1

$$\begin{cases} (4x^2-2y^2)\,dx = 2xy\,dy \\ y(1)=3 \end{cases} \rightarrow 4x^2-2y^2 = 2xy\frac{dy}{dx} \rightarrow 4x^2-2y^2 = 2xyy' \rightarrow$$

$$\rightarrow \boxed{y' + \frac{1}{x}y = 2xy^{-1}}$$ Ec. dif. Bernoulli

$$z = y^{1-\alpha} = y^{1+1} = y^2$$

$$z' = 2yy' = 2y\cdot\left(2xy^{-1} - \frac{y}{x}\right) = 4x - \frac{2y^2}{x} = 4x - \frac{2z}{x}$$

$$y' = 2xy^{-1} - \frac{y}{x}$$

$$\boxed{z' + \frac{2z}{x} = 4x}$$ ← Ec dif lineal de 1er orden

$$\boxed{z' + \frac{2z}{x} = 0}$$ ← Ec dif homogenea asociada

$$\hookrightarrow z' = \frac{-2z}{x} \rightarrow \frac{dz}{dx} = -\frac{2z}{x} \rightarrow \frac{-dz}{2z} = \frac{dx}{x} \rightarrow -\frac{1}{2}\int\frac{dz}{z} = \int\frac{dx}{x} \rightarrow$$

$$\rightarrow -\frac{1}{2}\log z = \log x + C \rightarrow \log z^{-1/2} = \log x + C \rightarrow z^{-1/2} = e^{\log(x)+C} = e^{\log(x)}\cdot \underset{K\in\mathbb{R}}{K}$$

$$z = x^{-2}k \rightarrow \boxed{z = x^{-2}k(x)} \; ; \; z' = k'(x)(x^{-2}) + k(x)(-2x^{-3})$$

$$k'(x)(x^{-2}) + \cancel{k(x)(-2x^{-3})} + \cancel{\frac{2k(x)(x^{-2})}{x}} = 4x \rightarrow k'(x) = 4x^3$$

$$K(x) = \int 4x^3\,dx = 4\int x^3\,dx = 4\frac{x^4}{4} + C = x^4 + C \; ; \; C\in\mathbb{R}$$

$$z = (x^4+C)(x^{-2}) = x^2 + \frac{C}{x^2} \rightarrow z = y^2 \rightarrow y = \sqrt{x^2 + \frac{C}{x^2}}$$

$$y(1) = 3 = \sqrt{1^2 + \frac{C}{1^2}} \rightarrow 3 = \sqrt{1+C} \rightarrow 9 = 1 + C \rightarrow \boxed{C = 8}$$

$$\boxed{y = \sqrt{x^2 + \frac{8}{x^2}}}$$

(3)

$$\left\{ y' - 6y = 5e^{6x}y^4 \right.$$ *Ec. dif. de Bernoulli*

$$z = y^{1-\alpha} = y^{-3}$$

$$z' = -3y^{-4}y' = -3y^{-4}(5e^{6x}y^4 + 6y) = -15e^{6x} - 18y^{-3} = -15e^{6x} - 18z$$

$$y' = 5e^{6x}y^4 + 6y$$

$$z' + 18z = -15e^{6x} \longleftarrow$$ Ec dif. lineal 1er orden

$$z' + 18z = 0 \longleftarrow$$ Ec dif homogénea asociada

$$z' = -18z \to \frac{dz}{dx} = -18z \to \frac{dz}{-18z} = dx \to \frac{-1}{18}\int\frac{dz}{z} = \int dx \to \frac{-1}{18}\log z = x + C$$

$$z^{-1/18} = e^{x+C} \to z^{-1/18} = e^x \cdot K \to z = e^{-18x} \cdot K$$

$$K \in \mathbb{R}$$

$$\boxed{z = e^{-18x} \cdot K(x)} \; ; \; \boxed{z' = e^{-18x}(-18) \cdot K(x) + K'(x)e^{-18x}}$$

$$e^{-18x} \cdot K'(x) = -15e^{6x} \to K'(x) = \frac{-15e^{6x}}{e^{-18x}} = -15e^{24x}$$

$$K(x) = \int -15e^{24x}dx = -15\int e^{24x}dx = \frac{-15}{24}\int 24e^{24x}dx = -\frac{15}{24} \cdot e^{24x} + C \; ; \; C \in \mathbb{R}$$

$$z = e^{-18x} \cdot \left(\frac{-15}{24}e^{24x} + C\right) = \frac{-15}{24}e^{6x} + e^{-18x} \cdot C$$

$$z = y^{-3} \to \boxed{y = z^{-1/3} = \left(\frac{-15}{24}e^{6x} + e^{-18x}C\right)^{-1/3} = \frac{-15}{24}e^{-2x} + e^{6x} \cdot C}$$

(5)

$$\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$$

Ec lineal de 1er orden

$\boxed{y' + xy = 0}$ Ec dif homogenea asoc.

$$y' = -xy \longrightarrow \frac{dy}{dx} = -xy \rightarrow \frac{dy}{y} = -x\,dx \rightarrow \int \frac{dy}{y} = \int -x\,dx$$

$$\log y = \frac{-x^2}{2} + C \longrightarrow y = e^{\frac{-x^2}{2}+C} = e^{\frac{-x^2}{2}} \cdot \underset{K \in \mathbb{R}}{K}$$

$$\boxed{y = e^{\frac{-x^2}{2}} \cdot K(x)} \; ; \; \boxed{y' = e^{\frac{-x^2}{2}}(-x) \cdot K(x) + e^{\frac{-x^2}{2}} K'(x)}$$

$$\cancel{e^{\frac{-x^2}{2}}(-x) \cdot K(x)} + e^{\frac{-x^2}{2}} K'(x) + \cancel{x \cdot e^{\frac{-x^2}{2}} K(x)} = 3xe^{x^2}$$

$$e^{\frac{-x^2}{2}} K'(x) = 3xe^{x^2} \longrightarrow K'(x) = \frac{3xe^{x^2}}{e^{\frac{-x^2}{2}}} = 3xe^{\frac{3}{2}x^2}$$

$$K(x) = \int 3x e^{\frac{3}{2}x^2} dx = e^{\frac{3}{2}x^2} + C$$

$$y = e^{\frac{-x^2}{2}} \cdot \left(e^{\frac{3}{2}x^2} + C\right) = e^{x^2} + e^{\frac{-x^2}{2}} C$$

$$y(0) = 2 = e^{0^2} + e^{\frac{-0^2}{2}} \cdot C = 1 + C \longrightarrow \boxed{C = 1}$$

$$\boxed{y = e^{x^2} + e^{\frac{-x^2}{2}}}$$

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$$\left\{ y' - \frac{2}{x+2} y = 2(x+2)^3 \right.$$

<u>Ec dif lineal 1er orden</u>

$$y' - \frac{2}{x+2} y = 0 \longleftarrow$$ <u>Ec dif homogénea asociada</u>

$$y' = \frac{2}{x+2} y \longrightarrow \frac{dy}{dx} = \frac{2}{x+2} y \longrightarrow \frac{dy}{y} = \frac{2dx}{x+2} \longrightarrow \int \frac{dy}{y} = \int \frac{2dx}{x+2}$$

$$\log|y| = 2\log(x+2) + c \rightarrow y = e^{\log(x+2)^2 + c} \Rightarrow y = (x+2)^2 \cdot K$$

$K \in \mathbb{R}$

$$\boxed{y = (x+2)^2 \cdot K(x)} \longrightarrow \boxed{y' = 2(x+2)\, k(x) + k'(x)(x+2)^2}$$

$$\cancel{2(x+2)k(x)} + k'(x)(x+2)^2 - \frac{\cancel{2(x+2)^2 \cdot k(x)}}{\cancel{(x+2)}} = 2(x+2)^3$$

$$k'(x)\cancel{(x+2)^2} = 2(x+2)^{\cancel{3}} \longrightarrow k'(x) = 2(x+2)$$

$$k(x) = \int 2(x+2)\,dx = 2\int (x+2)\,dx = 2\frac{x^2}{2} + 2\cdot 2x = x^2 + 4x + c$$

$$y = (x+2)^2 \cdot (x^2 + 4x + C) = (x^2 + 4 + 4x)(x^2 + 4x + c) :$$

$$y = x^4 + 4x^3 + Cx^2 + 16x + 4x^2 + 4C + 4x^3 + 16x^2 + 4Cx$$

$$\boxed{y = x^4 + 8x^3 + (20 + c)x^2 + (16 + 4C)x + 4C}$$