Wireless Power Optimisation

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Motivation

In a wireless environment there are numerous receivers and transmitters, sending information to each other. These systems contain multiple input, multiple output. We consider the simple case with transmitter-receiver (T-R) pairs, with inevitable interference from other transmitters.

Convex Optimisation

An optimisation problem can be written in the standard form:

$$\begin{aligned} \underset{x}{\text{minimise}} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \leq 0 \\ & h_i(x) = 0 \end{aligned} \tag{1}$$

A function f on the interval (x, y) is convex if it lies on or below the chord $(f(x), f(y))$

$$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y) \tag{2}$$

When all objective f_0 and constraint f_i, h_i functions are convex, it is a convex optimisation problem and any locally optimal solution can be shown to be globally optimal [1].

Signal Quality

The quality of a signal at a receiver is measured by the signal to interference and noise ratio (SINR):

$$\frac{S}{I+\sigma} \tag{3}$$

where S is the wanted signal, I is the interference and σ is the background noise.

Research Problem

We minimise the total power of all transmitters, given a minimum SINR for each receiver:

$$\begin{aligned} \text{minimise} \quad & \sum_{\forall i} p_i \\ \text{subject to} \quad & \text{SINR}_i \geq \alpha \\ & p_i \leq P_{\max} \\ & p_i \geq 0 \end{aligned} \tag{4}$$

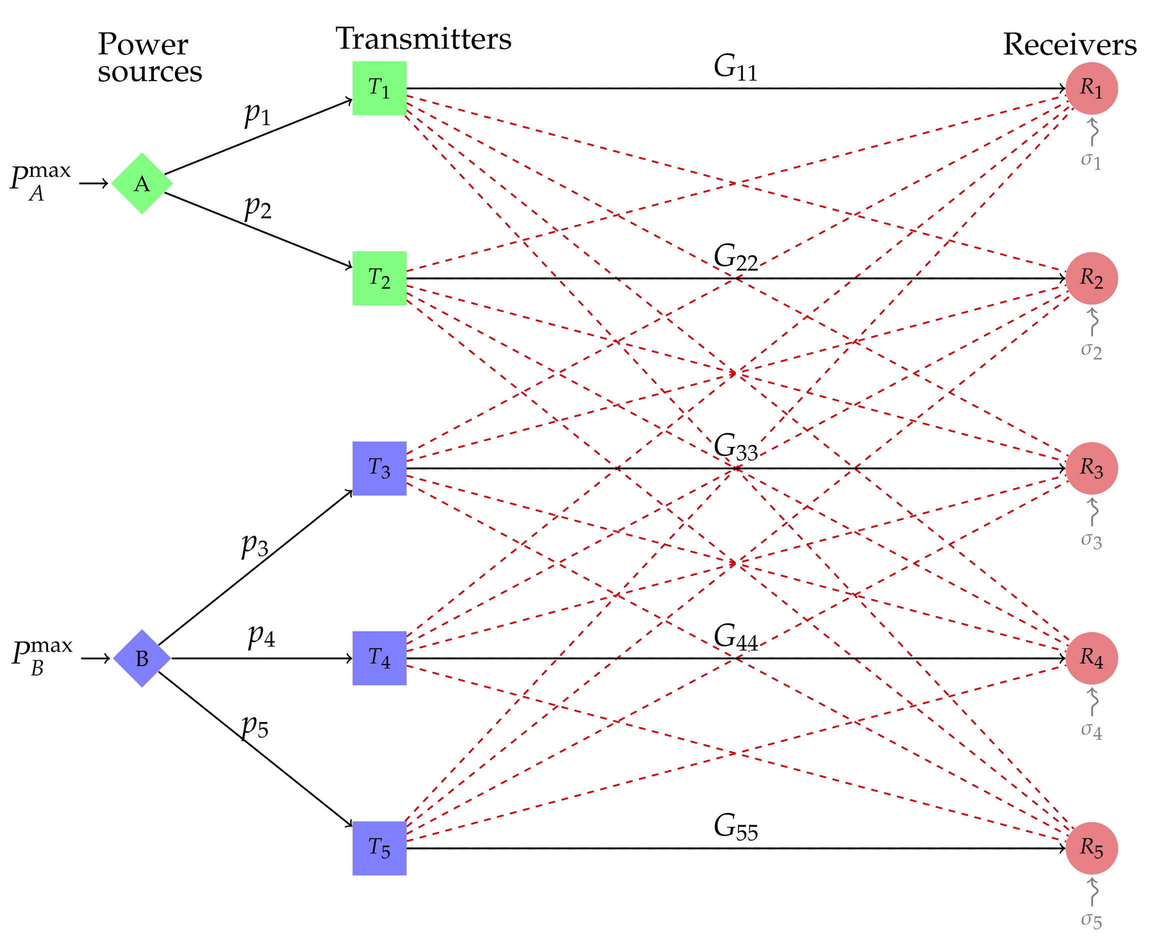


Figure 1

Wireless array of 5 transmitters T_i & receivers R_i. 2 power sources A & B supply distinct transmitter groups, limiting the group's power to $P^{\max}_{(\bullet)}$. Interference (····) & intended signal (–). The signal gain G_{ij} is the proportion of the input power signal from transmitter j reaching receiver i.

Pathloss and Rayleigh Fading

We developed a way of simulating a gain matrix, by including the drop in power due to pathloss and fading from random structure in the environment. For a T-R pair, the powers are related by:

$$\begin{aligned} & P_{\text{received}} = G \cdot P_{\text{transmitted}} \\ & \text{with} \quad G = \frac{k}{r^{\alpha}} x \end{aligned} \tag{5}$$

where k is a positive constant, r is the euclidean distance between the transmitter and receiver, α is some constant ($2 \leq \alpha \leq 5$) and $x \sim \text{Exp}(\lambda = 1)$ to mimic Rayleigh Fading [2].

Uniform Point Process (UPP)

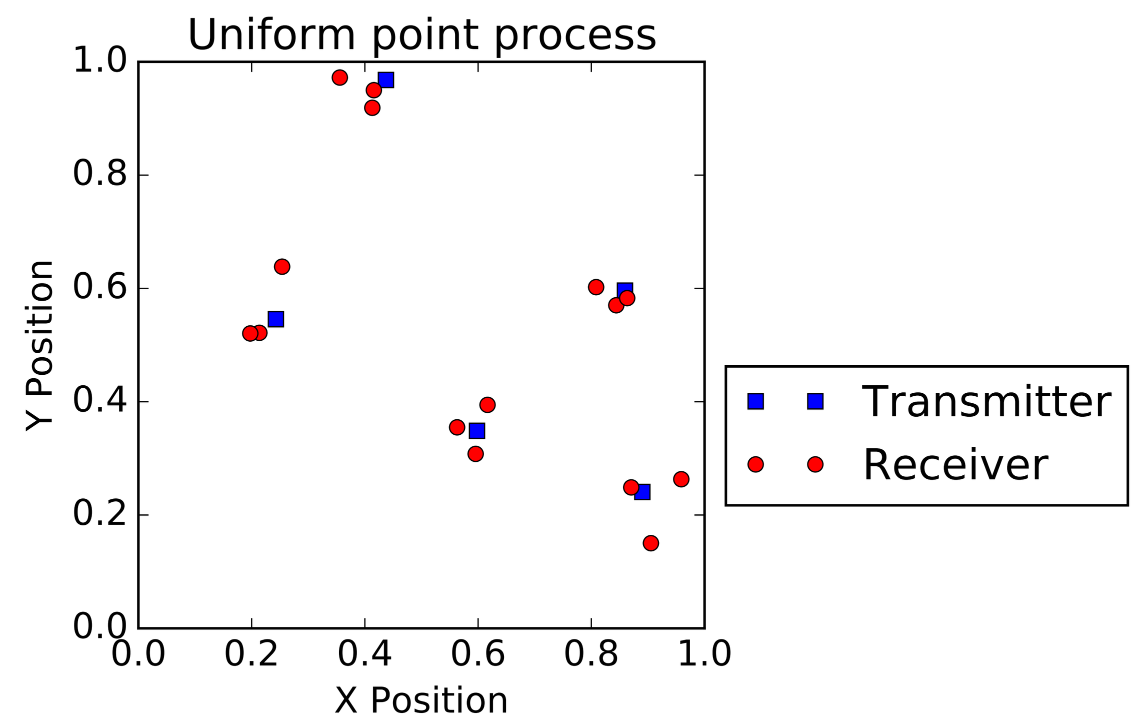
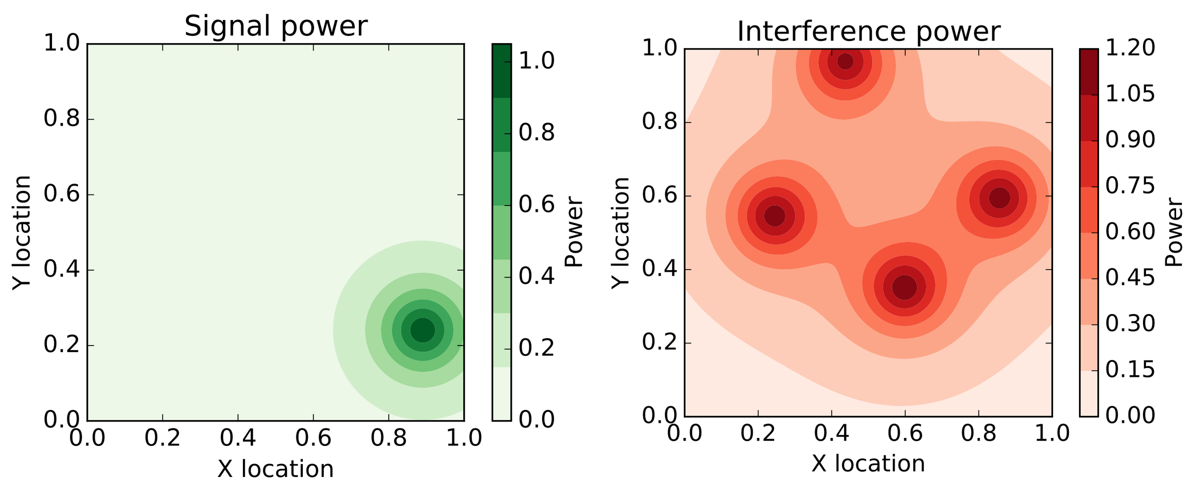


Figure 2

G can be generated by placing N transmitters down independently with uniform probability. M receivers for each transmitter were then placed within a circle of their transmitter. From the locations, G can be calculated using (5). Figure 2 shows an instance of the point process for $N = 5, M = 3$, while Figure 3 shows the signal and interference for receivers of transmitter 5.



(a) (b)

Figure 3

Grid Point Process (GPP)

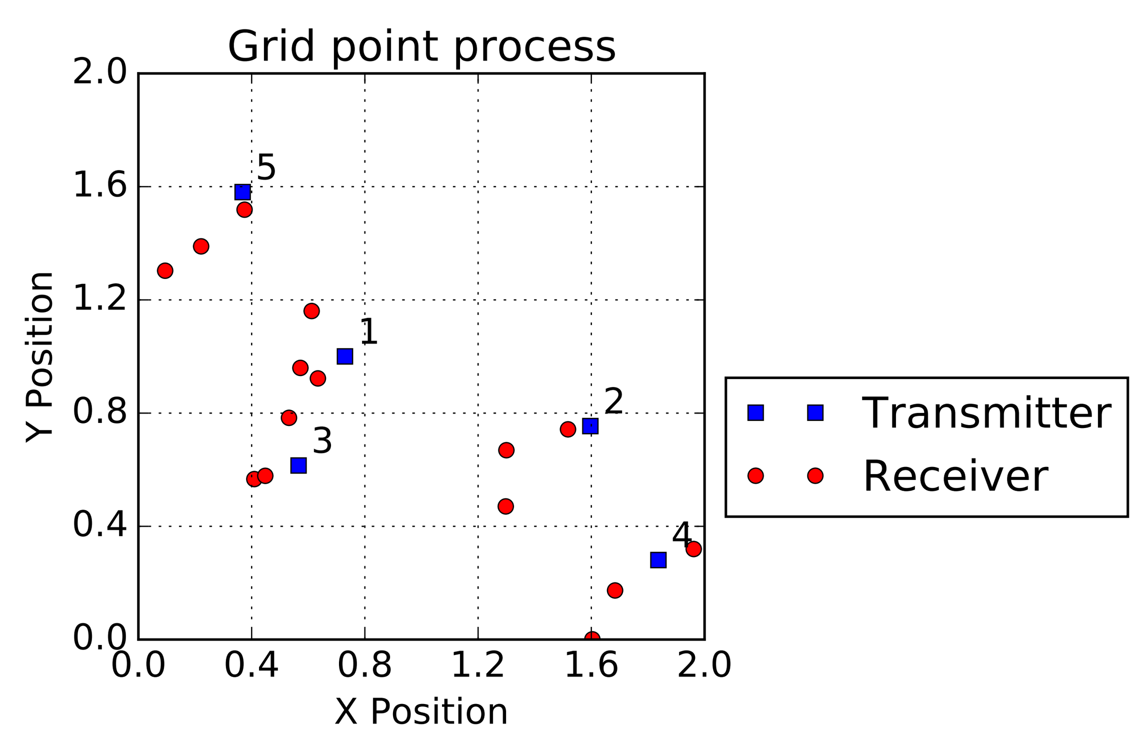


Figure 4

Another way to randomly populate the area is to divide it into cells. Each transmitter is randomly assigned a cell and then randomly placed within this cell. Then the receivers for each transmitter are also randomly placed within the cells. Figure 4 shows a $N = 5, M = 3$ case.

CVXPY library

Our main tool was the python library `cvxpy`. This library uses Interior Point Solver (IPS) methods to optimise the system, provided you can write your problem in Disciplined Convex Program (DCP) form. These rules allow the program to access convexity. The advantage of IPS is that it does not require continuous derivatives of the objective functions [3].

Results: Power

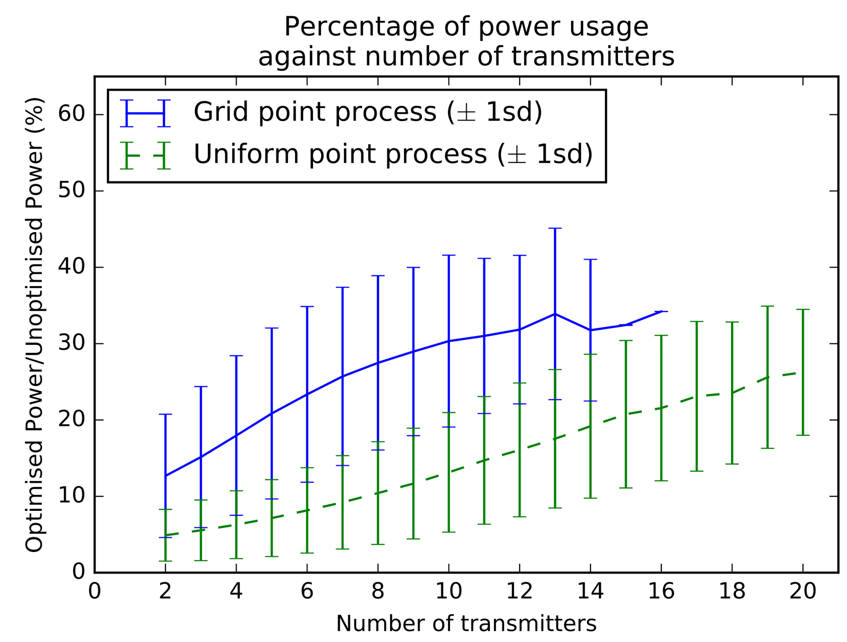


Figure 5

We reduced our model so each transmitter has its own maximum power. The simulation was optimised for varying numbers of transmitters (2 to 20) within a 2 by 2 space for both point processes. Figure 5 shows the average amount of total power used compared to having all the transmitters on full, with SINR ≥ 1. This average only considers feasible problems.

Results: Feasibility

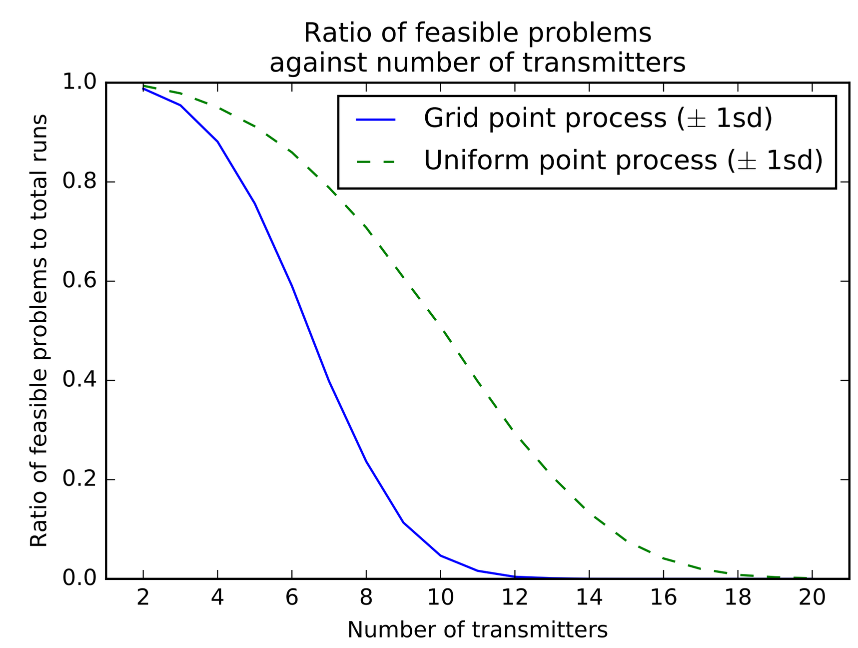


Figure 6

The simulation was run for the same set as above. Figure 6 shows the ratio of feasible problems to number of attempted runs. The problem is infeasible if the constraints and objective function are inconsistent i.e. there is no solution in the problem domain. It shows that the UPP is feasible for a higher number of transmitters than the GPP.

Future Work

So far we have concentrated on the interference of other transmitters, leaving noise constant. Investigating the robustness of the solution to variable noise would more realistic.

The wireless model we have investigated has each receiver only wanting signal from one transmitter. To generalise the model, each transmitter will be associated with M receivers and each receiver will be associated with T transmitters.

We will also implement a Poisson point process for positioning of the receivers and transmitters and use this to link our work with analytical results.

[1] S. Boyd and L. Vandenberghe, "Convex Optimization" 2004; [2] J. Linnartz, "Rayleigh Fading", 2010;
[3] S. Diamond and S. Boyd, "A Python-Embedded Modeling Language for Convex Optimization", 2016