(5) $\begin{cases} y'+xy = 3x\,e^{x^2} \\ y(0) = 2 \end{cases}$

Ec. dif. de orden 1

Ec. dif. lineal homogénea asociada

$y'+xy=0$

$$y' = -xy \rightarrow \frac{dy}{dx} = -xy \rightarrow \int \frac{dy}{y} = \int -x\,dx$$

$$\ln y = -\frac{x^2}{2}+C \rightarrow y = e^{-\frac{x^2}{2}+C} \rightarrow y = e^{-\frac{x^2}{2}} + \underbrace{e^{C}}_{K>0} \rightarrow \boxed{y = K\cdot e^{-\frac{x^2}{2}}}$$

Imponemos que sea solución $\rightarrow \underbrace{y = k(x)\cdot e^{-\frac{x^2}{2}}}_{\text{Derivamos}}$

$$y' = k'(x)\cdot e^{-\frac{x^2}{2}} + k(x)\cdot e^{-\frac{x^2}{2}}\cdot(-x) \rightarrow \boxed{y' = k'(x)\cdot e^{-\frac{x^2}{2}} + k(x)(-x)\cdot e^{-\frac{x^2}{2}}}$$

$$k'(x)\cdot e^{-\frac{x^2}{2}} \cancel{+ k(x)(-x)\cdot e^{-\frac{x^2}{2}}} \cancel{+ x\left(k(x)\cdot e^{-\frac{x^2}{2}}\right)} = 3x\,e^{x^2}$$

$$k'(x)\cdot e^{-\frac{x^2}{2}} = 3x\,e^{x^2} \rightarrow k'(x) = \frac{3x\,e^{x^2}}{e^{-\frac{x^2}{2}}} = 3x\,e^{x^2-\left(-\frac{x^2}{2}\right)}$$

$$k'(x) = 3x\,e^{\frac{3x^2}{2}}$$

$$k(x) = \int 3x\,e^{\frac{3x^2}{2}}\,dx = e^{\frac{3x^2}{2}} + C, \quad C\in\mathbb{R}$$

$$y = \left(e^{\frac{3x^2}{2}}+C\right)\cdot\left(e^{-\frac{x^2}{2}}\right) \rightarrow \boxed{y = e^{x^2} + e^{-\frac{x^2}{2}}\cdot C \quad C\in\mathbb{R}}$$

$$2 = y(0) \rightarrow 2 = e^{0^2} + e^{-\frac{0^2}{2}}\cdot C \rightarrow 2 = 2\cdot C \rightarrow \boxed{C=1}$$

$$\boxed{y = e^{x^2} + e^{-\frac{x^2}{2}}}$$

$\rightarrow$ Solución problema condiciones iniciales.

Escaneado con CamScanner