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$$e^x y^2 + y\,\text{sen}\,x + (3ye^x - 2\cos x)y' = 0$$

$$(-e^x y^2 - y\,\text{sen}\,x)\,dx = (3y\,e^x - 2\cos x)\,dy$$

$$(3ye^x - 2\cos x)\,dy + (e^x y^2 + y\,\text{sen}\,x)\,dx$$

$$\frac{2ye^x + \text{sen}\,x - 3ye^x - 2\,\text{sen}\,x}{e^x y^2 + y\,\text{sen}\,x} =$$

$$\frac{-3\,\text{sen}\,x - ye^x}{y(e^x y + \text{sen}\,x)} = \frac{-1}{y}$$

$$e^{\int -\left(\frac{-1}{y}\right)dy} = \boxed{e^{\ln y} = y}$$ Factor integrante

Así $\underbrace{e^{\ln y}(e^x y^2 + y\,\text{sen}\,x)}_{M(x,y)} + \underbrace{e^{\ln y}(3ye^x - 2\cos x)}_{N(x,y)}\,y' = 0$ Es exacta

$$\frac{df}{dx} = e^{\ln y}(e^x y^2 + y\,\text{sen}\,x) \to f = \int e^{\ln y}(e^x y^2 + y\,\text{sen}\,x)\,dx = \int y(e^x y^2 + y\,\text{sen}\,x)\,dx$$

$$\to f = y^3 e^x - y^2\cos x + K(y)$$

$$\frac{df}{dy} = \cancel{3y^2e^x} - \cancel{2y\cos x} + k'(y) = \cancel{3y^2e^x} - \cancel{2y\cos x} \Rightarrow \underbrace{k'(y) = 0}_{\text{Sólo buscamos una}} \quad K(y) = 0$$

$$f = y^3e^x - y^2\cos x \to y^3e^x - y^2\cos x = C,\ C \in \mathbb{R}$$

define de forma implícita el conjunto de soluciones y de la ecuación diferencial