industriales
etsii UPCT

Universidad Politécnica de Cartagena

EJERCICIOS ECUACIONES DIFERENCIALES

A continuación se plantean las siguientes ecuaciones diferenciales y problemas de condiciones iniciales. Tened en cuenta que:

a) Confiamos en que vais a trabajar individualmente los ejercicios, de hecho, pensad que son perfectos para comprobar que habéis entendido todo.

b) No obstante, os aconsejamos que contactéis para cualquier duda al profesor Rogelio Ortigosa (rogelio.ortigosa@upct.es) en el horario: **jueves de 15 a 20 horas**. El profesor os proporcionará un link para atender las tutorías a través de la aplicación **teams**.

c) La fecha máxima de entrega será el **17 de Abril**.

1. $\begin{cases} (4x^2 - 2y^2)\ dx = 2xy\, dy \\ y(1) = 3 \end{cases}$

2. $\begin{cases} y^{''} + 4y = -4sen(2x) \\ y(0) = -1 \\ y^{'}(0) = 4 \end{cases}$

3. $y^{'} - 6y = 5e^{6x}y^4$

4. $\begin{cases} y^{''} + 6y^{'} + 9y = 6e^{-3x} + 18 \\ y(0) = 2 \\ y^{'}(0) = 25 \end{cases}$

5. $\begin{cases} y^{'} + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$

6. $2xy - 3x^2y^2 + (x^2 - 2x^3y)\, y^{'} = 0$

7. $y^{'} - \frac{2}{x+2}y = 2\,(x+2)^3$

8. $e^xy^2 + ysenx + (3ye^x - 2cosx)\, y^{'} = 0$ (esta ecuación admite un factor integrante que depende de la variable y)

9. $y^{vii} + y^{vi} + 2y^v + 10y^{iv} + 13y^{'''} + 5y^{''} = 0$

1)

$$\begin{cases} (4x^2-2y^2)\,dx = 2xy\,dy \\ y(1)=3 \end{cases}$$

$$v = \frac{y}{x}$$

$$y = v\cdot x \rightarrow y' = v'\cdot x + v$$

$$(4x^2-2y^2) = 2xy\,\frac{dy}{dx}$$

$$(4x^2-2\cdot(v\cdot x)^2) = 2x\cdot(v\cdot x)\cdot(v'\cdot x+v)$$

$$\cancel{2x^2}\cdot(2-v^2) = \cancel{2x^2}\cdot v\cdot(v'\cdot x+v)$$

$$\frac{2-v^2}{v} = \frac{v}{1} - \frac{dv}{dx}\cdot x$$

$$\frac{2-2v^2}{v} = \frac{dv}{dx}\cdot x$$

$$\frac{dx}{x} = \frac{v}{2-2v^2}\cdot dv$$

$$\int\frac{dx}{x} = \int\frac{v}{2-2v^2}\cdot dv$$

$$\log x = -\frac{1}{4}\int\frac{-4v}{2-2v^2}\,dv = -\frac{1}{4}\cdot\log(2-2v^2)+K$$

$$\log x = -\frac{1}{4}\cdot\log(2-2v^2)+K$$

$$\log x = -\frac{1}{4}\cdot\log\ 2-2\left(\frac{y}{x}\right)^2+K$$

$$0\ \log x = -\frac{1}{4}\ \log 16 + K$$

$$K = \frac{\log 16}{4}$$

$$\boxed{\log x = -\frac{1}{4}\cdot\log\left|2-2\left(\frac{y}{x}\right)^2\right| + K}$$

3) $y' - 6y = 5\, e^{6x} y^4$

$y' - 6y = 5e^{6x}y^4$: $\quad y = \dfrac{2e^{6x}(-5e^{24x} + C_1)^{2/3}}{-5e^{24x} + C_1}$

$P(x) = -6$

$Q(x) = 5e^{6x}$

$n = 4$

$$-\frac{v'}{3} - 6v = 5e^{6x} \rightarrow v = \frac{-5e^{24x} + C_1}{8e^{18x}}$$

$$v - y^{-3} = \frac{-5e^{24x} + C_1}{8e^{18x}}$$

$$y = \frac{2e^{6x}(-5e^{24x} + C_1)^{2/3}}{-5e^{24x} + C_1}$$

$$y = \frac{2e^{6x}(-5e^{24x} + C_1)^{2/3}}{-5e^{24x} + C_1}$$

5) $y' + xy = 3x e^{x^2}$

$y(0) = 2$

$y' + xy = 0$

$\frac{dy}{dx} = -x \cdot y$

$\frac{dy}{y} = -x \cdot dx$

$\log y = \frac{-x^2}{2} + C$

$e^{\log y} = e^{\frac{-x^2}{2} + C} = e^{\frac{-x^2}{2}} \cdot e^C$

$y = e^{\frac{-x^2}{2}} \cdot K(y)$

$y' = -e^{\frac{-x^2}{2}} \cdot x \cdot K(y) + e^{\frac{-x^2}{2}} \cdot K'(y)$

$\cancel{-e^{\frac{-x^2}{2}} \cdot x \cdot K(y)} + e^{\frac{-x^2}{2}} \cdot K'(y) \cancel{+ x \cdot e^{\frac{-x^2}{2}} \cdot K(y)}$

$e^{\frac{-x^2}{2}} \cdot K'(y) = 3x e^{x^2}$

$K'(y) = \frac{3x \cdot e^2}{e^{\frac{-x^2}{2}}} = 3x \cdot e$

$K(y) = \int 3x \cdot e^{\frac{3x^2}{2}} dx = \int e^t dt = 3^{\frac{3x^2}{2}} + C$

$y = e^{-x^2/2} \cdot (e^{3x^2/2} + C) = e^{x^2} + e^{-x^2/2} : C$

$y(0) = 2$

$2 = 1 + 1 \cdot C \Rightarrow C = 1$

$y = e^{x^2} + e^{-x^2/2}$

6)

$2xy - 3x^2y^2 + (x^2 - 2x^3y)y' = 0 \Rightarrow y = \dfrac{-x^2 + \sqrt{x^4 - 4c_1x^3}}{2x^3}, \; y = \dfrac{-x^2 - \sqrt{x^4 - 4c_1x^3}}{2x^3}$

$M(x,y) + N(x,y)y' = 0$

$M(x,y) = 2xy - 3x^2y^2, \quad N(x,y) = x^2 - 2x^3y$

$x^2y - x^3y^2 + c_1 = c_2$

$x^2y - x^3y^2 = c_1$

$y = \dfrac{-x^2 + \sqrt{x^4 - 4c_1x^3}}{2x^3}, \; y = \dfrac{-x^2 - \sqrt{x^4 - 4c_1x^3}}{2x^3}$

$y = \dfrac{-x^2 + \sqrt{x^4 - 4c_1x^3}}{2x^3}, \; y = \dfrac{-x^2 - \sqrt{x^4 - 4c_1x^3}}{2x^3}$

7) $y' - \frac{2}{x+2} \cdot y = 2(x+2)$

$y' - \frac{2}{x+2} \cdot y = 0$

$\frac{dy}{dx} = \frac{2y}{x+2}$

$\frac{dy}{2y} = \left(\frac{2y}{x+2}\right) dx$

$\int \frac{1}{2y}\, dy = \int \frac{1}{x+2}\, dx$

$1/2 \log y = \log(x+2) + C$

$\log y^{1/2} = \log(x+2) + C$

$e^{\log y^{1/2}} = e^{\log(x+2)+C}$

$y^{1/2} = (x+2) \cdot e^C$

$y = (x+2)^2 \cdot k$

$y = (x+2)^2 \cdot k(x)$

$y' = 2(x+2) \cdot k(x) + (x+2)^2 \cdot k'(x) =$

$\frac{2}{\cancel{x+2}} (x+2)^{\cancel{2}} \cdot k(x) + 2(x+2)^3$

$(x+2)^2 \cdot k'(x) = 2(x+2)^3$

$k'(x) = 2 \frac{(x+2)^3}{(x+2)^2}$

$k'(x) = 2 \cdot (x+2)$

$k(x) = \frac{2x^2}{2} + 4x + C$

$k(x) = x^2 + 4x + C$

$y = (x+2)^2 \cdot x^2 + 4x + C$

d) $e^x y^2 + y \operatorname{sen} x) + (3ye^x - 2\cos x) y' = 0$

$N(y) \mid M(x,y) = e^x y^2 + y \operatorname{sen} x \mid \cdot N(y) \mid N(x,y) = (3ye^x - 2\cos x) \cdot M(y)$

$$\frac{dM}{dy} = (e^x \cdot 2y + \operatorname{sen} x) \cdot N(y) + (e^x y^2 + y \operatorname{sen} x) \cdot N'(y)$$

$$\frac{dN}{dx} = (3y e^x + 2 \operatorname{sen} x) N(y)$$

$e^x \cdot 2y + \operatorname{sen} x) \cdot N(y) + (e^x y^2 + y \cdot \operatorname{sen} x) N'(y) = 3y e^x + 2 \operatorname{sen} x) \cdot N(y)$

$(e^x y^2 + y \operatorname{sen} x) N'(y) = -(e^x \cdot 2y + \operatorname{sen} x) N(y)) + (3 y e^x + 2 \operatorname{sen} x) N(y)$

$(e^x y^2 + y \cdot \operatorname{sen} x) N'(y) = N(y) \cdot (y e^x + \operatorname{sen} x)$

$$\frac{dN}{dx} \cdot \frac{1}{N(y)} = \frac{y e^x + \operatorname{sen} x}{e^x \cdot y^2 + y \cdot \operatorname{sen} x}$$

$$\int dN = \frac{1}{N(y)} = \int \frac{1}{y} \, dy$$

$\log N(y) = \log y + C \to e^{\log N(y)} = e^{\log y + C} = e^{\log y} \cdot e^C$

$N(y) = y \cdot K \to K = 1 \quad N(y) = y$

$M(x,y) = y^3 \cdot e^x + y^2 \cdot \operatorname{sen} x$

$N(x,y) = 3y^2 \cdot e^x - 2y \cos x$

$$\frac{df}{dx} = y^3 \cdot e^x + y^2 \cdot \operatorname{sen} x$$

$df = y^3 \cdot e^x + y^2 \cdot \operatorname{sen} x) \, dx$

$f = \int y^3 \cdot e^x + y^2 \cdot \operatorname{sen} x) \, dx = y^3 \cdot e^x - y^2 \cos x + C(y)$

$\frac{df}{dy} = 3y^2 \cdot e^x - 2y \cos x \to 3y^2 \cdot e^x - 2y \cos x + C'(y) = 3y^2 \cdot e^x - 2y \cos x$

$C'(y) = 0 \to C(y) = 0$

$\boxed{y^3 \cdot e^x - y^2 \cdot \cos x + 0 = C}$

a/ $y^7+y^6+2y^5+10y^4+13y^3+5y^2=0$

$y^2(y^5+y^4+2y^3+10y^2+13y+5) = y^2\cdot(x+1)^3\cdot(x-(1+2i))\cdot(x-(1-2i))$

Raíces

$x=0$ Multiplicidad $2 \Rightarrow e^{0x},\ xe^{0x} \to 1, x$

$x=-1$ " $3 \Rightarrow e^{-x},\ x\cdot e^{-x},\ x^2\cdot e^{-x}$

$x=1\pm 2i \to e^{1x}\cdot \operatorname{sen} 2x = e^{x}\cdot \operatorname{sen} 2x$

$\to e^{1x}\cdot \cos 2x = e^{x}\cdot \cos 2x$

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
		0	2	8	5	0
-1		-1	1	-3	-5	
	1	-1	3	5	0	
-1		-1	2	-5		
	1	-2	5	0		

$x^2-2x+5=0$

$x=\dfrac{+2\pm\sqrt{4-20}}{2}=\dfrac{2\pm\sqrt{-16}}{2}=1\pm 2i$

$c(x)=C_1+C_2\cdot x+C_3\cdot e^{-x}+C_4\cdot x\cdot e^{-x}+C_5\cdot x^2\cdot e^{-x}+C_6\cdot e^{x}\cdot\cos 2x+C_7\cdot e^{x}\operatorname{sen} x$