1 $\begin{cases} (4x^2-2y^2)dx = 2xy \cdot dy \\ y(1)=3 \end{cases}$

$(4x^2-2y^2)\frac{dx}{dx} = 2\cancel{x} \cdot y \cdot \frac{dy}{dx} \Rightarrow y'(2xy) = 4x^2-2y^2 \Rightarrow y' = \frac{4x^2}{2xy} - \frac{2y^2}{2xy} \Rightarrow$

$y' = \frac{2x}{y} - \frac{y}{x} \Rightarrow y' + \frac{1}{x}y = 2xy^{-1} \rightarrow$ Bernoulli $(y'+p(x)y = q(x)y^n)$

$z = y^{1-\alpha} = y^{1-\alpha \cdot (-1)} = y^2 \Rightarrow \boxed{z=y^2}$ $\quad \boxed{\alpha = -1}$

$z' = 2y \cdot y' = 2y \cdot (2xy^{-1} - \frac{y}{x}) = 4x - \frac{2}{x} \cdot y^2$

$z' = 4x - 2/x \cdot z \Rightarrow z' + \frac{2}{x} z = 4x \quad z' + 2/x \cdot z = 4 \quad z' + \frac{2}{x} \cdot z = 0 \Rightarrow z' = -\frac{2}{x} \cdot z$

$\frac{dz}{z} = -\frac{2}{x} \cdot dx \Rightarrow \int \frac{dz}{z} = \int -\frac{2}{x} \cdot dx \Rightarrow \ln z = -2\ln x + C \Rightarrow z = e^{-2\ln(x)+c}$

$z = \cancel{e}^{\cancel{\ln} x^{-2}} \cdot \underbrace{e^c}_{k>0} = \boxed{x^{-2} \cdot k}$

$k'x^{-2} + k \cdot (-2x^{-3}) + \frac{2}{x} \cdot k \cdot x^{-2} = 4x \Rightarrow k'x^{-2} \cancel{+ k(-2x^{-3}) + k2x^{-3}} = 4x \Rightarrow$

$\Rightarrow k' \cdot x^{-2} = 4x \Rightarrow k' = 4x \cdot x^2 \Rightarrow \boxed{k' = 4x^3}$

$k(x) = \int 4x^3 = \boxed{x^4 + C}$

$z = (x^4+C) \cdot x^{-2} = x^2 + C \cdot x^{-2} \rightarrow z = y^2$

$y^2 = x^2 + \frac{C}{x^2} \Rightarrow \frac{x^4+C}{x^2} \Rightarrow y = \sqrt{\frac{x^4+C}{x^2}}$

$y(1) = \sqrt{\frac{1^4+C}{1^2}} \boxed{=3} \Rightarrow$

$\Rightarrow \sqrt{\frac{1+C}{1}} = 3 \Rightarrow \sqrt{1+C} = 3 \quad \sqrt{C} = 3-1 \quad \boxed{C = 2^2} \quad \boxed{C=4}$

Escaneado con CamScanner

2 $\begin{cases} y''+4y=-4\,\text{sen}(2x) \\ y(0)=-1 \\ y'(0)=4 \end{cases}$

1º Ec diferencial homogenea asociada

$y''+4y=0 \Rightarrow P(\lambda)=x^2+4=0 \Rightarrow \lambda=\sqrt{-4}=\pm 2i$ $\nearrow \cos 2x$ $\searrow \text{sen}\, 2x$

$y(x)=C_1\cos(2x+C_2\,\text{sen}(2x)$

Solucion

$y(x)=A_1\cos(2x)+B_1\,\text{sen}(2x) \Rightarrow y'(x)=A\cos(2x)-2Ax\,\text{sen}(2x)+B_1\,\text{sen}(2x)+2B_1\cos(2x)$

$y''=-2A\cdot\text{sen}(2x)-2A\,\text{sen}(2x)-4A\cos(2x)+2B\cos(2x)+2B\cos(2x)-4B_1\cdot\text{Sen}\,2x=$

$\Rightarrow 4\ \text{sen}(2x)-4Ax\cos(2x)+4B\cos(2x)-4B\,\text{sen}(2x)$

$y''+4y=-4\,\text{sen}(2x)$

$-4A\,\text{sen}(2x)-\cancel{4Ax\cos(2x)}+4B\cos(2x)-\cancel{4Bx\,\text{sen}(2x)}+4Ax\cdot\cos(2x)+4Bx\cdot\text{sen}(2x)=-4\,\text{sen}(2x)$

$-4A\,\text{sen}2x+4B\cos(2x)=-4\,\text{sen}(2x)$

$\begin{cases} -4A=-4 \Rightarrow A=1 \\ 4B=0 \Rightarrow B=0 \end{cases}$

$y(x)=x\cos(2x)$

$y(x)=x\cos(2x)+C_1\cos(2x)+C_2\,\text{sen}(2x)$

$y'=\cos(2x)-2x\,\text{sen}(2x)-2C_1\cdot\text{sen}(2x)+2C_2\cos(2x)$

$-1=y(0)=0\cdot\cos(2x)+C_1\cdot\cos(0)+C_2\,\text{Sen}(0) \Rightarrow \boxed{C_1=-1}$

$4=y'(0)=\cos 0-2\cdot 0\,\text{Sen}(2\cdot 0)-2C_1\cdot\text{sen}(0)+2C_2\cdot\cos(2\cdot 0) \Rightarrow C_2=3/2$

$y(x)=x\cdot\cos(2x)-\cos(2x)+3/2\ \text{sen}(2x$

$3.\ y' - 6y = 5e^{6x}y^4$

$z' = -3y^{-4}\cdot y' = -3y^{-4}\cdot(5e^{6x}y^4 + 6y) = -15e^{6x} - 18y^{-3} = -15e^{6x} - 18z$

$z' = -15e^{6x} - 18z \Rightarrow z' + 18z = 0 \Rightarrow \frac{dz}{z} = -18z \Rightarrow \int \frac{dz}{z} = \int -18\,dx \Rightarrow \ln z = -18x + C$

$z = e^{-18x+C} \Rightarrow z = e^{-18x} \cdot e^{C} \qquad z = k\cdot e^{-18x}$

$z' = k'(x)\cdot e^{-18x} + k(x) - 18e^{-18x} = k'(x)e^{-18x} - 18k(x)e^{-18x}$

$k'(x) = \frac{15e^{6x}}{e^{-18x}} \Rightarrow k'(x) = -15e^{6x}\cdot e^{18x} = -15e^{24x}$

$k(x) = \int -15e^{24x}dx = -15\int e^{24x}dx = -\frac{15}{24}e^{24x}$

$z = k(x)\cdot e^{-18x} = (-e^{24x} + Ce^{-18x}$

$z = 3y^{-3} \Rightarrow z = \frac{3}{y^3} \Rightarrow y^3 = \frac{3}{z} \quad y = \sqrt{\frac{3}{z}}$

$$\boxed{y = \sqrt[3]{\frac{3}{-e^{24x} + C_1e^{-18x}}}}$$

$4.\ y'' + 6y' + 9y = 6e^{-3x}18$

4. $\begin{cases} y''+6y'+9y=6e^{-3x}+18 \\ y(0)=2 \\ y'(0)=25 \end{cases}$

$y''+6y'+9y=0 \Rightarrow p(x)=x^2+6x+9=(x+3)^2 \Rightarrow y_h(x)=e^{-3x}+C_2xe^{-3x}$

$y(x)=\Delta x^2e^{-3x}$ $\quad y'(x)=2\Delta e^{-3x}-3\Delta x^2e^{-3x}$ $\quad y''(x)=2\Delta e^{-3x}-12\Delta xe^{-3x}+9\Delta x^2e^{-3x}$

$y''+6y'+9y=6e^{-3x}+18 \Rightarrow 2\Delta e^{-3x}-12\Delta xe^{-3x}+9\Delta x^2e^{-3x}+12\Delta xe^{-3x}-18\Delta x^2e^{-3x}+9\Delta x^2e^{-3x}=$
$=6e^{-3x}+18. \Rightarrow 2\Delta e^{-3x}=6e^{-3x}+18$; $2\Delta=6+18e^{3x}$; $\Delta=3+9e^{3x}$

$y_p(x)=(3+9e^{3x})x^2e^{-3x}=3x^2e^{-3x}+9x^2$

$y(x)=3x^2e^{-3x}+18x-3c_1e^{-3x}+C_2e^{-3x}-3C_2e^{-3x}$

$y(0)=2 \Rightarrow \boxed{C_1=2}$

$y'(x)=6xe^{-3x}-9x^2e^{-3x}+18x-3C_1e^{-3x}+C_2e^{-3x}-3C_2e^{-3x}$ $\quad y'(0)=25 \Rightarrow \boxed{C_2=31}$

$$\boxed{y(x)=3x^2e^{-3x}+9x^2+2e^{-3x}+31xe^{-3x}}$$

6. $\underbrace{2xy - 3x^2y^2}_{M} + \underbrace{(x^2 - y\,2x^3y)}_{N}y' = 0.$

$\frac{dM}{dy} = 2x - 6x^2y$; $\frac{dN}{dx} = 2x - 6x^2y$ $\quad \frac{dM}{dy} = \frac{dN}{dx} \Rightarrow$ Ec dif exacta.

$f / \frac{df}{dx} = M$; $\frac{df}{dy} = N$

$f(x,y) = C \quad C \in R$

$\frac{df}{dx} = M \Rightarrow f = \int 2xy - 3x^2y^2\,dx \Rightarrow$

$f = y\int 2x\,dx - y^2\int 3x^2y^2\,dx \rightarrow f = y\int 2x\,dx - y^2\int 3x^2\,dx = \boxed{y \cdot x^2 - y^2 \cdot x^3 + C}$

$\frac{df}{dy} = N \Rightarrow x^2 - 2y\,x^3 + C'(y) = x^2 - 2x^3 \cdot y \Rightarrow C'(y) = 0 \Rightarrow C(y) = 0$

$f(x,y) = yx^2 - y^2x^3$

$\boxed{C = yx^2 - y^2x^3}$

7. $y' - \frac{2}{x+2} y = 2(x+2)^3$ Ec lineal de 1er orden

$p(x) = -\frac{2}{x+2}$ $g(x) = 2(x+2)^3$

$y\mu = \int g \mu \cdot dx \Rightarrow \mu = e^{\int p(x) \cdot dx}$

$\mu = e^{\int -\frac{2}{x+2} dx} \Rightarrow \int -\frac{2}{x+2} dx = -2 \int \frac{1}{x+2} dx \Rightarrow$

$\Rightarrow -2 \int \frac{1}{x+2} = -2 \ln(x+2) + C.$

$\mu = e^{-2 \ln(x+2)} = e^{\ln (x+2)^{-2}} \Rightarrow \mu = (x+2)^{-2}$

$y \cdot (x+2)^{-2} = \int 2(x+2)^3 \cdot (x+2)^{-2} dx \Rightarrow y(x+2)^{-2} = \int 2x+4 \, dx = 2\frac{x^2}{2} + 4x + C$

$y \cdot (x+2)^{-2} = x \cdot (x+4) + C.$

$$\boxed{y = [(x(x+4)) \cdot (x+2)^2] + C(x+2)^2}$$

9. $y^{VIII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y^{III} + 5y^{II} = 0$

$r^2(r^5 + r^4 + r^3 + r^2 + r^1 + 5) = 0$

$r = -1$ mult 3
$r = 0$ mult 2

$(r^2 - 2r + 5) = 0$ mult 1 $\Rightarrow$

$r = 1 \pm 2i$ mult 1

$r = 0 \rightarrow C + X$

$r = -1 \Rightarrow e^{-x} + xe^{-x} + x^2e^{-x}$

$r = 1 \pm 2i \longrightarrow e^{1x}\cos 2x$

$\searrow e^{1x}\operatorname{sen} 2x$

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
	1	0	2	8	5	0
-1		-1	1	-3	-5	
	1	-1	3	5	0	
-1		-1	2	-5		
	1	-2	5	0		

$\dfrac{2 \pm \sqrt{4-20}}{2}$ ≠ No es p

$C_1 + C_2x + C_3e^{-x} + C_4xe^{-1} + C_5x^2e^{-x} + C_6e^x\cos 2x + C_7e^x\operatorname{sen} 2x = 0$

Siendo C_{1-7} pertenecientes a $\mathbb{R}$

$$\frac{1}{x^{+5}}$$

$$\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$$

$y' + xy = 0 \quad y' = -xy \quad \frac{dy}{dx} = -x \cdot y \Rightarrow \frac{dy}{y} = -x \cdot dx \Rightarrow \int \frac{dy}{y} = \int -x \, dx$

$\ln y = -\frac{x^2}{2} \Rightarrow \boxed{y = e^{-\frac{x^2}{2}+c}} \Rightarrow y = e^{-x^2} \cdot e^c \quad \boxed{y = k \cdot e^{-x^2}}$

$\boxed{y' = k(x)' \cdot e^{-x^2} + k \cdot 2x \cdot e^{-x^2}}$

$y' + x \cdot y = 3xe^{x^2}$

$k(x)' \cdot e^{-x^2} + k \cdot 2x \cdot e^{-x^2} + x \cdot k \cdot e^{-x^2} = 3xe^{x^2} \Rightarrow k(x)' \cdot \frac{1}{e^{x^2}} + \frac{k \cdot 2x}{e^{x^2}} + \frac{k \cdot x}{e^{x^2}} = 3x \cdot e^{x^2}$ $k>0$

$\Rightarrow k(x)' \cdot \frac{1}{e^{x^2}} + \frac{3x \cdot 2k}{e^{x^2}} = 3x \cdot e^{x^2} \Rightarrow \frac{k(x)' + 3x \cdot 2k}{e^{x^2} \cdot e^{x^2} \cdot 3x} = 3x \, e^{x^2} \quad \frac{k(x)' + 2k}{e^{x^2}}$

$\frac{k(x)'}{e^{x^2}} = 3xe^{x^2} - \frac{3x \cdot}{e^{x^2}} \Rightarrow \frac{k(x)'}{e^{x^2}} = \frac{\cancel{3x} \cdot 2e x^2 - \cancel{3x} \cdot 2k}{\cancel{e}^{x^2}} \Rightarrow \frac{k(x)'}{e^{x^2}} = e^{x^2}$

$k(x)' = 2e^{x^2} \quad k = 4xe^{x^2}$

$(3e^{x^2-x^2} + 4xe^{+x^2} + 2xe^{-x^2}) + x \cdot (4xe^{x^2}) = 3xe^{x^2}$

$y(0) = 2$