AST 322 - Introduction to Galactic and Extragalactic Astrophysics - HW 6

5.1 (5 pts)
i) Verify that Equation 5.9 is a valid solution of Equation 5.8 by plugging Equation 5.9 into Equation 5.8.
(EXTRA CREDIT) For an additional 1 point of extra credit, derive Equation 5.9 directly from Equation 5.8 by solving the differential equation with the appropriate initial conditions.
ii) Using Equation 5.9, which w component dominates the expansion of the universe for $a \to 0$ ($z \to \infty$)?
iii) Using Equation 5.9, which w component dominates the expansion of the universe for $a \to \infty$ ($t \to \infty$)?
iv) Discuss the cases $w = 0$ and $w = 1/3$ and solve for $\varepsilon(a)$ for each case by plugging in the appropriate w-value into Equation 5.9.

5.2 (4 pts)
i) Using Equation 5.9 and the appropriate w-values, calculate the redshift, $z_{\Lambda m}$, for which $\varepsilon_m = \varepsilon_\Lambda$.
ii) Discuss what this means for the growth of galaxies with time (no more than 1-2 paragraphs).
Hint: In preparation for a possible term project, do a little literature search (use ADS abstract link on Links) on the cosmic star-formation rate vs. redshift SFR(z). (e.g., Madau, P. & Dickinson, M. 2014, Ann. Rev. A&Ap, 52, p. 415). The PDF is available here: ***https://arxiv.org/abs/1403.0007*** *. Based on their Fig. 9, discuss what impact the expansion and Lambda (> 0) may have had on the growth of galaxies with cosmic time, and whether this is visible in the data.*
[For extra credit: You could also discuss how the galaxy merger rate has changed as a function of redshift, by comparing the mix of spiral and elliptical galaxies in high redshift clusters to those at low redshifts. (Hint: You need to know that galaxy mergers tend to transform spiral galaxies into ellipticals; Do a literature search of how the mix of spiral and elliptical galaxies has changed in galaxy clusters with redshift, as observed e.g. with Hubble).]

Questions continue on the next page...

5.3 (6 pts)
i) Derive Equation 5.38 from Equation 5.37 by making the ansatz that $a \propto t^q$:

$$q = \frac{2}{3+3w}$$

Discuss the meaning of this equation for $w = 0$ and/or $w = 1/3$.
ii) Derive Equation 5.39 from Equation 5.38 by applying the normalization condition that $a(t_0) = 1$:

$$a(t) = \left(\frac{t}{t_0}\right)^{2/(3+3w)}$$

Discuss the meaning of this equation for $w = 0$ and/or $w = 1/3$.
iii) Derive Equation 5.41 from Equation 5.39:

$$H_0 = \frac{2}{3(1+w)} t_0^{-1}$$

iv) Discuss the meaning of Equation 5.42 for $w = 0$ and/or $w = 1/3$:

$$t_0 = \frac{2}{3(1+w)} H_0^{-1}$$

5.4 (5 pts)
i) Derive Equation 5.48:

$$t_e(z) = \left(\frac{2}{3(1+w)H_0}\right)\left(\frac{1}{(1+z)^{3(1+w)/2}}\right)$$

ii) Derive Equation 5.49:

$$d_p(t_0) = ct_0 \frac{3(1+w)}{1+3w}\left[1 - \left(\frac{t_e}{t_0}\right)^{\frac{1+3w}{3+3w}}\right]$$

iii) Derive Equation 5.50:

$$d_p(t_0) = \frac{c}{H_0}\frac{2}{1+3w}\left[1 - (1+z)^{-(1+3w)/2}\right]$$

iv) Plot $d_p(t_0)$ and t_e as a function of redshift for both $w = 0$ and $w = 1/3$, and discuss their meaning.