1. PCA is a procedure that always reduces the dimensionality of the data. That is, it always reduces the number of features in the data. True or false. Explain in one sentence. (2 points)

2. For the following matrix, decide which, if any, of the following vectors are eigenvectors and give the corresponding eigenvalue. (4 points)

$$\begin{bmatrix} 3 & 0 & 1 \\ -4 & 1 & 2 \\ -6 & 0 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$(1) \quad (2) \quad (3) \quad (4) \quad (5)$$

3. An eigenvector is a vector whose values remain unchanged when a linear transformation is applied to it. True or false. Explain. (2 points)

4. I have the following eigenvalues corresponding to eigenvectors of the covariance matrix:  $\lambda_1 = 10, \ \lambda_2 = 6, \ \lambda_3 = 4, \ \lambda_4 = 3, \ \lambda_5 = 2$ If I want to explain at least 90% of the variance in my data, how many eigenvectors will I use? (2 points)

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$$(1) \quad (2) \quad (3) \quad (4) \quad (5)$$

3. Find the covariance matrix for the following data. Do not perform standardization. (4 points)  $s_1 = (-1, -1), s_2 = (-1, 1), s_3 = (1, -1), s_4 = (1, 1).$