regular expressions examples

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if r_1 is a r.e. then $L(r_1)$ denotes the language associated w/ r_1.

defn: $L(r)$ is defined as follows:

1. ϕ is a r.e. denoting the empty set.
2. λ is a r.e. denoting $\{\lambda\}$ - the empty string
3. $\forall a \in \Sigma$, a is r.e. denoting $\{a\}$

and if r_1, r_2 are r.e. :

4. $L(r_1 + r_2) = L(r_1) \cup L(r_2)$
5. $L(r_1 \cdot r_2) = L(r_1) L(r_2)$
6. $L((r_1)) = L(r_1)$
7. $L(r_1^*) = (L(r_1))^*$

ex $L(a^* \cdot (a+b))$

$= L(a^*) L(a+b)$

$= (L(a))^* [L(a) \cup L(b)]$

$= \{a\}^* [\{a\} \cup \{b\}]$

$= \{\lambda, a, aa, aaa, \dots\} \{a, b\}$

$L(r) = \{a, b, aa, ab, aaa, aab, \dots\}$

ex r_1 for even # of a's $= (aa)^*$

r_2 for odd # of b's $= (bb)^* b$ or $b(bb)^*$

$r_3 = (aa)^* (bb)^* b$

$L(r_3) = \{a^{2n} b^{2m+1} : n, m \geq 0\}$

ex Q: $\Sigma = \{a, b\}$ find a r.e., r, $\ni L(r) =$ $\{w \in \Sigma^* : w$ has @ least one pair of aa $\}$

A: $(a+b)^* aa (a+b)^*$

$r_1: (a+bc)^* \cdot (c+\phi)$

$r_2: (a+bc)^* \cdot (c+\lambda)$

is $L(r_1) = L(r_2)$?

$L(r_1) = L((a+bc)^* \cdot (c+\phi))$

$= L((a+bc)^*) L(c+\phi)$

$= (L(a) \cup L(bc))^* [L(c) \cup L(\phi)]$

$= (\{a\} \cup \{bc\})^* [\{c\} \cup \phi]$

$= \{a, bc\}^* \{c\}$

$= \{\lambda, a, bc, abc, aa, abcbc, \dots\} \{c\}$

$L(r_1) = \{c, ac, bcc, abcc, aac, abcbcc, \dots\}$

$L(r_2) = L(((a+bc)^*) \cdot (c+\lambda))$

$= L((a+bc)^*) L(c+\lambda)$

$= (L(a) \cup L(bc))^* [L(c) \cup L(\lambda)]$

$= (\{a\} \cup \{bc\})^* [\{c\} \cup \{\lambda\}]$

$= \{a, bc\}^* \{c, \lambda\}$

$= \{\lambda, a, bc, abc, \dots\} \{c, \lambda\}$

$L(r_2) = \{c, ac, bcc, abcc, \dots, \lambda, a, bc, abc, \dots\}$

$\therefore$ is $L(r_1) \stackrel{?}{=} L(r_2)$ no

what is the relationship between $L(r_1)$ and $L(r_2)$

$L(r_1) \subset L(r_2)$

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regular grammars

grammars: $G = (V, T, S, P)$

V = finite set of symbols = variables

T = finite set of symbols = terminals $\subset \Sigma$

S = start symbol, $S \in V$

P = finite set of productions $\Rightarrow$ generate strings

regular grammars:

all productions have following form:

right linear $\left\{ \begin{array}{l} A \to xB \\ \text{or } A \to x \end{array} \right.$ $A, B \in V$, $x \in T^*$

ex. $S \to aA$ $S, A \in V$ $a \in T^*$

or $S \to a$

or $S \to ababA$

or $S \to abab$

or $S \to \lambda$

or $S \to A$

left linear $\left\{ \begin{array}{l} A \to Bx \\ \text{or } A \to x \end{array} \right.$ also regular grammar

ex

linear

$S \to aB$

$S \to abab$ $\neq$ regular grammar

$S \to Bb$