2.

$$\begin{cases} y'' + 4y = -4\,\text{sen}(2x) \\ y(0) = -1 \\ y'(0) = 4 \end{cases}$$

• Raíces : $0 \pm 2i$ $\rightarrow e^{0x} \cdot \text{sen}(2x) = \text{sen}(2x)$, $\quad e^{0x} \cdot \cos(2x) = \cos(2x)$

Ec. dif. lineal. homogénea asociada

$$y'' + 4y = 0$$

$\hookrightarrow p(x) = x^2 + 4 = 0 \rightarrow x^2 = -4 \rightarrow x = \pm\sqrt{-4} = 0 \pm 2i$

$$y_h(x) = C_1\,\text{sen}(2x) + C_2 \cos(2x) \quad C_1, C_2 \in \mathbb{R}$$

• Sol particular

$$\boxed{y(x) = Ax\,\text{sen}(2x) + Bx\cos(2x)}$$

$$y'(x) = A \cdot \text{sen}(2x) + Ax \cdot 2\cos(2x) + B \cdot \cos(2x) + Bx \cdot (-2\,\text{sen}(2x))$$

$$\boxed{y'(x) = A\,\text{sen}(2x) + 2Ax\cos(2x) + B\cos(2x) - 2Bx\,\text{sen}(2x)}$$

$$\boxed{y''(x) = 2A\cos(2x) - 4Ax\,\text{sen}(2x) - 2B\,\text{sen}(2x) - 4Bx\cos(2x)}$$

$\hookrightarrow$ Sustituimos

$$2A\cos(2x) - \cancel{4Ax\,\text{sen}(2x)} - 2B\,\text{sen}(2x) - \cancel{4Bx\cos(2x)} + \cancel{4(Ax\,\text{sen}(2x) + Bx\cos(2x))} = -4\,\text{sen}(2x)$$

$$\begin{cases} 2A = 0 \rightarrow \boxed{A=0} \\ -2B = -4 \rightarrow \boxed{B=2} \end{cases}$$

Solución general

$$y(x) = 2x\cos(2x) + C_1\,\text{sen}(2x) + C_2\cos(2x) \quad C_1, C_2 \in \mathbb{R}$$

$$-1 = y(0) \rightarrow -1 = 2 \cdot 0 \cdot \cancel{\cos}(2\cdot 0) + C_1 \cdot \cancel{\text{sen}}(2\cdot 0) + C_2\cos(2\cdot 0) \rightarrow \boxed{C_2 = -1}$$

$$4 = y'(0) \rightarrow 4 = 4 \cdot 0(\cancel{-\text{sen}\,2\cdot 0}) + 2C_1\cos(2\cdot 0) + \cancel{2C_2(-\text{sen}\,2\cdot 0)}$$

$$4 = 2C_1 \rightarrow \boxed{C_1 = 2}$$

Solución Problema. C.I

$$\hookrightarrow \boxed{y(x) = 2x\cos(2x) + 2\,\text{sen}(2x) - \cos 2x}$$