# Efficient Visual-Inertial Navigation using IMU Pre-integration
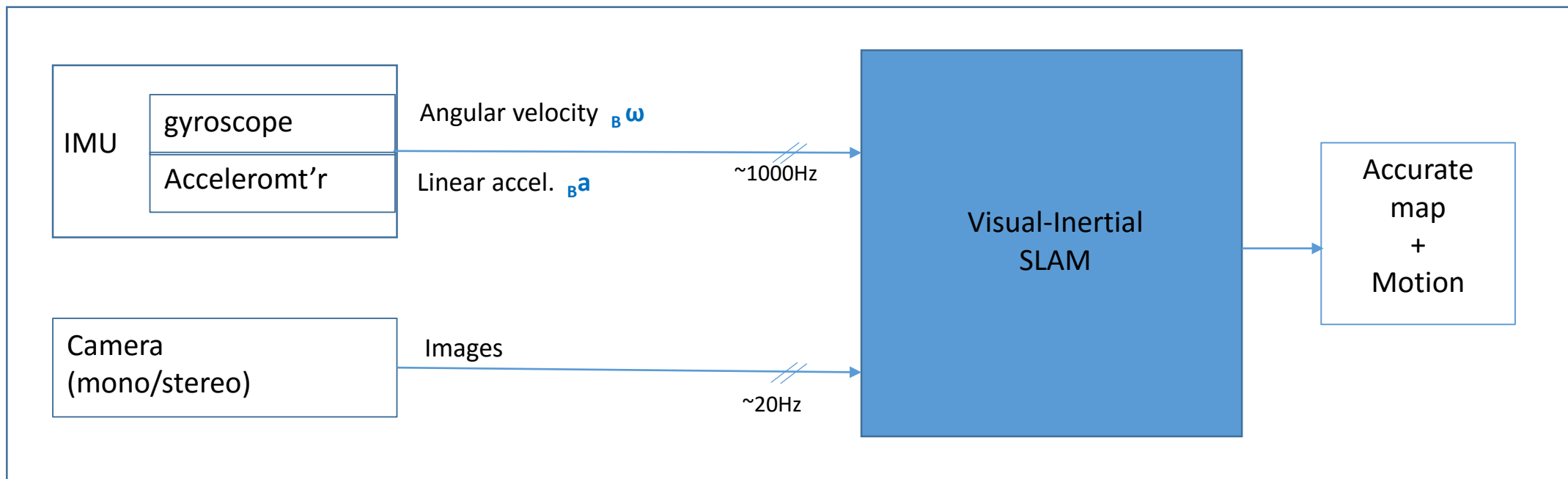
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# Visual-Inertial SLAM



Camera

IMU

IMU Measurements

Camera Frames

3D Landmark

| IMU | | Angular velocity $_B\omega$ |
|---|---|---|
| | gyroscope | |
| | Acceleromt'r | Linear accel. $_B a$ |

~1000Hz

Camera (mono/stereo)

Images

~20Hz

Visual-Inertial SLAM

Accurate map + Motion

# Graph SLAM revisited

| Gauss-Newton Framework | Needs to linearize f(**x**), given:<br>• Parameterization of **x**, **z**<br>• f(**x**)<br>• *Jacobian( f(**x**) )*<br>• $\Sigma_z$ |
|---|---|

⇑

A constrained optimization problem: assuming Gaussian observation

$$\min( \| z - f(x) \|^2_{\Sigma_z} )$$

⇑

ML (maximum likelihood) of state space

$$x_{1:T}, m = \underset{x_{1:T},\ m}{\operatorname{argmax}} (p(z_{1:T} \mid x_{1:T}, m, u_{1:T}, x_0))$$

# What's good parameterization of x, z
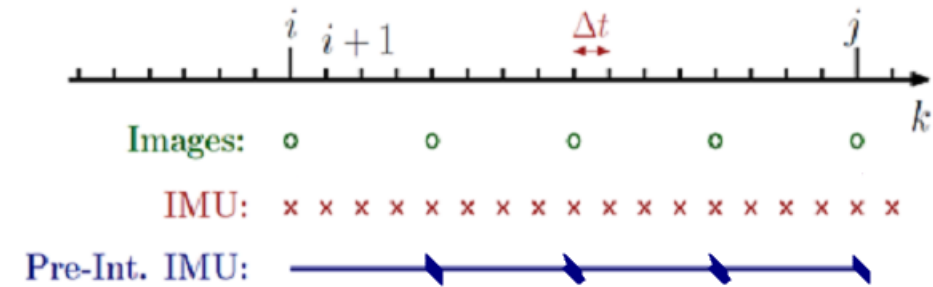


Fig. : Different rates for IMU and camera.

- Naïve parameterization:
  - **x**: pose, orientation, velocity $\{R_t,\ _wp_t,\ _wv_t\}$ for all IMU samples
  - **z**: all IMU data points $\{_B\omega_t,\ _Ba_t\}$, $t \in [0 \sim T_{total}]$ and all feature observations $\{z_{il}\}$, $i \in I_{total}$ $l \in C_i$
    $I_{total}$: all frms;  $C_i$ : correspondences in frm i;  $_B$ refers body frame
  - $\Sigma_z$ : angular rate noise: $\eta^g$, linear acc noise: $\eta^a$, image uv noise: $\eta^i$
  - Process model:
    $R(\ t + \Delta t\ ) = R(t)\ Exp(_B\omega(t)\ \Delta t\ )$
    $v(\ t + \Delta t\ ) = {}_wv(t) + R(t)\ _Ba(t)\ \Delta t$
    $p(\ t + \Delta t\ ) = {}_wp(t) + {}_wv(t)\ \Delta t + \frac{1}{2}R(t)\ _Ba(t)\ \Delta t^2$

- Infeasible: <span style="color:red">X</span>
  - too much observation data
  - Re-compute integration for each new linearization point change (estimating new rotation $R_t$ ).

# Pre-integration on IMU -- New form of state space and observations



**Camera Frames**
**IMU Measurements**

**3D Landmark**

**Preintegrated IMU Factor**

**Keyframes**

**3D Landmark**
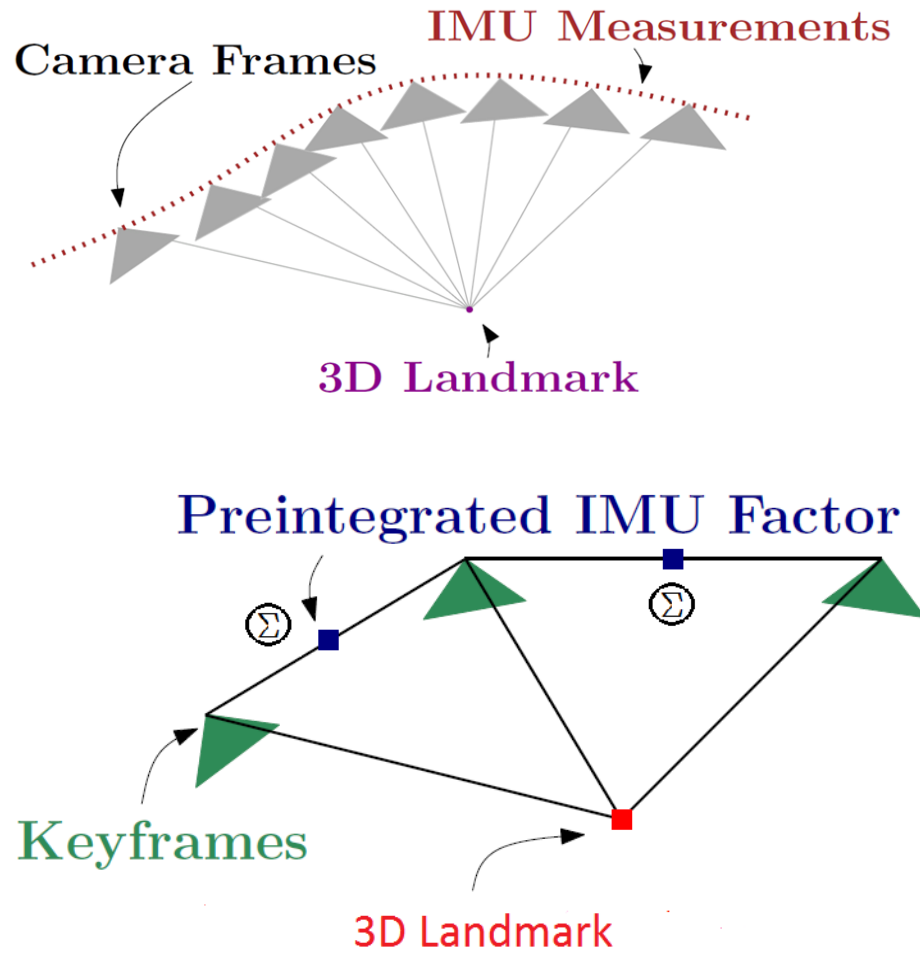
- **x**: pose, orientation, velocity for all key frame times: $\{R_t, {}_w\mathbf{p}_t, {}_w\mathbf{v}_t\}$

- **z**: Inertial delta -- integrated IMU data points (over key frame duration) $\{\Delta R_{ij}, \Delta \mathbf{v}_{ij}, \Delta \mathbf{p}^+_{ij}\}$, $i,j \in$ [adjacent keyframes]

$$\Delta R_{ij} = \prod_{k=i}^{j-1} Exp(\boldsymbol{\omega}_k \Delta t), \qquad \textit{delta rotation} \qquad \text{(Eq 1)}$$

$$\Delta \boldsymbol{v}_{ij} = \sum_{k=i}^{j-1} \Delta R^{ik} {}_{\mathbf{B}}\boldsymbol{a}_k \Delta t, \qquad \textit{delta velocity due to acceleration} \qquad \text{(Eq 2)}$$

$$p^+_{ij} = \sum_{k=i}^{j-1} \frac{3}{2} \Delta R^{ik} {}_{\mathbf{B}}\boldsymbol{a}_k \Delta t^2, \qquad \textit{corrective term to account for acceleration} \quad \text{(Eq3)}$$

- New observation model z = f(x):

$$\Delta R_{ij} = R_i^T R_j \qquad\qquad\qquad\qquad\qquad\qquad \text{(Eq 4)}$$

$$\Delta \boldsymbol{v}_i = R_i^T (\boldsymbol{v}_j - \boldsymbol{v}_i - \mathbf{g}\Delta t_{ij}) \qquad\qquad\qquad \text{(Eq 5)}$$

$$\Delta p^+_{ij} = R_i^T (\boldsymbol{p}_j - \boldsymbol{p}_i - \boldsymbol{v}_i \Delta t_{ij} - \tfrac{1}{2}\boldsymbol{g}\Delta t_{ij}^2) \qquad \text{(Eq 6)}$$

- Advantage:

  - Initial orientation is world frame. No explicit initialization stage or large uncertainty prior required.

  - Inertial observation integrated in body frame, K inertial observations treated as a single observation, reduces linearization error, Gauss-Newton more robust.

  - Much reduced state space and observation size, less computation
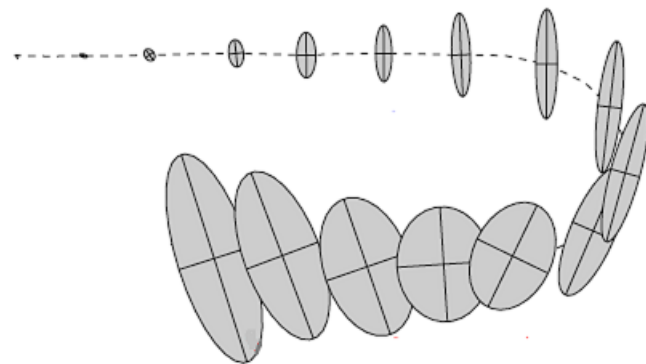
  - Can be implemented in real-time

# Σ Calculation in Pre-integration

Re-write (EQ1~3) to get pre-integration process model

$$\Delta\tilde{R}_{ij} = \prod_{k=i}^{j-1} Exp\big((_B\boldsymbol{\omega}_k + \boldsymbol{\eta}_k{}^g)\Delta t\big) \qquad\qquad \text{(Eq7)}$$

$$\Delta\tilde{\boldsymbol{v}}_{ij} = \sum_{k=i}^{j-1} \Delta\tilde{R}_{ik}(_B\boldsymbol{a}_k + \boldsymbol{\eta}_k{}^a)\Delta t \qquad\qquad \text{(Eq8)}$$
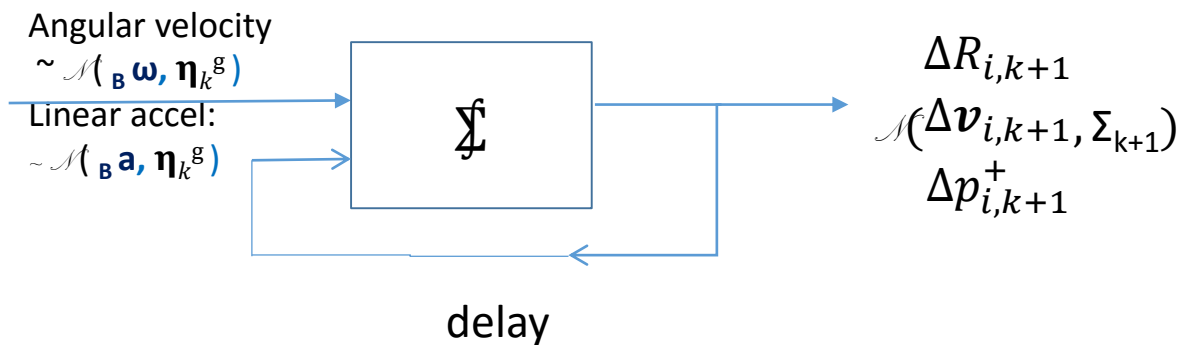
$$\Delta\tilde{\boldsymbol{p}}_{ij}^+ = \sum_{k=i}^{j-1} \tfrac{3}{2}\Delta\tilde{R}_{ik}(_B\boldsymbol{a}_k + \boldsymbol{\eta}_k{}^a)\Delta t^2 \qquad\qquad \text{(Eq9)}$$

$$\Sigma_{ii}=0$$

$$\Sigma_{i,k+1} = A_k\Sigma_k A_k^T + B_k\Sigma_\eta B_k^T, \qquad\qquad \text{(Eq10)}$$

$$A_k = \begin{bmatrix} \widetilde{\Delta R}_{k,k+1}^{\,T} & 0_{3\times3} & 0_{3\times3} \\ -\Delta\widetilde{R_{ik}}(\widetilde{\boldsymbol{a}}_k - \boldsymbol{b}_i^a)^\lnot & I_{3\times3} & I_{3\times3} \\ -\tfrac{1}{2}\Delta\widetilde{R_{ik}}(\widetilde{\boldsymbol{a}}_k - \boldsymbol{b}_i^a)^\lnot \Delta t^2 & I_{3\times3}\Delta t & I_{3\times3} \end{bmatrix} \quad \text{(Eq11)}$$

$$B_k = \begin{bmatrix} J_r^k\Delta t & 0_{3\times3} \\ 0_{3\times3} & \Delta\widetilde{R_{ik}}\Delta t \\ 0_{3\times3} & \tfrac{1}{2}\Delta\widetilde{R_{ik}}\Delta t^2 \end{bmatrix} \qquad\qquad \text{(Eq12)}$$

**Pre-integration in recursive form**

Angular velocity
$\sim \mathcal{N}(\,_B\boldsymbol{\omega}, \boldsymbol{\eta}_k{}^g\,)$

Linear accel:
$\sim \mathcal{N}(\,_B\mathbf{a}, \boldsymbol{\eta}_k{}^g\,)$

$\Sigma$

$\Delta R_{i,k+1}$
$\mathcal{N}(\Delta\boldsymbol{v}_{i,k+1}, \Sigma_{k+1})$
$\Delta p_{i,k+1}^+$

delay

# J Calculation in Pre-integration, from Eq4~6

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} = \left( \mathbf{R}_i^\mathsf{T} \left( \mathbf{p}_j - \mathbf{p}_i - \mathbf{v}_i \Delta t_{ij} - \tfrac{1}{2} \mathbf{g} \Delta t_{ij}^2 \right) \right)^\wedge$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{p}_i} = -\mathbf{I}_{3 \times 1}$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{v}_i} = -\mathbf{R}_i^\mathsf{T} \Delta t_{ij}$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{p}_j} = \mathbf{R}_i^\mathsf{T} \mathbf{R}_j$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{v}_j} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_a} = -\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}_i^a}$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_g} = -\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}_i^g}$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} = \left( \mathbf{R}_i^\mathsf{T} \left( \mathbf{v}_j - \mathbf{v}_i - \mathbf{g} \Delta t_{ij} \right) \right)^\wedge$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{p}_i} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{v}_i} = -\mathbf{R}_i^\mathsf{T}$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{p}_j} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{v}_j} = \mathbf{R}_i^\mathsf{T}$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_a} = -\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}_i^a}$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_g} = -\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}_i^g}$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} = -\mathbf{J}_r^{-1}(\mathbf{r}_{\Delta \mathbf{R}}(\mathbf{R}_i)) \mathbf{R}_j^\mathsf{T} \mathbf{R}_i$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \mathbf{p}_i} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \mathbf{v}_i} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} = \mathbf{J}_r^{-1}(\mathbf{r}_{\Delta \mathbf{R}}(\mathbf{R}_j))$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \mathbf{p}_j} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \delta \mathbf{v}_j} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_a} = 0$$

$$\frac{\partial \mathbf{r}_{\Delta \mathbf{R}_{ij}}}{\partial \tilde{\delta} \mathbf{b}_g} = -\mathbf{J}_r^{-1}\left( \mathbf{r}_{\Delta \mathbf{R}_{ij}}(\delta \mathbf{b}_i^g) \right) \mathrm{Exp}\left( \mathbf{r}_{\Delta \mathbf{R}_{ij}}(\delta \mathbf{b}_i^g) \right)^\mathsf{T} \mathbf{J}_r \left( \frac{\partial \Delta \bar{\mathbf{R}}_{ij}}{\partial \mathbf{b}^g} \delta \mathbf{b}_i^g \right) \frac{\partial \Delta \bar{\mathbf{R}}_{ij}}{\partial \mathbf{b}^g}$$

# Estimation of IMU non-zero bias



IMU1 cycle1 X-axis
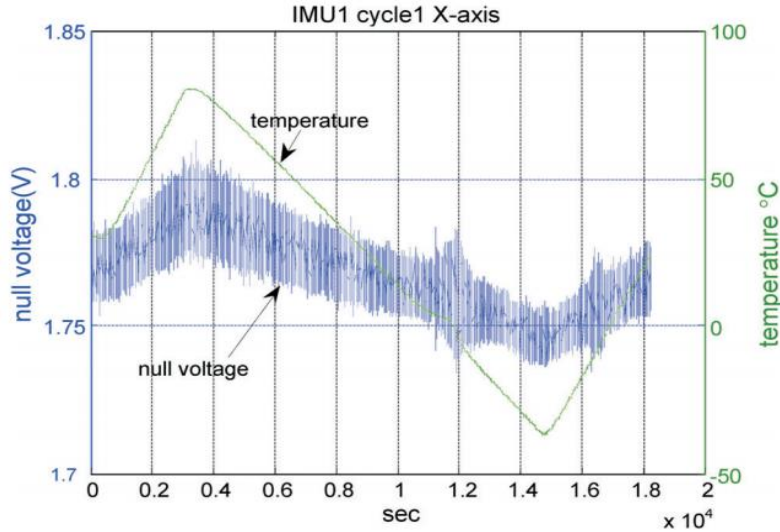
- $_B\boldsymbol{\omega}_t$, $_B\boldsymbol{a}_t$ consist of non-zero bias $\mathbf{b}^g$, $\mathbf{b}^a$(low pass) + gaussian noise $\boldsymbol{\eta}^g$, $\boldsymbol{\eta}^a$

- Solution:
  - assume bias is known, let $\bar{\mathbf{b}} = \mathbf{0}$, i.e. is part of observation z;
  - estimate $\boldsymbol{b} = \bar{\boldsymbol{b}} + \delta\boldsymbol{b}$ , i.e. part of state vector $\mathbf{x}$
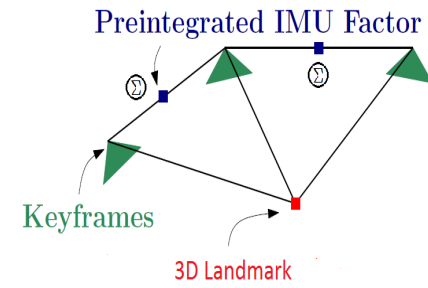  - Previous integrtion stays, except updated with 1st order expansion

Rewrite (Eq7~9) as:

$$\Delta\tilde{R}_{ij(\boldsymbol{b}_i^g)} = \prod_{k=i}^{j-1} Exp\big((\boldsymbol{w}_k + \mathbf{b}^g + \boldsymbol{\eta}_k{}^g)\Delta t\big)$$

$$= \Delta\tilde{R}_{ij}(\bar{\boldsymbol{b}}_i^g)Exp\big(\frac{\partial\Delta\bar{R}_{ij}}{\partial\boldsymbol{b}_g}\delta\boldsymbol{b}^g\big) \qquad (Eq\ 13)$$

$$\Delta\tilde{\boldsymbol{v}}_{ik}(\boldsymbol{b}_i^g, \boldsymbol{b}_i^a) = \sum_{k=i}^{j-1} \Delta\tilde{R}_{ik}\,(_B\boldsymbol{a}_k + \mathbf{b}^a + \boldsymbol{\eta}_k{}^a)\,\Delta t$$

$$= \Delta\tilde{\boldsymbol{v}}_{ik}(\bar{\boldsymbol{b}}_i^g, \bar{\boldsymbol{b}}_i^a) + \big(\frac{\partial\Delta\bar{\boldsymbol{v}}_{ij}}{\partial\boldsymbol{b}^a}\delta\boldsymbol{b}_i^g + \frac{\partial\Delta\bar{\boldsymbol{v}}_{ij}}{\partial\boldsymbol{b}^a}\delta\boldsymbol{b}_i^a\big) \quad (Eq14)$$

$$\Delta\tilde{\boldsymbol{p}}_{ij}^+(\boldsymbol{b}_i^g, \boldsymbol{b}_i^a) = \sum_{k=i}^{j-1}\frac{3}{2}\Delta\tilde{R}_{ik}(_B\boldsymbol{a}_k + \boldsymbol{b}^a + \boldsymbol{\eta}_k{}^a)\Delta t^2$$

$$= \Delta\tilde{\boldsymbol{p}}_{ij}^+(\bar{\boldsymbol{b}}_i^g, \bar{\boldsymbol{b}}_i^a) + \big(\frac{\partial\Delta\bar{\boldsymbol{p}}_{ij}^+}{\partial\boldsymbol{b}^a}\delta\boldsymbol{b}_i^g + \frac{\partial\Delta\bar{\boldsymbol{v}}_{ij}}{\partial\boldsymbol{b}^a}\delta\boldsymbol{b}_i^a\big) \quad (Eq15)$$

# Final $\mathbf{x}, \mathbf{z}, \sum, J$ for Pre-Integration Graph SLAM

- Observation preintegration $\mathbf{z}$ becomes :
  $\{ \Delta\tilde{R}_{ij}(\overline{\boldsymbol{b}}_i^g), \ \Delta\tilde{\boldsymbol{v}}_{ij}(\overline{\boldsymbol{b}}_i^g, \overline{\boldsymbol{b}}_i^a), \ \Delta\tilde{\boldsymbol{p}}_{ij}^+(\overline{\boldsymbol{b}}_i^g, \overline{\boldsymbol{b}}_i^a), \overline{\boldsymbol{b}}_i^g, \overline{\boldsymbol{b}}_i^a \}$,
  i,j ∈ [adjacent frames]

- State vector becomes:
  $\{R_t, {}_w\boldsymbol{v}_t, {}_w\boldsymbol{p}_t, \boldsymbol{b}_t^g, \boldsymbol{b}_t^a\}$

- Covariance computed same way as (Eq10), A, B in (Eq11,12) are expanded

$$\overline{A}_k = \begin{bmatrix} A_k & A_k^{bg} & A_k^{ba} \\ \mathbf{0}_{6\times 9} & I_{3\times 3} & I_{3\times 3} \end{bmatrix}, \overline{B}_k = \begin{bmatrix} B_k \\ \mathbf{0}_{6\times 6} \end{bmatrix}$$

$$A_k^{bg} = \begin{bmatrix} -J_r^k \Delta t \\ \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} \end{bmatrix}, \quad A_k^{ba} = \begin{bmatrix} \mathbf{0}_{3\times 3} \\ -\Delta\tilde{R}_{ik}\Delta t \\ \mathbf{0}_{3\times 3} \end{bmatrix}$$

- Jacobian for $\{\Delta R_{ij}, \Delta \mathbf{v}_{ij}, \Delta \mathbf{p}^+_{ij} \}$ over $R_t, {}_w\mathbf{v}_t, {}_w\mathbf{p}_t$ same as before

- Expand Jacobian for members over $\boldsymbol{b}_t^g, \boldsymbol{b}_t^a$ , see below

- Yet J can also be computed iteratively

$$\frac{\partial \Delta\bar{R}_{ij}}{\partial \mathbf{b}^g} = -\sum_{k=i}^{j-1} \left[ \Delta\tilde{R}_{k+1j}(\bar{\mathbf{b}}_i)^\top \, J_r^k \, \Delta t \right]$$
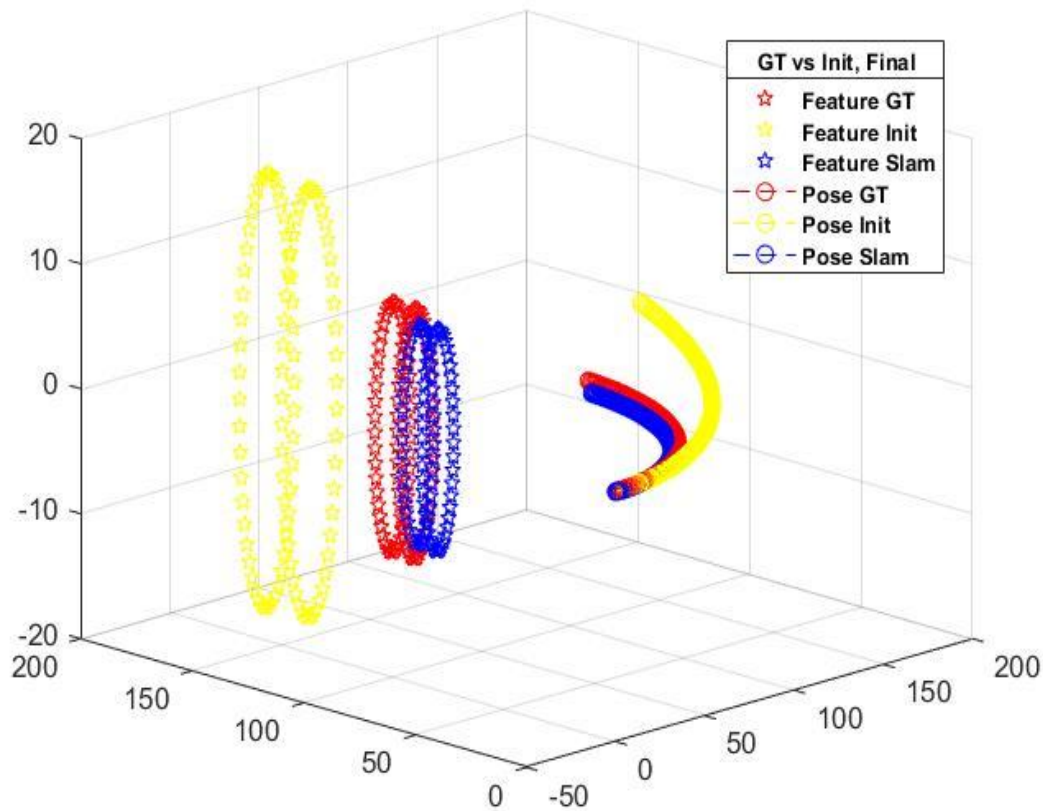
$$\frac{\partial \Delta\bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}^a} = -\sum_{k=i}^{j-1} \Delta\tilde{R}_{ij}(\bar{\mathbf{b}}_i)\Delta t$$

$$\frac{\partial \Delta\bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}^g} = -\sum_{k=i}^{j-1} \Delta\tilde{R}_{ij} \left(\tilde{\mathbf{a}}_k - \bar{\mathbf{b}}_i^a\right)^\wedge \frac{\partial \Delta\bar{R}_{ij}}{\partial \mathbf{b}^g}\Delta t$$

$$\frac{\partial \Delta\bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}^a} = -\sum_{k=i}^{j-1} \frac{3}{2}\Delta\tilde{R}_{ij}(\bar{\mathbf{b}}_i)\Delta t^2$$

# Experimental Results

## Simulation results for 150 camera poses



## Comparison of Naïve VIN vs Pre-integration

| | | Naïve VIN | Preintegration |
|---|---|---|---|
| Num camera poses= 10 | Num iterations | 1000 | 25 |
| | Total time | 1644.2 [sec] | 17.1[sec] |
| | Result/mdx | Not convergent yet, Max($\Delta\mathbf{x}$) = 4.76 | Yes converged, Max($\Delta\mathbf{x}$) = 17.6 |
| Num camera poses= 20 | Num iterations | 1000 | 19 |
| | Total time | 3403 [sec] | 25.3 [sec] |
| | Result/mdx | Not convergent yet, Max($\Delta\mathbf{x}$) = 7.64 | Yes converged, Max($\Delta\mathbf{x}$) = 3.87 |
| Num camera poses = 150 | Num iterations | NIL | 17 |
| | Total time | NIL | 171.4 [sec] |
| | Result/error$^2$ | Rank defiicient | Max($\Delta\mathbf{x}$) = 11.3 |

# Reference

- [1] T. Lupton and Sukkarieh, "Visual-Inertial-Aided Navigation for High-Dynamic Motion in Built Environments Without Initial Conditions", *IEEE Transactions on Robotics*, 2012

- [2] C. Forster, et. al, "IMU Preintegration on Manifold for Efficient Visual-Inertial Maximum-a-Posteriori Estimation", *Robotics: Science and Systems*, doi:10.15607/rss.2015.xi.006

- [3] C. Forster, et. Al., "Supplementary Material to IMU Preintegration on Manifold", 2015. [Online]. Available : http://rpg.ifi.uzh.ch/docs/RSS15_Forster_Supplementary.pdf