CS 422/622 Quiz 9 Name Solution

1. True/False: You must use PCA to reduce the dimensionality of your data. Briefly explain. (3 points)

You can use PCA to identify directions of greatest variance in your data.

2. Given N data points, with D dimensions, what size will the covariance matrix be and how many eigenvectors will you get? (2 points)

$D \times D$

3. Find the eigenvalues and eigenvectors of the following matrix. (5 points)

$$\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & -2 \\ 3 & -4-\lambda \end{vmatrix} = (1-\lambda)(-4-\lambda)+6 = 0$$

$$-4-\lambda+4\lambda+\lambda^2+6=0$$

$$\lambda^2+3\lambda+2=0$$

$$(\lambda+1)(\lambda+2)=0$$

$$\lambda_1 = -1$$

$$\lambda_2 = -2$$

$$\begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = -1\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$v_1 - 2v_2 = -v_1 \qquad -2v_2 = -2v_1$$

$\vec{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ normalize $\Rightarrow \vec{V}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

$|A - \lambda I| = 0$

8. $\begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = -2 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$v_1 - 2v_2 = -2v_1$

$-2v_2 = -3v_1$

$v_2 = \frac{3}{2} v_1$

$\vec{V}_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ normalize $\rightarrow \vec{V}_2 = \begin{pmatrix} \frac{2}{\sqrt{13}} \\ \frac{3}{\sqrt{13}} \end{pmatrix}$