1. $\begin{cases} (4x^2-2y^2)dx = 2xy\,dy \\ y(1) = 3 \end{cases}$

$u = \frac{y}{x}$; $y = u \cdot x$; $y' = u'x + u$; $y' = \frac{dy}{dx}$

$(4x^2-2y^2) = 2xy \cdot y'$

$(4x^2-2y^2) = 2xy(u'x+u)$

$(4x^2-2\cdot(u\cdot x)) = 2xy(u'x+u)$

$\frac{2-2u^2}{u} = u' \cdot x \longrightarrow \frac{2-2u^2}{u} = \frac{du}{dx} \cdot x$

$\int \frac{dx}{x} = \int \frac{u}{2-2u^2} \cdot du \longrightarrow \boxed{\log|x| = -\frac{\log(2-2\frac{y}{x})^2}{4} + K}$

$\log|x| = -\frac{\log 16}{4} + K$

$K - \frac{\log 16}{4} = 0 \longrightarrow K = \frac{\log 16}{4}$

$\boxed{\log|x| = \frac{\log 16}{4} - \frac{\log(2-\frac{2y}{x})^2}{4}} \longrightarrow$ Solución ecuación diferencial

$$
② \begin{cases} y'' + 4y = -4\,\text{sen}(2x) \\ y(0) = -1 \;;\; y'(0) = 4 \end{cases}
$$

$$y'' + 4y = 0$$

$$x^2 + 4 = 0 \longrightarrow \frac{0 \pm \sqrt{0 - 4\cdot 1\cdot 4}}{2\cdot 1} = \pm\sqrt{-4}$$

$$y(x) = Ax\,\text{sen}(2x) + Bx\cos(2x)$$

$$y'(x) = A\,\text{sen}(2x) + 2Ax\cos(2x) + B\cos(2x) - 2Bx\,\text{sen}(2x)$$

$$y''(x) = 2A\cos(2x) + 2A\cos(2x) - 4Ax\,\text{sen}(2x) - 2B\,\text{sen}(2x) - 2B\,\text{sen}(x) - 4Bx\cos(2x)$$

$$y'' + 4y = -4\,\text{sen}(2x) \longrightarrow \begin{cases} -4B = -4 \rightarrow B = 1 \\ 4A = 0 \longrightarrow A = 0 \end{cases}$$

$$y(x) = x\cos(2x) + c_1\,\text{sen}(2x) + c_2\cos(2x)$$

$y(0) = -1$ $\qquad -1 = 0 + 0 + c_2\cdot\cos(0)$

$$c_2 = -1$$

$y'(0) = 4$ $\qquad 4 = 1 - 0 + 2c_1 - 0$

$$c_1 = \frac{3}{2}$$

$$\boxed{y(x) = x\cos(2x) + \frac{3}{2}\,\text{sen}(2x) - \cos(2x)}$$

3. $y' - 6y = 5e^{6x}y^4 \Longrightarrow \alpha = 4$

$z = y^{1-4} \longrightarrow z = y^{-3}$

$y' - 6y = 5e^{6x}$

$\boxed{y' = 5e^{6x} + 6y}$

$z' = -3y^{-4} \cdot y' = -3y^{-4} \cdot (5e^{6x} + 6y) =$

$= -15e^{6x} - 18z = z'$

$z' + 18z = -15e^{6x}$ ec. dife. orden 1

Ec. dif. HOMOGENEA

$EH = z' + 18z = 0 \rightarrow \frac{dz}{dx} = -18z \longrightarrow \frac{dz}{z} = -18dx \longrightarrow$

$\int \frac{dz}{z} = \log|z| \quad \longrightarrow \log|z| = -18x + C \rightarrow$

$\longrightarrow z = ke^{-18x} \longrightarrow z = K(x)e^{-18x}$

$\int -18\,dx = -18x + C$

$z' = K'(x)e^{-18x} - 18K(x)e^{-18x}$

$\rightarrow K'(x)e^{-18x} - 18K(x)e^{-18x} + 18K(x)e^{-18x} = -15e^{6x}$

$K'(x)e^{-18x} = -15e^{6x} \longrightarrow K'(x) = -15e^{24x}$

$K(x) = \int K'(x) = \frac{-15}{24}e^{24x} + C$

$z = K(x) \cdot e^{-18x} = -\frac{15}{24}e^{6x} + c \cdot e^{-18x}$

$z = y^{-3} \longrightarrow \frac{1}{y^3} = -\frac{15}{24}e^{6x} + c \cdot e^{-18x} \longrightarrow \boxed{y^3 = \frac{5e^{6x}}{-8 + 5 \cdot c \cdot e^{-12x}}}$

4.

$$\begin{cases} y'' + 6y' + 9y = 6e^{-3x} + 18 \\ y(0) = 2 ; \; y'(0) = 25 \end{cases}$$

$$y'' + 6y' + 9y$$

$$x^2 + 6x + 9 = 0 \longrightarrow \frac{-6 \pm \sqrt{36 - 4 \cdot 9 \cdot 1}}{2 \cdot 1} = \begin{matrix} \nearrow -3 \\ \searrow -3 \end{matrix}$$

$$y = C_1 \cdot e^{-3x} + x C_2 e^{-3x} \quad C_1, C_2 \in \mathbb{R}$$

$$y(x) = Ax^2 e^{-3x} + B$$

$$y'(x) = 2xA \cdot e^{-3x} - 3Ax^2 e^{-3x} = e^{-3x}(2Ax - 3x^2 A)$$

$$y''(x) = e^{-3x}(2A - 12Ax + 9Ax^2)$$

$$e^{-3x}(2A - 12Ax + 9Ax^2) + 6e^{-3x}(2Ax - 3x^2 A) + 9(Ax^2 e^{-3x} + B) =$$

$$= 6e^{-3x} + 18$$

$$2Ae^{-3x} + 9B = 6e^{-3x} + 18 \quad \begin{cases} 2A = 6 \longrightarrow A = 3 \\ 9B = 18 \longrightarrow B = 2 \end{cases}$$

$$\boxed{y(x) = 3x^2 e^{-3x} + 2}$$

$$\boxed{y'(x) = e^{-3x}(6x - 6x^2)}$$

Escaneado con CamScanner

(5.) $\begin{cases} y' + x\cdot y = 3xe^{x^2} \\ y(0) = 2 \end{cases}$

$y' + x\cdot y = 0 \longrightarrow y' = -x\cdot y$

$\frac{dy}{dx} = -x\cdot y \longrightarrow \int \frac{dy}{y} = \int -x\,dx \longrightarrow$

$\longrightarrow \log|y| = \frac{-x^2}{2} + c$

$e^{\log|y|} = e^{\frac{-x^2}{2}+c}$

$\boxed{y = e^{-\frac{x^2}{2}} \cdot K(y)}$

$y' = -e^{-\frac{x^2}{2}} \cdot x \cdot K(y) + K'(y) \cdot e^{-\frac{x}{2}^2}$

$e^{\frac{-x^2}{2}} \cdot K'(y) = 3x\, e^{x^2}$

$K'(y) = 3x \cdot e^{\frac{3x^2}{2}}$

$K(y) = \int K'(y) = e^{\frac{3x^2}{2}} + C$

$\boxed{y = e^{\frac{-x^2}{2}} \cdot e^{\frac{3x^2}{2}} + 1}$

$y = e^{\frac{-x^2}{2}} \cdot e^{\frac{3x^2}{2}} + c$

$y(0) = 2 \qquad 2 = e^0 \cdot e^0 + c \longrightarrow c = 1$

Escaneado con CamScanner

(6.) $2xy - 3x^2y^2 + (x^2 - 2x^3y)y' = 0$

$$\left.\begin{array}{l} \dfrac{dM}{dy} = 2x - 6x^2y \\ \\ \dfrac{dN}{dy} = 2x - 6x^2y \end{array}\right\} \begin{array}{l}\text{coinciden} \\ \text{ec. dif. exacta}\end{array}$$

$$\frac{df}{dx} = M(x,y) = 2xy - 3x^2y^2$$

$$\int (2xy - 3x^2y^2)dx = x^2y - x^3 \cdot y^2 + K(y)$$

$$K'(y) = 0$$

$$f = x^2y - x^3y^2 \Rightarrow f(x,y) = C$$

$$\boxed{x^2y - x^3y^2 = C}$$

X

+2

+2

7.

$$y' - \frac{2}{x+2} \cdot y = 2(x+2)^3$$

$$y' - \frac{2}{x+2} \cdot y = 0$$

$$y' = \frac{2y}{x+2} \longrightarrow \frac{dy}{dx} = \frac{2y}{x+2} \longrightarrow \frac{dy}{2y} = \frac{dx}{x+2} \longrightarrow$$

$$\longrightarrow \int \frac{dy}{2y} = \int \frac{dx}{x+2} \longrightarrow \frac{1}{2} \log y = \log(x+2) + K$$

$$y^{1/2} = (x+2) \cdot K(x)$$

$$\boxed{y = (x+2)^2 \cdot K(x)}$$

$$y' = (x+2)^2 \cdot K'(x) + K(x)$$

$$K'(x) = \frac{2(x+2)^3}{(x+2)^2} \longrightarrow \boxed{K(x) = x^2 + 4x + C}$$

$$\boxed{y = (x+2)^2 (x^2 + 4x + C)}$$

(8) $(e^x y^2 + y\,\text{sen}x) + (3ye^x - 2\cos x)y' = 0$

$$\frac{dM}{dy} = (e^x \cdot 2y + \text{sen}x)\cdot N(y) + e^x y^2 + y\,\text{sen}x)\cdot N'(y)$$

$$\frac{dN}{dy} = (3xy \cdot e^x + 2\,\text{sen}x)\cdot N(y) + (e^x \cdot 2y + \text{sen}x) N(y) +$$
$$+ (e^x y^2 + y\,\text{sen}x) N'(y)$$

$$(e^x \cdot y^2 + y\,\text{sen}x) N'(y) = N(y)(ye^x + \text{sen}x)$$

$$\int \frac{dN}{N(y)} = \int \frac{1}{y}\,dy$$

$$\log|N(y)| = \log|y| + C \rightarrow e^{\log(N(y))} = e^{\log|y|} + e^c$$

$$N(y) = y \cdot K \rightarrow K = 1$$

$$N(y) = y$$

$$M(x,y) = y^3 \cdot e^x + y^2 \cdot \text{sen}x$$

$$N(x,y = 3y^2 \cdot e^x - 2y\cos x$$

$$\frac{df}{dx} = y^3 \cdot e^x + y^2 \cdot \text{sen}x$$

$$df = (y^3 e^x + y^2 \cdot \text{sen}x)dx$$

$$f = \int (y^3 e^x + y^2 \cdot \text{sen}x)dx$$

$$f = y^3 \cdot e^x - y^2 \cdot \cos x + C(y)$$

$$3y^2 \cdot e^x - 2y \cdot \cos x + C'(y) = 3y^2 e^x -$$
$$- 2y\cos x$$

$$C'(y) = 0 \rightarrow C(y) = 0$$

$$\boxed{y^3 \cdot e^x - y^2 \cdot \cos x = C}$$

(9.)

$$y^{vii} + y^{vi} + 2y^{v} + 10y^{iv} + 13y^{iii} + 5y^{ii} = 0$$

$$x^7 + x^6 + 2x^5 + 10x^4 + 13x^3 + 5x^2 = 0$$

$$x^2(x^5 + x^4 + 2x^3 + 10x^2 + 13x^1 + 5) = 0$$

	1	1	2	10	13	5
-1	1	-1	0	-2	-8	-5
	1	0	2	8	5	$\boxed{0}$
-1		-1	+1	-3	-5	
	1	-1	3	5	$\boxed{0}$	
-1		-1	2	-5		
	1	-2	5	$\boxed{0}$		

$x = -1$; multiplicidad $3 \rightarrow e^{-x}, e^{-x}\cdot x, e^{-x}\cdot x^2$

$x = 1 \pm 2i$ $\rightarrow e^{x}\cdot \operatorname{sen} 2x$, $\rightarrow e^{x}\cdot \cos 2x$

$x = 0$; multiplicidad $2 \rightarrow e^{0x}$; $x \cdot e^{0x}$

$$x^2 - 2x + 5 = 0 \longrightarrow \frac{+2 \pm \sqrt{4 - 4\cdot 1\cdot 5}}{2\cdot 1} = \frac{2 \pm \sqrt{-16}}{2} \rightarrow 1 \pm 2i$$

$$\boxed{C(x) = C_1 + C_2\cdot x + C_3\cdot e^{-x} + C_4 x\cdot e^{-x} + C_5\cdot x^2\cdot e^{-x} + C_6\cdot e^{x}\cdot \cos 2x + C_7\, e^{x}\cdot \operatorname{sen} 2x}$$