MASD (Probability Part) Assignment 5

Hand-in in groups of 2 or 3 before October 14, 2021 at 10:00
One submission per group
Remember to include the names of all group members

Problem 1 (Modelling Probabilities): Provide a suitable model (set of outcomes S and probability distribution $\mathbb{P}$) for the following probabilistic experiments. Answer the questions asked in each case.

1. We toss a coin and afterwards we roll a die. What is the probability that we observe heads and then roll a number smaller or equal to 4?

2. We are given an unfair die. We roll it 1000 times and record how often we see each number:

1	2	3	4	5	6
107	195	52	492	112	42

   How would you model the experiment of rolling this die again twice? Let X_1, X_2 be the outcome of the first and second roll, respectively. Compute the probability that the roll sum $X_1 + X_2$ equals 4.

3. We are given a deck of 52 (distinct) cards. They are well organised, starting with hearts $2, 3, 4, 5$. We shuffle them as best we can. What is the probability that after shuffling the first four cards of the deck are again the same, but in possibly different order?

Problem 2 (Exclusion-Inclusion Principle): Let $A = \{1, 2, \ldots, 50\}$. For any $n \in \mathbb{N}$ let A_n be the set of integers in A that are divisible by n. For example, $A_3 = \{3, 6, 9, \ldots, 48\}$ and $A_4 = \{4, 8, 12, \ldots, 48\}$.

1. Compute the number of elements in the sets A_2, A_3, A_4, i.e. the values of $|A_2|, |A_3|, |A_4|$.

2. Compute $|A_2 \cup A_3 \cup A_7|$.

Problem 3 (Combinatorics):

1. How many subsets of the numbers from 1 to 100 are there that have 10 elements and also contain the numbers 5 and 10?

2. An anagram is a word created from the letters of another word (with exactly the same number of appearances of each letter). How many (possibly nonsensical) anagrams are there of the word "STATISTICS"?

3. You have an even number $n \geq 20$ of friends, k of them are female and $n - k$ of them male, where $10 \leq k \leq n$. You invite half of them to a dinner party. How many different dinner party configurations can you organise with exactly 10 female friends present? How many can you organise with at least 10 female friends present? Express the outcome in terms of binomial coefficients.

4. How many distinct solutions $(x_1, x_2, \ldots, x_6)$ are there to $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 50$ with $x_i \in \mathbb{N}$?