$\frac{12}{10}$ ST

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2.1. $\frac{\Delta\rho}{\rho_0}$ where $\Delta\rho = \frac{M}{\frac{4}{3}\pi R^3}$ and $\rho_0 = 2.7\times10^{-27}\,kg/m^3$

a) $M = 100\,kg$, $R = 1\,m$ for a human

$$\frac{\Delta\rho}{\rho_0} = \frac{\frac{100}{\frac{4}{3}\pi 1^3}}{2.7\times10^{-27}} = 8.84\times10^{27} \simeq 10^{28}$$

b) $M = 2\times10^{30}\,kg$, $R = 1.5\times10^{11}\,m = 1\,AU$ for the sun

$$\frac{\Delta\rho}{\rho_0} = \frac{\frac{2\times10^{30}}{\frac{4}{3}\pi(1.5\times10^{11})^3}}{2.7\times10^{-27}} = 5.24\times10^{22} \simeq 10^{22}$$

c) $M = 5\times10^{11} M_\odot$ $R = 1\,Mpc = 3.1\times10^{22}\,m$ For the milky way and Andromeda

$$\frac{\Delta\rho}{\rho_0} = \frac{\frac{5\times10^{11}\times2\times10^{30}}{\frac{4}{3}\pi(3.1\times10^{22})^3}}{2.7\times10^{-27}} = 2.47 \simeq 10^1$$

d) $M \geq \rho_0 \times \frac{4}{3}\pi(100\,Mpc)^3 = 2.7\times10^{-27}\frac{kg}{m^3}\times\frac{4}{3}\pi(3.1\times10^{24})^3 = 3.37\times10^{47}\,kg$

$r = 100\,Mpc$ For universe inside 100 mpc sphere

$$\frac{\Delta\rho}{\rho_0} = \frac{\frac{3.37\times10^{47}}{\frac{4}{3}\pi(3.1\times10^{24})^3}}{2.7\times10^{-27}} = 1 \simeq 10^0$$

2.2. If $H_0 = 70\,km/s/Mpc$,

$$R_0 = \frac{c}{H_0} = \frac{km/s}{km/s/Mpc} = Mpc = \frac{3\times10^5}{70} = 4300\,Mpc = 4.3\,Gpc$$

$$t_0 = \frac{1}{H_0} = \frac{1}{70\,km/s/Mpc} = \frac{1}{70\,km/s/Mpc}, 3.1\times10^{19}\frac{km}{Mpc} = 4.42\times10^{17}\,s$$

$= 1.4\times10^{10}$ years

2.1 e) $M = 5M_\odot$ black hole

$$r = r_{sh} = \frac{2GM_\odot}{c^2} = \frac{2\cdot6.67\times10^{-11}\cdot5\times2\times10^{30}}{(3\times10^8)^2} = 1.5\times10^4\,m$$

$$\frac{\Delta\rho}{\rho_0} = \frac{5\times2\times10^{30}}{\frac{4}{3}\pi(1.5\times10^4)^3} / 2.7\times10^{-27} = 2.62\times10^{44} \simeq 10^{44}$$

2.3 a) Wien's Law: $\frac{hc}{\lambda_p} \simeq 4.97\,kT \to \lambda_p = \frac{hc}{4.97kT}$

$$= \frac{6.63\times10^{-34}\,m^2kgs^{-1} \cdot 3\times10^{8}\,ms^{-1}}{4.97 \cdot 1.38\times10^{-23}\,m^2kgs^{-2}K^{-1} \cdot T} \approx \frac{0.0029\,mK}{T} = \frac{0.29\,cm\cdot K}{T} = cm$$

$T_\odot = 5778K \to \lambda_p = \frac{0.29\,cm\cdot K}{5778K} \approx \underline{5.02\times10^{5}\,cm}$ This is the wavelength of green visible light, the 'color' of the sun

b) $$\bar{E}_{mean} = \frac{\varepsilon_\gamma}{n_\gamma} = \frac{\frac{\pi^2k^4}{15\hbar^3c^3}T^4}{\frac{2.4k^3}{\pi^2\hbar^3c^3}T^3} = \frac{\pi^4 kT}{15\cdot 2.4} = \frac{\pi^4 kT}{36} =$$

$\underline{2.7kT}$ $\quad T_\odot = 5778K$ for the sun

$\bar{E}_{mean\odot} = 2.7\times1.38\times10^{-23}\,JK^{-1}\cdot 5778K \approx \underline{2.15\times10^{-19}\,J}$

$T = 2.73K$ for CMB

$\bar{E}_{mean\,CMB} = 2.7\times1.38\times10^{-23}\,JK^{-1}\cdot 2.73K = \underline{1.02\times10^{-22}\,J}$

Extra Credit: If half the temperature of the sun can ionize hydrogen, and both the Sun and CMB are subject to the same hydrogen ionization physics, the redshift at which the CMB was generated is very high. The current observed temperature of the CMB is much less than half the Sun's temperature, but as it's known that the universe was opaque with electrons ionized from hydrogen or helium interacting with photons, the temperature must have been much higher, also supported by the Planck temperature. To have such a great decrease in temperature then, the CMB redshift must be high. Wien's law states $\lambda = \frac{0.29}{T}$ so high redshift would increase the wavelength, and so lower the temperature observed. Using the equation $T = T_0(1+z)$, where $T_0 = 2.73K$ and $T \approx 3000K$, $z \approx 1090$ for the CMB, high as expected +2