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contest

hash.sh

```
# Hashes a file, ignoring all whitespace and comments.
Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum
|cut -c-6
```

template.cpp

```
#pragma GCC optimize("03")
#pragma GCC optimize("unroll-loops")
#include <bits/stdc++.h>
#define pb push back
#define all(x)(x).begin(),(x).end()
#define debug(x) { cerr << #x << " = " << x << endl; }
#define IO { ios base::sync with stdio(false);
cin.tie(0); }
#define read(x) freopen(x. "r". stdin)
#define write(x) freopen(x, "w", stdout)
#define endl '\n'
#define int long long
using namespace std;
typedef long long ll;
typedef pair<int, int> ii;
typedef vector<int> vi;
void solve() {
signed main() {
IO:
int tc = 1;
 #ifndef ONLINE JUDGE
 read("input.txt");
write("output.txt");
 #endif
// cin >> tc;
 for (int cs = 1: cs <= tc: cs++) {
 solve();
return 0:
```

data-structures

```
FenwickTree.cpp
```

```
class maxseqtree {
 public:
 int n;
  vector<int> t;
  maxseqtree(vi &res) {
  n = res.size():
  t.assign(2 * n, -INF);
   for (int i = 0; i < n; i++) {</pre>
   t[n + i] = res[i];
  build();
 }
  void build() {
  for (int i = n - 1; i > 0; --i) t[i] = max(t[i << 1],
t[i<<1|1]):
 }
 void modify(int p, int value) {
  for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] =
\max(t[p], t[p^1]);
 int query(int l, int r) {
  int res = -INF;
  for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
   if (l&1) res = max(res.t[l++]):
   if (r&1) res = max(res,t[--r]);
  }
  return res;
};
```

HashMap.cpp

```
#pragma once

#include <bits/extc++.h> /** keep-include */
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
   const uint64_t C = ll(4e18 * acos(0)) | 71;
   ll operator()(ll x) const { return
   _builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash>
h({},{},{},{},{1<<16});</pre>
```

LazySegmentTree.cpp

```
struct LazySegTree {
   int n;
```

```
vector<int> tree, lazy;
    LazvSegTree(int size) {
        n = size;
        tree.assign(4 * n, 0); // stores max in a segment
        lazy.assign(4 * n, 0); // stores pending
additions
    void push(int node, int l, int r) {
        if (lazy[node] != 0) {
            tree[node] += lazy[node]; // apply the lazy
value
            if (l != r) { // not a leaf node}
                lazy[2*node] += lazy[node];
                lazy[2*node + 1] += lazy[node];
            lazv[node] = 0; // clear the lazv value
    }
    void range_add(int node, int l, int r, int ql, int
gr. int val) {
        push(node, l, r);
        if (qr < l || r < ql) return; // no overlap</pre>
        if (ql <= l && r <= qr) { // total overlap
            lazy[node] += val;
            push(node, l, r);
            return:
        // partial overlap
        int mid = (l + r) / 2;
        range add(2*node, l, mid, ql, qr, val);
        range_add(2*node+1, mid+1, r, ql, qr, val);
        tree[node] = max(tree[2*node], tree[2*node+1]);
    int range query(int node, int l, int r, int ql, int
qr) {
        push(node, l, r);
        if (qr < l || r < ql) return INT_MIN; // no
overlap
        if (ql <= l && r <= qr) return tree[node]; //</pre>
total overlap
        // partial overlap
        int mid = (l + r) / 2:
        int left = range_query(2*node, l, mid, ql, qr);
        int right = range query(2*node+1, mid+1, r, ql,
qr);
        return max(left, right);
    // Public API
    void add(int l, int r, int val) {
        range add(1, 0, n-1, l, r, val):
```

```
}
   int query(int l, int r) {
        return range_query(1, 0, n-1, l, r);
   }
};
LiChaoTree.cop
struct Line {
   long long m, b;
   Line(long long m = 0, long long b = LLONG MAX):
m(m), b(b) {}
   long long eval(long long x) const {
       return m * x + b:
   }
};
struct LiChaoTree {
   struct Node {
        Line line;
       Node *left = nullptr, *right = nullptr;
   };
   int low, high;
   Node* root:
   LiChaoTree(int l, int r) {
       low = l:
       hiah = r:
       root = nullptr;
   }
   void add line(Line newLine) {insert(root, low, high,
   void insert(Node* &node, int l, int r, Line newLine)
        if (!node) {
            node = new Node();
            node->line = newLine:
            return:
       int mid = (l + r) / 2;
        bool leftBetter = newLine.eval(l) <</pre>
node->line.eval(l);
        bool midBetter = newLine.eval(mid) <</pre>
node->line.eval(mid):
       if (midBetter) {swap(node->line, newLine);}
       if (r - l == 0) return:
       if (leftBetter != midBetter) {
            insert(node->left, l, mid, newLine);
       } else {
            insert(node->right, mid + 1, r, newLine);
   }
```

```
long long query(int x) {
        return query(root, low, high, x):
    long long query(Node* node, int l, int r, int x) {
        if (!node) return LLONG MAX;
        long long res = node->line.eval(x);
        if (l == r) return res;
        int mid = (l + r) / 2;
        if (x <= mid)</pre>
            return min(res, query(node->left, l, mid,
x));
        else
            return min(res, query(node->right, mid + 1,
r, x));
   }
};
int main() {
    LiChaoTree cht(-1e6, 1e6); // define x-range
    cht.add_line(\{2, 3\}); // add line y = 2x + 3
    cht.add_line(\{-1, 4\}); // add line y = -x + 4
    cout << cht.query(1) << '\n'; // returns min value at</pre>
x = 1
}
OrderStatisticTree.cpp
#include <bits/extc++.h>
using namespace __gnu_pbds;
// Pair<int, int> -> (value, unique id)
template<tvpename T>
using OrderedMultiSet = tree<</pre>
    pair<T, int>,
    null type,
    less<pair<T, int>>,
    rb tree tag,
    tree order statistics node update
>;
struct MultiSet {
    OrderedMultiSet<int> s;
    int uid = 0; // Unique ID to distinguish duplicates
    void insert(int x) {s.insert({x, uid++});}
    void erase(int x) {
        auto it = s.lower bound({x, -1}); // find first
occurrence
        if (it != s.end() && it->first == x) s.erase(it);
    int count(int x) {
```

```
return s.upper bound({x, INT MAX}) -
s.lower bound({x. -1}):
    int order of key(int x) {return s.order of key({x,
-1});}
    int find by order(int k) {
  if (k >= (int)s.size()) return -1;
        return s.find by order(k)->first;
    int size() {return s.size();}
RMQ.cpp
struct RMQ {
    vector<vector<int>> t;
    int n;
    void init(const vector<int>& a) {
        n = a.size();
        int K = 32 - builtin clz(n);
        t.assign(n + 20, vector<int>(K + 5, -inf));
        for (int i = 0; i < n; ++i) {t[i][0] = a[i];}</pre>
        for (int j = 1; (1 << j) <= n; ++j) {
            for (int i = 0; i + (1 << j) <= n; ++i) {
                t[i][j] = max(t[i][j - 1], t[i + (1 << (j
- 1))][j - 1]);
    int q(int l, int r) {
        int len = r - l + 1;
        int k = 31 - builtin clz(len);
        return max(t[l][k], t[r - (1 << k) + 1][k]);
};
SegmentTree.cpp
const pair<int. int> NEUTRAL = {INT MIN. -1}:
struct node {
    pair<int, int> val;
    node *L, *R;
    node() {
        val = NEUTRAL;
        L = R = nullptr:
```

```
}
};
pair<int, int> combine(pair<int, int> a, pair<int, int>
   return max(a, b);
struct SegmentTree {
   node* root:
   int lbound, rbound;
   SegmentTree(int l, int r) {
       lbound = l;
        rbound = r;
       root = new node();
   }
   void update(node* cur, int l, int r, int pos,
pair<int, int> value) {
       if (l == r) {
            cur->val = value:
            return:
       int mid = (l + r) / 2;
       if (pos <= mid) {</pre>
            if (!cur->L) cur->L = new node();
            update(cur->L, l, mid, pos, value);
       } else {
            if (!cur->R) cur->R = new node();
            update(cur->R, mid + 1, r, pos, value);
        pair<int, int> left = cur->L ? cur->L->val :
make pair(MIN, -1);
        pair<int, int> right = cur->R ? cur->R->val :
make pair(MIN, -1);
        cur->val = combine(left, right);
   }
   pair<int, int> query(node* cur, int l, int r, int ql,
int qr) {
       if (!cur || r < ql || l > qr) return {MIN, -1};
       if (ql <= l && r <= qr) return cur->val;
       int mid = (l + r) / 2:
        pair<int, int> left = query(cur->L, l, mid, ql,
qr);
        pair<int, int> right = query(cur->R, mid + 1, r,
ql, qr);
        return combine(left, right);
   void update(int pos, int val) {
        update(root. lbound. rbound. pos. {val. pos}):
```

```
}
    pair<int, int> query(int l, int r) {
        return query(root, lbound, rbound, l, r);
    }
};
UnionFind.com
struct dsu{
 vector<int> p;
 vector<int> sz;
 int comp;
 void init(int n){
  comp = n;
  p.resize(n);
  sz.resize(n);
  for(int i=0; i < n; i++)p[i] = i;
  for(int i=0; i < n; i++)sz[i] = 1;
 int parent(int at){
 if(p[at] == at) return at:
  return p[at] = parent(p[at]);
 void add(int u, int v){
  u = parent(u);
  v = parent(v);
  if(u == v) return:
  comp--;
  v = [u]q
  sz[v] += sz[u];
 int size(int u){
  return sz[parent(u)]:
 }
};
UnionFindRollback.com
struct RollbackUF {
 vi e; vector<pii> st;
 RollbackUF(int n) : e(n, -1) {}
 int size(int x) { return -e[find(x)]; }
 int find(int x) { return e[x] < 0 ? x : find(e[x]); }</pre>
 int time() { return sz(st); }
 void rollback(int t) {
  for (int i = time(); i --> t;)
   e[st[i].first] = st[i].second;
  st.resize(t):
 bool ioin(int a. int b) {
```

```
a = find(a), b = find(b);
  if (a == b) return false:
  if (e[a] > e[b]) swap(a, b);
  st.push back({a, e[a]});
  st.push back({b, e[b]});
  e[a] += e[b]; e[b] = a;
  return true;
};
mergeSortTree.cpp
const int MAXN = 1e5 + 5;
int a[MAXN]:
multiset<int> t[4 * MAXN];
void build(int v, int tl, int tr) {
    if (tl == tr) {
        t[v].insert(a[tl]);
    } else {
        int tm = (tl + tr) / 2;
        build(v*2, tl, tm);
        build(v*2+1, tm+1, tr);
        t[v].insert(t[v*2].begin(), t[v*2].end());
        t[v].insert(t[v*2+1].begin(), t[v*2+1].end());
    }
int query(int v, int tl, int tr, int l, int r, int x) {
    if (l > r) return INT MAX;
    if (tl == l && tr == r) {
        auto it = t[v].lower bound(x);
        return (it != t[v].end() ? *it : INT_MAX);
    int tm = (tl + tr) / 2;
    return min(
        query(v*2, tl, tm, l, min(r, tm), x),
        query(v*2+1, tm+1, tr, max(l, tm+1), r, x)
    );
}
void update(int v, int tl, int tr, int pos, int old val,
int new val) {
    t[v].erase(t[v].find(old val));
    t[v].insert(new_val);
    if (tl != tr) {
        int tm = (tl + tr) / 2;
        if (pos <= tm)</pre>
            update(v*2, tl, tm, pos, old val, new val);
        else
            update(v*2+1, tm+1, tr, pos, old_val,
new val);
```

```
}
int main() {
   // Initialize array
   int n = 5;
   a[0] = 5;
   a[1] = 1:
   a[2] = 7;
   a[3] = 3;
   a[4] = 6;
   // Build merge sort tree
   build(1, 0, n - 1);
   // Query: Find min element >= 4 in range [1, 4]
   int res = query(1, 0, n - 1, 1, 4, 4);
   if (res != INT MAX)
        cout << "Query result (>= 4 in [1,4]) = " << res</pre>
<< "\n";
   else
        cout << "No value >= 4 in [1.4]\n":
   update(1, 0, n - 1, 2, 7, 0);
   a[2] = 0;
   // Query again
   res = query(1, 0, n - 1, 1, 4, 4);
   if (res != INT_MAX)
        cout << "After update, query result = " << res <<</pre>
"\n";
   else
        cout << "After update, no value >= 4 in [1,4]\n";
```

geometry

Angle.cpp

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
```

```
};
bool operator<(Angle a, Angle b) {</pre>
// add a.dist2() and b.dist2() to also compare distances
 return make tuple(a.t, a.half(), a.y * (ll)b.x) <</pre>
        make tuple(b.t, b.half(), a.x * (ll)b.y);
// Given two points, this calculates the smallest angle
// them, i.e., the angle that covers the defined line
segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
if (b < a) swap(a, b);
 return (b < a.t180() ?</pre>
         make pair(a, b) : make pair(b, a.t360()));
Angle operator+(Angle a, Angle b) { // point a + vector b
 Angle r(a.x + b.x, a.y + b.y, a.t);
 if (a.t180() < r) r.t--;
 return r.t180() < a ? r.t360() : r;</pre>
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
int tu = b.t - a.t; a.t = b.t;
return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b <
a)};
}
```

CircleIntersection.cpp

CircleLine.cpp

```
#include "Point.h"
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
```

```
double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
if (h2 < 0) return {};
if (h2 == 0) return {p};
P h = ab.unit() * sqrt(h2);
return {p - h, p + h};
}</pre>
```

CirclePolygonIntersection.cpp

```
#pragma once
#include "../../content/geometry/Point.h"
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
 auto tri = [\&](Pp, Pq) {
 auto r2 = r * r / 2;
 Pd = q - p;
  auto a = d.dot(p)/d.dist2(), b =
(p.dist2()-r*r)/d.dist2();
 auto det = a * a - b;
 if (det <= 0) return arg(p, q) * r2;</pre>
 auto s = max(0., -a-sqrt(det)), t = min(1.,
-a+sqrt(det));
 if (t < 0 || 1 <= s) return arg(p, q) * r2;
 Pu = p + d * s, v = q + d * (t-1);
  return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
};
 auto sum = 0.0;
 rep(i,0,sz(ps))
 sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
return sum;
```

CircleTangents.cpp

```
#include "Point.h"

template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double
r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0) return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
```

```
return out;
ClosestPair.cpp
#include "Point.h"
typedef Point<ll> P:
pair<P, P> closest(vector<P> v) {
 assert(sz(v) > 1);
 set<P> S:
 sort(all(v), [](P a, P b) { return a.y < b.y; });</pre>
 pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
 int i = 0:
 for (P p : v) {
 P d{1 + (ll)sqrt(ret.first), 0};
  while (v[j].y \le p.y - d.x) S.erase(v[j++]);
  auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
d);
  for (; lo != hi; ++lo)
  ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
  S.insert(p);
 return ret.second;
ConvexHull.cpp
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
if (sz(pts) <= 1) return pts;</pre>
 sort(all(pts));
 vector<P> h(sz(pts)+1);
 int s = 0, t = 0;
 for (int it = 2; it--; s = --t, reverse(all(pts)))
 for (P p : pts) {
   while (t >= s + 2 \&\& h[t-2].cross(h[t-1], p) <= 0)
t--:
   h[t++] = p;
return {h.begin(), h.begin() + t - (t == 2 && h[0] ==
h[1])};
FastDelaunay.cpp
```

```
typedef Point<ll> P;
typedef struct Quad* Q;
```

```
typedef int128 t lll; // (can be ll if coords are <
2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other
struct Quad {
 Q rot, o; P p = arb; bool mark;
 P& F() { return r()->p; }
 Q& r() { return rot->rot; }
 O prev() { return rot->o->rot; }
 Q next() { return r()->prev(); }
} *H;
bool circ(P p, P a, P b, P c) { // is p in the
circumcircle?
 lll p2 = p.dist2(), A = a.dist2()-p2,
     B = b.dist2()-p2, C = c.dist2()-p2;
 return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B
> 0;
}
Q makeEdge(P orig, P dest) {
 0 r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
 H = r -> 0; r -> r() -> r() = r;
 rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r :
r->r();
 r->p = orig; r->F() = dest;
 return r;
void splice(Q a, Q b) {
 swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
Q connect(Q a, Q b) {
 Q q = makeEdge(a->F(), b->p);
 splice(q, a->next());
 splice(q->r(), b);
 return q;
pair<Q,Q> rec(const vector<P>& s) {
 if (sz(s) <= 3) {
  Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1],
s.back()):
  if (sz(s) == 2) return { a, a->r() };
  splice(a->r(), b);
  auto side = s[0].cross(s[1], s[2]);
  0 c = side ? connect(b, a) : 0;
  return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
#define H(e) e->F(). e->p
```

```
#define valid(e) (e->F().cross(H(base)) > 0)
O A. B. ra. rb:
int half = sz(s) / 2;
tie(ra, A) = rec({all(s) - half});
tie(B, rb) = rec({sz(s) - half + all(s)});
while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
       (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
Q base = connect(B->r(), A);
if (A->p == ra->p) ra = base->r();
if (B->p == rb->p) rb = base;
#define DEL(e, init, dir) 0 e = init->dir; if (valid(e))
 while (circ(e->dir->F(), H(base), e->F())) { \
  0 t = e->dir; \
  splice(e, e->prev()); \
  splice(e->r(), e->r()->prev()); \
  e->o = H; H = e; e = t; \
 }
for (;;) {
 DEL(LC, base->r(), o); DEL(RC, base, prev());
 if (!valid(LC) && !valid(RC)) break;
 if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
  base = connect(RC, base->r());
 else
  base = connect(base->r(), LC->r());
return { ra, rb };
vector<P> triangulate(vector<P> pts) {
sort(all(pts)); assert(unique(all(pts)) == pts.end());
if (sz(pts) < 2) return {};</pre>
0 e = rec(pts).first;
vector<Q> q = \{e\};
int qi = 0;
while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c -> mark = 1;
pts.push back(c->p); \
q.push_back(c->r()); c = c->next(); } while (c != e); }
ADD; pts.clear();
while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
return pts;
```

HullDiameter.cpp

#pragma once
#include "Point.h"

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
  for (;; j = (j + 1) % n) {
    res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
    if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >=
        break;
    }
  return res.second;
}
```

InsidePolygon.cpp

```
#pragma once
#include "Point.h"
#include "OnSegment.h"
#include "SegmentDistance.h"

template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
  cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

LineHullIntersection.cpp

- * Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points.
- * lineHull(line, poly) returns a pair describing the intersection of a line with the polygon:

```
#include "Point.h"

#define cmp(i,j)
sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) <
0</pre>
```

```
template <class P> int extrVertex(vector<P>& polv. P dir)
int n = sz(poly), lo = 0, hi = n;
if (extr(0)) return 0;
while (lo + 1 < hi) {</pre>
 int m = (lo + hi) / 2;
 if (extr(m)) return m;
 int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m):
 (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) =
m;
}
return lo;
#define cmpL(i) sqn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
int endA = extrVertex(poly, (a - b).perp());
int endB = extrVertex(poly, (b - a).perp());
if (cmpL(endA) < 0 \mid | cmpL(endB) > 0)
 return {-1, -1};
 array<int, 2> res;
 rep(i,0,2) {
 int lo = endB, hi = endA, n = sz(poly);
 while ((lo + 1) % n != hi) {
  int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n:
  (cmpL(m) == cmpL(endB) ? lo : hi) = m;
 res[i] = (lo + !cmpL(hi)) % n;
 swap(endA, endB);
 if (res[0] == res[1]) return {res[0], -1};
 if (!cmpL(res[0]) && !cmpL(res[1]))
 switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
  case 0: return {res[0], res[0]};
  case 2: return {res[1], res[1]};
return res:
```

LineProjectionReflection.cpp

- * Description: Projects point p onto line ab. Set refl=true to get reflection
- * of point p across line ab instead. The wrong point will be returned if P is
- * an integer point and the desired point doesn't have integer coordinates.

```
* Products of three coordinates are used in intermediate
steps so watch out
  * for overflow.

#pragma once

#include "Point.h"

template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

MinimumEnclosingCircle.cpp

```
* Description: Computes the minimum circle that encloses
a set of points.
* Time: expected O(n)
#pragma once
#include "circumcircle.h"
pair<P. double> mec(vector<P> ps) {
 shuffle(all(ps), mt19937(time(0)));
 P \circ = ps[0];
 double r = 0, EPS = 1 + 1e-8;
 rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
 o = ps[i], r = 0;
  rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
  o = (ps[i] + ps[i]) / 2;
  r = (o - ps[i]).dist();
   rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
   o = ccCenter(ps[i], ps[j], ps[k]);
   r = (o - ps[i]).dist();
 }
return {o, r};
```

Point.cpp

- * Description: Class to handle points in the plane.
- * T can be e.g. double or long long. (Avoid int.)

#pragma once

```
template <class T> int sgn(T x) \{ return (x > 0) - (x < 0) \}
0): }
template<class T>
struct Point {
typedef Point P;
T x, y;
explicit Point(T x=0, T y=0) : x(x), y(y) {}
bool operator<(P p) const { return tie(x,y) <</pre>
tie(p.x,p.v); }
bool operator==(P p) const { return
tie(x,y)==tie(p.x,p.y); 
P operator+(P p) const { return P(x+p.x, y+p.y); }
P operator-(P p) const { return P(x-p.x, y-p.y); }
 P operator*(T d) const { return P(x*d, y*d); }
P operator/(T d) const { return P(x/d, y/d); }
T dot(P p) const { return x*p.x + y*p.y; }
T cross(P p) const { return x*p.y - y*p.x; }
T cross(P a, P b) const { return
(a-*this).cross(b-*this); }
T dist2() const { return x*x + y*y; }
double dist() const { return sqrt((double)dist2()); }
// angle to x-axis in interval [-pi, pi]
double angle() const { return atan2(y, x); }
P unit() const { return *this/dist(); } // makes
dist()=1
P perp() const { return P(-v, x): } // rotates +90
dearees
P normal() const { return perp().unit(); }
// returns point rotated 'a' radians ccw around the
origin
P rotate(double a) const {
 return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
 friend ostream& operator<<(ostream& os, P p) {</pre>
 return os << "(" << p.x << "," << p.y << ")"; }
};
```

PointInsideHull.com

```
* Description: Determine whether a point t lies inside a convex hull (CCW
* order, with no collinear points). Returns true if
```

- point lies within
 * the hull. If strict is true, points on the boundary
 aren't included.
- * Time: O(\log N)

```
#pragma once
```

```
#include "Point.h"
#include "sideOf.h"
```

```
#include "OnSegment.h"
typedef Point<ll> P;
bool inHull(const vector<P>& l, P p, bool strict = true)
int a = 1, b = sz(l) - 1, r = !strict;
 if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);</pre>
 if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
 if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b],
p)<= -r)
 return false:
 while (abs(a - b) > 1) {
 int c = (a + b) / 2:
 (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
 return sqn(l[a].cross(l[b], p)) < r;</pre>
PolygonArea.cpp
 * Description: Returns twice the signed area of a
polvaon.
 * Clockwise enumeration gives negative area. Watch out
for overflow if using int as T!
#pragma once
#include "Point.h"
template<class T>
T polygonArea2(vector<Point<T>>& v) {
T = v.back().cross(v[0]):
```

PolvgonCenter.cpp

return a;

```
* Description: Returns the center of mass for a polygon.

* Time: O(n)

#pragma once

#include "Point.h"

typedef Point<double> P;

P polygonCenter(const vector<P>& v) {
    P res(0, 0); double A = 0;
    for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
        res = res + (v[i] + v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]);
```

rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);

```
}
return res / A / 3;
}
```

PolygonCut.cpp

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

```
#pragma once
#include "Point.h"

typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    auto a = s.cross(e, cur), b = s.cross(e, prev);
    if ((a < 0) != (b < 0))
      res.push_back(cur + (prev - cur) * (a / (a - b)));
    if (a < 0)
      res.push_back(cur);
}
return res;
}</pre>
```

PolygonUnion.cpp

- * Description: Calculates the area of the union of \$n\$ polygons (not necessarily
- * convex). The points within each polygon must be given in CCW order.
- * (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
- * Time: $$0(N^2)$$, where \$N\$ is the total number of points

```
#pragma once

#include "Point.h"
#include "sideOf.h"

typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x :
a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
   double ret = 0;
   rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
```

```
P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
 vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
  rep(j,0,sz(poly)) if (i != j) {
  rep(u,0,sz(poly[j])) {
   P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
   int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
   if (sc != sd) {
    double sa = C.cross(D, A), sb = C.cross(D, B);
    if (min(sc, sd) < 0)
     segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
   } else if (!sc && !sd && j<i &&
sgn((B-A).dot(D-C))>0){
     segs.emplace_back(rat(C - A, B - A), 1);
     segs.emplace back(rat(D - A, B - A), -1);
  }
 sort(all(seqs));
  for (auto& s : segs) s.first = min(max(s.first, 0.0),
1.0):
 double sum = 0;
 int cnt = seqs[0].second;
 rep(j,1,sz(segs)) {
  if (!cnt) sum += seqs[j].first - seqs[j - 1].first;
  cnt += segs[j].second;
 ret += A.cross(B) * sum;
return ret / 2;
```

SegmentDistance.cpp

Returns the shortest distance between point p and the line segment from point s to e.

```
#pragma once

#include "Point.h"

typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
   if (s==e) return (p-s).dist();
   auto d = (e-s).dist2(), t =
   min(d,max(.0,(p-s).dot(e-s)));
   return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.cpp

```
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned.
```

If no intersection point exists an empty vector is returned.

If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment.

The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

```
#include "Point.h"
#include "OnSegment.h"

template<class P> vector<P> segInter(P a, P b, P c, P d)
{
   auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
   // Checks if intersection is single non-endpoint point.
   if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)};
        set<P> s;
   if (onSegment(c, d, a)) s.insert(a);
   if (onSegment(a, b, c)) s.insert(b);
   if (onSegment(a, b, d)) s.insert(d);
   return {all(s)};
}
```

circumcircle.cpp

#pragma once

```
#pragma once

#include "Point.h"

typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
    abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A +
  (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
```

kdTree.cpp

```
#pragma once
#include "Point.h"
typedef long long T;
typedef Point<T> P;
const T INF = numeric limits<T>::max();
bool on x(const P& a, const P& b) { return a.x < b.x; }</pre>
bool on_y(const P& a, const P& b) { return a.y < b.y; }</pre>
struct Node {
 P pt; // if this is a leaf, the single point in it
T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
 Node *first = 0. *second = 0:
T distance(const P& p) { // min squared distance to a
point
 T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
 T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
 return (P(x,y) - p).dist2();
 Node(vector<P>&& vp) : pt(vp[0]) {
 for (P p : vp) {
  x0 = min(x0, p.x); x1 = max(x1, p.x);
  y0 = min(y0, p.y); y1 = max(y1, p.y);
  if (vp.size() > 1) {
  // split on x if width >= height (not ideal...)
   sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
   // divide by taking half the array for each child (not
  // best performance with many duplicates in the
middle)
   int half = sz(vp)/2;
   first = new Node({vp.begin(), vp.begin() + half});
   second = new Node({vp.begin() + half, vp.end()});
 }
}
};
struct KDTree {
Node* root:
 KDTree(const vector<P>& vp) : root(new Node({all(vp)}))
```

```
pair<T. P> search(Node *node. const P& p) {
  if (!node->first) {
  // uncomment if we should not find the point itself:
  // if (p == node->pt) return {INF, P()};
  return make pair((p - node->pt).dist2(), node->pt);
  Node *f = node->first. *s = node->second:
  T bfirst = f->distance(p), bsec = s->distance(p);
  if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);
  // search closest side first, other side if needed
  auto best = search(f, p):
  if (bsec < best.first)</pre>
  best = min(best, search(s, p));
  return best:
}
// find nearest point to a point, and its squared
distance
 // (requires an arbitrary operator< for Point)</pre>
 pair<T, P> nearest(const P& p) {
 return search(root, p);
};
lineDistance.cpp
Returns the signed distance between point p and the line
containing points a and b.
Positive value on left side and negative on right as seen
from a towards b. a==b gives nan.
P is supposed to be Point<T> or Point3D<T> where T is
e.g. double or long long.
It uses products in intermediate steps so watch out for
overflow if using int or long long.
Using Point3D will always give a non-negative distance.
For Point3D, call .dist on the result of the cross
product.
#include "Point.h"
template<class P>
```

double lineDist(const P& a, const P& b, const P& p) {

return (double)(b-a).cross(p-a)/(b-a).dist();

lineIntersection.cpp

```
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists \{1, \text{point}\} is returned. If no intersection point exists \{0, (0,0)\} is returned and if infinitely many exists \{-1, (0,0)\} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.
```

```
#pragma once
#include "Point.h"

template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
   return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

linearTransformation.cpp

```
* Author: Per Austrin, Ulf Lundstrom
 * Date: 2009-04-09
 * License: CCO
 * Source:
 * Description:\\
\begin{minipage}{75mm}
 Apply the linear transformation (translation, rotation
and scaling) which takes line p0-p1 to line q0-q1 to
point r.
\end{minipage}
\begin{minipage}{15mm}
\vspace{-8mm}
\includegraphics[width=\textwidth]{content/geometry/linea
rTransformation}
\vspace{-2mm}
\end{minipage}
* Status: not tested
#pragma once
#include "Point.h"
typedef Point<double> P:
P linearTransformation(const P& p0, const P& p1.
```

```
const P& q0, const P& q1, const P& r) {
P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
return q0 + P((r-p0).cross(num),
(r-p0).dot(num))/dp.dist2();
}
```

sideOf.cpp

```
* Description: Returns where $p$ is as seen from $s$
towards $e$. 1/0/-1 $\Leftrightarrow$ left/on line/right.
* If the optional argument $eps$ is given 0 is returned
if $p$ is within distance $eps$ from the line.
* P is supposed to be Point<T> where T is e.g. double or
lona lona.
* It uses products in intermediate steps so watch out
for overflow if using int or long long.
 * Usage:
 * bool left = sideOf(p1,p2,q)==1;
#pragma once
#include "Point.h"
template<class P>
int sideOf(P s, P e, P p) { return sqn(s.cross(e, p)); }
template<class P>
int sideOf(const P& s, const P& e, const P& p, double
eps) {
 auto a = (e-s).cross(p-s);
 double l = (e-s).dist()*eps:
 return (a > l) - (a < -l);
```

sphericalDistance.cpp

* Description: Returns the shortest distance on the sphere with radius radius between the points

* with azimuthal angles (longitude) f1 (\$\phi_1\$) and f2 (\$\phi_2\$) from x axis and zenith angles

* (latitude) t1 (\$\theta_1\$) and t2 (\$\theta_2\$) from z axis (0 = north pole). All angles measured

* in radians. The algorithm starts by converting the

* in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates

* so if that is what you have you can use only the two last rows. dx*radius is then the difference

* between the two points in the x direction and d*radius is the total distance between the points.
#pragma once

```
double sphericalDistance(double f1, double t1,
  double f2, double t2, double radius) {
 double dx = \sin(t2)*\cos(f2) - \sin(t1)*\cos(f1);
 double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
 double dz = cos(t2) - cos(t1);
 double d = sqrt(dx*dx + dy*dy + dz*dz);
return radius*2*asin(d/2):
graph
2sat.com
 * Description: Calculates a valid assignment to boolean
variables a, b, c,... to a 2-SAT problem,
* so that an expression of the type
$(a||b)\&\&(!a||c)\&\&(d||!b)\&\&...$
* becomes true, or reports that it is unsatisfiable.
* Negated variables are represented by bit-inversions
(\texttt{\tilde{}x}).
* Usage:
 * TwoSat ts(number of boolean variables);
 * ts.either(0, \tilde3); // Var 0 is true or var 3 is
false
 * ts.setValue(2); // Var 2 is true
* ts.atMostOne({0,\tilde1,2}); // <= 1 of vars 0,
\tilde1 and 2 are true
* ts.solve(); // Returns true iff it is solvable
* ts.values[0..N-1] holds the assigned values to the
 * Time: O(N+E), where N is the number of boolean
variables, and E is the number of clauses.
#pragma once
struct TwoSat {
int N:
 vector<vi> qr;
 vi values; // 0 = false, 1 = true
 TwoSat(int n = 0) : N(n), gr(2*n) {}
 int addVar() { // (optional)
  gr.emplace back();
  gr.emplace back();
  return N++:
```

void either(int f, int j) {

```
f = max(2*f, -1-2*f);
 i = max(2*i. -1-2*i):
  gr[f].push back(j^1);
 gr[j].push back(f^1);
 void setValue(int x) { either(x, x); }
 void atMostOne(const vi& li) { // (optional)
  if (sz(li) <= 1) return;</pre>
  int cur = ~li[0];
  rep(i,2,sz(li)) {
   int next = addVar();
   either(cur, ~li[i]);
   either(cur, next);
   either(~li[i], next);
   cur = ~next;
  either(cur, ~li[1]);
 vi val, comp, z; int time = 0;
 int dfs(int i) {
 int low = val[i] = ++time, x; z.push back(i);
  for(int e : qr[i]) if (!comp[e])
   low = min(low, val[e] ?: dfs(e));
  if (low == val[i]) do {
  x = z.back(); z.pop_back();
   comp[x] = low;
   if (values[x>>1] == -1)
   values[x>>1] = x&1;
 } while (x != i);
  return val[i] = low:
 bool solve() {
 values.assign(N, -1);
  val.assign(2*N, 0); comp = val;
  rep(i,0,2*N) if (!comp[i]) dfs(i);
 rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
 return 1;
};
BellmanFord.cpp
const ll inf = LLONG MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }};</pre>
struct Node { ll dist = inf; int prev = -1; };
```

```
void bellmanFord(vector<Node>& nodes. vector<Ed>& eds.
int s) {
nodes[s].dist = 0;
sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s();</pre>
});
 int \lim = sz(nodes) / 2 + 2; // /3+100 with shuffled
vertices
 rep(i,0,lim) for (Ed ed : eds) {
  Node cur = nodes[ed.a], &dest = nodes[ed.b];
  if (abs(cur.dist) == inf) continue;
  ll d = cur.dist + ed.w;
  if (d < dest.dist) {</pre>
   dest.prev = ed.a:
   dest.dist = (i < lim-1 ? d : -inf);</pre>
 }
 rep(i,0,lim) for (Ed e : eds) {
 if (nodes[e.a].dist == -inf)
   nodes[e.b].dist = -inf:
}
BinaryLifting.cpp
#pragma once
vector<vi> treeJump(vi& P){
 int on = 1, d = 1;
 while(on < sz(P)) on *= 2, d++;
 vector<vi> jmp(d, P);
 rep(i,1,d) rep(i,0,sz(P))
 jmp[i][j] = jmp[i-1][jmp[i-1][j]];
 return jmp;
int jmp(vector<vi>& tbl, int nod, int steps){
 rep(i,0,sz(tbl))
 if(steps&(1<<i)) nod = tbl[i][nod];</pre>
 return nod:
int lca(vector<vi>& tbl, vi& depth, int a, int b) {
 if (depth[a] < depth[b]) swap(a, b);</pre>
 a = imp(tbl, a, depth[a] - depth[b]);
 if (a == b) return a;
 for (int i = sz(tbl): i--:) {
  int c = tbl[i][a], d = tbl[i][b];
```

if (c != d) a = c. b = d:

```
return tbl[0][a]:
Dinic.cpp
 * Author: chilli
 * Date: 2019-04-26
* License: CC0
* Source: https://cp-algorithms.com/graph/dinic.html
* Description: Flow algorithm with complexity $0(VE\log
U) $ where $U = \max |\text{cap}|$.
* 0(\min(E^{1/2}, V^{2/3})E) if U = 1;
$0(\sqrt{V}E)$ for bipartite matching.
* Status: Tested on SPOJ FASTFLOW and SPOJ MATCHING,
stress-tested
*/
#pragma once
struct Dinic {
struct Edge {
 int to, rev;
 ll c, oc;
 ll flow() { return max(oc - c, OLL); } // if you need
flows
};
vi lvl, ptr, q;
vector<vector<Edge>> adj;
Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
 void addEdge(int a, int b, ll c, ll rcap = 0) {
 adj[a].push back({b, sz(adj[b]), c, c});
 adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
ll dfs(int v, int t, ll f) {
 if (v == t || !f) return f;
  for (int& i = ptr[v]; i < sz(adj[v]); i++) {</pre>
  Edge& e = adi[v][i]:
  if (lvl[e.to] == lvl[v] + 1)
   if (ll p = dfs(e.to, t, min(f, e.c))) {
    e.c -= p, adj[e.to][e.rev].c += p;
    return p:
 }
 return 0;
ll calc(int s, int t) {
 Il flow = 0: a[0] = s:
 rep(L,0,31) do { // 'int L=30' maybe faster for random
data
```

```
lvl = ptr = vi(sz(a)):
   int qi = 0, qe = lvl[s] = 1;
   while (qi < qe && !lvl[t]) {</pre>
   int v = q[qi++];
   for (Edge e : adj[v])
    if (!lvl[e.to] && e.c >> (30 - L))
      q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
   while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
 } while (lvl[t]);
  return flow:
 bool leftOfMinCut(int a) { return lvl[a] != 0; }
}:
DirectedMST.com
 * Source: https://github.com/spaghetti-source/algorithm/
blob/master/graph/arborescence.cc
* and https://github.com/bgi343/USACO/blob/42d177dfb9d6c
e350389583cfa71484eb8ae614c/Implementations/content/graph
s%20(12)/Advanced/DirectedMST.h for the reconstruction
 * Description: Finds a minimum spanning
 * tree/arborescence of a directed graph, given a root
node. If no MST exists, returns -1.
 * Time: O(E \log V)
#pragma once
#include "../data-structures/UnionFindRollback.h"
struct Edge { int a. b: ll w: }:
struct Node { /// lazy skew heap node
 Edge key:
 Node *1, *r;
 ll delta;
 void prop() {
 kev.w += delta:
  if (l) l->delta += delta:
 if (r) r->delta += delta;
 delta = 0:
 }
 Edge top() { prop(); return key; }
Node *merge(Node *a, Node *b) {
if (!a || !b) return a ?: b;
 a->prop(), b->prop();
 if (a->kev.w > b->kev.w) swap(a, b);
 swap(a->l, (a->r = merge(b, a->r)));
 return a:
```

```
void pop(Node*& a) { a->prop(): a = merge(a->l, a->r): }
pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
 RollbackUF uf(n):
 vector<Node*> heap(n);
 for (Edge e : g) heap[e.b] = merge(heap[e.b], new
Node{e}):
ll res = 0:
 vi seen(n, -1), path(n), par(n);
 seen[r] = r:
 vector<Edge> Q(n), in(n, \{-1,-1\}), comp;
 deque<tuple<int, int, vector<Edge>>> cycs;
 rep(s.0.n) {
 int u = s, qi = 0, w;
  while (seen[u] < 0) {</pre>
  if (!heap[u]) return {-1,{}};
   Edge e = heap[u]->top();
   heap[u]->delta -= e.w, pop(heap[u]);
   O[qi] = e. path[qi++] = u. seen[u] = s:
   res += e.w, u = uf.find(e.a);
   if (seen[u] == s) { /// found cycle, contract
    Node* cyc = 0;
    int end = qi, time = uf.time();
    do cyc = merge(cyc, heap[w = path[--qi]]);
    while (uf.ioin(u. w)):
    u = uf.find(u), heap[u] = cyc, seen[u] = -1;
    cycs.push front({u, time, {&O[qi], &O[end]}});
 rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
 for (auto& [u,t,comp] : cycs) { // restore sol
(optional)
 uf.rollback(t);
 Edge inEdge = in[u];
 for (auto& e : comp) in[uf.find(e.b)] = e;
 in[uf.find(inEdge.b)] = inEdge;
 rep(i,0,n) par[i] = in[i].a;
 return {res, par};
```

EdgeColoring.cpp

* Description: Given a simple, undirected graph with max degree \$D\$, computes a
* \$(D + 1)\$-coloring of the edges such that no neighboring edges share a color.

```
* ($D$-coloring is NP-hard, but can be done for
bipartite graphs by repeated matchings of
* max-degree nodes.)
* Time: O(NM)
vi edgeColoring(int N, vector<pii> eds) {
vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
 for (pii e : eds) ++cc[e.first], ++cc[e.second];
 int u, v, ncols = *max element(all(cc)) + 1;
 vector<vi> adj(N, vi(ncols, -1));
 for (pii e : eds) {
 tie(u, v) = e;
 fan[0] = v;
 loc.assign(ncols. 0):
 int at = u, end = u, d, c = free[u], ind = 0, i = 0;
 while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
  loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
 cc[loc[d]] = c;
  for (int cd = d; at != -1; cd ^= c ^ d, at =
adi[at][cd])
   swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
 while (adj[fan[i]][d] != -1) {
  int left = fan[i], right = fan[++i], e = cc[i];
   adi[u][e] = left;
   adj[left][e] = u;
   adi[right][e] = -1:
   free[right] = e;
 adj[u][d] = fan[i];
 adj[fan[i]][d] = u;
 for (int y : {fan[0], u, end})
  for (int& z = free[y] = 0; adj[y][z] != -1; z++);
rep(i,0,sz(eds))
 for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;)
++ret[i];
return ret;
```

EulerWalk.cop

- * Description: Eulerian undirected/directed path/cycle algorithm.
- * Input should be a vector of (dest, global edge index), where
- $\ ^{*}$ for undirected graphs, forward/backward edges have the same index.
- * Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or
- * empty list if no cycle/path exists.

```
* To get edge indices back, add .second to s and ret.
 * Time: O(V + E)
#pragma once
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int
src=0) {
int n = sz(qr):
vi D(n), its(n), eu(nedges), ret, s = {src};
 D[src]++; // to allow Euler paths, not just cycles
 while (!s.empty()) {
 int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
 if (it == end){ ret.push_back(x); s.pop_back();
continue: }
  tie(y, e) = gr[x][it++];
 if (!eu[e]) {
  D[x]--, D[y]++;
  eu[e] = 1; s.push back(y);
 }}
for (int x : D) if (x < 0 \mid | sz(ret) != nedges+1) return
{};
return {ret.rbegin(), ret.rend()};
```

FloydWarshall.cpp

```
#pragma once

const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>% m) {
  int n = sz(m);
  rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
  rep(k,0,n) rep(i,0,n) rep(j,0,n)
  if (m[i][k] != inf && m[k][j] != inf) {
    auto newDist = max(m[i][k] + m[k][j], -inf);
    m[i][j] = min(m[i][j], newDist);
  }
  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}</pre>
```

GlobalMinCut.cpp

```
* Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

* Time: O(V^3)

#pragma once

pair<int, vi> globalMinCut(vector<vi> mat) {
```

```
pair<int, vi> best = {INT MAX, {}};
 int n = sz(mat):
 vector<vi> co(n);
 rep(i,0,n) co[i] = {i};
 rep(ph,1,n) {
 vi w = mat[0];
 size t s = 0, t = 0;
  rep(it,0,n-ph) \{ // O(V^2) \rightarrow O(E \log V) \text{ with prio.} \}
aueue
   w[t] = INT MIN;
   s = t, t = max_element(all(w)) - w.begin();
   rep(i,0,n) w[i] += mat[t][i];
  best = min(best, \{w[t] - mat[t][t], co[t]\});
  co[s].insert(co[s].end(), all(co[t]));
  rep(i,0,n) mat[s][i] += mat[t][i];
  rep(i,0,n) mat[i][s] = mat[s][i];
  mat[0][t] = INT MIN;
return best:
```

HLD.cpp

- * Description: Decomposes a tree into vertex disjoint heavy paths and light
- * edges such that the path from any leaf to the root contains at most log(n)
- $\ ^{\ast}$ light edges. Code does additive modifications and \max queries, but can
- * support commutative segtree modifications/queries on paths and subtrees.
- * Takes as input the full adjacency list. VALS_EDGES being true means that
- * values are stored in the edges, as opposed to the nodes. All values
- st initialized to the segtree default. Root must be 0.

```
* Time: O((\log N)^2)
```

```
#pragma once
#include "../data-structures/LazySegmentTree.h"

template <bool VALS_EDGES> struct HLD {
  int N, tim = 0;
  vector<vi> adj;
  vi par, siz, rt, pos;
  Node *tree;
  HLD(vector<vi> adj_)
  : N(sz(adj )), adj(adj_), par(N, -1), siz(N, 1),
```

```
rt(N),pos(N),tree(new Node(0, N)){ dfsSz(0);
dfsHld(0): }
void dfsSz(int v) {
 for (int& u : adj[v]) {
   adj[u].erase(find(all(adj[u]), v));
   par[u] = v;
  dfsSz(u);
  siz[v] += siz[u];
  if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
 void dfsHld(int v) {
 pos[v] = tim++;
 for (int u : adj[v]) {
  rt[u] = (u == adj[v][0] ? rt[v] : u);
  dfsHld(u);
 template <class B> void process(int u, int v, B op) {
 for (;; v = par[rt[v]]) {
  if (pos[u] > pos[v]) swap(u, v);
  if (rt[u] == rt[v]) break;
  op(pos[rt[v]], pos[v] + 1);
 op(pos[u] + VALS_EDGES, pos[v] + 1);
 void modifyPath(int u, int v, int val) {
 process(u, v, [&](int l, int r) { tree->add(l, r, val);
});
int queryPath(int u, int v) { // Modify depending on
problem
 int res = -1e9;
 process(u, v, [&](int l, int r) {
   res = max(res, tree->query(l, r));
 });
 return res;
int querySubtree(int v) { // modifySubtree is similar
 return tree->query(pos[v] + VALS EDGES, pos[v] +
siz[v]);
}
};
LCA.cpp
#pragma once
```

#include "../data-structures/RMO.h"

```
struct LCA {
 int T = 0;
 vi time, path, ret;
 RMQ<int> rmq;
 LCA(vector<vi>& C): time(sz(C)), rmq((dfs(C,0,-1),
ret)) {}
 void dfs(vector<vi>& C, int v, int par) {
  time[v] = T++;
  for (int y : C[v]) if (y != par) {
   path.push_back(v), ret.push_back(time[v]);
   dfs(C, y, v);
 }
 }
 int lca(int a, int b) {
 if (a == b) return a;
  tie(a, b) = minmax(time[a], time[b]);
 return path[rmq.query(a, b)];
//dist(a,b){return depth[a] + depth[b] -
2*depth[lca(a,b)];}
};
LinkCutTree.cpp
 * Description: Represents a forest of unrooted trees.
You can add and remove
 * edges (as long as the result is still a forest), and
check whether
 * two nodes are in the same tree.
 * Time: All operations take amortized O(\log N).
#pragma once
struct Node { // Splay tree. Root's pp contains tree's
parent.
 Node *p = 0, *pp = 0, *c[2];
 bool flip = 0;
 Node() { c[0] = c[1] = 0; fix(); }
 void fix() {
 if (c[0]) c[0]->p = this;
 if (c[1]) c[1]->p = this;
 // (+ update sum of subtree elements etc. if wanted)
 void pushFlip() {
 if (!flip) return;
 flip = 0; swap(c[0], c[1]);
  if (c[0]) c[0]->flip ^= 1;
  if (c[1]) c[1]->flip ^= 1;
```

```
int up() { return p ? p->c[1] == this : -1; }
 void rot(int i, int b) {
 int h = i ^ b;
  Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y :
 if ((y->p = p)) p->c[up()] = y;
  c[i] = z - > c[i ^ 1];
  if (b < 2) {
   x \rightarrow c[h] = y \rightarrow c[h ^ 1];
   y - c[h ^ 1] = x;
  z \rightarrow c[i ^ 1] = this;
  fix(); x->fix(); y->fix();
  if (p) p->fix();
  swap(pp, y->pp);
 void splay() { /// Splay this up to the root. Always
finishes without flip set.
  for (pushFlip(); p; ) {
   if (p->p) p->p->pushFlip();
   p->pushFlip(); pushFlip();
   int c1 = up(), c2 = p - \sup();
   if (c2 == -1) p->rot(c1, 2);
   else p->p->rot(c2, c1 != c2);
 }
 Node* first() { /// Return the min element of the
subtree rooted at this, splayed to the top.
  pushFlip();
  return c[0] ? c[0]->first() : (splay(), this);
};
struct LinkCut {
 vector<Node> node;
 LinkCut(int N) : node(N) {}
 void link(int u, int v) \{ // \text{ add an edge } (u, v) \}
  assert(!connected(u, v));
  makeRoot(&node[u]);
  node[u].pp = &node[v];
 void cut(int u, int v) { // remove an edge (u, v)
  Node *x = &node[u], *top = &node[v];
  makeRoot(top); x->splay();
  assert(top == (x-pp : x-c[0]);
  if (x->pp) x->pp = 0;
  else {
   x - c[0] = top - p = 0:
```

```
x->fix();
 bool connected(int u, int v) { // are u, v in the same
tree?
  Node* nu = access(&node[u])->first();
  return nu == access(&node[v])->first();
 void makeRoot(Node* u) { /// Move u to root of
represented tree.
  access(u):
  u->splay();
  if(u->c[0]) {
   u - c[0] - p = 0;
   u - c[0] - flip ^= 1;
   u - > c[0] - > pp = u;
   u - > c[0] = 0;
   u->fix();
 Node* access(Node* u) { /// Move u to root aux tree.
Return the root of the root aux tree.
  u->splay();
  while (Node* pp = u->pp) {
   pp->splay(); u->pp = 0;
  if (pp->c[1]) {
   pp->c[1]->p = 0; pp->c[1]->pp = pp; }
   pp->c[1] = u; pp->fix(); u = pp;
  return u;
};
MinCostMaxFlow.cpp
* Description: Min-cost max-flow.
* If costs can be negative, call setpi before maxflow,
but note that negative cost cycles are not supported.
* To obtain the actual flow, look at positive values
only.
 * Status: Tested on kattis:mincostmaxflow, stress-tested
against another implementation
* Time: $0(F E \log(V))$ where F is max flow. $0(VE)$
for setpi.
#pragma once
// #include <bits/extc++.h> /// include-line,
keep-include
const ll INF = numeric limits<ll>::max() / 4:
```

```
struct MCMF {
 struct edge {
 int from, to, rev;
 ll cap, cost, flow;
 };
 int N:
 vector<vector<edge>> ed;
 vi seen:
 vector<ll> dist, pi;
 vector<edge*> par;
 MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N),
par(N) {}
 void addEdge(int from, int to, ll cap, ll cost) {
 if (from == to) return;
  ed[from].push back(edge{ from,to,sz(ed[to]),cap,cost,0
});
 ed[to].push back(edge{ to,from,sz(ed[from])-1,0,-cost,0
});
}
 void path(int s) {
  fill(all(seen), 0);
  fill(all(dist), INF);
  dist[s] = 0; ll di;
  gnu pbds::priority queue<pair<ll, int>> q;
  vector<decltype(q)::point_iterator> its(N);
  q.push({ 0, s });
  while (!q.empty()) {
  s = q.top().second; q.pop();
   seen[s] = 1; di = dist[s] + pi[s];
   for (edge& e : ed[s]) if (!seen[e.to]) {
   ll val = di - pi[e.to] + e.cost;
    if (e.cap - e.flow > 0 && val < dist[e.to]) {</pre>
     dist[e.to] = val;
     par[e.to] = &e;
     if (its[e.to] == q.end())
      its[e.to] = q.push({ -dist[e.to], e.to });
      q.modify(its[e.to], { -dist[e.to], e.to });
  }
 rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
```

```
pair<ll, ll> maxflow(int s, int t) {
  ll totflow = 0. totcost = 0:
  while (path(s), seen[t]) {
  ll fl = INF;
   for (edge* x = par[t]; x; x = par[x->from])
    fl = min(fl, x->cap - x->flow);
   totflow += fl:
   for (edge* x = par[t]; x; x = par[x->from]) {
   x->flow += fl;
    ed[x->to][x->rev].flow -= fl;
  rep(i,0,N) for(edge& e : ed[i]) totcost += e.cost *
e.flow:
  return {totflow, totcost/2};
 // If some costs can be negative, call this before
 void setpi(int s) { // (otherwise, leave this out)
  fill(all(pi), INF); pi[s] = 0;
  int it = N, ch = 1; ll v;
  while (ch-- && it--)
   rep(i,0,N) if (pi[i] != INF)
     for (edge& e : ed[i]) if (e.cap)
      if ((v = pi[i] + e.cost) < pi[e.to])</pre>
       pi[e.to] = v, ch = 1;
  assert(it >= 0); // negative cost cycle
};
MinimumVertexCover.cpp
 * Description: Finds a minimum vertex cover in a
bipartite graph.
 * The size is the same as the size of a maximum
matching, and
 * the complement is a maximum independent set.
#include "DFSMatching.h"
vi cover(vector<vi>& g, int n, int m) {
 vi match(m. -1):
 int res = dfsMatching(g, match);
 vector<bool> lfound(n, true), seen(m);
 for (int it : match) if (it != -1) lfound[it] = false;
 vi a. cover:
 rep(i,0,n) if (lfound[i]) q.push_back(i);
 while (!q.empty()) {
```

```
int i = q.back(); q.pop_back();
lfound[i] = 1;
for (int e : g[i]) if (!seen[e] && match[e] != -1) {
    seen[e] = true;
    q.push_back(match[e]);
}
rep(i,0,n) if (!lfound[i]) cover.push_back(i);
rep(i,0,m) if (seen[i]) cover.push_back(n+i);
assert(sz(cover) == res);
return cover;
}

SCC.cpp
#pragma once
vi val, comp, z, cont;
int Time, ncomps;
templatesclass G, class E> int dfs(int i, G& q, E& f) {
```

```
vi val, comp, z, cont;
int Time. ncomps:
template<class G, class F> int dfs(int j, G& g, F& f) {
int low = val[j] = ++Time, x; z.push back(j);
 for (auto e : g[j]) if (comp[e] < 0)
 low = min(low, val[e] ?: dfs(e,q,f));
if (low == val[i]) {
 do {
  x = z.back(); z.pop_back();
  comp[x] = ncomps;
  cont.push back(x);
 } while (x != j);
 f(cont); cont.clear();
 ncomps++;
return val[j] = low;
template<class G, class F> void scc(G& g, F f) {
int n = sz(a):
val.assign(n, 0); comp.assign(n, -1);
Time = ncomps = 0;
rep(i,0,n) if (comp[i] < 0) dfs(i, g, f);
```

WeightedMatching.cpp

- $\ ^{\star}$ Description: Given a weighted bipartite graph, matches every node on
- * the left with a node on the right such that no
- $\,$ * nodes are in two matchings and the sum of the edge weights is minimal. Takes

```
* cost[N][M], where cost[i][j] = cost for L[i] to be
matched with R[i] and
 * returns (min cost, match), where L[i] is matched with
 * R[match[i]]. Negate costs for max cost. Requires $N
\le M$.
 * Time: O(N^2M)
pair<int, vi> hungarian(const vector<vi> &a) {
 if (a.empty()) return {0, {}};
 int n = sz(a) + 1, m = sz(a[0]) + 1;
 vi u(n), v(m), p(m), ans(n - 1);
 rep(i,1,n) {
 p[0] = i;
  int j0 = 0; // add "dummy" worker 0
  vi dist(m, INT_MAX), pre(m, -1);
  vector<bool> done(m + 1);
  do { // dijkstra
   done[j0] = true;
   int i0 = p[j0], j1, delta = INT_MAX;
   rep(j,1,m) if (!done[j]) {
    auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
    if (cur < dist[i]) dist[i] = cur, pre[i] = i0;</pre>
   if (dist[j] < delta) delta = dist[j], j1 = j;</pre>
   rep(j,0,m) {
   if (done[j]) u[p[j]] += delta, v[j] -= delta;
    else dist[j] -= delta;
  }
   j0 = j1;
 } while (p[j0]);
  while (j0) { // update alternating path
  int j1 = pre[j0];
   p[j0] = p[j1], j0 = j1;
 rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
 return {-v[0], ans}; // min cost
math
```

Totient.cpp

```
}
    }
    if (n > 1)
        result -= result / n;
    return result;
}
void phi 1 to n(int n) { //O(nloglogn)
    vector<int> phi(n + 1);
    for (int i = 0; i <= n; i++)</pre>
        phi[i] = i;
    for (int i = 2; i <= n; i++) {</pre>
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                 phi[i] -= phi[i] / i;
    }
mobius.cpp
const int MAXN = 1e6 + 5;
int mu[MAXN]; // Möbius values
bool isPrime[MAXN];
vector<int> primes;
void computeMobius() {
    for (int i = 0; i < MAXN; i++) mu[i] = 1;</pre>
    vector<int> cnt(MAXN, 0);
    for (int i = 2; i < MAXN; ++i) {</pre>
        if (!cnt[i]) {
            for (int j = i; j < MAXN; j += i) {</pre>
                 cnt[i]++;
                 mu[j] *= -1;
            for (long long j = 1LL * i * i; j < MAXN; j +=</pre>
1LL * i * i) {
                 mu[j] = 0;
        }
    }
```

pollardRho.cpp

#pragma once
#include <bits/stdc++.h>
using namespace std;

```
using LL = long long:
mt19937 64 rng(chrono::steady clock::now().time since epo
ch().count());
LL modmul(LL a, LL b, LL mod) {
    return ( int128)a * b % mod; // safe for a * b up to
1e18
LL modpow(LL a, LL b, LL mod) {
   LL res = 1:
   while (b > 0) {
        if (b & 1) res = modmul(res, a, mod);
        a = modmul(a, a, mod):
        b >>= 1:
   }
   return res;
bool isPrime(LL n) {
   if (n < 2) return false;</pre>
   for (LL x: {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
37}) {
        if (x == n) return true;
        if (n % x == 0) return false;
        LL d = n - 1. s = 0:
        while (d % 2 == 0) d /= 2, ++s;
        LL cur = modpow(x, d, n);
        if (cur == 1 || cur == n - 1) continue;
        bool passed = false;
        for (int r = 0; r < s; ++r) {</pre>
            cur = modmul(cur. cur. n):
            if (cur == n - 1) {
                passed = true;
                break;
            }
        if (!passed) return false;
    return true;
LL pollard(LL n) {
   if (n % 2 == 0) return 2:
   if (isPrime(n)) return n;
   while (true) {
        LL c = uniform int distribution<LL>(1. n -
1)(rng);
```

```
LL x = uniform int distribution < LL > (2, n - 1)
1)(rna):
        LL y = x, d = 1;
        auto f = [\&](LL x) {
            return (modmul(x, x, n) + c) \% n;
        };
        while (d == 1) {
            x = f(x);
            y = f(f(y));
            d = gcd(abs(x - y), n);
        if (d != n) return d;
    }
vector<LL> factorize(LL n) {
    vector<LL> factors:
    function<void(LL)> factor = [&](LL x) {
        if (x == 1) return;
        if (isPrime(x)) {
            factors.push back(x);
            return;
        LL d = pollard(x);
        factor(d);
        factor(x / d);
    };
    factor(n);
    sort(factors.begin(), factors.end()):
    return factors;
int main() {
    LL n;
    cin >> n;
    vector<LL> factors = factorize(n):
    cout << "Factors of " << n << ": ";</pre>
    for (LL x : factors) cout << x << ' ';
    cout << "\n":
number-theory
Binomial.cpp
#pragma once
```

#include "ModOperations.h"

```
const int MX = 2e5 + 10:
int factorial[MX];
bool isPrecomputed = false;
void precomputeFactorials() {
    isPrecomputed = true:
    factorial[0] = 1:
    for (int i = 1; i < MX; i++) {</pre>
        factorial[i] = mul(factorial[i - 1], i);
}
int divide(int a. int b) {
    return mul(a, modInverse(b, MOD));
int nCr(int n, int r) {
    if (!isPrecomputed) precomputeFactorials();
    if (r < 0 || n < r) return 0:
    return divide(factorial[n], mul(factorial[r],
factorial[n - r]));
int nPr(int n, int r) {
    if (!isPrecomputed) precomputeFactorials();
    if (r < 0 || n < r) return 0;
    return divide(factorial[n], factorial[n - r]);
}
ExtendedGCD.cpp
#pragma once
// Returns gcd(a, b) and finds x, v such that: a*x + b*v
= \gcd(a, b)
int egcd(int a, int b, int &x, int &y) {
    x = 1, y = 0;
    int x1 = 0, y1 = 1;
    while (b) {
        int q = a / b;
        int t = b:
        b = a \% b;
        a = t:
        t = x1;
        x1 = x - q * x1;
        x = t:
        t = v1:
```

```
y1 = y - q * y1;
       v = t:
   return a;
ModOperations.cpp
#pragma once
const int MOD = 1e9 + 7;
inline int add(int a, int b) { return (a + b) % MOD; }
inline int sub(int a, int b) { return (a - b + MOD) % MOD;
inline int mul(int a, int b) { return (1LL * a * b) % MOD;
int power(int a, int b) {
   int res = 1:
   while (b > 0) {
       if (b & 1) res = mul(res, a);
       a = mul(a, a);
       b >>= 1;
   }
   return res;
numerical
Determinant.cpp
* Description: Calculates determinant of a matrix.
Destroys the matrix.
* Time: $0(N^3)$
#pragma once
double det(vector<vector<double>>& a) {
int n = sz(a); double res = 1;
rep(i,0,n) {
 int b = i:
 rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
 if (i != b) swap(a[i], a[b]), res *= -1;
 res *= a[i][i];
 if (res == 0) return 0;
  rep(j,i+1,n) {
```

double v = a[j][i] / a[i][i];

if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];

```
return res;
FastFourierTransform.cpp
* Description: fft(a) computes hat f(k) = \sum_{k=0}^{\infty} f(k) = \sum_{k=0}^{\infty} f(k)
\exp(2\pi i \cdot k \times / N) for all $k$. N must be a power
of 2.
   Useful for convolution:
   \text{texttt}\{\text{conv}(a, b) = c\}, \text{ where } c[x] = \text{sum}
a[i]b[x-i]$.
   For convolution of complex numbers or more than two
vectors: FFT, multiply
   pointwise, divide by n, reverse(start+1, end), FFT
back.
   Rounding is safe if (\sum a_i^2 + \sum a_i^2)
b i^2)\log 2{N} < 9\cdot10^{14}$
   (in practice $10^{16}$; higher for random inputs).
   Otherwise, use NTT/FFTMod.
* Time: O(N \setminus S) with SN = |A| + |B| $ ($\tilde 1s$ for
N=2^{22}
#pragma once
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
int n = sz(a), L = 31 - __builtin_clz(n);
 static vector<complex<long double>> R(2, 1);
 static vector<C> rt(2, 1); // (^ 10% faster if double)
 for (static int k = 2; k < n; k *= 2) {</pre>
 R.resize(n); rt.resize(n);
 auto x = polar(1.0L, acos(-1.0L) / k);
 rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
 vi rev(n);
 rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
 rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
for (int k = 1; k < n; k *= 2)
 for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
  // C z = rt[j+k] * a[i+j+k]; // (25% faster if
hand-rolled) /// include-line
   auto x = (double *)&rt[j+k], y = (double *)&a[i+j+k];
/// exclude-line
   C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]); ///
exclude-line
   a[i + j + k] = a[i + j] - z;
   a[i + j] += z;
 }
```

```
vd conv(const vd& a, const vd& b) {
 if (a.empty() || b.empty()) return {};
 vd res(sz(a) + sz(b) - 1);
 int L = 32 - __builtin_clz(sz(res)), n = 1 << L;</pre>
 vector<C> in(n), out(n);
 copv(all(a), begin(in));
 rep(i,0,sz(b)) in[i].imag(b[i]);
 fft(in):
 for (C& x : in) x *= x;
 rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
 rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
 return res;
FastFourierTransformMod.cpp
 * Description: Higher precision FFT, can be used for
convolutions modulo arbitrary integers
 * as long as $N\log 2N\cdot \text{mod} < 8.6 \cdot
10^{14}$ (in practice $10^{16}$ or higher).
* Inputs must be in $[0, \text{mod})$.
* Time: O(N \setminus log N), where SN = |A| + |B| S (twice as slow
as NTT or FFT)
#pragma once
#include "FastFourierTransform.h"
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
 if (a.empty() || b.empty()) return {};
 vl res(sz(a) + sz(b) - 1);
 int B=32-__builtin_clz(sz(res)), n=1<<B,</pre>
cut=int(sqrt(M));
 vector<C> L(n), R(n), outs(n), outl(n);
 rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] %
cut);
 rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] %
cut);
 fft(L), fft(R);
 rep(i,0,n) {
 int j = -i & (n - 1);
  outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
  outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
 fft(outl), fft(outs);
 rep(i,0,sz(res)) {
 ll av = ll(real(outl[i])+.5), cv =
ll(imag(outs[i])+.5);
 ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
```

```
FastSubsetTransform.cpp
* Description: Transform to a basis with fast
convolutions of the form
* $\displaystyle c[z] = \sum\nolimits {z = x \oplus y}
a[x] \cdot b[y]$,
* where $\oplus$ is one of AND, OR, XOR. The size of $a$
must be a power of two.
* Time: O(N \log N)
#pragma once
void FST(vi& a, bool inv) {
for (int n = sz(a), step = 1; step < n; step *= 2) {
 for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {</pre>
  int &u = a[j], &v = a[j + step]; tie(u, v) =
   inv ? pii(v - u, u) : pii(v, u + v); // AND
   // inv ? pii(v, u - v) : pii(u + v, u); // OR ///
include-line
   // pii(u + v, u - v); // XOR /// include-line
// if (inv) for (int& x : a) x /= sz(a); // XOR only ///
include-line
vi conv(vi a, vi b) {
FST(a, 0); FST(b, 0);
rep(i,0,sz(a)) a[i] *= b[i];
FST(a, 1); return a;
LinearRecurrence.cpp
* Description: Generates the $k$'th term of an $n$-order
* linear recurrence $S[i] = \sum j S[i-j-1]tr[j]$,
* given S[0 \mid n-1] and tr[0 \mid n-1].
* Faster than matrix multiplication.
* Useful together with Berlekamp--Massev.
* Usage: linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci
number
* Time: O(n^2 \log k)
const ll mod = 5; /** exclude-line */
typedef vector<ll> Poly;
ll linearRec(Polv S. Polv tr. ll k) {
```

res[i] = ((av % M * cut + bv) % M * cut + cv) % M:

}

return res;

```
int n = sz(tr):
 auto combine = [&](Poly a, Poly b) {
 Poly res(n * 2 + 1);
 rep(i,0,n+1) rep(j,0,n+1)
  res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
 for (int i = 2 * n; i > n; --i) rep(j,0,n)
  res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) %
mod;
 res.resize(n + 1);
 return res:
};
Poly pol(n + 1), e(pol);
 pol[0] = e[1] = 1;
 for (++k; k; k /= 2) {
 if (k % 2) pol = combine(pol, e);
 e = combine(e, e);
ll res = 0;
 rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
return res;
PolyInterpolate.cpp
* Author: Simon Lindholm
* Date: 2017-05-10
 * License: CC0
* Source: Wikipedia
* Description: Given $n$ points (x[i], y[i]), computes
an n-1-degree polynomial $p$ that
* passes through them: p(x) = a[0]*x^0 + ... +
a[n-1]*x^{n-1}$.
* For numerical precision, pick $x[k] =
c*\cs(k/(n-1)*\pi), k=0 \dots n-1$.
* Time: O(n^2)
*/
#pragma once
typedef vector<double> vd:
vd interpolate(vd x, vd y, int n) {
vd res(n), temp(n);
rep(k,0,n-1) rep(i,k+1,n)
 y[i] = (y[i] - y[k]) / (x[i] - x[k]);
 double last = 0; temp[0] = 1;
rep(k,0,n) rep(i,0,n) {
```

```
res[i] += y[k] * temp[i];
  swap(last. temp[i]):
  temp[i] -= last * x[k];
 return res;
PolyRoots.cpp
 * Description: Finds the real roots to a polynomial.
 * Usage: polyRoots({{2,-3,1}},-1e9,1e9) // solve
x^2-3x+2 = 0
 * Time: O(n^2 \log(1/\epsilon))
#pragma once
#include "Polynomial.h"
vector<double> polyRoots(Poly p, double xmin, double
xmax) {
 if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
 vector<double> ret;
 Poly der = p;
 der.diff():
 auto dr = polyRoots(der, xmin, xmax);
 dr.push back(xmin-1);
 dr.push back(xmax+1):
 sort(all(dr));
 rep(i,0,sz(dr)-1) {
  double l = dr[i], h = dr[i+1];
  bool sign = p(l) > 0;
  if (sign ^ (p(h) > 0)) {
   rep(it,0,60) { // while (h - l > 1e-8)
    double m = (l + h) / 2, f = p(m);
    if ((f \le 0) \land sign) l = m;
    else h = m:
   ret.push_back((l + h) / 2);
 }
 return ret;
Polvnomial.com
#pragma once
struct Polv {
 vector<double> a;
```

double operator()(double x) const {

```
for (int i = sz(a): i--:) (val *= x) += a[i]:
  return val;
 void diff() {
  rep(i,1,sz(a)) a[i-1] = i*a[i];
  a.pop back();
 void divroot(double x0) {
  double b = a.back(), c; a.back() = 0;
  for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b,
b=c;
  a.pop_back();
};
SolveLinear.cpp
* Author: Per Austrin. Simon Lindholm
 * Date: 2004-02-08
 * License: CC0
* Description: Solves A * x = b. If there are multiple
solutions, an arbitrary one is returned.
* Returns rank, or -1 if no solutions. Data in $A$ and
$b$ is lost.
* Time: O(n^2 m)
* Status: tested on kattis:equationsolver, and
bruteforce-tested mod 3 and 5 for n,m <= 3
*/
#pragma once
typedef vector<double> vd;
const double eps = 1e-12;
int solveLinear(vector<vd>& A, vd& b, vd& x) {
int n = sz(A), m = sz(x), rank = 0, br, bc;
if (n) assert(sz(A[0]) == m);
 vi col(m); iota(all(col), 0);
 rep(i,0,n) {
  double v, bv = 0;
  rep(r,i,n) rep(c,i,m)
  if ((v = fabs(A[r][c])) > bv)
   br = r, bc = c, bv = v;
  if (bv <= eps) {
  rep(j,i,n) if (fabs(b[j]) > eps) return -1;
   break:
  swap(A[i]. A[br]):
```

double val = 0:

```
swap(b[i], b[br]);
  swap(col[i], col[bc]);
  rep(j,0,n) swap(A[j][i], A[j][bc]);
  bv = 1/A[i][i];
  rep(j,i+1,n) {
   double fac = A[j][i] * bv;
  b[j] -= fac * b[i];
  rep(k,i+1,m) A[j][k] -= fac*A[i][k];
 }
  rank++;
 x.assign(m, 0);
 for (int i = rank; i--;) {
 b[i] /= A[i][i];
 x[col[i]] = b[i];
 rep(j,0,i) b[j] -= A[j][i] * b[i];
 return rank; // (multiple solutions if rank < m)</pre>
SolveLinear2.cpp
 * Description: To get all uniquely determined values of
$x$ back from SolveLinear, make the following changes:
#include "SolveLinear.h"
rep(j,0,n) if (j != i) // instead of <math>rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
 rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
x[col[i]] = b[i] / A[i][i];
fail:; }
SolveLinearBinarv.cop
 * Description: Solves $Ax = b$ over $\mathbb F 2$. If
there are multiple solutions, one is returned
arbitrarily.
 * Returns rank, or -1 if no solutions. Destroys $A$ and
$b$.
 * Time: O(n^2 m)
#pragma once
```

int solveLinear(vector<bs>& A. vi& b. bs& x. int m) {

typedef bitset<1000> bs:

```
assert(m \le sz(x)):
 vi col(m); iota(all(col), 0);
 rep(i,0,n) {
  for (br=i; br<n; ++br) if (A[br].any()) break;</pre>
 if (br == n) {
  rep(j,i,n) if(b[j]) return -1;
   break:
 }
  int bc = (int)A[br]._Find_next(i-1);
  swap(A[i], A[br]);
  swap(b[i], b[br]);
  swap(col[i], col[bc]);
  rep(j,0,n) if (A[j][i] != A[j][bc]) {
  A[j].flip(i); A[j].flip(bc);
  rep(j,i+1,n) if (A[j][i]) {
  b[i] ^= b[i];
   A[j] ^= A[i];
 rank++;
 x = bs();
 for (int i = rank; i--;) {
 if (!b[i]) continue;
 x[col[i]] = 1;
 rep(j,0,i) b[j] ^= A[j][i];
 return rank; // (multiple solutions if rank < m)</pre>
subset.cpp
Description: Various subset convolutions
Time: $0(2^K * K^2)$ for conv, $0(2^K * K)$ for others
bf4921. 91 lines
vector<LL> XorTransform(vector<LL> p, bool inverse) {
  int n = p.size();
  assert(((n & (n-1))==0)):
  for (int len = 1; 2*len <= n; len <<= 1) {
   for (int i = 0; i < n; i += len+len) {</pre>
      for (int j = 0; j < len; j++) {</pre>
        LL u = p[i+j], v = p[i+len+j];
        if (!inverse) p[i+j] = u+v, p[i+len+j] = u-v;
        else p[i+j] = (u+v)/2, p[i+len+j] = (u-v)/2;
   }
```

int n = sz(A), rank = 0, br;

```
return p:
vector<LL> SOS(vector<LL> p, bool inverse, bool subset) {
 int k = builtin ctz(p.size());
 assert(p.size() == (1<<k));
 for (int i=0: i<k: i++)</pre>
   for (int mask=0; mask<(1<<k); mask++)</pre>
      if (bool(mask & (1<<i)) == subset) {</pre>
        if (!inverse) p[mask] += p[mask^(1<<i)];</pre>
        else p[mask] -= p[mask^(1<<i)];</pre>
     }
 return p;
vector<LL> product(const vector<LL> &a, const vector<LL>
&b) {
 assert(a.size() == b.size());
 vector<LL> ans(a.size()):
 for (int i=0; i<a.size(); i++) ans[i] = a[i] * b[i];</pre>
 return ans;
vector<LL> XorConvolution(vector<vector<LL>> vs) {
   int n = vs.size():
   for (int i=0; i<n; i++) vs[i] = XorTransform(vs[i],</pre>
0);
   vector<LL> ans = vs[0];
   for (int i=1; i<n; i++) ans = product(ans, vs[i]);</pre>
    ans = XorTransform(ans, 1);
   return ans:
vector<LL> ORConvolution(vector<vector<LL>> vs) {
 int n = vs.size();
 for (int i=0; i<n; i++) vs[i] = SOS(vs[i], 0, 1);</pre>
 vector<LL> ans = vs[0];
 for (int i=1; i<n; i++) ans = product(ans, vs[i]);</pre>
 ans = SOS(ans, 1, 1);
 return ans:
vector<LL> AndConvolution(vector<vector<LL>> vs) {
 int n = vs.size();
 for (int i=0; i<n; i++) vs[i] = SOS(vs[i], 0, 0);</pre>
 vector<LL> ans = vs[0];
 for (int i=1: i<n: i++) ans = product(ans. vs[i]):</pre>
```

```
ans = SOS(ans, 1, 0);
 return ans:
vector<LL> SubsetConvolution(const vector<LL> &a, const
vector<LL> &b) {
 int k = builtin ctz(a.size());
  assert(a.size() == (1<<k) && b.size() == (1<<k));
  vector<vector<LL>> A(k+1, Z), B(k+1, Z), C(k+1, Z);
  for (int mask=0; mask<(1<<k); mask++) {</pre>
   A[__builtin_popcount(mask)][mask] = a[mask];
   B[__builtin_popcount(mask)][mask] = b[mask];
 }
  for (int i=0; i<k; i++) {</pre>
   A[i] = SOS(A[i], 0, 1);
    B[i] = SOS(B[i], 0, 1);
    for (int j=0; j<=i; j++)</pre>
      for (int mask = 0; mask < (1<<k); mask++)</pre>
        C[i][mask] += A[j][mask]*B[i-j][mask];
  vector<LL> ans(1<<k);</pre>
  for (int mask=0; mask<(1<<k); mask++) {</pre>
    ans[mask] = C[ builtin popcount(mask)][mask]:
 }
 return ans;
```

strings

Hashing-codeforces.cpp

```
#pragma once

typedef uint64_t ull;
static int C; // initialized below

// Arithmetic mod two primes and 2^32 simultaneously.
// "typedef uint64_t H;" instead if Thue-Morse does not apply.
template<int M, class B>
struct A {
  int x; B b; A(int x=0) : x(x), b(x) {}
  A(int x, B b) : x(x), b(b) {}
  A operator+(A o){int y = x+o.x; return{y - (y>=M)*M, b+o.b};}
  A operator-(A o){int y = x-o.x; return{y + (y< 0)*M, b-o.b};}</pre>
```

```
A operator*(A o) { return {(int)(1LL*x*o.x % M), b*o.b};
 explicit operator ull() { return x ^ (ull) b << 21; }</pre>
 bool operator==(A o) const { return (ull)*this ==
(ull)o: }
 bool operator<(A o) const { return (ull)*this < (ull)o;</pre>
};
typedef A<1000000007, A<1000000009, unsigned>> H;
struct HashInterval {
 vector<H> ha, pw;
 HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
  pw[0] = 1:
  rep(i,0,sz(str))
   ha[i+1] = ha[i] * C + str[i],
   pw[i+1] = pw[i] * C;
 }
 H hashInterval(int a, int b) { // hash [a, b)
  return ha[b] - ha[a] * pw[b - a]:
};
vector<H> getHashes(string& str, int length) {
 if (sz(str) < length) return {};</pre>
 H h = 0, pw = 1;
 rep(i,0,length)
 h = h * C + str[i], pw = pw * C;
 vector<H> ret = {h};
 rep(i,length,sz(str)) {
  ret.push_back(h = h * C + str[i] - pw * str[i-length]);
 return ret;
H hashString(string& s){H h{}; for(char c:s)
h=h*C+c;return h;}
#include <sys/time.h>
int main() {
 timeval tp:
 gettimeofdav(&tp. 0):
 C = (int)tp.tv_usec; // (less than modulo)
 assert((ull)(H(1)*2+1-3) == 0);
 // ...
```

Hashing.cpp

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and
тоге
// code, but works on evil test data (e.g. Thue-Morse,
// ABBA... and BAAB... of length 2^10 hash the same mod
2^64).
// "typedef ull H;" instead if you think test data is
random.
// or work mod 10^9+7 if the Birthday paradox is not a
typedef uint64 t ull;
struct H {
ull x; H(ull x=0) : x(x) {}
H operator+(H o) { return x + o.x + (x + o.x < x); }
H operator-(H o) { return *this + ~o.x; }
H operator*(H o) { auto m = (uint128 t)x * o.x;
 return H((ull)m) + (ull)(m >> 64); }
 ull get() const { return x + !~x; }
 bool operator==(H o) const { return get() == o.get(); }
bool operator<(H o) const { return get() < o.get(); }</pre>
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random
also ok)
struct HashInterval {
vector<H> ha, pw;
 HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
  pw[0] = 1;
  rep(i,0,sz(str))
  ha[i+1] = ha[i] * C + str[i],
   pw[i+1] = pw[i] * C;
H hashInterval(int a, int b) { // hash [a, b)
  return ha[b] - ha[a] * pw[b - a];
}
};
vector<H> getHashes(string& str, int length) {
if (sz(str) < length) return {};</pre>
H h = 0, pw = 1;
 rep(i.0.length)
 h = h * C + str[i], pw = pw * C;
 vector<H> ret = {h}:
 rep(i,length,sz(str)) {
  ret.push back(h = h * C + str[i] - pw * str[i-length]);
return ret:
```

```
H hashString(string& s){H h{}; for(char c:s)
h=h*C+c:return h:}
KMP.cpp
#pragma once
vi pi(const string& s) {
 vi p(sz(s));
 rep(i,1,sz(s)) {
 int g = p[i-1];
  while (g && s[i] != s[g]) g = p[g-1];
 p[i] = q + (s[i] == s[q]);
 return p;
vi match(const string& s, const string& pat) {
 vi p = pi(pat + '\0' + s), res;
 rep(i,sz(p)-sz(s),sz(p))
 if (p[i] == sz(pat)) res.push back(i - 2 * sz(pat));
 return res;
Manacher.cpp
 * Author: User adamant on CodeForces
 * Source: http://codeforces.com/blog/entry/12143
 * Description: For each position in a string, computes
p[0][i] = half length of
 * longest even palindrome around pos i, p[1][i] =
longest odd (half rounded down).
 * Time: O(N)
 * Status: Stress-tested
#pragma once
array<vi, 2> manacher(const string& s) {
 int n = sz(s):
 array<vi,2> p = \{vi(n+1), vi(n)\};
 rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
  int t = r - i + !z:
  if (i<r) p[z][i] = min(t, p[z][l+t]);</pre>
  int L = i-p[z][i], R = i+p[z][i]-!z;
  while (L>=1 && R+1<n && s[L-1] == s[R+1])
   p[z][i]++, L--, R++;
 if (R>r) l=L, r=R;
```

```
return p;
MinRotation.cpp
* Description: Finds the lexicographically smallest
rotation of a string.
 * Time: O(N)
#pragma once
int minRotation(string s) {
int a=0, N=sz(s); s += s;
 rep(b,0,N) rep(k,0,N) {
 if (a+k == b \mid | s[a+k] < s[b+k]) \{b += max(0, k-1);
break;}
 if (s[a+k] > s[b+k]) { a = b; break; }
return a;
SuffixArray.cpp
 * Author: , chilli
 * Date: 2019-04-11
 * License: Unknown
 * Source: Suffix array - a powerful tool for dealing
with strings
 * (Chinese IOI National team training paper, 2009)
 * Description: Builds suffix array for a string.
 * \texttt{sa[i]} is the starting index of the suffix
which
 * is $i$'th in the sorted suffix array.
 * The returned vector is of size $n+1$, and
\text{texttt}\{sa[0] = n\}.
 * The \texttt{lcp} array contains longest common
prefixes for
 * neighbouring strings in the suffix array:
 * \texttt{lcp[i] = lcp(sa[i], sa[i-1])}, \texttt{lcp[0]
 * The input string must not contain any nul chars.
 * Time: O(n \log n)
 * Status: stress-tested
 */
#pragma once
struct SuffixArrav {
vi sa, lcp;
 SuffixArrav(string s. int lim=256) { // or vector<int>
```

```
s.push back(0); int n = sz(s), k = 0, a, b;
  vi x(all(s)), y(n), ws(max(n, lim));
  sa = lcp = y, iota(all(sa), 0);
  for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim =
p) {
   p = j, iota(all(y), n - j);
   rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
   fill(all(ws), 0);
   rep(i,0,n) ws[x[i]]++;
   rep(i,1,lim) ws[i] += ws[i - 1];
   for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
   swap(x, y), p = 1, x[sa[0]] = 0;
   rep(i,1,n) = sa[i - 1], b = sa[i], x[b] =
   (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
  for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
  for (k \&\& k--, j = sa[x[i] - 1];
     s[i + k] == s[i + k]; k++);
};
Zfunc.cpp
 * Author: chilli
 * License: CC0
 * Description: z[i] computes the length of the longest
* except z[0] = 0. (abacaba -> 0010301)
 * Time: O(n)
* Status: stress-tested
 */
```

```
/**
 * Author: chilli
 * License: CC0
 * Description: z[i] computes the length of the longes
common prefix of s[i:] and s,
 * except z[0] = 0. (abacaba -> 0010301)
 * Time: O(n)
 * Status: stress-tested
 */
#pragma once

vi Z(const string& S) {
 vi z(sz(S));
 int l = -1, r = -1;
 rep(i,1,sz(S)) {
 z[i] = i >= r ? 0 : min(r - i, z[i - l]);
 while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
 z[i]++;
 if (i + z[i] > r)
 l = i, r = i + z[i];
}
return z;
```

various

DivideAndConquerDP.cpp

```
* Author: Simon Lindholm
 * License: CC0
 * Source: Codeforces
 * Description: Given $a[i] = \min {lo(i) \le k <
hi(i)}(f(i, k))$ where the (minimal)
 * optimal $k$ increases with $i$, computes $a[i]$ for $i
= L..R-1$.
 * Time: O((N + (hi-lo)) \log N)
 * Status: tested on
http://codeforces.com/contest/321/problem/E
 */
#pragma once
struct DP { // Modify at will:
 int lo(int ind) { return 0; }
 int hi(int ind) { return ind; }
 ll f(int ind, int k) { return dp[ind][k]; }
 void store(int ind, int k, ll v) { res[ind] = pii(k, v);
 void rec(int L, int R, int LO, int HI) {
 if (L >= R) return;
  int mid = (L + R) \gg 1;
  pair<ll, int> best(LLONG MAX, LO);
  rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
   best = min(best, make pair(f(mid, k), k));
  store(mid, best.second, best.first);
  rec(L, mid, L0, best.second+1);
  rec(mid+1, R, best.second, HI);
 void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX);
};
```

FastKnapsack.cpp

```
/**
  * Author: Marten Wiman
  * License: CC0
  * Source: Pisinger 1999, "Linear Time Algorithms for
Knapsack Problems with Bounded Weights"
  * Description: Given N non-negative integer weights w
and a non-negative target t,
  * computes the maximum S <= t such that S is the sum of
some subset of the weights.</pre>
```

```
* Time: O(N \max(w_i))
* Status: Tested on kattis:eavesdropperevasion,
```

stress-tested

```
*/
#pragma once
int knapsack(vi w, int t) {
int a = 0, b = 0, x;
while (b < sz(w) \&\& a + w[b] <= t) a += w[b++];
if (b == sz(w)) return a;
int m = *max element(all(w));
vi u, v(2*m, -1);
v[a+m-t] = b;
rep(i,b,sz(w)) {
 u = v;
 rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
 for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
  v[x-w[j]] = max(v[x-w[j]], j);
for (a = t; v[a+m-t] < 0; a--);
return a;
```

KnuthDP.cpp

```
* Efficiently solves interval DP of the form:
 * dp[i][j] = min(dp[i][k] + dp[k][j] + cost[i][j])
 * where the optimal `k` for dp[i][j] lies between
opt[i][j-1] and opt[i+1][j].
 * Requirements:
 * - cost(i, j) must satisfy the quadrangle inequality
(monotonicity).
* - opt[i][j] increases with i and j.
 * Time Complexity: O(N^2)
 * Works best when transitions are expensive (like O(1)
cost + nested loop).
*/
 const int INF = 1e9;
int dp[500][500], opt[500][500]; // adjust sizes as
needed
int cost(int i, int j); // define your cost function
 void knuthDP(int n) {
    for (int i = 0; i < n; ++i) {</pre>
         dp[i][i] = 0;
         opt[i][i] = i;
    for (int len = 1; len < n; ++len) {</pre>
         for (int i = 0; i + len < n; ++i) {</pre>
             int j = i + len;
             dp[i][i] = INF:
             int l = opt[i][j - 1];
             int r = opt[i + 1][j];
```

```
if (l > r) swap(l, r);
             for (int k = l; k <= r; ++k) {</pre>
                 int val = dp[i][k] + dp[k][j] + cost(i,
j);
                 if (val < dp[i][j]) {</pre>
                     dp[i][j] = val;
                     opt[i][j] = k;
                 }
             }
         }
trie.cpp
const int MAXN = 1000005;
struct trie {
    vector<vi> nxt;
    vi fin;
    int used = 0;
    trie() {
        nxt = vector<vi>(MAXN, vi(26, -1));
        fin = vi(MAXN);
   }
    void insert(string s) {
        int cur = 0;
        for (int i = 0; i < s.size(); i++) {</pre>
            int c = s[i] - 'a';
            if (nxt[cur][c] == -1) {
                used++;
                nxt[cur][c] = used;
            cur = nxt[cur][c];
        fin[cur]++;
   }
    bool search(string s) {
        int cur = 0;
        for (int i = 0; i < s.size(); i++) {</pre>
            int c = s[i] - 'a';
            if (nxt[cur][c] == -1) {
                return false;
            cur = nxt[cur][c];
        return fin[cur] > 0;
```

```
}
    int ans = 0;
    void traverse(int node, int level) {
         for (int i = 0; i < 26; i++) {
            if (nxt[node][i] == -1) continue;
            for (int j = i + 1; j < 26; j++) {
                if (nxt[node][j] == -1) continue;
        for (int i = 0; i < 26; i++) if (nxt[node][i] !=</pre>
-1) traverse(nxt[node][i], level + 1);
};
```