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I

$$\begin{cases} (4x^2 - 2y^2)\,dx = 2xy\,dy \\ y(1) = 3 \end{cases}$$

$v = \frac{y}{x}$;

$y = vx$;

$y' = v'x + v$

$4x^2 - 2y^2 = (2xy)\,y'$;

$$y' = v'x + v = \frac{\overset{2}{\cancel{4}}\cancel{x^2} - 2v^2\cancel{x^2}}{2v\cancel{x^2}} = \frac{2 - v^2}{v} ;$$

$$\frac{dv}{dx}\,x = \frac{2 - 2v^2}{v} \longrightarrow \int \frac{v\,dv}{2 - 2v^2} = \int \frac{dx}{x} ; \longrightarrow -\frac{1}{4}\ln|1 - v^2| = \ln|x| + C$$

$$1 - v^2 = \frac{1}{K^4 x^4} \longrightarrow v^2 = 1 - \frac{1}{K^4 x^4} \longrightarrow y^2 = x^2 - \frac{1}{K^4 x^2}$$

Para $y(1) = 3$:

$$9 = 1 - \frac{1}{K^4} \longrightarrow \frac{1}{K^4} = -8 \longrightarrow K = \sqrt[4]{-\frac{1}{8}}$$

II

$$\begin{cases} y'' + 4y = -4\,\text{sen}(2x) \\ y(0) = -1 \\ y'(0) = 4 \end{cases} \longrightarrow p(x) = x^2 + 4 = 0 \longrightarrow \overset{\text{raíces}}{\pm 2i} \begin{cases} \rightarrow \text{sen}\, 2x \\ \rightarrow \cos 2x \end{cases}$$

Sol. General: $y(x) = C_2 \cdot \cos 2x + C_1 \cdot \text{sen}\, 2x$

Sol. Particular:

$y = Ax\,\text{sen}\,2x + Bx\cos 2x$

$y' = A\,\text{sen}\,2x + 2Ax\cos 2x + B\cos 2x - 2Bx\,\text{sen}\,2x$

$y'' = 2A\cos 2x + 2A\cos 2x - 4Ax\,\text{sen}2x - 2B\,\text{sen}2x - 2B\,\text{sen}2x - 4Bx\cos 2x$

$\downarrow$

$4A\cos 2x \cancel{- 4Ax\,\text{sen}2x} - 4B\,\text{sen}2x \cancel{- 4Bx\cos 2x} \cancel{+ 4Ax\,\text{sen}\,2x} \cancel{+ 4Bx\cos 2x} =$

$= -4\,\text{sen}\,2x$

$\downarrow$

$$\left.\begin{matrix} 4A = 0 \\ -4B = -4 \end{matrix}\right\} \begin{matrix} A=0 \\ B=1 \end{matrix} \longrightarrow y(x) = x\cos 2x + C_1 \cdot \text{sen}\,2x + C_2 \cdot \cos 2x$$

$y'(x) = \cos 2x + 2x\,\text{sen}2x + 2C_1\cos 2x - 2C_2\,\text{sen}2x$

$y(0) = -1 = \cancel{0 \cdot \cos 2\cdot 0} + \cancel{C_1 \cdot \text{sen}\,2\cdot 0} + C_2 \cdot \cos 2\cdot 0\,;\quad C_2 = -1$

$y'(0) = 4 = \cos 2\cdot 0 \cancel{+ 2\cdot 0 \cdot \text{sen}\,2\cdot 0} + 2\cdot C_1 \cdot \cos 2\cdot 0 \cancel{- 2\cdot C_2 \cdot \text{sen}\,2\cdot 0}\,;\ C_1 = \frac{3}{2}$

$\downarrow$

$$\boxed{y(x) = \cos 2x + \frac{3}{2}\,\text{sen}\,2x}$$

III

$$y' - 6y = 5e^{6x} y^4$$

$$z = y^{-3}$$

$$z' = -3y^{-4} \cdot y'$$

$$\boxed{z'} = -3y^{-4}(5e^{6x}y^4 + 6y) =$$

$$\boxed{= -15e^{6x} - 18z} \quad \text{I}$$

$$\downarrow$$

$$z' + 18z = 0$$

$$\downarrow$$

$$\int \frac{dz}{z} = \int -18\,dx \to \ln|z| = -18x + C_1$$

$$\downarrow$$

$$z = K \cdot e^{-18x}$$

$$\boxed{z' = e^{-18x} \cdot (K'(x) - 18K(x))} \quad \text{II}$$

$$z = K(x) \cdot e^{-18x}$$

$$\text{I} \quad z' = -15e^{6x} - 18z$$

$$\text{II} \quad z' = K'(x)e^{-18x} - 18K(x)e^{-18x}$$

$$\downarrow$$

$$K'e^{-18x} = -15e^{6x}$$

$$K = \int -15e^{24x}\,dx = -\frac{15}{24}e^{24x} + C_1$$

$$z = Ke^{-18x} = -\frac{15}{24}e^{6x} + C_1e^{-18x}$$

$$\boxed{y = \sqrt[3]{\frac{1}{C_1e^{-18x} - \frac{5}{8}e^{6x}}}}$$

IV

$$\begin{cases} y'' + 6y' + 9y = 6e^{-3x} + 18 \\ y(0) = 2 \\ y'(0) = 25 \end{cases}$$

$\longrightarrow p(x) = x^2 + 6x + 9 = (x+3)^2$

Raíces $\rightarrow -3$ con mult. 2.

Sol. General:

$$y(x) = C_1 e^{-3x} + C_2 x e^{-3x}$$

Sol. Particular:

$$y(x) = Axe^{-3x} + Bx^2e^{-3x}$$

$$y'(x) = Ae^{-3x} - 3Axe^{-3x} + 2Bxe^{-3x} - 3Bx^2e^{-3x}$$

$$y''(x) = 9Axe^{-3x} - 6Ae^{-3x} + 9Bx^2e^{-3x} - 12Bxe^{-3x} + 2Be^{-3x}$$

$\downarrow$

$$\cancel{9Axe^{-3x}} \cancel{- 6Ae^{-3x}} + \cancel{9Bx^2e^{-3x}} - \cancel{12Bxe^{-3x}} + 2Be^{-3x} \cancel{+ 6Ae^{-3x}} \cancel{- 18Axe^{-3x}} + \cancel{12Bxe^{-3x}} - \cancel{18Bx^2e^{-3x}} + \cancel{9Axe^{-3x}} + \cancel{9Bx^2e^{-3x}} = 6e^{-3x} + 18$$

$\downarrow$

$$2Be^{-3x} = 6e^{-3x} + 18 \longrightarrow B = 3 + 9e^{3x}$$

$$y(x) = 3x^2e^{-3x} + 9x^2 + C_1e^{-3x} + C_2xe^{-3x}$$

$$y'(x) = 6xe^{-3x} - 9x^2e^{-3x} + 18x - 3C_1e^{-3x} + C_2e^{-3x} - 3C_2xe^{-3x}$$

$\downarrow$

$$y(0) = 0 + 0 + C_1 + 0 = 2 \longrightarrow C_1 = 2$$

$$y'(0) = 0 - 0 + 0 - 6 + C_2 - 0 = 25 \longrightarrow C_2 = 31$$

$$\boxed{y(x) = 2 \cdot e^{-3x} + 31x \cdot e^{-3x}}$$

V

$$\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases} \longrightarrow y' + xy = 0 \longrightarrow \frac{dy}{dx} = -xy \longrightarrow \ln|y| = -\frac{x^2}{2} + C_1$$

$$y = Ke^{-x^2/2}$$

$$y' = K'e^{-x^2/2} - xKe^{-x^2/2}$$

$$K'e^{-x^2/2} \cancel{- xKe^{-x^2/2}} \cancel{+ xKe^{-x^2/2}} = 3xe^{x^2}$$

$$K'(x) = 3xe^{3x^2/2} \longrightarrow K(x) = e^{\frac{3}{2}x^2} + C_1$$

$$y(x) = e^{x^2} + C_1 \cdot e^{-x^2/2} = e^{-x^2/2} \longrightarrow \boxed{y(x) = e^{-x^2/2}}$$

VI

$$2xy - 3x^2y^2 + (x^2 - 2x^3y)y' = 0 \longrightarrow \frac{dM}{dy} = \frac{dN}{dx}$$

$$\left.\begin{aligned} \frac{dM}{dy} &= 2x - 6x^2y \\ \frac{dN}{dx} &= 2x - 6x^2y \end{aligned}\right\} \text{Es exacta} \longrightarrow \frac{df}{dx} = M(x,y) \longrightarrow f = \int 2xy - 3x^2y^2\,dx = x^2y - x^3y^2 + K($$

$$\frac{df}{dy} = \cancel{x^2 - 2x^3y} + K'(y) = \cancel{x^2 - 2x^3y}$$

$$\boxed{C_1 = x^2y - x^3y^2}$$

VII

$$y' - \frac{2}{x+2}y = 2(x+2)^3 \to y' - \frac{2}{x+2}y = 0 \to \frac{dy}{dx} = \frac{2}{x+2}y \to y = K^2(x+2)^2$$

$$y' = 2K'(x+2)^2 + 2K^2(x+2)$$

$$2K'(x+2)^2 + 2K^2(x+2) - 2K^2(x+2) = 2(x+2)^3$$

$$\downarrow$$

$$K' = x+2 \to K = \frac{x^2}{2} + 2x + C_1$$

$$\boxed{y(x) = \left(\frac{x^2}{2} + 2x + C_1\right)(x+2)^2}$$

VIII

$$e^x y^2 + y\,\mathrm{sen}\,x + (3ye^x - 2\cos x)y' = 0 \qquad \frac{dM}{dy} = \frac{dN}{dx}$$

$$u(y)(e^x y^2 + y\,\mathrm{sen}\,x) + u(y)(3ye^x - 2\cos x)y' = 0$$

$$\left.\begin{aligned}\frac{dM}{dy} &= u'(y)(e^x y^2 + y\,\mathrm{sen}\,x) + u(y)(2e^x y + \mathrm{sen}\,x)\\ \frac{dN}{dx} &= u(x)(3ye^x + 2\,\mathrm{sen}\,x)\end{aligned}\right\} u'(y) = \frac{u(y)}{y}$$

$$\frac{du}{dy} = \frac{u(x)}{y} \to \ln|u| = \ln|y| + C_1 \to u = Ky$$

$$\downarrow$$

$$e^x y^3 + y^2\,\mathrm{sen}\,x + (3y^2e^x - 2y^2\cos x)y' = 0$$

$$\boxed{C_1 = y^2(ye^x - 2\cos x)}$$

$$\frac{df}{dy} = 3y^2e^x - 2y\cos x \to f = y^3e^x - y^2\cos x + K(x)$$

$$\frac{df}{dx} = \cancel{y^3e^x + y^2\,\mathrm{sen}\,x} + K'(x) = \cancel{y^3e^x + y^2\,\mathrm{sen}\,x} \to K'(x) = 0$$

IX

$$y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y''' + 5y'' = 0$$

$\Downarrow$

$$p(x) = x^2(x^5 + x^4 + 2x^3 + 10x^2 + 13x + 5) = x^2(x+1)^3(x^2-2x+5) = 0$$

Raíces
- $\to$ 0 con mult. 2 $\to 1, x$
- $\to$ -1 con mult. 3 $\to e^{-x}, xe^{-x}, x^2e^{-x}$
- $\to e^{x}\cos(4x)$
- $\to e^{x}\,\text{sen}(4x)$

$$\boxed{y(x) = C_1 + xC_2 + C_3 \cdot e^{-3x} + xC_4e^{-x} + x^2C_5e^{-x} + C_6e^{x}\cos 4x + C_7e^{x}\,\text{sen}\,4x}$$