

$$M = (\mathcal{Q}, \Sigma, \Gamma, \delta, q_0, z, F)$$

- \mathcal{Q} - finite set of internal states
- Σ - input alphabet (finite set of symbols)
- Γ - stack alphabet (finite set of symbols)
- δ - finite set of transition fnc(s)
 $(\mathcal{Q} \times \{\Sigma \cup \lambda\} \times \Gamma^*) \rightarrow (\mathcal{Q} \times \Gamma^*)$
↑ required to pop ↑ can be a string or λ or 1 symbol

q_0 - initial state, $\in \mathcal{Q}$

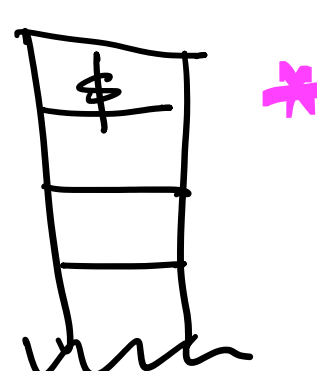
z - stack start symbol, $\in \Gamma$

F - finite set of internal final state(s), $\subset \mathcal{Q}$

$$\text{npda} : \delta(q_i, a, b) = \{ \underset{1.}{(q_j, d)}, \underset{2.}{(q_k, \lambda)} \}$$

ex $L = \{ w \mid n_a(w) = n_b(w) : w \in \{a, b\}^* \}$

$$\neg (q_0, \lambda, \lambda) = (q_1, \#) \Rightarrow$$



$$\begin{aligned} (q_1, a, \#) &= (q_1, a\#) \\ (q_1, a, a) &= (q_1, aa) \\ (q_1, a, b) &= (q_1, \lambda) \\ (q_1, b, \#) &= (q_1, b\#) \\ (q_1, b, b) &= (q_1, bb) \\ (q_1, b, a) &= (q_1, \lambda) \\ (q_1, \lambda, \#) &= (q_f, \lambda) = \end{aligned}$$

instantaneous description:

$$s = aabb \quad (q_1, aabb, \#) \vdash (q_1, abb, a\#) \vdash (q_1, bb, aa\#)$$

$$\vdash (q_1, b, a\#) \vdash (q_1, \lambda, \#) \vdash (q_f, \lambda, \lambda)$$

try: $s = a; baab; b; bba; \lambda; aabbab; \dots$

ex $L = \{ a^n b^n : n \geq 1 \}$

$$\begin{aligned} (q_0, a, \#) &= (q_0, a\#) \quad // \text{1st 'a'} \\ (q_0, a, a) &= (q_0, aa) \quad // \text{subsequent 'a's} \\ (q_0, b, a) &= (q_1, \lambda) \quad // \text{1st 'b'} \\ (q_1, b, a) &= (q_1, \lambda) \quad // \text{subsequent 'b's} \\ (q_1, \lambda, \#) &= (q_f, \lambda) \end{aligned}$$

wait!

$$\begin{aligned} (q_0, a, \#) &= (q_0, a\#) \\ (q_0, a, a) &= (q_0, aa) \\ (q_0, b, a) &= (q_0, \lambda) \\ (q_0, \lambda, \#) &= (q_f, \lambda) \end{aligned}$$

$s = aabb$

$$(q_0, aabb, \#) \vdash (q_0, abb, a\#) \vdash (q_0, bb, aa\#)$$

$$\vdash (q_0, b, a\#) \vdash (q_0, \lambda, \#) \vdash (q_f, \lambda, \lambda) \checkmark$$

$s = \lambda$

$$(q_0, \lambda, \#) \vdash (q_f, \lambda, \lambda) \times$$

$s = aabbab$

$$(q_0, aabbab, \#) \vdash (q_0, abbab, a\#) \vdash (q_0, bbab, aa\#)$$

$$\vdash (q_0, bab, a\#) \vdash (q_0, ab, \#) \vdash (q_0, b, a\#)$$

$$\vdash (q_0, \lambda, \#) \vdash (q_f, \lambda, \lambda) \times$$

wait wait!

$$(q_0, a, \#) = (q_1, a\#)$$

$$(q_1, a, a) = (q_1, aa)$$

$$(q_1, b, a) = (q_1, \lambda)$$

$$(q_1, \lambda, \#) = (q_f, \lambda)$$

$s = aabbab$

$$(q_0, aabbab, \#) \vdash (q_1, abbab, a\#) \vdash (q_1, bbab, aa\#)$$

$$\vdash (q_1, bab, a\#) \vdash (q_1, ab, \#) \times$$

try:

$$L = \{ a^n b^n : n \geq 0 \}$$

$$L = \{ a^n b^{n+1} : n \geq 0 \}$$