Notes

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1 Sampling configs from arbitrary quantum state on given basis

A pair of basis for a single qubit could be

$$\begin{aligned} |\epsilon_0\rangle &= \cos\theta|0\rangle + \sin\theta e^{i\phi}|1\rangle \\ |\epsilon_1\rangle &= -\sin\theta e^{-i\phi}|0\rangle + \cos\theta|1\rangle \end{aligned} \tag{1}$$

and their eigen value should be $\{-1, 1\}$, thus

$$P \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} P^{-1} \tag{2}$$

and P's colums are the its eigenvectors

$$P = \begin{pmatrix} \cos\theta & -\sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & \cos\theta \end{pmatrix} \tag{3}$$

and P^{-1} is

$$P^{-1} = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ -\sin\theta e^{i\phi} & \cos\theta \end{pmatrix} \tag{4}$$

Therefore, the operator is

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta e^{-i\phi} \\ \sin 2\theta e^{i\phi} & -\cos 2\theta \end{pmatrix} \tag{5}$$

2 Summary

Operator on bloch sphere

$$O = \begin{pmatrix} \cos 2\theta & \sin 2\theta e^{-i\phi} \\ \sin 2\theta e^{i\phi} & -\cos 2\theta \end{pmatrix} \tag{6}$$

its eigen vectors and corresponding eigen values are

$$
\begin{pmatrix} \cos\theta \\ \sin\theta e^{i\phi} \end{pmatrix}, 1
$$
$$
\begin{pmatrix} -\sin\theta e^{-i\phi} \\ \cos\theta \end{pmatrix}, -1 \tag{7}
$$

To transform a state vector $|\Psi\rangle$ on σ_z basis to O basis by matrix P in section 1:

$$
\begin{aligned}
|\Psi\rangle &= A_0|\epsilon_0\rangle + A_1|\epsilon_1\rangle \\
&= A_0 P|0\rangle + A_1 P|1\rangle
\end{aligned} \tag{8}
$$

or on σ_z basis

$$
P \begin{pmatrix} A_0 \\ A_1 \end{pmatrix} \tag{9}
$$

the inverse tranformation is:
given a state $|\Psi\rangle$ on σ_z basis, we have

$$
\begin{aligned}
|\Psi\rangle &= A_0|0\rangle + A_1|1\rangle \\
&= A_0 P^{-1}|\epsilon_0\rangle + A_1 P^{-1}|\epsilon_1\rangle
\end{aligned} \tag{10}
$$

3 Many-body case

σ_z basis: $|s_0\rangle, |s_1\rangle$ O^k basis: $|m_0\rangle, |m_1\rangle$
Give a quantum state $|\Psi\rangle$ on n particles on σ_z's basis

$$
\begin{aligned}
|\Psi\rangle &= \sum_{s_i \in \{0,1\}} A_{s_1,\cdots,s_n} \bigotimes_{i=1}^{n} |s_i\rangle \\
&= \sum_{m_i \in \{0,1\}} A_{m_1,\cdots,m_n} \bigotimes_{i=1}^{n} P_i^{-1}|m_i\rangle \\
&= \bigotimes_{i=1}^{n} P_i^{-1} \sum_{m_i \in \{0,1\}} A_{m_1,\cdots,m_n} \bigotimes_{i=1}^{n} |m_i\rangle
\end{aligned} \tag{11}
$$