Introduction to Quantum Information Processing

Roei Rosenzweig 313590937,
Roey Maor 205798440

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1 Mutual Information - Solution

1. We expand the expression to get:

$$I(X;Y) = H(X) - H(X|Y) = -\sum_{x} p(x)\log_2 p(x) + \sum_{x,y} p(x,y)\log_2 p(x|y)$$

We know that a probability distribution of variable X can be computed from the mutual probability of X and Y by summing all the values in range for Y:

$$p(x) = \sum_{y} p(x,y)$$

Plugging this into the previous equation, we get:

$$I(X;Y) = \sum_{x,y} p(x,y)\log_2 p(x|y) - \sum_{x,y} p(x,y)\log_2 p(x) = \sum_{x,y} p(x,y)\log_2\left(\frac{p(x|y)}{p(x)}\right)$$

From logarithm rules and conditional probability definition:

$$\log_2\left(\frac{p(x|y)}{p(x)}\right) = -\log_2\left(\frac{p(x)p(y)}{p(x,y)}\right)$$

Plugging it, we get:

$$\boxed{I(X;Y) = -\sum_{x,y} p(x,y)\log_2\left(\frac{p(x)p(y)}{p(x,y)}\right)}$$

2.