Class 9 : Kerr Black Holes

ASTR350 Black Holes (Spring 2020)
Prof. Richard Mushotzky

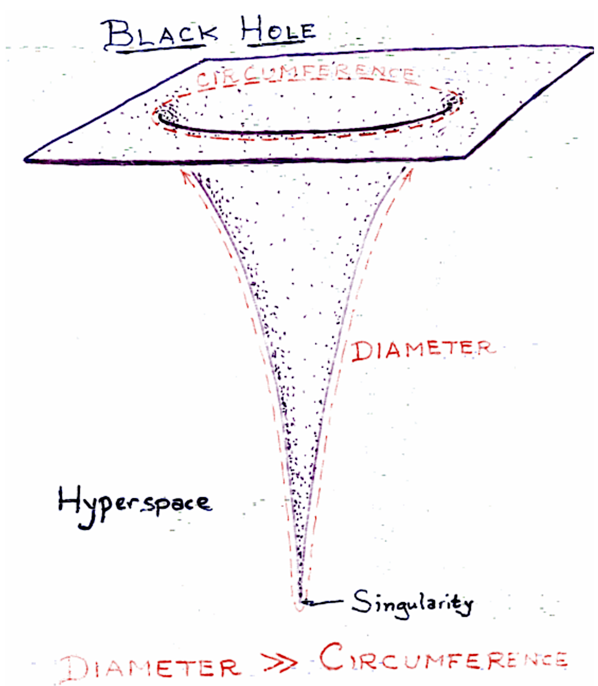
RECAP

- Schwarschild Solution
  - Describes **non-spinning**, uncharged black holes
  - Spherically Symmetric spacetime
  - External observers see infalling objects freeze/fade at the event horizon $r=2GM/c^2$
  - From point of view of infalling observer, pass through the event horizon and 'hit' the spacetime singularity at the center
  - Tidal forces will stretch (Spaghettify) observer before they reach the center (effect depends on mass of BH)

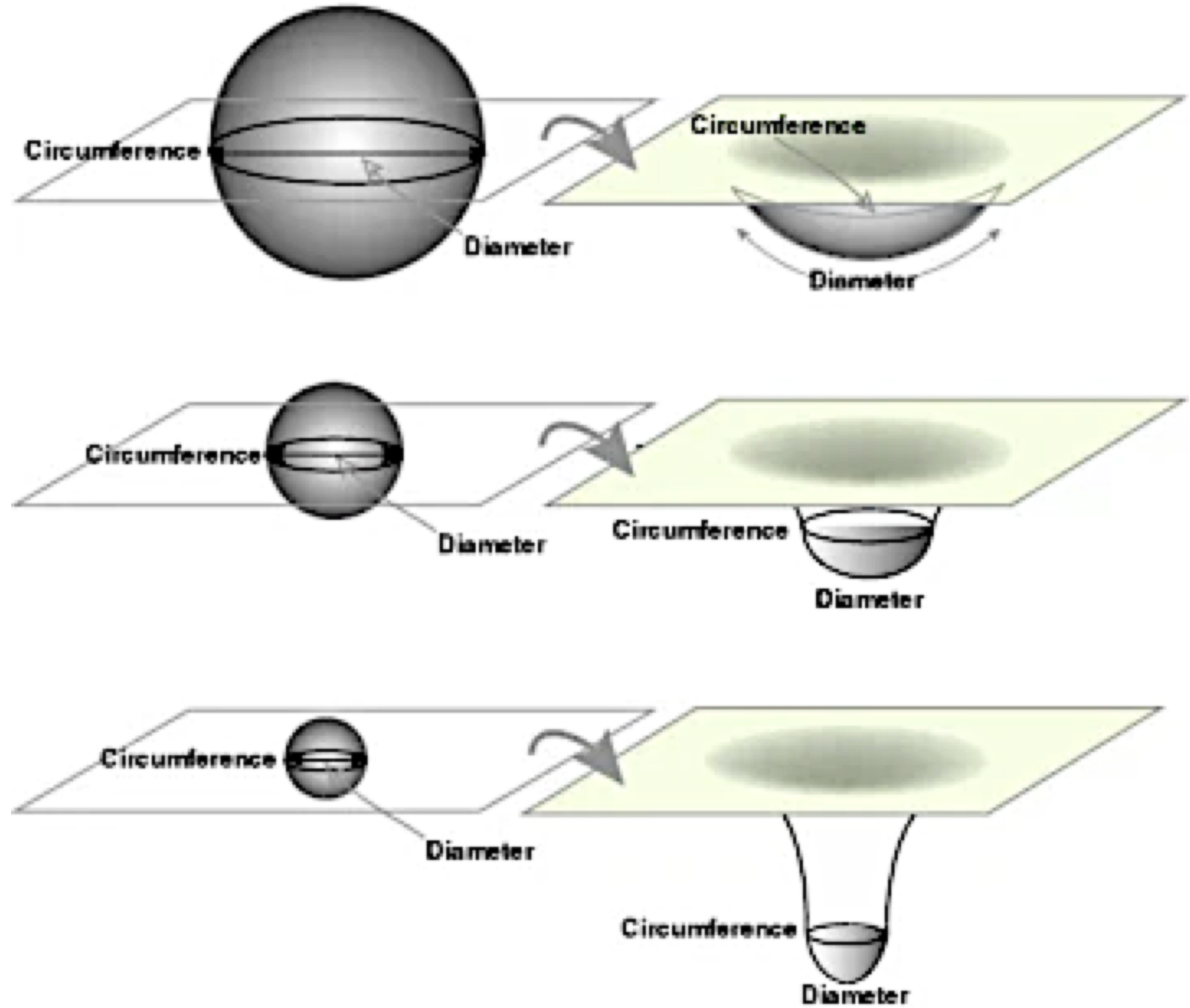
- Uncovered two aspects of the event horizon
  - Surface at which gravitational redshift is infinite
  - Region within horizon **cannot** causally affect outside-e.g. no information can be transmitted from inside horizon to outside world

Mass Bends Space Time

Ratio of circumference to diameter changes enormously

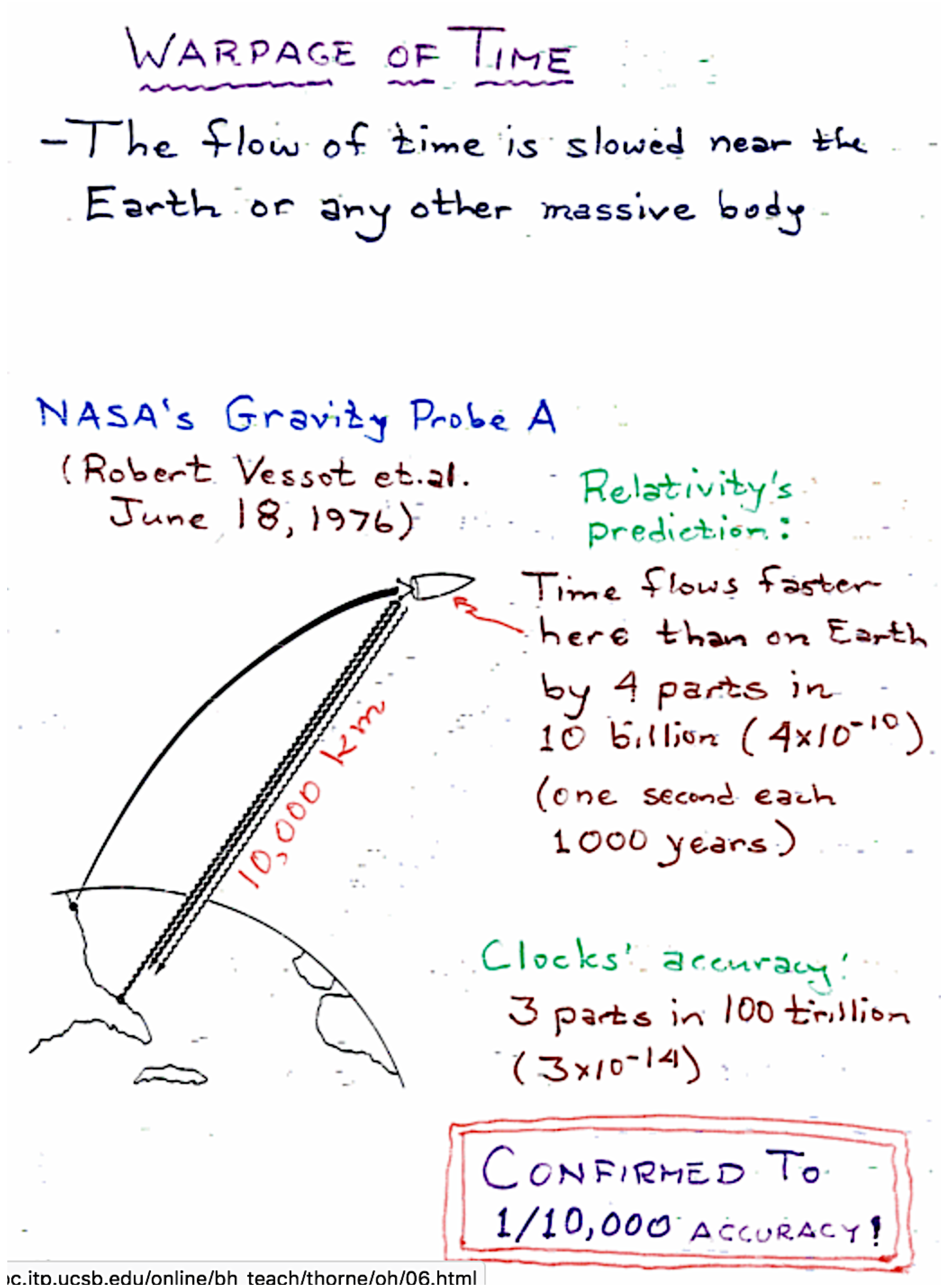


Stars with same mass but different sizes



Black Holes and Time Warps, Kip Thorne

Mass Slows Down Time



Kip Thorne-http://doug-pc.itp.ucsb.edu/online/bh_teach/thorne/oh/05.html

THIS CLASS

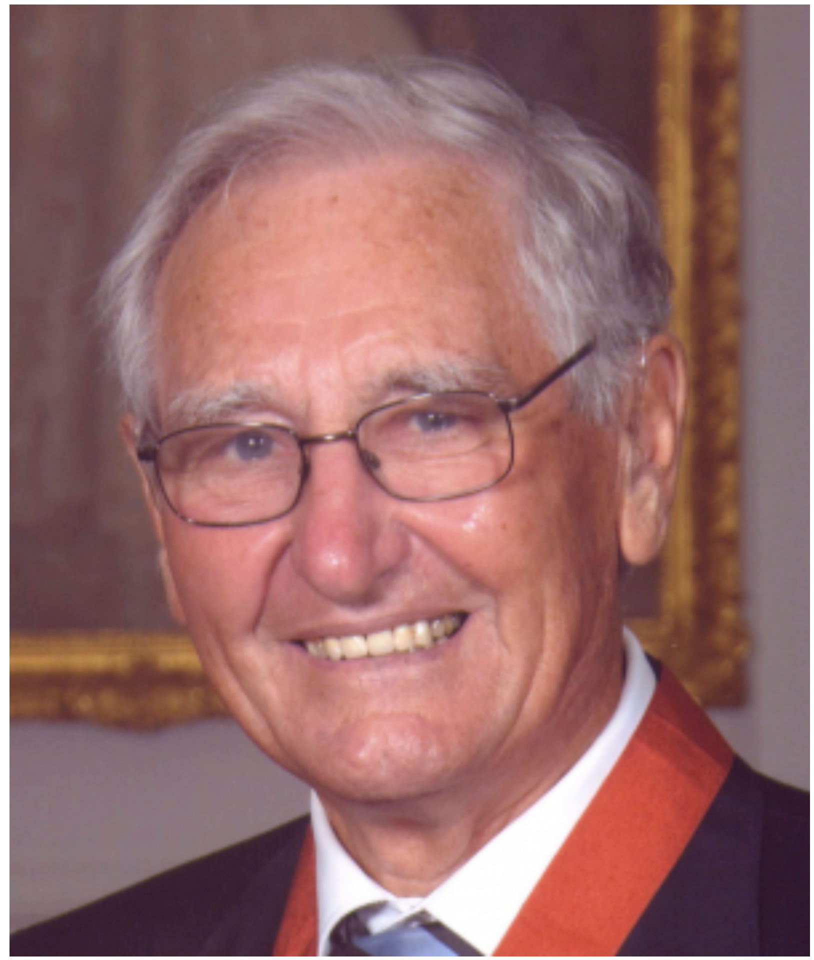
- Kerr (spinning) black holes!
  - No-hair theorem
  - Twisting of spacetime (“frame-dragging”)
  - Ergosphere and “black hole machines”
  - Orbits around black holes

For those having trouble with problem 4 of the homework see https://degiuli.com/en/the-balloon-in-the-car-and-general-relativity/

Last Homework- Monkey falling

I : Roy Kerr

- Roy Kerr (1934-)
  - Discovered exact solutions of Einstein's equations describing a spinning black hole **in 1963**
  - Was later shown that this solution is unique… any spinning (uncharged) black hole is described by the Kerr solution
  - Started a revolution in the theoretical understanding of *real* black holes



Real Black Holes

- Due to their origin expect 'stellar mass' black holes to be 'spinning'
  - collapse of massive star (more later)
- Not clear if supermassive black holes should have a 'lot' of spin

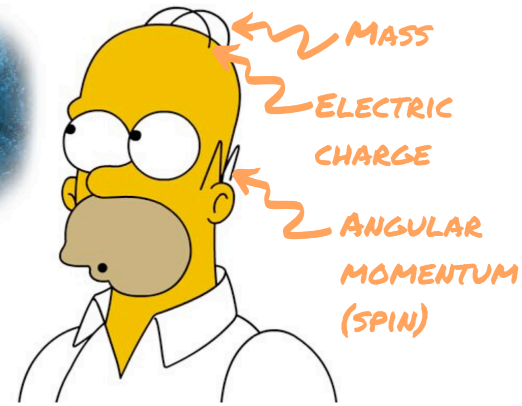
II : No hair theorem

- Any (isolated) black hole is described by just **three** quantities…
  - Mass
  - Spin
  - Electrical Charge
- Anything not measurable by a long range field is effaced

- Once these quantities are specified, the properties of the black hole exterior to the horizon (e.g. spacetime curvature) are uniquely determined.
  - There can be no lumps or bumps on a BH!

Why Mass,Charge and Spin

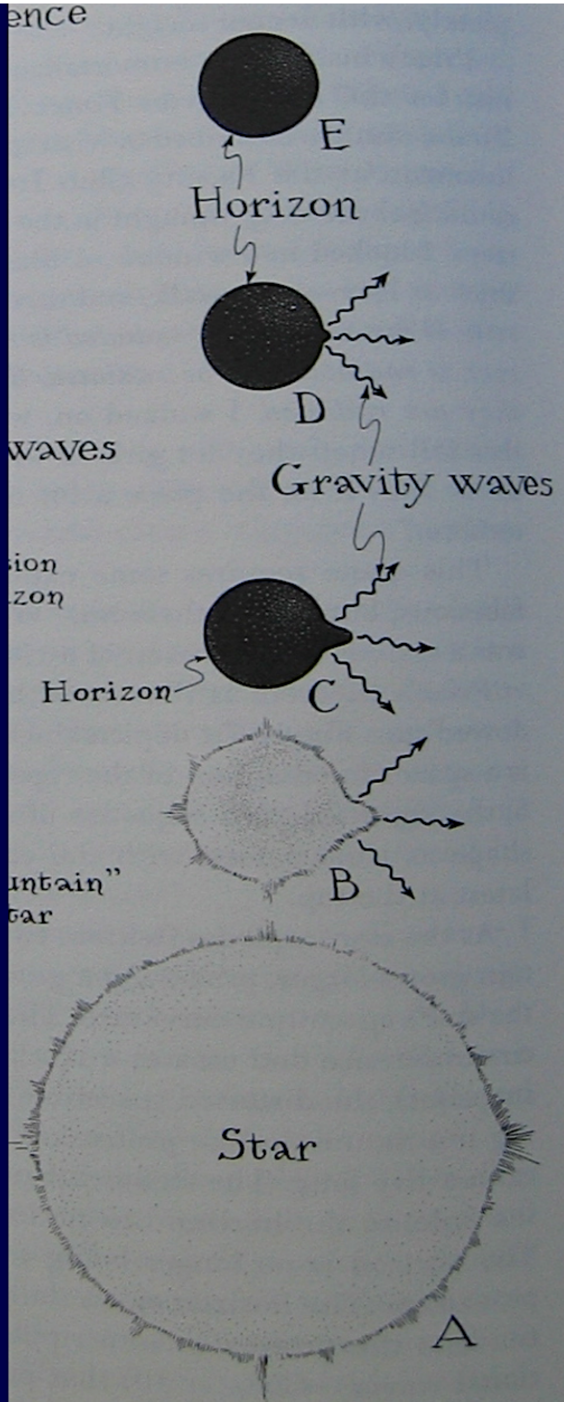
- The **no-hair theorem** states that all black holes can be completely characterized by only three *externally* observable classical parameters: mass, electric charge, and angular momentum.
- All other information ("hair" is a metaphor) about the matter which formed a black hole or is falling into it, "disappears" behind the black-hole event horizon and is therefore permanently inaccessible to external observers.



some hair theorem

Mass,Charge and Spin

- Suppose two black holes have the same masses, electrical charges, and angular momenta, but the first black hole was made by collapsing stars, whereas the second is made out of potatoes
  - *They will be completely indistinguishable to an observer outside the event horizon.*
- None of the special particle physics conserved quantities (baryonic number, leptonic number), all of which could be different for the material that collapsed and created the black hole) are conserved in the black hole and are they unobservable from the outside.



time

A black formed from an imploding star radiates its hair away, where its "hair" here is the mountain on top of the star

http://www.astro.sunysb.edu/rosalba/astro2030/KερρΒΗ.πδφ

[Figure from Thorne's "Black holes and time warps"]

- These are the only quantities which can be determined from a distance by examining its gravitational and electromagnetic fields.
  - (lumps and bumps go away- perfectly spherical)

- Astrophysical black holes are expected to have non-zero angular momentum, due to their formation via collapse of rotating stellar objects and growth via accretion, but effectively zero charge, since any net charge will quickly attract the opposite charge and neutralize.

Spin Can Have a Big Effect

- Remember that energy has mass
- Spinning black holes have a very different metric than non-spinning ones (much more complex)

Spinning BH-Metric

- A black hole with angular momentum ***J*** has a metric

$ds^2=(1-2GMr/\rho c^2)dr^2-\{(1/c^2)[4GMra\sin^2\theta/\rho c]\,drd\phi\}+(\rho/\Delta)dr^2+$
$\rho d\theta^2+(r^2+a^2+2GMra^2\sin^2\theta/\rho c^2)\sin^2\theta\, d\phi^2]$– Argh...

r,θ,ϕ– usual polar coordinates (NOT ON EXAM)

where $a=(J/Mc)$ is the angular momentum per unit mass (dimensions of distance) and
$\Delta=r^2-(2GMr/c^2)+a^2$; $\rho=r^2+a^2\cos^2\theta$

Just like Schwarzschild metric it becomes singular, but at a radius where
$r_+=GM/c^2+[(GM/c^2)^2-(J/Mc)^2]^{1/2}$

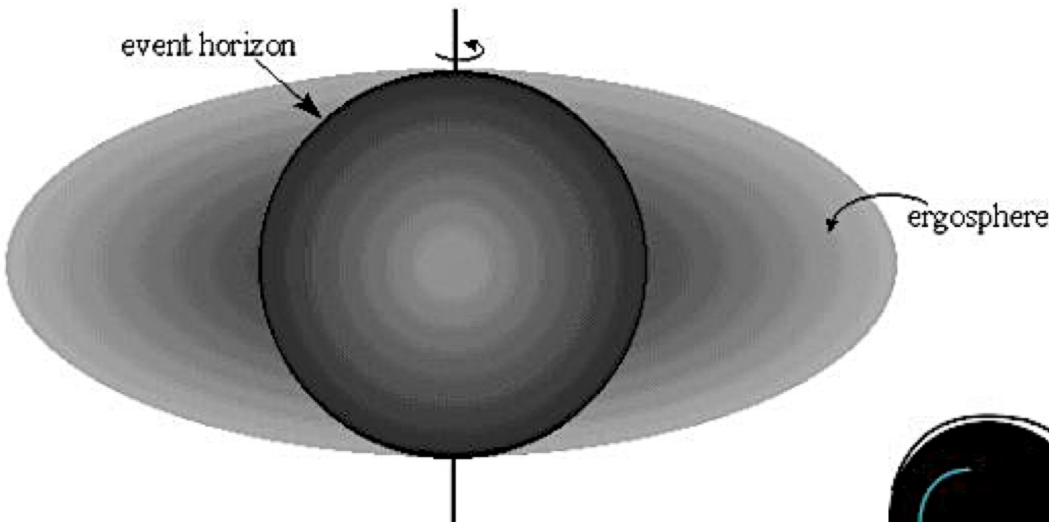
for J>0 this is smaller than the Schwarzschild radius

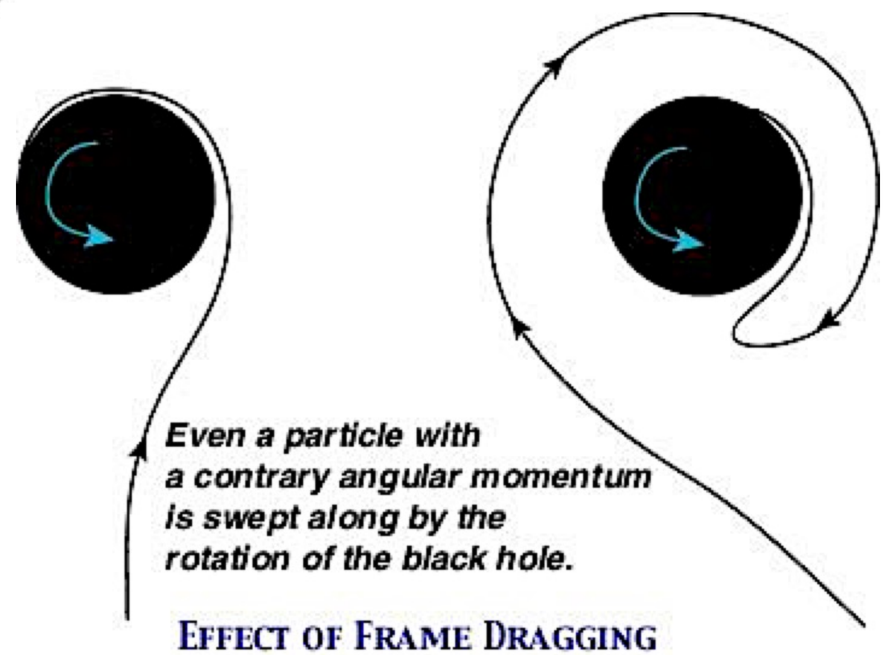
there is a maximum angular momentum ***$J=GM^2/c$;*** *for this value of J the horizon is at* ***$r_+=GM/c$; 1/2 of the Schwarzschild radius***

Schwarzschild metric

- $ds^2=-(1-2GM/c^2r)c^2dt^2-(1-2GM/c^2r)^{-1}dr^2+r^2(d\theta^2+\sin 2d\phi\theta^2)$- no mention of angular momentum

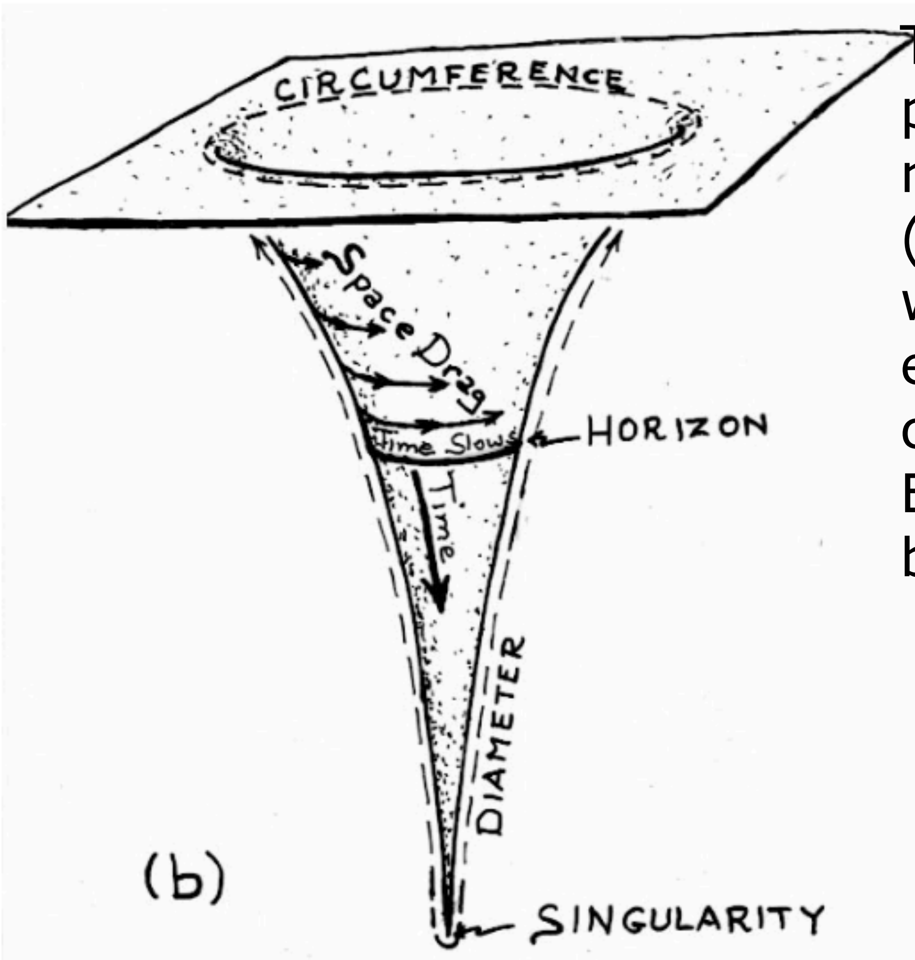
III : Frame dragging and the ergosphere





Graphics: University of Winnipeg, Physics Dept.

Space Being Dragged

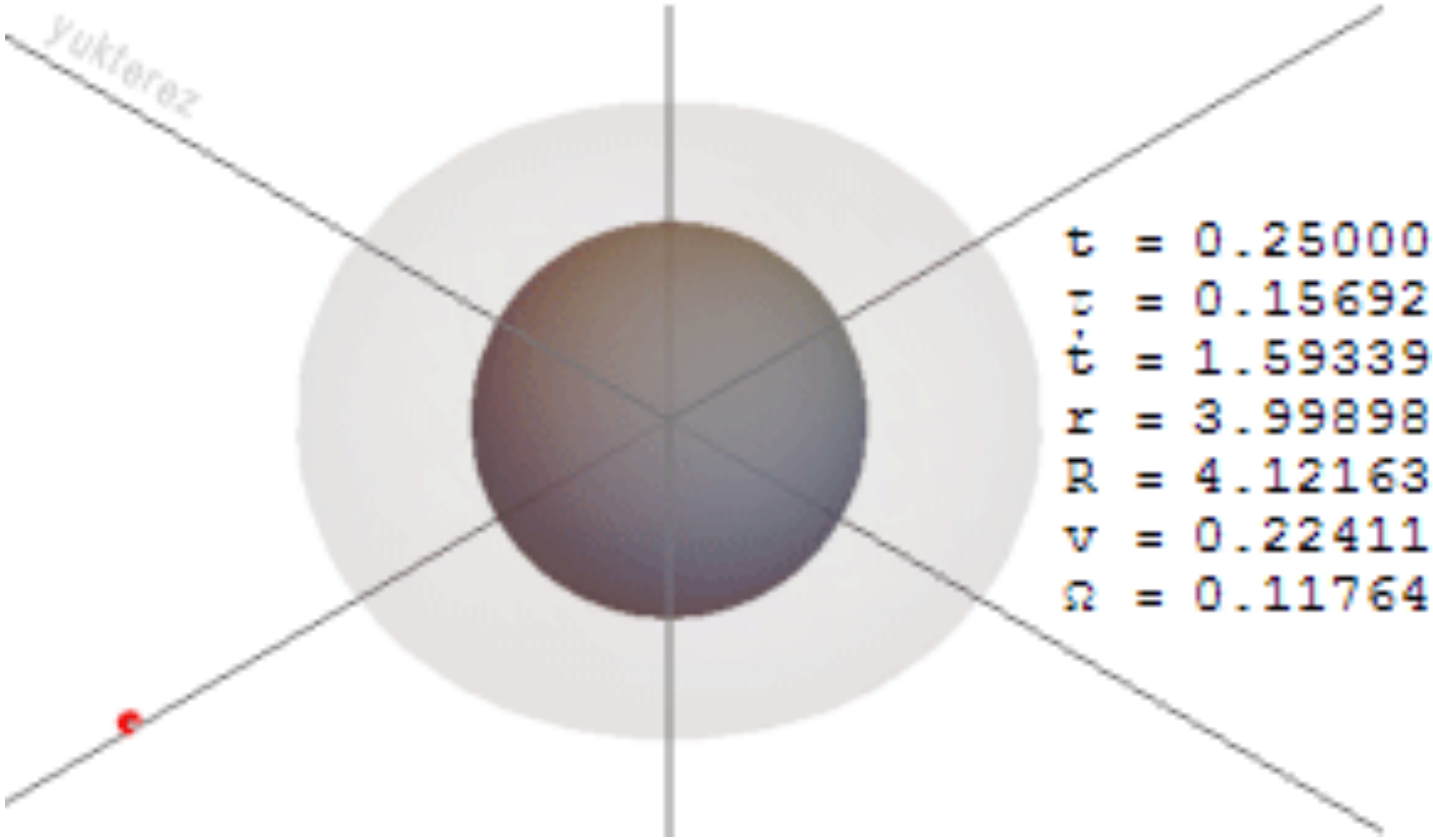


The whirl of spacetime is produced by the hole's huge rotational angular momentum (that resides in the spacetime whirl itself). The warpage's energy and angular momentum create the warpage, according to Einstein's equations. Warpage begets warpage. K. Thorne

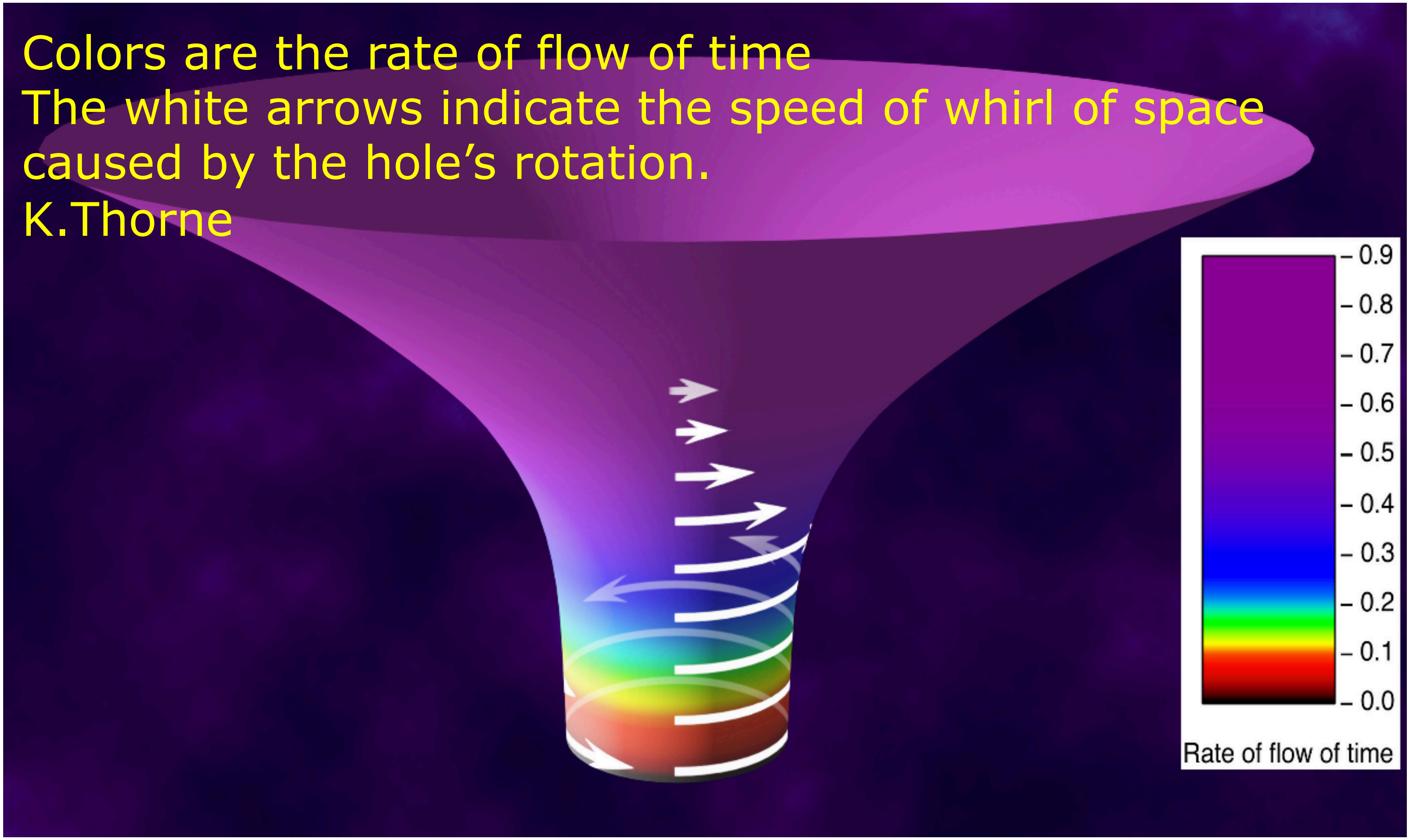
http://www.its.caltech.edu/~kip/index.html/PubScans/BlackHoles-Thorne-Starmus.pdf

Falling into a Kerr BH

https://degiuli.com/en/3-amazing-properties-of-rotating-black-holes/



Space Time Around a Kerr BH



Colors are the rate of flow of time
The white arrows indicate the speed of whirl of space caused by the hole’s rotation.
K.Thorne

Schwarzschild and Kerr Metric

- There is another 'fiducial' radius in the Kerr solution, that radius within which all light cones point in the direction of rotation, the 'static' radius, r_{static}.

- Between r_{static} and r_+ is a region called the 'ergosphere' within which particles must rotate with the black hole and from energy might be extracted (Penrose process).

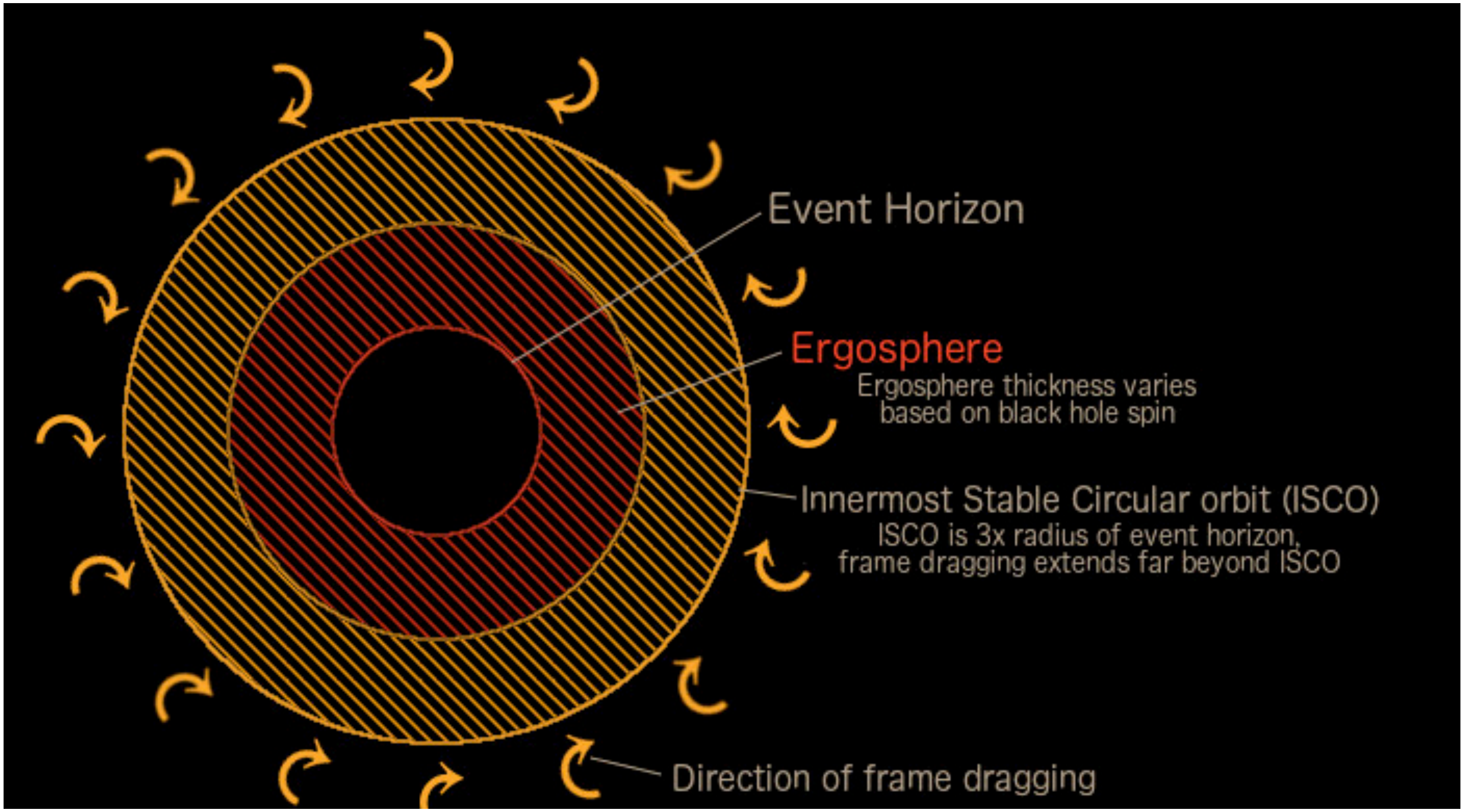
Kerr Solution

- Because of frame-dragging effect, there is a region of spacetime where static observers cannot exist, no matter how much external force is applied.
- This region is known as the "ergosphere" $r \leq \mathcal{M} + \sqrt{\mathcal{M}^2 - a^2 \cos 2\theta}$

  $\mathcal{M} = GM/c^2$
- How does the black hole get 'spun up'

  the distortion of curvature brought about by the presence of the infalling matter.

ErgoSphere



Event Horizon
Ergosphere
Ergosphere thickness varies based on black hole spin
Innermost Stable Circular orbit (ISCO)
ISCO is 3x radius of event horizon, frame dragging extends far beyond ISCO
Direction of frame dragging

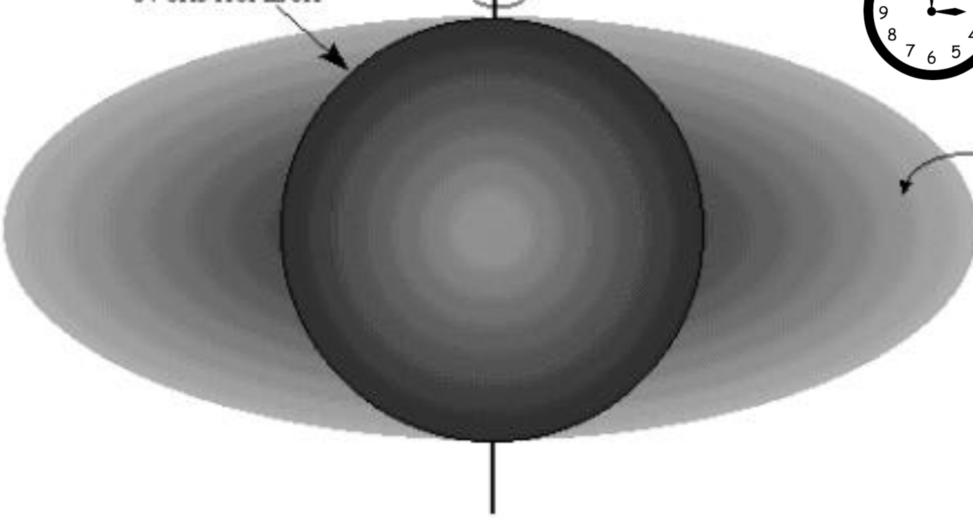
Differences Between Schwarzschild and Kerr Metric

- For a Schwarzschild BH the innermost stable radius is $3r_G=6GM/c^2$ - there are no stable circular orbits at smaller radii
  - The binding energy from this orbit is 0.0572 of the rest mass energy – the maximum that can be released
- For a Kerr the innermost stable radius is at $r_+=GM/c^2$ The spinning black hole drags the the inertial frame-
- The smaller critical radius allows more energy to be released by infalling matter
  - For a Kerr BH 0.423 of the energy can be released.

Ergosphere is not spherically symmetric
Faster black hole spins more oblate it becomes

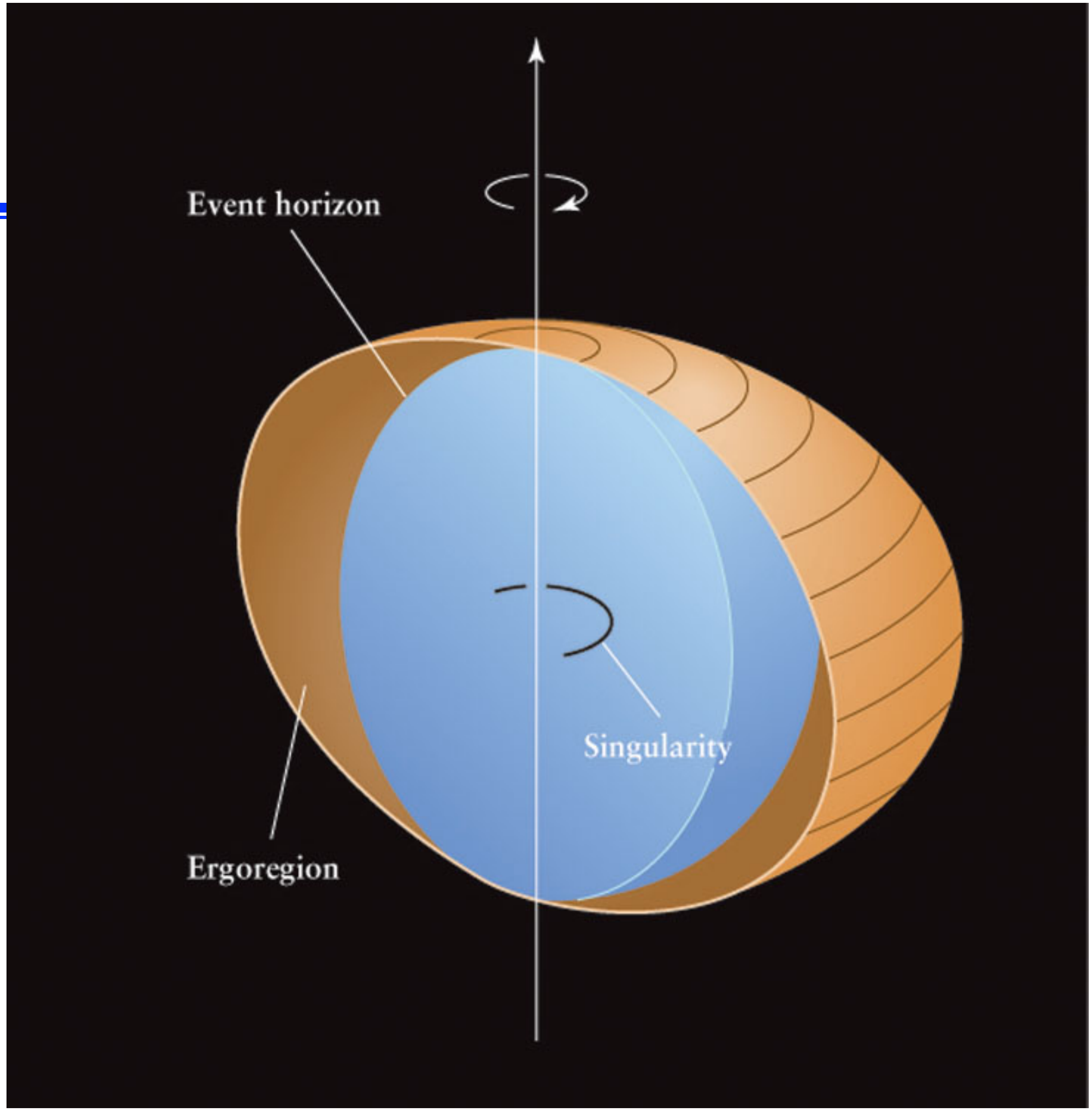
*Stationary clock appears to freeze/stop at the edge of the **ergosphere**.*

event horizon

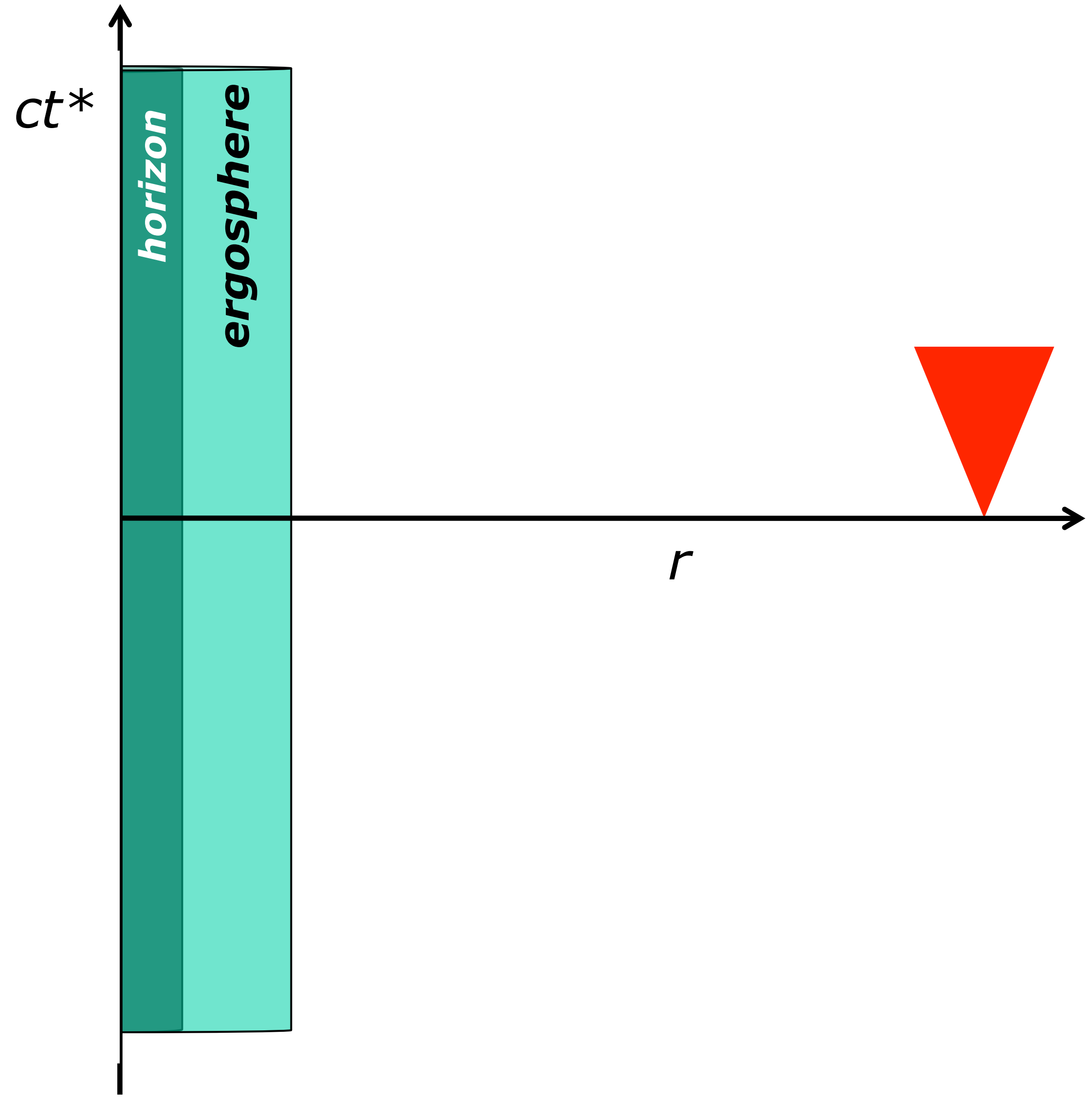


ergosphere

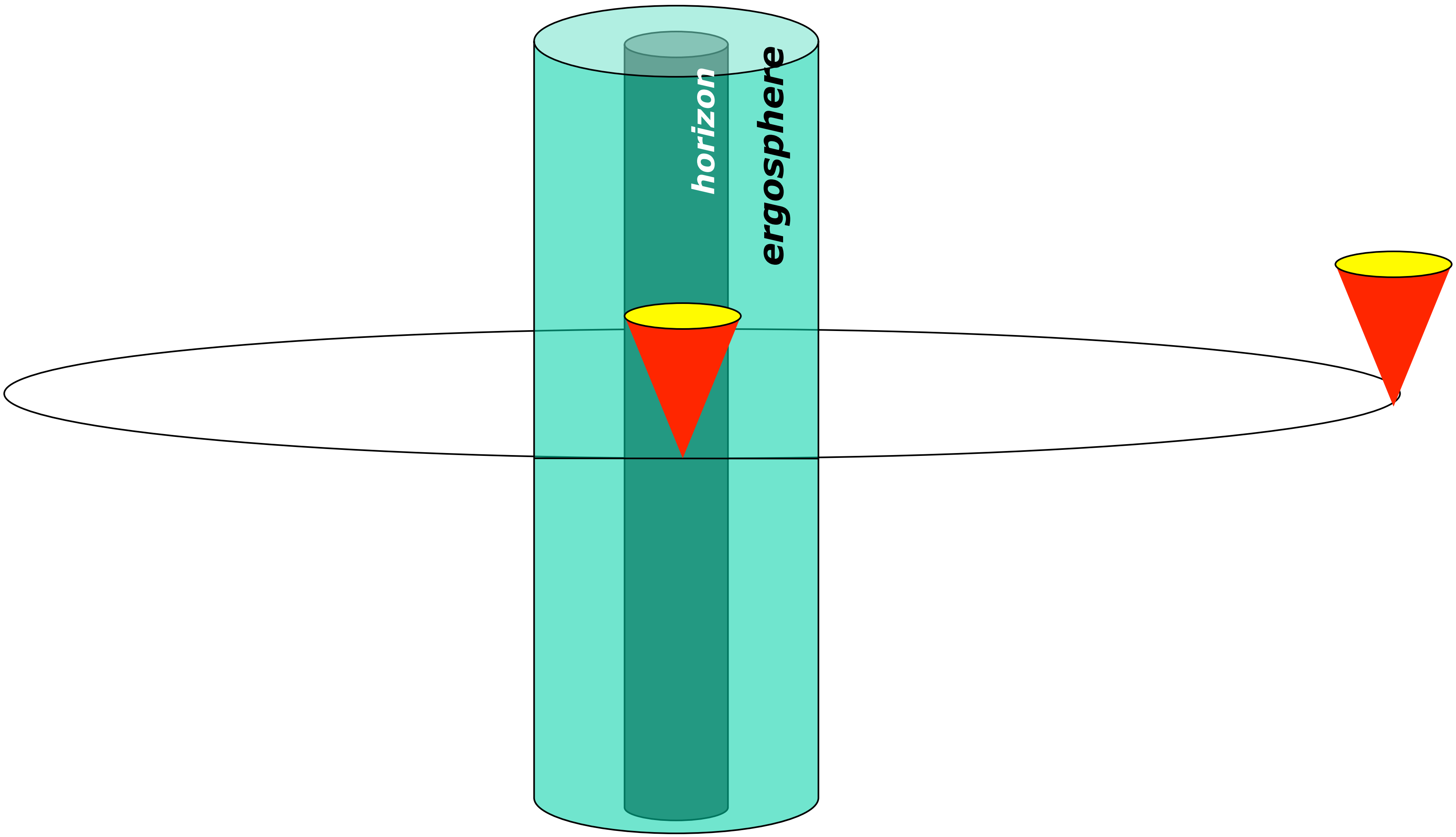
Graphics: University of Winnipeg, Physics Dept.


Event horizon
Singularity
Ergoregion

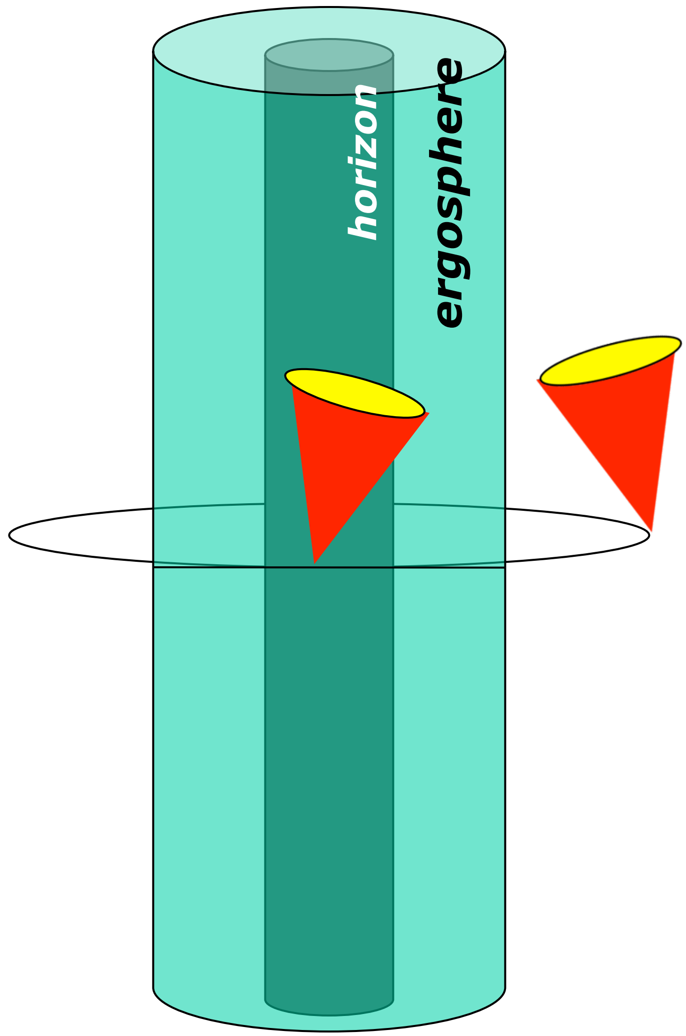
Space Time Diagram



Space Time Diagram- Orbiting Around a BH

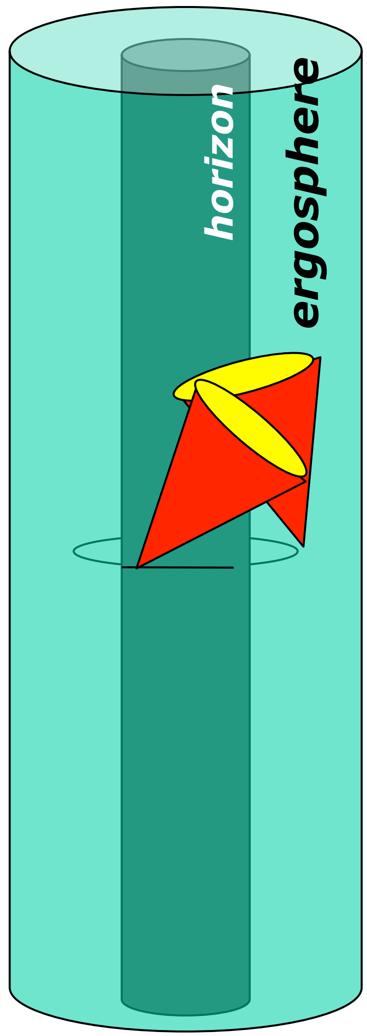


Space Time Diagram- Orbiting Around a BH – *Getting Closer to BH*



Frame dragging

When reaching close to the Kerr BHs, observers even with zero angular momentum will co-rotate with the BHs because of the swirling of spacetime from the rotating body.



Frame dragging effects tip over light cones in direction of rotation.

Within ergosphere, light cones tipped such that all <u>futures</u> rotate in sense of black hole.

In other words, within ergosphere it is impossible to stand still!

- Why is it called the ergosphere?

- It is the region of spacetime that holds the rotational energy of the black hole (Roger Penrose 1969) ergo is energy in Greek
  - Penrose conducted (rather artificial) thought experiments to show that the BH rotational energy can be extracted from within ergosphere
  - This energy is 0.29 Mc^2
  - Roger Blandford and Roman Znajek (1977) showed that realistic magnetic fields interacting with the BH can achieve the same affect.

IV : The event horizon

- Ergosphere is outside of the event horizon… we can travel in and out, and can see emission from within!
- Actual event horizon has familiar properties…
  - Surface of infinite redshift, even for clocks/sources that are co-rotating with the spacetime.
  - Seals off the interior space from view
- But rotation has an effect on the location of the event horizon.
  - Define spin parameter "***a***" (proportional to the angular momentum of the BH $a=J/M^2$ (where J is the Newtonian angular momentum)
  - ***a***=0 means non-spinning,
  - ***a***=1 means spinning at maximum rate

- Then the **event horizon** is at:

$$R_{evt} = \left(1 + \sqrt{1 - a^2}\right)\frac{GM}{c^2}$$

- **Smaller event horizons for spinning black holes**

Important later for how much energy can be extracted from accretion

$$a = 0 \Rightarrow R_{evt} = \frac{2GM}{c^2}$$

$$a = 1 \Rightarrow R_{evt} = \frac{GM}{c^2}$$

- What happens when a>1? Called **superspinners**. Kerr solution still gives an answer, but there is no event horizon! We have a **naked singularity**!
- **Cosmic Censorship Hypothesis** asserts that nature does not allow naked singularities and hence forbids a>1 black holes.

How Fast is the BH Spining

- Angular speed of BH is $\sim[10^5/M_{BH}/M_{sun}]\{a/(1+sqrt(1-a))\}$ radians/sec
- If a=0.9 and M_{BH}= mass of sun 10,000 revolutions per second (K. Thorne)
- If mass= 10^6suns 0.01 rev/sec

- How much energy in the spin ($0.29Mc^2$)- $5x10^{53}$ ergs for a 1 solar mass BH

So what is the actual size ?

$R_G \sim 1.5\ (M / M_\odot)$ km

So how close are neutron stars to being black holes ?

Neutron stars are only about a factor 2—3 larger than their event horizons

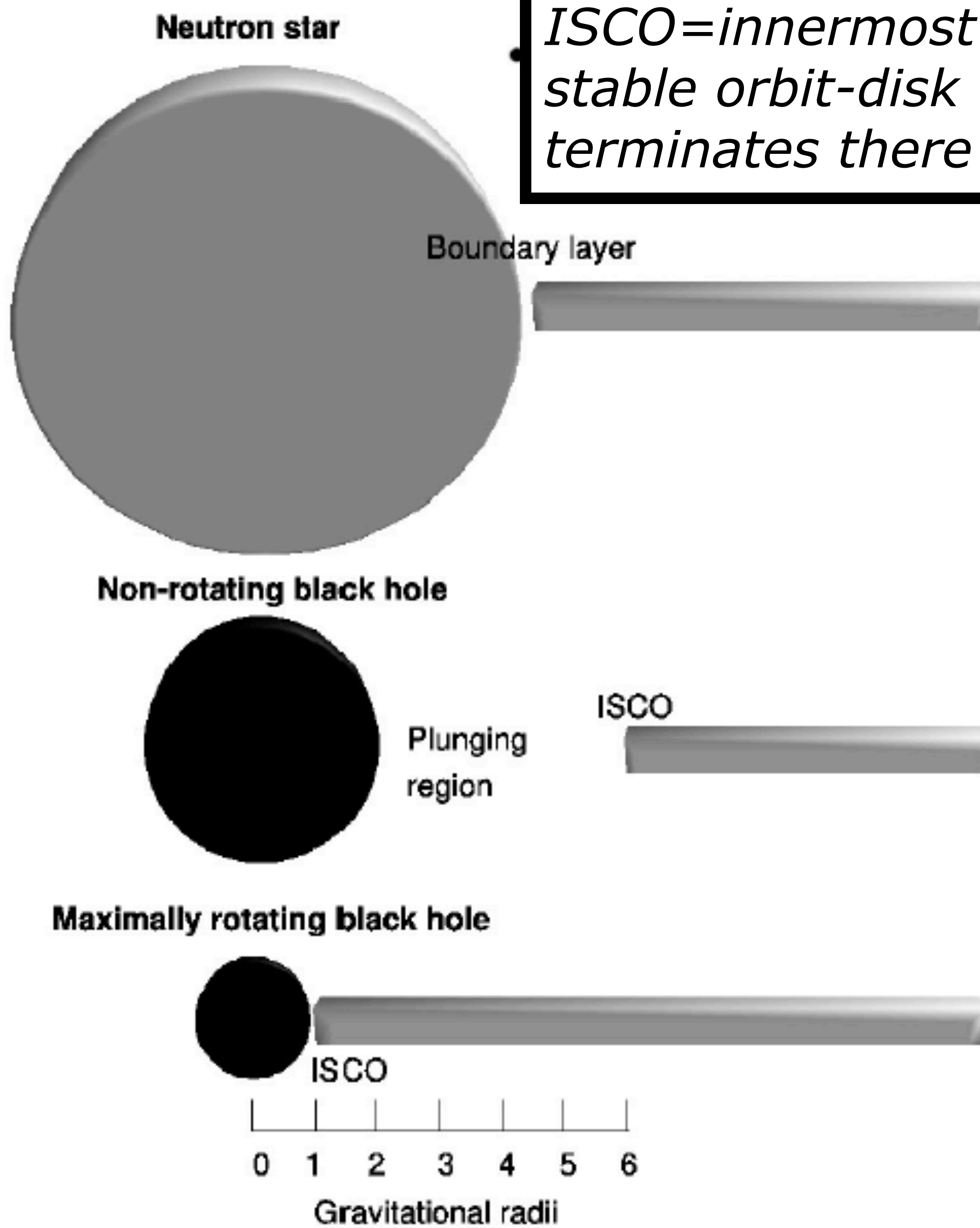
What about spin ?

A non-rotating ("Schwarzschild") black hole has its event horizon at $2\ R_G$ and its ISCO at $6\ R_G$

A maximally rotating ("Maximal Kerr") black hole has both its event horizon and ISCO at R_G

→ Spinning black holes are more compact → potentially more radiatively efficient

ISCO=innermost stable orbit-disk terminates there



R. Fender 2007

V : Orbits around black holes

- Very far from black hole…
  - Gravity behaves just as Newton says!
  - Velocity of a circular orbit is

$$V = \sqrt{\frac{GM}{r}}$$

  - Orbit is **stable**… if something on a circular orbit is nudged, the orbits just becomes slightly elliptical.
- As you get closer to the black hole, gravitational force becomes more and more non-Newtonian

Heading inwards…two special orbits…

- Innermost stable circular orbit (ISCO)
  - $R_{ISCO}=6GM/c^2$ (**Schwarzschild**)
  - $R_{ISCO}=2GM/c^2$ (**Kerr a=1**)
  - At and within this location, a particle with mass on a circular orbit is no longer stable… a small nudge and it will spiral into the black hole!
  - Very important for accretion disks (more later!)

- Photon circular orbit
  - $R_{ph}=3GM/c^2$ (Schwarzschild)
    - -A circular orbit can only exist in the equatorial plane (Kerr)
  - Only at this one radius, photons can travel in circles around the black hole
  - Nothing can be in orbit inside of this radius- must plunge.

Heading inwards…Meaning of Photon circular orbit

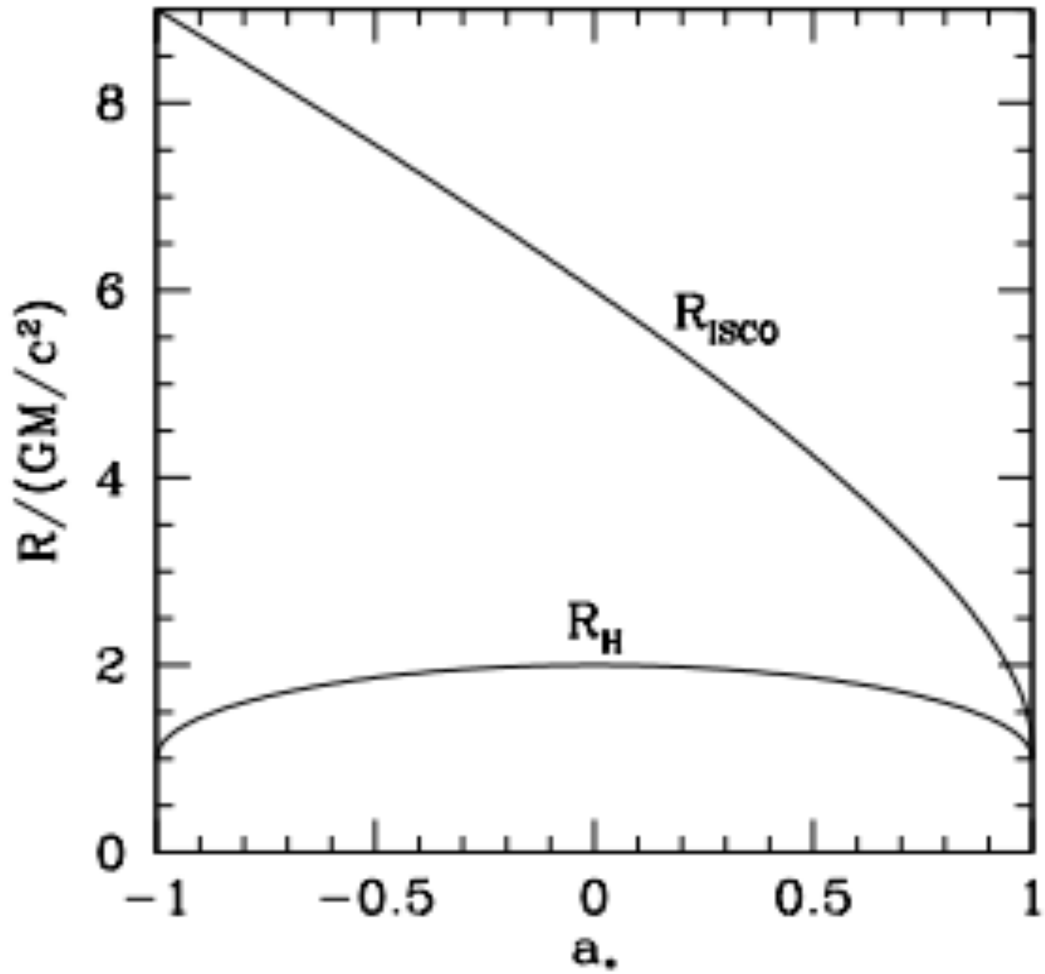
- Photons can orbit around the Schwarzschild black hole at a constant radius $r = 3GM/c^2$,
- Such an orbit is unstable:it defines the boundary between capture and non-capture of light rays by the Schwarzschild black hole
- For a Kerr black hole this orbit can be as small as

GM/c^2 for prograde a=1 :a prograde photon orbits the black hole at a smaller radius than a retrograde one for the same spin magnitude

to visualize the orbits go to https://duetosymmetry.com/tool/kerr-circular-photon-orbits/

How the Horizon and ISCO Change with Spin

As prograde spin increases R_{ISCO} and $R_{Horizon}$ get smaller

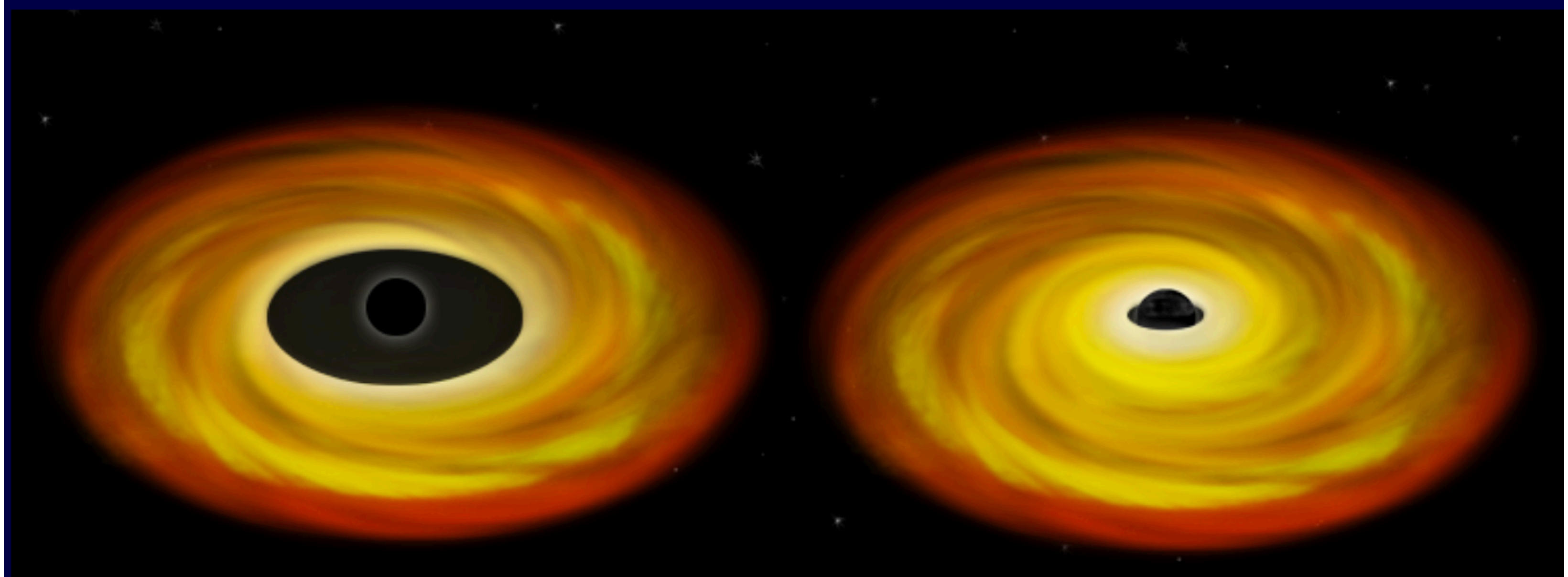


spin

How the Horizon and ISCO Change with Spin

Since gas as can orbit closer to a spinning black hole than to a non-rotating one

→ **Higher efficiency for energy extraction**



not spining　　　　　　　　spining

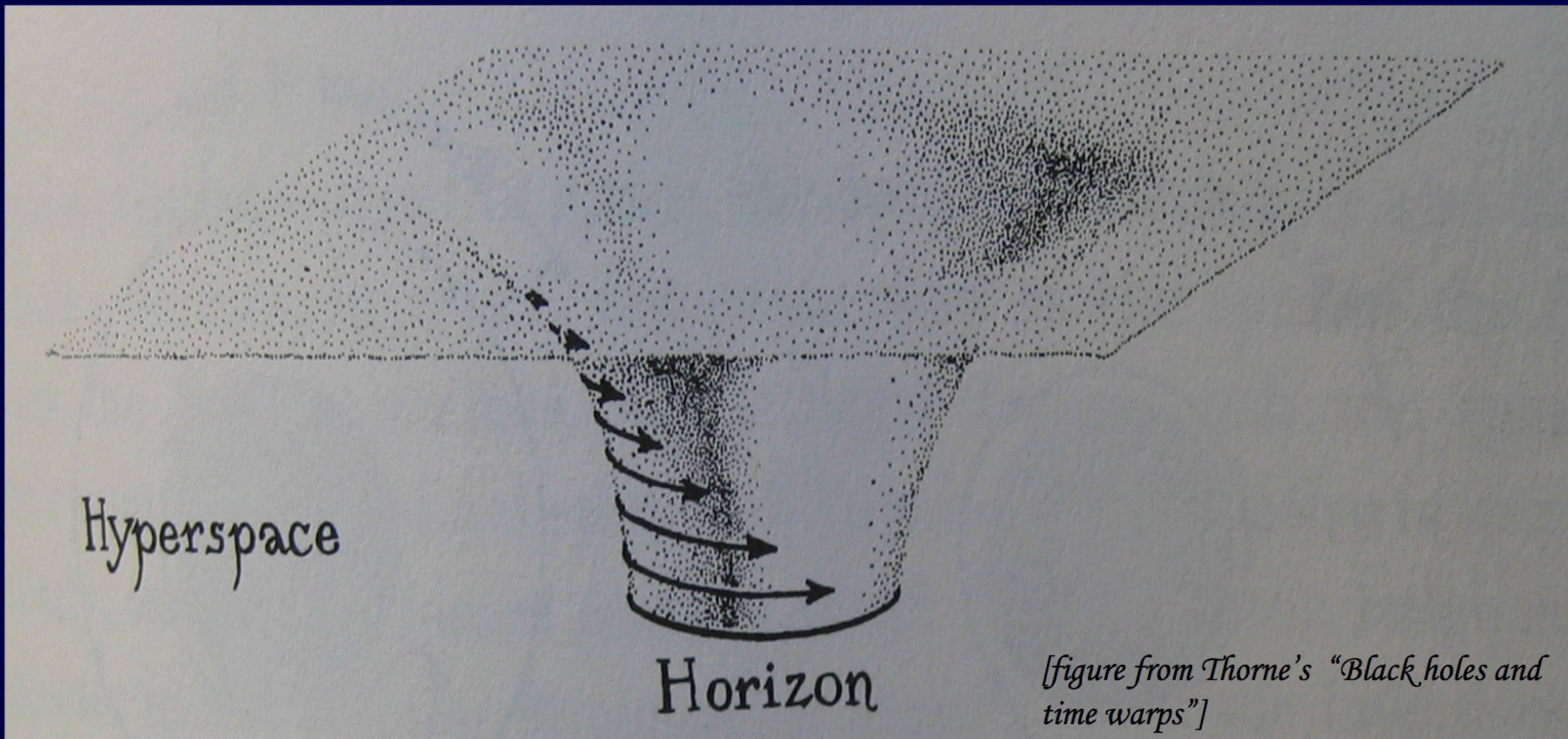
Summary:Kerr Solution

- Two “surfaces”:
  - The Horizon: region from which no signal can escape.
  - The Ergosphere: region inside which space rotates that is is impossible for a body to appear stationary to a distant observer

Summary Rotating black holes-

- Roy Kerr (1963)
  - Discovered solution to Einstein's equations corresponding to a *rotating* black hole
  - Kerr solution describes all black holes found in nature
- Features of the Kerr solution
  - Black Hole completely characterized by its mass and spin rate (no other features [except charge]; **no-hair theorem**)
  - Has space-time **singularity** and **event horizon** (like Schwarzschild solution)
  - Also has "**static surface**" inside of which nothing can remain motionless with respect to distant fixed coordinates
  - Space-time near rotating black hole is dragged around in the direction of rotation: "**frame dragging**".
  - ***Ergosphere*** – region where space-time dragging is so intense that its impossible to resist rotation of black hole.

Space around a spinning Black Hole



Because spacetime is "stuck" to the horizon, **space is dragged along with the spin.** This appears as a tornado-like swirl in hyperspace.

http://www.astro.sunysb.edu/rosalba/astro2030/KerrBH.pdf

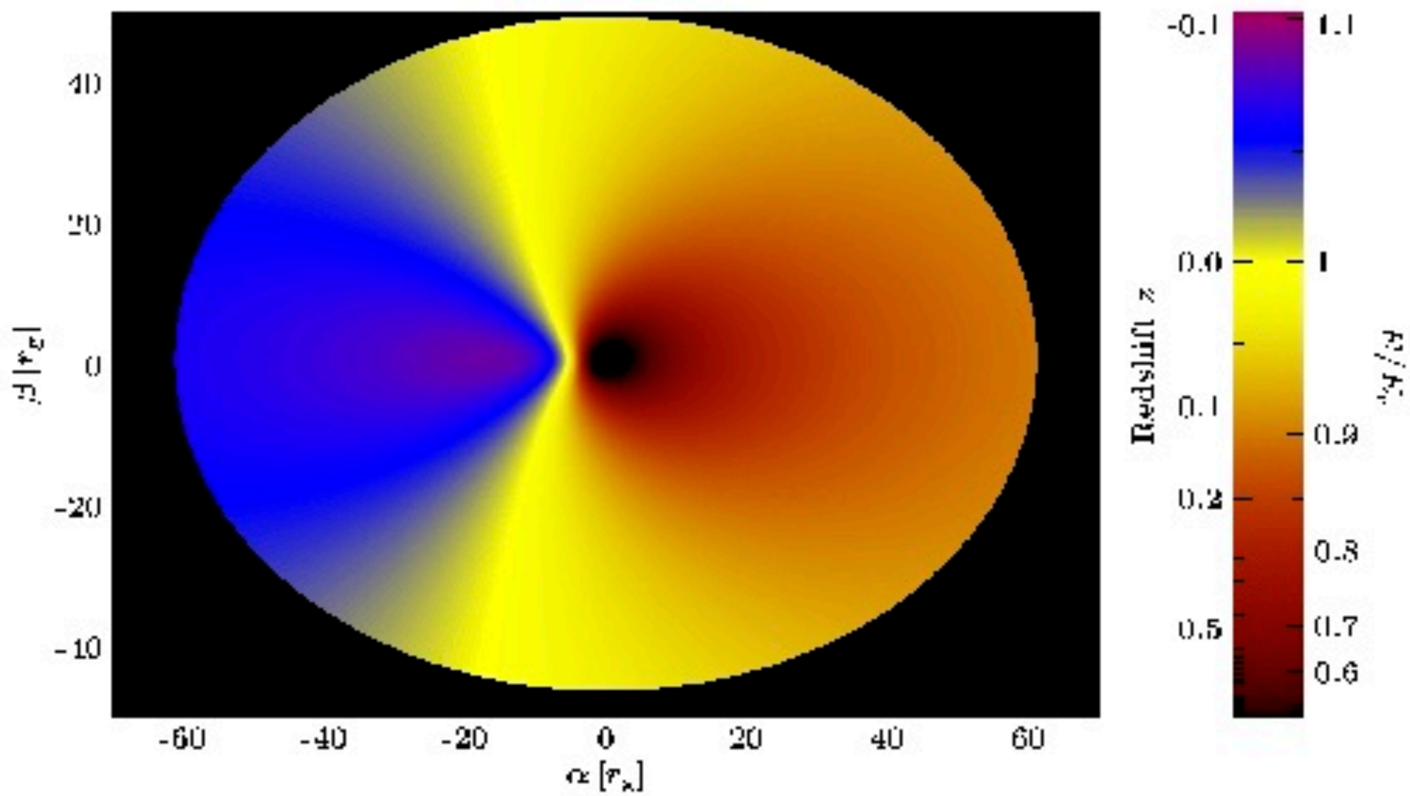
Next:Real black holes

So much for theory – what about reality

Thought to be two classes of black hole in nature

- ***"Stellar mass black holes"*** *– left over from the collapse/implosion of a massive star ($M>8M_{\odot}$)*
- ***"Supermassive black holes"*** *– giants that currently sit at the centers of galaxies (range of mass of BH from $10^{5.5}$-$10^{10}\ M_{\odot}$)*

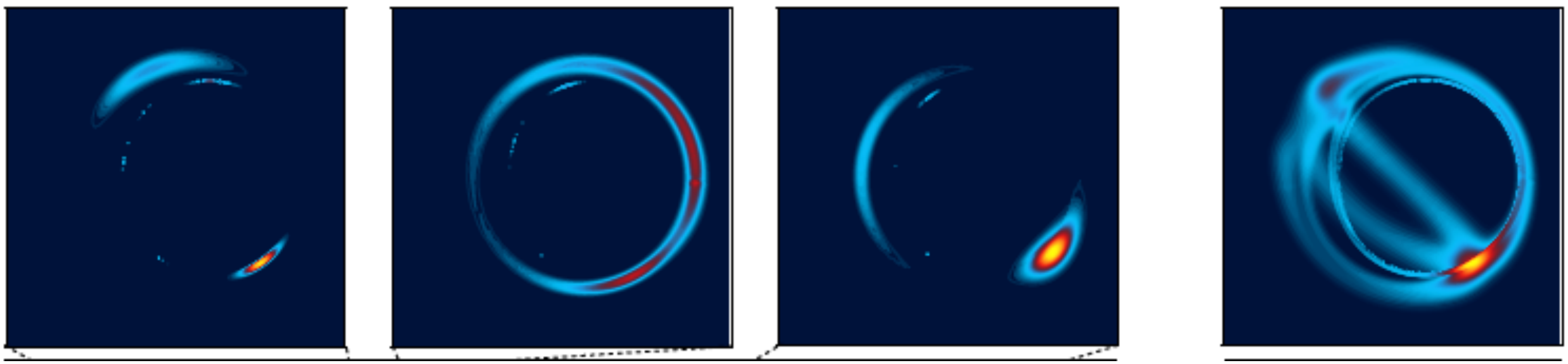
Relativistic disks



Courtesy T.Dauser

Orbital Motion near the Galactic Center Black Hole

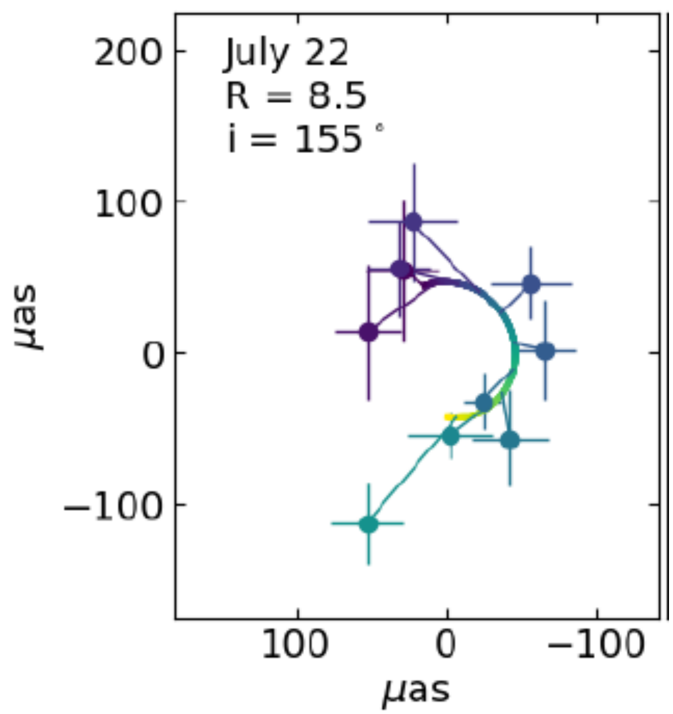
*Modeling the orbital motion of Sgr A**

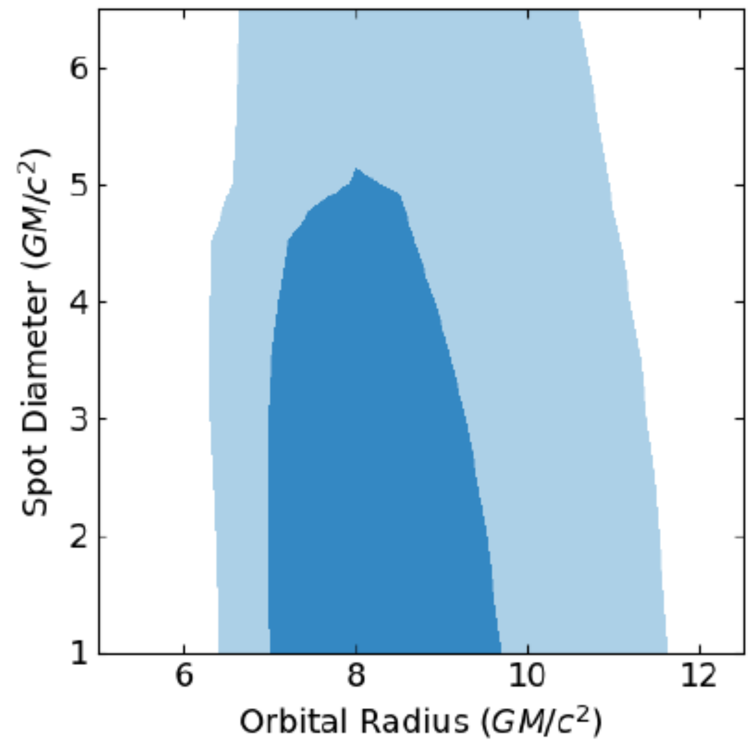


Hotspot rotating around a BH with $R=5GM/c^2$, spin $a=0.9$

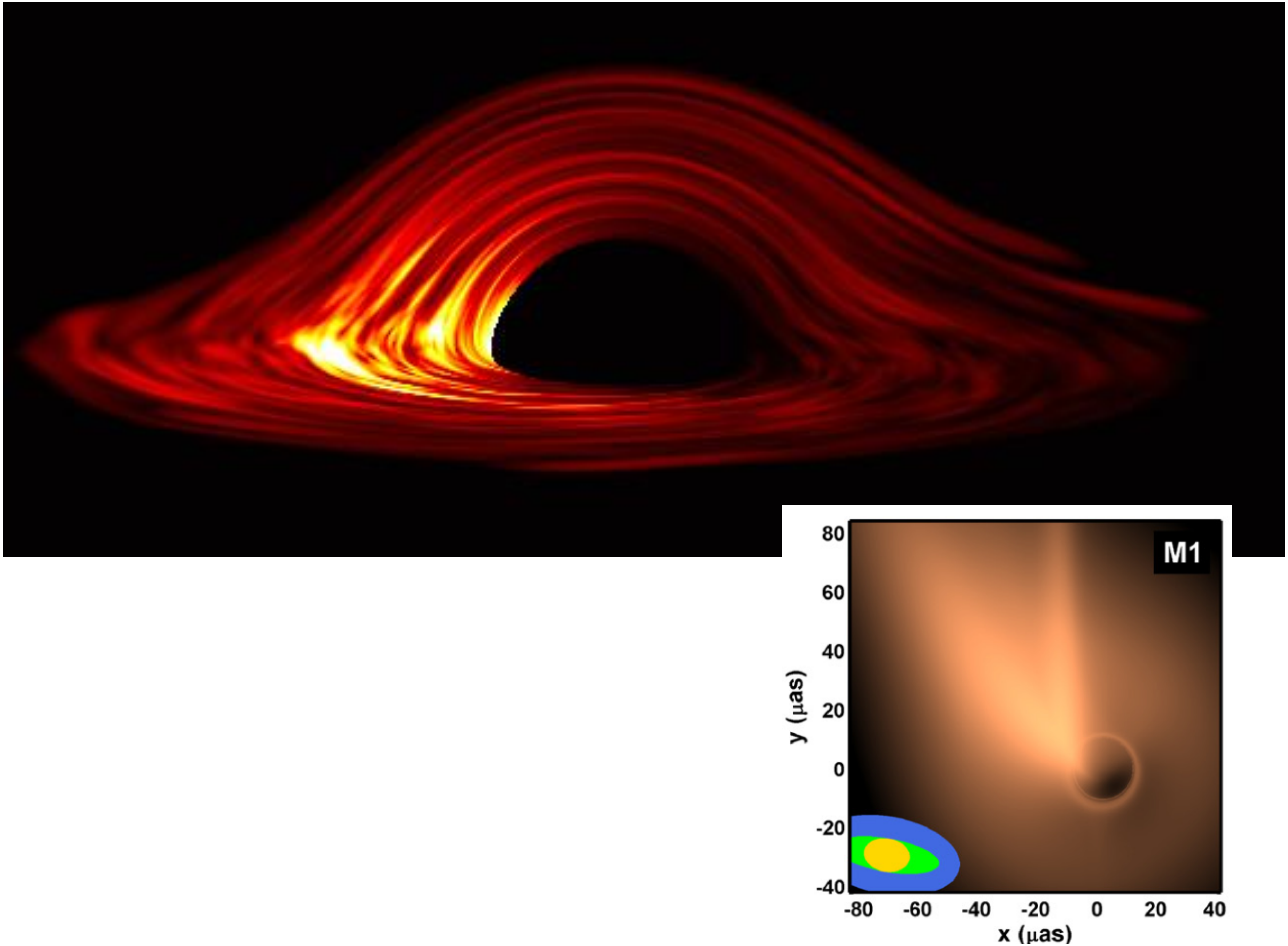
Orbital Motion near the Galactic Center Black Hole

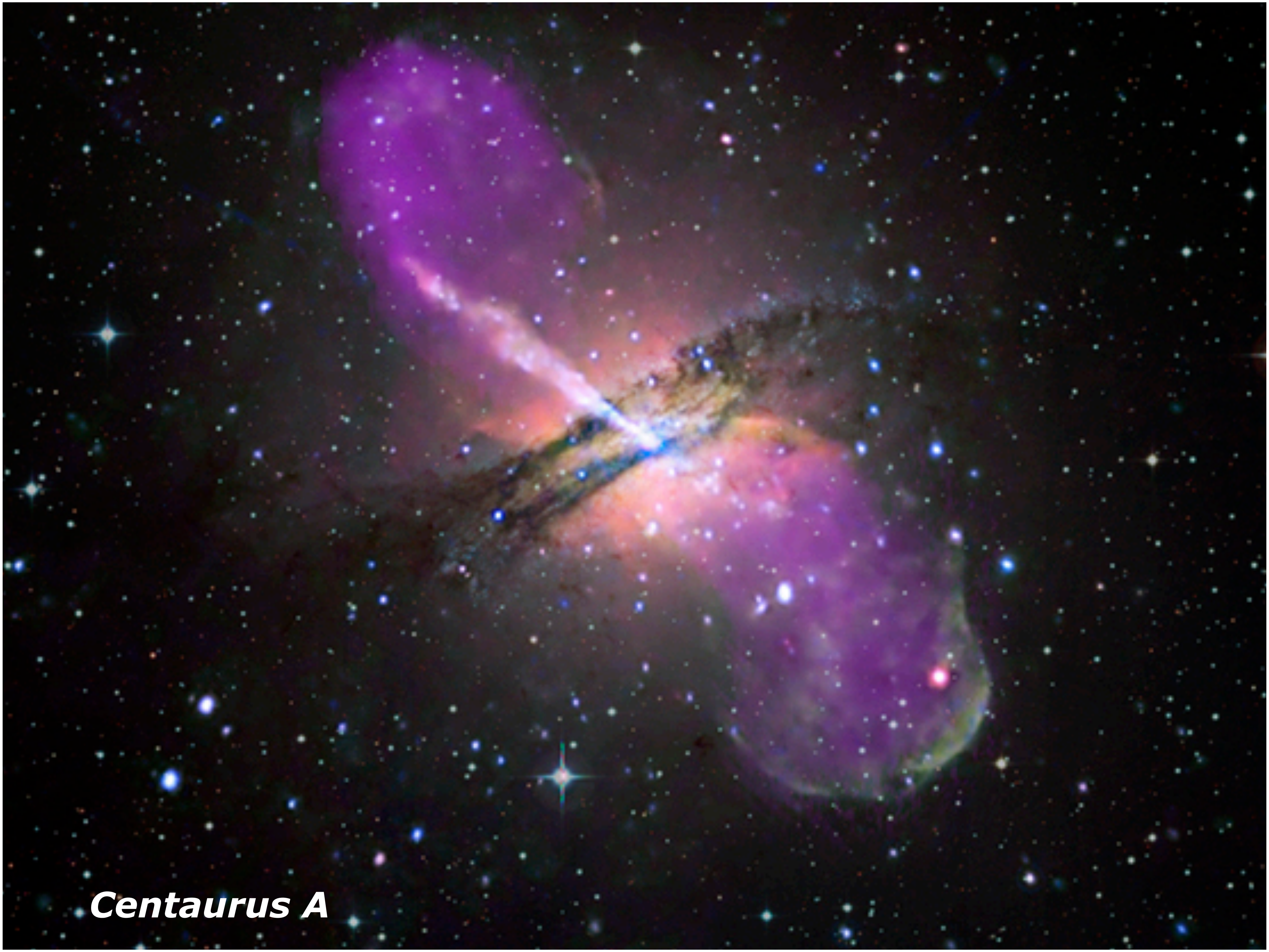
*Modeling the orbital motion of Sgr A**





Data showing rotation of hot spot- around SgrA* and model parameters
100μas=~10^{13}cm~ $10R_s$


M1
y (μas)
80
60
40
20
0
-20
-40
-80
-60
-40
-20
0
20
40
x (μas)


Centaurus A