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1.0 $l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} \Rightarrow \left(\frac{m^3 kg^{-1} s^{-2}\; kg\, m^2 s^{-2} s}{m^3 s^{-3}}\right)^{1/2} \Rightarrow \left(\frac{m^2}{1}\right)^{1/2} \Rightarrow m$ ✓

$$l_p = \left[\frac{(6.67\times10^{-11}\ m^3kg^{-1}s^{-2})(1.05\times10^{-34}\ Js)}{\cancel{(299792458\ m/s)^3}\ (3.00\times10^{8}\ m/s)^3}\right]^{1/2} = \underline{1.61\times10^{-35}\ m} \approx 1.62\times10^{-35}\ m$$ ✓

$$M_p = \left(\frac{\hbar c}{G}\right)^{1/2} \Rightarrow \left(\frac{kg\,m^2s^{-1}\ m s^{-1}}{m^3kg^{-1}s^{-2}}\right)^{1/2} \Rightarrow (kg^2)^{1/2} \Rightarrow kg$$ ✓

$$M_p = \left[\frac{(1.05\times10^{-34}\ Js)(3.00\times10^{8}\ m/s)}{(6.67\times10^{-11}\ m^3kg^{-1}s^{-2})}\right]^{1/2} = \underline{2.17\times10^{-8}\ kg} \approx 2.18\times10^{-8}\ kg$$ ✓

$$t_p = \left(\frac{G\hbar}{c^5}\right)^{1/2} \Rightarrow \left(\frac{m^3kg^{-1}s^{-2}\ kg\,m^2s^{-1}}{m^5s^{-5}}\right)^{1/2} \Rightarrow (s^2)^{1/2} \Rightarrow s$$ ✓

$$t_p = \left[\frac{(6.67\times10^{-11}\ m^3kg^{-1}s^{-2})(1.05\times10^{-34}\ Js)}{(3.00\times10^{8}\ m/s)^5}\right]^{1/2} = 5.37\times10^{-44}\ s \approx 5.39\times10^{-44}\ s$$ ✓

$$E_p = M_p c^2 \Rightarrow kg(m^2s^{-2}) \Rightarrow kg\,m^2s^{-2} \Rightarrow J$$ ✓ $= eV$

$$E_p = (2.18\times10^{-8}\ kg)(3.00\times10^{8}\ m s^{-1})^2 = \underline{1.96\times10^{9}\ J} \cdot \frac{6\times10^{18}\ eV}{1\ J} = \underline{1.18\times10^{28}\ eV}$$ ✓

$\approx 1.22\times10^{28}\ eV$

$$T_p = E_p/k \Rightarrow \frac{eV}{eV\,k^{-1}} \Rightarrow k$$ ✓

$$T_p = \frac{1.22\times10^{28}\ eV}{8.62\times10^{-5}\ eV k^{-1}} = \underline{1.42\times10^{32}\ k}$$ ✓

Meaning of planck numbers:

Planck length: length scale of the very early universe at the Planck time
- Smallest meaningful distance

Planck mass: ~~Mass of early universe~~ Mass equivalence of the energy present in the very early universe

Planck time: shortest meaningful time scale – Big Bang time $t = t_p$, time of the planck epoch.

Planck energy: energy of universe, at $t = t_p$ after Big Bang

Planck Temperature: Temperature of early universe at $t = t_p$

- Planck units – conditions of the universe right after the Big Bang, where QM and GR break down. – The planck epoch.

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1.1 $\rho_P = \frac{E_P}{V_P} = \frac{E_P}{l_P^3} = M_P c^2 l_P^{-3} = \left(\frac{\hbar c}{G}\right)^{1/2} c^2 \left(\frac{G\hbar}{c^3}\right)^{-3/2} = c^2\left(\frac{\hbar c}{G}\right)^{1/2}\left(\frac{c^9}{G^3\hbar^3}\right)^{1/2}$

$= c^2\left(\frac{c^{10}}{G^4 \hbar^2}\right)^{1/2} = \boxed{\frac{c^7}{G^2\hbar}}$

$\rho_P = \frac{c^7}{G^2\hbar} = \frac{(3.00\times 10^8\, ms^{-1})^7}{(6.67\times 10^{-11}\, m^3 kg^{-1}s^{-2})^2(1.05\times 10^{-34}\, \underbrace{Js}_{kg\,m^2 s^{-2}})} = \frac{46.817\times 10^{56}\, m^7 s^{-7}}{10^{-22}\,10^{-34}\, m^8 kg^{-1} s^{-5}}$

$\rho_P = 4.68\times 10^{113}\, m^{-1}s^{-2}kg = 4.68\times 10^{113}\, kg\,m^2 s^{-2} m^{-3} = \boxed{4.68\times 10^{113}\, J/m^3}$

matter density $\rho_0 = 2.7\times 10^{-27}\, kg/m^3 = \frac{M_0}{l_0^3}$

$E=mc^2$

energy density $\rho_0 = \rho_{0\,matter} c^2 = (2.7\times 10^{-27}\, kg/m^3)(3\times 10^8\, m/s)^2 = \boxed{2.43\times 10^{-10}\, J/m^3}$

$\frac{\rho_P}{\rho_0} = \frac{4.68\times 10^{113}\, Jm^{-3}}{2.43\times 10^{-10}\, Jm^{-3}} = 1.93\times 10^{123}$ $\qquad \frac{E/l_P^3}{E/l_0^3} = l_0^3/l_P^3$

Assuming conservation of energy holds, the difference in ρ_P and ρ_0 implies that the volume of the universe has increased by a factor of $\sim 10^{123}$ since the planck time, or that the length scale of the universe has increased by a factor of $\sim 10^{41}$.

1.2a $R_s = \frac{2GM}{c^2}$

$$R_{sp} = \frac{2GM_p}{c^2} = \frac{2G}{c^2}\left(\frac{\hbar c}{G}\right)^{1/2} = 2\left(\frac{\hbar G}{c^3}\right)^{1/2} = \boxed{2\ell_p \approx 3.24\times10^{-35}\ m}$$

1.2b $R_{sp} = 2\ell_p$ — The Schwarzschild radius of the universe at the planck epoch was twice the planck length.

The fallacy here is that based on this calculation, one would think that the Universe could have never expanded to its present-day state due to the universe essentially being within its own black hole.

1.2c A black hole remaining a black hole relies on nothing traveling faster than the speed of light. However, the above Schwarzschild radius problem can be solved if the expansion of the universe was able to overcome gravity.

- Inflation solves this problem since it was an incredibly rapid period of expansion that would have probably made the radius of the Universe greater than its Schwarzschild radius. Expansion of the universe allows matter to travel faster than c relative to each other.
- Antigravity could be another way out of the dilemma.

1.2d $M_{universe} \approx 10^{80}$ H atoms $\times\ 1.67\times10^{-27}$ kg/H atom $\approx 10^{53}$ kg

From lecture Notes $R_{S_{universe}} = \frac{2GM_{universe}}{c^2} = \frac{2(6.67\times10^{-11}\ m^3kg^{-1}s^{-2})(10^{53}\ kg)}{(3\times10^8\ ms^{-1})^2} = 1.48\times10^{26}\ m \approx 1.5\times10^{26}\ m$

$R_H = R_0 = 13.7$ Gly $\approx 1.3\times10^{26}$ m

$\therefore R_{S0} \gtrsim R_H$, so the Universe is technically within its own Schwarzschild radius, just like during the planck epoch. This would mean, in a static universe, that we would be in our own black hole and the Universe shouldn't be expanding/ wouldn't be able to expand past its R_S. However, the solution to this is the expansion of the universe. Galaxies on the "other side of the universe" are moving away from us at $v>c$. For the Schwarzschild radius to be relevant, it would have to be true that $v<c$. Because of dark energy / cosmological constant, the Universe can escape its so called black hole.