In the spaces given, re-write equations with the values you are calculating. Include units and make sure they match once fully simplified! *Example: to calculate gravitational force of the Sun on a 70kg person on Earth in CGS units (centimeter gram seconds), you'd write:*

$$F_G = \frac{G\ M_{sun}\ 70kg}{(1au)^2} = \frac{(6.67E-8\ cm^3\ g^{-1}\ s^{-2})(2E30\ g)(7E4\ g)}{(1.496E13\ cm)^2} \Rightarrow 41.7\ cm\ g\ s^{-2} \quad (1)$$

5 Estimating SMBH Masses from radio images

In this section we will measure the smallest observed radio size of a radio source surrounding a SMBH, and from there the SMBH mass. This was done with a network of ground-based radio interferometry dishes in Fig. 5, including an orbiting radio dish on the Japanese satellite Halca (also known as VSOP), which orbited up to 38,000 km away from Earth.

First, the wavelength and frequency of the observation are related as following for the radio image in Figure 5:

$$\lambda = \frac{c}{\nu}: \qquad c = 300,000\ km\ s^{-1}, \quad \nu\ =\ 5\ GHz. \quad (2)$$

(5a) Calculate the wavelength of this observations in cm:

$$\lambda = \frac{300,000\ km\ s^{-1}}{5\ \times\ 10^9 s^{-1}}\ =\ 0.00006\ km \times \frac{100,000\ cm}{1\ km}\ =\ 6\ cm. \quad (3)$$

Using this information, we can find the angular resolution of the telescope array used to make this observation. The resolution θ of any telescope is defined as the smallest detail or angle it can see, and the typical resolution of a telescope–including a radio interferometer–is given by:

$$\theta_{1,\ arcseconds} = 1.22 \times 206265 \times \frac{\lambda_{cm}}{B}, \quad (4)$$

where λ is the wavelength of the radio observations and B is the maximum diameter (or Baseline B) of the telescope (or radio interferometer) used during the observations, all converted to CGS units. Note B is twice the maximum distance of the satellite from Earth!

(5b) Calculate the maximum resolution θ_1 of the radio interferometer in arcseconds (as), and also converted to milli-arcseconds (mas):

$$\theta_{1,\ arcseconds} = 1.22 \times 206265 \times \frac{6\ cm}{2\ \times\ 38,000\ km \times\ 100,000\ cm\ /\ 1\ km}\ =\ 0.0001987\ as \quad (5)$$

$$\theta_{1,\ milliarcseconds}\ \Rightarrow 0.0001987\ as\ \times\ \frac{1,000\ mas}{1\ as}\ =\ 0.1987\ mas \quad (6)$$

Let's check this with Figure 5! Since the resolution θ is the *most* precise the telescope can be, we want to measure the narrowest part of a *point source* in Figure 5. (All shapes are distorted due to different baselines, and therefore different resolutions, in the two directions.)

(5c) Measure the width in the *narrowest direction* (either x or y) of the tiny tiny speck near (X,Y)=(0,0). We will assume this is the emitting region around our black hole. Also measure the width of two other point sources in the image. The point sources are the very skinny ellipses, and note that each tick mark on the Figure's axes is 1 milli-arcsecond.

(All θs measured should be around 0.2 milliarseconds. Definitely fractions of a tick mark on the X axis.)

(5d) Now average these three measurements and the angular resolution to get a θ_{avg}:

$\theta_{avg} \approx 0.2$ mas

The SMBH cannot be bigger than a given θ in angle on the sky. We now need to convert this angle with the known distance of the galaxy M87 to a physical size of the area immediately surrounding the SMBH, whose size was measured with this radio observation. For this we need to know the cosmological redshift z of the galaxy due to the expansion of the Universe, which is a direct measure of its distance D given Hubbles law:

$$D_{Mpc} = \frac{c}{H_0} \times z, \tag{7}$$

where c is again the speed of light (which in km/s is 3e5), and the expansion rate of the Universe is indicated by the Hubble Constant = 71 $km\ s^{-1}\ Mpc{-}1$ (see the Hubble expansion Lab at the end of the semester). The Hubble redshift of M87 is z = 0.0044.

(5e) Get the distance to M87 in Mpc (megaparsec), and then convert to parsec.

$$D_{Mpc} = \frac{300,000\ km\ s^{-1}}{71\ km\ s^{-1}\ Mpc{-}1} \times 0.0044 \ = \ 18.6\ MPc \times \frac{10^6\ parsec}{1\ Megaparsec} \ = \ 1.86 \times 10^7\ parsec \tag{8}$$

We can now use the small angle approximation to determine the linear diameter d of the emitting region around the black hole we measured in (5c):

$$d_{pc} = \theta_{as} \times D_{pc} \tag{9}$$

(5f) Calculate the linear diameter that the black hole would be, given the width θ_{avg} you derived above. Give d in parsecs, AU, and cm.

$d_{pc} = 1.86\ \times\ 10^7$ parsec
$a_{au} = 7.6\ \times\ 10^8$ astronomical units
$d_{cm} = 1.131\ \times\ 10^{22}$ centimeters

We know that the size of this region must be a bit bigger than the actual size of the SMBH, otherwise the radio emission could not have escaped. How much bigger we do not know, but from the model cartoons in the bottom right panels of Figure 2 we estimate that the SMBH radius rs could be a few to 10 × smaller (lets assume 5× smaller) than the linear diameter d of the radio source.

(5g) Calculate the radius of the black hole in centimeters given this assumption.

$r_s \approx d/5 = 2.262 \times 10^{21}$ centimeters

Our last step is to use the relation of the Schwarzschild radius of the SMBH to convert the measured linear diameter d into a mass for the SMBH. The Schwarzschild is essentially the same as what we get for Keplers third law of motion, except that in the relativistic case there is an extra factor of 2 in the equation:

$$M_{SMBH} = \frac{r_s^2\ c^2}{2\ G}, \tag{10}$$

where G is the gravitational constant in CGS units ($6.67\ \times 10^{-8}\ cm^3\ g^{-1}\ s^{-2}$), and c is the speed of light in CGS units ($2.998\ \times 10^{10}\ cm\ s^{-1}$). For this formula to work, everything must be in these cgs units, which are cm for length, cm/sec for speed.

(5h) Compute the mass of the black hole in grams. Be sure to write out your work and exactly what values with what units you're plugging in.

$$M_{SMBH} = \frac{(2.262\ \times\ 10^{21}\ cm)^2\ \times\ (2.99\ \times\ 10^{10}\ cm\ s^{-1})^2}{2\ \times\ 6.67\ \times 10^{-8}\ cm^3\ g^{-1}\ s^{-2}} = 1.53 \times 10^{49}\ grams \tag{11}$$

(5h) The masses of black holes are often measured and reported in units of solar masses. Calculate the mass of the SMBH knowing the mass of the Sun is $1.99\ \times 10^{33}\ gr$:

$$M_{SMBH} = 1.53\ \times\ 10^{49}\ grams\ \times\ \frac{1\ M_{\odot}}{1.99\ \times\ 10^{33}\ grams} = 7.63\ \times\ 10^{15}\ M_{\odot}$$

(5i) Take the log_{10} of this mass, write it into Table 2, and compare it to the literature value for the SMBH mass in M87, which is $M_{SMBH} = 3\ \times 10^9\ M_{\odot}$. What is your percentage error when working with logs, and when working with the regular numbers?

$$\text{Percentage error on } M_{SMBH} = 100\%\ \times \frac{log_{10}(M_{yours})\ -\ log_{10}(M_{literature})}{log_{10}(M_{literature})} \tag{12}$$

$log_{10}(7.63\ \times\ 10^{15}\ M_{\odot}) = 15.88$

Percent error, regular number space $= \frac{7.63\ \times\ 10^{15}\ -\ 3\ \times\ 10^9}{3\ \times\ 10^9} \times\ 100\% = 2.45\ \times\ 10^8\ \%$

Percent error, logspace$= \frac{15.88\ -\ 9.5}{9.5}\ \times\ 100\% = 67.6\%$

Note that such estimates are uncertain (after all, we cant see what we measure), and any measurement in astrophysics like these are considered a success if they are within a factor of 310!

(5j) Let's explore a little further what could be leading to such a discrepancy in our calculations. Using slightly modified version of all the previous equations and the same steps in logic, calculate what baseline you would need, in au, to measure a black hole with $M_{SMBH} = 3 \times 10^9 M_\odot$.

After lots'o'math, you get $\theta \approx 10^{-16}$ arseconds. If you are extra, have them try to get the baseline, which comes out to MANY MANY au.

(5k) Please discuss the uncertainties in this diameter-based method. What would be required to improve upon this mass estimate?

This assumes that the observable area is exactly the size of the black hole. It isn't–it's just the smallest thing we can see with this telescope. The SMBH itself is MUCH smaller than we can resolve, and if we treat our smallest pixel as its size, we're going to WAY overestimate the Schwarzschild radius and therefore the mass.

6 Estimating SMBH from the typical AGN variability time scale

In this section we will estimate the SMBH mass (Msmbh) from the variability seen in the light curves of up to five AGN measured with the ASU Braeside telescope near Flagstaff. The TA will tell you how many of the 5 AGN in Fig. 6a6e you will be measuring. In good weather conditions, one of the TAs may also take some live observations with the ASU Braeside telescope of one of the variable AGN that is visible during class night.

The typical time-span by which each AGN is referred to as the variability period Δt, even though the AGN feeding is not strictly a periodic phenomenon (unlike the case of regularly-varying stars like Cepheids).The SMBH mass can be derived from the average time-span values Δt in Tables 1a1e. Because of the finite speed of light, the actual region causing the flickering AGN light cannot be larger than the distance light can travel in the variability time-interval Δt. (Or: if the emitting region is a single object, all parts of it must be causally connected.) Therefore, the diameter d of the variability region cannot be larger than:

$$d = c \times \Delta t \tag{13}$$

The Schwarzschild of the SMBH must be less than half of this diameter:

$$r_s = 0.5 \times d = 0.5 \times c \times \Delta t \tag{14}$$

We use again the Schwarzschild equation above to convert our estimate of r_s into a direct estimate of (or upper limit to) the SMBH mass. (Again, work in CGS units, with the gravitational constant $G = 6.67 \times 10^{-8}\ cm^3\ g^{-1}\ s^{-2}$, and the speed of light $c = 2.998 \times 10^{10}\ cm\ sec^{-1}$.)

$$M_{SMBH} = \frac{r_s \times c^2}{2 \times G} \quad \text{or} \quad \frac{0.5 \times c^3 \times \Delta t}{2 \times G}. \tag{15}$$

Use these equations to evaluate your estimate of M_{SMBH} in terms of solar masses for each of the assigned AGN. (The mass of the Sun is $M_{\odot} = 1.99 \times 10^{33}\ g$). Take again the log_{10} of these masses, enter them in Table 2, and compare them to the literature values in Table 2 using percentage error. **Please show all your work, units, and inserted values for each calculation, either on a separate piece of paper or neatly at the margins.**

Please discuss the uncertainties in this variability method. What are the inherent uncertainties, and what would be required to improve upon this M_{SMBH} estimate? Why do you think this method provides a more accurate SMBH mass than the diameter method in Section 5?

Students get quite close in order of magnitude with this method; watch out for them not converting the au into cm, not using CGS, or keeping time in hours rather than converting to seconds. With the time series, they're looking for the change between local minima and maxima, not a repeating pattern. To save time, have them calculate the radius and mass of each black hole using the average time and magnitudes.