Project #4 Solutions

EECS 592

WN 2020

1. R&N Problem 13.10:

a. Compute the expected "payback" percentage of the machine. In other words, for each coin played, what is the expected coin return?

Each wheel can have one of five unique outcomes (BAR, BELL, LEMON, CHERRY, BLANK) where BLANK occurs (4/8) of the time because there are four of them and the others occur (1/8) of the time. When combining the three symbol wheels there can be $8^3 = 512$ different outcomes (considering each BLANK as unique). In terms of payouts, when the three symbol wheels are combined there can be a total of seven possible outcomes (6 ways to win as well as losing), each occurring with different probabilities and paying out different amounts. The expected payback is simply the sum of the expected value for each outcome, where the expected value is the probability multiplied by the value. We are given the values so we only need to compute the probabilities of each outcome.

The top four outcomes only have 1 possible symbol combination each and so we have

$$\begin{aligned} P(BAR/BAR/BAR) &= (1/512) \\ P(BELL/BELL/BELL) &= (1/512) \\ P(LEMON/LEMON/LEMON) &= (1/512) \\ P(CHERRY/CHERRY/CHERRY) &= (1/512). \end{aligned}$$

For the remaining $CHERRY$ based winning outcomes, the higher paying outcomes dominate them and must be accounted for. Additionally, the symbols can occur in any order. For example, CHERRY/CHERRY/? is the same as ?/CHERRY/CHERRY as well as CHERRY/?/CHERRY and similar things can be said for the one cherry case. So, we have

$$\begin{aligned} P(CHERRY/CHERRY/?) &= 3\left[(1/8)^2 - (1/512)\right] = (21/512) \\ P(CHERRY/?/?) &= 3\left[(1/8)^1 - (14/512) - (1/512)\right] = (147/512) \end{aligned}$$

Next, we can calculate the expected payback.

$$20(1/512) + 15(1/512) + 5(1/512) + 3(1/512) + 2(21/512) + 1(147/512) = 232/512$$

Since the cost to play is 1, the expected payback percentage is then $\boxed{45.3125\%}$.

b. Compute the probability that playing the slot machine once will result in a win.

We already determined each probability above so we just need to add them up as

$$P(Win) = (1/512) + (1/512) + (1/512) + (1/512) + (21/512) + (147/512) = (172/512)$$

c. Estimate the mean and median number of plays you can expect to make until you go broke, if you start with 10 coins. You can run a simulation to estimate this, rather than trying to compute an exact answer.

See slot_machine.py for python simulation code. I ran 10,000 trials and obtained a mean of 18.1893 and a median of 16.

2. R&N Problem 13.13:

Mathematically we can show this by defining V to be a patient has the virus, A as Test A returned positive, and B as Test B returned positive. Next, we can grab all of the information from the question as

$$\begin{aligned} P(V) &= 0.01 \\ P(A|V) &= 0.95 \\ P(A|\neg V) &= 0.10 \\ P(B|V) &= 0.90 \\ P(B|\neg V) &= 0.05 \end{aligned}$$

and then the question is really asking which is larger, $P(V|A)$ or $P(V|B)$?

$$\begin{aligned} P(V|A) &= \frac{P(A|V)P(V)}{P(A)} \\ &= \frac{P(A|V)P(V)}{P(A|V)P(V) + P(A|\neg V)P(\neg V)} \\ &= \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.10)(0.99)} \\ &= 0.0875576 \end{aligned}$$

$$\begin{aligned} P(V|B) &= \frac{P(B|V)P(V)}{P(B)} \\ &= \frac{P(B|V)P(V)}{P(B|V)P(V) + P(B|\neg V)P(\neg V)} \\ &= \frac{(0.90)(0.01)}{(0.90)(0.01) + (0.05)(0.99)} \\ &= 0.153846 \end{aligned}$$

$P(V|B) > P(V|A)$ therefore B is more indicative of someone really carrying the virus if the test returns positive.

3. Car Diagnosis Bayes Net (Modified from R&N Problem 14.8):

This problem can be interpreted and solved in slightly different ways. Here I present two reasonable ways although others might also be valid.

Version 1

a. Extend the network with Boolean variables $IcyWeather$ and $StarterMotor$. Sketch the updated Bayesian network with eight nodes

- $IcyWeather$ impacts the battery.
- $StarterMotor$ needs to work for the car to start.
- Figure 1 shows the extended network

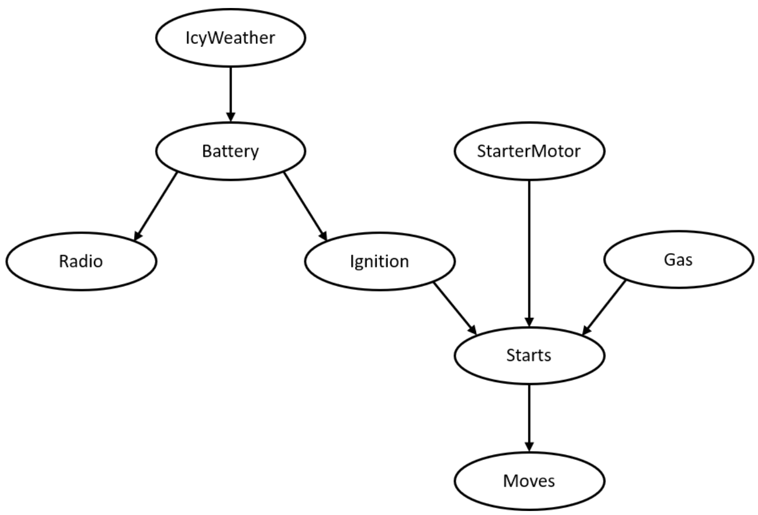

Figure 1: Version 1 for part a. The extended Bayes Net for car diagnosis.

b. How many independent values are contained in the full joint probability distribution for eight Boolean nodes, assuming that no conditional independence relations are known to hold among them?

If we assume no conditional independence of 8 nodes, the full joint distribution has $2^8 - 1 = \boxed{255 \text{ values}}$

c. How many independent probability values do your Bayesian network tables contain?

We have $IcyWeather$ (1), $StarterMotor$ (1), $Battery$ (2), $Radio$ (2), $Ignition$ (2), $Starts$ (8), Gas (1), and $Moves$ (2). This gives a total of $\boxed{19}$.

d. What nodes (if any) are conditionally independent of $Starts$ given $Moves$ and $Battery$?

$Radio$ and $IcyWeather$. They both have Markov blankets.

Version 2

a. Extend the network with Boolean variables $IcyWeather$ and $StarterMotor$. Sketch the updated Bayesian network with eight nodes

- $IcyWeather$ impacts the battery.
- $StarterMotor$ needs to work for the car to start.
- $Battery$ needs to work for the $StarterMotor$ to work
- Figure 2 shows the extended network

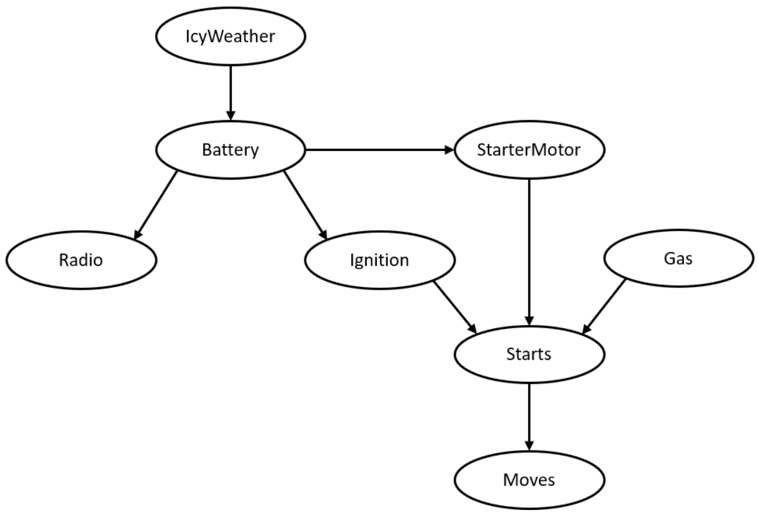

Figure 2: Version 2 for part a. The extended Bayes Net for car diagnosis.

b. How many independent values are contained in the full joint probability distribution for eight Boolean nodes, assuming that no conditional independence relations are known to hold among them?

If we assume no conditional independence of 8 nodes, the full joint distribution has $2^8 - 1 = \boxed{255 \text{ values}}$

c. How many independent probability values do your Bayesian network tables contain?

We have $IcyWeather$ (1), $StarterMotor$ (2), $Battery$ (2), $Radio$ (2), $Ignition$ (2), $Starts$ (8), Gas (1), and $Moves$ (2). This gives a total of $\boxed{20}$.

d. What nodes (if any) are conditionally independent of $Starts$ given $Moves$ and $Battery$?

$Radio$ and $IcyWeather$. They both have Markov blankets.

4. R&N Problem 14.11:

a. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.

Figure 3 shows a valid network.

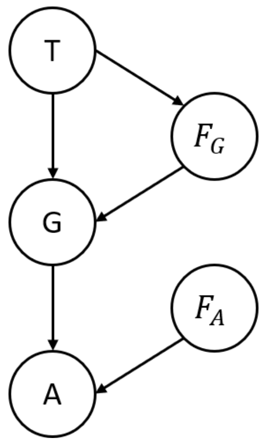

Figure 3

b. Is your network a polytree? Why or why not?

A polytree is defined as a network in which there is at most one undirected path between any two nodes in the network. In this case there are two undirected paths from T to G. Therefore, no, this is not a polytree.

c. Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with G.
Table 1 gives the conditional probabilities.

T	F_G	P(G=Normal)	P(G=High)
Normal	0	x	$1-x$
Normal	1	y	$1-y$
High	0	$1-x$	x
High	1	$1-y$	y

Table 1: CPT for the temperature Gauge

d. Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A
Table 2 gives the conditional probabilities.

G	F_A	P(A)	P(¬A)
Normal	0	0	1
Normal	1	0	1
High	0	1	0
High	1	0	1

Table 2: CPT for the Alarm

e. Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.

The question is asking to compute $P(T|\neg F_G, \neg F_A, A)$ (where we are considering Normal to be false and High to be true). Also, based on Table 2 we know that if the alarm isn't faulty and it is going off, that the gauge is always high so we are really trying to compute $P(T|\neg F_G, \neg F_A, A, G)$. Additionally, there is a Markov blanket around T so we can remove A and $\neg F_A$ from the evidence and simplify to $P(T|\neg F_G, G)$.

$$\begin{aligned}
P(T|\neg F_G, G) &= \frac{P(\neg F_G, G|T)P(T)}{P(\neg F_G, G)} \\
&= \frac{P(G|\neg F_G, T)P(\neg F_G|T)P(T)}{P(\neg F_G, G)} \\
&= \frac{P(G|\neg F_G, T)P(\neg F_G|T)P(T)}{P(\neg F_G, G|T)P(T) + P(\neg F_G, G|\neg T)P(\neg T)} \\
&= \frac{P(G|\neg F_G, T)P(\neg F_G|T)P(T)}{P(G|\neg F_G, T)P(\neg F_G|T)P(T) + P(G|\neg F_G, \neg T)P(\neg F_G|\neg T)P(\neg T)}
\end{aligned}$$

Next we can plug in the probability values from before and additionally define $P(T) = z$, $P(F_G|T) = w$, and $P(F_G|\neg T) = r$ and obtain

$$P(T|\neg F_G, G) = \frac{x(1-w)z}{x(1-w)z + (1-x)(1-r)(1-z)}$$

5. Simples Bayes Computations:

a. Compute $P(A, \neg B, \neg C, D)$

$$\begin{aligned}
P(A, \neg B, \neg C, D) &= P(\neg B, \neg C, D|A)P(A) \\
&= P(\neg C, D|A, \neg B)P(\neg B|A)P(A) \\
&= P(D|A, \neg B, \neg C)P(\neg C|A, \neg B)P(\neg B|A)P(A) \\
&= P(D|\neg B, \neg C)P(\neg C)P(\neg B|A)P(A) \\
&= (0.7)(1-0.8)(1-0.4)(0.3) \\
&\boxed{= 0.0252}
\end{aligned}$$

b. Compute $P(D|\neg A, B, C)$

$$\begin{aligned}
P(D|\neg A, B, C) = P(D|B, C) \\
\boxed{= 0.3}
\end{aligned}$$

c. Compute $P(\neg A|\neg B, C, \neg D)$

$$\begin{aligned}
P(\neg A|\neg B, C, \neg D) &= \frac{P(\neg B, C, \neg D|\neg A)P(\neg A)}{P(\neg B, C, \neg D)} \\
&= \frac{P(\neg B, C, \neg D|\neg A)P(\neg A)}{P(\neg B, C, \neg D|A)P(A) + P(\neg B, C, \neg D|\neg A)P(\neg A)} \\
&= \frac{P(\neg D|\neg B, C)P(C)P(\neg B|\neg A)P(\neg A)}{P(\neg D|\neg B, C)P(C)P(\neg B|A)P(A) + P(\neg D|\neg B, C)P(C)P(\neg B|\neg A)P(\neg A)} \\
&= \frac{(1-0.2)(0.8)(1-0.8)(1-0.3)}{(1-0.2)(0.8)(1-0.4)(0.3) + (1-0.2)(0.8)(1-0.8)(1-0.3)} \\
&\boxed{= 0.4375}
\end{aligned}$$

d. Compute $P(B|A, \neg C)$

$$\begin{aligned}
P(B|A, \neg C) = P(B|A) \\
\boxed{= 0.4}
\end{aligned}$$

e. Compute $P(\neg B)$

$$\begin{aligned}
P(\neg B) &= P(\neg B|A)P(A) + P(\neg B|\neg A)P(\neg A) \\
&= (1-0.4)(0.3) + (1-0.8)(1-0.3) \\
&\boxed{= 0.32}
\end{aligned}$$

6. Exam Bayes Net:

a. Given a high exam score (Ex), which variables are conditionally independent of intelligent (I)?

- $Success$
- $GainsPracticalSkills$

b. Given knows material (KM) as evidence, which node(s) are conditionally independent of success (S)?

- $Intelligent$
- $HardWorking$

c. Given success (S), which node(s) are conditionally independent of high exam score (Ex)?

- None

d. Given no evidence, compute P(KM)

$$\begin{aligned} P(KM) &= P(KM|I,H)P(I)P(H) + P(KM|I,\neg H)P(I)P(\neg H) \\ &+ P(KM|\neg I,H)P(\neg I)P(H) + P(KM|\neg I,\neg H)P(\neg I)P(\neg H) \\ &= (1.0)(0.75)(0.6) + (0.4)(0.75)(1-0.6) + (0.6)(1-0.75)(0.6) + (0.05)(1-0.75)(0.6) \\ &\boxed{= 0.6675} \end{aligned}$$

e. Compute $P(S|KM)$, the probability of Success given Knows Material

$$\begin{aligned} P(S|KM) &= P(S|PS,Ex,KM)P(PS|KM)P(Ex|KM) + P(S|PS,\neg Ex,KM)P(PS|KM)P(\neg Ex|KM) \\ &+ P(S|\neg PS,Ex,KM)P(\neg PS|KM)P(Ex|KM) + P(S|\neg PS,\neg Ex,KM)P(\neg PS|KM)P(\neg Ex|KM) \end{aligned}$$

$$\begin{aligned} P(Ex|KM) &= P(Ex|DP,KM)P(DP) + P(Ex,\neg DP,KM)P(\neg DP) \\ &= (0.85)(0.4) + (0.7)(1-0.4) \\ &= 0.76 \end{aligned}$$

$$\begin{aligned} P(S|KM) &= (0.8)(0.8)(0.76) + (0.7)(0.8)(1-0.76) + (0.7)(1-0.8)(0.76) + (0.3)(1-0.8)(1-0.76) \\ &\boxed{= 0.7416} \end{aligned}$$

f. Compute $P(PS|S)$, the probability of gaining Practical Skill given exam Success

$$\begin{aligned} P(Ex) &= P(Ex|DP,KM)P(DP)P(KM) + P(Ex|DP,\neg KM)P(DP)P(\neg KM) \\ &+ P(Ex|\neg DP,KM)P(\neg DP)P(KM) + P(Ex|\neg DP,\neg KM)P(\neg DP)P(\neg KM) \\ &= (0.85)(0.4)(0.6675) + (0.2)(0.4)(1-0.6675) + (0.7)(1-0.4)(0.6675) + (0.1)(1-0.4)(1-0.6675) \\ &= 0.55385 \end{aligned}$$

$$\begin{aligned} P(S) &= P(S|Ex,PS)P(Ex)P(PS) + P(S|Ex,\neg PS)P(Ex)P(\neg PS) \\ &+ P(S|\neg Ex,PS)P(\neg Ex)P(PS) + P(S|\neg Ex,\neg PS)P(\neg Ex)P(\neg PS) \\ &= (0.8)(0.55385)(0.8) + (0.7)(0.55385)(1-0.8) + (0.7)(1-0.55385)(0.8) + (0.3)(1-0.55385)(1-0.8) \\ &= 0.708616 \end{aligned}$$

$$\begin{aligned} P(S|PS) &= P(S|PS,Ex)P(Ex) + P(S|PS,\neg Ex)P(\neg Ex) \\ &= (0.8)(0.55385) + (0.7)(1-0.55385) \\ &= 0.755385 \end{aligned}$$

$$
\begin{aligned}
P(PS|S) &= \frac{P(S|PS)P(PS)}{P(S)} \\
&= \frac{(0.755385)(0.8)}{(0.708616)} \\
&\boxed{= 0.8528003884}
\end{aligned}
$$

g. Compute $P(KM|S)$, the probability of Knows Material given exam Success

$$
\begin{aligned}
P(KM|S) &= \frac{P(S|KM)P(KM)}{P(S)} \\
&= \frac{(0.7416)(0.6675)}{(0.708616)} \\
&\boxed{0.6985701706}
\end{aligned}
$$