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5. $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\cdots\cdots+\frac{1}{1+2+3+\cdots+n}=\frac{2n}{(n+1)}$

$P(n)=1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots\cdots+\frac{1}{1+2+3+\cdots+n}=\frac{2n}{n+1}$

Prove that it is true for $n=1$

LHS $=1$

RHS $=\frac{2n}{n+1}=\frac{2}{2}=\underline{\underline{1}}$

we assume that it is true for $n=k$

$1+\frac{1}{1+2}+\cdots\cdots+\frac{1}{1+2+3+\cdots+k}=\frac{2k}{k+1}$ ——— (1)

we need to prove it is true for $n=k+1$

$1+\frac{1}{1+2}+\cdots\cdots+\frac{1}{1+2+3+\cdots+k}+\frac{1}{1+2+3+\cdots+k+1}$

$=\frac{2k}{k+1}+\frac{1}{(k+1)(k+2)}=\frac{2k}{k+1}+\frac{1}{\frac{(k+1)(k+2)}{2}}$

$=\frac{2k}{(k+1)}$

$$= \frac{2k}{k+1} + \frac{1}{\frac{(k+1)(k+2)}{2}}$$

$$= \frac{2k}{k+1} + \frac{2}{(k+1)(k+2)}$$

$$= \frac{2}{k+1}\left[k + \frac{1}{k+2}\right]$$

$$= \frac{2}{k+1}\left[\frac{k^2+2k+1}{k+2}\right]$$

$$= \frac{2}{k+1}\left[\frac{(k+1)^2}{k+2}\right]$$

$$= \frac{2}{\cancel{k+1}} \times \frac{(k+1)^{\cancel{2}}}{k+2}$$

$$= \frac{2(k+1)}{(k+2)}$$

6. $1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + \cdots\cdots + n(n+1)(n+2)$

$$= \frac{n(n+1)(n+2)(n+3)}{4}$$

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$P(n) = 1\cdot2\cdot3 + 2\cdot3\cdot4 + \cdots\cdots + n(n+1)(n+2)$

$= \dfrac{n(n+1)(n+2)(n+3)}{4}$

We need to prove that it is true for $n=1$

$LHS = 1\cdot2\cdot3$

$= \underline{\underline{6}}$

$RHS = \dfrac{n(n+1)(n+2)(n+3)}{4}$

$= \dfrac{1(1+1)(1+2)(1+3)}{4} = \dfrac{1\times2\times3\times4}{4} = \underline{\underline{6}}$

We need to assume it is true for $n=k$

$P(k) = 1\cdot2\cdot3 + 2\cdot3\cdot4 + \cdots\cdots + k(k+1)(k+2) = \dfrac{k(k+1)(k+2)(k+3)}{4}$

We need to prove that it is true for $n=k+1$

$1\cdot2\cdot3 + 2\cdot3\cdot4 + \cdots\cdots + k(k+1)(k+2)$

$= \dfrac{\cancel{n(n+1)(n+2)}(n+3)}{4} + n(n+1)(n+2)$

$$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$$

$$= (k+1)(k+2)(k+3)\left[\frac{k}{4} + 1\right]$$

$$= (k+1)(k+2)(k+3)\left[\frac{k+4}{4}\right]$$

$$= \frac{(k+1)(k+2)(k+3)(k+4)}{4} = \text{RHS}$$

$\therefore$ $P(k+1)$ is true

It is true for all values of N.

7. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

$P(n) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

We need to prove it is true for $n = 1$

$\text{LHS} = \frac{1}{2^n} = \frac{1}{2}$

$\text{RHS} = 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

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We need to assume that it is true for $n=k$

$$P(k) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$$

We need to prove it is true for $n=k+1$

$$P(k+1) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{1}{2^k} + \frac{1}{2^k \cdot 2^1}$$

$$= 1 - \frac{1}{2^k}\left[1 - \frac{1}{2}\right]$$

$$= 1 + \frac{1}{2^k}\left[\frac{2-1}{2}\right] = 1 + \frac{1}{2^k}\left[\frac{1}{2}\right]$$

$$= 1 + \frac{1}{2^{k+1}} = RHS$$

$\therefore$ It is true for $P(k+1)$

$\therefore$ It is true for all values of N.

8. $1\cdot 2+2\cdot 3+3\cdot 4+\cdots\cdots+n(n+1)=\frac{n(n+1)(n+2)}{3}$

Let $P(n)=1\cdot 2+2\cdot 3+3\cdot 4+\cdots\cdots+n(n+1)=\frac{n(n+1)(n+2)}{3}$

We need to prove it is true for $n=1$

$$LHS = n(n+1) = 1(1+1) = \underline{2}$$

$$RHS = \frac{n(n+1)(n+2)}{3} = \frac{1(1+1)(1+2)}{3} = \frac{1\times 2\times 3}{3} = \underline{\underline{2}}$$

We need to assume it is true for $n=k$

$P(k) = 1\cdot 2+2\cdot 3+\cdots\cdots+k(k+1)=\frac{k(k+1)(k+2)}{3}$

We need to prove it is true for $n=k+1$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)+3(k+1)(k+2)}{3}$$

$$= \underline{\underline{\frac{(k+1)(k+2)(k+3)}{3}}}$$

$\therefore$ It is true for $P(k+1)$

$\therefore$ It is true for all values of N.

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9. $1 \cdot 3 + 2 \cdot 3^2 + \cdots\cdots + n \cdot 3^n = \frac{(2n-1)3^{n+1} + 3}{4}$

$P(n) = 1 \cdot 3 + 2 \cdot 3^2 + \cdots\cdots + n \cdot 3^n = \frac{(2n-1)3^{n+1} + 3}{4}$

We need to prove it is true for $n=1$

$LHS = 1 \cdot 3^1 = 3$

$RHS = \frac{(2-1)3^{1+1} + 3}{4} = \frac{1 \times 3^2 + 3}{4} = \frac{12}{4} = 3$

We need to assume it is true for $n=k$

$P(k) = 1 \cdot 3 + 2 \cdot 3^2 + \cdots\cdots + k \cdot 3^k = \frac{(2k-1)3^{k+1} + 3}{4}$

We need to prove it is true for $n=k+1$

$= \frac{(2k-1)\ 3^{k+1} + 3}{4} + k+1 \cdot 3^{k+1}$

$= \frac{(2k-1)3^{k+1} + 3 + 4(k+1) \cdot 3^{k+1}}{4}$

$= \frac{3^{k+1}\left[(2k-1) + 4(k+1)\right] + 3}{4}$

$= \frac{3^{k+1} \times [2k-1 + 4k + 4] + 3}{4}$

$$= \frac{3^{k+1} \times (6k-3)+3}{4}$$

$$= \frac{3^{k+1} \cdot 3(2k-1)+3}{4}$$

$$= \frac{3^{k+2} \times [2k-1]+3}{4} = \frac{3^{k+2} \times [2(k+1)-1]+3}{4}$$

$\therefore$ P(k+1) is true

$\therefore$ It is true for all values of N.

10. $1.5+3.5+5.7+\cdots\cdots+(2n-1)(2n+1)=\frac{n(4n^2+6n-1)}{3}$

$P(n) = 1.5+3.5+5.7+\cdots\cdots+(2n-1)(2n+1)=\frac{n(4n^2+6n-1)}{3}$

we need to prove it is true for $n=1$

$LHS = (2n-1)(2n+1) = (2-1)(2+1)$

$= 1 \times 3 = \underline{3}$

$RHS = \frac{n(4n^2+6n-1)}{3} = \frac{1 \times (4+6-1)}{3} = \frac{9}{3} = \underline{3}$

we need to assume it is true for $n=k$

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$P(k) = 1\cdot5 + 3\cdot5 + \cdots + (2k-1)(2k+1) = \frac{k(4k^2+6k-1)}{3}$

We need to prove it is true for $n = k+1$

$= \frac{k(4k^2+6k-1)}{3} + (2(k+1)-1)(2(k+1)+1)$

$= \frac{k(4k^2+6k-1)}{3} + (2k+2-1)(2k+2+1)$

$= \frac{k(4k^2+6k-1)}{3} + (2k+1)(2k+3)$

$= \frac{k(4k^2+6k-1)}{3} + ((2k)^2+6k+2k+3)$

$= \frac{k(4k^2+6k-1)}{3} + (4k^2+8k+3)$

$= \frac{k(4k^2+6k-1)+3(4k^2+8k+3)}{3}$

$= \frac{4k^3+6k^2-k+12k^2+24k+9}{3}$

$= \frac{4k^3+18k^2+23k+9}{3}$

$=$

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$$= \frac{k(4k^2+6k-1) + 12k^2+24k+9}{3}$$

$$= \frac{4k^3+6k^2-k+12k^2+24k+9}{3}$$

$$= \frac{4k^3+18k^2+23k+9}{3}$$

$$= \frac{4k^3+14k^2+4k^2+9k+14k+9}{3}$$

$$= \frac{4k^3+14k^2+9k+4k^2+14k+9}{3} = \frac{[k(4k^2+14k+9)]+[4k^2+14k+9]}{3}$$

$$= \frac{4k^2+14k+9\,[k+1]}{3}$$

$$= \frac{(k+1)\,[4k^2+8k+6k+4+6-1]}{3}$$

$$= \frac{(k+1)\,[(4k^2+8k+4)+(6k+6-1)]}{3}$$

$$= \frac{(k+1)\,[4(k^2+2k+1)+6(k+1)-1]}{3}$$

$$= \frac{(k+1)\,[4(k+1)^2+6(k+1)-1]}{3}$$

$\therefore P(k+1)$ is true

$\therefore$ It is true for all values of N.