Timothy ... zen   AST 322   1215293924   HW 3

3/3   3.1. $(\Delta s')^2 = -c^2(\Delta t')^2 + (\Delta \ell')^2$ ✓

$= -\gamma^2 [c(t_1 - t_2) - \frac{v}{c}(x_1 - x_2)]^2 + \gamma^2 [x_1 - x_2$ ✓

$v(t_1 - t_2)]^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$ (3.20 is proven below)

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ ✓   Then, ignoring the y and z terms for now,

$(\Delta s')^2 = \frac{-1}{1 - \frac{v^2}{c^2}} \left(c(t_1 - t_2) - \frac{v}{c}(x_1 - x_2) \right)^2$

$+ \frac{1}{1 - \frac{v^2}{c^2}} \left(x_1 - x_2 - v(t_1 - t_2) \right)^2 \ldots$

let $x_1 - x_2 = \Delta x$ and $t_1 - t_2 = \Delta t$. The equation becomes

$(\Delta s')^2 = \frac{-1}{1 - \frac{v^2}{c^2}} \left(c^2 \Delta t^2 + \frac{v^2}{c^2} \Delta x^2 - 2v \Delta t \Delta x \right) + \frac{1}{1 - \frac{v^2}{c^2}} \left(\Delta x^2 + v^2 \Delta t^2 - 2 v \Delta x \Delta t \right)$

$= \frac{1}{1 - \frac{v^2}{c^2}} \left(\Delta x^2 + v^2 \Delta t^2 - c^2 \Delta t^2 - \frac{v^2}{c^2} \Delta x^2 \right) \ldots$

$= \frac{1}{1 - \frac{v^2}{c^2}} \Delta x^2 \left(1 - \frac{v^2}{c^2} \right) - \frac{1}{1 - \frac{v^2}{c^2}} c^2 \Delta t^2 \left(1 - \frac{v^2}{c^2} \right)$

$= -c^2 \Delta t^2 + \Delta x^2 \ldots$

$= -c^2 (t_1 - t_2)^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 = (\Delta s)^2$ ✓

so that $(\Delta s')^2 = (\Delta s)^2$ and the 3.21 is shown from 3.20.

In showing 3.20, $+ c^2(\Delta t')^2 + (\Delta \ell')^2$

$= -\gamma^2 [c(t_1 - t_2) - \frac{v}{c}(x_1 - x_2)]^2 + \gamma^2 [x_1 - x_2 - v(t_1 - t_2)]^2$

$+ (y_1 - y_2)^2 + (z_1 - z_2)^2,$

Note that $x' = \gamma(x - vt)$, $y' = y$, $z' = z$, and the

$t(t - vx/c^2)$ (3.11)   so that

$(\Delta \ell')^2 = \gamma^2 [x_1 - x_2 - v(t_1 - t_2)]^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$ ✓

(3.15) and

$$\Delta t' = t_1' - t_2' = \gamma\left[(t_1 - t_2) - \frac{v}{c^2}(x_1 - x_2)\right] \quad (3\text{-}16) \checkmark$$

Subsituting these values:

$$(\Delta s')^2 = -c^2(\Delta t')^2 + (\Delta L')^2 = -c^2(t_1' - t_2')^2 + (x_1' - x_2')^2$$
$$+ (y_1' - y_2')^2 + (z_1' - z_2')^2$$
$$= -\gamma^2\left[c(t_1 - t_2) - \frac{v}{c}(x_1 - x_2)\right]^2 + \gamma^2\left[x_1 - x_2 - v(t_1 - t_2)\right]^2$$
$$+ (y_1 - y_2)^2 + (z_1 - z_2)^2 \checkmark$$

and 3.20 is shown, which 3.21 the follows as shown before.

2/2 **3.2.** For a box length $2M$, the time light takes to travel is $\Delta t = \frac{d}{v} = \frac{d}{c} = \frac{2m}{3\times10^8 m/s}$

With $a = -9.8 \, m/s^2$, during Δt the light travels downwards

$$\Delta x = \frac{1}{2}at^2 \checkmark = \frac{1}{2}g\frac{d^2}{c^2} = \frac{1}{2}(9.8)\left(\frac{2}{3\times10^8}\right)^2 \approx 2\times10^{-16} \, m \checkmark$$

and shown

5/5 **3.3.** If $dl^2 = dr^2 + S_k(r)^2 d\Omega^2$, show this equals $dl^2 = \frac{d\bar{x}^2}{1-k\bar{x}^2/R^2} + \bar{x}^2 d\Omega^2$ where $\bar{x} \equiv S_k(r)$

and $S_k(r) = \begin{cases} R\sin(r/R) & (k=+1) \\ r & (k=0) \\ R\sinh(r/R) & (k=-1) \end{cases}$

For $k=1$, using the second equation:

$$d\bar{x} = \cos\left(\frac{r}{R}\right)dr \Rightarrow dl^2 = \frac{\cos^2\left(\frac{r}{R}\right)dr^2}{1-\sin^2\left(\frac{r}{R}\right)} + \bar{x}^2 d\Omega^2 = \frac{\cos^2\left(\frac{r}{R}\right)dr^2 + \bar{x}^2 d\Omega^2}{\cos^2\left(\frac{r}{R}\right)}$$

$$= dr^2 + \bar{x}^2 d\Omega^2 = dr^2 + S_k(r)^2 d\Omega^2 \checkmark \text{ and the two equations are equal}$$

For $k=0$, $dl^2 = \frac{dr^2}{1-0\cdot\bar{x}^2/R^2} + \bar{x}^2 d\Omega^2 = dr^2 + S_k(r)^2 d\Omega^2 \checkmark$

and are equal)

For $k=+1$, $d\bar{x} = \cosh\left(\frac{r}{R}\right)dr \Rightarrow dl^2 = \frac{\cosh^2\left(\frac{r}{R}\right)dr^2}{1+\sinh^2\left(\frac{r}{R}\right)} + \bar{x}^2 d\Omega^2$

$$= \frac{\cosh^2\left(\frac{r}{R}\right)}{\cosh^2\left(\frac{r}{R}\right)}dr^2 + \bar{x}^2 d\Omega^2 = dr^2 + S_k(r)^2 d\Omega^2 \checkmark \text{ and are equal}$$