Interest Group Influence on Policy Proposals and Passage

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Abstract

Interest groups have various tools to influence policymaking. They can (i) shape the content of policy proposals and (ii) affect whether proposals become law. Recent empirical work documents bills with early interest group support pass at higher rates than those without. How does anticipation of a fight for passage affect group decisions to engage in such support? I study a two-stage model in which a legislator either accepts a proposal from a pro-change group or selects their own, costly proposal. Given a proposal, the pro-change and an anti-change interest group engage in an all-pay contest determining whether the bill passes. I show the group proposes in equilibrium when facing weak opposition, but selects out of proposing when facing strong opposition. As a result, equilibrium proposals from the group pass with high probability, while equilibrium proposals proposed by the legislator likely fail. The model provides a selection-based explanation for the empirical regularity, and highlights potential challenges in estimating an all-else-equal effect of group sponsorship on bill passage.

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Policymaking in the United States is a multistage process. To create new legislation, a bill must be proposed, pass through one or multiple legislative committees, survive floor votes in both legislative chambers, and avoid being vetoed by the executive. Interest groups engage in this process at multiple stages. In the early stages, groups aid legislators in developing policy proposals—for example, by helping draft bill language (Drutman, 2015) or even providing off-the-shelf model bills (Hertel-Fernandez, 2019). In later stages, interest groups deploy resources in support or opposition of proposed bills (McKay, 2022), e.g. by lobbying legislators on a floor vote, organizing grassroots campaigns, or appealing for or against an executive veto.[1]

In recent years, scholar have studied how interest group activity in early stages of the policy process, such as group bill sponsorship (Kroeger, 2022; Garlick, Kroeger, and Pellaton, 2025) or expressed group support for bills in Dear Colleague letters (Box-Steffensmeier, Christenson, and Craig, 2019), affects policy success. The core estimand of these studies is the effect of early-stage group support (or sponsorship) on bill passage. They show proposed bills with early-stage interest group support or sponsorship are more likely to pass than bills without, even when accounting for covariates (Kroeger, 2022; Box-Steffensmeier, Christenson, and Craig, 2019). The theoretical explanations for these findings are that interest group-sponsored bills are of higher quality (Kroeger, 2022) or that early group endorsements provide legislators with informative signals about support for the bill (Box-Steffensmeier, Christenson, and Craig, 2019).

Early-stage interest group support for or sponsorship of a proposed bill is not randomly assigned: behavior of both legislators and interest groups in early stages of the policy process is shaped by strategic anticipation of the bill's prospects. For instance, legislators deciding whether to include bill language proposed by an interest group try to anticipate how the inclusion of such language affects the likelihood the bill passes. Similarly, decisions by

[1] As an example, a lobbyist at the ACLU describes her job in the following way: "Our job here in Washington is not just to change policy one lawsuit at a time, but to try to reach thousands and millions of people by affecting the policy of the administration and the legislation of Congress. That means blocking bad bills and supporting bills that expand civil liberties." (Leech, 2013, p. 69).

interest groups on whether and how to get involved in shaping a bill's content are taken in anticipation of whether legislators will be receptive of such content and how opposing interest groups may respond, potentially affecting whether the proposal passes (Baumgartner et al., 2009; Levine, 2009; Lowery, 2013).[2]

Despite a widespread understanding in the interest group literature that such strategic anticipation is a ubiquitous force (Lowery, 2013), the mechanisms through which strategic anticipation shapes patterns of interest group activity across stages of the policy process remain poorly understood. Formal theories of interest group influence typically focus on a single stage or mechanism of interest group influence (see Schnakenberg and Turner, 2024), with limited exceptions (You, 2017; Wolton, 2021; de Figueiredo and de Figueiredo, 2002; Cardona, Freitas, and Rubí-Barceló, 2025).

In this paper, I provide a theoretical framework to gain insight into how anticipation of a fight for bill passage shapes the early-stage interaction between an interest group and a legislator determining the bill's content. I seek to shed light on several questions. When does the interest group—in equilibrium—engage in shaping the bill's content? How do proposed bills with and without interest group influence on the content differ in their equilibrium probability of passage?

To make progress on these questions, I study a formal model of policymaking with (potential) interest group involvement at two stages. In the proposal stage, a single legislator—e.g., a committee chair—has the power to propose a bill. They have three options: (i) propose their own alternative to the status quo, which is costly to the legislator; (ii) accept a proposal chosen by an aligned, but ideologically more extreme, (pro-change) interest group; or (iii) forego proposing, maintaining the status quo. If a proposal is made, the game moves to the policy passage stage. Whether the proposed bill succeeds depends on costly effort by the pro-change group in support of the proposal and an anti-change group's effort in opposition.

[2] See also Kroeger (2022, p. 208): "The benefits associated with introducing a bill may hinge upon the legislation's fate. (...) A legislator's extraction of policy goals from bill introduction are conditional upon passage."

I model the passage stage as an all-pay contest (Hillman and Riley, 1989).[3]

In the passage stage, each group's value of winning—and hence their effort choice—depends on the extremity of the proposed bill. Proposals further from the status quo raise the contest stakes for both groups and affect their relative valuations of winning (due to concave policy preferences). Hence, the passage stage equilibrium effort choices depend on the location of the proposed bill. Crucially, the equilibrium probability of passage decreases in proposal extremity.

In the proposal stage, the pro-change group and the legislator anticipate how proposal extremity affects passage stage behavior, shaping their preferences over proposals. The group's preferences over proposals reflect three considerations: their policy payoff conditional on passage, the probability of passage, and the cost of contest effort in the passage stage. By contrast, the legislator's preferences over proposals reflect only their payoff conditional on passage and the probability of passage. I show that if the pro-change group's cost of contest effort is low, they prefer a more extreme proposal than the legislator, while if the pro-change group's cost of contest effort is high, they prefer a more moderate proposal than legislator. Moreover, if the pro-change group's cost of contest effort is very high, they may prefer there to be no proposal at all, as the gain from any proposal would be competed away in the contest for passage.

These preferences over proposals, together with the legislator's cost of proposing, determine behavior in the policy proposal stage. The legislator accepts any group proposal inside an acceptance set around their optimal proposal, with the bounds pinned down by their cost of proposing. The pro-change group either proposes their favored proposal in the legislator's acceptance set—which may be more or less extreme than the legislator's optimal proposal—or does not propose, when their cost of contest effort is sufficiently high such that for any proposal inside the legislator's acceptance set, their expected policy gain is competed away

[3]Substantively, these efforts may capture a range of activities affecting policy passage, such as lobbying legislators directly on key floor votes, creating public pressure campaigns, mobilizing other interest groups, or appealing for or against an executive veto.

in the contest for passage. When the group does not propose, the legislator makes propose whenever their cost of proposing is not too high, as the legislator does not pay the cost of contest effort.

I show this implies an association between the equilibrium identity of the bill proposer (pro-change group or legislator) and the probability of bill passage. If the group proposes, the bill passes with high probability, whereas if the legislator proposes, the bill fails with high probability. This result is consistent with the empirical pattern that group-sponsored bills are more likely to pass. The core mechanisms are selection—as the group selects into proposing only if their cost of contest effort (relative to the cost of the anti-change group) is not too high—and strategic proposal moderation. This contrasts with explanations in the literature for the higher passage rates of group proposals, such as group creating higher-quality proposals (Kroeger, 2022) or group support serving as an informative signal to legislators (Box-Steffensmeier, Christenson, and Craig, 2019).

More generally, this model highlights the estimand of empirical studies—the average treatment effect of group sponsorship on bill passage—may be undefined when groups and legislators anticipate later stages in the policymaking process, because the dependent variable (bill passage) and the key explanatory variable (group sponsorship) are part of a dynamic strategic interaction (see also Wolton, 2021; Bueno De Mesquita and Tyson, 2020; Slough, 2023).

Related Literature

This paper builds on and contributes to a rich theoretical literature on interest group influence in policymaking, which focuses on three broad theoretical approaches: exchange theories, informational theories, and subsidy-based theories.[4] While each of these mechanisms have been studied extensively in isolation, less is known about how different forms of interest group influence across multiple stages of the policymaking process may interact

[4] Schnakenberg and Turner (2024) provide a useful overview of this literature.

to shape policy outcomes (Schnakenberg and Turner, 2024). I contribute to this literature by studying a model of interest group influence in which interest groups can affect both legislative proposals and proposal passage.

In my model, an interest group can shape legislative proposals in anticipation of future policymaking. Several other models also have this feature. For instance, Judd (2023) studies how interest group access—the opportunity for the group to make a binding offer to a proposing politician—affects policy outcomes, when policy is determined by a legislative interaction with multiple politicians. Levy and Razin (2013) analyze a repeated model in which a continuum of groups compete for the right to propose before a decisionmaker decides between the proposal and the status quo. Neither of these models allows for interest group activity across stages of the policy process.

I model interest group competition over whether a proposal passes using an all-pay auction (Hillman and Riley, 1989; Baye, Kovenock, and de Vries, 1996; Che and Gale, 1998). All-pay contests are a common approach to modeling lobbying competition for rents (Tullock, 1980; Siegel, 2009, 2014). One strand of the literature focuses on two-dimensional policy contests (Hirsch and Shotts, 2015; Hirsch, 2025; Hirsch and Shotts, 2025). In this framework, two contest participants simultaneously offer a proposal consisting of a spatial location and commonly valued policy quality to a policymaker, who selects their preferred option. By contrast, I study a sequential model in which a single proposal is selected prior to a contest over passage.[5]

Another strand of the literature focuses on one-dimensional policymaking contests (Epstein and Nitzan, 2004; Münster, 2006; Cardona, Freitas, and Rubí-Barceló, 2025). The key difference among these papers is how the alternatives over which the contest for policy passage is fought are selected. In Epstein and Nitzan (2004) and Münster (2006), two contest participants ex-ante each choose a spatial policy to be implemented if they win the contest.

[5] Interest group scholars show fighting to preserve the status quo against a specific proposal without offering a counterproposal is a common group strategy—e.g., see McKay (2012) and Baumgartner et al. (2009).

They show groups moderate their proposals relative to their ideal point to reduce the intensity of conflict—a force also present in my model. Similar to my paper, the alternatives in the policy passage in Cardona, Freitas, and Rubí-Barceló (2025) are an exogenous status quo and a single policy proposal determined by a prior strategic interaction.[6] In Cardona, Freitas, and Rubí-Barceló (2025), this proposal is determined by a tug-of-war between two competing interest groups (see Duggan and Gao, 2020), whereas in my paper it is determined by an interaction between a single (pro-change) interest group and a legislator.

Several papers also allow for different types of lobbying across two stages of the policy process, but study different mechanisms. You (2017) considers a model in which groups lobby policymakers for collective rents, before lobbying over the distribution of those rents, studying how groups allocate spending between ex-ante and ex-post lobbying. In de Figueiredo and de Figueiredo (2002), two competing groups offer a spatial policy and a transfer to a decisionmaker, where the losing interest group can choose to initiate a (litigation) contest. Anticipation of litigation has complex effects on the proposal stage: the possibility of policy failure pushes the proposing group to more extreme policies, while the cost of litigation efforts incentivizes conservation of resources by proposing a more moderate proposal and a more modest transfer to the decisionmaker. Lastly, Wolton (2021) also studies two-stage interest group influence where groups can affect both proposal content and passage. In his signaling model, groups can engage in costly spending to signal their resolve in the competition for policy passage, showing that in equilibrium, spending in the proposal stage (inside lobbying) is associated with policy compromise, while spending in the passage stage (outside lobbying) is associated with comprehensive reforms. Similar to Wolton (2021), my model seeks to understand how potentially observable interest group behavior (whether interest groups affect the proposal) in equilibrium is associated with observable outcomes (probability of proposal success). The key difference is the mechanism for interest group influence in the proposal stage: in my model, this is a subsidy-based mechanism where the group's power

[6] Bellani, Fabella, and Scervini (2023) also consider a spatial contest model where the policy outcome is either a proposal and an exogenous status quo, but with a single fixed proposer.

to affect proposals is due to their ability to offset the legislator's cost of developing a bill, whereas Wolton (2021) assumes an informational mechanism where groups burn money to inform legislators about their resolve in the policy passage stage.

Model

Players. The players are a legislator ℓ and two interest groups, denoted -1 and 1.

Policy Preferences. The policy space is $X = [-1, 1]$. Each player $i \in \{-1, \ell, 1\}$ has policy preferences represented by quadratic loss utility: given policy $y \in X$, player i's policy payoff is $u_i(y) = -(y - i)^2$.

I assume groups' ideal points are on opposite sides of legislator ideal point ℓ and status quo y_0: $-1 < y_0 < \ell < 1$. I refer to group 1 as the *pro-change* group, as they seek to shift policy in the same direction as the legislator, and group -1 as the *anti-change* group.

Timing. The game has two stages: (i) a proposal stage and (ii) a policy passage stage.

Proposal stage: First, the pro-change group offers a policy proposal, $y_1 \in X \setminus \{y_0\}$, or stays out, $y_1 = y_0$. Second, legislator ℓ either accepts the pro-change group's proposal, $y_\ell = y_1$; writes their own proposal $y_\ell \in X \setminus \{y_0, y_1\}$; or makes no proposal, $y_\ell = y_0$. If $y_\ell = y_0$, the game ends with the status quo in place, $y = y_0$. If $y_\ell \neq y_0$, the game moves to the passage stage.

Passage stage: The pro-change and anti-change group simultaneously choose efforts $e_1 \geq 0$ and $e_{-1} \geq 0$, respectively in favor and in opposition to proposal y_ℓ. If $e_1 \geq e_{-1}$, the proposal passes ($y = y_\ell$), while if $e_1 < e_{-1}$, the status quo remains in place ($y = y_0$).

Costs. The legislator incurs a cost of proposing $c \geq 0$ whenever they write their own proposal, i.e., when $y_\ell \in X \setminus \{y_0, y_1\}$. Each group $i \in \{-1, 1\}$ incurs a cost of effort in the passage stage, $\gamma_i \cdot e_i$. The (inverse of) cost parameter $\gamma_i > 0$ captures the relative strength of group i.

Equilibrium Concept and Restriction. The equilibrium concept is subgame perfect Nash equilibrium. Importantly, I restrict attention to strategy profiles in which the pro-change group does not propose when indifferent between making a proposal or not.[7] This restriction rules out (i) equilibria in which the pro-change group makes a proposal rejected by ℓ and (ii) equilibria in which the pro-change group's proposal is accepted by ℓ, but their expected policy gain of proposing equals their expected cost of contest effort.

Model Commentary

Three features of the policy passage stage merit discussion. First, the model abstracts from the precise mechanism(s) through which interest groups may affect whether policy proposals become law.[8] Interest groups have myriad ways to do so: e.g., lobbying legislators, staging grassroots protests, or buying ads to mobilize voters (see e.g. Mahoney, 2008). The all-pay contest is a tractable way to capture two core assumptions underlying these group activities: (i) all else equal, increasing group effort in favor (in opposition) of a proposal positively (negatively) affects the probability of policy success, and (ii) such efforts are costly to groups. The all-pay contest also yields clean predictions about groups' effort choices given any proposal, allowing analysis of how passage stage primitives affect equilibrium behavior in the proposal stage.[9]

Second, passage is a deterministic function of group efforts: the proposal passes if and only if the pro-change group exerts greater effort than the anti-change group. The results are robust to introducing *small noise* into the contest—i.e, a probabilistic contest where

[7]Equivalently, one can assume the pro-change group incurs an infinitesimal proposal cost $\epsilon > 0$ when offering a proposal $y_1 \in X \setminus \{y_0\}$.

[8]Wolton (2021) takes a similar, but distinct approach. In my model, pro- and anti-change groups simultaneously choose an effort level, whereas in Wolton (2021) two groups sequentially make a binary choice whether to engage in outside lobbying.

[9]For instance, if spending on negative lobbying (against proposals) is more effective than spending on lobbying for policy change (McKay, 2012), the model can capture this through a higher effort cost parameter of the pro-change group γ_1 relative to the anti-change group's parameter γ_{-1}.

groups' odds of winning are sufficiently dependent on their efforts (Ewerhart, 2017).[10] In extensions to the model, I relax the assumption that passage depends only on the two groups, by incorporating (exogenous) status quo bias or adding a veto player.

Third, I preclude the possibility of amendments to the proposal in the passage stage. While some scholars argue amendments and compromise throughout the policymaking process are common (Levine, 2009; Rosenthal, 2001), others find that—especially in the US context—proposals are generally either outright defeated or passed without substantial amending (Mahoney, 2008). Separating proposal content and passage into distinct stages allows me to tightly study how anticipation of opposition affects proposal behavior.

In the proposal stage, a single legislator has the power to propose. This legislator can represent a (sub-)committee chair, who typically holds substantial proposal power on a particular policy domain. In practice, groups may strategically choose which legislator to target as sponsor for their proposed legislation. While a model with a richer strategic interaction in the proposal stage may yield further insight into group and legislator behavior, I focus on the case of a single legislative proposer to cleanly disentangle core forces. I return to the question of group preferences over legislators at the end of the analysis.

Additionally, I assume the pro-change group has the opportunity to offer a proposal, while the anti-change group does not. As the legislator would never accept a proposal shifting policy from the status quo towards the anti-change group's ideal point, the only type of proposal by the anti-change group that would impact outcomes is preemptive, i.e., a proposal that if successful would shift policy away from their ideal point, but less so than the pro-change group's proposed policy. However, this would require the anti-change group to fight their own proposal in the passage stage, which may render such proposals not credible. Therefore, I restrict the ability to propose policies to the pro-change (aligned)

[10] In any equilibrium of a two-player all-pay contest with *small noise*, win probabilities and groups' expected utilities equal those in the unique equilibrium of the deterministic all-pay contest (Ewerhart, 2017). An example of a probabilistic contest with small noise is a Tullock contest in which player i wins with probability $e_i^R/(e_i^R + e_j^R)$, with decisiveness parameter $R > 2$. See Ewerhart (2017) for more details.

interest group.[11].

Analysis

The main analysis proceeds as follows. First, I analyze the policy passage stage, highlighting how the location of proposal y_ℓ shapes the probability it passes as well as the groups' effort costs. Second, I study two benchmark cases: one in which only the legislator proposes and one in which the pro-change group proposes. Third, I characterize equilibrium outcomes when proposals can come from either legislator or pro-change group. All proofs are contained in the Appendix.

Policy Passage Stage

First, I analyze how the policy passage stage unfolds given a proposal y_ℓ. The passage stage is modeled as a standard two-player all-pay contest (Hillman and Riley, 1989; Baye, Kovenock, and de Vries, 1996). Given group j's strategy, group i's incentives to exert effort depend (negatively) on their own unit cost of effort, γ_i, and (positively) on their stakes in the contest. These stakes equal the difference between i's policy payoff when they win compared to when they lose the contest: $s_i(y_\ell) = |u_i(y_\ell) - u_i(y_0)|$. For both groups, the stakes are increasing in the distance between status quo y_0 and proposal y_ℓ.

Define group i's effective valuation in the contest given a proposal y_ℓ as $v_i(y_\ell) = \frac{s_i(y_\ell)}{\gamma_i}$. The groups' effective valuations determine equilibrium efforts in the passage stage, the equilibrium probability the proposal is implemented and groups' net payoffs in the contest. Lemma 1 restates features of the unique mixed strategy Nash equilibrium of the two-player all-pay contest (see e.g. Vojnovic, 2015).

Lemma 1. *Given effective valuations $v_i \geq v_j > 0$, in the unique mixed strategy Nash equilibrium of the contest,*

[11] For a related model in which both groups can influence policy proposals, see Cardona, Freitas, and Rubí-Barceló (2025).

(i) the expected efforts satisfy $\mathbb{E}[e_i] = \frac{v_j}{2}$, $\mathbb{E}[e_j] = \frac{v_j}{v_i}\frac{v_j}{2}$;

(ii) the expected probability group i *wins the contest is* $1 - \frac{v_j}{2v_i}$;

(iii) the expected net contest payoffs are $v_i - v_j$ *for* i *and* 0 *for* j.

Whether the pro-change group or the anti-change group has a higher effective valuation in the contest depends on the extremity of proposal y_ℓ (relative to the status quo y_0) and the groups' effort cost parameters, γ_1 and γ_{-1}.

Lemma 2. *Let* $\tilde{y} = \frac{\gamma_{-1}-\gamma_1}{\gamma_1+\gamma_{-1}}$. *If* $y_\ell \in (y_0, \min\{2\tilde{y} - y_0, 1\}]$, *the pro-change group's effective valuation is greater than the anti-change group's effective valuation,* $v_1(y_\ell) \geq v_{-1}(y_\ell)$. *If* $y_\ell \in (\max\{y_0, 2\tilde{y} - y_0\}, 1]$, *then* $v_1(y_\ell) < v_{-1}(y_\ell)$.

Lemma 2 follows from strict concavity of group policy preferences. Combining Lemma 1 and 2, the equilibrium probability a proposal $y_\ell > y_0$ is implemented is

$$\rho(y_\ell) = \begin{cases} \frac{v_1(y_\ell)}{2v_{-1}(y_\ell)} & \text{if } y_\ell \in (\max\{y_0, 2\tilde{y} - y_0\}, 1], \\ 1 - \frac{v_{-1}(y_\ell)}{2v_1(y_\ell)} & \text{if } y_\ell \in (y_0, \min\{2\tilde{y} - y_0, 1\}]. \end{cases} \tag{1}$$

Two important features of passage probability $\rho(y_\ell)$ are: (i) $\rho(y_\ell)$ is continuous in the proposal location for all $y_\ell \in (y_0, 1]$; and (ii) $\rho(y_\ell)$ is strictly decreasing in the location of the proposal for all $y_\ell \in (y_0, 1]$, due to concavity of the groups' policy payoffs.

No Proposal Cost Benchmark

First, I consider the case when the legislator does not face a cost of proposing ($c = 0$). In this case, proposals always originate with the legislator, as they have no incentive to accept proposals from the pro-change group.

When $c = 0$, the legislator maximizes the following objective function:

$$V_\ell(y_\ell) = \rho(y_\ell) \cdot [u_\ell(y_\ell) - u_\ell(y_0)] + u_\ell(y_0).$$

The proposal y_ℓ affects both the probability of passage $\rho(y_\ell)$ and the legislator's payoff conditional on passage $u_\ell(y_\ell)$. A proposal further from status quo y_0 and closer to the legislator's ideal point ℓ increases their payoff conditional on the policy being passed, but decreases the probability of passage.

Lemma 3. *There exists a unique optimal proposal for ℓ, $y_\ell^* \in (y_0, \ell)$.*

The optimal proposal for the legislator balances the two aforementioned incentives. Absent the anti-change group, the legislator would simply propose their ideal policy ℓ. However, anticipating effort to block passage by the anti-change group, the legislator moderates their proposal compared to their ideal point: $y_\ell^* < \ell$. Note the legislator always attempts to change policy: $y_\ell^* > y_0$.

Corollary 1. *The legislator's optimal proposal y_ℓ^* is weakly decreasing in γ_1 and weakly increasing in γ_{-1}.*

Features of the institutional environment affect the legislator's optimal proposal. When the pro-change group's cost of contest effort γ_1 increases, the effective valuation of the pro-change group decreases for any proposal, lowering the probability a proposal $y_\ell > y_0$ passes. Moreover, the marginal effect of an increase in the proposal's location y_ℓ on the probability of passage becomes weakly more negative, and as a result the legislator's optimal proposal becomes weakly more moderate. The reverse logic applies when considering an increase in the anti-change group's cost of contest effort γ_{-1}.

Corollary 2. *The legislator's equilibrium expected policy utility $V_\ell(y_\ell^*)$ is strictly decreasing in γ_1 and strictly increasing in γ_{-1}.*

An increase in the pro-change group's contest costs γ_1 or a decrease in the anti-change group's contest costs γ_{-1} leave the legislator strictly worse off. Although the legislator modifies their proposal in response to changes in contest costs as outlined in Corollary 1, the overall effect on the legislator's equilibrium expected policy utility is strictly negative.

High Proposal Cost Benchmark

Second, I study the case when the legislator's cost of proposing is large, $c > V_\ell(y_\ell^*) - u_\ell(y_0)$, so that the legislator prefers the status quo to proposing themselves. Thus, any proposal must originate with the pro-change group—the legislator only acts as a veto player. If the group makes a proposal, it must pass the legislator's veto constraint, and balance three considerations: (i) the group's policy payoff conditional on policy passage, (ii) the probability the policy proposal passes, and (iii) the group's expected cost of contest effort.

Lemma 4. *Suppose $c > V_\ell(y_\ell^*) - u_\ell(y_0)$. The pro-change group's optimal proposal is*

$$y_1^* = \begin{cases} y_0 & \text{if } \tilde{y} \leq y_0, \\ \tilde{y} & \text{if } \tilde{y} \in (y_0, \min\{2\ell - y_0, 1\}), \\ 2\ell - y_0 & \text{if } \tilde{y} \in [2\ell - y_0, 1). \end{cases}$$

Moreover, y_1^* is weakly decreasing in γ_1 and weakly increasing in γ_{-1}.

When the pro-change group's effort costs are high relative to the anti-change group's effort costs, such that $\tilde{y} \leq y_0$, the pro-change group is the lower-valuation player in the contest for any proposal $y \in (y_0, 1]$. As a result, the group can never extract positive (expected) rents by proposing, and thus refrains from making a proposal ($y_1^* = y_0$).[12] Otherwise, the pro-change group would like to propose $\tilde{y}$, the policy maximizing their contest payoff. If the legislator prefers $\tilde{y}$ to the status quo, this proposal is accepted. If the legislator prefers the status quo to $\tilde{y}$, the group's (constrained) optimal proposal makes the legislator indifferent with the status quo.

Comparing Optimal Proposals

Why and how do the legislator's optimal proposal in the no proposal cost benchmark and the pro-change group's optimal proposal in the high proposal cost benchmark differ? There are

[12] This follows from the assumption the group never proposes when indifferent.

two key differences between the legislator and the pro-change group. First, the pro-change group is more ideologically extreme. This force pushes the pro-change group towards more extreme proposals than the legislator. Second, the pro-change group internalizes the cost of contest effort, whereas the legislator does not. This force pushes the pro-change group to seek more moderate policies than the legislator. Whether the group's optimal proposal is more or less extreme than the legislator's optimal proposal depends on the (relative) contest effort costs.

Proposition 1. *Fix γ_{-1}. There exist a cutoff $\hat{\gamma}_1$ such that for $\gamma_1 < \hat{\gamma}_1$, the pro-change group's optimal proposal is more extreme than ℓ's optimal proposal ($y_1^* > y_\ell^*$), and for $\gamma_1 > \hat{\gamma}_1$, the group's optimal proposal is less extreme than ℓ's optimal proposal ($y_1^* < y_\ell^*$).*

To illustrate the core results, I provide a numerical example. In the example, I set the status quo policy $y_0 = -\frac{1}{2}$, the legislator's ideal point $\ell = 0$, and the opposing group's contest effort cost $\gamma_{-1} = 1$. Figure 1 plots the legislator's optimal proposal y_ℓ^* and the group's optimal proposal y_1^* as a function of the pro-change group's cost parameter γ_1.

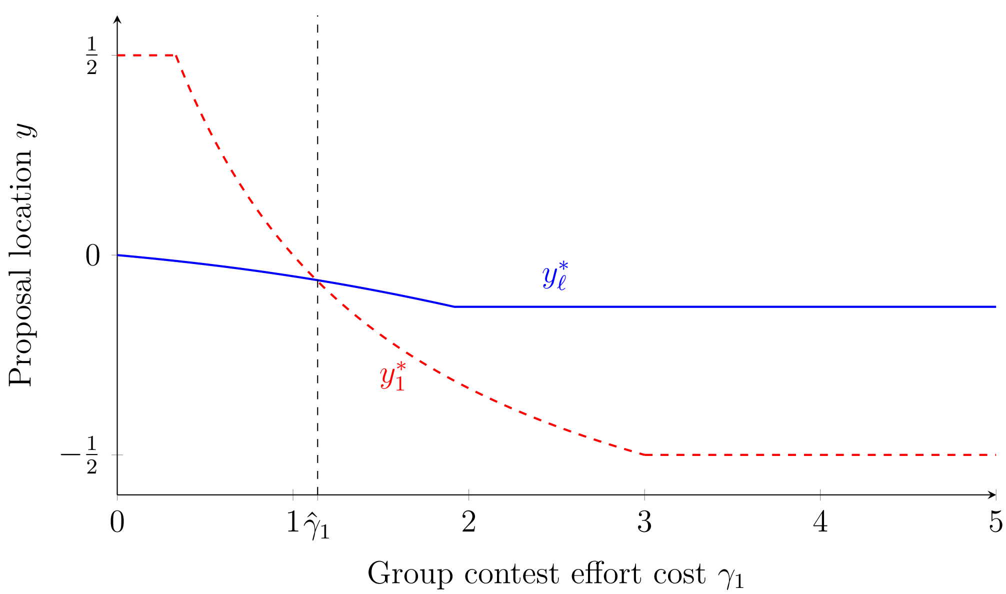

Figure 1: Optimal legislator proposal y_ℓ^* and group proposal y_1^* as a function of pro-change group's contest effort cost γ_1 for selected parameter values ($y_0 = -\frac{1}{2}$, $\ell = 0$, $\gamma_{-1} = 1$).

Figure 1 illuminates several features of the model. First, when the pro-change group's cost of contest effort is negligible—i.e., when $\gamma_1 \approx 0$—the model approximates the standard Romer-Rosenthal setter model. The legislator's optimal proposal approaches their ideal point, $y_\ell^* \approx 0$, and the group's optimal proposal approaches the inflection of the status quo about the legislator's ideal point, $y_1^* \approx 2\ell - y_0 = \frac{1}{2}$.

Increasing the pro-change group effort cost (weakly) moderates both the legislator's and the pro-change group's optimal proposal, as the effective valuation of the pro-change group in the contest decreases. Note that if $\gamma_1 < \hat{\gamma}_1 \approx 1.14$, the optimal proposal for the pro-change group is further from the status quo y_0 than the optimal proposal for the legislator, while the opposite is true for $\gamma_1 > \hat{\gamma}_1 \approx 1.14$. The key reason is that the group is directly affected by the increasing cost of contest effort, and as a result wants to lower the intensity of the contest through proposal moderation. When $\gamma_1 \geq 3$, for every proposal $y_\ell > y_0$, the pro-change group is the lower-valuation player in the contest, leaving them unable to extract positive rents from proposing. Therefore, their optimal proposal is to simply leave the status quo in place, $y_1^* = -\frac{1}{2}$, and avoid the contest altogether. Since the legislator does not internalize the costs of the contest, and any proposal will pass with positive probability, they still prefer to make a proposal $y_\ell^* > -\frac{1}{2}$.

Intermediate Proposal Cost

Now, I turn to intermediate proposal cost case: $c \in (0, V_\ell(y_\ell^*) - u_\ell(y_0))$. In this case, proposals may come from either the legislator or the pro-change group. When does the equilibrium proposal originate with the legislator, and when does it originate with the pro-change group? And are there any systematic differences between (equilibrium) proposals from the legislator and (equilibrium) proposals from the pro-change group?

The legislator accepts the group's proposal whenever their expected payoff of accepting is greater than proposing their own optimal proposal y_ℓ^* and paying the cost of proposing, c. In Appendix A, I show the legislator's expected payoff $V_\ell(y_\ell)$ is continuous in y_ℓ. Moreover, $V_\ell(y_\ell)$ is strictly increasing for all $y_\ell \in (y_0, y_\ell^*)$ and strictly decreasing for all $y_\ell \in (y_\ell^*, \min\{2\ell-$

$y_0, 1\}$), which guarantees the legislator's acceptance set is an interval.

Lemma 5. *Suppose* $0 < c < V_\ell(y_\ell^*) - u_\ell(y_0)$. *Legislator* ℓ*'s acceptance set* $A(c) = [\underline{a}(c), \overline{a}(c)]$ *is an interval, where* $\underline{a}(c) \in (y_0, y_\ell^*)$ *and* $\overline{a}(c) \in (y_\ell^*, 2\ell - y_0)$.

In equilibrium, the pro-change group either proposes a policy inside the legislator's acceptance set $A(c)$, or the legislator proposes their optimal proposal y_ℓ^* given by Lemma 3. Figure 2 shows how the bounds of the acceptance set depend on (relative) effort costs in the numerical example outlined above.

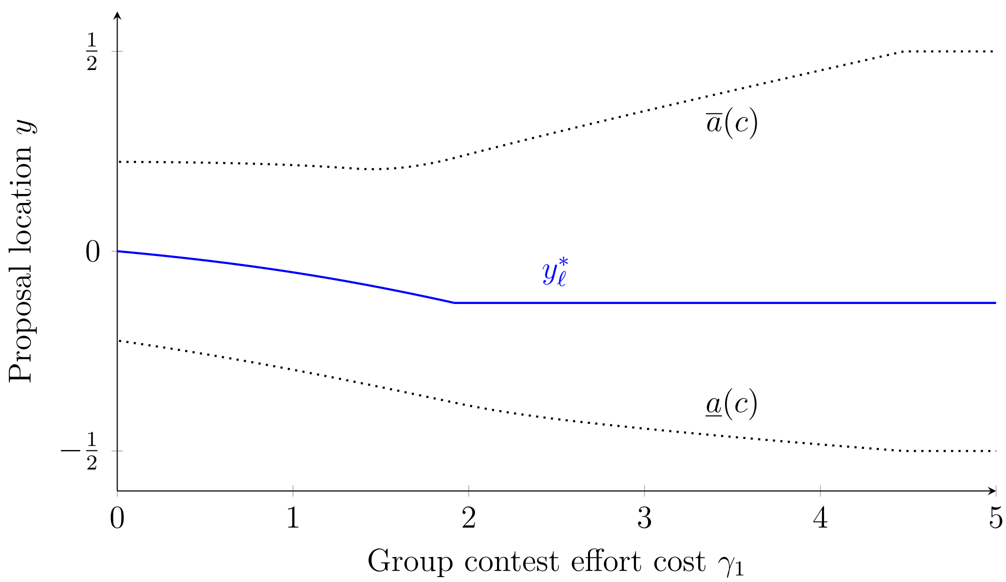

Figure 2: Legislator ℓ's acceptance set $[\underline{a}(c), \overline{a}(c)]$ as a function of the pro-change group's contest effort cost γ_1 for selected parameter values ($y_0 = -\frac{1}{2}$, $\ell = 0$, $\gamma_{-1} = 1$ and $c = 0.05$).

Proposition 2. *Suppose* $0 < c < V_\ell(y_\ell^*) - u_\ell(y_0)$.

(i) If $y_1^* = y_\ell^*$*, the equilibrium proposal is* y_ℓ^**, proposed by* ℓ.

(ii) If $y_1^* \in A(c) \setminus \{y_\ell^*\}$*, the equilibrium proposal is* y_1^**, proposed by the pro-change group.*

(iii) If $y_1^* > \overline{a}(c)$*, the equilibrium proposal is* $\overline{a}(c)$*, proposed by the pro-change group.*

(iv) If $y_1^* \in (y_0, \underline{a}(c))$ *and* $\underline{a}(c) < 2\tilde{y} - y_0$*, the equilibrium proposal is* $\underline{a}(c)$*, proposed by the pro-change group.*

(v) Otherwise, the equilibrium proposal is y_ℓ^* and is proposed by ℓ.

To illustrate Proposition 2, I again return to the numerical example. Figure 3 plots the equilibrium proposal location and proposer identity as a function of the pro-change group's contest effort cost γ_1.

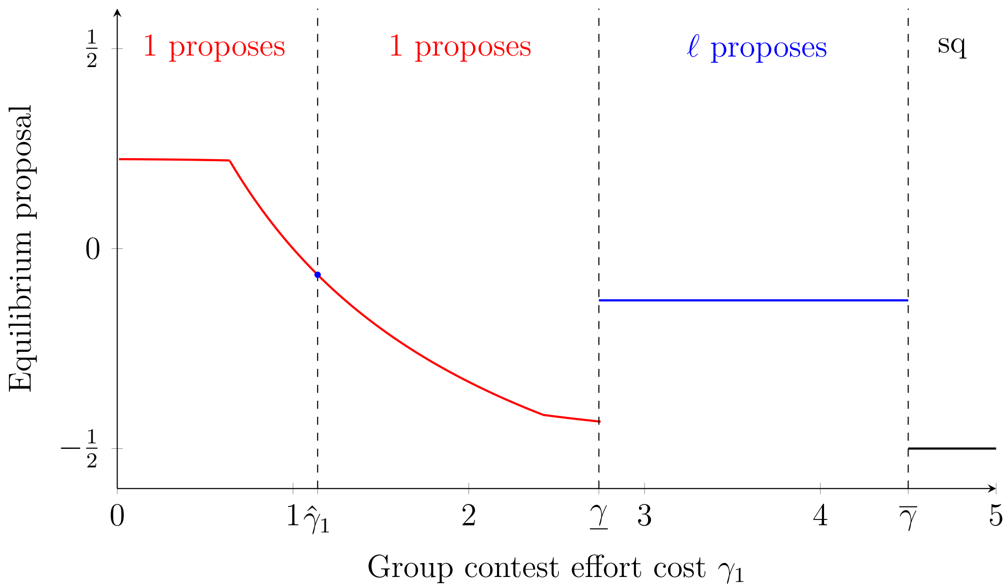

Figure 3: Equilibrium proposal and proposer identity as a function of the pro-change group's contest effort cost γ_1 for selected parameter values ($y_0 = -\frac{1}{2}$, $\ell = 0$, $\gamma_{-1} = 1$, and $c = 0.05$).

When the pro-change group's contest effort cost γ_1 is low, the upper bound of ℓ's acceptance set binds, and the group proposes $\overline{a}(c)$. When the effort cost is intermediate, such that $y_1^* \in A(c)$, the group proposes their optimal proposal y_1^*—except when optimal proposals exactly coincide (if $\gamma_1 = \hat{\gamma}_1 \approx 1.14$), when the group leaves proposing to the legislator. When γ_1 is large, such that $y_1^* < \underline{a}(c)$, the group proposes only if their expected payoff of proposing $\underline{a}(c)$ is strictly positive (i.e., they are the higher-valuation player in the contest given proposal $\underline{a}(c)$). In the example, this is the case when $\gamma_1 < \underline{\gamma}$. If $\gamma_1 \in [\underline{\gamma}, \overline{\gamma}]$, group 1 is the lower-valuation player for any proposal in $A(c)$, and hence they cannot gain from proposing. The legislator can benefit from proposing their own optimal proposal y_ℓ^*, since they do not pay for contest effort. Lastly, if $\gamma_1 > \overline{g}$, then $c > V_\ell(y_\ell^*) - u_\ell(y_0)$, and therefore no proposal is made and the status quo y_0 remains in place.

Proposer Identity and Policy Passage

A key implication from Proposition 2 is that equilibrium proposals authored by an interest group pass with high probability, while equilibrium proposals authored by the legislator typically pass with low probability.

Corollary 3. *Suppose $c > 0$.*

(i) If the group proposes in equilibrium, the proposal passes with probability $\rho(y_1^*) > \frac{1}{2}$.

(ii) If legislator ℓ proposes in equilibrium, then either (i) $y_\ell = y_1^* = y_\ell^*$ and it passes with probability $\rho(y_\ell^*) > \frac{1}{2}$, or (ii) it passes with probability $\rho(y_\ell^*) \leq \frac{1}{2}$.

The intuition for Corollary 3 is the following. The group only proposes if their payoff of the contest is strictly positive, which requires the group to win the contest with probability $\rho(y_1^*) > \frac{1}{2}$. The group elects not to propose in two cases. First, if the group and legislator optimal proposals exactly coincide, the group cannot improve on the legislator's proposal, and leaves it to the legislator to propose. Second, if the group is the low-valuation group in the contest for any proposal in the legislator's acceptance set, they cannot gain from proposing and hence refrain from doing so. The legislator, however, still benefits from proposing as long as the cost of proposing c is sufficiently low, since they do not internalize the cost of contest effort. Such legislator proposals pass with probability $\rho(y_\ell^*) < \frac{1}{2}$. Figure 4 illustrates this logic in the numerical example.

Preferences over Proposing Legislators

In the baseline model, the proposing legislator has a known, fixed ideal point. One question is how the contest for passage shapes the groups' preferences over legislators. Resources used in the contest for passage are ex-post wasteful from the groups' perspectives. Corollary 4 highlights this can result in agreement between the groups to select a more moderate legislator, in order to reduce the intensity of the contest.

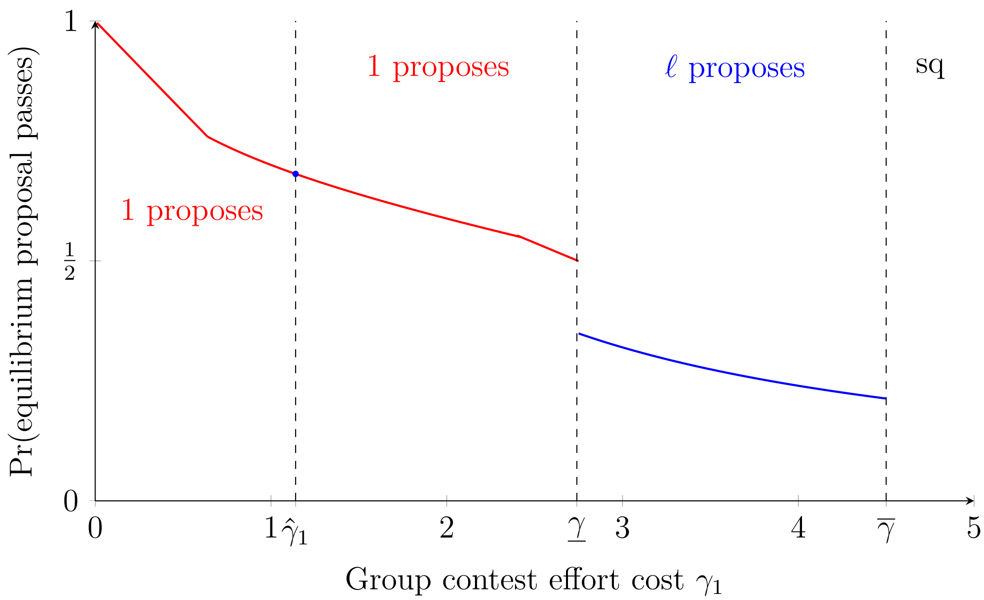

Figure 4: Probability equilibrium proposal passes and equilibrium proposer identity as a function of the pro-change group's contest effort cost γ_1 for selected parameter values ($y_0 = -\frac{1}{2}$, $\ell = 0$, $\gamma_{-1} = 1$, and $c = 0.05$).

Corollary 4. *If $y_0 < y_1^* < \underline{a}(c)$, there exists a more moderate legislator ideal point $\ell' \in (y_0, \ell)$ such that both the pro-change and the anti-change group would prefer ℓ' to ℓ.*

When the pro-change group's optimal proposal y_1^* lies between the status quo and the lower bound of ℓ's acceptance set, the pro-change group wants to propose y_1^* but cannot due to the legislator's acceptance constraint. A more moderate legislator would accept their optimal proposal y_1^*. The anti-change group would also prefer this proposal, which reduces contest intensity and yields a better outcome if they lose the contest. Thus, anticipation of later stages may align opposed interest groups' incentives in earlier stages of the policy process, such as selection of a proposing legislator.

Model Extensions

A key assumption in the main model is that only the relative efforts by the pro-change group in favor of and the anti-change group in opposition to the proposal affect proposal passage. In two extensions, I allow for sources of obstruction beyond the anti-change group. First, I consider an extension in which there is a possibility of exogenous obstruction (status quo bias). Second, I study how adding a veto player, whose obstruction decision is endogenous to the proposal, affects outcomes.

Exogenous Status Quo Bias

First, I extend the model to study how status quo bias affects outcomes. In the modified model, if the pro-change group wins the contest, the proposal passes with probability $\beta \in (0, 1)$ while the status quo remains with probability $1 - \beta$.[13]

Given any proposal y_ℓ, both groups' contest stakes are scaled by β compared to the baseline model, as the status quo persist with probability $1 - \beta$ irrespective of group efforts. As a result, both groups' contest efforts are (proportionally) lower relative to efforts in the baseline model, and the unconditional probability the proposal passes is also scaled by β. Therefore, the group's and the legislator's optimal proposals are unchanged.

Proposition 3. *With exogenous status quo bias, the legislator's optimal proposal y_ℓ^* and the pro-change group's optimal proposal y_1^*, are the same as in the baseline.*

While preferences over proposals are unchanged, exogenous status quo bias negatively affects the legislator's expected gain of proposing, since proposals are less likely to pass. As a result, the legislator is less willing to propose, expanding their acceptance set. As a result, the pro-change group may propose, which can result in more extreme proposals (if $y_1^* > \overline{a}(c)$) or less extreme proposals (if $y_1^* < \underline{a}(c)$).

Corollary 5. *Suppose $c > 0$. An increase in status quo bias (i) may switch the identity of the equilibrium proposer from legislator ℓ to pro-change group 1 and (ii) may increase or decrease the extremity of the equilibrium proposal.*

The linkage between proposer identity and policy passage from the baseline model persists: equilibrium proposals from the group typically pass at high rates relative to equilibrium

[13]This is equivalent to assuming the proposal fails to reach the contest stage with probability $1 - \beta$ and reaches the contest stage with probability β.

proposals originating with the legislator.

Corollary 6. *Suppose $c > 0$ and $\beta \in (0, 1)$.*

(i) If the pro-change group proposes in equilibrium, the proposal passes with probability $\rho^{\beta}(y_1^*) \in (\frac{\beta}{2}, \beta)$.

(ii) If legislator ℓ proposes in equilibrium, then either (i) $y_\ell^* = y_1^*$ and it succeeds with probability $\rho^{\beta}(y_\ell^*) \in (\frac{\beta}{2}, \beta)$, or (ii) it succeeds with probability $\rho^{\beta}(y_\ell^*) \leq \frac{\beta}{2}$.

Veto Player

In this extension, after legislator ℓ selects a proposal, a veto player with ideal point $z \in [-1, 1]$ and policy preferences $u_z(y) = -(y - z)^2$ chooses to either block the proposal, in which case y_0 remains in place, or allow it to move to the contest stage. As any proposal $y_\ell \in (y_0, 1]$ passes with positive probability if it reaches the passage stage, the veto player allows a proposal y_ℓ to move forward only if $|y_\ell - z| \leq |y_0 - z|$, i.e. when the proposal is closer to their ideal point than the status quo.

Proposition 4. *Let y^* denote the equilibrium proposal from the baseline model.*

(1) If $z \leq y_0$, the veto player's presence results in gridlock (no proposal).

(2) If $z \geq \frac{y^*+y_0}{2}$, the veto player's presence does not affect the proposal or outcomes.

(3) If $z \in (y_0, \frac{y^*+y_0}{2})$, the veto player's presence results in either (i) full gridlock or (ii) a proposal strictly closer to the status quo, increasing the probability of passage.

If the veto player is misaligned with legislator ℓ ($z \leq y_0$), the veto player's presence results in gridlock, as there exists no proposal preferred to the status quo by both legislator ℓ and veto player z. If the veto player is aligned with legislator ℓ and sufficiently extreme so that the equilibrium proposal from the baseline model is in the veto player's acceptance set ($z \geq \frac{y^*+y_0}{2}$), the veto player does not affect equilibrium outcomes.

The interesting case is when the veto player is aligned with the legislator, but moderate relative to the legislator, so that the optimal proposal from the baseline model would be rejected by the veto player ($y_0 < z < \frac{y^*+y_0}{2}$). In this case, the veto player poses a binding constraint. There are two possible cases: if the group's optimal proposal is $y_1^* = y_0$ and the policy gain for legislator ℓ of proposing the upper bound of the veto player's acceptance set, $2z - y_0$, is below the cost of proposing c, adding the veto player results in gridlock. Otherwise, either the group or the legislator proposes in equilibrium with the veto player, but the proposal is more moderate than in the baseline. Since the probability of passage is decreasing in the distance between the proposal and the status quo, such a proposal succeeds with higher probability in the passage stage.

Corollary 7. *The veto player's presence may (i) switch the proposer from* 1 *to* ℓ*; (ii) switch the proposer from* ℓ *to* 1*, or (iii) maintain the same proposer as in the baseline model.*

The veto player's presence can affect the identity of the proposer in two ways. First, the veto player's presence may fully align (constrained) preferences of the legislator and the group. If, for example, $y_1^* > y_\ell^* > 2z - y_0 > y_0$ and ℓ's cost of proposing c is not too high, then the constrained optimal proposal (with veto player z) for both 1 and ℓ is the upper bound of z's acceptance set, $2z - y_0$. As a result, the pro-change group leaves proposing to the legislator, whereas absent the veto, they propose themselves. Second, the veto player may affect whether the legislator can benefit from proposing. If $z \in (y_0, \frac{y_\ell^*+y_0}{2})$ and $V_\ell(2z - y_0) - u_\ell(y_0) < c < V_\ell(y_\ell^*) - u_\ell(y_0)$, then for any proposal in z's acceptance set, the legislator's cost of proposing is higher than their expected policy benefit. If, in addition, $y_1^* \in (y_0, \underline{a}(c))$ and $\underline{a}(c) \geq 2\tilde{y} - y_0$, the pro-change group does propose in equilibrium with the veto player. In this case, the proposer identity switches from the legislator to the pro-change group.

The veto player extension yields two key insights. First, whenever adding the veto player results in proposal moderation, the probability of the proposal passes increases, as proposal extremity and probability of passage are negatively related. Second, the effect of a veto player on the relationship between proposer identity and the probability of passage is challenging to disentangle: the veto player can both affect the value of proposing for the legislator and therefore whether the legislator wants to propose at all, as well as preference alignment between legislator and group. The net effect of these changes on the relationship between proposer identity and the probability of passage will depend on the location of the veto player and the distribution over model primitives.

Empirical Implications

Interest group scholars are often interested in estimating the causal effect of a particular type of interest group activity on bill success. For instance, Box-Steffensmeier, Christenson, and Craig (2019) show bills receiving more early-stage endorsements from interest groups in *Dear Colleague* letters are more likely to pass in Congress, even when controlling for bill and sponsor characteristics. Kroeger (2022) finds a positive effect of group sponsorship on bill passage using both selection on observables and matching designs.

The empirical strategies in this literature focus (primarily) on the bill as the unit of analysis and early stage group involvement as the treatment, aiming to estimate the effect of such involvement on bill passage. As an example, let T_i denotes bill i's treatment status, where $T_i = 1$ means bill i is group-sponsored and $T_i = 0$ means bill i is not, and define the potential outcomes of bill i—whether bill i would pass or fail depending on treatment status—as $Y_i(1)$ when group-sponsored and $Y_i(0)$ when not group-sponsored. The average treatment effect after controlling for observable bill characteristics X_i is then defined as $ATE = E[Y_i(1) - Y_i(0)|X_i]$.

The analysis of the model above highlights challenges for such a research designs when groups and legislators strategically anticipate the effects of proposals on the probability of passage: proposer identity, bill location, and (probability of) passage are, in equilibrium, jointly determined. Depending on goal of the researchers, this may causes estimation problems or issues in defining counterfactuals (Slough, 2023).

To illustrate, suppose legislative data is generated through repeated draws from a distribution over model primitives $(y_0, \ell, c, \gamma_1, \gamma_{-1})$ [14], i.e., the theoretical model above is the true data-generating process. Denote a single draw over these primitives as k (a policymaking instance). Proposition 2 implies a policymaking instance k has a unique equilibrium, in which there is either (i) no bill proposed; (ii) a bill proposed by the legislator, or (iii) a bill proposed by the interest group. Consider two policymaking instances, $k = 1, 2$, and assume that in $k = 1$, the researcher observes a bill i_1 proposed by the legislator (case (ii)), and in $k = 2$, they observe a bill i_2 proposed by the interest group (case (iii)).

I first consider a research design, assuming a bill i_k has a fixed location. Then treatment statuses of bills i_1, i_2 are classified as $T_{i_1} = 0$ and $T_{i_2} = 1$, and the observed outcomes are defined as $Y_{i_1}(0)$ and $Y_{i_2}(1)$. Defined at the the bill level, counterfactual outcomes $Y_{i_1}(1)$ and $Y_{i_2}(0)$ hold fixed all bill characteristics, including location, other than proposer identity. Then, the true ATE in the theoretical model is 0; proposer identity (by assumption) does not have a direct effect on bill passage if group and legislator propose identical bills. Equilibrium uniqueness in any policymaking instance k, however, means estimating this ATE would be impossible if perfectly controlling for covariates, as treated and untreated bills would have no common support. The result that group-sponsored bills do better would follow from any design that imperfectly controls for covariates, as observed treated bills pass are expected to pass at higher rates then observed untreated bills.

Alternatively, researchers may be interested in estimating the effect of group sponsorship of a particular bill holding fixed the characteristics of the policymaking instance but not the bill's location. Then treatment statuses of bills i_1, i_2 are still classified as $T_{i_1} = 0$ and $T_{i_2} = 1$, and the observed outcomes are defined as $Y_{i_1}(0)$ and $Y_{i_2}(1)$. How should counterfactual outcomes $Y_{i_1}(1)$ and $Y_{i_2}(0)$ be defined in this case? Consider $Y_{i_1}(1)$: the counterfactual outcome had the group sponsored the bill in policymaking instance $k = 1$.

[14] As a reminder, these model primitives are the status quo $y_0 \in (-1, 1)$, the legislator ideal point $\ell \in (-1, 1)$, the cost of proposing $c \in \mathbb{R}_+$, and the contest costs $\gamma_1 \in \mathbb{R}_+$ and $\gamma_{-1} \in \mathbb{R}_+$.

Based on the theoretical model, this quantity is in terms of Slough and Tyson (2023), a *phantom counterfactual*: if the group had been the proposer in policymaking instance $k = 1$, they may have elected not to propose a bill at all, and hence the passage probability would have been undefined.

A key implication for empirical work is that if legislators and interest groups strategically anticipate how proposals affect outcomes, the ATE of early group actions such as sponsorship on bill passage may not be identified—a common occurrence in research designs with sequential behavioral outcomes (Slough, 2023; Bueno De Mesquita and Tyson, 2020).

Conclusion

In this paper, I consider a model of policymaking in which interest groups can both shape the content and affect the passage of policy proposals. The analysis provides new insight into how anticipation of opposition in future stages of the policy process shapes proposal behavior, including who authors the proposal (interest group or legislator) and the extent of policy change sought in the proposal. Holding fixed the proposal, stronger opposition reduces the equilibrium probability the proposal succeeds and imposes higher effort costs on the pro-change group. These two forces affect the (relative) preferences over proposals for the pro-change group and the proposing legislator, and consequently whether proposing is valuable to the group.

The analysis shows that if the equilibrium proposal is written by the pro-change group, the proposal passes high probability, while if the proposal comes from the legislator, it typically fails with high probability. The mechanism driving this result is selection out of proposing by interest groups facing strong opposition. Since the pro-change group internalizes the cost of defending policy proposals, strong opposition makes it impossible for the group to extract gains through proposing. The legislator, on the other hand, can still benefit from proposing, as they are assumed not to incur costs for defending the proposal in later stages of the policy process. As such, the model provides a novel explanation for a descriptive finding that proposals backed by interest groups are more likely to become law (Kroeger, 2022).

More broadly, this paper contributes to the understanding of two-stage interest group influence on policymaking in a competitive environment, connecting to a growing and diverse literature that studies how groups on different sides of an issue may affect each others' behavior and outcomes (Baumgartner et al., 2009; Kang, 2016; Wolton, 2021; Egerod and Junk, 2022). In particular, the model shows anticipation of obstruction by an opposed group, which is recognized as an important but not well-understood factor in shaping group behavior (Lowery, 2013; Finger, 2019), affects what proposals are made and whether groups take an active role in policy design. The focus on anticipation as the key force and the identity of the proposer as a key observable outcome differentiates this paper from Wolton (2021), who focuses on an informational channel and only derives empirical predictions on the correlation between group spending and strength.

The main model abstracts from some of the institutional specifics of the policy process, as the contest for passage depends only on effort by two interest groups. I seek to relax this assumption in two extensions, allowing for status quo bias or an additional veto player. These extensions highlight that changing features of the policy passage stage can have counterintuitive implications: for instance, adding an additional veto point can increase the equilibrium probability of policy change. A more institutional approach to modeling the policy passage stage is left for future work.

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A Proofs Main Analysis

Lemma 1. *Given effective valuations $v_i \geq v_j > 0$, in the unique mixed strategy Nash equilibrium of the contest,*

(i) the expected efforts satisfy $\mathbb{E}[e_i] = \frac{v_j}{2}$, $\mathbb{E}[e_j] = \frac{v_j}{v_i}\frac{v_j}{2}$;

(ii) the expected probability group i wins the contest is $1 - \frac{v_j}{2v_i}$;

(iii) the expected net contest payoffs are $v_i - v_j$ for i and 0 for j.

Proof. Existence and uniqueness of a mixed strategy Nash equilibrium in the two-player all-pay contest follows from Hillman and Riley (1989). For a summary of equilibrium features, see e.g. Vojnovic (2015, p. 44). □

Lemma 2. *Let $\tilde{y} = \frac{\gamma_{-1}-\gamma_1}{\gamma_1+\gamma_{-1}}$. If $y_\ell \in (y_0, \min\{2\tilde{y} - y_0, 1\}]$, the pro-change group's effective valuation is greater than the anti-change group's effective valuation, $v_1(y_\ell) \geq v_{-1}(y_\ell)$. If $y_\ell \in (\max\{y_0, 2\tilde{y} - y_0\}, 1]$, then $v_1(y_\ell) < v_{-1}(y_\ell)$.*

Proof. Let $\Delta(y) \equiv v_1(y) - v_{-1}(y)$. Since $v_1(y)$ and $v_{-1}(y)$ are continuous over the interval $[y_0, 1]$, so is $\Delta(y)$. Furthermore, for $y \in (y_0, 1]$,

$$\frac{\partial \Delta(y)}{\partial y} \geq 0 \iff \frac{2}{\gamma_1}(1-y) - \frac{2}{\gamma_{-1}}(1+y) \geq 0 \qquad \iff y \leq \tilde{y}.$$

To obtain the result, there are two cases to consider. Case (i): $y_0 \in [\tilde{y}, 1]$. Since $\Delta(y_0) = 0$, and $\Delta(y)$ is continuous on $[y_0, 1]$ and strictly decreasing over $(y_0, 1]$, we have $\Delta(y) < 0$ for all $y \in (y_0, 1]$. Case (ii): $y_0 \in [-1, \tilde{y})$. Then $\Delta(y)$ is strictly increasing over $(y_0, \tilde{y})$, and strictly decreasing over $(\tilde{y}, 1]$. Either there is an interior solution on $(\tilde{y}, 1)$ to $\Delta(y) = 0$ or $\Delta(y) \geq 0$ for all $y \in [y_0, 1]$. Suppose an interior solution y^* exists. Solving using the quadratic formula yields $y^* = 2\tilde{y} - y_0$.

Hence, if $y \in (y_0, \min\{2\tilde{y} - y_0, 1\}]$, we have $\Delta(y) \geq 0$, and if $y \in (\max\{y_0, 2\tilde{y} - y_0\}, 1]$, we have $\Delta(y) < 0$. □

Claim 1. *The probability of implementation $\rho(y)$ is strictly decreasing and continuous in y for all $y \in (y_0, 1]$.*

Proof. By Equation (1), the probability of implementation for $y \in (y_0, 1]$ is given by

$$\rho(y) = \begin{cases} \frac{v_1(y)}{2v_{-1}(y)} & \text{if } y \in (\max\{y_0, 2\tilde{y} - y_0\}, 1] \\ 1 - \frac{v_{-1}(y)}{2v_1(y)} & \text{if } y \in (y_0, \min\{2\tilde{y} - y_0, 1\}]. \end{cases}$$

First, I show continuity. If $y_0 \in [\tilde{y}, 1]$ or $y_0 \in [-1, 2\tilde{y}-1)$, then $\rho(y)$ is clearly continuous. For $y_0 \in [\min\{-1, 2\tilde{y} - 1\}, \tilde{y})$, continuity at $y = 2\tilde{y} - y_0$ follows from $v_1(2\tilde{y} - y_0) = v_{-1}(2\tilde{y} - y_0)$.

Second, I show $\rho(y)$ is differentiable for all $y \in (y_0, 1]$. Clearly, differentiability of $v_1(y)$ and $v_{-1}(y)$ for all $y \in (y_0, 1]$ implies $\rho(y)$ is differentiable for all $y \in (y_0, 1)$ with the possible exception of $y = 2\tilde{y} - y_0$. Note that the following holds:

$$\begin{aligned} \lim_{y \to 2\tilde{y} - y_0^+} \frac{\partial \rho(y)}{\partial y} &= \lim_{y \to 2\tilde{y} - y_0^+} \frac{v_1'(y)v_{-1}(y) - v_1(y)v_{-1}'(y)}{2v_{-1}(y)^2} \\ &= \frac{v_1'(2\tilde{y} - y_0)v_{-1}(2\tilde{y} - y_0) - v_1(2\tilde{y} - y_0)v_{-1}'(2\tilde{y} - y_0)}{2v_{-1}(2\tilde{y} - y_0)^2} \\ &= \frac{v_1'(2\tilde{y} - y_0)v_{-1}(2\tilde{y} - y_0) - v_1(2\tilde{y} - y_0)v_{-1}'(2\tilde{y} - y_0)}{2v_1(2\tilde{y} - y_0)^2} \\ &= \lim_{y \to 2\tilde{y} - y_0^-} \frac{\partial \rho(y)}{\partial y} \end{aligned}$$

where the third line follows from $v_1(2\tilde{y} - y_0) = v_{-1}(2\tilde{y} - y_0)$.

Third, I show that $\frac{\partial \rho(y)}{\partial y} < 0$ for all $y \in (y_0, 1]$. Taking derivative yields:

$$
\begin{aligned}
\frac{\partial \rho(y)}{\partial y} &= \begin{cases} \frac{v_1'(y)v_{-1}(y)-v_1(y)v_{-1}'(y)}{2v_{-1}(y)^2} & \text{if } y \in (\max\{y_0, 2\tilde{y}-y_0\}, 1] \\ \frac{v_1'(y)v_{-1}(y)-v_1(y)v_{-1}'(y)}{2v_1(y)^2} & \text{if } y \in (y_0, \min\{2\tilde{y}-y_0, 1\}] \end{cases} \\
&= \begin{cases} -\frac{2\gamma_{-1}}{(2+y+y_0)^2\gamma_1} & \text{if } y \in (\max\{y_0, 2\tilde{y}-y_0\}, 1] \\ -\frac{2\gamma_1}{(2-y-y_0)^2\gamma_{-1}} & \text{if } y \in (y_0, \min\{2\tilde{y}-y_0, 1\}] \end{cases} \\
&< 0.
\end{aligned}
$$

Thus, $\rho(y)$ is strictly decreasing over $(y_0, 1]$. □

For any proposal $y \in [y_0, \ell]$, denote legislator ℓ's expected payoff as

$$
\begin{aligned}
V_\ell(y) &= \rho(y) \cdot (-(y-\ell)^2) + (1-\rho(y)) \cdot (-(y_0-\ell)^2) \\
&= \rho(y) \cdot s_\ell(y) - (y_0-\ell)^2
\end{aligned}
$$

where $s_\ell(y) = (y_0-\ell)^2 - (y-\ell)^2$.

Claim 2. *$V_\ell(y)$ is continuous and differentiable for all $y \in (y_0, 1]$.*

Proof. By Claim 1, the probability of implementation $\rho(y)$ is continuous and differentiable in y for all $y \in (y_0, 1]$. Moreover, ℓ's stakes of the contest are continuous and differentiable for all $y \in (y_0, 1]$. Hence $V_\ell(y)$ is continuous and differentiable for all $y \in (y_0, 1]$. □

Claim 3. *There exists a solution to the first-order condition $\frac{\partial V_\ell(y)}{\partial y} = 0$ on $y \in (y_0, \ell)$.*

Proof. First, note that $\frac{\partial V_\ell(y)}{\partial y}$ is continuous for all $y \in (y_0, \ell)$.

Moreover, note that since $\left.\frac{\partial s_\ell(y)}{\partial y}\right|_{y=\ell} = 0$ and $\left.\frac{\partial \rho(y)}{\partial y}\right|_{y=\ell} < 0$, we have

$$
\lim_{y\to\ell} \frac{\partial V_\ell(y)}{\partial y} = \left.\frac{\partial \rho(y)}{\partial y}\right|_{y=\ell} s_\ell(\ell) + \left.\frac{\partial s_\ell(y)}{\partial y}\right|_{y=\ell} \rho(y) = \left.\frac{\partial \rho(y)}{\partial y}\right|_{y=\ell} s_\ell(\ell) \qquad < 0,
$$

and that since $s_\ell(y_0) = 0$, we have

$$\lim_{y \to y_0^+} \frac{\partial V_\ell(y)}{\partial y} = \lim_{y \to y_0^+} \left(\frac{\partial \rho(y)}{\partial y} s_\ell(y) + \rho(y) \frac{\partial s_\ell(y)}{\partial y} \right) > 0.$$

By the intermediate value theorem, there exists at least one solution to the first-order condition $\frac{\partial V_\ell(y)}{\partial y} = 0$ on $y \in (y_0, \ell)$. □

Lemma 3. *There exists a unique optimal proposal for ℓ, $y_\ell^* \in (y_0, \ell)$.*

Proof. Any proposal $y < y_0$ is strictly dominated by y_0 and any proposal $y > \ell$ is strictly dominated by ℓ. By Claim 3, there exists at least one solution $\hat{y} \in (y_0, \ell)$ to the first order condition. Now, I show that at any such solution, we must have $\frac{\partial^2 V_\ell(y)}{\partial y^2} < 0$, implying $\hat{y}$ is a unique maximizer.

Suppose $\hat{y} \in (y_0, \ell)$ is a solution to $\frac{\partial V_\ell(y)}{\partial y} = 0$, which implies

$$\frac{\partial V_\ell(y)}{\partial y}\Big|_{y=\hat{y}} = 0 \iff (y_0 - \ell)^2 - (\hat{y} - \ell)^2 = \frac{1}{\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}}} 2\rho(\hat{y})(\hat{y} - \ell). \tag{2}$$

Taking the second derivative yields:

$$\begin{aligned}
\frac{\partial^2 V_\ell(y)}{\partial y^2}\Big|_{y=\hat{y}} &= \frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\hat{y}} [(y_0 - \ell)^2 - (\hat{y} - \ell)^2] - 4\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}} (\hat{y} - \ell) - 2\rho(\hat{y}) \\
&= \frac{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}}} 2\rho(\hat{y})(\hat{y} - \ell) - 4\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}} (\hat{y} - \ell) - 2\rho(\hat{y}) \\
&= -2\rho(\hat{y}) - 2(\hat{y} - \ell)\left(2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}} - \rho(\hat{y}) \frac{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}}} \right)
\end{aligned}$$

where the second line from substituting in based on Equation 2, and the third line from simplifying.

Since $\rho(\hat{y}) > 0$ and $\hat{y} - \ell < 0$, it suffices to show $2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}} - \rho(\hat{y}) \frac{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}}} \leq 0$. There are two cases to consider:

Case (i): $\hat{y} \geq 2\tilde{y} - y_0$. Then we have $\rho(\hat{y}) = \frac{\gamma_{-1}(2-\hat{y}-y_0)}{2\gamma_1(2+\hat{y}+y_0)}$, and $\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}} = -\frac{2\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2}$, and $\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\hat{y}} = \frac{4\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^3}$. Plugging in and simplifying:

$$\begin{aligned}
2\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}} - \rho(\hat{y})\frac{\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}}} &= -\frac{4\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2} - \frac{\gamma_{-1}(2-\hat{y}-y_0)}{2\gamma_1(2+\hat{y}+y_0)} \cdot \frac{\frac{4\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^3}}{-\frac{2\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2}} \\
&= -\frac{4\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2} + \frac{\gamma_{-1}(2-\hat{y}-y_0)}{2\gamma_1(2+\hat{y}+y_0)} \cdot \frac{2}{2+\hat{y}+y_0} \\
&= \frac{\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2}\Big(-4+2-\hat{y}-y_0\Big) \\
&= -\frac{\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)} \\
&< 0
\end{aligned}$$

Case (ii): $\hat{y} \leq 2\tilde{y} - y_0$. Then we have $\rho(\hat{y}) = 1 - \frac{\gamma_1(2+\hat{y}+y_0)}{2\gamma_{-1}(2-\hat{y}-y_0)}$, and $\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}} = -\frac{2\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^2}$, and $\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\hat{y}} = -\frac{4\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^3}$. Plugging in and simplifying:

$$\begin{aligned}
2\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}} - \rho(\hat{y})\frac{\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}}} &= -\frac{4\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^2} - \left(1 - \frac{\gamma_1(2+\hat{y}+y_0)}{2\gamma_{-1}(2-\hat{y}-y_0)}\right) \cdot \frac{-\frac{4\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^3}}{-\frac{2\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^2}} \\
&= -\frac{4\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^2} - \left(1 - \frac{\gamma_1(2+\hat{y}+y_0)}{2\gamma_{-1}(2-\hat{y}-y_0)}\right) \cdot \frac{2}{2-\hat{y}-y_0} \\
&= -\frac{2}{2-\hat{y}-y_0}\left(1 - \frac{\gamma_1(2+\hat{y}+y_0)}{2\gamma_{-1}(2-\hat{y}-y_0)} + \frac{2\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)}\right) \\
&= -\frac{2}{2-\hat{y}-y_0}\left(1 + \frac{\gamma_1}{2\gamma_{-1}}\right) \\
&< 0
\end{aligned}$$

Hence, any solution $\hat{y}$ to the first-order condition must be unique. □

Claim 4. *The legislator's optimal proposal y_ℓ^* is continuous in γ_1 for all $\gamma_1 \in (0, \infty)$ and continuous in γ_{-1} for all $\gamma_{-1} \in (0, \infty)$*

Proof. Fix γ_{-1}. For a given γ_1, Lemma 3 implies ℓ has a unique optimal proposal $y_\ell^*(\gamma_1)$, pinned down by

$$\frac{\partial V_\ell(y;\gamma_1)}{\partial y} = 0. \tag{3}$$

Denote $F(y,\gamma_1) \equiv \frac{\partial V_\ell(y;\gamma_1)}{\partial y} = \frac{\partial \rho(y)}{\partial y} s_\ell(y) + \frac{\partial s_\ell(y)}{\partial y}\rho(y)$

Part 1: Show continuity of $F(y,\gamma_1)$ with respect to y and γ_1. By Claim 1, $\rho(y)$ and $\frac{\partial \rho(y)}{\partial y}$ are continuous for all $y \in (y_0, 1]$. Moreover, since $s_\ell(y) = -(y-1)^2 + (y_0 - 1)^2$, both $s_\ell(y)$ and $\frac{\partial s_\ell(y)}{\partial y}$ are continuous in y. Hence, $F(y,\gamma_1)$ is continuous in y for all $y \in (y_0, 1]$.

To show continuity of $F(y,\gamma_1)$ in γ_1, note that none of the players' stakes ($s_1(y)$, $s_{-1}(y)$, and $s_\ell(y)$) depend on γ_1. Holding fixed y, we have

$$\rho(\gamma_1; y) = \begin{cases} \frac{\gamma_{-1} s_1(y)}{2\gamma_1 s_{-1}(y)} & \text{if } \gamma_1 \geq \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)} \\ 1 - \frac{\gamma_1 s_{-1}(y)}{2\gamma_{-1} s_1(y)} & \text{if } \gamma_1 < \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)} \end{cases}.$$

Note that $\rho(\gamma_1; y)$ is continuous for all $\gamma_1 \in (0,\infty)$, including at $\gamma_1 = \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)}$. In addition, we have

$$\frac{\partial \rho(\gamma_1; y)}{\partial y} = \begin{cases} \frac{\gamma_{-1}}{\gamma_1}\frac{1}{2s_{-1}(y)^2}\left(\frac{\partial s_1(y)}{\partial y}s_{-1}(y) - s_1(y)\frac{\partial s_{-1}(y)}{\partial y}\right) & \text{if } \gamma_1 \geq \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)} \\ \frac{\gamma_1}{\gamma_{-1}}\frac{1}{2s_1(y)^2}\left(\frac{\partial s_1(y)}{\partial y}s_{-1}(y) - s_1(y)\frac{\partial s_{-1}(y)}{\partial y}\right) & \text{if } \gamma_1 < \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)} \end{cases}.$$

Note that $\frac{\partial \rho(\gamma_1;y)}{\partial y}$ is also continuous in γ_1, including at $\gamma_1 = \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)}$. Hence, $F(y,\gamma_1)$ is continuous for all $\gamma_1 \in (0,\infty)$.

Part 2: Take an arbitrary $\gamma_1^0 \in (0,\infty)$ and consider a sequence $\{\gamma_1^n\}$ s.t. $\lim_{n\to\infty}\gamma_1^n = \gamma_1^0$. Let $\{y^n\}$ be the corresponding sequence such that $y^n = y_\ell^*(\gamma_1^n)$, so that each y^n satisfies $F(y^n, \gamma_1^n) = 0$. We want to show $\lim_{n\to\infty} y^n = y_\ell^*(\gamma_1^0)$.

Since $F(y^n, \gamma_1^n) = 0$ for all n, we have

$$\lim_{n\to\infty} F(y^n, \gamma_1^n) = \lim_{n\to\infty} 0 = 0. \tag{4}$$

Moreover, continuity of $F(y, \gamma_1)$ (established in part 1) implies that

$$\lim_{n\to\infty} F(y^n, \gamma_1^n) = F(\lim_{n\to\infty} y^n, \lim_{n\to\infty} \gamma_1^n) = F(\lim_{n\to\infty} y^n, \gamma_1^0) \tag{5}$$

Together, (4) and (5) imply that $F(\lim_{n\to\infty} y^n, \gamma_1^0) = 0$. By uniqueness of $y_\ell^*(\gamma_1)$ for any γ_1, we must have $\lim_{n\to\infty} y^n = \lim_{n\to\infty} y_\ell^*(\gamma_1^n) = y_\ell^*(\gamma_1^0)$. Hence, $y_\ell^*(\gamma_1)$ is continuous at γ_1^0, which was chosen arbitrarily, so $y_\ell^*(\gamma_1)$ is continuous in γ_1. An analogous argument shows y_ℓ^* is continuous in γ_{-1}. $\square$

Corollary 1. *The legislator's optimal proposal y_ℓ^* is weakly decreasing in γ_1 and weakly increasing in γ_{-1}.*

Proof. There are two cases to consider.

Case (i): $y_\ell^*(\gamma_1) \geq 2\tilde{y} - y_0$. In this case, y_ℓ^* is the solution to

$$\begin{aligned}
& \frac{\partial V_\ell(y;\gamma_1)}{\partial y} = 0 \\
\iff & \frac{\partial \rho(y)}{\partial y} s_\ell(y) + \frac{\partial s_\ell(y)}{\partial y}\rho(y) = 0 \\
\iff & \frac{\gamma_{-1}}{\gamma_1}\frac{1}{2s_{-1}(y)^2}\Big(\frac{\partial s_1(y)}{\partial y} s_{-1}(y) - s_1(y)\frac{\partial s_{-1}(y)}{\partial y}\Big) s_\ell(y) + \frac{\gamma_{-1} s_1(y)}{2\gamma_1 s_{-1}(y)}\frac{\partial s_\ell(y)}{\partial y} = 0 \\
\iff & \frac{1}{2s_{-1}(y)^2}\Big(\frac{\partial s_1(y)}{\partial y} s_{-1}(y) - s_1(y)\frac{\partial s_{-1}(y)}{\partial y}\Big) s_\ell(y) + \frac{s_1(y)}{2s_{-1}(y)}\frac{\partial s_\ell(y)}{\partial y} = 0
\end{aligned}$$

In this case, marginal changes in γ_1 (and γ_{-1}) do not affect the optimal proposal y_ℓ^*.

Case (ii): $y_\ell^*(\gamma_1) < 2\tilde{y} - y_0$. I show that in this case, the optimal proposal is strictly decreasing in γ_1. By the implicit function theorem, we have

$$\frac{\partial y_\ell^*(\gamma_1)}{\partial \gamma_1} = -\frac{\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\gamma_1)}}{\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=y_\ell^*(\gamma_1)}}$$

As shown in the proof of Lemma 3, the denominator is strictly negative: $\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\hat{y}} < 0$.

Next, I show $\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\gamma_1)} < 0$:

$$\begin{aligned}
\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\gamma_1)} &= \frac{\partial^2 \rho(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\gamma_1)} \cdot s_\ell(y_\ell^*) + \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=y_\ell^*(\gamma_1)} \cdot \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=y_\ell^*(\gamma_1)} \\
&= -\frac{2}{\gamma_{-1}(2 - y_\ell^* - y_0)^2} \cdot s_\ell(y_\ell^*) - \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=y_\ell^*(\gamma_1)} \cdot \frac{2 + y + y_0}{2\gamma_{-1}(2 - y - y_0)} \\
&< 0.
\end{aligned}$$

The inequality follows as $s_\ell(y_\ell^*) > 0$ and $\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=y_\ell^*(\gamma_1)} > 0$.

Then, we have $\frac{\partial y_\ell^*(\gamma_1)}{\partial \gamma_1} < 0$ whenever $y_\ell^* < 2\tilde{y} - y_0$. Similar proof structure shows for γ_{-1}. □

Lemma 4. *Suppose $c > V_\ell(y_\ell^*) - u_\ell(y_0)$. The pro-change group's optimal proposal is*

$$y_1^* = \begin{cases} y_0 & \textit{if } \tilde{y} \leq y_0, \\ \tilde{y} & \textit{if } \tilde{y} \in (y_0, \min\{2\ell - y_0, 1\}), \\ 2\ell - y_0 & \textit{if } \tilde{y} \in [2\ell - y_0, 1). \end{cases}$$

Moreover, y_1^* is weakly decreasing in γ_1 and weakly increasing in γ_{-1}.

Proof. The expected net contest payoffs for the aligned group given a proposal $y > y_0$ equal $\max\{v_1(y; y_0) - v_{-1}(y; y_0), 0\}$. Note $v_1(y; y_0) - v_{-1}(y; y_0)$ is maximized at $\tilde{y} = \frac{\gamma_{-1} - \gamma_1}{\gamma_1 + \gamma_{-1}}$, as FOC gives:

$$\frac{2}{\gamma_1}(1 - y) - \frac{2}{\gamma_{-1}}(1 + y) = 0 \iff y = \frac{\gamma_{-1} - \gamma_1}{\gamma_1 + \gamma_{-1}}.$$

If $y_0 \geq \tilde{y}$, then for any $y \geq y_0$, group 1's expected payoff is the same as the status quo, and hence $y_1^* = y_0$. If $y_0 \in (-1, \tilde{y})$, then $v_1(y; y_0) - v_{-1}(y; y_0) > 0$ for all $y \in (y_0, 2\tilde{y} - y_0]$. Therefore, group 1 either proposes their optimal proposal, $y_1^* = \tilde{y}$ if unconstrained by the legislator, or else the best proposal accepted by ℓ, $y_1^* = 2\ell - y_0$.

Moreover, note that $\frac{\partial \tilde{y}}{\partial \gamma_1} = -\frac{2\gamma_{-1}}{(\gamma_1 + \gamma_{-1})^2} < 0$ and $\frac{\partial \tilde{y}}{\partial \gamma_{-1}} = \frac{2\gamma_1}{(\gamma_1 + \gamma_{-1})^2} > 0$. $\square$

Proposition 1. *Fix γ_{-1}. There exist a cutoff $\hat{\gamma}_1$ such that for $\gamma_1 < \hat{\gamma}_1$, the pro-change group's optimal proposal is more extreme than ℓ's optimal proposal ($y_1^* > y_\ell^*$), and for $\gamma_1 > \hat{\gamma}_1$, the group's optimal proposal is less extreme than ℓ's optimal proposal ($y_1^* < y_\ell^*$).*

Proof. The proof has two parts: (1) show there must exist an $\hat{\gamma}_1 > 0$ such that $y_1^*(\hat{\gamma}_1) = y_\ell^*(\hat{\gamma}_1)$, and (2) show $\hat{\gamma}_1$ is unique.

Part 1: By Claim 4, the optimal proposal for the legislator y_ℓ^* is continuous in γ_1. The optimal proposal for the aligned group y_1^* is also continuous in γ_1. Hence, $y_1^*(\gamma_1) - y_\ell^*(\gamma_1)$ is continuous in γ_1.

For $\gamma_1 \leq \frac{1-\ell}{1+\ell}\gamma_{-1}$, we have $y_1^*(\gamma_1) \geq \ell > y_\ell^*(\gamma_1)$ since $y_\ell^*(\gamma_1) \in (y_0, \ell)$ for all γ_1 by Lemma 3.

Let $\check{\gamma}_1 = \min\{\gamma_1 : y_\ell^*(\gamma_1) \geq 2\tilde{y} - y_0\}$. Note that $\check{\gamma}_1$ is well-defined as $y_\ell^*(\gamma_1)$ is continuous and $\lim_{\gamma_1 \to \infty} y_\ell^*(\gamma_1) = \ell > -2 - y_0 = \lim_{\gamma_1 \to \infty} 2\tilde{y} - y_0)$. I show that if $\gamma_1 \geq \check{\gamma}_1$, then $y_\ell^*(\gamma_1) > y_1^*$. There are two cases to consider. First, if $\tilde{y} > y_0$, then $y_1^* = \tilde{y}$, and $y_1^*(\gamma_1) = \tilde{y} < 2\tilde{y} - y_0 \leq y_\ell^*(\gamma_1)$. Second, if $\tilde{y} \leq y_0$, then $y_1^*(\gamma_1) = y_0 < y_\ell^*(\gamma_1)$ since $y_\ell^* \in (y_0, \ell)$ by Lemma 3.

By intermediate value theorem, there exists an $\hat{\gamma}_1 \in (\frac{1-\ell}{1+\ell}\gamma_{-1}, \check{\gamma}_1)$ such that $y_1^*(\hat{\gamma}_1) - y_\ell^*(\hat{\gamma}_1) = 0$.

Part 2: The second part of the proof is to show $\hat{\gamma}_1$ is unique. In several steps, I show that if we have $y_1^*(\hat{\gamma}_1) = y_\ell^*(\hat{\gamma}_1)$, then

$$\left.\frac{\partial y_1^*(\gamma_1)}{\partial \gamma_1}\right|_{\gamma_1 = \hat{\gamma}_1} - \left.\frac{\partial y_\ell^*(\gamma_1)}{\partial \gamma_1}\right|_{\gamma_1 = \hat{\gamma}_1} < 0$$

implying the $\hat{\gamma}_1$ is unique.

Part 1 implies $y_\ell^*(\hat{\gamma}_1) = y_1^*(\hat{\gamma}_1) = \tilde{y}$. Hence, it follows that

$$\frac{\partial y_1^*(\gamma_1)}{\partial \gamma_1}\Big|_{\gamma_1=\hat{\gamma}_1} = -\frac{2\gamma_{-1}}{(\gamma_1+\gamma_{-1})^2}.$$

Second, as in case (ii) of the proof of Corollary 1, the implicit function theorem implies

$$\begin{aligned}
\frac{\partial y_\ell^*(\gamma_1)}{\partial \gamma_1}\Big|_{\gamma_1=\hat{\gamma}_1} &= -\frac{\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\hat{\gamma}_1)}}{\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=y_\ell^*(\hat{\gamma}_1)}} \\
&= -\frac{\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=\tilde{y}}}{\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}} \\
&= -\frac{\frac{\partial^2 \rho(y)}{\partial \gamma_1 \partial y}\Big|_{y=\tilde{y}} \cdot s_\ell(\tilde{y}) + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \cdot \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}}{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + 2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} + \rho(\tilde{y})\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}}.
\end{aligned}$$

Define $\psi(\gamma_1, \gamma_{-1}, \ell; y_0) = -\frac{\frac{\partial^2 \rho(y)}{\partial \gamma_1 \partial y}\Big|_{y=\tilde{y}} \cdot s_\ell(\tilde{y}) + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \cdot \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}}{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + 2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} + \rho(\tilde{y})\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}}$.

I show that when $\gamma_1 = \hat{\gamma}_1$, we must have $\frac{\partial \psi(\gamma_1, \gamma_{-1}, \ell; y_0)}{\partial \ell} < 0$. Taking this derivate, we have

$$\begin{aligned}
\frac{\partial \psi(\gamma_1, \gamma_{-1}, \ell; y_0)}{\partial \ell} = -\frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2} \Bigg(&\left(\frac{\partial^2 \rho(y)}{\partial y \partial \gamma_1}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(\tilde{y})}{\partial \ell} + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \frac{\partial}{\partial \ell}\left[\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\right]\right) \\
&\times \left(\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + 2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} + \rho(\tilde{y})\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}\right) \\
&- \left(\frac{\partial^2 \rho(y)}{\partial y \partial \gamma_1}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\right) \\
&\times \left(\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(\tilde{y})}{\partial \ell} + 2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial}{\partial \ell}\left[\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\right] + \rho(\tilde{y})\frac{\partial}{\partial \ell}\left[\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}\right]\right)\Bigg)
\end{aligned}$$

Plugging in $\frac{\partial s_\ell(\tilde{y})}{\partial \ell}\Big|_{y=\tilde{y}} = 2(\tilde{y}-y_0)$, $\frac{\partial}{\partial \ell}\left[\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\right] = 2$, $\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}} = -2$, and $\frac{\partial}{\partial \ell}\left[\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}\right] = 0$, we have

$$\begin{aligned}
\frac{\partial\psi(\gamma_1,\gamma_{-1},\ell;y_0)}{\partial\ell} &= -\frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\Bigg(\Big(2(\tilde{y}-y_0)\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}+2\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\Big) \\
&\qquad\times\Big(\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}}s_\ell(\tilde{y})+2\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}}\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}-2\rho(\tilde{y})\Big) \\
&\quad-\Big(\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}s_\ell(\tilde{y})+\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\Big) \\
&\qquad\times\Big(2(\tilde{y}-y_0)\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}}+4\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}}\Big)\Bigg) \\
&= -\frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\Bigg(-4\rho(\tilde{y})\Big((\tilde{y}-y_0)\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}+\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\Big) \\
&\quad+2\Big((\tilde{y}-y_0)\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}-s_\ell(\tilde{y})\Big)\Big(2\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}}-\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}}\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\Big)\Bigg)
\end{aligned}$$

where the last line follows from taking terms together and simplifying. Next, note that

$$\begin{aligned}
2\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}}-\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}}\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}} &= 2\Big(-\frac{2}{\gamma_{-1}(2-\tilde{y}-y_0)^2}\Big)\Big(-\frac{2\gamma_1}{\gamma_{-1}(2-\tilde{y}-y_0)^2}\Big) \\
&\quad-\Big(-\frac{4\gamma_1}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3}\Big)\Big(-\frac{2+\tilde{y}+y_0}{\gamma_{-1}(2-\tilde{y}-y_0)}\Big) \\
&= \frac{4\gamma_1}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3} \qquad (6)
\end{aligned}$$

and

$$\begin{aligned}
(\tilde{y}-y_0)\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}-s_\ell(\tilde{y}) &= (\tilde{y}-y_0)(-2(y-\ell)-(y_0-\ell)^2+(\tilde{y}-\ell)^2 \\
&= -(\tilde{y}-y_0)^2 \qquad (7)
\end{aligned}$$

Plugging in (6) and (7), we have:

$$\begin{aligned}
\frac{\partial\psi(\gamma_1,\gamma_{-1},\ell;y_0)}{\partial\ell} &= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\left(4\rho(\tilde{y})\Big((\tilde{y}-y_0)\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}+\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\Big)+\frac{4\gamma_1(\tilde{y}-y_0)^2}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3}\right)\\
&< \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\left(2(\tilde{y}-y_0)\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}+\frac{4\gamma_1(\tilde{y}-y_0)^2}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3}\right) \qquad (8)\\
&= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\left(-\frac{4(\tilde{y}-y_0)}{\gamma_{-1}(2-\tilde{y}-y_0)^2}+\frac{4\gamma_1(\tilde{y}-y_0)^2}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3}\right) \qquad (9)\\
&= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\frac{4(\tilde{y}-y_0)}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3}\Big(\gamma_1(\tilde{y}-y_0)-\gamma_{-1}(2-\tilde{y}-y_0)\Big) \qquad (10)\\
&= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\frac{4(\tilde{y}-y_0)}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3}\Big(-(\gamma_1+\gamma_{-1})+(\gamma_{-1}-\gamma_1)y_0\Big) \qquad (11)\\
&= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\frac{4(\tilde{y}-y_0)(\gamma_1+\gamma_{-1})}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3}\Big(-1+\tilde{y}\cdot y_0\Big) \qquad (12)\\
&< 0
\end{aligned}$$

where line (8) follows since $\rho(\tilde{y}) > \frac{1}{2}$, $\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}} < 0$, and $\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}} < 0$, line (9) follows from plugging in $\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}} = -\frac{2}{\gamma_{-1}(2-\tilde{y}-y_0)^2}$, line (10) follows from factoring terms, line (11) follows from substituting in $\tilde{y} = \frac{\gamma_{-1}-\gamma_1}{\gamma_{-1}+\gamma_1}$ and simplifying, and line (12) follows from factoring terms. The inequality follows since $1 > \tilde{y} > y_0 > -1$, and hence $\tilde{y}\cdot y_0 < 1$.

Since $\frac{\partial\psi(\gamma_1,\gamma_{-1},\ell;y_0)}{\partial\ell} < 0$, we know that for any $\ell \in (-1,1)$, we have $\psi(\gamma_1,\gamma_{-1},\ell;y_0) > \psi(\gamma_1,\gamma_{-1},1;y_0)$. Note that

$$\begin{aligned}
\psi(\gamma_1,\gamma_{-1},1;y_0) &= -\frac{\frac{\partial^2\rho(y)}{\partial\gamma_1\partial y}\Big|_{y=\tilde{y}}\cdot s_1(\tilde{y})+\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\cdot\frac{\partial s_1(y)}{\partial y}\Big|_{y=\tilde{y}}}{\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}}s_1(\tilde{y})+2\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}}\frac{\partial s_1(y)}{\partial y}\Big|_{y=\tilde{y}}+\rho(\tilde{y})\frac{\partial^2 s_1(y)}{\partial y^2}\Big|_{y=\tilde{y}}}\\
&= -\frac{\frac{1}{2\gamma_{-1}}\frac{\partial s_1(y)}{\partial y}\Big|_{y=\tilde{y}}}{2(1+\frac{\gamma_1}{2\gamma_{-1}})}\\
&= -\frac{2\gamma_{-1}}{(\gamma_1+\gamma_{-1})(\gamma_1+2\gamma_{-1})}
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\frac{\partial y_\ell^*(\gamma_1)}{\gamma_1}\bigg|_{\gamma_1=\hat{\gamma}_1} &= \psi(\hat{\gamma}_1, \gamma_{-1}, \ell; y_0) \\
&\geq \psi(\hat{\gamma}_1, \gamma_{-1}, 1; y_0) \\
&= -\frac{2\gamma_{-1}}{(\hat{\gamma}_1+\gamma_{-1})(\hat{\gamma}_1+2\gamma_{-1})} \\
&> -\frac{2\gamma_{-1}}{(\hat{\gamma}_1+\gamma_{-1})^2} \\
&= \frac{\partial y_1^*(\gamma_1)}{\gamma_1}\bigg|_{\gamma_1=\hat{\gamma}_1}
\end{aligned}$$

Thus, for any $\hat{\gamma}_1$ such that $y_1^*(\hat{\gamma}_1) = y_\ell^*(\hat{\gamma}_1)$, we must have $\frac{\partial y_1^*(\gamma_1)}{\gamma_1}\Big|_{\gamma_1=\hat{\gamma}_1} - \frac{\partial y_\ell^*(\gamma_1)}{\gamma_1}\Big|_{\gamma_1=\hat{\gamma}_1} < 0$, implying $\hat{\gamma}_1$ is unique. □

Lemma 5. *Suppose* $0 < c < V_\ell(y_\ell^*) - u_\ell(y_0)$. *Legislator* ℓ*'s acceptance set* $A(c) = [\underline{a}(c), \overline{a}(c)]$ *is an interval, where* $\underline{a}(c) \in (y_0, y_\ell^*)$ *and* $\overline{a}(c) \in (y_\ell^*, 2\ell - y_0)$.

Proof. By Claim 2, $V_\ell(y)$ is continuous and differentiable for all $y \in (y_0, 1]$. Moreover, by Lemma 3, we must have $\frac{\partial V_\ell(y)}{\partial y} > 0$ for all $y \in (y_0, y_\ell^*)$ and $\frac{\partial V_\ell(y)}{\partial y} < 0$ for all $y \in (y_\ell^*, \ell)$.

For $y \in [\ell, \min\{2\ell - y_0, 1\})$, we have:

$$\frac{\partial V_\ell(y)}{\partial y}\bigg|_{y\in[\ell,\min\{2\ell-y_0,1\})} = \frac{\partial \rho(y)}{\partial y}\bigg|_{y\in[\ell,\min\{2\ell-y_0,1\})}[u_\ell(y) - u_\ell(y_0)] + \rho(y)\frac{\partial u_\ell(y)}{\partial y}\bigg|_{y\in[\ell,\min\{2\ell-y_0,1\})}$$

Noting that $\frac{\partial \rho(y)}{\partial y}\Big|_{y\in[\ell,\min\{2\ell-y_0,1\})} < 0$, $[u_\ell(y) - u_\ell(y_0)] > 0$, $\rho(y) > 0$, and $\frac{\partial u_\ell(y)}{\partial y}\Big|_{y\in[\ell,\min\{2\ell-y_0,1}$)} < 0, we have $\frac{\partial V_\ell(y)}{\partial y} < 0$ for all $y \in [\ell, \min\{2\ell - y_0, 1\})$. □

Proposition 2. *Suppose* $0 < c < V_\ell(y_\ell^*) - u_\ell(y_0)$.

(i) If $y_1^* = y_\ell^*$, *the equilibrium proposal is* y_ℓ^*, *proposed by* ℓ.

(ii) If $y_1^* \in A(c) \setminus \{y_\ell^*\}$, *the equilibrium proposal is* y_1^*, *proposed by the pro-change group.*

(iii) If $y_1^* > \overline{a}(c)$, *the equilibrium proposal is* $\overline{a}(c)$, *proposed by the pro-change group.*

(iv) If $y_1^* \in (y_0, \underline{a}(c))$ and $\underline{a}(c) < 2\tilde{y} - y_0$, the equilibrium proposal is $\underline{a}(c)$, proposed by the pro-change group.

(v) Otherwise, the equilibrium proposal is y_ℓ^* and is proposed by ℓ.

Proof. Case (i): When the optimal proposals exactly coincide $y_1^* = y_\ell^*$, the expected payoff of the aligned group is the same irrespective of who proposes. By equilibrium selection outlined in the model section, when the group is indifferent, they prefer to leave proposing to the legislator.

Case (ii): When $y_1^* \in A(c) \setminus y_\ell^*$, the aligned group's optimal proposal is in the legislator's acceptance set. Since $y_1^* \neq y_\ell^*$ the group strictly prefers their own optimal proposal to leaving proposing to the legislator.

Case (iii): If $y_1^* > \overline{a}(c)$, then the group's payoff must be strictly increasing for all $y \in (y_0, y_1^*)$. Hence, the constrained optimal proposal is $\overline{a}(c)$, and since $\overline{a}(c) > y_\ell^*$, the group strictly prefers this proposal to y_ℓ^*, which the legislator would propose absent any group proposal.

Case (iv)/(v): If $y_1^* < \underline{a}(c)$, the choice of the group depends. When $v_1(\underline{a}(c)) > v_{-1}(\underline{a}(c)) \iff \underline{a}(c) < 2\tilde{y} - y_0$, the aligned group can extract a strictly positive expected value at the bound of the acceptance set $\underline{a}(c)$. If $v_1(\underline{a}(c)) \leq v_{-1}(\underline{a}(c))$, the group's expected payoff from proposing any policy in the acceptance set is 0, since they are the lower-valuation player for any such $y \in A(c)$. As a result, the group abstains from proposing, and the legislator proposes y_ℓ^* in equilibrium. □

B Proofs Extensions

Exogenous Status Quo Bias

Proposition 3. *With exogenous status quo bias, the legislator's optimal proposal y_ℓ^* and the pro-change group's optimal proposal y_1^*, are the same as in the baseline.*

Proof. In the extended model, given a proposal y, the status quo persists with probability $1-\beta$ even if 1 wins the contest. Hence, the stakes of the contest for group $i \in \{1,-1\}$ given parameter $\beta \in (0,1)$ are $s_i^\beta(y) = \beta \cdot s_i(y)$, where $s_i(y)$ are the stakes from the baseline model. As a result, the effective valuation of group i given β is $v_i^\beta = \beta \cdot v_i(y)$, where $v_i(y)$ is the valuation from the baseline model. Hence, the probability group 1 wins the contest equals $\rho(y)$, and the probability a proposal $y \in (y_0, 1]$ is implemented equals $\rho^\beta(y) = \beta \cdot \rho(y)$.

Given these valuations, Lemma 1 the expected net contest payoffs for group i when they are the higher-valuation group i are $v_i^\beta(y) - v_j^\beta(y) = \beta \cdot [v_i(y) - v_j(y)]$, and 0 otherwise. Hence, $\frac{\partial[v_i^\beta(y)-v_j^\beta(y)]}{\partial y} = 0 \iff \frac{\partial[v_i(y)-v_j(y)]}{\partial y} = 0$. Therefore, the group's optimal proposal is $y_1^{\beta *} = y_1^*$.

The legislator's optimal proposal maximizes

$$V_\ell^\beta(y) = \rho(y)s_\ell^\beta(y) - (y_0 - \ell) = \beta\rho(y)s_\ell(y) - (y_0 - \ell)$$

Hence, $\frac{\partial V_\ell^\beta(y)}{\partial y} = 0 \iff \frac{\partial V_\ell(y)}{\partial y} = 0$. Thus, $y_\ell^{\beta *} = y_\ell^*$. □

Corollary 5. *Suppose $c > 0$. An increase in status quo bias (i) may switch the identity of the equilibrium proposer from legislator ℓ to pro-change group 1 and (ii) may increase or decrease the extremity of the equilibrium proposal.*

Proof. Note that $V_\ell^\beta(y_\ell^*)$ is strictly decreasing in β. All else equal, a decrease in β expands ℓ's acceptance set: $\frac{\partial \underline{a}^\beta(c)}{\partial \beta} \leq 0$ and $\frac{\partial \overline{a}^\beta(c)}{\partial \beta} \geq 0$. When $y_0 < y_1^* < \underline{a}(c)$, an increase in β may switch proposer from ℓ to group 1. Moreover, the expansion of the acceptance set can both result in more extreme proposals (when $y_1^* > \overline{a}(c)$) or more moderate proposals (when $y_0 < y_1^* < \underline{a}(c)$ and 1 proposes). □

Veto Player Extension

Proposition 4. *Let y^* denote the equilibrium proposal from the baseline model.*

(1) If $z \leq y_0$, the veto player's presence results in gridlock (no proposal).

(2) If $z \geq \frac{y^*+y_0}{2}$, the veto player's presence does not affect the proposal or outcomes.

(3) If $z \in (y_0, \frac{y^*+y_0}{2})$, the veto player's presence results in either (i) full gridlock or (ii) a proposal strictly closer to the status quo, increasing the probability of passage.

Proof. Case (1): If $z \leq y_0$, any feasible proposal $y > y_0$ is vetoed by z. Hence, no proposal is made in equilibrium

Case (2): If $z \geq \frac{y^*+y_0}{2}$, the veto player accepts the baseline equilibrium proposal y^* since $y^* \leq 2z - y_0$. Hence, equilibrium is unchanged.

Case (3): Suppose $z \in (y_0, \frac{y^*+y_0}{2})$. First, suppose $y_\ell^* < 2z - y_0$, so ℓ's constrained optimal proposal is y_ℓ^* and their proposal strategy is to either propose y_ℓ^* (if $c \leq V_\ell(y_\ell^*) - u_\ell(y_0)$) or to never propose (if $c > V_\ell(y_\ell^*) - u_\ell(y_0)$). Then ℓ's acceptance set is $[\max\{\underline{a}(c), y_0\}, 2z - y_0]$. Moreover, by assumption, we must have $y^* = y_1^* > 2z - y_0$; hence, in equilibrium, aligned group 1 proposes $2z - y_0$ which is accepted by ℓ and z.

Second, suppose $y_\ell^* \geq 2z - y_0$. The constrained optimal proposal for ℓ is $2z - y_0$. If $V_\ell(2z - y_0) - u_\ell(y_0) < c$, the legislator never proposes and accepts any proposal $y_1 \in (y_0, 2z - y_0]$. In this case, if $y_1^* = y_0$, the equilibrium features gridlock, whereas if $y_1^* > y_0$, the equilibrium proposal is $\min\{y_1^*, 2z - y_0\}$ proposed by 1. If $V_\ell(2z - y_0) - u_\ell(y_0) \geq c$, the legislator accepts any proposal $y_1 \in [\underline{a}_v, 2z - y_0]$, where $\underline{a}_v$ is the solution to $V_\ell(\underline{a}_v) = V_\ell(2z - y_0) - c$ on $(y_0, 2z - y_0)$; otherwise, ℓ proposes $2z - y_0$ themselves. The equilibrium features the legislator proposing $2z - y_0$ whenever (i) $y_1^* \geq 2z - y_0$ or (ii) $y_1^* < \underline{a}_v$ and $\underline{a}_v \geq 2\tilde{y} - y_0$. Otherwise, the equilibrium features the aligned group proposing y_1^*.

Hence, in case (3), we either have gridlock or an equilibrium proposal strictly closer to the status quo. A proposal closer to the status quo must pass with strictly higher probability, since $\rho(y)$ is strictly decreasing in proposal location y over $(y_0, 1]$. □