Ejercicios ecuaciones diferenciables (entregables)

1. $\begin{cases} (4x^2 - 2y^2)\,dx = 2xy\,dy \\ y(1) = 3 \end{cases}$

$$y' = \frac{4x^2 - 2y^2}{2xy} \qquad y' = u'x + u$$

$$y = ux$$

$$u'x + u = \frac{4x^2 - 2y^2}{2x^2 u}$$

$$u'x + u = 2 - u$$

$$u'x = 2 - 2u$$

$$\frac{dx}{du}x = 2 - 2u \longrightarrow \int \frac{dx}{x} = \int 2 - 2u\,du$$

$$\log|x| + C = 2u - u^2$$

$$\log|x| + C = \frac{2y}{x} - \left(\frac{y}{x}\right)^2$$

- $y(1) = 3$

$$\frac{2y}{x} - \left(\frac{y}{3}\right)^2 = 3 \qquad \boxed{2x^{-1} - x^{-2} = 3}$$

2. $\begin{cases} y'' + 4y = -4\,\text{sen}(2x) \\ y(0) = -1 \\ y'(0) = 4 \end{cases}$

• Ec dif homogenea asociada

$y'' + 4y = 0 \longrightarrow P(x) = x^2 + 4 = 0 \longrightarrow x = \sqrt{-4} = \pm 2i$

Raices $\rightarrow 0 \pm 2i$ mult 1 $\longrightarrow e^{0x}\,\text{Cos}(2x) \rightarrow \text{Cos}(2x)$

$\qquad\qquad\qquad\qquad\qquad\searrow e^{0x}\,\text{Sen}(2x) \rightarrow \text{Sen}(2x)$

Solución ecuación diferencial homogénea asociada:

$y_h(x) = C_1\,\text{Cos}(2x) + C_2\,\text{Sen}(2x) \qquad C_1, C_2 \in \mathbb{R}$

• Solución particular:

$y(x) = Ax\,\text{Cos}(2x) + Bx\,\text{Sen}(2x)$

$y'(x) = A\,\text{Cos}(2x) - 2Ax\,\text{Sen}(2x) + B\,\text{Sen}(2x) + 2Bx\,\text{Cos}(2x)$

$y''(x) = -2A\,\text{Sen}(2x) - 2A\,\text{Sen}(2x) - 4Ax\,\text{Cos}(2x) + 2B\,\text{Cos}(2x) + 2B\,\text{Cos}(2x) - 4B + \text{Sen}\,2x =$

$= -4A\,\text{Sen}(2x) - 4Ax\,\text{Cos}(2x) + 4B\,\text{Cos}(2x) - 4Bx\,\text{Sen}(2x)$

• Sustituimos en $y'' + 4y = -4\,\text{Sen}(2x)$ -

$4A\,\text{Sen}(2x) - \cancel{4Ax\,\text{Cos}(2x)} + 4B\,\text{Cos}(2x) - \cancel{4Bx\,\text{Sen}(2x)} + \cancel{4Ax\,\text{Cos}(2x)} + \cancel{4B\,\text{Sen}(2x)}$

$-4A\,\text{Sen}\,2x + 4B\,\text{Cos}(2x) = -4\,\text{Sen}(2x)$

$\begin{cases} -4A = -4 \rightarrow A = 1 \\ 4B = 0 \rightarrow B = 0 \end{cases}$

$y_p(x) = x\,\text{Cos}(2x)$

Sol general $\rightarrow y(x) = x\,\text{Cos}(2x) + C_1\,\text{Cos}(2x) + C_2\,\text{Sen}(2x)$

$y'(x) = \text{Cos}(2x) - 2x\,\text{Sen}(2x) - 2C_1\,\text{Sen}(2x) + 2C_2\,\text{Cos}(2x) - 1 = y(0) = 0\,\text{Cos}(2x) +$

$C_1\,\text{Cos}(2\cdot 0) + C_2\,\text{Sen}(2\cdot 0) \rightarrow C_1 = -1$

$4 = y'(0) = \cos(0\cdot 2) - 2\cdot 0 \operatorname{Sen}(2\cdot 0) - 2C_1 \operatorname{Sen}(2\cdot 0) + 2C_2 \cos(2\cdot 0)$

$1 - 0 - 0 + 2C_2 = 4 \to 1 + 2C_2 = 4 \to 2C_2 = 3 \to C_2 = \frac{3}{2}$

$y(x) = x\cos(2x) - \cos(2x) + \frac{3}{2} \operatorname{sen}(2x)$

3. $y' - 6y = 5e^{6x} y^4$

$y' = 5e^{6x} y^4 + 6y$

$\alpha = 3 \quad z = y^{1-4} = y^{-3}$

$z' = -3y^{-4} \cdot y' = -3y^{-4}(5e^{6x} \cdot y^4 + 6y) = -15e^{6x} - 18z$

$z' = -15e^{6x} - 18z$

$z' + 18z = 0 \to \frac{dz}{dx} = -18z \to \int \frac{dz}{z} = \int -18dz \to \log z = -18x + c$

$z = e^{-18x + c} \to z = e^{-18x} + e^{c} \to z = K \cdot e^{-18K}$

Sea solución

$z' = K'(x)e^{-18x} + K(x) - 18e^{-18x} = K'(x)e^{-18x} - 18K(x)e^{-18x}$

$K'(x)e^{-18x} - \cancel{18K(x)e^{-18x}} + \cancel{18K(x)e^{-18x}} = -15e^{6x} \to K'(x)e^{-18x} = -15e^{6x}$

$K'(x) = \frac{-15e^{6x}}{e^{-18x}} \to K'(x) = -15e^{6x} - e^{18x} = -15e^{24x}$

$K(x) = \int -15e^{24x}dx = -15\int e^{24x}dx = -15e^{24x} \cdot 24 = -360e^{24x} + C$

$z = K(x)e^{-18x} = (-360e^{24x} + C)e^{-18x} = -360e^{6x} + Ce^{-18x}$

$z = -360e^{6x} + Ce^{-18x} \quad C \in \mathbb{R}$

$z = y^{-3} \to z = \frac{1}{y^3}$

$y^3 = \frac{1}{z} \to y = \frac{1}{\sqrt[3]{z}}$

$$\boxed{y = \frac{1}{\sqrt[3]{-360e^{6x} + Ce^{-18x}}}}$$

$$4\begin{cases} y''+6y'+9y=6e^{-3x}+18 \\ y(0)=2 \\ y'(0)=25 \end{cases}$$

- Ecuación diferencial homogénea asociada

$y''+6y'+9y=0$

$P(x)=x^2+6x+9=(x+3)^2$ — -3 mult $2 \rightarrow e^{-3x}, xe^{-3x}$

$y_h(x)=C_1e^{-3x}+C_2xe^{-3x}, \quad C_1,C_2\in\mathbb{R}$

- Solución particular

$y(x)=Ax^2e^{-3x}$

$y'(x)=2Ae^{-3x}-3Ax^2e^{-3x}$

$y''(x)=2Ae^{-3x}-6Axe^{-3x}-6Ae^{-3x}+9Ax^2e^{-3x}=2Ae^{-3x}-12Axe^{-3x}+9Ax^2e^{-3x}$

$\Downarrow$ sustituimos

$y''+6y'+9y=6e^{-3x}+18$

$2Ae^{-3x}-\cancel{12Axe^{-3x}}+\cancel{9Ax^2e^{-3x}}+\cancel{12Axe^{-3x}}-\cancel{18Ax^2e^{-3x}}+9Ax^2e^{-3x}=6e^{-3x}+18$

$2Ae^{-3x}=6e^{-3x}+18$

$2A=6+18e^{3x}$

$A=3+9e^{3x}$

$y_p(x)=(3+9e^{3x})x^2e^{-3x}=3x^2e^{-3x}+9x^2$

- Solución general

$y(x)=3x^2e^{-3x}+9x^2+C_1e^{-3x}+C_2xe^{-3x} \quad C_1,C_2\in\mathbb{R}$

$y(0)=2$; $3\cdot0\cdot e^0+9\cdot0+C_1\cdot e^0+C_2\cdot0\cdot e^0=2$; $C_1=2$

$y'(x)=6xe^{-3x}-9x^2e^{-3x}+18x-3C_1e^{-3x}+C_2e^{-3x}-3C_2e^{-3x}$

$y'(0)=25$; $6\cdot0\cdot e^0-9\cdot0\cdot e^0+18\cdot0-3\cdot C_1\cdot e^0+C_2\cdot e^0-3\cdot0\cdot e^0=25$

$-3C_1+C_2=25$; $-6+C_2=25$; $C_2=31$

$y(x)=3x^2e^{-3x}+9x^2+2e^{-3x}+31xe^{-3x}$

$$5 \begin{cases} y' + xy = 3x e^{x^2} \\ y(0) = 2 \end{cases}$$

• Ecuación diferencial lineal homogenea asociada

$y' + xy = 0$

$\frac{dy}{dx} = -xy \; ; \int \frac{dy}{y} = \int -x dx \; ; \log y = \frac{-x^2}{2} + c \; ; \; c \in \mathbb{R}$

$y = e^{-x^2/2 + c} = e^{-x^2/2} \cdot e^c = K e^{-x^2/2} \rightarrow y = K(x) e^{-x^2/2}$

$y = K(x) e^{-x^2/2} - K(x) x e^{-x^2/2}$

• sustituimos

$K'(x) e^{-x^2/2} - \cancel{K(x) x e^{-x^2/2}} + \cancel{K(x) x e^{-x^2/2}} = 3x e^{x^2}$

$K'(x) e^{-x^2/2} = 3x e^{x^2} \; ; \; K'(x) = 3x e^{x^2} / e^{-x^2/2}$

$K(x) = \int \frac{3x \cdot e^{x^2}}{e^{x^2}} dx = \int 3x dx = \frac{3x^2}{2} + C \; ; \; C \in \mathbb{R}$

• Sustituimos $y = K(x) e^{-x^2/2}$

$y = e^{\frac{3x^2}{2}} \cdot e^{-x^2/2} + C = e^{x^2} + C\sqrt{e^{x^2}} = \frac{C}{\sqrt{e^{x^2}}}$

6. $\underbrace{2xy - 3x^2y^2}_{M} + \underbrace{(x^2 - 2x^3y)}_{N} y' = 0$

$$\frac{dM}{dy} = \frac{dN}{dx}$$

$\frac{dM}{dy} = 2x - 6x^2y$; $\frac{dN}{dx} = 2x - 6x^2y$

$$\frac{dM}{dy} = \frac{dN}{dx}$$

• Ecuación diferencial

$$\frac{df}{dx} = M \rightarrow f = \int 2xy - 3x^2y^2\,dx$$

$$f = y\int 2x\,dx - y^2\int 3x^2\,dx = \boxed{y \cdot x^2 - y^2x^3 + C}$$

$$\frac{df}{dy} = N \rightarrow x^2 - \cancel{2yx^3} + C'(y) = x^2 - \cancel{2x^3y} \rightarrow C'y(0) = 0 \rightarrow C'(y) = 0$$

$$f(x,y) = yx^2 - y^2x^3$$

$$\boxed{yx^2 - y^2x^3 = C}$$

~~dejar~~ Recuadro

7 $y' - \frac{2}{x+2} y = 2(x+2)^3$ $y'(x) + p(x)y = q(x)$

$P(x) = -\frac{2}{x+2}$ $q(x) = 2(x+2)^3$

$\left[y\mu = \int q\mu \, dx \right] \longrightarrow \mu = e^{\int p(x)dx}$

$\mu = e^{\int -\frac{2}{x+2} dx} \longrightarrow \int -\frac{2}{x+2} dx = -2 \int \frac{1}{x+2} \cancel{dx} \left[= -2 \ln(x+2) + C \right]$

$\mu = e^{-2\ln(|x+2|)} = e^{\ln(x+2)^{-2}} \longrightarrow \underline{\mu = (x+2)^{-2}}$

$y(x+2)^{-2} = \int 2(x+2)^3 (x+2)^{-2} dx$

$y(x+2)^{-2} = -\int 2x + 4 dx = 2\int x dx + 4\int 1 dx = 2 \cdot \frac{x^2}{2} + 4x$

$y(x+2)^{-2} = x(x+4) + C$

$y = \left[x(x+4)(x+2)^2 \right] + C \cdot (x+2)^2$

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