Notes

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1 Approximate Energy derivative with local energy

Given the total energy is:

$$\langle E \rangle = \frac{1}{\sum_l |\Psi_l|^2} \sum_l |\Psi_l|^2 E_{loc} \tag{1}$$

and $E_{loc}(L) = \frac{\sum_r H_{lr}\overline{\Psi}_r}{\overline{\Psi}_l}$, denote $E_{loc}(L)$ as η_l and denote $\frac{1}{\Psi}\frac{\partial \Psi}{\partial \overline{\theta}}$ as Ω and $\frac{1}{\overline{\Psi}}\frac{\partial \overline{\Psi}}{\partial \overline{\theta}}$ as Ω^* and because network outputs Ψ, $\Omega = 0$

$$\begin{aligned}
\frac{\partial E}{\partial \overline{\theta}} &= -\frac{1}{Z^2}\partial_{\overline{\theta}} Z \sum_l |\Psi_l|^2 \eta_l + \frac{1}{Z}\partial_{\overline{\theta}} \sum_l |\Psi_l|^2 \eta_l \\
&= \langle \Omega + \Omega^* \rangle \langle E \rangle + \frac{1}{Z}\partial_{\overline{\theta}} \sum_l |\Psi_l|^2 \eta_l \\
&= \langle \Omega^* \rangle \langle E \rangle + \frac{1}{Z}\partial_{\overline{\theta}} \sum_l |\Psi_l|^2 \eta_l \\
&= \langle \Omega^* \rangle \langle E \rangle + \frac{1}{Z} \sum_l \partial_{\overline{\theta}} |\Psi_l|^2 \eta_l \\
&= \langle \Omega^* \rangle \langle E \rangle + \frac{1}{Z} \sum_l (\partial_{\overline{\theta}} \Psi_l \overline{\Psi}_l + \Psi_l \partial_{\overline{\theta}} \overline{\Psi}_l) \eta_l + |\Psi_l|^2 \partial_{\overline{\theta}} \eta_l \\
&= \langle \Omega^* \rangle \langle E \rangle + \langle \partial_{\overline{\theta}} \eta_l \rangle + \frac{1}{Z} \sum_l |\Psi_l|^2 \Omega^* \eta_l
\end{aligned} \tag{2}$$

$$\begin{aligned}
\sum_l |\Psi_l|^2 \partial_{\overline{\theta}} \eta_l &= \sum_l |\Psi_l|^2 \partial_{\overline{\theta}} \frac{\sum_r H_{lr}\overline{\Psi}_r}{\overline{\Psi}_l} = -\sum_l |\Psi_l|^2 \frac{\overline{\Psi}_l \sum_r H_{lr} \partial_{\overline{\theta}} \overline{\Psi}_r - \partial_{\overline{\theta}} \overline{\Psi}_l \sum_r H_{lr} \overline{\Psi}_r}{\overline{\Psi}_l^2} \\
&= -\sum_l |\Psi_l|^2 \frac{\Psi_l \sum_r H_{lr} \overline{\Psi}_r \Omega_r^* - \overline{\Psi}_l \Omega_l^* \sum_r H_{lr} \overline{\Psi}_r}{\overline{\Psi}_l^2} \\
&=
\end{aligned} \tag{3}$$

$$
\begin{aligned}
\sum_l |\Psi_l|^2 \Omega_l^* \eta_l &= \sum_l \Psi_l \overline{\Psi}_l \Omega_l^* \frac{\sum_r H_{lr} \overline{\Psi}_r}{\overline{\Psi}_l} \\
&= \sum_l \Psi_l \Omega_l^* \sum_r H_{lr} \overline{\Psi}_r
\end{aligned}
\tag{4}
$$