1) $(4x^2-2y^2)dx+(-2xy)dy=0$ $\quad y(1)=3$

$\frac{\partial P}{\partial y}=-4y$, $\frac{\partial Q}{\partial x}=-2y$ ¡no es exacta!

$\frac{Q_x-P_y}{Q}=\frac{-2y+4y}{-2xy}=\frac{2y}{-2xy}=\frac{-1}{x}$

$F.I.\ e^{-\int -\frac{1}{x}dx}=e^{\ln x}=x$

$(4x^3-2xy^2)dx+(-2x^2y)dy=0$

$\frac{\partial P}{\partial y}=-4xy$, $\frac{\partial Q}{\partial x}=-4xy$ ¡exacta!

$f(x,y)=\int(4x^3-2xy^2)dx=x^4-x^2y^2+g(y)$

$\frac{\partial f}{\partial y}=-2x^2y+g'(y)=-2x^2y$

$g'(y)=0$

$g(y)=K\text{te}$

solución $f(x,y)=x^4-x^2y^2+K=0$

$y(1)=3 \Rightarrow 1-1\cdot 9+K=0$

$K=8$

solución al PVI $f(x,y)=x^4-x^2y^2+8=0$

6) $(2xy-3x^2y^2)dx+(x^2-2x^3y)dy=0$

$\frac{\partial P}{\partial y}=2x-6x^2y$, $\frac{\partial Q}{\partial x}=2x-6x^2y$ ¡es exacta!

$f(x,y)=\int(2xy-3x^2y^2)dx=x^2y-x^3y^2+g(y)$

$\frac{\partial f}{\partial y}=x^2-2x^3y+g'(y)=x^2-2x^3y+0=Q$

$g'(y)=0$

$g(y)=K$

Sol. $f(x,y)=x^2y-x^3y^2+K=0$

2) $y''+4y=-4\,\text{sen}(2x)$ $\quad y(0)=-1 \quad y'(0)=4$

$y''+4y=0$

$\lambda^2+4=0$

$\lambda^2=-4 \quad \lambda=\sqrt{-4}=\begin{cases}0+2i\\0-2i\end{cases}$

$y_h=C_1e^{0x}\cdot\cos(2x)+C_2e^{0x}\cdot\text{sen}(2x)$

¿Has dicho $0+2i$ y $0-2i$ en y_h? ¡Sí! ¡una vez!

$y_p=Ax\cos(2x)+Bx\,\text{sen}(2x)$

$y_p'=A\cos(2x)-2Ax\cdot\text{sen}(2x)+B\,\text{sen}(2x)+2Bx\cos(2x)$

$y_p''=-2A\,\text{sen}(2x)-2A\,\text{sen}(2x)-4Ax\cdot\cos(2x)+2B\cos(2x)+2B\cdot\cos(2x)-4Bx\,\text{sen}(2x)$

$-4A\,\text{sen}(2x)+4B\cos(2x)-4Ax\cos(2x)-4Bx\,\text{sen}(2x)+4Ax\cos(2x)+4Bx\,\text{sen}(2x)=0\cos(2x)-4\,\text{sen}(2x)$

$+4B=0 \quad B=0$

$-4A=-4 \quad A=1$

$y_p=x\cos(2x)$

$y_G=C_1\cdot\cos(2x)+C_2\cdot\text{sen}(2x)+x\cos(2x)$

$y(0)=-1 \quad -1=C_1$

$y'=-2C_1\text{sen}(2x)+2C_2\cos(2x)+\cos(2x)-2x\,\text{sen}(2x)$ $\quad y'(0)=4 \quad 4=2C_2+1 \quad 2C_2=3 \quad C_2=\frac{3}{2}$

(3) $y' = 6y$

$\frac{1}{y} dy = 6x$

$Ly = 6x$

$e^{Ly} = e^{6x}$

$y = e^{6x} \cdot k$

$k' e^{6x} + 6k e^{6x} - 6k e^{6x} = 5 e^{6x} \cdot y^4 \cdot e^{24x}$

$k' = 5 e^{24x} \cdot k^4$

$\int \frac{1}{k^4} dk = \int 5 e^{24x} dx$

$\frac{k^{-3}}{-3} = \frac{5}{24} e^{24x} + cte$

$\frac{1}{k^3} = \frac{-15}{24} e^{24x} + cte$

$K = \frac{1}{-\frac{15}{24} e^{24x} + cte}$

4) $y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y^{III} + 5y^{II} = 0$

$\lambda^7 + \lambda^6 + 2\lambda^5 + 10\lambda^4 + 13\lambda^3 + 5\lambda^2 = 0$

$\lambda^2(\lambda^5 + \lambda^4 + 2\lambda^3 + 10\lambda^2 + 13\lambda + 5) = 0$

$\lambda_1 = 0 \quad m_1 = 2$

$\lambda^5 + \lambda^4 + 2\lambda^3 + 10\lambda^2 + 13\lambda + 5 = 0$

	1	1	2	10	13	5
		−1	0	−2	−8	−5
−1	1	0	2	8	5	0
		−1	1	−3	−5	
−1	1	−1	3	5	0	
		−1	2	−5		
−1	1	−2	5	0		

$\lambda_2 = -1 \quad m_1 = 3$

$\lambda^2 - 2\lambda + 5 = 0$

$\lambda = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = 1 \pm \sqrt{-4}$ $\begin{cases} 1+2i \\ 1-2i \end{cases}$

Solución: $y = C_1 e^{0x} + C_2 \cdot x \cdot e^{0x} + C_3 e^{-x} + C_4 \cdot x e^{-x} + C_5 x^2 e^{-x} + C_6 \cdot e^{x} \cdot \cos(2x) + C_7 e^{x} \cdot \operatorname{sen}(2x)$

$$\cancel{(x+2)^2 + 2K(x+2) - \frac{2}{x+1} \cdot K(x+2)^2} = 2(x+2)^3$$

$$K'(x+2)^2 = 2(x+2)$$

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(5) $y' + xy = 3xe^{x^2}$ $\quad y(0)=2$

$y' + xy = 0$

$y' = -xy$

$\int \frac{1}{y}\,dy = \int -x\,dx$

$Ly = -\frac{x^2}{2}$

$e^{Ly} = e^{-\frac{x^2}{2}}$

$y = e^{-\frac{x^2}{2}} \cdot K$

$K \cdot e^{-\frac{x^2}{2}} \cdot \cancel{(-x)} + \cancel{x \cdot K \cdot e^{-\frac{x^2}{2}}}$

$+ e^{-\frac{x^2}{2}} \cdot K' = 3xe^{x^2}$

$K' = 3x \frac{e^{x^2}}{e^{-\frac{x^2}{2}}}$

$K' = 3xe^{\frac{3}{2}x^2}$

$K = \int 3x e^{\frac{3}{2}x^2}\,dx = e^{\frac{3}{2}x^2} + cte$

Solución $y = \left(e^{\frac{3}{2}x^2} + K^{te}\right) \cdot e^{-\frac{x^2}{2}}$

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(8) $e^x y^2 + y\,\text{sen}\,x + (3ye^x - 2\cos x)y' = 0$ $\quad$ con F.I. $\mu(y)$.

$\frac{\partial P}{\partial y} = 2ye^x + \text{sen}\,x$

$\frac{\partial Q}{\partial x} = 3ye^x + 2\,\text{sen}\,x$

$$\frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{P} = \frac{2ye^x + \text{sen}\,x - 3ye^x - 2\,\text{sen}\,x}{e^x y^2 + y\,\text{sen}\,x} = \frac{-ye^x - \text{sen}\,x}{y(e^x y + \text{sen}\,x)} = \frac{-1}{y}$$

$$\text{F.I } \mu(y) = e^{\int \frac{1}{y}dy} = e^{Ly} = e^{Ly} = y$$

$\cancel{(e^x y + \text{sen}\,x)dx + (3e^x - \frac{2}{y}\cos x)dy}$

$(e^x \cdot y^3 + y^2\,\text{sen}\,x)\,dx + (3y^2e^x - 2y\cos x)\,dy = 0$

$\frac{\partial P}{\partial y} = 3y^2e^x + 2y\,\text{sen}\,x$

$\frac{\partial Q}{\partial x} = 3y^2e^x + 2y\,\text{sen}\,x$

$\quad\quad {}^{x\,3} \quad {}^{2}\cos x + g(y)$ $\quad\quad$ solución

$y' - 6y = 0$

$y' = 6y$

$\int \frac{1}{y}\,dy = \int 6\,dx$

$Ly = 6x + k$

$e^{Ly} = e^{6x} \cdot e^{k}$

$y = e^{6x} \cdot k$

$k' e^{6x} + \cancel{6ke^{6x}} - \cancel{6ke^{6x}} = 5 e \cdot e \cdot k$

$k' \cancel{e^{6x}} = 5 \cancel{e^{6x}} \cdot e^{24x} \cdot k^{4}$

$\int \frac{1}{k^4}\,dk = \int 5e^{24x}\,dx$

$\frac{k^{-3}}{-3} = \frac{5e^{24x}}{24} + cte$

$k = \frac{1}{-\frac{15}{24}e^{24x} + cte}$

Solución $y = \frac{1}{-\frac{15}{24}e^{24x} + cte} e^{6x}$

(4) $y'' + 6y' + 9y = 6e^{-3x} + 18$ $\begin{cases} y(0) = 2 \\ y'(0) = 25 \end{cases}$

y_h

$y'' + 6y' + 9y = 0$

$\lambda^2 + 6\lambda + 9 = 0$

$\lambda = \frac{-6 \pm \sqrt{36-36}}{2} = -3$ $\quad m_1 = 2$

$y_h = c_1 e^{-3x} + c_2 x \cdot e^{-3x}$

y_p ¿Ha salido -3 en y_h? sí, 2 veces

$y_p = Ax^2 \cdot e^{-3x} + B$

$y_p' = 2Axe^{-3x} - 3 \cdot Ax^2 e^{-3x}$

$y_p'' = 2Ae^{-3x} - 6Axe^{-3x} - 6Axe^{-3x} + 9Ax^2e^{-3x}$

$2Ae^{-3x} - \cancel{12Axe^{-3x}} + \cancel{9Ax^2e^{-3x}} + \cancel{12Axe^{-3x}} - \cancel{18Ax^2e^{-3x}} + \cancel{9Ax^2e^{-3x}} + 9B = 6e^{-3x} + 18$

$2A = 6$ $\quad A = 3$

$9B = 18$ $\quad B = 2$

$y_p = 3x^2e^{-3x} + 2$

SOLUCIÓN GENERAL

$y_G = c_1e^{-3x} + c_2xe^{-3x} + 3x^2e^{-3x} + 2$

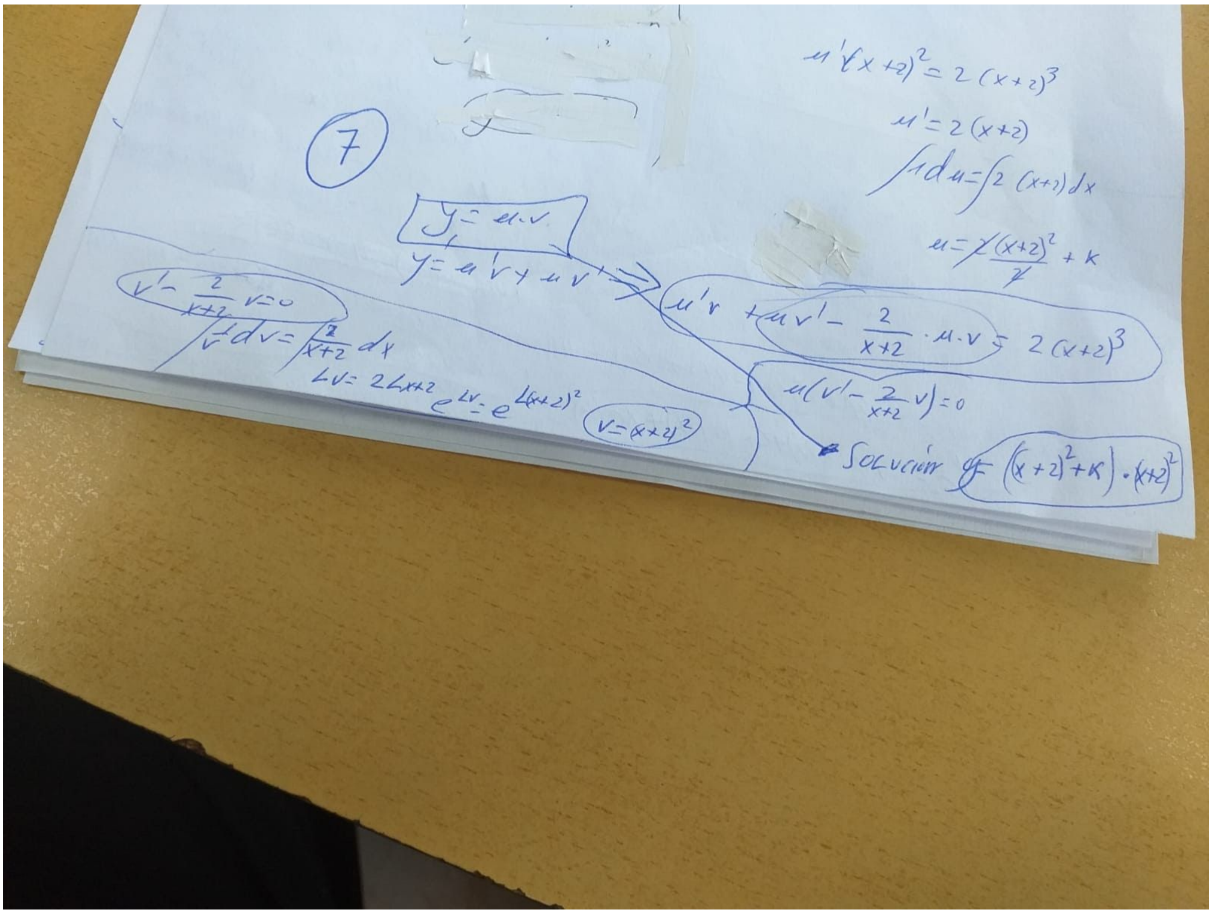
$y(0) = 2 \quad 2 = c_1 + 2 \quad c_1 = 0$

$y'(0) = 25$

$y_G' = -3c_1e^{-3x} + c_2e^{-3x} - 3c_2xe^{-3x} + 6xe^{-3x} - 9x^2e^{-3x}$

$25 = -3c_1 + c_2 \quad c_2 = 25$

SOLUCIÓN al P.V.I. - $y = 25xe^{-3x} + 3x^2e^{-3x} + 2$


7

$$\boxed{y = u\cdot v}$$

$$y' = u'v + uv'$$

$$u'v + uv' - \frac{2}{x+2}\cdot u\cdot v = 2(x+2)^3$$

$$u\left(v' - \frac{2}{x+2}v\right) = 0$$

$$v' - \frac{2}{x+2}v = 0$$

$$\frac{1}{v}dv = \frac{2}{x+2}dx$$

$$Lv = 2L(x+2)$$

$$e^{Lv} = e^{L(x+2)^2}$$

$$v = (x+2)^2$$

$$u'(x+2)^2 = 2(x+2)^3$$

$$u' = 2(x+2)$$

$$\int 1\,du = \int 2(x+2)\,dx$$

$$u = \cancel{2}\frac{(x+2)^2}{\cancel{2}} + K$$

Solución $y = \left((x+2)^2 + K\right)\cdot(x+2)^2$

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