pl for CFL

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Two cases:

prove $L = \{a^n b^{n+1} c^n : n \geq 0\}$ is not CFL.

Assume L is CFL $\Rightarrow \forall s \in L$, $|s| \geq p$ (pos int)
$s = uvxyz \Rightarrow |vxy| \leq p$, $|vy| \geq 1$, $s_i = uv^i x y^i z$
$\forall i \geq 0 \Rightarrow s_i \in L$.

let $s = a^p b^{p+1} c^p$ $\quad s \in L$ ✓ $\quad |s| = 3p+1 \geq p$ ✓

vy:
case 1: $a^k b^j$ $\quad 0 \leq k \leq p$, $0 \leq j \leq p$, $1 \leq k+j \leq p$
case 2: $b^k c^j$ $\quad 0 \leq k \leq p$, $0 \leq j \leq p$; $1 \leq k+j \leq p$

case 1: $s = a^k a^{p-k} b^j b^{p+1-j} c^p$
$s_i = a^{ki} a^{p-k} b^{ji} b^{p+1-j} c^p$

let $i = 0$
$s_0 = a^{p-k} b^{p+1-j} c^p$

if $k = 0 \Rightarrow j \geq 1$ $\quad \therefore n_b(s_0) \neq n_c(s_0)+1$
else if $j = 0 \Rightarrow k \geq 1$ $\quad \therefore n_a(s_0) \neq n_c(s_0)$
else if $k, j \geq 1$ $\quad \therefore n_a(s_0) \neq n_c(s_0)$
$\therefore s_0 \notin L$

case 2: $s = a^p b^k b^{p+1-k} c^j c^{p-j}$
$s_i = a^p b^{ki} b^{p+1-k} c^{ji} c^{p-j}$

let $i = 0$
$s_0 = a^p b^{p+1-k} c^{p-j}$

if $k = 0 \Rightarrow j \geq 1$ $\quad \therefore n_c(s_0) \neq n_a(s_0)$
else if $j = 0 \Rightarrow k \geq 1$ $\quad \therefore n_b(s_0) \neq n_a(s_0)+1$
else if $j, k \geq 1$ $\quad \therefore n_a(s_0) \neq n_c(s_0)$
$\therefore s_0 \notin L$

since in all possible decompositions
$s_i \notin L \Rightarrow L$ is not CFL.

prove $L = \{n_a(w) = n_b(w) \geq n_c(w) : w \in \{a,b,c\}^*\}$ is not CFL

Assume L is CFL $\Rightarrow \forall s \in L$, $|s| \geq p$ (pos int)
$s = uvxyz \Rightarrow |vxy| \leq p$, $|vy| \geq 1$ and
$s_i = uv^i x y^i z \in L$ $\forall i \geq 0$.

let $s = a^p b^p c^p$

vy:
case 1: $a^k b^j$ $\quad 0 \leq k \leq p$, $0 \leq j \leq p$, $1 \leq k+j \leq p$
$s = a^k a^{p-k} b^j b^{p-j} c^p$
$s_i = a^{ki} a^{p-k} b^{ji} b^{p-j} c^p$

let $i = 0$
$s_0 = a^{p-k} b^{p-j} c^p$

if $k = 0 \Rightarrow j \geq 1$ $\quad \therefore n_a(s_0) \neq n_b(s_0)$
~~if~~ if $j = 0 \Rightarrow k \geq 1$ $\quad \therefore n_b(s_0) \neq n_a(s_0)$
else $j, k \geq 1$ $\quad \therefore n_a/n_b(s_0) \ngeq n_c(s_0)$

case 2: $b^k c^j$ $\quad 0 \leq k \leq p$, $0 \leq j \leq p$, $1 \leq k+j \leq p$
$s = a^p b^k b^{p-k} c^j c^{p-j}$
$s_i = a^p b^{ki} b^{p-k} c^{ji} c^{p-j}$

~~let $i = 0$~~
~~$s_0 = a^p b^{p-k} c^{p-j}$~~
~~if $k = 0 \Rightarrow j \geq 1$ $\therefore \ldots$~~

let $i = 2$
$s_2 = a^p b^{2k} b^{p-k} c^{2j} c^{p-j}$
$= a^p b^{p+k} c^{p+j}$

if $k = 0 \Rightarrow j \geq 1$ $\quad \therefore n_c(s_2) \nleq n_a/n_b(s_2)$
if $j = 0 \Rightarrow k \geq 1$ $\quad \therefore n_a(s_2) \neq n_b(s_2)$
else $j, k \geq 1$ $\quad \therefore n_a(s_2) \neq n_b(s_2)$
$\therefore s_2 \notin L$

so for all decomps of s, $s_i \notin L$
$\therefore L$ is not CFL.

(1) prove $L = \{n_a(w) > n_b(w) > n_c(w) : w \in \{a,b,c\}^*\}$
is not CFL.

For fun: $L = \{n_a(w) = n_b(w) \neq n_c(w)\}$

(2) prove $L = \{n_a(w) < n_b(w) < n_c(w) : w \in \{a,b,c\}^*\}$
is not CFL