

Water!!!

TEAM #51350

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Abstract

The chief purpose of this study is to construct a model about the water consumption of a given area. In convention, water usage is divided into 3 parts: Industrial, Agricultural and Domestic. By considering each part of water usage as quantity which always adapt to the consumption of other human activities, We choose many fundamental quantities (such as population, PCGDP ...) to be fitted by proper functions as candidates to determine the each water consumption. Then specific correlation coefficients are calculated to verify the relations, in the process of which some irrelevant quantities are eliminated. By fitting the remaining quantities with each part of water usage, we can calculate the water usage per capita to predict the water usage situation in the following years.

After that, specific intervention plans will be designed under the simulation results to improve the future water supply ability. The simulation can then be conducted again under the new situations.

We put China as the main example of our analysis and come to the conclusion that China is in and will still be in water scarcity without intervention plan, and if proper intervention can be done, the situation will become less stress.

Key Words:

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1 Introduction

2 Nomenclatures

| | |
|-------|---|
| P | Population |
| GDP | Gross Domestic Product |
| PCGDP | Per Capita Gross Domestic Product |
| GAP | Cross Agricultural Product |
| AWC | Agricultural Water Consumption per year |
| IWC | Industrial Water Consumption per year |
| DWC | Domestic Water Consumption per year |
| TWR | Total Water Resource |
| SWR | total Surface Water Resource |
| UWR | total Underground Water Resource |
| PCGDP | Per Capita Gross Domestic Product |
| IA | Irrigation Area |
| ISP | Iron and Steel Production |
| ElP | Electricity Production. |
| EnC | Engel's Coefficient |
| A | Annual water supplies per person |

Table 1: Nomenclatures System

3 Model of water supply ability

When it come to the water supply ability of a region, a country or even the world. We often use the measurement called annual water supplies per person(A) for description[1]. We can set three levels to classify the ability of several regions (Table 2).

$$A = \left(\sum \text{WaterConsumption} \right) / \text{Population} \quad (1)$$

| | | |
|---------|-------------------|------------|
| level 1 | $A > 1700$ | Sufficient |
| level 2 | $1700 > A > 1000$ | stressful |
| level 3 | $1000 > A$ | scarce |

Table 2: water supply ability

To cover the internal dynamics of the water flow and the water storage change, we introduce following model.

3.1 Model Introduction

Water circulation is a rather complicated process, which make it almost impossible for us to design a purely fundamental model to include all the variables and their relations. Nevertheless, if we just collect all the data and using fitting method as our predicting model, it will be too trivial and old-fashioned. To solve this paradox, we introduce a phenomenological model which is quite normal in particle physics and other related field. Our model includes two main parts:

- prominent external parameters: a several statistical parameters about a region like population, GDP and so on.
- internal dynamic variables: find out the relation between prominent external parameters and internal variables including Agriculture Water Consumption, Industrial Water Consumption and Domestic Water consumption.

The prominent external parameters comes from the statistic numbers and it's fitness, while the internal dynamics variables is mainly based on prominent parameters, which makes them indirectly depend on time in our model.

3.2 Model Hypothesis

Our model is based on following hypothesis:

Hypothesis 1 We presume that the total water usage can be divided into three parts: Industrial, Agricultural and Domestic. And the main purpose of this model is to investigate these 3 quantities and relevant quantities. This hypothesis is plausible because the remaining fractions are relatively so small.

Hypothesis 2 Because that the water supply is one of the supporting industry in human society, we presume the water supply can always adjust itself to meet the need of the demand to a very large extent annually. Thus, each part of the water usage is treated as the function of more fundamental social quantities such as populations, PCGDP, irrigation area and so on.

Hypothesis 3 We presume that the fundamental social quantities mentioned above can specify the development information of human society. That is to say they are only the function of time itself. This hypothesis is plausible because in relatively short time period, say 15 years for example, the evolution of such fundamental quantities is determined by the intrinsic property of the society, but have little relationship with the water usage we interested in.

Hypothesis 4 We use specific model to fit the fundamental social quantities, and presume that the model will be legitimate in the following 15 years. This

hypothesis is plausible because that the model we choose to fit is proper and the data is plentiful.

Hypothesis 5 According to the above description, each part of the water usage is treated as the function of fundamental social quantities. We further presume that they act linearly on corresponding water usage. This hypothesis is plausible for mainly 2 reasons. Firstly, we will find in later discussion that all values taken into consideration change slowly with respect of time. So from the Taylor series viewpoint, the complex interactions exist in the system can be finely interpolated using a linear interacting model. Secondly, we intendedly choose the fundamental social quantities that can independently describe one aspect of the given society. So the interaction between these quantities are designed to be ignorable.

3.3 Model Structure

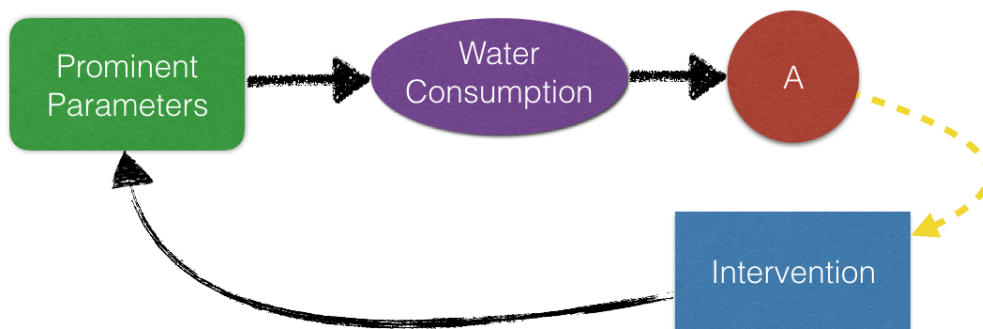


Figure 1: structure

Our model's structure is pretty clear. We using some prominent external parameters as the input of our dynamic system. To determine the water supply ability of a region, the most important part is the water flow which means the total amount of industrial, agricultural and residential consumption. The season we can use the consumption to measure the supply it that they must be the same during a long period just like the electricity use equal to the electricity produce.

$$\langle \text{Consumption} \rangle = \langle \text{Supply} \rangle$$

The W and \bar{W} is a crucial conception in our dynamic part, W stand for the total clean water resource over a period, while the \bar{W} is the total wasted water resource which can be transfered into W after several processing steps.

3.4 Model Prominent Parameters

The prominent parameters are some thing the dynamics part heavily rely on, to find the prominent parameters, we need to investigate the relation between some alternative parameters and consumptions according to the past data. In other words, prominent parameters are not given before a specific research object is given, we just give some alternative parameters and using the past data to determine which is better related and which is less related to our dynamic part variables.

Generally, there are several important alternative parameters like: Population, GDP Per Capita are of course important statistic numbers. Moreover, Agricultural Irrigation Area play an important role in agricultural consumption, Iron and Steel Production is also important in industrial consumption, and Engel's coefficient is an important statistical number about residential living which make it important in residential consumption.

3.5 Model Dynamics



Figure 2: water cycle[2]

The dynamics part of our model is inspired by the water cycle(Figure 2). Basically, our water is mainly refreshed though the precipitation. So it will be elegant if we using such water cycle process to build a mighty system about how the water flow and run with real data to determine the theoretical water consumption. However, after several data browse, we find that using precipitation as part of the water storage's changing rate is ridiculous—The total water resource is steadily a certain ratio of precipitation. Therefore using prominent parameters as arguments of internal variables and find the relation according to the past data will be a proper choice.

3.6 How to apply our model

With good structure of our model and program, the application of our model can be done in a pretty clear way.

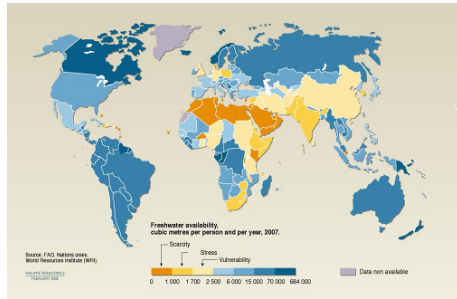
- steps 1** Using the past data of alternative prominent parameters, find proper relation between those parameters and time, therefore determine the time dependent equation of prominent parameters.
- steps 2** Using the data of alternative prominent parameters and consumption variables in the past to find the strength of correlation, Spearman or Pearson or other coefficients.

steps 3 According to the coefficients, structure the parameter dependent equations of consumption variables.

steps 4 With the equations construed above and the time dependent equation of prominent parameters, get the tendency of consumption variable therefore getting the prediction of water consumption and the water supply per person (A).

4 China's water scarcity

According to the UN water scarcity map[3], China is a country with water stress, which make it a region where water is moderately overloaded. In our consideration, level 2, will provide more abundant behavior in a dynamical model(will it become water scarce or water sufficient in the future?). Thus we pick up China as our research object. To make our model more predictable and more reality connected.



In order to explain why and how water is scarce in China, firstly, we state the water resources and water scarcity. Then we will give

social and environmental reasons to water scarcity.

Water resources China's water resources are geographically divided into nine major river basins, including Yangtze, Yellow (Huang), Hai-Luan, Huai, Song-Liao, Pearl, Southeast, Southwest, and Northwest. The average volume of internal renewable water resources is estimated to be approximately 2812 billion m^3 per year, which ranks fifth in the world. This volume includes both surface water and groundwater.

Approximately 98% of China's surface water is recharged by precipitation. The average precipitation decreases in a spatial gradient from more than 2000 mm at the southeastern coastline to less than 200 mm at the northwestern hinterlands.

4.1 Physical scarcity

Physical water scarcity is attributed to the shortfall in water resource volume to meet water needs.

We conclude physical water scarcity into 5 aspects as follow:

- Water shortages.

In normal water years, among 662 cities, 300 will have insufficient water supplies and 110 will experience severe water shortages; 30 out of 32 metropolitan areas with populations of more than 1 million people struggle to meet water demands. The total annual water shortage is estimated to be 30-40 billion m^3 and is even larger in dry years.

- Water resource use and overexploitation.

North China has experienced heavy demand for water, and groundwater is an important source for water supply in this area. The average rate of water resource use ranged from 31.0% to 91.7% for basins in the north compared to rates of 1.719.5% in the south.

- Reduced instream flow and degraded aquatic ecosystems.

The discharge from the river to the ocean dwindled from an annual average of 24 billion m^3 in the 1950s to 1 billion m^3 in 2001.

- Groundwater depletion.

It's recorded that groundwater depletion has increased in North China over the last two decades. Since the beginning of the 1980s, regions that overexploit groundwater have increased from 56 to 164 and the total area subject to groundwater overexploitation has increased from 87,000 km^2 to 180,000 km^2 .

- Seawater intrusion and ground subsidence.

Seawater intrusion has occurred in 72 locations in Hebei, Shandong, and Liaoning provinces, covering a total area of 142 km^2 in 1992. Cities such as Beijing, Tianjin, and Shanghai have been subject to ground subsidences of up to several meters.

4.2 Economic water scarcity

Economic water scarcity is caused by poor water quality that does not support any economic use of water rather than insufficient quantity.

We conclude economic water scarcity into two aspects as follow:

- Degrading water quality.

China's general water quality trend is characterized by extended water sections of poor quality. In North China, all major river basins experience water quality degradation, and the percentage of monitored water sections ranked poor ranges from 50% in the Yellow River basin to 78% in the Hai River basin.

China's lakes and reservoirs are experiencing accelerated eutrophication and degraded water quality.

- Socio-economic impact.

Degraded water quality has caused serious impacts on society. With a lack of clean, usable water, households, industries, and agriculture were forced to cut back their

water use. In 2003, economic losses attributed to poor water quality were at least 158 billion yuan or 1.16% of China's annual GDP.

The cancer mortality rates associated with poor water quality. The rates of stomach, liver, and bladder cancers are highest in rural areas and the mortality rates of liver and stomach cancers in China are well above the world averages.

4.3 Causes of water scarcity

There are many factors that contribute to China's water scarcity. We can classify them into social factors and environmental factors.

Social drivers: As for the social factors, we can consider the relationship between population and water scarcity, water resource management. List seven social reasons as follows:

Factor 1 Rapid industrialization and urbanization associated with a large population

Factor 2 Fragmented institutional system of water resource management

Factor 3 Supply-driven water resources management and inefficient water use

Factor 4 Underdeveloped water rights system

Factor 5 Inadequate water pricing

Factor 6 Insufficient investment in environmental protection and weak pollution control

Factor 7 Policies not well integrated with each other

Environmental drivers: As for the environmental factors, we can consider natural conditions, climate, distribution of water resource. List three factors as follows:

Factor 8 The spatial distribution of China's water resources is inconsistent with the local socioeconomic needs for water. The majority of China's water resources are located in the southern part of the country, whereas the greatest need for water comes from northern and eastern China.

Factor 9 Climate change aggravates the problems.

Factor 10 The loss of glaciers and wetlands upstream decreases river runoffs.

5 China's water scarcity: Further Investigation and Prediction

For further investigation, we study with the data from National Bureau of Statistics of the People's Republic China[4]. According to our models, we should predetermine the alternative prominent parameters and grab all the data and try to analyses them and they relation.

5.1 Prominent variables' tendency

Using proper assumption and the past data in databases. We can using fitness method to determine the time dependency of our alternative parameters. In our consideration, following factors especially eye-catching. For further convenience, we will investigate

| | |
|-------|-----------------------------------|
| P | Population |
| PCGDP | Per Capita Gross Domestic Product |
| IA | Irrigation Area |
| ISP | Iron and Steel Production |
| ELP | Electricity Production. |
| EnC | Engel's Coefficient |

their relation towards time at first.

Population The population growth model is various, for best concern, we use Logistic Model as the Population evolution model thus we can use the solution-Standard logistic sigmoid function as our target function by adding three basic coefficients.

$$P(t) = \frac{K^P}{1 + \exp[-\lambda^P(t - t_0^P)]}$$

Per Capita Gross Domestic Product As the largest developing country over the world, China has pretty high PCGDP growth rate. It seems like an exponential growth. However, according to our data browse, some developed countries have already reach some obstacles. Thus we think it will be suitable if we apply Logistic model in the fitness model of PCGDP.

$$\text{PCGDP}(t) = \frac{K^{\text{PCGDP}}}{1 + \exp[-\lambda^{\text{PCGDP}}(t - t_0^{\text{PCGDP}})]}$$

Irrigation Area According to our observation, there is a strong evidence about linear growth of Irrigation Area. Consider the complicity and the fact we are just required to predict about 15 years, a polynomial fitting with power 2 is sufficient.

$$\text{IA}(t) = A_0^{\text{IA}} + A_1^{\text{IA}}t + A_2^{\text{IA}}t^2$$

Iron and Steel Production The data of ISP perform in a similar way of IA, therefore we can use the same method to predict the ISP evolution.

$$\text{ISP}(t) = A_0^{\text{ISP}} + A_1^{\text{ISP}}t + A_2^{\text{ISP}}t^2$$

Electricity Production The behavior of ELP is every bit as same as two parameters above, thus we just apply the same evolution equation form.

$$\text{ELP}(t) = A_0^{\text{ELP}} + A_1^{\text{ELP}}t + A_2^{\text{ELP}}t^2$$

Engel's Coefficient Engel's Coefficient perform in a much different way. When it comes to the Engel, it make sense that with the development of society. It will keep decreasing as it used to be with a limit target: zero. Therefore we introduce a inverse proportion function to describe it's tendency.

$$\begin{cases} \text{Enc}_{\text{urban}}(t) = C^{\text{urban}}/(t - t_0^{\text{urban}}) \\ \text{Enc}_{\text{rural}}(t) = C^{\text{rural}}/(t - t_0^{\text{rural}}) \end{cases}$$

With the fitness method, we can get all the evolution of alternative prominent parameters in Figure 3, and the parameters in evolution equations in Tabl 3:

| | |
|--|--------------------------------------|
| $[t_0^P, \lambda^P, K^P]$ | [1968.14, 3.94%, 161162.404] |
| $[t_0^{\text{PCGDP}}, \lambda^{\text{PCGDP}}, K^{\text{PCGDP}}]$ | [2021.12, 14.9%, 187060.984] |
| $[A_0^{\text{IA}}, A_1^{\text{IA}}, A_2^{\text{IA}}]$ | [5.155e+07, -5.217e+04, 13.21] |
| $[A_0^{\text{ISP}}, A_1^{\text{ISP}}, A_2^{\text{ISP}}]$ | [1.027e+09, -1.031e+06, 258.6] |
| $[A_0^{\text{ELP}}, A_1^{\text{ELP}}, A_2^{\text{ELP}}]$ | [3.125e+08, -3.146e+05, 79.19] |
| $[C^{\text{urban}}, t_0^{\text{urban}}, C^{\text{rural}}, t_0^{\text{rural}}]$ | [1938.24, 2700.15, 1916.02, 4218.31] |

Table 3: evolution parameters

5.2 Relation between prominent parameters and variables

To investigate the relation between prominent parameters and variables, we firstly introduce the conception of correlation. In statistics, dependence is any statistical relationship between two random variables or two sets of data, and correlation refers to any of a broad class of statistical relationships involving dependence.

Pearson product-moment correlation coefficient [5] is a measure of the linear correlation between two variables X, Y . If there is two datasets $\{x_1, x_2, \dots, x_n\}$ and $\{y_1, y_2, \dots, y_n\}$, then the formula for r is[6]:

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

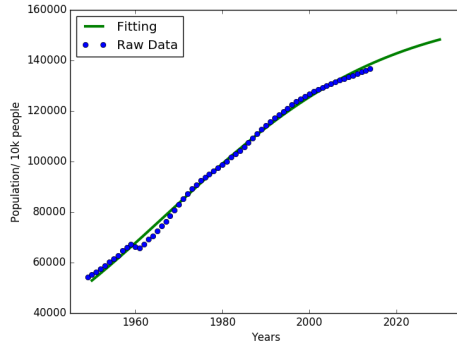
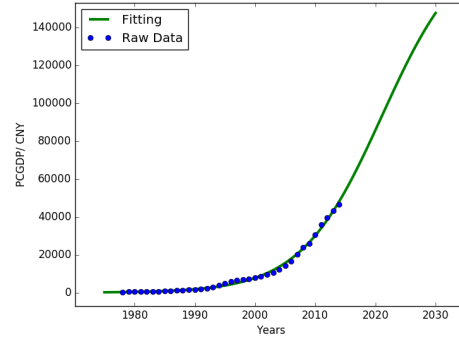
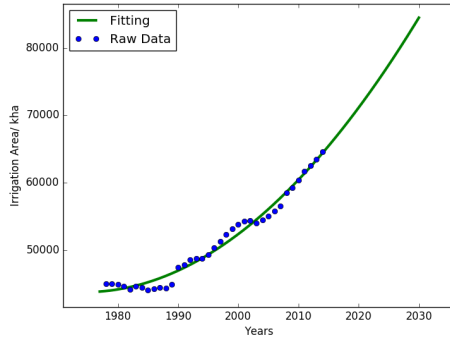
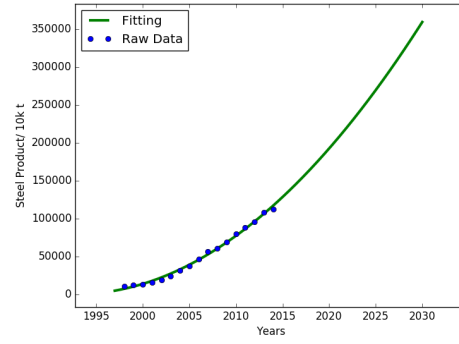
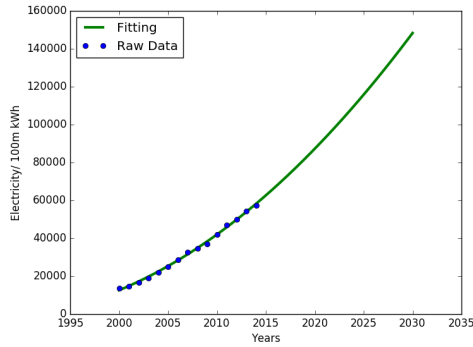
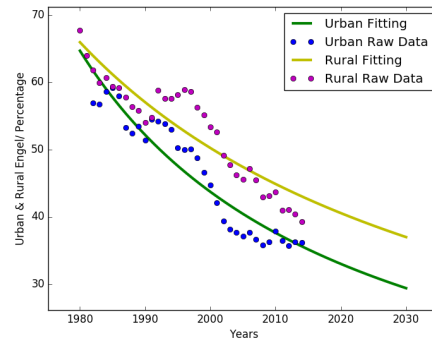
(a) $p(t)$ (b) $PCGDP(t)$ (c) $IA(t)$ (d) $ISP(t)$ (e) $ELP(t)$ (f) $Enc(t)$

Figure 3: Parameters fit: blue dots means the existed data which we refer from the national database, green lines stand for the optimized curve and the prediction value in the future

Spearman's rank correlation coefficient is a nonparametric measure of statistical dependence between two variables. It assesses how well the relationship between two variables can be described using a monotonic function. If there are no repeated data values, a perfect Spearman correlation of $+1$ or -1 occurs when each of the variables

is a perfect monotone function of the other[7][8]. The Spearman correlation coefficient is defined as the Pearson correlation coefficient between the ranked variables[?]. For a sample of size n , the n raw scores X_i, Y_i are converted ranks x_i, y_i , and ρ is computed from:

$$\rho = 1 - \frac{6 \sum (x_i - y_i)^2}{n(n^2 - 1)}$$

Kendall rank correlation coefficient is a statistic used to measure the association between two measured quantities. Let $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ be a set of observations of the joint random variables X and Y respectively, such that all the values of (x_i) and (y_i) are unique. Any pair of observations (x_i, y_i) and (x_j, y_j) , where $i \neq j$, are said to be concordant if the ranks for both elements agree: that is, if both $x_i > x_j$ and $y_i > y_j$ or if both $x_i < x_j$ and $y_i < y_j$. They are said to be discordant, if $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant. Thus the τ coefficient is defined as[10][11]:

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{\frac{1}{2}n(n-1)}$$

Using coefficients above, we get a map about the relation between prominent parameters and consumption variables.

| Data | Pearson (r_p, p) | Spearman (r_s, p) | Kendall (τ, p) | Average |
|---------------------------------|----------------------|-----------------------|-----------------------|---------|
| IWC $\sim P$ | (0.669,0.0245) | (0.627,0.0388) | (0.455,0.0516) | 0.584 |
| IWC \sim PCGDP | (0.614,0.0444) | (0.627,0.0388) | (0.455,0.0516) | 0.565 |
| IWC \sim ELP | (0.648,0.0311) | (0.627,0.0388) | (0.455,0.0516) | 0.577 |
| IWC \sim ISP | (0.653,0.0294) | (0.627,0.0388) | (0.455,0.0516) | 0.577 |
| AWC $\sim P$ | (0.924,4.95e-5) | (0.927,3.97e-5) | (0.782,0.000815) | 0.878 |
| AWC \sim PCGDP | (0.936,2.28e-5) | (0.927,3.97e-5) | (0.782,0.000815) | 0.882 |
| AWC \sim IA | (0.927,4.10e-5) | (0.927,3.97e-5) | (0.782,0.000815) | 0.879 |
| DWC $\sim P$ | (0.844,0.00108) | (0.836,0.00133) | (0.745,0.00141) | 0.808 |
| DWC \sim PCGDP | (0.806,0.00276) | (0.836,0.00133) | (0.745,0.00141) | 0.796 |
| DWC \sim Enc _{urban} | (-0.785,0.00417) | (-0.809,0.00256) | (-0.600,0.0102) | -0.731 |
| DWC \sim Enc _{rural} | (-0.419,0.199) | (-0.315,0.345) | (-0.204,0.383) | -0.313 |

Table 4: Coefficients: Coefficients of correlation in different standard

5.3 Fitness method in consumption variables

To investigate the evolution of consumption variables, We first investigate the correlation between one specific consumption with one specific prominent parameters.

Industry Water Consumption According to coefficients calculation result in Table 4, taking all $P, PCGDP, ELP, ISP$ in to account will be rational. Moreover, The

correlation between IWC and those factors are kinda weird, therefore using polynomial fit is unreasonable here. Further more, once we find that the IWC will never be negative and the shape of past data, it will be convenient for us to using Gaussian function here. At least we get the result in Table 5 and Figure 4.

| Parameters | p_0 | p_1 | p_2 |
|------------|--------|----------|--------|
| P | 1.43e3 | 7.62e-9 | 1.34e5 |
| PCGDP | 1.45e3 | 3.35e-10 | 3.31e4 |
| ELP | 1.44e3 | 3.10e-10 | 4.45e4 |
| ISP | 1.44e3 | 5.50e-11 | 8.40e4 |

Table 5: IWC - parameters: $p_0 \exp[-p_1(\text{para} - p_2)^2]$

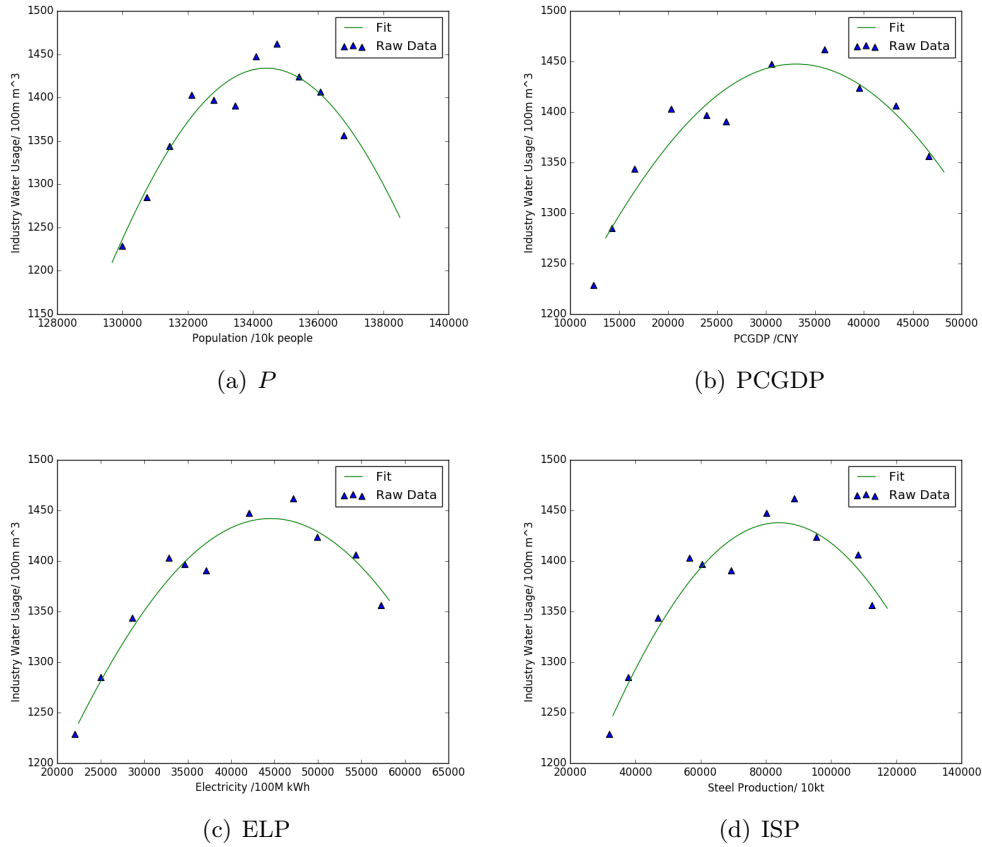


Figure 4: IWC - parameters

Agricultural Water Consumption According to coefficients calculation result in Table 4, taking all P , PCGDP, IAI in account will be reasonable. Due to the fact that we have no clue about the form of their relation, using polynomial fitting will be a proper choice. The results are in Table 6 and Figure 5

| Parameters | p_0 | p_1 | p_2 |
|------------|---------|---------|---------|
| P | -1.31e3 | 5.07e-2 | 3.42e4 |
| PCGDP | -1.50e3 | 9.54e-3 | -5.18e5 |
| IA | -1.09e3 | 3.20e-2 | -9.09e4 |

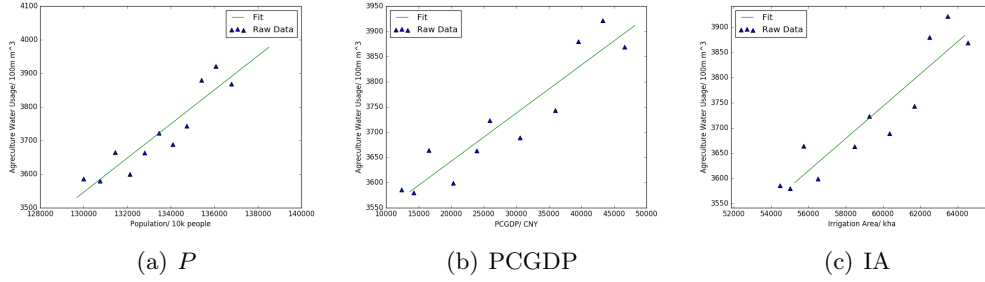
Table 6: AWC - parameters: $p_0 + p_1(\text{para} - p_2)$ 

Figure 5: AWC - parameters

Domestic Water Consumption The data about DWC and other parameters is kinda weird. Firstly, according to the coefficient in Table 4. We think the Engel's Coefficients is kinda useless and it will be reasonable to eliminate them from right now. Moreover, make the whole fit system more general, this time we try exponential fit, and the result are showed in Table 7 and Figure 6.

| Parameters | p_0 | p_1 | p_2 |
|------------|--------|----------|---------|
| P | 3.95e2 | -2.16e-5 | 1.05e5 |
| PCGDP | 3.98e2 | -3.82e-6 | -1.30e5 |

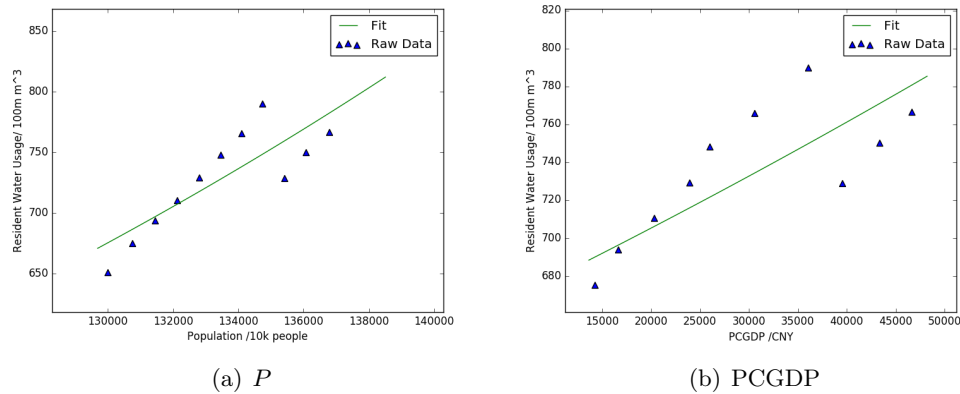
Table 7: DWC - parameters: $p_0 \exp[-p_1(\text{para} - p_2)]$ 

Figure 6: DWC - parameters

5.4 Prediction of future water supply ability

With the discussion above, now it's time to predict the future water supply ability A . Firstly, we introduce intermediate variables to separate the Water Consumption variables into several parts. For instance, $DWC = DWC(P, PCGDP)$, with the investigation above, we can write in a separated form:

$$DWC(t) = DWC(P(t), PCGDP(t)) = \frac{\alpha_P DWC(P) + \alpha_{PCGDP} DWC(PCGDP)}{\alpha_P + \alpha_{PCGDP}}$$

And the form of variables like $DWC(P)$ or $DWC(PCGDP)$ are exactly the correlation we got in Table 5 - 7. Others consumption variables are exactly same as this example. The analysis result are shown in Figure 7.

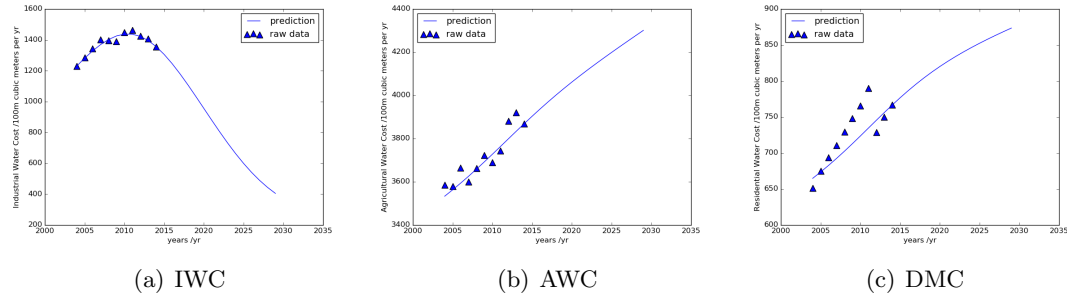


Figure 7: consumption predict

With the prediction value of water consumption, we can easily get the water supply ability through Equation 1. And the result is showed in Figure

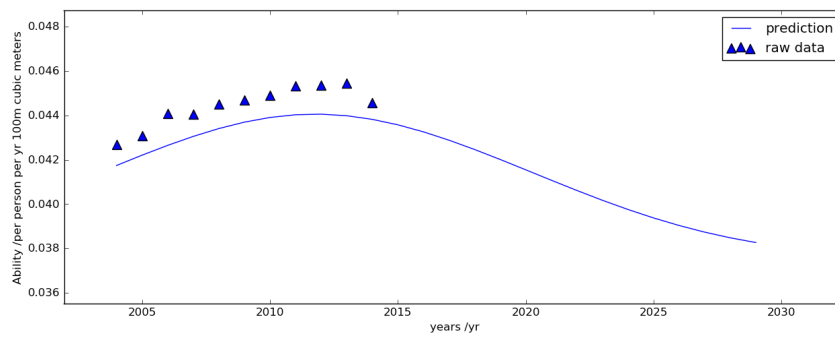


Figure 8: Water supply ability prediction

6 Intervention plan designing

7 Prediction with Intervention plan

8 Conclusion

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