ME03: Nonuniformly Charged Hollow Sphere with Gauss [50+5 points].

Consider a hollow charged sphere with inner radius R_1 and outer radius R_2, $R_2 > R_1$ (see figure). The solid part of the hollow sphere (i.e., between R_1 and R_2) is characterized by a nonuniform volume charge distribution with density $\rho(r) = \rho_0(r - R_1)/R_1$ for $r \in [R_1, R_2]$. Assuming the electrostatic field has only a radial component that depends only on r (you do NOT have to show any symmetry argument), use Gauss' theorem to find the field at any point in space. [50 points]

Very hard question if you aim at $100\,\%$. Try it only if you think you have already got more than 100 points in all the other questions. Assume R_2 gets closer and closer to R_1, eventually resulting in a very thin shell. In fact, assume that $R_2 = R_1 + \Delta r$, where Δr is a very small quantity ($\Delta r \ll R_1$). Find an equivalent *surface* charge density σ_{eq} that accurately represents the field when Δr is small. *Hint: You can effectively assume the field becomes discontinuous, with a jump between the inside and outside of the thin shell, for a small Δr. Then, Taylor the total* **volume** *charge contained within the thin shell with respect to a small Δr* [5 points].
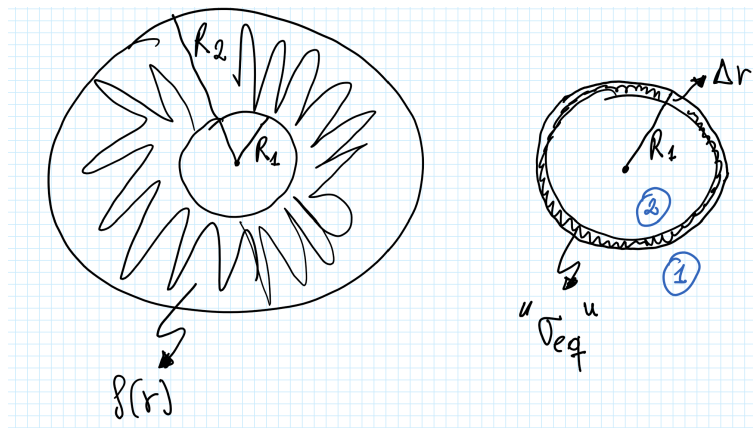

Figure M3