pumping lemma for CFL

Thursday, November 7, 2024 12:01 PM

pumping lemma for reg. lang:

$\forall s \in L,\ |s| \geq p \Rightarrow s = xyz \ni |xy| \leq p,\ |y| \geq 1$
$s_i = xy^iz \quad \forall i \geq 0,\ s_i \in L.$

pumping lemma for CFL:

$\forall s \in L,\ |s| \geq p \Rightarrow s = uvxyz \ni |vxy| \leq p,\ |vy| \geq 1$
$s_i = uv^ixy^iz \quad \forall i \geq 0,\ s_i \in L.$

ex prove $L = \{a^n b^n c^n : n \geq 0\}$ is not CFL.

Assume L is CFL. $\Rightarrow \forall s \in L,\ |s| \geq p,$
$s = uvxyz \ni |vxy| \leq p,\ |vy| \geq 1$ and
$s_i = uv^ixy^iz \in L \quad \forall i \geq 0.$

let $s = a^p b^p c^p \qquad s \in L$ ✓, $|s| = 3p \geq p$ ✓

$vy = ?$

1. $vy = a^k \quad 1 \leq k \leq p \qquad ??$ ~~$v = a^k \ y = a^j$~~
2. $vy = b^k \quad 1 \leq k \leq p$
3. $vy = c^k \quad 1 \leq k \leq p$
4. $vy =$ ~~ab~~ ~~$a^k b^k$~~ $a^k b^j \quad 0 \leq k \leq p,\ 0 \leq j \leq p,\ 1 \leq k+j \leq p$
5. $vy = b^k c^j \quad 0 \leq k \leq p,\ 0 \leq j \leq p,\ 1 \leq k+j \leq p$

case 1. $s = \underbrace{a^k}_{vy}\ \underbrace{a^{p-k} b^p}_{!vy} c^p$

$s_i = a^{ki} a^{p-k} b^p c^p$

let $i = 0 \Rightarrow s_0 = a^{p-k} b^p c^p$
since $k \geq 1 \Rightarrow n_a(s_0) \neq n_{b/c}(s_0)$
$\therefore s_0 \notin L$

case 2:
case 3:
case 4:
case 5: