JAIME CORREAS

1.- $\begin{cases} (4x^2 - 2y^2)\,dx = 2xy\,dy \\ y(1) = 3 \end{cases}$

$(4x^2 - 2y^2)\,dx = 2xy\,dy$ Ec. homogenea

$\begin{cases} y = vx \\ y' = v'x + v \end{cases}$

$\frac{4x^2 - 2y^2}{2xy} = \frac{dy}{dx}$; $\frac{4x^2 - 2y^2}{2xy} = y'$ *

* $\frac{4x^2 - 2y^2}{2xy} \Rightarrow \left(\frac{2x^2 - y^2}{xy} \right)$; $\frac{2x^2 - v^2x^2}{x^2 v} = v'x + v$;

$\frac{x^2(2 - v^2)}{x^2 \cancel{v}} = v'x + v \Rightarrow \frac{2 - v^2}{v} = \frac{dv}{dx} x + v$;

$\frac{2}{v} - \frac{v^2}{v} \Rightarrow \frac{2}{v} - v \Rightarrow \frac{2}{v} - v = \frac{dv}{dx} x + v$;

$\frac{2}{v} - v - v = \frac{dv}{dx} x$; $\frac{2}{v} - 2v = \frac{dv}{dx} x$;

$\frac{2 - 2v^2}{v} = \frac{dv}{dx} x$; $dx(2 - 2v^2) = dv \cdot v \cdot x$

$\frac{dx}{x} = \frac{v}{2 - 2v^2} dv \Rightarrow \int \frac{dx}{x} = \int \frac{v}{2 - 2v^2} dv \Rightarrow$ *

$\boxed{\operatorname{Log}|x| = \frac{1}{4} \log|1 - v^2| + C_1}$ * $x = e^{\frac{1}{4}\log\left|1 - \frac{y^2}{x^2}\right|} + C_1$

* $\int \frac{v}{2 - 2v^2} = \frac{1}{2} \int \frac{v}{1 - v^2} = \frac{1}{4}\frac{1}{2} \int (1 - v^2)\, v \cdot 2 = \frac{1}{4} \operatorname{Log}|1 - v^2|$

$\boxed{C_1 \simeq 2}$

* $3 = e^{\frac{1}{4}\log\left|1 - \frac{1}{9}\right|} + C_1$; $3 = 0'97 + C_1$

$3 - 0'97 = C_1$

(2)

$$\begin{cases} y'' + 4y = -4\,\mathrm{Sen}(2x) \\ y(0) = -1 \\ y'(0) = 4 \end{cases}$$

$\Rightarrow y'' + 4y = 0$ Ec. lineal. h. asociada

$x^2 + 4 = 0 \quad x = \pm\sqrt{-4} = \pm\sqrt{-1}\sqrt{4} = \pm i2 = 0 \pm 2i$

Dos raices con multi $\left\langle \begin{matrix} +0+2i \\ 0-2i \end{matrix} \right. \Rightarrow \begin{matrix} \to e^{0x}\,\mathrm{Sen}(2x) \\ \to e^{0x}\cos(2x) \end{matrix}$

$\to \mathrm{Sen}(2x)$ Sol. general:

$\to \cos(2x)$

$$y(x) = C_1\,\mathrm{Sen}(2x) + C_2\cos(2x)$$

$C_1, C_2 \in \mathbb{R}$

• $y'' + 4y = -4\,\mathrm{Sen}(2x)$

$y = A\,\mathrm{Sen}(2x) \qquad y' = 2A\cos(2x) \quad y'' = -4A\,\mathrm{Sen}(2x)$

$\cancel{-4A\,\mathrm{Sen}(2x)} + \cancel{4A\,\mathrm{Sen}(2x)} = -4\,\mathrm{Sen}(2x)$ Caso conflictivo

$0 = -4\,\mathrm{Sen}(2x)$

• $y = x(-4\,\mathrm{Sen}\,2x) \qquad y' = -4\,\mathrm{Sen}\,2x + (-8\cos(2x))$

$y' = -4\,\mathrm{Sen}\,2x - 8\cos(2x)$

$y'' = -8\cos 2x + 16\,\mathrm{Sen}(2x)$

$-8\cos(2x) + 16\,\mathrm{Sen}(2x) + 4x(-4\,\mathrm{Sen}\,2x) = -4\,\mathrm{Sen}(2x)$

Continuación ej. 2 JAIME CORBAS

$y'' + 4y = -4\operatorname{Sen}(2x)$

$y = Ax\cos(2x) + Bx\operatorname{Sen}(2x)$

$y' = A\cos(2x) - 2Ax\operatorname{Sen}2x + B\operatorname{Sen}(2x) + 2Bx\cos(2x)$

$y'' = -2A\operatorname{Sen}(2x) - 2A\operatorname{Sen}(2x) + 4Ax\cos(2x) + 2B\cos(2x)$

$+ 2B\cos(2x) - 4Bx\operatorname{Sen}(2x)$

$-4A\operatorname{Sen}(2x) + \cancel{4Ax\cos(2x)} + 4B\cos(2x) - \cancel{4Bx\operatorname{Sen}(2x)}$

$+ \cancel{4Ax\cos(2x)} + \cancel{4Bx\operatorname{Sen}(2x)} = -4\operatorname{Sen}(2x)$

$-4A\operatorname{Sen}(2x) + 4B\cos(2x) = -4\operatorname{Sen}(2x$

$-4A = -4 \quad \boxed{A = 1}$ Sol part:

$4B = 0 \quad \boxed{B = 0}$ ~~$y = \operatorname{Sen}(2x)$~~ $\boxed{y = x\cos(2x)}$

Sol general de $y'' + 4y = -4\operatorname{Sen}(2x)$

$y = x\cos(2x) + C_1\operatorname{Sen}(2x) + C_2\cos(2x)$

$y(0) = -1$

$-1 = \underbrace{0\cdot\cos(2\cdot0)}_{0} + \underbrace{C_1\operatorname{Sen}(2\cdot0)}_{0} + C_2\cos(0\cdot2)$ $\boxed{C_2 = -1}$

$y'(0) = 4$

$4 = \cos(2x) - 2x\operatorname{Sen}(2x) + 2C_1\cos(2x) - 2C_2\operatorname{Sen}(2x)$

$4 = 1 - 0 + 2C_1 - 0 \quad \boxed{C_1 = \frac{3}{2}}$

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SALME CORREAS

(3) $y' - 6y = 5e^{6x} y^4$

$z = y^{1-\alpha} \Rightarrow z = y^{-3} \Rightarrow z' = -3y^{-4} \cdot y'$

$z' = -3y^{-4}(5e^{6x}y^4 + 6y) = -15e^{6x} + (-18y^{-3})$

$z' = -15e^{6x} - 18y^{-3} \rightarrow z' = -15e^{6x} - 18z$

$\cancel{\frac{dz}{dx} = -15e^{6x} z}$; $\underline{z' + 18z = -15e^{6x}}$

$z' + 18z = 0 \rightarrow \frac{dz}{dx} = -18z$; $\int \frac{dz}{z} = \int -18\,dx$

$\log|z| = -18x + C'$; $z = ke^{-18x}$

$z = k(x)e^{-18x}$; $z' = k'(x)e^{-18x} - 18k(x)e^{-18x}$

$k'(x)e^{-18x} \cancel{-18k(x)e^{-18x}} + \cancel{18(k(x)e^{-18x})} = -15e^{6x}$

$k'(x) = \frac{-15e^{6x}}{e^{-18x}} = -15e^{24}$

$k(x) = \int -15e^{24} = -15\int e^{24} = \cancel{-360e}\ -15xe^{24}$

$z = k(x)e^{-18x} = -15xe^{24} \cdot e^{-18} = -15xe^{6}$

$z = y^{-3}$; $(-15xe^{6})^{-\frac{1}{3}} = y$

$$\boxed{y = \frac{1}{\sqrt[3]{-15xe^{6}}}}$$

(4)

JAIME CORREAS

$$\begin{cases} y'' + 6y' + 9y = 6^{-3x} + 18 \\ y(0) = 0 \ , \ y(0) = 15 \end{cases}$$

$y'' + y' + 9y = 6^{-3x} + 18$ Ec. lineal de 2° Orden

$y'' + y' + 9y = 0$

$x^2 + 6x + 9 = 0$

$x \ \frac{-6 \pm \sqrt{36-36}}{2} = -3$ con mult. 2.

Sol general : $\to$ e^{-3x} , $-3e^{-3x}$

$$y_h(x) = C_1 e^{-3x} + C_2 (x) e^{-3x} \quad C_1, C_2 \in \mathbb{R}$$

Sol. particular:

$y(x) = \cancel{Ae^{-3x}} = Ax^2 e^{-3x}$

$y'(x) = 2xAe^{-3x} + (-3Ax^2 e^{-3x})$

$y''(x) = 2Ae^{-3x} + 2x(-3)Ae^{-3x} - 6Axe^{-3x} + 9Ax^2e^{-3x}$

$= 2Ae^{-3x} + -6xAe^{-3x} - 6Axe^{-3x} + 9Ax^2e^{-3x}$

$= 2Ae^{-3x} - 12xAe^{-3} + 9Ax^2e^{-3x}$

$2Ae^{-3x} \cancel{-12xAe^{-3}} \cancel{+9Ax^2e^{-3x}} \cancel{+12xAe^{-3x}} \cancel{-18Ax^2e^{-3}} + 9Ax^2e^{-3x}$

$= 6^{-3x} + 18$

$2Ae^{-3x} = 6^{-3x} + 18$; $2A = 6 + 18e^{3x}$; $A = 3 + 9e^{3x}$

Sol: $\boxed{y(x) = 3x^2e^{-3x} + 9x^2 + C_1e^{-3x} + C_2xe^{-3x}}$, $C_1, C_2, \in \mathbb{R}$

$y(0) = 2$

$3 \cdot 0 \cdot e^0 + 9 \cdot 0 + C_1 e^0 + C_2 0 \cdot e^0 = 2$, $\boxed{C_1 = 2}$

$y'(x) = 6xe^{-3x} - 9x^2e^{-3x} + 18x - 3C_1e^{-3x} + C_2e^{-3x} - 3C_2e^{-3x}$

$y'(0) = 5$

$-3C_1 + C_2 = 25$; $-6 + C_2 = 25$; $C_2 = 31$

JAIME CONTRERAS

(5) $\begin{cases} y' + xy = 3x e^{x^2} \\ y(0) = 2 \end{cases}$

$y' + xy = 0 \Rightarrow y' = -xy \Rightarrow \frac{dy}{dx} = -xy$

$\frac{dy}{y} = -x\,dx$; $\log(y) = -\int x\,dx$; $\log|y| = -\frac{x^2}{2} + C_1$

$y = e^{-\frac{x^2}{2} + C_1} = K e^{-\frac{x^2}{2}}$; $y' = K'(x) e^{-\frac{x^2}{2}} + K(x) \cdot -x e^{-\frac{x^2}{2}}$

$K'(x) e^{-\frac{x^2}{2}} \cancel{- K(x) x e^{-\frac{x^2}{2}}} + x \cancel{K(x) e^{-\frac{x^2}{2}}} = 3x e^{x^2}$

$K'(x) = \frac{3x e^{x^2}}{e^{-\frac{x^2}{2}}} = 3x \left(e^{x^2 + \frac{x^2}{2}} \right) = 3x e^{\frac{3}{2}x^2}$

$K(x) = \int 3x e^{\frac{3}{2}x^2} = e^{\frac{3}{2}x} + C$

$y = e^{\frac{3}{2}x} \cdot e^{-\frac{x^2}{2}}$; $y = e^{\frac{x^2+3x}{2}}$

$y = \left(e^{\frac{3}{2}x} + C\right) e^{-\frac{x^2}{2}} \Rightarrow y = e^{\frac{x^2+3x}{2}} + C e^{-\frac{x^2}{2}}$

$y(0)$ $\boxed{2 = e^{\frac{0+0}{2}} + C e^{-\frac{0^2}{2}}} \rightarrow \boxed{C = 1}$

JAIME CORREAS

⑥ $2xy - 3x^2y^2 + (x^2 - 2x^3y)\,y' = 0$ Ec. dif. exacta

$$\underbrace{2xy - 3x^2y}_{M(x,y)} + \underbrace{(x^2 - 2x^3y)}_{N(x,y)}\,y' = 0$$

$$\frac{\delta M}{\delta y} = (2x \cdot 1 - 6x^2) \qquad \frac{\delta N}{\delta x} = (2x - 6x^2) \qquad f(x,y) = C,\ C \in \mathbb{R}$$

$$\frac{\delta f}{\delta x} = M \ ; \ \frac{\delta f}{\delta x} = 2xy - 3x^2y^2 \ // \ f(x) = \int 2xy - 3x^2y^2\,dx =$$

$$= \frac{2y}{2}x^2 - \frac{3y^2}{3}x^3 + K(y) = \underline{\underline{yx^2 - y^2x^3 + K(y)}}$$

$$\frac{\delta f}{\delta y} = N \Rightarrow x^2 - 2x^3y + K'(y) = N$$

$$x^2 - 2yx^3 + K'(y) = x^2 - 2x^3y$$

$$K'(y) = 0 \ // \ K(y) = 0$$

$$f(x,y) = yx^2 - y^2x^3$$

$yx^2 - yx^3 = C \quad C \in \mathbb{R}$ Define de forma implícita las soluciones ~~de~~ y de la ec. dif.

JAIME CORREAS

(7)

$y' - \frac{2}{x+2} y = 2(x+2)^3 \Rightarrow$ Ec. lineal. de 1er orden

$P(x) = x - \frac{2}{x+2}$; $y' - \frac{2}{x+2} y = 0 = y' = \frac{2}{x+2} y$

$\frac{dy}{dx} = \frac{2}{x+2} y \Rightarrow \int \frac{dy}{y} = \int \frac{2}{x+2} dx$

$\Rightarrow \log|y| = 2\log|x+2| + C_1$

$y = e^{2\log|x+2|} + C_1' \qquad y = k(x) e^{2\log|x+2|}$

$k \qquad y' = k'(x) e^{2\log|x+2|} + k(x) e^{2\log|x+2|} \cdot \frac{2}{x+2}$

$k'(x) e^{2\log|x+2|} + k(x) \cancel{\frac{2}{x+2}} e^{2\log|x+2|} - \cancel{\frac{2}{x+2}} k(x) e^{2\log|x+2|}$

$= 2(x+2)^3 \qquad k'(x) = \frac{2(x+2)^3}{e^{2\log|x+2|}}$

$k(x) = \int \frac{2(x+2)^3}{e^{\log|x+2|}} =$

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JAIME CORRALES

$$y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y^{III} + 5y^{II} = 0$$

$$r^7 + r^6 + 2r^5 + 10r^4 + 13r^3 + 5r^2 = 0$$

$$r^2(r^5 + r^4 + 2r^3 + 10r^2 + 13r + 5) = 0$$

	1	1	2	10	13	5
-1		-1	0	-1	-8	-5
	1	0	2	8	5	$\boxed{0}$

-1		-1	1	-3	-5
	1	-1	3	5	$\boxed{0}$

-1		-1	2	-5
	1	-2	5	$\boxed{0}$

$r = -1$ mult 3

$r = 0$ mult ~~a~~ 2

$r = 1 \pm 2i$ mult 1

$r = 0$ $\quad C + x$

$r = -1$ $\quad e^{-x}, xe^{-x} + x^2e^{-x}$

$r = 1 \pm 2i \rightarrow e^{1x}\cos 2x$

$\qquad\qquad \searrow e^{1x}\operatorname{Sen} 2x$

$\{ C, x, e^{-x}, xe^{-x}, e^{x}\cos x, e^{x}\operatorname{Sen} x \}$

Es una base del conjunto de todas las soluciones de la ec. diferencial.

Sol. General:

$$C_1 + C_2x + C_3e^{-x} + C_4xe^{-x} + C_5x^2e^{-x}$$

$$+ C_6e^{x}\cos 2x + C_7e^{x}\operatorname{Sen} 2x$$

$$C_1 \ldots C_7 \in \mathbb{R}$$