AST 322 - Introduction to Galactic and Extragalactic Astrophysics - HW 4

4.1 (5 pts) Derive from Equation 4.10, combined with Newton's 2nd Law, that Equation 4.12 is true:

$$\frac{1}{2}\left(\frac{dR_s}{dt}\right)^2 = \frac{GM_s}{R_s(t)} + U$$

Where U is a constant of integration.

4.3 (7 pts)

i) Show that Equation 4.28a is true (from 4.20):

$$H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{\kappa c^2}{R_0^2}$$

ii) What is the special meaning of the case $\kappa = 0$?

iii) What is the actual value of the critical density of the universe ρ_0 that you derive in that case? i.e., show that Equation 4.28b is true:

$$\rho_0 = \frac{3H_0^2}{8\pi G}$$

iv) Assuming the constants given in the text, calculate the numerical value of ρ_o from 4.28b. [Do 4.3 before 4.2]

4.2 (8 pts)

i) Derive Equation 4.17 from the equations that come before it:

$$\frac{1}{2}r_s^2\dot{a}^2 = \frac{4\pi}{3}Gr_s^2\rho(t)a(t)^2 + U$$

Then derive Equation 4.18 from that:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_s^2}\frac{1}{a(t)^2}$$

ii) Solve Equation 4.18 for $a(t)$ in the case of $\kappa = 0$, i.e., $U = 0$. Then sketch this solution.

iii) Discuss and sketch the solutions for the case of $\kappa = +1$ ($U < 0$) and $\kappa = -1$ ($U > 0$).

(EXTRA CREDIT)

i) (1 pt) What is the end behavior of your $a(t)$ for the case of $\kappa = 0$? What does this mean physically? Hint: calculate

$$\lim_{t\to\infty}\frac{da}{dt}$$

ii) (2 pts) Derive from Equation 4.18 that $a(t)$ has critical points (minima or maxima) determined by Equation 4.19:

$$a_{\max} = -\frac{GM_s}{Ur_s}$$

Discuss what this means for the case of $\kappa > 0$ and $\kappa < 0$.