1. Solution:

(a) $Occupation(Emily, Surgeon) \lor Occupation(Emily, Lawyer)$

(b) $Occupation(Joe, Actor) \land \exists o,\ o \neq Actor \land Occupation(Joe, o)$

(c) $\forall p,\ Occupation(p, Surgeon) \Rightarrow Occupation(p, Doctor)$

(d) $\neg\exists p\ Customer(Joe, p) \land Occupation(p, Lawyer)$

(e) $\exists p\ Boss(p, Emily) \land Occupation(p, Lawyer)$

(f) $\exists p\ Occupation(p, Lawyer) \land \forall q\ Customer(q, p) \Rightarrow Occupation(q, Doctor)$

(g) $\forall p\ Occupation(p, Surgeon) \Rightarrow \exists q\ Customer(p, q) \land Occupation(q, Lawyer)$

2. Solutions:

(a) $\forall x\ Even(x) \Leftrightarrow \exists y\ x = 2 * y$

(b) $\forall x\ Prime(x) \Leftrightarrow \forall y, z\ \ x = y * z \Rightarrow y = 1 \lor z = 1$

(c) $\forall x\ Even(x) \Rightarrow \exists y, z\ \ Prime(y) \land Prime(z) \land x = y + z$

3. Solutions:

(a) $Horse(x) \Rightarrow Mammal(x)$

$Cow(x) \Rightarrow Mammal(x)$

$Pig(x) \Rightarrow Mammal(x)$

(b) $Horse(y) \land Offspring(x, y) \Rightarrow Horse(x)$

(c) $Horse(Bluebeard)$

(d) $Parent(Bluebeard, Charlie)$

(e) $Offspring(x, y) \Rightarrow Parent(y, x)$

$Parent(y, x) \Rightarrow Offspring(x, y)$

(f) $Mammal(x) \Rightarrow \exists y\ Parent(y, x)$

4. Solution:

(a)

- Horse (h)
  - Horse (Bluebeard)
    - Offspring (Bluebeard, y)
      - Parent (y, Bluebeard)
        - Offspring (Bluebeard, y)
          - Parent (y, Bluebeard)
            - ⋮
  - Offspring (h, y)
    - Parent (y, h)
      Yes, {y / Bluebeard, h / Charlie}

(b) There is an infinite loop caused by rule (b), the branch with Offspring (Bluebeard, y) and Parent (y, Bluebeard) repeats infinitely, so that the rest of the proof will never be reached.

(c) There are two solutions: both Bluebeard and Charlie are horses.

(d) As suggested by Smith et al. whenever there is an infinite loop, we should suspend our effort to prove the subgoal. Instead, we jump to other branches of the proof of the supergoal, gathering all solutions. At last, we put those solutions into the suspended loop and try to find additional solutions if any.

5. Solution:

a. Translation into first-order logic:

Premise : $\forall x,\ Horse(x) \Rightarrow Animal(x)$

Conclusion : $\forall x, h\quad Horse(x) \land HeadOf(h, x) \Rightarrow \exists y\ Animal(y) \land HeadOf(h, y)$

b. Convert into CNF:

①: $\neg Horse(x) \lor Animal(x)$

②: $Horse(X)$

③: $HeadOf(H, X)$ Note: H and X are skolem constants

④: Refutation of conclusion: $\neg Animal(y) \lor \neg HeadOf(H, y)$

c. Proof

$\neg Horse(x) \lor Animal(x)$ $Horse(X)$ $HeadOf(H, X)$ $Animal(y) \lor \neg HeadOf(H, y)$

$Animal(X)$

$\neg Horse(X)$

FALSE. (Contradiction)

QED.

6. Solution:

(a) $\forall x\ (Hound(x) \Rightarrow Howl(x))$

(b) $\forall x\ \forall y\ (Have(x,y) \land Cat(y)) \Rightarrow \neg \exists z (HAVE(x,z) \land Mouse(z))$

(c) $\forall x\ (LightSleeper(x) \Rightarrow \neg \exists y\ (Have(x,y) \land Howl(y)))$

(d) $\exists x\ (Have(Sam, x) \land (Cat(x) \lor Hound(x)))$

(e) $Lightsleeper(Sam) \Rightarrow \neg \exists z\ (Have(Sam, z) \land Mouse(z))$

Proof:

① Convert into CNF:

(a) $\neg Hound(x) \lor Howl(x)$

(b) $\neg Have(x,y) \lor \neg Cat(y) \lor \neg Have(x,z) \lor \neg Mouse(z)$

(c) $\neg Lightsleeper(x) \lor \neg Have(x,y) \lor \neg Howl(y)$

(d) $Have(Sam, a) \land (Cat(a) \lor Hound(a))$

(e) Conclusion negation:

$\neg(\neg Lightsleeper(Sam) \lor \neg \exists z\ Have(Sam, z) \land Mouse(z))$

$Lightsleeper(Sam) \land Have(Sam, b) \land Mouse(b)$

② CNF clauses are:

1. $\neg Hound(x) \lor Howl(x)$
2. $\neg Have(x,y) \lor \neg Cat(y) \lor \neg Have(x,z) \lor \neg Mouse(z)$
3. $\neg Lightsleeper(x) \lor \neg Have(x,y) \lor \neg Howl(y)$
4. $Have(John, a)$
5. $Cat(a) \lor Hound(a)$
6. $Lightsleeper(John)$
7. $Have(John, b)$
8. $Mouse(b)$

③ Resolution:

1 & 5 → 9 : Cat(a) ∨ Howl(a)

2 & 8 → 10 : ¬Have(x, y) ∨ ¬Cat(y) ∨ ¬Have(x, b)

10 & 7 → 11 : ¬Have(John, y) ∨ ¬Cat(y)

9 & 11 → 12 : ¬Have(John, a) ∨ Howl(a)

4 & 12 → 13 : Howl(a) ∨

3 & 13 → 14 : ¬Lightsleeper(x) ∨ ¬Have(x, a)

4 & 14 → 15 : ¬Lightsleeper(John)

6 & 15 → 16 : FALSE.

7. Solution:

(a) $\forall x\ [FeelWarm(x) \Rightarrow Drunk(x) \lor \forall y\ (Custome(y) \land Have(x,y) \land Warm(y))]$

(b) $\forall y\ [Costume(y) \land Warm(y) \Rightarrow Furry(y)]$

(c) $\forall x\ [AIStudent(x) \Rightarrow CSstudent(x)]$

(d) $\forall x\ [AIStudent(x) \Rightarrow \exists z\ (Costume(z) \land Robot(z) \land Have(x,z))]$

(e) $\neg \exists z\ [Costume(z) \land Robot(z) \Rightarrow Furry(z)]$

(f) $\forall x\ [CSstudent(x) \land FeelWarm(x)] \Rightarrow \forall x\ [AIStudent(x) \Rightarrow Drunk(x)]$

Proof:

① Convert into CNF:

(a) $\neg FeelWarm(x) \lor Drunk(x) \lor (Custome(F(x)) \land Have(x, F(x)) \land Warm(F(x)))$

(b) $\neg Costume(y) \lor \neg Warm(y) \lor Furry(y)$

(c) $\neg AIStudent(x) \lor CSStudent(x)$

(d) $\neg AIStudent(x) \lor (Robot(G(x)) \land Costume(G(x)) \land Have(x, G(x)))$

(e) $\forall z\ \neg [Robot(z) \land Costume(z) \land Furry(z)]$

$\neg Robot(z) \lor \neg Costume(z) \lor \neg Furry(z)$

(f) Conclusion Negation:

$\neg [\neg CSstudent(x) \lor \neg FeelWarm(x) \lor \neg AIStudent(y) \lor Drunk(y)]$

$CSstudent(x) \land FeelWarm(x) \land AIStudent(y) \land \neg Drunk(y)$

CNF clauses are:

1(a) $\neg FeelWarm(x) \lor Drunk(x) \lor Custome(\ y\ )$

1(b) $\neg FeelWarm(x) \lor Drunk(x) \lor Have(x,\ y\ )$

1(c) $\neg FeelWarm(x) \lor Drunk(x) \lor Warm(\ y\ )$ ✓

2. $\neg Costume(y) \lor \neg Warm(y) \lor Furry(y)$ $\checkmark$

3. $\neg AIStudent(x) \lor CSStudent(x)$

4(a) $\neg AIStudent(x) \lor Robot(F(x))$

4(b) $\neg AIStudent(x) \lor Have(x, F(x))$ $\checkmark$

4(c) $\neg AIStudent(x) \lor Costume(F(x))$

5. $\neg Robot(z) \lor \neg Costume(z) \lor \neg Furry(z)$ $\checkmark$

6.(a) $CSStudent(x)$

6(b) $FeelWarm(x)$

6(c) $AIStudent(x)$ $\checkmark$

6(d) $\neg Drunk(x)$ $\checkmark$

4(a) + 6(c): $Robot(F(x))$ 7 $\checkmark$

7 + 5: $\neg Costume(F(x)) \lor \neg Furry(F(x))$ 8 $\checkmark$

8 + 2: $\neg Warm(F(x))$ 9 $\checkmark$

9 + 1(c): $\neg FeelWarm(x) \lor Drunk(x)$ 10 $\checkmark$

10 + 6(d): $\neg FeelWarm(x)$ 11 $\checkmark$

11 + 6(b): ϕ

QED.

8. Solution:

(a) $\forall x\ \forall y\ [\ Child(x) \land Candy(y) \Rightarrow Loves(x,y)\ ]$

(b) $\forall x\ [\ \exists y\ Candy(y) \land Loves(x,y) \Rightarrow \neg NF(x)\ ]$

(c) $\forall x\ [\ \exists y\ Pumpkin(y) \land Eats(x,y) \Rightarrow NF(x)\ ]$

(d) $\forall x\ \forall y\ [\ Pumpkin(y) \land Buys(x,y) \Rightarrow Carves(x,y) \lor Eats(x,y)\ ]$

(e) $\exists y\ Pumpkin(y) \land Buys(Stuart, y)$

(f) $Candy(Lifesavers)$

(g) $Child(Stuart) \Rightarrow \exists y\ Pumpkin(y) \land Carves(Stuart, y)$

Proof:

① Convert into CNF

(a) $\neg Child(x) \lor \neg Candy(y) \lor Loves(x,y)$

(b) $\neg Candy(F(x)) \lor \neg Loves(x, F(x)) \lor \neg NF(x)$

(c) $\neg Pumpkin(G(x)) \lor \neg Eats(x, G(x)) \lor NF(x)$

(d) $\neg Pumpkin(G(x)) \lor \neg Buys(x, G(x)) \lor Carves(x, G(x)) \lor Eats(x, G(x))$

(e) $Pumpkin(G(Stuart)) \land Buys(Stuart, G(Stuart))$

(f) $Candy(Lifesavers)$

(g) Conclusion Negation:

$\neg(\neg Child(Stuart) \lor Pumpkin(G(Stuart)) \land Carves(Stuart, G(Stuart)))$

$Child(Stuart) \land (\neg Pumpkin(G(Stuart)) \lor \neg Carves(Stuart, G(Stuart)))$

CNF Clauses are:

1. $\neg Child(x) \lor \neg Candy(y) \lor Loves(x,y)$
2. $\neg Candy(F(x)) \lor \neg Loves(x, F(x)) \lor \neg NF(x)$ ✓
3. $\neg Pumpkin(G(x)) \lor \neg Eats(x, G(x)) \lor NF(x)$ ✓

4. $\neg Pumpkin(G(x)) \lor \neg Buys(x, G(x)) \lor Carves(x, G(x)) \lor Eats(x, G(x))$ ✓

5(a) $Pumpkin(G(Stuart))$ ✓

5(b) $Buys(Stuart, G(Stuart))$ ✓

6. $Candy(Lifesavers)$ ✓

7(a) $Child(Stuart)$

7(b) $\neg Pumpkin(G(Stuart)) \lor \neg Carves(Stuart, G(Stuart))$

Resolution:

5(a) + 7(b): $\neg Carves(Stuart, G(Stuart))$ 8

5(a) + 4: $\neg Buys(Stuart, G(Stuart)) \lor Carves(Stuart, G(Stuart)) \lor Eats(Stuart, G(Stuart))$ 9

8+9: $\neg Buys(Stuart, G(Stuart)) \lor Eats(Stuart, G(Stuart))$ 10

10+5(b): $Eats(Stuart, G(Stuart))$ 11

3+5(a): $\neg Eats(Stuart, G(Stuart)) \lor NF(Stuart)$ 12

11+12: $NF(Stuart)$ 13

1+6: $\neg Child(x) \lor Loves(x, Lifesavers)$ 14

2+6: $\neg Loves(x, Lifesavers) \lor \neg NF(x)$ 15

14+15: $\neg Child(x) \lor \neg NF(x)$ 16

16+7(a): $\neg NF(Stuart)$ 17

13+17: ϕ

QED.

9. Solution:

(1) Fly (P_1, JFK, SFO)

(2) Fly (P_1, JFK, JFK)

(3) Fly (P_2, SFO, JFK)

(4) Fly (P_2, SFO, SFO)

10. Solution:

(1) Action (Go(x, y, r),
PRECOND: At(Shakey, x) $\wedge$ In(x, r) $\wedge$ In(y, r)
EFFECT: $\neg$At(Shakey, x) $\wedge$ At(Shakey, y))

Action (Push(b, x, y, r),
PRECOND: At(Shakey, x) $\wedge$ Movable(b) $\wedge$ At(b, x)
EFFECT: $\neg$At(Shakey, x) $\wedge$ $\neg$At(b, x) $\wedge$ At(Shakey, y) $\wedge$ At(b, y))

Action (ClimbUp(x, b),
PRECOND: At(Shakey, x) $\wedge$ At(b, x) $\wedge$ Climable (b)
EFFECT: On(Shakey, b) $\wedge$ $\neg$On(Shakey, Floor))

Action (ClimbDoan(b, x),
PRECOND: On(Shakey, b)
EFFECT: On(Shaky, Floor) $\wedge$ $\neg$On(Shakey, b))

Action (TurnOn (s, b)
PRECOND: On(Shakey, b) ∧ At(s, x) ∧ At(Shakey, x)
EFFECT: TurnedOn(s))

Action (TurnOff (s, b)
PRECOND: On (Shakey, b) ∧ At(s, x) ∧ At (Shakey, x)
EFFECT: TurnedOff(s))

(2) Initial State:

In(Switch1, Room1) ∧ In(Door1, Room1) ∧ In (Door1, Corridor)
In(Switch2, Room2) ∧ In (Door2, Room2) ∧ In (Door2, Corridor)
In(Switch3, Room3) ∧ In (Door3, Room3) ∧ In (Door3, Corridor)
In (Switch4, Room4) ∧ In (Door4, Room4) ∧ In (Door4, Corridor)
In (Shakey, Room3) ∧ At (Shakey, X_I)
In (Box1, Room1) ∧ At (Box1, X_1) ∧ Climable (Box1) ∧ Movable (Box1)
In (Box2, Room1) ∧ At (Box2, X_2) ∧ Climable (Box2) ∧ Movable (Box2)
In (Box3, Room1) ∧ At (Box3, X_3) ∧ Climable (Box3) ∧ Movable (Box3)
In (Box4, Room1) ∧ At (Box4, X_4) ∧ Climable (Box4) ∧ Movable (Box4)
TurnedOn (Switch1) ∧ TurnedOff (Switch2) ∧ TurnedOff (Switch3) ∧ TurnedOn(Switch4)

(3) Go(X_I, Door3, Room3)
Go (Door3, Door1, Corridor)
Go (Door1, X_2, Room1)
Push (Box2, X_2, Door1, Room1)
Push (Box2, Door1, Door2, Corridor)

Push(Box2, Door2, Switch2, Room2)