

1. We are training a neuron using the perceptron algorithm. We have one feature,  $x_1$ . For some sample,  $s$ , with label  $y = 1$ , we get  $a < 0$ . Using the update rules for the perceptron, show that we will do better on sample  $s$  after we update  $w$  and  $b$ . (3 points)

$$\cancel{w_{\text{old}}} \quad w_{\text{new}} = w_{\text{old}} + y \cdot x_1 = w_{\text{old}} + x_1$$

$$b_{\text{new}} = b_{\text{old}} + y = b_{\text{old}} + 1$$

$$a_{\text{old}} = w_{\text{old}} x_1 + b_{\text{old}}$$

$$a_{\text{new}} = w_{\text{new}} x_1 + b_{\text{new}} = (w_{\text{old}} + x_1) x_1 + b_{\text{old}} + 1 = w_{\text{old}} x_1 + b_{\text{old}} + x_1^2 + 1$$

2. Given a neuron with parameters  $w = [0, 1, 1]$  and  $b = 1$ , which of the following points are on the decision boundary for the neuron? How do you know? (3 points)

- $p_1 = (1, 2, 3)$   
 $p_2 = (1, 2, -3)$   
 $p_3 = (8, -1, 0)$   
 $p_4 = (2, 6, -8)$   
 $p_5 = (3, -2, 1)$

$$w \cdot p_1 = 0 + 2 + 3 = 5 + b = 6 > 0 \quad \text{X}$$

$$w \cdot p_2 = 0 + 2 - 3 + b = 0 \quad \checkmark$$

$$w \cdot p_3 + b = 0 - 1 + 0 + 1 = 0 \quad \checkmark$$

$$w \cdot p_4 + b = 0 + 6 - 8 + 1 = -1 < 0 \quad \text{X}$$

$$w \cdot p_5 + b = 0 + -2 + 1 + 1 = 0 \quad \checkmark$$

3. Using the training data below, train a neuron using the perceptron algorithm for two epochs. Use initial  $w = [0, 0]$  and  $b = 0$ . Give the final  $w$  and  $b$ . Iterate through the points in the order presented above. What are the  $w$  and  $b$  after two epochs? (4 points)

below

Sample	$x_1$	$x_2$	Label
$s_1$	0	0	-1
$s_2$	1	1	1
$s_3$	0	1	-1
$s_4$	2	2	1
$s_5$	1	0	-1
$s_6$	1	2	1

Epoch 1

$$a_1 = 0 + 0 + 0 = 0 \leq 0 \quad \text{update}$$

$$w_1 = 0 + 0 \cdot -1 = 0 \quad b = 0 - 1 = -1$$

$$w_2 = 0 + 0 \cdot -1 = 0$$

$$a_2 = 0 \cdot 1 + 0 \cdot 1 - 1 = -1 \leq 0 \quad \text{update}$$

$$w_1 = 0 + 1 \cdot 1 = 1 \quad w_2 = 0 + 1 \cdot 1 = 1$$

$$b = 1 - 1 = 0$$

$$a_3 = 1 \cdot 0 + 1 \cdot 1 + 0 = 1 \cdot -1 \leq 0 \quad \text{update}$$

$$w_1 = 1 + 0 = 1 \quad w_2 = 1 + 1 \cdot -1 = 0$$

$$b = 0 - 1 = -1$$

$$a_4 = 1 \cdot 2 + 0 \cdot 2 - 1 = 1 \cdot 1 > 0 \quad \checkmark$$

$$a_5 = 1 \cdot 1 + 0 \cdot 0 - 1 = 0 \leq 0 \quad \text{update}$$

$$w_1 = 1 + -1 = 0 \quad w_2 = 0 + 0 = 0 \quad b = -2$$

$$a_6 = 0 \cdot 1 + 0 \cdot 2 - 2 = -2 \leq 0 \quad \text{update}$$

$a_{\text{new}} > a_{\text{old}}$   
 So we will do better on  $s$  next time.

$> a_{\text{old}}$

activation is zero  $\sum w_i x_i + b$

Epoch 2

$a_1 = 0 + 0 - 1 = -1 \leq 0$  update  
 $w_1 = 0 + 0 \cdot -1 = 0$   
 $w_2 = 0 + 0 \cdot -1 = 0$   
 $b = -1 - 1 = -2$

$a_2 = 0 + 1 - 2 = -1 \leq 0$  update  
 $w_1 = 0 + 1 \cdot 1 = 1$   
 $w_2 = 0 + 1 \cdot 1 = 1$   
 $b = -2$

$a_3 = 1 + 1 - 2 = 0 \leq 0$  update  
 $w_1 = 1 + 0 = 1$   
 $w_2 = 1 + 1 \cdot -1 = 0$   
 $b = -2$

$a_4 = 1 + 2 - 2 = 1 > 0$   $\checkmark$

$a_5 = 1 + 0 - 2 = -1 \leq 0$  update  
 $w_1 = 1 + -1 = 0$   
 $w_2 = 0 + 0 = 0$   
 $b = -2$

$a_6 = 0 + 2 - 2 = 0 \leq 0$  update  
 $w_1 = 0 + 0 = 0$   
 $w_2 = 0 + 2 = 2$   
 $b = -2$

1. A model trained with the perceptron algorithm is guaranteed to give you 100% accuracy on the training data. True or false. Circle one and explain (2 points)

Only on linearly separable data.

2. Given  $D$  features, how many parameters does our neuron have? Explain. (2 points)

$D+1$  one parameter per feature (one  $w_i$  for each  $x_i$ ) plus the bias term.

3. We have trained a neuron using the perceptron algorithm until convergence. Our resulting weights are  $w = [10, 5, -4, 3, 2, 0]$  and  $b = 0$ . Which feature,  $x_1, x_2, x_3, x_4, x_5, x_6$  would you say is the most important? Why? (2 points)

$x_1$ , because it has the highest weight after training.

4. Using the training data below, train a neuron using the perceptron algorithm for two epochs. Use initial  $w = [0, 0]$  and  $b = 0$ . Give the final  $w$  and  $b$ . Iterate through the points in the order presented above. Give the  $w$  and  $b$  after two epochs. (4 points)

Same as other Side.

Sample	$x_1$	$x_2$	Label
$s_1$	0	0	-1
$s_2$	1	1	1
$s_3$	0	1	-1
$s_4$	2	2	1
$s_5$	1	0	-1
$s_6$	1	2	1

Epoch 2  $w_1=1$   $w_2=2$   $b=-1$

$$a_1 = 0 \cdot 1 + 0 \cdot 2 - 1 = -1 < 0 \checkmark$$

$$a_2 = 1 \cdot 1 + 2 \cdot 1 - 1 = 2 > 0 \checkmark$$

$$a_3 = 1 \cdot 0 + 2 \cdot 1 - 1 = 1 > 0 \text{ update}$$

$$w_1 = 1 + 0 \cdot -1 = 1 \quad w_2 = 2 + 1 \cdot -1 = 1 \quad b = -1 - 1 = -2$$

$$a_4 = 1 \cdot 2 + 1 \cdot 2 - 2 = 2 > 0 \checkmark$$

$$a_5 = 1 \cdot 1 + 1 \cdot 0 - 2 = -1 < 0 \checkmark$$

$$a_6 = 1 \cdot 1 + 1 \cdot 2 - 2 = 1 > 0 \checkmark \text{ done.}$$

$$w = [1, 1] \quad b = -2 \checkmark$$