Part (1)

Our choice of the environment is using set. Each player has a set that stores all the current location of each piece. The location is represented using a two digit integer, with the first (ten) digit representing the row No. and the other represents column No. E.g. 86 will represent a piece at row 8 column 6. If the parameter (8, 8, 2) is passes to the initial_state function, each set will have 16 elements. The reason we are using this representation is that we believe the integer will increase the performance when knowing the board will not be bigger than 9 by 9.

For transition function, we are using a four digit representation, with the first two digits the current location of a piece, the rest two the location after moving. E.g. 3222 represents a white piece moving straight from (3, 2) to (2, 2), as white piece can only move upward thus the last two digits will be smaller while black in reverse.

It is also for performance improving and easy operating as the last two digit will always be $\pm (9 \text{ or } 10 \text{ or } 11)$ of the first two before excluding illegal moves.

Part (2)

(A)

Evasive seems to play without losing any of it's pieces, as they will not make moves for the opponent to capture unless there is no choice. It could also be seen from the number of pieces captured when the game is done, which is almost always 0 on a small board size (5, 5, 1).

```
Black won.
 White captured 0 pieces.
Black captured 0 pieces.
Total number of moves was 13
Х...
X.X.X
0...0
.00..
..XO.
White won.
White captured 0 pieces.
Black captured 0 pieces.
Total number of moves was 10
.X.OX
Х....
.XX..
....0
000..
Black won.
White captured 0 pieces.
Black captured 0 pieces.
Total number of moves was 15
...X.
OX.X.
X..O.
0...0
.O..X
White won.
White captured 1 pieces.
Black captured 0 pieces.
Total number of moves was 16
...0.
Х...Х
O..X.
XO.O.
...0.
```

Conqueror tends to play more aggressively by capturing more pieces and it usually wins, by testing on four different board sizes. Evasive tried to escape but will finally run out of safe moves.

White uses Evasive, black uses Conqueror:

```
(5, 5, 1):
Black won.
White captured 1 pieces.
Black captured 3 pieces.
 Total number of moves was 18
. . . . .
X..XX
...0.
..0..
.X...
(6, 6, 1):
Black won.
White captured 1 pieces.
Black captured 5 pieces.
 Total number of moves was 30
...X..
..0...
.X..X.
.X...
. . . . . .
..X...
```

```
(7,7,2):

Black won.

White captured 4 pieces.

Black captured 10 pieces.

Total number of moves was 50

.XXXXX

X.....

...X...

...O.OX

....X...

...O.OX

....X...
```

(8, 8, 2):

Black won.

White captured 1 pieces.

Black captured 3 pieces.

Total number of moves was 32

XXXXXXXX.

XX.X...X

....XX...

O.OO...O

O.OO..O

OOO..OXO

(C)

- (1) Our first heuristic is called "winner takes all", which gives terminals states positive or negative infinite value depending on who is winning. In this case it will never give away a winning in one step and will try it's best to prevent opponent in the terminal state. Other than terminal states, the value is given by calculating number of its own pieces subtracts number of opponents pieces.
- (2) The second utility function, "tank", maintains the essential "one step away from winning" detecting, adding a more detailed value distributing for a good attaching formation and defending formation when there are more than two rows of pieces to start with.

White uses Evasive, black uses Winner:

```
(5,5,1):

Black won.

White captured 0 pieces.

Black captured 0 pieces.

Total number of moves was 18
...X.

XO.O.

XOX..

O....
..OX.
```

```
(6, 6, 1):
Black won.
White captured 0 pieces.
Black captured 0 pieces.
 Total number of moves was 32
....XX
00...
....00
XO....
...XX.
ОХ....
(7, 7, 2):
Black won.
White captured 10 pieces.
Black captured 3 pieces.
Total number of moves was 66
. . . . . . .
.0.0...
OX..X.O
х...о..
0...0.0
00...0
...X...
(8, 8, 2):
Black won.
White captured 8 pieces.
Black captured 1 pieces.
 Total number of moves was 74
. . . . . . . .
.00.X...
X....XX
..O.OX.X
.O.O.OXO
0.0.0.0.
0..0..0.
Х.....
```

White uses Conqueror, black uses Winner:

```
(5, 5, 1):
Black won.
White captured 4 pieces.
Black captured 1 pieces.
 Total number of moves was 22
. . . . .
.0.0.
....0
. . . . .
OX...
(6, 6, 1):
Black won.
White captured 5 pieces.
Black captured 2 pieces.
 Total number of moves was 30
. . . . . .
0...0.
. . . . . .
..0..0
. . . . . .
...X..
(7, 7, 2):
Black won.
White captured 13 pieces.
Black captured 5 pieces.
 Total number of moves was 58
. . . . . . .
0.....
....
. . . . . . .
..0...0
00..0.0
..X..O.
```

```
Black won.
White captured 15 pieces.
Black captured 4 pieces.
 Total number of moves was 72
. . . . . . . .
..00..0.
......
. . . . . . . .
....
. . . . . . . .
....000
OX.OO..O
White uses Evasive, black uses Tank:
(5, 5, 1):
Black won.
White captured 1 pieces.
Black captured 1 pieces.
 Total number of moves was 16
. . . . .
XO.XX
.0...
.0..0
..X..
(6, 6, 1):
Black won.
White captured 2 pieces.
Black captured 0 pieces.
 Total number of moves was 26
. . . . . .
X..OX.
00..X.
0....
...XO.
```

(8, 8, 2):

```
Black won.
White captured 6 pieces.
Black captured 4 pieces.
 Total number of moves was 54
....X
XO...XX
. . . . . X
00000XX
.....
...0.00
..X...
(8, 8, 2):
Black won.
White captured 3 pieces.
Black captured 3 pieces.
 Total number of moves was 66
. . . . . . X
.OXX.X..
X..X.OXX
00.X...
X..O.X.X
....000.
000.00..
....X.
White uses Conqueror, black uses Tank:
(5, 5, 1):
White won.
White captured 2 pieces.
Black captured 2 pieces.
 Total number of moves was 23
.O.X
0...
0...
..XX.
. . . . .
```

(7, 7, 2):

```
(6, 6, 1):
Black won.
White captured 4 pieces.
Black captured 2 pieces.
 Total number of moves was 34
. . . . . .
00...
O..OX.
. . . . . .
. . . . . .
...X..
(7, 7, 2):
Black won.
White captured 10 pieces.
Black captured 8 pieces.
Total number of moves was 64
..X...X
0.0..X
. . . . . . .
00...0.
. . . . . . .
....
..X...
(8, 8, 2):
Black won.
White captured 13 pieces.
Black captured 6 pieces.
 Total number of moves was 74
х...х..
...0....
. . . . . . . .
. . . . . . . .
000....
..00.0..
0..00...
....X..
```

White uses Winner, black uses Tank:

```
(5, 5, 1):
White won.
White captured 1 pieces.
Black captured 3 pieces.
 Total number of moves was 19
хо...
...X
хо...
.X...
. . . . .
(6, 6, 1):
White won.
White captured 1 pieces.
Black captured 4 pieces.
 Total number of moves was 29
O...X.
. . . . . .
...XX.
..X...
.X...O
. . . . . .
(7, 7, 2):
White won.
White captured 5 pieces.
Black captured 11 pieces.
 Total number of moves was 69
0....
O...X.X
. . . . . . .
X..XXX.
.X...X.
...OX..
. . . . . . .
```

```
(8, 8, 2):
White won.
White captured 4 pieces.
Black captured 14 pieces.
Total number of moves was 83
...XOX.X
...X...
..X...
..XX...
..XX...
X..XX...
```

Conclusion:

From the result we can see that, the two new utility functions perform better than the functions provided. To be specific, from our observation, "Winner" haven't lost any game so far, and "Tank" wins around 80% of the time, on various board sizes.

Between "Winner" and "Tank", "Winner" also haven't lost a single time on various board sizes. We assume that the remained randomness in "Winner" often beats the designed pattern of "Tank" near the end of the game, as the manually arranged position might give disadvantages, this can see from the result that "tank" is more willing to capture pieces since more protective to the end line, which will cause no protection remained in the end.

Thus "Winner takes all" is the champion, the weakness could be that the randomness remained in it might get beaten by a well designed strategy aimed to it.