1. $\begin{cases} (4x^2-2y^2)\, dx = 2xy \cdot dy \Rightarrow 4x^2-2y^2 = 2xy \cdot y' \Rightarrow y' = \dfrac{4x^2-2y^2}{2xy} \\ y(1)=3 \end{cases}$

$$V = \frac{y}{x} \rightarrow y = V \cdot x \rightarrow y' = v'x + v$$

$$v'x + v = \frac{4x^2 - 2(vx)^2}{2x\cancel{y}(vx)} = \frac{4-2v^2}{2v} \rightarrow v'x = \frac{4-2v^2}{2v} - \frac{2v^2}{2v}$$

$$v'x = \frac{4-4v^2}{2v} \;;\; \frac{dv}{dx}x = \frac{-4v^2+4}{2v} \Rightarrow \frac{dx}{x} = \frac{2v}{-4v^2+4}\, dv$$

$$\int \frac{2v}{-4v^2+4}\, dv = \int \frac{1}{x}\, dx = -\frac{1}{4} \ln|-4v^2+4| \cancel{+C} = \ln|x| + C$$

$$= -\frac{1}{4} \ln\left|-4\left(\frac{y}{x}\right)^2+4\right| = \ln|x| + C$$

$$\Rightarrow -\frac{1}{4} \ln|-4(3)^2+4| = \ln|1| + C$$

$$C = -\frac{1}{4} \ln|-32| - 0 \Rightarrow C = -0{,}86$$

$$\boxed{-\frac{1}{4} \ln\left|-4\left(\frac{y^2}{x}\right)+4\right| = \ln|x| - 0{,}86}$$

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2) $\begin{cases} y'' + 4y = -4\,\text{sen}(2x) \\ y(0) = -1,\ y'(0) = 4 \end{cases}$

Raices: $0 \pm 2i$ $\to e^{0x} \cdot \text{sen}(2x) = \text{sen}(2x)$; $\to e^{0x} \cdot \cos(2x) = \cos(2x)$

$y'' + 4y = 0$

$p(x): x^2 + 4 = 0 \to x^2 = -4 \to x = \pm\sqrt{-4} = \pm 2i$

$$y_h(x) = C_1 \cdot \text{sen}(2x) + C_2 \cos(2x)$$

$y(x) = Ax\,\text{sen}(2x) + Bx\cos(2x)$

$y'(x) = A \cdot \text{sen}(2x) + Ax \cdot 2\cos(2x) + B\cos(2x) + Bx \cdot (-2\,\text{sen}(2x))$

$y''(x) = 2A\cos(2x) - 4Ax\,\text{sen}(2x) - 2B\,\text{sen}(2x) - 4Bx\cos(2x)$

$2A\cos(2x) - \cancel{4Ax\,\text{sen}(2x)} - 2B\,\text{sen}(2x) - \cancel{4Bx\cos(2x)} + 4(Ax\,\text{sen}(2x), Bx\cos(2x))$

$= -4\,\text{sen}(2x) \Rightarrow 2A\cos(2x) - 2B\,\text{sen}(2x) = -4\,\text{sen}(2x)$

$\begin{cases} 2A = 0 \to A = 0 \\ -2B = -4 \to B = 2 \end{cases}$

$y(x) = 2x\cos(2x) + C_1\,\text{sen}(2x) + C_2\cos(2x)$

$\to -1 = y(0) \to -1 = \cancel{2 \cdot 0}\cos(2\cdot 0) + C_1\cancel{\text{sen}(2\cdot 0)} + C_2\cos(2\cdot 0)$ ($\to 0$)

$C_2 = -1$

$4 = y'(0) \to 4 = \cancel{4 \cdot 0(-\text{sen}\,2\cdot 0)} + 2 \cdot C_1\cos(2\cdot 0) + \cancel{2C_2(-\text{sen}\,2\cdot 0)}$ ($\to 0$)

$C_1 = 2$

$$y(x) = 2x\cos(2x) + 2\,\text{sen}(2x) - \cos 2x$$

③ $y' - 6y = 5e^{6x}y^4 \Rightarrow y' = 5e^{6x}y^4 + 6y$

$\alpha = 4 \qquad z = y^{1-\alpha} \rightarrow z = y^{-3}$

$z' = -3y^{-4} \cdot y' = -3y^{-4} \cdot (5e^{6x}y^4 + 6y) = -15e^{6x} - 18y^{-3}$

$z' + 18z = -15e^{6x}$ (Ec. dif. homogenea asociada)

$z' + 18z = 0 \rightarrow z' = -18z \rightarrow \frac{dz}{dx} = -18z \rightarrow \int \frac{dz}{z} = \int -18\,dx$

$\ln|z| = -18x + C \rightarrow z = e^{-18x+C} = e^{-18x} + e^C \rightarrow z = K \cdot e^{-18x}$

<u>z solución</u>

$z' = K'(x) \cdot e^{-18x} + K(x) \cdot e^{-18x} \cdot (-18)$

$K'(x) \cdot e^{-18x} + \cancel{K(x) - 18e^{-18x}} + \cancel{18 K(x)} e^{-18x} = -15e^{6x}$

$K'(x) e^{-18x} = -15e^{6x} \rightarrow K'(x) = \frac{-15e^{6x}}{e^{-18x}} = -15e^{6x+18x} \rightarrow K'(x) = -15e^{24x}$

$K(x) = \int -15e^{24x} = \frac{-15}{24}\int 24 \cdot e^{24x}\,dx = \frac{-5}{8}e^{24x} + C$

$z = \left(-\frac{5}{8}e^{24x} + C\right) \cdot e^{-18x} \rightarrow z = \frac{-5}{8}e^{+6x} + C\left(e^{-18x}\right)$

$y^{-3} = -\frac{5}{8}e^{6x} + C \cdot e^{-18x}$

$y = \sqrt[3]{\frac{1}{-\frac{5}{8}e^{6x} + Ce^{-18x}}}$

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4.

$$\begin{cases} y'' + 6y' + 9y = 6e^{-3x} + 18 \\ y(0) = 2 \\ y'(0) = 25 \end{cases}$$

$y'' + 6y' + 9y = 0$ (Ec. dif. hom. asociada)

$p(x) = x^2 + 6x + 9 = 0$

$$x = \frac{-6}{2} = -3$$

Raices (-3) mult 2 $\to e^{-3x}, \; x e^{-3x}$

$$Y_h(x) = C_1 e^{-3x} + C_2 x e^{-3x}$$

$y(x) = A e^{-3x} + B$

$y'(x) = -3A e^{-3x}$

$y''(x) = 9A e^{-3x}$

$\Rightarrow 9Ae^{-3x} + 6(-3Ae^{-3x}) + 9(Ae^{-3x} + B) = 6e^{-3x} + 18$

$9B = 6e^{-3x} + 18$

$$\begin{cases} A = 0 \\ B = 2 \end{cases} \to Y_p(x) = 2$$

$$Y(x) = 2 + C_1 e^{-3x} + C_2 x e^{-3x}$$

$2 = y(0) \to 2 = 2 + C_1 \cdot e^{-3\cdot 0} + \cancel{C_2 0 \cdot e^{-3\cdot 0}} \to 2 = 2 + C_1$

$\Downarrow$

$C_1 = 0$

$25 = y'(0) \to 25 = -3C_1 e^{-3\cdot 0} + C_2 (e^{-3\cdot 0} - \cancel{3 \cdot 0 \cdot e^{-3\cdot 0}})$

$\Downarrow$

$C_2 = 25$

$$Y(x) = 2 + 25x\, e^{-3x}$$

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5. $\begin{cases} y' + xy = 3x\, e^{x^2} \\ y(0) = 2 \end{cases}$

$y' + xy = 0$ (Ec. dif. hom. asociada)

$y' = -xy \Rightarrow \dfrac{dy}{dx} = -xy \Rightarrow \displaystyle\int \frac{dy}{y} = \int -x\, dx$

$\ln y = -\dfrac{x^2}{2} + C \Rightarrow y = e^{-\frac{x^2}{2} + C} \Rightarrow y = e^{-\frac{x^2}{2}} + e^{C} \Rightarrow y = K \cdot e^{-\frac{x^2}{2}}$

$y' = K'(x) \cdot e^{-\frac{x^2}{2}} + K(x) \cdot e^{-\frac{x^2}{2}} \cdot (-x) \Rightarrow y' = K'(x) \cdot e^{-\frac{x^2}{2}} + K(x)(-x) \cdot e^{-\frac{x^2}{2}}$

$K'(x) \cdot e^{-\frac{x^2}{2}} + \cancel{K(x)(-x) \cdot e^{-\frac{x^2}{2}}} + \cancel{x \cdot \left(K\, e^{-\frac{x^2}{2}}\right)} = 3x\, e^{x^2}$

$K'(x) = \dfrac{3x\, e^{x^2}}{e^{-\frac{x^2}{2}}} = 3x\, e^{x^2 - \left(-\frac{x^2}{2}\right)}$

$K(x) = \displaystyle\int 3x\, e^{\frac{3x^2}{2}}\, dx = e^{\frac{3x^2}{2}} + C$

$y = \left(e^{\frac{3x^2}{2}} + C\right) \cdot \left(e^{-\frac{x^2}{2}}\right) \Rightarrow y = e^{x^2} + e^{-\frac{x^2}{2}} \cdot C$

$2 = y(0) \Rightarrow 2 = e^{0} + e^{0} \cdot C \Rightarrow 2 = 2 \cdot C \Rightarrow C = 1$

$y = e^{x^2} + e^{-\frac{x^2}{2}}$

6. $\underbrace{2xy - 3x^2y^2}_{M(x,y)} + \underbrace{(x^2 - 2x^3y)}_{N(x,y)}y' = 0$

$$\left.\begin{aligned} \frac{dM}{dy} &= 2x - 3x^2 \cdot 2y = 2x - 6x^2y \\ \frac{dN}{dx} &= 2x - 6x^2 y \end{aligned}\right\} \text{Ec. dif. exacta}$$

$$\frac{df}{dx} = M(x,y) \to \frac{df}{dx} = 2xy - 3x^2y^2 \to f = \int (2xy - 3x^2y^2)\,dx$$

$$= 2y\int x\,dx - 3y^2\int x^2\,dx = 2y\frac{x^2}{2} - 3y^2\frac{x^3}{3} + K(y)$$

$$f = x^2y - x^3y^2 + K(y)$$

$$\frac{df}{dy} = x^2 - 2x^3y \to \cancel{x^2} - \cancel{2x^3y} + K'(x) = \cancel{x^2} - \cancel{2x^3y}$$

$$K'(y) = 0,\ K(y) = 0$$

$$f = x^2y - x^3y^3$$

$$\to x^2y - x^3y^2 = C$$

7. $y' - \frac{2}{x+2} y = 2(x+2)^3$

$$y' - \frac{2}{x+2} y = 0 \quad \text{(Ec. dif. homo. asociada)}$$

$$y' = \frac{2}{x+2} y \Rightarrow \frac{dy}{dx} = \frac{2y}{x+2} \Rightarrow \int \frac{1}{2y}\, dy = \int \frac{1}{x+2}\, dx$$

$$\frac{1}{2} \ln|2y| = \ln|x+2| + C$$

$$2y = e^{2\cdot(\ln|x+2|+C)} \Rightarrow 2y = e^{2\cdot\ln|x+2|} + e^{2\cdot C}$$

$$y = \frac{1}{2} \cdot \left(e^{2\cdot\ln|x+2|} + e^{2\cdot C}\right)$$

$$y = \frac{1}{2} \cdot K(x) \cdot e^{2\cdot\ln|x+2|}$$

$$y' = \frac{1}{2}\left(K(x)' \cdot e^{2\cdot\ln|x+2|} + K(x) \cdot \frac{2}{x+2}\, e^{2\cdot\ln|x+2|}\right)$$

$$y' = \frac{1}{2} K'(x) \cdot e^{2\cdot\ln|x+2|} + K(x) \cdot \frac{1}{x+2}\, e^{2\cdot\ln|x+2|}$$

$$\left(\frac{1}{2} K'(x) \cdot e^{2\ln|x+2|} + \cancel{K(x) \cdot \frac{1}{x+2}\, e^{2\ln|x+2|}}\right) - \cancel{\frac{2}{x+2}\left(\frac{1}{2} K(x) \cdot e^{2\ln|x+2|}\right)} = 2(x+2)^3$$

$$K'(x) = \frac{2\cdot 2(x+2)^3}{e^{2\ln|x+2|}} \Rightarrow K(x) = 4\int \frac{(x+2)^3}{e^{2\ln|x+2|}}\, dx = \frac{x^2}{2} + 2x + C$$

$$y = \frac{1}{2} \cdot \left(\frac{x^2}{2} + 2x + C\right) \cdot e^{2\cdot\ln|x+2|}$$

$$y = \frac{x^2}{4} \cdot e^{2\ln|x+2|} + x \cdot e^{2\cdot\ln|x+2|} + C \cdot e^{2\cdot\ln|x+2|}$$

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8. $e^x y^2 + y \operatorname{sen} x + (3ye^x - 2\cos x)\, y' = 0$

$M = \mu (e^x y^2 + y \operatorname{sen} x) \qquad \frac{dM}{dY} = \mu' (e^x y^2 + y \operatorname{sen} x) + \mu \cdot (2e^x y + \operatorname{sen} x)$

$N = \mu (3ye^x - 2\cos x) \qquad \frac{dN}{dx} = \mu (3y^2 e^x + 2 \operatorname{sen} x)$

$$\mu' (e^x y^2 + y \operatorname{sen} x) = \mu (y e^x + \operatorname{sen} x)$$

$$\mu' y = \mu$$

$$\int \frac{\mu'}{\mu} = \int \frac{1}{y}\, dy \Rightarrow \ln|\mu| = \ln y + C \Rightarrow C = 0 \Rightarrow \mu = y$$

$M = e^x y^3 + y^2 \operatorname{sen} x$

$N = 3y^2 e^x - 2y \cos x$

$f = \int \frac{df}{dx}\, dx = \int M\, dx = \int e^x y^3 + y^2 \operatorname{sen} x\, dx =$

$= e^x y^3 - y^2 \cos x + C_1(y)$

$f = \int \frac{df}{dy}\, dy = \int N\, dy = \int 3y^2 e^x - 2y \cos x\, dy =$

$= y^3 e^x - y^2 \cos x + C_2(x)$

$$\cancel{e^x y^3} - \cancel{y^2 \cos x} + C_1(y) = \cancel{y^3 e^x} - \cancel{y^2 \cos x} + C_2(x)$$

$$C_1(y) = C_2(x)$$

$$f = y^3 e^x - y^2 \cos x + C = 0$$