nfa -> dfa conversion

Tuesday, September 17, 2024 11:55 AM

prove dfa ≡ nfa. ⇒ dfa → nfa ∧ nfa → dfa
① ②

aside: $\delta_{nfa}: Q \times (\Sigma \cup \{\lambda\}) \to \boxed{2^Q}$

ex $\Sigma = \{a, b\}$ $\Sigma \cup \{\lambda\} = \{a, b, \lambda\}$

$Q = \{q_0, q_1\}$

$2^Q = \{\phi, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}$

$(q_0, a) = \sim$

$(q_0, b) = \sim$

$(q_0, \lambda) = \{q_0, q_1\}$

$(q_1, a) = \phi$

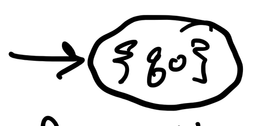
$(q_1, b) = \{q_0\}$

$(q_1, \lambda) = \{q_1\}$

② nfa → dfa ⇒ L(nfa) = L(dfa)

nfa → dfa conversion rules:

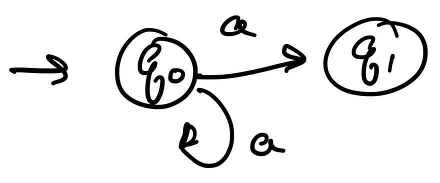
1. create a graph G_D w/ vertex $\{q_0\}$ = initial vertex



2. repeat for all edges:
   a. take any vertex, $\{q_i, \dots, q_k\}$, of G_D that has no outgoing edge for some $a \in \Sigma$.

   b. compute $\delta_n^*(q_i, a), \delta_n^*(q_j, a), \dots, \delta_n^*(q_k, a)$

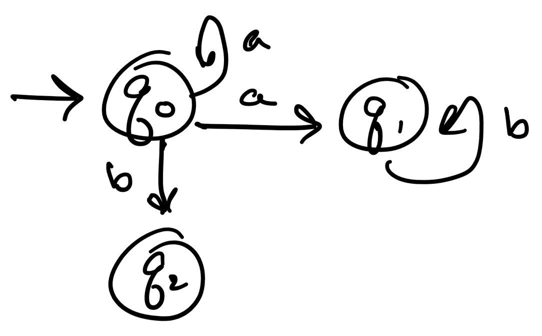
nfa



$\delta_n^*(q_0, a) = \{q_0, q_1\}$

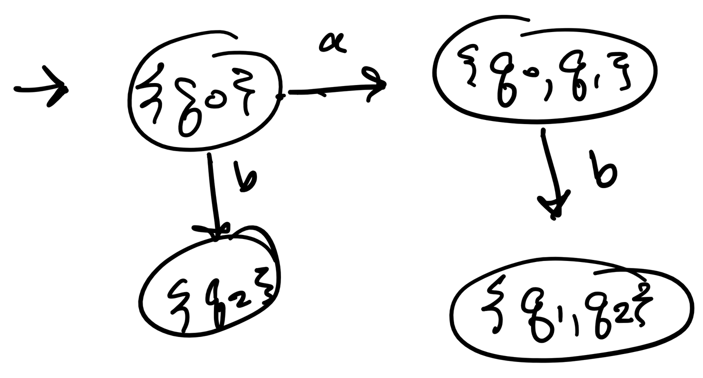
   c. form union of all $\delta_n^* \xrightarrow{a} \{q_l, \dots, q_n\}$
   d. create a vertex for G_D labeled $\{q_l, \dots, q_n\}$ if vertex does not already exist.
   e. add to G_D an edge from $\{q_i, \dots, q_k\} \xrightarrow{a} \{q_l, \dots, q_n\}$

nfa



$(q_0, a) = \{q_0, q_1\}$

G_D:



$(q_0, b) = \{q_2\}$

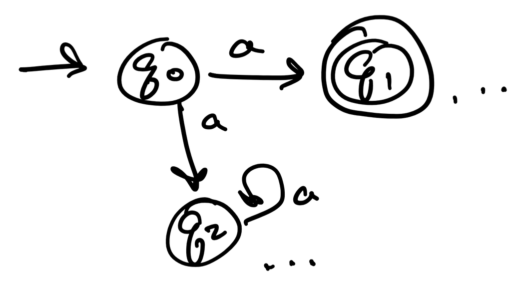
(2b) $(\{q_0, q_1\}, b) = \delta_n^*(q_0, b), \ \delta_n^*(q_1, b)$

(2e) $\{q_2\} \cup \{q_1\}$

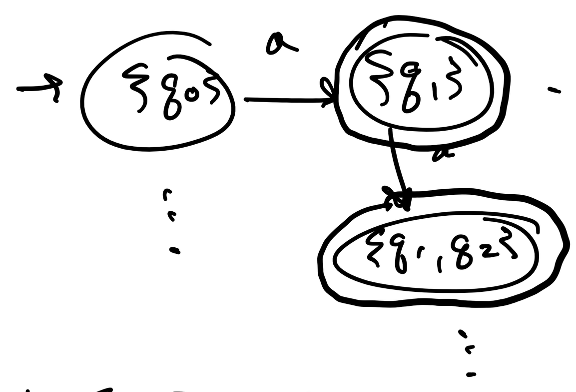
$= \{q_1, q_2\}$

3. every state of G_D w/ label $q_f \in F_N$ is final vertex

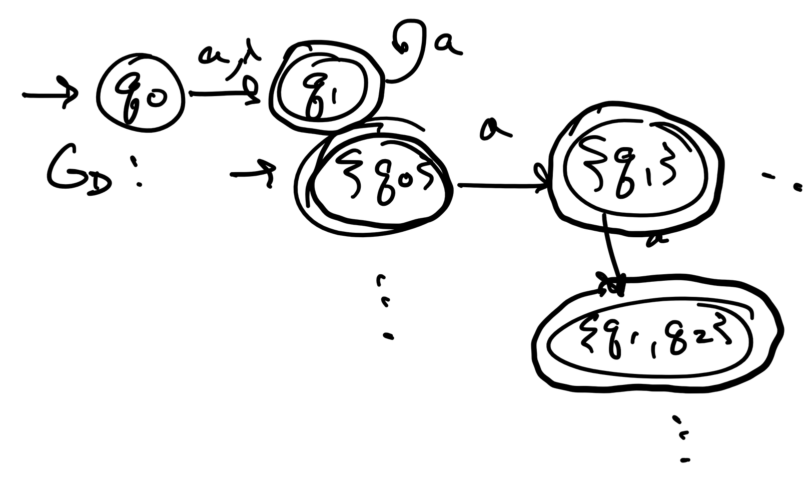
nfa:



G_D:



4. if M_n accepts λ, vertex $\{q_0\}$ in G_D is a final vertex.



∴ for ∀ nfa using nfa → dfa conversion rules ⇒ ∃ dfa, ∋ L(nfa) = L(dfa)

⇒ nfa → dfa

proven ② nfa → dfa

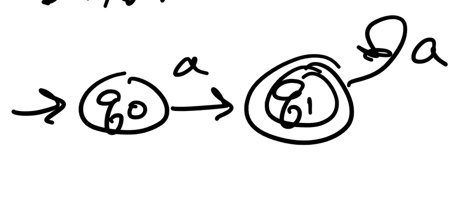
proved ① dfa → nfa and ② nfa → dfa

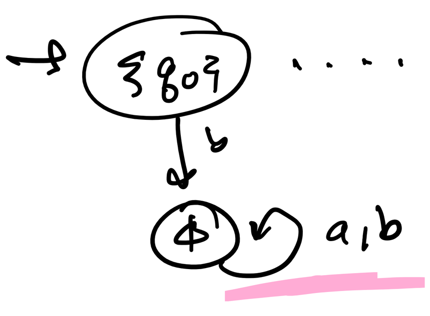
∴ nfa ≡ dfa.

remember:

$\Sigma = \{a, b\}$

nfa

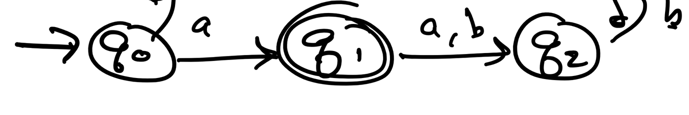




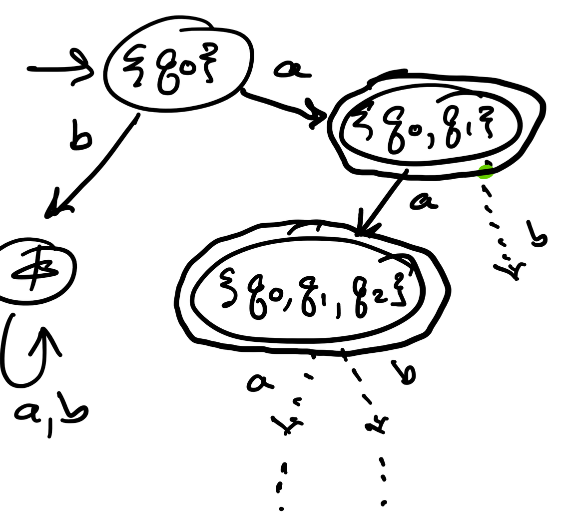
$(\phi, a) = \phi$

$(\phi, b) = \phi$

nfa



dfa



$\delta_n^*(q_0, a) = \{q_0, q_1\}$

$\delta_n^*(\{q_0, q_1\}, a)$

$= \delta_n^*(q_0, a) \cup \delta_n^*(q_1, a)$

$= \{q_0, q_1\} \cup \{q_2\}$

$= \{q_0, q_1, q_2\}$

$\delta_n^*(q_0, b) = \phi$

$\delta_n^*(\phi, a) = \phi$

$\delta_n^*(\phi, b) = \phi$