Chapter 10

Sinusoidal Steady-State Analysis (Problems)

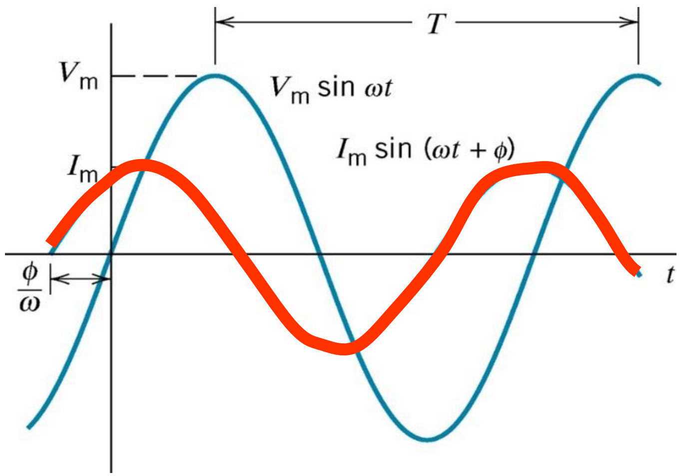
Example 10.2-1

- Consider the voltages $v_1 = 10\cos(200t+45^o)$ V and $v_2 = 8\sin(200t+15°)$ V. Determine the time by which $v_2(t)$ is advanced or delayed with respect to $v_1(t)$.

$$\boxed{\sin(\omega t) = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)}$$

$$\cos(\omega t) = \sin(90° - \omega t) = \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\sin(\omega t + 180°) = -\sin(\omega t)$$



Current ***leads*** the voltage OR Voltage ***lags*** the current

Example 10.2-1 Solution

$$v_2 = \sin(200t + 15^o)$$

$$v_2 = 8\cos(200t + 15^o - 90^o) = 8\cos(200t - 75^o)\,V$$

$$\theta_2 - \theta_1 = -75^o - 45^o = -120^o = -\frac{\pi}{3} rad$$

$$\frac{\phi}{\omega} = -\frac{\frac{\pi}{3}}{200} = -5.2\,ms$$

$$\theta_2 - \theta_1 < 0$$

This indicates a delay (lag)

Example 3

- Transform the current expression in time domain
  $i(t) = 5\sin(100t+120°)$ to the frequency domain.

$$\sin(\omega t) = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\cos(\omega t) = \sin(90° - \omega t) = \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\sin(\omega t + 180°) = -\sin(\omega t)$$

Example 3 Solution

$$i(t) = 5\sin(100t + 120^\circ)$$

$$[\sin(\omega t) = \cos(\omega t - 90^\circ)]$$

$$= 5\cos(100t + 120^\circ - 90^\circ)$$

$$= 5\cos(100t + 30^\circ)$$

$$i(t) = \mathrm{Re}\{5 e^{j100t} e^{j30^\circ}\} \Leftarrow$$

$$I = 5 e^{j30^\circ} = 5\angle 30^\circ$$

$$5\angle 30^\circ$$

Example 4

- Transform the voltage expression in frequency domain **V = 24 ∠125°** to time domain.

$$\sin(\omega t) = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\cos(\omega t) = \sin(90° - \omega t) = \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\sin(\omega t + 180°) = -\sin(\omega t)$$

Example 4 Solution

$V = 24\angle 125^\circ$ to time domain.

$$v(t) = \cancel{Re}\{24 \cdot \cancel{e^{j\omega t}} \cdot e^{j125^\circ}\}.$$

$$= 24\cos(\underline{\underline{\omega t}} + 125^\circ)$$

Example 10.3-2

Consider the phasors

$$\mathbf{V}_1 = 4.25\angle 115^\circ \text{ and } \mathbf{V}_2 = -4 + j3$$

Convert $\mathbf{V}_1$ to rectangular form and $\mathbf{V}_2$ to polar form.

<u>Polar to rectangular</u>

$$x = r cos\phi$$

$$y = r sin\phi$$

$$z = r(cos\phi + j sin\phi)$$

<u>Rectangular to polar</u>

$$r = \sqrt{x^2 + y^2} \quad \phi = \tan^{-1}\frac{y}{x}$$

$$z = \sqrt{x^2 + y^2}\angle\tan^{-1}\frac{y}{x}$$

Example 10.3-2 Solution

Ex10.3-2

$V_1 = 4.25\angle 115^\circ$ $\quad$ $V_2 = -4 + j3.$

rectangular

$x = r\cos\phi$

$y = r\sin\phi$

$V_1 \Rightarrow \begin{cases} 4.25\cos 115^\circ = -1.796 \\ 4.25\sin 115^\circ = 3.852 \end{cases}$

$V_1 = r(\cos\phi + j\sin\phi)$

$V_1 = -1.796 + j3.852$

$V_2 = -4 + j3.$

$\begin{cases} r = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1} y/x. \end{cases}$

$\cancel{V_2} =$

$r = \sqrt{(-4)^2 + (3)^2} = 5$

$\phi = \tan^{-1}(3/4) = -36.87^\circ = 143^\circ$

$V_2 = 5\angle 143^\circ$

Example 10.3-3

Consider the phasors

$$\mathbf{V}_1 = -1.796 + j3.852 = 4.25\angle 115^\circ \text{ and } \mathbf{V}_2 = -4 + j3 = 5\angle 143^\circ$$

Determine $\mathbf{V}_1 + \mathbf{V}_2$, $\mathbf{V}_1 \cdot \mathbf{V}_2$ and $\frac{\mathbf{V}_1}{\mathbf{V}_2}$.

Addition and Subtraction of Complex Number (rectangular)

$$Z_1 + Z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$Z_1 - Z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication/Division of Complex Number (polar)

$$Z_1 \cdot Z_2 = r_1 \cdot r_2 \angle(\phi_1 + \phi_2)$$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2} \angle(\phi_1 - \phi_2)$$

Example 10.3-3 Solution

$$V_1 = -1.796 + j3.852 = 4.25\angle 115^\circ$$

$$V_2 = -4 + j3 = 5\angle 143^\circ$$

$$V_1$$

$$V_1 + V_2 = (-1.796 + j3.852) + (-4 + j3)$$

$$V_1 + V_2 = (-1.796 - 4) + j(3.852 + 3)$$

$$V_1 \cdot V_2 = (4.25\angle 115^\circ)\cdot(5\angle 143^\circ)$$

$$= (4.25 \cdot 5)\angle(115^\circ + 143^\circ)$$

$$= 21.25\angle \cancel{102} 258^\circ \quad -360^\circ$$

$$= 21.25\angle -102^\circ$$

$$\frac{V_1}{V_2} = \frac{4.25\angle 115^\circ}{5\angle 143^\circ} = \left(\frac{4.25}{5}\right)\angle 115^\circ - 143^\circ$$

$$= 0.85\angle -28^\circ.$$

Example 10.3-4 – Kirchhoff's Law for AC Circuits

The input to the circuit shown in Figure 10.3-3 is the voltage source voltage,

$$v_s(t) = 25\cos(100t + 15°)\ \text{V}$$

The output is the voltage across the capacitor,

$$v_C(t) = 20\cos(100t - 22°)\ \text{V}$$

Determine the resistor voltage $v_R(t)$.

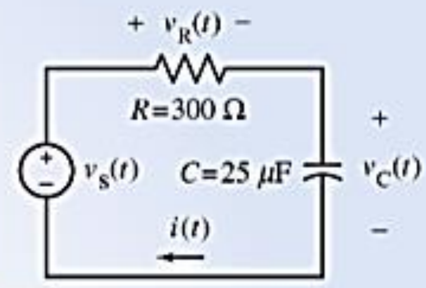


FIGURE 10.3-3 The circuit in Example 10.3-4

Addition and Subtraction of Complex Number (rectangular)

$$Z_1 + Z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$Z_1 - Z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

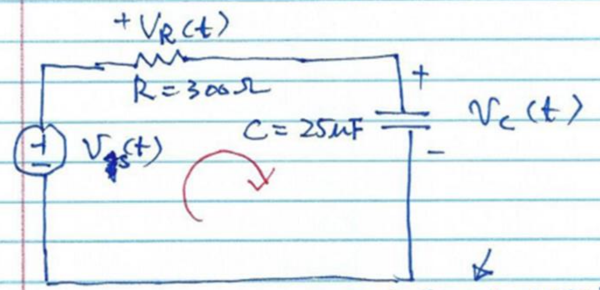
Multiplication/Division of Complex Number (polar)

$$Z_1 \cdot Z_2 = r_1 \cdot r_2 \angle(\phi_1 + \phi_2)$$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}\angle(\phi_1 - \phi_2)$$

Example 10.3-4 Solution

Ex 10.3-4.



$V_s(t) = 25\cos(100t + 15^\circ)\,V = 25\angle 15^\circ$

$V_c(t) = 20\cos(100t - 22^\circ)\,V = 20\angle -22^\circ$

$V_R(t) = ?$

$-V_s + V_R + V_c = 0. \quad V_R = V_s - V_c.$

KUL.

$$V_R(t) = V_s(t) - V_c(t)$$
$$= 25\cos(100t + 15^\circ) - 20\cos(100t - 22^\circ)$$

$$V_R(\omega) = V_s(\omega) - V_c(\omega)$$
$$= 25\angle 15^\circ - 20\angle -22^\circ$$
$$= (24.15 + j6.47) - (18.54 - j7.49)$$
$$= (24.15 - 18.54) + j(6.47 + 7.49)$$
$$= 5.61 + j13.96$$
$$= 15\angle 68.1^\circ V$$

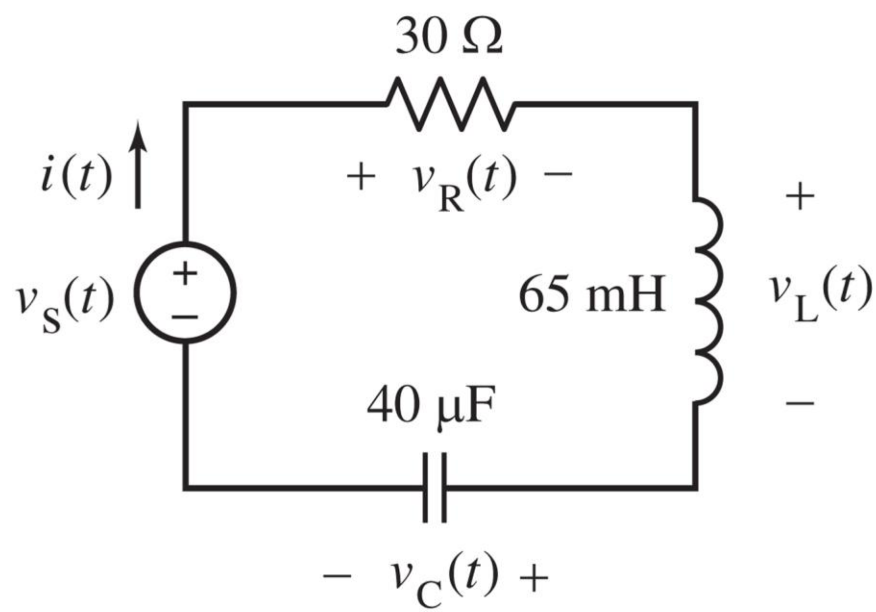
$$V_R(t) = 15\cos(100t + 68.1^\circ)$$

Example 10.4-1

The input to the AC circuit shown below is the source voltage

$$v_s(t) = 12\cos(1000t + 15^o)\,V$$

Determine (a) the impedances of the capacitor, inductor, and resistance and (b) the current $i(t)$



Example 10.4-1 Solution

Eample 10.4-1.

(a) The input frequency is $\omega = 1000$ rad/s. Using Eq. 10.4-4 shows that the impedance of the capacitor is

$$\mathbf{Z}_{\mathrm{C}}(\omega) = \frac{1}{j\omega C} = \frac{1}{j\,1000\,(40 \times 10^{-6})} = \frac{25}{j} = -j25 \ \ \Omega$$

Using Eq. 10.4-6 shows that the impedance of the inductor is

$$\mathbf{Z}_{\mathrm{L}}(\omega) = j\omega L = j\,1000\,(0.065) = j65 \ \ \Omega$$

Using Eq. 10.4-8, the impedance of the resistor is

$$\mathbf{Z}_{\mathrm{R}}(\omega) = R = 30 \ \ \Omega$$

(b) Apply KVL to write

$$12\cos(1000t + 15°) = v_{\mathrm{R}}(t) + v_{\mathrm{L}}(t) + v_{\mathrm{C}}(t)$$

Using phasors, we get

$$12\angle 15° = \mathbf{V}_{\mathrm{R}}(\omega) + \mathbf{V}_{\mathrm{L}}(\omega) + \mathbf{V}_{\mathrm{C}}(\omega) \qquad (10.4\text{-}10)$$

Using Eqs. 10.4-5, 10.4-7, and 10.4-9, we get

$$12\angle 15° = 30\,\mathbf{I}(\omega) + j65\,\mathbf{I}(\omega) - j25\,\mathbf{I}(\omega) = (30 + j40)\,\mathbf{I}(\omega) \qquad (10.4\text{-}11)$$

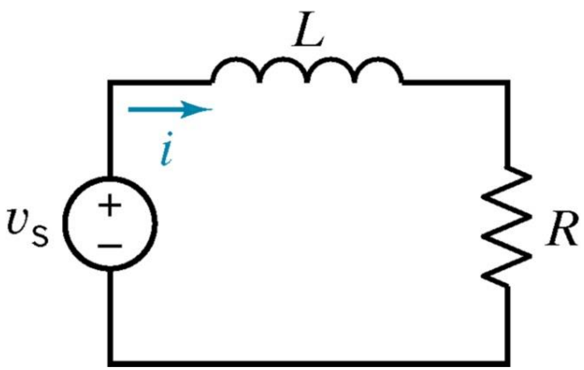
Solving for $\mathbf{I}(\omega)$ gives

$$\mathbf{I}(\omega) = \frac{12\angle 15°}{30 + j40} = \frac{12\angle 15°}{50\angle 53.13°} = 0.24\angle 38.13° \ \ \mathrm{A}$$

The corresponding sinusoid is

$$i(t) = 0.24\cos(1000t - 38.13°) \ \ \mathrm{A}$$

Example 7



$\omega = 100\ rad/s,\ R = 200\ \Omega,\ L = 2\,H,\ v(t) = V_m \cos\omega t\ \mathrm{V}$

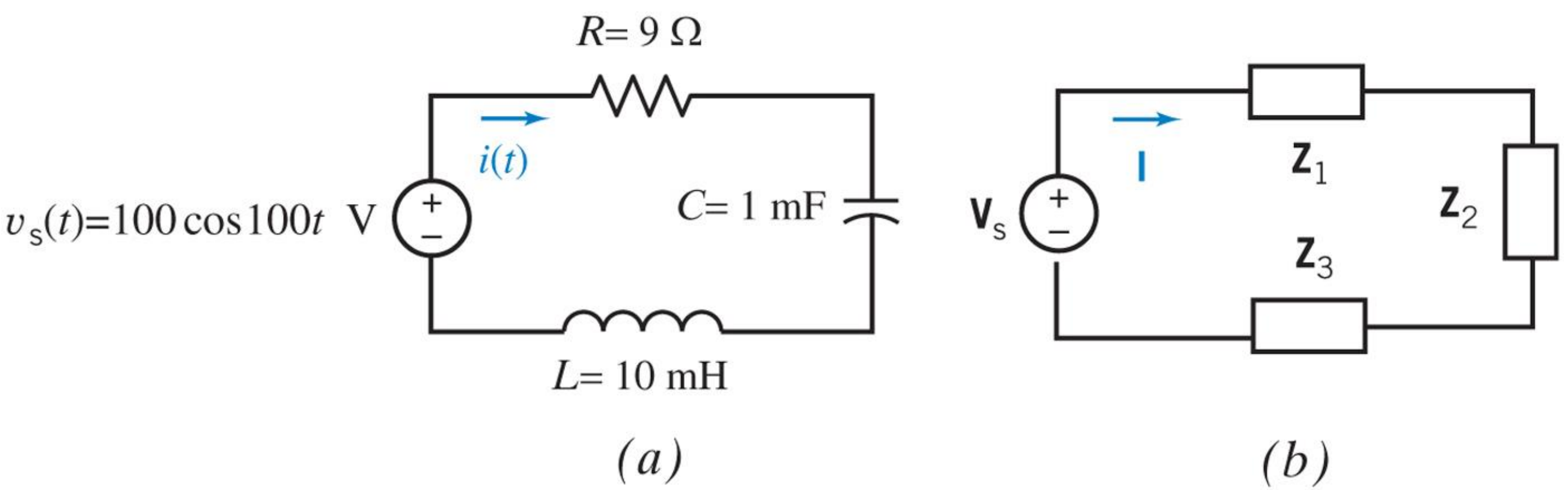
$\boldsymbol{i} = ?$

$$\mathrm{I} = \frac{\mathrm{V}}{j\omega L + R} = \frac{V_m\angle 0^\circ}{j\times 100\times 2 + 200} = \frac{V_m\angle 0^\circ}{j\times 200 + 200} = \frac{V_m\angle 0^\circ}{283\angle 45^\circ} = \frac{V_m\angle -45^\circ}{283}$$

$$\Rightarrow i(t) = \frac{V_m}{283}\cos\left(100t - 45^\circ\right)$$

Example 10.5-1 – KVL

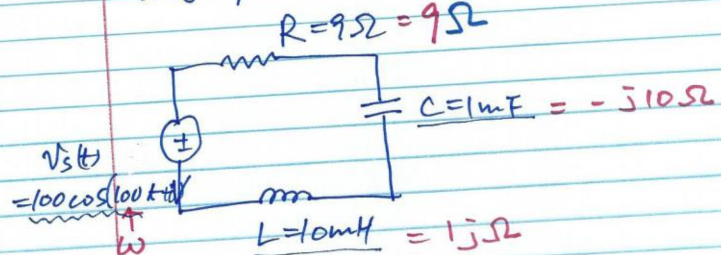
- Determine the steady state current $i(t)$ using phasor and impedances



Example 10.5-1 Solution

example 10.5-1.

Determine the steady state current $i(t)$ using phasor and impedances.



$$Z_R = R = 9\,\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{j\times 1}{j\times j\cdot 100\cdot 1m} = \frac{j}{-1\cdot 100\cdot 1m} = -j\cdot 10\,\Omega$$

$$Z_L = j\omega L = j100\cdot 10m = j = 1\cdot j\,\Omega.$$

$$\mathbf{V} = 100\angle 0^\circ \text{ V}$$

$$\mathbf{V} = \mathbf{I}\cdot \mathbf{Z}, \quad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{100\angle 0^\circ}{9-j10+j1} = \frac{100\angle 0^\circ}{9-9j}.$$

$$\left(\sqrt{9^2+(-9)^2} = 12.7, \quad \tan^{-1}(-9/9) = -45^\circ\right)$$

$$\mathbf{I} = \frac{100\angle 0^\circ}{12.7\angle -45^\circ} = \frac{100}{12.7}\angle 0-(-45^\circ)$$

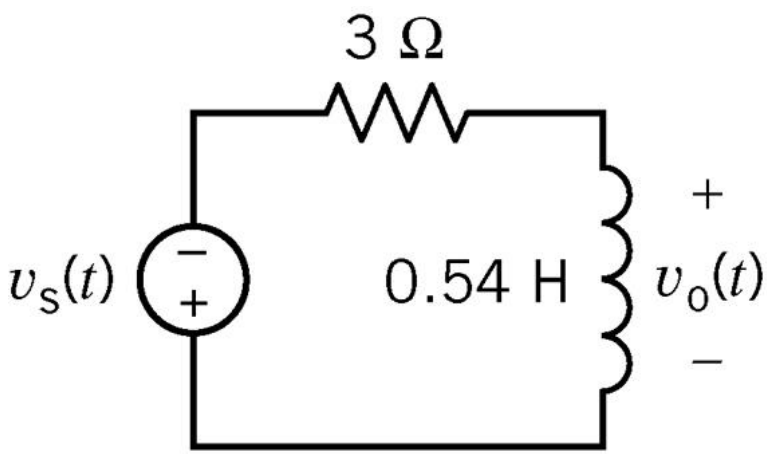
$$= 7.86\angle 45^\circ \text{ (A)}.$$

$$i(t) = 7.86\cos(100t + 45^\circ) \text{ (A)}.$$

Example 10.5-2 – Voltage Division

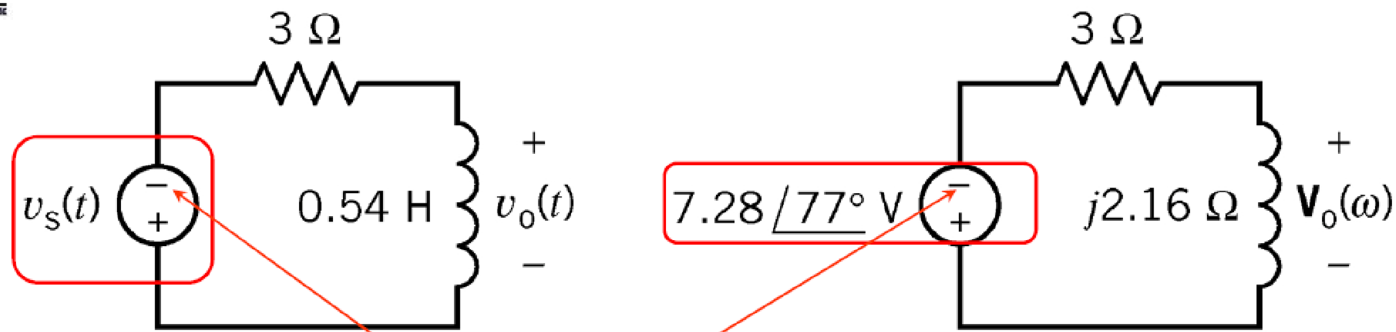
- Determine the steady-state output voltage, $v_o(t)$ if the source voltage is:

$$v_s(t) = 7.28\cos(4t + 77°)\ V$$



(a)

Example 10.5-2 Solution



When you use voltage division, you have to be careful with the direction:

$$V_o = -V_s \cdot Z_{0.54\,H} / (Z_{3\,\Omega} + Z_{0.54\,H})$$

$$\mathbf{V_o} = -7.28\angle 77^\circ \times \frac{j4\times 0.54}{3+j4\times 0.54} = -7.28\angle 77^\circ \times \frac{j2.16}{3+j2.16}$$

$$= e^{j180^\circ} \times 7.28 \times e^{j77^\circ} \times \frac{2.16\times e^{j90^\circ}}{3+j2.16}$$

$$= \frac{7.28\times 2.16}{3+j2.16} \times e^{j180^\circ} \times e^{j77^\circ} \times e^{j90^\circ}$$

$$= \frac{7.28\times 2.16}{3+j2.16} e^{j(180+77+90)} = \frac{7.28\times 2.16}{3.70\times e^{j36^\circ}} e^{j(180+77+90)}$$

$$= \frac{7.28\times 2.16}{3.70} e^{j(180+77+90-36)}$$

Example 10.5-3 – KVL

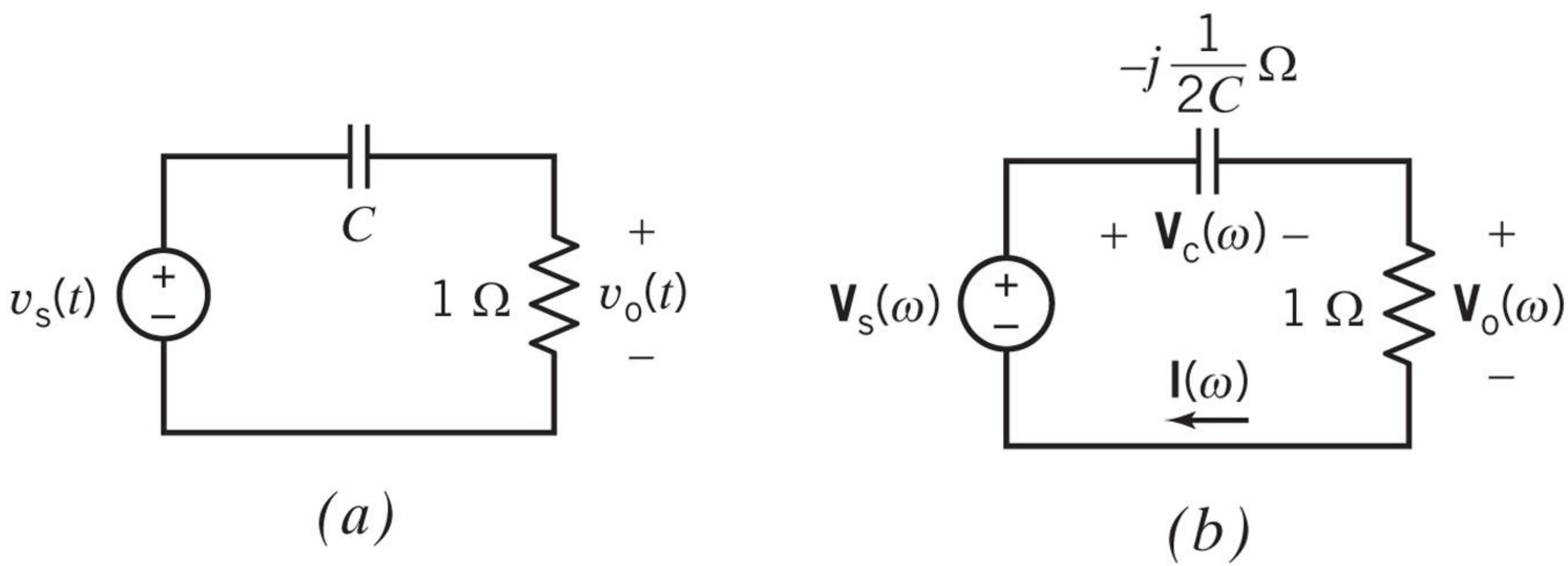
The input to the circuit is the voltage of the voltage source:

$$v_s(t) = 7.68\cos(2t + 47^o)\text{ V}$$

The output is the voltage across the resistor:

$$v_o(t) = 1.59\cos(2t + 125^o)$$

Determine capacitance C of the capacitor.

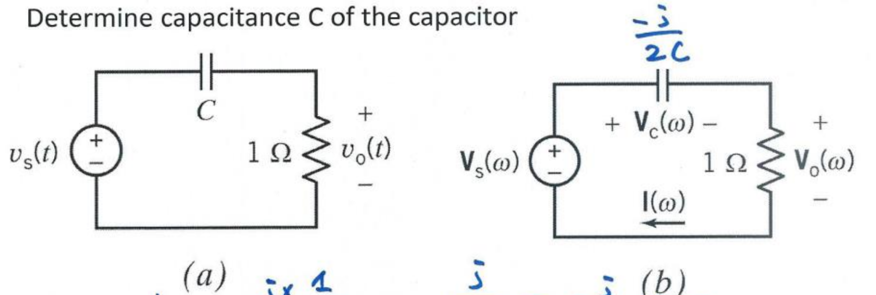


Example 10.5-3 Solution

Example 10.5-3

$v_s(t) = 7.68\cos(2t + 47^o)$ V $\quad v_o(t) = 1.59\cos(2t + 125^o)$

Determine capacitance C of the capacitor



$Z_c = \frac{1}{j\omega c} = \frac{j \times 1}{j \times j\omega c} = \frac{j}{-\omega c} = -\frac{j}{2c}\ \Omega.$

$Z_R = R = 1\Omega.\ V_s = 7.68\angle 47^\circ,\ V_o = 1.59\angle 125^\circ$

$I = \frac{V_o}{R} = \frac{1.59\angle 125^\circ}{1} = 1.59\angle 125^\circ\ (A).$

KVL $V_c = V_s - V_o = 7.68\angle 47^\circ - 1.59\angle 125^\circ$

Convert to Rect. $m(\cos\theta + j\sin\theta)$

$= (5.23 + j5.62) - (-0.91 + j1.3)$

$= (5.23 + 0.91) + (j5.62 - j1.3)$ $\sqrt{x^2+y^2}$

$= 6.14 + j4.32 = 7.51\angle 35^\circ (V)$ $\tan^{-1}\frac{Im}{Re}$.

The impedance of capacitor

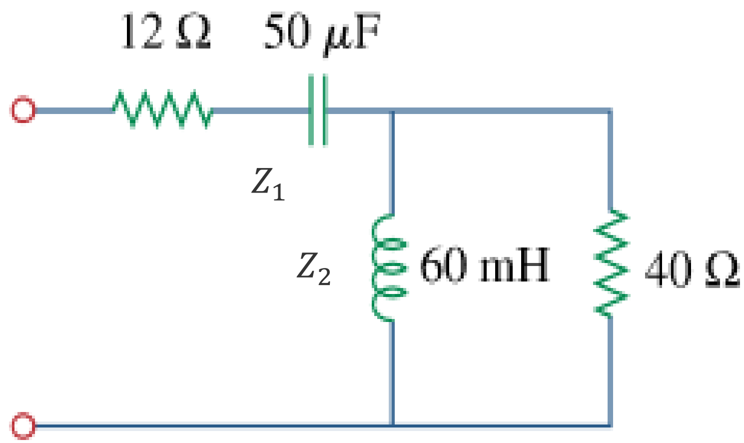
$-\frac{j}{2C} = \frac{V_c}{I} = \frac{7.51\angle 35^\circ}{1.59\angle 125^\circ} = \boxed{4.72\angle -90^\circ}$

$C = \frac{-j1}{2(4.72\angle -90^\circ)} = \frac{1\angle -90^\circ}{2(4.72\angle -90^\circ)}$

$= 0.106F.$

Example 8 – Z_{eq}

- At $\omega = 377\ rad/s$, find the input impedance of the circuit.

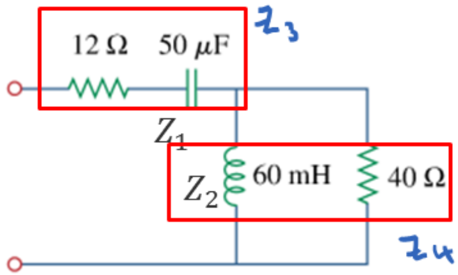


R: $Z = R$

L: $Z = j\omega L$

C: $Z = -j\dfrac{1}{\omega C}$

Example 8 Solution



$$Z_1 = -j\frac{1}{(377)\cdot(50\times10^{-6})} = -j\,53.1\,\Omega$$

$$Z_2 = j(377)\cdot(60\times10^{-3}) = j\,22.6\,\Omega$$

$$Z_3 = 12\,\Omega + (-j\,53.1\,\Omega)$$

$$Z_3 = 12 - 53.1\,\Omega$$

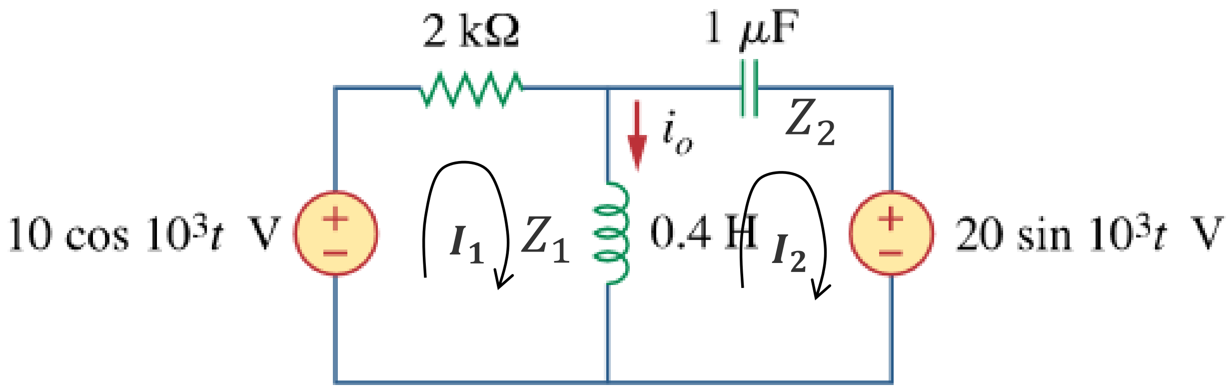
$$Z_4 = \frac{(40\,\Omega)(j\,22.6\,\Omega)}{(40\,\Omega)+(j\,22.6\,\Omega)} = 9.7 + j\,17.1\,\Omega$$

$$Z_{eq} = Z_3 + Z_4 = (12 - j\,53.1) + 9.7 + j\,17.1$$

$$= 21.7 - j\,36$$

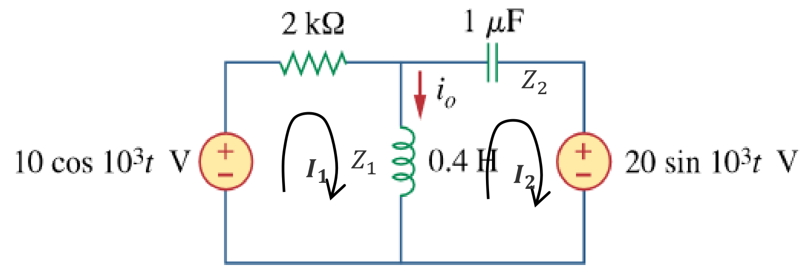
Example 9 – Mesh Analysis

- Calculate i_o



$$\sin(\omega t) = \cos(\omega t - 90°) = 1\angle -90° = -1j$$

Example 9 Solution



$w = 1000$ rad/s

$Z_1 = jwL = j400$

$Z_2 = -j\frac{1}{wC} = -j\frac{1}{(10^3)(10^{-6})} = -j\frac{1}{10^{-3}}$

Mesh analysis

$I_1 - I_o - I_2 = 0 \qquad I_o = I_1 - I_2.$ (KCL)

$-10 + 2000I_1 + j400(I_1 - I_2) = 0$

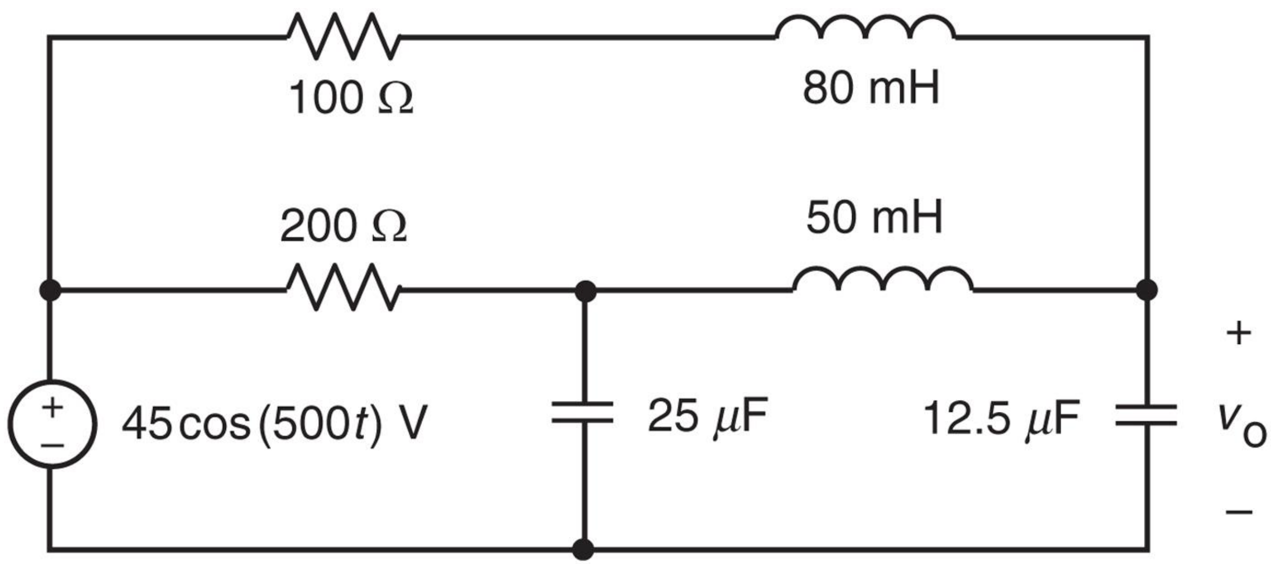
$$\boxed{-10 + (2000 + j400)I_1 - j400I_2 = 0}$$

$j400(I_2 - I_1) - j1000I_2 - 20j = 0$

$$\boxed{+j400I_1 + (+600j)I_2 + 20j = 0}$$

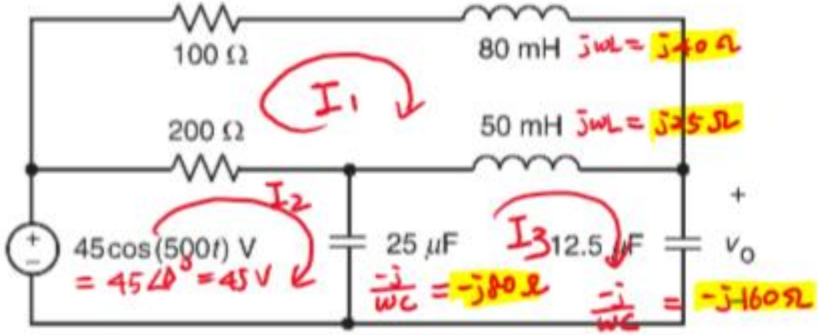
Example 10.6-2 – Mesh Analysis

- Determine the mesh currents for the circuit shown below.



Example 10.6-2

Determine the mesh currents for the circuit shown in the figure



$$100 \cdot I_1 + j40 \cdot I_1 + j25(I_1 - I_3) + 200(I_1 - I_2) = 0 \quad \text{—} \ 1$$

$$-45\angle 0^\circ + 200(I_2 - I_1) + (-j80)(I_2 - I_3) = 0 \quad \text{—} \ 2$$

$$-j80(I_3 - I_2) + j25(I_3 - I_1) + (-j160)(I_3) = 0 \quad \text{—} \ 3$$

$$\begin{bmatrix} 360 + j65 & -200 & -j25 \\ -200 & 200 - j80 & j80 \\ -25 & j80 & -j215 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 45 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0.374 \angle 115^\circ \\ 0.575 \angle 25^\circ \\ 0.171 \angle 28^\circ \end{bmatrix}$$

Exercise 10.6-3 – Node Analysis

The input to the circuit shown in Figure 10.6-10 is the voltage source voltage

$$v_s(t) = 10 \cos(10t) \text{ V}$$

The output is the current $i(t)$ in resistor R_1. Determine $i(t)$.

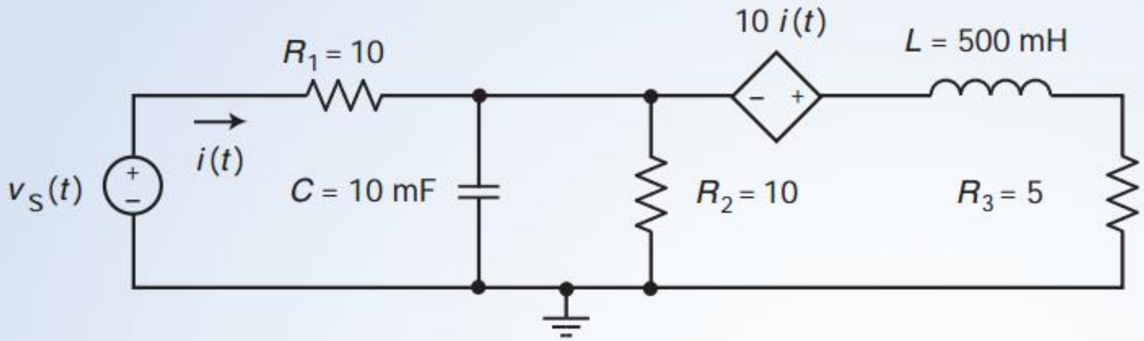


FIGURE
Example

Example 10.6-3 Solution

The output is the current $i(t)$ in resistor R_1. Determine $i(t)$.

Node analysis.

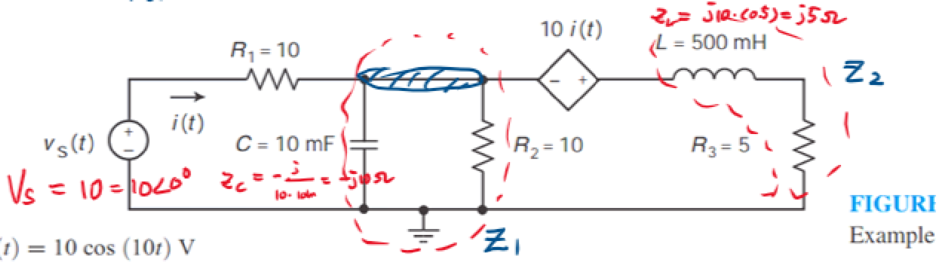


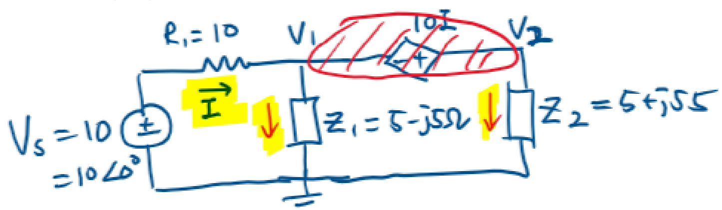
FIGURE
Example

$v_s(t) = 10\cos(10t)$ V

$Z_1 = 10 \,//\, -j10 = \frac{10\cdot(-j10)}{10-j10} = \frac{10\cdot(-j10)(10+j10)}{(10-j10)(10+j10)} = \frac{1000-1000j}{100+100}$

$= 5-5j\,\Omega$

$Z_2 = 5+j5\,\Omega.$



For Dep. source $\Rightarrow \boxed{I = \frac{10-V_1}{10}}$

Supernode

$V_2 - V_1 = 10\cdot I$, $\quad V_2 - \cancel{V_1} = \cancel{10}\cdot\frac{10-\cancel{V_1}}{\cancel{10}}$, $\quad V_2 = 10$ or $10\angle 0^\circ$

$$\frac{10-V_1}{10} - \frac{V_1}{Z_1} - \frac{\cancel{V_2}^{\,10}}{Z_2} = 0$$

$$\frac{10-V_1}{10} - \frac{V_1}{5-j5} - \underbrace{\frac{10}{5+j5}}_{=1-j1} = 0$$

$$1 - \frac{V_1}{10} - \frac{V_1}{5-j5} - (1-j) = 0$$

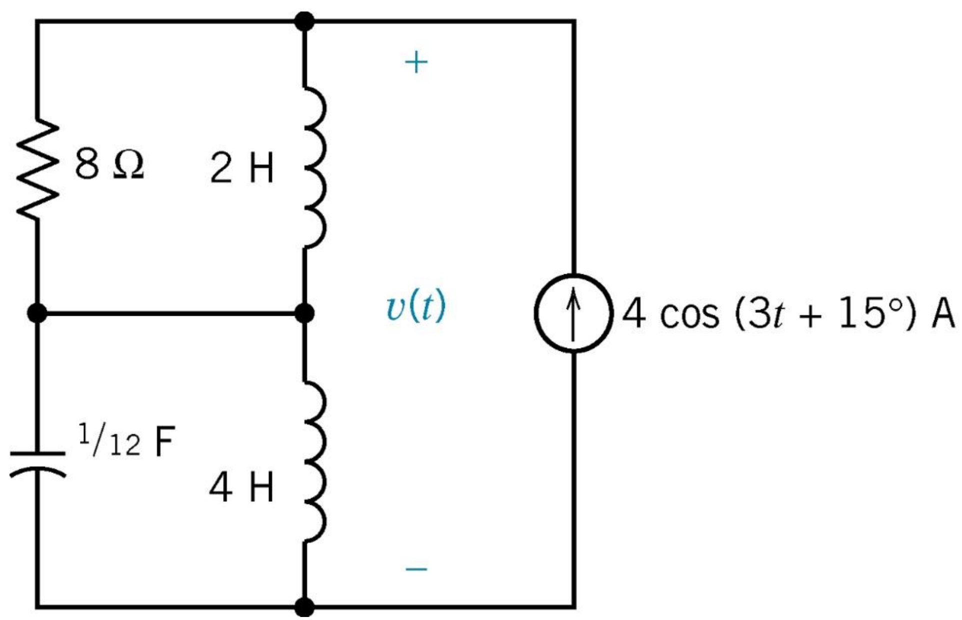
$$-V_1\left(\frac{1}{10} + \frac{1}{5-j5}\right) = \cancel{-1}\cancel{+1} - j$$

$$V_1 = -j \Big/ \left(\frac{1}{10} + \frac{1}{5-j5}\right) = 2+4j$$

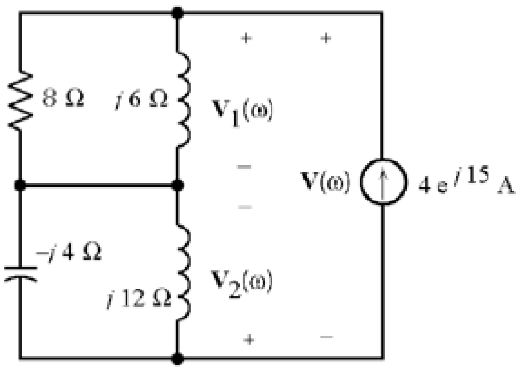
$$I = \frac{10-(2+4j)}{10} = 0.89\angle -26.6^\circ, \qquad i(t) = 0.89\cos(10t - 26.6^\circ)$$

Exercise 10.7-2

- Determine the phasor representation of each circuit element. Calculate *v(t)*.



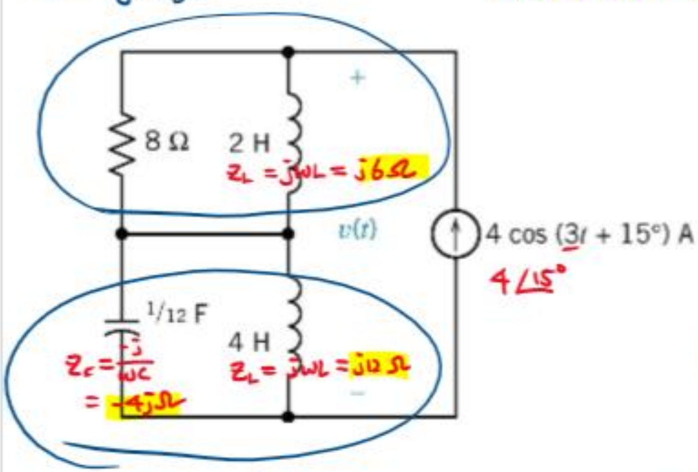
Exercise 10.7-2 Solution

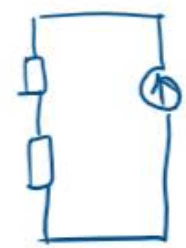


v(t) =

$$V_1 = \frac{8 \cdot j6}{8 + j6} \cdot 4\angle 15^\circ = 19.2\angle 68^\circ$$

Exercise 10.7 – 2





$$V_1 + V_2 = 19.2\angle 68^\circ + 24\angle -75^\circ$$

$$= 14.4\angle -22^\circ$$

$$V_2 = \frac{j12(-j4)}{j12 - j4} \cdot 4\angle 15^\circ = 24\angle -75^\circ$$

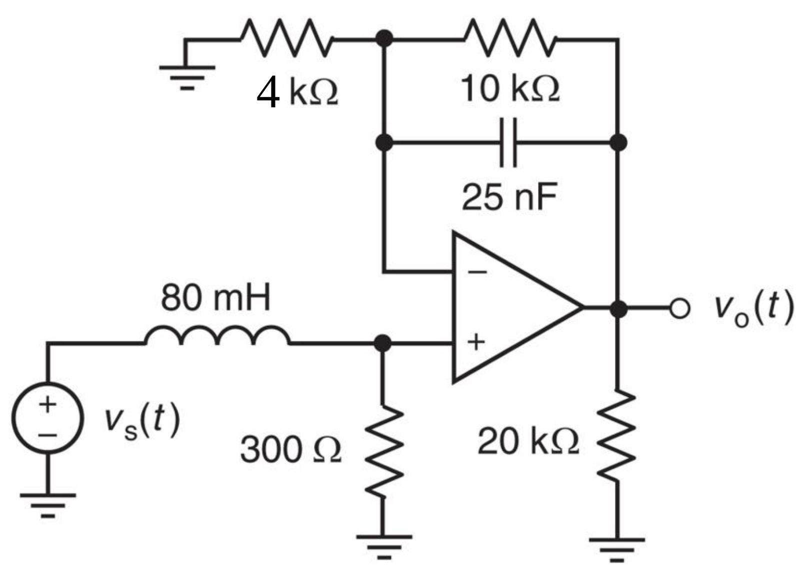
$$v(t) = 14.4\cos(3t - 22^\circ)\ [V]$$

Example 10.6-4 – OpAmp

The input to the ac circuit shown in Figure 10.6-13 is the voltage source voltage

$$v_s(t) = 125 \cos(5000t + 15°) \text{ mV}$$

Determine the output voltage $v_o(t)$.



Example 10.6-4 Solution

Solution

The impedances of the capacitor and inductor are

$$\mathbf{Z}_C = -j\frac{1}{5000(25\times10^{-9})} = -j8000\ \Omega \text{ and } \mathbf{Z}_L = j5000(80\times10^{-3}) = j400\ \Omega$$

Figure 10.6-14 show the circuit represented in the frequency domain using phasors and impedances.

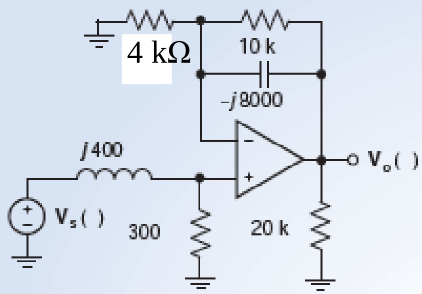


FIGURE 10.6-14 The frequency domain representation of the circuit from Figure 10.6-13.

Applying KCL at the noninverting node of the op amp, we get

$$\frac{\mathbf{V}_s - \mathbf{V}_a}{j400} = \frac{\mathbf{V}_a}{300} + 0 \quad \Rightarrow \quad \mathbf{V}_s = \mathbf{V}_a\left(1 + \frac{j400}{300}\right)$$

Solving for $\mathbf{V}_a$ gives

$$\mathbf{V}_a = \left(\frac{300}{300 + j400}\right)\mathbf{V}_s = \left(0.6\angle{-53.1^\circ}\right)\left(0.125\angle{15^\circ}\right) = 0.075\angle{-38.1^\circ}\ \text{V}$$

Next, apply KCL at the inverting node of the op amp to get

$$\frac{\mathbf{V}_a}{4000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{10,000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{-j8000} = 0$$

Multiplying by 80,000 gives

$$0 = 20\mathbf{V}_a + 8(\mathbf{V}_a - \mathbf{V}_o) + j10(\mathbf{V}_a - \mathbf{V}_o)$$

Solving for $\mathbf{V}_o$ gives

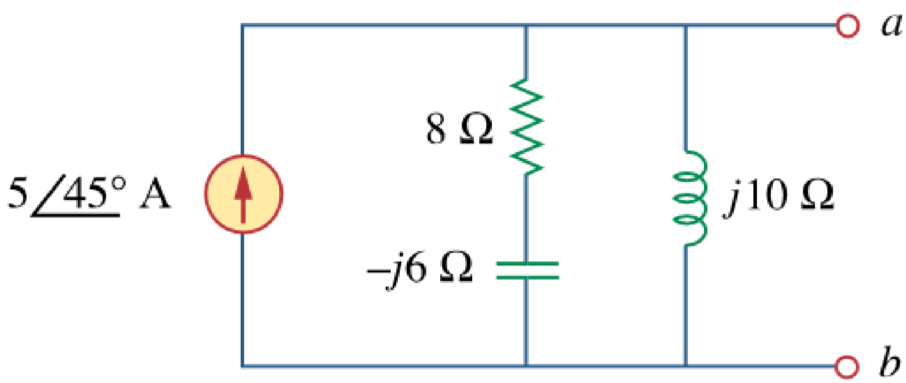
$$\mathbf{V}_o = \frac{28 + j10}{8 + j10}\mathbf{V}_a = \frac{29.73\angle{19.65^\circ}}{12.81\angle{51.34^\circ}}\left(0.075\angle{-38.1^\circ}\right) = 0.174\angle{-69.79^\circ}$$

In the time domain, the output voltage is

$$v_o(t) = 174\cos(5000t - 69.79^\circ)\ \text{mV}$$

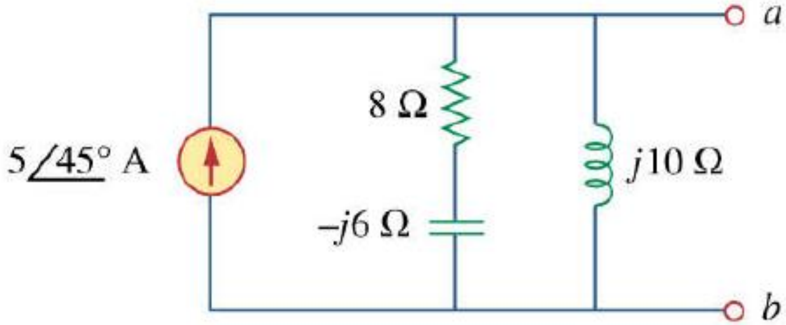
Example 10

For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a*-*b*.

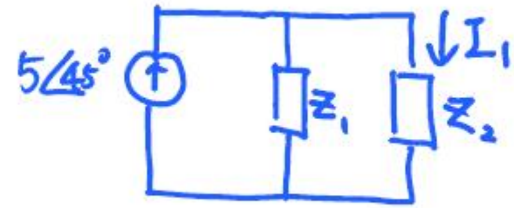


Example 10 Solution

For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a*-*b*.



$V_{Th} = ?$



$Z_1 = 8 - j6$

$Z_2 = j10$

$$I_1 = (5\angle 45^\circ)\frac{Z_1}{Z_1 + Z_2}$$

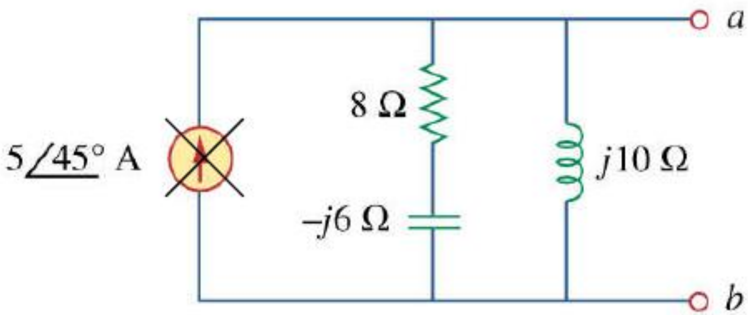
$$I_1 = (5\angle 45^\circ)\frac{8 - j6}{8 - j6 + j10} = 5.6 - 0.5j$$

$$V_{Th} = I_1 Z_2 = (5.6 - 0.5j)(j10)$$

$$V_{Th} = 5 + j\,55.7$$

Example 10 Solution

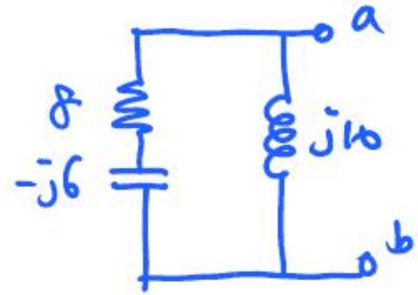
For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a*-*b*.



$V_{Th} = ?$

$V_{Th} = 5 + j\,55.7$

$Z_{th} = ?$



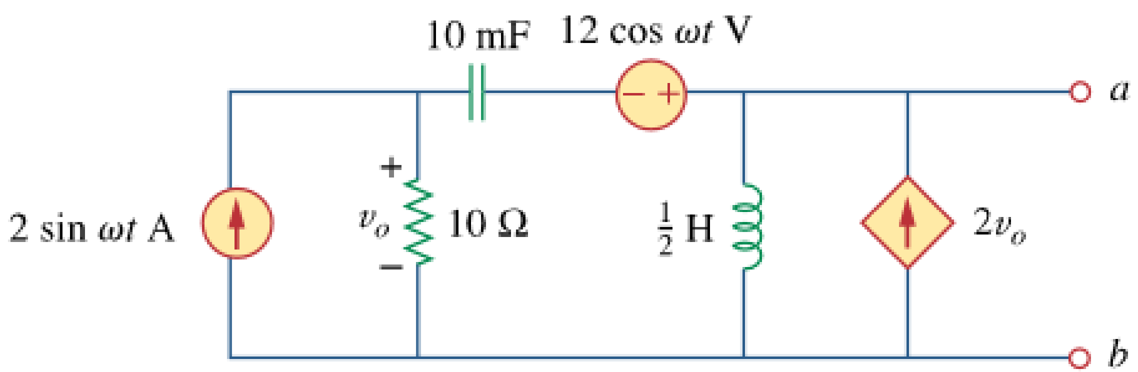
$$Z_{eq} = Z_1 // Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\rightarrow Z_{eq} = \frac{(8 - j6)(j10)}{8 - j6 + j10}$$

$$\rightarrow Z_{eq} = 10 + j5$$

Example 11 – Thévenin/Norton

10.66 At terminals *a*-*b*, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.109. Take $\omega = 10$ rad/s.



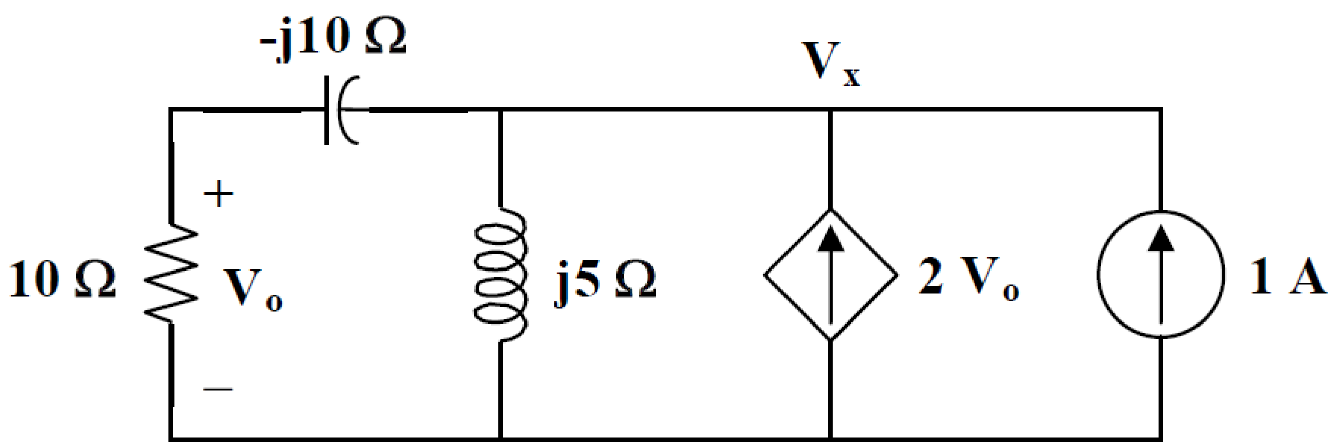
Example 11 Solution

$$\omega = 10$$

$$0.5 \text{ H} \longrightarrow j\omega L = j(10)(0.5) = j5$$

$$10 \text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(10 \times 10^{-3})} = -j10$$

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

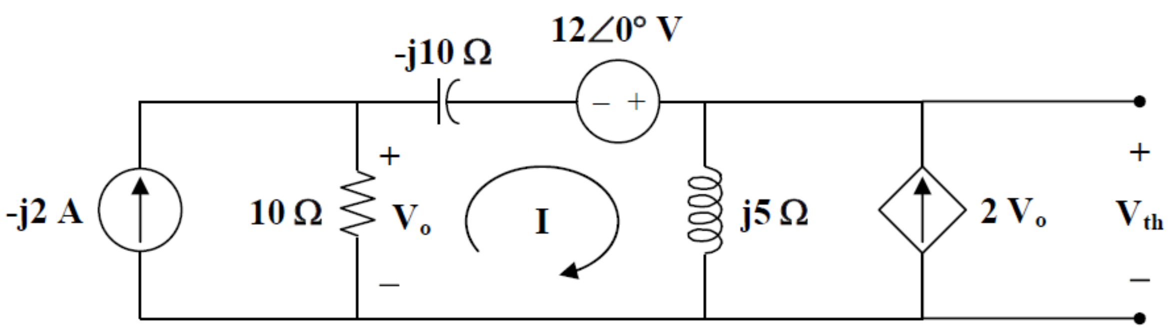
Example 11 Solution

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_x}{j5} + \frac{\mathbf{V}_x}{10 - j10}, \qquad \text{where} \quad \mathbf{V}_o = \frac{10\mathbf{V}_x}{10 - j10}$$

$$1 + \frac{19\mathbf{V}_x}{10 - j10} = \frac{\mathbf{V}_x}{j5} \longrightarrow \mathbf{V}_x = \frac{-10 + j10}{21 + j2}$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \frac{\mathbf{V}_x}{1} = \frac{14.142\angle 135^\circ}{21.095\angle 5.44^\circ} = \mathbf{670\angle 129.56^\circ\ m\Omega}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$(10 - j10 + j5)\mathbf{I} - (10)(-j2) + j5(2\mathbf{V}_o) - 12 = 0$$

where $\mathbf{V}_o = (10)(-j2 - \mathbf{I})$

Example 11 Solution

Thus,

$$(10 - j105)\mathbf{I} = -188 - j20$$

$$\mathbf{I} = \frac{188 + j20}{-10 + j105}$$

$$\mathbf{V}_{th} = j5(\mathbf{I} + 2\mathbf{V}_o) = j5(-19\mathbf{I} - j40) = -j95\mathbf{I} + 200$$

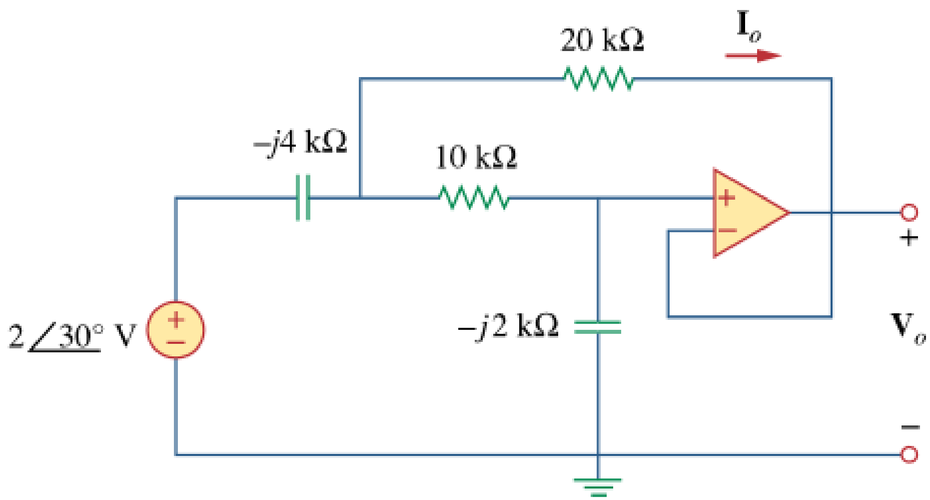
$$\mathbf{V}_{th} = \frac{-j95(188 + j20)}{-10 + j105} + 200 = \frac{(95\angle -90°)(189.06\angle 6.07°)}{105.48\angle 95.44} + 200$$

$$= 170.28\angle -179.37° + 200 = -170.27 - j1.8723 + 200 = 29.73 - j1.8723$$

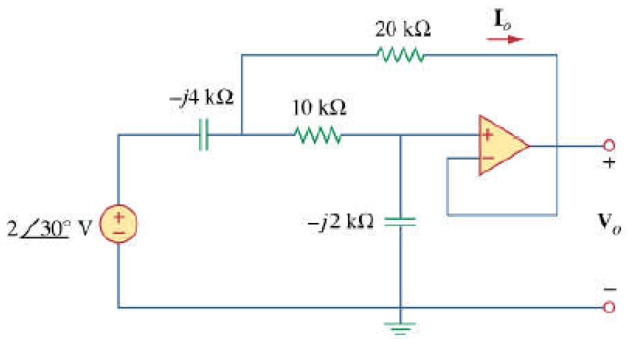
$$\mathbf{V}_{th} = \mathbf{29.79\angle{-3.6°}\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{29.79\angle -3.6°}{0.67\angle 129.56°} = \mathbf{44.46\angle{-133.16°}\ A}$$

Example 12



Example 12 Solution



$$I_o = \frac{V_1 - V_o}{20k}$$

Nodal @ V_1 :

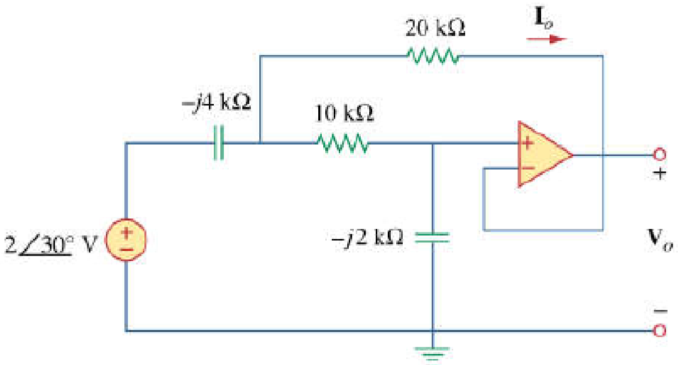
$$0 = -\frac{V_1 - (2\angle 30^\circ)}{-j4{,}000} - \frac{V_1 - V_o}{10{,}000} - \frac{V_1 - V_o}{20{,}000}$$

$$2\angle 30^\circ \Rightarrow 1.73 + j1$$

$$V_1\left(-\frac{1}{-j4{,}000} - \frac{1}{10{,}000} - \frac{1}{20{,}000}\right) + V_o\left(\frac{1}{10{,}000} + \frac{1}{20{,}000}\right) + \frac{1.73+j1}{-j4{,}000} = 0$$

$$① \rightarrow (1.73 + 1j) = (1 - j0.6)V_1 + (0 + j0.6)V_o$$

Example 12 Solution



Nodal @ V_o

$$-\frac{V_o - V_1}{10{,}000} - \frac{V_o}{-j2{,}000} = 0$$

$$\rightarrow V_o\left(-\frac{1}{10k} + \frac{1}{j2{,}000}\right) + V_1\left(\frac{1}{10{,}000}\right) = 0$$

② $\rightarrow V_1 = V_o(1 + j5)$

$\therefore (1.73 + 1j) = (4 + j5)\, V_o$

$\rightarrow V_o = 0.31 \angle -21.3° \ V$

$$\rightarrow I_o = \frac{V_1 - V_o}{20{,}000} = 78 \angle 69° \ \mu A.$$