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1) $\begin{cases}(4x^2-2y^2)\,dx = 2xy\,dy \\ y(1)=3\end{cases}$

ec. homogénea

$(4t^2x^2 - 2t^2y^2)\,dx = 2tx\cdot ty\,dy$

$t^2(4x^2-2y^2)\,dx = 2t^2xy\,dy$

$(4x^2-2y^2)\,dx = 2xy\,dy$,

$y = ux \quad dy = u\,dx + x\,du$

$(4x^2 - 2(ux)^2)\,dx = 2x\,ux(u\,dx + x\,du)$

$(4x^2 - 2u^2x^2)\,dx = 2ux^2(u\,dx + x\,du)$

$x^2(4-2u^2)\,dx = 2ux^2(u\,dx + x\,du)$

$(4-2u^2)\,dx = 2ux\,dx + 2ux\,du$

$(4-2u^2-2u^2)\,dx = 2ux\,du$

$(4-4u^2)\,dx = 2ux\,du$

$\frac{du}{x}\cdot\frac{2u}{4-4u^2}\,du \qquad \int\frac{dx}{x} = \int\frac{2u}{1-4u^2}\,du$

$\int\frac{dx}{x} = \ln v$

$\int\frac{2u}{4-4u^2}\,du = 2\int\frac{dt/-8}{t} = -\frac{1}{4}\int\frac{dt}{t} = \frac{1}{4}\ln|t| =$

$= -\frac{1}{4}\ln|4-4u^2|$

$4-4u^2 = t$

$-8u\,du = dt$

$u\,du = \frac{dt}{-8}$

$\ln x = -\frac{1}{4}\ln|4-4u^2| \quad$ con $u = y/x$

$\ln x = -\frac{1}{4}\ln\left|4-4\left(\frac{y^2}{x^2}\right)\right|$

$-4\ln x = \ln\left|4-4\frac{y^2}{x^2}\right|$

$\ln x^{-4} = \ln\left|4-4y^2/x^2\right|$

$\frac{1}{x^4} = 4 - \frac{4y^2}{x^2} + C$

$\frac{1}{1^4} = 4 - \frac{4(-3)^2}{1^2} + C$

$1 = 4-36+C$

$C = 33$

Solución $\frac{1}{x^4} = 4 - \frac{4y^2}{x^2} + 33$

2) $y'' + 4y = -4\sin 2x$

$y(0) = -1$

$y'(0) = 4$

$r^2+4=0 \quad r^2=-4 \quad r = \sqrt{-4} = \pm 2i$

$y_c = C_1\cos 2x + C_2\sin 2x$ ← Solución general.

Solución particular

$y_p = Ax\cos 2x$

$y_p' = A\cos 2x + Ax(-2\sin 2x) = A\cos 2x - 2Ax\sin 2x$

$y_p'' = -2A\sin 2x - [2A\sin 2x + 2Ax\cos 2x] = -2A\sin 2x - 2A\sin 2x - 4Ax\cos 2x =$

$= -4A\sin 2x - 4Ax\cos 2x$

$= -4A\sin 2x - 4Ax\cos 2x + 4Ax\cos 2x = -4\sin 2x - 4A = -4$ $\boxed{A=1}$

$y_p = x\cos 2x$

Solución: $y = C_1\cos 2x + C_2\sin 2x + x\cos 2x$

$y(0) = -1 \rightarrow y = C_1\cdot\cos 0 + C_2\sin 0 + 0\cos 0 = -1$

$\boxed{C_1 = -1}$

$y(0) = 4 \Rightarrow y' = -2C_1\sin 2x + 2C_2\cos 2x + \cos 2x + x(-\sin 2x) = 2C_2 + 1 = 4$ $\boxed{C_2 = 3/2}$

Solución

$\boxed{y = -\cos 2x + \frac{3}{2}\cdot\sin 2x + x\cos 2x}$