pumping lemma CFL examples

Thursday, November 14, 2024 12:02 PM

pumping lemma CFL:

$\forall s \in L$, $|s| \geq p$ (pos int), $s = uvxyz$,
$|vxy| \leq p$, $|vy| \geq 1$, $\Rightarrow s_i = uv^ixy^iz \in L \ \forall i \geq 0$.

$L_1 = \{a^{2n} b^{2m} : n, m \geq 0\}$ regular - proved by creating nfa.

$L_2 = \{ n_a(w) = 2n ; n_b(w) = 2m : n, m \geq 0\}$
$w \in \{a, b\}^*$

$\left(\cancel{(a+b)(a+b)}\right)^*$
regular - proved by creating nfa.

$L_3 = \{a^n b^{n+1} c^n, n \geq 0\}$ CFL?? prove not using pumping lemma CFL.

Assume L_3 is CFL. $\Rightarrow \forall s \in L_3$, $|s| \geq p$ $s = uvxyz$
$\ni |vxy| \leq p$, $|vy| \geq 1 \Rightarrow s_i = uv^ixy^iz \in L_3 \ \forall i \geq 0$.

let $s = a^p b^{p+1} c^p$ $\quad |s| = p+p+1+p = 3p+1 \geq p$ ✓
$s \in L_3$ ✓

$vy = ?$
case 1: $vy = a^k$ $\quad 1 \leq k \leq p$
case 2: $vy = b^k$ $\quad 1 \leq k \leq p$
case 3: $vy = c^k$ $\quad 1 \leq k \leq p$
case 4: $vy = a^k b^j$ $\quad 1 \leq k \leq p$, $1 \leq j \leq p$, $2 \leq k+j \leq p$
case 5: $vy = b^k c^j$ $\quad 1 \leq k \leq p$, $1 \leq j \leq p$; $2 \leq k+j \leq p$

Case 1: $s = a^k a^{p-k} b^{p+1} c^p$
$s_i = a^{ki} a^{p-k} b^{p+1} c^p$
let $i = 0$
$s_0 = a^{p-k} b^{p+1} c^p$ $\quad$ since $k \geq 1 \Rightarrow$
$n_a(s_0) \neq n_c(s_0)$
and $n_a(s_0) + 1 \neq n_b(s_0)$

$\therefore s_0 \notin L_3$

Case 2: $s = a^p b^k b^{p+1-k} c^p$
$s_i = a^p b^{ki} b^{p+1-k} c^p$
let $i = 0$
$s_0 = a^p b^{p+1-k} c^p$ $\quad$ since $k \geq 1$
$\Rightarrow n_b(s_0) \leq n_a(s_0) / n_c(s_0)$
$\therefore s_0 \notin L_3$

Case 3: Do on own time.

Case 4: $s = a^k a^{p-k} b^j b^{p+1-j} c^p$
$s_i = a^{ki} a^{p-k} b^{ji} b^{p+1-j} c^p$

let $i = 0$
$s_0 = a^{p-k} b^{p+1-j} c^p$
$\Rightarrow$ since $k \geq 1$, $\Rightarrow n_a(s_0) \neq n_c(s_0)$
$j \geq 1$ and $n_b(s_0) \leq n_c(s_0)$

Case 5: $s = a^p b^k b^{p+1-k} c^j c^{p-j}$
$s_i = a^p b^{ki} b^{p+1-k} c^{ji} c^{p-j}$

let $i = 0$
$s_0 = a^p b^{p+1-k} c^{p-j}$

$\Rightarrow$ since $k \geq 1 \Rightarrow n_b(s_0) \leq n_a(s_0)$
$j \geq 1 \Rightarrow n_a(s_0) \neq n_c(s_0)$
$\therefore s_0 \notin L_3$

$\therefore$ since in all possible decompositions (cases 1-5)
$s_i \notin L_3 \Rightarrow L_3$ is not CFL.

prove L_3 is not CFL w/ 2 cases.

blah, blah

case 1: $vy = a^k b^j$ $\quad 0 \leq k \leq p$, $0 \leq j \leq p$, $1 \leq k+j \leq p$

case 2: $vy = b^k c^j$ $\quad 0 \leq k \leq p$, $0 \leq j \leq p$, $1 \leq k+j \leq p$

will finish next time.