Chapter 8

The Complete Response of RL and RC Circuits

First-Order Circuits

- Circuits that contain only one inductor and no capacitors or only one capacitor and no inductors can be represented by a first-order differential equation. These circuits are called *first-order circuits*.
- Thévenin's & Norton's equivalent circuits are used to simplify the analysis with one energy storage element.

First-Order Circuits

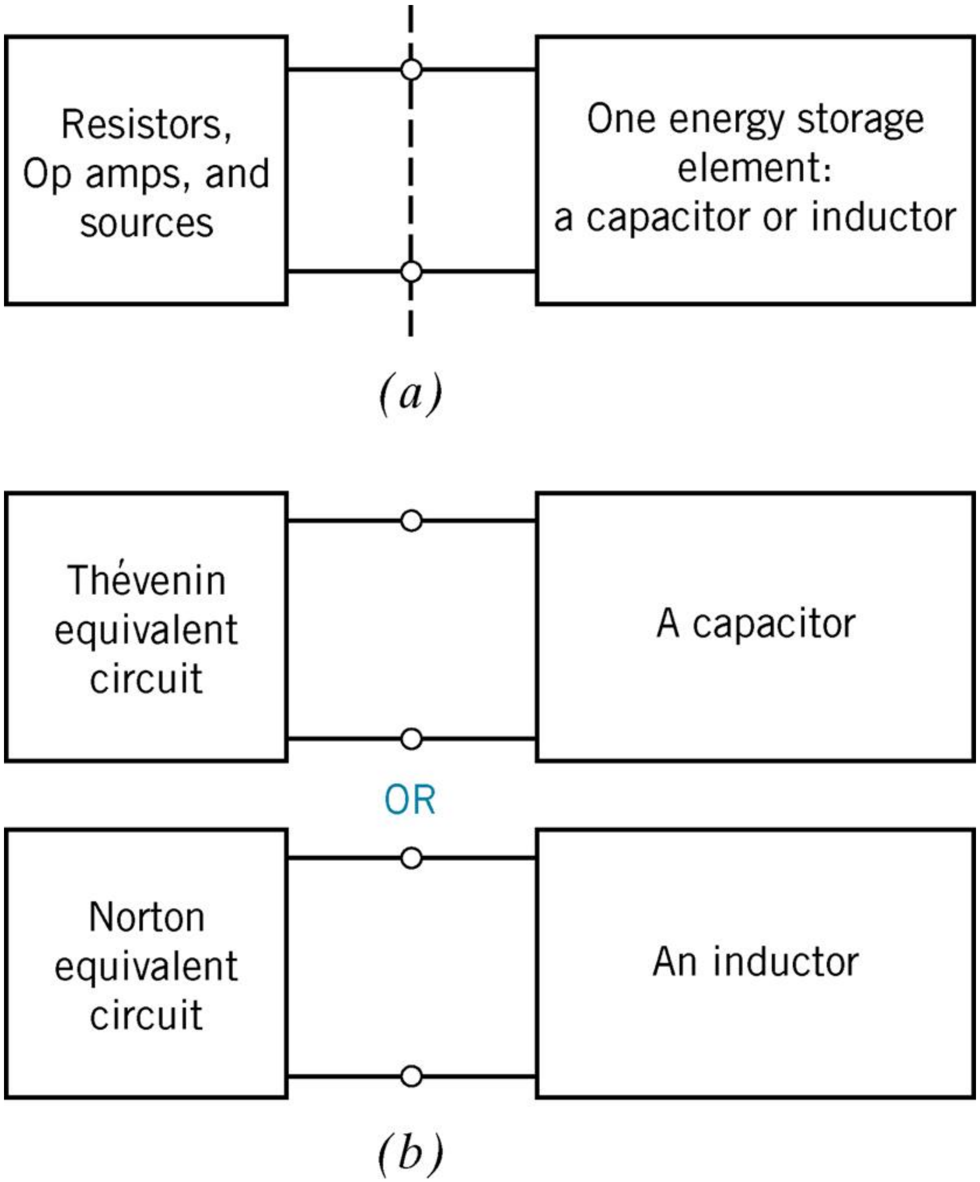
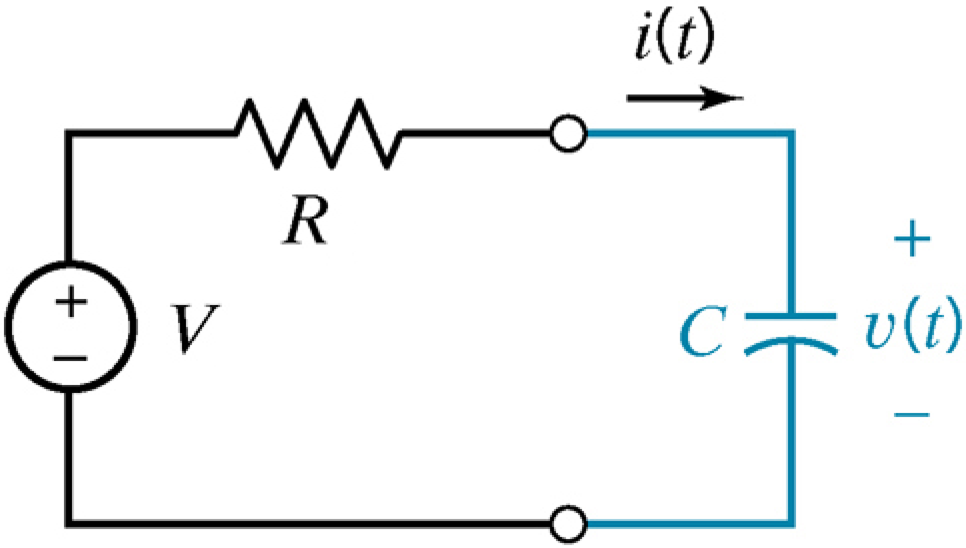


FIGURE 8.2-1 A plan for analyzing first-order circuits. (*a*) First, separate the energy storage element from the rest of the circuit. (*b*) Next, replace the circuit connected to a capacitor by its Thévenin equivalent circuit or replace the circuit connected to an inductor by its Norton equivalent circuit.

FIRST-ORDER RC CIRCUITS

1st Order RC Circuit



$$-V + R\cdot i(t) + v(t) = 0$$

$$\Rightarrow -V + R\cdot C\frac{dv(t)}{dt} + v(t) = 0$$

$$\Rightarrow R\cdot C\frac{dv(t)}{dt} + v(t) = V$$

$$\Rightarrow \frac{dv(t)}{dt} + \frac{v(t)}{RC} = \frac{V}{RC}$$

1st Order Differential Equation

- Complete Response of the differential equation =

  Transient Response (t is small) + Steady State Response ($t \longrightarrow \infty$)

  (Natural Response) (Forced Response)

$$\int \left(\frac{dv(t)}{dt} + \frac{v(t)}{RC} \right) = \int \frac{V}{RC}$$

- If input voltage V is constant (DC), the answer will be:

$$\boxed{v(t) = V + \left(v(0) - V\right) e^{-\frac{t}{RC}}}$$

1st Order Differential Equation

- If input voltage V is constant (DC), the answer will be:

$$v(t) = V + \boxed{\left(v(0) - V\right)e^{-\frac{t}{RC}}}$$

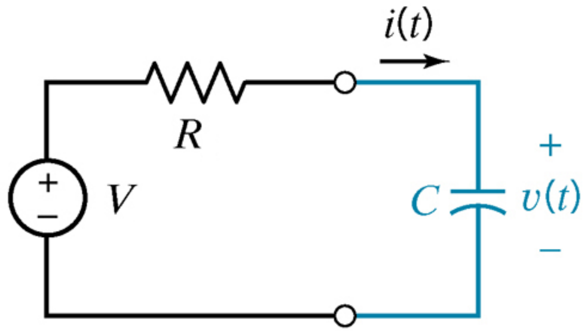
Time constant
τ = ***RC***

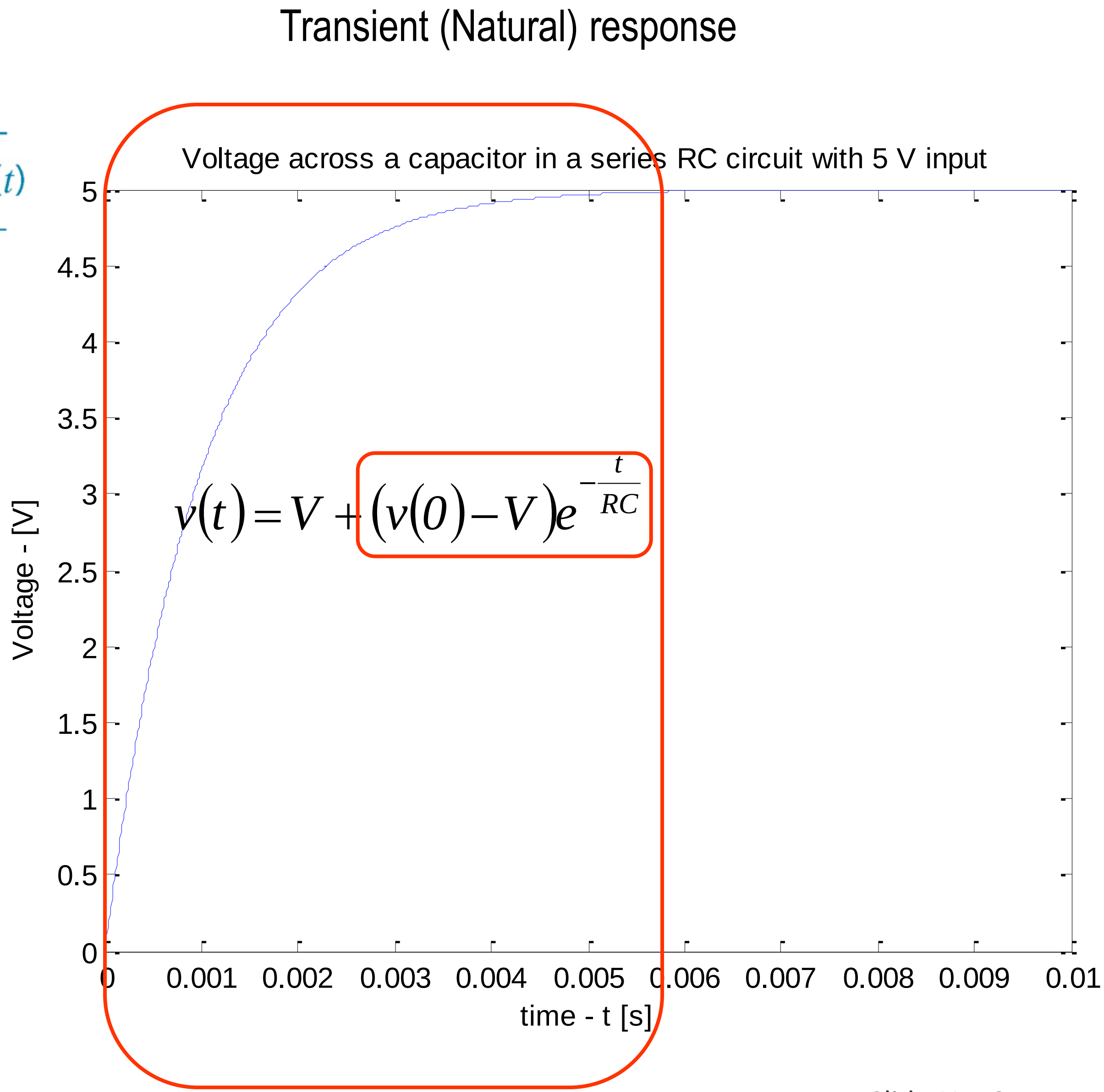
Transient (Natural) Response: The part of the response which disappears after long enough time

$$v(t) = \boxed{V} + \left(v(0) - V\right)e^{-\frac{t}{RC}}$$

Steady State (Forced) Response: The part of the response which stays after long enough time passes.

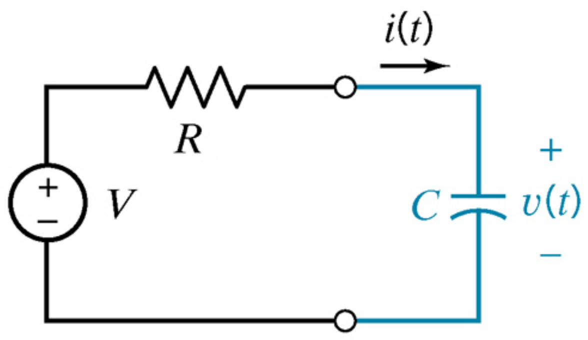
Transient (Natural) Response



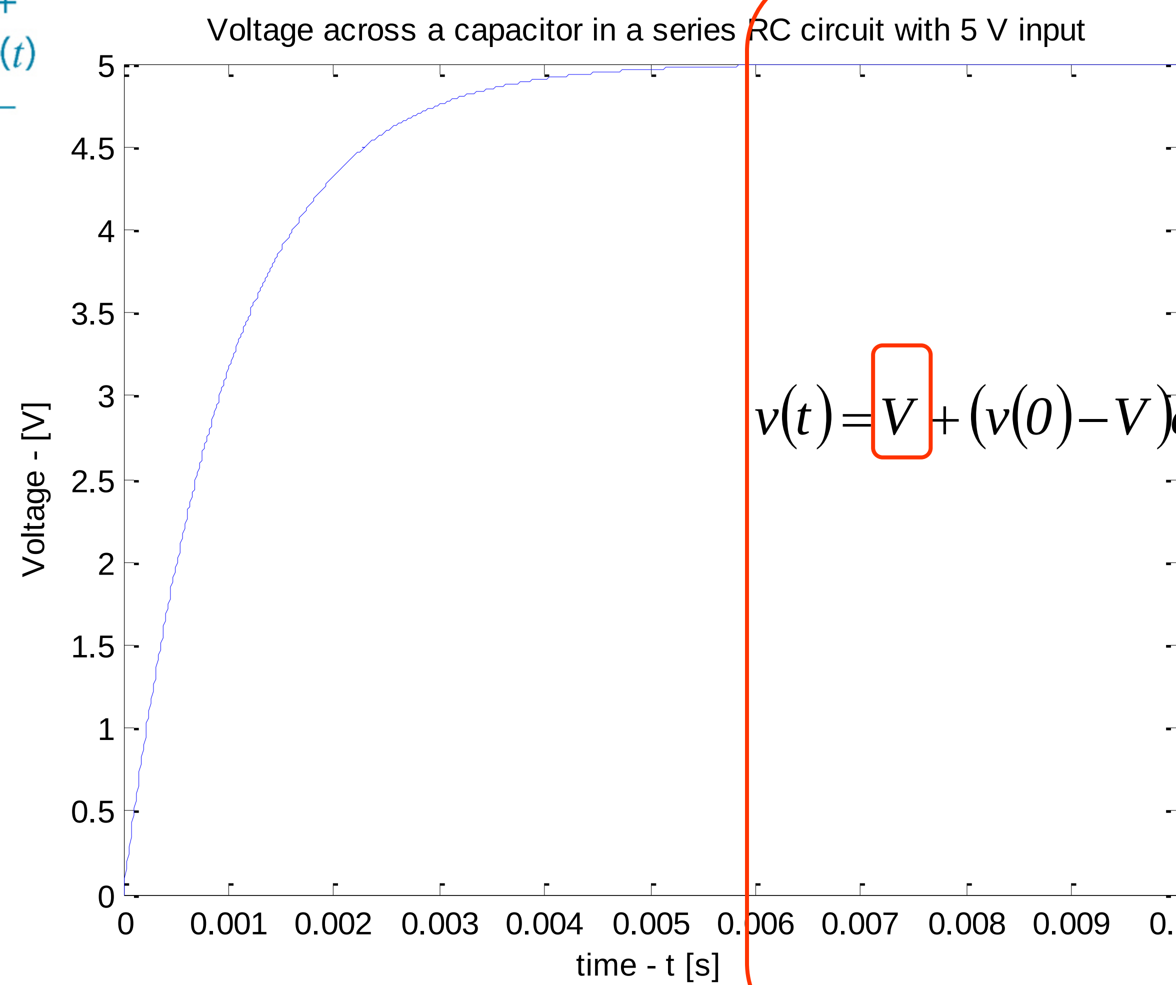


$$v(t) = V + \boxed{(v(0) - V)e^{-\frac{t}{RC}}}$$

Steady-State (Forced) Response



Steady-State (Forced) Response



$$v(t) = \boxed{V} + \left(v(0) - V\right)e^{-\frac{t}{RC}}$$

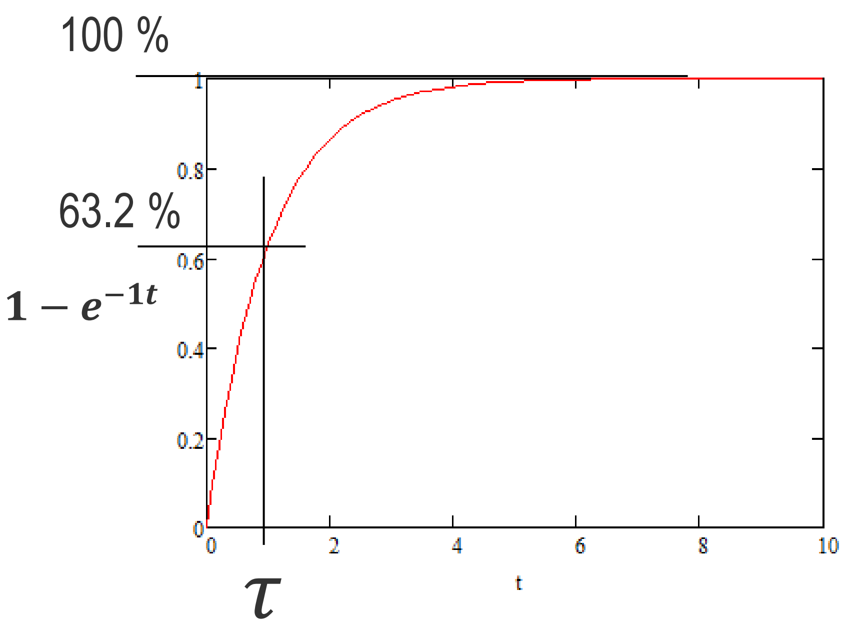
Time Constant

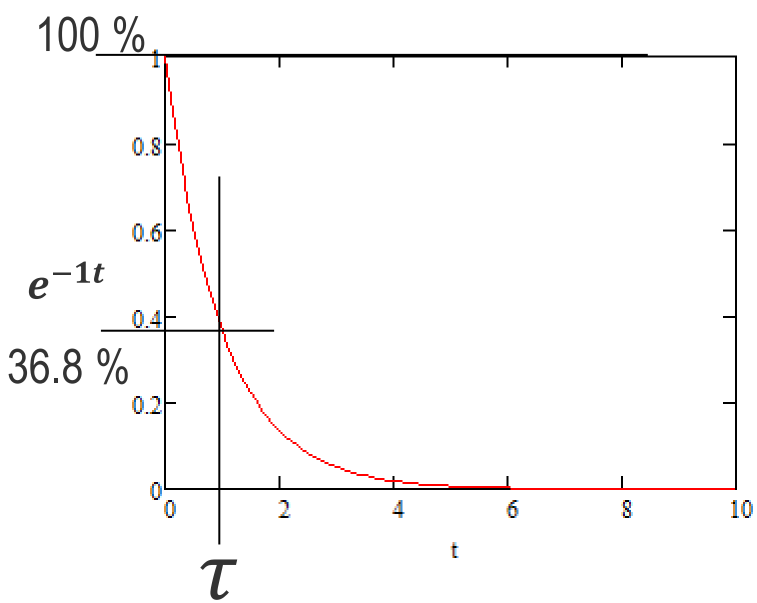
- At t = τ = RC,

$$Ve^{-\frac{\tau}{RC}} = Ve^{-1} = 0.368 \cdot V$$

Setting for $t = \tau$ for <u>the rise sets $V(t)$</u> equal to $0.63V_{max}$. This means that the time constant is the time elapsed after 63% of V_{max} has been reached

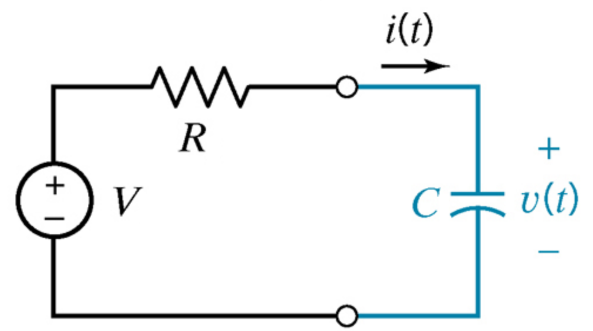
Setting for $t = \tau$ for <u>the fall sets $V(t)$</u> equal to $0.37V_{max}$, meaning that the time constant is the time elapsed after it has fallen to 37% of V_{max}.





Example 1

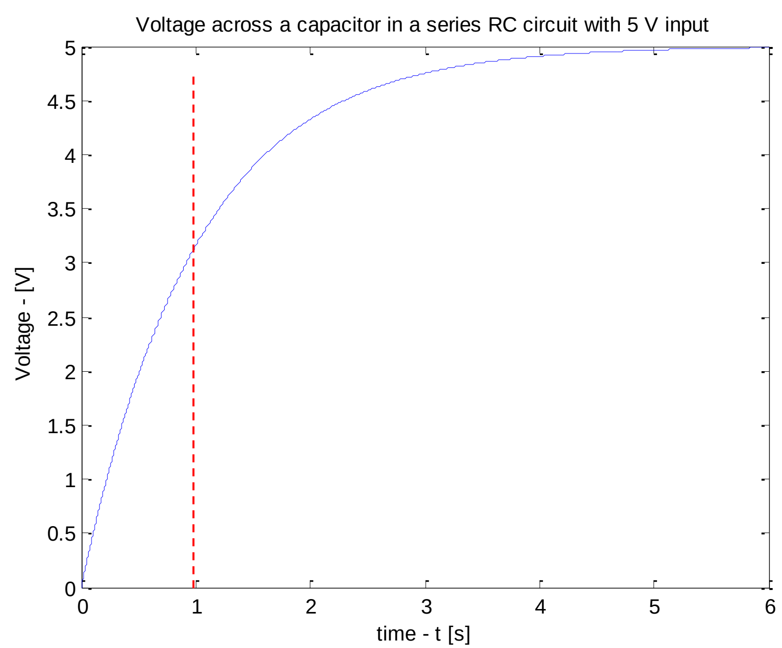
- R = 1 Ω, C = 1 F, initial capacitor voltage, $v(0) = 0$. 5 V DC is applied to an RC circuit. What is the capacitor voltage?



$$v(t) = V + \left(v(0) - V\right)e^{-\frac{t}{RC}}$$

$$v(t) = 5 + (0 - 5)e^{-\frac{t}{1\times1}}$$

$$= 5\left(1 - e^{-t}\right)$$



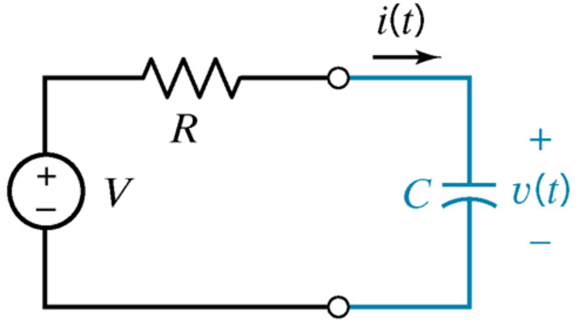
Example 1

- R = 1 Ω, C = 1 mF, initial capacitor voltage, $v(0) = 0$. 5 V DC is applied to an RC circuit. What is the capacitor voltage?

$$v(t) = V + \left(v(0) - V\right)e^{-\frac{t}{RC}}$$

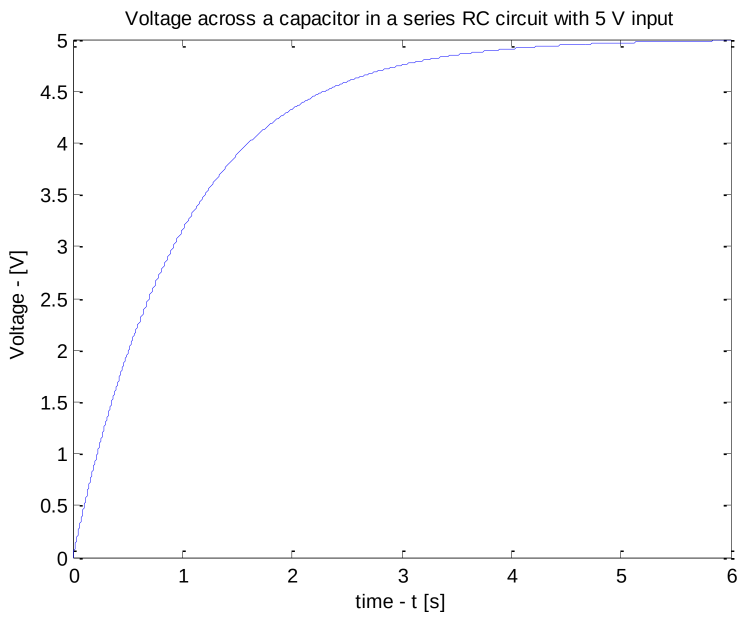
$$v(t) = 5 + (0 - 5)e^{-\frac{t}{1\times 1m}}$$

$$= 5\left(1 - e^{-10^3 t}\right)$$

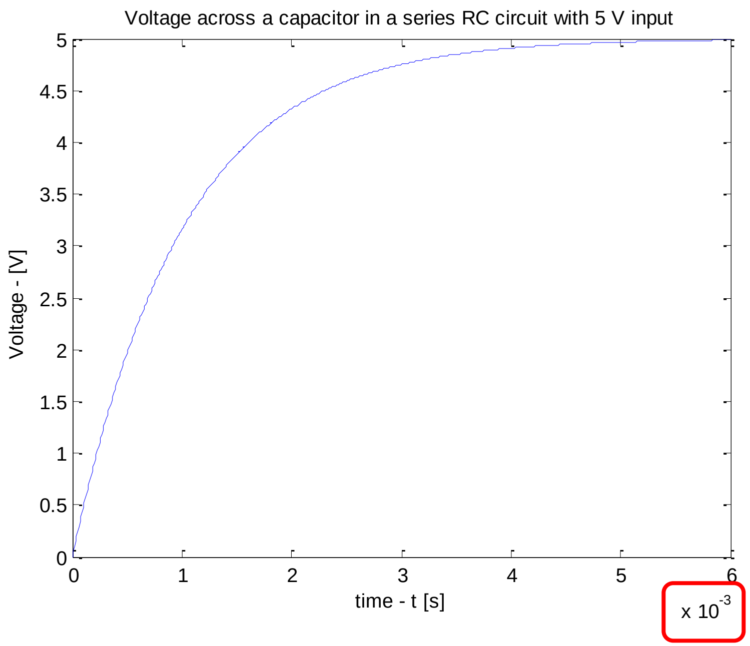


Example 1

R = 1 Ω, C = 1 F

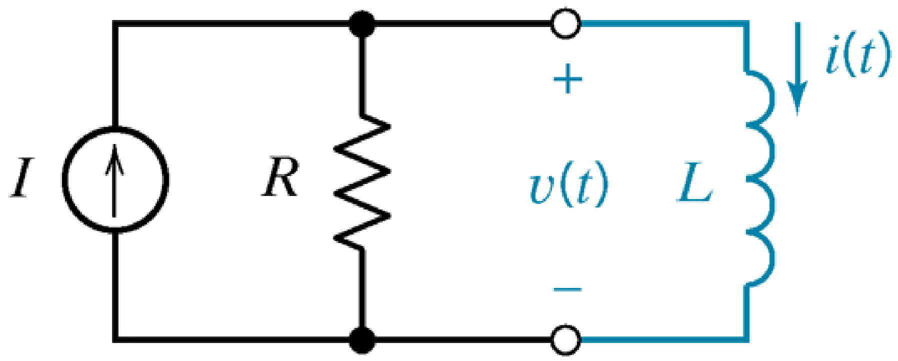


R = 1 Ω, C = 1 mF



FIRST-ORDER RL CIRCUITS

1st Order RL Circuit



$$I = i_R + i(t)$$

$$\Rightarrow I = \frac{v(t)}{R} + i(t)$$

$$\Rightarrow I = \frac{L\frac{di(t)}{dt}}{R} + i(t)$$

$$\Rightarrow I = \frac{L}{R}\frac{di(t)}{dt} + i(t)$$

$$\Rightarrow \frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{R}{L}$$

1st Order Differential Equation

- Complete Response of the differential equation =

  Transient Response (t is small) + Steady State Response ($t \rightarrow \infty$)

  (Natural Response) (Forced Response)

$$\int \left(\frac{di(t)}{dt} + \frac{R}{L} i(t) \right) = \int \frac{R}{L} I$$

- If input current ***I*** is constant (DC), the answer will be:

$$\boxed{i(t) = I + \left(i(0) - I\right) e^{-\frac{R}{L}t}}$$

1st Order Differential Equation

- If input current ***I*** is constant (DC), the answer will be:

$$i(t) = I + \boxed{\left(i(0) - I\right)e^{-\frac{R}{L}t}}$$

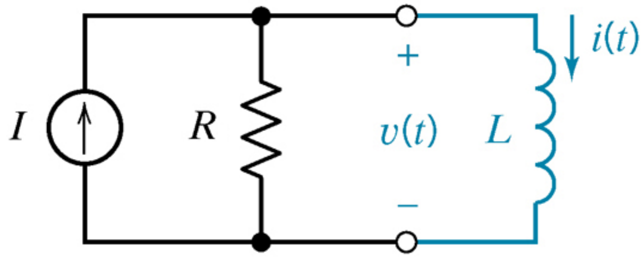
Transient (Natural) Response: The part of the response which disappears after long enough time

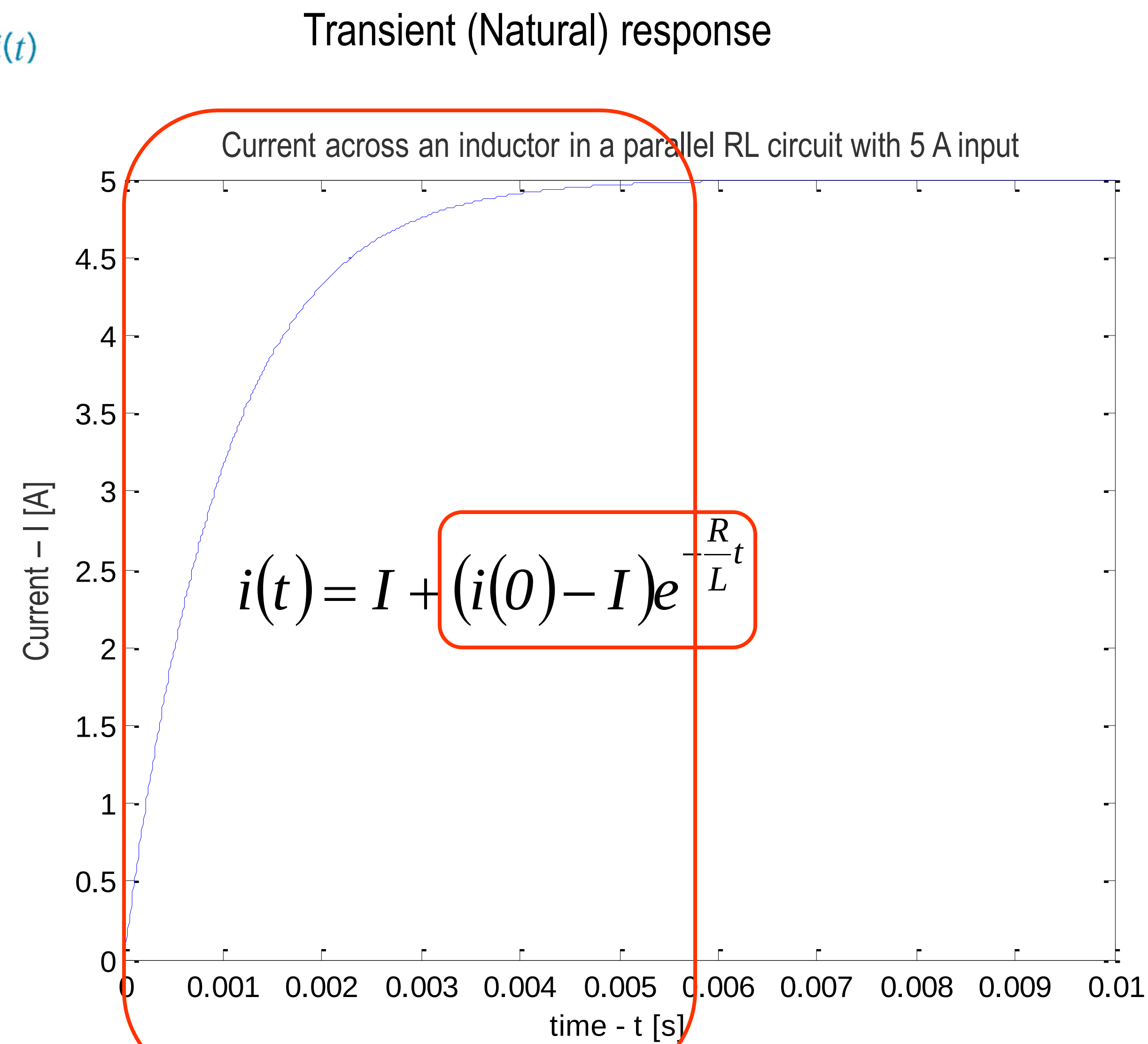
$$i(t) = \boxed{I} + \left(i(0) - I\right)e^{-\frac{t}{\boxed{\tau}}}$$

Time constant
τ = ***L / R***

Steady State (Forced) Response: The part of the response which stays after long enough time passes.

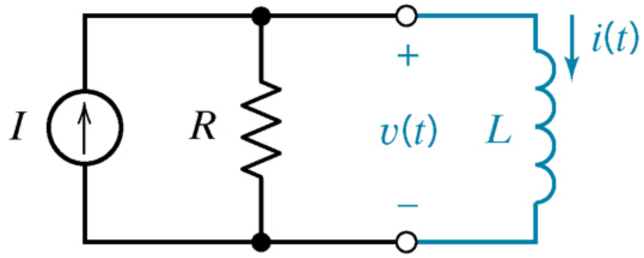
Transient (Natural) Response



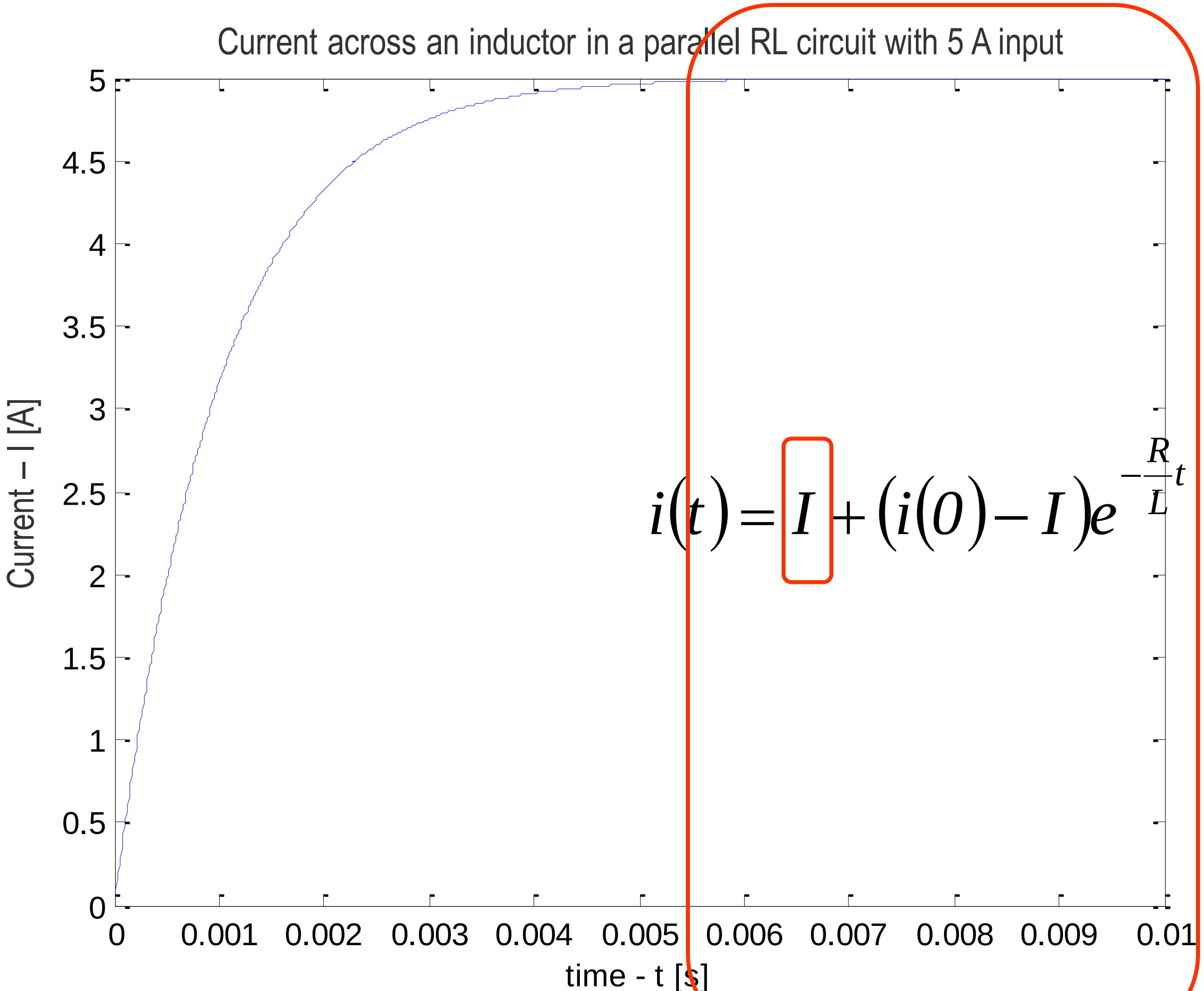


$$i(t) = I + \boxed{(i(0) - I)e^{-\frac{R}{L}t}}$$

Steady-State (Forced) Response



Steady-State Forced Response



$$i(t) = \boxed{I} + \left(i(0) - I\right)e^{-\frac{R}{L}t}$$

REMEMBER: Initial Conditions

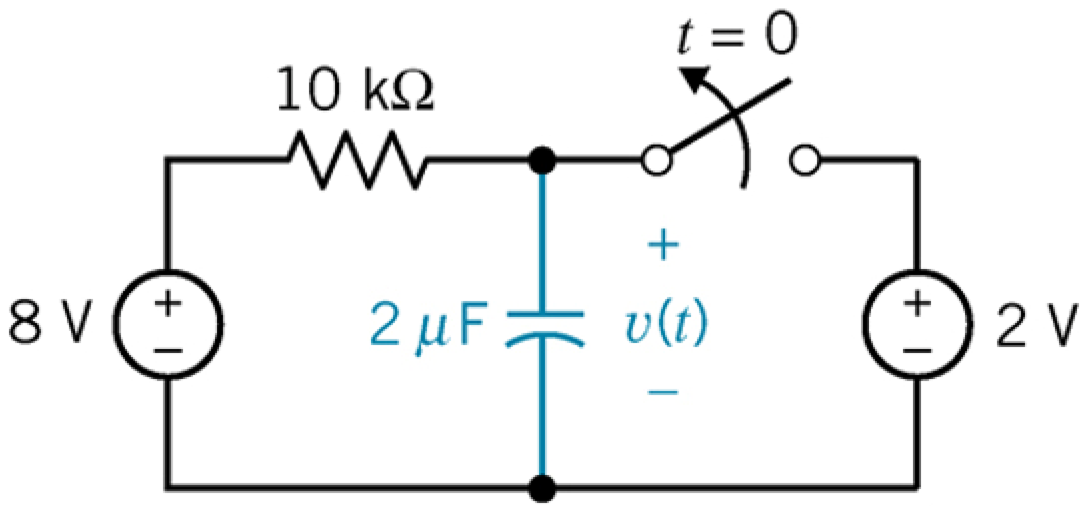
- **DC circuits**
  - Independent voltage and current sources are DC (constant) – They do not change with time.
- The circuit includes at least one capacitor or one inductor (If multiple capacitors/inductors find the equivalent)
- The change in capacitor voltage or in inductor current is NOT instantaneous. It is continuous.
  - We denote the time immediately before the switch opens/closes as t_0^-
  - We denote the time immediately after the switch opens/closes as t_0^+
  - The capacitor voltage or inductor current have the same values right before and right after the switch closes

REMEMBER: Initial Conditions

- A **capacitor** in a DC circuit behaves like an **open circuit** in *steady state*.
- An **inductor** in a DC circuit behaves like a **short circuit** in *steady state*.
- **Draw** the circuit for *before the switch operation* and *after the switch operation*!

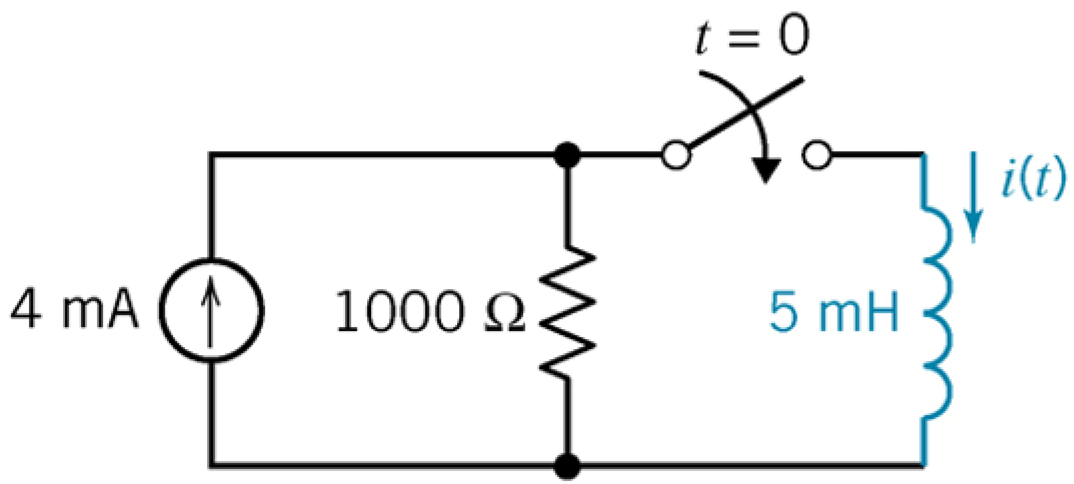
Example 8.3-1

- What is the value of the ***capacitor voltage*** 50 ms after the switch opens? $t \geq 0$



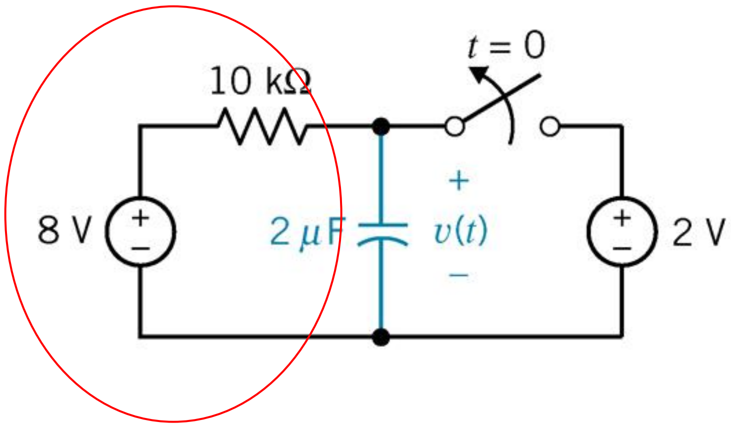
Example 8.3-2

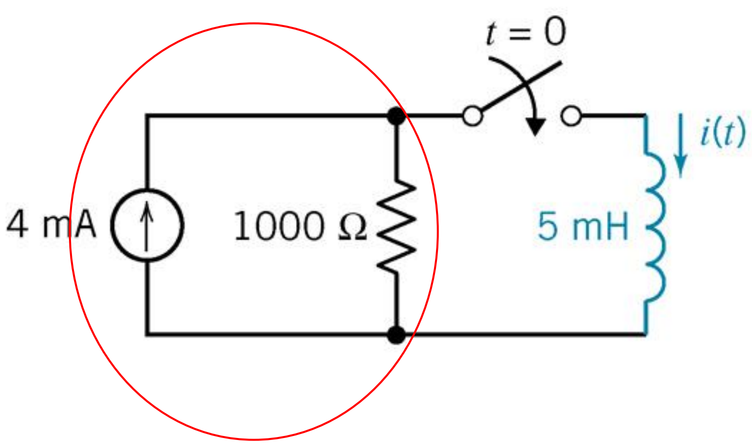
- Find the inductor current after the switch closes. How long will it take for the ***<u>inductor current</u>*** to reach **2 mA**? $t \geq 0$



In the previous problems

- At t ≥ 0, the circuits are already Thévenin or Norton Eq. Circuits

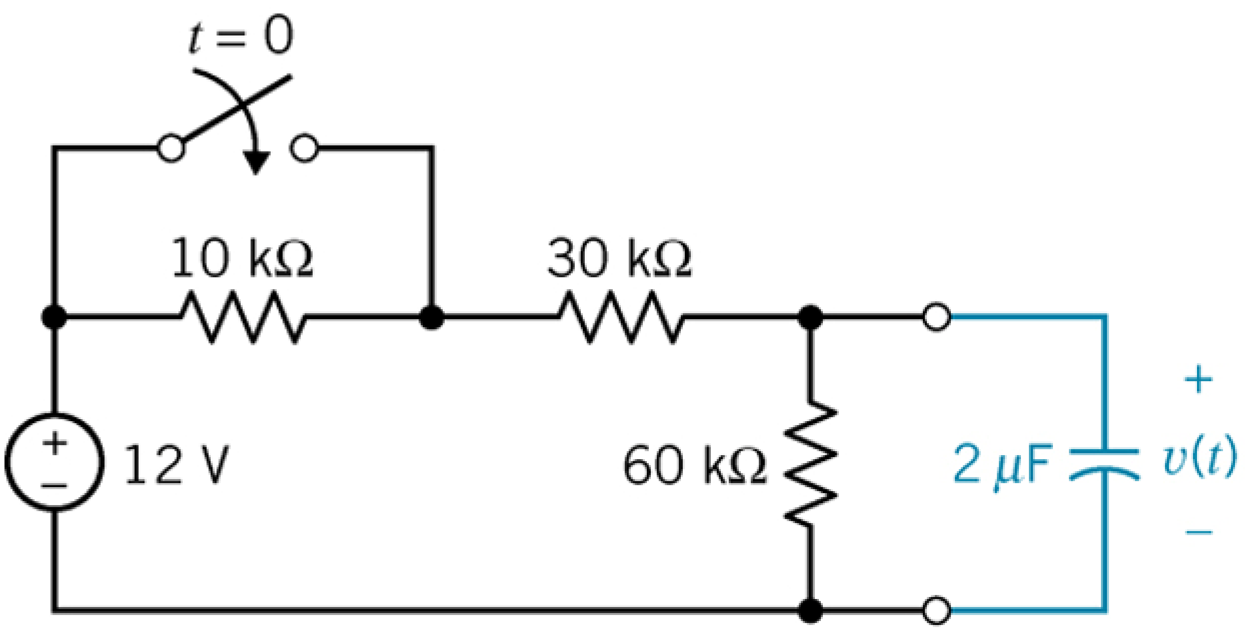




See next problem…

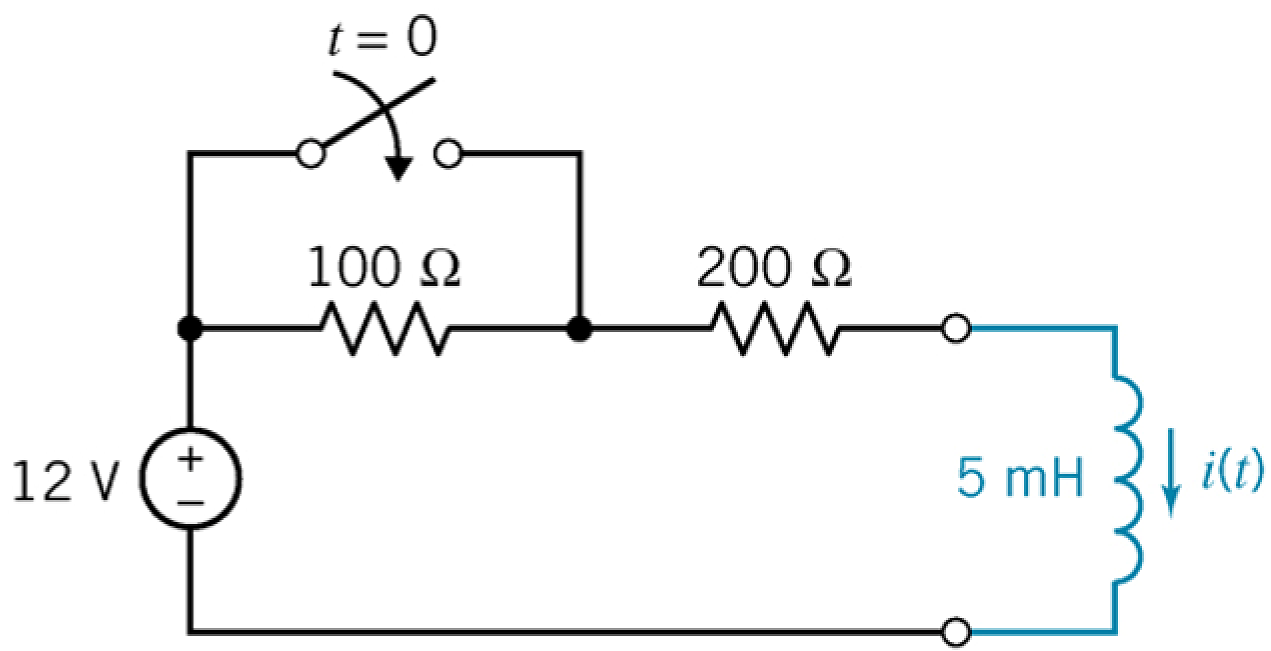
Example 8.3-3

- The switch has been open for a long time and the circuit has reached steady state before the switch closes at time t = 0. Find the ***capacitor voltage*** for $t \geq 0$.



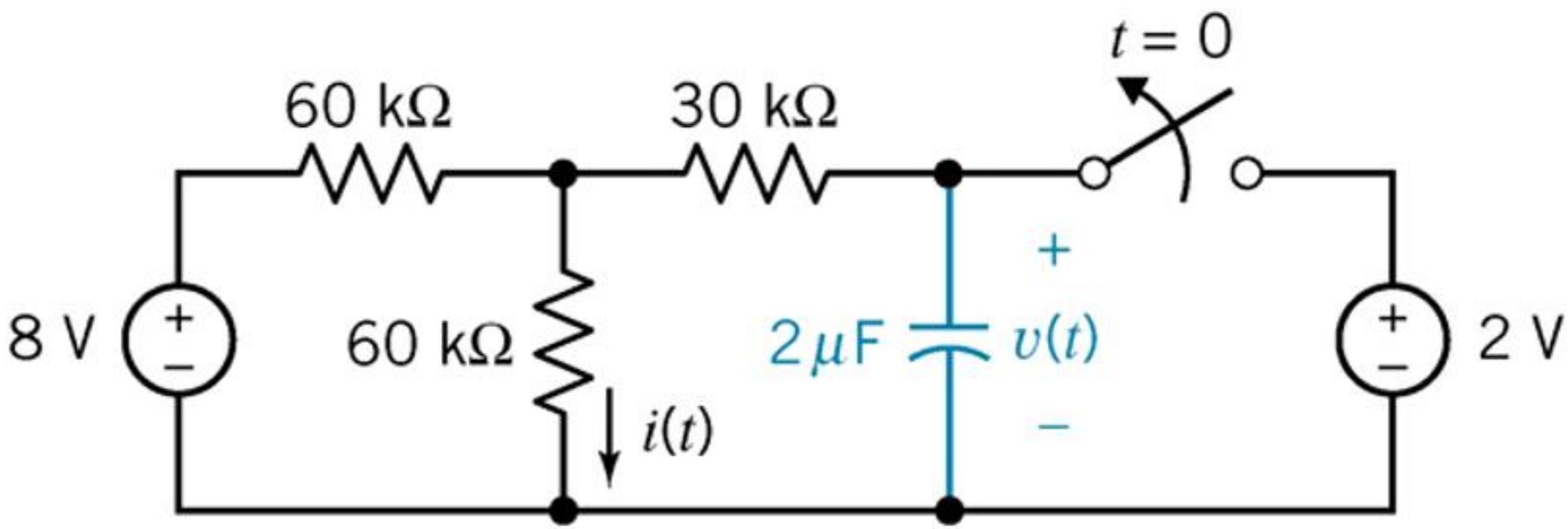
Example 8.3-4

- The switch has been open for a long time and the circuit has reached steady state before the switch closes at time t = 0. Find the ***inductor current*** for $t \geq 0$.



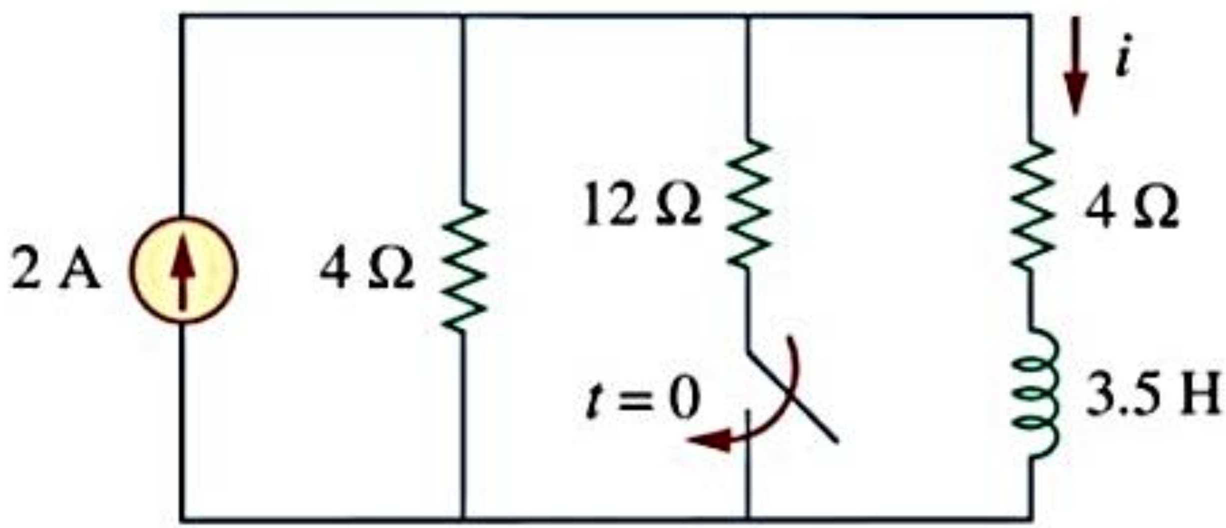
Example 8.3-5

- The circuit is at steady state before the switch opens. Find the current ***i*(t)** for t > 0. What is the voltage $v(t)$ at t = 60 ms?



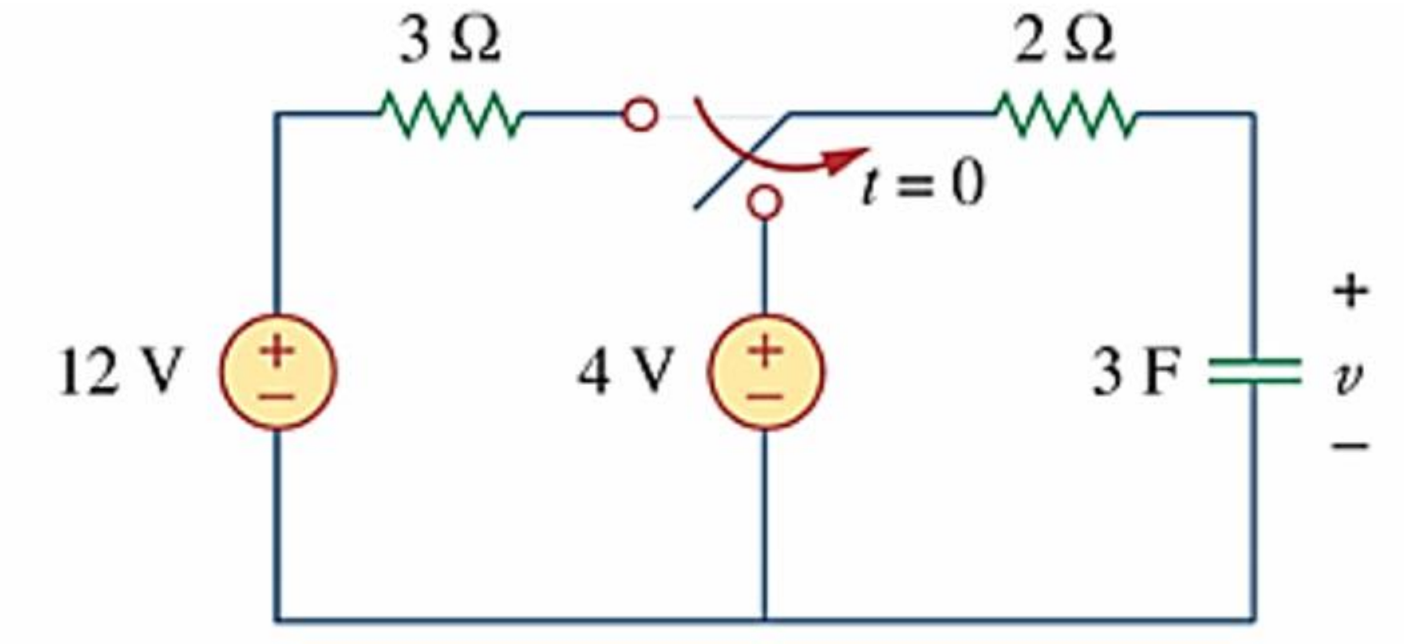
Example 2

Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120.



Example 3

Find the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in the figure



Example 8.5-1

- The first order switch is at steady state before the switch closes at $t = 0$. Find the capacitor voltage, $v(t)$, for $t > 0$.