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## Look at parametric substitutions for Matter - Curvature only universe

Original diff eq for Matter – Curvature only universe

$$\frac{da}{dt} = H_o \left(\frac{\Omega_m}{a} + (1 - \Omega_m)\right)^{\frac{1}{2}}$$

Look at parametric solution for $\Omega_m > 1.0$

The following are ' given ' :

In[48]:= $a_\theta = \frac{1}{2} \frac{\Omega_m}{(\Omega_m - 1)} (1 - \text{Cos}[\theta]);$

In[51]:= $da_\theta = \partial_\theta (a_\theta)$

Out[51]= $\frac{\text{Sin}[\theta]\ \Omega_m}{2\ (-1 + \Omega_m)}$

In[54]:= $t_\theta = \frac{1}{2\ H_0} \frac{\Omega_m}{(\Omega_m - 1)^{\frac{3}{2}}} (\theta - \text{Sin}[\theta]);$

In[55]:= $dt_\theta = \partial_\theta (t_\theta)$

Out[55]= $\frac{(1 - \text{Cos}[\theta])\ \Omega_m}{2\ H_0\ (-1 + \Omega_m)^{3/2}}$

So

In[56]:= $da_t = \frac{da_\theta}{dt_\theta}$

Out[56]= $\frac{\text{Sin}[\theta]\ H_0 \sqrt{-1 + \Omega_m}}{1 - \text{Cos}[\theta]}$

Substitute into the Original equation (the "/." operator means substitute)

In[59]:= $\frac{da}{dt} == H_o \left(\frac{\Omega_m}{a} + (1 - \Omega_m)\right)^{\frac{1}{2}}$ /. $a \to a_\theta$ /. da -> $da_\theta$ /. $dt \to dt_\theta$

Out[59]= $\frac{\text{Sin}[\theta]\ H_0 \sqrt{-1 + \Omega_m}}{1 - \text{Cos}[\theta]} == H_o \sqrt{1 + \frac{2\ (-1 + \Omega_m)}{1 - \text{Cos}[\theta]} - \Omega_m}$

In[61]:= $\text{Assuming}\Big[2\pi \geq \theta \geq 0\ \&\&\ \Omega_m > 1,$

$\text{FullSimplify}\Big[\frac{\text{Sin}[\theta]\ H_0 \sqrt{-1 + \Omega_m}}{1 - \text{Cos}[\theta]} == H_o \sqrt{1 + \frac{2\ (-1 + \Omega_m)}{1 - \text{Cos}[\theta]} - \Omega_m}, \text{Reals}\Big]\Big]$

Out[61]= $\text{Cot}\left[\frac{\theta}{2}\right] H_0 \sqrt{-1 + \Omega_m} == H_o \sqrt{\text{Cot}\left[\frac{\theta}{2}\right]^2 (-1 + \Omega_m)}$

Assuming

$$\mathrm{Cot}\left[\frac{\theta}{2}\right] = \sqrt{\mathrm{Cot}\left[\frac{\theta}{2}\right]^2}$$

we get

$$\mathrm{Cot}\left[\frac{\theta}{2}\right] H_0 \sqrt{-1+\Omega_m} == \mathrm{Cot}\left[\frac{\theta}{2}\right] H_0 \sqrt{-1+\Omega_m}$$

which is always TRUE.

In[63]:= ParametricPlot[

$$\left\{H_0 \frac{(\theta - \mathrm{Sin}[\theta])\,\Omega_m}{2\,H_0\,(-1+\Omega_m)^{3/2}},\ \frac{1}{2}\,\frac{\Omega_m}{(\Omega_m - 1)}\,(1-\mathrm{Cos}[\theta])\right\} /.\ \Omega_m \to \{1.1\} /.\ H_0 \to \mathrm{H0},$$

$\{\theta, 0, 2\pi\}$, AspectRatio $\to$ .4]

Out[63]=