pumping lemma examples

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proof by contradiction:

1. Assume L is regular
2. $\Rightarrow$ state the pumping lemma
3. * pick string $\in L$, $|s| \geq p$
4. show all possible decompositions of s (ie, $y = ?$) based $|xy| \leq p$, $|y| \geq 1$
5. * pick i - how many times pumping where $i \geq 0$
6. pump up or down based on your i.
7. show pumped string $\notin L$
8. $\therefore$ L must <u>not</u> be regular.

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ex show $L = \{a^n b^n : n \geq 0\}$ is not regular.

Assume L is regular $\Rightarrow \forall s \in L$, $|s| \geq p$
(p = pos. int. called pumping length),
$s = xyz$, $|xy| \leq p$, $|y| \geq 1$, and

$\rightarrow$ $s_i = xy^iz \in L \quad \forall i \geq 0.$

let $s = a^p b^p$ $s \in L$ ✓, $|s| = p + p = 2p \geq p$. ✓

$s = a^{2p} b^{2p}$
$s = a^{p+2r} b^{p+2r}$

aside:

$a\,a\,a\,a\,a\;b\,b\,b\,b\,b$

$x \rightarrow y \rightarrow z$

$\leq p$

≥ 1

since $|xy| \leq p$, $|y| \geq 1$

$\Rightarrow$ $y = a^k$ $1 \leq k \leq p$ $x = a^m$ $0 \leq m \leq p-1$

$\Rightarrow$ $s = \underbrace{a^m}_{x} \underbrace{a^k}_{y} \underbrace{a^{p-m-k} b^p}_{z}$

$s_i = a^m a^{ki} a^{p-m-k} b^p$

let $i = 0$

$s_0 = a^m a^{p-m-k} b^p = a^{p-k} b^p$

$\Rightarrow$ since $k \geq 1$ $\Rightarrow n_a(s_0) < n_b(s_0)$

$\Rightarrow$ $s_0 \notin L$ because $\# a \neq \# b$

$\therefore$ L must not be regular!

or

$s_i = a^m a^{ki} a^{p-m-k} b^p$

let $i = 2$

$s_2 = a^m a^{2k} a^{p-m-k} b^p = a^{p+k} b^p$

since $k \geq 1 \Rightarrow n_a(s_2) > n_b(s_2)$

$\therefore$ $s_2 \notin L \Rightarrow L$ is not regular.

$s = \underbrace{a^k}_{y} \underbrace{a^{p-k} b^p}_{y}$

$s_i = a^{ki} a^{p-k} b^p$

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ex show $L = \{w \in \{a,b\}^* : n_a(w) \neq n_b(w)\}$

⋮

let $s = a^p b^{p+1}$ $s \in L$ ✓, $|s| = p + p + 1 = 2p + 1 \geq p$ ✓

⋮

$y = a^k$ $1 \leq k \leq p$