GV 12/10 +1

Junhong Sun AST 322 1215243924

2/2

1. $G = 6.67 \times 10^{-11} \frac{m^3}{kg s^2}$ $\hbar = 1.05 \times 10^{-34} J \cdot s = 1.05 \times 10^{-34} \frac{kg m^2}{s}$

$c = 3.00 \times 10^8 m/s$ $k = 8.62 \times 10^{-5} eV K^{-1}$

1.1 $l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} = \sqrt{\frac{\cancel{m^3}}{\cancel{kg s^2}} \cdot \frac{kg m^2}{s} \cdot \frac{s^3}{\cancel{m^3}}} = m$ and is a length ✓

$\sqrt{(6.67 \times 10^{-11} \times 1.05 \times 10^{-34}) / (3 \times 10^8)^3} = 1.62 \times 10^{-35} m$ ✓

1.2 $M_p = \left(\frac{\hbar c}{G}\right)^{1/2} = \sqrt{\frac{kg \cancel{m^2}}{\cancel{s}} \cdot \frac{\cancel{m}}{\cancel{s}} \cdot \frac{kg \cancel{s^2}}{\cancel{m^3}}} = kg$ and is a weight ✓

$\sqrt{(1.05 \times 10^{-34} \cdot 3 \times 10^8) / (6.67 \times 10^{-11})} = 2.18 \times 10^{-8}$ ✓

1.3 $t_p = \left(\frac{G\hbar}{c^5}\right)^{1/2} = \sqrt{\frac{\cancel{m^3}}{\cancel{kg s^2}} \cdot \frac{\cancel{kg m^2}}{\cancel{s}} \cdot \frac{s^{8\,2}}{\cancel{m^5}}} = s$ and is a time ✓

$\sqrt{(6.67 \times 10^{-11} \cdot 1.05 \times 10^{-34}) / (3 \times 10^8)^5} = 5.39 \times 10^{-44} s$ ✓

1.4 $E_p = M_p c^2 = kg \cdot \frac{m^2}{s^2} = J$ and is energy ✓

$2.18 \times 10^{-8} \times (3 \times 10^8)^2 = 1.96 \times 10^9 J$ ✓

$1.96 \times 10^9 J / (1.6 \times 10^{-19} eV/J) = 1.22 \times 10^{28} eV$ ✓

1.5 $T_p = E_p / k = \cancel{eV} \cdot \frac{K}{\cancel{eV}} = K$ and is temperature ✓

$1.22 \times 10^{28} / 8.62 \times 10^{-5} = 1.42 \times 10^{32} K$ ✓

✓ Physically, these values show the properties of the universe near its starting ~~at the~~ planck time, showing how small ($10^{-35} m$), and how hot ($10^{32} K$) and energetic ($10^9 J$) it was.

Equation derivation on next page.

1/2

1.6 $\rho_p = \frac{c^7}{\hbar G^2} = \frac{\cancel{m^7}}{s^{\cancel{7}2}} \cdot \frac{\cancel{s}}{kg m^2} \cdot \frac{kg^2 s^{\cancel{4}}}{\cancel{m^6}} = \frac{kg}{s^2 m} \quad \frac{kg\, m^2}{s^2\, m^3} = \frac{J}{m^3}$ ✓

$= \frac{(3 \times 10^8)^7}{1.05 \times 10^{-34} \cdot (6.67 \times 10^{-11})^2} = 4.68 \times 10^{113} \frac{J}{m^3}$ ✓ ✓ Comparing this

Converting the energy density to mass density using $E=mc^2$,

to the mass density of the universe today of $2.7\times10^{-27}\ kg/m^3$,

$\Rightarrow \rho_p/c^2 = \frac{?}{m^3}\cdot\frac{s^2}{m^2} = \frac{kg}{s^2 m}\cdot\frac{s^2}{m^2} = \frac{kg}{m^3}$ ✓

$\Rightarrow 4.62\times10^{113}/(3\times10^8)^2 = 5.2\times10^{96}\ kg/m^3$, with the difference of 10^{123} in magnitude, the early universe at planck time, the universe was much denser than today, and if the universe is a closed system, this would indicate that the universe expanded. ✓

$\frac{10^{96}}{10^{-27}} \neq 10^{...}$ (-1)

(2/2) 1.2a. $r_{sp} = \frac{2GM_p}{c^2} = \frac{2\cdot6.67\times10^{-11}\cdot2.18\times10^{-8}}{(3\times10^8)^2} = 3.24\times10^{-35}\ m$ ✓

(2/2) 1.2.b. $r_{sp} = 3.24\times10^{-35}\ m$ compared to l_p of $1.62\times10^{-35}\ m$, $r_{sp} \approx 2l_p$ approximately. ✓ Looking at the equations themselves,

$r_{sp} = \frac{2GM_p}{c^2} = \frac{2G}{c^2}\cdot\left(\frac{\hbar c}{G}\right)^{1/2} = \frac{2G^{1/2}\hbar^{1/2}}{c^{3/2}} = 2\left(\frac{G\hbar}{c^3}\right)^{1/2} = 2l_p$ ✓

The small difference in the actual values calculated come from rounding errors in between calculations.

Equation derivation: ✓ $E=mc^2$, $\rho = \frac{E}{\frac{4}{3}\pi R^3} \approx \frac{E}{L^3}$

$\rho_p = \frac{M_p c^2}{l_p^3} = \left(\frac{\hbar c}{G}\right)^{1/2}\left(\frac{c^3}{G\hbar}\right)^{1/2\cdot3}\cdot c^2 = \frac{c^7}{\hbar G^2}$ and the formula is shown

1.2.b cont. The fallacy that $r_{sp} = 2l_p$ is that at the Planck time, the universe should have been a black hole ✓ itself with its length or l_p being smaller than r_{sp} or the radius needed for nothing to be able to escape. However, clearly the universe changed and expanded. ✓

(2/2) 1.2.c. Possible ways out of this fallacy are that gravity might not work at such small distances like the planck length. Gravity is a long ranged force, and the weak and the strong forces dominate at such small scales. The schwarzschild radius assumes that all the mass inside the radius has a gravitational pull as expected which might not be the case in the extremely small universe that started to expand very rapidly after the big bang at extreme temperature, using

1.2 d. For the universe, $M_u = 8.8 \times 10^{52}$ kg,

EC +2

$$\frac{2GM}{c^2} = \frac{2 \cdot 8.8 \times 10^{52} \cdot 6.67 \times 10^{-11}}{(3 \times 10^8)^2} \approx 1.3 \times 10^{26}\ \text{m}$$ ✓

with $H_0 = 68\ \text{km}\ s^{-1}\ \text{Mpc}^{-1}$, $t_0 = \frac{1}{H_0}$ s and

$r_H = ct_0$, or the velocity of light times the time light traveled in the age of the universe is the Hubble radius.

$$r_H = \frac{c}{H_0} = \frac{3 \times 10^8}{68} \approx 4.4 \times 10^6\ \text{Mpc} = 1.36 \times 10^{20}\ \text{m}$$ ✓

Again, r_H is smaller than r_{su}, indicating that the universe should be a black hole. ✓

This fallacy may be solved similarly as before, that the universe is expanding so that the condition for a normal black hole, or ~~dense~~ matter having a large gravitational pull, does not apply. Matter in the universe doesn't moving further apart from each other in a static space, but the universe itself is expanding. Expected behavior of gravitational forces or relativity then won't be the same over a black hole and the whole universe.