P 7.5-11 Determine $i(t)$ for $t \geq 0$ for the circuit of Figure P 7.5-11*a* when $i(0)$ = 25 mA and $v_s(t)$ is the voltage shown in Figure P 7.5-11*b*.

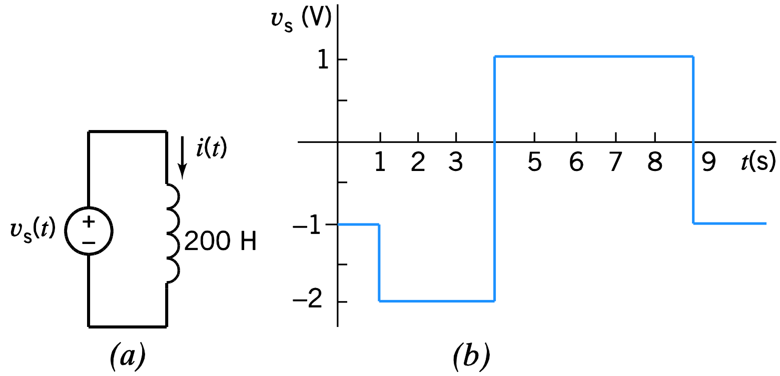


Figure P 7.5-11

Solution:

$$i(t) = \frac{1}{200}\int_0^t -d\tau + 0.025 = \frac{-t}{200} + 0.025 \qquad \text{for} \qquad 0 < t < 1$$

$$i(t) = \frac{1}{200}\int_1^t -2\ d\tau + 0.02 = \frac{-2(t-1)}{200} + 0.02 \qquad \text{for} \qquad 1 < \text{t} < 4$$

$$i(t) = \frac{1}{200}\int_4^t d\tau - 0.01 = \frac{t-4}{200} - 0.01 \qquad \text{for} \qquad 4 < \text{t} < 9$$

$$i(t) = 0.015 = 15\ \text{mA} \qquad t < 9$$

P 7.5-12 The inductor current in the circuit shown in Figure P 7.5-12 is given by

$$i(t) = 6 + 4e^{-8t}\ \text{A} \quad \text{for}\ t \geq 0$$

Determine $v(t)$ for $t > 0$.

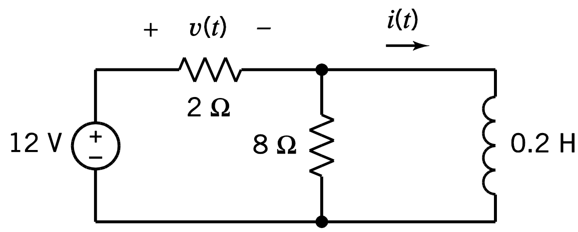
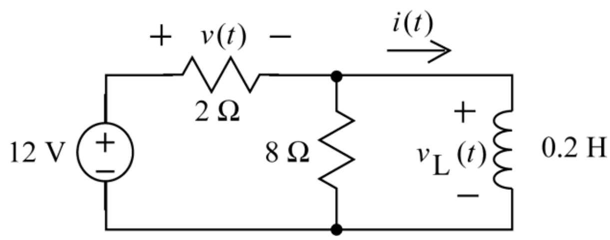


Figure P 7.5-12

Solution:

$$v_L(t) = 0.2\frac{d}{dt}i(t)$$
$$= -6.4e^{-8t}\ \text{V} \quad \text{for}\ t > 0$$



Use KVL to get

$$v(t) = 12 - \left(-6.4e^{-8t}\right) = 12 + 6.4e^{-8t}\ \text{V} \quad \text{for}\ t > 0$$

(checked: LNAP 6/25/04)

P 7.5-13 The inductor current in the circuit shown in Figure P 7.5-13 is given by

$$i(t) = 5 - 3e^{-4t} \text{ A} \quad \text{for } t \ge 0$$

Determine $v(t)$ for $t > 0$.

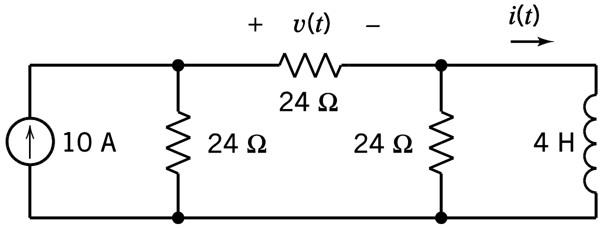
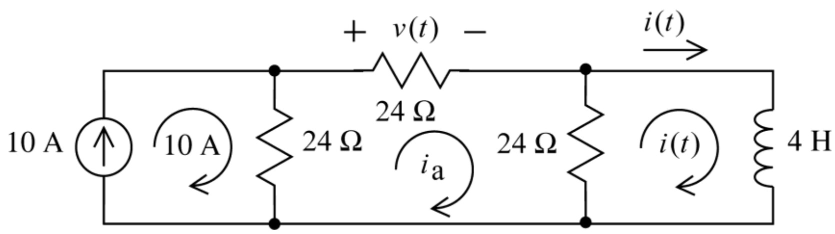


Figure P 7.5-13

Solution:



We'll write and solve a mesh equation. Label the meshes as shown. Apply KVL to the center mesh to get

$$24i_a + 24(i_a - i(t)) + 24(i_a - 10) = 0 \quad \Rightarrow \quad i_a = \frac{i(t)+10}{3} = 5 - e^{-4t} \text{ A} \quad \text{for } t > 0$$

Then $$v(t) = 24i_a = 120 - 24e^{-4t} \text{ V} \quad \text{for } t > 0$$

(checked: LNAP 6/25/04)

P 7.5-14 The inductor current in the circuit shown in Figure P 7.5-14 is given by

$$i(t) = 3 + 2e^{-3t} \text{ A} \quad \text{for } t \ge 0$$

Determine $v(t)$ for $t > 0$.

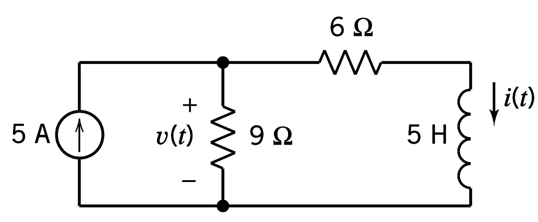


Figure P 7.5-14

Solution: Apply KVL to get

$$v(t) = 6i(t) + 5\frac{d}{dt}i(t) = 6(3 + 2e^{-3t}) + 5\frac{d}{dt}(3 + 2e^{-3t}) = 18(1 - e^{-3t}) \text{ V} \quad \text{for } t > 0$$

P 7.7-1 Find the current $i(t)$ for the circuit of Figure P 7.7-1.
Answer: $i(t)$ = 15 sin 100t mA

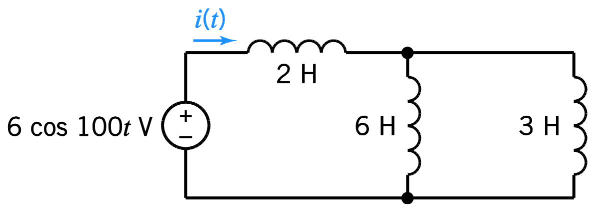


Figure P 7.7-1

Solution:

$$6\text{ H} \parallel 3\text{ H} = \frac{6\times 3}{6+3} = 2\text{ H} \quad \text{and} \quad 2\text{ H} + 2\text{ H} = 4\text{ H}$$

$$i(t) = \frac{1}{4}\int_0^t 6\cos 100\tau \; d\tau = \frac{6}{4\times 100}\left[\sin 100\tau\Big|_0^t\right] = 0.015\sin 100t \text{ A} = 15\sin 100t \text{ mA}$$

P 7.7-2 Find the voltage $v(t)$ for the circuit of Figure P 7.7-2.
Answer: $v(t) = -6e^{-250t}$ mV

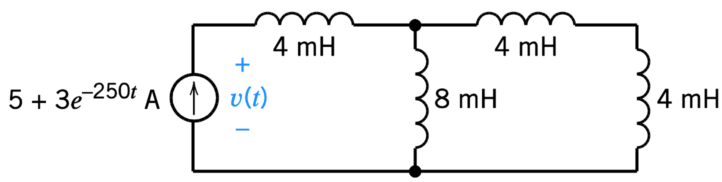


Figure P 7.7-2

Soluton:

$$4\text{ mH} + 4\text{ mH} = 8\text{ mH} \quad , \quad 8\text{mH} \parallel 8\text{mH} = \frac{\left(8\times 10^{-3}\right)\times\left(8\times 10^{-3}\right)}{8\times 10^{-3}+8\times 10^{-3}} = 4\text{ mH}$$

and $4\text{ mH} + 4\text{ mH} = 8\text{ mH}$

$$v(t) = (8\times 10^{-3})\frac{d}{dt}(5+3e^{-250t}) = (8\times 10^{-3})(0+3(-250)\,e^{-250t}) = -6\,e^{-250t} \text{ V}$$

P7.7-11

Consider the combination of circuit elements shown in Figure P7.7-11.

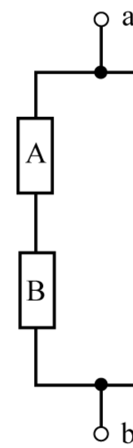


Figure P7.7-11

(a) Suppose element A is a 20 μF capacitor, element B is a 5 μF capacitor and element C is a 20 μF capacitor. Determine the equivalent capacitance.

(b) Suppose element A is a 50 mH inductor, element B is a 30 mH inductor and element C is a 20 mH inductor. Determine the equivalent inductance.

(c) Suppose element A is a 9 kΩ resistor, element B is a 6 kΩ resistor and element C is a 10 kΩ resistor. Determine the equivalent resistance.

Answers: (a) C_{eq} = 20 μF, (b) L_{eq} = 16 mH, (c) R_{eq} = 6 kΩ

Solution:

(a) $C_{eq} = \frac{20(5)}{20+5} + 20 = 24\ \mu\text{F}$ (b) $L_{eq} = \frac{(50+30)(20)}{(50+30)+20} = 16\ \text{mH}$ (c) $R_{eq} = \frac{(9+6)(10)}{(9+6)+10} = 6\ \text{k}\Omega$

P7.8-11

The circuit shown in Figure 7.8-11 has reached steady state before the switch opens at time t = 0. Determine the values of $i_L(t)$, $v_C(t)$ and $v_R(t)$ immediately before the switch opens and the value of $v_R(t)$ immediately after the switch opens.

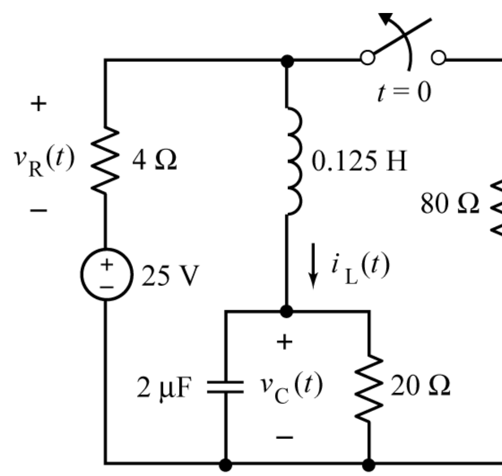
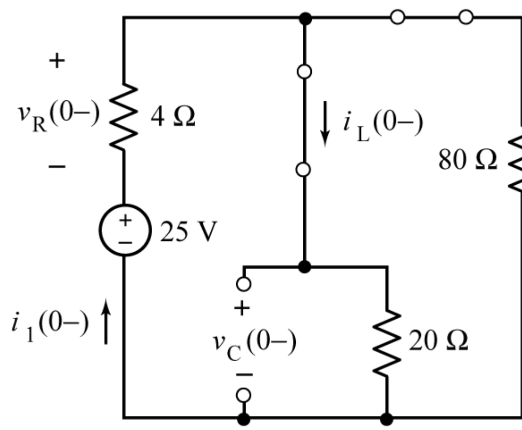


Figure 7.8-11

Answers: $i_L(0-)$ = 1.25 A, $v_C(0-)$ = 20 V, $v_R(0-)$ = –5 V and $v_R(0+)$ = –4 V

Solution: Because

- This **circuit has reached steady state** before the switch opens at time $t = 0$.
- The only source is a **constant** voltage **source**.



At t=0–, t**he capacitor acts like an open circuit** and **the inductor acts like a short circuit**. From the circuit

$$i_1(0-) = \frac{25}{4+(20\|80)} = \frac{25}{4+16} = 1.25 \text{ A},$$

$$i_L(0-) = \left(\frac{80}{20+80}\right) i_1(0-) = 1 \text{ A},$$

$$v_C(0-) = 20 i_L(0-) = 20 \text{ V}$$

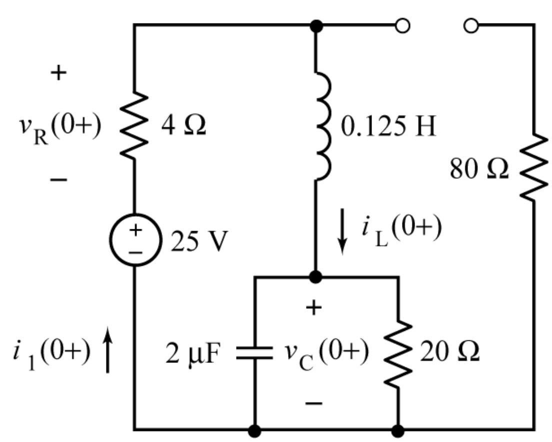
and

$$v_R(0-) = -4 i_1(0-) = -5 \text{ V}$$

The **capacitor voltage and inductor current don't change instantaneously** so

$$v_C(0+) = v_C(0-) = 20 \text{ V} \text{ and}$$

$$i_L(0+) = i_L(0-) = 1 \text{ A}$$



Apply KCL at the top node to see that

$$i_1(0+) = i_L(0+) = 1 \text{ A}$$

From Ohm's law

$$v_R(0+) = -4 i_1(0+) = -4 \text{ V}$$

(Notice that the resistor voltage did change instantaneously.)

P 8.3-2 The circuit shown in Figure P 8.3-2 is at steady state before the switch opens at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the current in the inductor, $i(t)$. Determine $i(t)$ for $t > 0$.

Answer: $i(t) = 1 + e^{-0.5t}$ A for $t > 0$

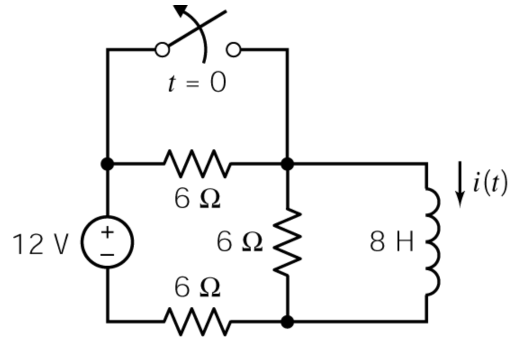
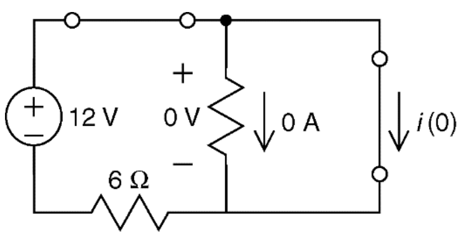


Figure P 8.3-2

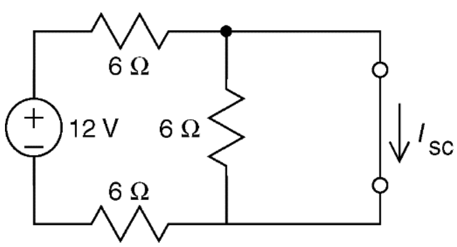
Solution:



Here is the circuit before $t = 0$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit.

An inductor in a steady-state dc circuit acts like an short circuit, so a short circuit replaces the inductor. The current in that short circuit is the initial inductor current, $i(0)$.

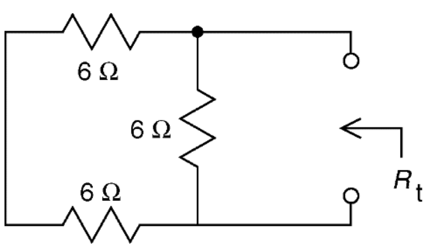
$$i(0) = \frac{12}{6} = 2 \text{ A}$$



Next, consider the circuit after the switch opens. The open switch is modeled as an open circuit.

We need to find the Norton equivalent of the part of the circuit connected to the inductor. Here's the circuit used to calculate the short circuit current, I_{sc}.

$$I_{sc} = \frac{12}{6+6} = 1 \text{ A}$$



Here is the circuit that is used to determine R_t. An open circuit has replaced the open switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by an short circuit.

$$R_t = 6 \,||\, (6+6) = \frac{(6+6)(6)}{(6+6)+6} = 4 \ \Omega$$

Then $$\tau = \frac{L}{R_{\mathrm{t}}} = \frac{8}{4} = 2 \;\; \mathrm{s}$$

Finally, $$i(t) = I_{\mathrm{sc}} + \left(i(0) - I_{\mathrm{sc}}\right) e^{-t/\tau} = 1 + e^{-0.5\,t} \;\; \mathrm{A} \quad \text{for } t > 0$$

P 8.3-3 The circuit shown in Figure P 8.3-3 is at steady state before the switch closes at time $t = 0$. Determine the capacitor voltage, $v(t)$, for $t > 0$.

Answer: $v(t) = -6 + 18e^{-6.67t}$ V for $t > 0$

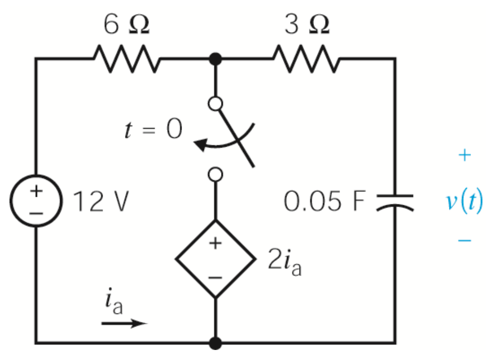
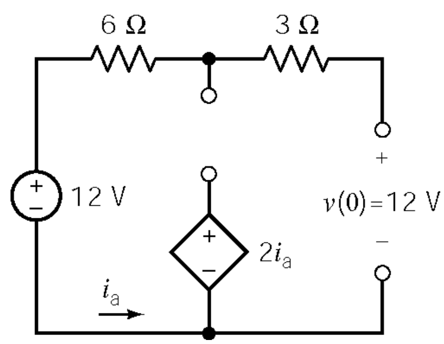
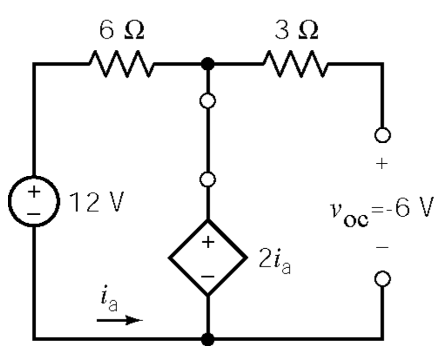


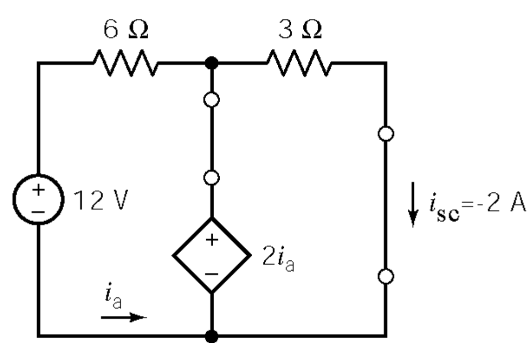
Figure P 8.3-3

Solution: Before the switch closes:



After the switch closes:





Therefore $R_t = \dfrac{-6}{-2} = 3\,\Omega$ so $\tau = 3(0.05) = 0.15$ s .

Finally, $v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = -6 + 18\, e^{-6.67t}$ V for $t > 0$

P 8.3-4 The circuit shown in Figure P 8.3-4 is at steady state before the switch closes at time $t = 0$. Determine the inductor current, $i(t)$, for $t > 0$.

Answer: $i(t) = -2 + \dfrac{10}{3} e^{-0.5t}$ A for $t > 0$

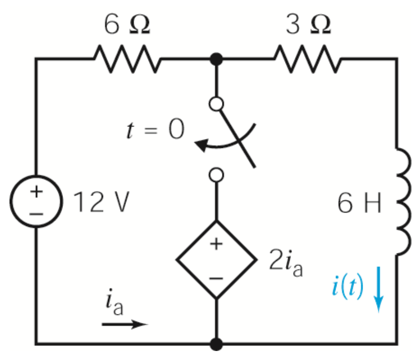
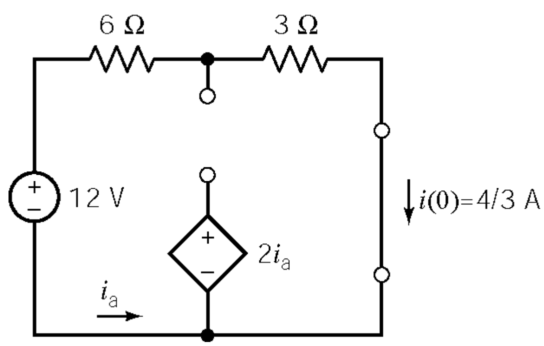
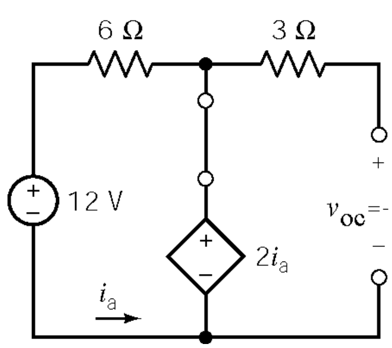


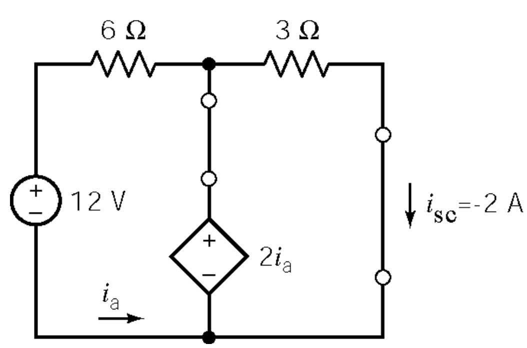
Figure P 8.3-4

Solution: Before the switch closes:



After the switch closes:





Therefore $R_t = \dfrac{-6}{-2} = 3\,\Omega$ so $\tau = \dfrac{6}{3} = 2$ s.

Finally, $i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = -2 + \dfrac{10}{3} e^{-0.5t}$ A for $t > 0$

P 8.3-8 The circuit shown in Figure P 8.3-8 is at steady state before the switch opens at time t = 0. The input to the circuit is the voltage of the voltage source, V_s. This voltage source is a dc voltage source; that is, V_s is a constant. The output of this circuit is the voltage across the capacitor, $v_o(t)$. The output voltage is given by

$$v_o(t) = 2 + 8e^{-0.5t}\text{ V} \quad \text{for} \quad t > 0$$

Determine the values of the input voltage, V_s, the capacitance, C, and the resistance, R.

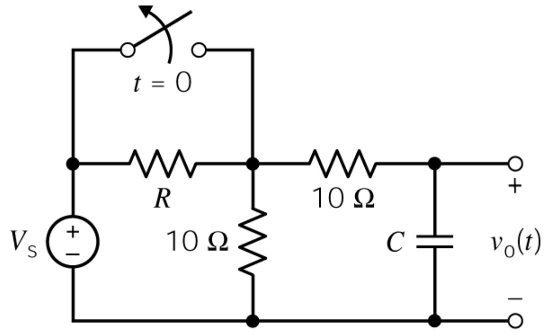
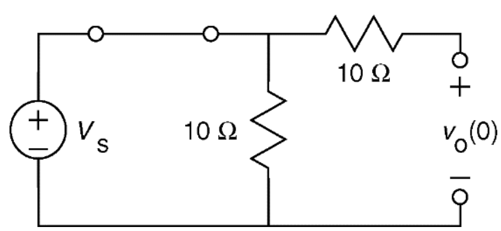


Figure P 8.3-8

Solution: Before the switch opens, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the capacitor voltage, will have constant values. Opening the switch disturbs the circuit. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch opened.



Here is the circuit before t = 0, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit. The combination of resistor and a short circuit connected is equivalent to a short circuit. Consequently, a short circuit replaces the switch and the resistor R. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the capacitor voltage, $v_o(t)$.

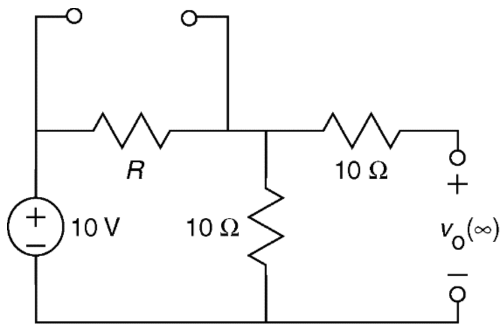
Because the circuit is at steady state, the value of the capacitor voltage will be constant. This constant is the value of the capacitor voltage just before the switch opens. In the absence of unbounded currents, the voltage of a capacitor must be continuous. The value of the capacitor voltage immediately after the switch opens is equal to the value immediately before the switch opens. This value is called the initial condition of the capacitor and has been labeled as $v_o(0)$. There is no current in the horizontal resistor due to the open circuit. Consequently, $v_o(0)$ is equal to the voltage across the vertical resistor, which is equal to the voltage source voltage. Therefore

$$v_o(0) = V_s$$

The value of $v_o(0)$ can also be obtained by setting t = 0 in the equation for $v_o(t)$. Doing so gives

$$v_o(0) = 2 + 8\,e^0 = 10\text{ V}$$

Consequently, $$V_s = 10\text{ V}$$



Next, consider the circuit after the switch opens. Eventually (certainly as $t \to \infty$) the circuit will again be at steady state. Here is the circuit at $t = \infty$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the steady-state capacitor voltage, $v_o(\infty)$. There is no current in the horizontal resistor and $v_o(\infty)$ is equal to the voltage across the vertical resistor. Using voltage division,

$$v_o(\infty) = \frac{10}{R+10}(10)$$

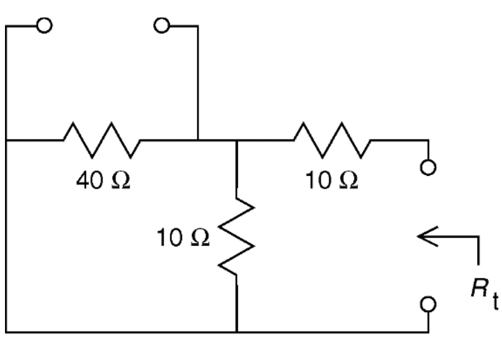
The value of $v_o(\infty)$ can also be obtained by setting $t = \infty$ in the equation for $v_o(t)$. Doing so gives

$$v_o(\infty) = 2 + 8\,e^{-\infty} = 2 \text{ V}$$

Consequently,

$$2 = \frac{10}{R+10}(10) \quad \Rightarrow \quad 2\,R + 20 = 100 \quad \Rightarrow \quad R = 40\,\Omega$$

Finally, the exponential part of $v_o(t)$ is known to be of the form $e^{-t/\tau}$ where $\tau = R_t\,C$ and R_t is the Thevenin resistance of the part of the circuit connected to the capacitor.



Here is the circuit that is used to determine R_t. An open circuit has replaced the open switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by a short circuit.

$$R_t = 10 + \frac{(40)(10)}{40+10} = 18\ \Omega$$

so

$$\tau = R_t\,C = 18\,C$$

From the equation for $v_o(t)$

$$-0.5\,t=-\frac{t}{\tau} \quad\Rightarrow\quad \tau=2\text{ s}$$

Consequently,

$$2=18\,C \quad\Rightarrow\quad C=0.111=111\text{ mF}$$

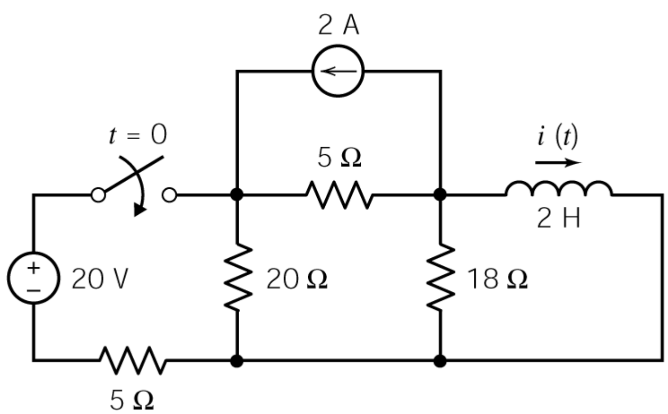
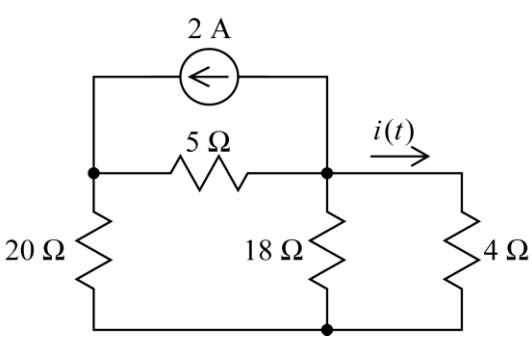


Figure P 8.3-14

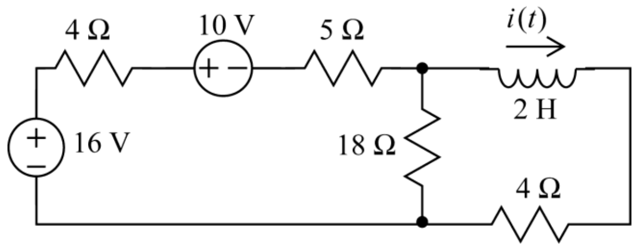
P 8.3-14 The circuit shown in Figure P 8.3-14 is at steady state when the switch closes at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

Solution: Before $t = 0$, with the switch open and the circuit at steady state, the inductor acts like a short circuit so we have

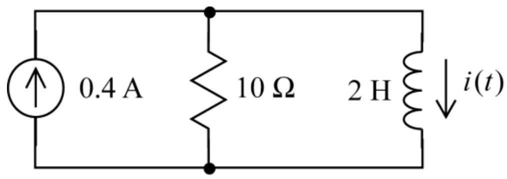


$$i(t)=-\frac{18}{4+18}\left[\frac{5}{5+20+(18\,||\,4)}2\right]=0.29\text{ A}$$

After $t = 0$, we can replace the part of the circuit connected to the inductor by its Norton equivalent circuit. First, performing a couple of source transformations reduces the circuit to



Next, replace the series voltage sources by an equivalent voltage source, replace the series resistors by an equivalent resistor and do a couple of source transformations to get



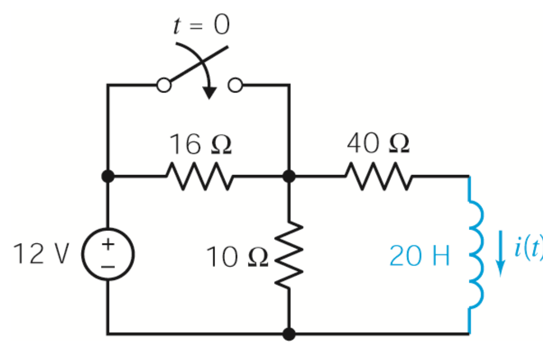
so $$\tau = \frac{2}{10} = 0.25 \quad \Rightarrow \quad \frac{1}{\tau} = 5\frac{1}{\text{s}}$$

The current is given by $$i(t) = [0.29 - 0.4]e^{-5t} + 0.4 = 0.4 - 0.11e^{-5t} \text{ A} \quad \text{for } t \geq 0$$

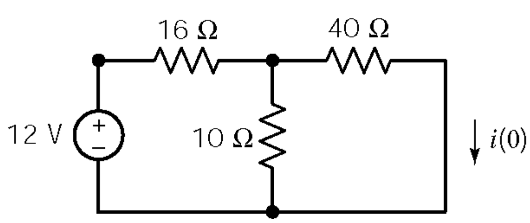
P 8.3-15 The circuit in Figure P 8.3-15 is at steady state before the switch closes. Find the inductor current after the switch closes.

Hint: $i(0) = 0.1$ A, $I_{sc} = 0.3$ A, $R_t = 40\ \Omega$

Answer: $i(t) = 0.3 - 0.2e^{-2t}$ A $t \geq 0$

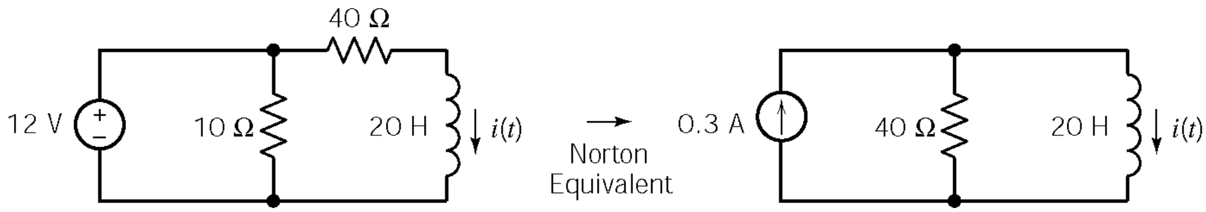


Solution: At steady-state, immediately before $t = 0$:



$$i(0) = \left(\frac{10}{10+40}\right)\left(\frac{12}{16+40\|10}\right) = 0.1 \text{ A}$$

After $t = 0$, the Norton equivalent of the circuit connected to the inductor is found to be



$$\text{so } I_{sc} = 0.3\text{ A},\ R_t = 40\,\Omega,\ \tau = \frac{L}{R_t} = \frac{20}{40} = \frac{1}{2}\text{ s}$$

$$\text{Finally:}\qquad i(t) = (0.1-0.3)e^{-2t} + 0.3 = 0.3-0.2e^{-2t}\text{ A}$$

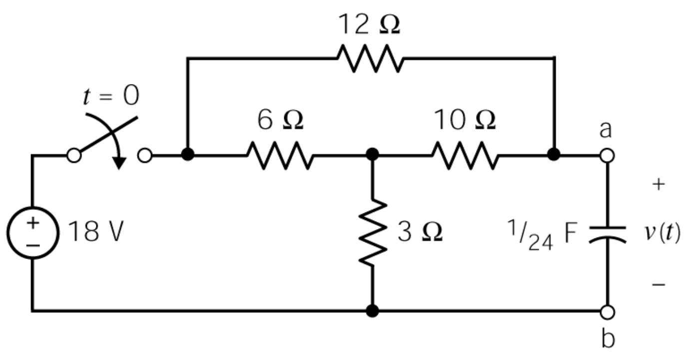


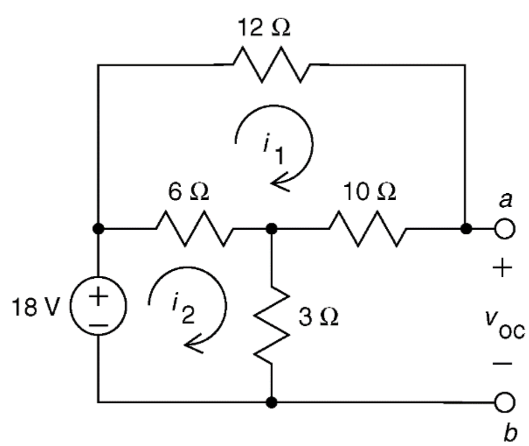
Figure P 8.3-19

P 8.3-19 The circuit shown in Figure P 8.3-19 is at steady state before the switch closes. Find $v(t)$ for $t \geq 0$.

Solution: Before the switch closes $v(t) = 0$ so $v(0+) = v(0-) = 0 \text{ V}$.

For $t > 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor, i.e. the part of the circuit to the left of the terminals $a - b$.

Write mesh equations to find v_{oc}:



Mesh equations:

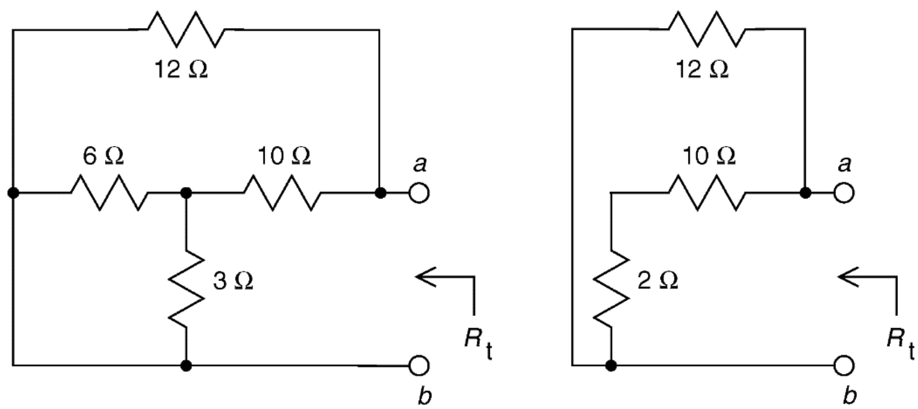
$$12\, i_1 + 10\, i_1 - 6\left(i_2 - i_1\right) = 0$$
$$6\left(i_2 - i_1\right) + 3\, i_2 - 18 = 0$$

$$28\, i_1 = 6\, i_2$$
$$9\, i_2 - 6\, i_1 = 18$$

$$36\, i_1 = 18 \quad \Rightarrow \quad i_1 = \frac{1}{2} \text{ A}$$
$$i_2 = \frac{14}{3}\left(\frac{1}{2}\right) = \frac{7}{3} \text{ A}$$

Using KVL, $$v_{oc} = 3\, i_2 + 10\, i_1 = 3\left(\frac{7}{3}\right) + 10\left(\frac{1}{2}\right) = 12 \text{ V}$$

Find R_t:



$$R_t = \frac{12(10+2)}{12+(10+2)} = 6\ \Omega$$

Then $$\tau = R_t C = 6\left(\frac{1}{24}\right) = \frac{1}{4}\ \text{s} \quad \Rightarrow \quad \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$

and $$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (0-12)e^{-4t} + 12 = 12\left(1-e^{-4t}\right)\ \text{V} \quad \text{for } t \ge 0$$

(checked: LNAP 7/15/04)

P 8.3-20 The circuit shown in Figure P 8.3-20 is at steady state before the switch closes. Determine $i(t)$ for $t \geq 0$.

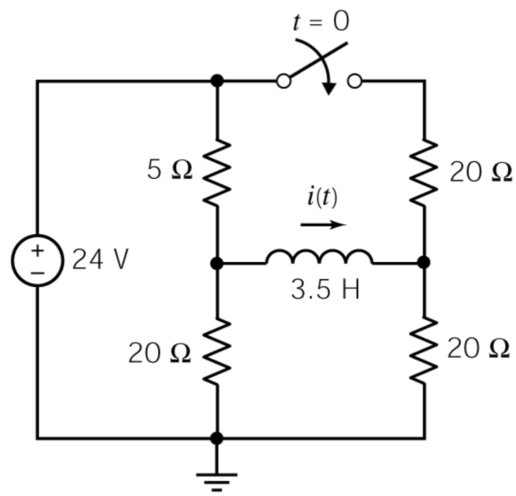
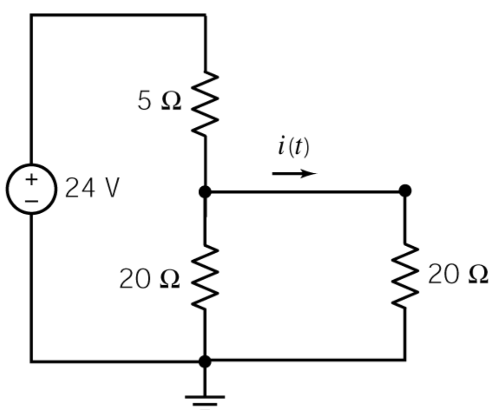


Figure P 8.3-20

Solution: Before the switch closes the circuit is at steady state so the inductor acts like a short circuit. We have

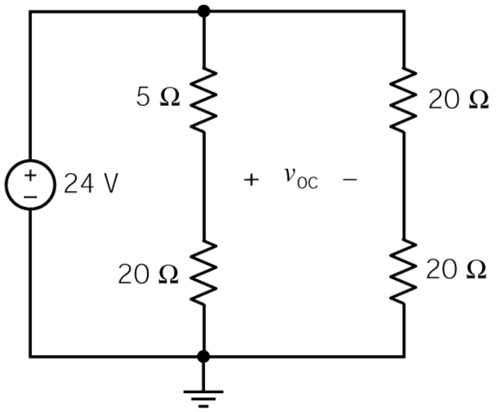


$$i(t) = \frac{1}{2}\left(\frac{24}{5+(20 \,||\, 20)}\right) = 0.8 \text{ A}$$

so

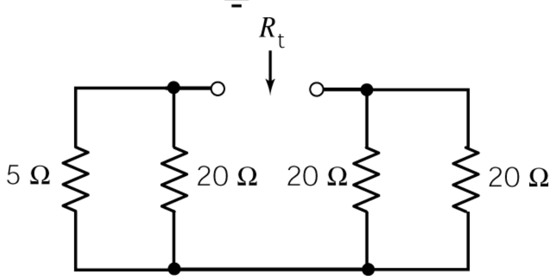
$$i(0+) = i(0-) = 0.8 \text{ A}$$

After the switch closes, find the Thevenin equivalent circuit for the part of the circuit connected to the inductor.



Using voltage division twice

$$v_{oc} = \left(\frac{20}{25} - \frac{1}{2}\right) 24 = 7.2 \text{ V}$$



$$R_t = (5 \,||\, 20) + (20 \,||\, 20) = 14 \ \Omega$$

$$i_{sc} = \frac{v_{oc}}{R_t} = \frac{7.2}{14} = 0.514 \text{ A}$$

Then

$$\tau = \frac{L}{R_t} = \frac{3.5}{14} = \frac{1}{4} \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 4 \ \frac{1}{\text{s}}$$

and

$$i(t) = \left(i(0+) - i_{sc}\right) e^{-t/\tau} + i_{sc} = (0.8 - 0.514) e^{-4t} + 0.514 = 0.286 e^{-4t} + 0.514 \text{ A} \quad \text{for } t \ge 0$$

P8.3-27

The circuit shown in Figure P8.3-27 is at steady state before the switch closes at time t = 0. After the switch closes, the inductor current is given by

$$i(t) = 0.6 - 0.2e^{-5t} \text{ A} \quad \text{for } t \ge 0$$

Determine the values of R_1, R_2 and L.

Answers: R_1 = 20 Ω, R_2 = 10 Ω and L = 5 H

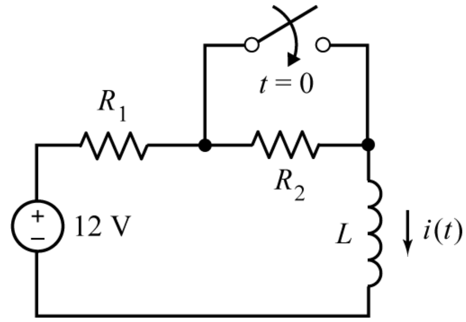
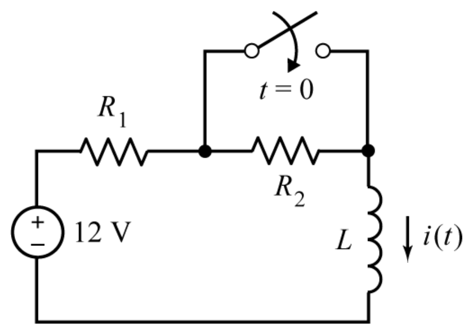


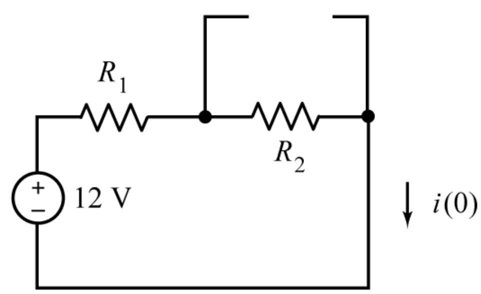
Figure P8.3-27

Solution:



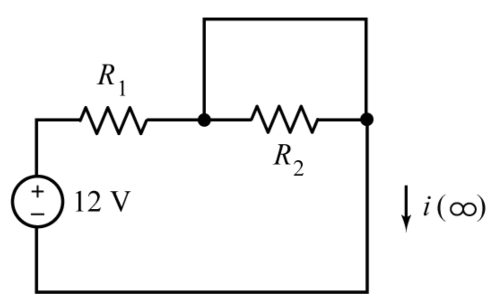
The steady state current before the switch closes is equal to $i(0) = 0.6 - 0.2e^{-5(0)} = 0.4 \text{ A}$.

The inductor will act like a short circuit when this circuit is at steady state so



$$0.4 = i(0) = \frac{12}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 = 30 \ \Omega$$

After the switch has been open for a long time, the circuit will again be at steady state. The steady state inductor current will be $i(\infty) = 0.6 - 0.2e^{-5(\infty)} = 0.6 \text{ A}$



The inductor will act like a short circuit when this circuit is at steady state so

$$0.6 = i(\infty) = \frac{12}{R_1} \quad \Rightarrow \quad R_1 = 20\ \Omega$$

Then $R_2 = 10\ \Omega$.

After the switch is closed, the Thevenin resistance of the part of the circuit connected to the inductor is $R_t = R_1$. Then

$$5 = \frac{1}{\tau} = \frac{R_t}{L} = \frac{R_1}{L} = \frac{20}{L} \quad \Rightarrow \quad L = 4\ \text{H}$$

P 8.4-2 The circuit shown in Figure P 8.4-2 is at steady state before the switch closes at time t = 0. The switch remains closed for 1.5 s and then opens. Determine the inductor current, $i(t)$, for $t > 0$.

Answer: $v(t) = \begin{cases} 2 + e^{-0.5t}\ \text{A} & \text{for } 0 < 1 < 1.5\ \text{s} \\ 3 - 0.53e^{-0..667(t-1.5)}\ \text{A} & \text{for } 1.5\ \text{s} < t \end{cases}$

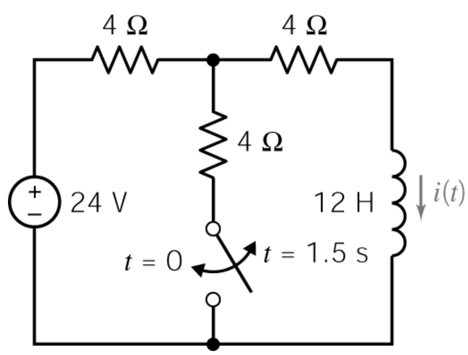
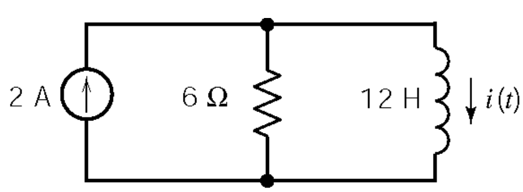


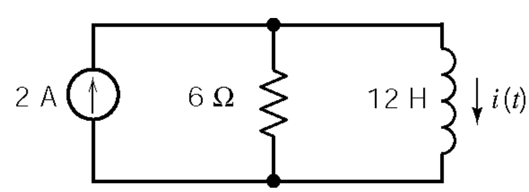
Figure P 8.4-2

Solution:

Replace the part of the circuit connected to the inductor by its Norton equivalent circuit to get:



$t < 0$ and $t > 1.5$ s



$0 < t < 1.5$ s

Before the switch closes at t = 0 the circuit is at steady state so $i(0)$ = 3 A. For 0 < t < 1.5s, i_{sc} = 2 A and R_t = 6 Ω so $\tau = \frac{12}{6} = 2$ s. Therefore

$$i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-t/\tau} = 2 + e^{-0.5t} \text{ A} \quad \text{for } 0 < t < 1.5 \text{ s}$$

At t =1.5 s, $i(1.5) = 2 + e^{-0.5(1.5)} = 2.47$ A. For 1.5 s < t, i_{sc} = 3 A and R_t = 8 Ω so $\tau = \frac{12}{8} = 1.5$ s.

Therefore

$$i(t) = i_{sc} + (i(1.5) - i_{sc})\, e^{-(t-1.5)/\tau} = 3 - 0.53\, e^{-0.667(t-1.5)} \text{ V} \quad \text{for } 1.5 \text{ s} < t$$

Finally

$$i(t) = \begin{cases} 2 + e^{-0.5\,t} \text{ A} & \text{for } 0 < t < 1.5 \text{ s} \\ 3 - 0.53\, e^{-0.667(t-1.5)} \text{ A} & \text{for } 1.5 \text{ s} < t \end{cases}$$