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2/8/22

HW 2

2.1) a) $\frac{\Delta\rho}{\rho} \approx \frac{100\ kgm^{-3}}{3\times10^{-27}\ kgm^{-3}} = 3.3\times10^{28} \approx \underline{10^{28}}$

b) $\frac{\Delta\rho}{\rho} \approx \frac{4\times10^{-5}\ kgm^{-3}}{3\times10^{-27}\ kgm^{-3}} = 1.33\times10^{22} \approx \underline{10^{22}}$

c) $\frac{\Delta\rho}{\rho} \approx \frac{3\times10^{-26}\ kgm^{-3} - 3\times10^{-27}\ kgm^{-3}}{3\times10^{-27}\ kgm^{-3}} = 9 \approx \underline{10^{1}}$

d) $\frac{\Delta\rho}{\rho} \approx \frac{6\times10^{-27}\ kgm^{-3} - 3\times10^{-27}\ kgm^{-3}}{3\times10^{-27}\ kgm^{-3}} = 1 = \underline{10^{0}}$

$5M_\odot$ BH

e) $\frac{\Delta\rho}{\rho}$ $R_s = \frac{2GM}{c^2}$

$R_s = \frac{2(6.67\times10^{-11}\ m^3kg^{-1}s^{-2})(10^{31}\ kg)}{(3.00\times10^{8}\ ms^{-1})^2}$

$R_s = 14822\ m$

$\Delta\rho \approx \frac{M}{V} = \frac{10^{31}\ kg}{\frac{4}{3}\pi(14822)^3}$

$\Delta\rho \approx 7.33\times10^{17}\ kgm^{-3}$

$\frac{\Delta\rho}{\rho} = \frac{7.33\times10^{17}\ kgm^{-3}}{3\times10^{-27}\ kgm^{-3}}$

$\frac{\Delta\rho}{\rho} \approx 2\times10^{44} \sim \underline{\underline{10^{44}}}$

2.2) $H_0 = 68\ kms^{-1}Mpc^{-1} \pm 2\ kms^{-1}Mpc^{-1}$

$R_0 = \frac{c}{H_0} = \frac{299{,}792.458\ \cancel{km}s^{-1}}{68\pm2\ \cancel{kms^{-1}}Mpc^{-1}} = 4408.113 \pm 125.963\ Mpc$

$\approx 4400 \pm 100\ Mpc \cdot \frac{1\ Gpc}{1000\ Mpc} = 4.4\ Gpc \pm 0.1\ Gpc$

$\boxed{R_0 \approx 4.4 \pm 0.1\ GPC}$

$t_0 = H_0^{-1} = \frac{1}{68\pm2\ kms^{-1}Mpc^{-1}} \cdot \frac{3.086\times10^{19}\ km}{1\ Mpc} = 4.538\times10^{17} \pm 1.297\times10^{16}\ s$

$4.538 \pm 0.130\times10^{17}\ s \cdot \frac{1\ min}{60\ s} \cdot \frac{1\ hr}{60\ min} \cdot \frac{1\ day}{24\ hr} \cdot \frac{1\ yr}{365.25\ days} = 1.438 \pm 0.041\times10^{10}\ yrs$

$\boxed{t_0 = 14{,}380{,}000{,}000 \pm 400{,}000{,}000\ yrs} \rightarrow 1.438 \pm 0.41\ Gyrs$

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2.3 A. $\mathcal{E}(f)\,df = \frac{8\pi h}{c^3}\frac{f^3\,df}{e^{hf/kT}-1}$ $\qquad \lambda = \frac{c}{f} \quad f = \frac{c}{\lambda}$

$$df = \frac{-c}{\lambda^2}d\lambda$$

$$\mathcal{E}(\lambda)\cancel{d\lambda} = \frac{8\pi h}{c^3}\frac{\frac{c^3}{\lambda^3}\cdot\frac{c}{\lambda^2}d\lambda}{e^{hc/\lambda kT}-1} = \frac{8\pi hc}{\lambda^5}\frac{\cancel{d\lambda}}{e^{hc/\lambda kT}-1}$$

$$\frac{d\mathcal{E}}{d\lambda} = \frac{-40\pi hc}{\lambda^6}\cdot\frac{1}{e^{hc/\lambda kT}-1} + \frac{8\pi hc}{\lambda^5}\left[\frac{-1}{(e^{hc/\lambda kT}-1)^2}e^{hc/\lambda kT}\left(\frac{-hc}{\lambda^2 kT}\right)\right]$$

let $u = \frac{hc}{\lambda kT}$, let $\frac{d\mathcal{E}}{d\lambda} = 0$ $\qquad \lambda = \frac{hc}{ukT}$

$$0 = -40\pi hc\left(\frac{ukT}{hc}\right)^6\cdot\frac{1}{e^u-1} + 8\pi hc\left(\frac{ukT}{hc}\right)^5\left(\frac{+\cancel{hc}}{\cancel{kT}}\cdot\frac{u^2k^2T^2}{h^2c^2}\right)\left(\frac{e^u}{(e^u-1)^2}\right)$$

$$0 = \frac{-40\pi k^6T^6u^6}{h^5c^5}\cdot\frac{1}{e^u-1} + \frac{8\pi k^6T^6u^7}{h^5c^5}\left(\frac{e^u}{(e^u-1)^2}\right)$$

$$0 = \frac{8\pi k^6T^6u^6}{h^5c^5}\cdot\frac{1}{(e^u-1)^2}\left[ue^u - 5(e^u-1)\right]$$

$$0 = ue^u - 5e^u + 1 \implies$$ calculator result $u = -1.937,\ 4.9932162$

$$4.993 = \frac{hc}{\lambda kT} \implies \lambda_{peak} = \frac{hc}{4.993kT} = \frac{(6.63\times10^{-34}\,Js)(3.00\times10^{8}\,ms^{-1})}{(4.993)(1.381\times10^{-23}\,JK^{-1})\,T}$$

$$\lambda_{peak} = \frac{0.002884\,mK}{T} \approx \boxed{\frac{0.29\,cmK}{T} = \lambda_{peak}}$$

$$\lambda_{peak\odot} = \frac{0.29\,cmK}{5800\,K} = 4.97\times10^{-5}\,cm \approx \boxed{0.50\ \mu m}$$

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2.3 A. $hf_{peak} = 2.82kT$

$\lambda = \frac{c}{f}\left[\frac{m/s}{1/s}\right] \Rightarrow f = \frac{c}{\lambda}$

$$\frac{hc}{\lambda_{peak}} = 2.82kT$$

$$\lambda_{peak} = \frac{hc}{2.82kT} = \frac{(2\pi \times 6.58\times10^{-16}\ eVs)(3\times10^{8}\ m\,s^{-1})}{2.82(8.62\times10^{-5}\ eVK^{-1})T} = \frac{0.0051\ mK}{T}\cdot\frac{100\ cm}{1\ m}$$

$$\lambda_{peak} = \frac{0.51\ cmK}{T}$$

B. $E_{mean} = \frac{\varepsilon_\gamma}{n_\gamma} = \frac{\alpha T^4}{\beta T^3} = \frac{\pi^2}{15}\frac{k^4}{\hbar^3 c^3}\cdot\frac{\pi^2\hbar^3 c^3}{2.4041\,k^3}T = \frac{\pi^4 kT}{15\cdot 2.4041}$

$$\boxed{E_{mean} = 2.70kT}$$

$T_\odot \approx 5800K \quad E_{mean\,\odot} = 2.70(1.38\times10^{-23}\ JK^{-1})(5800K) = \underline{2.16\times10^{-19}\ J}$

$2.16\times10^{-19}\ J\cdot\frac{1\ eV}{1.60\times10^{-19}\ J} = \underline{1.35\ eV}$ FROM SUN

$T_0 = T_{CMB} \approx 2.726\ K$

$E_{mean\,0} = 2.70(1.38\times10^{-23}\ JK^{-1})(2.726\ K) = \underline{1.02\times10^{-22}\ J}$

$1.02\times10^{-22}\ J\cdot\frac{1\ eV}{1.60\times10^{-19}\ J} = \underline{6.35\times10^{-4}\ eV}$ CMB Photons

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E.C. $T_{ionization} = \frac{T_\odot}{2} = \frac{5800K}{2} = \underline{2900K}$

If the CMB is from the temperature of H ionization, then that light was redshifted to the current state, $z \propto 1100$.

$$\lambda_{peak-ionization} = \frac{0.29\,cm}{(2900K)} = 10^{-4}\,cm = 10^{-6}\,m$$

$$\lambda_{peak-CMB} = \frac{0.29\,cm}{(2.7255K)} = 0.106\,cm = 1.064\times10^{-3}\,m$$

$$z_{CMB} = \frac{\Delta\lambda}{\lambda_{em}} = \frac{1.064\times10^{-3}m - 10^{-6}m}{10^{-6}m} = \underline{1063 \approx 1100}$$

2.4

$$dQ = dE + PdV$$
$$0 = dE + PdV$$
$$\frac{dE}{dt} = -P\frac{dV}{dt} \implies \frac{d}{dt}\left[\alpha T^4 V\right] = -\frac{1}{3}\alpha T^4 \frac{dV}{dt}$$
$$\implies 4\alpha T^3 \frac{dT}{dt} V + \alpha T^4 \frac{dV}{dt} = \frac{-1}{3}\alpha T^4 \frac{dV}{dt}$$
$$4T^3 V \frac{dT}{dt} = \frac{dV}{dt} T^4 \overbrace{\left(\frac{-1}{3} - 1\right)}^{-4/3}$$
$$\int \frac{1}{T}\frac{dT}{dt} = \int \frac{-1}{3V}\frac{dV}{dt} \implies \frac{d}{dt}\int \frac{1}{T}dT = \frac{d}{dt}\int \frac{-1}{3V}dV$$
$$\implies \frac{d}{dt}\ln|T| = -\frac{d}{dt}\ln|V^{1/3}| \implies \frac{d}{dt}\ln(T) = -\frac{d}{dt}\ln(a) \quad \text{bc } V \propto a^3,\ V^{1/3} \propto a$$

$a = \frac{1}{1+z}$

$$\rightarrow T \propto a^{-1}$$
$$T = Ka^{-1} = T_0\left(\frac{1}{1+z}\right)^{-1} = \boxed{T_0(1+z) = T(z)}$$
$$V \propto a^3$$
$$V = Ka^3 = V_0\left(\frac{1}{1+z}\right)^3 = \boxed{\frac{R_0^3}{(1+z)^3} = V(z)}$$

$$\varepsilon_\gamma = \alpha T^4 \quad \varepsilon_\gamma(z) = \alpha\left[T_0(1+z)\right]^4 = \boxed{\alpha T_0^4 (1+z)^4 = (4.17\times10^{-14})(1+z)^4 = \varepsilon_\gamma(z)}$$