MASD (Probability Part) Assignment 6

Hand-in in groups of 2 or 3 before October 28, 2021 at 10:00
One submission per group
Remember to include the names of all group members

Problem 1 (Conditional Probabilities):

1. Let B be an event with positive probability, $\mathbb{P}(B) > 0$, on some probability space $(S, \mathbb{P})$ and define $\widehat{\mathbb{P}}(A) := \mathbb{P}(A|B)$ for any event $A \subset S$. Show that $\widehat{\mathbb{P}}$ is also a probability distribution on S.

2. We are presented with a hat with $B = 2$ blue and $R = 3$ red balls in it. In the first round we blindly pick a ball from the hat. Then we return to the hat the ball we picked and add another one of the same colour (increasing the number of balls in the hat by 1). In the second round we proceed in the same way, but we add two balls of the picked colour (increasing the number of balls in the hat by 2 compared to before the second round). Afterwards we pick a ball blindly from the hat. What is the probability that this last ball is red?

   Hint: Use the law of total probability.

Problem 2 (Discrete and Continuous Random Variables): Determine and draw the CDF of the following random variable X:

1. Let X be a roll of the unfair dice from Problem 1.2 on Assignment sheet 5.

2. Let X be a continuous random variable with PDF $f_X(x) = \frac{1}{2}\mathrm{e}^{-|x|}$ for $x \in \mathbb{R}$.

Determine the PDF (for continuous random variables) or PMF (for discrete random variables) for the distribution of the following random variable X.

3. Let X have CDF $F_X(x) = 4x$ for $x \in [0, 1/4]$.

4. Let X have CDF $F_X(x) = 1 - \frac{1}{\lfloor x \rfloor^k}$ for any $x \geq 1$, where $\lfloor x \rfloor := \max\{n \in \mathbb{Z} : n \leq x\}$ is rounding down and $k \in \mathbb{N}$.

Problem 3 (Expectation and Variance): Determine the expectation and variance of the following random variable X:

1. Let X be a roll of the unfair dice from Problem 1.2 on Assignment sheet 5.

2. Let $X = 4Y + 3$, where $Y \sim \text{Bernoulli}(p)$

3. Let $X \sim \text{Binomial}(n, p)$.

4. Let $X = \log Y$, where Y is uniformly distributed on $[0, 1]$.

5. Let X have CDF $F_X(x) = 1 - \mathrm{e}^{-\alpha x}$ for $x \geq 0$ with some constant $\alpha > 0$.