Matemáticos

$\boxed{1}$ $\begin{cases} (4x^2 - 2y^2)\,dx = 2xy\,dy \\ y(1) = 3 \end{cases}$ —

$2x^2 + 2x = 4x^3$

$(4x^2 - 2y^2)\cancel{\frac{dx}{dx}} = 2xy\frac{dy}{dx}$

$y'(2xy) = 4x^2 - 2y^2$

$y' = \frac{\cancel{4}x^{\cancel{2}}}{\cancel{2}xy} - \frac{\cancel{2}y^{\cancel{2}}}{\cancel{2}xy} \Rightarrow y' = \frac{2x}{y} - \frac{y}{x} \Rightarrow$

$\Rightarrow y' + \frac{1}{x} \cdot y = 2xy^{-1}$ $\rightarrow$ Bernoulli $\left(y' + p(x)\,y = q(x)\,y^{\alpha}\right)$

$\boxed{\alpha = -1}$

$z = y^{1-\alpha} = y^{1-1(-1)} = y^2 \Rightarrow \boxed{z = y^2}$

$z' = 2y \cdot y' = 2y \cdot \left(2xy^{-1} - \frac{y}{x}\right) = 4x - \frac{2}{x} \cdot y^2 \longrightarrow$

$y' = 2xy^{-1} - \frac{y}{x}$

$\rightarrow z' = 4x - \frac{2}{x} \cdot z \Rightarrow z' + \frac{2}{x} \cdot z = 4x$

$z' + \frac{2}{x} \cdot z = 4x$

$z' + \frac{2}{x} \cdot z = 0 \Rightarrow z' = -\frac{2}{x} \cdot z \rightarrow \frac{dz}{dx} = -\frac{2}{x} \cdot z \Rightarrow$

$\Rightarrow \frac{dz}{z} = -\frac{2}{x} \cdot dx \Rightarrow \int \frac{dz}{z} = \int -\frac{2}{x}\,dx \Rightarrow \ln(z) = -2 \cdot \ln(x) + C \Rightarrow$

$\Rightarrow z = e^{-2\ln(x) + C} \rightarrow z = \cancel{e}^{\cancel{\ln}(x)^{-2}} \cdot \underbrace{e^{C}}_{K} = x^{-2} \cdot K$

$z = K \cdot x^{-2}$

$\longrightarrow$

$\underline{1}$

$\longrightarrow$

$z' = k' \cdot x^{-2} + k \cdot (-2 \cdot x^{-3}) \quad ; \quad z = k\, x^{-2}$

$k' \cdot x^{-2} + k \cdot (-2 \cdot x^{-3}) + \frac{2}{x} k \cdot x^{-2} = 4x$

$k' x^{-2} + \cancel{k\, (-2 \cdot x^{-3})} + \cancel{k\, 2\, x^{-3}} = 4x$

$k' x^{-2} = 4x$

$k' = 4x \cdot x^{2} \Rightarrow \boxed{k' = 4x^3}$

$k(x) = \int 4x^3 = \boxed{x^4 + C} \quad C \in \mathbb{R}$

$z = (x^4 + C) \cdot x^{-2} = x^2 + C \cdot x^{-2} \rightarrow z = y^2$

$y^2 = x^2 + \frac{C}{x^2} \rightarrow \frac{x^4 + C}{x^2} \rightarrow$

$\rightarrow y = \sqrt{\frac{x^4 + C}{x^2}} \quad C \in \mathbb{R}$

$y(1) = \sqrt{\frac{1^4 + C}{1^2}} = 3 \rightarrow$

$\rightarrow \sqrt{\frac{1 + C}{1}} = 3 \rightarrow \sqrt{1 + C} = 3$

$1 + \sqrt{C} = 3 \rightarrow \sqrt{C} = 2 \rightarrow \boxed{C = 4}$

Matemáticas

$\boxed{2}$ $\begin{cases} y'' + 4y = -4 \cdot sen(2x) \\ y(0) = -1 \quad , \quad y'(0) = 4 \end{cases}$

Ec. dif homogenea lineal 2º orden

$y'' + 4y = 0 \rightarrow P(\lambda) = \lambda^2 + 4 = 0 \rightarrow \lambda = \sqrt{4} = \pm 2i$

Raices $\rightarrow 0 \pm 2$ mult. 1 $\rightarrow e^x \cos(2x) \rightarrow \cos(2x)$
$\rightarrow e^x sen(2x) \rightarrow sen(2x)$

Sol. ecuación dif. homogenea asociada:

$y_h(x) = C_1 \cos(2x) + C_2 \cdot sen(2x) \quad C_1, C_2 \in \mathbb{R}$

Sol. particular:

$y(x) = Ax \cos(2x) + Bx\, sen(2x)$

$y'(x) = A \cos(2x) - 2Ax \cdot sen(2x) + B \cdot sen(2x) + 2Bx \cdot \cos(2x)$

$y''(x) = -2A\, sen(2x) - 2A\, sen(2x) - 4Ax \cos(2x) + 2B\cos(2x) + 2B\cos(2x) - 4Bx\, sen\, 2x =$

$= -4A \cdot sen(2x) - 4Ax\cos(2x) + 4B\cos(2x) - 4Bx\, sen(2x)$

Sustituimos en $y'' + 4y = -4\, sen(2x)$

$-4A \cdot sen(2x) - \cancel{4Ax \cdot \cos(2x)} + 4B \cdot \cos(2x) - \cancel{4Bx\, sen(2x)} + \cancel{4Ax\cos(2x)} + \cancel{4Bx\, sen(2x)} =$

$= -4 \cdot sen(2x)$

$-4A \cdot sen(2x) + 4B\cos(2x) = -4 \cdot sen(2x)$

$\begin{cases} -4A = -4 \rightarrow A = 1 \\ 4B = 0 \rightarrow B = 0 \end{cases}$ $\quad y_m = x \cdot \cos(2x)$

Sol general $\to$ $y(x) = x\cos(2x) + C_1 \cdot \cos(2x) + C_2 \cdot sen(2x)$

$y'(x) = \cos(2x) - 2x \cdot sen(2x) - 2C_1 sen(2x) + 2C_2 \cos(2x)$

$-1 = y(0) = 0 \cdot \cos(2x) + C_1 \cos(2 \cdot 0) + C_2\, sen(2 \cdot 0) \to C_1 = -1$

$4 = y'(0) = \cos(0 \cdot 2) - 2 \cdot 0 \cdot sen(2 \cdot 0) - 2 \cdot C_1 \cdot sen(2 \cdot 0) + 2 \cdot C_2 \cdot \cos(2 \cdot 0)$

$1 - 0 - 0 + 2C_2 = 4 \to 1 + 2C_2 = 4 \to 2C_2 = 3 \to C_2 = 3/2$

$$\boxed{y(x) = x\cos(2x) - \cos(2x) + 3/2\, sen(2x)}$$

Escaneado con CamScanner

Matemáticas

$$\frac{e^{2x}}{e^{6x}} = \frac{e^{2x}}{e^{2\cdot 3x}}$$

$\boxed{3}$ $y' - 6y = 5e^{6x}y^4$ Bernoulli

$p(x) = -6$ $q(x) = 5e^{6x}$ $\alpha = 4$

$z = y^{1-\alpha} \rightarrow \boxed{z = y^{-3}}$ $y' = 5e^{6x}y^4 + 6y$

$z' = -3y^{-4} \cdot y' = -3y^{-4} \cdot (5e^{6x} \cdot y^4 + 6y) =$

$= -15e^{6x} - 18y^{-3} \rightarrow z' = -15e^{6x} - 18z$

$z' + 18z = -15e^{6x}$

$z' + 18z = 0 \Rightarrow z' = -18z \rightarrow \frac{dz}{dx} = -18z \Rightarrow$

$\Rightarrow \frac{dz}{z} = -18 \cdot dx \Rightarrow \int \frac{dz}{z} = -18\int dx \Rightarrow$

$\Rightarrow \ln z = -18x + C \rightarrow z = e^{-18x + C} = e^{-18x} \cdot k$

$z = e^{-18x} \cdot K$

$z' = -18 \cdot e^{-18x} \cdot K + e^{-18x} \cdot k'$

$6x - (-18x) =$
$= 24x$

$\cancel{-18e^{-18x}} \cdot K + e^{-18x} \cdot k' + \cancel{18 \cdot e^{-18x}} \cdot k = -15e^{6x}$

$e^{-18x} \cdot k' = -15e^{6x} \rightarrow k' = \frac{-15e^{6x}}{e^{-18x}} = -15\frac{e^{6x}}{e^{-18x}}$

$k' = -15e^{24x}$

$k = -15\int e^{24x}\,dx = \frac{-15}{24} \cdot e^{24x} + C \rightarrow \boxed{z = e^{-18x} \cdot \left(\frac{-15}{24} \cdot e^{24x} + C\right)}$

$$\boxed{y = \frac{1}{\sqrt[3]{e^{-18x} \cdot \left(\frac{-15}{24}\right) \cdot e^{24x} + C}} \quad C \in \mathbb{R}}$$

Escaneado con CamScanner

Matemáticas

$\boxed{4}$ $\begin{cases} y'' + 6y' + 9y = 6e^{-3x} + 18 \\ y(0) = 2 \quad , \; y'(0) = 25 \end{cases}$

$\boxed{y = e^{rx}}$

Ec. lineal homogenea asociada

$$r^2 + 6r + 9 = 0 \qquad \frac{-6 \pm \sqrt{36-36}}{2} = \left\langle \begin{matrix} -3 \\ -3 \end{matrix} \right.$$

$r = -3$ mult. 2

$$y_h(x) = C_1 e^{-3x} + C_2 x e^{-3x} \qquad C_1 \text{ y } C_2 \in \mathbb{R}$$

Particular:

$$y(x) = Ax^2 \cdot e^{-3x}$$

$$y'(x) = 2Ae^{-3x} - 3Ax^2 e^{-3x}$$

$$y''(x) = 2Ae^{-3x} - 6A \cdot xe^{-3x} - 6A \cdot xe^{-3x} + 9Ax^2 e^{-3x} =$$

$$= 2A \cdot e^{-3x} - 12Axe^{-3x} + 9Ax^2 e^{-3x}$$

$$y'' + 6y' + 9y = 6e^{-3x} + 18$$

$$2Ae^{-3x} - \cancel{12Axe^{-3x}} + \cancel{9Ax^2e^{-3x}} + \cancel{12Axe^{-3x}} - \cancel{18Ax^2e^{-3x}} + \cancel{9Ax^2e^{-3x}} =$$

$$= 6e^{-3x} + 18$$

$$2Ae^{-3x} = 6e^{-3x} + 18 \Rightarrow \frac{2A \cdot \cancel{e^{-3x}}}{\cancel{e^{-3x}}} = \frac{6\cancel{e^{-3x}}}{\cancel{e^{-3x}}} + \frac{18}{e^{-3x}} \Rightarrow$$

$$\Rightarrow 2A = 6 + 18 \cdot e^{-3x} \rightarrow A = 3 + 9e^{-3x}$$

$$y_e(x) = (3 + 9e^{3x})x^2 e^{-3x} = 3x^2 e^{-3x} + 9x^2$$

General:

$$y(x) = 3x^2 e^{-3} + 9x^2 + C_1 e^{-3x} + C_2 x e^{-3x}$$

$$y(0) = 3 \cdot 0 \cdot e^0 + 9 \cdot 0 + C_1 \cdot e^0 + C_2 \cdot 0 \cdot e^0 = 2 \rightarrow y(0) = 2 \qquad C_1, C_2 \in \mathbb{R}$$

$$y'(x) = 6xe^{-3x} - 9x^2 e^{-3x} + 18 \cdot x - 3 \cdot C_1 e^{-3x} + C_2 e^{-3x} - 3C_2 e^{-3x}$$

$$y'(0) = 6 \cdot 0 e^0 - 9 \cdot 0 e^0 + 18 \cdot 0 - 3 \cdot C_1 \cdot e^0 + C_2 e^0 - 3C_2 \underbrace{e^0}_{25} \rightarrow y(0) = 25$$

$-3C_1 + C_2 = 25$; $-6 + C_2 = 25$; $C_2 = 31$

$$y(x) = 3x^2 e^{-3x} + 9x^2 + 2e^{-3x} + 31xe^{-3x}$$

Matemáticas

(5) $\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$

Primer orden
$y'(x) + p(x)y = q(x)$

$p(x) = x \qquad q(x) = 3xe^{x^2}$

$y\mu = \int q\mu \, dx \rightarrow \mu = e^{\int p(x)\,dx}$

$x^2 + \frac{x^2}{2} = \frac{3x^2}{2}$

$\mu = e^{\int x\,dx} \rightarrow \int x\,dx = \frac{x^2}{2}$

$\mu = e^{x^2/2}$

$ye^{x^2/2} = \int 3xe^{x^2} \cdot e^{x^2/2}\,dx = 3\int xe^{x^2} \cdot e^{x^2/2}\,dx \Rightarrow$

$ye^{x^2/2} = 3\int xe^{\frac{3}{2}x^2}\,dx = \cancel{3} \cdot \frac{e^{3x^2/2}}{\cancel{3}} + C$

$ye^{x^2/2} = e^{3x^2/2} + C$

$\boxed{y = \frac{e^{3x^2/2} + C}{e^{x^2/2}}}$

$C \in \mathbb{R}$

Escaneado con CamScanner

Matemáticas

6. $2xy - 3x^2y^2 + (x^2 - 2x^3y)y' = 0$

$$\underbrace{(2xy - 3x^2y^2)}_{M(x,y)} + \underbrace{(x^2 - 2x^3y)}_{N(x,y)}\, y' = 0 \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = 2x - 6x^2y \ ; \ \frac{\partial N}{\partial x} = 2x - 6x^2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \to \text{Ec dif exacta}$$

$$\exists f / \frac{\partial f}{\partial x} = M, \ \frac{\partial f}{\partial y} = N$$

$$f(x,y) = C \quad C \in \mathbb{R}$$

$$\frac{\partial f}{\partial x} = M \to f = \int 2xy - 3x^2y^2 \, dx \to$$

$$f = y\int 2x\,dx - y^2\int 3x^2\,dx = \boxed{y \cdot x^2 - y^2 \cdot x^3 + C(y)}$$

$$\frac{\partial f}{\partial y} = N \to \cancel{x^2} - 2y\cancel{x^3} + C'(y) = \cancel{x^2} - 2\cancel{x^3}y \to$$

$$\to C'(y) = 0 \to C(y) = 0$$

$$f(x,y) = yx^2 - y^2x^3$$

$\boxed{yx^2 - y^2x^3 = C}$ $C \in \mathbb{R} \to$ Define de forma implícita los sol. de la ec. diferencial

Matemáticas

lineal de primer orden

7. $y' - \frac{2}{x+2} y = 2(x+2)^3$ $\boxed{y'(x) + p(x) y = q(x)}$

$p(x) = -\frac{2}{x+2}$ $q(x) = 2(x+2)^3$

$\boxed{y \mu = \int q \mu \, dx}$ lineales $\rightarrow \mu = e^{\int p(x) \cdot dx}$

$\mu = e^{\int -\frac{2}{x+2} dx}$ $\rightarrow \int -\frac{2}{x+2} dx = -2 \int \frac{1}{x+2} dx \rightarrow$

$\rightarrow -2 \int \frac{1}{x+2} = \boxed{-2 \ln(x+2) + C}$

$\mu = e^{-2\ln(|x+2|)} = e^{\ln(x+2)^{-2}} \Rightarrow \boxed{\mu = (x+2)^{-2}}$

$y \cdot (x+2)^{-2} = \int 2(x+2)^3 \cdot (x+2)^{-2} dx \rightarrow$

$y \cdot (x+2)^{-2} = \int 2x + 4 \, dx = 2 \int x \, dx + 4 \int 1 \, dx = \cancel{2} \frac{x^2}{\cancel{2}} + 4x$

$y \cdot (x+2)^{-2} = x(x+4) + C_1$

$$\boxed{y = \left(x(x+4)\right) \cdot (x+2)^2 + C_1 \cdot (x+2)^2}$$

Matemáticas

(admite factor integrante que depende de y)

⑧ $e^x y^2 + y\cdot \operatorname{sen} x + (3ye^x - 2\cos x)\, y' = 0$

$(e^x y^2 + y \operatorname{sen} x) + (3ye^x - 2\cos x)\, y' = 0$

factor Integrante $y \to \underbrace{(y)(e^x y^2 + y \operatorname{sen} x)}_{M(x,y)} + \underbrace{(y)(3ye^x - 2\cos x)}_{N(x,y)} y' = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = 3y^2 e^x + 2y\cdot \operatorname{sen} x$

$\frac{\partial N}{\partial x} = 3y^2 e^x + 2y \operatorname{sen} x$

$\to\ =$ Ec. dif exacta

$\Rightarrow \exists f \,/\, \frac{\partial f}{\partial x} = M,\ \frac{\partial f}{\partial y} = N$

$f(x,y) = C \quad C \in \mathbb{R}$

$\frac{\partial f}{\partial x} = M \Rightarrow \frac{\partial f}{\partial x} = e^x y^3 + y^2 \cdot \operatorname{sen} x \Rightarrow f = \int (e^x y^3 + y^2 \cdot \operatorname{sen} x)\, dx \to$

$\to f = y^3 \int e^x \, dx + y^2 \int \operatorname{sen} x \, dx = \boxed{y^3 e^x - y^2 \cdot \cos x + C(y)}$

$\frac{\partial f}{\partial y} = N \Rightarrow \cancel{3y^2 e^x} - 2y \cos x + C'(y) = \cancel{3y^2 e^x} - 2\cos x\, y \to$

$\to C'(y) = 0 \to C(y) = 0$

$f(x,y) = y^3 e^x - y^2 \cos x$

$\boxed{y^3 e^x - y^2 \cdot \cos x = C}$ $C \in \mathbb{R}$, define de forma implícita las sol. (y) de la ec. diferencial

Matemáticas

9) $y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y^{III} + 5y^{II} = 0$ $\boxed{y = e^{rx}}$

$r^7 + r^6 + 2r^5 + 10r^4 + 13r^3 + 5r^2 = 0$

$r^2(r^5 + r^4 + 2r^3 + 10r^2 + 13r + 5) = 0$

-5 5, 1, 7

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
	1	0	2	8	5	0/
-1		-1	1	-3	-5	
	1	-1	3	5	0/	
-1		-1	2	-5		
	1	-2	5	0/		

$r = -1$ mult. 3

$r = 0$ mult 2

$(r^2 - 2r + 5) = 0$ mult 1 $\Rightarrow$

$\Rightarrow$ $r = 1 \pm 2i$ mult 1

$\frac{2 \pm \sqrt{4-20}}{2} \notin \mathbb{R}$

$r = 0 \rightarrow c + x$

$r = -1 \rightarrow e^{-x} + xe^{-x} + x^2e^{-x}$

$r = 1 \pm 2i \rightarrow$ $\rightarrow e^{1x} \cos 2x$

$\rightarrow e^{1x} \operatorname{sen} 2x$

$\{ c, x, e^{-x}, xe^{-x}, x^2e^{-x}, e^{x}\cos 2x, e^{x} \operatorname{sen} 2x \}$

Es una $\mathbb{R}$ base del conjunto de todas la sol de la ec. dif.

Así la sol general es:

$c_1 + c_2x + c_3e^{-x} + c_4xe^{-x} + c_5x^2e^{-x} + c_6e^{x}\cdot\cos 2x + c_7e^{x}\operatorname{sen} 2x$

$c_1 \ldots c_7 \in \mathbb{R}$