EJERCICIOS ECUACIONES DIFERENCIALES → LIMPIO

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1- $\begin{cases}(4x^2-2y^2)\,dx = 2xy\,dy \\ y_{(1)}=3\end{cases}$

$dy/dx = \frac{2(2x^2-y^2)}{2\cdot x\cdot y}$; $y' = \frac{2x^2-y^2}{x\cdot y}$; $v = y/x$; $y = vx$; $y' = v'x + v$;

$v'x + v = \frac{2x^2 - v^2x^2}{x\cdot v\cdot x} = \frac{x^2(2-v^2)}{x^2\, v} = \frac{2-v^2}{v}$; $v'x = \frac{2(1-v^2)}{v} = \frac{dv}{dx}\cdot x$; $e^{C} = K$

$\int \frac{dx}{x} = \int \frac{v}{2\cdot(1-v^2)}\cdot dv$; $\ln|x| = -1/4 \int \frac{-2v}{1-v^2}\,dv = -1/4 \ln|1-v^2| + C$; $x = (1-v^2)^{-1/4}\cdot K$

$x = \frac{K}{\sqrt[4]{1-y^2/x^2}}$; $y_{(1)}=3$; $1 = \frac{K}{\sqrt[4]{1-9}}$; $K = \sqrt[4]{-8}$; $x = \sqrt[4]{\frac{-8\cdot x^2}{x^2-y^2}}$;

$x^4 = \frac{-8x^2}{x^2-y^2}$; $x^2 = \frac{-8}{x^2-y^2}$; $\boxed{x^4-(x\cdot y)^2 = -8}$ ← Esta ecuación lleva implícita la solución de la ecuación diferencial.

$x^2\cdot y^2 - x^4 = +8$; $x^2y^2 = 8 + x^4$; $\boxed{y = \sqrt{\frac{8+x^4}{x^2}}}$ ← Esta es LA solución

2- $\begin{cases}y'' + 4y = -4\,\text{sen}(2x) \\ y_{(0)} = -1 \\ y'_{(0)} = 4\end{cases}$

$p(x) = x^2+4 = 0$; $x = \sqrt{-4} = \pm 2i$

$y = C_1\,\text{sen}(2x) + C_2\cos(2x)$, $C_1, C_2 \in \mathbb{R}$

$y_p = A\cdot x\cdot \text{sen}(2x) + B\cdot x\cdot\cos(2x)$; $y'_p = A(\text{sen}(2x) + x\cdot 2\cdot\cos(2x)) + B(\cos(2x) - 2x\,\text{sen}(2x))$

~~$y''_p = A(\cos(2x) + B\cos(2 \quad y''_p = A(\cos$~~

$y''_p = A(2\cos(2x) + 2(\cos(2x) - 2x\,\text{sen}(2x))) + B(-2\,\text{sen}(2x) - 2(\text{sen}(2x) + 2x\cos(2x)))$

$y''_p = (2A)\cos(2x) + (2A)\cos(2x) - (4xA)\,\text{sen}(2x) - (2B)\,\text{sen}(2x) - (2B)\,\text{sen}(2x) - (4xB)\cos(2x)$

$y''_p = (2A+2A-4xB)\cos(2x) - (2B+2B+4xA)\,\text{sen}(2x) = (4A-4xB)\cos(2x) - (4B+4xA)\,\text{sen}(2x)$

SUSTITUIR → $(4A-4xB)\cos(2x) - (4B+4xA)\,\text{sen}(2x) + 4(Ax\,\text{sen}(2x) + Bx\cos(2x)) = -4\,\text{sen}(2x)$

$4A - \cancel{4xB} + \cancel{4xB} = 0$; $A = 0$

$-4B - \cancel{4xA} + \cancel{4xA} = -4$; $B = 1$

$y_p = Ax\,\text{sen}(2x) + Bx\cos(2x)$; $y_p = x\cos(2x)$

$y_{(x)} = C_1\,\text{sen}(2x) + C_2\cos(2x) + x\cos(2x) \Rightarrow y_{(0)} = -1 \Rightarrow -1 = C_1\cdot 0 + C_2\cdot 1 + 0$; $\boxed{C_2 = -1}$

$y'_{(x)} = 2\cdot C_1\cos(2x) - 2C_2\,\text{sen}(2x) + \cos(2x) - x\cdot 2\cdot\text{sen}(2x) \Rightarrow y'_{(0)} = 4 \Rightarrow 2\cdot C_1 + 1 = 4$; $\boxed{C_1 = 1'5 = 3/2}$

$$\boxed{y_{(x)} = 3/2\cdot\text{sen}(2x) + (x-1)\cos(2x)}$$

$\boxed{3-}$ $\{ y'-6y=5\cdot e^{6x}\cdot y^4$ $\quad \alpha=4;\ z=y^{1-\alpha}=y^{-3};\ z'=-3\cdot y^{-4}\cdot y'$

$y'=y^4 5\cdot e^{6x}+6y;\ z'=-3\left(\frac{5e^{6x}\cdot y^4}{y^4}+6\frac{y}{y^4}\right)=-15\cdot e^{6x}+18y^{(1-\alpha)}=-15e^{6x}-18z$

$z'+18z=-15e^{6x} \leftarrow$ Ec. lin. 1ª Orden $\rightarrow$ Ec. dif. lin. hom. asoc. $\rightarrow z'+18z=0$

$\frac{dz}{dx}=-18z;\ \ln|z|=-18x+c;\ z=1/e^{18x}\cdot K;\ \underline{z=K(x)/e^{18x}};\ \underline{z'=K(x)\cdot(e^{-18x})'+K'(x)\cdot e^{-18x}}$

SUSTITUIR $\rightarrow$ $\cancel{K(x)\cdot(e^{-18x})'}+K'(x)\cdot e^{-18x}\cancel{+18\cdot K(x)\cdot e^{-18x}}=-15e^{6x}$

$K'(x)=-15\,e^{6x}\cdot e^{18x}=-15\,e^{24x};\ K(x)=-15\int e^{24x}dx=-15\left(\frac{e^{24x}}{24}+C_1\right)$

$z=\frac{-15\left(\frac{e^{24x}}{24}+C_1\right)}{e^{18x}}=\frac{-15}{24}e^{(24-18)x}-\frac{15C_1}{e^{18x}};\ \boxed{z=\frac{-5}{8}e^{6x}-\frac{15C_1}{e^{18x}}}$

$z=y^{-3};\ \boxed{y=\left(\frac{-5}{8}e^{6x}-\frac{15C_1}{e^{18x}}\right)^3}\quad C_1\in\mathbb{R}$

$\boxed{4-}$ $\begin{cases} y''+6y'+9y=6e^{-3x}+18 \\ y(0)=2 \\ y'(0)=25 \end{cases}$ $\quad P(x)=x^2+6x+9=0;\ x=\frac{-6\pm\sqrt{\cancel{36-4\cdot 9}}^{\,0}}{2}=\boxed{-3\text{ mult } 2}$

$y=C_1/e^{3x},\ C_1\in\mathbb{R}$

$\underline{y_p=A\cdot x/e^{3x}+B};\ y_p'=\underline{\frac{Ae^{3x}-Ax\cdot 3e^{3x}}{e^{6x}}};\ \underline{y_p'=\frac{A}{e^{3x}}(1-3x)};\ \cancel{y_p''=A\cdot -3e^{-3x}}$

$y_p''=3A\cdot\left(\frac{-\cancel{3}e^{3x}-(1-3x)\cdot\cancel{3}e^{3x}}{(e^{3x})^2}\right)=3A\left(\frac{-1-1+3x}{e^{3x}}\right);\ \underline{y_p''=3A\left(\frac{3x-2}{e^{3x}}\right)};$ SUSTITUIMOS

$(9Ax-6A)\cdot t+6\cdot(A-3Ax)\cdot t+9(Ax\,t+B)=6t+18$

$At(9x-6+6-3x+9x)+9\cdot B=6t+18;\ 9B=18;\ \boxed{B=2}$

$15Ax=6;\ 5Ax=2$

$y(x)=c/e^{3x}+Ax/e^{3x}+2;\ \frac{Ax+c}{e^{3x}}+2=y(x);\ y(0)=2=2+C;\ \boxed{C=0}$

$y'(0)=25=\frac{A\cdot e^{3x}+(Ax+\cancel{c})\cdot 3\cdot e^{3x}}{e^{6x}}=A+3A+\cancel{3c};\ \cancel{4A=25};\ \cancel{A=(5/2)^2}\ \boxed{A=25}$

$$\boxed{y(x)=\frac{25x}{e^{3x}}+2}$$

5.- $\begin{cases} y' + xy = 3x e^{x^2} \\ y(0) = 2 \end{cases}$

Ec. diferencial lineal homogenea asociada $\rightarrow y' + x\cdot y = 0;\ \frac{dy}{dx} = -x\cdot y;\ \frac{dy}{y} = -x\cdot dx;$

$\ln|y| = -\frac{x^2}{2} + C_1;\ y = \frac{K(x)}{\sqrt{e^{x^2}}};$ ~~[crossed out]~~ ; $y' = \frac{K'(x)\cdot\sqrt{e^{x^2}} - K(x)\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{e^{x^2}}}\cdot e^{x^2}\cdot 2x}{e^{x^2}}$; SUSTITUIMOS

$\frac{K'(x)}{\sqrt{e^{x^2}}}\left[-\frac{K(x)}{\sqrt{e^{x^2}}}\cdot x + x\cdot\frac{K(x)}{\sqrt{e^{x^2}}}\right] = 3x e^{x^2};\ K'(x) = 3x\left(e^{x^2}\right)^{3/2};$

$K(x) = \int 3x\left(e^{x^2}\right)^{3/2} dx = \int 3x\cdot e^{\frac{3x^2}{2}} dx =$ [Función por su derivada] $= e^{\frac{3}{2}x^2} + C_1',\ C_1' \in \mathbb{R}$

$y = \frac{K(x)}{\sqrt{e^{x^2}}} = \frac{\sqrt{e^{x^2}}\cdot e^{x^2} + C_1}{\sqrt{e^{x^2}}} = e^{x^2} + \frac{C_1}{\sqrt{e^{x^2}}};\ \boxed{y = e^{x^2} + \frac{C_1}{\sqrt{e^{x^2}}},\ C_1' \in \mathbb{R}}$

6.- $\{\underbrace{2xy - 3x^2y^2}_{M(x,y)} + \underbrace{(x^2 - 2x^3y)}_{N(x,y)}\cdot y' = 0$

$\frac{df}{dx} = 2xy - 3x^2y^2;$

$f = \int 2xy\,dx - \int 3x^2y^2\,dx = 2y\cdot\frac{x^2}{2} - 3y^2\frac{x^3}{3} + K(y) = yx^2 - y^2x^3 + K(y)$

$\frac{dM}{dy} = 2x\cdot 1 - 3x^2\cdot 2y = 2x - 6x^2y$

$\frac{dN}{dx} = 2x - 6x^2y \leftarrow$ son iguales $\Rightarrow$ EXACTAS

$f = x^2y - y^2x^3 + K(y);\ \frac{df}{dy} = N(x,y) \Rightarrow x^2 - 2x^3y + K'(y) = x^2 - 2x^3y;\ K'(y) = 0;\ K(y)$ cte.

Por lo que $\boxed{x^2y - x^3y^2 = C_1}$, $C_1 \in \mathbb{R}$, define de forma implícita las soluciones de la ec. diferencial.

7.- $\{y' - \frac{2y}{x+2} = 2(x+2)^3 \Rightarrow$ denotaremos $(x+2) = t$, para operar mas fácil

Ec dif lineal homogenea asociada $\rightarrow y' - \frac{2}{t}\cdot y = 0;\ \frac{dy}{dx} = \frac{2y}{t};\ \ln|y| = 2\cdot\ln|x+2| + C_1$

SUSTITUYENDO $\Rightarrow$

$y = (x+2)^2\cdot K(x);\ y' = 2\cdot(x+2)\cdot K(x) + K'(x)(x+2)^2;$

$2\cdot(x+2)\cdot K(x) + K'(x)(x+2)^2 - \frac{2\cdot(x+2)^2\cdot K(x)}{(x+2)} = 2(x+2)^3;\ K'(x) = 2\cdot(x+2);\ K(x) = 2\cdot\left(\frac{x^2}{2} + 2x\right) + C_1$

$y = (x+2^2)\left(2x\left(\frac{x}{2}+2\right) + C_1\right) = (x+2)^2\cdot(x^2+4x) + C_1 = (x+2)^2\cdot\left((x+2)^2 + (C_1 - 4)\right)$

$y = (x+2)^4 + (x^2 + 4x + 4 - 4 + C_1) = (x+2)^4 + (x^2 + 4x + C_1);\ \boxed{y = (x+2)^4 + (x^2 + 4x + C_1),\ C_1 \in \mathbb{R}}$

$\boxed{8-)}\ \{\underbrace{e^x y^2 + y\,\text{sen}\,x}_{M(x,y)/\mu(y)} + \underbrace{(3y\,e^x - 2\cos x)}_{N(x,y)/\mu(y)}\, y' = 0$ (Por fact. integrante $\mu(y)$)

$$\frac{d(M(x,y)\cdot\mu(y))}{dy} = \frac{d(N(x,y)\cdot\mu(y))}{dx};\ \frac{(2y\,e^x + \text{sen}x)\mu(y) + (e^x y^2 + y\,\text{sen}x)\mu'(y)}{(3y\,e^x + 2\,\text{sen}\,x)\mu(y) + (3y\,e^x - 2\cos x)\cancel{\mu'(y)}^{\,0}} =$$

$(2y\,e^x + \text{sen}\,x - 3y\,e^x - 2\,\text{sen}\,x)\mu(y) = \cancel{(3y\,e^x - 2\cos x)}(-e^x y^2 - y\,\text{sen}\,x)\mu'(y)$

$(-1)(e^x \cdot y + \text{sen}x)\mu(y) = (-1)(e^x y^2 + y\,\text{sen}x)\mu'(y)$; $\mu(y) = y\cdot\mu'(y) = y\cdot\frac{d\mu}{dy}$; $\underline{\underline{\mu(y) = y}}$

$f = \int e^x y^3\,dx + \int y^2\,\text{sen}x\,dx = \underline{e^x y^3 - y^2\cos x + C_1}$

$\frac{df}{dy} = N(x,y) \Rightarrow 3y^2 e^x + 2y\,\cancel{\text{sen}x} + K'(y) = 3y^2 e^x - 2y\cos x$; $K'(y) = 0$; $\underline{K\ \text{cte}}$, por lo que

$\boxed{e^x y^3 - y^2\cos x = C_1,\ C_1 \in \mathbb{R}}$, es el conjunto de las soluciones de la ec. dif.

$\boxed{9-)}\ \{y^{(VII)} + y^{(VI)} + 2y^{V} + 10y^{(IV)} + 13y''' + 5y'' = 0$

Polinómio característico $\overset{\text{pol:}}{\Longrightarrow} x^7 + x^6 + 2x^5 + 10x^4 + 13x^3 + 5x^2 = 0$; $\boxed{x_1 = 0 \text{ mult } 2}$ $\boxed{x_2 = -1 \text{ mult } 3}$

$p(x) = x^5 + x^4 + 2x^3 + 10x^2 + 13x + 5 = 0$

	1	1	2	10	13	5
-1		-1	0	-2	-8	-5
	1	0	2	8	5	0
-1		-1	1	-3	-5	
	1	-1	3	5	0	
-1		-1	2	-5		
	1	-2	5	0		

$x^2 - 2x + 5 = 0$; $x = \frac{2 \pm \sqrt{4 - 4\cdot 5}}{2} = \frac{2 \pm 4i}{2} = \boxed{1 \pm 2i} = x_3$

$$\boxed{y_{(x)} = C_1 + C_2 x + C_3/e^x + \frac{x\cdot C_4}{e^x} + \frac{C_5 x^2}{e^x} + C_6\cdot e^x\,\text{sen}(2x) + C_7 e^x\cos(2x),}$$

$C_1, C_2, C_3, C_4, C_5, C_6, C_7 \in \mathbb{R}$