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Hans R. Stoll*

I. Introduction

THE GROWTH IN THE VOLUME of stock market activity and the increased sophistication of investors has brought with it greater interest and activity in the related, albeit more complicated, put and call option market. The put and call market is a kind of futures market in stocks (with important differences) which has never generated the volume of trading activity of the commodity futures markets. As the stock market continues to grow, this situation is, however, likely to change.

Options are "negotiable contracts in which the writer, for a certain sum of money called the 'premium,' gives the buyer the right to demand within a specified time the purchase or sale by the writer of a specified number of shares of a stock at a fixed price called the 'contract price.' "[1] A put is an option to sell. A call is an option to buy.[2] Options are written for units of 100 shares, and the contract price is typically the market price of 100 shares of stock at the time at which the option is written. The standard maturities are 30, 60 and 90 days, and 6 months 10 days, although any maturity less than one year can usually be negotiated.

The aim of this paper is to formulate and test empirically some hypotheses about the relationship between put and call prices (or premiums). No attempt will be made to formulate a theory of the determinants of the *level* of put and/or call prices. This more difficult problem has been investigated by others.[3] It is clear, however, that, in a general way, the level of a put or call price will depend on the probability distribution of stock price changes expected during the option period. If the stock price has a chance of increasing greatly during the option period, calls will sell at a relatively high price. If

* Board of Governors of the Federal Reserve System. The author is a Visiting Professor from the University of Pennsylvania where class discussion generated the idea for this paper. He is grateful for suggestions from the Finance Workshop at the University of Pennsylvania and from his colleagues at the Board. Needless to say responsibility for the remaining errors is his alone, and the conclusions are his and not necessarily those of the Board of Governors.

1. Securities and Exchange Commission, *Report on Put and Call Options*, 1961, p. 5.

2. The principal difference between the put and call market and the commodity futures market is that options are transacted at the option of the buyer at any time during the contract period while commodity futures specify much more narrowly the transaction date.

3. See e.g. Baumol, Malkiel, Quandt [1966], Sprenkle [1961].

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the stock price has a chance of decreasing greatly, puts will sell at a high price.

The nature of the relationship between put and call prices has not been investigated in detail nor subjected to empirical tests. It is, for example, a popular misconception that call prices are higher than put prices simply because the demand for call options is greater.[4] The purpose of this paper is to show that, in theory, an arbitrage mechanism exists which ought to keep put and call prices in line with each other irrespective of the demands of buyers of options. The efficacy of this theoretical arbitrage mechanism in maintaining put and call price parity will be examined empirically to see if institutional restrictions and other frictions prevent the equilibrium relationship from being reached.

II. THE THEORY OF PUT AND CALL PARITY

A. *Mechanics*

As in all contracts, there are two sides to the option contract—the writer side and the buyer side. The writer receives a premium paid by the buyer (less a transaction cost going to the put and call dealer who brings the two together). The buyer of a call and the writer of a put believe stock prices will increase. The buyer of a put and the writer of a call believe stock prices will decrease. Possible profits of option buyers are unlimited while their losses are limited. They, therefore, pay a premium. Possible losses of option writers are unlimited while their gains are limited. They, therefore, receive a premium.

A buyer of a call on 100 shares of XYZ costing $400 (say) will, for example, profit to the extent that the per share price increases more than $4, and his profits can be large if the stock price increases greatly. He can lose no more than the original $400, however. The writer of a call receives $400 (less a transaction cost), and this is his maximum profit. His losses can, however, be as large as the profits of the call purchaser; and, since in theory there is no limit to the possible profits of the buyer, there is no limit to his possible losses. Puts are handled in the same way. In all cases, option buyer profits (losses) are option writer losses (profits).

Buyers of options need not be concerned with margin requirements on the stock; they are required only to supply the premium in cash. Option writers, who have obligations to receive or to deliver stock, must, under New York Stock Exchange (not Federal Reserve Board) regulations, supply margin to guarantee their contracts. The contract must also be endorsed by a New York Stock Exchange member firm. Writers of calls must deposit, at a minimum, a margin of 30 per cent of the value of the stock, and writers of puts must deposit, at a minimum a margin of 25 per cent. A writer of a call possessing the stock and a writer of a put short in the stock need not deposit margin in addition to that for their long or short position. Dividend payments and the issuance of rights or warrants cause the contract price to be reduced by the amount of the dividend or the value of the right or warrant.[5] Changes

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4. The Securities and Exchange Commission Report, *op. cit.*, at one point, takes this general line of reasoning: "In a rising market, calls will be more in demand and will sell for more than puts. In a falling market, puts will increase in price." (p. 83)

5. This convention has resulted in other misconceptions about the relation between put and call prices. Sarnoff (1968) reports, "Therefore, it follows that puts are usually cheaper than calls because of possible dividend reductions." (p. 102) Since the stock price falls with the issuance of dividends

in capitalization such as stock splits and stock dividends change the contract so that it refers to the number of new shares equivalent to the old number in value.

There are a variety of ways in which puts, calls, long positions, and short positions, may be combined to bring about positions of varying degrees of risk and expected return. An excellent and more complete discussion of these possible combinations is to be found in Kruizenga [1964]. In this paper, using the vector notation presented by Kruizenga, only those contract combinations important to the understanding of put-call parity will be discussed.

Each of the basic positions in the put and call market and in the stock market can be represented by a vector which gives the profit before premium to be derived from a unit increase or a unit decrease in the stock price. A call can, for example, be described as follows:

$$\begin{array}{cc} & \text{profit before premium} \\ \text{unit price increase} & \begin{bmatrix} +1 \\ 0 \end{bmatrix} \\ \text{unit price decrease} & \end{array}$$

Profits (before premium) go up directly with increases in share price and this is indicated by "+1." No loss or gain accrues to the call buyer from price declines, and this is indicated by "0."

Omitting the column and row labels, one can now describe the six basic positions as follows:

$$\begin{array}{ccc} \text{Long} & \text{Short} & \text{Buy call} \\ \begin{bmatrix} +1 \\ -1 \end{bmatrix} & \begin{bmatrix} -1 \\ +1 \end{bmatrix} & \begin{bmatrix} +1 \\ 0 \end{bmatrix} \\ \text{Write call} & \text{Buy put} & \text{Write put} \\ \begin{bmatrix} -1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ +1 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \end{bmatrix} \end{array}$$

A number of possible combinations can now readily be illustrated by adding vectors. This is done by adding corresponding positions in each vector:

$$\begin{array}{ccccc} \text{Long} & & \text{Short} & & \text{No position} \\ \begin{bmatrix} +1 \\ -1 \end{bmatrix} & + & \begin{bmatrix} -1 \\ +1 \end{bmatrix} & = & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \text{Buy call} & & \text{Write put} & & \text{Long position} \\ \begin{bmatrix} +1 \\ 0 \end{bmatrix} & + & \begin{bmatrix} 0 \\ -1 \end{bmatrix} & = & \begin{bmatrix} +1 \\ -1 \end{bmatrix} \\ \text{Long} & & \text{Buy put} & & \text{Buy call} \\ \begin{bmatrix} +1 \\ -1 \end{bmatrix} & + & \begin{bmatrix} 0 \\ +1 \end{bmatrix} & = & \begin{bmatrix} +1 \\ 0 \end{bmatrix} \\ \text{Short} & & \text{Buy call} & & \text{Buy put} \\ \begin{bmatrix} -1 \\ +1 \end{bmatrix} & + & \begin{bmatrix} +1 \\ 0 \end{bmatrix} & = & \begin{bmatrix} 0 \\ +1 \end{bmatrix} \end{array}$$

It is well known that a long position and a short position in the stock for the same number of shares is equivalent to no position at all (a perfect hedge).

or rights and warrants, the buyer of a put would have received unfair advantage without this convention.

Buying a call and writing a put is equivalent to a long position because the call buyer gains if the stock price increases while the put writer loses if the stock price decreases. A put purchased against a long position yields downside protection. A call purchased against a short position yields protection against a stock price increase.

The last two illustrations bring out the very important fact that there are two ways to purchase options: calls can be purchased either directly or as a combination of a long position and a put purchase; puts can be purchased either directly or as a combination of a short position and a call purchase. These possibilities set up a direct relationship between put and call prices. If investors judge call prices to be too high relative to put prices, they will achieve a "buy a call" position by purchasing a put and going long, and they will achieve a "buy a put" position directly. If put prices are judged too high relative to call prices, investors will go short and buy a call in order to achieve a "buy a put" position, and they will achieve a "buy a call" position directly. The operation described should in the aggregate, to the extent that investors are rational and aware of these possibilities, and to the extent that new capital is flowing in, keep put and call prices closely related.

The mechanism described so far does depend, however, on a flow of new capital and is not an arbitrage mechanism that would ensure put-call parity. There exists such a mechanism, called *conversion* by means of which a put can be converted into a call and/or a call converted into a put at no risk to the converter and, theoretically at least, with no capital investment.

Suppose that, in the opinion of an individual converter, put and call prices are out of equilibrium, call prices being too high. He can write calls (receiving a premium), buy the stock to be able to meet a possible obligation to the buyers of the calls, and buy puts (paying a premium) to protect himself against a decline in the value of the stock he has purchased. In vector notation this operation is given as follows:

$$\underset{\text{Write a call}}{\begin{bmatrix} -1 \\ 0 \end{bmatrix}} + \underset{\text{Go long}}{\begin{bmatrix} +1 \\ -1 \end{bmatrix}} + \underset{\text{Buy a put}}{\begin{bmatrix} 0 \\ +1 \end{bmatrix}} = \underset{\text{No position}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} \qquad (1)$$

If, on the other hand, put prices are judged by the converter to be too high, he will write puts, go short, and buy calls. By going short he takes no loss if the stock price does decline and he is forced to meet his put contract— what is lost on the put contract is gained on the short sale. The purchase of a call protects his short position against an increase in the price of the stock. In vector notation:

$$\underset{\text{Write a put}}{\begin{bmatrix} 0 \\ -1 \end{bmatrix}} + \underset{\text{Go short}}{\begin{bmatrix} -1 \\ +1 \end{bmatrix}} + \underset{\text{Buy a call}}{\begin{bmatrix} +1 \\ 0 \end{bmatrix}} = \underset{\text{No position}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} \qquad (2)$$

Each of these contracts has a cost or benefit associated with it. The next section discusses these costs and benefits and develops the equilibrium, or parity, relationship between put and call prices.

B. *Put and Call Parity*

Consider first the relationship between put and call prices in a competitive and frictionless (but uncertain) world in which there are no transaction costs (the writer of an option receiving what the buyer pays) and in which the benefit to being short is equal to the cost of being long.[6] Subsequently, any modifications necessitated by the introduction of transaction costs and institutional restrictions on short selling will be introduced.

The following notation is used:

V_t — Dollar value of 100 shares of the stock in question at time t.
P_t — Dollar price of a put option at time t.
C_t — Dollar price of a call option at time t.
i_t — Interest rate on a riskless security at time t.
$p_t, c_t,$ — $\dfrac{P_t}{V_t}, \dfrac{C_t}{V_t}$. Relative put and call prices.

It is assumed that all option contracts are for 100 shares and are written at the market price of the stock at time t. The interest rate and the option prices refer to contracts of the same maturity. Typically, that maturity is 30, 60, 90, or 190 days.[7] Subscripts will be omitted when the meaning is clear.

1. *Options are held to maturity.* Theoretical put and call price parity can be deduced from the arbitrage opportunities which are available to the individual investor and which are represented by equations (1) and (2). The results are most clear if it is assumed that all options are held to maturity.

Consider first the cash flows generated for an individual, with no initial capital, from the operation of converting puts into calls [given by (1)]. The writing of a call yields a positive cash flow of C, and the purchase of a put results in a negative cash flow, P. To go long, V must be borrowed for the length of the outstanding option. The interest cost is $V \cdot i$ and the present value of the interest cost is $\dfrac{V \cdot i}{1+i}$. The certainty rate of interest is used because there is no risk involved to going long; the combination of writing a call and buying a put constitutes an offsetting short position and produces a perfect hedge.[8] It is assumed that the lender is willing, under these conditions, to lend the full amount at the riskless rate. It is also assumed that, for the same reason, no margin deposit is required on the writing of a call. The profit that results from the arbitrage operation is denoted by M. The following equation summarizes the cash flows:

$$C - \frac{V \cdot i}{1+i} - P = M. \tag{3}$$

Consider now the cash flows generated in the conversion of calls into puts [given by (2)]. The sale of a put and the purchase of a call results in a cash flow of P—C. It is assumed that a short position is the obverse of a long

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6. The meaning of the last clause will become clearer below.

7. 190 days is used to represent the contract for 6 months and 10 days.

8. It is, in fact, true that no margin is required if one is both short and long in the same stock.

position, that the short seller receives the interest on the proceeds of the sale of the borrowed stock. In addition, because of his fully hedged position, it is again assumed that no margin is required of the converter either on his short sale or on the sale of the put.[9] The present value of the cash flow on the short position is then $\frac{V \cdot i}{1 + i}$. Denote the profit resulting from this arbitrage operation by N. The cash flows may be summarized as follows:

$$P + \frac{V \cdot i}{1 + i} - C = N. \tag{4}$$

In a competitive world without transaction costs, market equilibrium is reached when $M = N = 0$. Since conversion is assumed to be a costless and riskless operation, individuals will be committed to arbitraging the abnormal difference between put and call prices until all profits are eliminated. Therefore, it can quickly be deduced from either (3) or (4) that

$$C - P = \frac{V \cdot i}{1 + i}, \tag{5}$$

the difference between the put and call price is equal to the present value of the interest cost of borrowing on a sure thing. This position is termed put-call parity.

More simply, put and call prices may be expressed as a fraction of the value of the contract, and these relative premiums may be denoted by small letters c and p, so that

$$c - p = \frac{i}{1 + i} \doteq i. \tag{6}$$

In equilibrium, then, relative put and call prices differ approximately by the sure rate of interest.

In a competitive and frictionless world, put and call prices are closely and simply related, irrespective of investor demands, by the sure rate of interest. If the interest cost is constant, any change in call prices is immediately and fully offset by an equal change in put prices, the call price always exceeding the put price by the interest cost. If the demand for calls is indeed greater than the demand for puts, calls will be created through the conversion process until the difference in option prices reflects only the sure rate of interest.

Since the put and call market is a small fraction of the capital markets and since puts and calls for a particular stock generally represent only a small fraction of the shares outstanding, any adjustment toward put-call parity is likely to involve changes in P and C rather than changes in the sure rate of interest or the price of the stock. Therefore, it can be said that the sure rate of interest determines the difference between put and call prices expressed as a fraction of the shares' value and that the sure rate of interest and the stock price determine the absolute difference between put and call prices.

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9. Note, however, that the short must pay any dividends declared on the stock. The long receives dividends.

Nothing in the theoretical discussion presented here specifies, however, what the level of either absolute or relative put and call prices ought to be. The basic equation (6) specifies only that whatever c and p may be, their difference should equal i. Presumably more volatile issues will have both higher c's and higher p's than less volatile issues. For example, if the sure rate of interest is 3 per cent per one-half year and if 190 day call premiums are 20 per cent on very volatile issues and 10 per cent on very stable issues, the corresponding put premiums ought to be 17 per cent and 7 per cent, respectively.
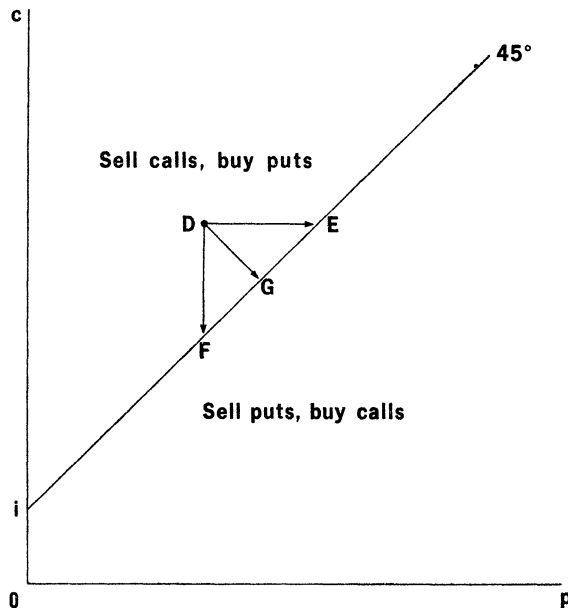
PUT-CALL PARITY

FIGURE 1

Put and call parity is illustrated in Figure 1. The 45° line represents points of put-call parity. As option contracts are currently written (the writer taking the greater risk), premiums must be positive, and therefore all points must lie in the first quadrant. The intercept represents the sure rate of interest. Points below the line imply that put prices are high relative to call prices; and, in that case, there is an incentive to sell puts and buy calls (and go short to complete the hedge) which will reduce put premiums and raise call premiums. Points above the line imply that call prices are high relative to put prices; and, in that case, there is an incentive to sell calls and buy puts (and go long to complete the hedge) which will reduce call premiums and raise put premiums.

The path of adjustment toward equilibrium cannot be specified with certainty, and only some of the possible routes can be discussed here. Consider the disequilibrium point D in Figure 1. If put prices were to bear the burden of adjustment, the new equilibrium would be point E. If call prices were to

bear the burden of adjustment, the new equilibrium would be point F. In either of these cases adjustment would have to come through the inflow or outflow of new capital affecting only the one option—put or call. Conversion would tend to affect both premiums to an equal and opposite extent. If one can make the assumption that expectations remain unchanged during the adjustment process, one can argue that the net capital flow into puts and into calls ought to be about equal and that neither put nor call prices ought to bear the burden of adjustment. In that case the final position is likely to be point G.

If the probability distribution of expected percentage stock price changes does not change greatly over time, for an individual stock, relative put and call prices will remain at approximately the same level; and, therefore, all observations for that specific stock will tend to lie at approximately the same point on the parity line. Different stocks with different chances of loss and gain will lie at different points along the line.

2. *Options exercised before maturity.* The put-call parity line may no longer be uniquely defined if the possibility that options are exercised before maturity is allowed. This is due to the fact that all the cash flows resulting from positions (1) and (2) are not, in that case, given by (3) and (4). First, the present value of the interest paid (or received) is now less since the funds are not needed for the full length of the option period. Second, if the option written by the converter is exercised against him before maturity, the option purchased by him may still have a realizable value which is not included in (3) or (4). Consider, for example, the case of conversion of puts into calls. If the stock price rises, the call may be exercised against the converter (who uses his long position to cover) even though a considerable time remains until maturity of the contract. In that case, the put which the converter holds is likely to have some value because the probability is likely to be greater than 0 that the stock price will decline below the exercise price before maturity. A similar situation may arise for the case of the conversion of calls into puts if the stock price declines. When the put written by the converter is exercised against him before maturity, the call contract will still have a value greater than zero.

The effect of these considerations is that the cash flows, $-\dfrac{V \cdot i}{1 + i}$ and $-P$ in (3) and $\dfrac{V \cdot i}{1 + i}$ and $-C$ in (4), are overstated by a factor which depends on the probability that the option written by the converter will be exercised before maturity. In addition, profits are no longer certain; and the position, (1) or (2), is not, strictly speaking, a perfect hedge (even though the only "risk" is that the converter does better than contracted for). If the analysis is carried through, two parity lines on either side of the line given by (6) will result, the relevant line depending on the initial disequilibrium situation.

The above conclusions depend, however, on a certain amount of irrationality on the part of the buyer of the options written by the converter. If, in the circumstances described above, the option purchased by the converter

has value at a date before maturity, then the option sold by him is likely to have a value over and above its current exercise value. This follows from intuitive considerations based on the assumption of symmetry in percentage stock price changes as well as from the put-call parity theory. The customer of the converter would therefore be irrational to exercise his option. Instead, if he wishes to liquidate his position, he would sell the option itself, leaving the converter's position unaffected.

Consider again the case of the conversion of puts into calls. Take a point in time, t, after the hedge has been established and n days before maturity. Suppose that the price of the 100 shares of stock has risen by ΔV_t from the initial value of V_0 to the current value, V_t. Suppose also that the put purchased by the converter is still worth $_nP_t > 0$. Then, in a perfect market, the value of the call must be greater than its simple exercise value of ΔV_t. One way to see this intuitively is to remember that a positive put price implies that there is a chance for the stock value to decline below the contract price; that is, by more than ΔV_t. If the market believes that the stock value has a chance of falling by more than ΔV_t, it must, in equilibrium, also believe that it has a chance of increasing more than ΔV_t above the contract price. (Otherwise, the current value would not be V_t.)

A second way to show this is to consider again the conversion mechanism, which now is concerned with options written "away from the market"—at V_0 rather than V_t. If calls sold only for their exercise price and if $_nP_t > 0$, there would be an incentive to convert calls into puts as per equation (2). The costs and benefits are

$$-_nC_t + _nP_t + \frac{n i_t V_t}{1 + _n i_t} + \frac{\Delta V_t}{1 + _n i_t} = _nN_t.$$

The term, $\dfrac{\Delta V_t}{1 + _n i_t}$, is required to reflect the profit which is assured by selling short at V_t at the same time that the long position, represented by the purchase of a call and the sale of a put, is at V_0. In equilibrium $N = 0$, and the resulting relationship between put and call prices (dropping $1 + i$ in the denominator) is

$$_nC_t = \Delta V_t + _n i_t V_t + _nP_t. \tag{7}$$

The same equilibrium would result if the conversion of puts into calls were considered.

Clearly it would be irrational for the holder of a call to exercise for a profit of ΔV_t when he could sell his call at a value equal to the sum of the right hand side of (7). Only when $_nP_t = 0$ is the exercise of the call option rational.[10] In addition, as long as there is uncertainty about future stock price movements, such a situation is most likely to arise at maturity of the options.[11]

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10. The interest cost does not enter in because the new holder of the stock who exercised his call must bear this opportunity cost. If the call is sold and not exercised, its price is $_nC_t = _n i_t.V_t + \Delta V_t$ because the holder of the call does not bear the opportunity cost of having funds tied up in the stock.

11. Transaction costs and matters of convenience will, of course, in actuality sometimes cause the hedge position to be liquidated before maturity.

In the same way, the converter who buys calls and writes puts is unlikely to find his put contract exercised against him under conditions where the call has market value greater than the interest that would be foregone on the liquidation of his short sale. Suppose the stock price falls by ΔV_t so that the exercise value of the put is ΔV_t. If, in that case, the call has market value greater than $_n i_t V_t$, the put must also have market value greater than ΔV_t. The put would then be sold and not exercised.

The analysis can now safely (or at least with justification) proceed on the assumption that the hedge position is maintained to maturity; and if it is not, with the knowledge that, in a rational market, the hedge will never be liquidated, so that the option purchased by the converter yields him additional profit.[12] Therefore, the proper cash flows have been considered in (3) and (4), and the put-call parity line is unique.

C. *Institutional Restrictions*

In the real world, a number of institutional complications may make it difficult for a position of put-call parity to be reached. If trading activity is light due to a lack of understanding of the put and call market (say), arbitragers may find it difficult to locate buyers or sellers of puts or calls. In general, such market thinness will show up in the form of high transaction costs (since, at some costs, a buyer or seller is usually to be found).

Transaction costs are indeed high and are probably the principal reason for divergences from put-call parity. The arbitrage operation involves the certain costs of brokerage fees on the sale or purchase of stock and options, the fee to the put and call dealer for bringing buyer and seller of options together, and the endorser's fee. For a nonmember of the New York Stock Exchange, these transaction costs are extremely high. Therefore, most of the arbitrage is carried out by "conversion houses," who are N.Y.S.E. members and specialize in this activity, and who are not required to pay nonmember commission rates or endorsement fees. The conversion costs on 100 shares of a $50 stock, if a member firm undertakes the operation, are approximately the following:[13]

Buy stock (and sell before maturity) or sell short (and cover before maturity)

Transfer taxes	$ 6.00
Floor brokerage	13.20
Put and call dealer's spread	40.00
	$59.20

The put and call dealer's spread is an uncertain quantity. The figure shown is the approximate median difference between what the writer of a call received and the buyer paid, determined for calls selling in the $400-$600

12. The referee has pointed out that Boness (1962) and Samuelson (1965) have proved that, in a perfect market, calls will be held until maturity if there is no differential risk aversion between calls and the underlying common stock.

13. The examples take the point of view of the converter. Such a point of view may not be familiar to the individual investor who thinks of conversion as the exchange of one option for another. See, for example, Sarnoff (1968), pp. 141-146.

price range.[14] If the market is active, buyers and sellers will abound, and this "middlemen" fee will decrease.[15] If the market is very inactive, the dealer's spread may be greater than $40.

In addition to the costs already mentioned, the arbitrager also bears the costs of borrowing funds to assume a long position or of lending the funds received from a short sale. Such costs are likely to be small.

In total, the transaction costs described make it possible for put and call prices to diverge from parity by one to two per cent on either side of the parity line shown in Figure 1. The difference, that is, between relative put and call prices can be greater or less than the sure rate of interest by the amount of transaction cost expressed as a per cent of the value of the stock.

In addition to transaction costs, institutional restrictions on short selling may also cause a divergence from parity in those cases in which parity would be brought about by going short; i.e., in cases in which an incentive existed to convert calls into puts (equation 2; points below the parity line in Figure 1). Since most buyers desire calls, and since equal quantities of puts and calls tend to be written,[16] the need for a conversion of calls into puts is unlikely to arise. But were it to arise, it is not clear that the restoration of equilibrium would be easy since the benefit of going short is often not equal to the cost of going long. Interest is never explicitly earned on the proceeds of a short sale—or any sale—held with the broker. There is no benefit to having a credit balance. Interest may, however, be saved if the seller (short or long) has an already existing debit balance against which the proceeds of a sale may be credited.[17] Therefore, the individual investor who is likely to be faced with these institutional restrictions is not likely to convert calls into puts. Sophisticated "conversion houses" will, however, know how to reap the full benefit of short sales and so will not be affected by the possible restrictions discussed above. Since these arbitragers are also members of the N.Y.S.E. they are unaffected by the Exchange's regulations with respect to short sales and may, therefore, use the proceeds of the short sale.

Finally a brief consideration should be given to the taxation of options. If puts and calls are taxed differently or if taxes change the interest cost, the slope and/or intercept of the put-call parity line may change. This is a complicated matter—first, because the tax treatment of options is by no means clear, and second, because it is difficult to predict the equilibrium put-call parity line when taxes, even if well known, differ significantly among individuals.[18]

14. Reported in the SEC study, *Report on Put and Call Options*, (1961), p. 90.

15. Sarnoff (1968) reports that when holders of options desire to convert, the put and call dealer's fee is often only $12.50 (p. 142). In such a situation a ready market involving no risk is available.

16. SEC, *Report on Put and Call Options*, (1961).

17. The principal difference between a short sale and a long sale appears to be that the proceeds of a short sale may never be withdrawn, whereas the proceeds of a long sale may be withdrawn so long as the margin requirement is satisfied. An account not already in debt, therefore, has a considerable disincentive to sell short, and only accounts with debit balances (which are more "speculative" [?]) would not face such a disincentive.

18. This analysis of tax effects was motivated and helped a great deal by a reading of B. Malkiel and R. Quandt (1968). That monograph presents a more detailed discussion of the taxation of options than is possible here.

The effect of taxes on the parity line depends on the disequilibrium position from which one starts, on whether stock prices are expected to increase or decrease, and on whether the holding period is expected to be greater or less than six months.

Let us now summarize the effect of taxes in these various states of the world. Let t_c = capital gains tax rate, t_o = ordinary income tax rate which is assumed to be equal to the effective short term capital gains rate.[19]

Consider first the disequilibrium situation in which call prices are too high relative to put prices. (Incentive to sell calls, buy puts, go long.) If stock prices increase (and calls are exercised), the effect of taxes is the following:

The interest cost of carrying the stock is unaffected.	$-V \cdot i$
The seller of the call delivers his stock against the call. The loss on the call offsets the gain on the long position. His selling price is the contract price plus the call premium, which is therefore taxed as capital gains.	$C(1 - t_c)$
The put is worthless and is sold to the dealer for \$1 to generate a capital loss to be offset against the gain on writing of the call.	$-P(1 - t_c)$

Adding the cash flows yields[20]

$$C(1 - t_c) - P(1 - t_c) - V \cdot i = M_1 \qquad (8)$$

where "1" refers to a stock price increase.

If stock prices decrease (and puts are exercised) the effect of taxes is the following:

Interest cost	$-V \cdot i$
The call is not exercised and the writer's premium is, in this case, taxed as ordinary income no matter what the maturity of the call.	$C(1 - t_o)$
The put is sold at a capital gain so that he is left with $(-\Delta V - P) \cdot (1 - t_c)$ where $\Delta V < o$. The long position is liquidated at a loss — $\Delta V(1 - t_c)$. If there is income elsewhere against which to offset $\Delta V t_c$, the effective cost of the put premium is	$-P(1 - t_c)$

Adding the cash flows yields

$$C(1 - t_o) - P(1 - t_c) - V \cdot i = M_2 \qquad (9)$$

where "2" refers to a stock price decrease.

Consider now the disequilibrium situation in which put prices are too high relative to call prices (incentive to sell puts, buy calls, go short). Since this situation is less likely and since the derivation of the effect of taxes is similar, only the final results are presented. If stock prices were certain to increase, the arbitrage profit would be given as follows:

$$-C(1 - t_c) + P(1 - t_o) + V \cdot i = N_1. \qquad (10)$$

19. This is not strictly true because capital losses can only be offset against a maximum of \$1000 ordinary income while there is no limit on the amount of short or long term gains against which capital losses may be offset.

20. The tax benefits can be greater than indicated by (8) because the seller of the call can generate higher paper losses to be offset against gains elsewhere in his portfolio. See Malkiel and Quandt (1968).

the obverse of (9). If stock prices were certain to decrease, these arbitrage profits would be

$$-C(1 - t_c) + P(1 - t_o) + V \cdot i = N_2. \tag{11}$$

Equation (11) is not the obverse of (8) because of the tax treatment of capital gains on short sales. No matter what the holding period, these gains are treated as short term. Therefore t_o instead of t_c enters.

If q_1 and q_2 are the probabilities of a stock price increase and a stock price decrease respectively, the expected *percentage* arbitrage profits for the two initial disequilibria are given by

$$c(1 - q_1 t_c - q_2 t_o) - p(1 - t_c) - i = E(m) \tag{12}$$

and

$$-c(1 - t_c) + p(1 - t_o) + i = E(n). \tag{13}$$

If $E(m)$ and $E(n)$ go to zero (no risk aversion), two different parity lines result:

$$c = \frac{1}{1 - q_1 t_c - q_2 t_o} i + \frac{(1 - t_c)}{1 - q_1 t_c - q_2 t_o} p \tag{12.a}$$

and

$$c = \frac{1}{1 - t_c} i + \frac{(1 - t_o)}{(1 - t_c)} p. \tag{13.a}$$

When the option maturity is less than 6 months so that $t_o = t_c$, the same parity line results whatever the initial disequilibrium position:

$$c = \frac{1}{1 - t} \cdot i + p. \tag{14}$$

In summary, the effect of taxes should, in all cases, be to make the coefficient of i greater than one. For maturities less than 6 months, the coefficient of p is unchanged. For maturities greater than 6 months the effect of taxes is to create two boundary lines. Since $t_c < t_o$, one boundary line will have a slope greater than 1 [line (12.a)]; and one, a slope less than 1 [line (13.a)]. Observable put and call prices are likely to fall within these boundaries. In that case an empirically estimated parity line may be little different from the line to be expected in a no-tax world.

The analysis turns now to empirical tests of the put-call parity theory which do not take consideration of the institutional restrictions discussed here, partly because they would be difficult to formulate and partly because it is not clear that these restrictions materially affect the equilibrium relationship between put and call prices developed earlier.

III. EMPIRICAL TESTS OF PUT-CALL PARITY

There are two principal formulations that may be used to test the put-call parity theory: the "absolute" formulation given by (5) or the "relative" formulation given by (6). Because a relative option price is more meaningful, especially when inter-company comparisons are made, and because it was felt

that this was the variable considered by market participants, the "relative" formulation is relied upon.[21] Empirical tests using the "absolute" formulation are, however, also carried out and will be discussed if findings conflict.

The basic hypothesis is then tested by running the following regression:

$$c = a_0 + a_1 i + a_2 p + u \qquad (15)$$

where c and p are relative prices of 90 or 190 options, i is the certainty rate of interest now stated on an annual basis, and u is a random error term. Probably the most interesting question that can be answered by the empirical tests is whether the slope of the put-call parity line shown in Figure 1 is actually unitary. For this reason (15) is formulated somewhat more loosely than is implied by (6), and the dependent variable is not (c-p), which would constrain the slope to one. However, what the dependent variable ought to be is now less clear since the causation cannot be said to run in any particular direction between c and p. Of the two, c has a larger variance and was therefore chosen as the dependent variable.

The theory is fully supported if

$$a_0 = 0, a_2 = 1, a_1 = \begin{cases} .25 \text{ for the 90 day maturity} \\ .50 \text{ for the 190 day maturity.} \end{cases}$$

Since the interest rate is in annual terms, the adjustment for maturity comes in the coefficient which is not now expected to equal one.[22]

The data are first described and then the regression results are presented, in turn, for two sets of data—"Regular" companies and "New Business" companies.

A. *The Data*

Each week the Put and Call Dealers Association submits, to the Securities and Exchange Commission, put and call prices and trading volumes for 15 "Regular" companies and 10 "New Business" companies. The 15 "Regular" companies remain unchanged from week to week while the "New Business" companies vary, presumably representing the stocks with the greatest amount of activity. In each case the option price is a representative price for the week.

Two years (1966-67) or 104 weeks of put and call prices—90 and 190 day— were available for each of the "Regular" companies.[23] Since the prices represent nominal quotations,[24] the data on the volume of options were visually

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21. In addition, heteroscadasticity may be a problem in the "absolute" formulation.

22. Because of uncertainty as to the time period for which options are held, the interest rate was not adjusted directly, before running the regression. In addition i, instead of the theoretically specified $i/1+i$, is used. However, if i is not adjusted before hand, it can be shown that, for the maturities and interest rate levels dealt with here, i is preferable. If maturities are known, the correct variable is $\dfrac{i/s}{1+i/s} = h_1$ where s is the factor required to put i on the same maturity basis as the options. The adjustment implied by running a regression on i or $i/1+i$ gives $i/s = h_2$ or $1/s(i/1+i) = h_3$. For the magnitudes involved here, h_2 is closer to h_1 than is h_3. (For $i = .04$ and the 90 day maturity where $s=4$, $h_1 = .00991$ $h_2 = .01$, $h_3 = .00962$.)

23. The author is grateful to Mr. James Drenning for making the data available to him.

24. Any new put or call is always negotiated. The final price cannot be determined until a writer

examined for a one-half-year sample-period in order to identify those stocks with inactive markets whose prices would tend to be less reliable. On this basis, five companies were immediately excluded from further consideration.[25] The remaining companies appeared to have a relatively active option market.[26]

One year (1967) or 52 weeks of put and call prices—90 day and 190 day— were available for the "New Business" companies.[27] Ten stocks were quoted in every week but one (which had 9), so that a total of 519 observations representing 110 different companies were available. The volume of transactions was also examined and, indeed, showed that these companies possessed active option markets.

In order to test the put-call parity theory, the stock price for each company and each week and the certainty rate of interest for each week were also required. The Friday-closing stock price was chosen, and the end-of-week three-month Treasury bill rate (at market) was used to represent the short-run certainty rate of interest. Needless to say, considerable error may enter because the option price need not correspond directly to the stock price in existence at the Friday closing. In addition, it is assumed that the three-month bill rate is representative of the cost of short-term funds and could be used to explain both the 90 day and 190 day put-call price difference. Collection of the 6 month bill rate was considered; but since the 3 and 6 month bill rates are highly correlated and since the interest variable was not expected to be significant in this rather imperfect market, the additional work was not undertaken.

B. *"Regular" Companies*

1. The "Regular" companies are first considered as 10 separate sets of time series (each with 104 observations), and regression results for (15) for these 10 companies are presented in Table 1. To eliminate serial correlation, the variables are transformed on the assumption of a first order auto-regressive scheme; and only these results are presented.[28]

For both maturities there is strong support for the basic relationship between put and call prices hypothesized earlier. Relative put and call prices are strongly positively correlated, and the coefficient, a_2, is close to one as predicated, although only the asterisked coefficients (14 of 20) are not more than 2 standard errors from one.[29] For data of this quality there is likely to be

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is found and buyer and writer agree. Only "special" option prices (on outstanding options) which are quoted in Put and Call Dealers advertisements in the daily papers reflect true transactable prices, and these are subject to prior sale or to other market changes.

25. Republic Steel, Northern Pacific RR, Southern Pacific RR, Lockhead, Int. Tel. and Tel.

26. U.S. Steel, Standard Oil of N.J., Chrysler, General Motors, General Electric, New York Central RR, Radio Corp. of America, General Dynamics, Anaconda, Western Union.

27. The author is grateful to Professor Robert Doede for making these data available to him.

28. It is assumed that the residuals of (15) follow the following pattern:
$$u_t = \gamma u_{t-1} + e_t$$
where e_t is independently distributed. If the assumption is correct, the following regression will not show serial correlation:
$$c - \gamma c = a_0(1 - \gamma) + a_1(i - \gamma i) + a_2(p - \gamma p) + e.$$
If $\gamma = 1$ the appropriate variables are first differences. See, e.g., Johnston (1963), Ch. 7, for a discussion of serial correlation.

29. Standard error $=$ t/coefficient. Two standard errors is the (two tailed) 5% level of significance.

TABLE 1

PUT-CALL PARITY REGRESSION COEFFICIENTS AND t VALUES,[1] "REGULAR" COMPANIES

$$(c - \gamma c) = a_0(1 - \gamma) + a_1(i - \gamma i) + a_2(p - \gamma p)$$

	90 Day						190 Day					
	a_0	a_1	a_2	γ	r^2	DW[2]	a_0	a_1	a_2	γ	r^2	DW[2]
U.S. Steel	.8774 (1.490)	.0135 (.119)	1.0529* (21.42)	.910	.957	1.90	.3290 (.685)	.1171 (1.34)	1.1002 (35.96)	.909	.969	1.94
St. Oil, N.J.	.6478 (1.837)	−.0563 (−.837)	1.0859 (30.38)	.875	.977	1.90	1.4294 (2.839)	−.0871 (−1.240)	1.0535* (22.59)	.543	.944	2.41
Chrysler	2.2208 (2.296)	−.0462 (−.255)	.9180* (16.96)	.925	.958	1.80	1.5465 (1.080)	.3263 (1.265)	.9064* (15.45)	.940	.926	1.94
G.M.	.1315 (.219)	.1005 (.868)	1.0410* (18.30)	.878	.958	1.88	2.3999 (2.377)	−.1550 (−.806)	.9250* (12.24)	.862	.931	1.91
G.E.	2.9390 (3.991)	−.1765 (−1.286)	.8200 (13.77)	.890	.957	1.98	3.5656 (3.4014)	−.2218 (1.184)	.8549 (15.12)	.939	.960	2.01
N.Y. Central	3.3616 (3.895)	−.1899 (−1.131)	.8262 (17.67)	.925	.948	2.02	1.4899 (1.681)	−.0294 (−.176)	1.0194* (24.36)	.860	.963	2.07
R.C.A.	2.0561 (3.155)	−.1766 (−1.352)	1.0124* (26.29)	.901	.952	1.75	1.6866 (2.480)	−.1799 (−1.310)	1.0807 (38.83)	.912	.967	1.82
Gen. Dynamics	.1034 (.122)	.2600 (1.632)	.9481* (21.68)	.785	.899	2.38	.9889 (.726)	.3526 (1.376)	.8885* (15.07)	.678	.881	2.13
Anaconda	.3803 (.462)	.1061 (.616)	1.0193* (22.57)	.854	.950	1.77	.0132[3] (.214)	.0843 (.520)	1.0128* (35.79)	1.000	.928	2.15
West. Union	.3222 (.198)	.3302 (1.121)	.9492* (11.10)	.903	.861	2.39	2.1822 (1.722)	−.0193 (−.082)	.9769* (22.58)	.920	.951	2.07

[1] In parentheses.
[2] Durbin-Watson statistic.
[3] This is not estimated a_0 but the constant resulting if all other variables are in first difference form.

measurement error which will tend to bias downward the coefficient of p.[30] Therefore, and especially because the direction of causation is not clear, all regressions were run with the positions of c and p reversed. The implied slope of c on p was, in this case, always greater than one. Thus, the regression estimates either bracket the expected coefficient of one or are not significantly different from one in every case but three (those values of a_2 significantly greater than none).

The findings for the interest rate coefficient, a_1, are not in accord with the theory. That coefficient is almost uniformly insignificant (except, perhaps, in the case of General Dynamics); and, in addition, the sign is negative about half the time. Different formulations of the "relative" model, such as implied by (6), produced no improvement. This finding should, however, not be too surprising in light of the fact that the put and call market for the "Regular" companies is a relatively inactive one in which the change in market interest rates may be inconsequential when compared to transaction costs and the imperfectness of the data. Note that the constant term, a_0, is consistently positive and is significantly different from zero for half the regressions, and this occurs despite the fact that γ is close to one. (For $\gamma = 1$, the constant should drop out.) Apparently, a_0 (which is in percentage points) picks up the interest costs which are not reflected in the coefficient of the treasury bill rate.

The average difference between the level of put and call prices is in the reasonable range approximating interest costs for each of the contract periods, although it is lower than expected when the high transaction costs and the market's presumed preference for calls is considered. Columns 11 and 12 of Table 2 present the average differences. The grand average for all companies stated on an annual basis is 4.36 per cent for the 90 day maturity and 3.08 per cent for the 190 day maturity. Each of these figures is less than the corresponding average treasury bill rate for the same period (4.58 per cent and 4.84 per cent). The apparent low interest cost may be due to the fact that options are not held to maturity. The fact that apparent interest costs are not the same for the two maturities may be due to differential transaction costs—on an annual basis four transactions are required in the 90 day maturity, only two in the 190 day maturity.

Running regressions in "absolute" terms raises r^2 (because high priced stocks have high priced calls), tends to lower the coefficient of p and increases the significance of the constant term. But as before, the theoretical coefficient of one was generally bracketed by running the regression in both directions.

The findings reported thus far support the put-call parity theory and show that relative put and call prices are closely related regardless of stock price movements or expectations about future movements.

2. A difficulty with the "Regular" data is the infrequency of changes in C or in P.[31] On the assumption that the most accurate reporting occurred at these "change" dates, the observations reflecting such changes were analyzed separately.

30. See Johnston (1963), Ch. 6.

31. Of the 1040 observations, a total of 200 cases existed in which the 90 day P or C or the 190 day P or C changed.

It should be noted that there were several observations contradictory to the theory. In six cases (six companies), absolute 90 day put and call prices moved in opposite directions. In nine cases (five companies), 190 day put and call prices moved in opposite directions. These findings could conceivably be consistent with the put-call parity theory if interest rates changed drastically. But the interest rate never fluctuated enough to warrant such a conclusion. They may also be explained by lags in reporting and/or in market adjustment.

Because few observations remained for any single company (a minimum of 13 and a maximum of 29), all 200 observations were pooled in order to test the put-call parity theory.[32] The results are presented below. Standard errors of coefficients (not t ratios) are given in the parentheses.

$$90 \text{ Day:} \quad c = 1.046 + .1129i + .9419 p \quad r^2 = .852$$
$$\qquad\qquad\quad (.493) \ (.099) \quad (.028)$$

$$190 \text{ Day:} \quad c = 1.488 + .2253i + .9070 p \quad r^2 = .852$$
$$\qquad\qquad\quad (.670) \ (.135) \quad (.027)$$

The results continue to support put-call parity, perhaps even more strongly. The correlation is strongly positive. The fact that the coefficient of p is less than one may again be due to measurement error. At any rate, running the regression with the positions of c and p interchanged again brackets the expected coefficient of one. The interest rate coefficient is more uniformly positive and, in the 190 day maturity, significant; but it does not appear to capture all the difference between c and p, the remainder apparently appearing as before in the constant term which is now significant. Although the interest rate coefficients are lower than expected, note that the 190 day coefficient is approximately twice that of the 90 day coefficient, as predicted.

Running the "absolute" formulation produced little change except that the interest cost coefficient becomes uniformly significant and closer to the theoretical expectation. The regression results follow (standard errors in parentheses):

$$90 \text{ Day:} \quad C = 14.361 + .2918iV + .9383 P \quad r^2 = .952$$
$$\qquad\qquad\quad (8.114) \ (.034) \qquad (.026)$$

$$190 \text{ Day:} \quad C = 30.208 + .4542iV + .8985 P \quad r^2 = .960$$
$$\qquad\qquad\quad (10.599) \ (.046) \qquad (.023)$$

The constant term now represents the dollar difference between call and put prices unexplained by the interest cost variable.

3. If it can be assumed that the conditions affecting c and p remained, for each company, relatively unchanged over time, the observations for each company may be averaged and compared. These figures are presented in Table 2 and plotted in Figure 2. The lines in the figure represent the parity lines for the 90 day and 190 day options. The slopes are one, and the intercepts are ¼ and ½ of the average three-month and six-month treasury bill rates for 1966-1967.

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32. Serial correlation is not meaningful in this context and was not a problem. Therefore, no transformations of the data were necessary.

TABLE 2

Average Stock Price, Average Relative Option Prices and Standard Deviations 1966-1967

Company	Stock Price Mean (1)	Coef. of var. (2)	90 Day Call Mean (3)	St. Dev. (4)	90 Day Put Mean (5)	St. Dev. (6)	190 Day Call Mean (7)	St. Dev. (8)	190 Day Put Mean (9)	St. Dev. (10)	Av. Call-Av. Put 90 Day (11)	190 Day (12)
U.S. Steel	44.01	9.50	7.27	.36	6.12	.17	10.37	.30	8.67	.22	1.15	1.70
St. Oil, N.J.	67.46	8.21	6.10	.27	5.25	.26	9.02	.52	7.59	.40	.85	1.43
Chrysler	43.79	18.25	8.12	.92	6.67	.81	11.72	1.05	9.47	1.41	1.45	2.25
G.M.	81.94	10.96	5.91	.44	5.10	.59	8.45	.76	7.30	.78	.81	1.15
G.E.	99.23	10.48	6.78	.57	5.70	1.04	9.83	1.01	8.46	2.50	1.08	1.37
N.Y. Central	73.72	10.73	8.34	.63	7.14	.47	11.97	1.08	10.40	.64	1.20	1.57
RCA	51.05	10.07	8.23	.45	7.01	.40	11.44	.69	9.83	.68	1.22	1.61
Gen. Dynamics	57.62	14.86	9.04	.51	8.16	.62	12.70	1.80	11.46	1.54	.88	1.30
Anaconda	87.97	9.90	7.75	.87	6.81	.63	11.36	1.43	9.96	1.48	.94	1.40
West. Union	39.00	15.35	8.96	.77	7.68	.72	12.61	1.36	10.92	1.29	1.28	1.69
Grand Average			7.65		6.56		10.95		9.41		1.09	1.54
Equivalent Annual Basis											4.36	3.08

All points appear to lie along a line of common slope, 1; however, the observations of 190 days maturity lie slightly but consistently below their parity line. These findings are in full agreement with the regression results for each of the companies and for the pooled "change" observations. In all cases the slope is close to one while the difference between relative put and call prices, particularly for the 190 day maturity, tends to understate the true costs of arbitrage even in a frictionless world.
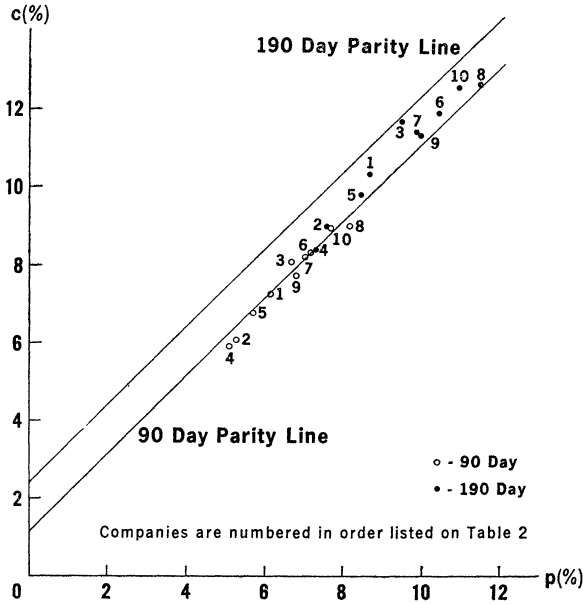

FIGURE 2

4. The dispersion of the various companies along the parity line reflects the different expected volatilities of the stocks in the sample. General Dynamics was, on the average, apparently expected to undergo the greatest variability; General Motors was expected to be the most stable. Past variability as measured by the coefficient of variation of the stock price appears to have a slight relationship to expected variability as measured by the level of the relative option price.[33] But the relationship is not strong and a complete investigation of this matter is beyond the scope of this paper.

Even individual companies were found to have considerable variability over time in relative option prices. The standard deviations of c and p are given for each company in Table 2. Chrysler, which had one of the more active put and call markets, shows a range in the relative 190 day call price from 9.84 per cent to 14.29 per cent. A great deal of this variability is undoubtedly due to the fact that option prices are nominal and change much less frequently than stock prices, the ratio of the two often changing only because the stock price changes. If, however, there is variability in the "true" relative option prices,

33. The data are in Table 2.

interesting implications about investor expectations as to the behavior of stock prices result; namely, that the distribution of percentage stock price changes is not expected to be stationary through time. The evidence available so far appears to indicate, to the contrary, that the distribution of past percentage stock price changes has been stationary.[34] Again, the consistency of this indirect evidence on market estimates of stock price variability with the direct evidence from past stock price behavior is an interesting area for future research which cannot be carried out here.

C. *"New Business" Companies*

1. The "New Business" set of data represents a much more active put and call market and, in general, a much more volatile group of stocks than the "Regular" set of data. As a result, put and call prices ought to be more meaningful. They will also be much larger. Average relative put and call prices are as follows for the two sets of data.

	90 Day		190 Day	
	p	c	p	c
"Regular" (1040)	6.56%	7.65%	9.41%	10.95%
"New Business" (519)	9.55%	11.17%	13.55%	15.70%

For example the market expected, in the case of "New Business" companies, an average upward fluctuation of at least 15.70 per cent during a 190 day period but of only 10.95 per cent in the case of "Regular" companies. The variability of relative option prices was also greater. Some 190 day calls were greater than 25 per cent; and since a sprinkling of "Regular" companies appeared in the "New Business" sample, some calls were quite low priced.

The average difference between relative put and call prices is also greater in the more volatile market, but the figures are in the reasonable range. The annualized 90 day difference is 6.48 per cent and the annualized 190 difference is 4.30 per cent, which compares with average 3 and 6 month treasury bill rates of 4.30 per cent and 4.61 per cent during 1967. The greater difference in the 90 day maturity is again probably due to the higher annual transaction costs in that market.

2. The regression results for the pooled "New Business" companies are presented in the first part of Table 3. The highly significant positive correlation between relative put and call prices is again demonstrated.

Some differences from the "Regular" regressions are immediately noticeable. First, the interest rate is now significantly positive as the theory predicts. In addition, the coefficient is of the proper magnitude in the 190 day maturity, but it is approximately twice the predicted value in the 90 day maturity. In general, however, the significance of the treasury bill rate, which is a proxy for the interest costs of conversion and could be swamped by other factors such as transaction costs, is encouraging. Second, the constant is significantly negative in contradiction to the theoretical expectation of zero.

Third, the coefficient of the relative put price, although close to one, is significantly greater than one in the case of both maturities. A number of

34. See Blume (1968).

alternative formulations of the basic regression equation (15) were tried to see if the results would be sensitive to such changes. As before, the positions of p and c were reversed in the regression. However, the implied coefficient of p was greater than one, as before, and therefore the two regressions did not bracket the expected coefficient, 1, but spanned a value slightly greater than that shown in Table 3. Constraints were imposed on some coefficients to see if other co-efficients would improve, but again no significant improvement occurred. Forc-ing the regression through the origin only reduced the coefficient of the 90 day p from 1.114 to 1.083, and that of the 190 day p from 1.133 to 1.106. Both

TABLE 3
PUT-CALL PARITY REGRESSIONS, "NEW BUSINESS" COMPANIES
(standard errors are in parentheses)

	"Relative" Formulation					r^2
90 Day:	c =	−1.742 (.385)	+	.5301i (.081)	+	1.114p (.017)	.899
190 Day:	c =	−2.115 (.457)	+	.5743i (.097)	+	1.133p (.014)	.924

	"Absolute" Formulation					r^2
90 Day:	C =	33.914 (2.374)	+	.1476 iV (.024)	+	.9992P (.011)	.989
190 Day:	C =	44.510 (2.689)	+	.1423 iV (.027)	+	1.0170P (.009)	.993

are still significantly different from one. At the same time, the coefficient of i changed radically away from the expected value (to .196 and .171 for the 90 and 190 day maturities, respectively). Alternatively, constraining the co-efficient of p to 1 produced no change in the interest rate coefficient.

The "absolute" formulation produced results apparently much closer to theoretical expectations, at least as far as the slope of the put-call parity line is concerned (see Table 3). The coefficient of P is not significantly different from one, and the cost of funds term, iV, has a highly significant coefficient which is, however, lower than in the "relative" formulation. The constant term, which is also significant, represents the dollar difference between put and call prices not explained by the interest cost variable. The nature of the differ-ence between the coefficients of p and of P is the same for "New Business" and "Regular" companies: the coefficient tends to be less in the "absolute" formulation. What is different in the case of "New Business" data is the fact that the coefficients of both p and P are larger and that the difference between the coefficients of p and P is now much more significant. It is not clear that the "absolute" formulation, which comes closer to theory, should be believed in this particular instance. In addition, running the "absolute" formulation with the positions of C and P reversed results in coefficients of P which span an average coefficient greater than one.

It appears then that, although the put-call parity theory again receives

strong support, the "New Business" regressions indicate a slope of the put-call parity line which may be too high and which is not consistent with the "Regular" data. In other words, the "New Business" regressions appear to indicate that whenever relative option prices are high (above average) the difference between relative call and put prices is greater than when relative option prices are low (below average). Transaction costs may be a function of the level of relative option prices and might, therefore, explain the higher slope, but it is not obvious why that should be the case, nor why the regression results should be different for the "Regular" data.

Second, existing tax laws, as discussed in section II. C., bring about two boundary lines in the 190 day maturity. If puts are being converted into calls (as market practitioners say is the case), the upper boundary line with a slope greater than one is relevant and may explain the findings. However, there is no difference between the 90 day and 190 day regressions as the tax hypothesis would suggest; and, in addition, the same results are not evident for the "Regular" companies.

A final explanation of the difference between the "Regular" and "New Business" findings relies on the greater activity in the "New Business" market and on the assumption that call prices lead put prices. Since high relative call prices are more likely to have risen recently than low relative call prices, larger gaps between c and p are to be expected when relative option prices are above average (because put prices have not yet adjusted). Since, conversely, low relative call prices are more likely to have fallen recently, smaller gaps between c and p are to be expected when relative option prices are below average. The bias posited here did not exist in the case of "Regular" companies because they constituted a relatively stable market with much less frequent changes in stock prices and option prices. The "New Business" companies are much more likely to experience frequent changes in option prices that take time to adjust.[35]

In summary, considering the nature of the companies in the "New Business" sample and the quality of the reporting, the data strongly support the put-call parity theory.

IV. CONCLUSIONS

The purpose of this paper has been to formulate the put-call parity theory,[36] and to test it using data on nominal option prices. By and large the theory is supported by the time series and cross-section regression analysis carried out.

The existence of put-call parity is consistent with the evidence on the "random walk" hypothesis which implies (1) that all information as to the direction of future stock price movements is impounded in the current price, and (2) that relative stock price changes follow a symmetric probability distribution. Put and call prices can therefore have no predictive power. A rise in the relative call price ($c = \dfrac{C}{V}$) in no way implies that the expected value of the stock price

35. It is, of course, impossible to distinguish from the data alone whether the upward bias is due to a true market lag or to a reporting lag. Probably both are the case.

36. The reader familiar with the international finance literature will notice the not unintentional similarity in terminology with "the interest rate parity theory."

change is greater than before. It implies only that the probability distribution of price changes has widened. The fact that relative put and call prices move together, aside from affirming put-call parity, implies only that the probability distribution of relative stock price changes is symmetrical.

Some of the more interesting paths for additional research lie in investigating further than could be done in this paper the relationship between the level of option prices and investor expectations about stock price changes.

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