1. $\begin{cases} (4x^2-2y^2)\,dx = 2xy\,dy \\ y(1)=3 \end{cases}$ $\qquad \dfrac{dy}{dx} = y' = \dfrac{(4x^2-2y^2)}{2xy}$

$$f(tx,ty) = \frac{4t^2x^2-2t^2y^2}{2tx\,ty} = \frac{t^2(4x^2-2y^2)}{t^2\cdot 2xy}$$

Cambio: $y=ux \rightarrow y'=u'x+u \rightarrow u'x+u = \dfrac{4x^2-2u^2x^2}{2xux} \rightarrow u'x+u = \dfrac{x^2(4-2u^2)}{2ux^2} \rightarrow$

$$u'x+u = \frac{4-2u^2}{2u} \rightarrow u'x = \frac{4-2u^2}{2u} - u \rightarrow u'x = \frac{4-2u^2-2u^2}{2u} \rightarrow u'x = \frac{4-4u^2}{2u} \rightarrow$$

$$u'x = \frac{2-2u^2}{u} \rightarrow u' = \frac{2-2u^2}{u}\cdot\frac{1}{x} \rightarrow \frac{du}{dx} = \frac{2-2u^2}{u}\cdot\frac{1}{x} \rightarrow \int\frac{u}{2-2u^2}\,du = \int\frac{1}{x}\,dx \rightarrow$$

$$-\frac{1}{2}\ln|2-2u^2| = \ln x + C \rightarrow \ln|2-2u^2| = -2\ln x + C \rightarrow \ln|2-2u^2| = \ln x^{-2} + C \rightarrow$$

$$2-2u^2 = \frac{C}{x^2} \rightarrow 2u^2 = 2-\frac{C}{x^2} \rightarrow u = \sqrt{1-\frac{C}{2x^2}}$$

Deshago el cambio: $y=ux \rightarrow u=\dfrac{y}{x}$

$$\frac{y}{x} = \sqrt{1-\frac{C}{2x^2}} \rightarrow y = x\sqrt{1-\frac{C}{2x^2}}$$

$$y(1)=3 \rightarrow 3 = 1\cdot\sqrt{1-\frac{C}{2}} \rightarrow 9 = 1-\frac{C}{2} \rightarrow \frac{C}{2} = -8 \rightarrow C=-16$$

$$y = x\sqrt{1-\frac{8}{x^2}}$$

3. $y' - 6y = 5e^{6x} y^4$

$y' - 6y = 0 \to y' = 6y \to \frac{dy}{dx} = 6y \to \int \frac{1}{y} dy = \int 6\, dx \to \ln y = 6x + C \to$

$y = e^{6x} \cdot C$

$y' = 6e^{6x} \cdot C + e^{6x} \cdot C' \to 6e^{6x} \cdot C + e^{6x} \cdot C' - 6e^{6x} C = 5e^{6x} (e^{6x} \cdot C)^4 \to$

$C' = 5e^{24x} C^4 \to \int \frac{1}{C^4} dC = \int 5e^{24x} dx \to \int C^{-4} dC = \int 5e^{24x} dx \to$

$\frac{C^{-3}}{-3} = \frac{5}{24} e^{24x} + K \to \frac{1}{C^3} = \frac{-15}{24} e^{24x} + K \to C = \sqrt{\frac{1}{-\frac{15}{24} e^{24x} + K'}}$

$$y = e^{6x} \left(\sqrt{\frac{1}{-\frac{15}{24} e^{24x} + K}} \right)$$

4. $y'' + 6y' + 9y = 6e^{-3x} + 18$

$y'' + 6y' + 9y = 0 \to \lambda^2 + 6\lambda + 9 = 0 \to \lambda = \frac{-6 \pm \sqrt{36-36}}{2} = -3$

$y_h = C_1 e^{-3x} + x C_2 e^{-3x}$

$y_p = x^2 \cdot A e^{-3x} + B \to y_p' = 2x \cdot A e^{-3x} + x^2 (-3) e^{-3x} = e^{-3x} (2Ax - 3x^2) \to$

$y_p'' = -3e^{-3x} (2Ax - 3x^2) + e^{-3x} (2A - 6x) = e^{-3x} (-3x^2 - 6x + 2Ax + 2A)$

$e^{-3x} (-3x^2 - 6x + 2Ax + 2A) + 6e^{-3x} (2Ax - 3x^2) + 9x^2 A e^{-3x} + 9B = e^{-3x} + 18$

Términos con e^{-3x} : $-3x^2-6x+2Ax+2A+12Ax-18x^2+9x^2=1$

$\hookrightarrow$ Términos sin x: $2A=1 \rightarrow A=1/2$

Términos sin e^{-3x} : $9B=18 \rightarrow B=2$

$$\left.\right\} y_p = \frac{1}{2}x^2e^{-3x}+2$$

$$y_g = C_1e^{-3x}+xC_2e^{-3x}+\frac{1}{2}x^2e^{-3x}+2$$

$$y_g' = -3C_1e^{-3x}+C_2e^{-3x}+xC_2e^{-3x}(-3)+\frac{1}{2}\cdot 2x+e^{-3x}+\frac{1}{2}x^2e^{-3x}(-3) \rightarrow$$

$$y_g' = e^{-3x}\left(-3C_1+C_2-3xC_2+x-\frac{3}{2}x^2\right)$$

$y(0)=2 \Rightarrow 2=C_1+2 \Rightarrow C_1=0$

$y'(0)=25 \Rightarrow 25=-3\cdot C_1+C_2 \Rightarrow C_2=25$

$$y_g = 25xe^{-3x}+\frac{1}{2}x^2e^{-3x}+2$$

5. $\begin{cases} y'+xy=3xe^{x^2} \\ y(0)=2 \end{cases}$

$$y'+xy=0 \rightarrow \frac{dy}{dx}=-xy \rightarrow \int\frac{1}{y}dy=\int-x\,dx \rightarrow \ln y=-\frac{x^2}{2}+C \rightarrow y=c\cdot e^{-\frac{x^2}{2}}$$

$$y'=c'\cdot e^{-\frac{x^2}{2}}+c(-x)e^{-\frac{x^2}{2}} \rightarrow c'e^{-\frac{x^2}{2}}-cxe^{-\frac{x^2}{2}}+xce^{-\frac{x^2}{2}}=3xe^{x^2} \rightarrow$$

$$c'=3xe^{x^2}\cdot e^{-\frac{x^2}{2}} \rightarrow c'=3xe^{\frac{x^2}{2}} \rightarrow \frac{dc}{dx}=3xe^{\frac{x^2}{2}} \rightarrow \int dc=\int 3xe^{\frac{x^2}{2}} \rightarrow$$

$$c=3e^{\frac{x^2}{2}}+K \Rightarrow y=\left(3e^{\frac{x^2}{2}}+K\right)\cdot e^{-\frac{x^2}{2}}$$

$y(0)=2 \rightarrow 2=(3+K) \rightarrow K=-1$

$$y=\left(3e^{\frac{x^2}{2}}-1\right)e^{-\frac{x^2}{2}}$$

6. $2xy - 3x^2y^2 + (x^2 - 2x^3y)y' = 0$

$(x^2 - 2x^3y)y' = -(2xy - 3x^2y^2) \rightarrow (x^2 - 2x^3y)\frac{dy}{dx} = -(2xy - 3x^2y^2) \rightarrow$

$(x^2 - 2x^3y)dy = -(2xy - 3x^2y^2)dx \rightarrow \underbrace{(2xy - 3x^2y^2)}_{P}dx + \underbrace{(x^2 - 2x^3y)}_{Q}dy = 0$

$\frac{dP}{dy} = 2x - 6x^2y \qquad \frac{dQ}{dx} = 2x - 6x^2y$

$\int 2xy - 3x^2y^2\,dx = x^2y - x^3y^2 + g(y) = f(x,y)$

$f'_y(x,y) = \cancel{x^2} - \cancel{2x^3y} + g'(y) = \cancel{x^2} - \cancel{2x^3y} = Q \qquad g'(y) = 0 \quad g(y) = K$

$f(x,y) = x^2y - x^3y^2 + K$

7. $y' - \frac{2}{x+2}y = 2(x+2)^3$

$y' - \frac{2}{x+2}y = 0 \rightarrow y' = \frac{2}{x+2}y \rightarrow \frac{dy}{dx} = \frac{2}{x+2}y \rightarrow \int \frac{1}{y}dy = \int \frac{2}{x+2}dx \rightarrow$

$\ln y = 2\ln|x+2| + C \rightarrow \ln y = \ln|(x+2)^2| + C \rightarrow y = (x+2)^2 \cdot C$

$y' = 2(x+2)C + (x+2)^2C' \rightarrow \cancel{2(x+2)C} + (x+2)^2 \cdot C' - \cancel{\frac{2}{x+2}(x+2)^2 \cdot C} = 2(x+2)^3$

$\rightarrow (x+2)^2C' = 2(x+2)^3 \rightarrow C' = 2(x+2) \rightarrow \frac{dc}{dx} = 2(x+2) \rightarrow \int dc = \int 2(x+2)\,dx \rightarrow$

$C = x^2 + 4x + K$

$y = (x+2)^2 \cdot (x^2 + 4x + K)$

9. $y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y''' + 5y'' = 0$

$\lambda^7 + \lambda^6 + 2\lambda^5 + 10\lambda^4 + 13\lambda^3 + 5\lambda^2 = 0$

$\lambda^2(\lambda^5 + \lambda^4 + 2\lambda^3 + 10\lambda^2 + 13\lambda + 5) = 0 \rightarrow \lambda = 0$

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
	1	0	2	8	5	0
-1		-1	1	-3	-5	
	1	-1	3	5	0	
-1		-1	2	-5		
	1	-2	5	0		

$x^2 - 2x + 5 = 0 \rightarrow x = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$

$\lambda = 0 \quad m = 2$

$\lambda = -1 \quad m = 3$

$\lambda = 1 \pm 2i$

$y_h = y_q = C_1 + xC_2 + C_3 e^{-x} + C_4 x e^{-x} + C_5 x^2 e^{-x} + C_6 e^{x} \operatorname{sen}(2x) + C_7 e^{x} \cos(2x)$