

1. What would the neural network below predict for the following test data? That is, what is  $net$  for each test input? (3 points)

$$t_1 = (1, 1, 1)$$

$$t_2 = (-5, -2, 1)$$

$$t_3 = (3, 1, 4)$$

$$\textcircled{1} \quad h_1 = 2 \quad h_2 = 4$$

$$= 1 \quad = 1$$

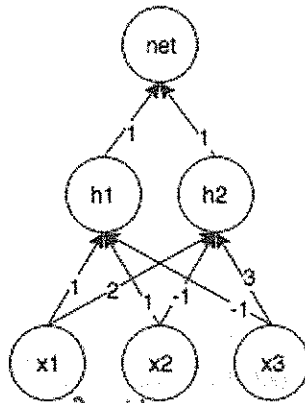
$$net = 2$$

$$\textcircled{2} \quad h_1 = -1 \quad h_2 = -1$$

$$net = -2$$

$$\textcircled{3} \quad h_1 = -1 \quad h_2 = 1$$

$$net = 0 \text{ or } 2 \text{ if } \text{sign}(0) = 1$$



$$h_1 = \text{sign}(\langle w_1, x \rangle)$$

$$h_2 = \text{sign}(\langle w_2, x \rangle)$$

$$net = \langle v, h \rangle$$

$\langle a, b \rangle$  is a dot product

2. Give an equation for the output of the network above,  $net$ , in terms of the inputs  $x_1, x_2, x_3$ . Do not use dot product notation. (2 points)

$$net = \text{sign}(x_1 + x_2 - x_3) + \text{sign}(2x_1 - x_2 + 3x_3)$$

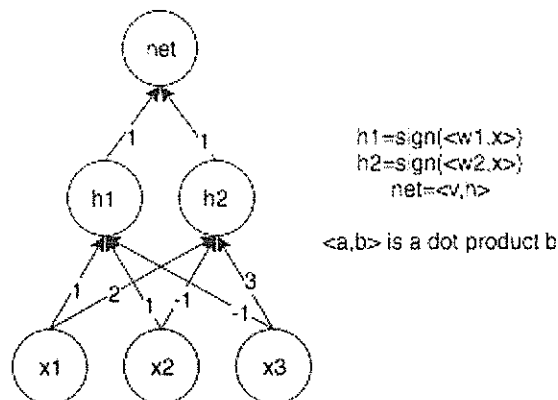
3. Let's say I have a two layer neural network, with  $w_{ij}$  being the weight between the  $j^{\text{th}}$  input and the  $i^{\text{th}}$  hidden unit, and  $v_i$  being the weight between the  $i^{\text{th}}$  hidden unit and the final output unit. Each hidden unit applies the same non-linear function to its input:  $h_i = f(w_i \cdot x)$ . The loss function we are trying to minimize is  $L = e^{-y \cdot net}$  where  $net$  is the output of the network:  $net = v \cdot h$ . What are  $\nabla_w L$  and  $\nabla_v L$ ? Show your work. (5 points)

$$L = e^{-y \cdot net} \quad net = v \cdot h = v \cdot f(w \cdot x)$$

$$\nabla_v L = -y f(w \cdot x) e^{-y \cdot net} = -y h e^{-y \cdot net}$$

$$\nabla_w L = \frac{\partial L}{\partial f} \cdot \frac{\partial f}{\partial w} = -y v f'(w \cdot x) \cdot x e^{-y \cdot net}$$

For the following problems, use the network given below.



1. What would the neural network below predict for the following test data? That is, what is *net* for each test input? (2 points)

$$t_1 = (1, 1, 1)$$

$$t_2 = (-5, -2, 1)$$

2 -2 Same as other side

2. Give an equation for the output of the network above, *net*, in terms of the inputs  $x_1, x_2, x_3$ . Do not use dot product notation. (2 points)

Same as other side

3. We are trying to train the network above. Ignore the weights on the figure. We start with all  $w_{ij} = 0$  and  $v_i = 0$ . Remember the  $w$  are the weights for the first layer and the  $v$  are the weights for the second layer. Assume the loss function is  $L = \frac{1}{2}(y - net)^2$ .

- (a) How do  $h_1$  and  $h_2$  behave in this setting? Why? (2 points)

$h_1$  &  $h_2$  will do the same thing. Their inputs have same values & backprop same gradient.

- (b) What is the value of *net* for the first input sample? (1 points)

$$net = h_1 + h_2 = 0 \quad \text{if } \text{sign}(0) = 0 \quad \text{if } \text{sign}(0) = -1 \quad net = -2$$

- (c) Give the equation for the weight update for  $v$ . (3 points)

$$\nabla_v L = (y - net) \cdot h$$

$$v = v - \eta (y - net) h \quad \text{gradient descent.}$$