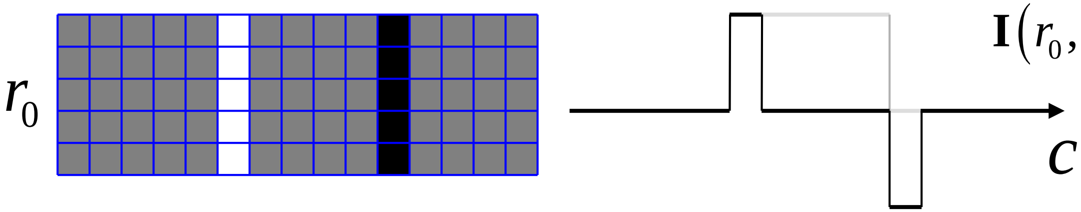


Image



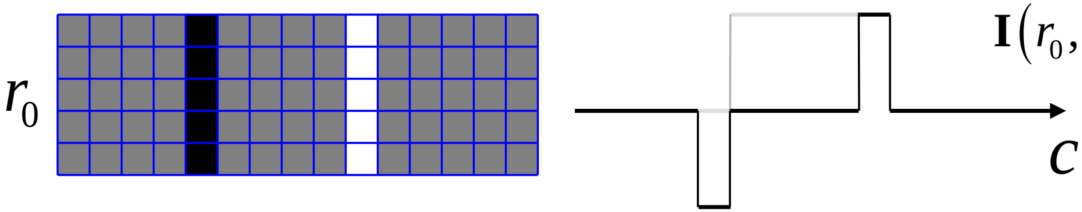
$\mathbf{I}(r_0, c)$

Left (back) Difference



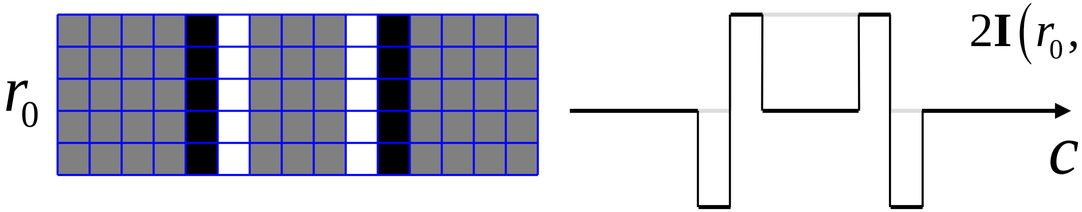
$\mathbf{I}(r_0, c) - \mathbf{I}(r_0, c-1)$

Right (fwd) Difference



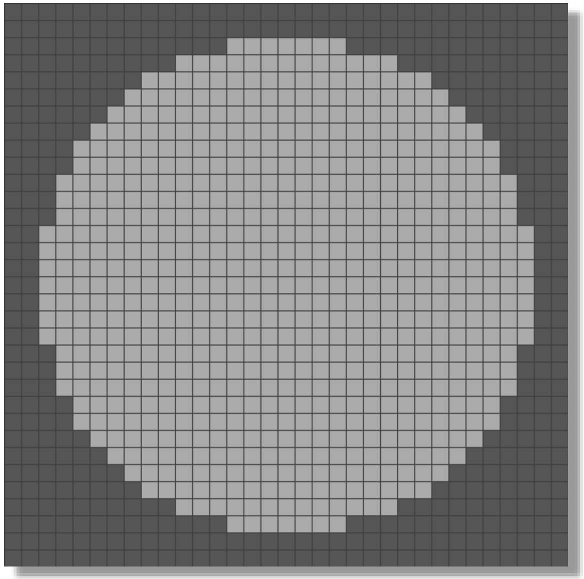
$\mathbf{I}(r_0, c) - \mathbf{I}(r_0, c+1)$

Sum of Differences

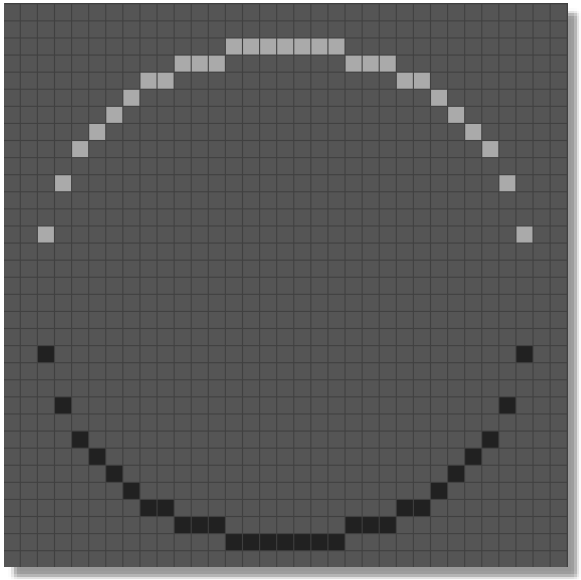


$$2\mathbf{I}(r_0, c) - \mathbf{I}(r_0, c+1) - \mathbf{I}(r_0, c+1)$$

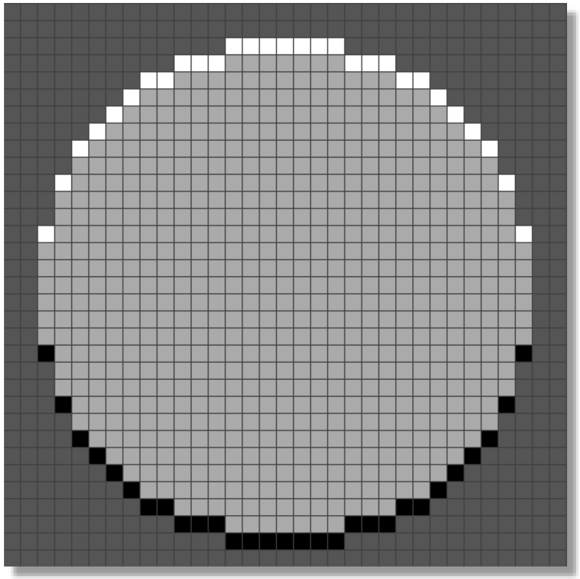
Horizontal Differencing / Sharpening



original: $\mathbf{I}(r,c)$

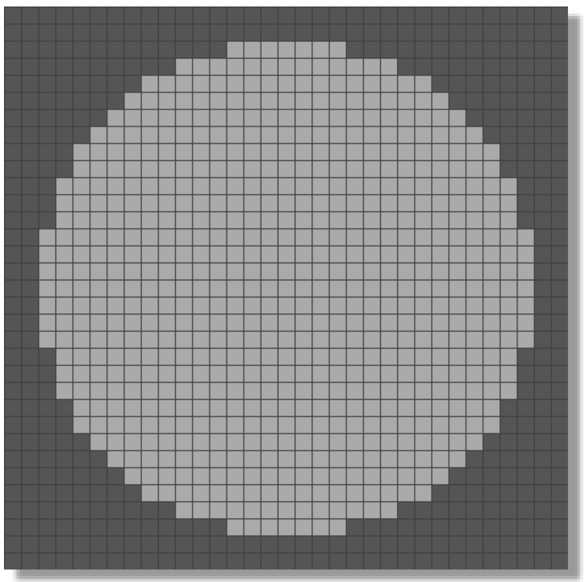


upward diff: $\mathbf{I}(r,c)-\mathbf{I}(r-1,c)$

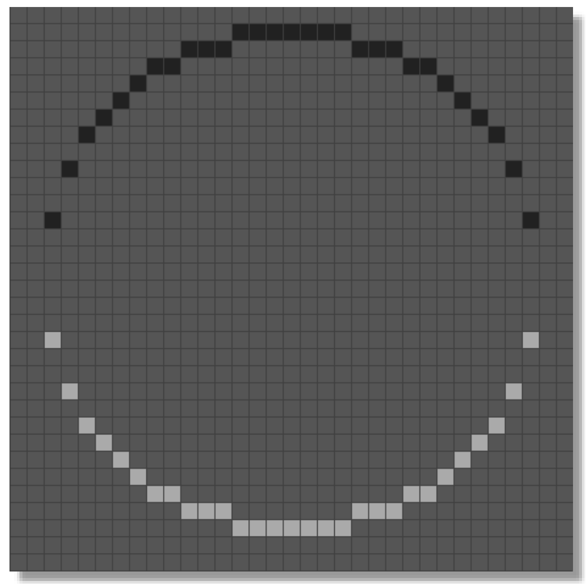


sharpened: $2\mathbf{I}(r,c)-\mathbf{I}(r-1,c)$

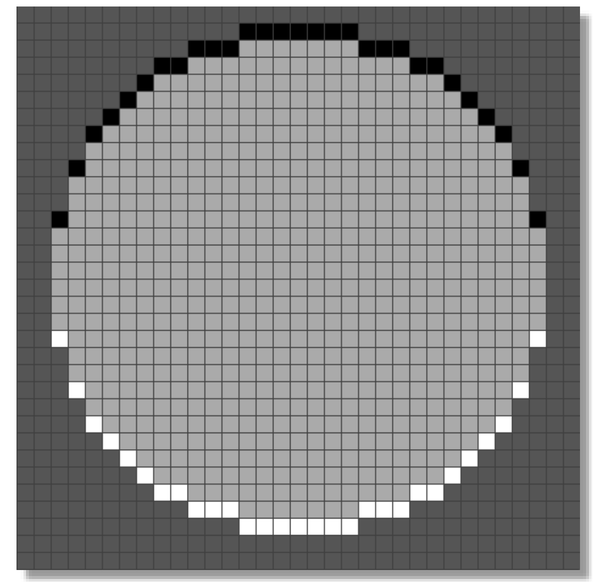
Horizontal Differencing / Sharpening



original: $\mathbf{I}(r,c)$

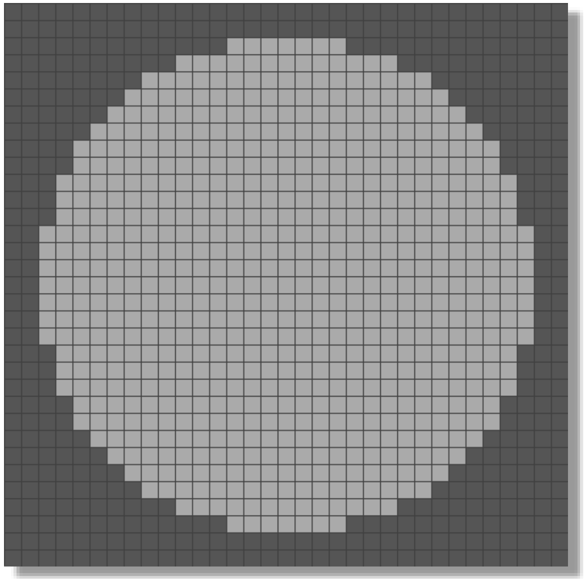


downward diff: $\mathbf{I}(r,c)\text{-}\mathbf{I}(r+1,c)$

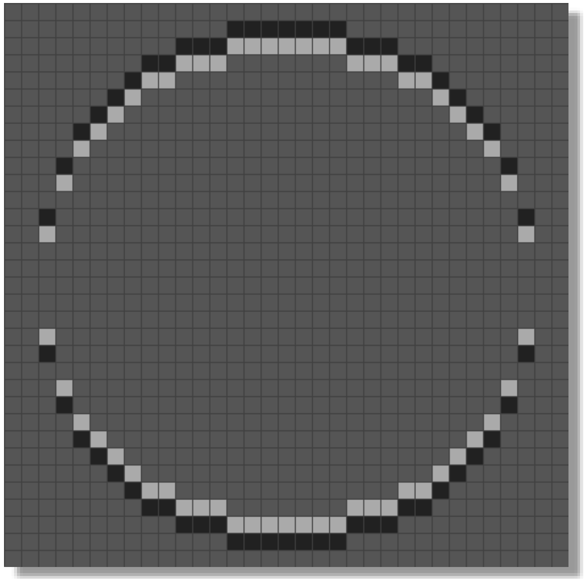


sharpened: $2\mathbf{I}(r,c)\text{-}\mathbf{I}(r+1,c)$

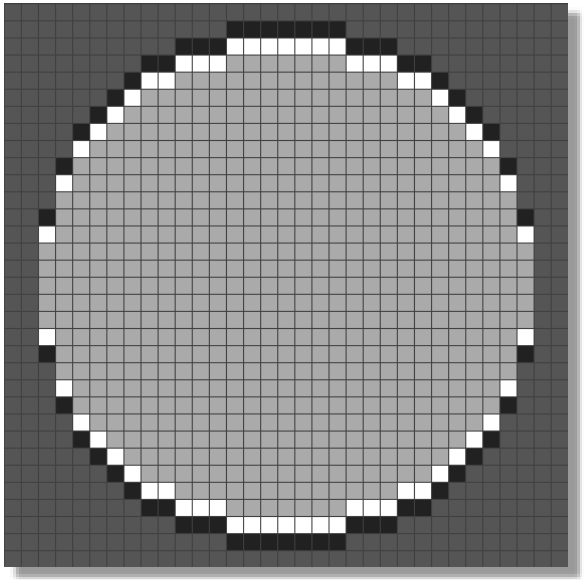
Horizontal Differencing / Sharpening



original: $\mathbf{I}(r,c)$

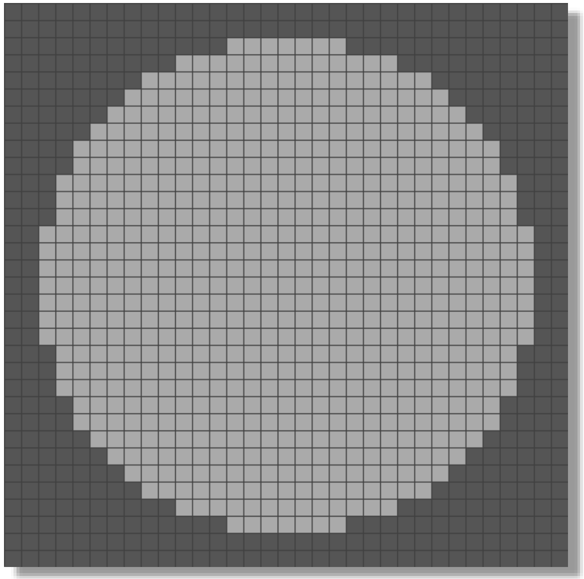


$2\mathbf{I}(r,c)$-$\mathbf{I}(r$-$1,c)$-$\mathbf{I}(r+1,c)$

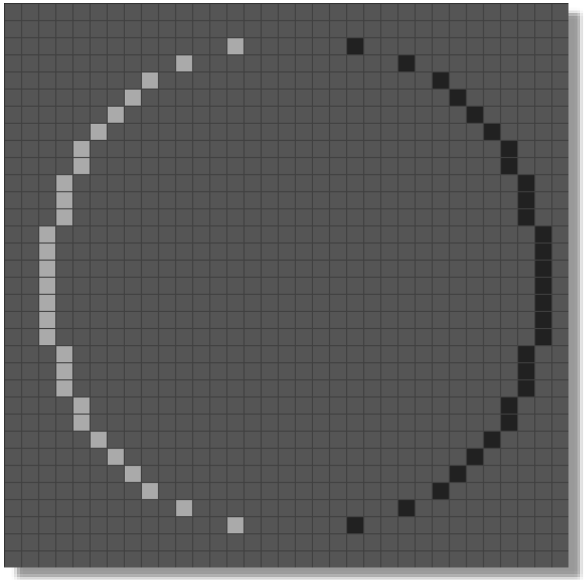


$3\mathbf{I}(r,c)$-$\mathbf{I}(r$-$1,c)$-$\mathbf{I}(r+1,c)$

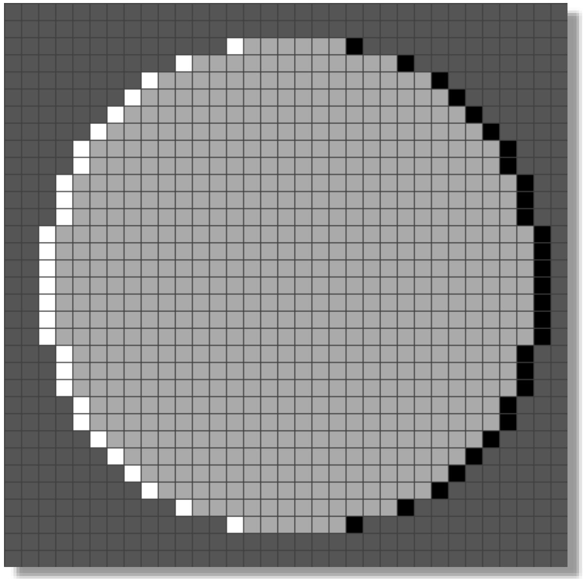
Vertical Differencing / Sharpening



original: $\mathbf{I}(r,c)$

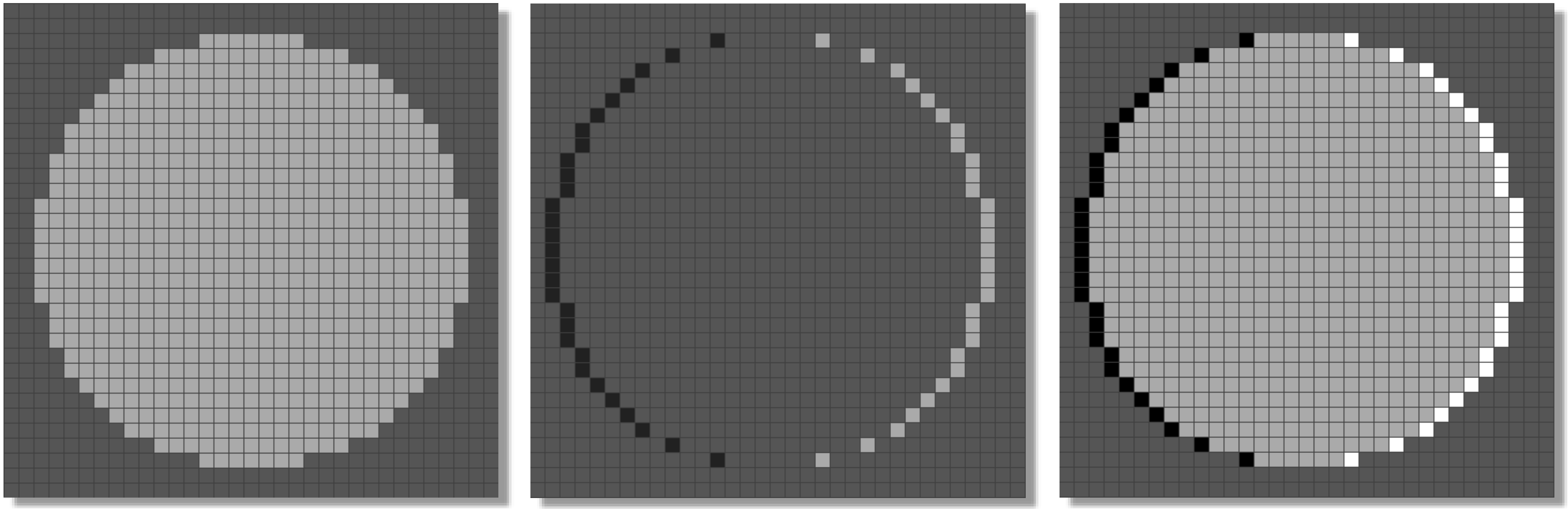


backward diff: $\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$



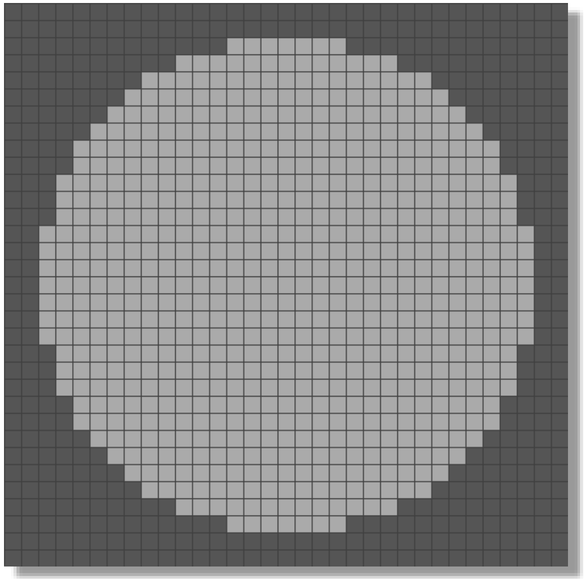
sharpened: $2\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$

Vertical Differencing / Sharpening

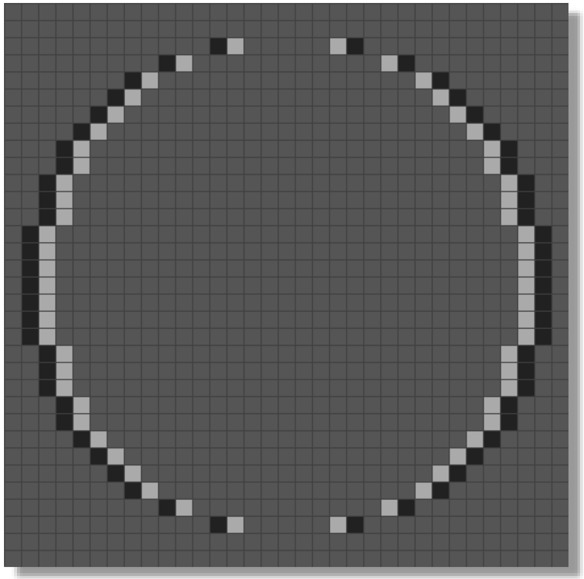


original: $\mathbf{I}(r,c)$ forward diff: $\mathbf{I}(r,c)$-$\mathbf{I}(r,c+1)$ sharpened: $2\mathbf{I}(r,c)$-$\mathbf{I}(r,c+1)$

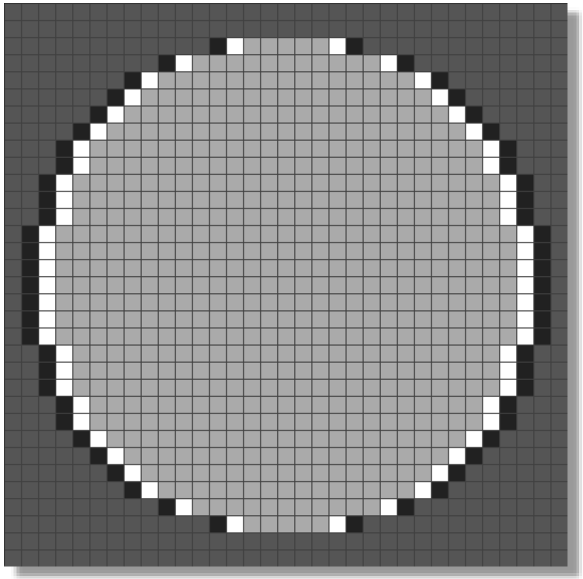
Vertical Sharpening



original: $\mathbf{I}(r,c)$

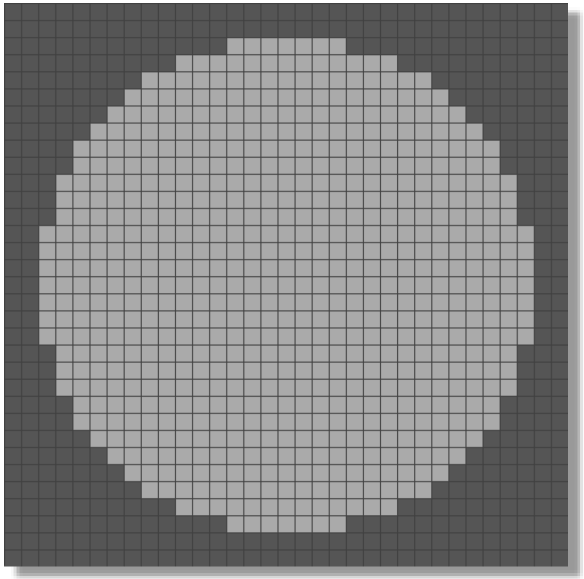


$2\mathbf{I}(r,c)\text{-}\mathbf{I}(r,c\text{-}1)\text{-}\mathbf{I}(r,c+1)$

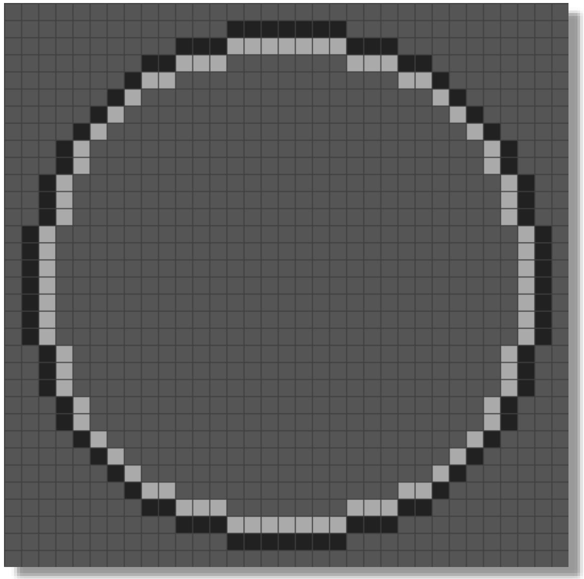


$3\mathbf{I}(r,c)\text{-}\mathbf{I}(r,c\text{-}1)\text{-}\mathbf{I}(r,c+1)$

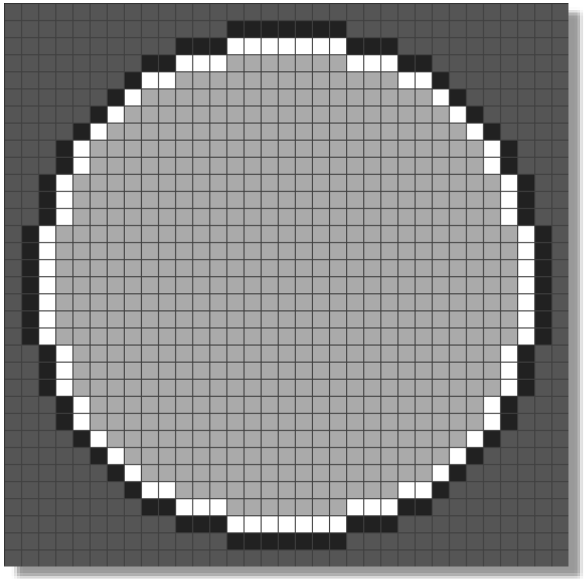
Horizontal + Vertical Sharpening



original: $\mathbf{I}(r,c)$



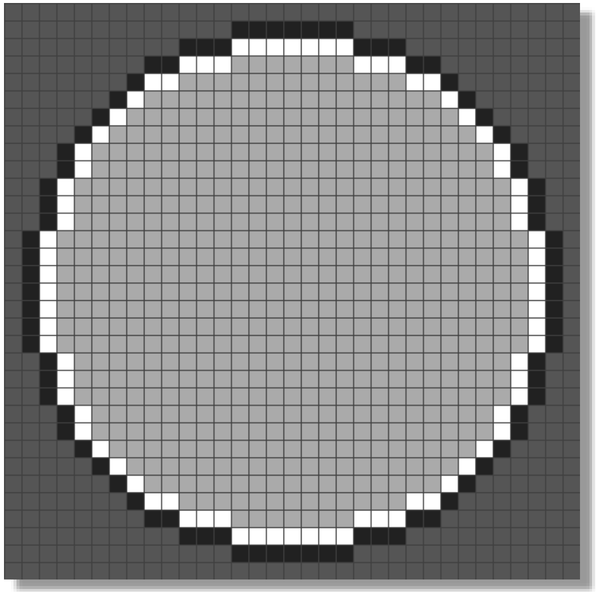
$4\mathbf{I}(r,c)-\mathbf{I}(r,c+1)-\mathbf{I}(r,c-1)-\mathbf{I}(r+1,c)-\mathbf{I}(r-1,c)$



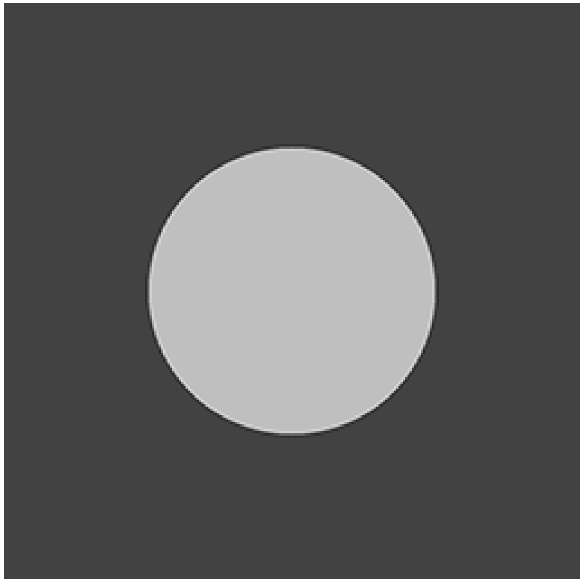
$5\mathbf{I}(r,c)-\mathbf{I}(r,c+1)-\mathbf{I}(r,c-1)-\mathbf{I}(r+1,c)-\mathbf{I}(r-1,c)$

Perceptual Note

That our visual system does something like the edge enhancement of the disk on the left is strongly suggested by the appearance of the disk on the right. It contains only 2 intensity levels. But, we see 4 – the background, the disk, and concentric dark and bright circles surrounding the disk.



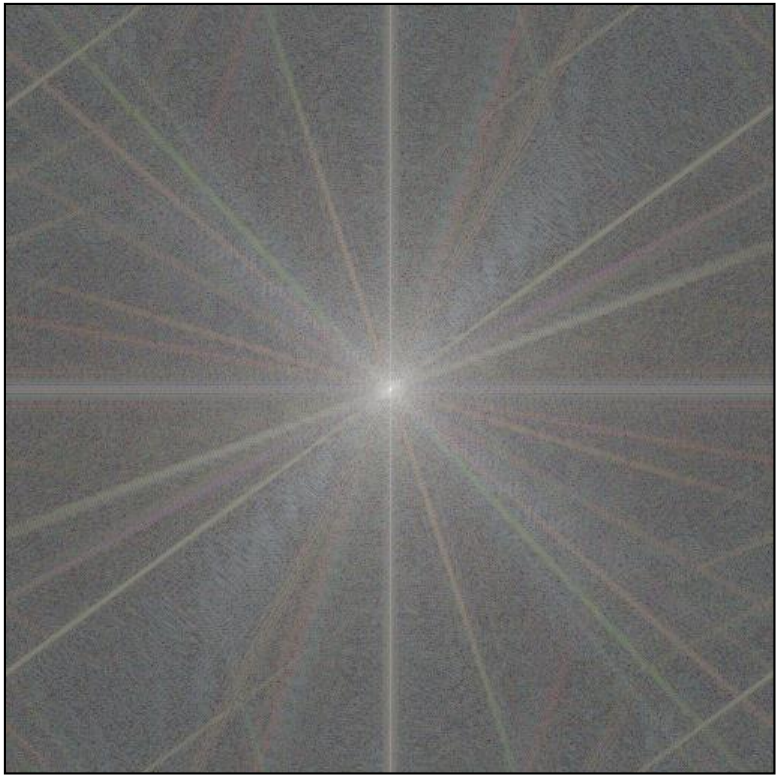
$5\mathbf{I}(r,c)-\mathbf{I}(r,c+1)-\mathbf{I}(r,c-1)-\mathbf{I}(r+1,c)-\mathbf{I}(r-1,c)$



a two-level image: $\mathbf{I}(r,c) \in \{64,192\}$

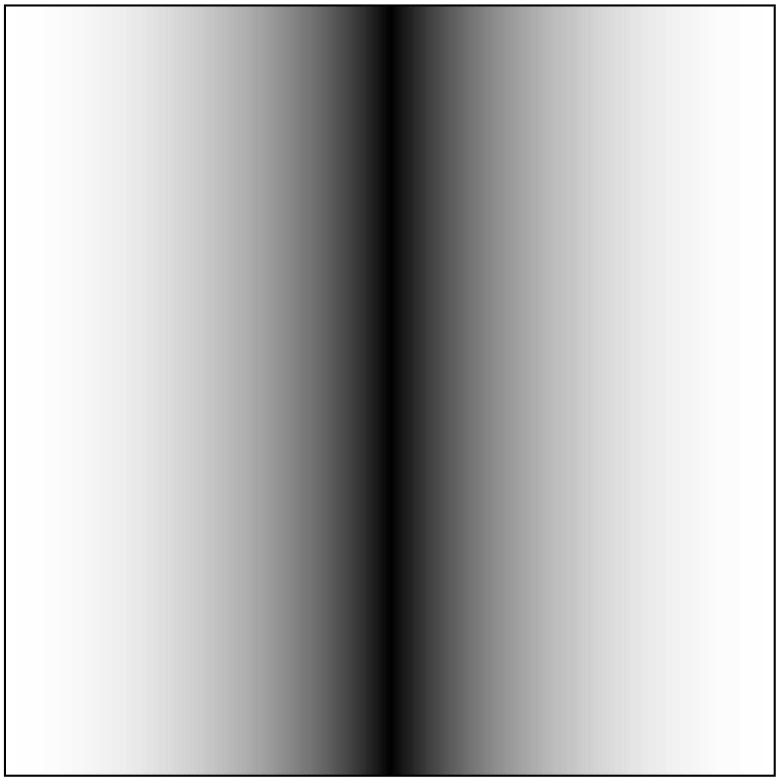
Differencing / Highpass Filtering

original image, **I**

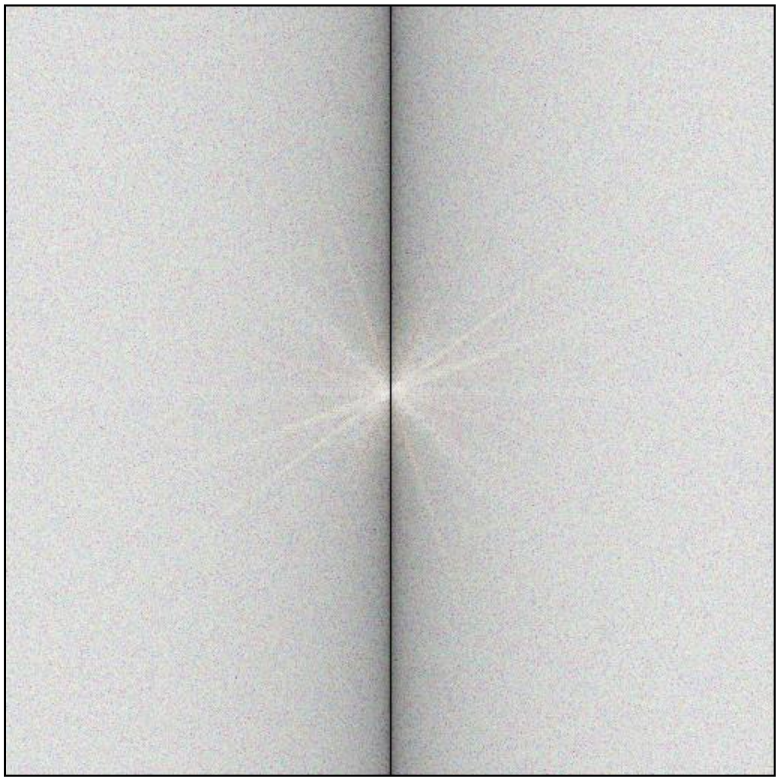


power spectrum

Differencing / Highpass Filtering

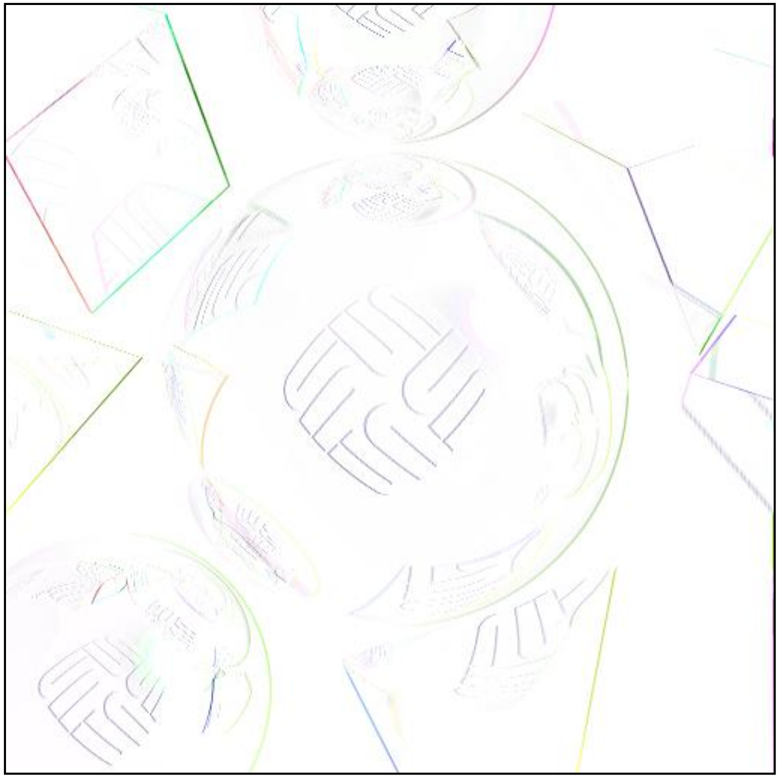


power spectrum of $\mathbf{h}$ = [-1 1 0]



power spectrum of $\mathbf{I} * \mathbf{h} = \mathbf{I}(r,c) - \mathbf{I}(r,c+1)$

Differencing / Highpass Filtering

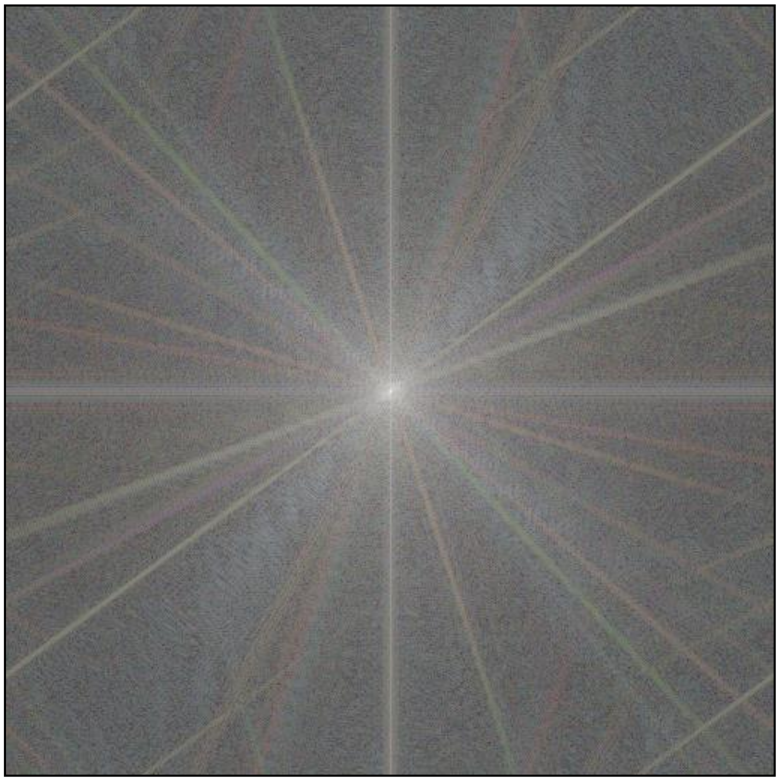


negative pixels in differenced image

positive pixels in differenced image

Differencing / Highpass Filtering

original image, **I**

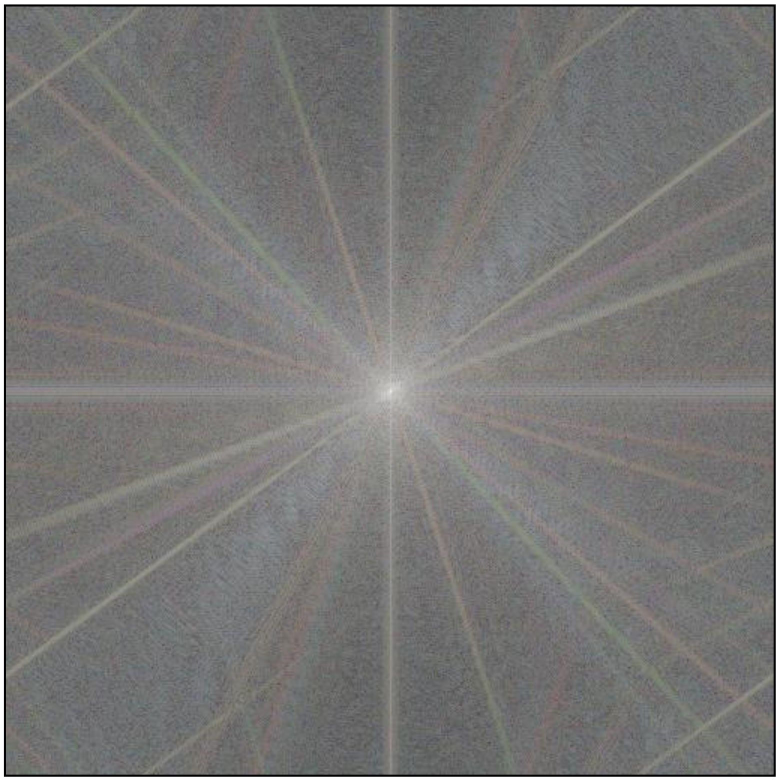


power spectrum

Add the differenced image, I(r,c)-I(r,c+1), back to the original to get a HF enhanced version. It is a “sharper” version of the original.

[...]arpening

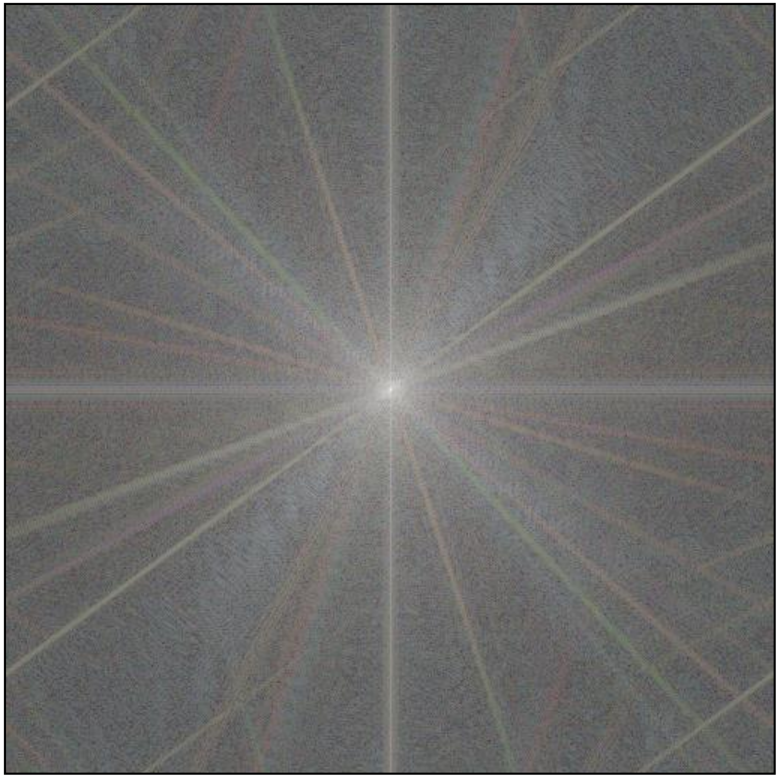
sharpened image, $2\mathbf{I}(r,c)-\mathbf{I}(r,c+1)$



power spectrum

Image Sharpening

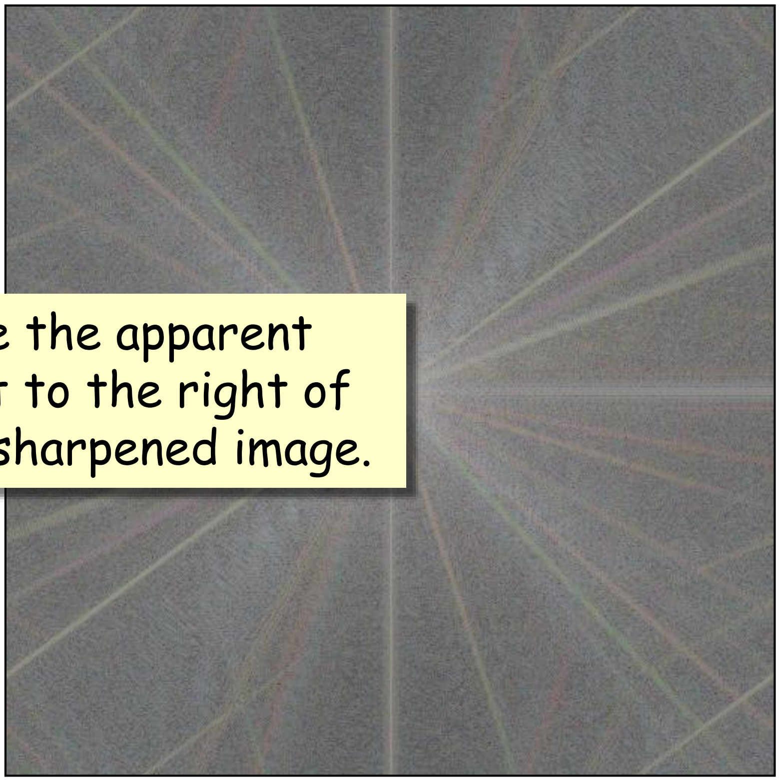
original image, **I**



power spectrum

Image Sharpening

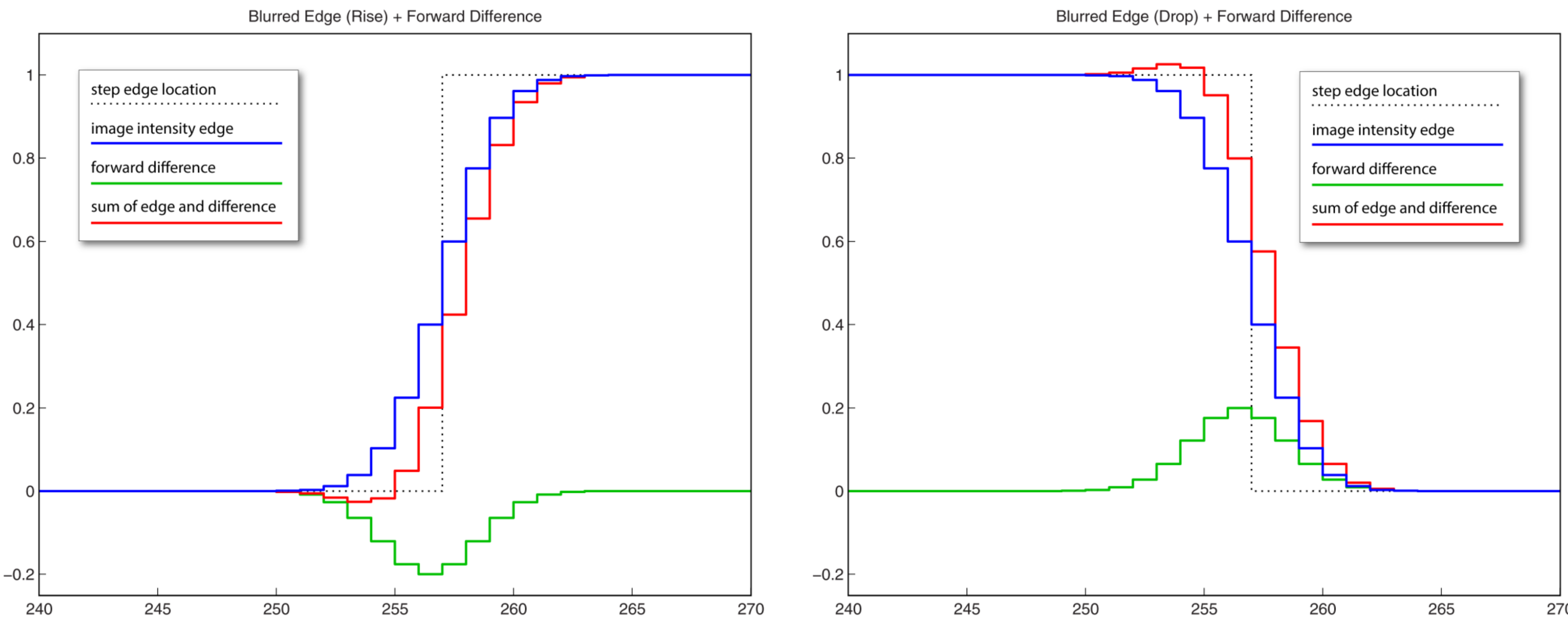
sharpened image, $2\mathbf{I}(r,c)-\mathbf{I}(r,c+1)$



Note the apparent shift to the right of the sharpened image.

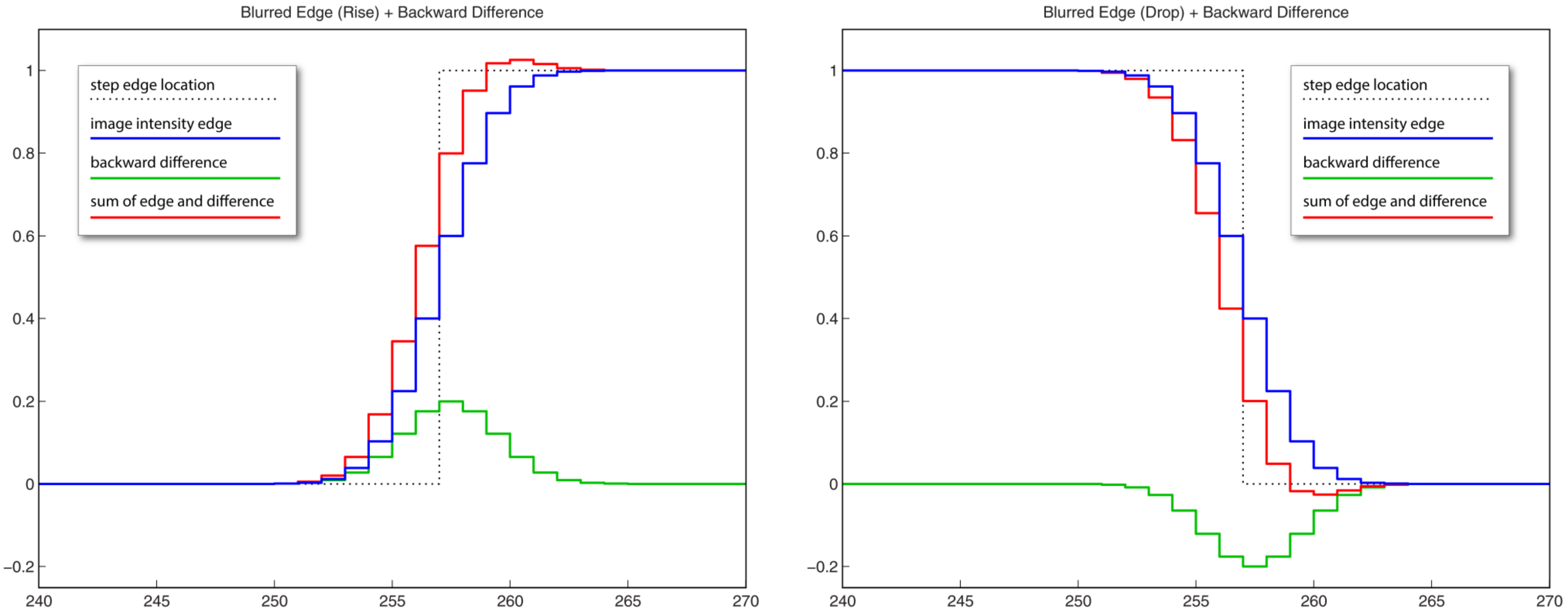
power spectrum

Image Sharpening: Edge Enhancement



Adding a differenced image back to the original increases the high frequency content. It steepens the slopes of the edges which makes the image look "sharper." Note also that a forward difference, **I**(r,c)-**I**(r,c+1), causes the apparent edge to shift to the right.

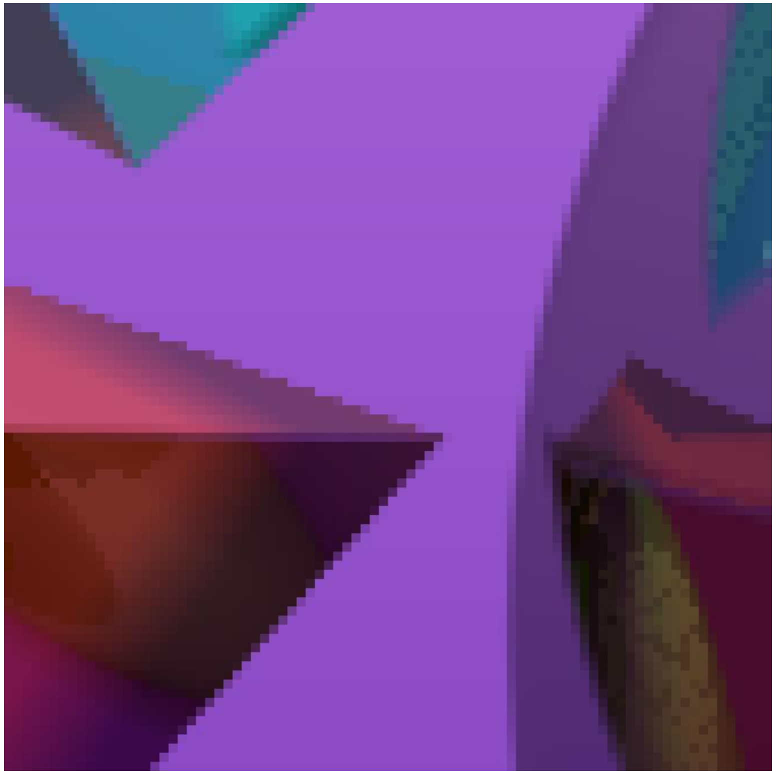
Image Sharpening: Edge Enhancement



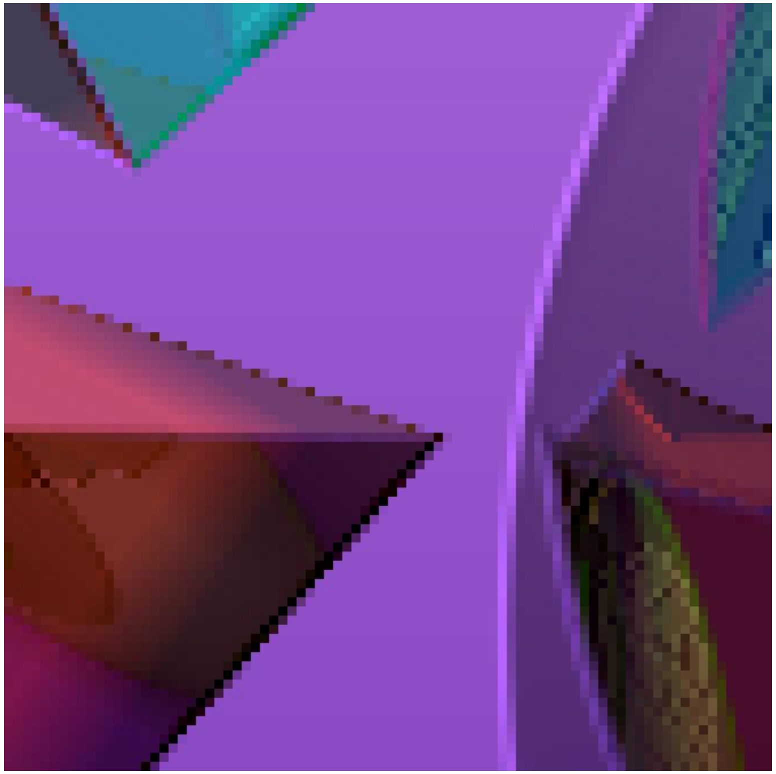
Adding a differenced image back to the original increases the high frequency content. It steepens the slopes of the edges which makes the image look “sharper.” Note also that a backward difference, **I**(r,c)-**I**(r,c-1), causes the apparent edge to shift to the left.

The shift occurs because the direction of the differencing operation pushes edges in the same direction.

Sharpening: Differencing / Highpass Filtering



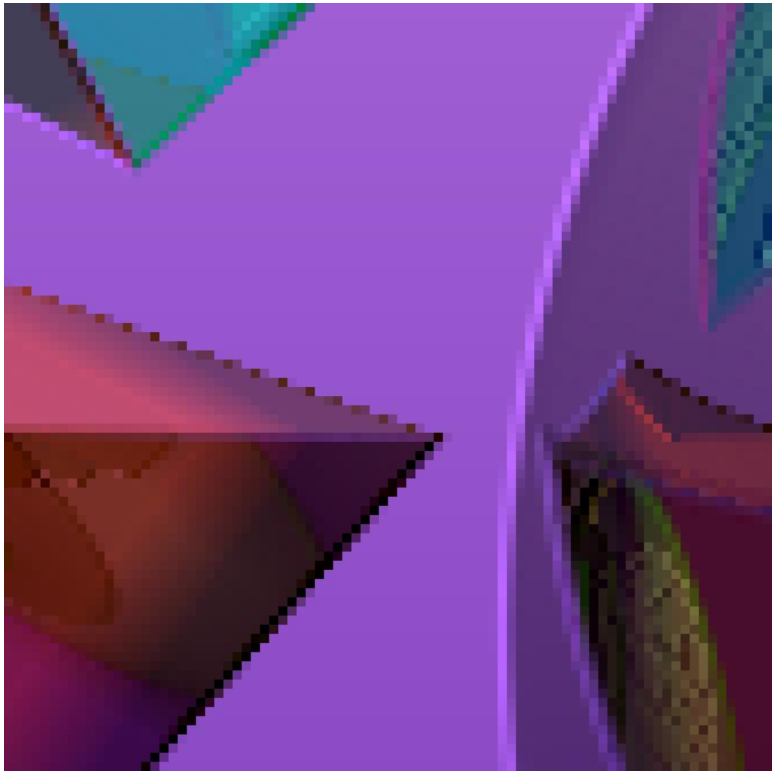
original image



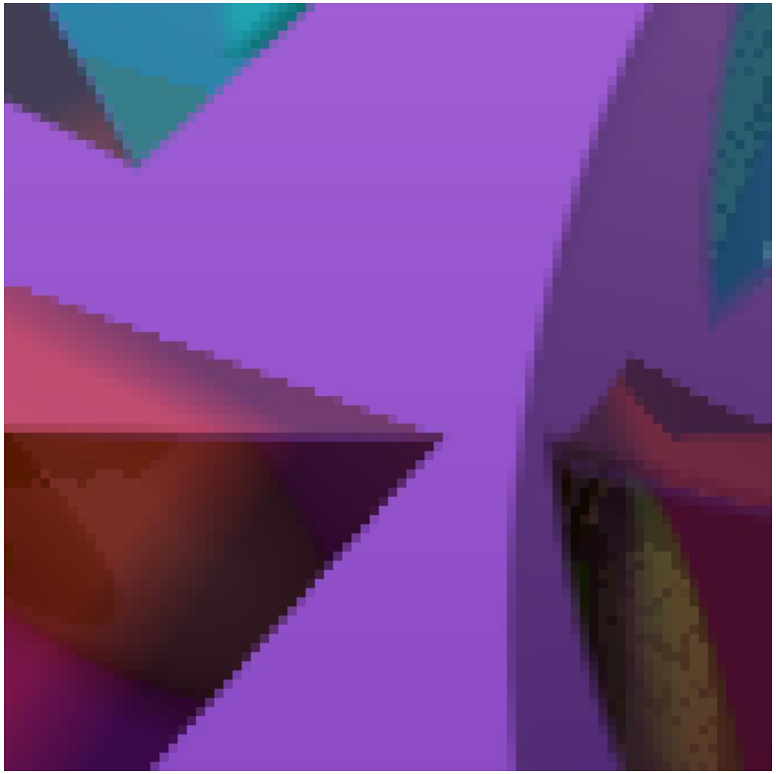
sharpened image, $2\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$

The shift occurs because the direction of the differencing operation pushes edges in the same direction. (see pp.7-8)

Sharpening: Differencing / Highpass Filtering

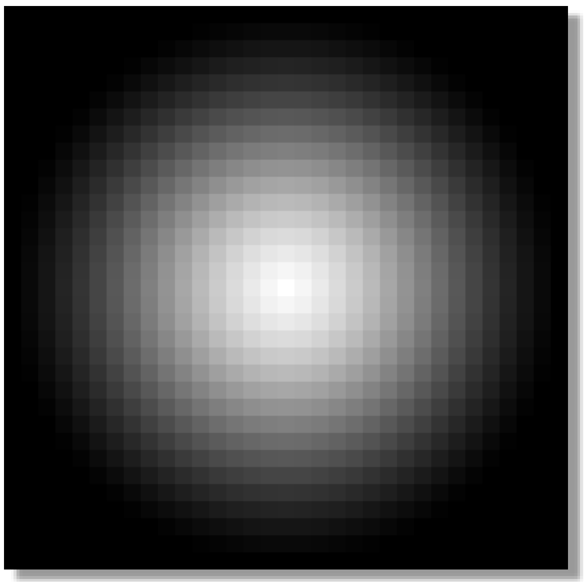


sharpened image, $2\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$

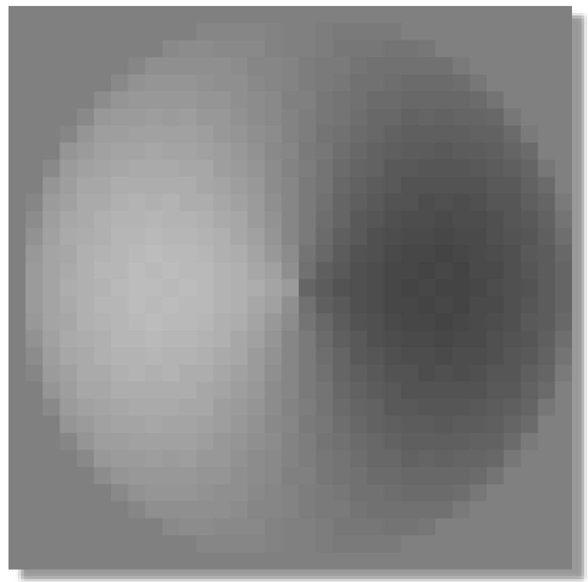


original image

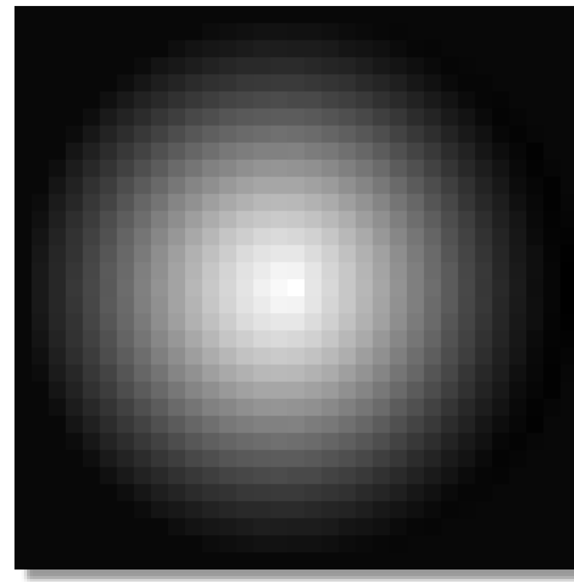
Apparent Shift due to HF Enhancement



original

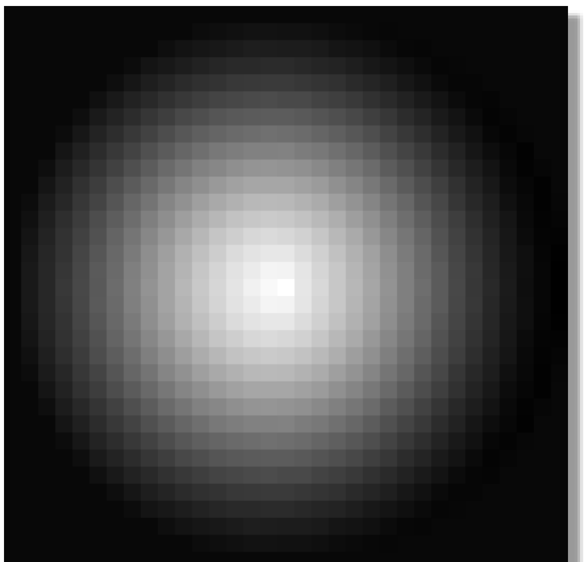


backward diff: $\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$

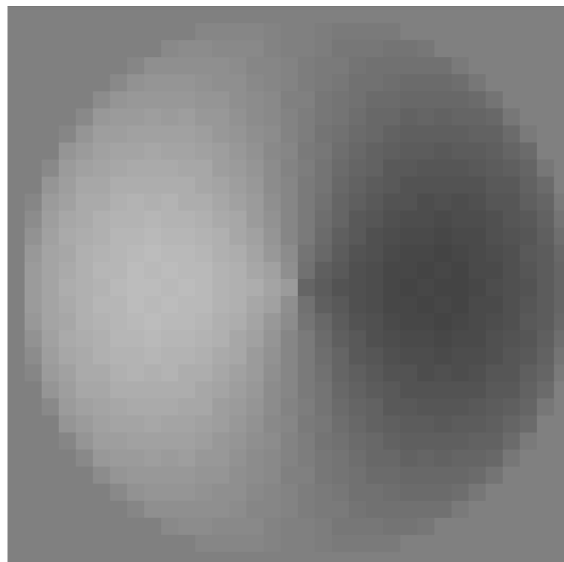


enhanced: $2\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$

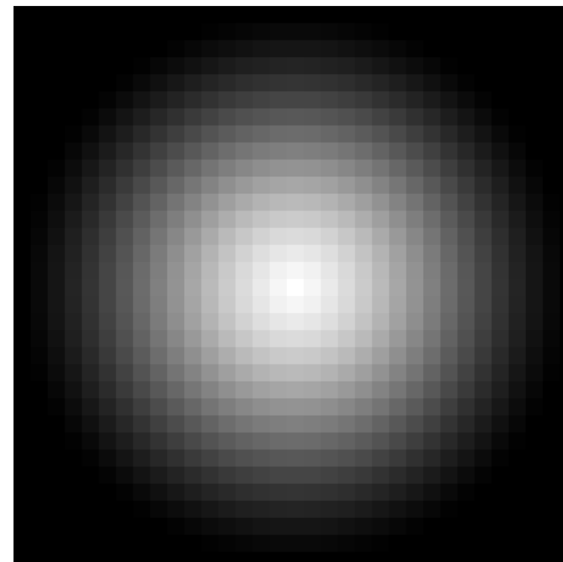
Apparent Shift due to HF Enhancement



enhanced: $2\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$



backward diff: $\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$



original

Differentiation Through Integration

1. $$\frac{\partial}{\partial w}[\mathbf{I}*\mathbf{h}](r,c)=\frac{\partial}{\partial w}\iint_{\mathrm{supp}(\mathbf{I})}\mathbf{I}(\rho-r,\chi-c)\,\mathbf{h}(\rho,\chi)\,d\rho\,d\chi$$
$$w=\alpha x+\beta y,\quad \alpha^2+\beta^2=1$$

Assume that h(ρ,χ) = δ(ρ,χ). Then I∗h = I. and ∂I/∂w = ∂(I∗h)/∂w.

2. $$\mathfrak{F}\left\{\frac{\partial}{\partial w}\mathbf{J}(r,c)\right\}=jz\mathfrak{F}\left\{\mathbf{J}(r,c)\right\}$$
$$z=\alpha u+\beta v,\quad \alpha^2+\beta^2=1$$

Differentiation property of the Fourier Transform.

3. $$\mathfrak{F}\{\mathbf{I}*\mathbf{h}\}=\mathfrak{F}\{\mathbf{I}\}\cdot\mathfrak{F}\{\mathbf{h}\}$$
$$\mathbf{I}*\mathbf{h}=\mathfrak{F}^{-1}\left\{\mathfrak{F}\{\mathbf{I}\}\cdot\mathfrak{F}\{\mathbf{h}\}\right\}$$

Convolution property of the Fourier Transform.

∂/∂w is a directional derivative with direction vector $[\alpha\ \beta]^T$.

Differentiation Through Integration

4.

$$\mathcal{F}\left\{\frac{\partial}{\partial w}[\mathbf{I}*\mathbf{h}]\right\} = jz\cdot\mathcal{F}\{\mathbf{I}\}\cdot\mathcal{F}\{\mathbf{h}\}$$
$$=\left[jz\cdot\mathcal{F}\{\mathbf{I}\}\right]\cdot\mathcal{F}\{\mathbf{h}\}$$
$$=\mathcal{F}\{\mathbf{I}\}\cdot\left[jz\cdot\mathcal{F}\{\mathbf{h}\}\right]$$

$$w=\alpha x+\beta y,\quad \alpha^2+\beta^2=1$$

$$z=\alpha u+\beta v,\quad \alpha^2+\beta^2=1$$

5.

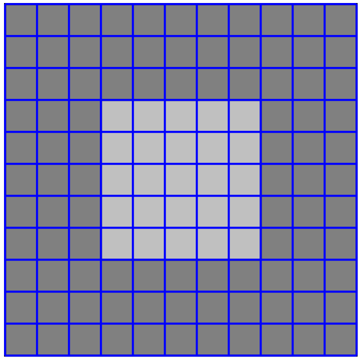
$$\frac{\partial}{\partial w}[\mathbf{I}*\mathbf{h}](r,c)=\left[\mathbf{I}*\frac{\partial}{\partial w}\mathbf{h}\right](r,c)$$

Apply 2 and 3 to 1 to get 4 & 5.

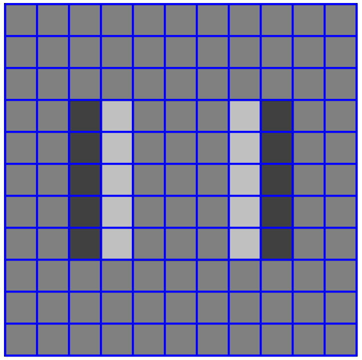
$[\alpha\ \beta]^T$ is a direction in the plane. *w* and *z* are projections along that direction.

The derivative of a convolution of I by h is the convolution of I by the derivative of h.

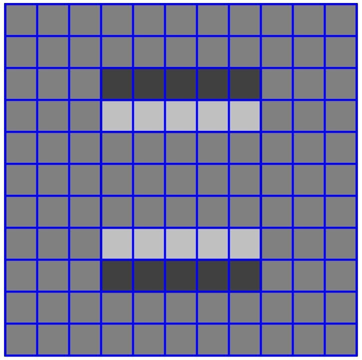
Symmetric Differences



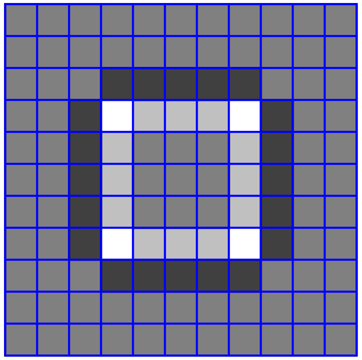
$$\mathbf{I}(r_0, c)$$



$$2\mathbf{I}(r_0, c) - \mathbf{I}(r_0, c-1) - \mathbf{I}(r_0, c+1)$$



$$2\mathbf{I}(r_0, c) - \mathbf{I}(r-1, c_0) - \mathbf{I}(r+1, c_0)$$



$$4\mathbf{I}(r, c) - \mathbf{I}(r-1, c) - \mathbf{I}(r+1, c) - \mathbf{I}(r, c-1) - \mathbf{I}(r, c+1)$$

- 510
- 255
- 0
- -255

legend

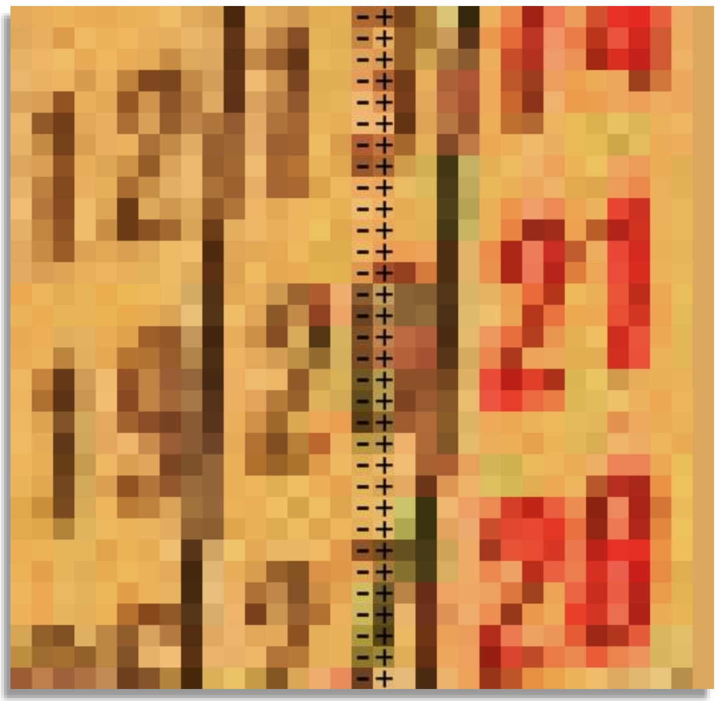
-1	2	-1

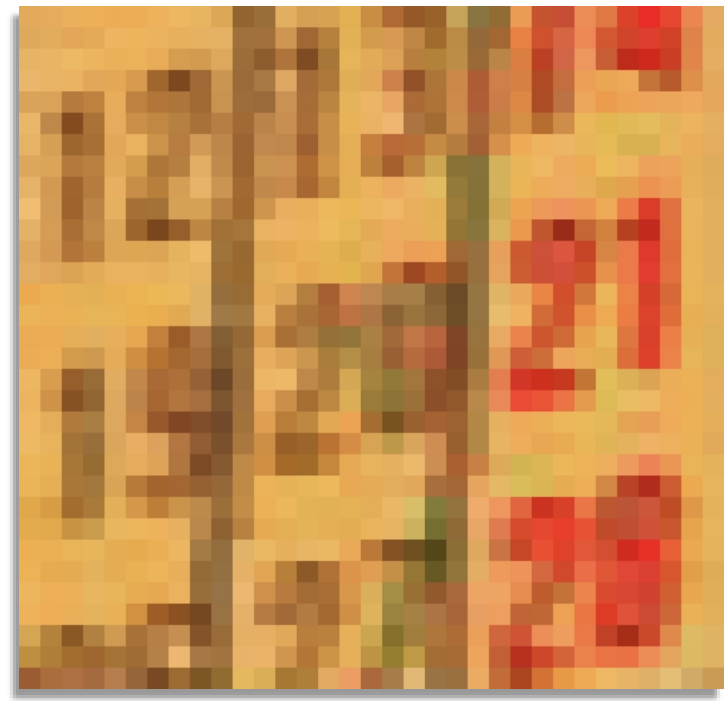
	-1	
	2	
	-1	

	-1	
-1	4	-1
	-1	

convolution matrices

A note on FBUD* differences & convolution

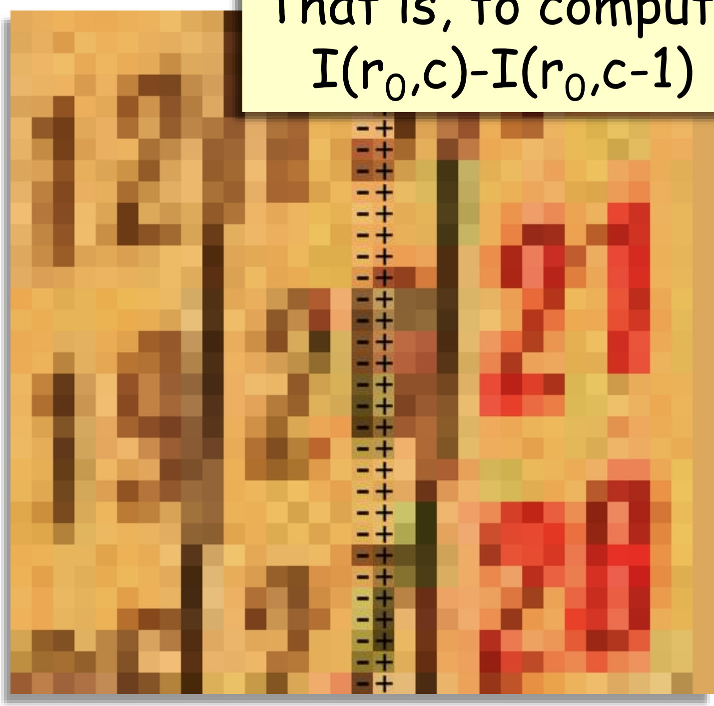




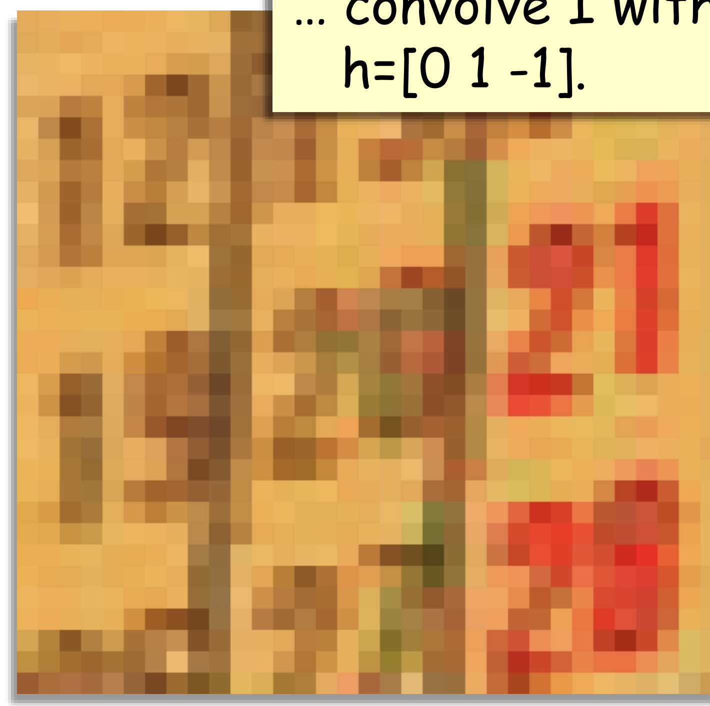
A backward difference on **I** is the same as right shifting a copy of **I** by one pixel and subtracting it from **I**.

*forward, backward, up, down

A note on FBUD* differences & convolution



A backward difference is the same as…

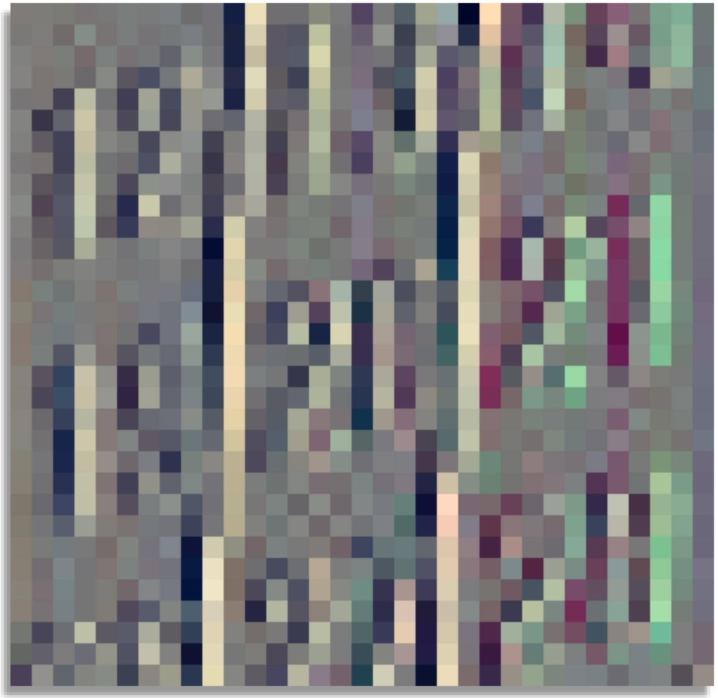


… right shifting a copy and subtracting it.

h_b=[0 1 -1] is a backward difference whereas h_f=[-1 1 0] is a forward difference.

*forward, backward, up, down

A note on FBUD* differences & convolution



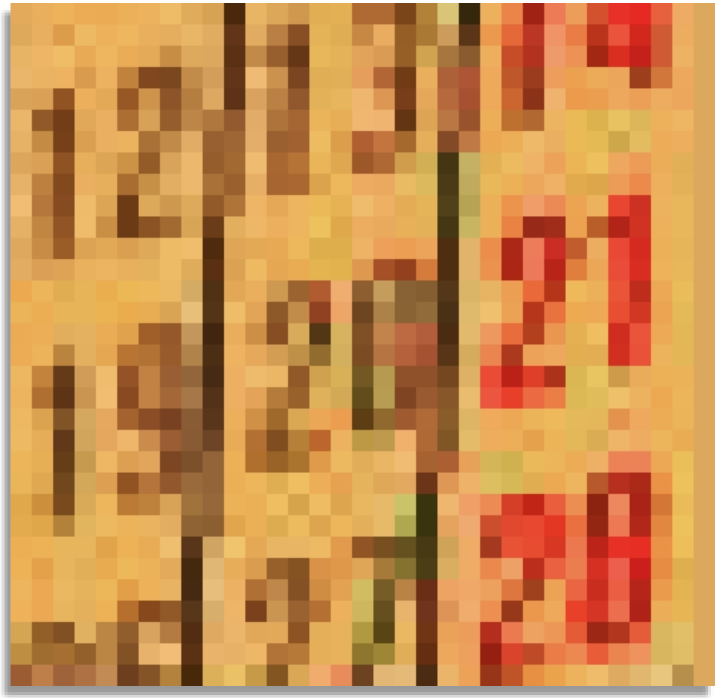
backward diff: $\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$

enhanced: $2\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$

*forward, backward, up, down

A note on FBUD* differences & convolution

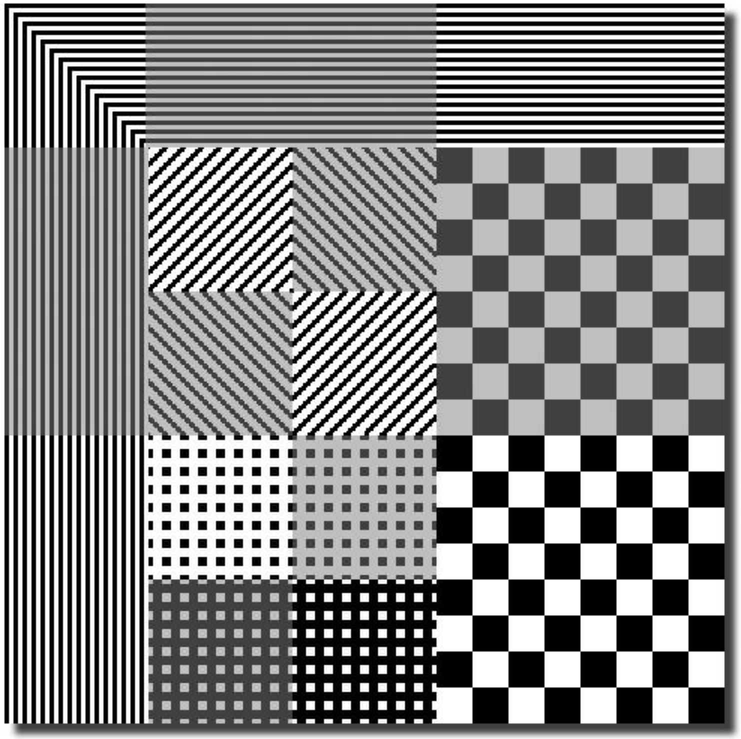
enhanced: $2\mathbf{I}(r,c)-\mathbf{I}(r,c-1)$



original

*forward, backward, up, down

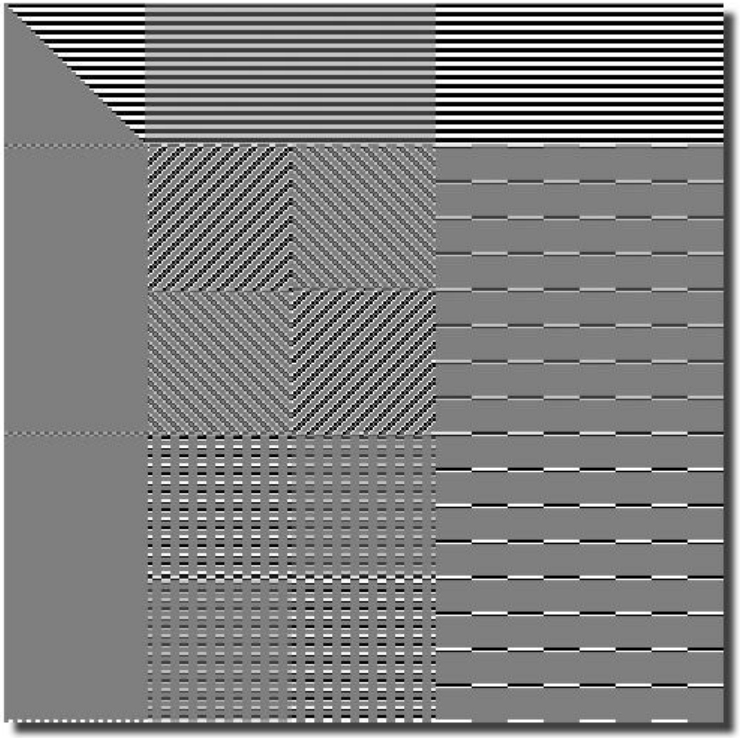
Convolution Examples: Original Images

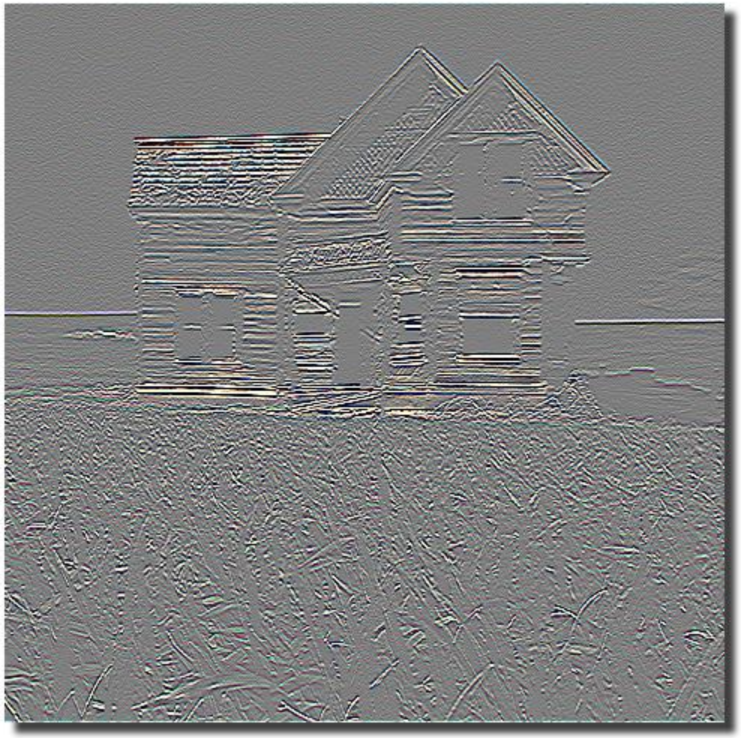


$$\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

signed image;
0 is middle gray

Convolution Examples: Vertical Difference

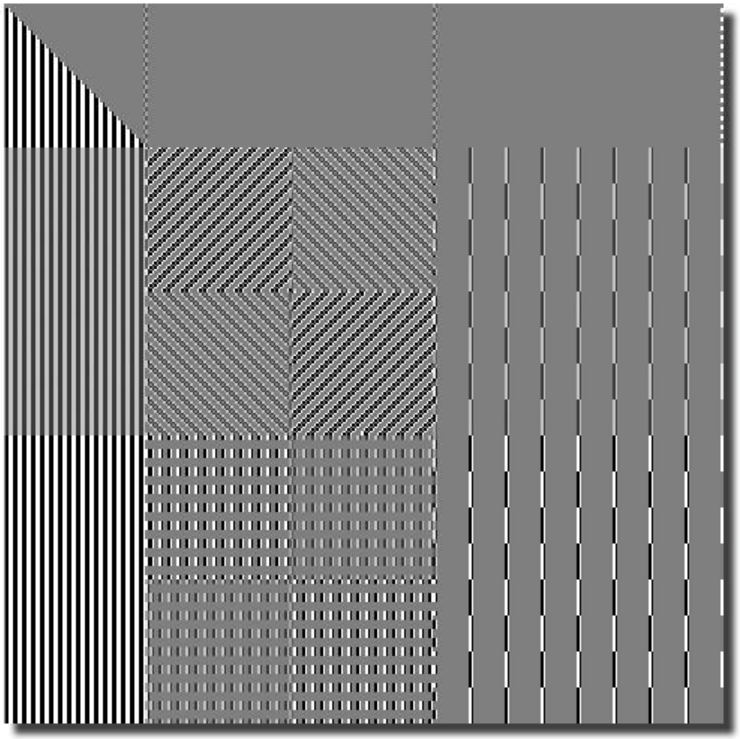
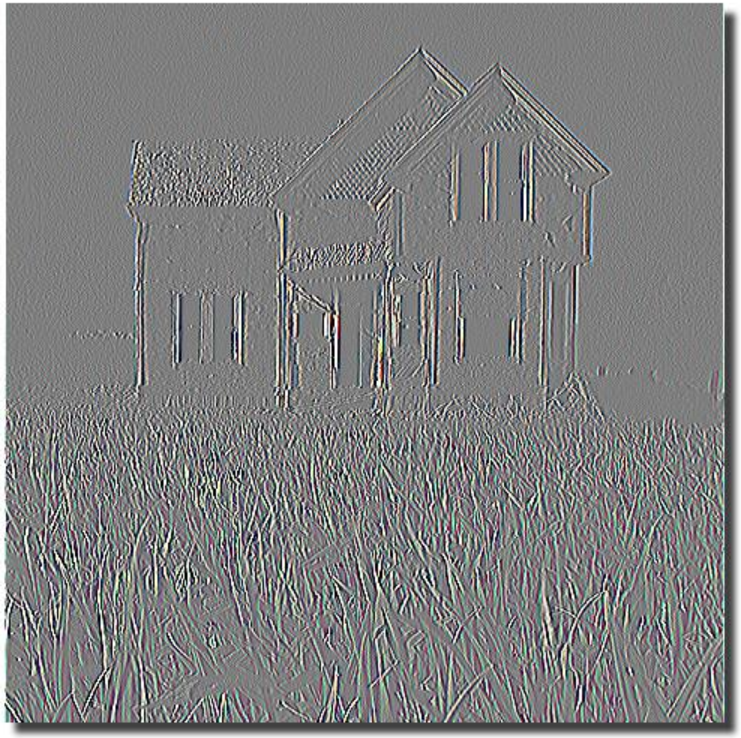




$\begin{bmatrix} -1 & 2 & -1 \end{bmatrix}$

signed image;
0 is middle gray

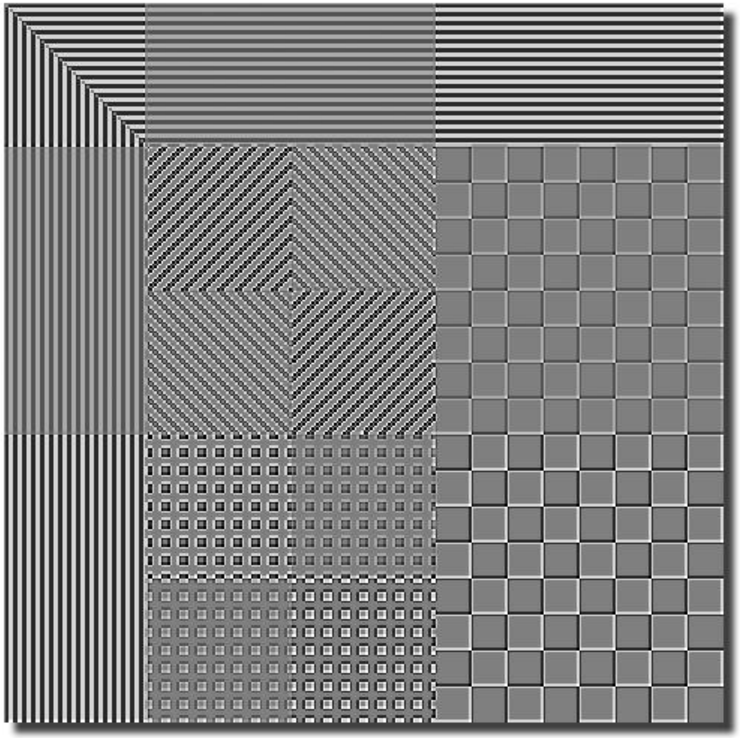
Convolution Examples: Horizontal Difference

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

signed image;
0 is middle gray

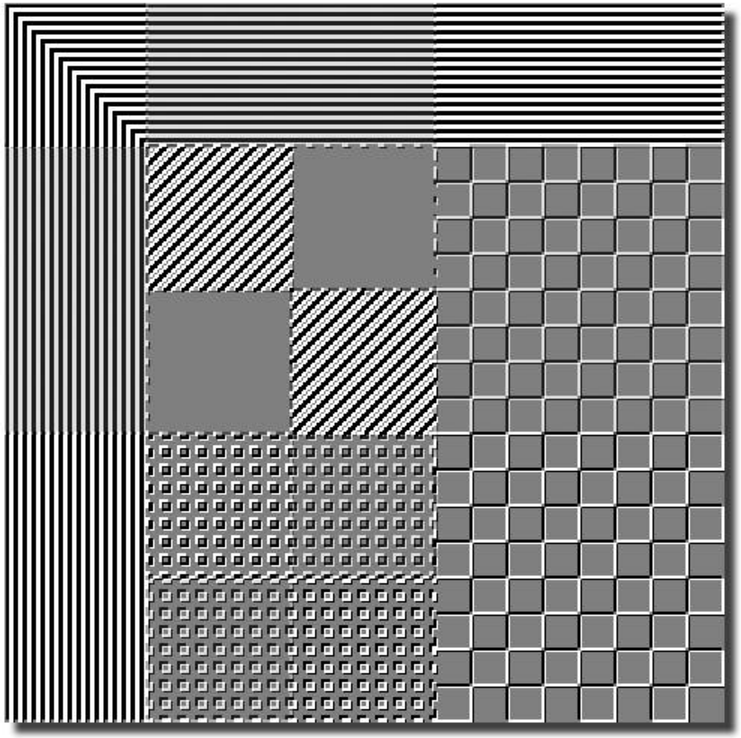
Convolution Examples: H + V Diff.



$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

signed image;
0 is middle gray

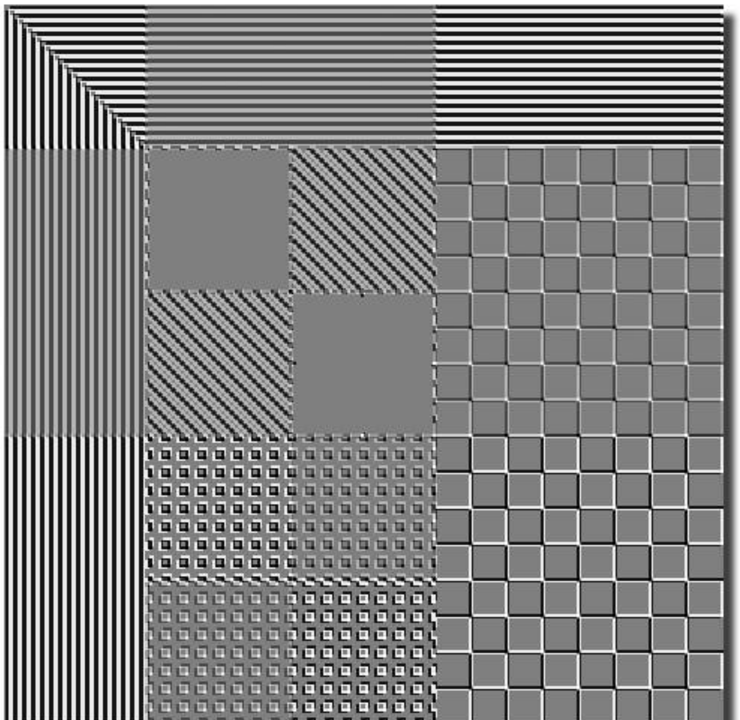
Convolution Examples: Diagonal Difference



$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

signed image;
0 is middle gray

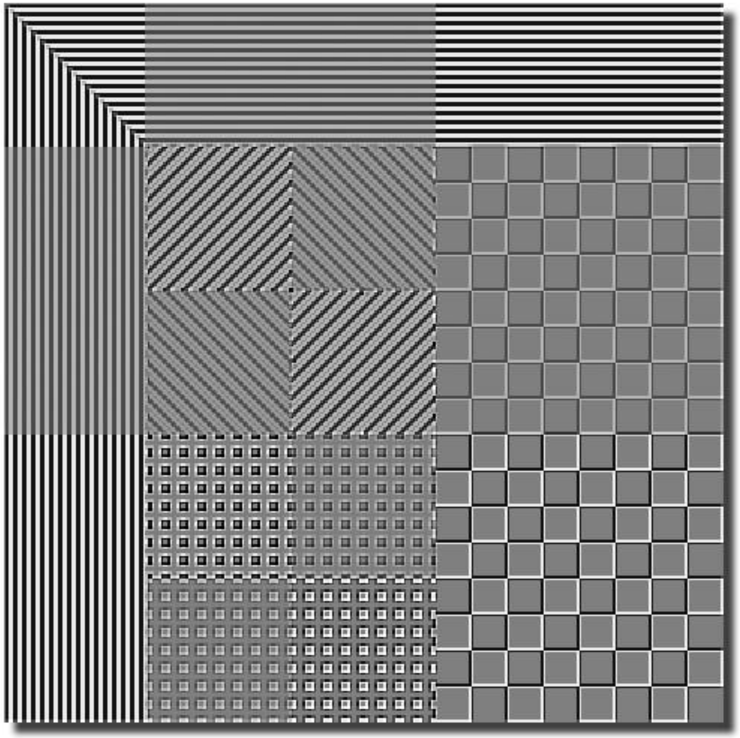
Convolution Examples: Diagonal Difference



$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

signed image;
0 is middle gray

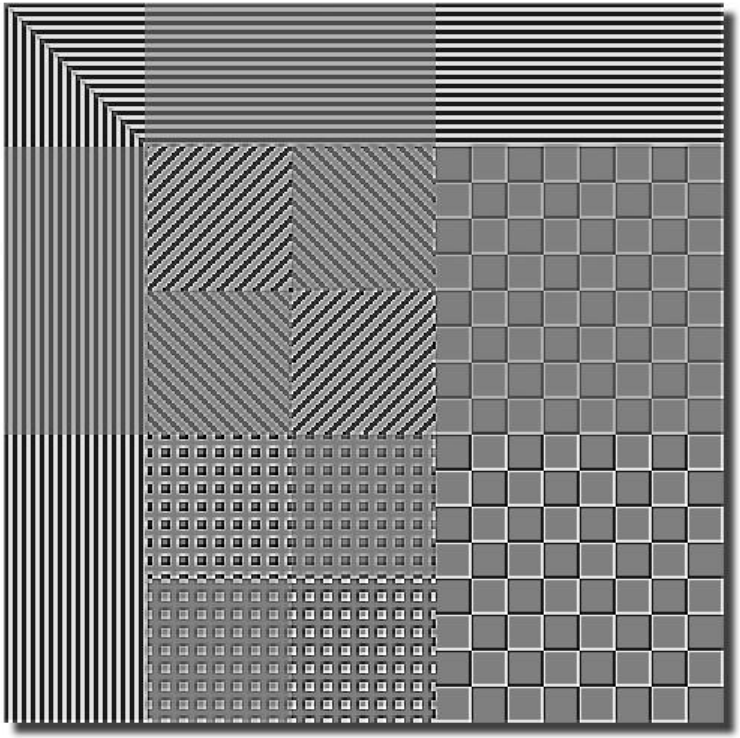
Convolution Examples: D + D Difference



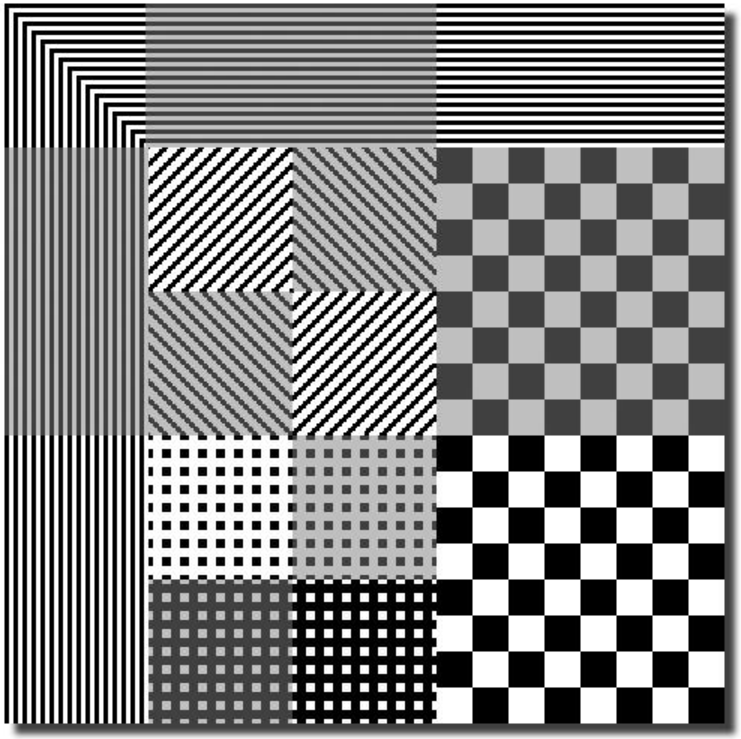
$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

signed image;
0 is middle gray

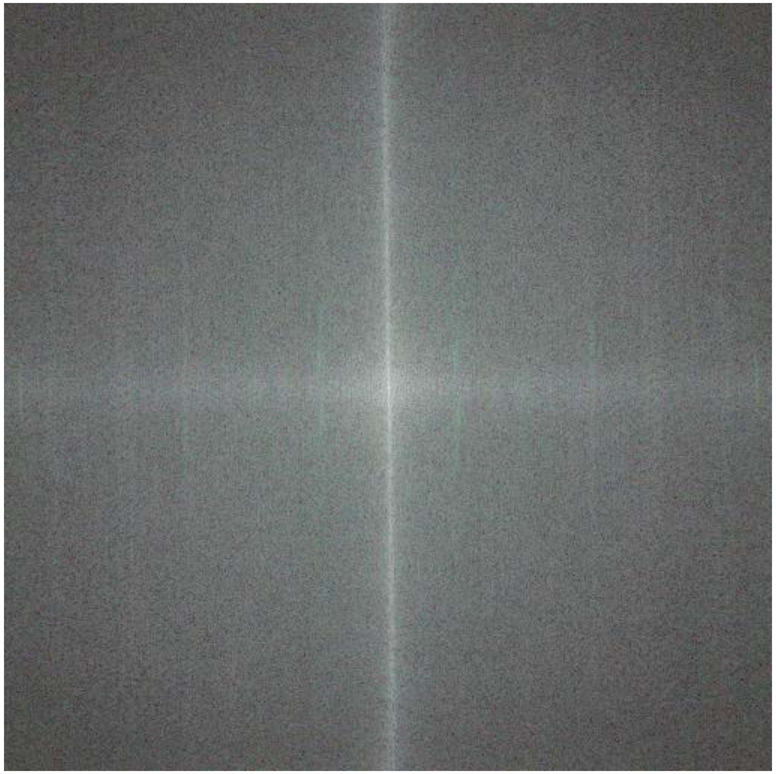
Convolution Examples: H + V + D Diff.



Convolution Examples: Original Images



Original Image



power spectrum of **I**

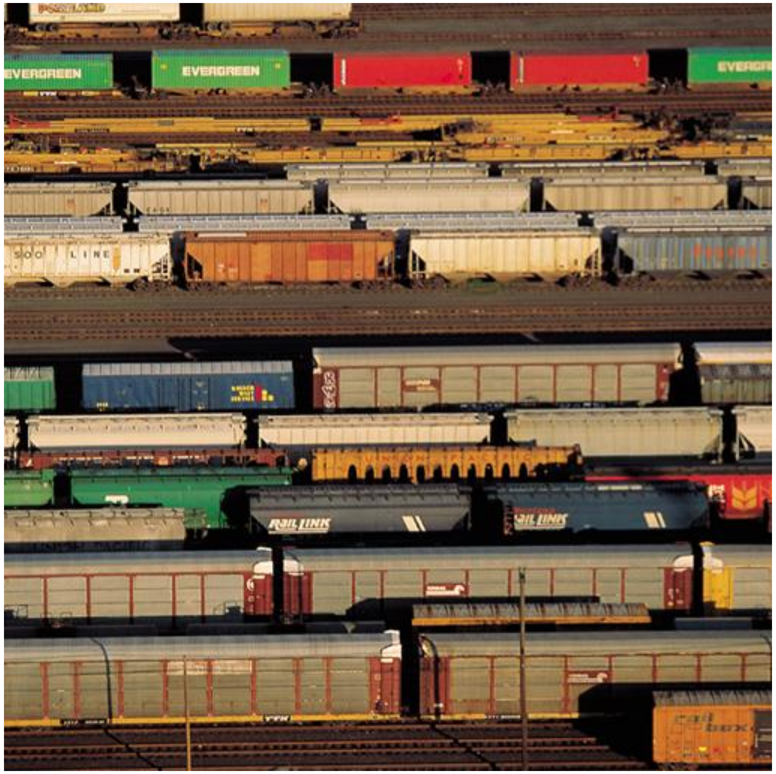
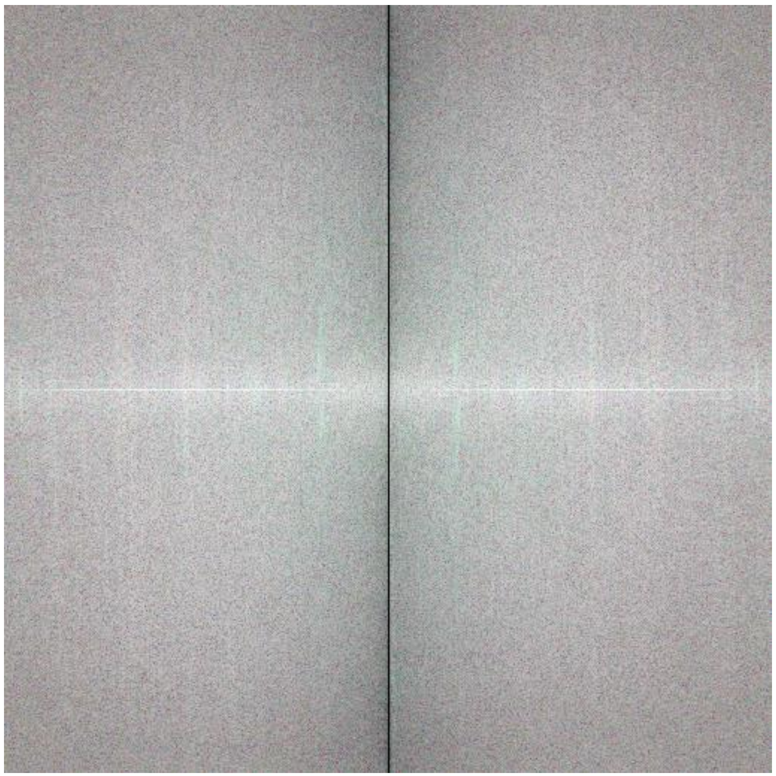
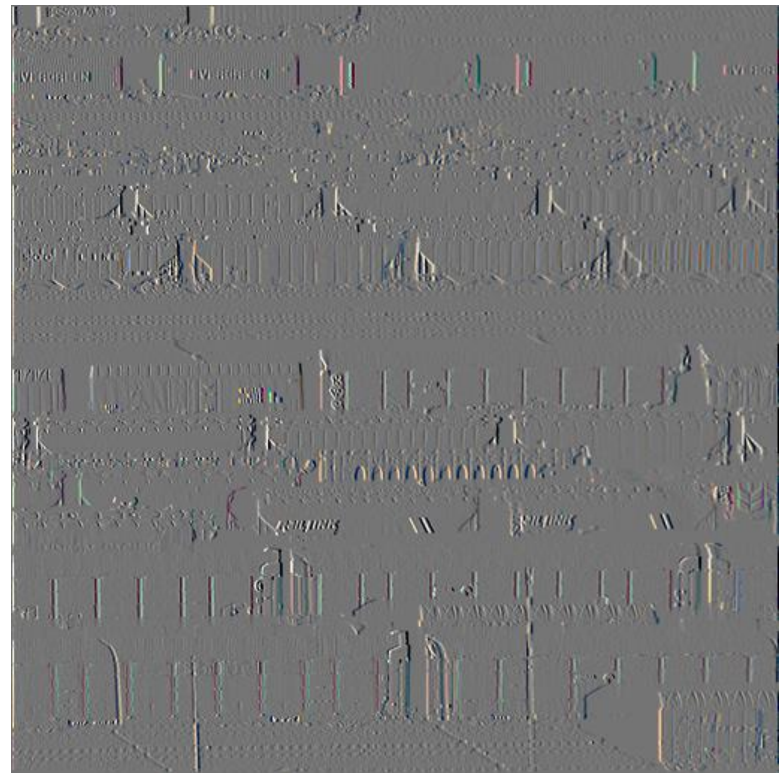


image **I**

Left Difference



power spectrum of $\mathbf{I}*\mathbf{h} = \mathbf{I}*[-1\ 1\ 0]$



$\mathbf{I}*\mathbf{h} = \mathbf{I}*[0\ 1\ -1]$

Original Image + Left Difference

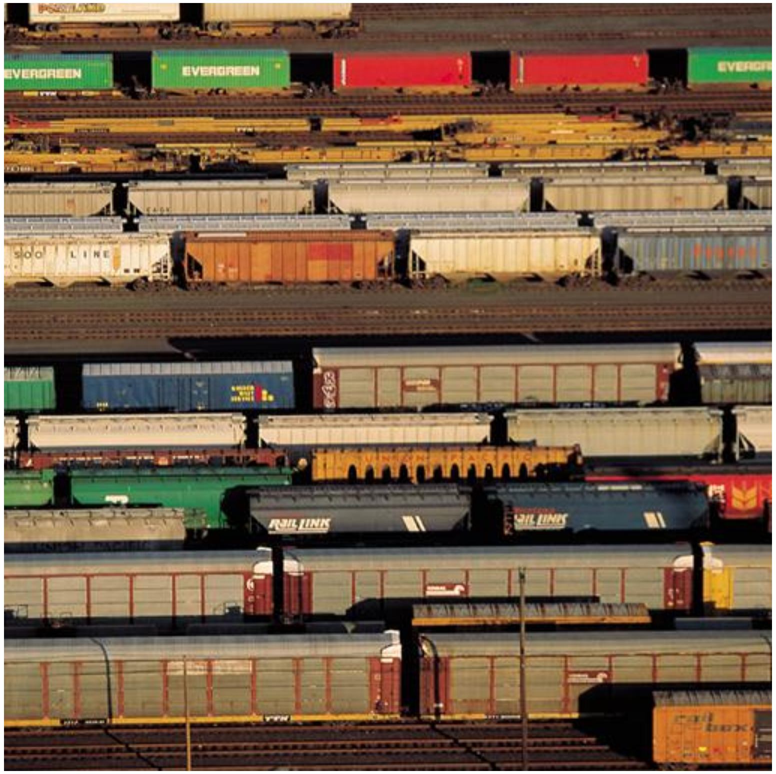
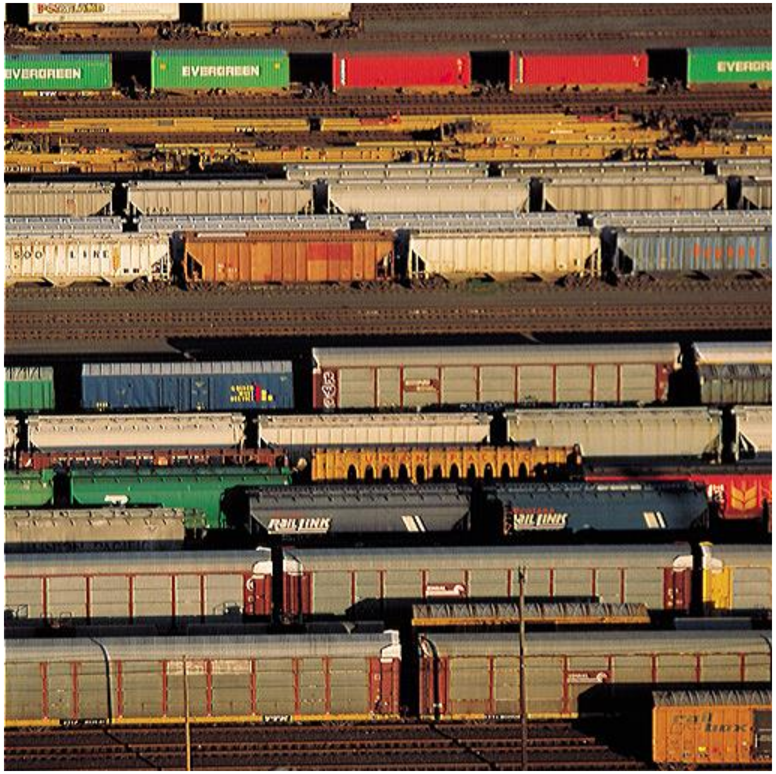
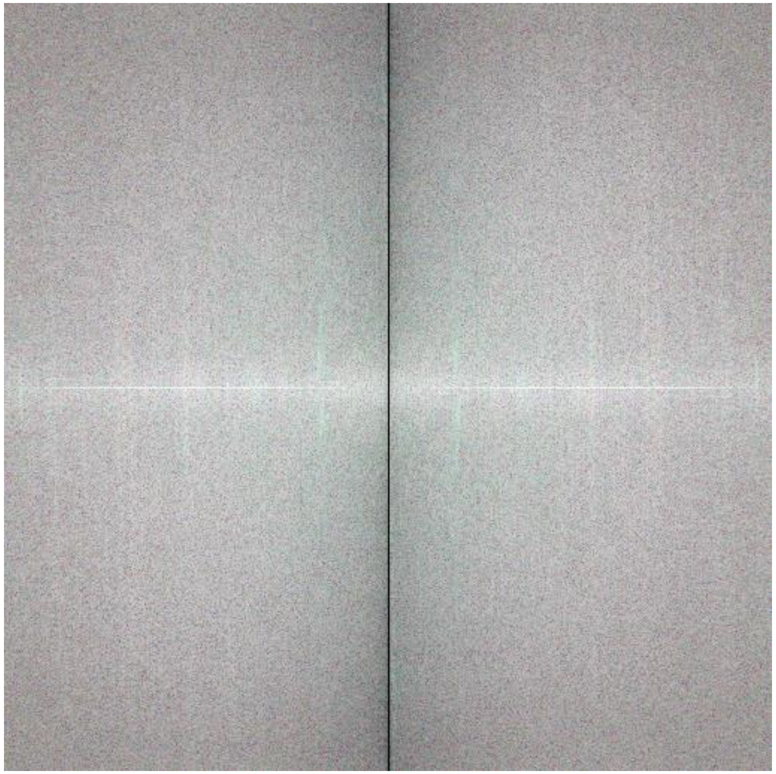


image **I**

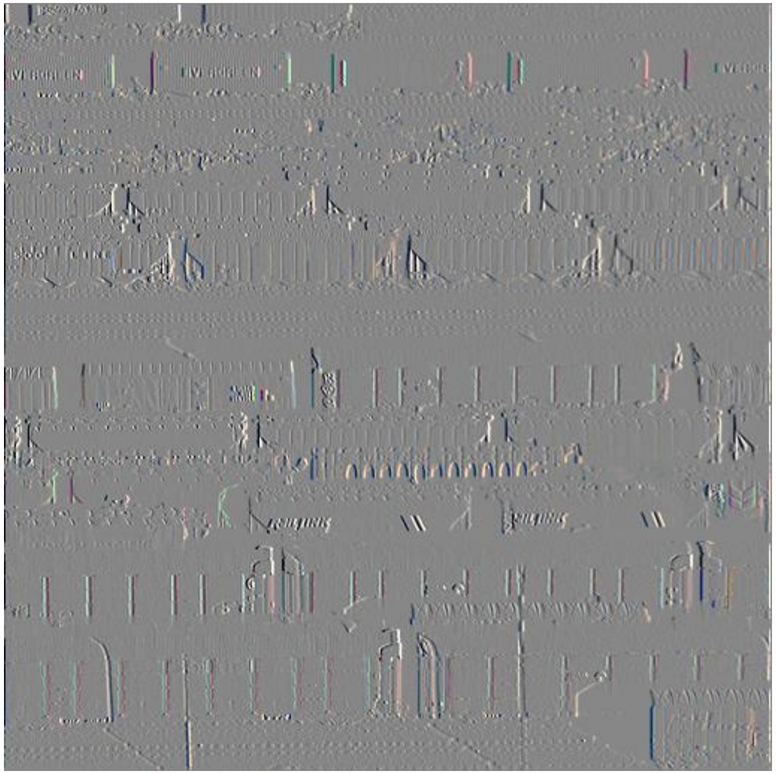


$\mathbf{I}+(\mathbf{I}*\mathbf{h}) = \mathbf{I}+(\mathbf{I}*[0\ 1\ \text{-}1])$

Right Difference



power spectrum of **I**∗**h** = **I**∗[1 -1]



I∗**h** = **I**∗[-1 1 0]

Original Image + Right Difference

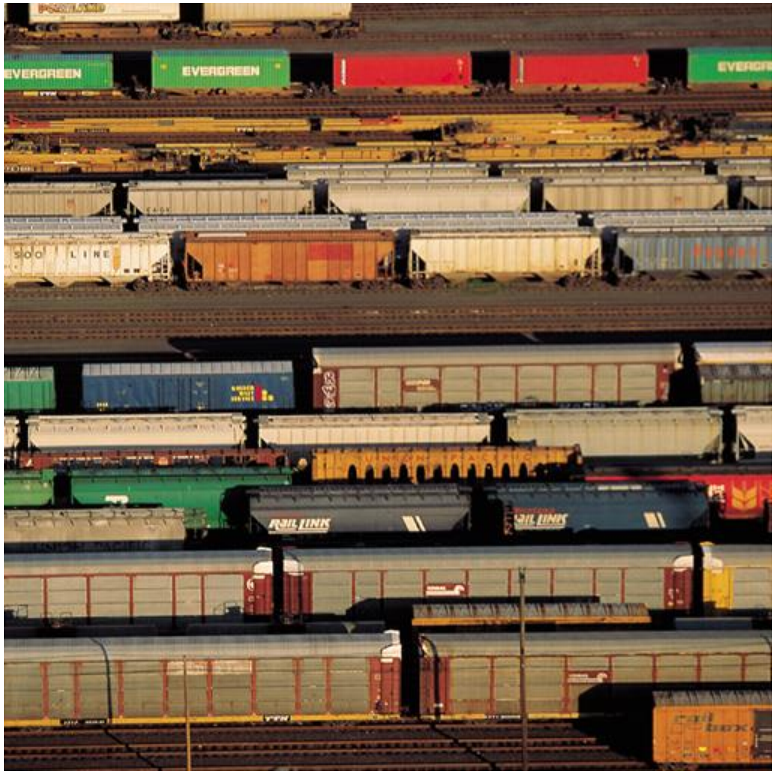
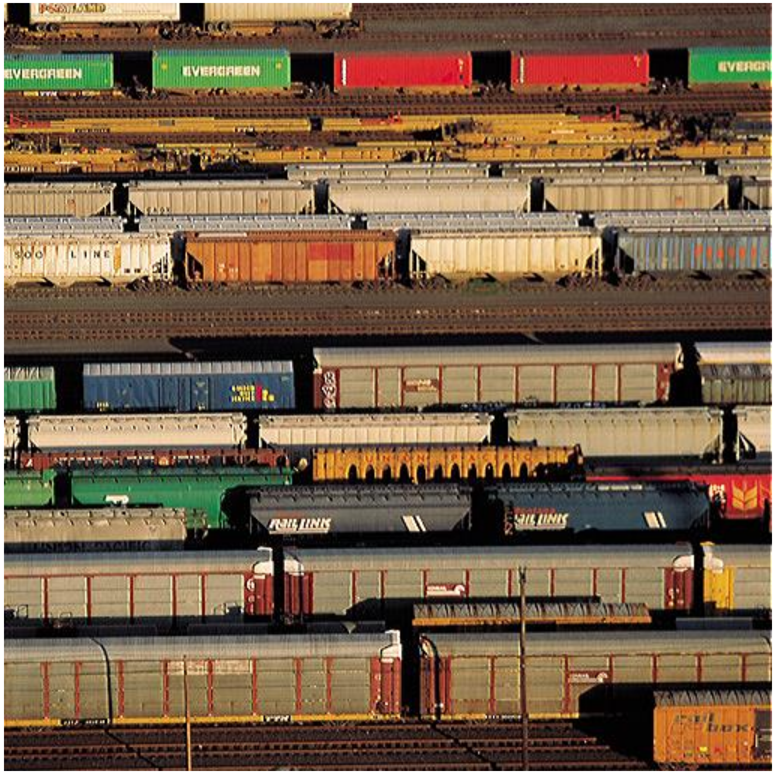
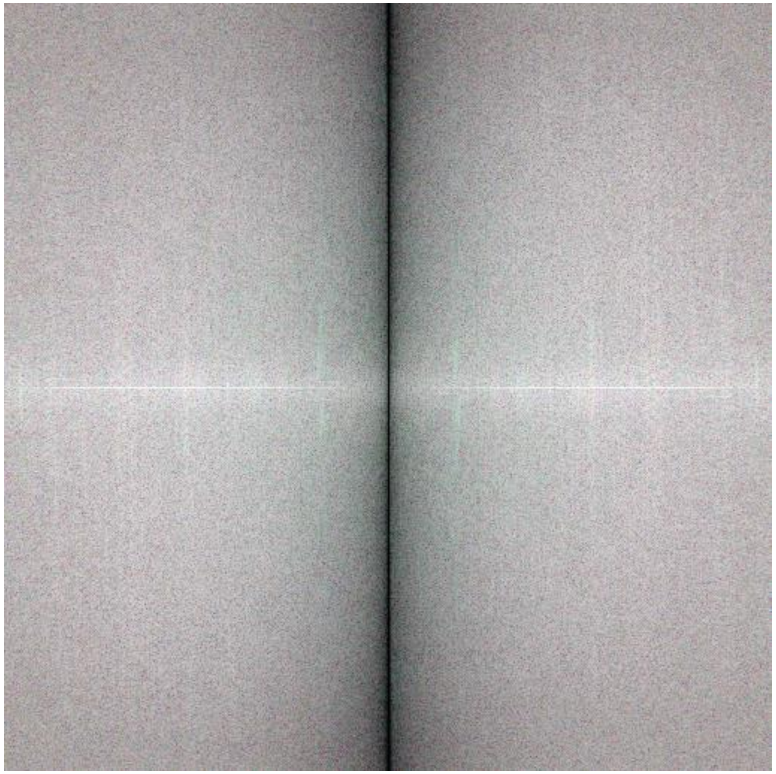


image **I**

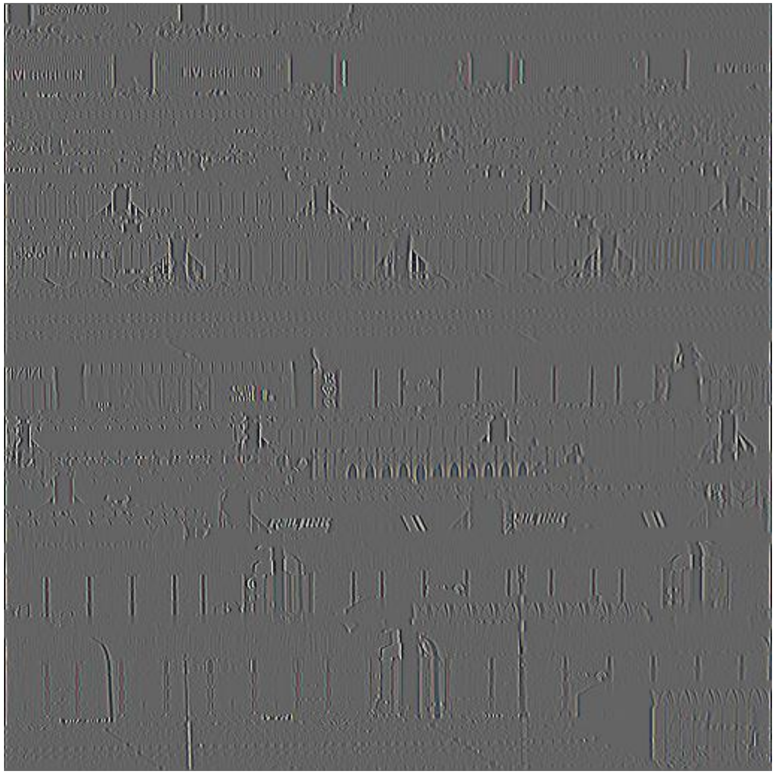


$\mathbf{I}+(\mathbf{I}*\mathbf{h}) = \mathbf{I}+(\mathbf{I}*[-1\ 1\ 0])$

Vertical Edges (L+R Diffs)



power spectrum of $\mathbf{I}*\mathbf{h} = \mathbf{I}*[-1\ 2\ -1]$



$\mathbf{I}*\mathbf{h} = \mathbf{I}*[-1\ 2\ -1]$

Original Image + Vertical Edges

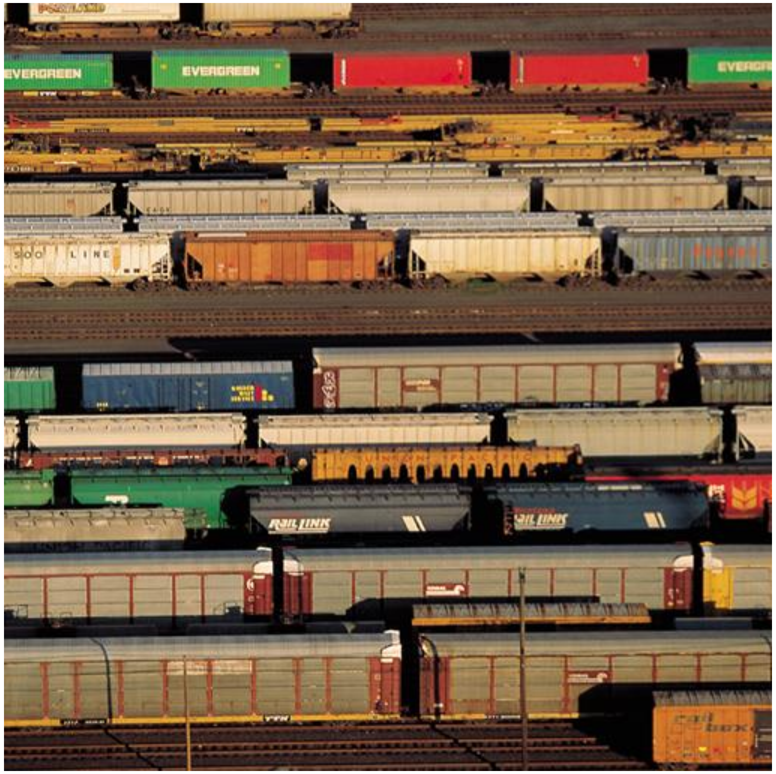
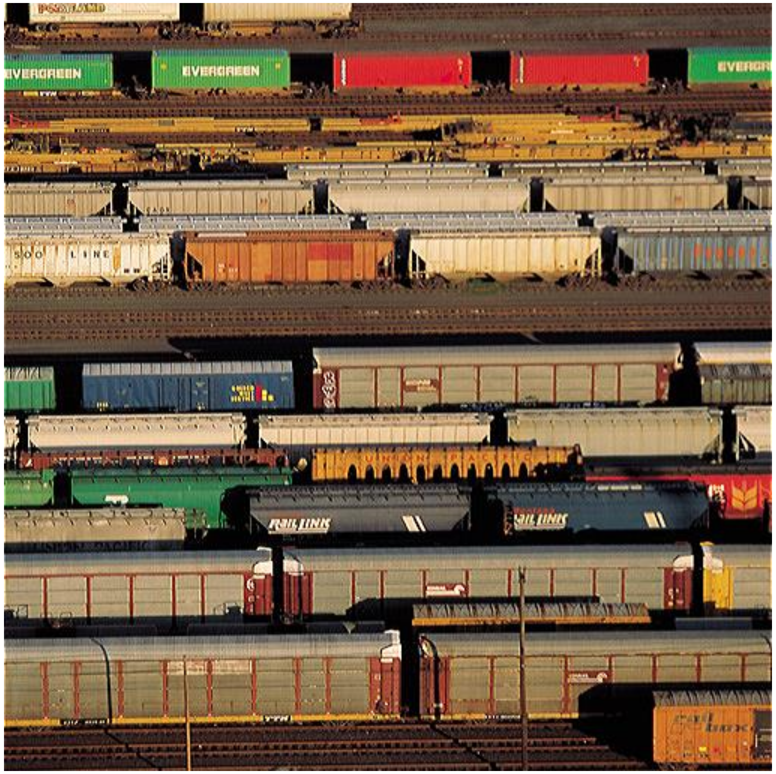
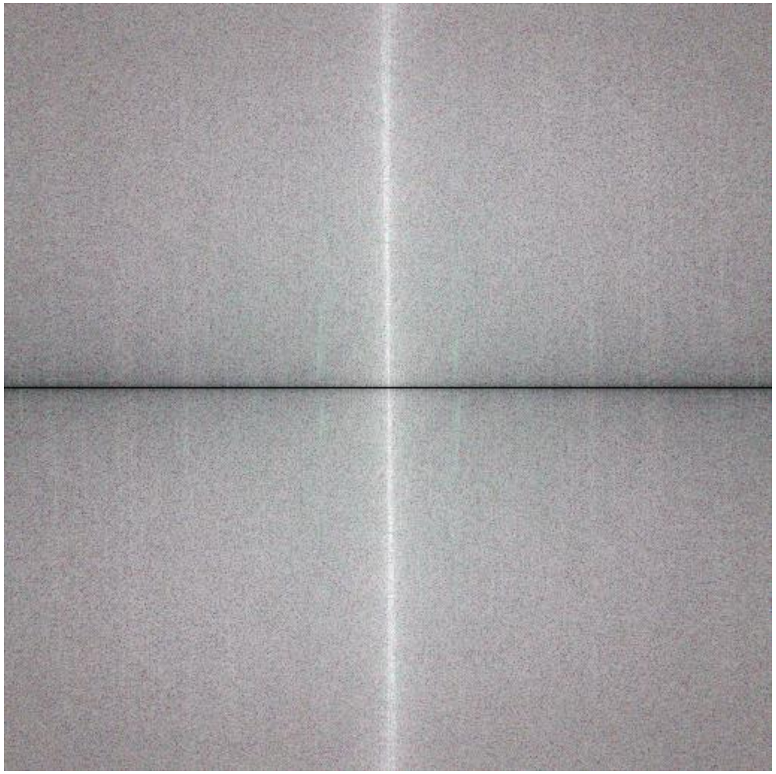


image **I**

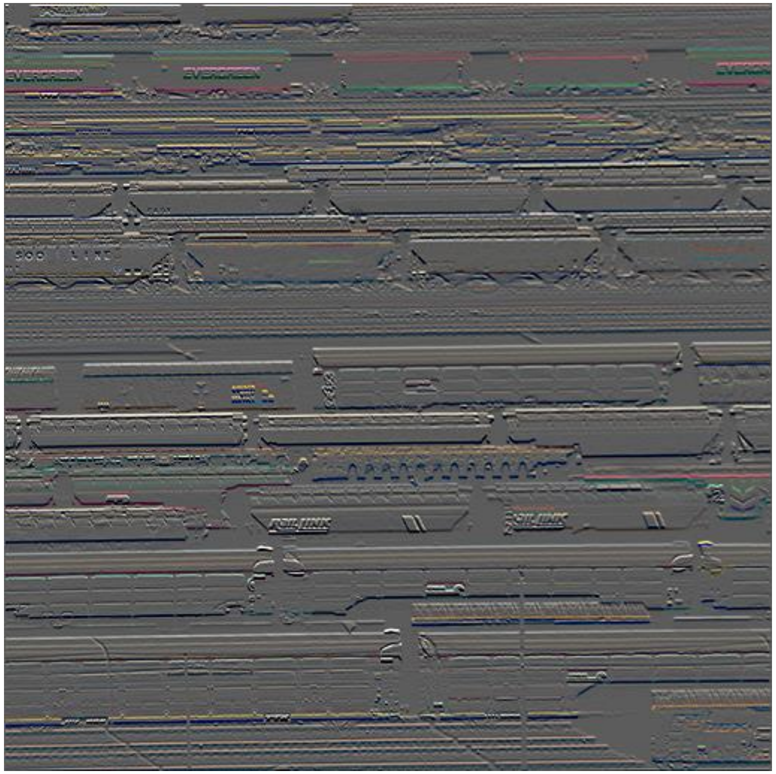


I+(**I**∗**h**) = **I**+(**I**∗[-1 2 -1])

Down Difference



power spectrum of $\mathbf{I}*\mathbf{h} = \mathbf{I}* \begin{bmatrix} 1 \\ -1 \end{bmatrix}$



$\mathbf{I}*\mathbf{h} = \mathbf{I}* \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Original Image + Down Difference

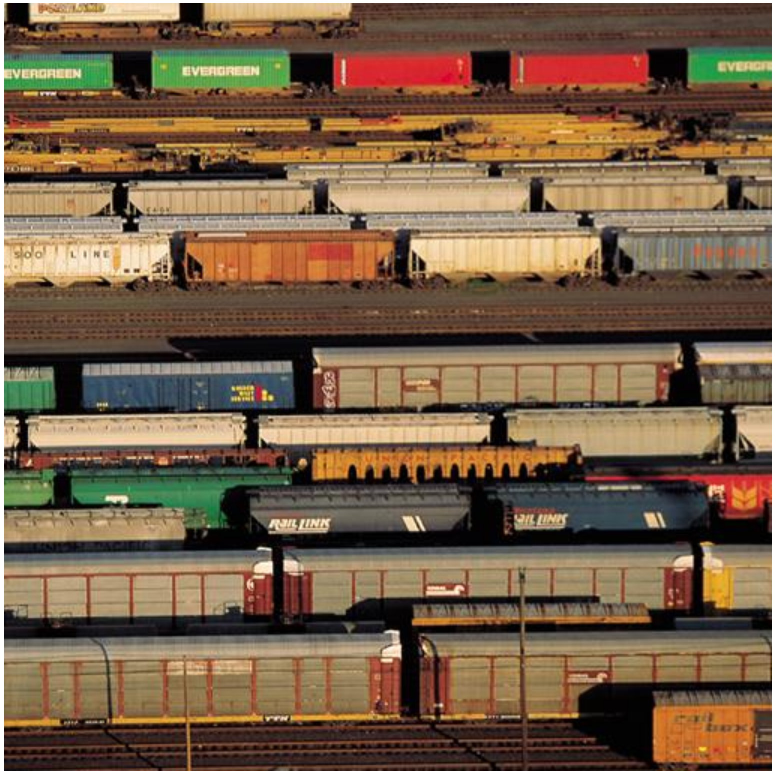
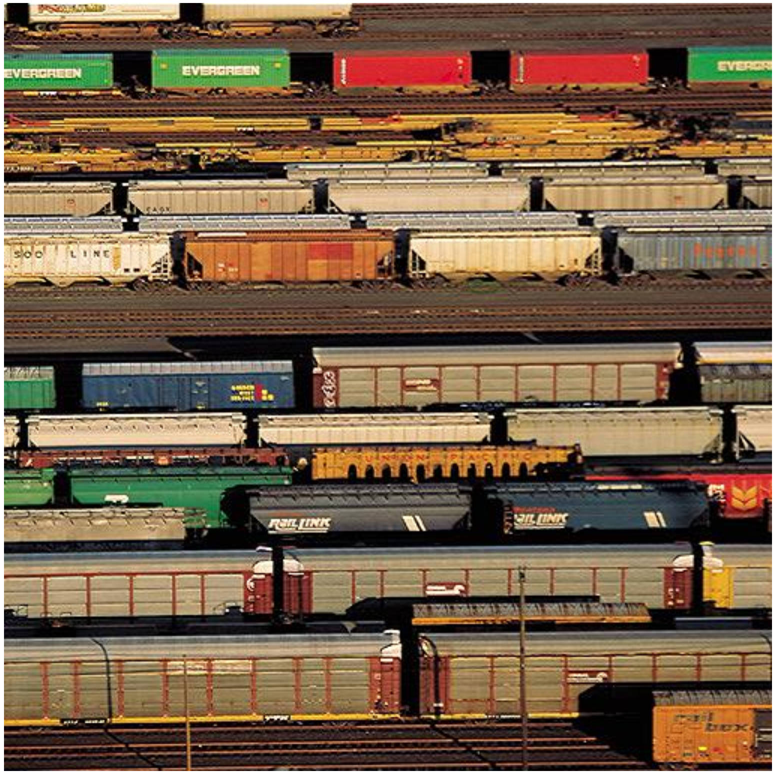
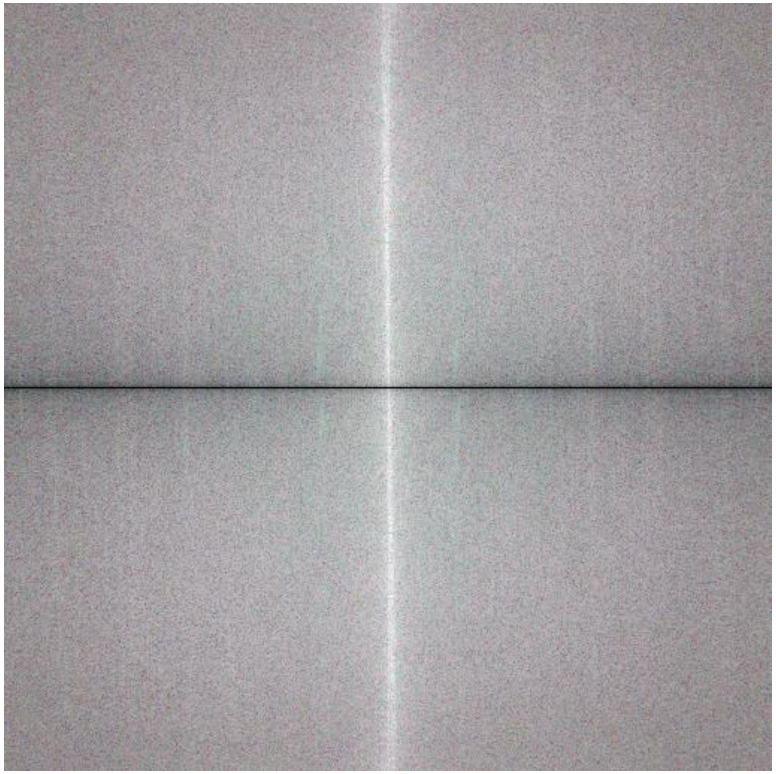


image $\mathbf{I}$

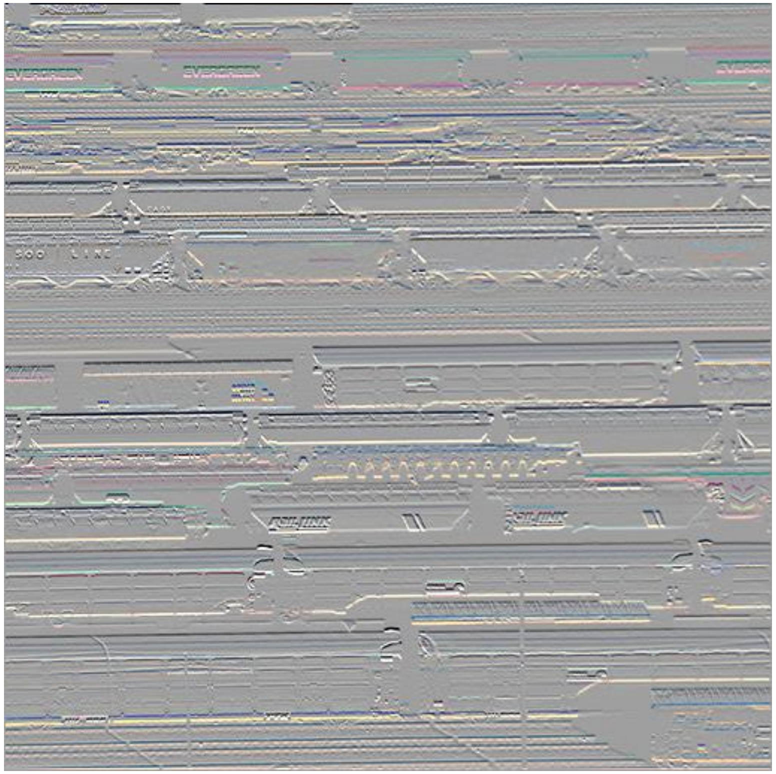


$\mathbf{I}+(\mathbf{I}*\mathbf{h}) = \mathbf{I}+\left(\mathbf{I}*\begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$

Up Difference



power spectrum of $\mathbf{I} * \mathbf{h} = \mathbf{I} * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$



$\mathbf{I} * \mathbf{h} = \mathbf{I} * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Original Image + Up Difference

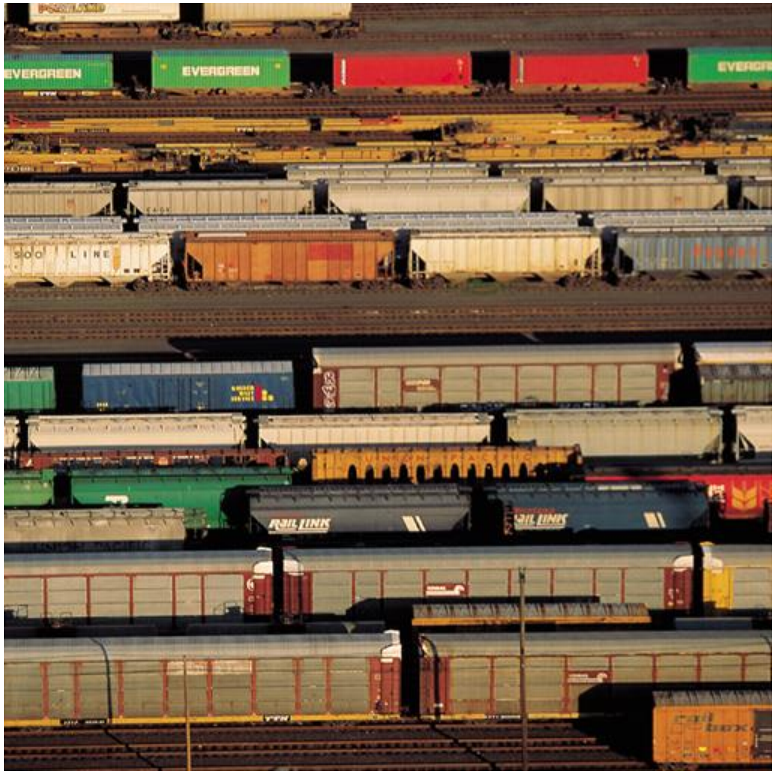
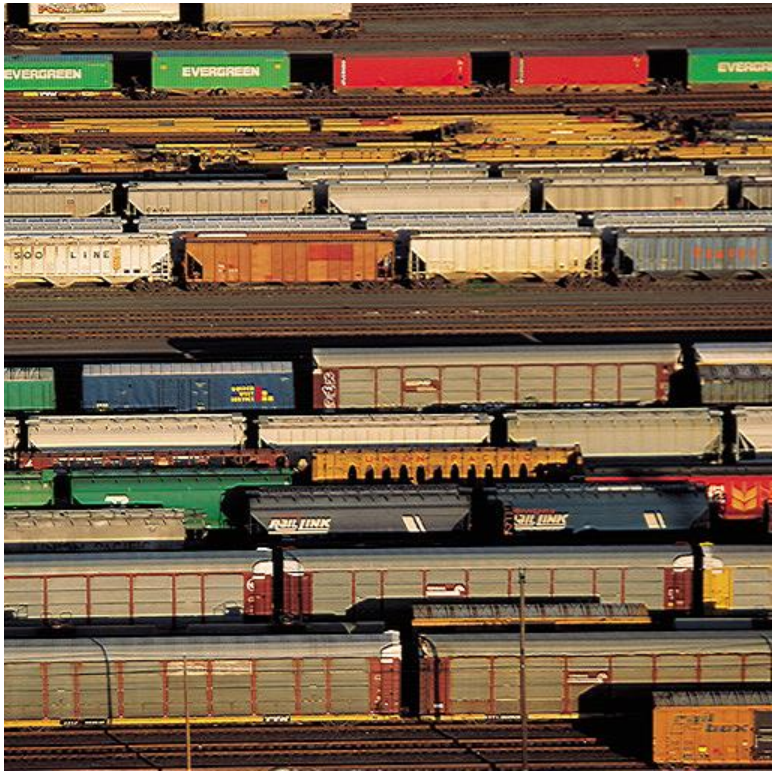
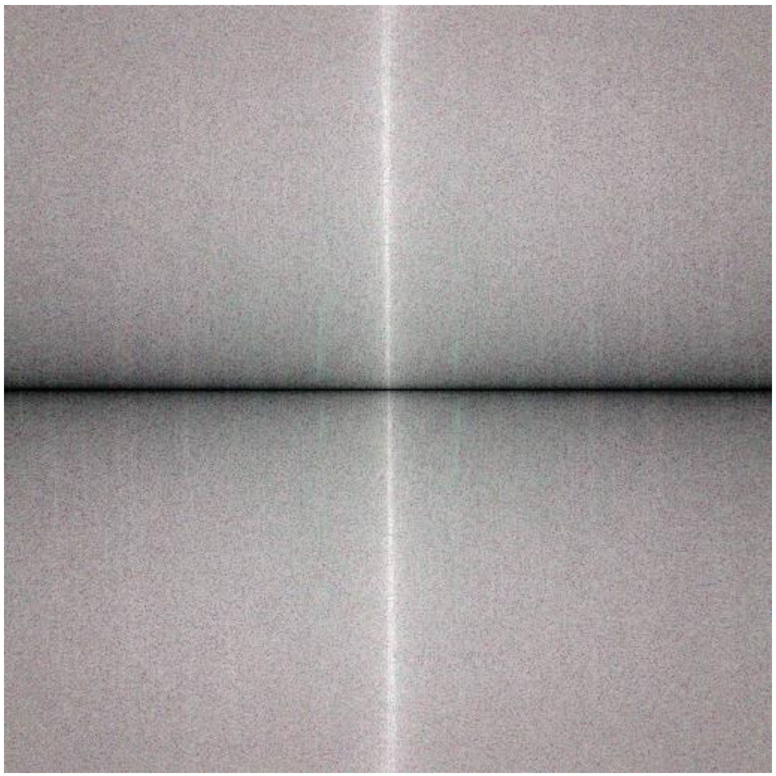


image $\mathbf{I}$



$\mathbf{I}+(\mathbf{I}*\mathbf{h}) = \mathbf{I}+\left(\mathbf{I}*\begin{bmatrix}1\\-1\end{bmatrix}\right)$

Horizontal Edges (D+U Diffs)



power spectrum of $\mathbf{I} * \mathbf{h} = \mathbf{I} * \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$

$\mathbf{I} * \mathbf{h} = \mathbf{I} * \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$

Original Image + Horizontal Edges

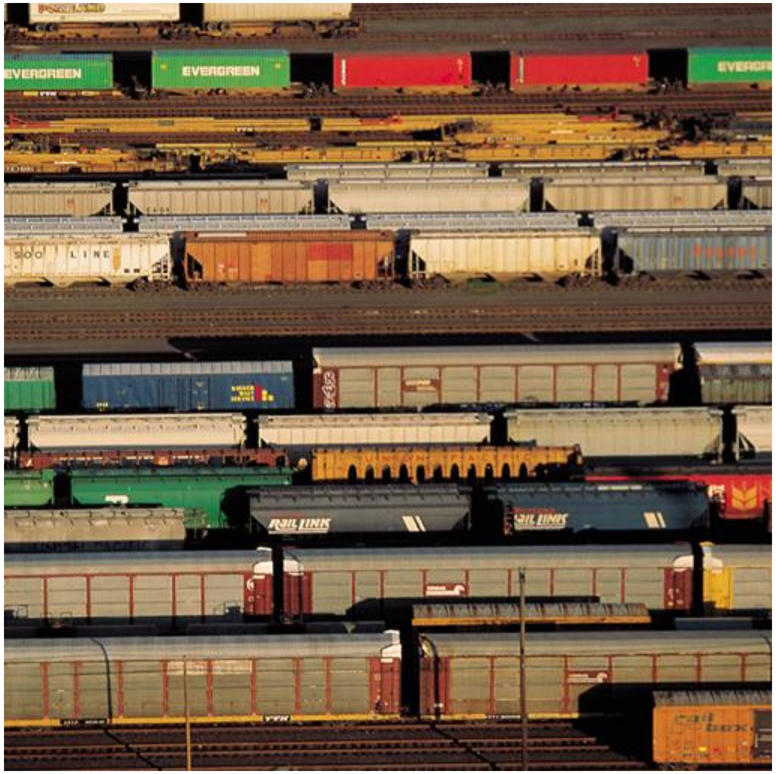
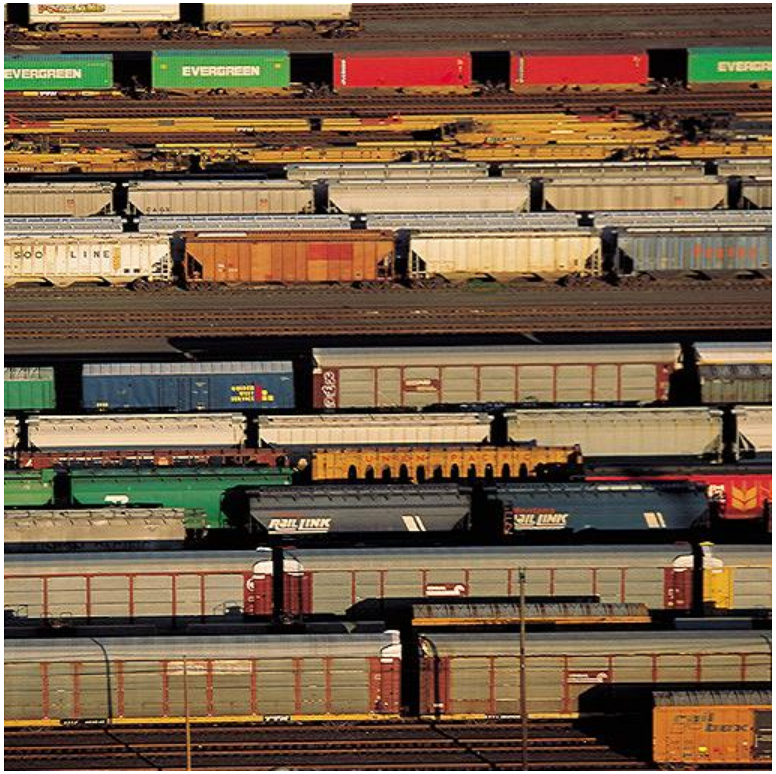
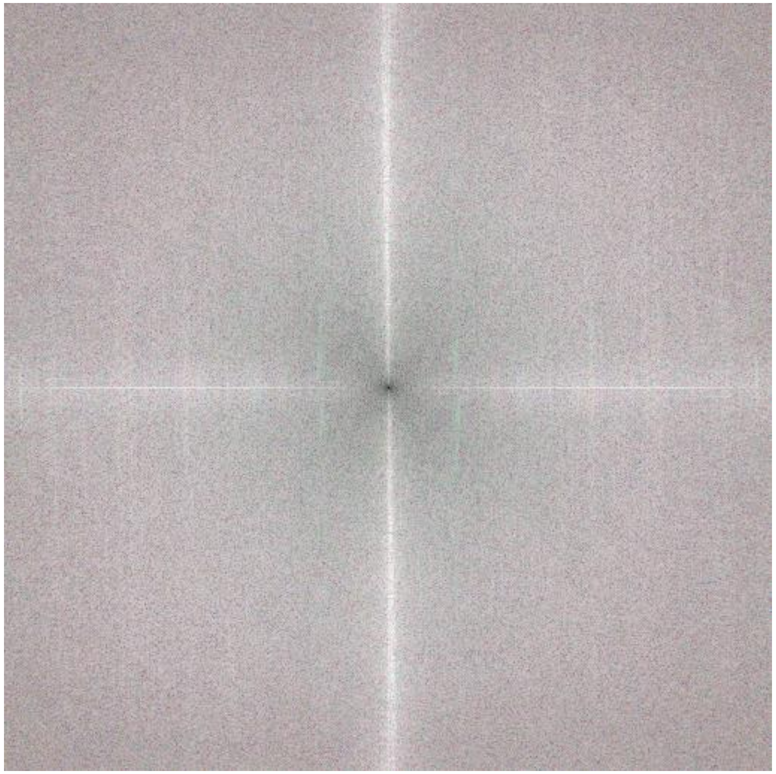


image **I**



$\mathbf{I}+(\mathbf{I}*\mathbf{h}) = \mathbf{I}+\left(\mathbf{I}*\begin{bmatrix}-1\\2\\-1\end{bmatrix}\right)$

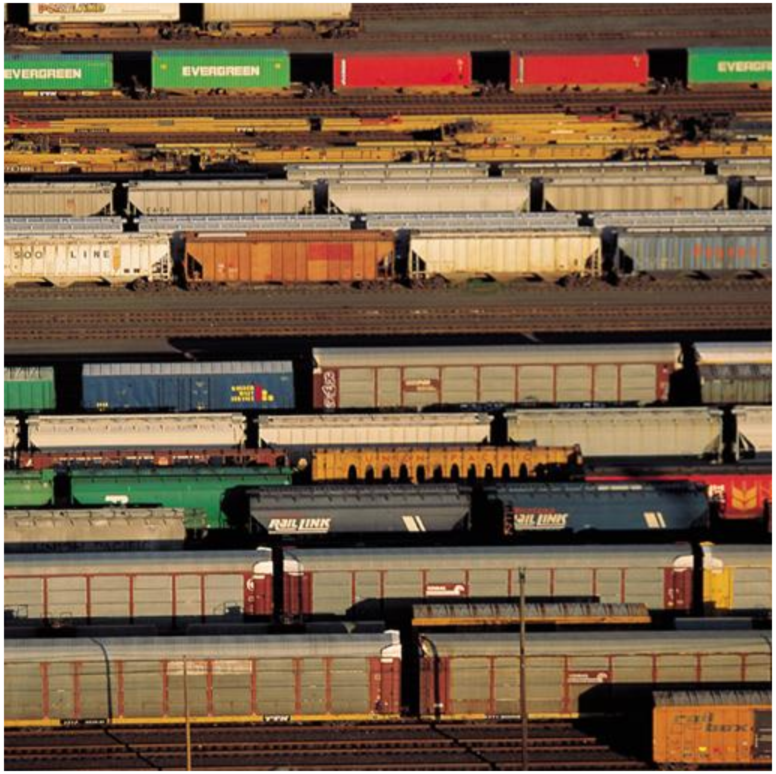
Horiz. + Vert. Edges (L+R+D+U Diffs)



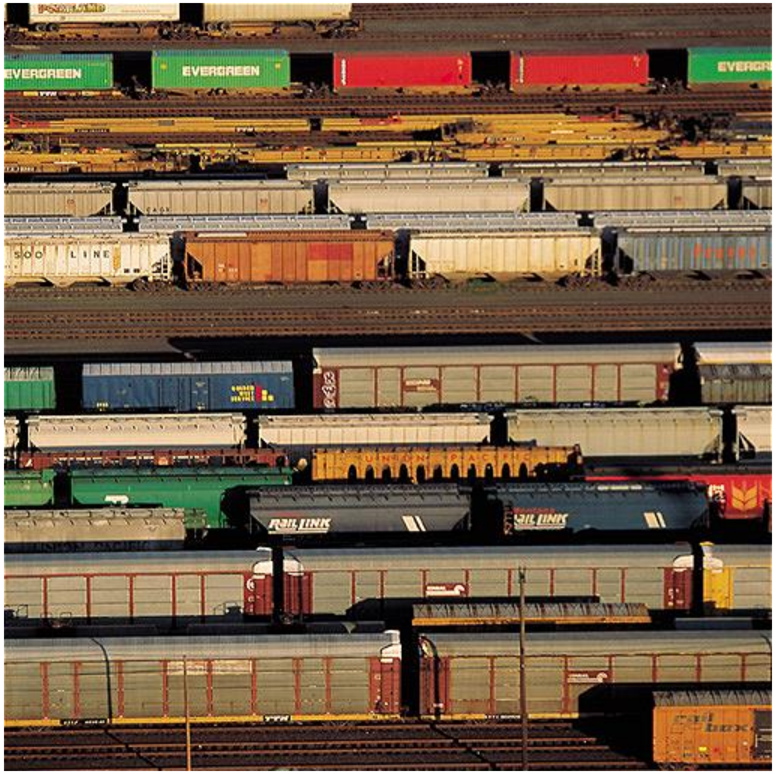
power spectrum of $\mathbf{I}*\mathbf{h} = \mathbf{I}* \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

$\mathbf{I}*\mathbf{h} = \mathbf{I}* \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

Original Image + Horiz. + Vert. Edges

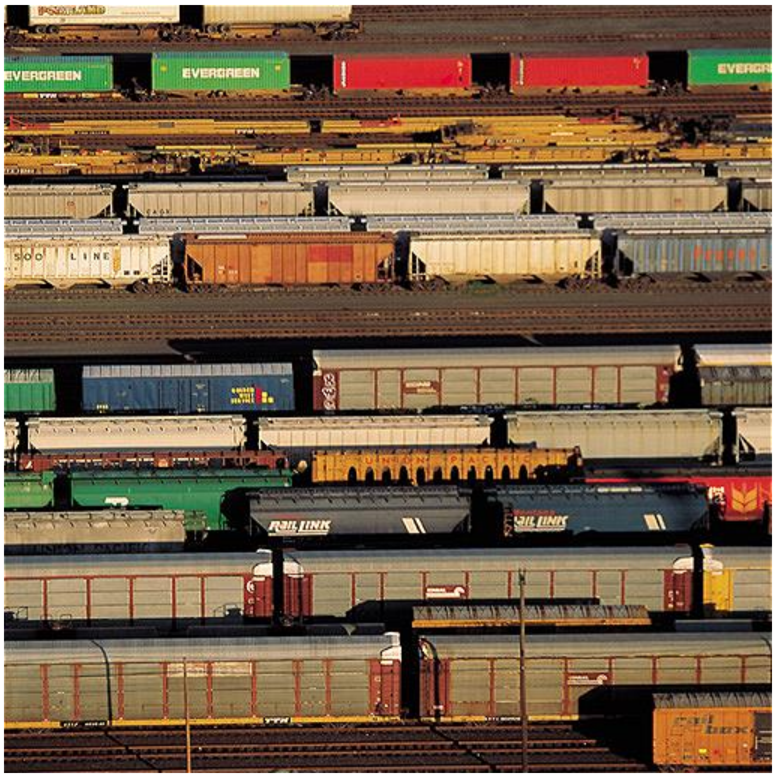


original



sharpened

Original Image + Horiz. + Vert. Edges

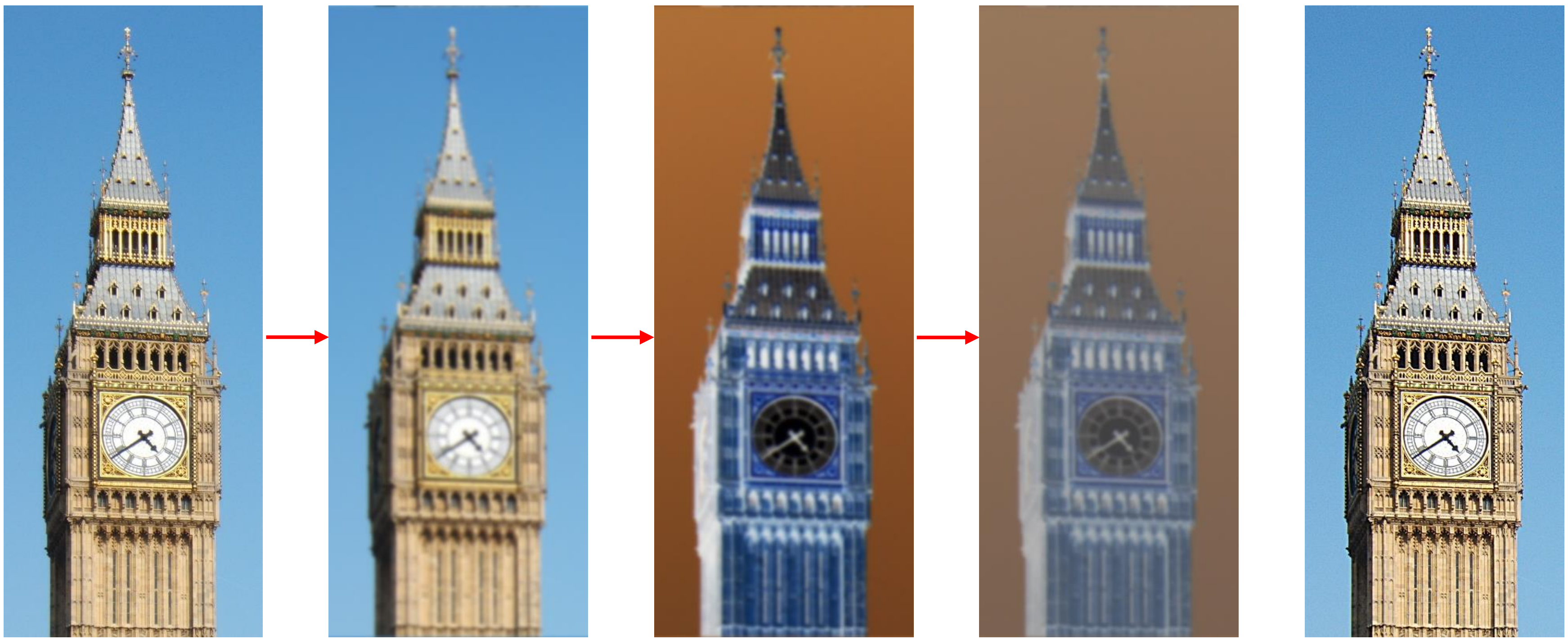


sharpened

original

Unsharp Masking

is a film-photography darkroom technique for sharpening an image. A blurred copy of the photonegative is contrast reduced and used to mask the original image.



original ○ blurred + negative lo-contrast ○ = USM ●

Sharpening Through Blurring: Unsharp Masking

Let I be an image.

Let $\mathbf{G}_\sigma$ be a Gaussian convolution mask.

Then $\mathbf{J} = \mathbf{I} * \mathbf{G}_\sigma$ is a blurred image and $\mathbf{K} = \mathbf{I} - \mathbf{J}$ contains all the high spatial frequencies from I.

Define:

$$\mathbf{U} = (1+\alpha)\,\mathbf{K} + \mathbf{J} = \alpha\,\mathbf{K} + \mathbf{I},$$

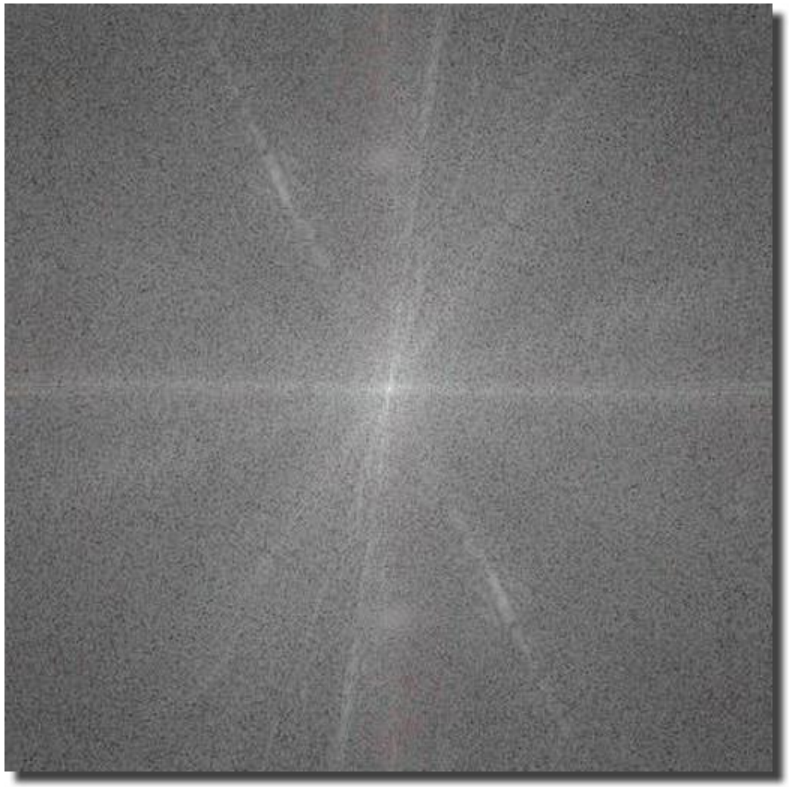
where, typically $0 < \alpha < 2$.

U is called the *unsharp masking* of image **I**.

Often, the control, α, is given as a percent value.
Then the formula is $(\alpha/100)*K+I$.

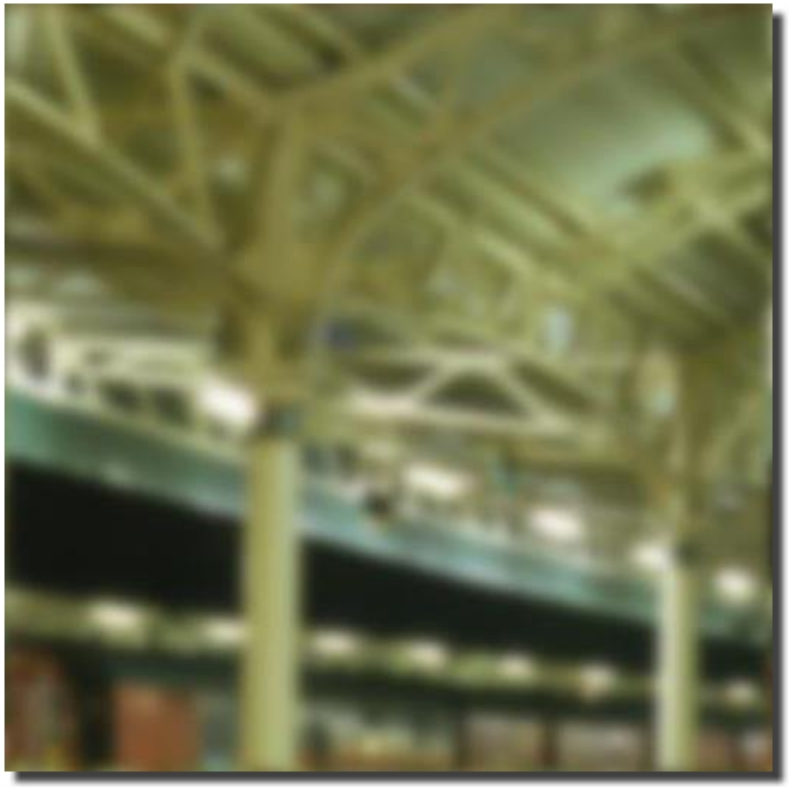
Sharpening Through Blurring: Unsharp Masking

original image

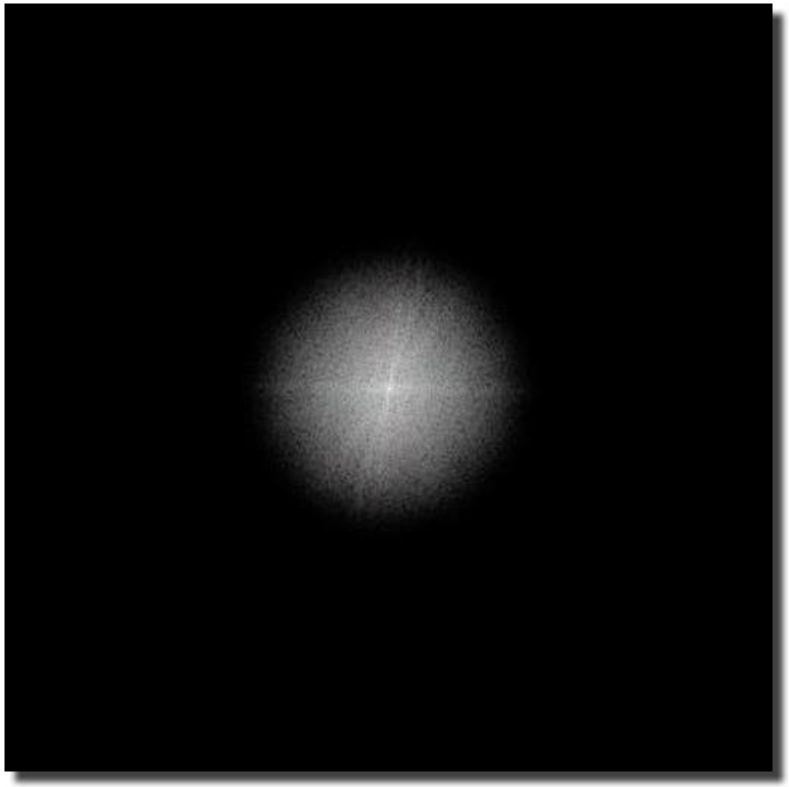


log power spectrum

Sharpening Through Blurring: Unsharp Masking



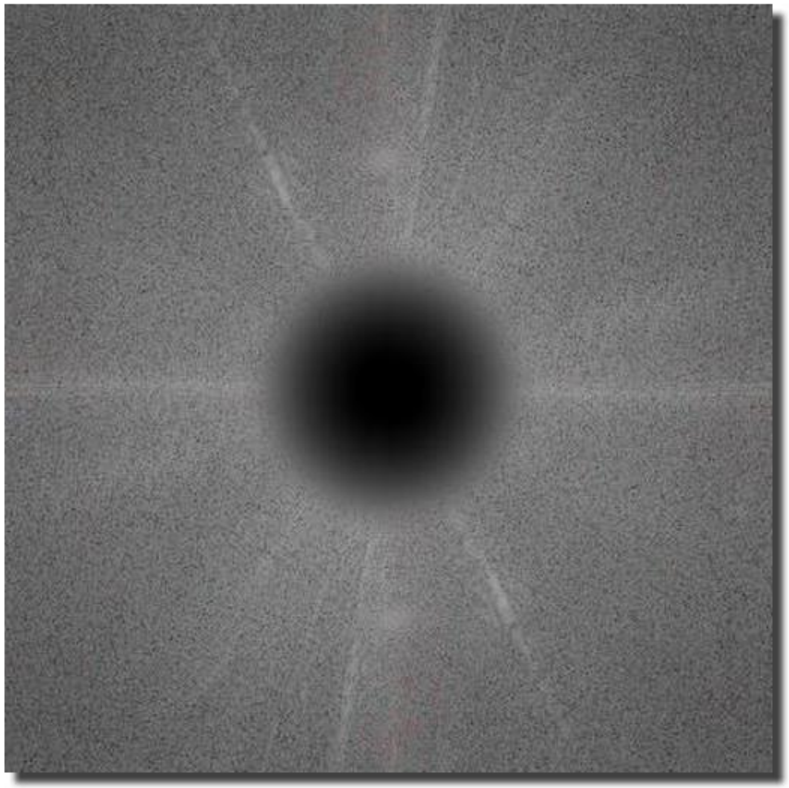
Gaussian blur σ=4



log power spectrum

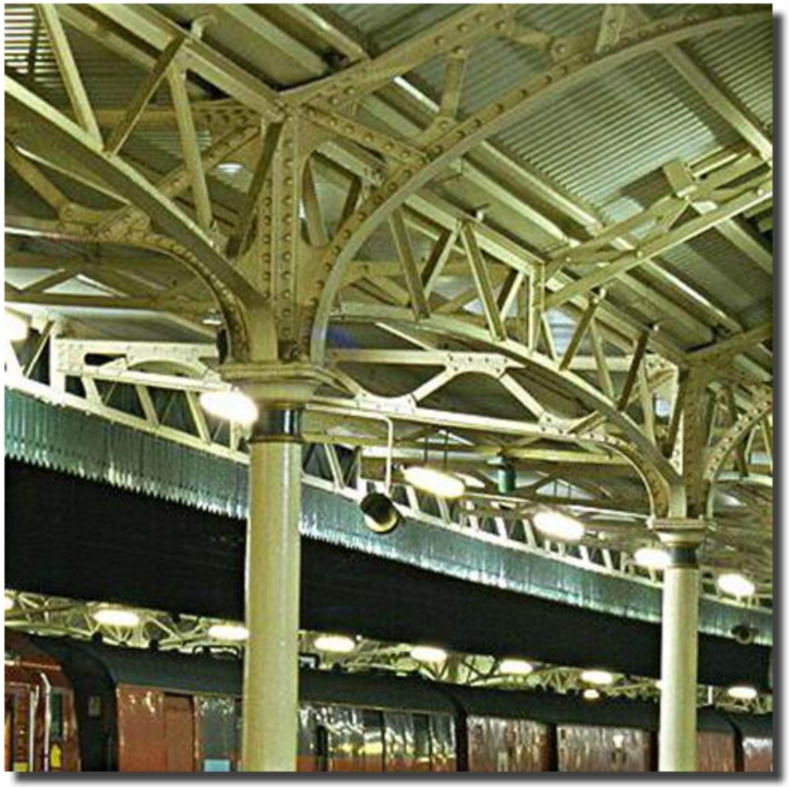
Sharpening Through Blurring: Unsharp Masking

original minus Gaussian blur

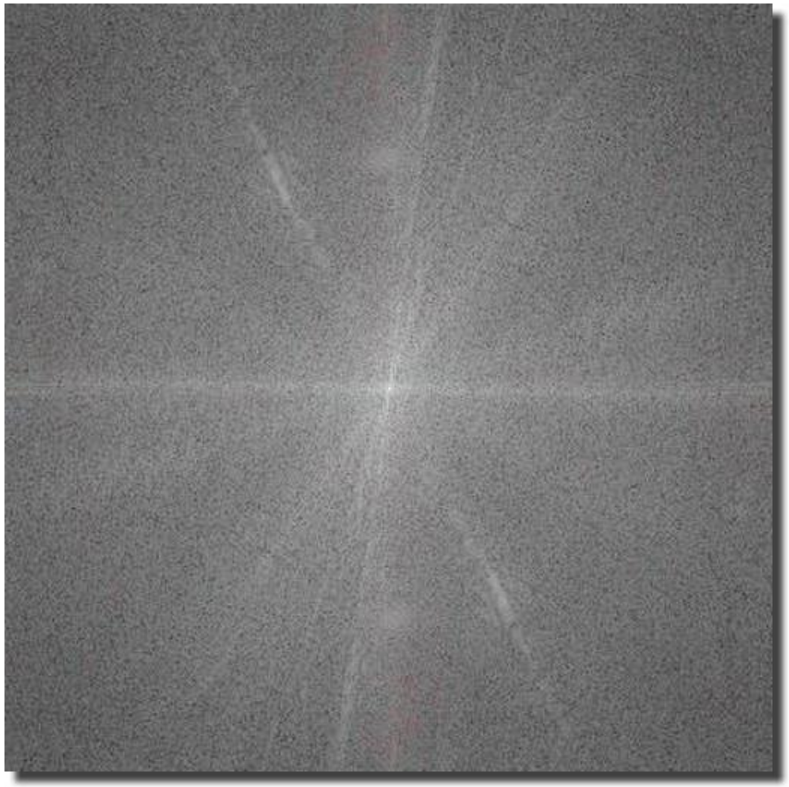


log power spectrum

Sharpening Through Blurring: Unsharp Masking



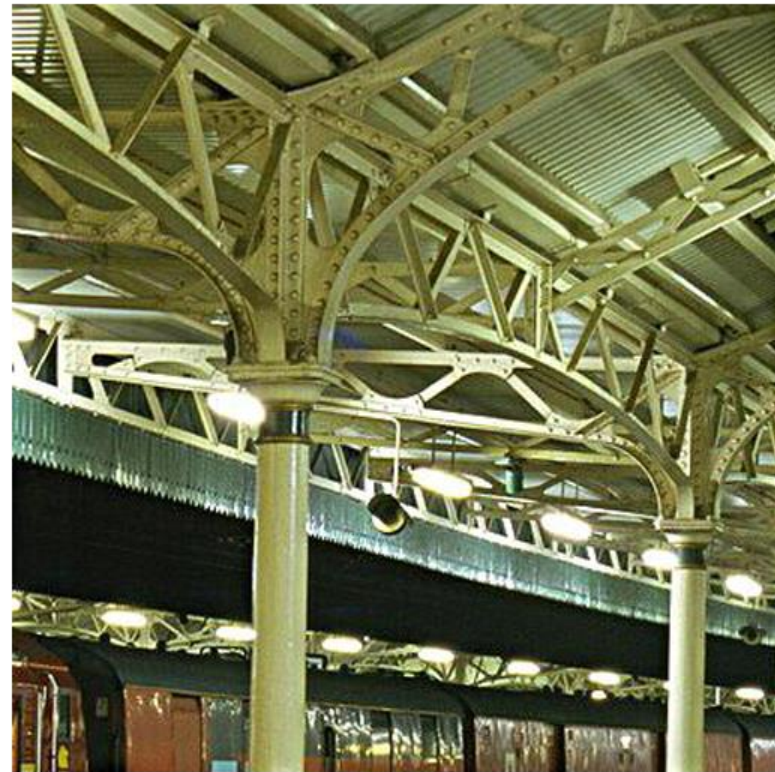
unsharp masked image



log power spectrum

Sharpening Through Blurring: Unsharp Masking

original image



unsharp masked image

Sharpening above a Specific Scale.

An image is sharpened by taking a linear combination of the image and a highpass filtered version of itself. The scale of the sharpening can be controlled via the cutoff of the HPF. In the following examples the image has been sharpened via

$$\mathbf{I}_{\mathrm{hfe},\sigma} = \mathbf{I} + \alpha \mathbf{I}_{\mathrm{hpf},\sigma} = \mathbf{I} + \alpha\Big(\mathbf{I} - \big[\mathbf{I} * g(\sigma)\big]\Big) = (1+\alpha)\mathbf{I} - \alpha\big[\mathbf{I} * g(\sigma)\big],$$

where g is a 2D Gaussian with $\sigma \in \{1, 2, 4, 8, 16, 32, 64, 128, 256\}$ and α is a scale factor, usually in $(0,2)$. After the computation, each image was histogram matched to $\mathbf{I}$.

$\sigma_0 = 0$ Original Image

Sharpening above a Specific Scale

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$\sigma_0 = 8, \alpha=1$

Sharpening above a Specific Scale

Sharpening above a Specific Scale

Sharpening above a Specific Scale

Sharpening above a Specific Scale

$\sigma_0 = 128, \alpha=1$

Sharpening above a Specific Scale

Sharpening above a Specific Scale

Sharpening above a Specific Scale

Emphasizing a Specific Pass Band.

An image can be bandpass filtered by subtracting two differently Gaussian filtered copies of it. That specific band can be emphasized in the image by adding it back to the image. In the following examples the image has been emphasized via

$$\begin{aligned}\mathbf{I}_{\mathrm{bpe},\sigma_0,\sigma_1} &= \mathbf{I} + \alpha \mathbf{I}_{\mathrm{bpf},\sigma_0,\sigma_1} \\ &= \mathbf{I} + \alpha\Big[\mathbf{I} - \Big(\big[\mathbf{I} * g(\sigma_0)\big] - \big[\mathbf{I} * g(\sigma_1)\big]\Big)\Big] \\ &= (1+\alpha)\mathbf{I} - \alpha\Big(\mathbf{I} * \big[g(\sigma_0) - g(\sigma_1)\big]\Big),\end{aligned}$$

where $\sigma_0, \sigma_1 \in \{1, 2, 4, 8, 16, 32, 64, 128, 256\}$ and α is a scale factor, usually in $(0,2)$. After the computation, each image was histogram matched to $\mathbf{I}$.

$(\sigma_1, \sigma_0) = (1,0),\ \alpha=1$

Emphasizing a Specific Pass Band

Emphasizing a Specific Pass Band

$(\sigma_1, \sigma_0) = (4,2)$, $\alpha=1$

Emphasizing a Specific Pass Band

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Noise Enhancement: the Problem with Sharpening

- Noise occurs in every natural imaging device
  - Quantum effects in CCD arrays
  - Random distribution of silver halide grains in film
  - Neuronal noise in the retina
- Spatially independent noise
  - The noise in one sensor has no effect on that in its neighbors
  - $\Rightarrow$ the autocorrelation of the signal is an impulse at the origin
  - The chances of getting repeated patterns of any frequency are virtually nil
  - $\Rightarrow$ the frequency spectrum of the noise is flat

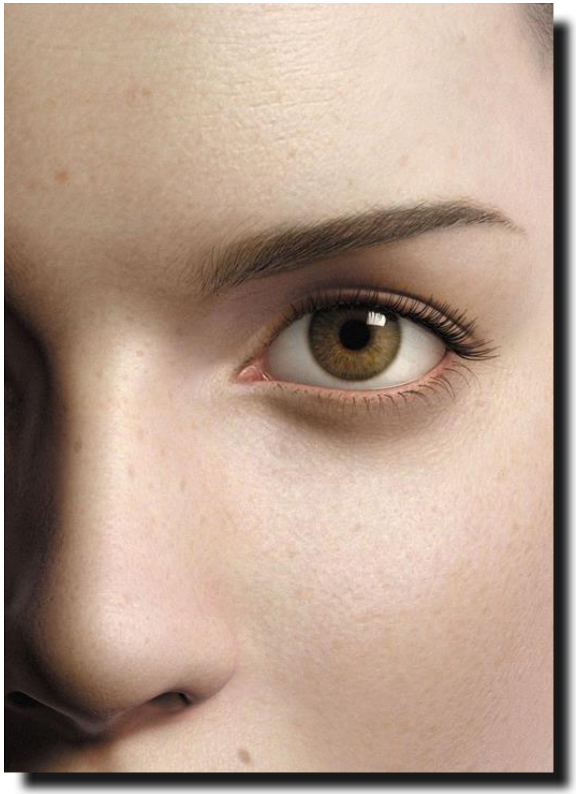
Recall: Autocorrelation = inverse Fourier transform of power spectrum; Fourier transform of an impulse at (0,0) is a constant.

Noise Enhancement: the Problem with Sharpening

- The spectra of most natural images fall-off toward the high frequencies.
- IID noise has a flat spectrum.
- Therefore, at some relatively high frequency (HF) the energy in the noise is greater than that in the uncorrupted image.
- Sharpening multiplies the FT of the image by u and v (or by linear combinations of them) which, at HF, increases the noise more than the uncorrupted image.

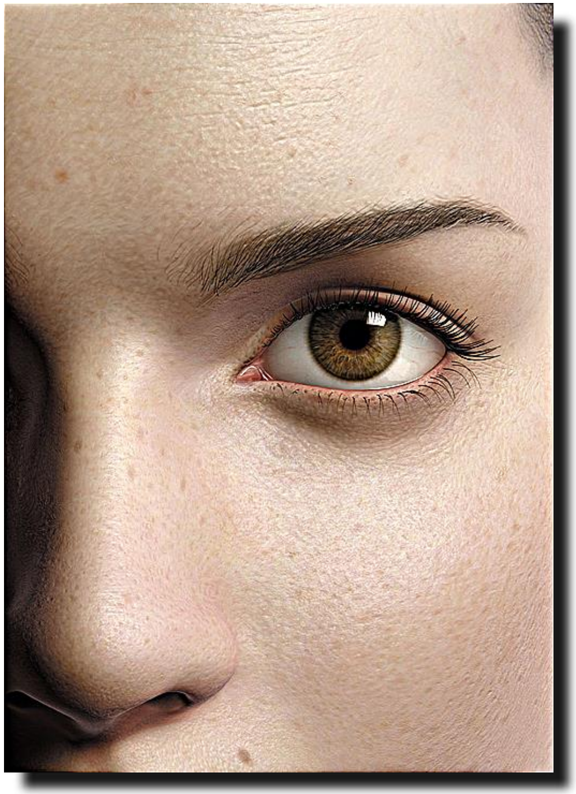
Effects of Noise on Linear Enhancement of HF

original image

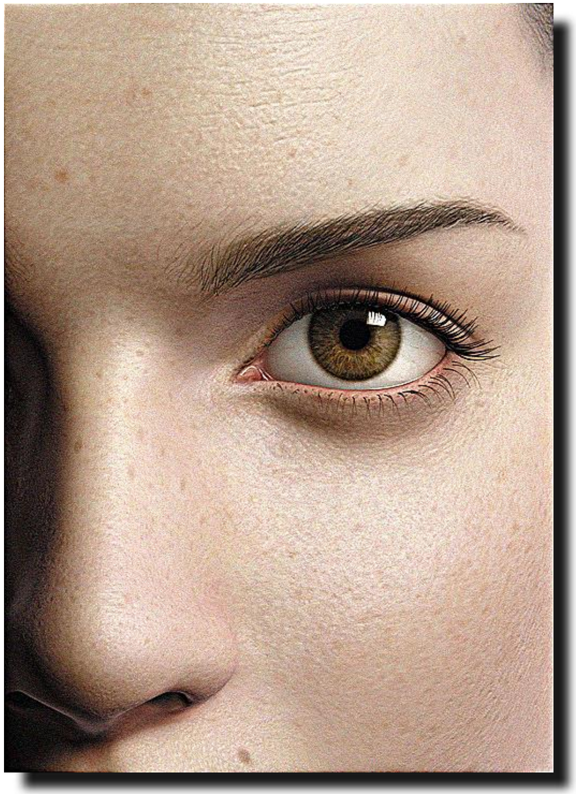


noisy image

Effects of Noise on Linear Enhancement of HF



HF enhanced original



HF enhanced noisy image

Effects of Noise on Linear Enhancement of HF

original image

noisy image

Effects of Noise on Linear Enhancement of HF

HF enhanced original

HF enhanced noisy image

Effects of Noise on Linear Enhancement of HF

original image

HF enhanced original

Effects of Noise on Linear Enhancement of HF

noisy image

HF enhanced noisy image