# Cosmic Discordance: Planck and luminosity distance data exclude LCDM.

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We show that a combined analysis of CMB anisotropy power spectra obtained by the Planck satellite and luminosity distance data simultaneously excludes a flat universe and a cosmological constant at 99% C.L.. These results hold separately when combining Planck with three different datasets: the two determinations of the Hubble constant from Riess et al. 2019 and Freedman et al. 2020, and the Pantheon catalog of high redshift supernovae type-Ia. We conclude that either LCDM needs to be replaced by a drastically different model, or else there are significant but still undetected systematics. Our result calls for new observations and stimulates the investigation of alternative theoretical models and solutions.



## I. INTRODUCTION

Most current theories of cosmological structure formation are essentially based on three ingredients: an early stage of accelerated expansion (i.e. *Inflation*, see [1–3] for reviews), a clustering matter component to facilitate structure formation (i.e., *Dark Matter*, see [4]), and an energy component to explain the current stage of accelerated expansion (i.e, *Dark Energy*, see [5, 6]). While there is still no direct experimental evidence for this 'cosmic trinity', numerous viable theoretical candidates have been developed.

Among them, the currently most popular paradigm of structure formation is the Lambda Cold Dark Matter (LCDM) model, recently even acclaimed as the 'standard model' of cosmology (see e.g., [7] and references therein). The LCDM model is based on the choice of three, very specific, solutions: Inflation is given by a single, minimally coupled, slow-rolling scalar field; Dark Matter is a pressureless fluid made of cold, i.e., with low momentum, and collisionless particles; Dark Energy is a cosmological constant term.

It is important to note that these choices are mostly motivated by *computational* simplicity, i.e., the theoretical predictions under LCDM for several observables are, in general, easier to compute and include fewer free parameters than most other solutions. However, computational simplicity does not imply naturalness. Indeed, while the cosmological constant is described by one single parameter (its current energy density), its physical nature could be much more fine-tuned than a scalar field represented by (at least) two parameters (energy density and equation of state). At the same time, CDM is assumed to be always cold and collisionless during all the many evolutionary phases of the Universe. Some form of interaction or decay must exist for CDM, but this aspect is not considered in the LCDM model, other than freezeout from a thermal origin at high temperature. Finally, the primordial spectrum of inflationary perturbations is described by a power-law, therefore parametrized by only two numbers: the amplitude $A_s$ and the spectral index $n_s$ of adiabatic scalar modes. However, since inflation is a dynamical process that, at some point, must end, the scale-dependence of perturbations could be more complicated (see e.g., [8]).

For these reasons, the 6 parameter LCDM model (that, we recall, is not motivated by any fundamental theory) can be rightly considered, at best, as a first-order approximation to a more realistic scenario that still needs to be fully explored. With the increase in experimental sensitivity, observational evidence for deviations from LCDM is, therefore, *expected*.

Despite its status as a conjecture, the LCDM model has been, however, hugely successful in describing most of the cosmological observations. Apart from a marginally significant mismatch with Cosmic Microwave Background (CMB) observations at large angular scales (see e.g., [9]), LCDM provided a nearly perfect fit to the measurements made by the WMAP satellite mission, also in combination with complementary observational data such as those coming from Baryon Acoustic Oscillation (BAO) surveys, supernovae type-Ia (SN-Ia), and direct measurements of the Hubble constant (see, e.g. [10]).

More recent data, however, are starting to show some interesting discrepancies with the LCDM model. Under LCDM, the Planck CMB anisotropies seem to prefer a value of the Hubble constant that is significantly smaller than values derived in a more direct way from luminosity distances of supernovae (see e.g. [11]). At the same time, the combination of the amplitude $\sigma_8$ of the dark matter fluctuations on scales of 8 Mpc $h^{-1}$ and the matter density $\Omega_m$, parametrized by the $S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$ parameter, is significantly smaller in recent cosmic shear

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surveys than the value derived from Planck data under LCDM ([12]).

Systematics can play a role, and the LCDM model still produces a reasonable fit to the data. However, the main ambition of modern cosmology is to identify a cosmological model that can be used as an ideal laboratory to test fundamental physics and possible 'extensions' thereof. For example, stringent constraints on neutrino physics have been placed using Planck data in combination with BAO and other observables (see e.g. [13–15]). The possibility of constraining fundamental physics with such high precision is challenged if the underlying cosmological model does not produce an excellent fit to current data. In practice, current tensions are already presenting a serious limitation to what in recent years has been defined as *precision* cosmology.

Recently, it has been shown that these tensions are exacerbated when the possibility of a closed universe is considered [18, 19]. Planck CMB data alone, indeed, prefers a closed universe at 99% C.L. (see e.g. page 40 of [13]) and this translates to an even lower Hubble constant and an even larger $S_8$ parameter. Moreover, significant tensions at about three standard deviations now emerge between Planck and BAO data ([18],[19]). In practice, not only tensions with cosmological data exist, but even larger discordances may be hidden by the assumption of the LCDM model itself.

The main problem for a closed Universe is the lack of concordance with other observables. Apart from BAO, indeed, the closed model preferred by Planck does not agree with luminosity distance measurements of supernovae type Ia and predicts too large a matter density $\Omega_m \sim 0.5$ in striking contrast with local measurements of galaxy clustering ([18]). However, as we discussed above, many of the assumptions in LCDM, such as the assumption of a cosmological constant, are questionable and lack any robust justification. Hence it is useful to pose the question whether a further increase in the number of parameters, in addition to curvature, can help reconcile the Planck result with other observations. The main goal of this *Letter* is to answer this question and, to investigate whether an alternative cosmological model exists wherein current independent cosmological observables are in better agreement than in the LCDM model. In brief, given current tensions with luminosity data and the CMB preference for a closed universe, we search for a new cosmological concordance model that significantly differs from LCDM.

Our approach is simply based on an extension of the cosmological parameter space as we have done before in [20, 21]. Instead of the usual six parameters, we also allow variations of the dark energy equation of state, the curvature of the universe, the neutrino mass, and the running of the spectral index of primordial fluctuations.

## II. METHOD

The LCDM model is based on six free parameters (see e.g. [13]): the angular size of the sound horizon at decoupling $\theta_{MC}$, the cold dark matter and baryon densities $\Omega_c h^2$ and $\Omega_b h^2$, the optical depth at reionization $\tau$, and the amplitude $A_s$ and the spectral index $n_s$ of inflationary scalar perturbations. As discussed in the introduction, several tensions between cosmological observables are starting to emerge when this model is assumed. We, therefore, investigate the following extensions (considering them all *simultaneously*):

- The **curvature parameter** $\Omega_k$. The possibility of a curved universe is fully compatible with General Relativity and is also allowed in some non-standard, inflationary models. Moreover, as stressed in [18], Planck angular spectral data alone prefer models with positive curvature ($\Omega_k < 0$, see page 40 of [13]).
- The **running of the spectral** index of inflationary perturbations $\alpha_s = dn_s/dlnk$. A sizable running is expected in many inflationary models, ranging from $\alpha_s \sim (1-n_S)^2 \sim 10^{-3}$ in slow-roll models (see e.g. [22]) to higher values (see e.g.[23–25]). An indication at about $\sim 3$ standard deviations for a negative running has been recently claimed by [14] combining Planck with BAO and Lyman-$\alpha$ forest data.
- The **dark energy equation of state** $w$. We consider a dark energy equation of state of the form $P = w\rho$, where $P$ and $\rho$ are the dark energy pressure and density and $w$ is a free parameter, constant with redshift. $w = -1$ corresponds to a cosmological constant.
- **The sum of neutrino masses** $\Sigma m_\nu$. We know from oscillation and long-baseline neutrino experiments that neutrinos have to be massive. However, the total mass is still unknown. In the LCDM model, a minimal mass of $\Sigma m_{nu} = 0.06$ eV is assumed. However, the total mass of neutrinos can be higher.

Concerning the experimental data, we consider:

- The Planck 2018 temperature and polarization CMB angular power spectra. In this paper we use the **reference likelihood** from the Planck 2018 release that is given by the multiplication of the `Commander`, `SimALL`, and `PlikTT,TE,EE` likelihoods (see page 3 of [28]). This corresponds to the reference dataset used in the Planck papers. We refer to this data simply as **Planck**.
- The Baryon Acoustic Oscillation data from the compilation used in [13]. This consists of data from the 6dFGS [29], SDSS MGS [35], and BOSS

DR12 [36] surveys. We refer to this dataset as **BAO**.

- The luminosity distance data of 1048 type Ia supernovae from the PANTHEON catalog [37]. We refer to this dataset as **Pantheon**.

- The most recent determination of the Hubble constant from Riess et al. 2019. This is assumed as a gaussian prior on the Hubble constant of $H_0 = 74.03 \pm 1.42$ km/s/Mpc. We refer to this prior as **R19** [38].

- The recent determination of the Hubble constant from the Tip-of-the-Red-Giant-Branch approach (Freedmann et al. 2020). This is assumed as a gaussian prior on the Hubble constant of $H_0 = 69.6 \pm 2.0$ km/s/Mpc (we sum statistical and systematic errors in quadrature). We refer to this prior as **F20** [39].

The comparison between theory and data is made adopting the public available `CosmoMC` code based [40] on a Monte Carlo Markov chain algorithm. The theoretical predictions are made using the `CAMB` Boltzmann integrator [41].

## III. RESULTS

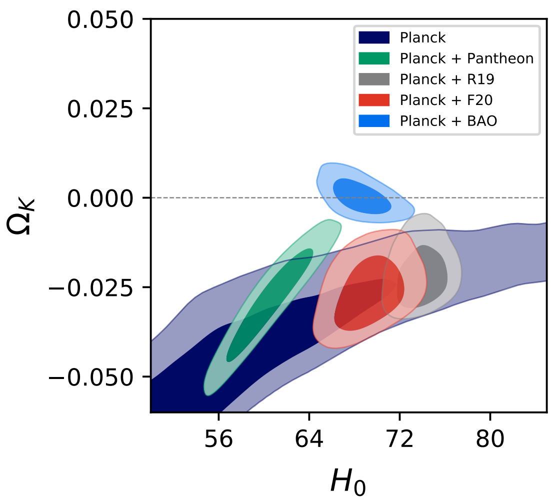


Figure 1. **Cosmic Discordance**. Constraints at the 68% and 95% C.L. on the $\Omega_k$ vs $H_0$ plane for the Planck, Planck+R19, Planck+F20, Planck+BAO, and Planck+Pantheon datasets. A 10 parameters model, LCDM+$w$+$\Omega_k$+$\alpha_S$+$\Sigma m_\nu$, is assumed in the analysis.

The parameter constraints are reported in Table I while in Figure 1 and Figure 2 we report the 2D contours at 68% and 95% confidence levels in the $\Omega_k$ vs $H_0$ and $H_0$ vs $w$ planes. As one can see from Figure 1 and the constraints in Table I, the preference from the Planck measurements for a closed universe at more than 95% C.L. is also clearly present in the extended parameter space we are considering. The confidence levels from Planck plotted in Figure 1, while very broad and virtually unable to constrain the Hubble constant, are clearly below the $\Omega_k = 0$ line that describes a flat universe. On the other hand, inclusion of the equation of state $w$ now allows the Planck data to be in perfect agreement with the Pantheon, R19, and F20 measurements. As we can see from Figure 1, all the 95% confidence regions from the Planck+Pantheon, Planck+F20, and Planck+R19 datasets are well below the $\Omega_k = 0$ line. This clearly shows that the recent claims of a closed universe as being incompatible with luminosity distance measurements are simply due to the assumption of a cosmological constant. Indeed, as we can see from Figure 2, where we show the 2D contour plots in the $H_0$ vs $w$ plane, all the three datasets, combined with Planck, exclude a cosmological constant, clearly preferring a value of $w < -1$. In practice, integrating the marginalized posterior, we have found that Planck+Pantheon, Planck+R19, and Planck+F20 all exclude a cosmological constant and a flat universe at more than 99% C.L.

It is, however, important to notice that the luminosity distance measurements, when combined with Planck, provide values of the Hubble constant that are in tension between themselves. Indeed, it is evident from Figure 1 that the confidence regions from Planck+Pantheon, Planck+F20, and Planck+R19 are inconsistent at more than 95% C.L., providing different constraints on the Hubble constant.

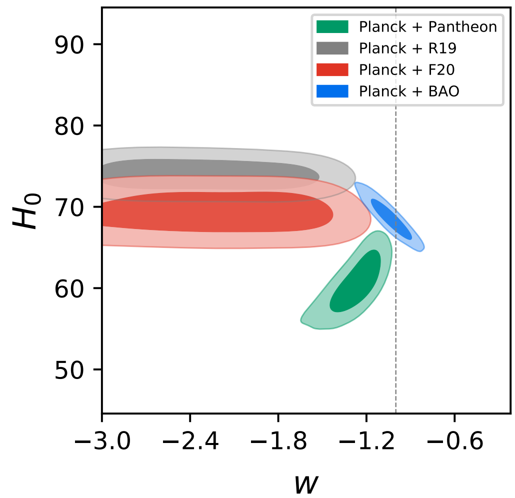


Figure 2. **Cosmic Discordance**. Constraints at the 68% and 95% C.L. on the $w$ vs $H_0$ plane for the Planck+R19, Planck+F20, Planck+BAO, and Planck+Pantheon datasets. A 10 parameter model, LCDM+$w$+$\Omega_k$+$\alpha_S$+$\Sigma m_\nu$, is assumed in the analysis.

Finally, we have also considered a Planck+BAO dataset. As already discussed in [18, 19], when curvature is considered, BAO data are in strong tension with the Planck measurements. This can also be seen from the best fit $\chi^2$ value reported in Table 1, which increases by $\Delta\chi^2 \sim 15$ when the BAO are combined with Planck. The BAO dataset consists of 8 measurements. However, the 2dF measurements are statistically irrelevant, given the larger error bars, and the DR12 6 determinations are correlated. If we consider 5 degrees of freedom, this $\Delta\chi^2$ value suggests tensions at around three standard deviations, in agreement with the findings of [18, 19] but in the case of a LCDM+$\Omega_k$ 7 parameter model. We therefore conclude that the tension between Planck and BAO persists in this extended parameter space, and we stress again that the combination of the Planck and BAO dataset should be considered with some caution. Nonetheless, if we force a combined analysis of the two measurements, we can see from Figure 1 that Planck+BAO prefers a flat universe and a Hubble constant compatible with the F20 value. However, Planck+BAO is now not only in tension with Planck, but also with Planck+R19, Planck+F20, and Planck+Pantheon. Figure 1 clearly reveals the significant tensions between the current cosmological datasets in our extended cosmological model scenario.

From the results in Table I we also note that none of the data combinations considered suggests a negative running. A running of $\alpha_s \sim -0.01$, as claimed in [14], is however compatible within two standard deviations with all the datasets.

Finally, it is interesting to note that the bounds on the neutrino masses from Planck and luminosity distance measurements are all much more relaxed with respect to the Planck+BAO case, with the Planck+F20 case even mildly suggesting a neutrino mass of $\Sigma \sim 0.28$ eV.

## IV. CONCLUSIONS

In this *Letter*, we have shown that a combined analysis of the recent Planck angular power spectra with different luminosity distance measurements is in strong disagreement (at more than 99% C.L.) with the two main expectations of the standard LCDM model, i.e., a flat universe and a cosmological constant.

The first question we have to address is whether any of the Planck+luminosity distance cosmologies, despite being incompatible with the BAO dataset, could agree with other, independent measurements. As we can see from Table I, the constraints obtained in the case of Planck+F20, Planck+R19, and Planck+Pantheon for the remaining cosmological parameters are reasonable. For example, a value of the matter density in the range $0.25 < \Omega_m < 0.35$, acceptable in the case of galaxy cluster analyses, is compatible to one standard deviation with all cases. The derived age of the Universe is now around $t_0 \sim 15$ Gyrs, allowing better compatibility with the ages of the oldest Population II stars [16]. The Planck+Pantheon result is fully compatible with the luminosity distances of high redshift quasars as presented in [17]. There is also one other issue worthy of mention. The CMB low multipole values and alignments ([32]) and the 2-point angular correlation function ([30]) present possible discrepancies with the standard LCDM model. These anomalies are claimed to be significant ([31]) but disputed by the Planck collaboration ([34]), with any difference in these results depending on masking model uncertainties, among other issues. Nor does this effect depend on galactic plane orientation (see e.g. [33]). However, the only generic explanation for lack of large angular scale correlation invokes a closed universe.

The second question is whether experimental systematics can explain the observed discrepancies. The answer to this question is affirmative, although less obvious is where these systematics could be. In this work, we use the Planck 2018 nominal (official) likelihood based on `Plik`. An alternative likelihood code exists, `CamSpec`, that gives results more compatible with the LCDM scenario, especially in its last version, as presented in [42]. When the `CamSpec` code is adopted, current tensions at about 99% C.L. could shift by one standard deviation to about 95% C.L., i.e., in the realm of a possible statistical fluctuation. While the indication for a closed universe is also present in `CamSpec`, this shows that small shifts in the parameters could be expected when considering a different approach to the Planck likelihood. However, we note here that the `Plik` likelihood is the official likelihood validated by the Planck team. There is, therefore, no motivation at the moment to choose `CamSpec` over `Plik` apart from any theoretical prejudice for LCDM. We also note that the `CamSpec` is not public available. Systematics can be undoubtedly present in luminosity distance data, and the tension between the values on the Hubble constant from F20 and R19 seems to point in this direction. A change in how systematics are considered in the Pantheon dataset could affect our results (see, e.g. [43]). Nevertheless, Planck+BAO data, despite the tension between the two measurements, clearly prefers a flat LCDM model. However, the BAO datapoints used in our analysis have been derived under flat LCDM and are, therefore, not strictly model-independent as are the CMB and luminosity distances data. For example, if the dark energy component is different from a cosmological constant and interacts with the dark matter, then non-linearities could behave differently from what is expected in LCDM and consequently affect the BAO result (see e.g. [44, 45]). We are, therefore, in a situation where there is no apparent reason to trust one dataset more than another. Let us, however, recall that the CMB anisotropy spectra are well inside the linear regime and probably have the highest theoretical justification.

The final question is whether a closed model with a phantom ($w < -1$) dark energy component is theoretically appealing. Closed inflationary models have been

| Parameters | Planck | Planck +R19 | Planck +F20 | Planck +BAO | Planck + Pantheon |
|---|---|---|---|---|---|
| $\Omega_b h^2$ | $0.02253 \pm 0.00019$ | $0.02253^{+0.00020}_{-0.00016}$ | $0.02255^{+0.00019}_{-0.00017}$ | $0.02243 \pm 0.00016$ | $0.02255 \pm 0.00018$ |
| $\Omega_c h^2$ | $0.1183 \pm 0.0016$ | $0.1187^{+0.0015}_{-0.0018}$ | $0.1184 \pm 0.0015$ | $0.1198 \pm 0.0014$ | $0.1186 \pm 0.0015$ |
| $100\theta_{\rm MC}$ | $1.04099 \pm 0.00035$ | $1.04103^{+0.00034}_{-0.00031}$ | $1.04105 \pm 0.00034$ | $1.04095 \pm 0.00032$ | $1.04107 \pm 0.00034$ |
| $\tau$ | $0.0473 \pm 0.0083$ | $0.052^{+0.009}_{-0.011}$ | $0.0491 \pm 0.0079$ | $0.0563 \pm 0.0081$ | $0.0506 \pm 0.0082$ |
| $\Sigma m_\nu$ [eV] | $0.43^{+0.16}_{-0.37}$ | $< 0.513$ | $0.28^{+0.11}_{-0.23}$ | $< 0.194$ | $< 0.420$ |
| $w$ | $-1.6^{+1.0}_{-0.8}$ | $-2.11^{+0.35}_{-0.77}$ | $-2.14 \pm 0.46$ | $-1.038^{+0.098}_{-0.088}$ | $-1.27^{+0.14}_{-0.09}$ |
| $\Omega_k$ | $-0.074^{+0.058}_{-0.025}$ | $-0.0192^{+0.0036}_{-0.0099}$ | $-0.0263^{+0.0060}_{-0.0077}$ | $0.0003^{+0.0027}_{-0.0037}$ | $-0.029^{+0.011}_{-0.010}$ |
| $\ln(10^{10}A_s)$ | $3.025 \pm 0.018$ | $3.037^{+0.016}_{-0.026}$ | $3.030 \pm 0.017$ | $3.049 \pm 0.017$ | $3.034 \pm 0.017$ |
| $n_s$ | $0.9689 \pm 0.0054$ | $0.9686^{+0.0056}_{-0.0050}$ | $0.9693 \pm 0.0051$ | $0.9648 \pm 0.0048$ | $0.9685 \pm 0.0051$ |
| $\alpha_S$ | $-0.0005 \pm 0.0067$ | $-0.0012 \pm 0.0066$ | $-0.0010 \pm 0.0068$ | $-0.0054 \pm 0.0068$ | $-0.0023 \pm 0.0065$ |
| $H_0$[km/s/Mpc] | $53^{+6}_{-16}$ | $73.8 \pm 1.4$ | $69.3 \pm 2.0$ | $68.6^{+1.5}_{-1.8}$ | $60.5 \pm 2.5$ |
| $\sigma_8$ | $0.74^{+0.08}_{-0.16}$ | $0.932 \pm 0.040$ | $0.900 \pm 0.039$ | $0.821 \pm 0.027$ | $0.812^{+0.031}_{-0.018}$ |
| $S_8$ | $0.989^{+0.095}_{-0.063}$ | $0.874 \pm 0.032$ | $0.900^{+0.034}_{-0.031}$ | $0.826 \pm 0.016$ | $0.927 \pm 0.037$ |
| $Age$[Gyr] | $16.10^{+0.92}_{-0.80}$ | $14.90^{+0.72}_{-0.32}$ | $15.22^{+0.054}_{-0.038}$ | $13.77 \pm 0.10$ | $14.98 \pm 0.39$ |
| $\Omega_m$ | $0.61^{+0.21}_{-0.34}$ | $0.264^{+0.010}_{-0.013}$ | $0.300^{+0.017}_{-0.020}$ | $0.305 \pm 0.016$ | $0.393^{+0.030}_{-0.036}$ |
| $\Delta\chi^2_{bestfit}$ | 0.0 | 0.62 | 0.88 | 14.77 | 1037.82 |

Table I. Constraints at 68% CL errors on the cosmological parameters in case of the 12 parameters model using different combinations of the datasets. The quoted upper limits are at 95% CL. In the bottom line we quote the difference in the best-fit $\chi^2$ values with respect to the Planck data alone result.

proposed in the literature [46] and a closed universe is expected in several scenarios (see e.g. [47], [48],[49]). An experimental indication for a phantom dark energy component could hint for interaction between dark matter and a $w > -1$ dark energy component (see e.g. [50, 51]). In this respect, it is interesting to note that if a closed universe increases the fine-tuning of the theory, the removal of a cosmological constant, on the other hand, reduces it. It is, therefore difficult to decide whether a phantom closed model is less or more theoretically convoluted than LCDM.

Our conclusions provide a significant indication against the LCDM scenario when Planck is combined with luminosity distance measurements. Not fitting practically half of the current cosmological data is undoubtedly a significant blow to the LCDM model. Moreover, the tensions that we have found significantly affect the ability of cosmology to test fundamental physics. For example, considering Table I, we can see that a Planck+F20 analysis indicates a neutrino mass of $\Sigma m_\nu \sim 0.3$ eV at the level of one standard deviation, while Planck+BAO rules this out with a 95% C.L. limit of $\Sigma m_\nu < 0.194$ eV. This however also means that future laboratory measurements of a neutrino mass could play a key role in resolving current cosmological tensions.

Our result calls for new observations and stimulates the investigation of alternative theoretical models and solutions.

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