MASD assignment 1

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Exercise 1

a)

We are to prove that:

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon$$

for all values $\epsilon \geq \epsilon_0$ where the above statement holds for ϵ_0

This could be put together, to get the following expression:

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon_0 \leq \epsilon$$

From this expression we can see, that since the statement is true for ϵ_0 and that ϵ is always greater than ϵ_0, that the statement holds true for all $\epsilon \geq \epsilon_0$ since the statements has the only requirement of ϵ_0 being greater than $|f(x) - L|$ which is also true for all ϵ greater than ϵ_0

b)

To prove this, we're going to use proof by contradiction by way of an example:
we have the following statement that would hold true for a unique inverse of a one-to-one function:

$$f(f^{-1}(y)) = y \text{ for all } y \in R_f$$

For this example we will use proof by contradiction, and thus have the following assumptions for this proof:
1. The one-to-one function is $1y = x$
2. The inverse of the function in assumption 1 is $-1y = x$

Since the above statement holds for *all* y, if we can find one value where above statements do not hold, we have proven that the other option (the inverse of assumption one is $1y = x$) must be the case.

This will be proven by using a singular value, considering that will be all that is needed to prove that the statements do not hold of all values of y. For this example we will use the value of y_0.

$-1 * y_0 = -y_0$
$1 * -y_0 = -y_0$

Since $-y_0 \neq y_0$ then it can be said that we have found a case in which the above assumptions do not hold, and the opposite is proven.

c)

To prove this, proof by contradiction will once again be used. For this proof, the following expression will be used:

$f(f^-1(y)) = y$

The following assumptions are used:
1. The function f, is: 1x = y
2. The inverse of the function, f^{-1}, is follows the following rules:
   $f(y) = y, y \neq 2; f(y) = 5, y = 2$

What we want to show is that there is some y, following the assumptions, that do not hold true for the expression. For this we will use the value of y = 2. The values will be input into the above expression:

$f^{-1}(2) = 5$
$f(5) = 5$

Since $5 \neq 2$, that means that the expression does not hold true for a function that is discontinuous, and therefore f^{-1} has to be continous.

Exercise 2

a)

python code for plot

```
import math
x = [i * 0.001 for i in range(-5000, 0)]
y = [i**2.0 for i in x]
plt.plot(x, y)
x2 = [k for k in range (0,3)]
y2 = [k for k in x2]
plt.plot(x2, y2)
x3 = [g * 0.1 for g in range (21,50)]
```

```
y3 = [5 for g in x3]
plt.plot(x3, y3)
```

plot

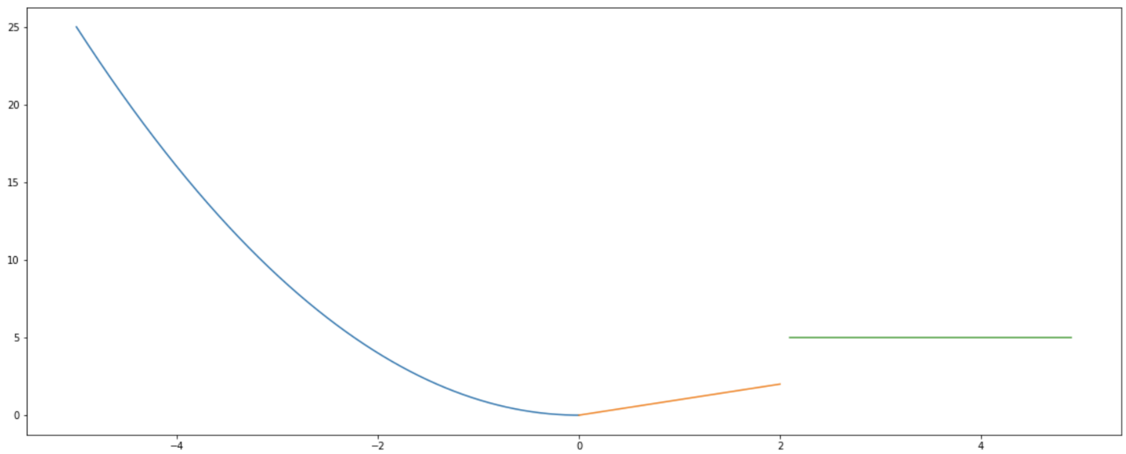

Figure 1: blue: f(x) = x^2, orange: f(x) = x, green: f(x) = 5

f is discontinuous when a = 2, the reason for the discontinuity is that $\lim_{x \to a} f(x)$ does not exist because the left and right limits are different. Right limit is 5, and left limit is 2 in the point a = 2.

b)

now i will Prove that f is not continuous for all a $\in [-5, 5]$
we know for a function can be called continuous the following must be satisfied:

1. f(a) is defined for all a $\in f(a)$
2. $\lim_{x \to a} f(x)$ exist
3. $\lim_{x \to a} f(x) = f(a)$

When proving that f is not continuous, only one of the statements above has to be false.
When looking at condition 2, we know that $\lim_{x \to a} f(x)$ has to exist for f to be continous.

lets see if $\lim_{x \to 2} f(x)$ exist
$\lim_{x \to 2^-} = 2\}$
$\lim_{x \to 2^+} = 5\}$
$2 \neq 5$
$\lim_{x \to 2} f(x) = DNE$

Exercise 3

a)

to prove that $\lim_{x\to\infty} S_n = \pi \cdot r^2$
we first know that $n \geq 3$ is number of vertex in our polygon. So we can multiply n by 2 to get the numbers of right triangles in the polygon, and lets call them 'rt'.

our first expression we need to prove this is $\theta = \frac{2\cdot\pi}{2\cdot n} = \frac{\pi}{n}$ where $2 \cdot \pi$ is the angle around the whole polygon, and then we divide that with $2 \cdot n$ to find the angle for every single right triangle.

So now we can say that $l = sin(\theta) \cdot h \in$ rt

So our expression for the area of the polygon is following: $A = h \cdot l \cdot \frac{1}{2} \cdot 2 \cdot n$
now lets put the expression $\pi \cdot r^2$ into our equation
$\pi \cdot r^2 = h \cdot l \cdot n$ now lets put our expression for l into the equation
$\pi \cdot r^2 = sin(\theta) \cdot h \cdot h \cdot n$ now lets divide by r in both sides of the equation
$\pi \cdot r^2 = sin(\frac{\pi}{n}) \cdot h^2 \cdot n$ now we divide by n in both sides
$\frac{\pi}{n} \cdot r^2 = sin(\frac{\pi}{n}) \cdot h^2$ now we divide both sides with $\frac{\pi}{n}$
$r^2 = \frac{sin(\frac{\pi}{n})}{\frac{\pi}{n}} \cdot h^2$ now when looking at the expression $\lim_{x\to 0} \frac{sin(x)}{x} = 1$ it says that when x approaches 0 the expression becomes 1

so in this expression: $r^2 = \frac{sin(\frac{\pi}{n})}{\frac{\pi}{n}} \cdot h^2$ we have that n approaches ∞ then it will become pi over a very big number, and that will get closer and closer to zero, and as we saw in this expression: $\lim_{x\to 0} \frac{sin(x)}{x} = 1$ we then know that we will end up with 1. So $r^2 = 1 \cdot h^2 \Rightarrow r = h$ So we have proven that because l approaches r with the assumption that the polygons area becomes the circles area, that $\lim_{x\to\infty} S_n = \pi \cdot r^2$