**Lecture 5. Features and Fitting**

# Harris corner detector

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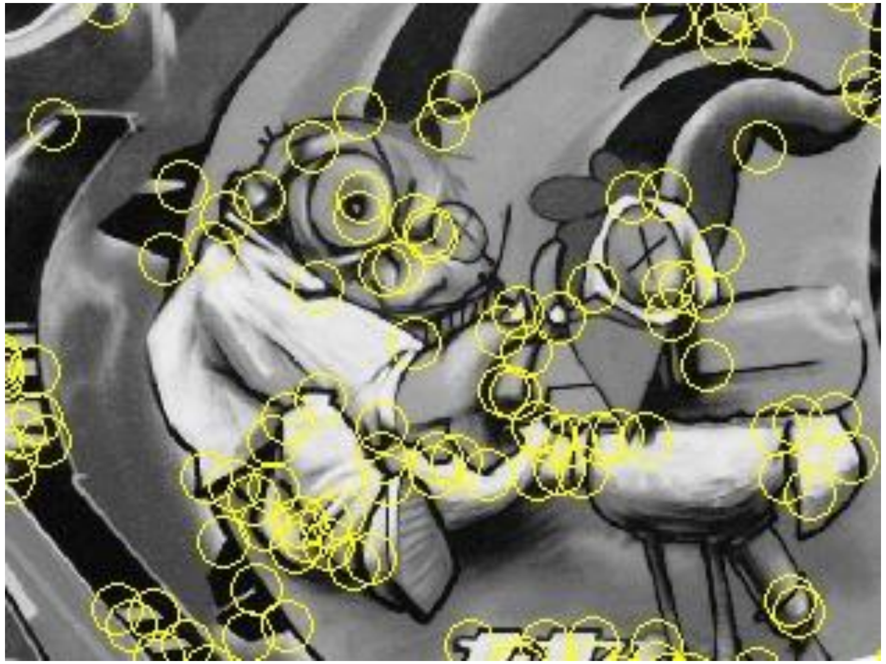
CS131 Computer Vision: Foundations and Applications

# What will we learn today?

- Keypoint localization: Harris corner detector

Some background reading:
Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

# Keypoint Localization

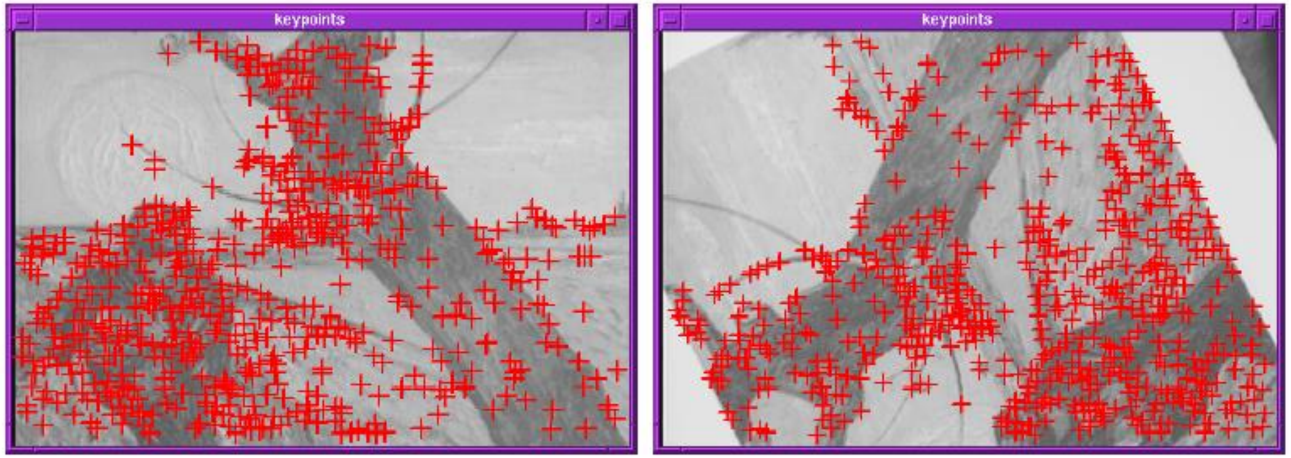


- Goals:
  - Repeatable detection
  - Precise localization
  - Interesting content

$\Rightarrow$ *Look for two-dimensional signal changes*

Slide credit: Bastian Leibe

# Finding Corners



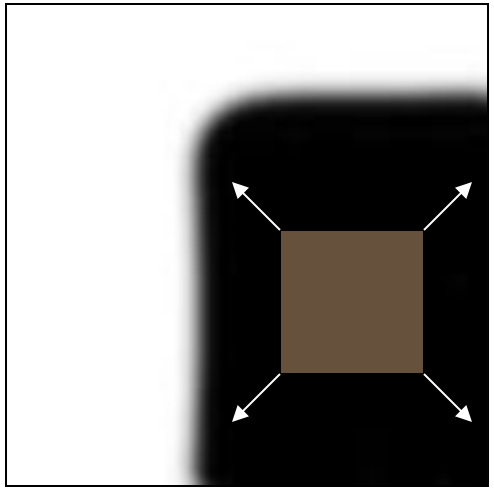

- Key property:
  - In the region around a corner, the image gradient has two or more dominant directions
- Corners are *repeatable* and *distinctive*

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
*Proceedings of the 4th Alvey Vision Conference*, 1988.

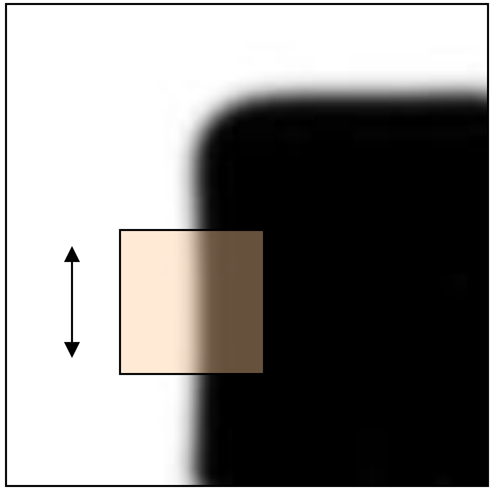
Slide credit: Svetlana Lazebnik

# Corners as Distinctive Interest Points

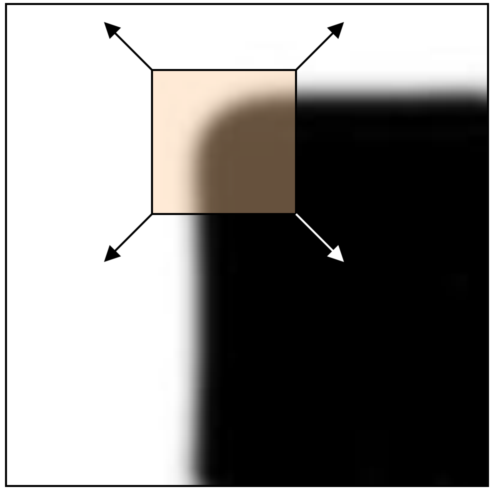
- Design criteria
  - We should easily recognize the corner point by looking through a small window (*locality*)
  - Shifting the window in *any direction* should give *a large change* in intensity (*good localization*)



**"flat" region:**
**no change in all directions**



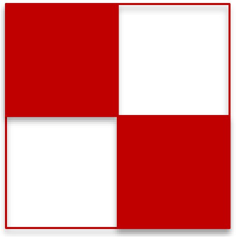
**"edge":**
**no change along the edge direction**



**"corner":**
**significant change in all directions**

Slide credit: Alyosha Efros

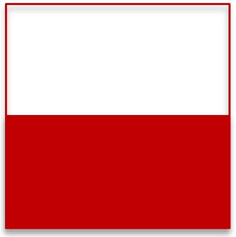
# Corners versus edges



$\sum I_x^2 \longrightarrow$ Large

$\sum I_y^2 \longrightarrow$ Large

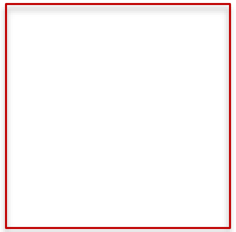
Corner



$\sum I_x^2 \longrightarrow$ Small

$\sum I_y^2 \longrightarrow$ Large

Edge

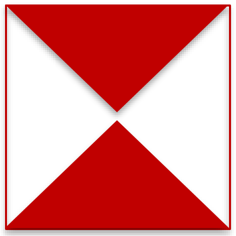


$\sum I_x^2 \longrightarrow$ Small

$\sum I_y^2 \longrightarrow$ Small

Flat

# Corners versus edges



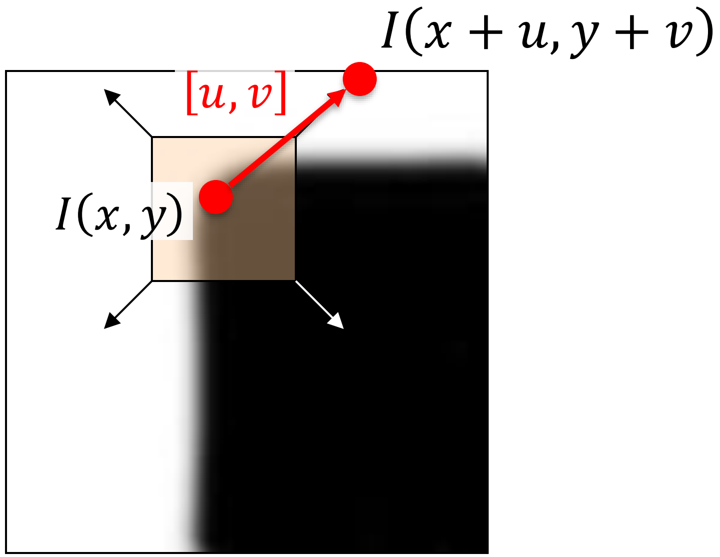
$$\sum I_x^2 \longrightarrow ??$$

$$\sum I_y^2 \longrightarrow ??$$

Corner

# Harris Detector Formulation

- Localize patches that result in large change of intensity when shifted in *any direction.*
- When we shift by $[u, v]$, the intensity change at the center pixel is:




**"corner":**
**significant change in all directions**

- Measure change as intensity difference:

$$(I(x+u, y+v) - I(x, y))$$

- That's for a single point, but we have to accumulate over the patch or "small window" around that point…

# Harris Detector Formulation

- When we shift by $[u, v]$, the change in intensity for the "small window" is:

$$E(u,v) = \boxed{\sum_{x,y}} \boxed{w(x,y)} \boxed{\left[\boxed{I(x+u, y+v)} - \boxed{I(x,y)}\right]^2}$$

Sum over window

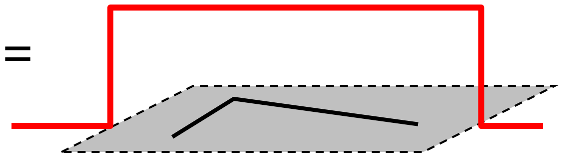
Window function

Shifted intensity

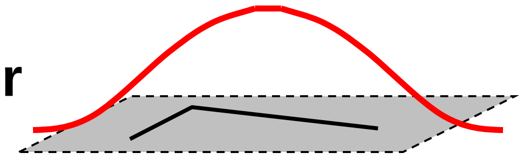
Intensity

Intensity change

Window function $w(x,y)$ =



or



1 in window, 0 outside

Gaussian

Slide credit: Rick Szeliski

# Harris Detector Formulation

- This measure of change can be approximated by (Taylor expansion):

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where $M$ is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

**Gradient with respect to $x$, times gradient with respect to $y$**

Sum over image region – the area we are checking for corner

Slide credit: Rick Szeliski

# Harris Detector Formulation

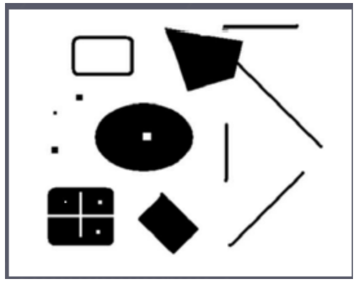
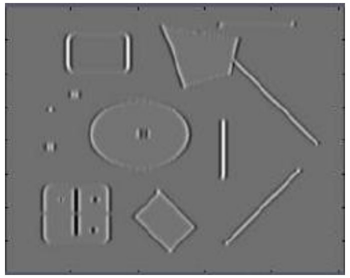
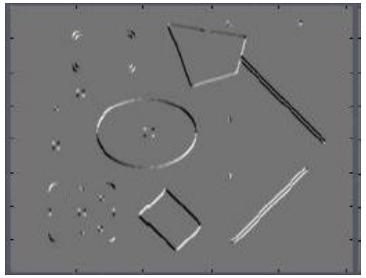


Image $I$



$I_x$

$I_y$



$I_xI_y$

where $M$ is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

**Gradient with respect to $x$, times gradient with respect to $y$**

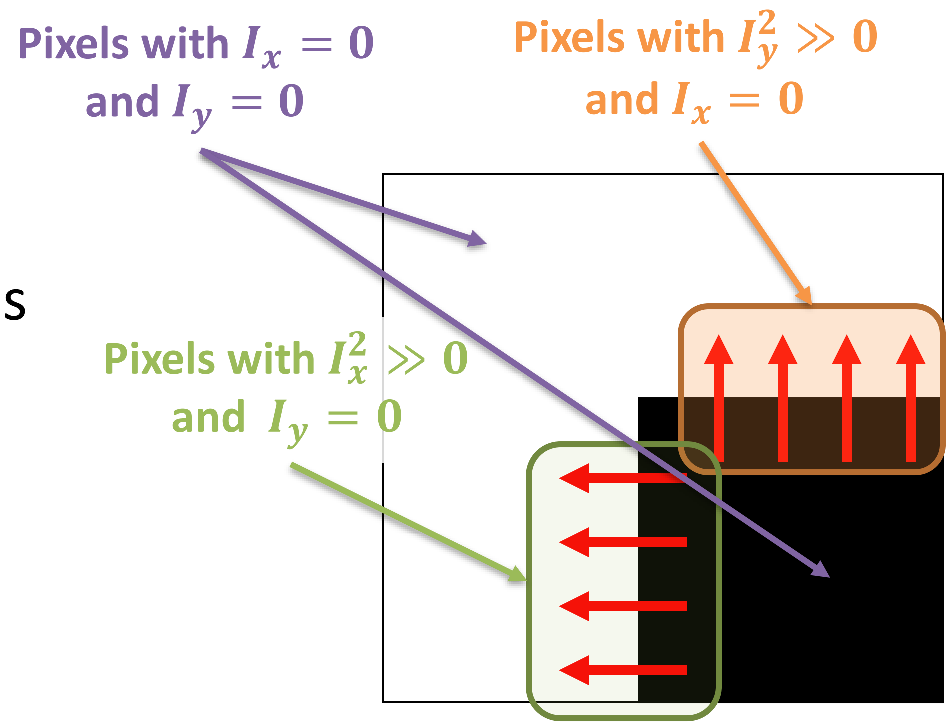
Sum over image region – the area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y]$$

Slide credit: Rick Szeliski

# What Does This Matrix Reveal?

- First, let's consider an axis-aligned corner.
- In that case, the dominant gradient directions align with the $x$ or the $y$ axis
- $M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$
- This means: if either $\lambda$ is close to 0, then this is not a corner, so look for image windows where both lambdas are large.
- What if we have a corner that is not aligned with the image axes?



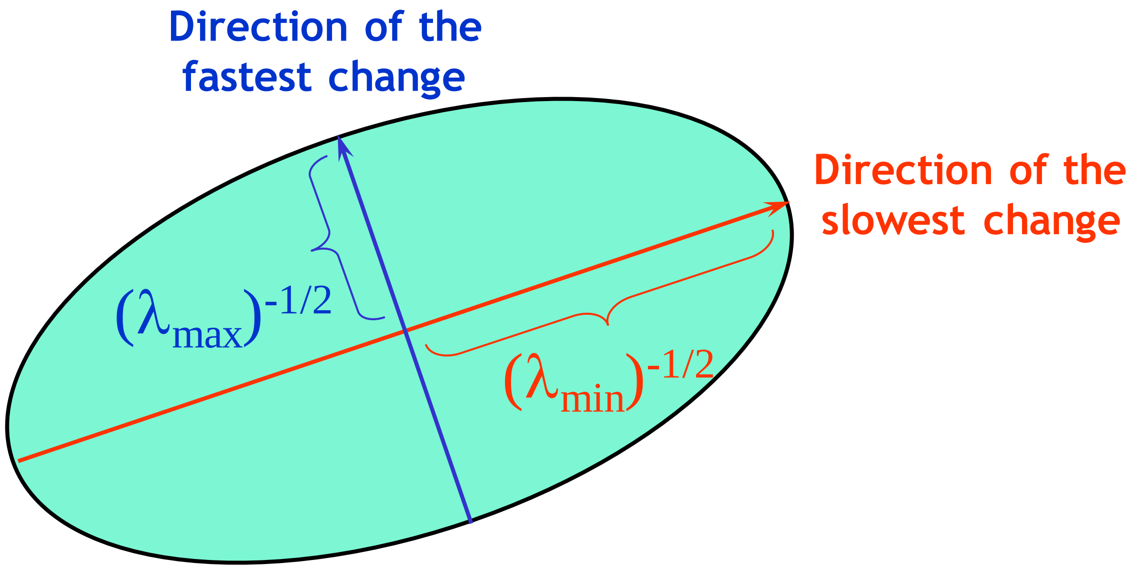

Slide credit: David Jacobs

# General Case

- Since $M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$ is symmetric, we can re-rewrite $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

**(Eigenvalue decomposition)**

- We can think of $M$ as an ellipse with its axis lengths determined by the eigenvalues $\lambda_1$ and $\lambda_2$; and its orientation determined by $R$

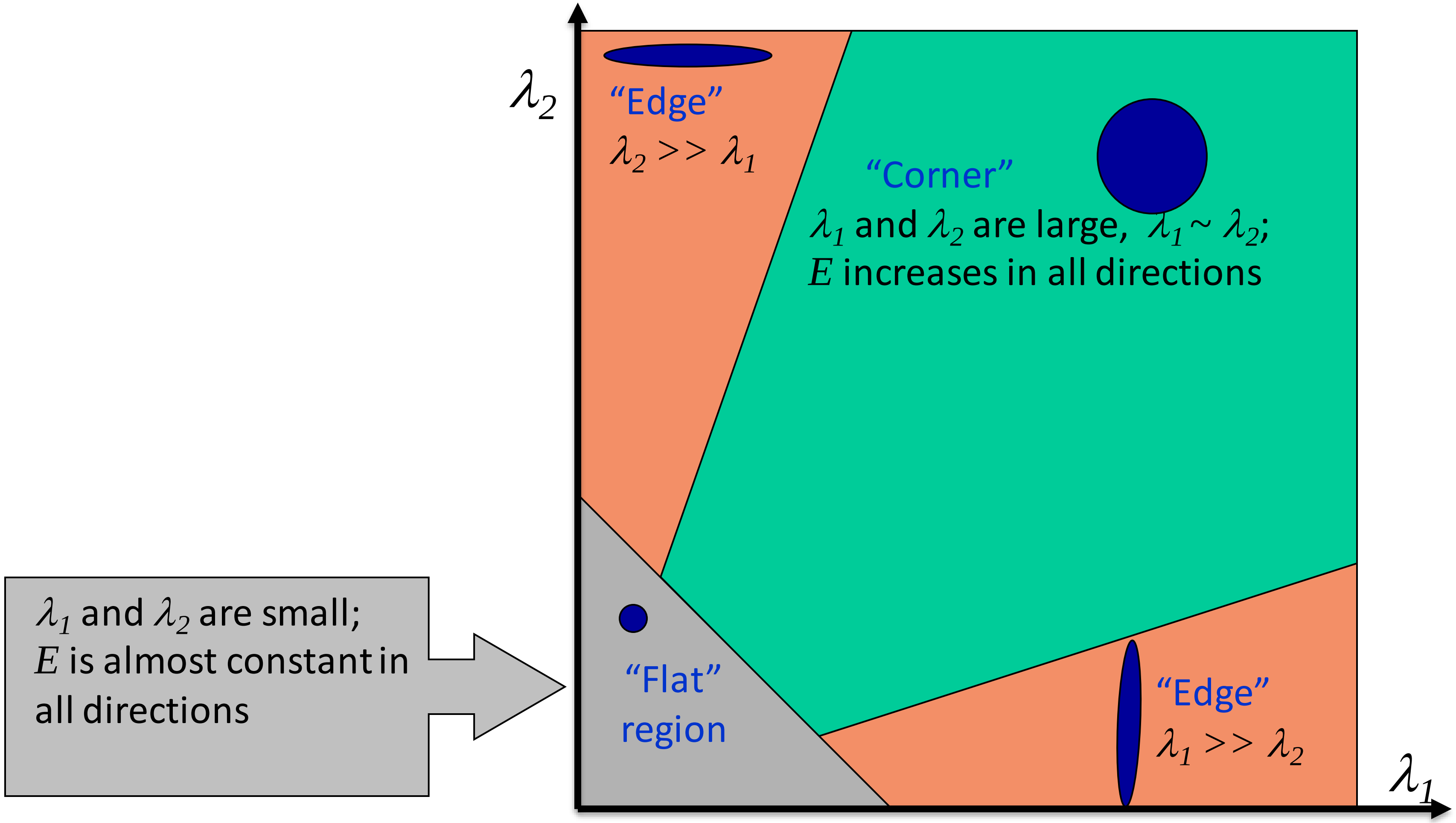



- A rotated corner would produce the same eigenvalues as its non-rotated version.

adapted from Darya Frolova, Denis Simakov

# Interpreting the Eigenvalues

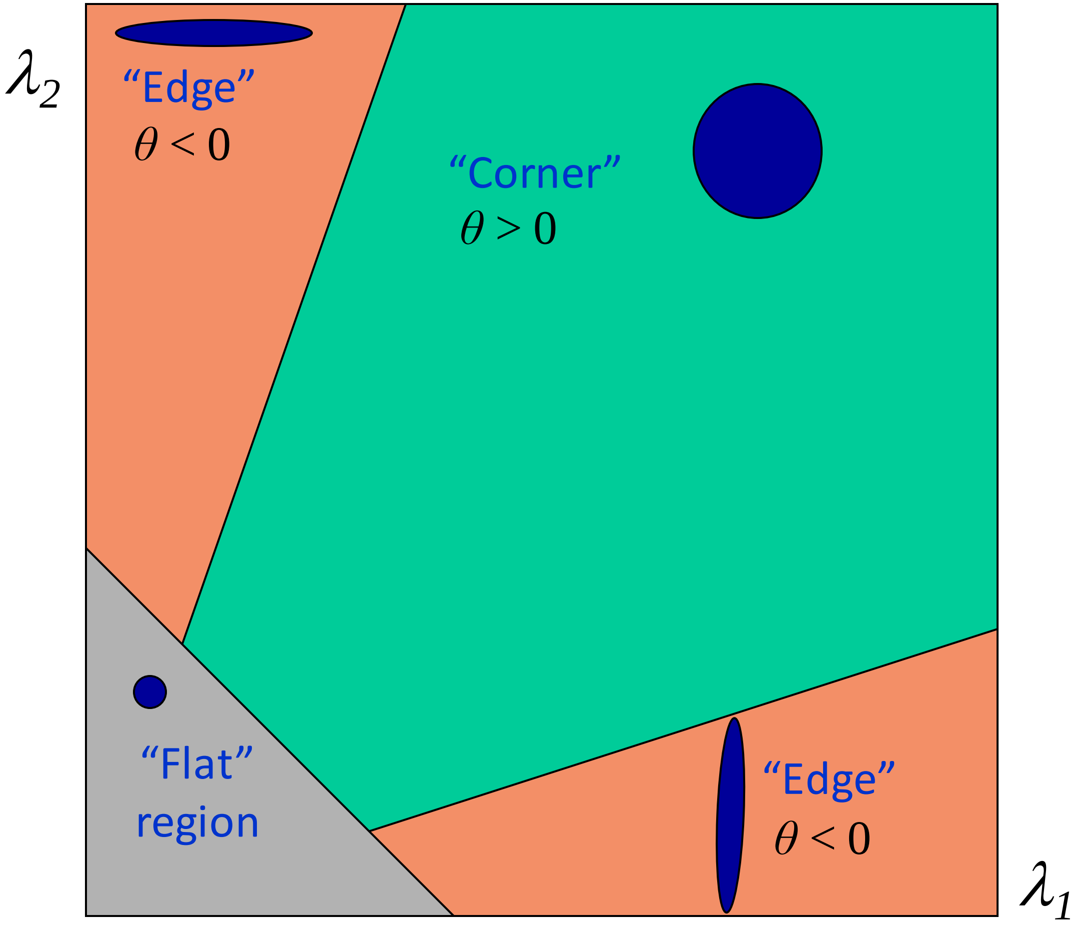
- Classification of image points using eigenvalues of $M$:




Slide credit: Kristen Grauman

# Corner Response Function

$$q = \det(M) - a\,\mathrm{trace}(M)^2 = l_1 l_2 - a(l_1 + l_2)^2$$




- Fast approximation
  - Avoid computing the eigenvalues
  - $\alpha$: constant (0.04 to 0.06)

Slide credit: Kristen Grauman

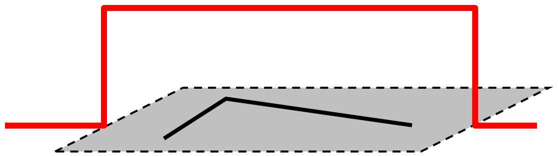
# Window Function $w(x,y)$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
  - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

  - Problem: not rotation invariant

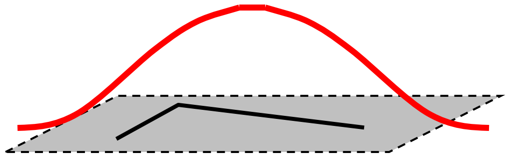


**1 in window, 0 outside**

- Option 2: Smooth with Gaussian
  - Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

  - Result is rotation invariant



**Gaussian**

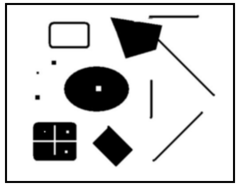
Slide credit: Bastian Leibe

# Summary: Harris Detector [Harris88]

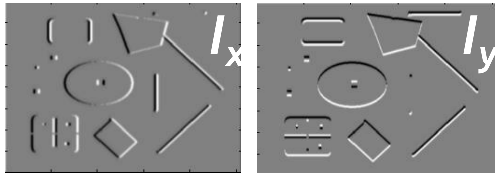
- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I,\sigma_D)=g(\sigma_I)*\begin{bmatrix} I_x^2(\sigma_D) & I_xI_y(\sigma_D) \\ I_xI_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

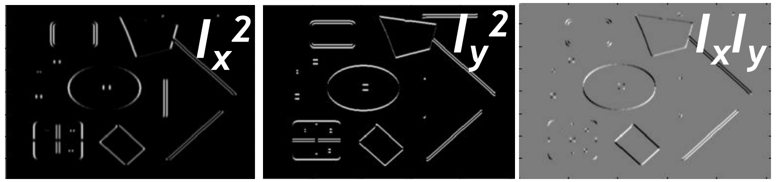
$\sigma_D$: for Gaussian in the derivative calculation
$\sigma_I$: for Gaussian in the windowing function



**1. Image derivatives**




**2. Square of derivatives**




**3. Gaussian filter $g(\sigma_I)$**


**4. Cornerness function - two strong eigenvalues**

$$q=\det[M(s_I,s_D)]-a[\mathrm{trace}(M(s_I,s_D))]^2$$
$$=g(I_x^2)g(I_y^2)-[g(I_xI_y)]^2-\alpha[g(I_x^2)+g(I_y^2)]^2$$


**5. Perform non-maximum suppression**

Slide credit: Krystian Mikolajczyk

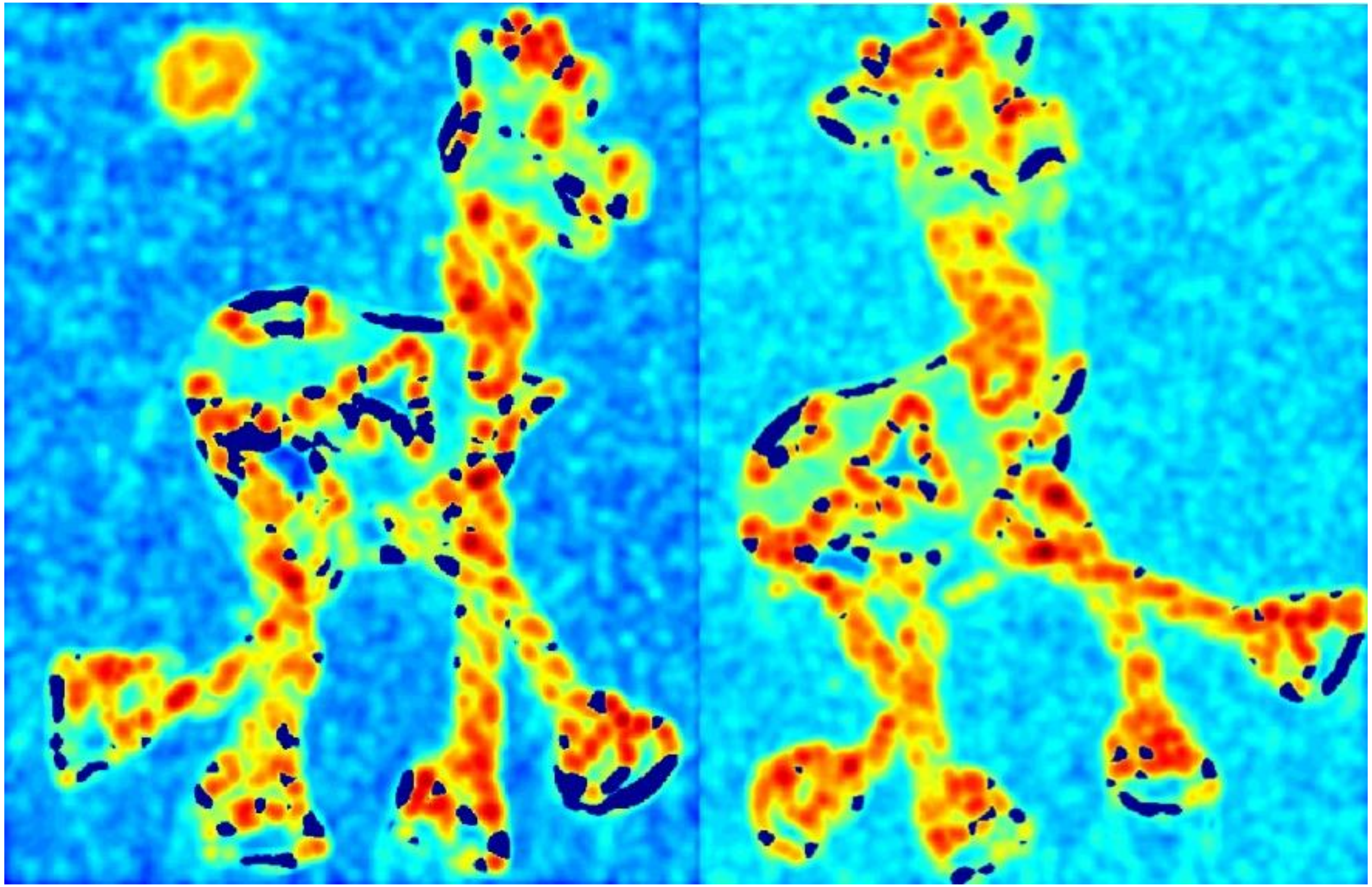
# Harris Detector: Example

- Input Image

Slide adapted from Darya Frolova, Denis Simakov

# Harris Detector: Example

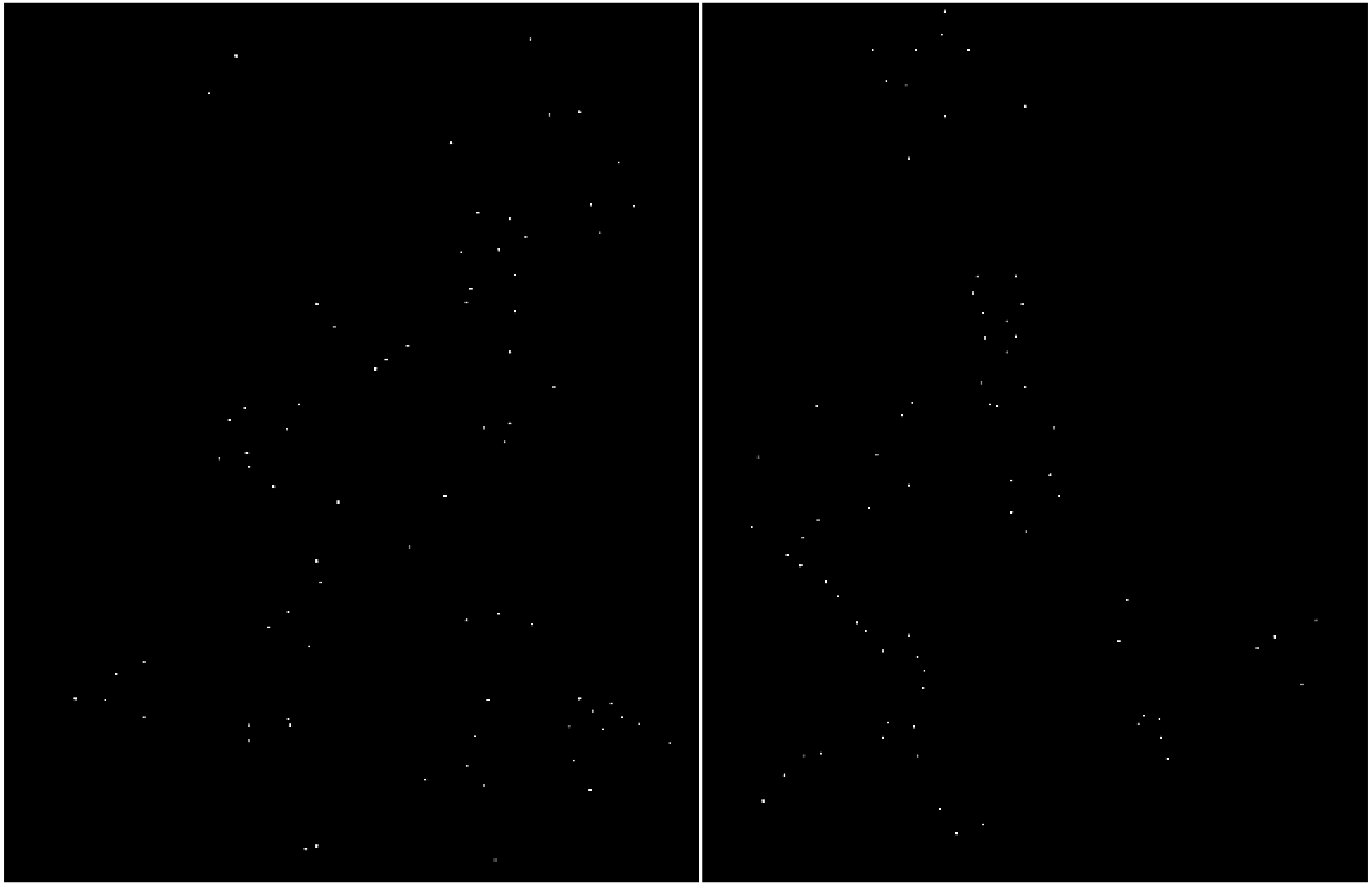
- Input Image
- Compute corner response function $\theta$



Slide adapted from Darya Frolova, Denis Simakov

# Harris Detector: Example

- Input Image
- Compute corner response function $\theta$
- Take only the local maxima of $\theta$, where $\theta >$ threshold



Slide adapted from Darya Frolova, Denis Simakov

# Harris Detector: Example

- Input Image
- Compute corner response function $\theta$
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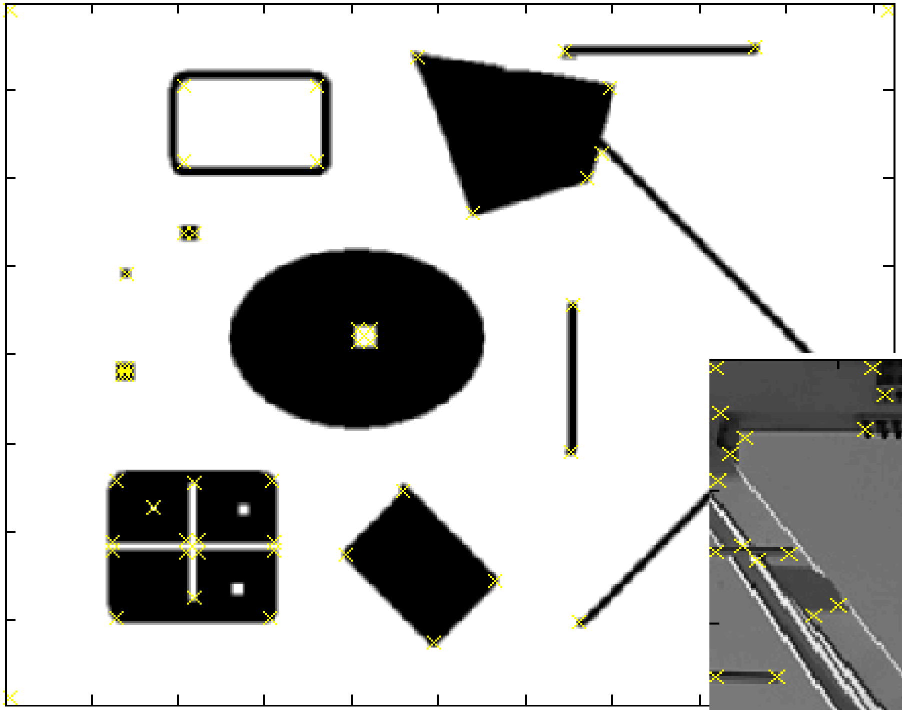
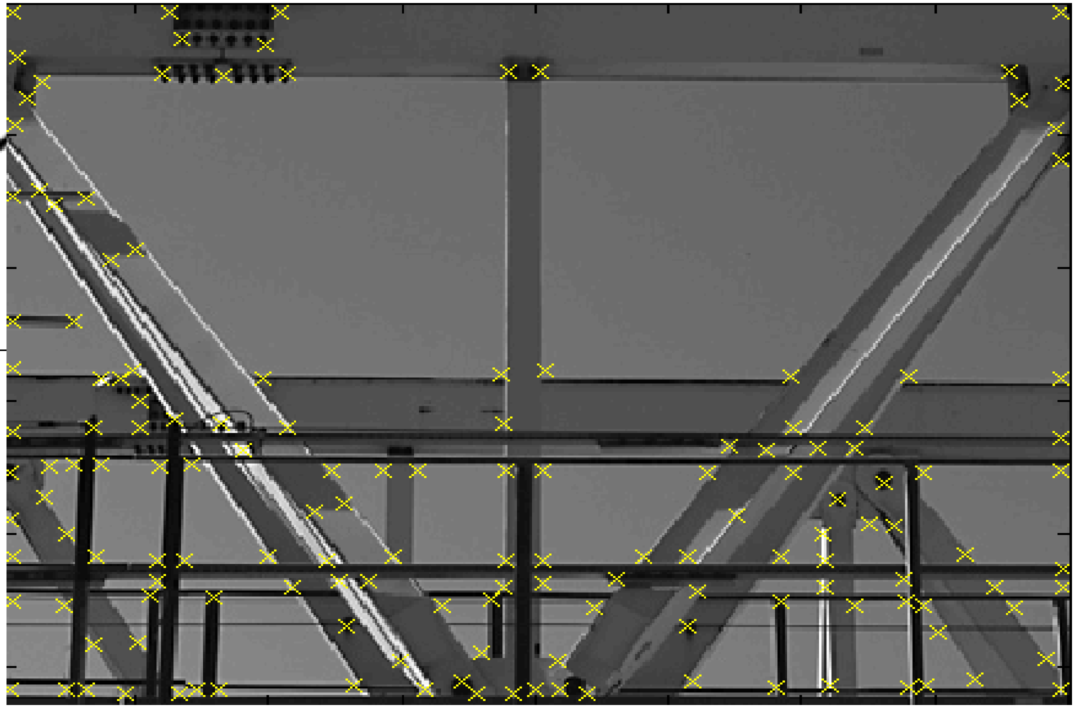
Slide adapted from Darya Frolova, Denis Simakov

# Harris Detector – Responses [Harris88]





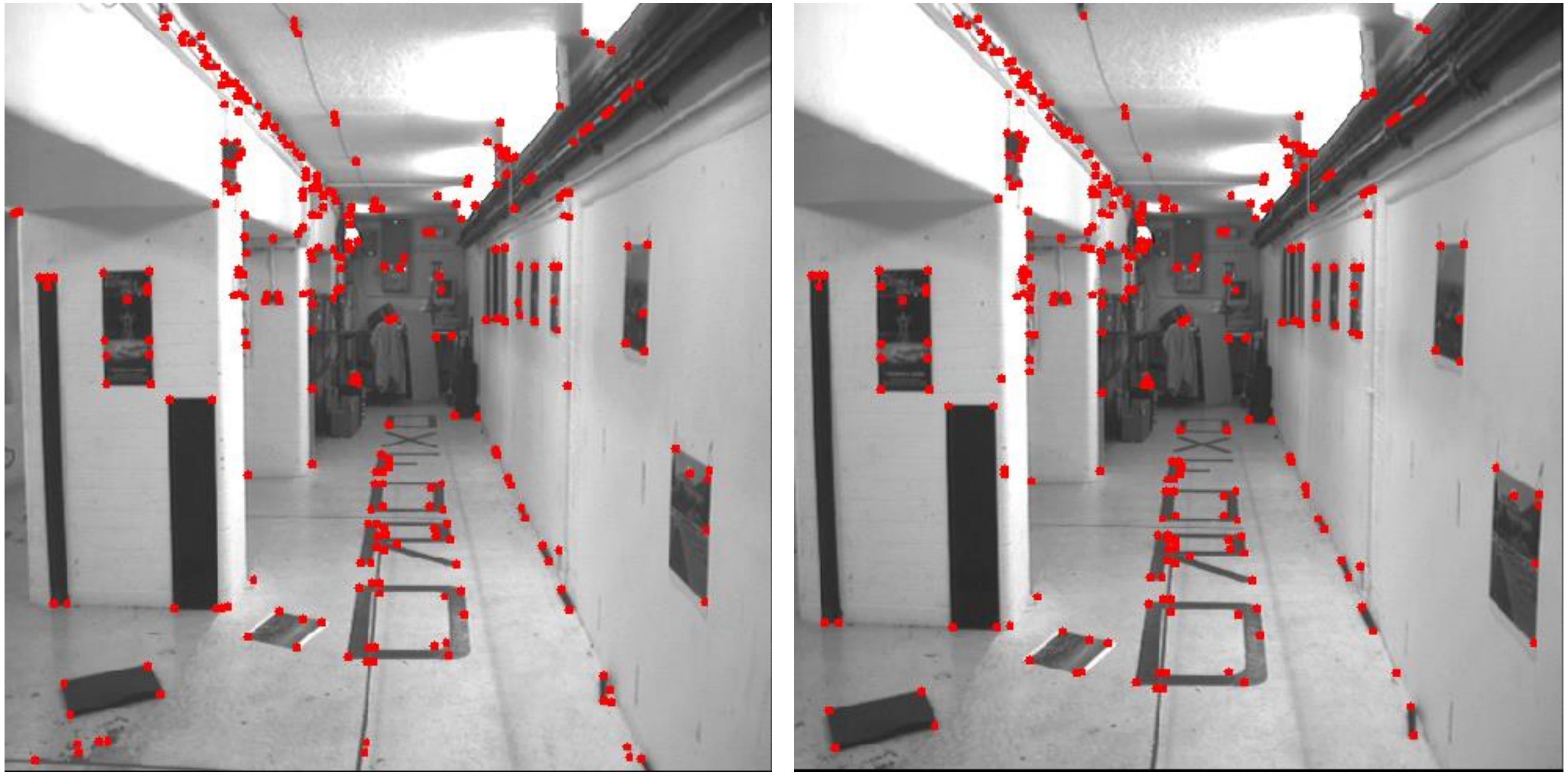
Effect: A very precise corner detector.

Slide credit: Krystian Mikolajczyk

# Harris Detector – Responses [Harris88]

Slide credit: Krystian Mikolajczyk

# Harris Detector – Responses [Harris88]



- Results are well suited for finding stereo correspondences

Slide credit: Kristen Grauman

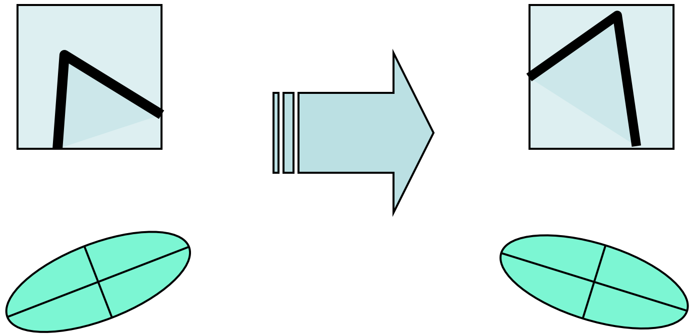
# Harris Detector: Properties

- Translation invariance?

Slide credit: Kristen Grauman

# Harris Detector: Properties

- Translation invariance
- Rotation invariance?



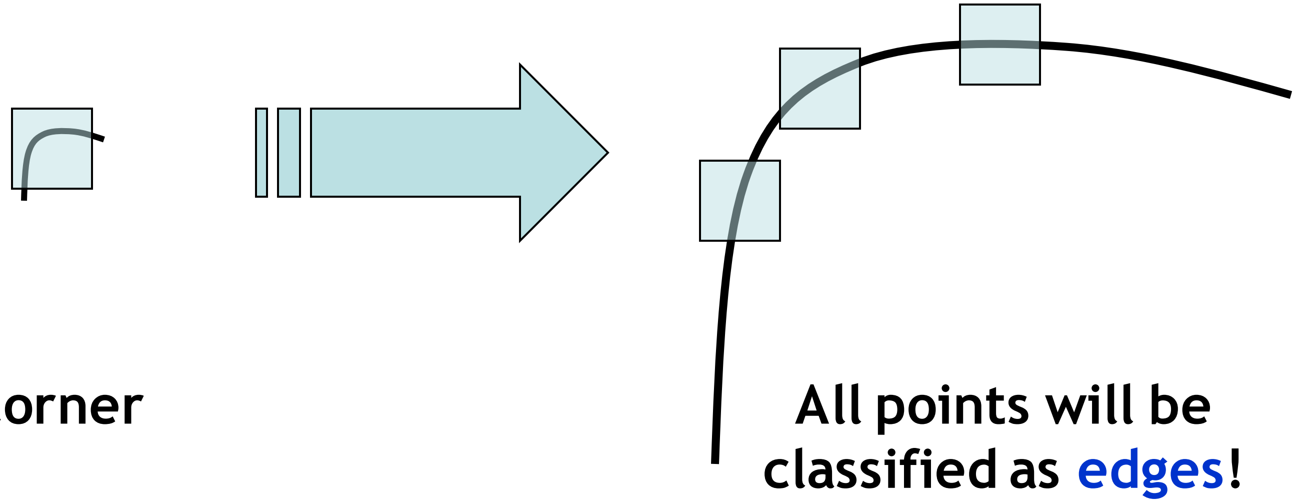
**Ellipse rotates but its shape (i.e. eigenvalues) remains the same**

***Corner response*** $\theta$ **is invariant to image rotation**

Slide credit: Kristen Grauman

# Harris Detector: Properties

- Translation invariance
- Rotation invariance
- Scale invariance?




**Not invariant to image scale!**

Slide credit: Kristen Grauman

# Summary

- Harris corner detector
  - Formulation
  - Examples