Efficient Visual-Inertial Navigation using IMU Pre-integration

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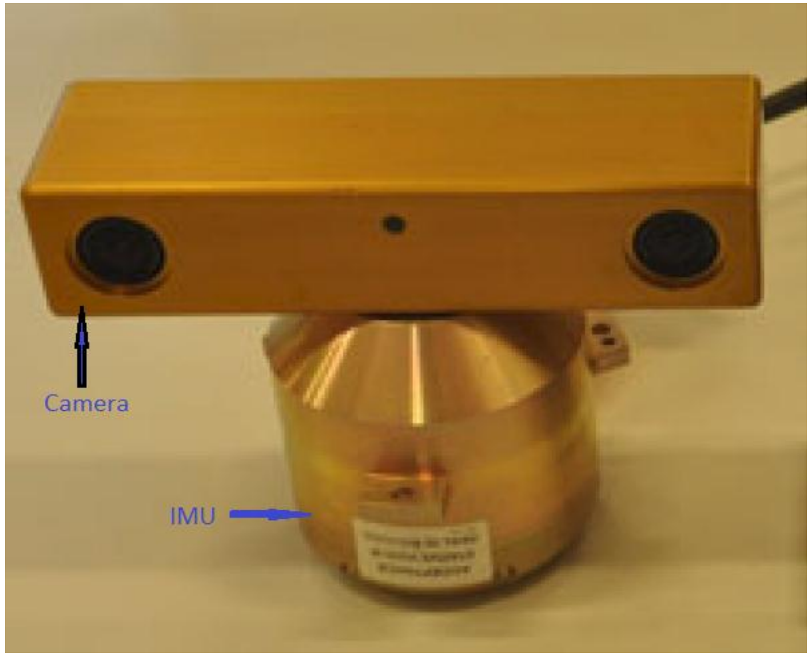
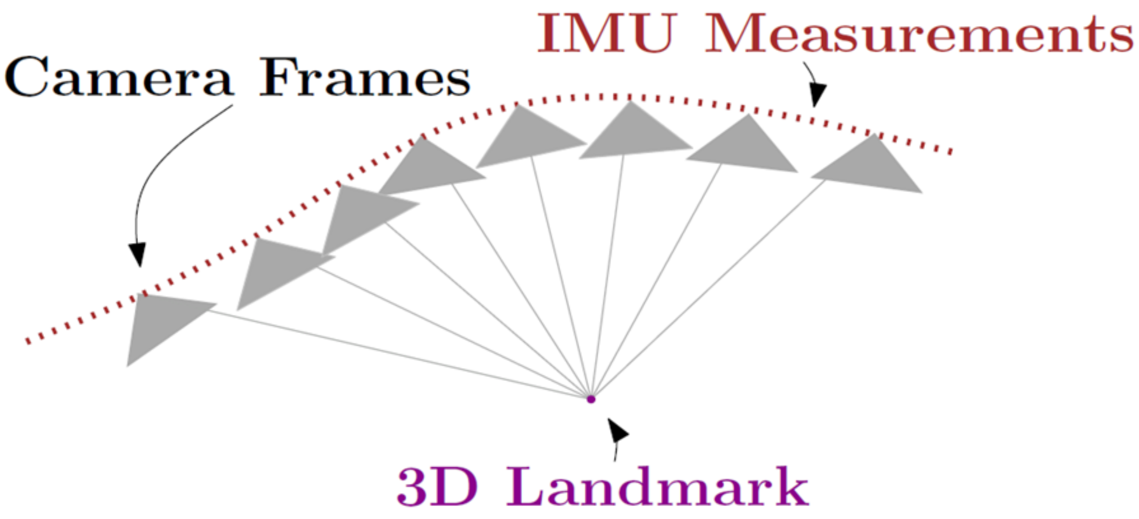
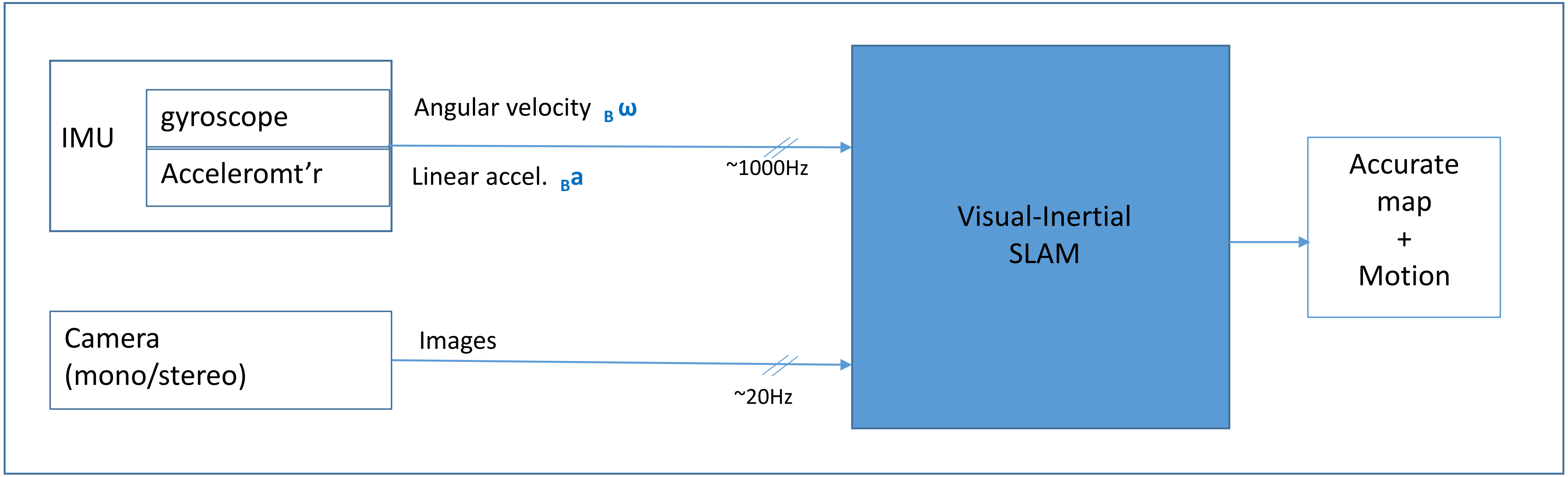
Visual-Inertial SLAM







Graph SLAM revisited

Gauss-Newton Framework

Needs to linearize f(**x**), given:

- Parameterization of **x**, **z**
- f(**x**)
- *Jacobian(* f(**x**) *)*
- $\boldsymbol{\Sigma}_z$

⇧

A constrained optimization problem: assuming Gaussian observation

$$\min(\, \|\boldsymbol{z} - f(\boldsymbol{x})\|^{2}_{\boldsymbol{\Sigma}_z} \,)$$

⇧

ML (maximum likelihood) of state space

$$\boldsymbol{x}_{1:T}, \boldsymbol{m} = \underset{\boldsymbol{x}_{1:T},\ \boldsymbol{m}}{\operatorname{argmax}} (p(\boldsymbol{z}_{1:T} \mid \boldsymbol{x}_{1:T}, \boldsymbol{m}, \boldsymbol{u}_{1:T}, \boldsymbol{x}_0))$$

What's good parameterization of x, z

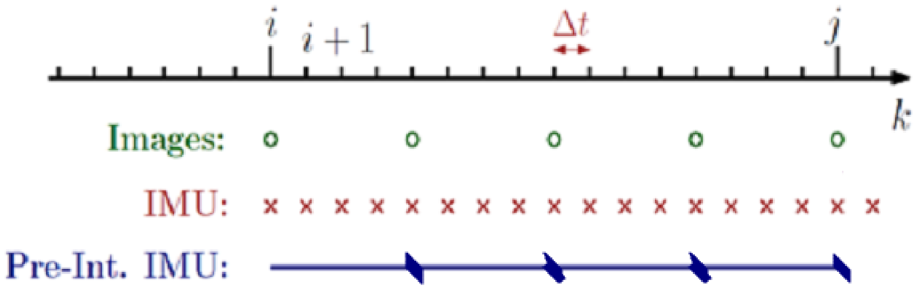
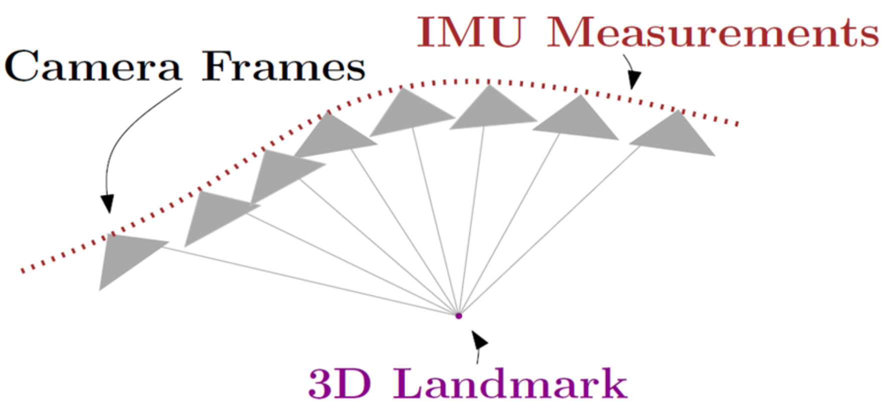
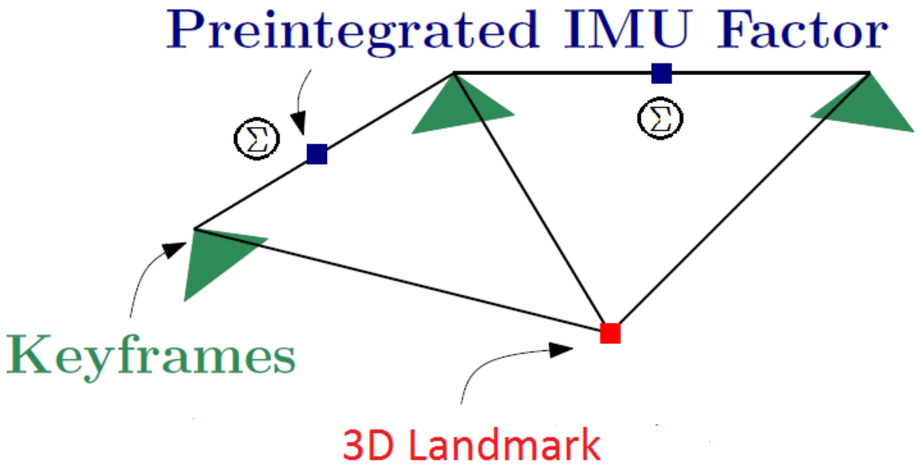


Fig. : Different rates for IMU and camera.

- Naïve parameterization:
  - **x**: pose, orientation, velocity $\{R_t,\ {}_{w}\boldsymbol{p}_t,\ {}_{w}\boldsymbol{v}_t\}$ for all IMU samples
  - **z**: all IMU data points $\{{}_{B}\boldsymbol{\omega}_t,\ {}_{B}\boldsymbol{a}_t\}$, $t \in [0 \sim T_{total}]$ and all feature observations $\{\boldsymbol{z}_{il}\}$, $i \in I_{total}$ $l \in C_i$
    I_{total}: all frms; C_i : correspondences in frm i; ${}_{B}$ refers body frame
  - $\boldsymbol{\Sigma}_z$: angular rate noise: $\boldsymbol{\eta}^g$, linear acc noise: $\boldsymbol{\eta}^a$, image uv noise: $\boldsymbol{\eta}^i$
  - Process model:
    $R(\, t + \Delta t\,) = R(t)\, Exp({}_{B}\boldsymbol{\omega}(t)\, \Delta t\,)$
    $\boldsymbol{v}(\, t + \Delta t\,) = {}_{w}\boldsymbol{v}(t) + R(t)\, {}_{B}\boldsymbol{a}(t)\, \Delta t$
    $\boldsymbol{p}(\, t + \Delta t\,) = {}_{w}p(t) + {}_{w}\boldsymbol{v}(t)\, \Delta t + \frac{1}{2} R(t)\, {}_{B}\boldsymbol{a}(t)\, \Delta t^2$
- Infeasible: X
  - too much observation data
  - Re-compute integration for each new linearization point change (estimating new rotation R_t).

Pre-integration on IMU -- New form of state space and observations





- **x**: pose, orientation, velocity for all key frame times: $\{R_t, {}_w\mathbf{p}_t, {}_w\mathbf{v}_t\}$
- **z**: Inertial delta -- integrated IMU data points (over key frame duration) $\{\Delta R_{ij}, \Delta \mathbf{v}_{ij}, \Delta \mathbf{p}^+_{ij}\}$, $i,j \in$ [adjacent keyframes]

$$\Delta R_{ij} = \prod_{k=i}^{j-1} Exp(\boldsymbol{\omega}_k \Delta t), \quad \textit{delta rotation} \quad \text{(Eq 1)}$$

$$\Delta \boldsymbol{v}_{ij} = \sum_{k=i}^{j-1} \Delta R^{ik}\, {}_{\mathrm{B}}\boldsymbol{a}_k \Delta t, \quad \textit{delta velocity due to acceleration} \quad \text{(Eq 2)}$$

$$p^+_{ij} = \sum_{k=i}^{j-1} \tfrac{3}{2} \Delta R^{ik}\, {}_{\mathrm{B}}\boldsymbol{a}_k \Delta t^2, \quad \textit{corrective term to account for acceleration} \quad \text{(Eq3)}$$

- New observation model $z = f(x)$:

$$\Delta R_{ij} = R_i^T R_j \quad \text{(Eq 4)}$$

$$\Delta \boldsymbol{v}_i = R_i^T (\boldsymbol{v}_j - \boldsymbol{v}_i - \mathbf{g}\Delta t_{ij}) \quad \text{(Eq 5)}$$

$$\Delta p^+_{ij} = R_i^T (\boldsymbol{p}_j - \boldsymbol{p}_i - \boldsymbol{v}_i \Delta t_{ij} - \tfrac{1}{2}\boldsymbol{g}\Delta t_{ij}^2) \quad \text{(Eq 6)}$$

- Advantage:
  - Initial orientation is world frame. No explicit initialization stage or large uncertainty prior required.
  - Inertial observation integrated in body frame, K inertial observations treated as a single observation, reduces linearization error, Gauss-Newton more robust.
  - Much reduced state space and observation size, less computation
  - Can be implemented in real-time

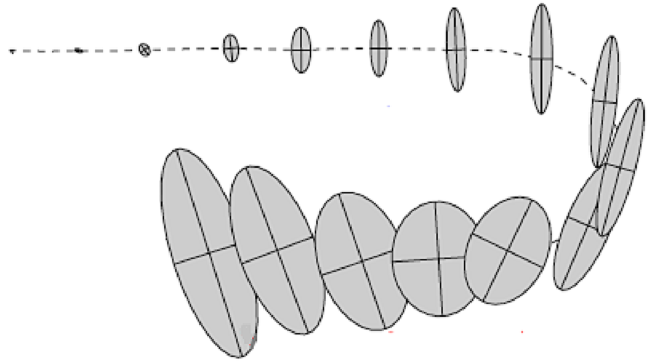
Σ Calculation in Pre-integration

Re-write (EQ1~3) to get pre-integration process model

$$\Delta\tilde{R}_{ij} = \prod_{k=i}^{j-1} Exp\big(({}_{\mathrm{B}}\boldsymbol{\omega}_k + \boldsymbol{\eta}_k{}^{g})\Delta t\big) \qquad \text{(Eq7)}$$

$$\Delta\tilde{\boldsymbol{v}}_{ij} = \sum_{k=i}^{j-1} \Delta\tilde{R}_{ik}({}_{\mathrm{B}}\boldsymbol{a}_k + \boldsymbol{\eta}_k{}^{a})\Delta t \qquad \text{(Eq8)}$$

$$\Delta\tilde{\boldsymbol{p}}_{ij}^{+} = \sum_{k=i}^{j-1} \tfrac{3}{2}\Delta\tilde{R}_{ik}({}_{\mathrm{B}}\boldsymbol{a}_k + \boldsymbol{\eta}_k{}^{a})\Delta t^2 \qquad \text{(Eq9)}$$



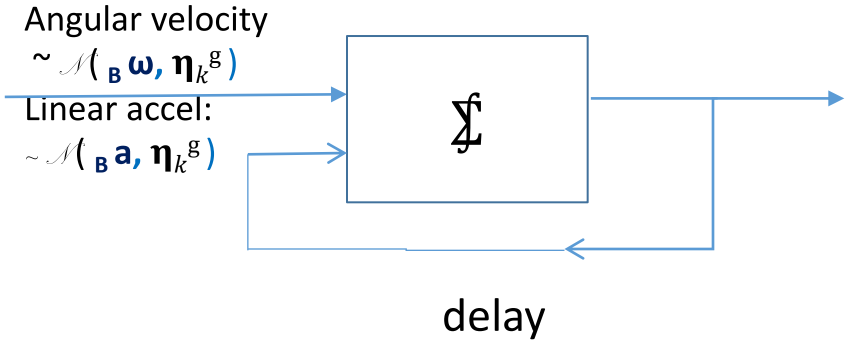
$\Sigma_{ii}=0$

$$\Sigma_{i,k+1} = A_k \Sigma_k A_k^T + B_k \Sigma_\eta B_k^T, \qquad \text{(Eq10)}$$

$$A_k = \begin{bmatrix} \widetilde{\Delta R^T_{k,k+1}} & 0_{3\times3} & 0_{3\times3} \\ -\Delta\widetilde{R_{ik}}(\widetilde{\boldsymbol{a}_k} - \boldsymbol{b}_i^a)^{\neg} & I_{3\times3} & I_{3\times3} \\ -\tfrac{1}{2}\Delta\widetilde{R_{ik}}(\widetilde{\boldsymbol{a}_k} - \boldsymbol{b}_i^a)^{\neg}\,\Delta t^2 & I_{3\times3}\Delta t & I_{3\times3} \end{bmatrix} \qquad \text{(Eq11)}$$

$$B_k = \begin{bmatrix} J_r^k \Delta t & 0_{3\times3} \\ 0_{3\times3} & \Delta\widetilde{R_{ik}}\Delta t \\ 0_{3\times3} & \tfrac{1}{2}\Delta\widetilde{R_{ik}}\Delta t^2 \end{bmatrix} \qquad \text{(Eq12)}$$

Pre-integration in recursive form



$$\mathcal{N}\left(\begin{matrix}\Delta R_{i,k+1} \\ \Delta \boldsymbol{v}_{i,k+1} \\ \Delta p^{+}_{i,k+1}\end{matrix}, \Sigma_{k+1}\right)$$

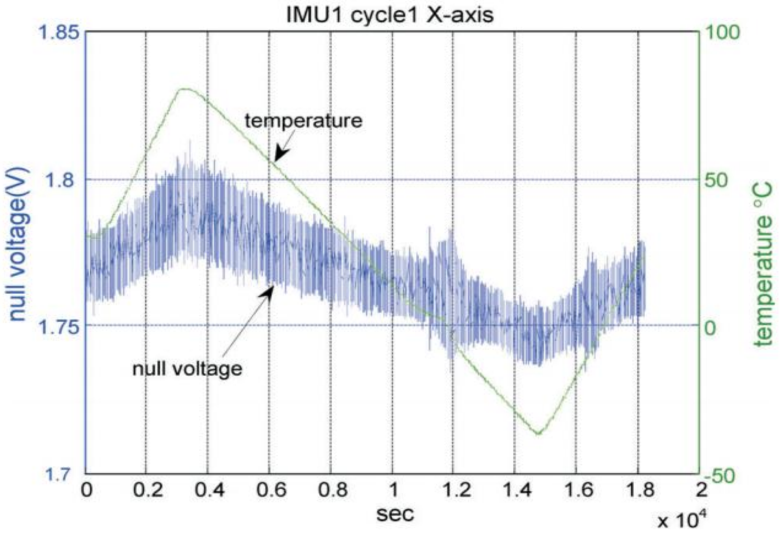
J Calculation in Pre-integration, from Eq4~6

$$
\begin{aligned}
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} &= \left(\mathrm{R}_i^\top \left(\mathbf{p}_j - \mathbf{p}_i - \mathbf{v}_i \Delta t_{ij} - \frac{1}{2}\mathbf{g}\Delta t_{ij}^2\right)\right)^\wedge \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{p}_i} &= -\mathrm{I}_{3\times 1} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{v}_i} &= -\mathrm{R}_i^\top \Delta t_{ij} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} &= 0 \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{p}_j} &= \mathrm{R}_i^\top \mathrm{R}_j \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{v}_j} &= 0 \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \tilde{\mathbf{b}}_a} &= -\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}_i^a} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \tilde{\mathbf{b}}_g} &= -\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}_i^g}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} &= \left(\mathrm{R}_i^\top \left(\mathbf{v}_j - \mathbf{v}_i - \mathbf{g}\Delta t_{ij}\right)\right)^\wedge \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{p}_i} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{v}_i} &= -\mathrm{R}_i^\top \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{p}_j} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{v}_j} &= \mathrm{R}_i^\top \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \tilde{\mathbf{b}}_a} &= -\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}_i^a} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \tilde{\mathbf{b}}_g} &= -\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}_i^g}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathbf{r}_{\Delta \mathrm{R}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} &= -\mathrm{J}_r^{-1}\left(\mathbf{r}_{\Delta \mathrm{R}}(\mathrm{R}_i)\right)\mathrm{R}_j^\top \mathrm{R}_i \\
\frac{\partial \mathbf{r}_{\Delta \mathrm{R}_{ij}}}{\partial \delta \mathbf{p}_i} &= 0 \\
\frac{\partial \mathbf{r}_{\Delta \mathrm{R}_{ij}}}{\partial \delta \mathbf{v}_i} &= 0 \\
\frac{\partial \mathbf{r}_{\Delta \mathrm{R}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} &= \mathrm{J}_r^{-1}\left(\mathbf{r}_{\Delta \mathrm{R}}(\mathrm{R}_j)\right) \\
\frac{\partial \mathbf{r}_{\Delta \mathrm{R}_{ij}}}{\partial \delta \mathbf{p}_j} &= 0 \\
\frac{\partial \mathbf{r}_{\Delta \mathrm{R}_{ij}}}{\partial \delta \mathbf{v}_j} &= 0 \\
\frac{\partial \mathbf{r}_{\Delta \mathrm{R}_{ij}}}{\partial \delta \tilde{\mathbf{b}}_a} &= 0 \\
\frac{\partial \mathbf{r}_{\Delta \mathrm{R}_{ij}}}{\partial \delta \tilde{\mathbf{b}}_g} &= -\mathrm{J}_r^{-1}\left(\mathbf{r}_{\Delta \mathrm{R}_{ij}}(\delta \mathbf{b}_i^g)\right) \mathrm{Exp}\left(\mathbf{r}_{\Delta \mathrm{R}_{ij}}(\delta \mathbf{b}_i^g)\right)^\top \mathrm{J}_r\left(\frac{\partial \Delta \bar{\mathrm{R}}_{ij}}{\partial \mathbf{b}^g}\delta \mathbf{b}_i^g\right)\frac{\partial \Delta \bar{\mathrm{R}}_{ij}}{\partial \mathbf{b}^g}
\end{aligned}
$$

Estimation of IMU non-zero bias



- $_B\boldsymbol{\omega}_t$, $_B\mathbf{a}_t$ consist of non-zero bias $\mathbf{b}^g$, $\mathbf{b}^a$(low pass) + gaussian noise $\boldsymbol{\eta}^g$, $\boldsymbol{\eta}^a$
- Solution:
  - assume bias is known, let $\bar{\mathbf{b}} = \mathbf{0}$, i.e. is part of observation z;
  - estimate $\boldsymbol{b} = \bar{\boldsymbol{b}} + \delta\boldsymbol{b}$, i.e. part of state vector $\mathbf{x}$
  - Previous integrtion stays, except updated with 1st order expansion

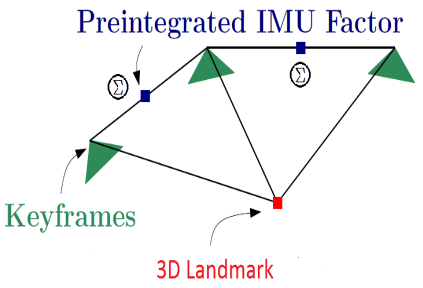
Rewrite (Eq7~9) as:

$$\Delta\tilde{R}_{ij(\boldsymbol{b}_i^g)} = \prod_{k=i}^{j-1} Exp\big((\boldsymbol{w}_k + \mathbf{b}^g + \boldsymbol{\eta}_k{}^g)\Delta t\big)$$
$$= \Delta\tilde{R}_{ij}\big(\bar{\boldsymbol{b}}_i^g\big)Exp\big(\frac{\partial\Delta\bar{R}_{ij}}{\partial\boldsymbol{b}_g}\delta\boldsymbol{b}^g\big) \quad \text{(Eq 13)}$$

$$\Delta\tilde{\boldsymbol{v}}_{ik}\big(\boldsymbol{b}_i^g, \boldsymbol{b}_i^a\big) = \sum_{k=i}^{j-1}\Delta\tilde{R}_{ik}\left({}_{\mathrm{B}}\boldsymbol{a}_k + \mathbf{b}^a + \boldsymbol{\eta}_k{}^a\right)\Delta t$$
$$= \Delta\tilde{\boldsymbol{v}}_{ik}\left(\bar{\boldsymbol{b}}_i^g, \bar{\boldsymbol{b}}_i^a\right) + \left(\frac{\partial\Delta\bar{\boldsymbol{v}}_{ij}}{\partial\boldsymbol{b}^a}\delta\boldsymbol{b}_i^g + \frac{\partial\Delta\bar{\boldsymbol{v}}_{ij}}{\partial\boldsymbol{b}^a}\delta\boldsymbol{b}_i^a\right) \quad \text{(Eq14)}$$

$$\Delta\tilde{\boldsymbol{p}}_{ij}^+(\boldsymbol{b}_i^g, \boldsymbol{b}_i^a) = \sum_{k=i}^{j-1}\frac{3}{2}\Delta\tilde{R}_{ik}({}_{\mathrm{B}}\boldsymbol{a}_k + \boldsymbol{b}^a + \boldsymbol{\eta}_k{}^a)\Delta t^2$$
$$= \Delta\tilde{\boldsymbol{p}}_{ij}^+(\bar{\boldsymbol{b}}_i^g, \bar{\boldsymbol{b}}_i^a) + \left(\frac{\partial\Delta\bar{\boldsymbol{p}}_{ij}^+}{\partial\boldsymbol{b}^a}\delta\boldsymbol{b}_i^g + \frac{\partial\Delta\bar{\boldsymbol{v}}_{ij}}{\partial\boldsymbol{b}^a}\delta\boldsymbol{b}_i^a\right) \quad \text{(Eq15)}$$

Final **x**, **z**, Σ ,J for Pre-Integration Graph SLAM



- Observation preintegration **z** becomes : $\{ \Delta\tilde{R}_{ij}(\overline{\boldsymbol{b}}_i^g),\ \Delta\tilde{\boldsymbol{v}}_{ij}(\overline{\boldsymbol{b}}_i^g, \overline{\boldsymbol{b}}_i^a),\ \Delta\tilde{\boldsymbol{p}}_{ij}^{+}(\overline{\boldsymbol{b}}_i^g, \overline{\boldsymbol{b}}_i^a), \overline{\boldsymbol{b}}_i^g, \overline{\boldsymbol{b}}_i^a \}$, i,j ∈ [adjacent frames]
- State vector becomes: $\{R_t, {}_{\mathrm{w}}\boldsymbol{v}_t, {}_{\mathrm{w}}\boldsymbol{p}_t, \boldsymbol{b}_t^g, \boldsymbol{b}_t^a\}$
- Covariance computed same way as (Eq10), A, B in (Eq11,12) are expanded

$$\overline{A_k} = \begin{bmatrix} A_k & A_k^{bg} & A_k^{ba} \\ \mathbf{0}_{6\times 9} & I_{3\times 3} & I_{3\times 3} \end{bmatrix}, \overline{B_k} = \begin{bmatrix} B_k \\ \mathbf{0}_{6\times 6} \end{bmatrix}$$

$$A_k^{bg} = \begin{bmatrix} -J_r^k \Delta t \\ \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} \end{bmatrix}, \quad A_k^{ba} = \begin{bmatrix} \mathbf{0}_{3\times 3} \\ -\Delta\tilde{R}_{ik}\Delta t \\ \mathbf{0}_{3\times 3} \end{bmatrix}$$

- Jacobian for $\{\Delta R_{ij}, \Delta \mathbf{v}_{ij}, \Delta \mathbf{p}^{+}_{ij}\}$ over R_t, ${}_{\mathrm{w}}\mathbf{v}_t$, ${}_{\mathrm{w}}\mathbf{p}_t$ same as before
- Expand Jacobian for members over $\boldsymbol{b}_t^g$, $\boldsymbol{b}_t^a$, see below
- Yet J can also be computed iteratively

$$\frac{\partial \Delta \bar{\mathrm{R}}_{ij}}{\partial \mathbf{b}^g} = -\sum_{k=i}^{j-1} \left[\Delta\tilde{\mathrm{R}}_{k+1j}(\bar{\mathbf{b}}_i)^{\mathsf{T}}\ \mathrm{J}_r^k\ \Delta t \right]$$

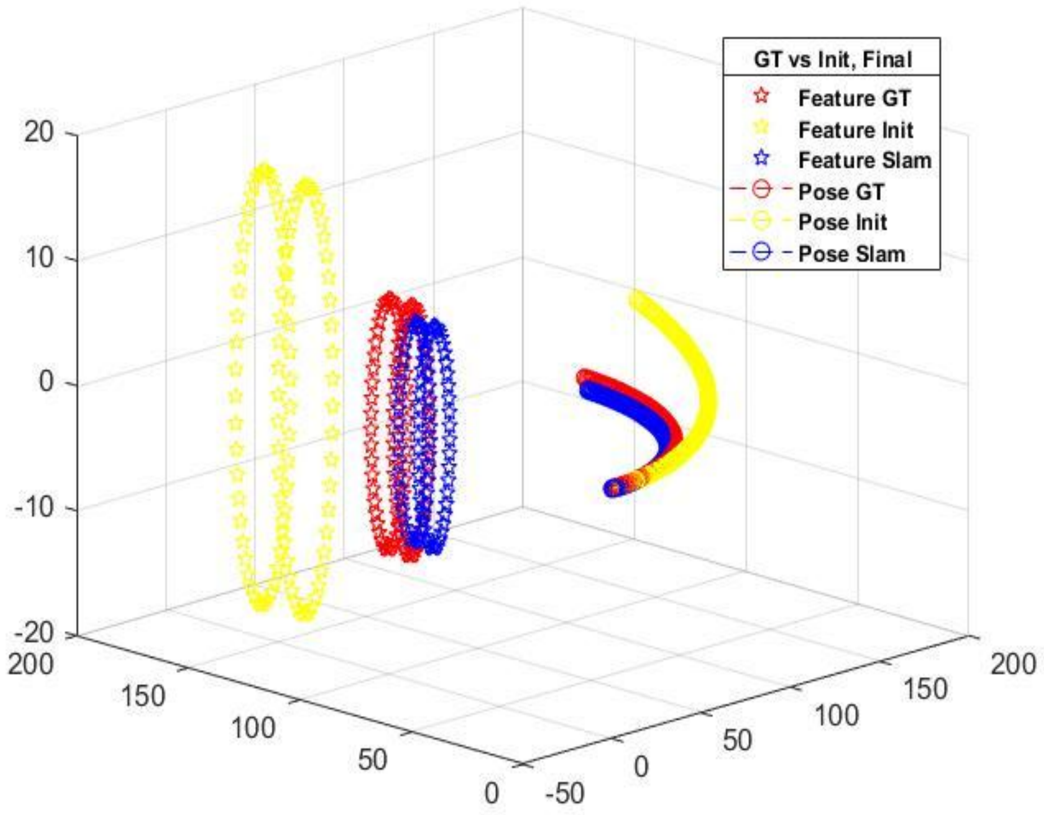
$$\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}^a} = -\sum_{k=i}^{j-1} \Delta\tilde{\mathrm{R}}_{ij}(\bar{\mathbf{b}}_i)\Delta t$$

$$\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}^g} = -\sum_{k=i}^{j-1} \Delta\tilde{\mathrm{R}}_{ij}\left(\tilde{\mathbf{a}}_k - \bar{\mathbf{b}}_i^a\right)^{\wedge} \frac{\partial \Delta \bar{\mathrm{R}}_{ij}}{\partial \mathbf{b}^g} \Delta t$$

$$\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}^a} = -\sum_{k=i}^{j-1} \frac{3}{2} \Delta\tilde{\mathrm{R}}_{ij}(\bar{\mathbf{b}}_i)\Delta t^2$$

Experimental Results

Simulation results for 150 camera poses

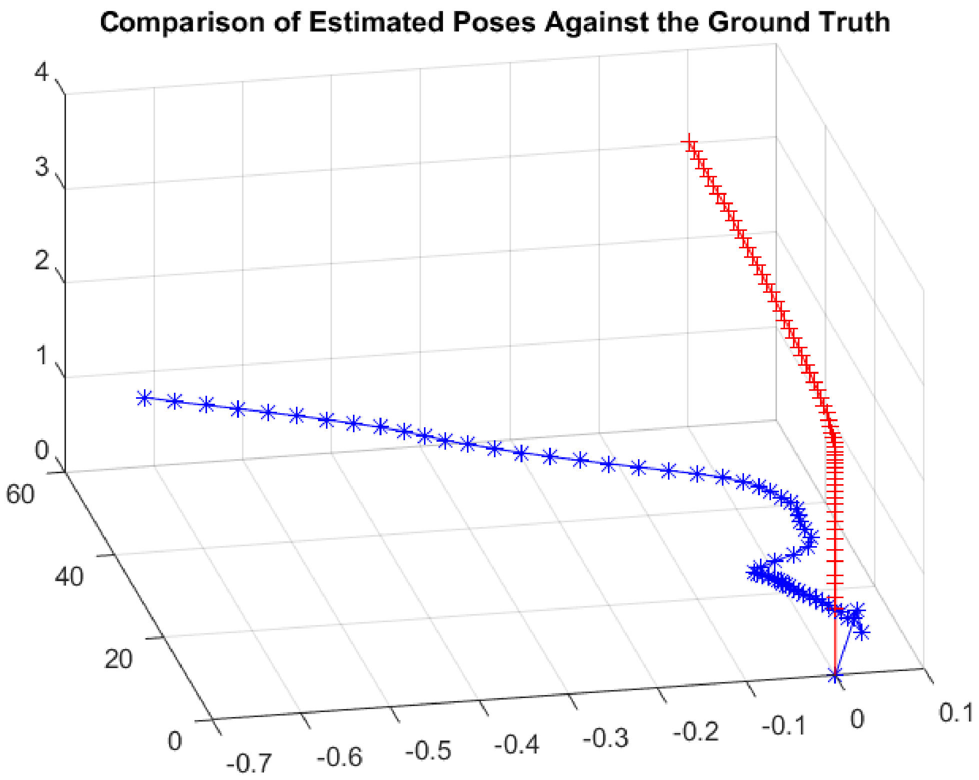


Comparison of Naïve VIN vs Pre-integration

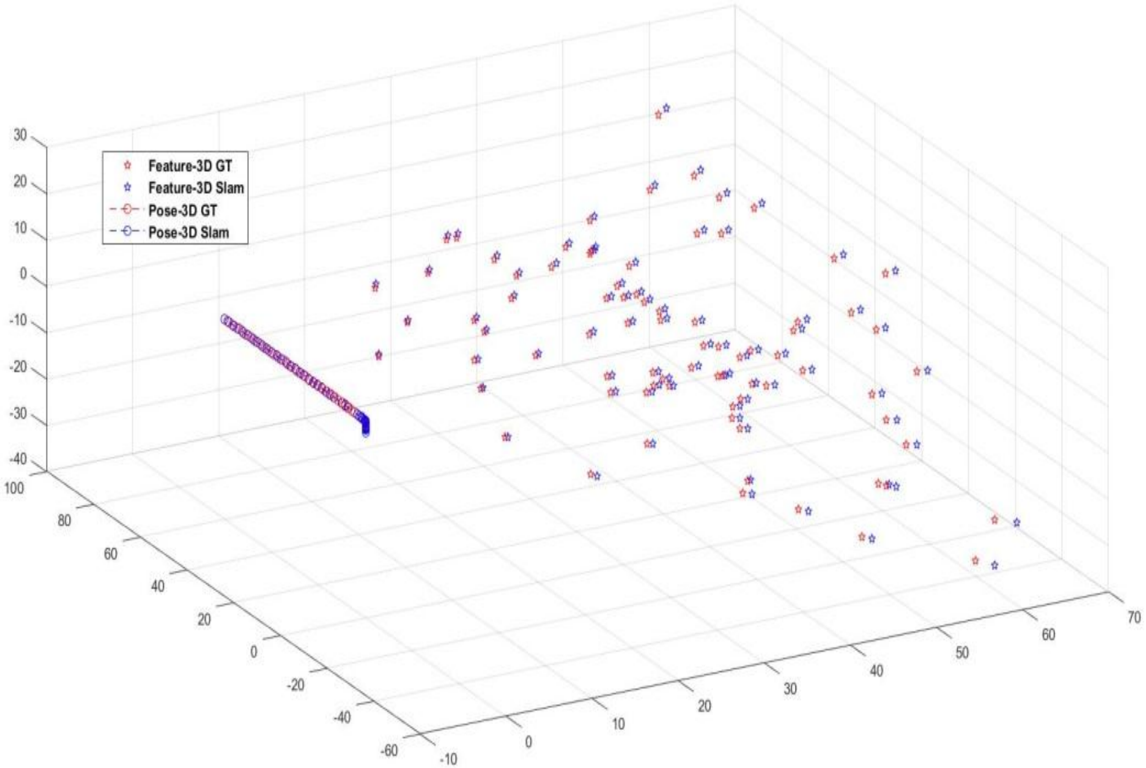
		Naïve VIN	Preintegration
Num camera poses= 10	Num iterations	1000	25
	Total time	1644.2 [sec]	17.1[sec]
	Result/mdx	Not convergent yet, Max(Δ**x**) = 4.76	Yes converged, Max(Δ**x**) = 17.6
Num camera poses= 20	Num iterations	1000	19
	Total time	3403 [sec]	25.3 [sec]
	Result/mdx	Not convergent yet, Max(Δ**x**) = 7.64	Yes converged, Max(Δ**x**) = 3.87
Num camera poses = 150	Num iterations	NIL	17
	Total time	NIL	171.4 [sec]
	Result/error^2	Rank defiicient	Max(Δ**x**) = 11.3

Smoothing versus Incremental changes

Poor initial values lead to errors in smoothing



Solution: Incrementally add new frame



Reference

- [1] T. Lupton and Sukkarieh, “Visual-Inertial-Aided Navigation for High-Dynamic Motion in Built Environments Without Initial Conditions”, *IEEE Transactions on Robotics*, 2012
- [2] C. Forster, et. al, “IMU Preintegration on Manifold for Efficient Visual-Inertial Maximum-a-Posteriori Estimation”, *Robotics: Science and Systems*, doi:10.15607/rss.2015.xi.006
- [3] C. Forster, et. Al., “Supplementary Material to IMU Preintegration on Manifold”, 2015. [Online]. Available : http://rpg.ifi.uzh.ch/docs/RSS15_Forster_Supplementary.pdf