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AST 422 Final

Problem A5

Part A

The Malmquist bias refers to the overrepresentation of intrinsically bright objects in a survey. It occurs when a survey is limited by the faintest magnitude (i.e. flux) it can detect, and so the flux from the faintest objects at large distances may be below the detection threshold while the flux from the brightest objects at the same distance is above it, allowing the intrinsically more luminous objects to be detected preferentially. This leads to a luminosity function (number of objects per luminosity bin) which underestimates the number of low luminosity objects. Thus if a collection of objects observed in a survey with a Malmquist bias is used as standard candles for measurement of the Hubble constant, the understanding of their instrinsic luminosity which allows them to be used as standard candles may be misunderstood, leading to incorrect distance measurements.

Part B

If there is significant large scale structure such as superclusters in the direction in which H_0 is measured, then the galaxies being used for the measurement are in a region of large overdensity compared to the average density of the universe. Thus the large amounts of mass near them may cause significant deviations from their overall Hubble flow movement. Since these structure related motions are essentially random, it will cause many galaxies to sit either above or below on the “line” described by the true Hubble law, which will introduce uncertainty into the measured value of H_0.

Part C

Similarly to the problem of large scale structure in direction in which H_0 is measured, large scale streaming motions of galaxies (or rather the entire clusters/superclusters in which they reside) also introduces deviations from the Hubble flow. However rather than being random these deviations are coherent, and so the galaxies will be found to be moving systematically faster or slower than the Hubble flow, which will lead to a value of the Hubble constant which is larger or smaller than the true value, even if uncertainties in the measurements are very small.

Problem B3

Part A

The Mattig equation for the proper distance D is

$$D = \frac{2c}{H_0 \Omega_0^2 (1+z)} \left\{ \Omega_0 z + (\Omega_0 - 2) \left[(\Omega_0 z + 1)^{\frac{1}{2}} - 1 \right] \right\}.$$

Angular size distance for an object is defined by

$$d_A = \frac{D}{1+z} = \frac{Y}{\theta},$$

where Y is its physical size. Thus the angular size of the object is

$$\theta = \frac{Y(1+z)}{D}.$$

Part B

The expanded form of the angular size is

$$\theta = \frac{YH_0\Omega_0^2(1+z)^2}{2c\left\{\Omega_0 z + (\Omega_0 - 2)\left[(\Omega_0 z + 1)^{\frac{1}{2}} - 1\right]\right\}}.$$

For $\Omega_m = \Omega_0 = 0$, θ is undefined because the denominator is equal to zero. For $\Omega_{0m} = 1$ the angular size is

$$\theta = \frac{1}{2}\left(\frac{YH_0}{c}\right)\left[\left(\frac{1}{1+z}\right) - \left(\frac{1}{1+z}\right)^{\frac{3}{2}}\right]^{-1},$$

and for $\Omega_m = 2$ it is

$$\theta = \left(\frac{YH_0}{c}\right)\frac{(1+z)^2}{z}.$$

Part C

Using the expression for angular size when $\Omega_m = 1$, we have

$$\frac{d\theta}{dz} = -\frac{1}{2}\left(\frac{YH_0}{c}\right)\left[\left(\frac{1}{1+z}\right) - \left(\frac{1}{1+z}\right)^{\frac{3}{2}}\right]^{-2}\left[\frac{3}{2}(1+z)^{-\frac{5}{2}} - (1+z)^{-2}\right]^{-1}.$$

Setting this equal to zero to find the critical redshift at which θ is minimized gives

$$\frac{3}{2}(1+z)^{-\frac{5}{2}} - (1+z)^{-2} = 0,$$

which reduces to

$$\frac{3}{2}(1+z)^{-\frac{1}{2}} = 1,$$

which has the solution $z = \frac{5}{4}$.

Differentiating the angular size for $\Omega_m = 2$ gives

$$\frac{d\theta}{dz} = \frac{2z(1+z) - (1+z)^2}{z^2}.$$

Equating this to zero yields

$$2z(z+1) - (1+z)^2 = 0,$$

which is satisfied when $z = 1$. This is the redshift at which angular size is minimized.

Part D

Solving for the physical size Y in the equation for angular size when $\Omega_m = 1$ we find

$$Y = \frac{2\theta c}{H_0}\left[\frac{1}{1+z} - \frac{1}{(1+z)^{\frac{3}{2}}}\right].$$

Using this, the size of a $10'$ fluctuation in the CMB at $z = 1100$ is $21\,kpc$.

Solving for the physical size when $\Omega_m = 2$ we find

$$Y = \frac{\theta c}{H_0}\frac{z}{(1+z)^2},$$

which for the same CMB fluctuation gives a size of $11\,kpc$.

Problem B6

Part A

Redshift is related to the scale factor by the relation

$$\frac{a(t_0)}{a(t_e)} = 1 + z.$$

Differentiating with respect to t_0 gives

$$\frac{dz}{dt_0} = \frac{1}{a^2(t_e)}\left[a(t_e)\frac{da(t_0)}{dt_0} - a(t_0)\frac{da(t_e)}{dt_0}\right].$$

Writing $\frac{da(t_e)}{dt_0}$ as $\frac{da(t_e)}{dt_e}\frac{dt_e}{dt_0}$ and expanding we have

$$\frac{dz}{dt_0} = \frac{1}{a(t_e)}\frac{da(t_0)}{dt_0} - \frac{a(t_0)}{a^2(t_e)}\frac{da(t_e)}{dt_e}\frac{dt_e}{dt_0}.$$

The first term on the right is

$$\frac{1}{a(t_e)}\frac{da(t_0)}{dt_0} = \frac{a(t_0)}{a(t_e)}\left[\frac{1}{a(t_0)}\frac{da(t_0)}{dt_0}\right] = (1+z)H_0.$$

The second term is

$$\frac{a(t_0)}{a^2(t_e)}\frac{da(t_e)}{dt_e}\frac{dt_e}{dt_0} = \frac{a(t_0)}{a(t_e)}\left[\frac{1}{a(t_e)}\frac{da(t_e)}{dt_e}\right]\frac{dt_e}{dt_0} = (1+z)H(t_e)\left(\frac{1}{1+z}\right) = H(t_e).$$

To evaluate this term we use the equation for the evolution of the scale factor in a single component universe,

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3+3w}},$$

from which $H(t) = \frac{1}{a(t)}\frac{da(t)}{dt}$ is found to be

$$H(t) = \frac{2}{3(1+w)}t^{-1}.$$

In a single component universe, t_e is given by

$$t_e = \frac{2}{3(1+w)H_0}(1+z)^{-\frac{3(1+w)}{2}}.$$

Therefore we have

$$H(t_e) = H_0(1+z)^{\frac{3(1+w)}{2}},$$

giving the final expression

$$\frac{dz}{dt_0} = H_0(1+z) - H_0(1+z)^{\frac{3(1+w)}{2}}.$$

In order to find which values of w give positive/negative values of $\frac{dz}{dt_0}$, we set it equal to zero giving

$$H_0(1+z) = H_0(1+z)^{\frac{3(1+w)}{2}}$$

$$1 = (1+z)^{\frac{1+3w}{2}}$$

$$w = -\frac{1}{3},$$

which is the critical value at which its sign changes. Plugging in $w = 0$ i.e. a value greater than the critical value we find

$$\left(\frac{dz}{dt_0}\right)_{w=0} = H_0(1+z)\left[1-(1+z)^{\frac{1}{2}}\right],$$

which is less than zero because $z \geq 0$, making the term in brackets negative for all z. Therefore the redshift decreases with time for $w > -\frac{1}{3}$, and thus increases with time for $w < -\frac{1}{3}$.

Part B

For a matter only universe we have $w = 0$, thus

$$\frac{dz}{dt_0} = H_0(1+z)\left[1-(1+z)^{\frac{1}{2}}\right]$$

which can be written as

$$dt_0 = \frac{dz}{H_0(1+z)\left[1-(1+z)^{\frac{1}{2}}\right]}.$$

For a galaxy with $z = 1.000000$, the time taken for the redshift to change by one part in 10^6 (i.e. $dz = -10^{-6}$, note dz is negative since $w > -\frac{1}{3}$) can be found directly from this equation without integrating since $\mid dz \mid \ll 1$. Plugging in the values of z and dz gives

$$dt_0 = 16,180\,yr.$$

Problem C6

If the quasar luminosity varies on a timescale of $\delta t_0 = 3$ days, then since

$$\frac{dt_e}{dt_0} = \frac{1}{1+z}$$

we have

$$\delta t_e = \frac{\delta t_0}{1+z}$$

so for $z = 5$, $\delta t_e = 12\,hr$. Thus the maximum size of the quasar $R_{max} = c\delta t_e$ is $1.30 \times 10^{13}\,m = 4.21 \times 10^{-10}\,Mpc$.

The angular size distance d_A to a redshift 5 object in the benchmark model is approximately $0.3c/H_0 = 1230\,Mpc$ (Ryden, Figure 7.4).

Using $d_A = l/\delta\theta$ where $l = R_{max}$ we have the angular size of the quasar is

$$\delta\theta = \frac{R_{max}}{d_A} = 7.0 \times 10^{-8}\,arcsec.$$

An object with an angular size this small must be observed using interferometry, so that D in the angular resolution $\theta = \lambda/D$ can be made very large compared to the observing wavelength to make θ extremely small.

Problem C7

The proper distance to a galaxy at the current time in a single component universe is

$$d_p(t_0) = \frac{c}{H_0}\frac{2}{(1+3w)}\left[1-(1+z)^{-\frac{(1+3w)}{2}}\right].$$

This will be used in parts A and B of this problem.

Part A

For radiation $w = \frac{1}{3}$, and so

$$d_p(t_0) = \frac{c}{H_0}\left[1 - (1+z)^{-1}\right] = \frac{c}{H_0}\left(\frac{z}{1+z}\right).$$

Thus for a galaxy with $z = 6$ we have

$$d_p(t_0) = \frac{6}{7}\frac{c}{H_0} = 3523\,Mpc,$$

where $H_0 = 73\frac{km/s}{Mpc}$ or $\frac{c}{H_0} = 4110\,Mpc$ has been used.

Part B

For matter $w = 0$ and so

$$d_p(t_0) = \frac{2c}{H_0}\left[1 - (1+z)^{-\frac{1}{2}}\right],$$

which at $z = 6$ is

$$d_p(t_0) = 2\left(1 - \frac{1}{\sqrt{7}}\right)\frac{c}{H_0} = 5113\,Mpc.$$

Part C

For a cosmological constant ($w = -\frac{1}{3}$), the proper distance at the current time t_0 is

$$d_p(t_0) = \frac{c}{H_0}z,$$

which for $z = 6$ is

$$d_p(t_0) = 6\frac{c}{H_0} = 24,660\,Mpc.$$

Part D

The proper distance at time of emission is related to the proper distance at the time of observation by

$$d_p(t_e) = \frac{d_p(t_0)}{1+z}.$$

Thus for radiation

$$d_p(t_e) = \frac{c}{H_0}\frac{z}{(1+z)^2},$$

which at $z = 6$ is

$$d_p(t_e) = \frac{6}{49}\frac{c}{H_0} = 503\,Mpc.$$

Part E

For matter

$$d_p(t_e) = \frac{2c}{H_0}\left[\frac{1}{1+z} - \frac{1}{(1+z)^{\frac{3}{2}}}\right],$$

which at $z = 6$ is

$$d_p(t_e) = 2\left(\frac{1}{7} - \frac{1}{7^{\frac{3}{2}}}\right)\frac{c}{H_0} = 730\,Mpc.$$

Part F

For a cosmological constant

$$d_p(t_e) = \frac{c}{H_0}\left(\frac{z}{1+z}\right),$$

which at $z = 6$ is

$$d_p(t_e) = \frac{6}{7}\frac{c}{H_0} = 3523\,Mpc.$$