context-free grammars

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Regular grammar: production forms

$A \to xB$ $\quad A, B \in V$

or $A \to x$ $\quad x \in T^*$

context-free grammar: production forms

$A \to x$ $\quad A \in V$

$x \in (T \cup V)^*$

$G = (V, T, S, P)$

ex $L = \{ww^R : w \in \{a,b\}^*\}$

$= \{\lambda, aa, bb, abba, baab, \ldots\}$

P: $S \to aSa$

$S \to bSb$ $\qquad G = (\{S\}, \{a,b\}, S, P)$

$S \to \lambda$

ex. $G = (\{S, A, B\}, \{a, b\}, S, P)$

P: (1) $S \to AB$

(2) $A \to aaA$

(3) $A \to \lambda$

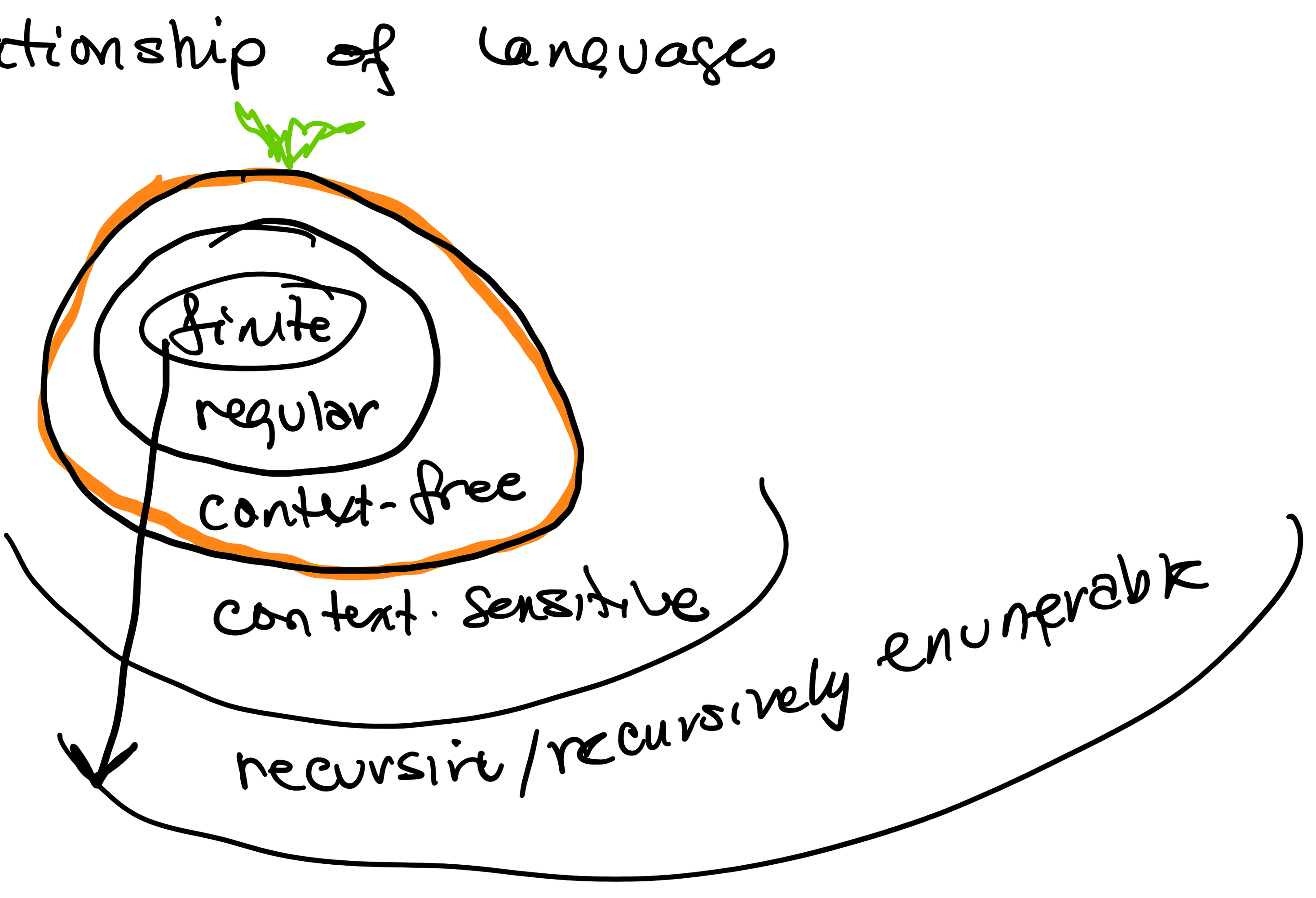
(4) $B \to Bb$

(5) $B \to \lambda$

$S \to AB \to \lambda B \to \lambda\lambda \to \lambda$

$S \to AB \to A\lambda \to \lambda\lambda \to \lambda$

relationship of languages



type	language	production form	device
3	regular	$A \to xB$, $A \to x$ $A, B \in V$, $x \in \Sigma^*$	fa
2	context-free	$A \to \alpha$ $A \in V$, $\alpha \in (V \cup \Sigma)^*$	pda
1	context-sensitive	$\alpha \to \beta$ $\alpha, \beta \in (V \cup \Sigma)^*$ $\lvert\beta\rvert \geq \lvert\alpha\rvert$ α contain a variable	lba
0	recursively enumerable [unrestricted]	$\alpha \to \beta$ $\alpha, \beta \in (V \cup \Sigma)^*$ α contains a variable	tm

ex of context-sensitive productions

$A \to \lambda$ ✓ $\qquad AB \to bA$ ✓

$A \to BB$ ✓

$A \to aaab$ ✓

$A \to BaA$ ✓

~~$b \to aa$~~ ✓

Chomsky Normal Form (CNF)

$A \to BC$ $\quad A, B, C \in V$

or $A \to x$ $\quad x \in T$

what if had $\lambda \in L$, where L is CFL ?

$L_1 = L - \{\lambda\}$

$L(G_1) = L_1$ where G_1 is in CNF

$G = (V_1 \cup \{S_{new}\}, T_1, S_{new}, P)$

where P: $S_{new} \to S_1 \mid \lambda$

$S_1 \to \vdots$ $\Big\}$ CNF

P: $(\underbrace{\{S_{new} \to S_1}_{\neq CNF} \mid \underbrace{\lambda\}}_{\neq CNF} \cup \underbrace{P_1}_{= CNF})$

Steps to convert into CNF . . .

$S \to \lambda \mid aA$

$A \to \lambda$