CS 422/622 Quiz 1 Name Solution

1. Show that the following two vectors are not perpendicular. (2 points)

$$\underset{A}{\begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix}} \quad \underset{B}{\begin{bmatrix} 8 \\ -1 \\ 2 \end{bmatrix}}$$

$A \cdot B = 1 \cdot 8 + 7 \cdot -1 + 3 \cdot 2$

$= 8 + -7 + 6 = 7 \neq 0$

Since the dot product isn't zero, A & B are not perpendicular.

2. Find the inverse of the following matrix (5 points)

$AA^{-1} = I$ Lets create a system of eq. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 1 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$a + 2d = 1 \rightarrow a = 1 - 2d$

$4d + g = 0 \rightarrow g = -4d$

$3a + 4g = 0 \rightarrow 3(1-2d) + 4(-4d) = 0$

$3 - 6d - 16d = 0$

$d = \frac{3}{22}$ $a = 1 - 2\left(\frac{3}{22}\right) = \frac{8}{11}$ $g = -4\left(\frac{3}{22}\right) = \frac{-6}{11}$

More work on back →

$$\begin{bmatrix} \frac{8}{11} & \frac{-4}{11} & \frac{1}{11} \\ \frac{3}{22} & \frac{2}{11} & \frac{-1}{22} \\ \frac{-6}{11} & \frac{3}{11} & \frac{2}{11} \end{bmatrix}$$

3. Find the gradient of f. (3 points)

$$f(x, y, z) = x^2y^3z + yz^2 + 3x^4 + z + 1$$

$\nabla f = (2xy^3z + 12x,\ 3x^2y^2z + z^2,\ x^2y^3 + 2yz + 1)$

$b+2e=0 \rightarrow b=-2e$

$4e+h=1 \rightarrow h=1-4e$

$3b+4h=0 \rightarrow 3(-2e)+4(1-4e)=0$

$-6e+4-16e=0 \quad e=\frac{4}{22}=\frac{2}{11}$

$b=\frac{-4}{11} \quad h=1-\frac{8}{11}=\frac{3}{11}$

$C+2f=0 \rightarrow c=-2f$

$4f+i=0 \rightarrow i=-4f$

$3c+4i=1$

$3(-2f)+4(-4f)=1$

$-6f-16f=1$

$f=-\frac{1}{22}$

$C=\frac{1}{11}$

$i=\frac{2}{11}$