$$1. \begin{cases} (4x^2 - 2y^2)\,dx = 2xy\,dy \\ y(1) = 3 \end{cases}$$

$$y' = \frac{2x^2 + y^2}{xy} = \frac{2x}{y} + \frac{y}{x} \rightarrow \text{Homogenea} \qquad v = \frac{y}{x} \; ; \quad \begin{array}{l} y = vx \\ y' = v'x + v \end{array} \; ; \quad v' = \frac{dv}{dx}$$

$$v'x + v = \frac{2x}{vx} + \frac{vx}{x} \; ; \quad v'x + v = \frac{2}{v} + v \; ; \quad v'x = \frac{2}{v} + v - v \; ;$$

$$; \; \frac{dv}{dx} \cdot x = \frac{2}{v} \; ; \quad \int \frac{1}{2} v\,dv = \int \frac{1}{x}\,dx \; , \quad \frac{1}{2} \cdot \frac{v^2}{2} = \ln|x| + C \; ;$$

$$\frac{y^2/x^2}{4} = \ln|x| + C \; ; \quad \frac{y^2}{4x^2} = \ln|x| + C \; ; \quad y = 2x\sqrt{\ln|x| + C}$$

$$y(1) = 3 \longrightarrow \quad 3 = 2 \cdot 1 \sqrt{\ln|1| + C} \rightarrow \quad C = \frac{9}{4}$$

$$\boxed{y = 2x\sqrt{\ln|x| + 9/4}} \quad \text{Solución}$$

2. $\begin{cases} y'' + 4y = -4\,\text{sen}(2x) \\ y(0) = -1, \quad y'(0) = 4 \end{cases}$

$y_h(x) \longrightarrow \quad y'' - 4y = 0 \;; \qquad r^2 + 4 = 0 \;; \quad r = \pm 2i$

Raices $\rightarrow$ $0 \pm 2i$ $\begin{cases} e^{0x} \cos(2x) = \cos(2x) \\ e^{0x} \,\text{sen}(2x) = \text{sen}(2x) \end{cases}$

$y_h(x) = C_1 \cos(2x) + C_2 \,\text{sen}(2x) \qquad C_1, C_2 \in \mathbb{R}$

Como $y_h(x) = y_p(x)$, multiplico $y_p(x) \cdot x$

$y_p(x) \longrightarrow \quad y_p(x) = Ax\cos(2x) + Bx\,\text{sen}(2x)$

$y_p'(x) = A\cos(2x) - 2 \cdot Ax\,\text{sen}(2x) + B\,\text{sen}(2x) + 2Bx\cos(2x) =$

$= (A + 2Bx)\cos(2x) + (-2Ax + B)\,\text{sen}(2x)$

$y_p''(x) = 2B\cos(2x) - 2(A + 2Bx)\,\text{sen}(2x) - A\,\text{sen}(2x) + 2(-2Ax + B)\cos(2x) =$

$= (2B - 4Ax + 2B)\cos(2x) + (-2A - 4Bx - 2A)\,\text{sen}(2x) =$

$= (4B - 4Ax)\cos(2x) + (-4A - 4Bx)\,\text{sen}(2x)$

Sustituyo en $y'' + 4y = -4\,\text{sen}(2x)$

$(4B - 4Ax)\cos(2x) + (-4A - 4Bx)\,\text{sen}(2x) + 4(Ax\cos(2x) + Bx\,\text{sen}(2x)) = -4\,\text{sen}\,2x$

$\left.\begin{array}{l} \cos(\cancel{2x}) \rightarrow 4B - 4Ax + 4Ax = 0 \\ \text{sen}(\cancel{2x}) \rightarrow -4A - 4Bx + 4Bx = -4 \end{array}\right\} \quad \begin{array}{l} B = 0 \\ A = 1 \end{array}$

Sustituyo en la $y_p(x)$

$y_p(x) = x\cos(2x)$

$y(x) = x\cos(2x) + C_1\cos(2x) + C_2\,\text{sen}(2x), \qquad C_1, C_2 \in \mathbb{R}$

$y'(x) = \cos(2x) - 2x\,\text{sen}(2x) - 2C_1\,\text{sen}(2x) + 2C_2\cos(2x), \qquad C_1, C_2 \in \mathbb{R}$

$\left.\begin{array}{l} y(0) = -1 \rightarrow \quad -1 = C_1 \\ y'(0) = 4 \rightarrow \quad 4 = 1 + 2C_2 \end{array}\right\} \quad C_2 = 3/2$

$y(x) = x\cos(2x) - \cos(2x) + \frac{3}{2}\,\text{sen}(2x)$

$y(x) = (x - 1)\cos(2x) + \frac{3}{2}\,\text{sen}(2x)$ Sol. del problema de cond. iniciales.

3. $y' - 6y = 5e^{6x} \cdot y^4$ Bernoulli $\alpha = 4$, $z = y^{1-\alpha}$

$z = y^{-3}$

$z' = -3y^{-4} \cdot y'$ $y' = 6y + 5e^{6x} y^4$

$z' = -3y^{-4} \cdot (6y + 5e^{6x} \cdot y^4)$

$z' = -18y^{-3} - 15e^{6x}$; $z = y^{-3}$

$z' = -18z - 15e^{6x}$; $z' + 18z = -15e^{6x}$ E.d. lineal

$z' + 18z = 0$; $z' = -18z$; $\frac{dz}{dx} = -18z$; $\int \frac{1}{z}\, dz = \int -18\, dx =$

$\ln|z| = -18x + C$; $z = e^{-18x + C} = Ke^{-18x} \longrightarrow z = Ke^{-18x}$

$K \longrightarrow k(x)$

$z = k(x)e^{-18x}$; $z' = k'(x) \cdot e^{-18x} - 18\, k(x)e^{-18x}$

$k'(x)e^{-18x} - \cancel{18k(x)e^{-18x}} + \cancel{18k(x)e^{-18x}} = -15e^{6x}$

$k'(x)e^{-18x} = -15e^{6x}$; $k'(x) = -15e^{24x}$

$k(x) = \int -15e^{24x} dx = \frac{-15}{24} \int 24\, e^{24x} dx = \frac{-5}{8} e^{24x} + c$

$z = \left(\frac{-5}{8} e^{24x} + c\right) e^{-18x} = \frac{-5}{8} e^{6x} + c e^{-18x} \longrightarrow z = y^{-3}$

$y^{-3} = \frac{-5}{8} e^{6x} + ce^{-18x}$

$\frac{1}{y^3} = -\frac{5}{8} e^{6x} + ce^{-18x} \longrightarrow \boxed{y = \frac{1}{\sqrt[3]{-\frac{5}{8} e^{6x} + ce^{-18x}}}}$

4. $\begin{cases} y'' + 6y' + 9y = \underbrace{6e^{-3x}}_{exp\ y_{p_1}} + \underbrace{18}_{pol.\ y_{p_2}} \\ y(0) = 2, \quad y'(0) = 25 \end{cases}$

$y(x) = y_{p_1}(x) + y_{p_2}(x) + y_h(x)$

Homogenea $\rightarrow$ $y'' + 6y' + 9y = 0$, $r^2 + 6r + 9 = 0$

$r = -3(d)$ raices $\rightarrow$ -3 Mult. 2 $\begin{cases} e^{-3x} \\ xe^{-3x} \end{cases}$

$y_h(x) = C_1 e^{-3x} + C_2 x e^{-3x} = (C_1 + C_2) e^{-3x}$ $\quad C_1, C_2 \in \mathbb{R}$

$y_{p_1} \rightarrow$ $y_{p_1}(x) = Ax^2 e^{-3x}$ Caso problemático

$y'_{p_1}(x) = 2Axe^{-3x} - 3Ae^{-3x}x^2 = (2Ax - 3Ax^2)e^{-3x} =$

$y''_{p_1}(x) = (2A - 6Ax)e^{-3x} - 3(2Ax - 3Ax^2)e^{-3x} =$

$= (2A - 6Ax - 6Ax + 9Ax^2)e^{-3x} = (2A - 12Ax + 9Ax^2)e^{-3x}$

Sustituyo en $y'' + 6y' + 9y = 6e^{-3x}$

$(2A - 12Ax + 9Ax^2)e^{-3x} + 6(2Ax - 3Ax)e^{-3x} + 9Ax^2 e^{-3x} = 6e^{-3x}$

$(2A - 12Ax + 9Ax^2 + 12Ax - 18Ax^2 + 9Ax^2)e^{-3x} = 6e^{-3x}$

$2Ae^{-3x} = 6e^{-3x}$; $\quad 2A = 6 \longrightarrow A = 3$; $\quad y_{p_1}(x) = 3x^2 e^{-3x}$

$y_{p_2} \rightarrow$ $y_{p_2}(x) = A$; $\quad y'_{p_2} = 0$

Sustituyo en $y'' + 6y' + 9y = 18$

$0 + 6 \cdot 0 + 9A = 18 \longrightarrow A = 2$; $\quad y_{p_2}(x) = 2$

$y(x) = 3x^2 e^{-3x} + 2 + (C_1 + C_2 x)e^{-3x}$

$y(x) = (3x^2 + C_1 + C_2 x)e^{-3x} + 2$ $\quad C_1, C_2 \in \mathbb{R}$

$\left.\begin{matrix} y(0) = 2 \\ y'(0) = 25 \end{matrix}\right\}$ $\quad y'(x) = (6x + C_2)e^{-3x} - 3(3x^2 + C_1 + C_2 x)e^{-3x}$

$y'(x) = (6x + C_2 - 9x^2 - 3C_1 - 3C_2 x)e^{-3x}$

$\left.\begin{matrix} y(0) = 2 \rightarrow 2 = C_1 + 2 \\ y'(0) = 25 \rightarrow 25 = C_2 - 3C_1 \end{matrix}\right\}$ $\quad \begin{matrix} C_1 = 0 \\ C_2 = 25 \end{matrix}$

5. $\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$

La convierto en una ec. diferencial de variables separadas

$y' = -xy$; $\frac{dy}{dx} = -xy$; $\int \frac{dy}{y} = -\int -x\,dx$; $\ln|y| = -x^2 + C$

$y = ke^{-x^2/2}$

La cte. la hacemos variable ; $k \to k(x)$

$y = k(x)e^{-x^2/2}$

$y' = k'(x)e^{-x^2/2} - xk(x)e^{-x^2/2}$

Sustituyo

$k'(x)e^{-x^2/2} - \cancel{xk(x)e^{-x^2/2}} + \cancel{xk(x)e^{-x^2/2}} = 3xe^{-x^2}$

$k'(x)e^{-\frac{x^2}{2}} = 3xe^{-x^2}$

$k'(x) = \frac{3xe^{-x^2}}{e^{-x^2/2}} = 3xe^{-x^2/2}$

$k(x) = \int 3xe^{-x^2/2}\,dx = -3\int -xe^{-x^2/2}\,dx = -3e^{-x^2/2} + C$

$\boxed{y = (-3e^{-x^2/2} + c)e^{-x^2/2}}$ Solución de la ec. dif. lineal.

$y(0) = 2$ (x, y) $\longrightarrow$ $2 = (-3e^0 + c)e^0$

$2 = -3 + C \longrightarrow C = 5$

$\boxed{y = (-3e^{-x^2/2} + 5)e^{-x^2/2}}$

6. $(2xy - 3x^2y^2) + (x^2 - 2x^3y) \cdot y' = 0$

$$\left.\begin{array}{l} \dfrac{dM}{dy} = 2x - 6x^2y \\[2ex] \dfrac{dN}{dx} = 2x - 6x^2y \end{array}\right\} \quad \frac{dM}{dy} = \frac{dN}{dx} \qquad \text{Exacta}$$

$\dfrac{df}{dx} = M$; $f(x,y) = \int M\,dx = \int 2xy - 3x^2y^2\,dx = \dfrac{2yx^2}{2} - \dfrac{3y^2x^3}{3} + k(y)$

$f(x,y) = x^2y - x^3y^2 + k(y)$

$\dfrac{df}{dy} = N$, $\dfrac{df}{dy} = x^2 - 2x^3y + k'(y)$

$\cancel{x^2} - 2\cancel{x^3}y + k'(x) = \cancel{x^2} - 2\cancel{x^3}y$; $k'(x) = 0$

$k(y) = \int 0\,dy = \text{cte.}$ Sustituyo $k(y)$ en $f(x,y)$

$f(x,y) = x^2y - x^3y^2 + \text{cte.} \rightarrow \boxed{x^2y - x^3y = C}$ Define de forma implícita las soluciones de y de la e.d.

7. $y' - \dfrac{2}{x+2} y = 2(x+2)^3$

La convierto en ec. dif. de V.S.

$y' - \dfrac{2}{x+2} y = 0$; $y' = \dfrac{2}{x+2} y$; $\dfrac{dy}{dx} = \dfrac{2}{x+2} y$;

$\int \dfrac{dy}{y} = \int \dfrac{2}{x+2} dx$; $\ln|y| = 2\ln|x+2| + C$

$y = e^{2\ln|x+2|+C} = Ke^{\ln|x+2|^2} = K(x+2)^2$

$y = k(x+2)^2$ Convierte la cte. en variable ; $k \rightarrow k(x)$

$y = k(x)(x+2)^2$

$y' = k'(x)(x+2)^2 + 2k(x)(x+2)$

Sustituyo

$k'(x)(x+2)^2 + 2k(x)(x+2) - \dfrac{2}{x+2} k(x)(x+2)^2 = 2(x+2)^3$

$k'(x)(x+2)^2 = 2(x+2)^3$

$k'(x) = 2(x+2)$

$k(x) = \int 2(x+2)dx = (x+2)^2 + C$

Sustituyo

$y = ((x+2)^2 + C)(x+2)^2$

8. $e^{x}y^{2} + y\operatorname{sen}(x) + (3ye^{x} - 2\cos x)\,y' = 0$

$$\mu(y)(e^{x}y^{2} + y\operatorname{sen}(x)) + \mu(y)(3ye^{x} - 2\cos(x))\,y' = 0$$

$$\left.\begin{aligned} \frac{dM}{dy} &= \mu'(y)(e^{x}y^{2} + y\operatorname{sen} x) + \mu(y)(2e^{x}y + \operatorname{sen} x) \\ \frac{dN}{dx} &= \mu(y)(3ye^{x} + 2\operatorname{sen} x) \end{aligned}\right\}$$

$$\mu'(y)(e^{x}y^{2} + y\operatorname{sen} x) = \mu(y)(3ye^{x} + 2\operatorname{sen} x - 2e^{x}y - \operatorname{sen} x)$$

$$\frac{\mu'(y)}{\mu(y)} = \frac{ye^{x} + \operatorname{sen} x}{y^{2}e^{x} + y\operatorname{sen} x} = \frac{ye^{x} + \operatorname{sen} x}{y(ye^{x} + \operatorname{sen} x)} = \frac{1}{y}$$

$$\int \frac{\mu'(y)}{\mu(y)} = \int \frac{1}{y}\,dy = \operatorname{Ln}|y| \longrightarrow \ln|y| = \ln|\mu(y)| \longrightarrow \mu(y) = y$$

Factor integrante

$$y(e^{x}y^{2} + y\operatorname{sen} x) + y(3ye^{x} - 2\cos x)\,y' = 0$$

$$(e^{x}y^{2} + y\operatorname{sen} x)\,dx + (3ye^{x} - 2\cos x)\,dy = 0$$

$$\left.\begin{aligned} \frac{dM}{dy} &= 3y^{2}e^{x} + 2y\operatorname{sen} x \\ \frac{dN}{dx} &= 3y^{2}e^{x} + 2y\operatorname{sen} x \end{aligned}\right\} \text{ Exacta.}$$

$$\frac{df}{dx} = M \longrightarrow f(x,y) = \int (e^{x}y^{3} + y^{2}\operatorname{sen} x)\,dx = e^{x}y^{3} - y^{2}\cos x + k(y)$$

$$\frac{df}{dy} = N \longrightarrow \frac{df}{dy} = 3y^{2}e^{x} - 2y\cos x + k'(y)$$

$$\longrightarrow 3y^{2}e^{x} - 2y\cos x + k'(y) = 3y^{2}e^{x} - 2y\cos x \;;\quad k'(y) = 0$$

$$k(y) = \int 0\,dy = \text{cte.}$$

$$f(x,y) = e^{x}y^{3} - y^{2}\cos x + C$$

9. $y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y''' + 5y'' = 0$

$$P(x) = x^7 + x^6 + 2x^5 + 10x^4 + 13x^3 + 5x^2 = x^2(x^5 + x^4 + 2x^3 + 10x^2 + 13x + 5) = 0$$

Raices $\rightarrow$ 0. Multiplicidad 2 $\rightarrow$ e^{0x}, xe^{0x}

$$x^5 + x^4 + 2x^3 + 10x^2 + 13x + 5 = 0$$

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
	1	0	2	8	5	0
-1		-1	1	-3	-5	
	1	-1	3	5	0	
-1		-1	+2	-5		
	1	-2	5	0		

Raices $\rightarrow$ -1. Multip. 3 $\rightarrow$ $e^{-x}, xe^{-x}, x^2e^{-x}$

$$x^2 - 2x + 5 = 0 \; ; \quad x = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

Raices $\rightarrow$ $1 + 2i$ $\rightarrow$ $e^x \cos(2x), e^x \operatorname{sen}(2x)$

$$y(x) = C_1 e^{0x} + C_2 x e^{0x} + C_3 e^{-x} + C_4 x e^{-x} + C_5 x^2 e^{-x} + C_6 e^x \cos(2x) + C_7 e^x \operatorname{sen}(2x) =$$

$$= (C_1 + C_2 x) e^{0x} + (C_3 + C_4 x + C_5 x^2) e^{-x} + (C_6 \cos(2x) + C_7 \operatorname{sen}(2x)) e^x$$

$$y(x) = (C_1 + C_2 x) + e^{-x}(C_3 + C_4 x + C_5 x^2) + e^x (C_6 \cos(2x) + C_7 \operatorname{sen}(2x))$$

$C_1, C_2, C_3, C_4, C_5, C_6, C_7 \in \mathbb{R}$