4-5 Hermite and spline interpolation

Files: The accompanying files for this assignment are `assignment4_1.html` and `assignment4_2.html`.
Delivery: upload the modified HTML files and any other necessary files to the Racó. All explanations and/or answers to the problems should be included in the HTML file, or in a separate PDF document.

Problem 1. Write a program to draw a **cubic Hermite polynomial** that interpolates two points P_0, P_1, allowing the user to see and control the two points and the two tangent vectors at P_0 and P_1.

Problem 2. Consider the particular case $P_0 = (200, 200)$, $P_1 = (400, 300)$, and tangent vectors $\vec{v_0} = (100, 100)$ and $\vec{v_1} = (100, 0)$.

(a) Compute the position of the curve at $t = 1/2$. How should the tangent vectors be modified in order to: keep the same directions at P_0 and P_1, and at the same time go through $(300, 300)$ at $t = 1/2$? Solve the problem first, and then illustrate your result.

(b) What happens as you enlarge the tangent vectors?

(c) What happens if you invert the direction of the tangent vectors.

Problem 3. With the help of your program, describe what happens in the following special cases:

1. The two points coincide (i.e., $P_0 = P_1$).
2. The two points coincide and the two tangent vectors are $(0, 0)$.
3. The two tangent vectors are equal to the vector $P_1 - P_0$.

Problem 4. Write a program to draw a **cubic spline** that interpolates four points P_0, P_1, P_2, P_3. Implement the following two variants: clamped (allowing the user to see and control the four points and the two tangent vectors at P_0 and P_3) and relaxed. Compare the result of the two, commenting and justifying the differences.

Show the result for the particular case $P_0 = (300, 200)$, $P_1 = (500, 200)$, $P_2 = (500, 400)$, $P_3 = (300, 400)$, and for the tangent vectors $\vec{v_0} = (200, -200)$ and $\vec{v_3} = (-200, -200)$.