

(a) Proof:

$$\text{Commutative: } g * f[n] = \sum_{k=0}^{n-1} g(n-k) f(k) = \sum_{k=0}^{n-1} f(k) g(n-k)$$

$$\text{Let } n-k = m \quad g * f[n] = \sum_{m=0}^{n-1} f(n-m) g(m) = f * g$$

$$\begin{aligned} \text{Associative: } ((f * g) * h)[n] &= \sum_{k=0}^n (f * g)(k) h(n-k) \\ &= \sum_{k=0}^n \sum_{l=0}^k f(l) g(k-l) h(n-k) = \sum_{l=0}^n \sum_{k=l}^n f(l) g(k-l) h(n-k) \\ &= \sum_{l=0}^n \sum_{k=0}^{n-l} f(l) g(k) h(n-k-l) = \sum_{l=0}^n f(l) (g * h)(n-l) = (f * (g * h))[n] \end{aligned}$$

(b) Solution:

$$H = \begin{bmatrix} H[0,0] & H[0,1] & \dots & H[0,N-1] \\ \vdots & & & \\ H[M-1,0] & H[M-1,1] & \dots & H[M-1,N-1] \end{bmatrix}$$

(c) Proof:

$$\begin{aligned} g[m,n] &= g[m,n] * f[m,n] = f[m,n] * g[m,n] \\ &= \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} f[i,j] \cdot g[m-i, n-j] \\ &= \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} f_1(i) \cdot f_2(j) \cdot g[m-i, n-j] \\ &= \sum_{j=-\infty}^{\infty} f_2(j) \sum_{i=-\infty}^{\infty} f_1(i) \cdot g[m-i, n-j] \\ &= f_2[n] * (f_1[m] * g[m,n]) \\ &= f_1[m] * (f_2[n] * g[m,n]) \end{aligned}$$