

1. A and B are 2x2 matrices.
- $A \neq B$
- .

Does $AB=BA$, always, sometimes, or never? Circle one and prove it. (3 points)

~~always/never~~
-3

10/wrong positive example \rightarrow

10/wrong neg example \rightarrow

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} b & a \\ a & b \end{bmatrix} = \begin{bmatrix} 2ab & a^2+b^2 \\ b^2+a^2 & 2ab \end{bmatrix}$$

A B

any a & b work.

$$\begin{bmatrix} b & a \\ a & b \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2ab & a^2+b^2 \\ a^2+b^2 & 2ab \end{bmatrix}$$

also need negative example

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

2. Find the gradient of
- f
- . Show your work. (2 points)

$$f(x, y, z) = x^2yz^3 + 5xz + 3y^2 + z + 10$$

one mistake \rightarrow

-1

no work \rightarrow

-1

$$\frac{\partial f}{\partial x} = 2xyz^3 + 5z$$

$$\frac{\partial f}{\partial y} = x^2z^3 + 6y$$

$$\frac{\partial f}{\partial z} = 3x^2yz^2 + 5x + 1$$

$$\nabla f = (2xyz^3 + 5z, x^2z^3 + 6y, 3x^2yz^2 + 5x + 1)$$

3. I want a model that can predict the cost of gas. Is this a classification or regression problem? Why? (2 points)

classification

Predict a real value given past data.

-2

no "real value" -1

4. Show that A and B are orthogonal. Find another vector C that is orthogonal to A, such that
- $\|C\|_2 = 1$
- . Show your work. (3 points)

answer correct but no work -1

$$A = \begin{bmatrix} 8 \\ 4 \\ 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 8 \\ 4 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = 8 - 8 + 0 = 0 \quad \checkmark$$

Dot product 0
therefore $A \perp B$.

do not give 1/3
in this problem.

Find vector parallel to B w/ $\|C\|_2 = 1$.
Make B a unit vector.

$$C = \frac{B}{\|B\|_2} = \frac{\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}}{\sqrt{1+2^2}} = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \\ 0 \end{bmatrix} \quad \checkmark$$

Any vector $\perp A$
and unit would

1. Are the following matrices inverses? Prove it. (3 points)

-1 for each
math mistake

$$\begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

(2)

$$\begin{bmatrix} -24+25 & -48+18+30 & -72+72+0 \\ 20-20 & 40-15-24 & 60-60 \\ -5+5 & -10+4+6 & -15+16+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{yes} \checkmark$$

2. Find the gradient of f at $(1,2,1)$. Show your work. (3 points)

Same #2

$$f(x,y,z) = x^2yz^3 + 5xz + 3y^2 + z + 10$$

← ~~df/dx~~ Same as #2 691

-1 if they don't get this $\rightarrow (9, 13, 12)$

3. Are the following vectors orthogonal? How do you know? Show your work. (2 points)

$$6 \cdot 1 + 2 \cdot -3 + 3 \cdot 1$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$$

(3)

$$6 - 6 + 3 = 3$$

-1 yes

-1 how do you know

-1 no show work

no Because the dot product is nonzero.

minimum: 0/2

4. What does it mean for a model to be *generalizable*? (2 points)

-1 if they don't Perform well on unseen data.

Say anything about either unseen or held out or test data.

-2 if nonsense