Quiz 8 Solutions

$$h_1 = (w_1 x_1 + w_2 x_2)^2 = (1 \times 0 + 2 \times 1)^2 = 4$$

$$h_2 = (w_3 x_1 + w_4 x_2)^2 = (-1 \times 0 + 1 \times 1)^2 = 1$$

$$\hat{y} = w_5 h_1 + w_6 h_2 = -1 \times 4 + 2 \times 1 = -2$$

$$L = (y - \hat{y})^2 = (-1 - -2)^2 = 1$$

$$\frac{\partial L}{\partial \hat{y}} = -2(y - \hat{y}) = -2(-1 - -2) = -2$$

$$\frac{\partial \hat{y}}{\partial w_6} = h_2 \quad 1 \qquad \frac{\partial \hat{y}}{\partial w_5} = h_1 \quad 4$$

$$\frac{\partial \hat{y}}{\partial h_1} = w_5 \quad -1 \qquad \frac{\partial \hat{y}}{\partial h_2} = w_6 \quad 2$$

$$\frac{\partial h_1}{\partial w_1} = 2x_1(w_1 x_1 + w_2 x_2) = 2(0)(2) = 0$$

$$\frac{\partial h_1}{\partial w_2} = 2x_2(w_1x_1 + w_2x_2) = 2(1)(2) = 4$$

$$\frac{\partial h_1}{\partial x_1} = 2w_1(w_1x_1 + w_2x_2) = 2(1)(2) = 4$$

$$\frac{\partial h_1}{\partial x_2} = 2w_2(w_1x_1 + w_2x_2) = 2(2)(2) = 8$$

$$\frac{\partial h_2}{\partial w_3} = 2x_1(w_3x_1 + w_4x_2) = 2(0)(1) = 0$$

$$\frac{\partial h_2}{\partial w_4} = 2x_2(w_3x_1 + w_4x_2) = 2(1)(1) = 2$$

$$\frac{\partial h_2}{\partial x_1} = 2w_3(w_3x_1 + w_4x_2) = 2(-1)(1) = -2$$

$$\frac{\partial h}{\partial x_2} = 2w_4(w_3x_1 + w_4x_2) = 2(1)(1) = 2$$

$$\frac{dL}{dw_6} = \frac{dL}{d\hat{y}} \frac{d\hat{y}}{dw_6} = -2(1) = -2$$

$$\frac{dL}{dw_5} = \frac{dL}{d\hat{y}} \frac{d\hat{y}}{dw_6} = -2(4) = -8$$

$$\frac{dL}{dw_4} = \frac{dL}{d\hat{y}} \frac{d\hat{y}}{dh_2} \frac{dh_2}{dw_4} = -2(2)(2) = -8$$

$$\frac{dL}{dw_3} = \frac{dL}{d\hat{y}} \frac{d\hat{y}}{dh_2} \frac{dh_2}{dw_3} = -2(2)(0) = 0$$

$$\frac{dL}{dw_2} = \frac{dL}{d\hat{y}} \frac{d\hat{y}}{dh_1} \frac{dh_1}{dw_2} = -2(-1)(4) = 8$$

$$\frac{dL}{dw_1} = \frac{dL}{d\hat{y}} \frac{d\hat{y}}{dh_1} \frac{dh_1}{dw_1} = -2(-1)(0) = 0$$

$$w_1 = w_1 - \eta \frac{\partial L}{\partial w_1} = 1 - 0.1(0) = \boxed{1}$$

$$w_2 = w_2 - \eta \frac{\partial L}{\partial w_2} = 2 - 0.1(8) = \boxed{1.2}$$

$$w_3 = w_3 - \eta \frac{\partial L}{\partial w_3} = -1 - 0.1(0) = \boxed{-1}$$

$$w_4 = w_4 - \eta \frac{\partial L}{\partial w_4} = 1 - 0.1(-8) = \boxed{1.8}$$

$$w_5 = w_5 - \eta \frac{\partial L}{\partial w_5} = -1 - 0.1(-8) = \boxed{0.2}$$

$$w_6 = w_6 - \eta \frac{\partial L}{\partial w_6} = 2 - 0.1(-2) = \boxed{2.2}$$