Quiz 9 Solutions

Question 1

$$A = \begin{pmatrix} 1 & 8 \\ 1 & 3 \end{pmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{pmatrix} 1 & 8 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 1-\lambda & 8 \\ 1 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) - 8 = 0$$

$$3 - 4\lambda + \lambda^2 - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

$$(\lambda - 5)(\lambda + 1) = 0$$

$$\lambda_1 = 5 \quad \lambda_2 = -1$$

$$Av = \lambda v$$

$$\begin{pmatrix} 1 & 8 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 5 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$v_1 + 8v_2 = 5v_1 \longrightarrow 4v_1 = 8v_2$$

set $v_1 = 2$

then $v_2 = 1$

need $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ to be length 1

$$\left\| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\|_2 = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \quad \lambda_1 = 5$$

On to the 2nd eigenvector . . .

$\lambda_2 = -1 \qquad Av = \lambda v$

$$\begin{pmatrix} 1 & 8 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = -1\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$v_1 + 8v_2 = -v_1 \longrightarrow -2v_1 = 8v_2$

if $v_2 = 1$

then $v_1 = -4$

$\begin{pmatrix} -4 \\ 1 \end{pmatrix} \longrightarrow$ need length $= 1$

$\sqrt{1^2 + (-4)^2} = \sqrt{17}$

$$\begin{pmatrix} \frac{-4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \end{pmatrix} \qquad \lambda_2 = -1$$

Note: you don't <u>have</u> to normalize here since you aren't projecting any points. Any scalar multiple of an eigenvector is an eigenvector.

I just normalized out of habit.

Question 2

V_1 & V_2 are the first two P.C.S

need to normalize them first

$$\frac{V_1}{\|V_1\|} = \frac{\begin{pmatrix}1\\1\\1\end{pmatrix}}{\sqrt{3}} = \begin{pmatrix}\frac{1}{\sqrt{3}}\\\frac{1}{\sqrt{3}}\\\frac{1}{\sqrt{3}}\end{pmatrix}$$

$$\frac{V_2}{\|V_2\|} = \frac{\begin{pmatrix}1\\-2\\1\end{pmatrix}}{\sqrt{6}} = \begin{pmatrix}\frac{1}{\sqrt{6}}\\\frac{-2}{\sqrt{6}}\\\frac{1}{\sqrt{6}}\end{pmatrix}$$

$$\begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\\\frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}}\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}\frac{6}{\sqrt{3}}\\0\end{pmatrix} = \begin{pmatrix}2\sqrt{3}\\0\end{pmatrix}$$