Project #3 Solutions

EECS 592

WN 2020

1. R&N Problem 8.10:

a. Emily is either a surgeon or a lawyer

$$Occupation(Emily, Surgeon) \lor Occupation(Emily, Lawyer)$$

b. Joe is an actor, but he also holds another job

$$Occupation(Joe, Actor) \land [\exists x\ Occupation(Joe, x) \land x \neq Actor]$$

c. All surgeons are doctors

$$\forall x\ Occupation(x, Surgeon) \Rightarrow Occupation(x, Doctor)$$

d. Joe does not have a lawyer (i.e. is not a customer of any lawyer

$$\forall x\ Occupation(x, Lawyer) \Rightarrow \neg Customer(Joe, x)$$

e. Emily has a boss who is a lawyer

$$\exists x\ Boss(x, Emily) \land Occupation(x, Lawyer)$$

f. There exists a lawyer all of whose customers are doctors

$$\exists x\ Occupation(x, Lawyer) \land [\forall y\ Customer(y, x) \Rightarrow Occupation(y, Doctor)]$$

g. Every surgeon has a lawyer

$$\forall x\ Occupation(x, Surgeon) \Rightarrow [\exists y\ Customer(x, y) \land Occupation(y, Lawyer)]$$

2. R&N Problem 8.20:

For this problem we assume we are working with natural numbers and equality is defined. Equality can be defined as $\forall x, y\ x = y \iff \neg(x < y) \land \neg(y < x)$ and natural number axioms are shown in the book (8.3.3).

a. "x is an even number"

$$\forall x\ Even(x) \iff \exists y\ x = y + y$$

b. "x is prime"

$$\forall x\ Prime(x) \iff [(1 < x) \land (\forall y, z\ x = y * z \Rightarrow (y = 1) \lor (z = 1))]$$

c. Golbach's conjecture

$$\forall x\,[Even(x) \wedge (2 < x)] \Rightarrow [\exists y, z\ Prime(y) \wedge Prime(z) \wedge x = y + z]$$

3. R&N Problem 9.6:

$Horse(x)$: x is a horse
$Cow(x)$: x is a cow
$Pig(x)$: x is a pig
$Mammal(x)$: x is a mammal
$Offspring(x, y)$: x is the offspring of y
$Parent(x, y)$: x is the parent of y

a. Horses, cows, and pigs are mammals

$$\forall x\ Horse(x) \Rightarrow Mammal(x)$$
$$\forall x\ Cow(x) \Rightarrow Mammal(x)$$
$$\forall x\ Pig(x) \Rightarrow Mammal(x)$$

b. An offspring of a horse is a horse

$$\forall x, y\ \ [Offspring(x, y) \wedge Horse(y)] \Rightarrow Horse(x)$$

c. Bluebeard is a horse

$$Horse(Bluebeard)$$

d. Bluebeard is Charlie's parent

$$Parent(Bluebeard, Charlie)$$

e. Offspring and parent are inverse relations

$$\forall x, y\ Parent(x, y) \iff Offspring(y, x)$$

f. Every mammal has a parent

$$\forall x\ Mammal(x) \Rightarrow \exists y\ Parent(y, x)$$

4. R&N Problem 9.13:

a. Draw the proof tree generated by an exhaustive backward-chaining algorithm for the query $\exists h\ Horse(h)$, where clauses are matched in the order given.

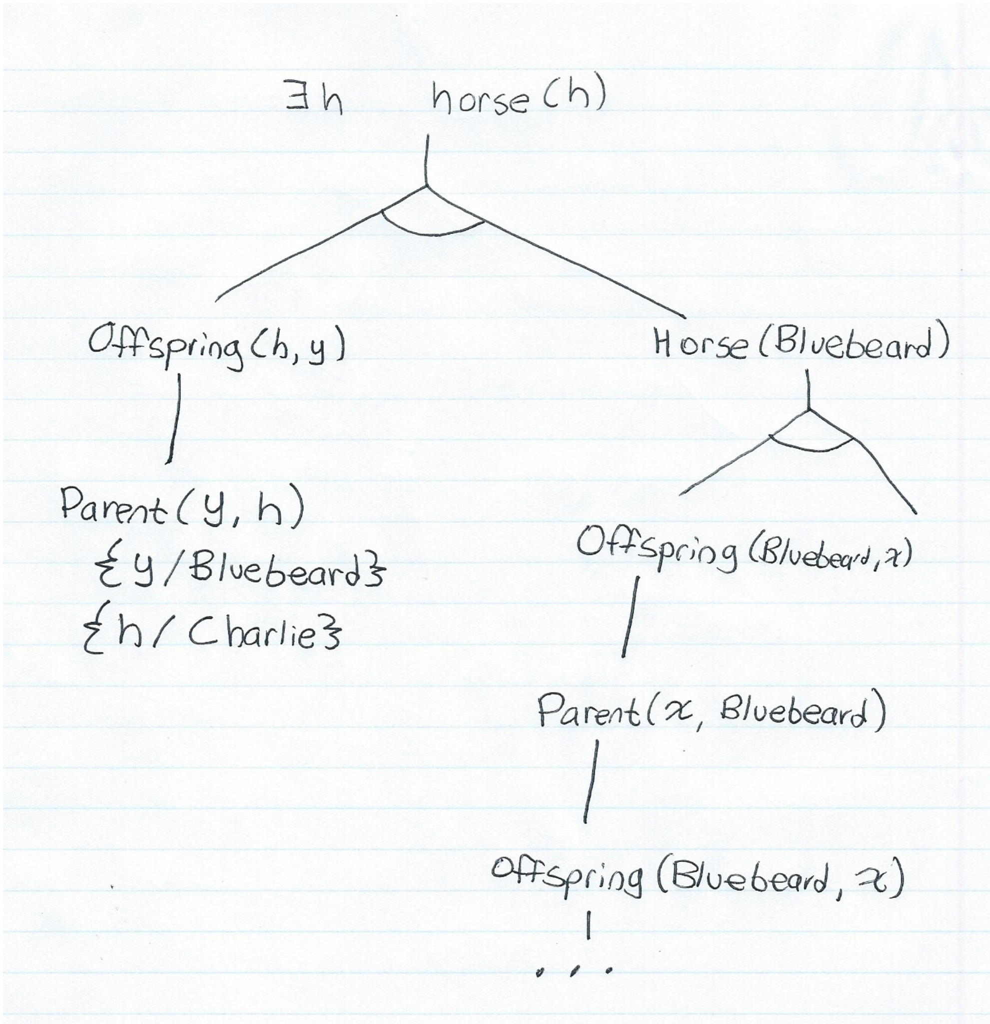

Figure 1: Backwards chaining for problem 4

b. What do you notice about this domain?

The domain is recursive and the backwards chaining algorithm will never finish since it is stuck in a cycle.

c. How many solutions for h actually follow from your sentences?

Based on our knowledge base there are 2 solutions, *Charlie* and *Bluebeard*.

d. Can you think of a way to find all of them?

Ref. [1] details methods and proves their optimality and completeness for finding multiple solutions to a repeating inference problem. Theorems 3.1 and 3.5 handle the single answer and multiple answer cases respectively. The main idea is that repeated states can be pruned from the search tree, since they will never provide an answer.

In our tree, $Offspring(Bluebeard, x)$ appearing a second time would count as a repeated state, causing the algorithm to recurse back up to $Horse(Bluebeard)$ and try a different implication to satisfy it, which in this case would be $Horse(Bluebeard)$ exactly and then $Horse(Charlie)$ would be solved for. On an additional call (via higher level control), the goal could be satisfied directly with $Horse(Bluebeard)$ since that would be next statement in the specified order and thus we would get our 2 solutions.

5. R&N Problem 9.23: From "Horses are animals," it follows that "The head of a horse is the head of an animal." Demonstrate that this inference is valid by carrying out the following steps

$$\begin{aligned}&HeadOf(h, x) : h \text{ is the head of } x\\ &Horse(x) : x \text{ is a Horse}\\ &Animal(x) : x \text{ is an animal}\end{aligned}$$

a. Translate the premise and the conclusion into the language of first-order logic.

Premise:

$$\forall x \; Horse(x) \Rightarrow Animal(x)$$

Conclusion:

$$\forall x, h \; Horse(x) \wedge HeadOf(h, x) \Rightarrow [\exists a \; Animal(a) \wedge HeadOf(h, a)]$$

6. Write the following statements in first-order logic. Complete all steps of FOL resolution to prove the conclusion

$$\begin{aligned}&Hound(x) : x \text{ is a hound}\\ &Howl(x) : x \text{ howls at night}\\ &Cat(x) : x \text{ is a cat}\\ &Mouse(x) : x \text{ is a mouse}\\ &Has(x, y) : x \text{ has a } y\\ &LightSleeper(x, y) : x \text{ is a light sleeper}\\ &Sam : constant\end{aligned}$$

a. All hounds howl at night

$$\begin{aligned}&\forall x \; Hound(x) \Rightarrow Howl(x)\\ &\neg Hound(x) \vee Howl(x)\\ &\boxed{\mathbf{R}_1 : \neg Hound(x) \vee Howl(x)}\end{aligned}$$

b. Anyone who has any cats will not have any mice

$$\forall r, c, m \; Has(r, c) \wedge Cat(c) \wedge Mouse(m) \Rightarrow \neg Has(r, m)$$
$$\neg Has(r, c) \vee \neg Cat(c) \vee \neg Mouse(m) \vee \neg Has(r, m)$$
$$\boxed{\mathbf{R}_2 : \neg Has(r, c) \vee \neg Cat(c) \vee \neg Mouse(m) \vee \neg Has(r, m)}$$

c. Light sleepers do not have anything that howls at night

$$\forall q, h \; LightSleeper(q) \wedge Howl(h) \Rightarrow \neg Has(q, h)$$
$$\neg LightSleeper(q) \vee \neg Howl(h) \vee \neg Has(q, h)$$
$$\boxed{\mathbf{R}_3 : \neg LightSleeper(q) \vee \neg Howl(h) \vee \neg Has(q, h)}$$

d. Sam has either a cat or a hound

$$\exists p \; Has(Sam, p) \wedge (Cat(p) \vee Hound(p))$$
$$Has(Sam, C_1) \wedge (Cat(C_1) \vee Hound(C_1))$$
$$\boxed{\mathbf{R}_4 : Has(Sam, C_1)}$$
$$\boxed{\mathbf{R}_5 : Cat(C_1) \vee Hound(C_1)}$$

e. (Conclusion) If Sam is a light sleeper, then Sam does not have any mice

$$\forall n \; LightSleeper(Sam) \wedge Mouse(n) \Rightarrow \neg Has(Sam, n)$$
$$\neg LightSleeper(Sam) \vee \neg Mouse(n) \vee \neg Has(Sam, n)$$

¬ Conclusion:

$$\neg \left[\neg LightSleeper(Sam) \vee \neg Mouse(n) \vee \neg Has(Sam, n)\right]$$
$$LightSleeper(Sam) \wedge Mouse(n) \wedge Has(Sam, n)$$
$$\boxed{\mathbf{R}_6 : LightSleeper(Sam)}$$
$$\boxed{\mathbf{R}_7 : Mouse(n)}$$
$$\boxed{\mathbf{R}_8 : Has(Sam, n)}$$

Resolution steps to prove conclusion shown in Figure 2.

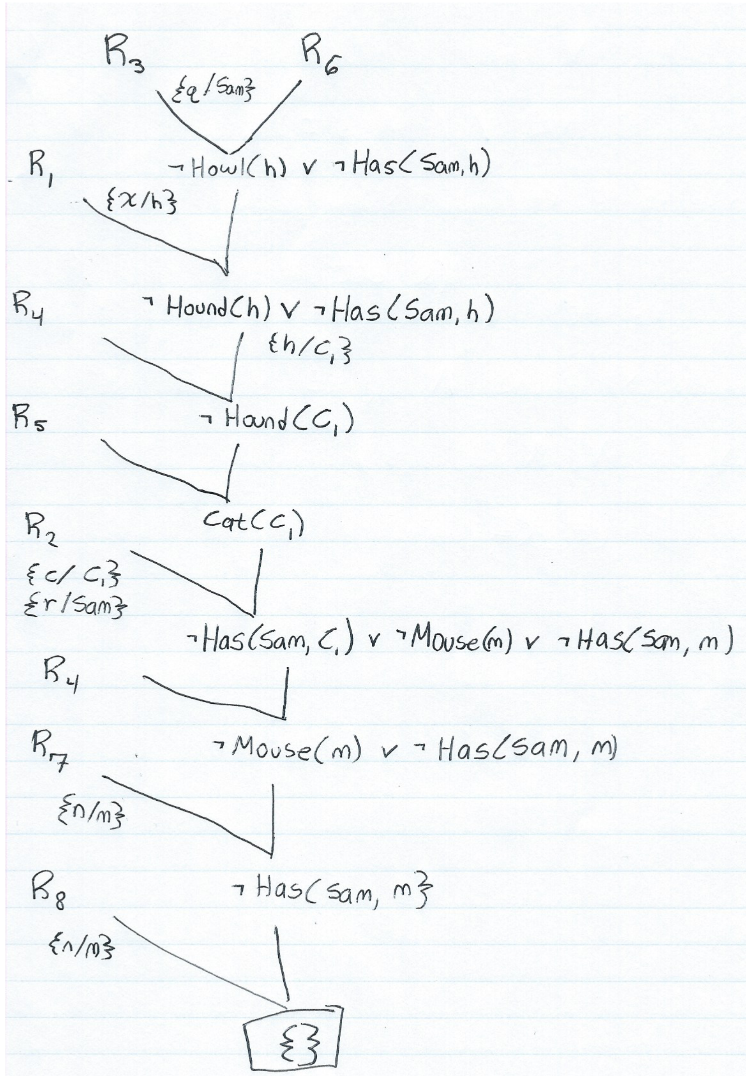

Figure 2: Resolution steps for problem 6

7. Write the following statements in first-order logic. Complete all steps of FOL resolution to prove the conclusion

$$
\begin{aligned}
&Has(x, y) : x \text{ has a } y \\
&FeelWarm(x) : x \text{ feels warm} \\
&Drunk(x) : x \text{ is drunk} \\
&IsWarm(x) : x \text{ is warm} \\
&Costume(x) : x \text{ is a costume} \\
&Furry(x) : x \text{ is furry} \\
&AI(x) : x \text{ is an AI student} \\
&CS(x) : x \text{ is a CS student} \\
&Robot(x) : x \text{ is a robot costume}
\end{aligned}
$$

a. Everyone who feels warm either is drunk, or every costume they have is warm

$$\forall x, y\ FeelWarm(x) \Rightarrow [Drunk(x) \vee (Has(x, y) \wedge Costume(y) \Rightarrow IsWarm(y))]$$
$$\neg FeelWarm(x) \vee [Drunk(x) \vee (\neg Has(x, y) \vee \neg Costume(y) \vee IsWarm(y))]$$
$$\boxed{\mathbf{R}_1 : \neg FeelWarm(x) \vee Drunk(x) \vee \neg Has(x, y) \vee \neg Costume(y) \vee IsWarm(y)}$$

b. Every costume that is warm is furry

$$\forall c\ Costume(c) \wedge IsWarm(c) \Rightarrow Furry(c)$$
$$\boxed{\mathbf{R}_2 : \neg Costume(c) \vee \neg IsWarm(c) \vee Furry(c)}$$

c. Every AI student is a CS student

$$\forall s\ AI(s) \Rightarrow CS(s)$$
$$\boxed{\mathbf{R}_3 : \neg AI(s) \vee CS(s)}$$

d. Every AI student has some robot costume

$$\forall p, \exists r\ AI(p) \Rightarrow [Has(p, r) \wedge Robot(r)]$$
$$\neg AI(p) \vee [Has(p, Rof(p)) \wedge Robot(Rof(p))]$$
$$(\neg AI(p) \vee Has(p, Rof(p))) \wedge (\neg AI(p) \vee Robot(Rof(p)))$$
$$\boxed{\mathbf{R}_4 : \neg AI(p) \vee Has(p, Rof(p))}$$
$$\boxed{\mathbf{R}_5 : \neg AI(p) \vee Robot(Rof(p))}$$

Also, we need to say every robot costume is a costume (to make things work).

$$\forall q,\ Robot(q) \Rightarrow Costume(q)$$

$$\boxed{\mathbf{R}_6 : \neg Robot(q) \vee Costume(q)}$$

e. No robot costume is furry

$$\forall m,\ Robot(m) \Rightarrow \neg Furry(m)$$

$$\boxed{\mathbf{R}_7 : \neg Robot(m) \vee \neg Furry(m)}$$

f. (Conclusion) If every CS student feels warm, then every AI student is drunk

$$[\forall n\ CS(n) \Rightarrow FeelWarm(n)] \Rightarrow [\forall t\ AI(t) \Rightarrow Drunk(t)]$$
$$[\forall n\ \neg CS(n) \vee FeelWarm(n)] \Rightarrow [\forall t\ \neg AI(t) \vee Drunk(t)]$$
$$\neg\,[\forall n\ \neg CS(n) \vee FeelWarm(n)] \vee [\forall t\ \neg AI(t) \vee Drunk(t)]$$

$\neg$ **Conclusion:**

$$[\forall n\ \neg CS(n) \vee FeelWarm(n)] \wedge \neg\,[\forall t\ \neg AI(t) \vee Drunk(t)]$$
$$[\forall n\ \neg CS(n) \vee FeelWarm(n)] \wedge [\exists t\ AI(t) \wedge \neg Drunk(t)]$$
$$[\neg CS(n) \vee FeelWarm(n)] \wedge AI(T_1) \wedge \neg Drunk(T_1)$$

$$\boxed{\mathbf{R}_8 : \neg CS(n) \vee FeelWarm(n)}$$

$$\boxed{\mathbf{R}_9 : AI(T_1)}$$

$$\boxed{\mathbf{R}_{10} : \neg Drunk(T_1)}$$

Resolution steps to prove conclusion shown in Figure 3.

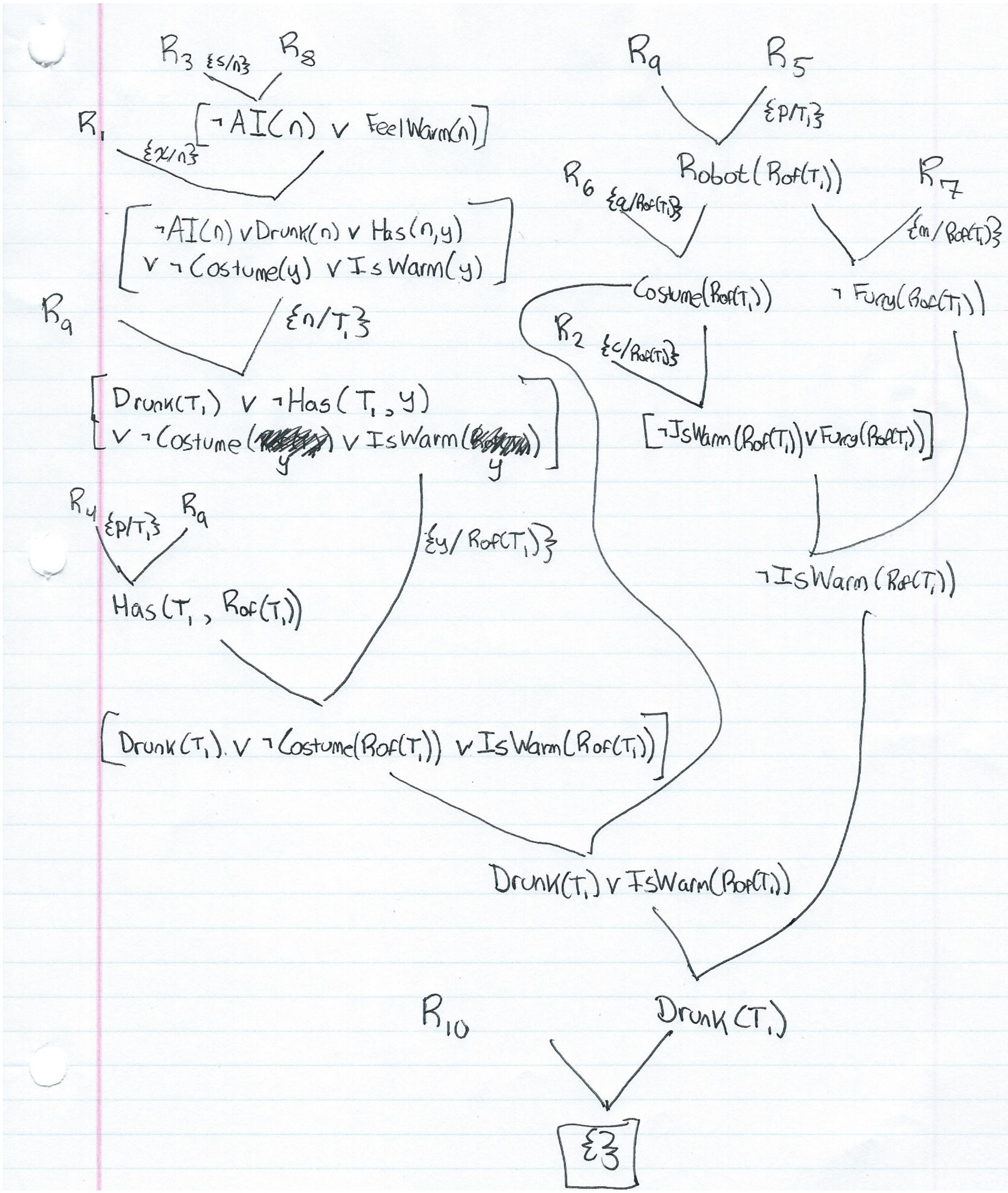

Figure 3: Resolution steps for problem 7

8. Write the following statements in first-order logic. Complete all steps of FOL resolution to prove the conclusion

$Child(x)$: x is a child
$Loves(x, y)$: x loves y
$Candy(x)$: x is a candy
$Nutr(x)$: x is a nutrition fanatic
$Eats(x, y)$: x eats y
$Pumpkin(x)$: x is a pumpkin
$Buys(x, y)$: x buys y
$Carves(x, y)$: x carves y
$Stuart$: constant
$Lifesavers$: constant

a. Every child loves every candy

$$\forall x, c\ Child(x) \land Candy(c) \Rightarrow Loves(x, c)$$

$$\boxed{\mathbf{R}_1 : \neg Child(x) \lor \neg Candy(c) \lor Loves(x, c)}$$

b. Anyone who loves some candy is not a nutrition fanatic

$$\forall a\ \ [\exists b\ Candy(b) \land Loves(a, b)] \Rightarrow \neg Nutr(a)$$
$$\forall a\ \neg\,[\exists b\ Candy(b) \land Loves(a, b)] \lor \neg Nutr(a)$$
$$\forall a\ \ [\forall b\ \neg Candy(b) \lor \neg Loves(a, b)] \lor \neg Nutr(a)$$

$$\boxed{\mathbf{R}_2 : \neg Candy(b) \lor \neg Loves(a, b) \lor \neg Nutr(a)}$$

c. Anyone who eats any pumpkin is a nutrition fanatic

$$\forall y, p\ Pumpkin(p) \land Eats(y, p) \Rightarrow Nutr(y)$$

$$\boxed{\mathbf{R}_3 : \neg Pumpkin(p) \lor \neg Eats(y, p) \lor Nutr(y)}$$

d. Anyone who buys any pumpkin either carves it or eats it

$$\forall z, q\ Pumpkin(q) \land Buys(z, q) \Rightarrow [Carves(z, q) \lor Eats(z, q)]$$

$$\boxed{\mathbf{R}_4 : \neg Pumpkin(q) \lor \neg Buys(z, q) \lor Carves(z, q) \lor Eats(z, q)}$$

e. Stuart buys a pumpkin

$$\exists r\ Pumpkin(r) \land Buys(Stuart, r)$$
$$Pumpkin(R_1) \land Buys(Stuart, R_1)$$
$$\boxed{\mathbf{R}_5 : Pumpkin(R_1)}$$
$$\boxed{\mathbf{R}_6 : Buys(Stuart, R_1)}$$

f. Lifesavers is a candy

$$\boxed{\mathbf{R}_7 : Candy(Lifesavers)}$$

g. (Conclusion) If Stuart is a child, then Stuart carves some pumpkin

$$Child(Stuart) \Rightarrow [\exists d\ Pumpkin(d) \land Carves(Stuart, d)]$$
$$\neg Child(Stuart) \lor [\exists d\ Pumpkin(d) \land Carves(Stuart, d)]$$

¬ Conclusion:

$$Child(Stuart) \land \neg\, [\exists d\ Pumpkin(d) \land Carves(Stuart, d)]$$
$$Child(Stuart) \land [\forall d\ \neg Pumpkin(d) \lor \neg Carves(Stuart, d)]$$
$$\boxed{\mathbf{R}_8 : Child(Stuart)}$$
$$\boxed{\mathbf{R}_9 : \neg Pumpkin(d) \lor \neg Carves(Stuart, d)}$$

Resolution steps to prove conclusion shown in Figure 4.

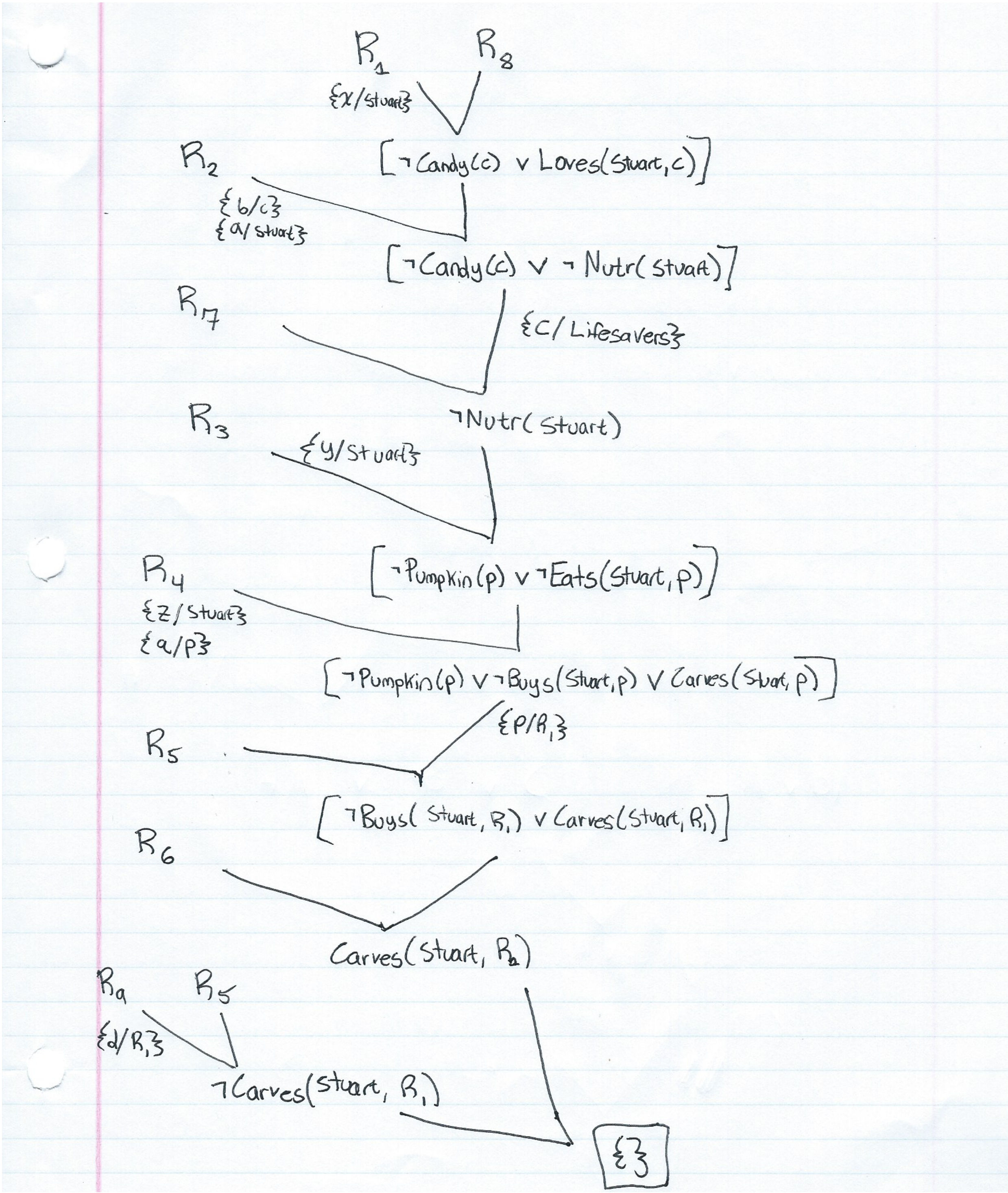

Figure 4: Resolution steps for problem 8

9. R&N 10.2 (Flying)

$$
\begin{aligned}
&Fly(P1, JFK, JFK)\\
&Fly(P1, JFK, SFO)\\
&Fly(P2, SFO, SFO)\\
&Fly(P2, SFO, JFK)
\end{aligned}
$$

10. R&N 10.4 (Shakey the Robot)

$$
\begin{aligned}
&Action(Go(x, y, r),\\
&\quad PRECOND: At(Shakey, x) \land In(x, r) \land In(y, r)\\
&\quad EFFECT: At(Shakey, y) \land \neg At(Shakey, x))\\
&Action(Push(b, x, y, r),\\
&\quad PRECOND: At(Shakey, x) \land Pushable(b) \land At(b, x) \land In(x, r) \land In(y, r)\\
&\quad EFFECT: At(b, y) \land At(Shakey, y) \land \neg At(b, x) \land \neg At(Shakey, x))\\
&Action(ClimbUp(x, b),\\
&\quad PRECOND: At(Shakey, x) \land At(b, x) \land Climbable(b) \land On(Shakey, Floor)\\
&\quad EFFECT: On(Shakey, b) \land \neg On(Shakey, Floor))\\
&Action(ClimbDown(b, x),\\
&\quad PRECOND: On(Shakey, b)\\
&\quad EFFECT: On(Shakey, Floor) \land \neg On(Shakey, b))\\
&Action(TurnOn(s, b, x),\\
&\quad PRECOND: On(Shakey, b) \land At(Shakey, x) \land At(s, x) \land TurnedOff(s)\\
&\quad EFFECT: TurnedOn(s) \land \neg TurnedOff(s)\\
&Action(TurnOff(s, b, x),\\
&\quad PRECOND: On(Shakey, b) \land At(Shakey, x) \land At(l, x) \land TurnedOn(s)\\
&\quad EFFECT: TurnedOff(s) \land \neg TurnedOn(s)
\end{aligned}
$$

Initial State:

$$
\begin{aligned}
&In(Switch_1, Room_1) \land In(Door_1, Room_1) \land In(Door_1, Corridor)\\
&In(Switch_2, Room_2) \land In(Door_2, Room_2) \land In(Door_2, Corridor)\\
&In(Switch_3, Room_3) \land In(Door_3, Room_3) \land In(Door_3, Corridor)\\
&In(Switch_4, Room_4) \land In(Door_4, Room_4) \land In(Door_4, Corridor)\\
&In(Shakey, Room_3) \land At(Shakey, X_{Start}) \land On(Shakey, Floor)\\
&In(Box_1, Room_1) \land In(Box_2, Room_1) \land In(Box_3, Room_1) \land In(Box_4, Room_1)\\
&Climbable(Box_1) \land Climbable(Box_2) \land Climbable(Box_3) \land Climbable(Box_4)\\
&Pushable(Box_1) \land Pushable(Box_2) \land Pushable(Box_3) \land Pushable(Box_4)\\
&At(Box_1, X_1) \land At(Box_2, X_2) \land At(Box_3, X_3) \land At(Box_4, X_4)\\
&TurnedOn(Switch_1) \land TurnedOff(Switch_2) \land TurnedOff(Switch_3) \land TurnedOn(Switch_4)
\end{aligned}
$$

Plan:

$Go(X_S, Door_3)$
$Go(Door_3, Door_1)$

$Push(Box_2, X_2, Door_1)$

$Push(Box_2, Door_1, Door_2)$
$Push(Box_2, Door_2, Switch2)$

References

[1] D. E. Smith, M. R. Genesereth, and M. L. Ginsberg, "Controlling recursive inference," *Artificial Intelligence*, vol. 30, no. 3, pp. 343–389, 1986.