FBA:
$$\max\ \mu = f'v \quad \text{s.t.} \quad S \cdot V = 0, \quad V_L \le V \le V_U$$

$\xrightarrow[\text{Transformation}]{}$

$$\max\ \mu = f'v \quad \text{s.t.} \quad \begin{bmatrix} S \\ I \\ I \end{bmatrix} V \begin{bmatrix} = 0 \\ \ge V_L \\ \le V_U \end{bmatrix}$$

Regular FBA, shown for comparison to to the formulations below

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~~find small~~ For each growth rate, find the smallest number of reaction bounds that must be relaxed to achive that growth rate

Formulation #1: minimum number of changes

$$\min \sum_i \alpha^i$$

s.t.

$$Sv = 0$$
$$v^u = \cancel{\mu}\ \mu^* = \{1, 1.01, 1.02, \ldots\}$$
$$v^i \ge V_L^i - \alpha^i \cdot 1000$$
$$v^i \le V_U^i + \alpha^i \cdot 1000$$
$$\alpha^i = \{0, 1\}$$

$\xrightarrow[\text{Transformation}]{}$

$$\min \sum_i \alpha^i \quad \text{s.t.} \quad \begin{bmatrix} S & 0 \\ I & 1000\,I \\ I & -1000\,I \end{bmatrix} \begin{bmatrix} V \\ \alpha \end{bmatrix} \begin{bmatrix} = 0 \\ \ge V_L \\ \le V_U \end{bmatrix}$$

For each growth rate, find the smallest total change in the reaction bounds required to achive that growth rate

Formulation #1: minimum magnitude of changes

$$\min \sum_i \alpha^i$$

s.t.

$$Sv = 0$$
$$v^\mu = \cancel{\mu}\ \mu^* = \{1, 1.01, 1.02, \ldots\}$$
$$v^i \ge V_L^i - \alpha^i$$
$$v^i \le V_U^i + \alpha^i$$
$$\alpha^i \in \mathbb{R}$$

$\xrightarrow[\text{Transformation}]{}$

$$\min \sum_i \cancel{\alpha}\ \alpha^i \quad \text{s.t.} \quad \begin{bmatrix} S & 0 \\ I & I \\ I & -I \end{bmatrix} \begin{bmatrix} V \\ \alpha \end{bmatrix} \begin{bmatrix} = 0 \\ \ge V_L \\ \le V_U \end{bmatrix}$$