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1. From Neurons to Networks

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From Neurons to Networks

biological neurons (very brief)
- single neurons
- synapses and networks
- synaptic plasticity and learning

simplified description
- modelling of biological neurons/networks
- **inspiration for artificial neural networks**

artificial neural networks
- unrealistic simplifications
- architectures and types of networks:
  recurrent **attractor neural networks** (associative memory)
  Example: **The Hopfield model**
  feed-forward neural networks (classification/ regression)

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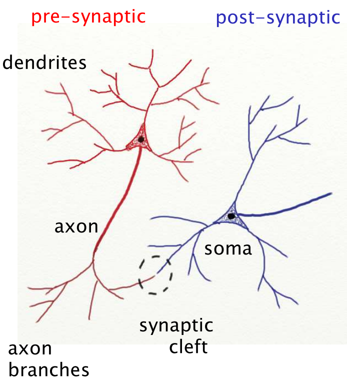
From Neurons to Networks



neurons:
highly specialized cells
- cell body **soma**
- incoming **dendrites**
- branched **axon**

many neurons !
$\gtrsim 10^{12}$ in the brain

highly inter-connected !
$\gtrsim 1000$ *neighbors*

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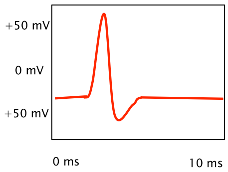
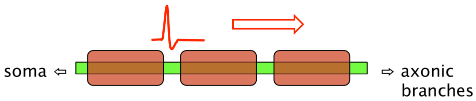
From Neurons to Networks

action potentials / spikes:



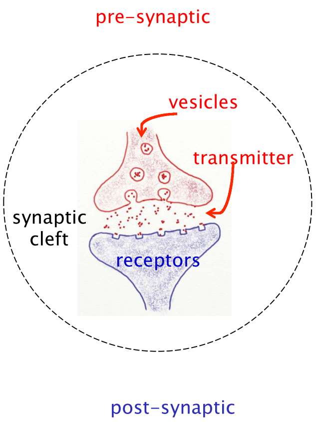
- cells generate localized electric pulses
  membrane is charged / discharged
  by means of ion transport
- class of models (electro-chemistry):
  Hodgkin-Huxley neurons + extensions
- signals travel along the axon
  (serves as *cable*)



From Neurons to Networks

synapse:

- arriving pre-synaptic spike triggers release of **neuro-transmitters**
- different kinds of synapses play specific roles:
- pre-synaptic pulse at **excitatory** /**inhibitory** synapse triggers / hinders post-synaptic spike generation



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- incoming pulses { increase (excitatory) / decrease (inhibitory) } membrane potential

- all or nothing response

potential exceeds **threshold** ⇨ postsynaptic neuron *fires*

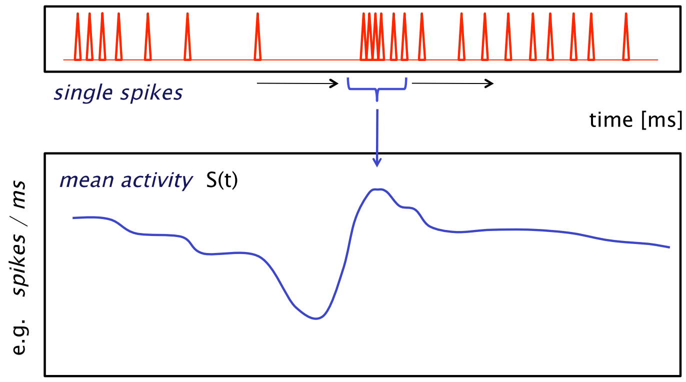
potential is sub-**threshold** ⇨ **postsynaptic neuron** *rests*

- important class of models:
  Integrate-and-Fire neurons and networks
  - no details of spike generation and signal propagation
  - neuron as simple threshold element,
  - excitatory and inhibitory connections with other units

(see additional material)

From Neurons to Networks

simplified description of neural activity: **firing rates**



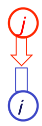
From Neurons to Networks

(mean) **local potential** at neuron i (with activity S_i)

$$\sum_j w_{ij} S_j$$ **weighted sum** of incoming activities

synaptic weights

$$w_{ij} = \begin{cases} > 0 & \textbf{excitatory} \text{ synapse} \\ = 0 & \text{no incoming synapse} \\ < 0 & \textbf{inhibitory} \text{ synapse} \end{cases}$$



Note: $w_{ii} = 0, \quad w_{ij} \neq w_{ji}$ (in general)

connections *i→j and j→i* are unrelated.

Activation Function

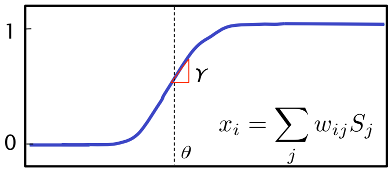
non-linear response: $S_i \;=\; h\left[\sum_j w_{ij} S_j\right]$

- minimal activity $\quad h(x\to -\infty) \equiv 0$
- maximal activity $\quad h(x\to +\infty) \equiv 1$
- monotonic increase $\quad h'(x) > 0$

important class of fcts.: ***sigmoidal activation***

just one example: $h(x_i) = \frac{1}{2}\Big(1 + \tanh\left[\gamma(x_i - \theta)\right]\Big)$

gain parameter γ
local threshold θ
Remark:
do not confuse θ with threshold for single spike generation!



Symmetrization (I)

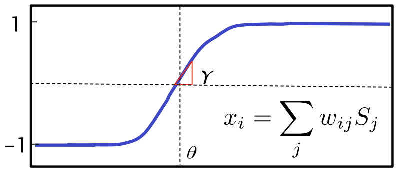
non-linear response: $S_i \;=\; g\left[\sum_j w_{ij} S_j\right]$

- minimal activity $\quad g(x\to -\infty) \equiv -1$
- maximal activity $\quad g(x\to +\infty) \equiv 1$
- monotonic increase $\quad g'(x) > 0$

sigmoidal activation

just one example: $g(x_i) = \tanh\left[\gamma(x_i - \theta)\right]$

gain parameter γ
local threshold θ



Symmetrization (I)

$$-1 \;\leq\; S_i \;=\; g\left[\sum_j w_{ij} S_j\right] \;\leq +1$$

$S_j > 0, w_{ij} > 0$	**excitation**	$S_j < 0, w_{ij} < 0$
$S_j > 0, w_{ij} < 0$	**inhibition**	$S_j < 0, w_{ij} > 0$
(plausible)		**(unrealistic)**

symmetric treatment of low / high activity

- not suitable for realistic modelling of biological neurons, absence of spikes cannot excite activity etc. (further consequences discussed below)
- convenient in bio-inspired **artificial neural networks** (more drastic simplifications to come)

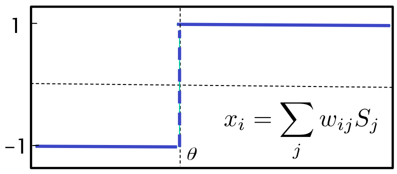
McCulloch Pitts Neurons

an extreme case: **infinite gain** $\gamma\to\infty$

$$g(x_i) = \tanh\left[\gamma(x_i - \theta)\right] \;\to\; \text{sign}\left[x - \theta\right] \;=\; \begin{cases} +1 & \text{for } x \geq \theta \\ -1 & \text{for } x < \theta \end{cases}$$

McCulloch Pitts [1943]:
model neuron is either quiescent or maximally active
do not consider graded response

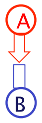
local threshold θ
Remark:
this is not the all-or-nothing response in single spike generation!



Synaptic Plasticity

D. Hebb [1949]

consider
- presynaptic neuron A
- postsynaptic neuron B
- excitatory synapse w_{BA}



Hypothesis: **Hebbian Learning**

If A and B (frequently) fire at the same time
the excitatory synaptic strength w_{BA} increases

→ memory-effect will favor joint activity in the future
high activity of A will trigger high activity at B
correlations of activity stored in the network

Symmetrization (II)

$$-1 \leq S_A, S_B \leq +1$$

change of synaptic strength $\Delta w_{BA} \propto S_A S_B$

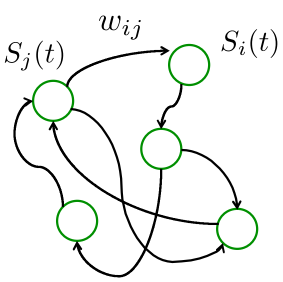
beyond Hebb's hypothesis: **symmetric treatment**
- of high/low activity
- of pre- postsynaptic activity

S_A	S_B		
high	high	strengthens	excitatory
low	low	weakens	inhibitory
high	low	weakens	excitatory
low	high	strenghtens	inhibitory

synapse w_{BA}

Artificial Neural Networks

in the following:
- assembled from simple *firing rate neurons*
- connected by *weights,* real valued synaptic strenghts
- various architectures and types of networks
  e.g.: **attractor neural networks, feed-forward networks**

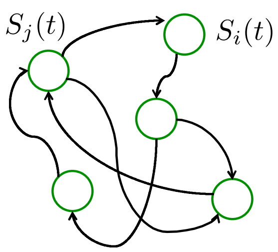


here:
number of neurons N=5
partial connectivity:
7 directed synapses

Attractor Neural Networks

training phase:
dynamics of weights (synapses), given the network activity

$$w_{ij}(t+\Delta t) = w_{ij} + \eta S_i(t) S_j(t)$$ **(Hebbian Learning)**

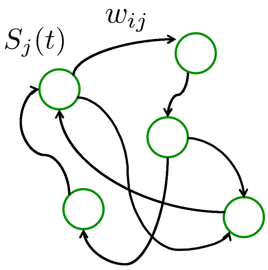


for example:
sequence of **activity patterns,**
memorization of frequent states,
external stimuli etc.

$$\mathbf{S}(t) \in \{\boldsymbol{\xi}^\mu\}_{\mu=1}^P$$

Attractor Neural Networks

working phase:
dynamics of neurons, given the weights



$$S_i(t+\Delta t) = g\left[\sum_j w_{ij} S_j(t)\right]$$

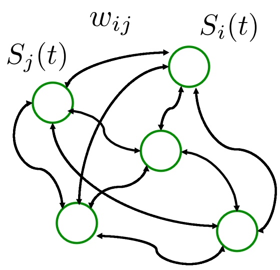
variants:
- sequential update *i=1,2,3,...*
- random sequential update
- parallel update

retrieval of patterns: from incomplete / noisy state $\mathbf{S}(0)$
approach original pattern $\mathbf{S}(t\rightarrow\infty) = \boldsymbol{\xi}^{\nu}$

Attractor Neural Networks

extreme example: **Little-Hopfield model**

- symmetric Hebbian interactions $w_{ij} = w_{ji} = \dfrac{1}{P}\sum_{\mu=1}^{P} \xi_i^{\mu}\,\xi_j^{\mu}$



- McCulloch Pitts neurons

$$S_i(t+\Delta t) = \text{sign}\left[\sum_j w_{ij} S_j(t)\right]$$

- parallel (Little)
  random sequential (Hopfield)
  dynamics of neurons

Attractor Neural Networks

consider a single *pattern* $\boldsymbol{\xi}^1$ $w_{ij} \propto \xi_i^1 \xi_j^1$

show: $\mathbf{S}(t) = \boldsymbol{\xi}^1$ is reproduced under the dynamics!

$$S_i(t+\Delta t) = \text{sign}\left(\sum_j w_{ij}\xi_j^1\right) = \text{sign}\left(\xi_i^1 \sum_j \underbrace{\xi_j^1 \xi_j^1}_{>0}\right) = \xi_i^1$$

fixed point: unchanged under system dynamics

(?) stable fixed point: small deviations from patterns vanish
(?) attractor: retrieved from similar initial states

much more difficult to show: set of random (zero mean) patterns

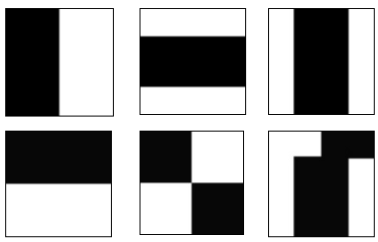
$\{\boldsymbol{\xi}^{\mu}\}_{\mu=1}^{P}$ with $P = \alpha N$ is retrieved, if α is not too big

N neurons (with ~N^2 weights) *memorize* ~*O(N)* activity patterns

Demo: Associative Memory

- consider a **Hopfield network** of 2500 neurons
- visualized as array of 50*50 neurons
  note: there is no spatial structure imposed, every neuron is connected with every other neuron!

training patterns:



+ (994) random patterns with activity +/- 1 with equal probability

saved as 'pattern.mat'

Demo: Associative Memory

- calculate synaptic matrices saved in jmats.mat (load by `load jmats.mat`)

$$w_{ij} = w_{ji} = \frac{1}{P} \sum_{\mu=1}^{P} \xi_i^\mu \, \xi_j^\mu$$

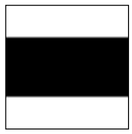
P=10 (jmat10)
P=100 (jmat100)
P=1000 (jmat1000)

hopf (jmat, inipat, q, stps) e.g. hopf(jmat10,1,0.4,100)

- jmat synaptic matrix to be used
- inipat index of pattern used as initial state
- q fraction of flipped spins in initial state
- stps number of steps per neuron in the system

Demo: Associative Memory

regime of low 'memory load' (here: *P=10*)



noisy initial state dynamics (almost) perfect retrieval

- distributed memory
- robust system!

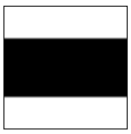
one can show: set of random (zero mean) patterns

$\{\xi^\mu\}_{\mu=1}^{P}$ with $P = \alpha N$ is retrieved *successfully* for $\alpha \leq 0.14$

N neurons (with ~N^2 weights) *memorize* ~$\mathcal{O}(N)$ activity patterns

Demo: Associative Memory

intermediate load (e.g. *P=100*)



noisy initial state dynamics noise dependent retrieval state potential 'confusion'

- distributed memory
- robust system!

one can show: besides the original patterns, 'spurious states' are also stabilized, e.g. mixtures of different patterns

Demo: Associative Memory

overloaded memory (e.g. *P=1000*)

?

noisy initial state dynamics no retrieval patterns unstable

- distributed memory
- robust system!

Remarks

- the above is, strictly speaking, only true for 'uncorrelated patterns'
- one can improve the Hopfield network by *decorrelating,* i.e.

$$w_{ij} = \frac{1}{P} \sum_{\mu=1}^{P} \left(\xi_i^\mu - \langle \xi_i^\mu \rangle\right) \left(\xi_i^\mu - \langle \xi_j^\mu \rangle\right)$$

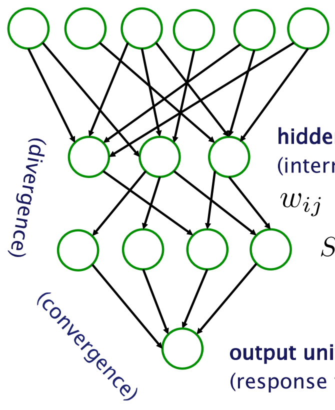
- improved training prescriptions:
  iterative learning, e.g. perceptron algorithm

- Hopfield model serves as a basic model of learning and retrieval
- other 'recurrent networks' are used in applications
- more sophisticated and detailed models for biological modelling

- different architectures, e.g.
  'Feed-forward' networks for regression/classification tasks

Feed-Forward Networks

input layer (external stimulus)



layered architecture
(here: 6-3-4-1)

directed connections
(here: only to next layer)

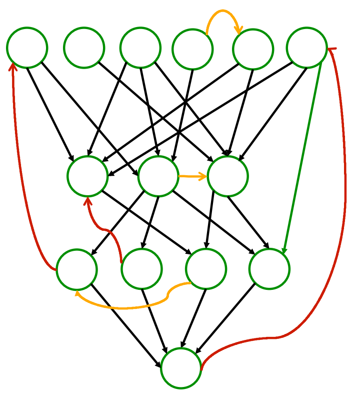
hidden units
(internal representation)

w_{ij}

$$S_i = g\left(\sum_j w_{ij} S_j\right)$$

output unit(s)
(response to the stimulus)

Feed-Forward Networks



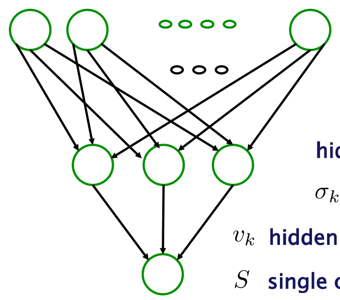
modifications:

(still) feed-forward weights
input/output relation

intra-layer connections
internal dynamics

feed-back connections
recurrent networks,
intermediate connectivity

Feed-Forward Networks



input units $\xi_j \in I\!R$

$w_j^{(k)}$ **input to hidden weights**

hidden layer units

$\sigma_k = g\left(\sum_j w_j^{(k)} \xi_j\right)$

v_k **hidden to output weights**

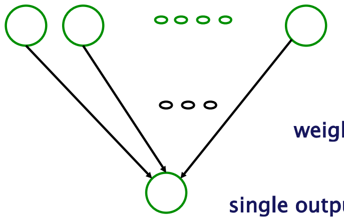
S **single output unit**

output = **non-linear function** of input variables:

$$S = g_{out}\left(\sum_{k=1}^{K} v_k \sigma_k\right) = g_{out}\left(\sum_{k=1}^{K} v_k \, g\left(\sum_j w_j^{(k)} \xi_j\right)\right)$$

parameterized by set of all weights (and threshold)

The Perceptron



input units $\xi_j \in \mathbb{R}$

weights $w_j \in \mathbb{R}, \mathbf{w} \in \mathbb{R}^N$

single output unit

$$S = \operatorname{sign}\left(\sum_{j=1}^{N} w_j \xi_j - \theta\right)$$

output = ***"linear separable function"*** of input variables

parameterized by the weight vector $\mathbf{w}$ and threshold θ

Feed-Forward Networks

training phase:
adaptation of parameters
based on a set of example data $D = \{\boldsymbol{\xi}^\mu, S^\mu\}_{\mu=1}^{P}$

working phase
generalization, i.e. appl. of hypothesis to novel input $\boldsymbol{\xi} \to S$

possible aims:

classification: S is a discrete label (e.g. $S \in \{1, 2, \ldots, C\}$)
which assigns vectors ξ to one of **C** classes

regression: S is a continuous response (e.g $S \in \mathbb{R}$)
which describes a property of ξ quantitatively

Feed-Forward Networks

supervised learning (e.g. in feedforward networks)

classification / **regression** problems

- which are difficult to formulate as a simple s*et of rules* (unknown or too complex)
- for which it is relatively simple to obtain *example data* (by observation or consulting an expert)

just a few examples:

- handwritten character (digit) recognition
- prediction of protein structure from amino acid sequences
- analysis of spectra (e.g. infrared spectra -> fat content of meat)
- prediction of customer interests (e.g. at amazon.com)
- adaptive engine control (driver dependent)
- medical data/image analysis (diagnosis, risk prediction etc.)
- fault identification in technical processes

. . . many more . . .