HA04 1.1) Let Σ = surface for $\Omega = \bar{B}(r)$, the sphere w/ radius r

1° $0 < r \leq R_1$: $\oint_\Sigma \vec{E} \cdot \hat{n} \, dA = \frac{q}{\varepsilon_0} = 0$

Note $\vec{E}(r) = E(r) \cdot \hat{r}$ on Σ

$\therefore E(r) \cdot 4\pi r^2 = 0 \Rightarrow E(r) = 0 \Rightarrow \vec{E}(r) = 0$

2° $R_1 < r \leq R_2$: $E(r) = \frac{q(\Omega)}{4\pi\varepsilon_0 r^2}$

$q(\Omega) = \oint_\Omega \rho \, dv = \int_{R_1}^{r} \rho \cdot 4\pi \tilde{r}^2 d\tilde{r} = 4\pi \int_{R_1}^{r} \frac{\rho_0 R_2}{\tilde{r}} \tilde{r}^2 d\tilde{r}$

$= 4\pi \rho_0 R_2 (r^2 - R_1^2) \cdot \frac{1}{2} = 2\pi \rho_0 R_2 (r^2 - R_1^2)$

$E(r) = \frac{1}{4\pi\varepsilon_0 r^2} 2\pi \rho_0 R_2 (r^2 - R_1^2)$

$= \frac{r^2 - R_1^2}{2\varepsilon_0 r^2} \rho_0 R_2$

$\vec{E}(r) = \frac{r^2 - R_1^2}{2\varepsilon_0 r^2} \rho_0 R_2 \hat{r}$

3° $r \geq R_2$: $q(\Omega) = \int_{R_1}^{R_2} \rho 4\pi \tilde{r}^2 d\tilde{r} = 2\pi \rho_0 R_2 (R_2^2 - R_1^2)$

$E(r) = \frac{1}{4\pi\varepsilon_0 r^2} 2\pi \rho_0 R_2 (R_2^2 - R_1^2)$

$= \frac{R_2^2 - R_1^2}{2\varepsilon_0 r^2} \rho_0 \cdot R_2 \Rightarrow \vec{E}(r) = \frac{R_2^2 - R_1^2}{2\varepsilon_0 r^2} \rho_0 R_2 \hat{r}$

2) 1° $r \geq R_2$: sps $V(r) = 0$ at infinity $r \to \infty$

$V(r) = -\int \vec{E}(\tilde{r}) \, d\vec{\tilde{r}} = -\int \frac{R_2^2 - R_1^2}{2\varepsilon_0} \rho_0 R_2 \cdot \frac{1}{\tilde{r}^2} d\tilde{r}$

$= \frac{R_2^2 - R_1^2}{2\varepsilon_0} \rho_0 R_2 \cdot \frac{1}{r} + C$

Since $\lim_{r\to\infty} V(r) = 0$, $C = 0$

$\therefore V(r) = \frac{R_2^2 - R_1^2}{2\varepsilon_0} \rho_0 R_2 \cdot \frac{1}{r}$

2° $R_1 \leq r \leq R_2$

$V(r) = -\int \vec{E}(\tilde{r}) \, d\vec{\tilde{r}} = -\int \frac{\rho_0 R_2}{2\varepsilon_0} - \frac{R_1^2}{2\varepsilon_0} \rho_0 R_2 \cdot \frac{1}{\tilde{r}^2} \tilde{r}$

$= -\left(\frac{\rho_0 R_2}{2\varepsilon_0} r + \frac{R_1^2}{2\varepsilon_0} \rho_0 R_2 \cdot \frac{1}{r} \right) + C_1$

Since $V(R_2) = \frac{R_2^2 - R_1^2}{2\varepsilon_0} \rho_0 \cancel{R_2} \frac{\cancel{1}}{\cancel{R_2}} = -\left(\frac{\rho_0}{2\varepsilon_0} R_2^2 + \frac{\rho_0}{2\varepsilon_0} R_1^2 \right) + C_1$

$= -\frac{R_2^2 + R_1^2}{2\varepsilon_0} \rho_0 + C_1$

$\Rightarrow C_1 = \frac{\rho_0}{\varepsilon_0} R_2^2 \Rightarrow V(r) = -\frac{\rho_0 R_2}{2\varepsilon_0} \left(r + \frac{R_1^2}{r} \right) + \frac{\rho_0}{\varepsilon_0} R_2^2$

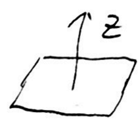
3° $0 < r \leq R_1$, $V(r) = -\int \vec{E}(\tilde{r}) \, d\vec{\tilde{r}} = 0 + C_2$

$C_2 = V(R_1) = -\frac{\rho_0 R_2}{2\varepsilon_0} (R_1 + R_1) + \frac{\rho_0}{\varepsilon_0} R_2^2$

$= -\frac{\rho_0}{\varepsilon_0} R_2 R_1 + \frac{\rho_0}{\varepsilon_0} R_2^2 = \frac{\rho_0}{\varepsilon_0} R_2 (R_2 - R_1)$

$\therefore V(r) = \frac{\rho_0}{\varepsilon_0} R_2 (R_2 - R_1)$

For an infinite plane (uniformly charged w/ density σ), the field at height h is given by $\frac{\varepsilon_0}{2}\sigma\hat{z}$



$\therefore$ Pontential $V(h) = -\int \frac{\varepsilon_0}{2}\sigma dz = -\frac{\varepsilon_0}{2}\sigma h + C$

Assume $\lim_{h\to 0} V(h) = 0 \Rightarrow C = 0$

Thus for the horizontal line, $U_{ei} = \oint V(d)\,dq = V(d)\int dq$

$= \frac{\varepsilon_0}{4}\sigma d \cdot q$

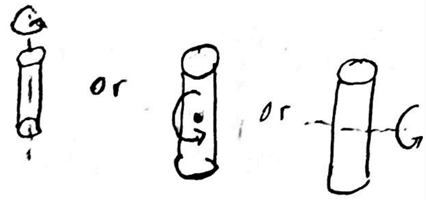
$dq = \lambda dl$, due to the plane field only

for the rotated line, $U_{ef} = \oint V(h)\,dq = \int_0^L -\frac{\varepsilon_0}{2}\sigma \cdot (d+l)\lambda \cdot dl$

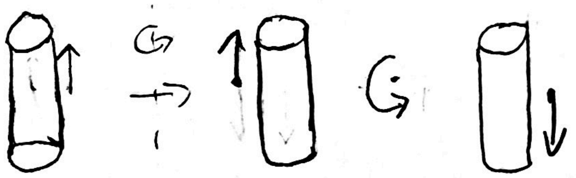
$= -\frac{\varepsilon_0}{2}\sigma\lambda \int_0^L (d+l)\,dl = -\frac{\varepsilon_0}{2}\sigma \cdot \frac{q}{L}\cdot\left(dL + \frac{L^2}{2}\right)$

$W = U_{ei} - U_{ef} = +\frac{\varepsilon_0}{4}\sigma q L$

4.3 1) Note cylinders are the same if rotated π for

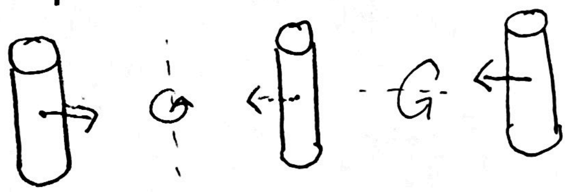


1° Consider E_z on $\hat{z}$



$\Rightarrow E_z = -E_z \Rightarrow E_z = 0$

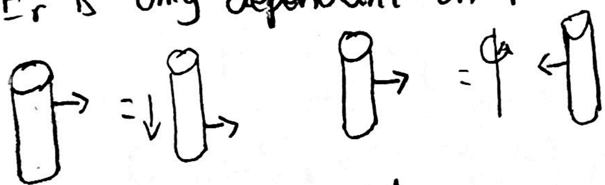
2° E_φ on $\hat{\varphi}$



$\Rightarrow E_\varphi = -E_\varphi \Rightarrow E_\varphi = 0$

3° Since immune to translation in $\hat{z}$ & rotation around z (in $\hat{\varphi}$)

E_r is only dependant on r



Let Ω be a height h cylinder with radius r, and same axis as the ones given

1° $0<r<R_1$, $\oint_\Sigma \vec{E}\cdot\hat{u}\,dA = \frac{q_\Omega}{\varepsilon_0} = 0$, where Σ is surface of Ω

Note since $\vec{E} = E_r(r)\hat{r}$, $\vec{E}\cdot\hat{u} = 0$ for the top & bottom circle

$\vec{E}\cdot\hat{u} = E_r$ for the horizontal surface

$$E_r(r)\,2\pi r h = \frac{q_\Omega}{\varepsilon_0} \Rightarrow E(r) = \frac{q_\Omega}{2\pi r h \varepsilon_0} = 0 \Rightarrow \vec{E}(r) = 0$$

2° $R_1 < r < R_2$, $E_r(r) = \frac{q_\Omega}{2\pi r h \varepsilon_0} = \frac{\cancel{2\pi} R_1 \cancel{h}(-\sigma)}{\cancel{2\pi} r \cancel{h} \varepsilon_0} = -\frac{R_1\sigma}{\varepsilon_0}\cdot\frac{1}{r} \Rightarrow \vec{E}(r) = -\frac{\sigma R_1}{\varepsilon_0 r}\hat{r}$

3° $r > R_2$, $E_r(r) = \frac{q_\Omega}{2\pi r h \varepsilon_0} = \frac{+\sigma 2\pi R_2 h - 2\pi R_1 h \sigma}{2\pi r h \varepsilon_0} = \frac{(R_2 - R_1)\sigma}{r\varepsilon_0} \Rightarrow \vec{E}(r) = \frac{(R_2-R_1)\sigma}{\varepsilon_0 r}\hat{r}$

2° $V_{out} - V_{in} = -\int_{R_1}^{R_2} \vec{E}(r)\cdot\hat{r}\,dr = -\int_{R_1}^{R_2} -\frac{\sigma R_1}{\varepsilon_0 r}\,dr = +\frac{\sigma R_1}{\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r} = +\frac{\sigma R_1}{\varepsilon_0}\cdot(\ln R_2 - \ln R_1)$