# Final Exam

*AST 422 Spring 2007*

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last updated: Apr 30, 2007

DUE: Wednesday May 9, 10:00 am in my mailbox in room PSF-686

Rules:

In the allotted time, please complete 6 questions of your choosing from the set of 15 listed below. To make sure that all topics are covered fairly, I ask you that you select at least a minimum number of questions from each of the three main groups, as indicated below. So please read all questions before you start.

All questions have equal weight, to be roughly equally distributed among each of the sub-questions. Hence, out of a total of 150 points, each of the 5 questions that you choose will be worth 30 points, and each sub-question typically 6-10 points (for typically 5-3 sub-questions per question).

As Always: Brevity is as much appreciated as the correct answer. Please do not give me more than 203 pages for each main question, if at all possible.

NOTE: $H_0$ indicates the current value of the Hubble constant throughout, $q_0$ the current value of the deceleration parameter, $\Omega_0$ or $\Omega_m$ the current value of the density parameter, and $t_0$ the current age of the Universe. For some questions you must assume a cosmology, I suggest we all use $H_0 = 73\, km\, s^{-1}\, Mpc^{-1}$, $\Omega_m = 0.24$, $\Omega_\Lambda = \Lambda = 0.76$, unless given otherwise. In most FRW type questions, I will assume $\Omega_\Lambda = 0$ for simplicity, so the answers can be found analytically – in other questions we will use the "real" cosmology $H_0 = 73$, $\Omega_m = 0.24$, $\Omega_\Lambda = \Lambda = 0.76$. If you don't like solving these questions for $\Omega_\Lambda = 0$ you may solve them for $\Omega_\Lambda = \Lambda = 0.76$, but then you have to use the numerical integration path. You may solve the questions for either $\Omega_\Lambda$, but please tell us upfront which $\Omega_\Lambda$ you have chosen.

Group A: Select 1∼2 Questions

A-1) Consider the following 5 galaxies: A with $v = 5000\, km/s$ at distance $R = 100\, Mpc$; B with $v = 10,000\, km/s$ and $R = 20\, Mpc$; C with $v = 5000\, km/s$ and $R = 50\, Mpc$; D with $v = 10,000\, km/s$ and $R = 100\, Mpc$; and E with at $z = 1.5$ with a measured age of $3.5 \sim 4.0\, Gyr$ at that redshift.

A-1.a) If your Universe only consists of galaxies A and B, what would you conclude about the value of the Hubble constant and its error?

A-1.b) If your Universe only consists of galaxies C and D, what would you conclude about the value of the Hubble constant and its error?

A-1.c) If your Universe consisted of galaxies A, B, C, and D, what would you derive as the most likely value of the Hubble constant, if you also knew that the distance measurements to the galaxies were uncertain by at least 35%?

A-1.d) If your Universe consisted of galaxies A, B, C, and D, what would you be able to conclude about large scale motions and about the Hubble expansion ($H_0$), if you knew that HST had measured their distances to better than 15% accuracy? How large could large-scale streaming motions be in such a Universe?

A-1.e) Compute the age of the Universe if $H_0 = 73\,km/s/Mpc$ and $\Omega = 0$. Roughly what would this age be for $\Omega_m = 0.24$, 1, and 2 ($\Omega_\Lambda = 0$ in all cases)? Please sketch the expansion or scale-factor of the Universe as a function of cosmic time for these different values of $\Omega_m$. For which value of $\Omega_m$ do you get the oldest Universe?

A-1.f) Galaxy E is at a redshift $z = 1.5$, where the Universe was only 25% of its current age (for $\Omega_m = 1$, $\Omega_\Lambda = 0$), and was measured to have stellar population age of 3.5 $\sim$ 4.0 Gyr at that redshift. Discuss what problems this poses for certain values of $H_0$. How does this compare to the globular cluster age problem (assuming the same values of $H_0$ and $\Omega_m$)? If $\Omega_m = 1$, what values of $H_0$ would be implied by galaxy E? Is this consistent with the most recent $H_0$ measurements? What other solutions to this problem can you think of?

A-2.a) Give a brief discussion (telegram style, no more than two pages) about what the major uncertainties are in determining the Hubble constatn. Discuss in particular how Large Scale Structure can and has afected the value of $H_0$.

A-2.b) Out to what distances/on what scales do you know for sure that the Universe is not homogeneous and isotropic, and at what distances/on what scales should the Universe be homogeneous and isotropic? Documnet your answer.

A-2.c) Describe briefly the differences and merits of the Cold Dark Matter (CDM) models of galaxy formation. Discuss briefly the major observational pieces of evidence in favor of/against CDM and in favor of/against HDM.

A-3.a) Derive from scratch a period - Luminosity relation for Cepheids, using what you know about stellar structure (polytrope models etc), the stellar mass-luminosity relation, the relation between bolometric absolute magnitude and $M_V$, and the Hertzsprung Russell diagram. Give the relevant intermediate steps, and derive:
$\log P = (0.5h + 0.3) \cdot M_V + (0.5hd + 0.3d + 3a) \cdot (B - V) + const$

A-3.b) Discuss this result. What does each term mean, and how does it apply to the distance scale problem?

A-3.c) Approximately how is the zero-point in the Cepheid distance scale affected by dust? How does dust quantitatively affect the zero-point in the Cepheid distance scale? How do (HST) distance measurements that use Cepheids attempt to correct for this?

A-3.d) Derive for a simply rotating spiral galaxy disk, using Kepler's laws, the Tully-Fischer relation: Luminosity $L$ proportional to $(\Delta v)^4$, where $v$ in km/sec. Discuss your assumptions, how does this apply to the distance scale problem?

A-4) Describe briefly FOUR DIRECT methods for the determination of $H_0$. Describe for each method roughly how it works, what the % uncertainties are, and what the major causes for these uncertainties are. When describing these four methods, discuss at least the following:

A-4.a) A method based on standard candles.

A-4.b) A method based on standard rods.

A-4.c) A method based on a physical model of hot gases.

A-4.d) A method based on the effects of large masses on the path of light rays.

A-5) Describe how the determination of the Hubble constant may be affected by:

A-5.a) The Malmquist bias. You may assume a Schechter or a Gaussian galaxy LF.

A-5.b) Significant Large Scale Structure in the direction in which you are measuring $H_0$.

A-5.c) Significant large scale streaming motions (or a "Great Attractor") in the galaxy distribution.

A-6) Define a problem of your choosing in nearby extragalactic astronomy and cosmology (topics: distance scale, large scale structure, etc.), and solve it. You will be graded on this question for how well you pose the problem (25%), how relevant it is to cosmology (25%), and how well you solve it (50%).

Group B: Select 2$\sim$3 Questions

B-1.a) Show that for all world models with $\Omega_\Lambda = 0$, we have essentially $q_0 = 1/2$ or $\Omega_m = 1$ shortly after creation, i.e. when $R(t) \to 0$ for $t \to 0$. Show that this is the case NOT ONLY for $\kappa = 0$, but also for $\kappa = -1$ and $\kappa = +1$. Discuss the relevance of this result. (You may assume reasonable results from previous homework questions. HINT: this issue is related to the flatness problem in inflation).

B-1.b) Sketch the look-back time $\tau(z)$ for $\Omega_m = 0$, 0.1, 1.0, 2.0.

B-1.c) Show that the Age of the Universe:

$t_0 = \frac{1.0}{H_0}$ for $\Omega_m = 0.0$

$t_0 = \frac{2}{3H_0}$ for $\Omega_m = 1.0$

$t_0 = \frac{0.577}{H_0}$ for $\Omega_m = 2.0$

assuming $\Omega_\Lambda = 0$ in all cases for simplicity.

B-2.a) Assume that the cosmic time can be parametrized as function of the "epicyclic" angle $\theta$, which is a monotonic function of cosmic time, and assume that:

$COS(\theta_0) = (2 - \Omega_m)/\Omega_m$
$COS(\theta_1) = (z + COS(\theta_0))/(1 + z)$,

where $COS \equiv \cos$ for $\kappa = +1$ ($\Omega_m = 2$ or $q_0 = 1$; $\Omega_\Lambda = 0$) and $\equiv \cosh$ for $\kappa = -1$ ($\Omega_m = 0$ or $q_0 = 0$; $\Omega_\Lambda = 0$). Show that the look-back time as function of the total age of the Universe can be written as:

$$\frac{\tau(z)}{t_0} = 1 - \left[\frac{\theta_1 - \sin(\theta_1)}{\theta_0 - \sin(\theta_0)}\right]$$

for $\kappa = +1$, and

$$\frac{\tau(z)}{t_0} = 1 - \left[\frac{\sinh(\theta_1) - \theta_1}{\sinh(\theta_0) - \theta_0}\right]$$

for $\kappa = -1$,

and derive the product $t_0 H_0$ in both cases.

B-2.b) For general values of $\Omega_m$ or $q_0$, show that the look-back time as function of redshift can be written as:

$$\tau(z) = \frac{q_o}{H_0(2q_0 - 1)^{3/2}} \cdot [\theta_0 - \theta_1 + \sin(\theta_1) - \sin(\theta_0)]$$

for $\kappa = +1$, and

$$\tau(z) = \frac{q_0}{H_0(1 - 2q_0)^{3/2}} \cdot [\theta_1 - \theta_0 + \sinh(\theta_0) - \sinh(\theta_1)]$$

for $\kappa = -1$

B-2.c) Derive the special cases $\tau(z) = \frac{2}{3Ho}\frac{(1+z)^{3/2}-1}{(1+z)^{3/2}}$ for $\Omega_m = 1$ or $q_0 = 1/2$ (assuming $\Omega_\Lambda = 0$) and $\tau(z) = \frac{z}{H_0(1+z)}$ for $\Omega_m = 0$ or $q_0 = 0$ (assuming $\Omega_\Lambda = 0$). Sketch $\tau(z)$ for relevant values of $\Omega_m$ in the range $\Omega_m = 0 \sim 2$ (or $q_0 = 0 \sim 1$).

B-3.a) Using the Mattig equation (Longair eq 7.38), show that the observed angular size $\theta$ (in arcsec) of an object of physical size $Y$, or a proper length $Y$, (in kpc) at redshift $z$ goes as:

$$\theta = k\frac{Y(1+z)}{R_0 r}$$

where $(R_0 r)$ is the distance of the object as seen by the observer, and $k$ is some arbitrary constant.

B-3.b) Express $\theta$ in units of $(\frac{YH_0}{c})$ as a function of redshift $z$ for the case $\Omega_m = 0$ or $q_0 = 0$ and the case $\Omega_m = 1$ or $q_0 = 0.5$ (assuming $\Omega_\Lambda = 0$ in both cases). Use Longair eq 5.56, 5.57 or Ryden eq 7.36.

B-3.c) Sketch $\theta(z)$ for the cases $\Omega_m = 0$, 1 and 2 (or $q_0 = 0$, 0.5, and 1), and determine for $\Omega_m = 0$ and 1 (or $q_0 = 0$ and $q_0 = 0.5$) the value of the redshift at which $\theta$ will appear the smallest (assuming $\Omega_\Lambda = 0$ in all cases).

B-3.d) What is the physical size scale of a CBR fluctuation of $10'$ diameter at $z = 1100$ for $\Omega_m = 0$, 1 and 2 (or $q_0 = 0$, 0.5, and 1; $\Omega_\Lambda = 0$ in all cases)? What is it for the cosmology $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$? At what angular scales do you expect the next generation of microwave satellites PLANCK to see most structure in the CBR, in terms of $\Delta T/T$? Document your answer.

B-4) In all of the following sub-questions, make drawings of the run of the relevant parameter with redshift for various values of $\Omega_m = 0 \sim 2$ (or $q_0 = 0 \sim 1$; assuming $\Omega_\Lambda = 0$) and discuss the results. If you wish, you may do this for the more complicated case of $\Omega_m + \Omega_\Lambda = 1$ for various values of $\Omega_m$. Where necessary, you may assume that objects have a power-law spectrum with spectral index alpha, $\alpha$.

B-4.a) Discuss how the luminosity distance depends on redshift.

B-4.b) Discuss how the volume element available per unit redshift depends on redshift. Discuss at least two reasons why there may be a decline in the observed quasar and young galaxy density for $z > 2 \sim 3$.

B-4.c) Discuss how the observed integrated bolometric and monochromatic flux of an object of fixed standard spectrum depends on redshift.

B-4.d) Discuss how the observed bolometric and monochromatic surface brightness of an object of fixed standard spectrum depends on redshift.

B-4.e) Discuss how the angular size of an object of fixed metric linear size depends on redshift.

B-5.a) Show why the matter density in the post-recombination Universe scales as $(1+z)^3$, and why the radiation density has scaled since then – and even before then – as $(1+z)^4$. What does this mean for the shape of the Cosmic Background Radiation spectrum and for the CBR temperature as function of $(1+z)$?

B-5.b) Show qualitatively (i.e. never mind the exact physical constants) why the Jeans mass of a spherical clouds of hot matter goes as $T^{-3}$ before recombination, and as $T^{3/2}$ after recombination. Discuss what – as a consequence – the typical masses are of gas clouds that first fragment into stars or groupings of stars in the early Universe.

B-5.c) Discuss briefly why the faint galaxy two-point correlation function has an amplitude that is so low. What does this tell you about the redshift distribution of the faint blue galaxy population and about the clustering of faint galaxies (and its evolution, or lack thereof).

B-6.a) A light source in a flat universe has a redshift $z$ when observed at a time $t_0$. Show that the observed redshift changes with time at a rate of:

$$\frac{dz}{dt_0} = H_0(1+z) - H_o(1+z)^{3(1+\omega)/2}$$

where $\omega = 0$ for matter+vacuum-energy dominated universes (you may assume $\omega = 0$). For what values of $\omega$ does the redshift decrease with time? For what values of $\omega$ does the redshift increase with time?

B-6.b) Suppose you are in a flat, matter-only universe that has a Hubble constant $H_0 = 73\, km/sec/Mpc$. You observe a galaxy wth redshift $z = 1.000000$. How long will you have to be observing this galaxy from Earth to see its redshifts change by one part in $10^6$? [Use th results from 6.a]

B-7) Define a problem of your choosing in cosmology and the early universe (topics: expansion, FRW models, geometry, observational distinction between the various cosmological models, etc), and solve it. You will be graded on this question for how well you pose the problem (25%), how relevant it is to cosmology (25%), and how well you solve it (50%).

Group C: Select 2$\sim$3 Questions

C-1.a) For a population of cosmic radio sources emitting radiation with an assumed constant luminosity or radio power $P$ (in $W/Hz$) and constant space density (in $N_0/Mpc^3$), derive for the Euclidean case the source count $N(S)$, where $N$ is the number of sources (per ster rad) with observed flux density greater than some value $S$ (measured in $Jy = 10^{-26} W/Hz/m^2$).

C-1.b) What is the logarithmic slope of the source counts in flux units ($S$), and in magnitude units $[m = -2.5\log(S) + const]$ for this Euclidean case? Sketch this together the radio source counts observed by Ryle et al. and the optical galaxy counts by many recent observers. What are the actual observed slopes at the bright end of the counts in both radio and optical? How do you interpret the slopes being different from question a? Discuss all possible reasonable answers.

C-1.c) At approximately what magnitude do the B-band galaxy counts make a significant change in slope (slope changing from approximately what value to what other value)? Calculate schematically the contribution of each magnitude bin of the B-band galaxy counts to the Extragalactic Background Light (EBL) for $10 < B < 30mag$. At which magnitude do the B-band galaxy counts contribute maximally to the EBL? How can this be interpreted in terms of their redshift distribution, and what does this imply for galaxy formation/evolution scenarios?

C-2.a) Discuss why it was – in hindsight – not surprising that in the deepest available HST fields ( 50 hr exposures in both HDF's), faint galaxies were seen down to $V, I = 30mag$ which are generally quite compact, with scale-lengths $r_{hl} = 0.1 \sim 0.3''$. Discuss why the most compact objects are relatively easy to detect with HST, what observational selection effects have to be overcome, and how their flux depends on the various factors of $(1+z)$.

C-2.b) The Next Generation Space Telescope (NGST) is an instrument currently planned for launch in 2007. It will have an 8 meter (segmented mirror) and a rich array of optical and infrared camera's and spectrographs. Estimate how faint this telescope can see in a 1000 hr exposure (Hint: compare to what the 2.4 meter HST can do with its 50 hr exposures in the HDF in V and I).

C-2.c) Assume the IR detectors are good enough that similar sensitivities can be obtained in the near-IR JHK bands. For $H_0 = 73$ and $\Omega_m = 0.24$, compute out to which redshift this telescope can see AGN ($M_V = -25$ and $r_{hl} < 0.1\,pc$), $L^*$ galaxies ($M_V = -22$ and $r_{hl} = 5\,kpc$), sub-galactic objects ($M_V = -17$ and $r_{hl} = 0.1\,kpc$), supernovae ($M_V = -19$), and globular clusters ($M_V = -10$ and $r_{hl} = 1\,pc$). Assume for simplicity that all these objects are young and therefore blue with flat power-law spectra ($\alpha = 0$ or 1), so that K-corrections are straightforward. [Note: You need to get the answer to question b right to answer c correctly.]

C-2.d) What are likely the only objects that the NGST will be able to see at $z > 10$? Argue your answer and mention at least two reasons why this category of object is favored above all the others. (You may assume that AGN form after at least one galactic free fall time has passed).

C-3.a) Consider a stellar population of $10^{10}\,M_\odot$ in total, all born at the same time in the very early history of a galaxy, and following a Salpeter IMF between $0.08\,M_\odot < M < 100\,M_\odot$ – with no stars forming outside this range. Roughly how large a fraction of the stars in this galaxy will go off in Supernovae? How many stars is this? How often do they go off as SNe?

C-3.b) Roughly when did this Supernovae phase happen during the evolution of the galaxy? What redshift range does this roughly correspond to, if the entire galaxy formed within one free fall time after the Big Bang?

C-3.c) Sketch qualitatively how the various stellar populations that develop in the HR diagram (i.e. the MS, RGB, HB, AGB, P-AGB, etc) during the evolution of this galaxy contribute to its total galaxy light in the Johnson filters UBVRI+JHK, as well as in the far UV (around $1500$ and $2500\mathring{A}$). Briefly discuss which component dominates each filter and why.

C-3.d) Compute the probability that this galaxy is seen with a supernovae, if it were discovered at very high redshift by the HST or NGST. [You need to know the answer to question a and b to answer d].

C-4.a) Discuss briefly how the integrated optical/near-IR spectra of galaxies depend on the age of their dominant stellar population. Make a schematic drawing of their spectra. How does this affect the K-correction we derive for such galaxies?

C-4.b) Discuss briefly how the integrated optical/near-IR spectra of galaxies depend on the past star-formation history. Make a schematic drawing of their spectra for various assumed (cartoon-like) star-formation histories.

C-4.c) Discuss how the answers of questions a and b are connected: what is the physical cause for this connection? How can we best measure galaxy ages?

C-5.a) Sketch and discuss how the galaxy counts determined as function of Hubble type enable us to constrain the galaxy luminosity function (LF) and its evolution with cosmic epoch.

C-5.b) What is the major uncertainty in these exercises? How can we break this degeneracy by measuring the redshifts or the redshift distribution of faint galaxies?

C-5.c) Discuss evidence in favor of, and evidence against the premise that faint blue galaxies are primarily lower-luminosity (i.e. $L < L^*$ in the LF) objects of irregular morphology and with sizes typically smaller than those of NGC galaxies seen today. What does this mean for the formation scenario of galaxies?

C-6) You observe a quasar at a redshift of $z = 5.0$ and determine that the observed light flux from the quasar varies on a timescale $\Delta t_0 = 3$ days. If the observed variation in flux is due to a variation in the intrinsic luminosity of the quasar, what was the variation timescale $\delta t_e$ at the time the light was emitted? For the light of the quasar to vary on a timescale $\delta t_e$, the bulk of the quasar light must come from a region of physical size $R \leq R_{max} = c\,\delta t_e$ . What is the angular size of $R_{max}$ in the standard model? How would you go about observing this?

C-7) In a flat Universe with $H_0 = 73km/s/Mpc$, you observe a galaxy at a $z = 6$.

C-7.a) What is the current proper distance to the galaxy, $d_p(t_0)$, if the universe only contains radiation?

C-7.b) What is $d_p(t_0)$ if the Universe only contains matter?

C-7.c) What is $d_p(t_0)$ if the Universe only contains a cosmological constant?

C-7.d) What was the proper distance at the time the light was emitted, $d_p(t_e)$, if the universe only contains radiation?

C-7.e) What was the proper distance at the time the light was emitted, $d_p(t_e)$, if the universe only contains matter?

C-7.f) What was the proper distance at the time the light was emitted, $d_p(t_e)$, if the universe only contains a cosmological constant?

C-8) The surface brightness SB of an astronomical object is defined as its observed flux divided by its angular area: $SB = f/(\delta theta)^2$.

C-8.a) For a class of objects that are both standard candles and standard yardstick, what is SB as a function of redshift in the case $z << 1$?

C-8.b) For a class of objects that are both standard candles and standard yardstick, what is SB as a function of redshift in the case $z >> 1$?

C-8.c) Would the SB of this class of object be useful way of determining the deceleration parameter qo? Why or why not?

C-8.d) Discuss the grave cosmological consequences of your answer in C-8.b. Why do you think observations with HST at $z >> 5$ are so hard? What do you think objects that JWST will see at $z = 15$ will look like to this telescope — resolved or unresolved? Document your answer carefully.

C-9) Define a problem of your choosing in galaxy formation and evolution (topics: galaxy formation, galaxy evolution, AGN evolution, structure formation, clustering evolution, the Intergalactic Medium etc), and solve it. You will be graded on this question for how well you pose the problem (25%), how relevant it is to cosmology (25%), and how well you solve it (50%).