1. $\begin{cases} (4x^2 - 2y^2)\,dx = 2xy\,dy \\ y(1) = 3 \end{cases}$

$$(4x^2 - 2y^2)\frac{dx}{dx} = 2xy\frac{dy}{dx}$$

$$y'(2xy) = 4x^2 - 2y^2$$

$$y' = \frac{4x^2}{2xy} - \frac{2y^2}{2xy} \Rightarrow y' = \frac{2x}{y} - \frac{y}{x} \Rightarrow$$

$$\Rightarrow y' + \frac{1}{x}y = 2xy^{-1} \rightarrow \text{Berroulli } (y' + p(x)y = q(x)y^{\alpha})$$

$$\boxed{\alpha = -1}$$

$$z = y^{1-\alpha} = y^{1-1(-1)} = y^2 \Rightarrow \boxed{z = y^2}$$

$$z' = 2y \cdot y' = 2y \cdot \left(2xy^{-1} - \frac{y}{x}\right) = 4x - \frac{2}{x}y^2 \rightarrow$$

$$y' = 2xy^{-1} - \frac{y}{x}$$

$$\rightarrow z' = 4x - \frac{2}{x} \cdot z \Rightarrow z' + \frac{2}{x} \cdot z = 4x$$

$$z' + \frac{2}{x}z = 4x$$

$$z' + \frac{2}{x} \cdot z = 0 \rightarrow z' = -\frac{2}{x} \cdot z \rightarrow \frac{dz}{dx} = -\frac{2}{x}z \rightarrow$$

$$\rightarrow \frac{dz}{z} = -\frac{2}{x}dx \rightarrow \int \frac{dz}{z}dx = \int -\frac{2}{x}dx \rightarrow \ln(z) = -2\ln(x) + C_1 \Rightarrow$$

$$z = e^{-2\ln(x) + C_1} \rightarrow z = e^{\ln(x)^{-2}} \cdot \underbrace{e^{C_1}}_{k} = x^{-2} \cdot k$$

$$z = k \cdot x^{-2}$$

$$z' = k' \cdot x^{-2} + k \cdot (-2 \cdot x^{-3}) \quad ; \quad z = k \cdot x^{-2}$$

$$k' \cdot x^{-2} + k \cdot (-2\ x^{-3}) + \frac{2}{x}\ k \cdot x^{-2} = 4x$$

$$k'\ x^{-2} + k \cdot (-2 \cdot x^{-3}) + k\ 2\ x^{-3} = 4x$$

$$k'\ x^{-2} = 4x$$

$$k' = 4x * x^2 \Rightarrow \boxed{k' = 4x^3}$$

$$k(x) = \int 4x^3 = \boxed{x^4 + C_1} \quad C \in \mathbb{R}$$

$$z = (x^4 + C_1) \cdot x^{-2} = x^2 + C \cdot x^{-2} \rightarrow z = y^2$$

$$y^2 = x^2 + \frac{C}{x^2} \rightarrow \frac{x^4 + C}{x^2} \rightarrow$$

$$\rightarrow y = \sqrt{\frac{x^4 + C_1}{x^2}} \qquad C_1 \in \mathbb{R}$$

$$y(1) = \sqrt{\frac{1^4 + C_1}{1^2}} = 3 \rightarrow$$

$$\rightarrow \sqrt{\frac{1 + C_1}{2}} = 3 \longrightarrow \sqrt{1 + C} = 3$$

$$1 + \sqrt{C} = 3 \longrightarrow \sqrt{C} = 2 \longrightarrow \boxed{C = 4}$$

2. $\begin{cases} y'' + 4y = -4 \cdot \text{sen}(2x) \\ y(0) = -1 \ , \ y'(0) = 4 \end{cases}$

- Ec dif. homogenea asociada:

$y'' + 4y = 0 \rightarrow P(x) = x^2 + 4 = 0 \rightarrow x = \sqrt{-4} = \pm 2i$

Raices $\rightarrow 0 \pm 2$, mult 1 $\begin{cases} \nearrow e^{ex} \cos(2x) \rightarrow \cos(2x) \\ \searrow e^{ex} \text{sen}(2x) \rightarrow \text{sen}(2x) \end{cases}$

Sol. ec dif. homogenea asociada:

$y(x) = C_1 \cdot \cos(2x) + C_2 \cdot \text{sen}(2x) \quad C_1, C_2 \in \mathbb{R}$

- Sol. particulares

$y(x) = Ax \cos(2x) + Bx \cdot \text{sen}(2x)$

$y'(x) = A \cdot \cos(2x) - 2Ax \cdot \text{sen}(2x) + B \cdot \text{sen}(2x) + 2Bx \cdot \cos(2x)$

$y''(x) = -2A \cdot \text{sen}(2x) - 2A \ \text{sen}(2x) - 4A \cdot \cos(2x) + 2B\cos(2x) +$

$+ 2B\cos(2x) - 4Bx \cdot \text{sen}\, 2x = -4A \cdot \text{sen}(2x) - 4Ax \cos(2x) + 4B\cos(2x)$

$- 4Bx \cdot \text{sen}(2x)$

Sustituimos en $y'' + 4y = -4 \cdot \text{sen}(2x)$

$-4A\,\text{sen}(2x) - 4Ax \cdot \cos(2x) + 4B\cos(2x) - 4Bx \cdot \text{sen}(2x) + 4Ax\cos(2x)$

$+ 4Bx\,\text{sen}(2x) = -4 \cdot \text{sen}(2x) \quad -4A \cdot \text{sen}\, 2x + 4B\cos(2x) = -4 \cdot \text{sen}(2x)$

$$\begin{cases} -4A = -4 \rightarrow A = \frac{-4}{-4} = 1 \\ 4B = 0 \rightarrow B = 0 \end{cases}$$

$$y(x) = x \cdot \cos(2x)$$

Sol. general $\rightarrow$ $y(x) = x \cdot \cos(2x) + C_1 \cdot \cos(2x) + C_2 \cdot \operatorname{sen}(2x)$

$$y'(x) = \cos(2x) - 2x \cdot \operatorname{sen}(2x) - 2C_1 \cdot \operatorname{sen}(2x) + 2C_2 \cos(2x)$$

$$-1 = y(0) = 0 \cdot \cos(2x) + C_1 \cdot \cos(2 \cdot 0) + C_2 \cdot \operatorname{sen}(2 \cdot 0) \rightarrow C_1 = -1$$

$$4 = y'(0) = \cos(0 \cdot 2) - 2 \cdot 0 \cdot \operatorname{sen}(2 \cdot 0) - 2C_1 \cdot \operatorname{sen}(2 \cdot 0) + 2C_2 \cdot \cos(2 \cdot 0)$$

$$1 - 0 - 0 + 2 \cdot C_2 = 4 \rightarrow 1 + 2C_2 = 4 \rightarrow 2C_2 = 3 \rightarrow C_2 = \frac{3}{2}$$

$$\boxed{y(x) = x \cdot \cos(2x) - \cos(2x) + \frac{3}{2} \operatorname{sen}(2x}$$

Soluciones problema condiciones iniciales

3.- $y' - 6y = 5e^{6x}y^4 \rightarrow y' = 5e^{6x}y^4 + 6y$

$\alpha = 3 \qquad z = y^{1-4} = y^{-3}$

$z' = -3y^{-4} \cdot y' = -3y^{-4} \cdot (5e^{6z} y^4 + 6y) = -15e^{6x} - 18y^{-3} =$

$= -15e^{6x} - 18z$

$z' = -15e^{6x} - 18z$ (Ec. diferencial 1er orden)

$z' + 18z = 0 \rightarrow \frac{dz}{dx} = -18z \rightarrow \int \frac{dz}{z} = \int -18\,dx \rightarrow \log z = -18x$

$\rightarrow z = e^{-18x+c} - 2z = e^{-18x} + e^c \rightarrow z = K \cdot e^{-18x}$

$z' = k'(x)\, e^{-18x} + k(x) - 18e^{-18x} = k'(x)\, e^{-18x} - 18k(x)\, e^{-18x}$

$k'(x)\, e^{-18x} - 18k(x)\, e^{-18x} + 18k(x)\, e^{-18x} = -15e^{6x} \rightarrow k'(x)\, e^{-18x} =$

$= -15e^{6x}$

$k'(x) = \frac{-15e^{6x}}{e^{-18x}} \rightarrow k'(x) = -15e^{6x} \cdot e^{18x} = -15e^{24x}$

$k(x) = \int -15e^{24x}\,dx = -15\int e^{24x}\,dx = -15e^{24x} \cdot 24 = -360e^{24x} + C,\ C \in \mathbb{R}$

$z = k(x) \cdot e^{-18x} = (-360e^{24x} + C)\, e^{-18x} = -360e^{6x} + Ce^{-18x}$

$z = -360e^{6x} + Ce^{-18x} \quad C \in \mathbb{R}$

$z = y^{-3} \rightarrow z = \frac{1}{y^3},\ y^3 = \frac{1}{z} \rightarrow y = \frac{1}{\sqrt[3]{z}},\ \boxed{y = \frac{1}{\sqrt[3]{-360e^{6x} + Ce^{-18x}}}\ C \in \mathbb{R}}$

$$\begin{cases} y'' + 6y' + 9y = 6e^{-2x} + 18 \\ y(0) = 2 \quad , \quad y'(0) = 25 \end{cases}$$

- Ec. dif. lineal homogénea asociada

$y'' + 6y' + 9y = 0$

$p(x) = x^2 + 6x + 9 = (x+3)^2$

RAICES $\rightarrow -3$ mult 2 $\rightarrow e^{-3x} , \; xe^{-3x}$

$yh(x) = C_1 e^{-3x} + C_2 xe^{-3x} \quad , \quad C1, C_2 \in \mathbb{R}$

- sol particular

$y(x) = Ax^2 e^{-3x}$

$y'(x) = 2Ae^{-3x} - 3Ax^2 e^{-3x}$

$y''(x) = 2Ae^{-3x} - 6Axe^{-3x} - 6Axe^{-3x} + 9Ax^2e^{-3x} =$

$= 2Ae^{-3x} - 12Axe^{-3x} + 9Ax^2e^{-3x}$

sustituimos:

$y'' + 6y' + 9y = 6e^{-3x} + 18$

$2Ae^{-3x} - 12Axe^{-3x} + 9Ax^2e^{-3x} + 12A\cancel{x}e^{-3x} - 18Ax^2e^{-3x} + 9Ax^2e^{-3x} =$

$= 6e^{-3x} + 18$

$2Ae^{-3x} = 6e^{-3x} + 18 \; ; \; 2A = 6 + 18e^{3x} \; ; \; A = 3 + 9e^{3x}$

$yp(x) = (3 + 9e^{3x})\, x^2 e^{-3x} = 3x^2 e^{-3x} + 9x^2$

- sol general

$$y(x) = 3x^2 e^{-3x} + 9x^2 + C_1 e^{-3x} + C_2 x e^{-3x}, \quad C_1, C_2 \in \mathbb{R}$$

$y(0) = 2$

$$3 \cdot 0 \cdot e^0 + 9 \cdot 0 + C_1 \cdot e^0 + C_2 \cdot 0 \cdot e^0 = 2; \quad C_1 = 2$$

$$y'(x) = 6x e^{-3x} - 9x^2 \cdot e^{-3x} + 18x - 3C_1 e^{-3x} + C_2 e^{-3x} - 3C_2 e^{-3x}$$

$y'(0) = 25$

$$6 \cdot 0 \cdot e^0 - 9 \cdot 0 \cdot e^0 + 18 \cdot 0 - 3 \cdot C_1 \cdot e^0 + C_2 \cdot e^0 - 3\, 0 e^0 = 25$$

$$-3C_1 + C_2 = 25; \quad -6 + C_2 = 25; \quad C_2 = 31$$

$$y(x) = 3x^2 e^{-3x} + 9x^2 + 2e^{-3x} + 31\, x e^{-3x}$$

5) $\begin{cases} y' + xy = 3xe^{x^2} \\ y(0) = 2 \end{cases}$

$\boxed{\begin{array}{l} \text{Primer orden} \\ y'(x) + p(x)\,y = q(x) \end{array}}$

$p(x) = x \qquad q(x) = 3xe^{x^2}$

$y\mu = \int q\mu \, dx \longrightarrow \mu = e^{\int p(x)dx}$

$\mu = e^{\int x dx} \longrightarrow \int x\,dx = \frac{x^2}{2}$

$\mu = e^{x^2/2}$

$y\, e^{x^2/2} = \int 3xe^{x^2} \cdot e^{x^2/2} dx = 3 \int x \cdot e^{x^2} \cdot e^{x^2/2} dx \Rightarrow$

$y e^{x^2/2} = 3 \int x \cdot e^{3/2\, x^2} dx = 3 \cdot \frac{e^{3\, x^2/2}}{3} + C_1$

$y\, e^{x^2/2} = e^{3\, x^2/2} + C_1$

$\boxed{y = \frac{e^{3\, x^2/2} + C}{e^{x^2/2}}} \qquad C \in \mathbb{R}$

$6.-\ 2xy - 3x^2y^2 + (x^2 - 2x^3y)y' = 0$

$$\underbrace{(2xy - 3x^2y^2)}_{M(x,y)} + \underbrace{(x^2 - 2x^3y)}_{N(x,y)}\, y' = 0$$

$$\frac{dM}{dy} = 2x - 6x^2y \ ; \quad \frac{dN}{dx} = 2x - 6x^2y$$

$$\frac{dM}{dy} = \frac{dN}{dx} \Rightarrow \text{Ec. diferencial}$$

$$\frac{df}{dx} = M \rightarrow f = \int 2xy - 3x^2y^2\,dx \rightarrow$$

$$f = y\int 2x\,dx - y^2\int 3x^2dx = \boxed{y \cdot x^2 - y^2 \cdot x^3 + C(y)}$$

$$\frac{df}{dy} = N \rightarrow x^2 - 2yx^3 + C'(y) = x^2 - 2x^3y \rightarrow$$

$$\rightarrow C'(y) = 0 \longrightarrow C(y) = 0$$

$$f(x,y) = yx^2 - y^2 \cdot x^3$$

$\boxed{yx^2 - y^2x^3 = C}$ $C \in \mathbb{R}$ $\Rightarrow$ Define de forma implícita la sol de la ecuación diferencial

7. $y' - \frac{2}{x+2} y = 2(x+2)^3$

Lineal de primer grado

$$\boxed{y'(x) + p(x)y = q(x)}$$

$p(x) = -\frac{2}{x+2}$ $\quad q(x) = 2(x+2)^3$

$$\boxed{y\mu = \int q \cdot \mu \, dx}$$ lineales $\rightarrow \mu = e^{\int p(x)\,dx}$

$$\mu = e^{\int -\frac{2}{x+2} dx} \longrightarrow \int -\frac{2}{x+2} dx = -2\int \frac{1}{x+2} dx \rightarrow$$

$$\rightarrow -2\int \frac{1}{x+2} = \boxed{-2 \ln(x+2) + C_1}$$

$$\mu = e^{-2\ln(|x+2|)} = e^{\ln(x+2)^{-2}} \rightarrow \boxed{\mu = (x+2)^{-2}}$$

$$y \cdot (x+2)^{-2} = \int 2(x+2)^3 \cdot (x+2)^{-2} dx \rightarrow$$

$$y(x+2)^{-2} = -\int 2x + 4 \, dx = 2\int x \, dx + 4\int 1 \, dx = 2\frac{x^2}{2} + 4x$$

$$y(x+2)^{-2} = x(x+4) + C_1$$

$$\boxed{y = ((x(x+4)) \cdot (x+2)^2) + C_1 (x+2)^2}$$

8. $e^x y^2 + y \cdot \operatorname{sen} x + (3y e^x - 2\cos x) y' = 0$

farctor integrante $y \to \underbrace{(y)(e^x y^2 \cdot y \cdot \operatorname{sen} x)}_{M(x,y)} + \underbrace{(y)(3y e^x - 2\cos x)}_{N(x,y)} y' = 0$

$\frac{dM}{dy} = +3y^2 e^x + 2y \cdot \operatorname{sen} x$

$\frac{dN}{dy} = 3y^2 e^x + 2y \cdot \operatorname{sen} x$

= Ec. dif. exacta

$f / \frac{df}{dx} = M, \frac{df}{dy} = N$

$f(x,y) = C \quad C \in R$

$\frac{df}{dx} = M \Rightarrow \frac{df}{dx} = e^x y^3 + y^2 \cdot \operatorname{sen} x \to f = \int (e^x y^2 + y^2 \cdot \operatorname{sen} x) dx \to$

$\to f = y^3 \int e^x \, dx + y^2 \int \operatorname{sen} x \, dx = \boxed{y^3 e^x - y^2 \cdot \cos x + C}$

$\frac{df}{dy} = N \Rightarrow 3y^2 e^x - 2y \cdot \cos x + C'(y) = 3y^2 e^x - 2\cos x y \to$

$\to C'(y) = 0 \to C(y) = 0$

$f(x,y) = y^3 e^x - y^2 \cdot \cos x$

$\boxed{y^3 e^x - y^2 \cdot \cos x = C}$ $C \in \mathbb{R}$, define la forma implicita las sol (y) de la ec. diferencial

9. $y^{VII} + y^{VI} + 2y^{V} + 10y^{IV} + 13y''' + 5y'' = 0$ $\boxed{y = e^{rx}}$

$r^7 + r^6 + 2r^5 + 10r^4 + 13r^3 + 5r^2 = 0$

$r^2 \cdot (r^5 + r^4 + 2r^3 + 10r^2 + 13r + 5) = 0$

$r = -1$ mult 3

$r = 0$ mult 2

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
	1	0	2	8	5	0
-1		-1	1	-3	-5	
	1	-1	3	5	0	
-1		-1	2	-5		
	1	-2	5	0		

$(r^2 - 2r + 5) = 0$ mult 1

$\Rightarrow r = 1 \pm 2i$ mult 1

$\frac{2 \pm \sqrt{4 - 20}}{2} \notin \mathbb{R}$

$r = 0 \longrightarrow c + x$

$r = -1 \longrightarrow e^{-x} + xe^{-x} + x^2 e^{-x}$

$r = 1 \pm 2i \longrightarrow e^{1x} \cos 2x$

$\quad\quad\quad\quad \searrow e^{1x} \operatorname{sen} 2x$

$\{c, x, e^{-x}, x e^{-x}, x^2 e^{-x}, e^{x} \cdot \cos 2x, e^{x} \cdot \operatorname{sen} 2x\}$

Es una $\mathbb{R}$ base del conjunto de todas la sol de la ecuación diferencial

Así como la sol general es:

$C_1 + C_2 x + C_3 e^{-x} + C_4 x e^{-x} + C_5 x^2 e^{-x} + C_6 e^{x} \cdot \cos 2x + C_7 e^{x} \operatorname{sen} 2x$

$C_1 \ldots C_x \in \mathbb{R}$