

1. Run gradient descent on the function $f(x_1, x_2) = \frac{1}{2}x_1^2 + x_2^2$ starting at the point $s_0 = (2, 2)$ and using a step size of $\eta = \frac{1}{2}$. Run three steps of the algorithm. What is the output? (5 points)

$$\nabla f = \begin{bmatrix} x_1 \\ 2x_2 \end{bmatrix}$$

$$\textcircled{1} \quad \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

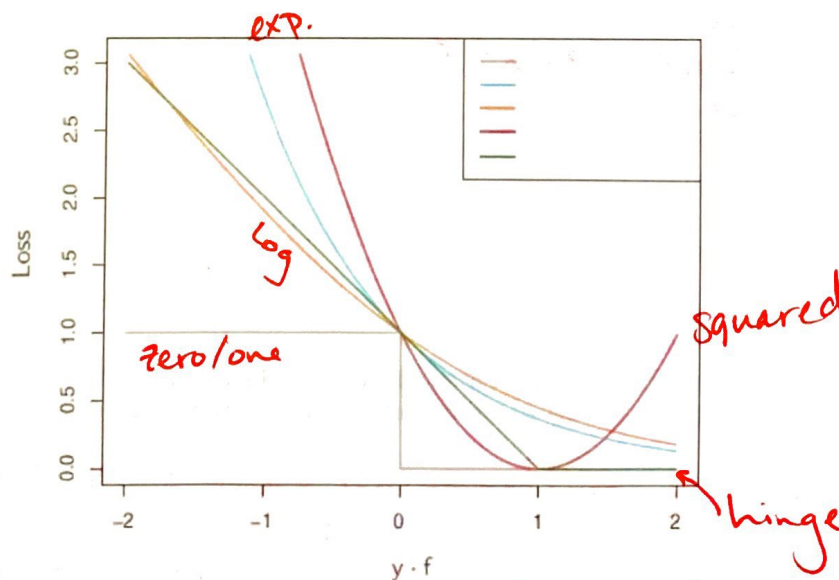
$$\textcircled{2} \quad \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$$

$$\textcircled{3} \quad \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} \\ 0 \end{bmatrix}$$

Output

$$\begin{bmatrix} \frac{1}{4} \\ 0 \end{bmatrix}$$

2. Label each of the following loss functions using these options: 0/1, hinge, sigmoid, squared, absolute, exponential, log. (5 points)



1. We want to find the w that minimizes the following objective function $L(w)$. What values of w might gradient descent return? Explain. (3 points)

$$L(x) = x^3$$

$$w = 0$$

$$w = \infty$$

$$\frac{dL}{dw} = 3w^2 = 0 @ w = 0$$

But if we pass $w = 0$
gradient descent could take us
down the curve forever.

2. Recall our regularized loss function. In this particular loss function, we have used an exponential loss $e^{y\hat{y}}$ with a $\|w\|^2$ regularizer. Find ∇L_w and $\frac{\partial L}{\partial b}$. (5 points)

$$L(w, b) = \sum_n e^{y_n(w \cdot x_n + b)} + \frac{\lambda}{2} \|w\|^2$$

$$\frac{\partial L}{\partial b} = e^{y_n \hat{y}_n} \cdot y_n$$

$$\nabla L_w = e^{y_n \hat{y}_n} \cdot y_n x_n + \lambda \vec{w}$$

$$\hat{y}_n = w \cdot x_n + b$$

3. If I am performing gradient descent on the following function: $L(w) = w^2$ starting with $w=3$. What would cause gradient descent to not reach the minimum and return $w = 0$? (2 points)

If my η (learning rate) is too high,
I will never reach the minimum