pumping lemma - regular languages

Tuesday, October 1, 2024 12:00 PM

$L_1 = \{a^n b^n : n \geq 0\}$ ⇒ not regular

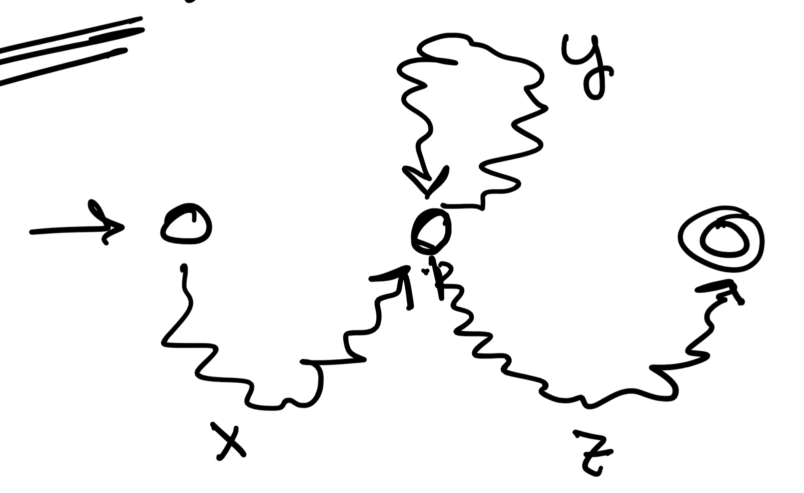
$L_2 = \{w : w$ has an equal # of a's & b's$\}$ ⇒ not regular

$L_3 = \{w : w$ has equal occurrences of ab and ba as substrings$\}$ ⇒ regular

Q: prove a L is regular ⇒
1. create a dfa
2. create a nfa
3. create a r.e.
4. create a regular grammar.
[5.] finite language.

Q: prove a L is not regular ⇒
1. using the pumping lemma for regular lang. create a proof by contradiction.

in general: pumping lemma for regular languages is a set of characteristic of infinite regular languages that are guaranteed.



$\boxed{s_1 = xyz \in L}$

pumping lemma
$s_2 = xyyz \in L$
$s_3 = xyyyz = xy^3z \in L$
$s_0 = xz = xy^0z \in L$

pumping down

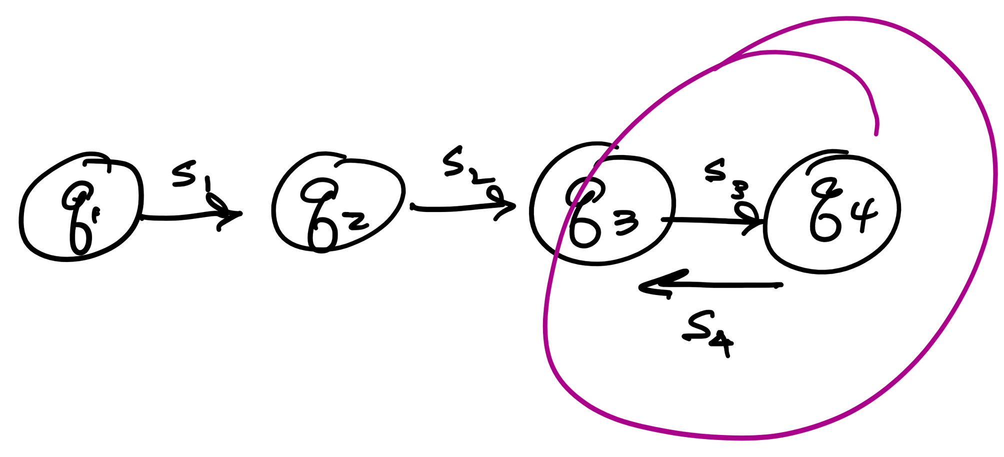
$\forall i \geq 2,\ s_i \in L$

$i = 0,\ s_0 \in L$

} $\forall i \geq 0,\ s_i \in L$

p = number of states
$|s| \geq p$

$p = 4$
$|s| \geq 4$



pumping lemma for regular languages:

$\forall s \in L$, where L is infinite regular language,
$|s| \geq p$, where p is positive integer,
$s = xyz \ni |xy| \leq p,\ |y| \geq 1$ ⇒
$s_i = xy^iz \in L\ \ \forall\ i \geq 0.$

proof using pumping lemma: [proof by contradiction]

1. Assume L is regular ⇒ pumping lemma for r.e. holds i.e., $\forall s \in L,\ |s| \geq p$ (pos. int.) ⇒ $s = xyz \ni |xy| \leq p,\ |y| \geq 1$ ⇒ $s_i = xy^iz \in L$ $\forall i \geq 0.$

2. pick $s \in L,\ |s| \geq p.$

   ex. $L = \{a^n b^n : n \geq 0\}$

   pick $s \in L,\ |s| \geq p$

   let $s = a^p b^p$ $\quad [|s| = p + p = 2p \geq p,\ s \in L]$

   ~~bad ex. of s: $s = a^{p-1} b^{p-1}$~~

3. show xy equal to.

   show: $|xy| \leq p,\ |y| \geq 1$

   ex. $y = a^k\ \ 1 \leq k \leq p$

   $x = \lambda$
   $y = a^p$
   $z = b^p$

   $x = a^{p-2}$
   $y = a$
   $z = ab^p$

   $x = \lambda$
   $y = a$
   $z = a^{p-1}b^p$

   ~~$x = a^{p-1}$~~
   ~~$y = ab$~~
   ~~$z = b^{p-1}$~~

   ⇒ $\boxed{y = a^k\ \ 1 \leq k \leq p}$

4. pick i and pump string

   ex. let $i = 2$

   $s_2 = xy^2z$ w/ $y = a^k$

   $s = \underbrace{a^k}_{y} \underbrace{a^{p-k} b^p}_{xz\ [!y]}$

   $s_i = xy^iz$

   $s_i = a^{ki} a^{p-k} b^p$

   $s_2 = \underbrace{a^{2k}}_{y} \underbrace{a^{p-k} b^p}_{!y} = a^{p+k} b^p$

   but $k \geq 1$

   ⇒ $n_a(s_2) > n_b(s_2)$

   ∴ $s_2 \notin L$

∴ L is not regular