HOMEWORK - 1

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H/W 1.0: Verify (NOT prove from first principles) that Ry Eq. 1.1-1.5 are correct. Then given the values of G, c, h_bar, verify that the listed values of l_p (m), M_P (kg), t_P (s), E_p (eV) and T_P (K) are correct. What do these extremely small or large values mean physically?

→ Verification (using unit analysis)

$$G \rightarrow m^3 kg^{-1} s^{-2} \,,\quad c \rightarrow m s^{-1} \,,\quad \hbar \rightarrow Js = kg \cdot m^2 \cdot s^{-1}$$

$$k \rightarrow J K^{-1}$$

$$l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} \xrightarrow{\text{units}} \left(\frac{m^3}{kg\, s^2} \times \frac{kg\, m^2}{s} \times \frac{s^3}{m^3}\right)^{1/2} = (m^2)^{1/2} = m$$

$$m_p = \left(\frac{\hbar c}{G}\right)^{1/2} \xrightarrow{\text{units}} \left(\frac{kg\, m^2}{s} \times \frac{m}{s} \times \frac{kg\, s^2}{m^3}\right)^{1/2} = (kg^2)^{1/2} = kg$$

$$t_p = \left(\frac{G\hbar}{c^5}\right)^{1/2} \xrightarrow{\text{units}} \left(\frac{m^3}{kg \cdot s^2} \times \frac{kg\, m^2}{s} \times \frac{s^5}{m^5}\right)^{1/2} = (s^2)^{1/2} = s$$

$$E_p = m_p c^2 \xrightarrow{\text{units}} \left(kg \times \frac{m^2}{s^2}\right) = J$$

$$T_p = E_p/k \xrightarrow{\text{units}} \left(kg \times \frac{m^2}{s^2}\right) \times \left(\frac{K s^2}{kg\, m^2}\right) = K$$

This shows that Ry Eq. 1.1 – 1.5 are correct.

→ verifying the values

$$l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} = \left(\frac{6.67 \times 10^{-11} \times 1.05 \times 10^{-34}}{(3 \times 10^8)^3}\right)^{1/2} = 1.611 \times 10^{-35}\ m$$

$$m_p = \left(\frac{\hbar c}{G}\right)^{1/2} = \left(\frac{1.05\times10^{-34} \times 3\times10^{8}}{6.67\times10^{-11}}\right)^{1/2} = 2.173\times10^{-8}\,kg$$

$$t_p = \left(\frac{G\hbar}{c^5}\right)^{1/2} = \left(\frac{6.67\times10^{-11} \times 1.05\times10^{-34}}{(3\times10^{8})^{5}}\right)^{1/2} = 5.369\times10^{-44}\,s$$

$$E_p = m_p c^2 = (2.173\times10^{-8})(3\times10^{8})^2 = 1.956\times10^{9}\,J = 1.221\times10^{28}\,eV$$

$$T_p = E_p/k = (1.221\times10^{28})/(8.62\times10^{-5}) = 1.417\times10^{32}\,K$$

These Planck values give certain values to represent the beginning of the universe. These small and large values accurately represent the values and conditions at the beginning of the universe. The small values show how small and compact the universe was at the beginning but the larger values show how hot and dense the universe was.

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H/W 1.1: Derive from the order-of-magnitude equations for the Planck epoch
the energy density during the Planck time: rho_P = c^7 / ( h_bar . G^2 ).
What is rho_P in physical units? Compare this to the rho_o value today of
2.7 x 10^-27 kg/m^3 (see Ry 2.2 pg. 11). What does this imply?
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Deriving the formula for energy density during planck time.

$$\rho_p = \frac{E_p}{l_p^3} \Rightarrow \frac{m_p c^2}{l_p^3} = \left(\frac{\hbar c}{G}\right)^{1/2} \cdot c^2 \times \left(\frac{c^3}{G\hbar}\right)^{3/2} = \hbar^{-1} c^7 G^{-2}$$

$$\Rightarrow \rho_p = \frac{c^7}{\hbar G^2}$$

Units $\rightarrow$ $$\frac{m^7}{s^7} \times \frac{s^2}{kg\,m^2 s} \times \frac{kg^2 s^4}{m^6} = \frac{kg}{s^2\,m} \times \frac{m^2}{m^2} = \frac{kg\,m^2}{s^2} \cdot \frac{1}{m^3} = \frac{J}{m^3}$$

Inserting these values, $\rho_p = \frac{(3\times10^8)^7}{(1.05\times10^{-34})(6.67\times10^{-11})^2} = \frac{2.187\times10^{59}}{4.671\times10^{-55}}$

$= 4.682\times10^{113}\ J/m^3$

$E = mc^2,\ m = \frac{E}{c^2} = \frac{4.682\times10^{113}}{(3\times10^8)^2} \times \frac{kg\,m^2}{s^2 m^3} \times \frac{s^2}{m} = 5.202\times10^{96}\ kg/m^3$

mass density during planck time $= 5.202\times10^{96}\ kg/m^3$

ρ_0 for today is $2.7\times10^{-27}\ kg/m^3$.

It is seen from these values that the mass density during Planck time is higher by a huge order of magnitude when compared to this mass density of the universe today. This implies that the early universe was very compact and dense.

H/W 1.2a: The Schwarzschild radius of a black hole is r_s = 2 G M_BH / c^2. Derive the Schwarzschild radius of the Universe during the Planck time, r_s,P.

During planck time, $m = 2.18\times10^{-8}\ kg$; $r_s = \frac{2GM_{BH}}{c^2}$

$= \frac{2\times(6.67\times10^{-11})(2.18\times10^{-8})}{(3\times10^8)^2} \approx \quad r_{s,P} = 3.23\times10^{-35}\ m$

Unit Analysis $\rightarrow$ $r_s = \left(\frac{m^3}{kg\,s^2} \times kg \times \frac{s^2}{m^2}\right) = m$

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H/W 1.2b: How does r_s,P compare to l_P? Describe the fallacy here.
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$$r_{s,P} = 3.23 \times 10^{-35}\ m \qquad \frac{r_{s,P}}{l_P} \approx 2 \Rightarrow r_{s,P} = 2 l_P$$

$$l_P = 1.62 \times 10^{-35}\ m$$

It is seen that the Schwarzschild radius of the black during plank time is twice the value of Planck length. Usually when an object/body is smaller than the Schwarzschild radius, it must exist as a black hole but the universe during Planck time did not become a black despite it being 2 times smaller than its Schwarzschild radius. So the main question is that why did the universe in its early stages not collapse into a blackhole.

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H/W 1.2c: Briefly suggest possible ways out of this dilemma.
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We know that a black hole is static and in comparison to this the early universe was expanding. This early universe was a hot dense compact region that had high amounts of energy and due to big bang it was rapidly expanding and due to this the universe is still expanding to this day. Since it was so compact previously, there was a lot of energy and this energy was greater than the inward gravitational force that would condense the universe into a black hole. This outward acting energy was able to overcome this inward force, hence the universe kept expanding despite the Schwarzschild radius during Planck time being twice the Planck length. Additionally, to collapse into a blackhole, there must be a center and for the early universe (or the universe in general) we know there was no center. The matter was distributed and the space was expanding. This could also suggest why the early universe did not collapse into a blackhole.

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H/W 1.2d (EXTRA CREDIT): Later on this semester, we will ask you to compute
r_s,U for the universe today, and compare it to the Hubble radius r_H today.
Describe the same issue here as in 1.2a--1.2c.
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The same logic can be applied here as applied to the previous question. The Schwarzschild radius of the universe today is the same as the hubble radius. The same question arises here that why does not the universe collapse into a black hole as it is the same size as its Schwarzschild radius. Again, we know that a black hole is static and the universe is expanding. The expansion of the universe is happening because there is a lot of energy acting outwards and this energy is greater than the inward force which is preventing it from collapsing into a black hole. We also know that the expansion in the universe is not happening around a point. Due to this, we cannot describe a center of mass for the universe or a point where all the gravitational force acts. So, this prevents it from collapsing into a black hole as well as there is no single point over which the universe will collapse into.