1. Solution:

a. $P(BAR/BAR/BAR) = (\frac{1}{8})^3 = \frac{1}{512}$

$P(BELL/BELL/BELL) = (\frac{1}{8})^3 = \frac{1}{512}$

$P(LEMON/LEMON/LEMON) = (\frac{1}{8})^3 = \frac{1}{512}$

$P(CHERRY/CHERRY/CHERRY) = (\frac{1}{8})^3 = \frac{1}{512}$

$P(CHERRY/CHERRY/?) = (\frac{1}{8})^2(1-\frac{1}{8}) \cdot C_3^2 = \frac{21}{512}$

$P(CHERRY/?/?) = (\frac{1}{8})(1-\frac{1}{8})(1-\frac{1}{8}) \cdot C_3^1 = \frac{147}{512}$

$E[Coin\_return] = 20 \times \frac{1}{512} + 15 \times \frac{1}{512} + 5 \times \frac{1}{512} + 3 \times \frac{1}{512} + 2 \times \frac{21}{512} + 1 \times \frac{147}{512} = \frac{29}{64}$

b. $P(Win) = \frac{1}{512} + \frac{1}{512} + \frac{1}{512} + \frac{1}{512} + \frac{21}{512} + \frac{147}{512} = \frac{43}{128}$

c. The program is attatched within the folder. Please refer to it.
After running the program, the mean is 18.3205
the median is 16.0

2. Solution:

We denote Test A recognizing virus as T_A

We denote Test B recognizing virus as T_B

We denote the virus is present as P

From the information, we could know:

$P(T_A \mid P) = 0.95$ $\quad P(\neg T_A \mid P) = 1 - 0.95 = 0.05$

$P(T_A \mid \neg P) = 0.1$ $\quad P(\neg T_A \mid \neg P) = 1 - 0.1 = 0.9$

$P(T_B \mid P) = 0.9$ $\quad P(\neg T_B \mid P) = 1 - 0.9 = 0.1$

$P(T_B \mid \neg P) = 0.05$ $\quad P(\neg T_B \mid \neg P) = 1 - 0.05 = 0.95$

$P(P) = 0.01$ $\quad P(\neg P) = 1 - 0.01 = 0.99$.

The test whose positive result is more indicative is the one whose posterior probability $P(P \mid T_A)$ or $P(P \mid T_B)$ is largest.

According to Bayes' Rule:

$$P(P \mid T_A) = \frac{P(T_A \mid P) \cdot P(P)}{P(T_A)} = \frac{P(T_A \mid P) \cdot P(P)}{P(T_A \mid P) \cdot P(P) + P(T_A \mid \neg P) \cdot P(\neg P)}$$

$$= \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.1 \times 0.99} = 0.087$$

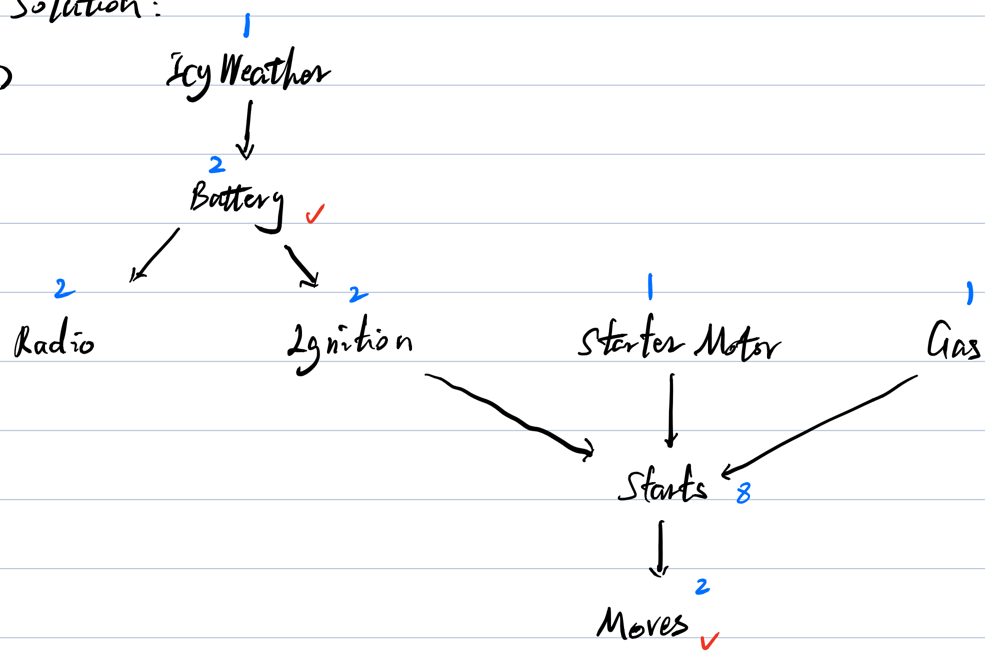
$$P(P \mid T_B) = \frac{P(T_B \mid P) \cdot P(P)}{P(T_B)} = \frac{P(T_B \mid P) \cdot P(P)}{P(T_B \mid P) \cdot P(P) + P(T_B \mid \neg P) \cdot P(\neg P)}$$

$$= \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.05 \times 0.99} = 0.154$$

Hence, the information returned by test B is much more indicative.

3. Solution:

(a)



(b) With 8 Boolean variables, the joint has $2^8 - 1 = 255$ independent values.

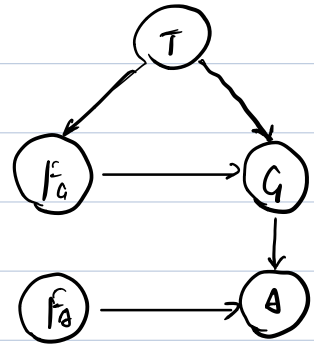
(c) Given the topology above, the number of independent probability value is:

$1+2+2+2+1+1+8+2 = 19$

(d) Since Node is conditionally independent of all other nodes in network given its Markov blanket, only radio is independent

4. Solution:

(a)



(b) It is not a polytree, because the temperature is influencing the gauge in two ways, which means there are more than one undirected / direct way from gauge to temperature.

(c)

	T = Normal		T = High	
	F_G	$\neg F_G$	F_G	$\neg F_G$
G = Normal	y	x	$1-y$	$1-x$
G = High	$1-y$	$1-x$	y	x

(d)

	G = Normal		G = High	
	F_A	$\neg F_A$	F_A	$\neg F_A$
A	0	0	0	1
$\neg A$	1	1	1	0

(e) We denote $T=High$ as T and $G=High$ as G.

From the information, we know the probability of interest is:

$$P(T \mid A, \neg F_G, \neg F_A)$$

Since the behavior of alarm is deterministic, we could know that if the alarm sounds and it is working properly, then G must be High.

Hence $P(T \mid A, \neg F_G, \neg F_A) = P(T \mid \neg F_G, G)$

According to Bayes' Rule:

$$P(T \mid \neg F_G, G) = \frac{P(T, \neg F_G, G)}{P(\neg F_G, G)}$$

$$= \frac{P(T) \cdot P(\neg F_G \mid T) \cdot P(G \mid T, \neg F_G)}{P(T) \cdot P(\neg F_G \mid T) \cdot P(G \mid T, \neg F_G) + P(\neg T) \cdot P(\neg F_G \mid \neg T) \cdot P(G \mid \neg T, \neg F_G)}$$

Let $P(T) = a$, $P(F_G \mid T) = b$ $\quad P(F_G \mid \neg T) = c$

Then $P(T \mid \neg F_G, G) = \dfrac{a \cdot (1-b) x}{a(1-b)x + (1-a)(1-c)(1-x)}$

5. Solution:

(a). $P(A=T, B=F, C=F, D=T)$

$= P(A=T) \cdot P(B=F \mid A=T) \cdot P(C=F) \cdot P(D=T \mid B=F, C=F)$

$= 0.3 \cdot (1-0.4) \cdot (1-0.8) \cdot 0.7$

$= 0.0252$

(b). $P(D=T \mid A=F, B=T, C=T)$

$$= \frac{P(D=T, A=F, B=T, C=T)}{P(A=F, B=T, C=T)}$$

$$= \frac{P(A=F)P(B=T \mid A=F) \cdot P(C=T) \cdot P(D=T \mid B=T, C=T)}{P(A=F)P(B=T \mid A=F) \cdot P(C=T) \cdot P(D=T \mid B=T, C=T) + \cdots P(A=F)P(B=T \mid A=F) \cdot P(C=T) \cdot P(D=F \mid B=T, C=T)}$$

$$= \frac{(1-0.3) \cdot 0.8 \cdot 0.8 \cdot 0.3}{(1-0.3) \cdot 0.8 \cdot 0.8 \cdot 0.3 + (1-0.3) \cdot 0.8 \cdot 0.8 \cdot (1-0.3)}$$

$= 0.3$

(c) $P(A=F \mid B=F, C=T, D=F)$

$$= \frac{P(A=F, B=F, C=T, D=F)}{P(B=F, C=T, D=F)}$$

$$= \frac{P(A=F) \cdot P(B=F \mid A=F) \cdot P(C=T) \cdot P(D=F \mid B=F, C=T)}{P(A=F) \cdot P(B=F \mid A=F) \cdot P(C=T) \cdot P(D=F \mid B=F, C=T) + \cdots + P(A=T)P(B=F \mid A=T) \cdot P(C=T) \cdot P(D=F \mid B=F, C=T)}$$

$$= \frac{(1-0.3) \cdot (1-0.8) \cdot 0.8 \cdot (1-0.2)}{(1-0.3) \cdot (1-0.8) \cdot 0.8 \cdot (1-0.2) + 0.3 \cdot (1-0.4) \cdot 0.8 \cdot (1-0.2)}$$

$= \frac{7}{16}$

(d) $P(B=T \mid A=T, C=F)$

$$= \frac{P(A=T, B=T, C=F)}{P(A=T, C=F)}$$

$$= \frac{P(A=T, B=T, C=F, D=T) + P(A=T, B=T, C=F, D=F)}{P(A=T, B=T, C=F, D=T) + P(A=T, B=F, C=F, D=T) + \cdots}$$

$$\cdots + P(A=T, B=T, C=F, D=F) + P(A=T, B=F, C=F, D=F)$$

where

$P(A=T, B=T, C=F, D=T) = P(A=T) \cdot P(B=T \mid A=T) \cdot P(C=F) \cdot P(D=T \mid B=T, C=F)$

$= 0.3 \cdot 0.4 \cdot (1-0.8) \cdot 0.6 = 0.0144$

$P(A=T, B=T, C=F, D=F) = P(A=T) \cdot P(B=T \mid A=T) \cdot P(C=F) \cdot P(D=F \mid B=T, C=F)$

$= 0.3 \cdot 0.4 \cdot (1-0.8) \cdot (1-0.6) = 0.0096$

$P(A=T, B=F, C=F, D=T) = P(A=T) \cdot P(B=F \mid A=T) \cdot P(C=F) \cdot P(D=T \mid B=F, C=F)$

$= 0.3 \cdot (1-0.4) \cdot (1-0.8) \cdot 0.7 = 0.0252$

$P(A=T, B=F, C=F, D=F) = P(A=T) \cdot P(B=F \mid A=T) \cdot P(C=F) \cdot P(D=F \mid B=F, C=F)$

$= 0.3 \cdot (1-0.4) \cdot (1-0.8) \cdot (1-0.7) = 0.0108$.

Hence, $P(B=T \mid A=T, C=F) = \dfrac{0.0144 + 0.0096}{0.0144 + 0.0252 + 0.0096 + 0.0108} = 0.4$

(e) $P(B=F) = P(B=F \mid A=T) \cdot P(A=T) + P(B=F \mid A=F) \cdot P(A=F)$

$= (1-0.4) \cdot 0.3 + (1-0.8) \cdot (1-0.3)$

$= 0.32$

6. Solution:

(a) Hardworking, Gains practical skills and Success are conditionally independent of intelligent (I), given High exam score.

(b) Given Knows material as evidence, Intelligent and Hardworking are independent of success.

(c) Given Success, no node is conditionally independent of High exam score.

(d) $P(KM) = P(KM \mid I, H) \cdot P(I) \cdot P(H) + P(KM \mid I, \neg H) \cdot P(I) \cdot P(\neg H)$

$+ P(KM \mid \neg I, H) \cdot P(\neg I) \cdot P(H) + P(KM \mid \neg I, \neg H) \cdot P(\neg I) \cdot P(\neg H)$

$= 1 \cdot 0.75 \cdot 0.6 + 0.4 \cdot 0.75 \cdot (1-0.6) + 0.6 \cdot (1-0.75) \cdot 0.6$

$+ 0.05 \cdot (1-0.75) \cdot (1-0.6) = 0.665$

(e) $P(S \mid KM) = P(DP) \cdot P(Ex \mid DP, KM) \cdot P(PS) \cdot P(S \mid PS, Ex)$

$+ P(DP) \cdot P(Ex \mid DP, KM) \cdot P(\neg PS) \cdot P(S \mid \neg PS, Ex)$

$+ P(DP) \cdot P(\neg Ex \mid DP, KM) \cdot P(PS) \cdot P(S \mid PS, \neg Ex)$

$+ P(DP) \cdot P(\neg Ex \mid DP, KM) \cdot P(\neg PS) \cdot P(S \mid \neg PS, \neg Ex)$

$+ P(\neg DP) \cdot P(Ex \mid \neg DP, KM) \cdot P(PS) \cdot P(S \mid PS, Ex)$

$+ P(\neg DP) \cdot P(Ex \mid \neg DP, KM) \cdot P(\neg PS) \cdot P(S \mid \neg PS, Ex)$

$+ P(\neg DP) \cdot P(\neg Ex \mid \neg DP, KM) \cdot P(PS) \cdot P(S \mid PS, \neg Ex)$

$+ P(\neg DP) \cdot P(\neg Ex \mid \neg DP, KM) \cdot P(\neg PS) \cdot P(S \mid \neg PS, \neg Ex)$

$= 0.4 \cdot 0.\cancel{665} \cdot 0.85 \cdot 0.8 \cdot 0.8 + 0.4 \cdot 0.\cancel{665} \cdot 0.85 \cdot (1-0.8) \cdot 0.7$

$+ 0.4 \cdot 0.\cancel{665} \cdot (1-0.85) \cdot 0.8 \cdot 0.7 + 0.4 \cdot 0.\cancel{665} \cdot (1-0.85) \cdot (1-0.8) \cdot 0.3$

$+ (1-0.4)\cdot\cancel{0.665}\cdot 0.7\cdot 0.8\cdot 0.8 + (1-0.4)\cdot\cancel{0.665}\cdot 0.7\cdot(1-0.8)\cdot 0.7$

$+(1-0.4)\cdot\cancel{0.665}\cdot(1-0.7)\cdot 0.8\cdot 0.7 + (1-0.4)\cdot\cancel{0.665}\cdot(1-0.7)\cdot(1-0.8)\cdot 0.3$

$= 0.7416$

(f) $P(PS|S) = \dfrac{P(PS, S)}{P(S)} = \dfrac{P(S|PS)\cdot P(PS)}{P(S)}$

where $P(S|PS) = P(S|PS, Ex)\cdot P(Ex) + P(S|PS, \neg Ex)\cdot P(\neg Ex)$

$P(Ex) = P(Ex|DP, KM)\cdot P(DP)\cdot P(KM) + P(Ex|\neg DP, KM)\cdot P(\neg DP)\cdot P(KM)$

$+ P(Ex|DP, \neg KM)\cdot P(DP)\cdot P(\neg KM) + P(Ex|\neg DP, \neg KM)\cdot P(\neg DP)\cdot P(\neg KM).$

$= 0.85\cdot 0.4\cdot 0.665 + 0.7\cdot(1-0.4)\cdot 0.665 + 0.2\cdot 0.4\cdot(1-0.665) + 0.1\cdot(1-0.4)\cdot(1-0.665)$

$= 0.5523.$

$P(\neg Ex) = 1 - P(Ex) = 0.4477$

$P(S|PS) = 0.8\cdot 0.5523 + 0.7\cdot 0.4477 = 0.75523$

$P(S) = P(S|PS, Ex)\cdot P(PS)\cdot P(Ex) + P(S|\neg PS, Ex)\cdot P(\neg PS)\cdot P(Ex)$

$+ P(S|PS, \neg Ex)\cdot P(PS)\cdot P(Ex) + P(S|\neg PS, \neg Ex)\cdot P(\neg PS)\cdot P(\neg Ex)$

$= 0.8\cdot 0.8\cdot 0.5523 + 0.7\cdot(1-0.8)\cdot 0.5523 + 0.7\cdot 0.8\cdot 0.4477 + 0.3\cdot(1-0.8)\cdot 0.4477$

$= 0.708368.$

Hence $P(PS|S) = \dfrac{0.75523\cdot 0.8}{0.708368} \approx 0.853.$

(5) $P(KM|S) = \dfrac{P(S, KM) \cdot}{P(S)} = \dfrac{P(S|KM) \cdot P(KM)}{P(S)}$

$= \dfrac{0.493164 \cdot 0.665}{0.708368} \approx 0.463$

$$P(S \mid KM) = \underbrace{P(S \mid Ex, PS, KM)}_{= \; P(S \mid Ex, PS)} \cdot \underbrace{P(Ex \mid KM)}_{\substack{\Downarrow \\ P(Ex \mid KM, DP) \cdot P(DP) \\ + \, P(Ex \mid KM, \neg DP) \cdot P(\neg DP)}} \cdot \overbrace{P(PS \mid KM)}^{\substack{P(PS) \\ \Uparrow}}$$