University of Nevada – Reno
Computer Science & Engineering
Department

CS454/654 Reliability and Security
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Lecture 12

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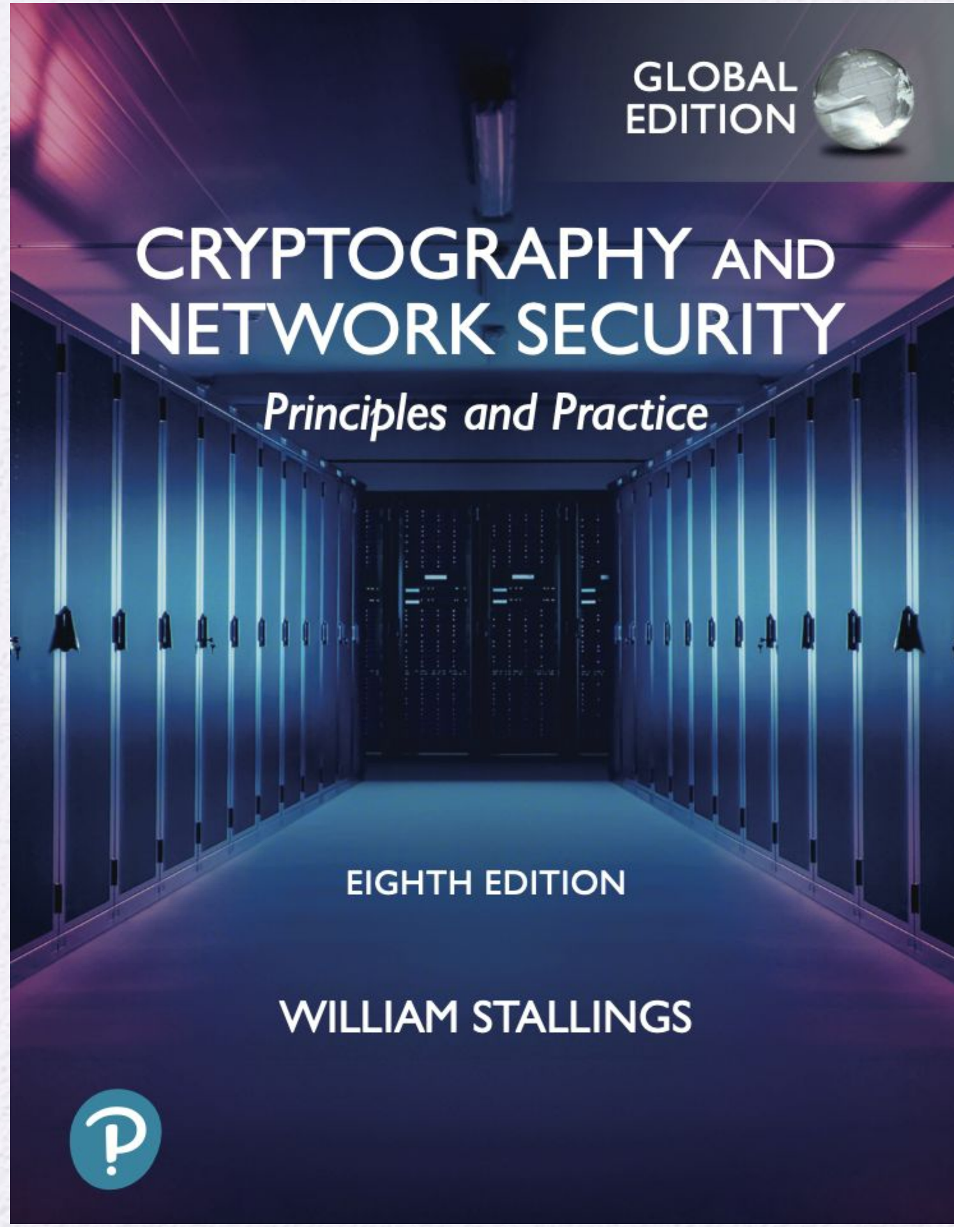

CHAPTER 6

ADVANCED ENCRYPTION STANDARD

6.1 Finite Field Arithmetic

6.2 AES Structure

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Appendix 6A Polynomials with Coefficients in GF(2^8)

Advanced Encryption Standard (AES)

- The Advanced Encryption Standard (AES) was published by the National Institute of Standards and Technology (NIST) in 2001.
- AES is a symmetric block cipher that is intended to replace Data Encryption Standard (DES) as the approved standard for a wide range of applications.
- Key length 16, 24, 32, bytes (128, 192, or 256 bits) referred to as AES-128, AES-192, or AES-256, depending on the key length.
- Input plaintext is 16 bytes (128 bits).

Table 6.1 AES Parameters

Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240

The key (16 bytes) that is provided as input is expanded into an array of 44 words (each of which is 4 bytes).

Both Encryption/Decryption starts with AddRoundKey, followed by set of Rounds.

For each round, 4 following stages are performed:

- Substitute bytes
- ShiftRows
- MixColumns
- AddRoundKey

Only AddRoundKey makes use of the key.

No. of rounds	Key Length (bytes)
10	16
12	24
14	32

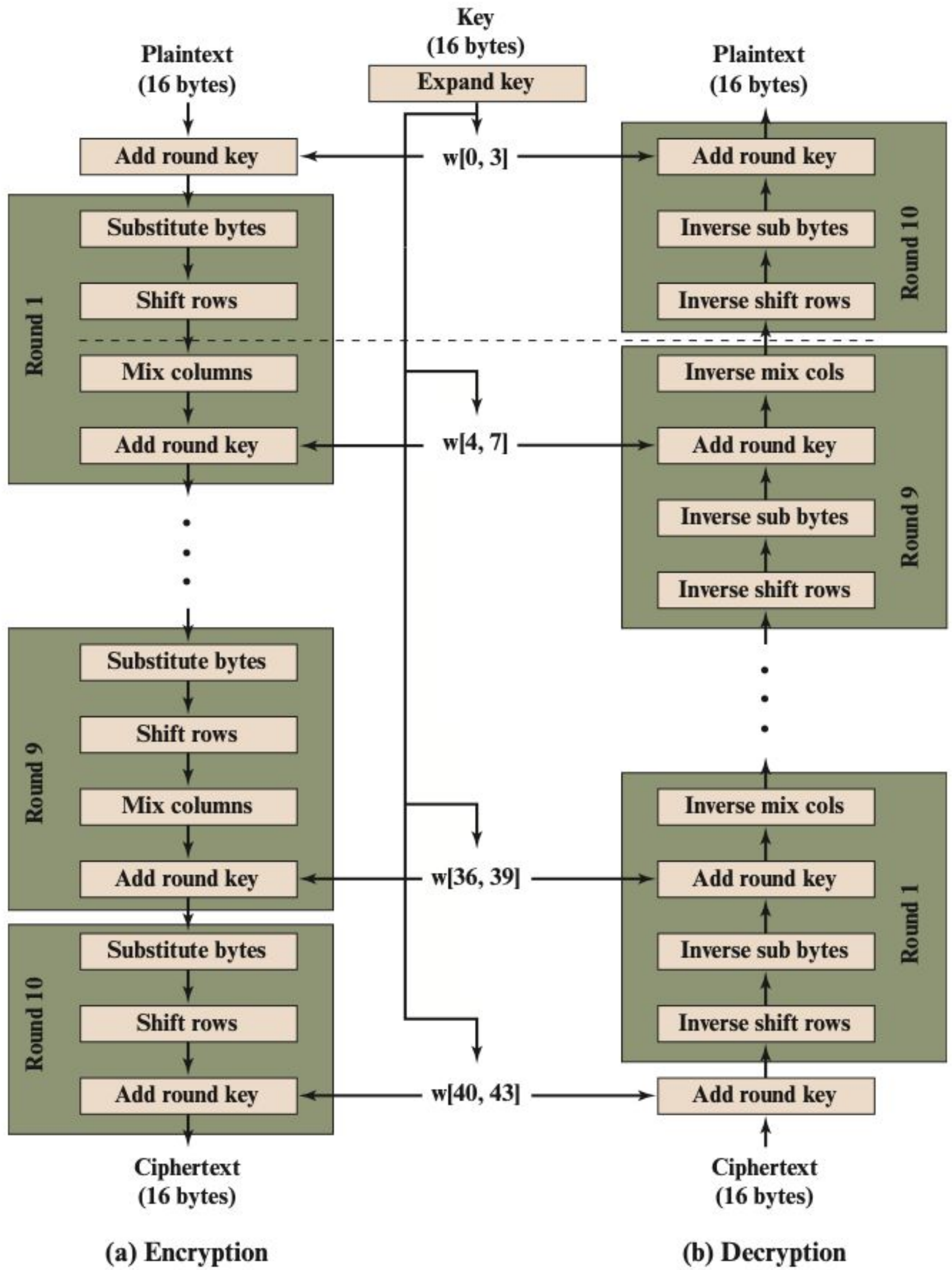


Figure 6.3 AES Encryption and Decryption

Key Expansion Algorithm

Takes as input a 4-word (16-byte) key and produces a linear array of 44 words each 4 bytes and total is 176 bytes.

Table 6.1 AES Parameters

Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240

The input key of size 16-byte converted into matrix of 4x4.

k_0	k_4	k_8	k_{12}
k_1	k_5	k_9	k_{13}
k_2	k_6	k_{10}	k_{14}
k_3	k_7	k_{11}	k_{15}

```
KeyExpansion (byte key[16], word w[44])
{
    word temp
    for (i = 0; i < 4; i++) w[i] = (key[4*i], key[4*i+1],
                                    key[4*i+2],
                                    key[4*i+3]);
    for (i = 4; i < 44; i++)
    {
    temp = w[i - 1];
    if (i mod 4 = 0) temp = SubWord (RotWord (temp))
                                ⊕ Rcon[i/4];
    w[i] = w[i-4] ⊕ temp
    }
}
```

iteration	word
0	EA D2 73 21
1	B5 8D BA D2
2	31 2B F5 60
3	7F 8D 29 2F

RotWord performs a one-byte circular left shift on a word. Input word [B0, B1, B2, B3] is transformed into [B1, B2, B3, B0].

SubWord performs a byte substitution on each byte of its input word, using the S-box (Table 6.2a - given in next slide).

The result of the two from **RotWord** and **SubWord** is XORed with a round constant, Rcon[j].

The leftmost 4 bits of the byte are used as a row value and the rightmost 4 bits are used as a column value.

The hexadecimal value {95} references row 9, column 5 of the S-box, which contains the value {2A}.

EA	04	65	85
83	45	5D	96
5C	33	98	B0
F0	2D	AD	C5

→

87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

Table 6.2 AES S-Boxes

x \ y	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

(a) S-box

x \ y	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

(b) Inverse S-box

```
KeyExpansion (byte key[16], word w[44])
{
    word temp
    for (i = 0; i < 4; i++) w[i] = (key[4*i], key[4*i+1],
                                     key[4*i+2],
                                     key[4*i+3]);
    for (i = 4; i < 44; i++)
    {
    temp = w[i - 1];
    if (i mod 4 = 0) temp = SubWord (RotWord (temp))
                              ⊕ Rcon[i/4];
    w[i] = w[i-4] ⊕ temp
    }
}
```

The result of the two from **RotWord** and **SubWord** is XORed with a round constant, Rcon[j].

Three rightmost bytes of round constant is always 0. Thus, Rcon[j]=[RC[j], 0, 0, 0]

RC[1]=1, RC[j]=2*RC[j-1] a multiplication defined over the field GF(2^8).

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

Values for 16-byte key

No. of rounds	Key Length (bytes)
10	16
12	24
14	32

Note, in j=5 the RC[j]= 0x10 (not integer 10)
When RC[9] is computed from 2*RC[8] the reduction is needed.
2*RC[8]= 0001 0000 0000 (but we have only 8 bits so reduction is needed). The irreducible polynomial used in AES is $m(x) = x^8 + x^4 + x^3 + x + 1$ which is 0x11B (1 0001 1011)

```
KeyExpansion (byte key[16], word w[44])
{
    word temp
    for (i = 0; i < 4; i++) w[i] = (key[4*i], key[4*i+1],
                                     key[4*i+2],
                                     key[4*i+3]);
    for (i = 4; i < 44; i++)
    {
    temp = w[i - 1];
    if (i mod 4 = 0) temp = SubWord (RotWord (temp))
                                ⊕ Rcon[i/4];
    w[i] = w[i-4] ⊕ temp
    }
}
```

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

When RC[9] is computed from 2*RC[8] the reduction is needed.
2*RC[8]= 0001 0000 0000 (but we have only 8 bits so reduction is needed). The irreducible polynomial used in AES is $m(x) = x^8 + x^4 + x^3 + x + 1$ which is 0x11B (1 0001 1011)

2*RC[8]= 0001 0000 0000 = $x^8 = f(x)$ $m(x)= x^8 + x^4 + x^3 + x + 1$

$$x^8 \bmod m(x) = [m(x) - x^8] = (x^4 + x^3 + x + 1)$$

$x^4 + x^3 + x + 1$ = 0001 1011 = 0x1B (a value of RC[9]).

Key EA D2 73 21 B5 8D BA D2 31 2B F5 60 7F 8D 29 2F

Iteration is i=36 meaning we are generating the word 36

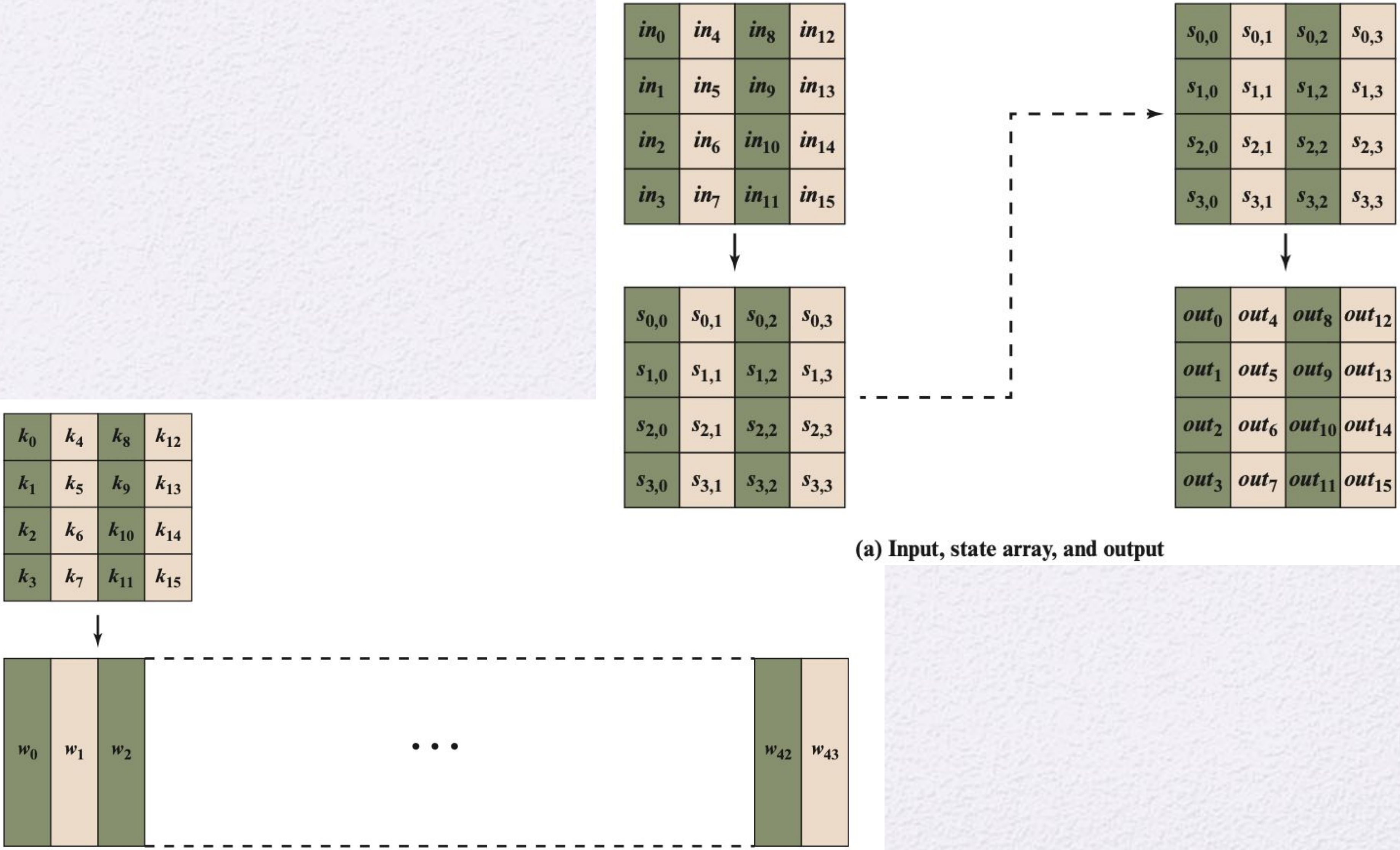
Table 6.3 Example Round Key Calculation

Description	Value
i (decimal)	36
temp = w[i − 1]	7F8D292F
RotWord (temp)	8D292F7F
SubWord (RotWord (temp))	5DA515D2
Rcon (9)	1B000000
SubWord (RotWord (temp)) ⊕ Rcon (9)	46A515D2
w[i − 4]	EAD27321
w[i] = w[i − 4] ⊕ SubWord (RotWord (temp)) ⊕ Rcon (9)	AC7766F3

```
KeyExpansion (byte key[16], word w[44])
{
    word temp
    for (i = 0; i < 4; i++) w[i] = (key[4*i], key[4*i+1],
                                    key[4*i+2],
                                    key[4*i+3]);
    for (i = 4; i < 44; i++)
    {
    temp = w[i - 1];
    if (i mod 4 = 0) temp = SubWord (RotWord (temp))
                                ⊕ Rcon[i/4];
    w[i] = w[i-4] ⊕ temp
    }
}
```

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

Input plaintext 16 bytes converted to 4x4 square of bytes and copied to State array, modified at each step and after final iteration copied to output matrix.

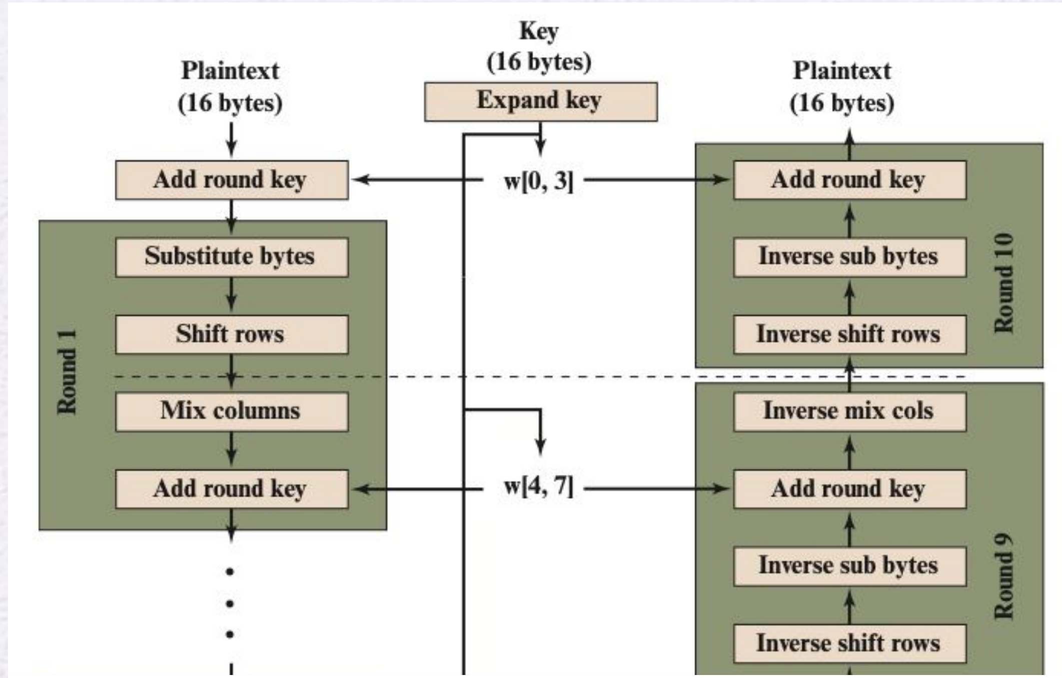


(b) Key and expanded key

Figure 6.2 AES Data Structures

AddRoundKey Transformation

The 16-bytes of State array are bitwise XORed with the 16-bytes the round key.



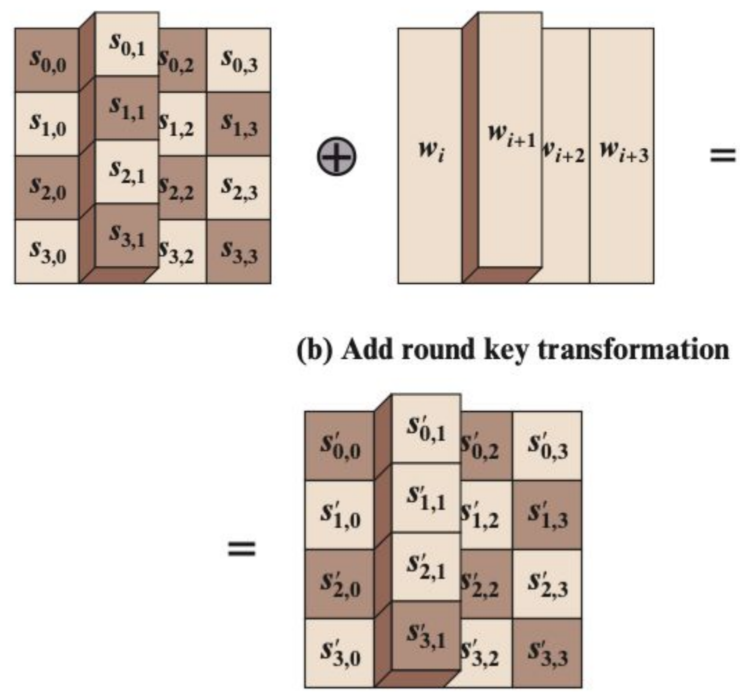
47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

$\oplus$

AC	19	28	57
77	FA	D1	5C
66	DC	29	00
F3	21	41	6A

=

EB	59	8B	1B
40	2E	A1	C3
F2	38	13	42
1E	84	E7	D6



(b) Add round key transformation

Substitute bytes.

The leftmost 4 bits of the byte are used as a row value and the rightmost 4 bits are used as a column value.

the hexadecimal value {95} references row 9, column 5 of the S-box, which contains the value {2A}

EA	04	65	85			87	F2	4D	97
83	45	5D	96			EC	6E	4C	90
5C	33	98	B0	→		4A	C3	46	E7
F0	2D	AD	C5			8C	D8	95	A6

There also exist inverse S-Box for decryption.

Rationale: The S-box is designed to be resistant to known cryptanalytic attacks. Specifically, there should be a low correlation between input bits and output bits and and output should not be the linear function of input.

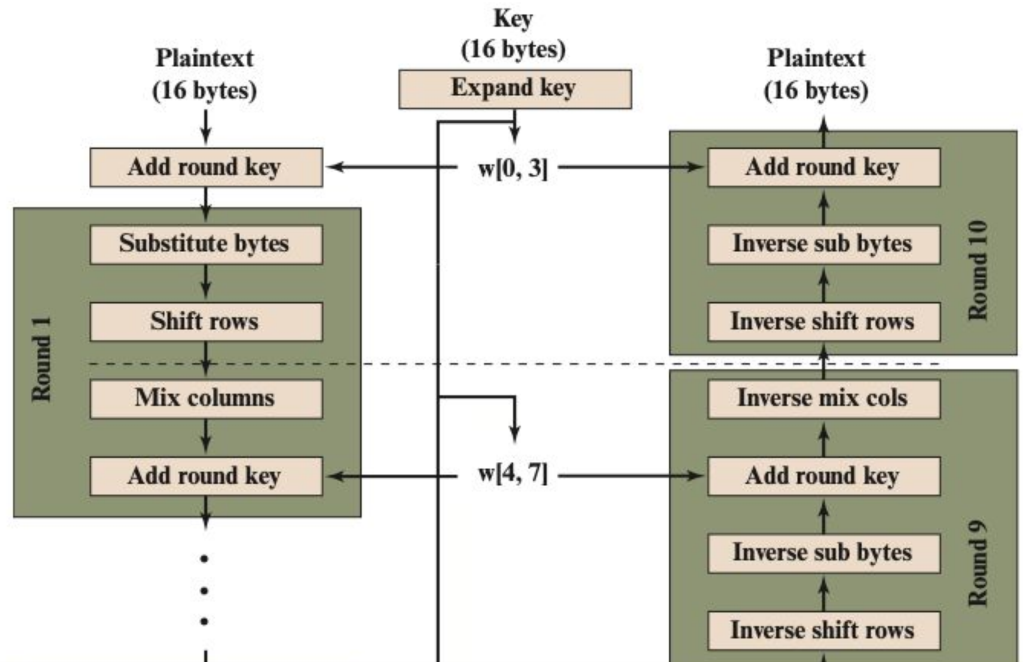


Table 6.2 AES S-Boxes

x \ y	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

(a) S-box

ShiftRows Transformation

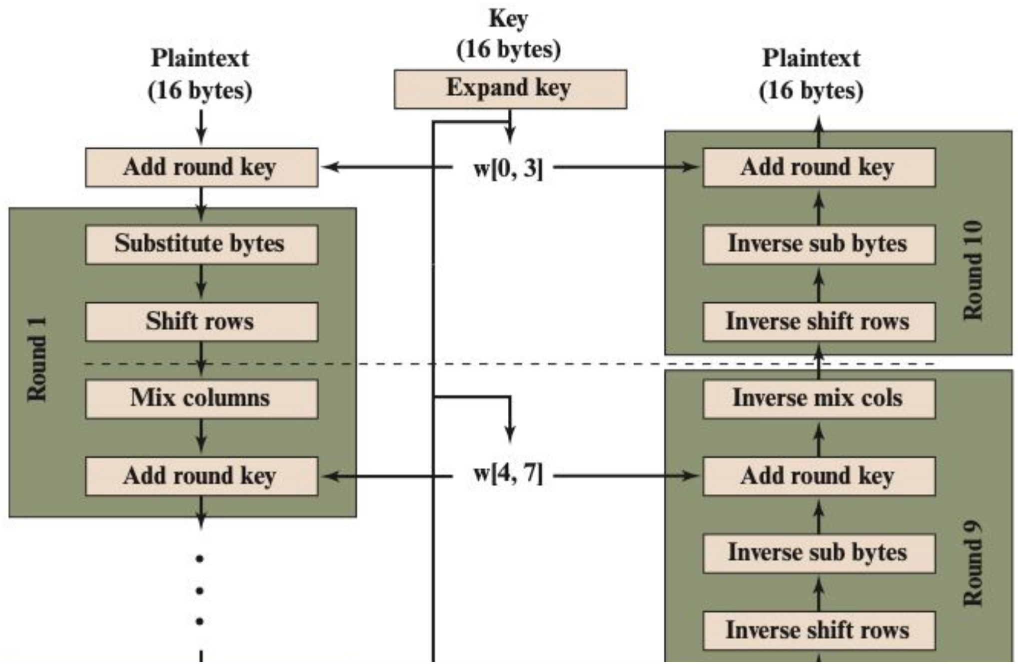
For the 1st row of State is not altered.
For the 2nd row, a 1-byte circular left shift is performed.
For the 3rd row, a 2-byte circular left shift is performed.
For the 4th row, a 3-byte circular left shift is performed.

87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

→

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

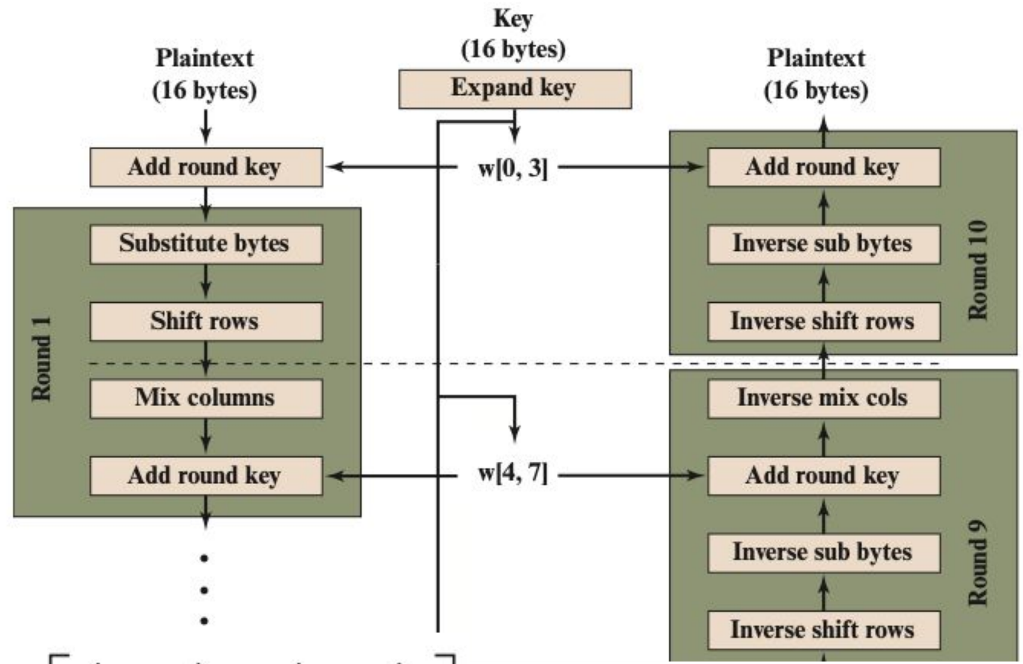
Inverse shift row transformation performs the circular shifts in the opposite direction.



Rationale: transformation ensures that the 4 bytes of one column are spread out to four different columns (diffusion)

MixColumns Transformation

Each byte of a column is mapped into a new value that is a function of all four bytes in that column.



The transformation can be defined by the following matrix multiplication on State.

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix}\begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

$$\begin{aligned} s'_{0,j} &= (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j} \\ s'_{1,j} &= s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j} \\ s'_{2,j} &= s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j}) \\ s'_{3,j} &= (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j}) \end{aligned}$$

Individual addition and multiplication are performed in GF(2^8) field.

Inverse MixColumns Transformation

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix}\begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

The Advanced Encryption Standard (AES) uses arithmetic in the finite field GF(2^8), with the irreducible polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$. Consider the two polynomials $f(x) = x^6 + x^4 + x^2 + x + 1$ and $g(x) = x^7 + x + 1$. Then

$$\begin{aligned} f(x) + g(x) &= x^6 + x^4 + x^2 + x + 1 + x^7 + x + 1 \\ &= x^7 + x^6 + x^4 + x^2 \end{aligned}$$

$$\begin{aligned} f(x) \times g(x) &= x^{13} + x^{11} + x^9 + x^8 + x^7 \\ &\quad + x^7 + x^5 + x^3 + x^2 + x \\ &\quad + x^6 + x^4 + x^2 + x + 1 \\ &= x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \end{aligned}$$

$$\begin{array}{r l} & x^5 + x^3 \\ x^8 + x^4 + x^3 + x + 1 \,\big) & x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \\ & \underline{x^{13} + x^9 + x^8 + x^6 + x^5} \\ & x^{11} + x^4 + x^3 \\ & \underline{x^{11} + x^7 + x^6 + x^4 + x^3} \\ & x^7 + x^6 + 1 \end{array}$$

Therefore, $f(x) \times g(x) \bmod m(x) = x^7 + x^6 + 1$.

$$\begin{aligned} x \times f(x) = {} & (b_6x^7 + b_5x^6 + b_4x^5 + b_3x^4 + b_2x^3 + b_1x^2 + b_0x) \\ & + (x^4 + x^3 + x + 1) \end{aligned}$$

It follows that multiplication by x (i.e., 00000010) can be implemented as a 1-bit left shift followed by a conditional bitwise XOR with (00011011), which represents $(x^4 + x^3 + x + 1)$. To summarize,

$$x \times f(x) = \begin{cases} (b_6b_5b_4b_3b_2b_1b_00) & \text{if } b_7 = 0 \\ (b_6b_5b_4b_3b_2b_1b_00) \oplus (00011011) & \text{if } b_7 = 1 \end{cases} \tag{5.6}$$

Multiplication by a higher power of x can be achieved by repeated application of Equation (5.6). By adding intermediate results, multiplication by any constant in GF(2^8) can be achieved.

$f(x) * x^2$, $b_7 = 1$, so left shift
And xor with m(x) = 00011011

In an earlier example, we showed that for $f(x) = x^6 + x^4 + x^2 + x + 1$, $g(x) = x^7 + x + 1$, and $m(x) = x^8 + x^4 + x^3 + x + 1$, we have $f(x) \times g(x) \bmod m(x) = x^7 + x^6 + 1$. Redoing this in binary arithmetic, we need to compute (01010111) × (10000011). First, we determine the results of multiplication by powers of x:

$$\begin{aligned} (01010111) \times (00000010) &= (10101110) && f(x) * x \quad b_7\text{=0 so just left shift} \\ (01010111) \times (00000100) &= (01011100) \oplus (00011011) = (01000111) \\ (01010111) \times (00001000) &= (10001110) \\ (01010111) \times (00010000) &= (00011100) \oplus (00011011) = (00000111) \\ (01010111) \times (00100000) &= (00001110) \\ (01010111) \times (01000000) &= (00011100) \\ (01010111) \times (10000000) &= (00111000) && f(x) * x^7. \quad b_7\text{=0 so just left shift} \end{aligned}$$

So,

$$\begin{aligned} (01010111) \times (10000011) &= (01010111) \times [(00000001) \oplus (00000010) \oplus (10000000)] \\ &= \underbrace{(01010111)}_{*1} \oplus \underbrace{(10101110)}_{*x} \oplus \underbrace{(00111000)}_{*x^7} = (11000001) \end{aligned}$$

which is equivalent to $x^7 + x^6 + 1$.

MixColumns Transformation

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

used $m(x) = x^8 + x^4 + x^3 + x + 1$

The following is an example of MixColumns:

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

→

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

$$s'_{0,j} = (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j}$$
$$s'_{1,j} = s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j}$$
$$s'_{2,j} = s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j})$$
$$s'_{3,j} = (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j})$$

$$\begin{array}{lllllllll}
(\{02\}\cdot\{87\}) & \oplus & (\{03\}\cdot\{6E\}) & \oplus & \{46\} & \oplus & \{A6\} & = & \{47\} \\
\{87\} & \oplus & (\{02\}\cdot\{6E\}) & \oplus & (\{03\}\cdot\{46\}) & \oplus & \{A6\} & = & \{37\} \\
\{87\} & \oplus & \{6E\} & \oplus & (\{02\}\cdot\{46\}) & \oplus & (\{03\}\cdot\{A6\}) & = & \{94\} \\
(\{03\}\cdot\{87\}) & \oplus & \{6E\} & \oplus & \{46\} & \oplus & (\{02\}\cdot\{A6\}) & = & \{ED\}
\end{array}$$

For the first equation, we have $\{02\}\cdot\{87\} = (0000\ 1110) \oplus (0001\ 1011) = (0001\ 0101)$ and $\{03\}\cdot\{6E\} = \{6E\} \oplus (\{02\}\cdot\{6E\}) = (0110\ 1110) \oplus (1101\ 1100) = (1011\ 0010)$. Then,

$$\begin{array}{lll}
\{02\}\cdot\{87\} & = & 0001\ 0101 \\
\{03\}\cdot\{6E\} & = & 1011\ 0010 \\
\{46\} & = & 0100\ 0110 \\
\{A6\} & = & \underline{1010\ 0110} \\
 & & 0100\ 0111 = \{47\}
\end{array}$$

$02 \cdot 87$

$0000\ 0010$ $\quad$ $1000\ 0111$

$f(x) = x$ $\quad$ $g(x) = x^7 + x^2 + x + 1$

$f(x) \cdot g(x) = x^8 + x^3 + x^2 + x$

reduce with $m(x) = x^8 + x^4 + x^3 + x + 1$

using formula $x^8 \bmod m(x) = m(x) - x^8$

We didn't have to use it

$x^8 + x^3 + x^2 + x \;\div\; x^8 + x^4 + x^3 + x + 1 = 1$

$x^8 + x^4 + x^3 + x + 1$

$-x^4 + x^2 + 1$ → remainder

$(x^2 + 1) - x^4$

$0000\ 0101$ $\quad$ $0001\ 0000$

addition and subtraction in $GF(2)$ is XOR operation

$0000\ 0101$

XOR $0001\ 0000$

$0001\ 0101$

$02 \cdot 87 =$

$= 87 \cdot 02$

$\cancel{b_7 x^2 + b_8}$

$1000\ 0111$ $\quad$ $0000\ 0010$

$b_7 x^7 + b_2 x^2 + b_1 x^1 + b_0$ $\quad$ $b_1 x^1$

$b_7 = 1$ so left shift

$1000\ 0111 \Rightarrow 0000\ 1110$

XOR with $m(x) = 0001\ 1011$ XOR

$0001\ 0101$

6.5 AN AES EXAMPLE

Plaintext:	0123456789abcdeffedcba9876543210
Key:	0f1571c947d9e8590cb7add6af7f6798
Ciphertext:	ff0b844a0853bf7c6934ab4364148fb9

Table 6.4 Key Expansion for AES Example

Key Words	Auxiliary Function
w0 = 0f 15 71 c9 w1 = 47 d9 e8 59 w2 = 0c b7 ad d6 w3 = af 7f 67 98	RotWord (w3) = 7f 67 98 af = x1 SubWord (x1) = d2 85 46 79 = y1 Rcon (1) = 01 00 00 00 y1 ⊕ Rcon (1) = d3 85 46 79 = z1
w4 = w0 ⊕ z1 = dc 90 37 b0 w5 = w4 ⊕ w1 = 9b 49 df e9 w6 = w5 ⊕ w2 = 97 fe 72 3f w7 = w6 ⊕ w3 = 38 81 15 a7	RotWord (w7) = 81 15 a7 38 = x2 SubWord (x2) = 0c 59 5c 07 = y2 Rcon (2) = 02 00 00 00 y2 ⊕ Rcon (2) = 0e 59 5c 07 = z2
w8 = w4 ⊕ z2 = d2 c9 6b b7 w9 = w8 ⊕ w5 = 49 80 b4 5e w10 = w9 ⊕ w6 = de 7e c6 61 w11 = w10 ⊕ w7 = e6 ff d3 c6	RotWord (w11) = ff d3 c6 e6 = x3 SubWord (x3) = 16 66 b4 83 = y3 Rcon (3) = 04 00 00 00 y3 ⊕ Rcon (3) = 12 66 b4 8e = z3

Key Words	Auxiliary Function
w12 = w8 ⊕ z3 = c0 af df 39 w13 = w12 ⊕ w9 = 89 2f 6b 67 w14 = w13 ⊕ w10 = 57 51 ad 06 w15 = w14 ⊕ w11 = b1 ae 7e c0	RotWord (w15) = ae 7e c0 b1 = x4 SubWord (x4) = e4 f3 ba c8 = y4 Rcon (4) = 08 00 00 00 y4 ⊕ Rcon (4) = ec f3 ba c8 = 4
w16 = w12 ⊕ z4 = 2c 5c 65 f1 w17 = w16 ⊕ w13 = a5 73 0e 96 w18 = w17 ⊕ w14 = f2 22 a3 90 w19 = w18 ⊕ w15 = 43 8c dd 50	RotWord (w19) = 8c dd 50 43 = x5 SubWord (x5) = 64 c1 53 1a = y5 Rcon(5) = 10 00 00 00 y5 ⊕ Rcon (5) = 74 c1 53 1a = z5
w20 = w16 ⊕ z5 = 58 9d 36 eb w21 = w20 ⊕ w17 = fd ee 38 7d w22 = w21 ⊕ w18 = 0f cc 9b ed w23 = w22 ⊕ w19 = 4c 40 46 bd	RotWord (w23) = 40 46 bd 4c = x6 SubWord (x6) = 09 5a 7a 29 = y6 Rcon(6) = 20 00 00 00 y6 ⊕ Rcon(6) = 29 5a 7a 29 = z6
w24 = w20 ⊕ z6 = 71 c7 4c c2 w25 = w24 ⊕ w21 = 8c 29 74 bf w26 = w25 ⊕ w22 = 83 e5 ef 52 w27 = w26 ⊕ w23 = cf a5 a9 ef	RotWord (w27) = a5 a9 ef cf = x7 SubWord (x7) = 06 d3 bf 8a = y7 Rcon (7) = 40 00 00 00 y7 ⊕ Rcon(7) = 46 d3 df 8a = z7
w28 = w24 ⊕ z7 = 37 14 93 48 w29 = w28 ⊕ w25 = bb 3d e7 f7 w30 = w29 ⊕ w26 = 38 d8 08 a5 w31 = w30 ⊕ w27 = f7 7d a1 4a	RotWord (w31) = 7d a1 4a f7 = x8 SubWord (x8) = ff 32 d6 68 = y8 Rcon (8) = 80 00 00 00 y8 ⊕ Rcon(8) = 7f 32 d6 68 = z8
w32 = w28 ⊕ z8 = 48 26 45 20 w33 = w32 ⊕ w29 = f3 1b a2 d7 w34 = w33 ⊕ w30 = cb c3 aa 72 w35 = w34 ⊕ w32 = 3c be 0b 3	RotWord (w35) = be 0b 38 3c = x9 SubWord (x9) = ae 2b 07 eb = y9 Rcon (9) = 1B 00 00 00 y9 ⊕ Rcon (9) = b5 2b 07 eb = z9
w36 = w32 ⊕ z9 = fd 0d 42 cb w37 = w36 ⊕ w33 = 0e 16 e0 1c w38 = w37 ⊕ w34 = c5 d5 4a 6e w39 = w38 ⊕ w35 = f9 6b 41 56	RotWord (w39) = 6b 41 56 f9 = x10 SubWord (x10) = 7f 83 b1 99 = y10 Rcon (10) = 36 00 00 00 y10 ⊕ Rcon (10) = 49 83 b1 99 = z10
w40 = w36 ⊕ z10 = b4 8e f3 52 w41 = w40 ⊕ w37 = ba 98 13 4e w42 = w41 ⊕ w38 = 7f 4d 59 20 w43 = w42 ⊕ w39 = 86 26 18 76	

Table 6.5 AES Example

Start of Round	After SubBytes	After ShiftRows	After MixColumns	Round Key
01 89 fe 76 23 ab dc 54 45 cd ba 32 67 ef 98 10				0f 47 0c af 15 d9 b7 7f 71 e8 ad 67 c9 59 d6 98
0e ce f2 d9 36 72 6b 2b 34 25 17 55 ae b6 4e 88	ab 8b 89 35 05 40 7f f1 18 3f f0 fc e4 4e 2f c4	ab 8b 89 35 40 7f f1 05 f0 fc 18 3f c4 e4 4e 2f	b9 94 57 75 e4 8e 16 51 47 20 9a 3f c5 d6 f5 3b	dc 9b 97 38 90 49 fe 81 37 df 72 15 b0 e9 3f a7
65 0f c0 4d 74 c7 e8 d0 70 ff e8 2a 75 3f ca 9c	4d 76 ba e3 92 c6 9b 70 51 16 9b e5 9d 75 74 de	4d 76 ba e3 c6 9b 70 92 9b e5 51 16 de 9d 75 74	8e 22 db 12 b2 f2 dc 92 df 80 f7 c1 2d c5 1e 52	d2 49 de e6 c9 80 7e ff 6b b4 c6 d3 b7 5e 61 c6
5c 6b 05 f4 7b 72 a2 6d b4 34 31 12 9a 9b 7f 94	4a 7f 6b bf 21 40 3a 3c 8d 18 c7 c9 b8 14 d2 22	4a 7f 6b bf 40 3a 3c 21 c7 c9 8d 18 22 b8 14 d2	b1 c1 0b cc ba f3 8b 07 f9 1f 6a c3 1d 19 24 5c	c0 89 57 b1 af 2f 51 ae df 6b ad 7e 39 67 06 c0
71 48 5c 7d 15 dc da a9 26 74 c7 bd 24 7e 22 9c	a3 52 4a ff 59 86 57 d3 f7 92 c6 7a 36 f3 93 de	a3 52 4a ff 86 57 d3 59 c6 7a f7 92 de 36 f3 93	d4 11 fe 0f 3b 44 06 73 cb ab 62 37 19 b7 07 ec	2c a5 f2 43 5c 73 22 8c 65 0e a3 dd f1 96 90 50
f8 b4 0c 4c 67 37 24 ff ae a5 c1 ea e8 21 97 bc	41 8d fe 29 85 9a 36 16 e4 06 78 87 9b fd 88 65	41 8d fe 29 9a 36 16 85 78 87 e4 06 65 9b fd 88	2a 47 c4 48 83 e8 18 ba 84 18 27 23 eb 10 0a f3	58 fd 0f 4c 9d ee cc 40 36 38 9b 46 eb 7d ed bd
72 ba cb 04 1e 06 d4 fa b2 20 bc 65 00 6d e7 4e	40 f4 1f f2 72 6f 48 2d 37 b7 65 4d 63 3c 94 2f	40 f4 1f f2 6f 48 2d 72 65 4d 37 b7 2f 63 3c 94	7b 05 42 4a 1e d0 20 40 94 83 18 52 94 c4 43 fb	71 8c 83 cf c7 29 e5 a5 4c 74 ef a9 c2 bf 52 ef
0a 89 c1 85 d9 f9 c5 e5 d8 f7 f7 fb 56 7b 11 14	67 a7 78 97 35 99 a6 d9 61 68 68 0f b1 21 82 fa	67 a7 78 97 99 a6 d9 35 68 0f 61 68 fa b1 21 82	ec 1a c0 80 0c 50 53 c7 3b d7 00 ef b7 22 72 e0	37 bb 38 f7 14 3d d8 7d 93 e7 08 a1 48 f7 a5 4a
db a1 f8 77 18 6d 8b ba a8 30 08 4e ff d5 d7 aa	b9 32 41 f5 ad 3c 3d f4 c2 04 30 2f 16 03 0e ac	b9 32 41 f5 3c 3d f4 ad 30 2f c2 04 ac 16 03 0e	b1 1a 44 17 3d 2f ec b6 0a 6b 2f 42 9f 68 f3 b1	48 f3 cb 3c 26 1b c3 be 45 a2 aa 0b 20 d7 72 38
f9 e9 8f 2b 1b 34 2f 08 4f c9 85 49 bf bf 81 89	99 1e 73 f1 af 18 15 30 84 dd 97 3b 08 08 0c a7	99 1e 73 f1 18 15 30 af 97 3b 84 dd a7 08 08 0c	31 30 3a c2 ac 71 8c c4 46 65 48 eb 6a 1c 31 62	fd 0e c5 f9 0d 16 d5 6b 42 e0 4a 41 cb 1c 6e 56
cc 3e ff 3b a1 67 59 af 04 85 02 aa a1 00 5f 34	4b b2 16 e2 32 85 cb 79 f2 97 77 ac 32 63 cf 18	4b b2 16 e2 85 cb 79 32 77 ac f2 97 18 32 63 cf		b4 ba 7f 86 8e 98 4d 26 f3 13 59 18 52 4e 20 76
ff 08 69 64 0b 53 34 14 84 bf ab 8f 4a 7c 43 b9				

Avalanche Effect

Avalanche Effect when 8th bit of the plaintext is changed.
The 2nd column of the table shows the value of the State matrix (as a vector) at the end of each round for the two plaintexts.

Table 6.6 Avalanche Effect in AES: Change in Plaintext

Round		Number of Bits that Differ
	0123456789abcdeffedcba9876543210 0023456789abcdeffedcba9876543210	1
0	0e3634aece7225b6f26b174ed92b5588 0f3634aece7225b6f26b174ed92b5588	1
1	657470750fc7ff3fc0e8e8ca4dd02a9c c4a9ad090fc7ff3fc0e8e8ca4dd02a9c	20
2	5c7bb49a6b72349b05a2317ff46d1294 fe2ae569f7ee8bb8c1f5a2bb37ef53d5	58
3	7115262448dc747e5cdac7227da9bd9c ec093dfb7c45343d689017507d485e62	59
4	f867aee8b437a5210c24c1974cffeabc 43efdb697244df808e8d9364ee0ae6f5	61
5	721eb200ba06206dcbd4bce704fa654e 7b28a5d5ed643287e006c099bb375302	68
6	0ad9d85689f9f77bc1c5f71185e5fb14 3bc2d8b6798d8ac4fe36a1d891ac181a	64
7	db18a8ffa16d30d5f88b08d777ba4eaa 9fb8b5452023c70280e5c4bb9e555a4b	67
8	f91b4fbfe934c9bf8f2f85812b084989 20264e1126b219aef7feb3f9b2d6de40	65
9	cca104a13e678500ff59025f3bafaa34 b56a0341b2290ba7dfdfbddcd8578205	61
10	ff0b844a0853bf7c6934ab4364148fb9 612b89398d0600cde116227ce72433f0	58

Avalanche Effect

Avalanche Effect when 8th bit of the key is changed.
The 2nd column of the table shows the value of the State matrix (as a vector) at the end of each round for the two plaintexts.

Table 6.7 Avalanche Effect in AES: Change in Key

Round		Number of Bits that Differ
	0123456789abcdeffedcba9876543210 0123456789abcdeffedcba9876543210	0
0	0e3634aece7225b6f26b174ed92b5588 0f3634aece7225b6f26b174ed92b5588	1
1	657470750fc7ff3fc0e8e8ca4dd02a9c c5a9ad090ec7ff3fc1e8e8ca4cd02a9c	22
2	5c7bb49a6b72349b05a2317ff46d1294 90905fa9563356d15f3760f3b8259985	58
3	7115262448dc747e5cdac7227da9bd9c 18aeb7aa794b3b66629448d575c7cebf	67
4	f867aee8b437a5210c24c1974cffeabc f81015f993c978a876ae017cb49e7eec	63
5	721eb200ba06206dcbd4bce704fa654e 5955c91b4e769f3cb4a94768e98d5267	81
6	0ad9d85689f9f77bc1c5f71185e5fb14 dc60a24d137662181e45b8d3726b2920	70
7	db18a8ffa16d30d5f88b08d777ba4eaa fe8343b8f88bef66cab7e977d005a03c	74
8	f91b4fbfe934c9bf8f2f85812b084989 da7dad581d1725c5b72fa0f9d9d1366a	67
9	cca104a13e678500ff59025f3bafaa34 0ccb4c66bbfd912f4b511d72996345e0	59
10	ff0b844a0853bf7c6934ab4364148fb9 fc8923ee501a7d207ab670686839996b	53