EJERCICIOS EC. DIFERENCIALES

① $\begin{cases}(4x^2-2y^2)\,dx = 2xy\,dy \\ y(1)=3\end{cases} \longrightarrow y' = \dfrac{4x^2-2y^2}{2xy} = v'x+v = \dfrac{4x^2-2(vx)}{2x(vx)}$

$y' = v'x+v$

$y = vx$

$v'x+v = 2-v \; ; \; v'x = 2-2v \longrightarrow \dfrac{dx}{dv}\, x = 2-2v \longrightarrow \int \dfrac{dx}{x} = \int 2-2v\, dv =$

$= \text{Log}|x| + K = 2v - v^2 \longrightarrow \log|x| + K = 2\frac{y}{x} - \left(\frac{y}{x}\right)^2 ; \; y(1)=3 ; \; 2\frac{y}{x} - \left(\frac{y}{x}\right)^2 = 3 ;$

$\boxed{2x^{-1} - x^{-2} = 3}$

② $\begin{cases} y'' + 4y = -4\,\text{sen}(2x) \\ y(0) = -1 \\ y'(0) = 4 \end{cases}$

- Ecuación diferencial ~~asociada~~ homogénea asociada

$y'' + 4y = 0 \rightarrow P(x) = x^2 + 4 = 0 \rightarrow x = \sqrt{-4}$

Raices $\rightarrow 0 \pm 2i$ mult 1 $\begin{cases} e^{0x}\cos(2x) = \cos(2x) \\ e^{0x}\,\text{sen}(2x) = \text{sen}(2x) \end{cases}$

- Solucion ecuación diferencial homogénea asociada

$y_h(x) = C_1\cos(2x) + C_2\,\text{sen}(2x) \qquad C_1, C_2 \in \mathbb{R}$

- Solución particular.

$y(x) = A_x\cos(2x) + B_x\,\text{sen}(2x)$

$y'(x) = A\cos(2x) - 2A_x\,\text{sen}(2x) + B\,\text{sen}(2x) + 2B_x\cos(2x)$

$y''(x) = -2A\,\text{sen}(2x) - 2A\,\text{sen}(2x) - 4A_x\cos(2x) + 2B\cos(2x) +$
$+ 2B\cos(2x) - 4B_x\,\text{sen}(2x) = -4A\,\text{sen}(2x) - 4A_x\cos(2x) +$
$+ 4B\cos(2x) - 4B_x\,\text{sen}(2x)$

$\longrightarrow y'' + 4y = -4\,\text{sen}(2x)$

$\begin{cases} -4A = -4 \rightarrow A = 1 \\ 4B = 0 \rightarrow B = 0 \end{cases}$

$y_p(x) = x\cos(2x);$

- Solución general

$y(x) = x\cos(2x) + C_1\cos(2x) + C_2 \operatorname{sen}(2x)$

$y'(x) = \cos(2x) - 2x\operatorname{sen}(2x) - 2C_1\operatorname{sen}(2x) + 2C_2\cos(2x)$

$y(0) = -1 \rightarrow C_1\cos(0) \rightarrow C_1 = -1$

$y'(0) = 4 \rightarrow \cos(0) - 2C_1\operatorname{sen}(0) + 2C_2\cos(0)$

$1 - 0 - 0 + 2C_2 = 4 \rightarrow 1 + 2C_2 = 4 \rightarrow 2C_2 = 3 \rightarrow C_2 = \frac{3}{2}$

$$\boxed{y(x) = x\cos(2x) - \cos(2x) + 3/2\operatorname{sen}(2x)}$$

(3) $y' - 6y = 5e^{6x}y^4$

$y' = 5e^{6x}y^4 + 6y$

$\alpha = 3 \quad z = y^{1-4} = y^{-3}$

$z' = -3y^{-4} \cdot y' = -3y^{-4}\cdot(5e^{6x}y^4 + 6y) = -15e^{6x} - 18y^{-3} = -15e^{6x} - 18z$

$z' = -15e^{6x} - 18z \rightarrow z' + 18z = -15e^{6x} \rightarrow z' + 18z = 0 \rightarrow \frac{dz}{dx} = -18z \rightarrow$

$\rightarrow \int\frac{dz}{z} = \int -18\,dx \rightarrow \log|z| = -18x + c \rightarrow z = e^{-18x+c} \rightarrow e^{-18x} + e^{c} \rightarrow$

$\rightarrow z = K \cdot e^{-18x}$

$* \ z' = K'(x)e^{-18x} + K'(x) - 18e^{-18x} = K'(x)e^{-18x} - 18K(x)e^{-18x}$

$K'(x)e^{-18x} \cancel{-18K(x)e^{-18x}} \cancel{+18K(x)e^{-18x}} = -15e^{6x}$

$K'(x) = \frac{-15e^{6x}}{e^{-18x}} \Rightarrow K'(x) = -15e^{6x}\cdot e^{18x} = -15e^{24x}$

$K(x) = \int -15e^{24x}\,dx = -15\int e^{24x}\,dx = -15e^{24x}\cdot 24 = -360e^{24x} + c \quad c \in \mathbb{R}$

$z = K(x)\cdot e^{-18x} = (-360e^{24x} + c)\,e^{-18x} = -360e^{6x} + c(e^{18x}) + c \quad c \in \mathbb{R}$

$z = y^{-3} \Rightarrow z = \frac{1}{y^3} \rightarrow y = \quad y = \frac{1}{\sqrt[3]{z}}$

$$\boxed{y = \frac{1}{\sqrt[3]{-360e^{6x} + c(e^{-18x})}}, \ c \in \mathbb{R}}$$

Escaneado con CamScanner

(4) $\begin{cases} y'' + 6y' + 9y = 6e^{-3x} + 18 \\ y(0) = 2 \\ y'(0) = 25 \end{cases}$

• Ec. diferencial lineal homogenea asociada

$y'' + 6y' + 9y = 0$

$p(x) = x^2 + 6x + 9 = (x+3)^2 \rightarrow$ mult. 2 $\rightarrow -3 \rightarrow e^{-3x}, \; xe^{-3x}$

$y_h(x) = c_1 e^{-3x} + c_2 x e^{-3x}, \quad c_1, c_2 \in \mathbb{R}$

• solución particular

$y(x) = Ax^2 e^{-3x}$

$y'(x) = 2Axe^{-3x} - 3Ax^2 e^{-3x}$

$y''(x) = 2Ae^{-3x} - 6Axe^{-3x} - 6Axe^{-3x} + 9Ax^2 e^{-3x} = 2Ae^{-3x} - 12Axe^{-3x} +$

$+ 9Ax^2 e^{-3x}$

sustituimos:

$y'' + 6y' + 9y = 6e^{-3x} + 18$

$2Ae^{-3x} - \cancel{12Axe^{-3x}} + \cancel{9Ax^2} e^{-3x} + \cancel{12Ax} e^{-3x} - 18\cancel{Ax^2} e^{-3x} + \cancel{9Ax^2} e^{-3x} = 6e^{-3x} + 18$

$2Ae^{-3x} = 6e^{-3x} + 18 \; ; \; 2A = 6 + 18e^{3x} \; ; \; A = 3 + 9e^{3x}$

$y_p(x) = (3 + 9e^{3x})x^2 e^{-3x} = 3x^2 e^{-3x} + 9x^2$

• solución general

$y(x) = 3x^2 e^{-3x} + 9x^2 + c_1 e^{-3x} + c_2 x e^{-3x} \quad c_1 c_2 \in \mathbb{R}$

$y(0) = 2$

$3 \cdot 0 \cdot e^0 + 9 \cdot 0 + c_1 e^{-3 \cdot 0} + c_2 \cdot 0 e^0 \rightarrow \boxed{c_1 = 2}$

$y'(x) \; 6xe^{-3x} - 9x^2 e^{-3x} + 18x - 3c_1 e^{-3x} + c_2 e^{-3x} - 3c_2 e^{-3x}$

$y'(0) = 25$

$\cancel{6(0)e^0} - \cancel{90 \cdot e^0} + 18(0) - 3c_1 e^0 + c_2 e^0 - 3c_2 e^0 = 25$

$-3c_1 + c_2 - 3c_2 = 25 \rightarrow -6 - 2c_2 = 25 \rightarrow c_2 = \frac{31}{-2}$

$$\boxed{y(x) = 3x^2 e^{-3x} + 9x^2 + 2e^{-3x} + \frac{31}{2} x e^{-3x}}$$

5 $\begin{cases} y' + xy = 3x e^{x^2} \\ y(0) = 2 \end{cases}$

- Ec. diferencial lineal homogenea asociada

$y' + xy = 0$

$\frac{dy}{dx} = -xy; \int \frac{dy}{y} = \int -x\,dx; \ \log y = -\frac{x^2}{2} + c \to c \in \mathbb{R}$

$y = e^{-x^2/2 + c} = e^{-x^2/2} \cdot e^c = K e^{-x^2/2} \to y = K(x) e^{-x^2/2}$

$y' = K'(x) e^{-x^2/2} - K(x)\, x\, e^{-x^2/2}$

$\to K'(x) e^{-x^2/2} - K(x)\, x\, e^{-x^2/2} + K(x)\, x\, e^{-x^2/2} = 3x e^{x^2} \to K'(x) = \frac{3x e^{x^2}}{e^{-x^2/2}} \to$

$\to K(x) = \int \frac{3x e^{x^2}}{e^{-x^2/2}} dx = 3/2\, x^2 e^{x^2 + x^2/2} + c \quad c \in \mathbb{R}$

- Sustituimos

$y = \frac{3}{2} x^2 e^{x^2 + x^2/2} \cdot e^{-x^2/2} + c = e^{-x^2/2 + c} \to y = 3/2\, x^2 e^{x^2} = e^{-x^2/2} \cdot e^c$

$\cancel{3/2\, 0^2 e^{0^2}}\ 3/2\, 0^2 e^0 - e^0 \cdot e^c = 2 \to e^c = 2 \to$

$$\boxed{y = \cancel{2}\ 3/2\, x^2 e^{x^2} - e^{-x^2/2} \cdot e^c \ ; c \in \mathbb{R}}$$

6 $\underbrace{2xy - 3x^2y^2}_{F(x,y)} + \underbrace{(x^2 - 2x^3 y)}_{G(x,y)} y' = 0 \longrightarrow \frac{dF}{dy} = \frac{dG}{dx} = 0$

$\frac{dF}{dy} = 2x - 6x^2 y \ ; \ \frac{dG}{dx} = 2x - 6x^2 y$

$f / \frac{df}{dy} = F; \ \frac{df}{dy} = G \to f(x,y) = c, \ c \in \mathbb{R}$

$\frac{df}{dx} = F \to f = \int 2xy - 3x^2y^2\, dx \to f = y\int 2x\,dx - y\int 3x^2\,dx =$

$= \boxed{yx^2 - y^2x^3 + c}$

⑦ $y' - \frac{2}{x+2} y = 2(x+2)^3 \longrightarrow y'(x) + p(x)\,y = q(x)$

$p(x) = \frac{-2}{x+2}$; $q(x) = 2(x+2)^3$

$\int p(x)\,dx \to \int \frac{-2}{x+2}\,dx = -2\int \frac{1}{x+2}\,dx \to -2\ln(x+2) + c = \mu$

$y\mu\ \int q\mu\,dx \to y(x+2)^{-2} = \int 2(x+2)^3 \cdot (x+2)^{-2}\,dx \to$

$= y(x+2)^{-2} = \int 2x + 4\,dx = x^2 + 4x \to$

$\to \boxed{y = (x^2 + 4x)(x+2)^2 + c}$

⑧ $\underbrace{e^x y^2 + y\,\mathrm{sen}\,x}_{F(x,y)} + \underbrace{(3y e^x - 2\cos x)}_{G(x,y)}\, y' = 0$

$\frac{dF}{dy} = \frac{dG}{dx} \longrightarrow f \,/\, \frac{df}{dx} = F\ ;\ \frac{df}{dy} = G \to f(x,y) = c \quad c \in \mathbb{R}$

$\to \frac{df}{dx} = e^x y^3 + y^2\,\mathrm{sen}\,x \to f = \int e^x y^3 + y^2\,\mathrm{sen}\,x\,dx \to f = y^3 \int e^x\,dx + y^2 \int \mathrm{sen}\,dx$

$= \boxed{y^3 e^x - y^2 \cos x + c}$

$\to 3y^2 e^x - 2y\cos x + c'(y) = 3y e^x - 2\cos x\ y \to$

$c' = 0 \quad c(y) = 0$