CS 691 Final Exam Name _____

Short Problems (20 points)

1. Let's say we have two polynomial feature maps: $\phi_1(x) = \{x, x^2\}$, and $\phi_2(x) = \{2x, 2x^2\}$. In general, is the margin we would attain using $\phi_2(x)$ greater, equal, or smaller, in comparison to the margin resulting from $\phi_1(x)$? Explain. (5 points)

2. Give one similarity and one difference between feature selection and PCA. (5 points)

3. True/False: We would expect the support vectors to remain the same in general as we move from a linear kernel to higher order polynomial kernels. Explain. (5 points)

4. Which of the following statements are true? No need to explain. (5 points)

   (a) Training a k-nearest neighbors classifier takes more computational time than applying it.

   (b) The more training examples, the more accurate the prediction of a k-nearest neighbors classifier.

   (c) k-nearest neighbors cannot be used for regression (to predict a real-value).

   (d) A k-nearest neighbors is sensitive to outliers.

   (e) K=1 in K-NN always give 100% accuracy on the training data.

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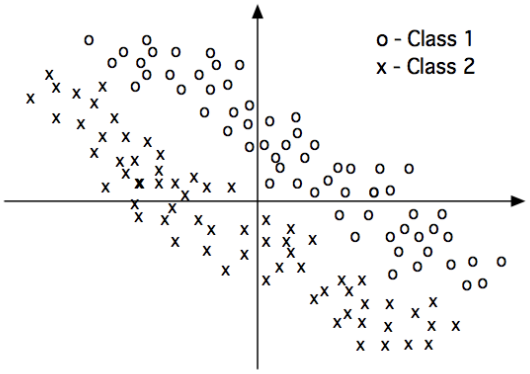
PCA (10 Points)

5. Explain how you could combine PCA with a supervised method we have covered this semester to classify the following data. Clearly state the steps you would take to combine the two methods and identify the particular supervised method you would use.



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Neural Networks (20 Points)

6. Consider two types of Neural Network activations:

   - linear: $h = w \cdot x + b$
   - hard threshold: $h = 1$ if $w \cdot x + b \geq 0$, and $h = 0$ otherwise.

   Which of the following functions can be exactly represented by a neural network with one hidden layer which uses linear and/or hard threshold activation functions? For each case, justify your answer.

   (a) Polynomials of degree one (5 points)

   (b) Hinge loss: $h(x) = max(1 - x, 0)$ (5 points)

   (c) Polynomials of degree two (5 points)

   (d) Piece-wise constant functions (5 points)

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Decision Boundaries (20 Points)

7. For each of the following situations, **indicate whether the classifier will produce a linear decision boundary always, sometimes or never. Give a short explanation of your answer.** In every case, assume that the input has two features x_1 and x_2, which have continuous values ranging from -1 to 1. For example, the value of x_1 is any real number between -1 and 1. And suppose the label, y has two possible values, 1 or -1. In all cases, assume there is plenty of training data (at least 100 examples for each class).

   (a) We build a decision tree with two levels. (3 points)

   (b) We build a decision tree with one level. (3 points)

   (c) We train a Perceptron. (3 points)

   (d) We use K-nearest neighbor, with K = 1. (3 points)

   (e) We train an SVM using a quadratic kernel. (4 points)

   (f) We use a neural network. The network has two input units, two hidden units and one output unit. We use ReLU in the hidden units (remember ReLU(x) = max(0,x)) and a sigmoid after the output unit. The input is classified as belonging to class 1 if the output is greater than $\frac{1}{2}$, and as class -1 if the output is less than $\frac{1}{2}$. (4 points)

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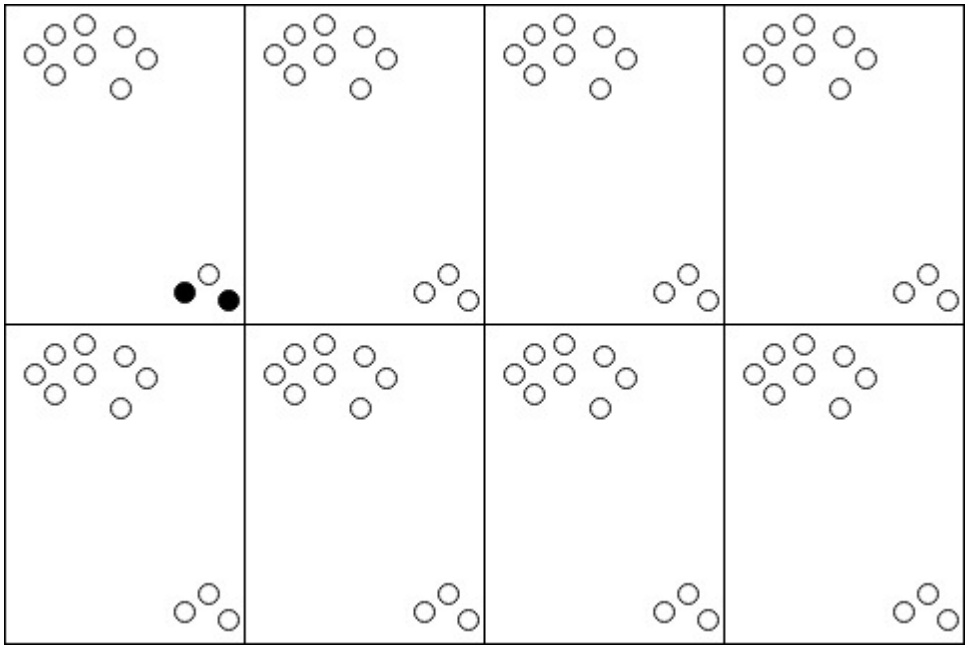
K-Means (10 Points)

8. Consider the following questions regarding K-Means.

   (a) Run K-Means manually for the following dataset. Circles are data points and the filled-in circles in the first panel are the initial cluster centers. Draw the cluster centers and the decision boundaries that define each cluster. Use a different panel for each iteration. Use as many panels as you need until convergence. (6 points)



   (b) True/False: It is possible that K-Means (K=2) gives the same decision boundary as 1-NN for a binary classification problem. If true, give an example. False, briefly explain. (4 points)

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SVM (10 Points)

9. In class we learned that SVM can be used to classify linearly inseparable data by transforming it to a higher dimensional space with a kernel $K(x, z) = \phi(x) \cdot \phi(z)$ where $\phi(x)$ is a feature mapping. Let K_1 and K_2 be kernels, an c be a positive constant. Let ϕ_1 be the mapping for K_1 and ϕ_2 be the mapping for K_2. Explain how to use ϕ_1 and ϕ_2 to obtain the following kernels. That is, give me the ϕ in terms of ϕ_1 and ϕ_2 that is the feature mapping for K.

   (a) $K(x, z) = cK_1(x, z)$

   (b) $K(x, z) = K_1(x, z)K_2(x, z)$

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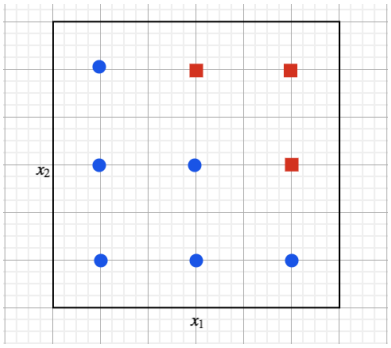
Ensembles (10 Points)

10. Recall that Adaboost learns a classifier f using a weighted sum of weak learners f_t as follows:

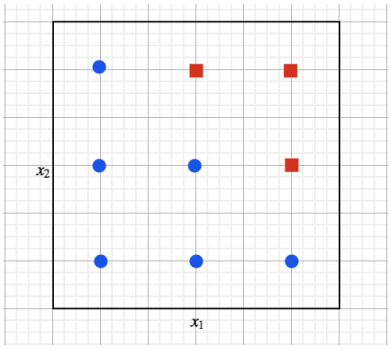
$$f(x) = sign(\sum_k \alpha_k f_k(x))$$

In this question, we will use decision trees as our weak learners, which classify a point as $\{1, -1\}$ based on a sequence of threshold splits on its features (x_1 and x_2). Red squares are negative points and blue circles are positive points. **In the questions below, be sure to mark which regions are classified positive/negative.** Assume that our weak learners are decision trees of maximum depth 2, which minimize the weighted training error. That is, the set of classifiers f_k are all depth-1 or depth-2 decision trees with minimum training set error.

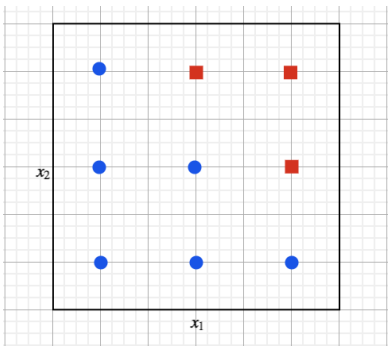
(a) Using the dataset below, draw the decision boundary learned by f_1 (the first decision tree). There are many possible answers for this.



(b) On the dataset below, circle the point(s) with the highest weights on the second iteration, and draw the decision boundary learned by f_2 (the second decision tree).



(c) On the dataset below, draw the decision boundary $f = sign(\alpha_1 f_1 + \alpha_2 f_2)$. (Hint: you do not need to explicitly compute the α's).



Algorithm 32 AdaBoost($\mathcal{W}$, $\mathcal{D}$, K)

1: $\boldsymbol{d}^{(0)} \leftarrow \langle \frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N} \rangle$ // Initialize
2: **for** $k = 1 \ldots K$ **do**
3: $f^{(k)} \leftarrow \mathcal{W}(\mathcal{D}, \boldsymbol{d}^{(\text{k-1})})$ //
4: $\hat{y}_n \leftarrow f^{(\text{k})}(\boldsymbol{x}_n), \forall n$
5: $\hat{\epsilon}^{(k)} \leftarrow \sum_n d_n^{(\text{k-1})} [y_n \neq \hat{y}_n]$
6: $\alpha^{(k)} \leftarrow \frac{1}{2} \log \left(\frac{1-\hat{\epsilon}^{(\text{k})}}{\hat{\epsilon}^{(\text{k})}} \right)$
7: $d_n^{(k)} \leftarrow \frac{1}{Z} d_n^{(\text{k-1})} \exp[-\alpha^{(\text{k})} y_n \hat{y}_n], \forall n$
8: **end for**
9: **return** $f(\hat{\boldsymbol{x}}) = \text{sgn}\left[\sum_k \alpha^{(\text{k})} f^{(\text{k})}(\hat{\boldsymbol{x}})\right]$