EJERCICIOS ECUACIONES DIFERENCIALES

1. $$\begin{cases} (4x^2 - 2y^2)dx = 2xdy \\ y(1) = 3 \end{cases} \rightarrow y' = \frac{4x^2 - 2y^2}{2xy}$$

$$y' = u'x + u$$
$$y = ux$$

Sustit.

$$u'x + u = \frac{4x^2 - 2u^2}{2x^2 u} \Rightarrow u'x + u = 2 - u \Rightarrow u'x = 2 - 2u \rightarrow$$

$$\rightarrow \frac{dx}{du} x = 2 - 2u \rightarrow \int \frac{dx}{x} = \int 2 - 2u \, du$$

$$\log|x| + C = 2u - u^2$$
$$\log|x| + C = 2\frac{y}{x} - \left(\frac{y}{x}\right)^2$$

$$\cdot\ y(1) = 3 \rightarrow 2\frac{1}{x} - \left(\frac{1}{x}\right)^2 = 3$$

$$\underline{2x^{-1} - x^{-2} = 3}$$

Escaneado con CamScanner

② $\begin{cases} y'' + 4y = -4 \operatorname{sen}(2x) \\ y(0) = -1 \ , \ y'(0) = 4 \end{cases}$ - Ec. diferencial homogénea asociada.

$y'' + 4y = 0 \rightarrow P(x) = x^2 + 4 = 0 \rightarrow x = \sqrt{-4} = \pm 2i$

Raíces $\rightarrow 0 \pm 2 * i \begin{cases} e^{0x} \cos(2x) = \cos(2x) \\ e^{0x} \operatorname{sen}(2x) = \operatorname{sen}(2x) \end{cases}$

Solución ecu. diferencial homogénea asociada:

$y_h(x) = C_1 \operatorname{Cos}(2x) + C_2 \operatorname{Sen}(2x)$

• Sol. particular $\rightarrow$

$y(x) = A_x \operatorname{Cos}(2x) + Bx \operatorname{Sen}(2x)$

$y'(x) = A \operatorname{Cos}(2x) - 2Ax \operatorname{Sen}(2x) + B \operatorname{Sen}(2x) + 2Bx \operatorname{Cos}(2x)$

$y''(x) = -2A \operatorname{Sen}(2x) - 2A \operatorname{Sen}(2x) - 4Ax \operatorname{Cos}(2x) + 2B \operatorname{Cos}(2x) + 2B \operatorname{Cos}(2x) - 4Bx \operatorname{Sen} 2x =$

$= -4A \operatorname{Sen}(2x) - 4Ax \operatorname{Cos}(2x) + 4B \operatorname{Cos}(2x) - 4Bx \operatorname{Sen}(2x)$

- Sustiamos en $y'' + 4y = -4 \operatorname{Sen}(2x)$

$-4A \operatorname{Sen}(2x) - \cancel{4Ax \operatorname{Cos}(2x)} + 4B \operatorname{Cos}(2x) - \cancel{4Bx \operatorname{Sen}(2x)} + \cancel{4Ax \operatorname{Cos}(2x)} + \cancel{4Bx \operatorname{Sen}(2x)} = -4 \operatorname{Sen}(2x)$

$\rightarrow -4A \operatorname{Sen} 2x + 4B \operatorname{Cos}(2x) = -4 \operatorname{sen}(2x)$

$\begin{cases} -4A = -4 \rightarrow A = \frac{-4}{-4} = 1 \\ 4B = 0 \rightarrow B = 0 \end{cases}$

$y_p(x) = x \operatorname{Cos}(2x)$

• Solu. general $\rightarrow$

$y(x) = x \operatorname{Cos}(2x) + C_1 \operatorname{Cos}(2x) + C_2 \operatorname{Sen}(2x)$

$y'(x) = \operatorname{Cos}(2x) - 2x \operatorname{Sen}(2x) - 2C_1 \operatorname{Sen}(2x) + 2C_2 \operatorname{Cos}(2x)$

$-1 = y(0) = 0 \cdot \operatorname{Cos}(2x) + C_1 \operatorname{Cos}(2 \cdot 0) + C_2 \operatorname{Sen}(2 \cdot 0) \rightarrow \underline{C_1 = -1}$

$4 = y'(0) = \operatorname{Cos}(2 \cdot 0) - 2 \cdot 0 \operatorname{Sen}(2 \cdot 0) - 2C_1 \operatorname{Sen}(2 \cdot 0) + 2C_2 \operatorname{Cos}(2 \cdot 0) \rightarrow$

$\rightarrow 1 - 0 - 0 + 2C_2 = 4 \rightarrow 1 + 2C_2 = 4 \rightarrow \underline{C_2 = \frac{3}{2}}$

$$\underline{y(x) = x \operatorname{Cos}(2x) - \operatorname{Cos}(2x) + \frac{3}{2} \operatorname{Sen}(2x)}$$

③ $y' - 6y = 5e^{6x}y^4 \rightarrow y' = 5e^{6x}y^4 + 6y$

$\alpha = 3 \quad ; \quad z = y^{1-4} \rightarrow y^{-3}$

$z' = -3y^{-4} \cdot y' = -3y^{-4} \cdot (5e^{6x}y^4 + 6y) = -15e^{6x} - 18y^{-3} = -15e^{6x} - 18z$

$z' = -15e^{6x} - 18z \rightarrow z' + 18z = -15e^{6x}$

$z' + 18z = 0 \rightarrow \frac{dz}{dx} = -18z \rightarrow \int \frac{dz}{z} = \int -18\,dx \rightarrow \log z = -18x + C \rightarrow$

$\rightarrow z = e^{-18x+C} \rightarrow z = k \cdot e^{-18x}$

$z' = k'(x)e^{-18x} + k(x) - 18e^{-18x} = k'(x)e^{-18x} - 18k(x)e^{-18x}$

$k'(x)e^{-18x} - \cancel{18k(x)e^{-18x}} + \cancel{18k(x)e^{-18x}} = 15e^{6x} \rightarrow k'(x)e^{-18x} = -15e^{6x}$

$k(x) = \int -15e^{24x}dx = 15\int e^{24x}dx = -15e^{24x} \cdot 24 = -360e^{24x} + C$

$z = k(x) \cdot e^{-18x} = (-360e^{24x} + C)e^{-18x} = -360e^{6x} + Ce^{-18x}$

$z = -360e^{6x} + Ce^{-18x}$

$z = y^{-3} \rightarrow z = \frac{1}{y^3} \quad ; \quad y^3 = \frac{1}{z} \rightarrow y = \frac{1}{\sqrt[3]{z}}$

$$y = \frac{1}{\sqrt{-360e^{6x} + Ce^{-18x}}}$$

Escaneado con CamScanner

④ $\begin{cases} y'' + 6y' + 9y = 6e^{-3x} + 18 \\ y(0) = 2 \\ y'(0) = 25 \end{cases}$

- Ecu. dif lineal homogénea asociada

$y'' + 6y' + 9y = 0 \rightarrow P(x) = x^2 + 6x + 9 = (x+3)^2$

Raíces $\rightarrow -3$ mult. $2 \rightarrow e^{-3x}, \ xe^{-3x}$

$y_h(x) = C_1 e^{-3x} + C_2 xe^{-3x}$

- Sol. particular:

$y(x) = Ax^2 e^{-3x}$

$y'(x) = 2Ae^{-3x} - 3Ax^2 e^{-3x}$

$y''(x) = 2Ae^{-3x} - 6Axe^{-3x} - 6Axe^{-3x} + 9Ax^2e^{-3x} = 2Ae^{-3x} - 12Axe^{-3x} + 9Ax^2e^{-3x}$

- Sust:

$y'' + 6y' + 9y = 6e^{-3x} + 18$

* $2Ae^{-3x} = 6e^{-3x} + 18 \ ; \ 2A = 6 + 18e^{3x} \ ; \ A = 3 + 9e^{3x}$

$y_P(x) = (3 + 9e^{3x})x^2 e^{-3x} = 3x^2 e^{-3x} + 9x^2$

~~Solución general~~

$2Ae^{-3x} - 12\cancel{A}xe^{-3x} + 9\cancel{Ax^2}e^{-3x} + 12\cancel{A}xe^{-3x} - 18\cancel{Ax^2}e^{-3x} + 9\cancel{Ax^2}e^{-3x} = 6e^{-3x} + 18;$

- Solución general:

$y(x) = 3x^2e^{-3x} + 9x^2 + C_1 e^{-3x} + C_2 xe^{-3x}$

$y(0) = 2 \rightarrow 3 \cdot 0 \cdot e^0 + 9 \cdot 0 + C_1 \cdot e^0 + C_2 \cdot 0 \cdot e^0 = 2 \ ; \ C_1 = 2$

$y'(x) = 6xe^{-3x} - 9x^2 e^{-3x} + 18x - 3C_1 e^{-3x} + C_2 e^{-3x} - 3C_2 e^{-3x}$

$y'(0) = 25 \rightarrow 6 \cdot 0 \cdot e^0 - 9 \cdot 0 \cdot e^0 + 18 \cdot 0 - 3 \cdot C_1 \cdot e^0 + C_2 \cdot e^0 - 3 \cdot 0 \cdot e^0 = 25$

$-3C_1 + C_2 = 25 \ ; \ -6 + C_2 = 25 \ ; \ C_2 = 31$

$$\underline{y(x) = 3x^2 e^{-3x} + 9x^2 + 2e^{-3x} + 31xe^{-3x}}$$

⑤ $\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$ — Ec. diferencial lineal homogenea asociada

$y' + xy = 0$

$\frac{dy}{dx} = -xy$; $\int \frac{dy}{y} = \int -x dx$; $\log y = -\frac{x^2}{2} + C$

$y = e^{-x^2/2 + c} = e^{-x^2/2} \cdot e^c = K e^{-x^2/2} \rightarrow y = K(x) e^{-x^2/2}$

$y' = K'(x) e^{-x^2/2} - K(x) x e^{-x^2/2}$

Sust:

$K'(x) e^{-x^2/2} - K(x) x e^{-x^2/2} + K(x) x e^{-x^2/2} = 3xe^{x^2}$

$K'(x) e^{-x^2/2} = 3xe^{x^2}$; $K'(x) = 3xe^{x^2} / e^{-x^2/2}$

$K(x) = \int \frac{3x \cdot \cancel{e^{x^2}}}{\cancel{e^{x^2}}} dx = \int 3x dx = \frac{3x^2}{2} + C$

Sust $\rightarrow y = K(x) e^{-x^2/2}$

$y = e^{\frac{3x^2}{2}} \cdot e^{-x^2/2} + C = e^{x^2} + C\sqrt{e^{x^2}} = \frac{C}{\sqrt{e^{x^2}}}$

$$y = \frac{C}{\sqrt{e^{x^2}}}$$

⑥ $\underbrace{2xy - 3x^2y^2}_{M(x,y)} + \underbrace{(x^2 - 2x^3y)}_{N(x,y)}y' = 0$

$\frac{dM}{dy} = \frac{dN}{dx}$

- $\frac{dM}{dy} = 2x + 6x^2y$; $\frac{dN}{dx} = 2x - 6x^2y$

$\frac{dM}{dy} = \frac{dN}{dx} \rightarrow$ Ecuación diferencial exa.

$\exists f / \frac{df}{dx} = M \quad \frac{df}{dy} = N$

$f(x,y) = C$

$\frac{df}{dx} = M \rightarrow f = \int 2xy - 3x^2y^2\,dx \rightarrow f = y\int 2x\,dx - y^2\int 3x^2\,dx =$

$= y \cdot x^2 - y^2 \cdot x^3 + C(y)$

$\frac{df}{dy} = N \rightarrow \cancel{x^2} - 2yx^3 + C'(y) = \cancel{x^2} - 2x^3y \rightarrow C'(y) = 0 \rightarrow C(y) = 0$

$f(x,y) = yx^2 - y^2x^3$

$\boxed{yx^2 - y^2x^3 = C}$ en $C \in \mathbb{R}$

(7) $y' - \frac{2}{x+2} y = 2(x+2)^3$ - Lineal de primer orden

$P(x) = -\frac{2}{x+2}$ $\quad q(x) = 2(x+2)^3$

$y\,\mu = \int q\,\mu\,dx \rightarrow \mu = e^{\int p(x)\cdot dx}$

$\mu = e^{\int -\frac{2}{x+2} dx} \rightarrow \int -\frac{2}{x+2} dx = -2\int \frac{1}{x+2} dx \rightarrow -2\int \frac{1}{x+2} = -2\log(x+2) + C$

$\mu = e^{-2\log(|x+2|)} = e^{\ln(x+2)^{-2}} \rightarrow \mu = (x+2)^{-2}$

$y \cdot (x+2)^{-2} = \int 2(x+2)^3 \cdot (x+2)^{-2} dx \rightarrow$

$\rightarrow y \cdot (x+2)^{-2} = \int 2x + 4\,dx = 2\int x\,dx + 4\int 1\,dx = \cancel{2}\frac{x^2}{\cancel{2}} + 4x$

$y \cdot (x+2)^{-2} = x(x+4) + G$

$y = ((x(x+4)) \cdot (x+2)^2) + G \cdot (x+2)^2$

$⑧ (y) e^x y^2 + y \operatorname{sen} x + (3 y e^x - 2 \cos x) y' = 0$

factor integrante $y \rightarrow \underbrace{(y) e^x y^2 + y \operatorname{sen} x}_{M(x,y)} + \underbrace{(3 y e^x - 2 \cos x)}_{N(x,y)} y' = 0$

$$\frac{dM}{dy} = \frac{dN}{dx}$$

$$\frac{dM}{dy} = 3y^2 e^x + 2y \operatorname{sen} x$$

$$\frac{dN}{dx} = 3y^2 e^x + 2y \operatorname{sen} x$$

$\Rightarrow$ = Ec. dif exact.

$$\exists f \,/\, \frac{df}{dx} = M \,,\ \frac{df}{dy} = N$$

$$f(x,y) = C$$

$$\frac{df}{dx} = M \rightarrow \frac{df}{dx} = e^x y^3 + y^2 \cdot \operatorname{sen} x \rightarrow f = \int (e^x y^3 + y^2 \cdot \operatorname{sen} x)\, dx \rightarrow$$

$$\rightarrow f = y^3 \int e^x \, dx + y^2 \int \operatorname{sen} x \, dx = \underline{y^3 e^x - y^2 \cdot \cos x + C}$$

$$\frac{df}{dy} = N \rightarrow 3y^2 e^x - 2y \cdot \cos x + C'(y) = 3y^2 e^x - 2 \cos x \; y \rightarrow C'(y) = 0 \rightarrow C(y) = 0$$

$$f(x,y) = y^3 e^x - y^2 \cos x$$

$\boxed{y^3 e^x - y^2 \cdot \cos x = C}$ en $C \in \mathbb{R}$

9 $y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y''' + 5y'' = 0$

$y = e^{rx}$

$r^7 + r^6 + 2r^5 + 10r^4 + 13r^3 + 5r^2 = 0$

$r^2(r^5 + r^4 + 2r^3 + 10r^2 + 13r + 5) = 0$

$r = -1$ multi 3

$r = 0$ multi 2

$$\begin{array}{c|cccccc}
 & 1 & 1 & 2 & 10 & 13 & 5 \\
-1 & & -1 & 0 & -2 & -8 & -5 \\
\hline
 & 1 & 0 & 2 & 8 & 5 & 0/ \\
-1 & & -1 & 1 & -3 & -5 & \\
\hline
 & 1 & -1 & 3 & 5 & 0/ & \\
-1 & & -1 & 2 & -5 & & \\
\hline
 & 1 & -2 & 5 & 0/ & &
\end{array}$$

$(r^2 - 2r + 5) = 0$ multi 1 $\rightarrow$

$\rightarrow$ $r = 1 \pm 2i$ multi 1

$$\frac{2 \pm \sqrt{4-20}}{2} \notin \mathbb{R}$$

$r = 0 \rightarrow c + x$

$r = -1 \rightarrow e^{-x} + xe^{-x} + x^2e^{-x}$

$r = 1 \pm 2i \begin{cases} e^{1x} \cos 2x \\ e^{1x} \operatorname{sen} 2x \end{cases}$

$\{c, x, e^{-x}, xe^{-x}, x^2e^{-x}, e^x \cos 2x, e^x \operatorname{sen} 2x\}$

Sol. general:

$$C_1 + C_2x + C_3e^{-x} + C_4xe^{-x} + C_5x^2e^{-x} + C_6e^x \cdot \cos 2x + C_7e^x \operatorname{sen} 2x$$

C_1 a $C_7 \in \mathbb{R}$