EJERCICIO 1   LAURA ZAMORA AROCA . GIM 1

$$\begin{cases} (4x^2-2y^2)dx = 2xy\,dy \\ y(1)=3 \end{cases}$$

$$(4x^2-2y^2) = 2xy\cdot y' \; ; \; y' = \frac{4x^2-2y^2}{2xy} = \frac{\cancel{2}(2x^2-y^2)}{\cancel{2}xy} = \frac{2x^2-y^2}{xy}$$

$$v = y/x \; ; \; y = vx \; ; \; y' = v'x + v$$

$$v'x + v = \frac{2x^2 - v^2x^2}{x^2 v} = \frac{x^2(2-v^2)}{\cancel{x^2}\, v} = \frac{2-v^2}{v}$$

$$v'x = \frac{2-v^2}{v} - v = \frac{2-v^2-v^2}{v} = \frac{2-2v^2}{v} = \frac{2(1-v^2)}{v}$$

$$\frac{dv}{dx}x = \frac{2(1-v^2)}{v} \; ; \; \int \frac{v}{2(1-v^2)}dv = \int \frac{x}{dx} \; ; \; \frac{1}{2}\int \frac{v}{1-v^2}dv \; (*) = \log x \; ;$$

$$-1/4 \log(1-v^2) + c = \log x , \; c \in \mathbb{R}$$

$$(*)\; \frac{1}{2}\int \frac{v}{1-v^2}dv = \begin{vmatrix} t = 1-v^2 \\ dt = -2v\,dv \\ dv = dt/-2v \end{vmatrix} = \frac{1}{2}\int \frac{\cancel{v}}{t}\cdot\frac{dt}{-2\cancel{v}} = \frac{-1}{4}\int \frac{dt}{t} = -\frac{1}{4}\log t =$$

$$= -1/4 \log(1-v^2) + c \; ; \; c \in \mathbb{R}$$

$$-1/4 \log(1-v^2) + c = \log x$$

$$e^{(1-v^2)^{-1/4} + c} = \log x \; ; \; x = (1-v^2)^{-1/4}\cdot e^c = (1-v^2)^{-1/4}\cdot K$$

$$x = \frac{K}{\sqrt[4]{1-v^2}} = \frac{K}{\sqrt[4]{1-y^2/x^2}} \; , \; y(1) = 3 \to 1 = \frac{K}{\sqrt[4]{1-9}} \; ; \; K = \sqrt[4]{-8}$$

$$x = \sqrt[4]{\frac{-8x^2}{x^2-y^2}} \; ; \; x^4 = \frac{-8x^2}{x^2-y^2} \; ; \; x^2 = \frac{-8}{x^2-y^2} \; ; \; x^2\cdot(x^2-y^2) = -8 \; ;$$

$$x^4 - x^2y^2 = -8$$

$$x^2y^2 - x^4 = 8 \; ; \; x^2y^2 = 8 + x^4$$

$$\boxed{y = \sqrt{\frac{8+x^4}{x^2}}}$$

EJERCICIO 2: LAURA ZAMORA AROCA

$$\begin{cases} y''+4y=-4\sin(2x) \\ y(0)=-1 \ , \ y'(0)=4 \end{cases}$$

- Ec. dif. lineal homogénea asociada:

$y''+4y=0 \rightarrow p(x)=x^2+4=0$ $\qquad x=\dfrac{0\pm\sqrt{16}\cdot\sqrt{-1}}{2}=0\pm 2i$

RAÍCES $\rightarrow 0\pm 2i$ mult. 1 $\rightarrow e^{0x}\sin(2x)\rightarrow \sin(2x)$

$\rightarrow e^{0x}\cos(2x)\rightarrow\cos(2x)$

- solución:

$$\boxed{y_h(x)=c_1\sin(2x)+c_2\cos(2x) \quad , \ c_1,c_2\in\mathbb{R}}$$

- Solución particular:

$y(x)=Ax\sin(2x)+Bx\cos(2x)$

$y'(x)=A\sin(2x)+2Ax\cos(2x)+B\cos(2x)-2Bx\sin(2x)$

$y''(x)=2A\cos(2x)+2A\cos(2x)-4Ax\sin(2x)-2B\sin(2x)$
$-2B\sin(2x)-4Bx\cos(2x)=4A\cos(2x)-4Ax\sin(2x)$
$-4B\sin(2x)-4Bx\cos(2x)$

sostituimos: $y''+4y=-4\sin(2x)$

$4A\cos(2x)-\cancel{4Ax\sin(2x)}-4B\sin(2x)-\cancel{4Bx\cos(2x)}+\cancel{4Ax\sin(2x)}+$
$+\cancel{4Bx\cos(2x)}=-4\sin(2x)$

$4A\cos(2x)-4B\sin(2x)=-4\sin(2x)$

$$\begin{cases} 4A=0 \rightarrow A=0 \\ -4B=-4 \rightarrow B=1 \end{cases}$$

solución: $\boxed{y_p=\sin(2x)}$

- Solución general: $y(x)=\sin(2x)+c_1\sin(2x)+c_2\cos(2x)$

$y(0)=-1$

$\sin(0)+c_1\cdot\sin(0)+c_2\cdot\cos(0)=-1$; $\underline{c_2=-1}$

$y'(x)=2\cos(2x)+2c_1\cos(2x)-2c_2\sin(2x)$

$y'(0)=4$

$2\cos(0)+2c_1\cos(0)-2c_2\sin(0)=4$; $2+2c_1=4$

Solución problema condiciones iniciales $\qquad c_1=1$

$$\boxed{y(x)=\sin(2x)+\sin(2x)-\cos(2x)}$$

EJERCICIO 3

LAURA ZAMORA AROCA

$y' - 6y = 5e^{6x} y^4$ Ec. dif. Bernoulli

$\alpha = 4$

$z = y^{1-4} = y^{-3}$

$z' = -3y^{-4} \cdot y' = -3y^{-4}(5e^{6x}y^4 + 6y) = -15e^{6x} - 18y^{-3} = -15e^{6x} - 18z$

$z' + 18z = -15e^{6x}$ (ec. dif. 1er orden)

↳ $z' + 18z = 0$ (ec. dif. homogenea asociada)

$\frac{dz}{dx} = -18z$; $\int \frac{dz}{z} = \int -18dx$; $\log z = -18x + C$

$z = e^{-18x+C} = e^{-18x} \cdot e^{C} = Ke^{-18x}$

$z = K(x)e^{-18x}$

$z' = K'(x)e^{-18x} - 18K(x)e^{-18x}$

sustituimos en: $z' + 18z = -15e^{6x}$

$K'(x)e^{-18x} - 18\cancel{K(x)}e^{-18x} + 18\cancel{K(x)}e^{-18x} = -15e^{6x}$

$K'(x)e^{-18x} = -15e^{6x}$; $K'(x) = \frac{-15e^{6x}}{e^{-18x}} = -15e^{6x} \cdot e^{18x} = -15e^{24x}$

$K(x) = \int -15e^{24x}dx = -15\int e^{24x}dx = -15/24\, e^{24x} = -5/8\, e^{24x} + C$, $C \in \mathbb{R}$

$z = (-5/8\, e^{24x} + C)e^{-18x} = -5/8 e^{6x} + Ce^{-18x}$

$z = -5/8\, e^{6x} + Ce^{-18x}$; $C \in \mathbb{R}$

$z = y^{-3}$; $z = 1/y^3$; $y^3 = 1/z$; $y = 1/\sqrt[3]{z}$

$$\boxed{y = \frac{1}{\sqrt[3]{-5/8e^{6x} + Ce^{-18x}}} , \ C \in \mathbb{R}}$$

EJERCICIO 4

LAURA ZAMORA AROCA

$$\begin{cases} y'' + 6y' + 9y = 6e^{-3x} + 18 \\ y(0) = 2 \quad , \quad y'(0) = 25 \end{cases}$$

- Ec. dif. lineal homogénea asociada

$y'' + 6y' + 9y = 0$

$P(x) = x^2 + 6x + 9 = (x+3)^2$

RAÍCES $\rightarrow -3$ mult. 2 $\rightarrow e^{-3x}$, xe^{-3x}

$y_h(x) = c_1 e^{-3x} + c_2 x e^{-3x}$, $c_1, c_2 \in \mathbb{R}$

- Sol. particular

$y(x) = A x^2 e^{-3x}$

$y'(x) = 2A e^{-3x} - 3Ax^2 e^{-3x}$

$y''(x) = 2Ae^{-3x} - 6Axe^{-3x} - 6Axe^{-3x} + 9Ax^2e^{-3x} =$

$= 2Ae^{-3x} - 12Axe^{-3x} + 9Ax^2e^{-3x}$

sustituimos:

$y'' + 6y' + 9y = 6e^{-3x} + 18$

$2Ae^{-3x} - \cancel{12Ax}e^{-3x} + \cancel{9Ax^2}e^{-3x} + \cancel{12Ax}e^{-3x} - \cancel{18Ax^2}e^{-3x} + \cancel{9Ax^2}e^{-3x} =$

$= 6e^{-3x} + 18$;

$2Ae^{-3x} = 6e^{-3x} + 18$; $2A = 6 + 18e^{3x}$; $A = 3 + 9e^{3x}$

$y_p(x) = (3 + 9e^{3x})x^2 e^{-3x} = 3x^2 e^{-3x} + 9x^2$

- Sol. general:

$y(x) = 3x^2 e^{-3x} + 9x^2 + c_1 e^{-3x} + c_2 x e^{-3x}$, $c_1, c_2 \in \mathbb{R}$

$y(0) = 2$

$3 \cdot 0 \cdot e^0 + 9 \cdot 0 + c_1 \cdot e^0 + c_2 \cdot 0 \cdot e^0 = 2$; $c_1 = 2$

$y'(x) = 6xe^{-3x} - 9x^2 e^{-3x} + 18x - 3c_1 e^{-3x} + c_2 e^{-3x} - 3c_2 e^{-3x}$

$y'(0) = 25$

$6 \cdot 0 \cdot e^0 - 9 \cdot 0 \cdot e^0 + 18 \cdot 0 - 3 \cdot c_1 \cdot e^0 + c_2 \cdot e^0 - 3 \cdot 0 \cdot e^0 = 25$

$-3c_1 + c_2 = 25$; $-6 + c_2 = 25$; $c_2 = 31$

$$\boxed{y(x) = 3x^2 e^{-3x} + 9x^2 + 2e^{-3x} + 31 x e^{-3x}}$$

Escaneado con CamScanner

EJERCICIO 5 LAURA ZAMORA AROCA

$$\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$$

- Ec. dif. lineal. homogenea asociada

$y' + xy = 0$

$\frac{dy}{dx} = -xy$; $\int \frac{dy}{y} = \int -x\,dx$; $\log y = -\frac{x^2}{2} + c$; $c \in \mathbb{R}$

$y = e^{-x^2/2 + c} = e^{-x^2/2} \cdot e^c = Ke^{-x^2/2} \to y = K(x)e^{-x^2/2}$

$y' = K'(x)e^{-x^2/2} - K(x)xe^{-x^2/2}$

sustituimos:

$K'(x)e^{-x^2/2} - \cancel{K(x)xe^{-x^2/2}} + \cancel{K(x)xe^{-x^2/2}} = 3xe^{x^2}$

$K'(x)e^{-x^2/2} = 3xe^{x^2}$; $K'(x) = 3xe^{x^2}/e^{-x^2/2}$

$K(x) = \int \frac{3x \cdot \cancel{e^{x^2}}}{\cancel{e^{x^2}}}dx = \int 3x\,dx = \frac{3x^2}{2} + c$; $c \in \mathbb{R}$

sustituimos: $y = K(x)e^{-x^2/2}$

$y = e^{\frac{3x^2}{2}} \cdot e^{-x^2/2} + c = e^{x^2} + c\sqrt{e^{x^2}} = \frac{c}{\sqrt{e^{x^2}}}$

$$\boxed{y = \frac{c}{\sqrt{e^{x^2}}} \; ; \; c \in \mathbb{R}}$$

LAURA ZAMORA AROCA

EJERCICIO 6

$$\underbrace{2xy-3x^2y^2}_{M(x,y)}+\underbrace{(x^2-2x^3y)}_{N(x,y)}y'=0$$

Ec. dif. exacta

$$\frac{dM}{dy}=2x-6x^2y=\frac{dN}{dx}=2x-6x^2y \quad ✓$$

$$\frac{df}{dx}=M(x,y)\ ;\ \frac{df}{dx}=2xy-3x^2y^2\ ;\ f=\int 2xy-3x^2y^2\,dx=$$
$$=2y\int x\,dx-3y^2\int x^2\,dx=$$
$$=x^2y-x^3y^2+K(y)$$

$$\frac{df}{dy}=N(x,y)$$

$$\cancel{x^2}-2\cancel{x^3y}+K'(y)=\cancel{x^2}-2\cancel{x^3y} \to K'(y)=0$$

elijo : $K(y)=0$

$f=x^2y-x^3y^2$

entonces $x^2y-x^3y^2=C$, $C\in\mathbb{R}$ define de forma implícita las soluciones 'y' de la ec. diferencial

EJERCICIO 7

$$y'-\frac{2}{x+2}y=2(x+2)^3$$

Ec. dif. lineal 1^er orden

$$y'-\frac{2}{x+2}y=0$$ (ec. dif. lineal homogenea asociada)

$$\frac{dy}{dx}=\frac{2}{x+2}y\ ,\ \int\frac{dy}{y}=\int\frac{2}{x+2}dx\ ,\ \log y=2\log(x+2)+C$$

$$y=e^{\log(x+2)^2+C}=e^{\log(x+2)^2}\cdot e^C=K(x+2)^2$$

$$y=K(x)(x+2)^2$$
$$y'=K'(x)(x+2)^2+2K(x)(x+2)$$

sustituimos:

$$K'(x)(x+2)^2+2\cancel{K(x)(x+2)}-\frac{2}{\cancel{x+2}}\left(\cancel{K(x)(x+2)^2}\right)=2(x+2)^3$$

$$K'(x)(x+2)^2=2(x+2)^3$$

$$K(x)=\int\frac{2(x+2)^{\cancel{3}}}{\cancel{(x+2)^2}}dx=\int 2(x+2)dx=2\int x+2\,dx=2\left(\int x\,dx+\int 2\,dx\right)=x^2+4x+C\ ,\ C\in\mathbb{R}$$

$$\boxed{y=x^2+4x(x+2)^2+C\ ,\ C\in\mathbb{R}}$$

Escaneado con CamScanner

EJERCICIO 8 — LAURA ZAMORA AROCA

$e^x y^2 + y\,\text{sen}\,x + (3ye^x - 2\cos x)\,y' = 0$ (Factor integrante $\mu(y)$)

$\underbrace{\mu(y)(e^x y^2 + y\,\text{sen}\,x)}_{M(x,y)} + \underbrace{\mu(y)(3ye^x - 2\cos x)}_{N(x,y)}\,y' = 0$ (ec. dif. exacta)

$\dfrac{dM}{dy} = \dfrac{dN}{dx}$; $\dfrac{dM}{dy} = \mu'(y)(e^x y^2 + y\,\text{sen}\,x) + \mu(y)(2ye^x + \text{sen}\,x)$

$\dfrac{dN}{dx} = \mu(y)(3ye^x + 2\,\text{sen}\,x)$

$\mu'(y)(e^x y^2 + y\,\text{sen}\,x) + \mu(y)(2ye^x + \text{sen}\,x) = \mu(y)(3ye^x + 2\,\text{sen}\,x)$

$\mu(y)(2ye^x + \text{sen}\,x - 3ye^x - 2\,\text{sen}\,x) = (-e^x y^2 - y\,\text{sen}\,x)\,\mu'(y)$

$\mu(y)(-e^x y - \text{sen}\,x) = \mu'(y)(-e^x y^2 - y\,\text{sen}\,x)$

$\mu(y)(-1)(e^x y + \text{sen}\,x) = \mu'(y)(-1)(e^x y^2 + y\,\text{sen}\,x)$

$\mu(y) = y \cdot \mu'(y) = y \cdot d\mu/dy$; $\mu(y) = y$

$f = \int e^x y^3\,dx + \int y^2\,\text{sen}\,x\,dx = e^x y^3 - y^2 \cos x + C$; $C \in \mathbb{R}$

$df/dy = N(x,y) \rightarrow 3y^2 e^x - 2y\cos x + K'(y) = 3y^2 e^x - 2y \cos x$

tomamos: $K'(y) = 0$; $K = cte$

$\boxed{e^x y^3 - y^2 \cos x = C \ ; \ C \in \mathbb{R}}$ conjunto de soluciones de la ecuación diferencial

EJERCICIO 9 LAURA ZAMORA AROCA

$$y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y^{III} + 5y^{II} = 0$$

$$p(x) = x^7 + x^6 + 2x^5 + 10x^4 + 13x^3 + 5x^2 =$$
$$= x^2(x^5 + x^4 + 2x^3 + 10x^2 + 13x + 5) =$$
$$= (x+1)^3(x^2 - 2x + 5)$$

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
	1	0	2	8	5	$\boxed{0}$
-1		-1	1	-3	-5	
	1	-1	3	5	$\boxed{0}$	
-1		-1	2	-5		
	1	-2	5	$\boxed{0}$		

$$x^2 - 2x + 5 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 - 4 \cdot 5}}{2} = \frac{2 \pm \sqrt{16}\sqrt{-1}}{2} =$$
$$= \frac{2 \pm 4i}{2} = 1 \pm 2i$$

RAÍCES $\longrightarrow$ -1 mult. 3 $\longrightarrow e^{-x}, xe^{-x}, x^2e^{-x}$ (2)

$\longrightarrow$ 0 mult. 2 $\longrightarrow e^0, xe^0 = 1, x$ (1)

$\longrightarrow$ $1 \pm 2i$ mult. 1 $\longrightarrow e^x \operatorname{sen}(2x)$

$\longrightarrow e^x \cos(2x)$ (3)

$$\boxed{\begin{aligned} y(x) &= c_1 + c_2x + c_3e^{-x} + c_4xe^{-x} + c_5x^2e^{-x} + c_6e^x\operatorname{sen}(2x) + \\ &+ c_7e^x\cos(2x) \quad , c_1, c_2, c_3, c_4, c_5, c_6, c_7 \in \mathbb{R} \end{aligned}}$$