Converting a C.F.G. into C.N.F.

C.N.f.	→ C.F.G.	Regular Grammar
$A \rightarrow BC$ or $A \rightarrow x$ where	$A \rightarrow \alpha$ where $A \in V$ and $\alpha \in (V \cup \varepsilon)^{*}$	$A \rightarrow x$ $A \rightarrow x\beta$ where $A, B \in V$ and $x \in T^{*}$
$A, B, C \in V$ and $x \in T$ (not T^{*})		

Method of Conversion from a C.F.G. to C.N.f.:

in order!

1) λ: (completely empty string is immediately produced)

2) λ-Production: (empty string produced somewhere in grammar)

3) Unit Production: (If $A \rightarrow B$, and $B \rightarrow ab$, reduce $A \rightarrow B$ into $A \rightarrow ab$)

4) Useless Production: (If a production can never be reached, or it can never terminate in an infinite recursive loop, ∴ never producing any terminal symbols, it is useless. Simply eliminate it.)

Example

Convert the grammar:

$$G_{C.F.G} = (V_1, T_1, S_1, P_1) \quad P = \begin{cases} S \to AB \mid aB \\ A \to abb \mid \lambda \\ B \to bbA \end{cases}$$

Step 1) λ ?

- This grammar does not immediately produce the empty string, so move to step 2

Step 2) λ - Production ?

- For every **Variable** **Production** **Symbol** use Substitution in order to remove λ - productions.

a)
$$S \to AB \mid aB$$
$$A \to abb \mid \lambda$$
$$B \to bbA$$

$\longrightarrow$

b)
$$S \to AB \mid aB$$
$$A \to abb \mid \cancel{\lambda}$$
$$B \to bbA$$

$\longrightarrow$

c)
$$S \to AB \mid aB \mid B$$
$$A \to abb$$
$$B \to bbA \mid bb$$

d)
$$S \to AB \mid aB$$
$$A \to abb$$
$$B \to bbA \mid bb\lambda$$

$\longrightarrow$

e)
$$S \to AB \mid aB$$
$$A \to abb$$
$$B \to bbA \mid bb\cancel{\lambda}$$

$\longrightarrow$

f)
$$S \to AB \mid aB \mid \cancel{\lambda}B$$
$$A \to abb$$
$$B \to bbA \mid bb$$

3) $S \to AB \mid aB \mid B$
$A \to abb$
$B \to bbA \mid bb$
✓   No more λ-Productions!

Step 3) Unit - Production?

a) $S \to AB \mid aB \mid B$ → "B" is a Unit Production!
   $A \to abb$
   $B \to bbA \mid bb$

b) $S \to AB \mid aB \mid bbA \mid bb$ → Substitute Unit-Production with what the Variable Produces
   $A \to abb$
   $B \to bbA \mid bb$

c) $S \to AB \mid aB \mid bbA \mid bb$ → No more Unit-Productions.
   $A \to abb$
   $B \to bbA \mid bb$

Step 4) Useless Productions?

- All P. Symbols are reachable from "S," and all P. Symbols produce terminal Symbols, and can terminate.

∴ No useless productions!

Step 5) Convert into C.N.f. such that all productions are of the form:

$$A \to BC$$
$$\text{or}$$
$$A \to X$$

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a)

$S \to AB \mid aB \mid bbA \mid bb$ } Mark productions that are NOT in C.N.f.

$A \to abb$

$B \to bbA \mid bb$

b) $S \to AB \mid X_a B \mid X_{bb} A \mid X_b X_b$

$A \to X_a X_{bb}$

$B \to X_{bb} A \mid X_b X_b$

} Now all productions are of the form: $A \to BC$, but we do not have definitions for X_a, X_{bb}, X_b, etc.

c)

$S \rightarrow AB \mid X_a B \mid X_{bb} A \mid X_b X_b$

$A \rightarrow X_a X_{bb}$

$B \rightarrow X_{bb} A \mid X_b X_b$

$X_a \rightarrow a$

$X_{bb} \rightarrow X_b X_b$

$X_b \rightarrow b$

Done!

All productions are of the form:

$A \rightarrow BC$

or

$A \rightarrow X$

where $L(G_{C.F.G}) = L(G_{C.N.F.})$

[a]