Introduction to Quantum Information Processing

Roei Rosenzweig 313590937,
Roey Maor 205798440

May 16, 2017

1 Harmonic Quantum Oscillator - Solution

1.

$$a + a^\dagger = 2 \cdot X \cdot \sqrt{\frac{m\omega}{2\hbar}} \Rightarrow X = \sqrt{\frac{2\hbar}{m\omega}} \frac{a + a^\dagger}{2}$$

$$a - a^\dagger = 2 \cdot \frac{i}{m\omega} P \cdot \sqrt{\frac{m\omega}{2\hbar}} \Rightarrow P = \sqrt{\frac{2\hbar}{m\omega}} \frac{m\omega}{i} \frac{a - a^\dagger}{2}$$

2. We begin by calculating an expression for the operator $aa^\dagger$, by using $[X, P] = i\hbar$:

$$\begin{aligned} aa^\dagger &= \frac{m\omega}{2\hbar} \left(X^2 + \frac{i}{m\omega} PX - \frac{i}{m\omega} XP + \frac{1}{m^2\omega^2} P^2 \right) \\ &= \frac{m\omega}{2\hbar} \left(X^2 - i\hbar \frac{i}{m\omega} + \frac{1}{m^2\omega^2} P^2 \right) \\ &= \frac{m\omega}{2\hbar} \left(X^2 + \frac{1}{m^2\omega^2} P^2 \right) + \frac{m\omega}{2\hbar} \frac{h}{m\omega} = C + \frac{1}{2} \end{aligned}$$

From symmetry, we get that $a^\dagger a = C - \frac{1}{2} = N$, thus $aa^\dagger = N + 1$. Now we can calculate the hamiltonian:

$$\begin{aligned} \mathcal{H} &= \frac{P^2}{2m} + \frac{1}{2} m\omega^2 X^2 = \frac{1}{2m} \left(\sqrt{\frac{2\hbar}{m\omega}} \frac{m\omega}{i} \frac{a - a^\dagger}{2} \right)^2 + \frac{1}{2} m\omega^2 \left(\sqrt{\frac{2\hbar}{m\omega}} \frac{a + a^\dagger}{2} \right)^2 \\ &= \frac{1}{2m} \frac{2\hbar}{m\omega} \frac{m^2\omega^2}{-1} \frac{a^2 - (N+1) - N + \left(a^\dagger\right)^2}{4} + \frac{1}{2} m\omega^2 \frac{2\hbar}{m\omega} \frac{a^2 + (N+1) + N + \left(a^\dagger\right)^2}{4} \\ &= -\omega\hbar \frac{a^2 - (N+1) - N + \left(a^\dagger\right)^2}{4} + \omega\hbar \frac{a^2 + (N+1) + N + \left(a^\dagger\right)^2}{4} \\ &= \hbar\omega \frac{4N + 2}{4} = \hbar\omega \left(N + 1\right) \end{aligned}$$