Diego López Pérez

1.

$$\begin{cases} (4x^2-2y^2)\,dx = 2xy\,dy \rightarrow y' = \dfrac{4x^2-2y^2}{2xy} \\ y(1)=3 \end{cases}$$

$v = \frac{y}{x} \rightarrow y = vx \rightarrow y' = v'\cdot x + v\cdot 1 = v'x + v$

$y = vx, \quad y' = v'x + v$

$v'x + v = \dfrac{4x^2 - 2(vx)^2}{2x^2 v} \Rightarrow v'x + v = 2 - v \Rightarrow v'x = 2 - 2v$

$\dfrac{dx}{dv}\, x = 2 - 2v \rightarrow \int \dfrac{dx}{x} = \int 2 - 2u\, du \rightarrow \log|x| + C = 2u - u^2 \rightarrow \log|x| + C = 2\,\dfrac{y}{x} - \left(\dfrac{y}{x}\right)^2$

$3 = y(1) = 2\cdot\dfrac{1}{x} - \left(\dfrac{1}{x}\right)^2 = 3 \rightarrow \boxed{2x^{-1} - x^{-2} = 3}$

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2.

$$\begin{cases} y''+4y=-4\,Sen(2x) \\ y(0)=-1\ ,\ y'(0)=4 \end{cases}$$

- Ec dif homogénea asociada:

$y''+4y=0 \to P(x)=x^2+4=0 \to x=\sqrt{-4}=\pm 2i$

Raíces $\to 0\pm 2i$ mult 1 $\begin{cases} \to e^{0x}\,Cos(2x) \to Cos(2x) \\ \to e^{0x}\,Sen(2x) \to Sen(2x) \end{cases}$

Sol. ec. dif. homogénea asociada:

$y_h(x)=C_1\,Cos(2x)+C_2\,Sen(2x) \quad C_1,C_2\in\mathbb{R}$

- Sol. particular:

$y(x)=Ax\,Cos(2x)+Bx\,Sen(2x)$

$y'(x)=A\,Cos(2x)-2Ax\,Sen(2x)+B\,Sen(2x)+2Bx\,Cos(2x)$

$y''(x)=-2A\,Sen(2x)-2A\,Sen(2x)-4Ax\,Cos(2x)+2B\,Cos(2x)+2B\,Cos(2x)-4Bx\,Sen\,2x=$

$=-4A\,Sen(2x)-4Ax\,Cos(2x)+4B\,Cos(2x)-4Bx\,Sen(2x)$

Sustituimos en: $y''+4y=-4\,Sen(2x)$

$-4A\,Sen(2x)-\cancel{4Ax\,Cos(2x)}+4B\,Cos(2x)-\cancel{4Bx\,Sen(2x)}+\cancel{4Ax\,Cos(2x)}+\cancel{4Bx\,Sen(2x)}=-4\,Sen(2x)\to$

$\to -4A\,Sen\,2x+4B\,Cos(2x)=-4\,Sen(2x)$

$$\begin{cases} -4A=-4 \to A=\frac{-4}{-4}=1 \\ 4B=0 \to B=0 \end{cases}$$

$y_p(x)=x\,Cos(2x)$

Sol general $\to y(x)=x\,Cos(2x)+C_1\,Cos(2x)+C_2\,Sen(2x)$

$y'(x)=Cos(2x)-2x\,Sen(2x)-2C_1\,Sen(2x)+2C_2\,Cos(2x)$

$-1=y(0)=\cancel{0\cdot Cos(2x)}+C_1\,Cos(2\cdot 0)+\cancel{C_2\,Sen(2\cdot 0)}\to \underline{C_1=-1}$

$4=y'(0)=Cos(0\cdot 2)-2\cdot 0\,Sen(2\cdot 0)-2C_1\,Sen(2\cdot 0)+2C_2\,Cos(2\cdot 0)$

$1-0-0+2C_2=4 \to 1+2C_2=4 \to 2C_2=3 \to \underline{C_2=\frac{3}{2}}$

$$\boxed{y(x)=x\,Cos(2x)-Cos(2x)+\frac{3}{2}\,Sen(2x)}$$

Solución problema condiciones iniciales

3.-

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$y' - 6y = 5e^{6x}y^4 \to y' = 5e^{6x}y^4 + 6y$

$\alpha = 3 \quad z = y^{1-4} = y^{-3}$

$z' = -3y^{-4} \cdot y' = -3y^{-4} \cdot (5e^{6x}y^4 + 6y) = -15e^{6x} - 18y^{-3} = -15e^{6x} - 18z$

$z' = -15e^{6x} - 18z \to z' + 18z = -15e^{6x}$ (Ec dif 1^er orden)

$z' + 18z = 0 \to \frac{dz}{dx} = -18z \to \int \frac{dz}{z} = \int -18\,dx \to \log z = -18x + C \to z = e^{-18x+C} \to z = e^{-18x} + e^{C} \to$

$\to z = K \cdot e^{-18x}$

Imponemos que sea solución

$z' = k'(x)e^{-18x} + k(x) \cdot -18e^{-18x} = k'(x)e^{-18x} - 18k(x)e^{-18x}$

$k'(x)e^{-18x} \cancel{-18k(x)e^{-18x}} \cancel{+18k(x)e^{-18x}} = -15e^{6x} \to k'(x)e^{-18x} = -15e^{6x}$

$k'(x) = \frac{-15e^{6x}}{e^{-18x}} \to k'(x) = -15e^{6x} \cdot e^{18x} = -15e^{24x}$

$k(x) = \int -15e^{24x}\,dx = -15\int e^{24x}\,dx = -15e^{24x} \cdot 24 = -360e^{24x} + C,\ C \in \mathbb{R}$

$z = k(x) \cdot e^{-18x} = (-360e^{24x} + C)e^{-18x} = -360e^{6x} + Ce^{-18x}$

$z = -360e^{6x} + Ce^{-18x} \quad C \in \mathbb{R}$

$z = y^{-3} \to z = \frac{1}{y^3} / y^3 = \frac{1}{z} \to y = \frac{1}{\sqrt[3]{z}}$

$$\boxed{y = \frac{1}{\sqrt[3]{-360e^{6x} + Ce^{-18x}}} \quad C \in \mathbb{R}}$$

Escaneado con CamScanner

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4:
$$\begin{cases} y''+6y'+9y=6e^{-3x}+18 \\ y(0)=2\,,\quad y'(0)=25 \end{cases}$$

- Ec dif lineal homogénea asociada

$y''+6y'+9y=0$

$p(\lambda)=x^2+6x+9=(x+3)^2$

Raíces $\to -3$ mult. $2 \to e^{-3x}, xe^{-3x}$

$y_h(x)=C_1e^{-3x}+C_2xe^{-3x}\,,\ C_1,C_2\in\mathbb{R}$

- Sol. particular

$y(x)=Ax^2e^{-3x}$

$y'(x)=2Ae^{-3x}-3Ax^2e^{-3x}$

$y''(x)=2Ae^{-3x}-6Axe^{-3x}-6Axe^{-3x}+9Ax^2e^{-3x}=2Ae^{-3x}-12Axe^{-3x}+9Ax^2e^{-3x}$

- Sustituimos

$y''+6y'+9y=6e^{-3x}+18$ $\quad 9Ax^2e^{-3x}+12Axe^{-3x}-18Ax^2e^{-3x}+9Ax^2e^{-3x}=$

$2Ae^{-3x}-12Axe^{-3x}+9Ax^2e^{-3x}$

$=6e^{-3x}+18;$

$2Ae^{-3x}=6e^{-3x}+18;\ 2A=6+18e^{3x};\ A=3+9e^{3x}$

$y_p(x)=(3+9e^{3x})x^2e^{-3x}=3x^2e^{-3x}+9x^2$

- Sol general:

$y(x)=3x^2e^{-3x}+9x^2+C_1e^{-3x}+C_2xe^{3x};\ C_1,C_2\in\mathbb{R}$

$y(0)=0$

$3\cdot0\cdot e^0+9\cdot0+C_1\cdot e^0+C_2\cdot0\cdot e^0=2;\ C_1=2$

$y'(x)=6xe^{-3x}-9x^2e^{-3x}+18x-3C_1e^{-3x}+C_2e^{-3x}-3C_2e^{-3x}$

$y'(0)=25$

$6\cdot0\cdot e^0-9\cdot0\cdot e^0+18\cdot0-3\cdot C_1\cdot e^0+C_2\cdot e^0-3\cdot0\cdot e^0=25$

$-3C_1+C_2=25;\ -6+C_2=25;\ C_2=31$

$$\boxed{y(x)=3x^2e^{-3x}+9x^2+2e^{-3x}+31xe^{-3x}}$$

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5:

$$\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$$

$y' + xy = 3xe^{x^2}$

Ec dif lineal homogenea asociada

$y' + xy = 0 \to \frac{dy}{dx} = -xy \to \int \frac{dy}{y} = \overbrace{\int -x \; dx}^{*} \to \log y = -\frac{x^2}{2} + C \to$

$* \int -x \; dx = -\int x \, dx = -\frac{x^2}{2} + C = -\frac{x^2}{2} + C$

$\to y = e^{-\frac{x^2}{2} + C} = e^{-\frac{x^2}{2}} \cdot \underbrace{e^{C}}_{k} = \boxed{k e^{-\frac{x^2}{2}}}$ sol ec dif lin hom asociada

$y = k e^{-\frac{x^2}{2}}$ e imponemos que sea sol de la ec diferencial

$y' = k'(x) \cdot e^{-\frac{x^2}{2}} + k(x) \cdot (-x) e^{-\frac{x^2}{2}}$

imponemos que sea solución

$k'(x) \cdot e^{-\frac{x^2}{2}} + \cancel{k(x) \cdot (-x) e^{-\frac{x^2}{2}}} + \cancel{x k e^{-\frac{x^2}{2}}} = 3xe^{x^2} \to k'(x) \cdot e^{-\frac{x^2}{2}} = 3xe^{x^2} \to$

$\to k(x) = \int \frac{3x\cancel{e^{x^2}}}{\cancel{e^{x^2}}} = \int 3x \, dx = \frac{3x^2}{2} + C; \; C \in \mathbb{R}$

Sustituimos: $y = k(x) e^{-x^2/2}$

$y = e^{\frac{3x^2}{2}} \cdot e^{-x^2/2} + C = e^{x^2} + C\sqrt{e^{x^2}} = \frac{C}{\sqrt{e^{x^2}}}$

$y = \frac{C}{\sqrt{e^{x^2}}}; \; C \in \mathbb{R}$

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6.

$2xy - 3x^2y^2 + (x^2 - y2x^3y)y' = 0$

$\underbrace{(2xy - 3x^2y^2)}_{M(x,y)} + \underbrace{(x^2 - 2x^3y)}_{N(x,y)} y' = 0 \qquad \frac{dM}{dy} = \frac{dN}{dx}$

$\frac{dM}{dy} = 2x - 6x^2y; \ \frac{dN}{dx} = 2x - 6x^2y$

$\frac{dM}{dy} = \frac{dN}{dx} \to$ Ec dif exacta $\exists f / \frac{df}{dx} = M, \ \frac{df}{dy} = N$

$f(x,y) = C \ C \in \mathbb{R}$

$\frac{df}{dx} = M \to f = \int 2xy - 3x^2y^2 dx \to f = y\int 2x\,dx - y^2\int 3x^2 dx = y \cdot x^2 - y^2 \cdot x^3 + C(y)$

$\frac{df}{dy} = N \to x^2 - 2yx^3 + C'(y) = x^2 - 2x^3y \to C'(y) = 0 \to C(y) = 0$

$f(x,y) = yx^2 - y^2x^3$

$\boxed{yx^2 - y^2x^3 = C}$ $C \in \mathbb{R} \to$ Define de forma implícita las sol. de la ec. diferencial.

7.

$y' - \frac{2}{x+2}y = 2(x+2)^3 \qquad \boxed{y'(x) + p(x)y = q(x)} \to$ Lineal primer orden

$p(x) = -\frac{2}{x+2} \quad q(x) = 2(x+2)^3$

$y\mu = \int q\mu\,dx \to$ lineales $\to \mu = e^{\int p(x)\cdot dx}$

$\mu = e^{\int \frac{-2}{x+2} dx} \to \int -\frac{2}{x+2} dx \Rightarrow -2\int \frac{1}{x+2} dx \to -2\int \frac{1}{x+2} = \boxed{-2\ln(x+2) + C}$

$\mu = e^{-2\ln(|x+2|)} = e^{\ln(x+2)^{-2}} \to \mu = (x+2)^{-2}$

$y \cdot (x+2)^{-2} = \int 2(x+2)^3 \cdot (x+2)^{-2} dx \to y(x+2)^{-2} = \int 2x + 4\,dx = 2\int x\,dx + 4\int dx = \frac{2x^2}{2} + 4x$

$y \cdot (x+2)^{-2} = x(x+4) + C$

$\boxed{y = ((x(x+4)) \cdot (x+2)^2) + C(x+2)^2}$

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8:

$e^x y^2 + y\,\mathrm{Sen}\,x + (3ye^x - 2\mathrm{Cos}\,x)\,y' = 0$

factor integrante $y \to \underbrace{(y)(e^x y^2 + y\,\mathrm{Sen}\,x)}_{M(x,y)} + \underbrace{(y)(3ye^x - 2\mathrm{Cos}\,x)}_{N(x,y)}\,y' = 0$

$\frac{dM}{dy} = \frac{dN}{dx}$

$\frac{dM}{dy} = 3y^2 e^x + 2y \cdot \mathrm{Sen}\,x$

$\frac{dN}{dx} = 3y^2 e^x + 2y\,\mathrm{Sen}\,x$

$\Rightarrow$ = Ec dif exacta

$\exists f / \frac{df}{dx} = N,\ \frac{df}{dy} = N$

$f(x,y) = C \quad C \in \mathbb{R}$

$\frac{df}{dx} = M \to \frac{df}{dx} = e^x y^3 + y^2 \cdot \mathrm{Sen}\,x \to f = \int (e^x y^3 + y^2 \cdot \mathrm{Sen}\,x)\,dx \to$

$\to f = y^3 \int e^x dx + y^2 \int \mathrm{Sen}\,x\,dx = y^3 e^x - y^2 \cdot \mathrm{Cos}\,x + C$

$\frac{df}{dy} = N \to 3y^2 e^x - 2y \cdot \mathrm{Cos}\,x + C'(y) = 3y^2 e^x - 2\mathrm{Cos}\,x\,y \to C'(y) = 0 \to C(y) = 0$

$f(x,y) = y^3 e^x - y^2 \mathrm{Cos}\,x$

$\boxed{y^3 e^x - y^2 \mathrm{Cos}\,x = C}$ $C \in \mathbb{R}$, define de forma implícita las sol (y) de la ec. diferencial.

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4 -

$y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y''' + 5y'' = 0$

$r^7 + r^6 + 2r^5 + 10r^4 + 13r^3 + 5r^2 = 0$

$r^2(r^5 + r^4 + 2r^3 + 10r^2 + 13r + 5) = 0$

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
	1	0	2	8	5	0
-1		-1	1	-3	-5	
	1	-1	3	5	0	
-1		-1	2	-5		
	1	-2	5	0		

$r = -1$ mult 3

$r = 0$ mult 2

$(r^2 - 2r + 5) = 0$ mult 1 $\to r = 1 \pm 2i$ mult 1

$r = 0 \to C + x$

$r = -1 \to e^{-x} + xe^{-x} + x^2 e^{-x}$

$r = 1 \pm 2i \to e^{1x}\cos 2x$, $e^{1x}\,\mathrm{Sen}\,2x$

$\{1, x, e^{-x}, xe^{-x}, x^2e^{-x}, e^x\cos 2x, e^x\,\mathrm{Sen}\,2x\}$

Es una $\mathbb{R}$ base del conjunto de todas las sol de la ec. dif.

Así la sol general es:

$C_1 + C_2x + C_3e^{-x} + C_4xe^{-x} + C_5x^2e^{x} + C_6e^{x}\cdot\cos 2x + C_7e^{x}\,\mathrm{Sen}\,2x \quad C_1, \ldots, C_7 \in \mathbb{R}$

$\to z' = k'\cdot x^{-2} + k(-2\cdot x^{-3})$; $z = kx^{-2}$

$k'\cdot x^{-2} + k\cdot(-2\cdot x^{-3}) + \frac{2}{x}k\cdot x^{-2} = 4x$

$k'x^{-2} + k(-2\cdot x^{-3}) + k2x^{-3} = 4x$

$k'x^{-2} = 4x \to k' = 4x\cdot x^2 \to k' = 4x^3$

$k(x) = \int 4x^3 = \boxed{x^4 + C}\ C \in \mathbb{R}$

$z = (x^4 + C)\cdot x^{-2} = x^2 + C\cdot x^{-2} \to z = y^2$

$y^2 = x^2 + \frac{C}{x^2} \to \frac{x^4 + C}{x^2} \to y = \sqrt{\frac{x^4 + C}{x^2}} \quad C \in \mathbb{R}$

$y(1) = \sqrt{\frac{1^4 + C}{1^2}} = 3 \to \sqrt{\frac{1 + C}{1}} = 3 \to \sqrt{1 + C} = 3$

$1 + \sqrt{C} = 3 \to \sqrt{C} = 2 \to \boxed{C = 4}$

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