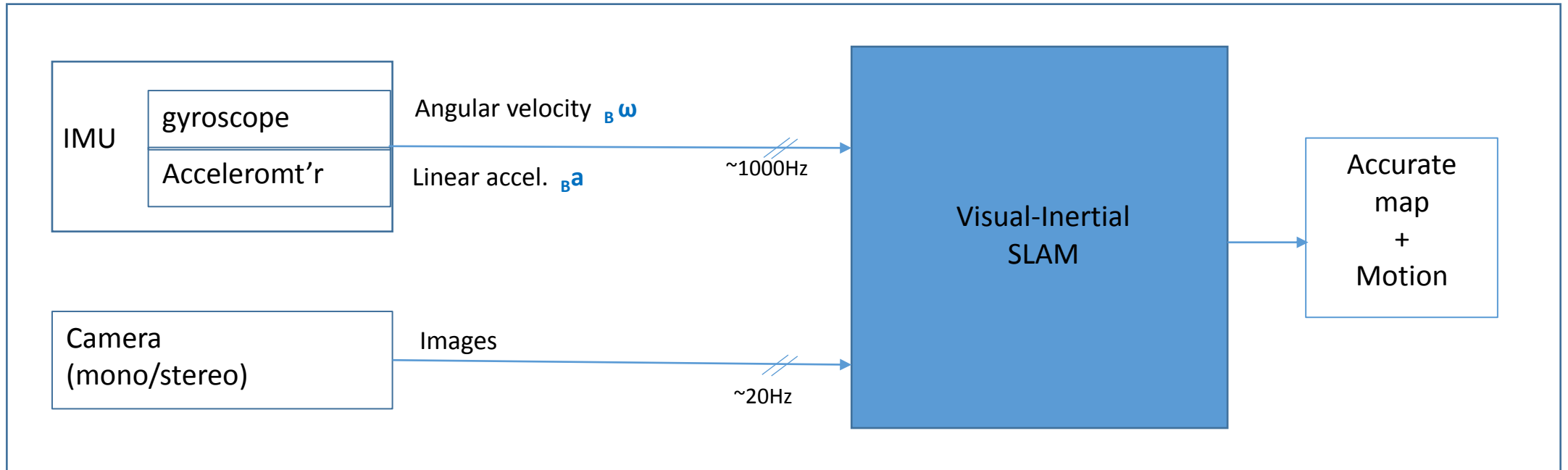
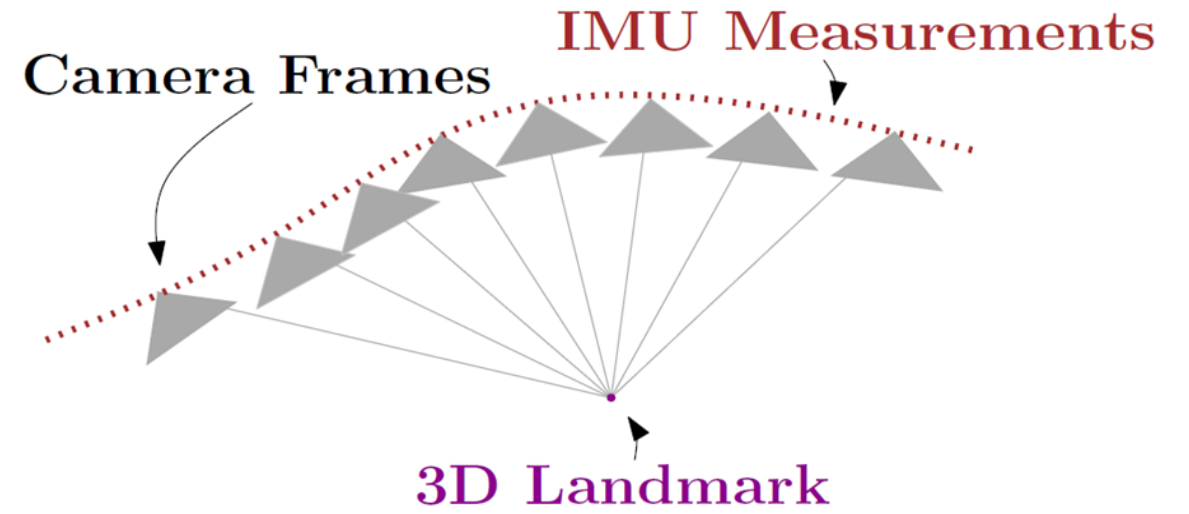
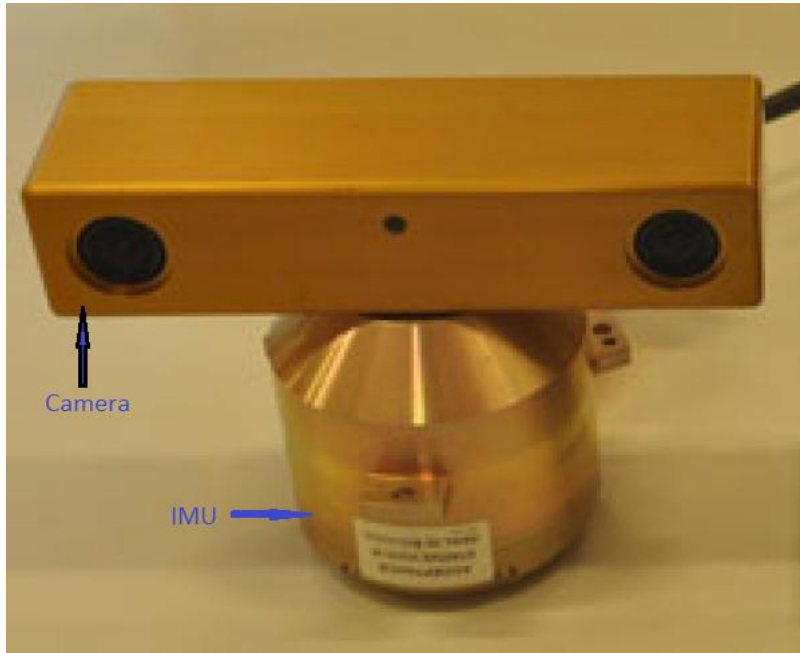


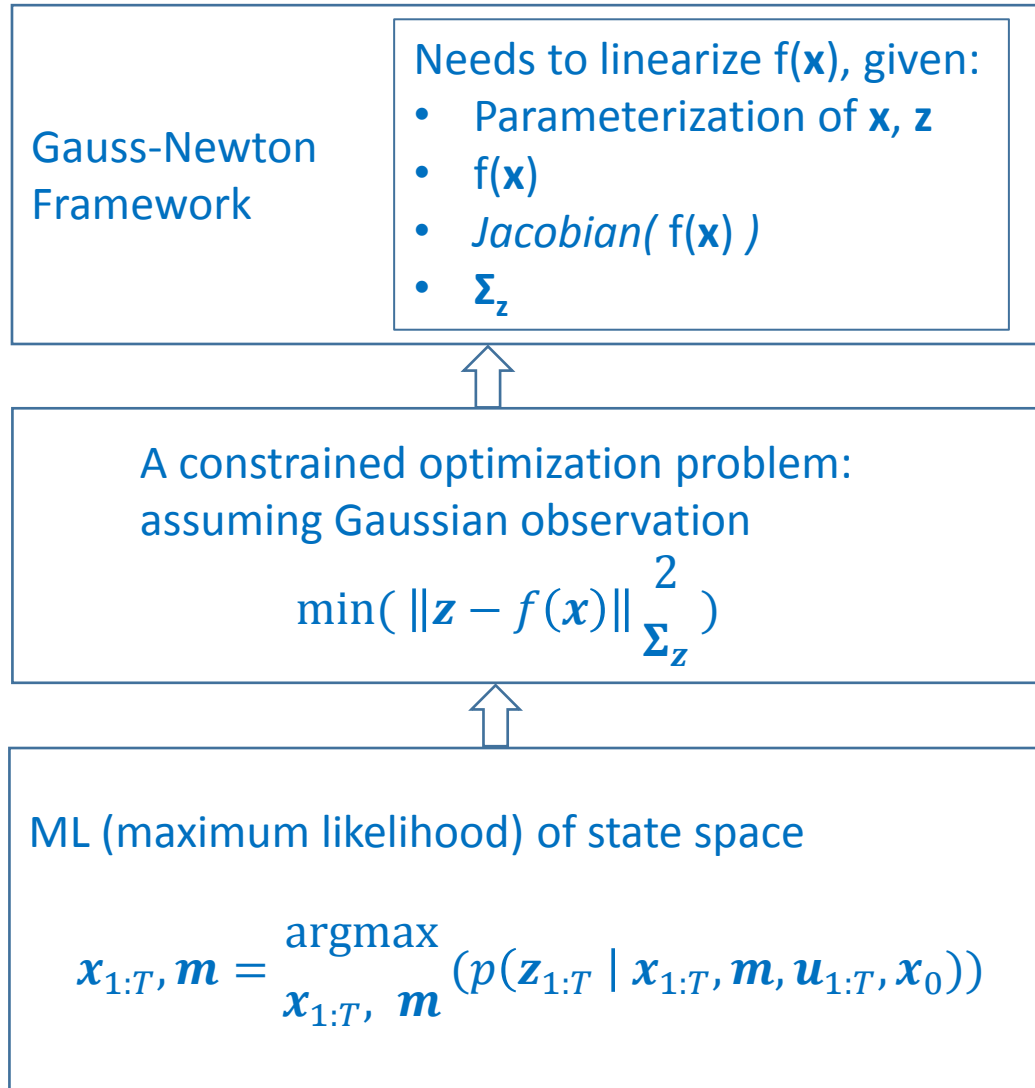
Efficient Visual-Inertial Navigation using IMU Pre- integration

Liyang Liu 2016-05-13

Visual-Inertial SLAM



Graph SLAM revisited



What's good parameterization of \mathbf{x}, \mathbf{z}

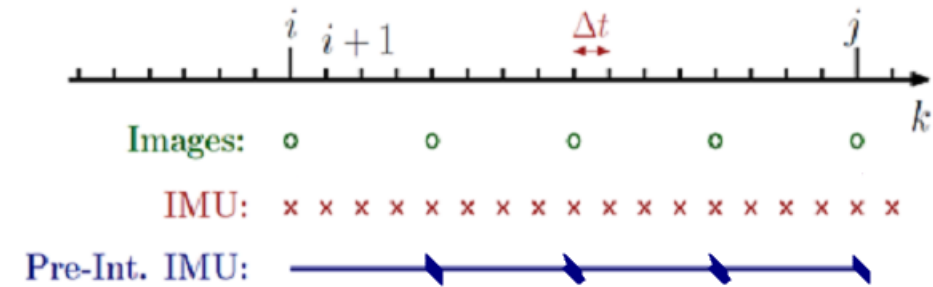


Fig. : Different rates for IMU and camera.

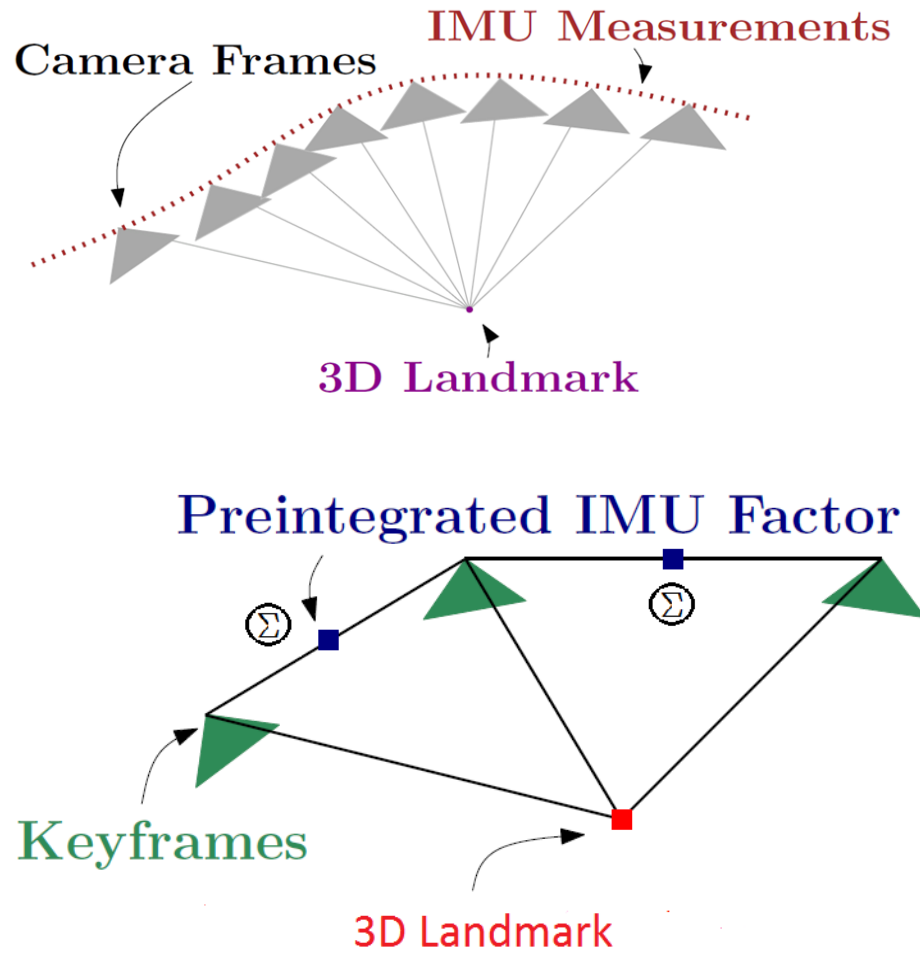
- Naïve parameterization:
 - \mathbf{x} : pose, orientation, velocity $\{R_t, {}_w\mathbf{p}_t, {}_w\mathbf{v}_t\}$ for all IMU samples
 - \mathbf{z} : all IMU data points $\{{}_B\boldsymbol{\omega}_t, {}_B\mathbf{a}_t\}$, $t \in [0 \sim T_{\text{total}}]$ and all feature observations $\{\mathbf{z}_{il}\}$, $i \in I_{\text{total}} \mid \in C_i$
 I_{total} : all frms; C_i : correspondences in frm i ; $_B$ refers body frame
 - Σ_z : angular rate noise: $\boldsymbol{\eta}^g$, linear acc noise: $\boldsymbol{\eta}^a$, image uv noise: $\boldsymbol{\eta}^i$
 - Process model:

$$R(t + \Delta t) = R(t) \text{Exp}({}_B\boldsymbol{\omega}(t) \Delta t)$$

$$\mathbf{v}(t + \Delta t) = {}_w\mathbf{v}(t) + R(t) {}_B\mathbf{a}(t) \Delta t$$

$$\mathbf{p}(t + \Delta t) = {}_w\mathbf{p}(t) + {}_w\mathbf{v}(t) \Delta t + \frac{1}{2} R(t) {}_B\mathbf{a}(t) \Delta t^2$$
- Infeasible: X
 - too much observation data
 - Re-compute integration for each new linearization point change (estimating new rotation R_t).

Pre-integration on IMU -- New form of state space and observations



- \mathbf{x} : pose, orientation, velocity for all key frame times: $\{R_t, {}_w\mathbf{p}_t, {}_w\mathbf{v}_t\}$
- \mathbf{z} : Inertial delta -- integrated IMU data points (over key frame duration) $\{\Delta R_{ij}, \Delta \mathbf{v}_{ij}, \Delta \mathbf{p}_{ij}^+\}$, $i, j \in [\text{adjacent keyframes}]$

$$\Delta R_{ij} = \prod_{k=i}^{j-1} \text{Exp}(\boldsymbol{\omega}_k \Delta t), \quad \text{delta rotation} \quad (\text{Eq 1})$$

$$\Delta \mathbf{v}_{ij} = \sum_{k=i}^{j-1} \Delta R_{ik}^T \mathbf{a}_k \Delta t, \quad \text{delta velocity due to acceleration} \quad (\text{Eq 2})$$

$$\Delta \mathbf{p}_{ij}^+ = \sum_{k=i}^{j-1} \frac{1}{2} \Delta R_{ik}^T \mathbf{a}_k \Delta t^2, \quad \text{corrective term to account for acceleration} \quad (\text{Eq 3})$$
- New observation model $\mathbf{z} = f(\mathbf{x})$:
$$\Delta R_{ij} = R_i^T R_j \quad (\text{Eq 4})$$

$$\Delta \mathbf{v}_{ij} = R_i^T (\mathbf{v}_j - \mathbf{v}_i - \mathbf{g} \Delta t_{ij}) \quad (\text{Eq 5})$$

$$\Delta \mathbf{p}_{ij}^+ = R_i^T (\mathbf{p}_j - \mathbf{p}_i - \mathbf{v}_i \Delta t_{ij} - \frac{1}{2} \mathbf{g} \Delta t_{ij}^2) \quad (\text{Eq 6})$$
- Advantage:
 - Initial orientation is world frame. No explicit initialization stage or large uncertainty prior required.
 - Inertial observation integrated in body frame, K inertial observations treated as a single observation, reduces linearization error, Gauss-Newton more robust.
 - Much reduced state space and observation size, less computation
 - Can be implemented in real-time

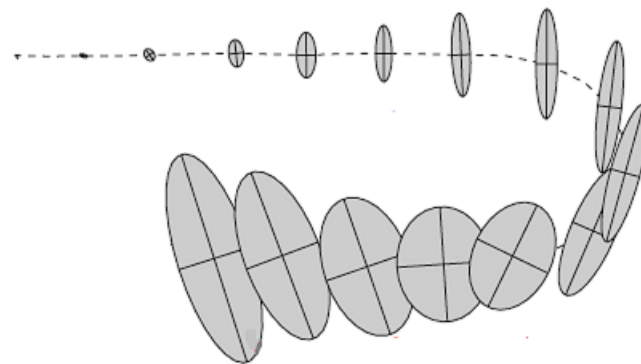
Σ Calculation in Pre-integration

Re-write (EQ1~3) to get pre-integration process model

$$\Delta \tilde{R}_{ij} = \prod_{k=i}^{j-1} \text{Exp}((\mathbf{B} \boldsymbol{\omega}_k + \boldsymbol{\eta}_k^g) \Delta t) \quad (\text{Eq7})$$

$$\Delta \tilde{\mathbf{v}}_{ij} = \sum_{k=i}^{j-1} \Delta \tilde{R}_{ik} (\mathbf{B} \mathbf{a}_k + \boldsymbol{\eta}_k^a) \Delta t \quad (\text{Eq8})$$

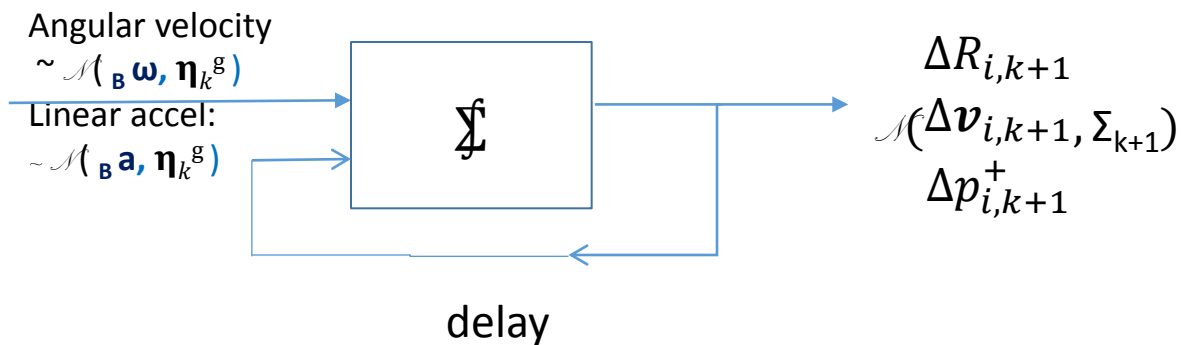
$$\Delta \tilde{\mathbf{p}}_{ij}^+ = \sum_{k=i}^{j-1} \frac{3}{2} \Delta \tilde{R}_{ik} (\mathbf{B} \mathbf{a}_k + \boldsymbol{\eta}_k^a) \Delta t^2 \quad (\text{Eq9})$$



$$\Sigma_{ii} = 0$$

$$\Sigma_{i,k+1} = A_k \Sigma_k A_k^T + B_k \Sigma_\eta B_k^T, \quad (\text{Eq10})$$

Pre-integration in recursive form



$$A_k = \begin{bmatrix} \widetilde{\Delta R_{k,k+1}^T} & 0_{3 \times 3} & 0_{3 \times 3} \\ -\Delta \widetilde{R}_{ik} (\widetilde{\mathbf{a}}_k - \mathbf{b}_i^a)^\top & I_{3 \times 3} & I_{3 \times 3} \\ -\frac{1}{2} \Delta \widetilde{R}_{ik} (\widetilde{\mathbf{a}}_k - \mathbf{b}_i^a)^\top \Delta t^2 & I_{3 \times 3} \Delta t & I_{3 \times 3} \end{bmatrix} \quad (\text{Eq11})$$

$$B_k = \begin{bmatrix} J_r^k \Delta t & 0_{3 \times 3} \\ 0_{3 \times 3} & \Delta \widetilde{R}_{ik} \Delta t \\ 0_{3 \times 3} & \frac{1}{2} \Delta \widetilde{R}_{ik} \Delta t^2 \end{bmatrix} \quad (\text{Eq12})$$

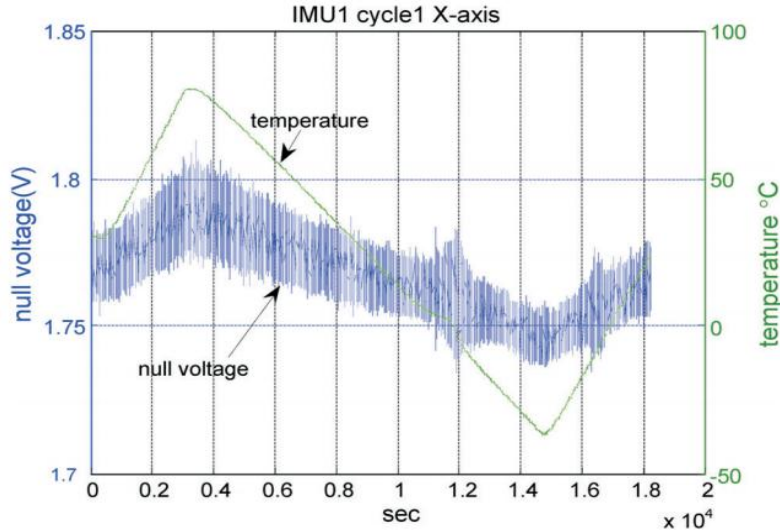
J Calculation in Pre-integration, from Eq4~6

$$\begin{aligned}
 \frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \phi_i} &= \left(\mathbf{R}_i^\top (\mathbf{p}_j - \mathbf{p}_i - \mathbf{v}_i \Delta t_{ij} - \frac{1}{2} \mathbf{g} \Delta t_{ij}^2) \right)^\wedge \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{p}_i} &= -\mathbf{I}_{3 \times 1} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{v}_i} &= -\mathbf{R}_i^\top \Delta t_{ij} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \phi_j} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{p}_j} &= \mathbf{R}_i^\top \mathbf{R}_j \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{v}_j} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_a} &= -\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}_i^a} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_g} &= -\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}_i^g}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \phi_i} &= \left(\mathbf{R}_i^\top (\mathbf{v}_j - \mathbf{v}_i - \mathbf{g} \Delta t_{ij}) \right)^\wedge \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{p}_i} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{v}_i} &= -\mathbf{R}_i^\top \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \phi_j} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{p}_j} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{v}_j} &= \mathbf{R}_i^\top \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_a} &= -\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}_i^a} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_g} &= -\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}_i^g}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \phi_i} &= -\mathbf{J}_r^{-1}(\mathbf{r}_{\Delta \mathbf{R}}(\mathbf{R}_i)) \mathbf{R}_j^\top \mathbf{R}_i \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \mathbf{p}_i} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \mathbf{v}_i} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \phi_j} &= \mathbf{J}_r^{-1}(\mathbf{r}_{\Delta \mathbf{R}}(\mathbf{R}_j)) \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \mathbf{p}_j} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \mathbf{v}_j} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_a} &= \mathbf{0} \\
 \frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_g} &= -\mathbf{J}_r^{-1}(\mathbf{r}_{\Delta \mathbf{R}_{ij}}(\delta \mathbf{b}_i^g)) \text{Exp}(\mathbf{r}_{\Delta \mathbf{R}_{ij}}(\delta \mathbf{b}_i^g))^\top \mathbf{J}_r \left(\frac{\partial \Delta \bar{\mathbf{R}}_{ij}}{\partial \mathbf{b}^g} \delta \mathbf{b}_i^g \right) \frac{\partial \Delta \bar{\mathbf{R}}_{ij}}{\partial \mathbf{b}^g}
 \end{aligned}$$

Estimation of IMU non-zero bias



- $\mathbf{w}_t, \mathbf{a}_t$ consist of non-zero bias $\mathbf{b}^g, \mathbf{b}^a$ (low pass) + gaussian noise $\boldsymbol{\eta}^g, \boldsymbol{\eta}^a$
- Solution:
 - assume bias is known, let $\bar{\mathbf{b}} = \mathbf{0}$, i.e. is part of observation \mathbf{z} ;
 - estimate $\mathbf{b} = \bar{\mathbf{b}} + \delta\mathbf{b}$, i.e. part of state vector \mathbf{x}
 - Previous integration stays, except updated with 1st order expansion

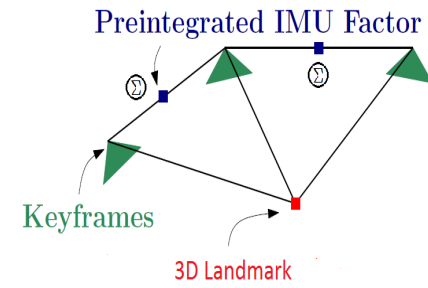
Rewrite (Eq7~9) as:

$$\begin{aligned}\Delta\tilde{R}_{ij}(\mathbf{b}_i^g) &= \prod_{k=i}^{j-1} \text{Exp}((\mathbf{w}_k + \mathbf{b}^g + \boldsymbol{\eta}_k^g)\Delta t) \\ &= \Delta\tilde{R}_{ij}(\bar{\mathbf{b}}_i^g) \text{Exp}\left(\frac{\partial\Delta\tilde{R}_{ij}}{\partial\mathbf{b}_i^g} \delta\mathbf{b}_i^g\right)\end{aligned}\quad (\text{Eq 13})$$

$$\begin{aligned}\Delta\tilde{\mathbf{v}}_{ik}(\mathbf{b}_i^g, \mathbf{b}_i^a) &= \sum_{k=i}^{j-1} \Delta\tilde{R}_{ik} (\mathbf{a}_k + \mathbf{b}^a + \boldsymbol{\eta}_k^a) \Delta t \\ &= \Delta\tilde{\mathbf{v}}_{ik}(\bar{\mathbf{b}}_i^g, \bar{\mathbf{b}}_i^a) + \left(\frac{\partial\Delta\tilde{\mathbf{v}}_{ij}}{\partial\mathbf{b}_i^g} \delta\mathbf{b}_i^g + \frac{\partial\Delta\tilde{\mathbf{v}}_{ij}}{\partial\mathbf{b}_i^a} \delta\mathbf{b}_i^a\right)\end{aligned}\quad (\text{Eq 14})$$

$$\begin{aligned}\Delta\tilde{\mathbf{p}}_{ij}^+(\mathbf{b}_i^g, \mathbf{b}_i^a) &= \sum_{k=i}^{j-1} \frac{3}{2} \Delta\tilde{R}_{ik} (\mathbf{a}_k + \mathbf{b}^a + \boldsymbol{\eta}_k^a) \Delta t^2 \\ &= \Delta\tilde{\mathbf{p}}_{ij}^+(\bar{\mathbf{b}}_i^g, \bar{\mathbf{b}}_i^a) + \left(\frac{\partial\Delta\tilde{\mathbf{p}}_{ij}^+}{\partial\mathbf{b}_i^g} \delta\mathbf{b}_i^g + \frac{\partial\Delta\tilde{\mathbf{p}}_{ij}^+}{\partial\mathbf{b}_i^a} \delta\mathbf{b}_i^a\right)\end{aligned}\quad (\text{Eq 15})$$

Final \mathbf{x} , \mathbf{z} , Σ , \mathbf{J} for Pre-Integration Graph SLAM



- Observation preintegration \mathbf{z} becomes :
 $\{ \Delta \tilde{\mathbf{R}}_{ij}(\bar{\mathbf{b}}_i^g), \Delta \tilde{\mathbf{v}}_{ij}(\bar{\mathbf{b}}_i^g, \bar{\mathbf{b}}_i^a), \Delta \tilde{\mathbf{p}}_{ij}^+(\bar{\mathbf{b}}_i^g, \bar{\mathbf{b}}_i^a), \bar{\mathbf{b}}_i^g, \bar{\mathbf{b}}_i^a \}$,
 $i, j \in [\text{adjacent frames}]$
- State vector becomes:
 $\{R_{t, w} \mathbf{v}_{t, w}, \mathbf{p}_t, \mathbf{b}_t^g, \mathbf{b}_t^a\}$
- Covariance computed same way as (Eq10), \mathbf{A} , \mathbf{B} in (Eq11,12) are expanded

$$\overline{\mathbf{A}}_k = \begin{bmatrix} \mathbf{A}_k & \mathbf{A}_k^{bg} & \mathbf{A}_k^{ba} \\ \mathbf{0}_{6 \times 9} & \mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix}, \overline{\mathbf{B}}_k = \begin{bmatrix} \mathbf{B}_k \\ \mathbf{0}_{6 \times 6} \end{bmatrix}$$

$$\mathbf{A}_k^{bg} = \begin{bmatrix} -\mathbf{J}_r^k \Delta t \\ \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \end{bmatrix}, \mathbf{A}_k^{ba} = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ -\Delta \tilde{\mathbf{R}}_{ik} \Delta t \\ \mathbf{0}_{3 \times 3} \end{bmatrix}$$

- Jacobian for $\{\Delta \mathbf{R}_{ij}, \Delta \mathbf{v}_{ij}, \Delta \mathbf{p}_{ij}^+\}$ over $R_{t, w} \mathbf{v}_{t, w}, \mathbf{p}_t$ same as before
- Expand Jacobian for members over $\mathbf{b}_t^g, \mathbf{b}_t^a$, see below
- Yet \mathbf{J} can also be computed iteratively

$$\frac{\partial \Delta \tilde{\mathbf{R}}_{ij}}{\partial \mathbf{b}^g} = - \sum_{k=i}^{j-1} [\Delta \tilde{\mathbf{R}}_{k+1j}(\bar{\mathbf{b}}_i)^T \mathbf{J}_r^k \Delta t]$$

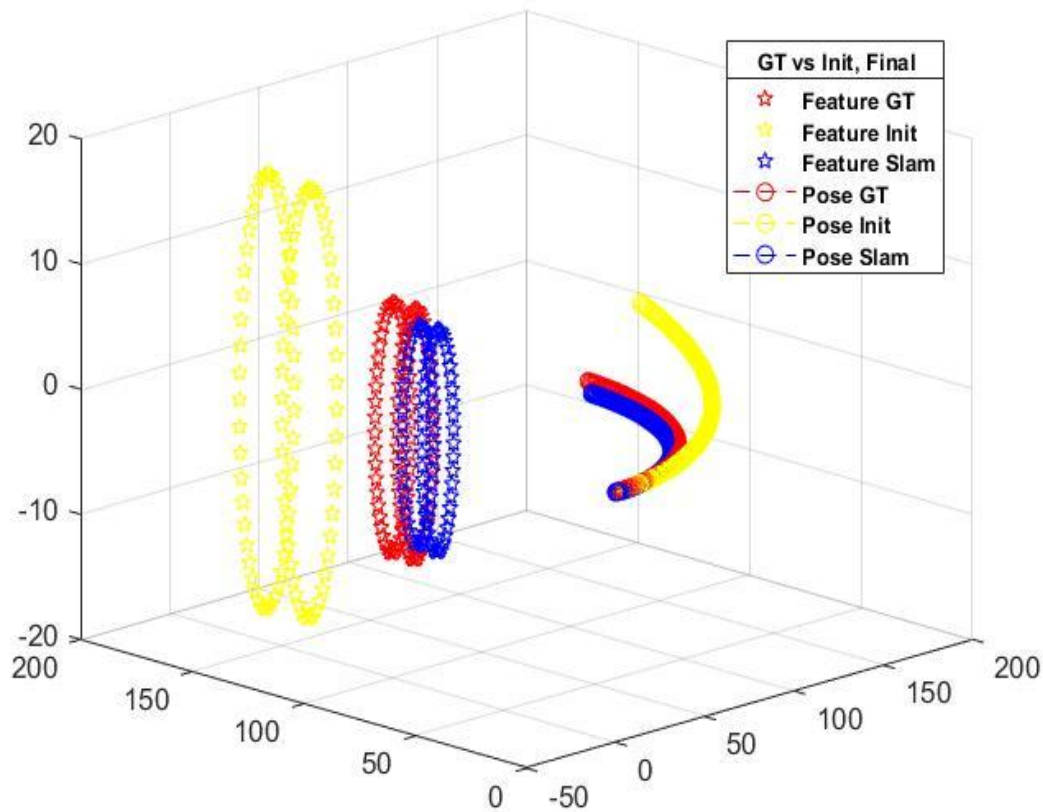
$$\frac{\partial \Delta \tilde{\mathbf{v}}_{ij}}{\partial \mathbf{b}^a} = - \sum_{k=i}^{j-1} \Delta \tilde{\mathbf{R}}_{ij}(\bar{\mathbf{b}}_i) \Delta t$$

$$\frac{\partial \Delta \tilde{\mathbf{v}}_{ij}}{\partial \mathbf{b}^g} = - \sum_{k=i}^{j-1} \Delta \tilde{\mathbf{R}}_{ij}(\tilde{\mathbf{a}}_k - \bar{\mathbf{b}}_i^a)^\wedge \frac{\partial \Delta \tilde{\mathbf{R}}_{ij}}{\partial \mathbf{b}^g} \Delta t$$

$$\frac{\partial \Delta \tilde{\mathbf{p}}_{ij}}{\partial \mathbf{b}^a} = - \sum_{k=i}^{j-1} \frac{3}{2} \Delta \tilde{\mathbf{R}}_{ij}(\bar{\mathbf{b}}_i) \Delta t^2$$

Experimental Results

Simulation results for 150 camera poses

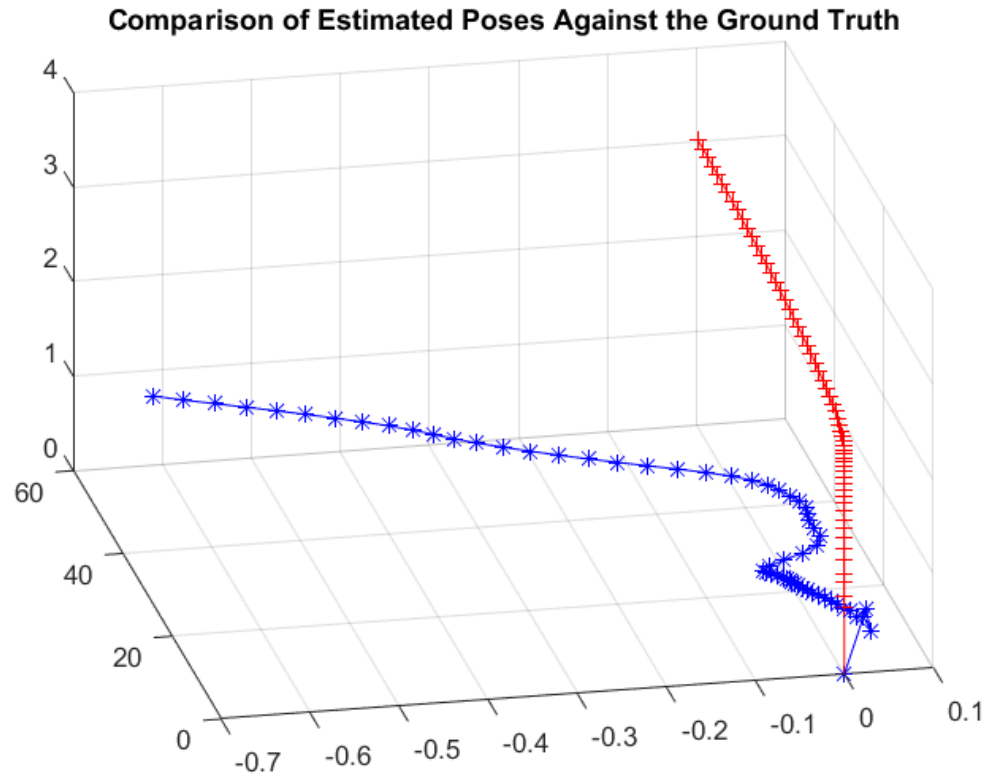


Comparison of Naïve VIN vs Pre-integration

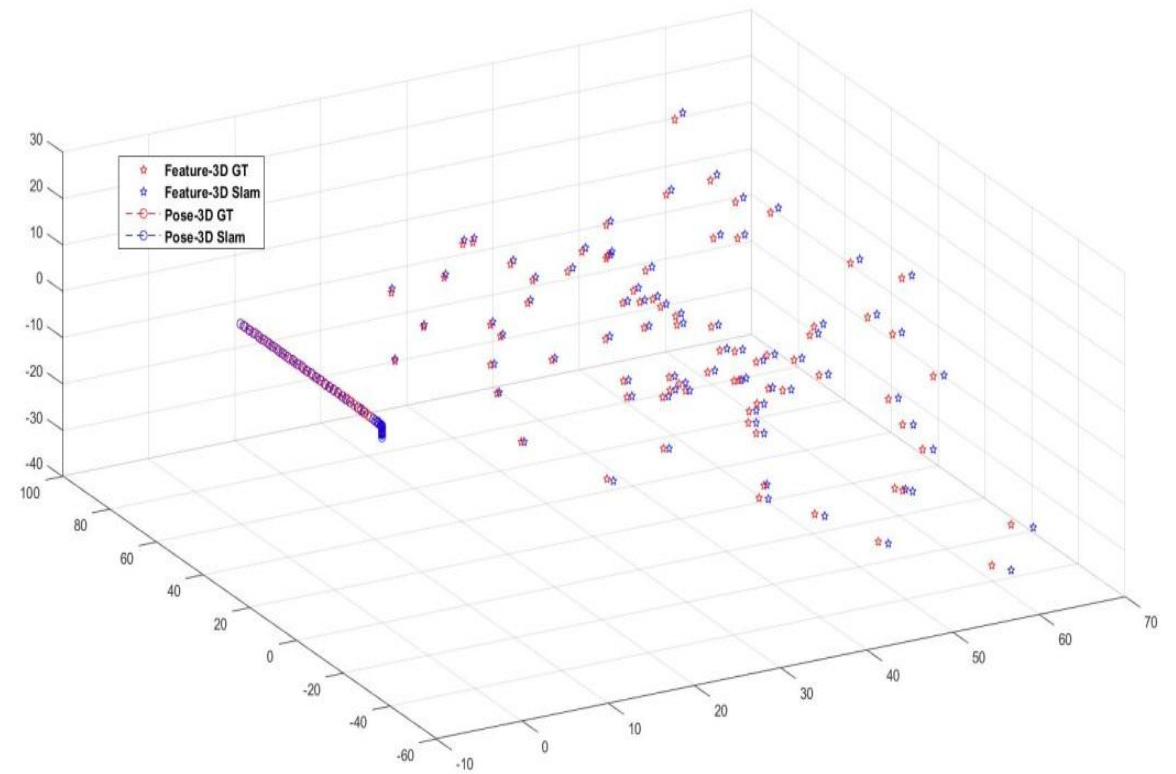
		Naïve VIN	Preintegration
Num camera poses= 10	Num iterations	1000	25
	Total time	1644.2 [sec]	17.1[sec]
	Result/mdx	Not convergent yet, $\text{Max}(\Delta \mathbf{x}) = 4.76$	Yes converged, $\text{Max}(\Delta \mathbf{x}) = 17.6$
Num camera poses= 20	Num iterations	1000	19
	Total time	3403 [sec]	25.3 [sec]
	Result/mdx	Not convergent yet, $\text{Max}(\Delta \mathbf{x}) = 7.64$	Yes converged, $\text{Max}(\Delta \mathbf{x}) = 3.87$
Num camera poses = 150	Num iterations	NIL	17
	Total time	NIL	171.4 [sec]
	Result/error ²	Rank deficient	$\text{Max}(\Delta \mathbf{x}) = 11.3$

Smoothing versus Incremental changes

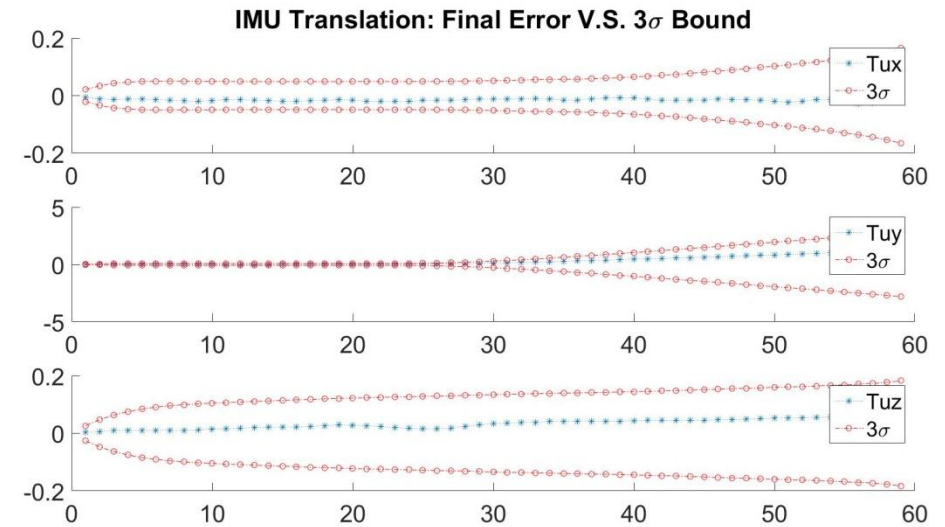
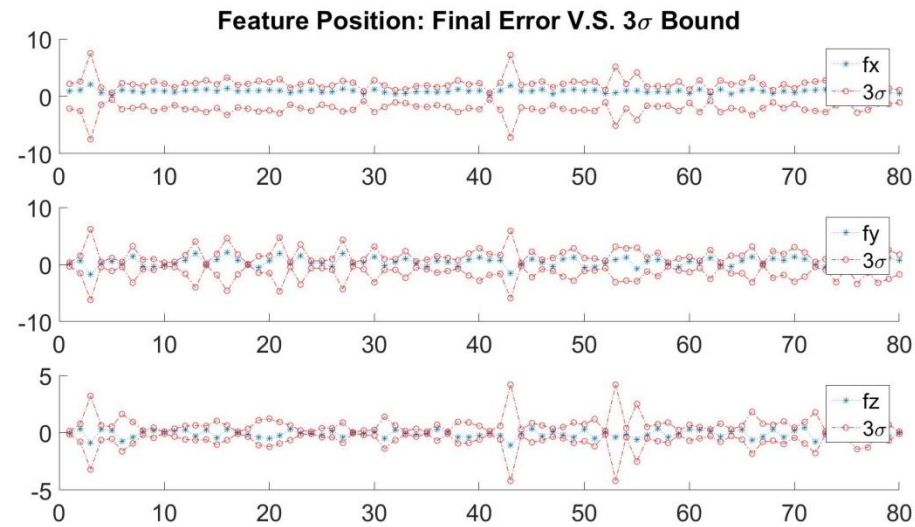
Poor initial values lead to errors in smoothing



Solution: Incrementally add new frame



Error analysis



Reference

- [1] T. Lupton and Sukkarieh, “Visual-Inertial-Aided Navigation for High-Dynamic Motion in Built Environments Without Initial Conditions”, *IEEE Transactions on Robotics*, 2012
- [2] C. Forster, et. al, “IMU Preintegration on Manifold for Efficient Visual-Inertial Maximum-a-Posteriori Estimation”, *Robotics: Science and Systems*, doi:10.15607/rss.2015.xi.006
- [3] C. Forster, et. Al., “Supplementary Material to IMU Preintegration on Manifold”, 2015. [Online]. Available : http://rpg.ifi.uzh.ch/docs/RSS15_Forster_Supplementary.pdf