

1. Are the following two vectors perpendicular? Explain briefly. (2 points)

$$\begin{bmatrix} 0 \\ 7 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$$

Same as 622 #1

2. Is the following matrix invertible? Briefly explain. (3 points)

All you need to do is calculate the determinant.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$$

$$\frac{1}{1 \cdot 4 - 0 \cdot 2} = \frac{1}{4}$$

If the denominator was 0 then we couldn't invert.

So yes, because the determinant is ~~4~~ 4.

3. Find the gradient of  $f$  at  $(1, 1, 1)$ . (5 points)

$$f(x, y, z) = x^3 z^2 + xy^2 + 3z^4 + x + 5$$

$\nabla f$  on 622 side.

Plug in  $(1, 1, 1)$

$$\nabla f(1, 1, 1) = (3+1+1, 2, 2+12) = (5, 2, 14)$$

#### Useful Formulas

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

1. Are the following two vectors perpendicular? Explain briefly. (2 points)

$$\begin{bmatrix} 0 \\ 7 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$$

No, because their dot product is not 0.

$$0 \cdot 3 + 7 \cdot (-1) + 2 \cdot 4 = 1 \neq 0$$

2. Show that matrix multiplication is not commutative. That is  $AB \neq BA$ . (5 points)

To show this, you just have to give an example!

$$\begin{bmatrix} 1 & 4 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 19 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 17 \\ 11 & 37 \end{bmatrix}$$

already I've shown  
 $AB \neq BA$ .

3. Find the gradient of  $f$ . (3 points)

$$f(x, y, z) = x^3 z^2 + xy^2 + 3z^4 + x + 5$$

$$\frac{\partial f}{\partial x} = 3x^2 z^2 + y^2 + 1$$

$$\frac{\partial f}{\partial y} = 2xy$$

$$\frac{\partial f}{\partial z} = 2x^3 z + 12z^3$$

$$\nabla f = (3x^2 z^2 + y^2 + 1, 2xy, 2x^3 z + 12z^3)$$