1)

$$\begin{cases} (4x^2-2y^2)\,dx = 2xy\,dy \\ y(1)=3 \end{cases}$$

$$v = \frac{y}{x} \to y = vx$$
$$y' = v'x + v$$

$$4x^2-2y^2 = 2xy\,y' \to y' = \frac{4x^2-2y^2}{2xy}$$

$$v'x + v = \frac{4\cancel{x^2} - 2v^2\cancel{x^2}}{2\cancel{x^2}v} \to v'x+v = \frac{4-2v^2}{2v} \to v'x = \frac{4-2v^2}{2v} - v$$

$$v'x = \frac{4-2v^2-2v}{2v} \to \frac{dv}{dx}x = \frac{4-2v^2-2v}{2v} \to \int \frac{2v}{-2v^2-2v+4}\,dv = \int \frac{dx}{x}$$

$$\cancel{\int \frac{2v}{-2v^2-2v+4} = -\frac{1}{2}\int \frac{-2v+2+2}{-v^2-2v+2}}$$

$$\int \frac{v}{-v^2-v+2} = \int \frac{A}{x-1} + \frac{B}{-x-2} = \int \frac{-\frac{1}{3}}{v-1} + \int \frac{\frac{2}{3}}{-v-2} \to$$

$$-Av-2A+Bv-B = v \quad \begin{cases} -A+B = 1 \\ -2A-B = 0 \end{cases}$$
$$-3A = 1 \qquad A = -\frac{1}{3} \qquad B = \frac{2}{3}$$

$$\to -\frac{1}{3}\ln(v-1) - \frac{2}{3}\ln(-v-2) = \ln x + C_1 \to$$

$$\to \boxed{-\frac{1}{3}\ln\left(\frac{y}{x}-1\right) - \frac{2}{3}\ln\left(-\frac{y}{x}-2\right) = \ln x + C_1}$$

define de manera implícita las soluciones y de la ecuación

$$e^{-\frac{1}{3}\ln(\frac{y}{x}-1)} + e^{-\frac{2}{3}\ln(-\frac{y}{x}-2)} = e^{\ln x + C} + e^{c}$$

2) $y'' + 4y = -4\,\text{sen}(2x)$ $\quad p(x) = x^2 + 4 \to x = \pm 2i$

$y(0) = -1$

$y'(0) = 4$

$\boxed{y(x) = C_2 \cos(2x) + C_1\,\text{sen}(2x)}$

Misma estructura

$y_p = Ax\,\text{sen}(2x) + Bx\cos(2x)$

$y_p' = A\,\text{sen}(2x) + 2Ax\cos(2x) + B\cos(2x) - 2Bx\,\text{sen}(2x)$

$y_p'' = A\,\text{sen}(2x) + 2A\cos(2x) - 4Ax\,\text{sen}(2x) + B\cos(2x) - 2B\,\text{sen}(2x) - 4Bx\cos(2x)$

$A\,\text{sen}(2x) + 2A\cos(2x) - \cancel{4Ax\,\text{sen}(2x)} + B\cos(2x) - 2B\,\text{sen}(2x) - \cancel{4Bx\cos(2x)} +$
$+ \cancel{4Ax\,\text{sen}(2x)} + \cancel{4Bx\cos(2x)} = -4\,\text{sen}(2x)$

$A - 2B = -4 \to A + 4A = -4 \to A = -\frac{4}{5}$

$\cancel{2A - 4B} = 2A + B = 0 \to B = -2A \quad B = -\frac{8}{5}$

$y(x) = -\frac{4}{5}x\,\text{sen}(2x) - \frac{8}{5}x\cos(2x) + C_1\,\text{sen}(2x) + C_2\cos(2x)$

$\boxed{-1 = C_2}$

$y'(x) = -\frac{4}{5}\,\text{sen}(2x) - \frac{8}{5}x\cos(2x) \quad -\frac{8}{5}\cos(2x) + \frac{16}{5}x\,\text{sen}(2x) + 2C_1\cos(2x) - 2C_2\,\text{sen}(2x)$

$4 = -\frac{8}{5} + 2C_1$

$C_1 = \frac{14}{5}$

$\boxed{y(x) = -\frac{4}{5}x\,\text{sen}(2x) - \frac{8}{5}x\cos(2x) + \frac{14}{5}\text{sen}(2x) - \cos(2x)}$

3) $y' - 6y = 5e^{6x} y^4$

$y' - 6y = 0 \rightarrow \frac{dy}{dx} = 6y \rightarrow \int \frac{dy}{y} = \int 6\, dx \rightarrow \ln y = 6x$

$y = K(x) e^{6x}$ $\quad y' = K'(x) e^{6x} + 6K(x) e^{6x}$

$\cancel{6K(x) e^{6x}} + K'(x) e^{6x} - \cancel{6K(x) e^{6x}} = 5e^{6x} y^4$

$K'(x) \cancel{e^{6x}} = 5\cancel{e^{6x}} y^4 \rightarrow K'(x) = 5y^4 \rightarrow K(x) = \int 5y^4 = y^5$

$\boxed{\cancel{y = y^5 e^{6x}}}$ $\quad \boxed{y = (y^5 + C) e^{6x}}$ $\quad C \in \mathbb{R}$

1) $y'' + 6y' + 9y = 6e^{-3x} + 18$

$y(0) = 2$

$y'(0) = 25$

$p(x) = x^2 + 6x + 9 \qquad \frac{-6 \pm \sqrt{6^2 - 4 \cdot 9}}{2} = \frac{-6 \pm 0}{2} = -3$ m.2

$y(x) = C_1 e^{-3x} + C_2 x e^{-3x}$

$y_p = Ax^2 e^{-3x} \cancel{+B} + B$

$y'_p = 2Axe^{-3x} - 3Ax^2 e^{-3x}$

$y''_p = 2Ae^{-3x} - 6Axe^{-3x} - 6Axe^{-3x} + 9Ax^2 e^{-3x}$

$2Ae^{-3x} \cancel{-6Axe^{-3x}} \cancel{-6Axe^{-3x}} \cancel{+9Ax^2e^{-3x}} \cancel{+12Axe^{-3x}} \cancel{-18Ax^2e^{-3x}} \cancel{+9Ax^2e^{-3x}} + 9B$

$2Ae^{-3x} + 9B = 6Ae^{-3x} + 18 \qquad \boxed{A = 3} \quad \boxed{B = 2}$

$y(x) = 3x^2 e^{-3x} + 9 + C_1 e^{-3x} + C_2 x e^{-3x}$

$2 = 9 + C_1 \rightarrow \boxed{C_1 = -7}$

$y'(x) = 6\cancel{x}e^{-3x} - 9x^2 e^{-3x} - 3C_1 e^{-3x} - 3\cancel{C_2 x e^{-3}} + C_2 e^{-3x}$

$25 = -3C_1 + C_2 \rightarrow 25 = 21 + C_1 \rightarrow \boxed{C_2 = 4}$

$\boxed{y(x) = 3x^2 e^{-3x} + 9 - 7e^{-3x} + 4x e^{-3x}}$

5) $\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$

$y' + xy = 0 \rightarrow y' = -xy \rightarrow \frac{dy}{dx} = -xy \rightarrow \int \frac{dy}{y} = \int -x \, dx$

$\ln y = -\frac{x^2}{2} + C \rightarrow y = e^{-\frac{x^2}{2}} \cdot k(x)$

$y' = k'(x) e^{-\frac{x^2}{2}} + k(x) \cdot e^{-\frac{x^2}{2}}(-x)$

$k'(x) e^{-\frac{x^2}{2}} - \cancel{k(x) x e^{-\frac{x^2}{2}}} + \cancel{k(x) x e^{-\frac{x^2}{2}}} = 3xe^{x^2}$

$k'(x) = \frac{3xe^{x^2}}{e^{-\frac{x^2}{2}}} \rightarrow k'(x) = 3xe^{x^2} \cdot (e^{\frac{x^2}{2}})^3$

$K(x) = \int 3xe^{x^2} e^{\frac{x^2}{2}} dx \rightarrow K(x) = (e^{\frac{x^2}{2}})^3 + C$

$y(x) = e^{-\frac{x^2}{2}} \left((e^{\frac{x^2}{2}})^3 + C \right) \rightarrow y(x) = e^{-\frac{x^2}{2}} C + (e^{\frac{x^2}{2}})^2$

$2 = C \rightarrow \boxed{y(x) = 2e^{-\frac{x^2}{2}} + (e^{\frac{x^2}{2}})^2}$

6) $\underbrace{2xy - 3x^2y^2}_{M} + \underbrace{(x^2 - 2x^3y)}_{N}y' = 0$

$\frac{dM}{dy} = 2x - 6x^2y$ Integral exacta

$\frac{dN}{dx} = 2x - 6x^2y$

$\frac{df}{dx} = M \rightarrow \frac{df}{dx} = 2xy - 3x^2y^2 \rightarrow f = \int 2xy - 3x^2y^2\,dx$

$f = \int 2xy - \int 3x^2y^2 \rightarrow f = 2y\int x - y^2\int 3x^2 \rightarrow$

$\underline{f = yx^2 - y^2x^3 + K(y)}$

$\frac{df}{dy} = N \rightarrow \cancel{x^2 - x^3 + K'(y) = x^2 - 2x^3y}$

$\cancel{K'(y) = x^2 - 2x^3y + x^3 - x^2}$

$\cancel{K(y) = \int -2x^3y + x^3 \rightarrow K(y) = -2x^3\int 2y}$

$K(y) = -x^3y^2$

$\frac{df}{dy} \rightarrow \cancel{x^2} - \cancel{2yx^3} + K'(y) = \cancel{x^2} - \cancel{2x^3}y$

$K'(y) = 0 \rightarrow \underline{K(y) = 0}$

$f = yx^2 - y^2x^3 \rightarrow \boxed{C = yx^2 - y^2x^3}$ (Define de forma implícita las soluciones y de la ecuación)

7) $y' - \frac{2}{x+2} y = 2(x+2)^3$

$y' - \frac{2}{x+2} y = 0 \rightarrow y' = \frac{2}{x+2} y \rightarrow \int \frac{dy}{y} = \int \frac{2dx}{x+2}$

$\ln y = 2 \ln (x+2) + C \qquad y = e^{2\ln(x+2)} + \underset{K}{e^{c}}$

$y = K(x) \cdot (x+2)^2$

$y' = K'(x)(x+2)^2 + K(x) \cdot 2 \cdot (x+2)$

$K'(x)(x+2)^2 = \cancel{\frac{2}{x+2}} + \cancel{K(x) \cdot 2(x+2)} - \cancel{\frac{2}{(x+2)} K(x)(x+2)^2} = 2(x+2)^3$

$K'(x) \cancel{(x+2)^2} = 2(x+2)^{\cancel{3}} \rightarrow K(x) = \int 2(x+2)$

$K(x) = 2\int x+2 \rightarrow \underline{\underline{K(x) = x^2 + 4x}} + C$

$\boxed{y = (x^2 + 4x + C)(x+2)^2}$ (Define de forma implícita las soluciones y de la ecuación)

8) $\underbrace{e^x y^2 + y\,\text{sen}\,x}_{M} + \underbrace{(3ye^x - 2\cos x)}_{N} y' = 0$

$\frac{dM}{dy} = \frac{dN}{dx} \rightarrow \frac{\mu(y)(e^x y^2 + y\,\text{sen}\,x)}{dy} = \mu'(y)(e^x y^2 + y\,\text{sen}\,x) + \mu(y)\cdot(2e^x y + \text{sen}\,x)$

$\frac{dN}{dx} \rightarrow \frac{\mu(y)(3ye^x - 2\cos x)}{dx} = \mu(y)\cdot(3ye^x + 2\,\text{sen}\,x)$

$\mu'(e^x y^2 + y\,\text{sen}\,x) + \mu(2e^x y + \text{sen}\,x) = \mu(3ye^x + 2\,\text{sen}\,x)$

$\mu'(e^x y^2 + y\,\text{sen}\,x) = \mu(3ye^x + 2\,\text{sen}\,x) - \mu(2e^x y + \text{sen}\,x)$

$\mu'(e^x y^2 + y\,\text{sen}\,x) = (3ye^x + 2\,\text{sen}\,x - 2e^x y - \text{sen}\,x)\mu$

$\frac{\mu'}{\mu} = \frac{ye^x + \text{sen}\,x}{e^x y^2 + y\,\text{sen}\,x} \rightarrow \frac{\mu'}{\mu} = \frac{\cancel{(ye^x + \text{sen}\,x)}}{\cancel{(ye^x + \text{sen}\,x)}\,y} \rightarrow \frac{d\mu}{dy} = \frac{\mu}{y}$

$\int \frac{d\mu}{\mu} = \int \frac{dy}{y} \rightarrow \cancel{\ln}\ \boxed{\mu = \cancel{\ln}\, y}$

$\frac{df}{dx} = M \rightarrow \frac{df}{dx} = e^x y^3 + y^2\,\text{sen}\,x \rightarrow \int df = \int e^x y^3 + y^2\,\text{sen}\,x\ dx \rightarrow$

$\rightarrow f = y^3 e^x + y^2 \cos x + K(y)$

$\frac{df}{dy} = N \rightarrow \cancel{3y^2 e^x} - \cancel{2y\cos x} + K'(y) = \cancel{3y^2 e^x} - \cancel{2y\cos x}$

$K'(y) = 0 \rightarrow K(y) = 0$

$\cancel{y(x)} = \boxed{y^3 e^x - y^2 \cos x = C}$ (define de forma implícita las soluciones y de la ecuación)

9) $y^{vii} + y^{vi} + 2y^{v} + 10y^{iv} + 13y''' + 5y'' = 0$

$p(x) = x^7 + x^6 + 2x^5 + 10x^4 + 13x^3 + 5x^2$

	5	4	3	2	1	0		
	1	1	2	10	13	5	$\cancel{0}$	0
-1		-1	0	-2	-8	-5		
	1	0	2	8	5	$\boxed{0}$		
-1		-1	1	-3	-5			
	1	-1	3	5	$\boxed{0}$			
-1		-1	2	-5				
	1	-2	5	$\boxed{0}$				

$x = 0 \quad m.2$

$x = -1 \quad m.3$

$$\frac{2 \pm \sqrt{4 - 4 \cdot 5}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$

$x = 1 \pm 2i$

$$\boxed{y(x) = C_1 + C_2 x + C_3 e^{-x} + C_4 x e^{-x} + C_5 x^2 e^{-x} + C_6 e^{x} \operatorname{sen}(2x) + C_7 e^{x} \cos(2x)}$$