3.1 Note that a sphere is symetric if rotate in θ & φ of a spherical system with O = center of shell

$\therefore$ The $\hat\theta$ & $\hat\varphi$ component must be 0

and $\vec{E}$ only depends on r

$\therefore\ \vec{E} = E_r(r)\,\hat{r}$, rotate by $\Delta\theta = \pi$ then $\Delta\varphi = \pi$ will result in oppsite vector at same point for both direction. By semetry, they must be 0 since same

Consider any point (θ, φ, r) on the surface of the sphere $\vec{B}(r) = \Omega$

Let Σ' be the surface of Ω

By Gauss' Thm, $\oiint_{\Sigma'} \vec{E}\cdot\hat{u}\,dA = \frac{q_\Omega}{\varepsilon_0}$ ($q_\Omega = \iiint \rho\,dv$)

$$= \oiint_{\Sigma'} E_r(r)\,\hat{r}\cdot\hat{r}\,dA = E_r(r)\cdot\oiint_{\Sigma'} dA = E_r(r)\cdot 4\pi r^2$$

($E_r(r)$) const on Σ', where r does not charge

$$\therefore\ E_r(r) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_\Omega}{r^2}$$

while $r > R$, $q_\Omega = q$, $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{r}$

$r < R$, $q_\Omega = 0$, $\vec{E} = 0$

3.2. S° be surface of sphere $\bar{B}(r) := \Omega$ $q_\Omega = \frac{4}{3}\pi r^3 \rho_0$

1) $\oiint_S \vec{E}\cdot\hat{u}\, dA = \frac{q_\Omega}{\varepsilon_0} \Rightarrow \vec{E} = \frac{q_\Omega}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{4}{3}\cancel{\pi} r^3 \rho_0 \cdot \frac{1}{\cancel{4\pi}\varepsilon_0 \cancel{r^2}}\hat{r}$

$\oiiint_\Omega \rho\, dv$

$= \frac{\rho_0 r}{3\varepsilon_0}\cdot\hat{r}$

2) 1° $r \in [0, R_0]$, $q_\Omega = \oiiint_\Omega \rho\, dv = 4\pi\int_0^r \tilde{r}^2 \cdot \rho_0 \frac{\tilde{r}}{R_0} d\tilde{r}$

$= \frac{4\pi\rho_0}{R_0}\int_0^r \tilde{r}^3 d\tilde{r} = \frac{\cancel{4}\pi}{R_0}\rho_0 \cdot \frac{r^4}{\cancel{4}}$

$= \pi \cdot \frac{\rho_0}{R_0}\cdot r^4$

$\therefore \vec{E} = \frac{q_\Omega \hat{r}}{4\pi\varepsilon_0 r^2} = \frac{\hat{r}}{4\cancel{\pi}\varepsilon_0 \cancel{r^2}}\cdot \cancel{\pi}\cdot\frac{\rho_0}{R_0} r^{\cancel{4}\,2} = \frac{\rho_0 r^2}{4\varepsilon_0 R_0}\hat{r}$ $\frac{\rho_0}{3}[\tilde{r}^3]_{R_0}^{r}$

2° $r = [R_0, R]$, $q_\Omega = 4\pi\int_0^{R_0}\tilde{r}^2 \cdot \rho_0 \frac{\tilde{r}}{R_0} d\tilde{r} + 4\pi\int_{R_0}^{r}\rho_0 \tilde{r}^2 d\tilde{r}$

$= \pi\rho_0 R_0^3 + \frac{4\pi}{3}\rho_0 (r^3 - R_0^3) = \frac{\pi}{3}\rho_0(4r^3 - R_0^3)$

$\vec{E} = \frac{1}{4\cancel{\pi}\varepsilon_0 r^2}\cdot\frac{\cancel{\pi}}{3}\rho_0(4r^3 - R_0^3)\hat{r} = \frac{\rho_0(4r^3 - R_0^3)\,\hat{r}}{12\varepsilon_0 r^2}$

3.3 1) a) $\int_\gamma \vec{E}\hat{t}\,dl = \int_A^C \vec{E}\hat{t}\,dl + \int_C^B \vec{E}\hat{t}\,dl$

$$= \int_0^1 \vec{E}(1,y,0)\cdot\hat{u}_y\,dy + \int_1^0 \vec{E}(x,1,0)\,\hat{u}_x\,dx$$

$$= \int_0^1 -(1^2+0)+0\,dy + \int_1^0 (-2x\cdot 1+0)+0\,dx$$

$$= \int_0^1 -dy + \int_1^0 -2x\,dx$$

$$= [-y]_0^1 + [-x^2]_1^0 = (-1-0)-(0-1) = -1+1 = 0$$

b) Note: γ' is the form $(x, 1-x, 0)$, when x is from 1 to 0

$$\int_{\gamma'} \underbrace{\vec{E}\,\hat{t}}_{\frac{1}{\sqrt{2}}(\vec{u}_y-\vec{u}_x)}\,\underbrace{dl}_{\sqrt{2}dx} = \int_1^0 [(-2xy+3\cdot 0)\vec{u_x} - (x^2+0)\vec{u_y} + (3x-0)\cdot\vec{u_z}]\cdot\frac{1}{\sqrt{2}}(-\vec{u_x}+\vec{u_y})\cdot\sqrt{2}\,dx$$

$$= -\int_0^1 +2x(1-x)\,\vec{u_x}\cdot\vec{u_x} - x^2\,\vec{u_y}\cdot\vec{u_y} + 0\,dx$$

$$= -\int_0^1 2x-2x^2+x^2\,dx = \int_0^1 2x-3x^2\,dx = [x^2-x^3]_0^1 = 0$$

2) a) $\text{Flux} = \iint_{OABCB} \vec{E}\cdot\hat{z}\,dA = \int_0^1\int_0^1 \vec{E}\cdot\hat{z}\,dx\,dy$

$$= \int_0^1\int_0^1 3x-4y\cancel{z}^{\,0}\,dx\,dy = \int_0^1 [\tfrac{3}{2}x^2]_0^1\,dy = \frac{3}{2}$$

b) $\text{Flux} = \iint_{ABC} \vec{E}\cdot\hat{z}\,dA = \int_0^1\int_{1-y}^1 \vec{E}\cdot\hat{z}\,dx\,dy$ ← given any y $x\in[1-y, 1]$ is in trinagle

$$= \int_0^1\int_{1-y}^1 3x\,dx\,dy = \int_0^1 [\tfrac{3}{2}x^2]_{1-y}^1\,dy = \int_0^1 \frac{3}{2} - \frac{3}{2}\underbrace{(1-y)^2}_{1-2y+y^2}\,dy$$

$$= \int_0^1 \cancel{\tfrac{3}{2}-\tfrac{3}{2}} + 3y - \tfrac{3}{2}y^2\,dy = \int 3y - \tfrac{3}{2}y^2\,dy$$

$$= [\tfrac{3}{2}y^2 - \tfrac{1}{2}y^3]_0^1 = \frac{3}{2} - \frac{1}{2} = 1$$