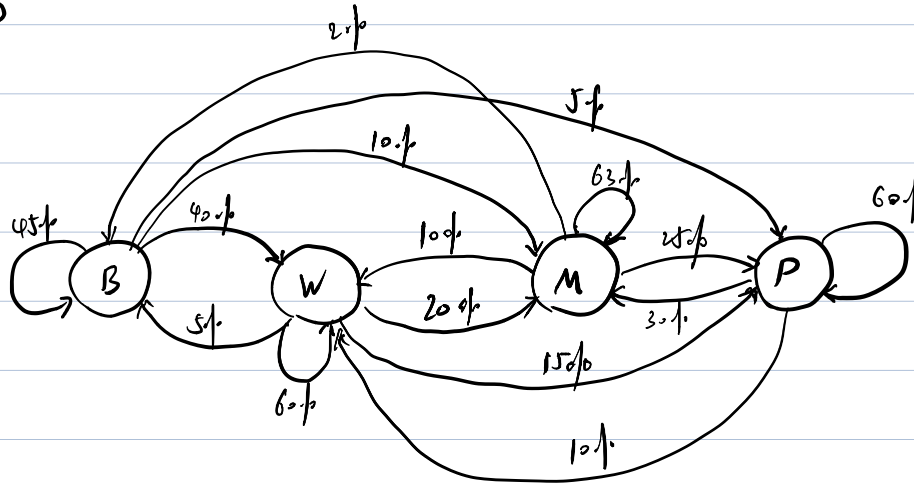
Problem 1 Solution:

(a)



(b) The transition probability matrix would be:

$$
T = \begin{array}{c} \\ B \\ W \\ M \\ P \end{array}
\begin{array}{c}
\begin{array}{cccc} B & W & M & P \end{array} \\
\begin{bmatrix}
0.45 & 0.4 & 0.1 & 0.05 \\
0.05 & 0.6 & 0.2 & 0.15 \\
0.02 & 0.1 & 0.63 & 0.25 \\
0 & 0.1 & 0.3 & 0.6
\end{bmatrix}
\end{array}
$$

(c) Let $b_0 = [0, 0, 1, 0]$ represents the initial state

Then the final state would be

$b_4 = b_0 \cdot T^4 = [0.0315, 0.2019, 0.4214, 0.3453]$

Hence the probability for a middle-class person to become wealthy after 4 steps is 0.2019.

(d) Let $b_0 = [0, 1, 0, 0]$ represents the initial state

Then the final state would be

$b_\phi = b_0 \cdot T^{10} = [0.0354, 0.2229, 0.4032, 0.3386]$

Hence the probability for a wealthy person to become middle-class after 10 steps is 0.4032

(e) Assuming $V = [v_1, v_2, v_3, v_4]$ is the stationary distribution

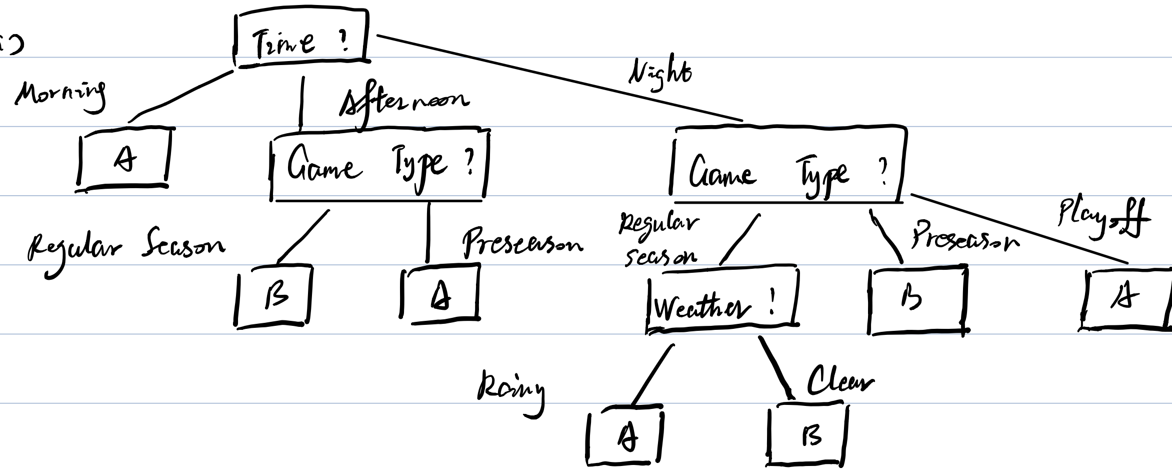
Then V is a left Eigenvector of M with Eigenvalue 1,

say, $VM = V$. Take transpose of both sides, $M^T V^T = V^T$.

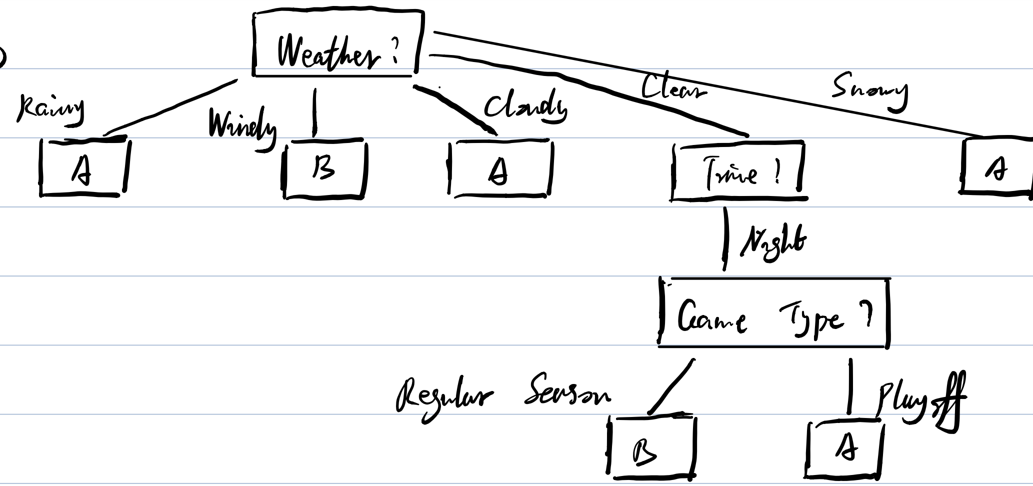
Then we got $V = [0.0348, 0.2209, 0.4044, 0.3399]$.

Problem 2. Solution:

(a)



(b)



(c) For a playoff game at a clear night, it has 3 examples which is AAB, from the decision tree, the out come is likely to be A.