



# EECE 4353 Image Processing

## High Dynamic Range Imaging and Images

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# Primary References

[1] Paul E. Debevec and Jitendra Malik. “Recovering high dynamic range radiance maps from photographs.” In ACM SIGGRAPH 2008 classes (SIGGRAPH '08). ACM, New York, NY, USA, Article 31, 10 pages.

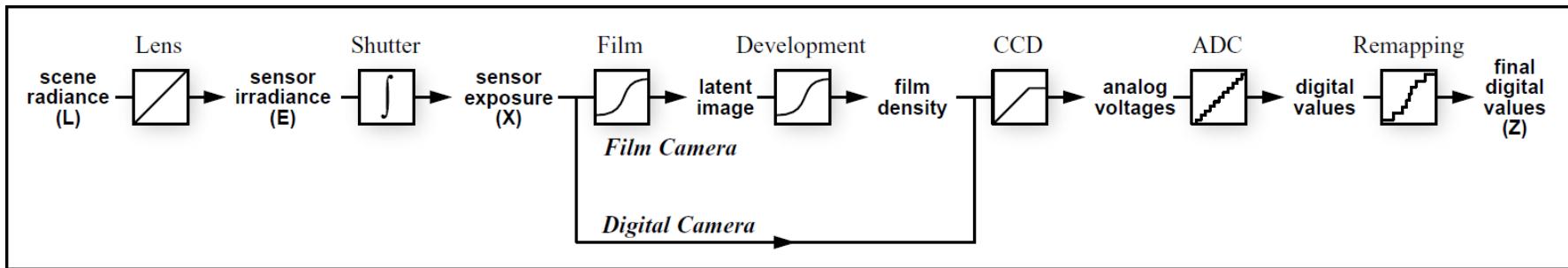
[2] Paul E. Debevec’s web pages on HDR:

<http://ict.debevec.org/~debevec/Research/HDR/>

[3] Raanan Fattal, Dani Lischinski, and Michael Werman. “Gradient domain high dynamic range compression.” In *Proceedings of the 29th annual conference on Computer graphics and interactive techniques* (SIGGRAPH '02). ACM, New York, NY, USA, 249-256.



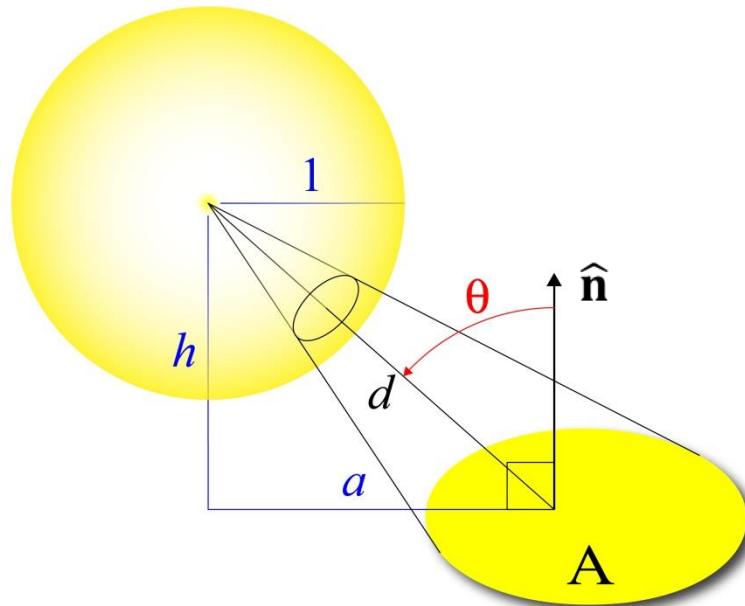
# The Process of Imaging



**Image Acquisition Pipeline.** From scene radiance to pixel values for both film and digital cameras. Usually all quantities are unknown except for the pixel values and the exposure time. Unknown nonlinear mappings include exposure, development, scanning, digitization, and remapping.



# The Process of Imaging — Radiometry



**Solid Angle:** The area traced out on the surface of a unit sphere by a cone with vertex on the center of the sphere.

Units: steradians.

The total solid angle of a single point in space is  $4\pi$  steradians.

If a surface patch of area  $A$  is a distance,  $d$ , away from the source point and the surface normal,  $\hat{\mathbf{n}}$ , makes an angle,  $\theta$ , with the cone axis, and if  $d \gg \sqrt{A}$ , then the solid angle,  $\Omega$ , subtended by the patch is approximately  $\Omega = d^{-2} A \cos \theta$ .



# The Process of Imaging — Radiometry

$L$  – radiance: the amount of light emitted from a surface; the power per unit foreshortened area emitted into a unit solid angle.

Units: (watts/meter<sup>2</sup>)/steradian.

$E$  – irradiance: the amount of light impinging on a surface; the power per unit area. Units: watts/meter<sup>2</sup>. In the case of imaging, the surface being irradiated is the camera's image plane.

The irradiance at a surface due to a point source radiator of constant intensity,  $I_0$ , has power per unit area  $\Omega = d^{-2} I_0 \cos \theta$ . If the patch is at a rectangular distance of  $d = \sqrt{h^2 + a^2}$ , then the surface irradiance is

$$E = I_0 \frac{h}{\sqrt{h^2 + a^2}} \frac{1}{h^2 + a^2} = \frac{I_0}{h^2} \left[ \frac{h}{\sqrt{h^2 + a^2}} \right]^3 = \frac{I_0}{h^2} \cos^3 \theta.$$



# High Dynamic Range Images

Require more than 8-bits per pixel to view.

Cannot be displayed properly on conventional monitors nor printed on conventional printers.

Created through computer graphic rendering,  
specialized imaging devices (*e.g.* medical imagers), or  
through the compositing of multiple ordinary images of the  
same scene each made with different exposure times — a.k.a.  
*Bracketed Images* .



# Example: Bracketed Image Set



exposure:  
1/20s



# Example: Bracketed Image Set





# Example: Bracketed Image Set





# Example: Bracketed Image Set





# Make an HDR Image from Bracketed Images



1. Estimate the camera characteristic for each color band.
2. Remap each band of the BI set through the characteristic.
3. Tone-map the result of step 2 into a viewable image.





# The Camera Characteristic Curve

$$Z_{\mathbf{p},i} = f \left( \Delta t_i \int E_{\mathbf{p}}(\lambda) R_{\mathbf{p}}(\lambda) d\lambda \right)$$

$\lambda$  = wavelength of incident radiation,

$R_{\mathbf{p}}$  = spectral response of sensing element at pixel  $\mathbf{p} = (r,c)$ ,

$\Delta t_i$  = exposure time of image  $i$  from the bracketed image set,

$E_{\mathbf{p}}$  = irradiance at pixel  $\mathbf{p}$ ,<sup>1</sup>

$f(\cdot)$  = nonlinear mapping function (characteristic curve), and

$Z_{\mathbf{p},i}$  = numerical value of pixel  $\mathbf{p}$  in image  $i$ .

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<sup>1</sup>The images are assumed to be taken in succession quickly enough that the scene lighting does not change.



# The Camera Characteristic Curve

This is usually simplified by assuming a uniform spectral response so that the equation becomes

$$Z_{\mathbf{p},i} = f(E_{\mathbf{p}} \Delta t_i)$$

$\Delta t_i$  = exposure time of image  $i$  from the bracketed image set,

$E_{\mathbf{p}}$  = irradiance at pixel  $\mathbf{p} = (r,c)$ ,

$f(\cdot)$  = nonlinear mapping function (characteristic curve), and

$Z_{\mathbf{p},i}$  = numerical value of pixel  $\mathbf{p}$  in image  $i$ .

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Slides 13-21 & 24 were derived from reference [1].



# The Camera Characteristic Curve

We can assume that we know both the pixel values and the exposure times for each image in the bracketed image set. To form an HDR image we must estimate  $f$  globally and  $\mathbf{E}$  at each pixel location  $\mathbf{p} = (r, c)$ . [ $\mathbf{E}$  is the irradiance map –  $E_{\mathbf{p}}$  for all  $\mathbf{p}$ .] We assume that  $f$  is monotonic, therefore invertible, and write

$$f^{-1}(Z_{\mathbf{p},i}) = E_{\mathbf{p}} \Delta t_i. \quad (1)$$

Take the natural logarithm of both sides to transform multiplication into addition.

$$\ln f^{-1}(Z_{\mathbf{p},i}) = \ln E_{\mathbf{p}} + \ln \Delta t_i.$$



# The Camera Characteristic Curve

Lump the logarithm with the inverse mapping to define  $g$ .

$$g(Z_{p,i}) = \ln E_p + \ln \Delta t_i, \quad (2)$$

where

$$g(\cdot) = \ln f^{-1}(\cdot).$$

In the equation,  $Z_{p,i}$  and  $\Delta t_i$  are assumed to be known,  $E_p$  and  $g$  are unknown. But  $g$  is assumed to be smooth and monotonic.

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# Computing the Camera Characteristic Curve

The idea is to recover the  $g$  and  $E_p$  that best satisfy (2) in a least-squared error sense. In a digital image,  $g$  can take on only a finite number of values from  $Z_{\min}$  to  $Z_{\max}$ .

Let  $N = R \times C$  be the number of pixels in each image and let  $M$  be the number of images.

Then we want to find the  $Z_{\max} - Z_{\min} + 1$  values of  $g(Z)$  and the  $N$  values of  $\ln E_p$  that minimize:<sup>2</sup>

$$O = \sum_{p=1}^N \sum_{i=1}^M \left[ g(Z_{p,i}) - \ln E_p + \ln \Delta t_i \right]^2 + \lambda \sum_{z=Z_{\min}+1}^{Z_{\max}-1} \ddot{g}(z)^2. \quad (3)$$

---

<sup>2</sup>The variable,  $p$ , is a linear index through the set of pixels,  $\mathbf{p}$ .



# Computing the Camera Characteristic Curve

The first term of (3) satisfies (2). The second term — the 2<sup>nd</sup> derivative of  $g$  — ensures that  $g$  is smooth. Here we let

$$\ddot{g}(z) = g(z-1) - 2g(z) + g(z+1).$$

$\lambda \in \mathbb{R}$  is a Lagrange multiplier. It weights the smoothness term relative to the data fitting term. It was found in [1] that the additional constraint,

$$g\left(\frac{1}{2}(Z_{\min} + Z_{\max})\right) = 0,$$

causes the midpoint of  $g$  to have unit exposure and ...

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# Computing the Camera Characteristic Curve

...that the following intensity weighting function,

$$w(z) = \begin{cases} z - Z_{\min} & \text{for } z \leq \frac{1}{2}(Z_{\min} + Z_{\max}) \\ Z_{\max} - z & \text{for } z > \frac{1}{2}(Z_{\min} + Z_{\max}) \end{cases}, \quad (4)$$

emphasizes the smoothness of  $g$  near the middle of the curve.  
Then (3) becomes:

$$O = \sum_{p=1}^N \sum_{i=1}^M \left\{ w(Z_{p,i}) \left[ g(Z_{p,i}) - \ln E_p + \ln \Delta t_i \right] \right\}^2 + \lambda \sum_{z=Z_{\min}+1}^{Z_{\max}-1} [w(z) \ddot{g}(z)]^2. \quad (5)$$



# Computing the Camera Characteristic Curve

Notes:

1. Not all pixels need to be used in the minimization. It is sufficient to have  $N$  such that  $N(P-1) > Z_{\max} - Z_{\min}$ .
2. The  $g$  found by solving (5) may not be monotonic non-decreasing (MND). It can be forced to be so with a hold function.
3. We find  $\mathbf{E} = \{E_p \mid p \in \text{supp}(\mathbf{I})\}$  by mapping the pixel values from the bracketed image set through  $g$ , weighting them and summing them.
4. Matlab code is included in [1] and is reproduced on the next pages.



# Computing the Camera Characteristic Curve

```
%  
% gsolve.m - Solve for imaging system response function  
%  
% Given a set of pixel values observed for several pixels in several  
% images with different exposure times, this function returns the  
% imaging system's response function g as well as the log film irradiance  
% values for the observed pixels.  
%  
% Assumes:  
%  
% Zmin = 0  
% Zmax = 255  
%  
% Arguments:  
%  
% Z(i,j) is the pixel values of pixel location number i in image j  
% B(j) is the log delta t, or log shutter speed, for image j  
% l is lamdba, the constant that determines the amount of smoothness  
% w(z) is the weighting function value for pixel value z  
%  
% Returns:  
%  
% g(z) is the log exposure corresponding to pixel value z  
% lE(i) is the log film irradiance at pixel location i
```

Code: Debevec and Malik [1]



# Computing the Camera Characteristic Curve

```
function [g,lE]=gsolve(Z,B,l,w)
n = 256;
A = zeros(size(Z,1)*size(Z,2)+n+1,n+size(Z,1));
b = zeros(size(A,1),1);
%% Include the data-fitting equations
k = 1;
for i=1:size(Z,1)
    for j=1:size(Z,2)
        wij = w(Z(i,j)+1);
        A(k,Z(i,j)+1) = wij; A(k,n+i) = -wij; b(k,1) = wij * B(i,j);
        k=k+1;
    end
end
%% Fix the curve by setting its middle value to 0
A(k,129) = 1;
k=k+1;
%% Include the smoothness equations
for i=1:n-2
    A(k,i)=l*w(i+1); A(k,i+1)=-2*l*w(i+1); A(k,i+2)=l*w(i+1);
    k=k+1;
end
%% Solve the system using SVD
x = A\b;
g = x(1:n);
lE = x(n+1:size(x,1));
```

Code:Debevec and Malik [1]



# Example: Sample the Bracketed Image Set

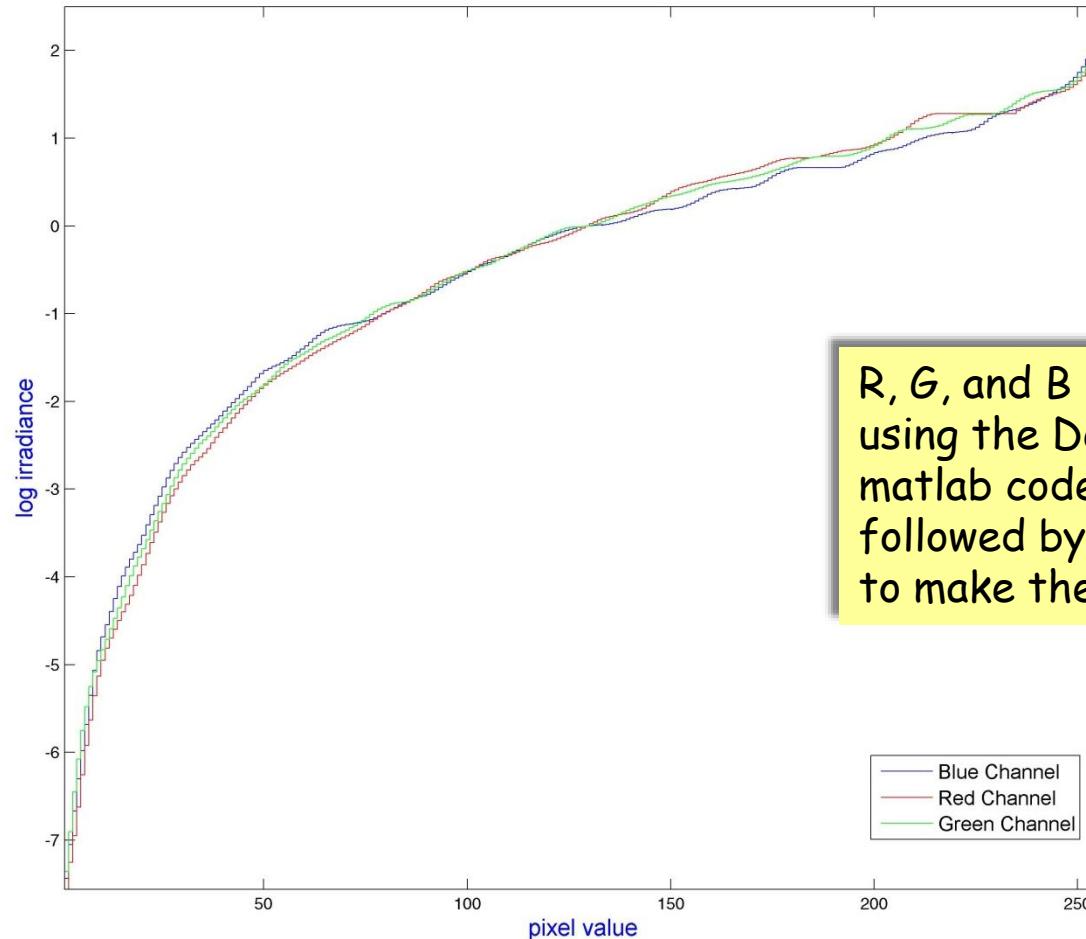
1000 pixel locations are selected at random. Pixel values are taken at those locations from each band of each image in the bracketed set .





# Example: Camera Characteristic Curves

Camera Characteristic Curves





# Constructing the HDR Irradiance Image

From (2)  $E_p$ , the image irradiance at pixel  $p$ , is given by

$$\ln E_p = g(Z_{p,i}) - \ln \Delta t_i. \quad (6)$$

Create the log irradiance map by summing from the bracketed image set using

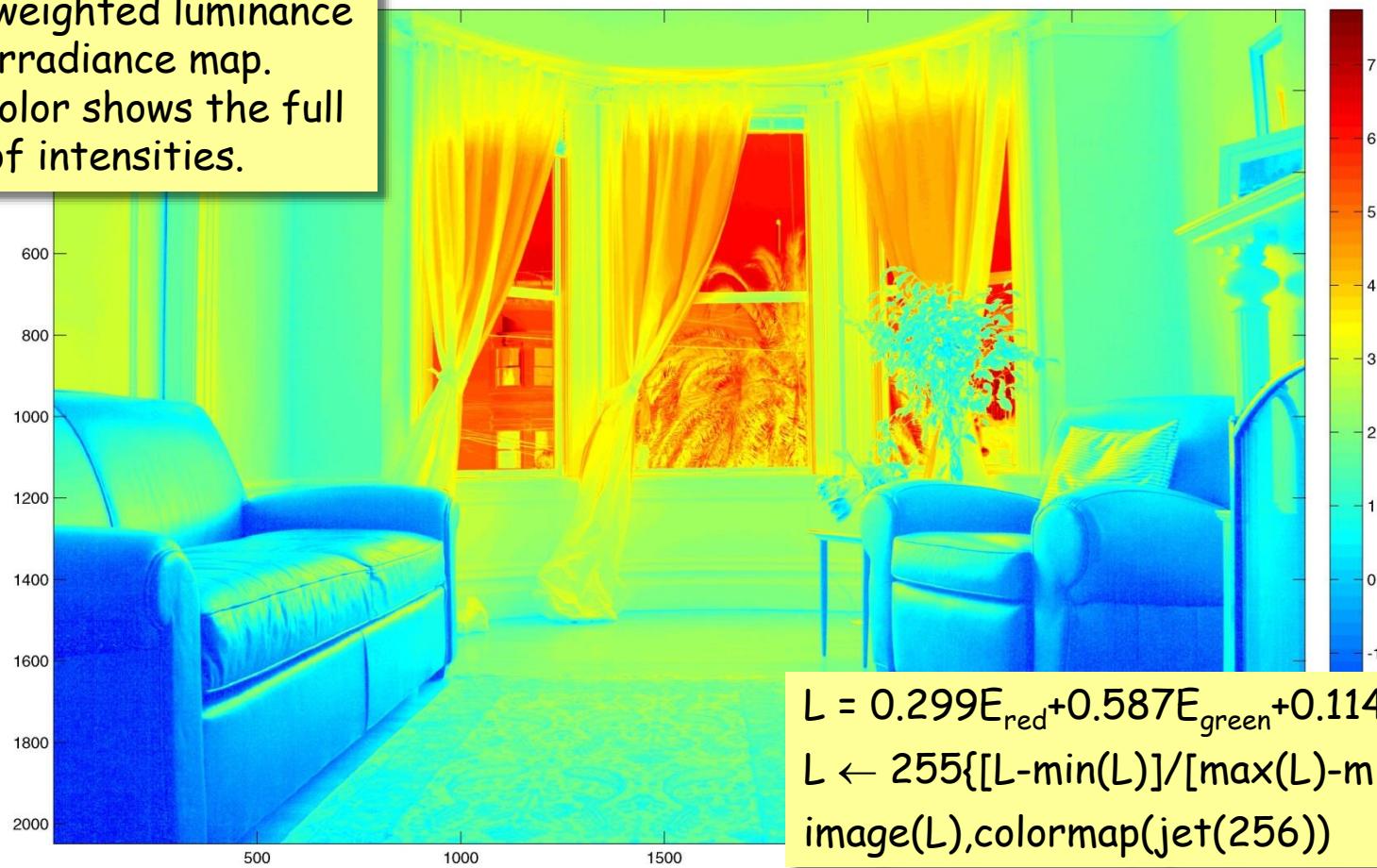
$$\ln E_p = \frac{\sum_{i=1}^P w(Z_{p,i})(g(Z_{p,i}) - \ln \Delta t_i)}{\sum_{i=1}^P w(Z_{p,i})}. \quad (7)$$

for each pixel,  $p$ .



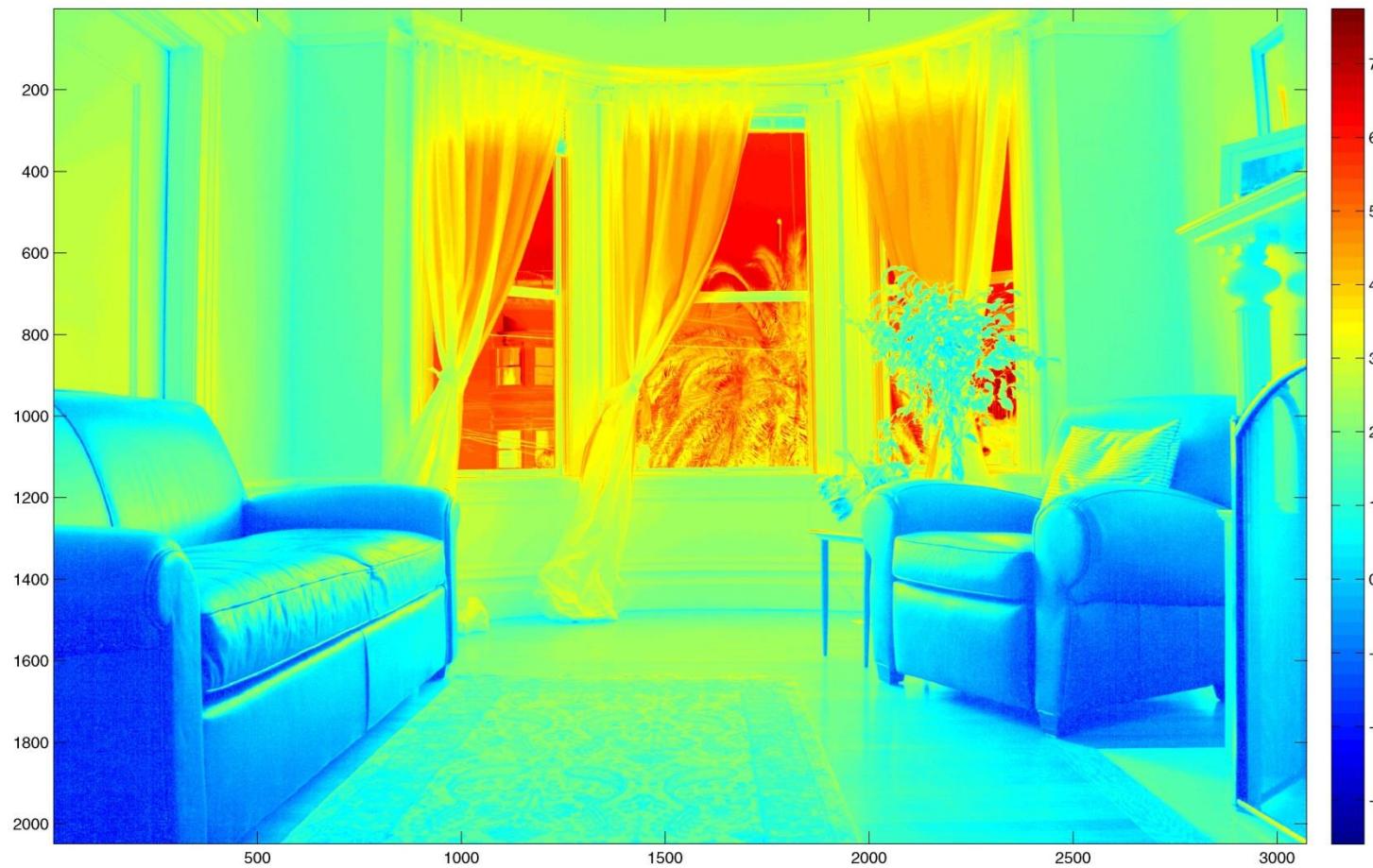
# Example: HDR Log Irradiance Map

NTSC weighted luminance  
of log irradiance map.  
False color shows the full  
range of intensities.





# Example: HDR Log Irradiance Map





# Original Image, 1/20 s Exposure



exposure:  
1/20s



# Tonemapped HDR Combination of BI Set

Constructed  
via (4).





# Tonemapped HDR Combination of BI Set

Constructed  
via Adobe  
Photoshop  
CS5 v12.0.4.





# Procedure: BI Set → Camera Curves

1. Load  $N$  exposure-bracketed images,  $I_1, \dots, I_N$ .
2. Load  $N$  exposure times,  $t(1), \dots, t(N)$ .
3. Extract color bands,  $R_1, \dots, R_N, G_1, \dots, G_N, B_1, \dots, B_N$ .
4. Get  $n \approx 1000$  random cdt pairs  $(r(j), c(j))$  for  $j=1:n$ .
5. Convert coordinates to indices:  $p = \text{sub2ind}(\text{size}(R_1), r, c)$ .
6. Let  $ZR = [R_1(p) \ R_2(p) \ \dots \ R_N(p)]$ ; size:  $n \times N$ . Sim:  $ZG$  and  $ZB$ .
7. Compute weights,  $w = [1:128 \ 256-(128:255)]$ ;
8. Make a row vector of the log exposure times,  
 $lt = [\ln(t(1)) \ \dots \ \ln(t(N))]$ ;
9. Replicate it for use by `gsolve`:  $T = \text{repmat}(lt, [n \ 1])$ ; size:  $n \times N$ .
10. Let smoothness weight  $l=10$ ; (This is  $\lambda$ , the Lagrange mult.)



# Procedure: BI Set → Camera Curves

10. Solve for the r, g, and b camera characteristics:

```
[gR,lER]=gsolve(ZR,T,l,w);  
[gG,lEG]=gsolve(ZG,T,l,w);  
[gB,lEB]=gsolve(ZB,T,l,w);
```

11. Make the curves monotone nondecreasing:

```
mgR=MakeMonotone(gR);  
mgG=MakeMonotone(gG);  
mgB=MakeMonotone(gB);
```

12. Consolidate the curves:  $g=[mgR \ mgG \ mgB]$ ; size:  $256 \times 3$ .

13. Consolidate the images into a cell array:  $I=\{I1, I2, \dots, IN\}$ ;



## Camera Curves → HDR Log Irradiance Image

14. Construct a log irradiance image,  $E$ , using (7) and (4):

```
[R,C,B] = size(I1);
E = zeros(R,C,B);
for d = 1:3      % for each colorband
    gd = g(:,:,d);
    Dn = zeros(R,C); Nm = Dn;
    for j = 1:N    % for each image in the BI set
        Nm = Nm + w(I{j}(:,:,d)+1).*(gd(I{j}(:,:,d)+1)-lt(j));
        Dn = Dn + w(I{j}(:,:,d)+1);
    end
    z = Dn == 0;           % find zeros in the denominator
    Dn(z) = 1; Nm(z) = 0; % make the output 0 there
    E(:,:,:,d) = Nm./Dn; % log irradiance image
end
```



# HDR → 8-Bit Image: Tone Mapping

The dynamic range of an HDR image is much greater than the 256 levels that a standard digital image can represent and much greater than that of a standard monitor. The conversion of an HDR image to 8-bits requires tone mapping.

This can be done with point operators like a contrast mapping, gamma correction, or histogram remapping. But those results are often not very good.

Better are local operators that remap as a function of the contrast of edges .

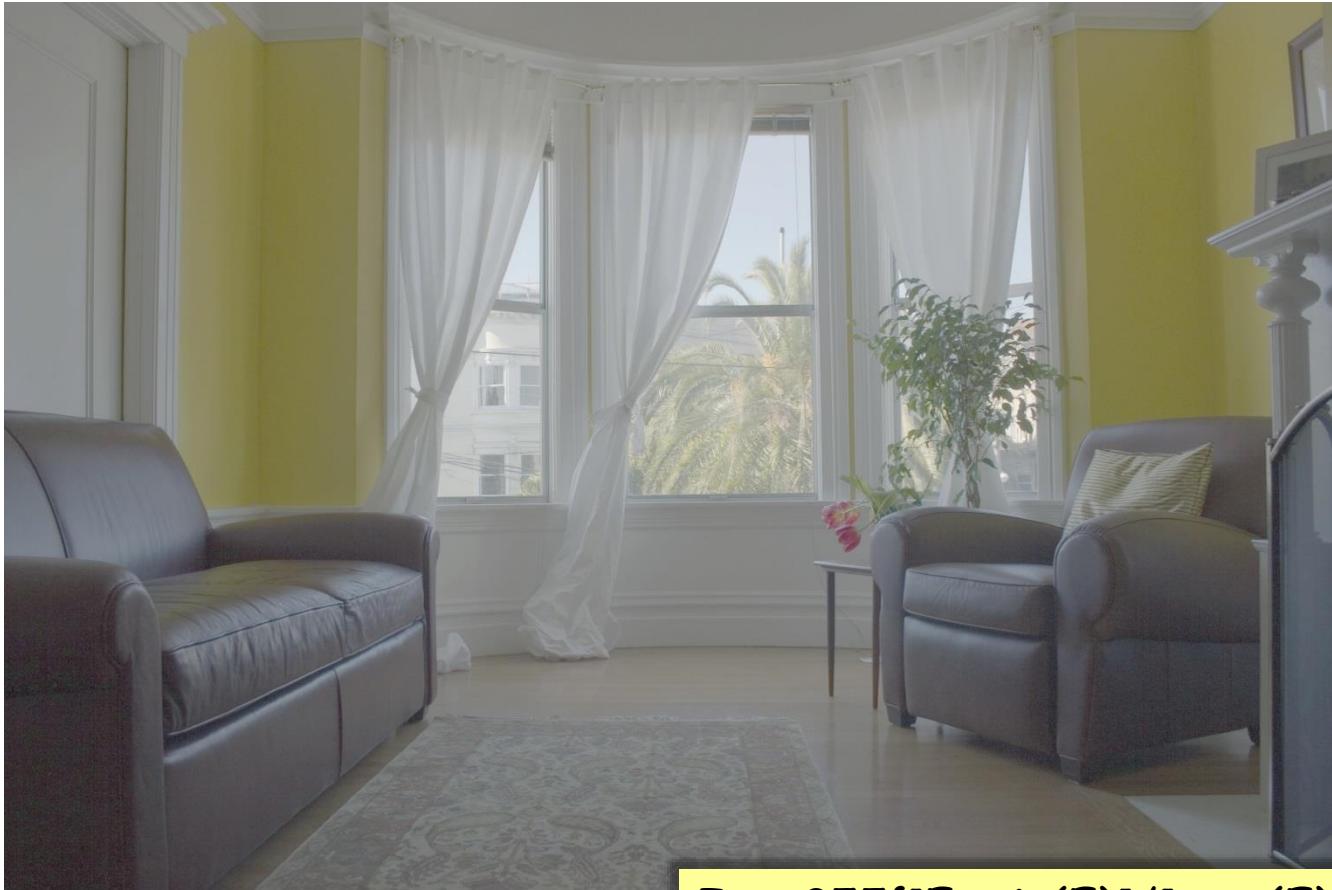


# HDR → 8-Bit Image: Photoshop Algorithm





# HDR → 8-Bit Image: Linear Contrast Map



$$I \leftarrow 255\{[E - \min(E)] / [\max(E) - \min(E)]\}$$



# HDR → 8-Bit Image: Gamma Correction

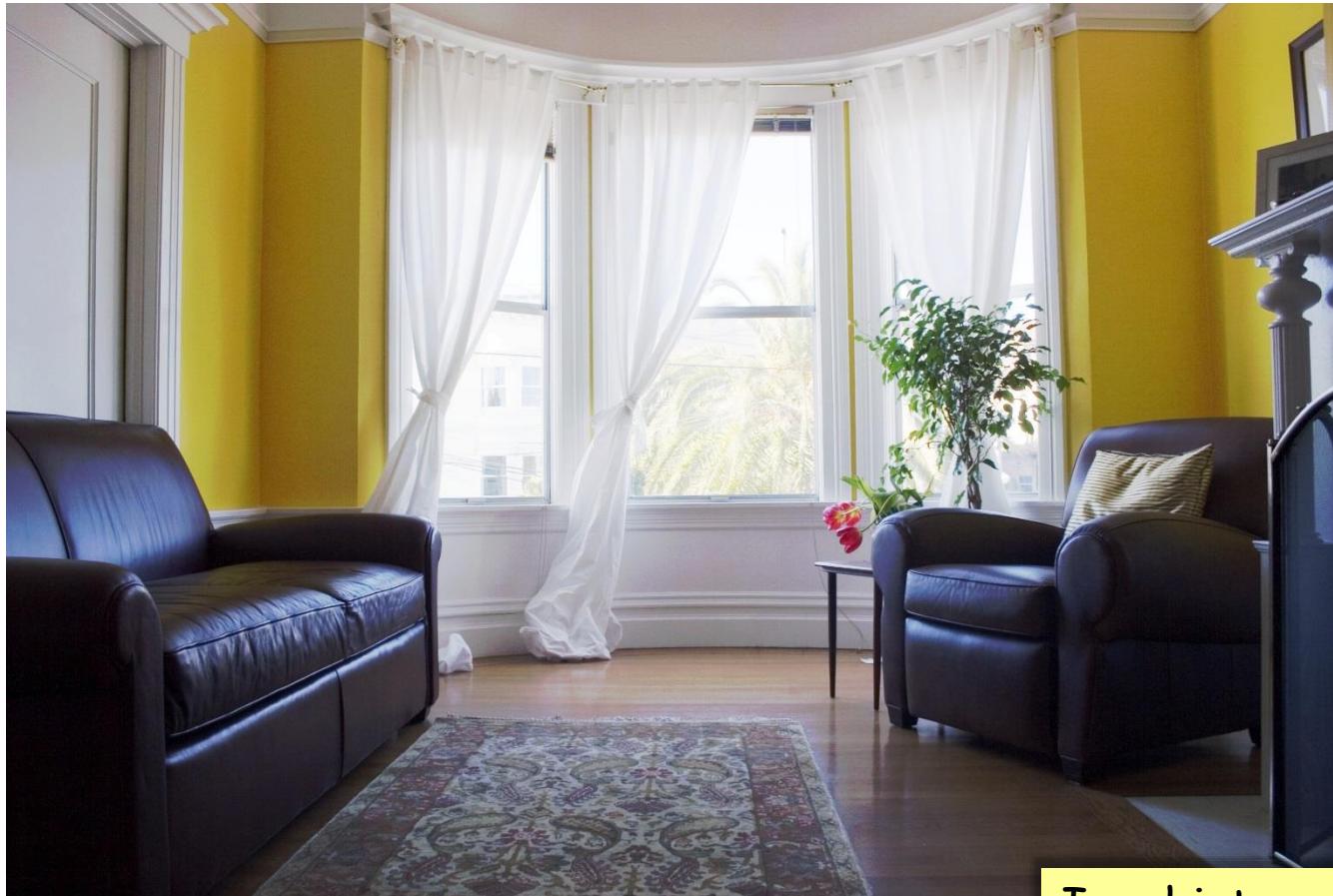


$$\gamma = 1/2$$

$$I \leftarrow 255\{([E-\min(E)]/[\max(E)-\min(E)])^{1/\gamma}\}$$



# HDR → 8-Bit Image: Histogram EQ



$I \leftarrow \text{histogram\_eq}(E)$



# HDR → 8-Bit Image: FLW Algorithm<sup>1,2</sup>



<sup>1</sup>R. Fattal, D. Lischinski, M. Werman, "Gradient domain high dynamic range compression," In *Proceedings of the 29th annual conference on Computer graphics and interactive techniques* (SIGGRAPH '02). ACM, New York, NY, USA, 249-256.

<sup>2</sup>Computed by the freeware open-source program *Luminance HDR 2.4.0*. (Windows x64 version) URL: <http://qtpfsgui.sourceforge.net> retrieved 2015-10-29.