MASD (Probability Part) Assignment 7

Hand-in in groups of 2 or 3 before **November 2, 2021 at 14:00**
One submission per group
Remember to include the names of all group members

No resubmissions are permitted for this assignment!

Problem 1 (Random variables and their distributions):

1. Determine $\mathbb{E}|X|$ for $X \sim N(0,1)$.
2. Let X, Y be independent and uniformly distributed on the interval $[-1,1]$. Determine the PDF and CDF of $X+Y$ and $\min\{X,Y\}$.
3. Let X, Y be i.i.d. and uniformly distributed on $[0,1]$. Compute the variance of the random variable $3X + Y - 1$.

Problem 2 (Joint distribution):

1. Let X, Y be i.i.d. uniformly distributed on $\{-1,0,1\}$. Compute the covariance $\mathrm{Cov}(X, X-Y)$. What is the joint PMF for the joint distribution of X and $X-Y$?
2. Compute $\mathbb{E}e^{\frac{1}{2}(X^2-Y^2)}$, where X, Y are jointly Gaussian with $(X,Y) \sim N(0,A)$ and
$$A = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$
3. Let X and Y be jointly continuous random variables with joint PDF
$$f_{X,Y}(x,y) = \frac{6}{7}(x+y)^2 \mathbb{1}_{[0,1]}(x)\, \mathbb{1}_{[0,1]}(y)\,.$$
Determine the PDF of the distribution of X.

Problem 3* **(Statistics):** We consider the statistical model $(\mathbb{P}_\theta)_{\theta>0}$ with $\mathbb{P}_\theta = \mathrm{Exp}(\theta)$ and are given a sample $X_1, \ldots, X_n$ of n independent observations distributed as $\mathbb{P}_\theta$. Determine the MLE $T(x)$ for this model.

*This problem is optional. It will not be contribute to your score on this assignment sheet.