pumping lemma continued

Tuesday, October 8, 2024   12:02 PM

ex show $L = \{w \in \{a,b\}^* : n_a(w) < n_b(w)\}$ is not regular.

Assume L is regular $\Rightarrow \forall s \in L$, $|s| \geq p$ (pos int), $s = xyz$ where $|xy| \leq p$, $|y| \geq 1$ $\Rightarrow$ $s_i = xy^iz, \forall i \geq 0$ $s_i \in L$.

let $s = a^p b^{p+1}$

$\cancel{s = b^p}$
$\cancel{s = ab^p}$
$\boxed{s = ab^{p+1}}$

since $|xy| \leq p$, $|y| \geq 1$ $\Rightarrow$

$y = a^k \quad 1 \leq k \leq p$

$s_u = \underbrace{a^k}_{y} \underbrace{a^{p-k} b^{p+1}}_{!y}$

$s_i = a^{ki} a^{p-k} b^{p+1}$

let $i = 2$

$s_2 = a^{2k} a^{p-k} b^{p+1}$

$= a^{p+k} b^{p+1}$

$\Rightarrow$ since $k \geq 1 \Rightarrow n_a(s_2) \geq n_b(s_2)$

$\therefore s_2 \notin L \Rightarrow L$ is not regular!!

aside [wrong]:

let $i = 0$

$s_0 = \cancel{a^{k \cdot 0}} a^{p-k} b^{p+1}$

$= a^{p-k} b^{p+1}$

$\Rightarrow n_a(s_0) < n_b(s_0)$

$s_0 \in L$

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$y = a^k \quad 1 \leq k \leq p$
$x = a^m \quad 0 \leq m \leq p-k$
$z = a^{p-k-m} b^{p+1}$

$s = \underbrace{a^m}_{x} \underbrace{a^k}_{y} \underbrace{a^{p-k-m} b^{p+1}}_{z}$

$s_i = a^m a^{ki} a^{p-k-m} b^{p+1} = a^{ki} a^{p-k-m+m} b^{p+1} = a^{ki} a^{p-k} b^{p+1}$

$\Rightarrow n_a(s_i) = a^{ki+p-k}$
$n_b(s_i) = b^{p+1}$

let $i = 2$
$n_a(s_2) = a^{2k+p-k} = a^{p+k}$
$n_b(s_2) = b^{p+1}$

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show $L = \{ww^R : w \in \{a,b\}^*\}$ is not regular.

Assume L is regular $\Rightarrow \forall s \in L$, $|s| \geq p$ (pos int) $s = xyz \Rightarrow |xy| \leq p$, $|y| \geq 1$ and $s_i = xy^iz \in L$ $\forall i \geq 0$.

~~let $s_T = a^p a^p$~~ $\quad s \in L$, $|s| = 2p \geq p$ ✓

~~$y = a^k \quad 1 \leq k \leq p$~~

~~$s_u = a^k a^{p-k} a^p$~~

~~$s_i = a^k a^{p-k} a^p$~~

~~let $i = 0$~~

~~$s_0 = a^{p-k} a^p$~~

$a^p \to a^{p-2} \qquad a^p$

$\cancel{a}aaa\ldots a \,|\, aaaa\ldots a$

$\underbrace{\qquad}_{a^{p-1}} \quad \underbrace{\qquad}_{a^{p-1}}$

$\in L$

let $s_L = a^p bba^p \qquad s \in L$, $|s| = 2p+2 \geq p$ ✓

$y = a^k \quad 1 \leq k \leq p$ since $|xy| \leq p$, $|y| \geq 1$

$s_u = \underbrace{a^k}_{y} \underbrace{a^{p-k} bba^p}_{!y}$

$s_i = \cancel{a^{ki}}^{\,0} a^{p-k} bba^p$

let $i = 0$

$s_0 = \underbrace{a^{p-k} b}_{w} \underbrace{ba^p}_{w_R}$ $\quad$ since $k \geq 1 \Rightarrow n_a(w) \neq n_a(w^R)$

$\therefore s_0 \notin L$.

$\therefore L$ is not regular.

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ex prove $L = \{a^n b^m : n \neq m\}$ is not regular

⋮

let $s = a^{p!} b^{(p+1)!}$

target! $n_a(s_i) = n_b(s_i) \therefore s_i \notin L$