Homework 1 Solution AST422

(a)

$$Assume: P(z) = L(z) = const$$
$$\rho(z) = const \Rightarrow n(z) = const, assume, m = const$$

1.

$$Euclidian: z$$
$$S(r) = \frac{L(r)}{4\pi r^2} \propto \frac{1}{r^2}$$
$$N(>S) = \int_0^r n(r) 4\pi r^2 dr = 4\pi n \cdot \frac{1}{3} r^3 = \frac{4\pi n}{3} (\frac{L}{4\pi S})^{3/2} \propto S^{-3/2}$$

Discussion:

$$simply, assume:$$
$$L(r) \sim r^{-\alpha}, n(r) \sim r^{-\beta}$$
$$S(r) = \frac{L(r)}{4\pi r^2} \propto \frac{1}{r^{2+\alpha}}$$

1)

$$2 + \alpha > 0:$$
$$r \uparrow, L(r) \uparrow, or \downarrow slower, than, \frac{1}{r^2}, S(r) \downarrow$$
$$N(>S) = \int_0^r n(r) 4\pi r^2 dr \sim \int_0^r r^{-\beta} r^2 dr \sim \frac{1}{3-\beta} r^{3-\beta} \Big|_\varepsilon^r \sim \frac{1}{3-\beta} S^{-\frac{3-\beta}{2+\alpha}}$$
$$3 - \beta > 0:$$
$$r \uparrow, n(r) \uparrow, or \downarrow slower, than, \frac{1}{r^3}, N(>S) \uparrow$$
$$S \uparrow, N(>S) \downarrow$$
$$3 - \beta < 0:$$
$$r \uparrow, n(r) \downarrow faster, than, \frac{1}{r^3}, N(>S) \uparrow$$
$$S \uparrow, N(>S) \downarrow$$

2)

$2+\alpha<0$:

$r\uparrow, L(r)\uparrow\ faster, than, r^2, S(r)\uparrow$

$$N(>S)=\int_r^\infty n(r)4\pi r^2 dr \sim \int_r^\infty r^{-\beta} r^2 dr \sim \frac{1}{3-\beta} r^{3-\beta}\Big|_r^\infty \sim \frac{1}{3-\beta} S^{-\frac{3-\beta}{2+\alpha}}\Big|_S^\infty$$

$3-\beta>0$:

$r\uparrow, n(r)\uparrow, or\downarrow\ slower, than, \frac{1}{r^3}, N(>S)\uparrow$

$S\uparrow, N(>S)\downarrow$

$3-\beta<0$:

$r\uparrow, n(r)\downarrow\ faster, than, \frac{1}{r^3}, N(>S)\uparrow$

$S\uparrow, N(>S)\downarrow$

Other assumptions:
If not Euclidian space, geometry f~1/r^2 no longer hold.
Expanding/Contracting universe, the light would be redshifted/blueshifted.
Finite age of the universe changes the integration upper limit to horizon distance.

(b)

$N(>S)\propto S^{-\gamma}$

$$n(S)=\frac{dN(>S)}{dS}\propto S^{-\gamma-1}$$

$$B=\int_0^\infty n(S)SdS \propto \int_0^\infty S^{-\gamma-1}SdS \propto \frac{1}{1-\gamma} S^{-\gamma+1}\Big|_0^\infty$$

$1-\gamma>0, i.e., \gamma<1, diverge, for, bright, fluxes$

$1-\gamma<0, i.e., \gamma>1, diverge, for, faint, fluxes$

$\gamma=1, B=\ln S\Big|_0^\infty \to \infty$

$for, finite, upper\ limit, of, S, in, reality, observation,$

$\Rightarrow \gamma>1, infinite$

$\Rightarrow \gamma<1, finite$

(c)

1)

$$m = -2.5\log S + C$$

$$\Rightarrow S = C \cdot 10^{-0.4m}$$

$$from,(a)$$

$$N(<m) = N(>S) \propto S^{-3/2} \propto 10^{-3/2\cdot(-0.4m)} \propto 10^{0.6m}$$

2)

$$for, N(<m) \propto 10^{\alpha m}$$

$$B = \int_{-\infty}^{\infty} n(m)S(m)dm$$

$$n(m) = \frac{dN(<m)}{dm} \propto 10^{\alpha m}$$

$$S(m) \propto 10^{-0.4m}$$

$$\Rightarrow B = \int_{-\infty}^{\infty} n(m)S(m)dm \propto \int_{-\infty}^{\infty} 10^{-0.4m} \cdot 10^{\alpha m}\, dm \propto \int_{-\infty}^{\infty} 10^{(\alpha-0.4)m}\, dm \propto 10^{(\alpha-0.4)m}\Big|_{-\infty}^{+\infty}$$

$$similarly, constrain, the, bright, end$$

$$\alpha > 0.4, \inf inite$$

$$\alpha < 0.4, finite$$

3)

dark night, finite age of the universe