.pumping lemma again

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review: pre-processing

articulate language characteristics

$L_1 = \{a^n b^n : n \geq 0\}$

$L_2 = \{w \in \{a,b\}^* : n_a(w) \geq n_b(w)\}$

$L_3 = \{ww^R : w \in \{a,b\}^*\}$

$L_4 = \{a^n b^m : n \neq m\}$

list some strings in L.

$L_1 = \{\lambda, ab, aabb, aaabbb, \ldots\}$

$L_2 = \{a, aa, aab, ab, aba, ba, babaaaa, \ldots\}$

$L_3 = \{aa, \lambda, abba, babaabab, \ldots\}$

$L_4 = \{a, b, aab, abb, \ldots\}$

state your goal ⇒ contradiction

$L_1 = n_a(s_i) \neq n_b(s_i)$

$L_2 = n_a(s_i) < n_b(s_i)$

$L_3 =$ w is not concatenated w/ w^R

$L_4 = n_a(s_i) = n_b(s_i)$

order of processing

1. Assume L is regular.
2. pumping lemma for r.l. holds ie,
3. pick a string $\in L$; $|string| \geq p$.

you pick $s \in L$, $|s| \geq p$

L_1: $s = a^p b^p$ $\quad s = a^{p+1} b^{p+1}$ $\quad s = a^{2p} b^{2p}$ $\quad \cancel{s = a^{p/2} b^{p/2}}$ $\quad \cancel{s = a^{p-1} b^{p-1}}$

L_2: $s = a^{p+1} b^p$ $\quad s = a^p b^p$

L_3: $s = a^p b^p b^p a^p$ $\quad s = a^p bb a^p$

L_4: $s = a^{p!} b^{(p+1)!}$

4. show all possible decompositions of s given $|xy| \leq p$, $|y| \geq 1$.

L_1: $y = a^k \quad 1 \leq k \leq p$

L_2: $y = a^k \quad 1 \leq k \leq p$

L_3: $y = a^k \quad 1 \leq k \leq p$ $\qquad \underbrace{a\,aa \ldots a}_{p} \mid bb \underbrace{aaa \ldots a}_{p}$

L_4: $y = a^k \quad 1 \leq k \leq p$ $\qquad \underbrace{aaa \ldots a}_{p!} \underbrace{bbb \ldots bbb}_{(p+1)!}$

5. show s, s_i

L_1: $s = \underbrace{a^k}_{y} \underbrace{a^{p-k} b^p}_{!y}$

$s_i = a^{ki} a^{p-k} b^p$

L_2: $s = \underbrace{a^k}_{y} \underbrace{a^{p-k} b^p}_{!y}$

$s_i = a^{ki} a^{p-k} b^p$

L_3: $s = a^k a^{p-k} bb a^p$

$s_i = a^{ki} a^{p-k} bb a^p$

L_4: $s = \underbrace{a^k}_{y} a^{p!-k} b^{(p+1)!}$

$s_i = a^{ki} a^{p!-k} b^{(p+1)!}$

you pick an i

6. L_1: $i = 2$

L_2: $i = 0$

L_3: $i = 0$

L_4: find pos int $i \ni n_a(s_i) = n_b(s_i)$

$n_a(s_i) = ki + p! - k$

$n_b(s_i) = (p+1)!$

$ki - k + p! = (p+1)!$

$k(i-1) + p! = (p+1)!$

$k(i-1) = (p+1)! - p!$

$k(i-1) = (p+1)p! - p!$

$k(i-1) = p!\,(p+1-1)$

$i - 1 = \frac{p!\,(p)}{k}$

$i = \frac{p!\,(p)}{k} + 1$ $\qquad [\ p! = p \cdot (p-1) \cdot (p-2) \cdot k \cdot 1$

$i = \frac{p!}{k}(p) + 1$

$=$ int $\cdot$ int $+$ int

$=$ int $+$ int

$i =$ int pos int.

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