A Generalized Correlation for One-component Homogeneous Equilibrium Flashing Choked Flow

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This paper presents, for the first time, a generalized correlation for one-component homogeneous equilibrium choked flow. This particular critical flow model has long been used as a reference because it is merely an extension of the standard gas-dynamic model. In this respect the flashing two-phase mixture is treated much like a single-phase compressible fluid while undergoing an isentropic expansion with equal phasic velocities and temperature. In many engineering applications, this model in fact provides a best-estimate calculation (Moody, 1975; Lahey and Moody, 1977; Fauske, 1985). The calculational procedure requires seeking a maximum in the mass velocity expression derived from the first law of thermodynamics.

$$G = [2(h_o - h)]^{1/2}/v \qquad (1)$$

where subscript o denotes a stagnation condition. Here the specific enthalpy and specific volume are expressed in terms of quality and the phasic properties, i.e., $h_o = h_{fo} + x_o h_{fgo}$, $h = h_f + x h_{fg}$, and $v = v_f + x v_{fg}$. The static quality is determined along a constant entropy line,

$$x = (s_o - s_f)/s_{fg} = x_o + \frac{s_{fo} - s_f}{s_{fg}} \qquad (2)$$

For an isentropic process the second law allows Eq. 1 to be written as

$$G = \left[2 \int_{P_o}^{P} - v dP \right]^{1/2} \bigg/ v \qquad (3)$$

Calculational methods using either Eq. 1 or Eq. 3 generally require extensive thermophysical data as well as trial-and-error procedures.

Derivation of Correlating Parameter

To obtain simple approximate results for the isentropic flashing process, Epstein et al. (1983) proposed a simple explicit expression that relates local pressure to local specific volume in the form of

$$\frac{v}{v_{fo}} = \omega \left(\frac{P_o}{P} - 1 \right) + 1 \qquad (4)$$

for an isentropic two-phase expansion from an all-liquid (saturated) stagnation condition. The parameter ω was found to be a unique function of the stagnation pressure for a given fluid, which was expressed in graphical form. Grolmes and Leung (1984) later utilized this $P - v$ relationship in Eq. 3 to obtain an analytical solution for choked flow, again for an all-liquid stagnation condition. Specifically, the critical pressure ratio, η, satisfies the following relation,

$$\eta^2 + (\omega^2 - 2\omega)(1 - \eta)^2 + 2\omega^2 \ln \eta + 2\omega^2(1 - \eta) = 0 \qquad (5)$$

while the critical flow rate is given by

$$G = \eta \left(\frac{P_o}{\omega v_{fo}} \right)^{1/2} \qquad (6)$$

The parameter ω was presented in terms of stagnation properties. (As defined here ω is equivalent to $1/\beta$ in the paper by Grolmes and Leung and Eq. 7 is an approximate solution for an isentropic expansion.)

$$\omega = \frac{C_{fo} T_o P_o}{v_{fo}} \left(\frac{v_{fgo}}{h_{fgo}} \right)^2 = \frac{P_o}{v_{fo} G_L^2} \qquad (7)$$

where

$$G_L = \frac{h_{fgo}}{v_{fgo}} \left(\frac{1}{C_{fo} T_o} \right)^{1/2} \tag{8}$$

which is called the limiting flow by Grolmes and Leung, and is also known (Fauske, 1985) as the critical flow rate for the equilibrium-rate model for the all-liquid inlet condition.

The present study extended the above analysis to a two-phase inlet condition and, as such, ω is more appropriately defined in terms of the stagnation two-phase specific volume, $v_o = v_{fo} + x_o v_{fgo}$, i.e.,

$$\omega = \left(\frac{v}{v_o} - 1 \right) \Big/ \left(\frac{P_o}{P} - 1 \right) \tag{9}$$

In this two-phase regime, Eq. 5 for the critical pressure ratio still applies, while the critical mass flux is now given more generally by

$$G = \eta \left(\frac{P_o}{\omega v_o} \right)^{1/2} \tag{10}$$

Thus the above equation suggests a correlation scheme by plotting the normalized mass flux $G/\sqrt{P_o/v_o}$ against the parameter ω. Since one is not required to obtain an exact isentropic expansion formula for ω in such a correlation method, an approximate result based on an isenthalpic (constant enthalpy) process may be justified. As will be demonstrated, this approach allows ω to be expressed more readily in terms of the stagnation properties and proves to be successful in correlating a wide range of fluids and inlet conditions. Thus with the isenthalpic quality given by

$$x = \frac{h_o - h_f}{h_{fg}} = x_o + \frac{h_{fo} - h_f}{h_{fg}} = x_o + \frac{\overline{C_f}(T_o - T)}{h_{fg}} \tag{11}$$

and together with the following approximations and the Clapeyron relation:

$$v_g \gg v_f, \, v_{go} \gg v_{fo},$$

$$h_{fg} \simeq h_{fgo},$$

$$\overline{C_f} \simeq C_{fo}, \, Pv_{fg} \simeq P_o v_{fgo}$$

$$\frac{P_o - P}{T_o - T} = \frac{\overline{h}_{fg}}{\overline{v}_{fg} T} \simeq \frac{h_{fgo}}{v_{fgo} T_o} \tag{12}$$

one arrives at a simple expression for the correlating parameter

$$\omega = \frac{x_o v_{fgo}}{v_o} + \frac{C_{fo} T_o P_o}{v_o} \left(\frac{v_{fgo}}{h_{fgo}} \right)^2 = \frac{x_o v_{fgo}}{v_o} + \frac{P_o}{v_o G_L^2} \tag{13}$$

Perhaps not too surprising, this result reduces to the more restricted form in Eq. 7 for zero quality inlet.

Results and Comparisons

Hall and Czapary (1980) have compiled the most extensive tables of homogeneous equilibrium critical flow for water. Their calculational procedure was based on Eq. 1 together with the

well-established steam-water property tables. Figure 1 shows that Hall and Czapary's data up to a reduced thermodynamic temperature of 0.9 can be correlated successfully by the proposed scheme. Because of the available space in the figure, only selected quality (x_o) values of 0.0, 0.01, 0.03, 0.1, 0.3, 0.6, and 1.0 in their table were used. Even then more than half of these data points cannot be shown due to overlapping. Figure 1 illustrates that both the flow rate and the critical pressure ratio can be correlated remarkably well for $\omega > 4.0$ by the analytical solution. The discrepancies at the higher quality end (i.e., ω approaching 1.0) are due to increasing departure from the assumed isenthalpic solution for ω. However, the points in this region do not exhibit any greater scatter, suggesting that the correlating parameter is still effective. Thus in equation form, the solution for η in Eq. 5 can be more conveniently approximated by

$$\eta = 0.6055 + 0.1356(\ln \omega) - 0.0131(\ln \omega)^2 \tag{14}$$

while for $\omega \geq 4.0$, the best fit to the calculated flow data follows from Eqs. 10 and 14

$$G/\sqrt{P_o/v_o} = [0.6055 + 0.1356(\ln \omega) \\ - 0.0131(\ln \omega)^2]/\omega^{0.5} \tag{15a}$$

and for $\omega < 4.0$, a best fit yields

$$G/\sqrt{P_o/v_o} = 0.66/\omega^{0.39} \tag{15b}$$

As for the goodness of fit, the selected water data (150 points) in Figure 1 were correlated with a standard deviation of 0.7%.

Comparison of this correlation with ten common fluids at a stagnation pressure of 5 bar (absolute) is shown in Figure 2. The calculational method is based on the expanded integral form of
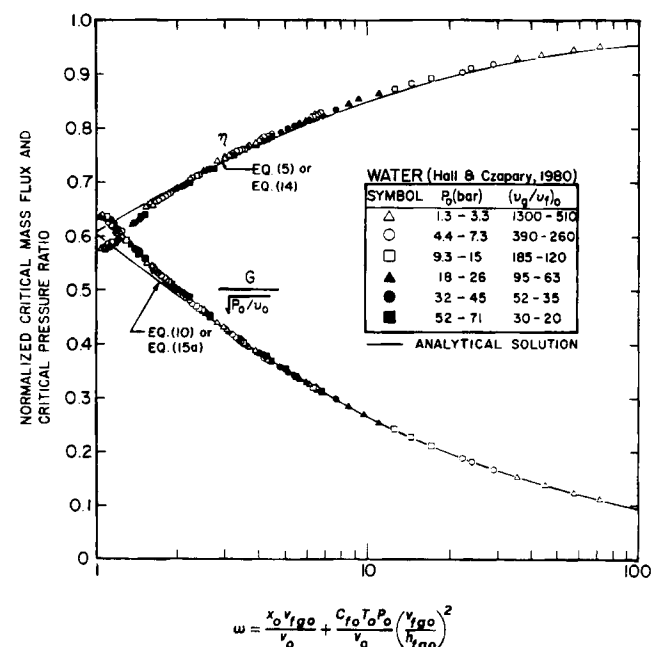

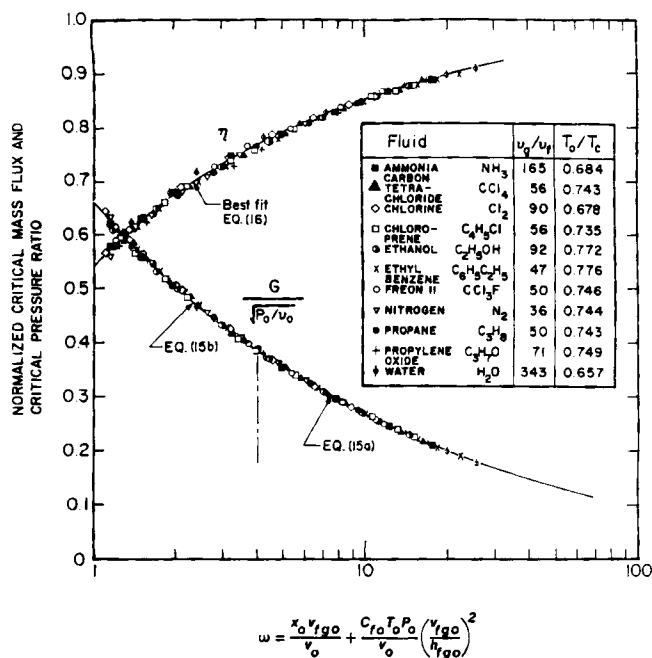Figure 1. Correlation of critical flow parameters for water.

$$\omega = \frac{x_o v_{fgo}}{v_o} + \frac{C_{fo}T_oP_o}{v_o}\left(\frac{v_{fgo}}{h_{fgo}}\right)^2$$

Figure 2. Correlation of critical flow parameters for ten common fluids and water at 5 bar.

$$\omega = \frac{x_o v_{fgo}}{v_o} + \frac{C_{fo}T_oP_o}{v_o}\left(\frac{v_{fgo}}{h_{fgo}}\right)^2$$

Figure 3. Suggested correlation at near-critical region.

Eq. 3 as presented by Grolmes and Leung. Here Freon-11 properties are taken from a DuPont (1965) publication, while the rest are taken from Yaws (1977). The critical flow data calculations of Figure 2 were correlated successfully by Eq. 15 with a standard deviation of 0.8%. A similar comparison was performed at a higher stagnation pressure of 15 bar, yielding a standard deviation of 2.0% using Eq. 15. While the calculated data for critical pressure ratio exhibited somewhat more scatter than that for the flow data calculations, the correlation can still be regarded as good. In equation form, the best fit for η can be given by,

$$\eta = 0.55 + 0.217(\ln \omega) - 0.046(\ln \omega)^2 + 0.004(\ln \omega)^3 \quad (16)$$

Applicability of the Correlation

The present critical flow correlation is accurate enough for most engineering application up to a reduced temperature of 0.9 or a reduced pressure of about 0.5. As the critical point is approached, property variation becomes significant; thus most of the assumptions made in Eq. 12 are violated. However, as shown in Figure 3, the departure can be reasonably correlated using the reduced temperature as an additional parameter. Besides the water data of Hall and Czapary, only two other fluids, ammonia and Freon-11, which possess extensive tabulated data (Reynolds, 1979, and DuPont, 1965, respectively), were correlated and the calculations for G and η were performed using Eq. 1. (The expanded integral form of Eq. 3 as presented by Grolmes and Leung is inaccurate in this region due to neglecting liquid compressibility effect.) In this near-critical region, the C_{fo} value in the ω term is more correctly given by $(dh_f/dT)_o$ rather than the specific heat at constant pressure. As a practical matter, the ability to evaluate critical flow rate in this region is most often hampered by the lack of reliable or consistent thermodynamic data.
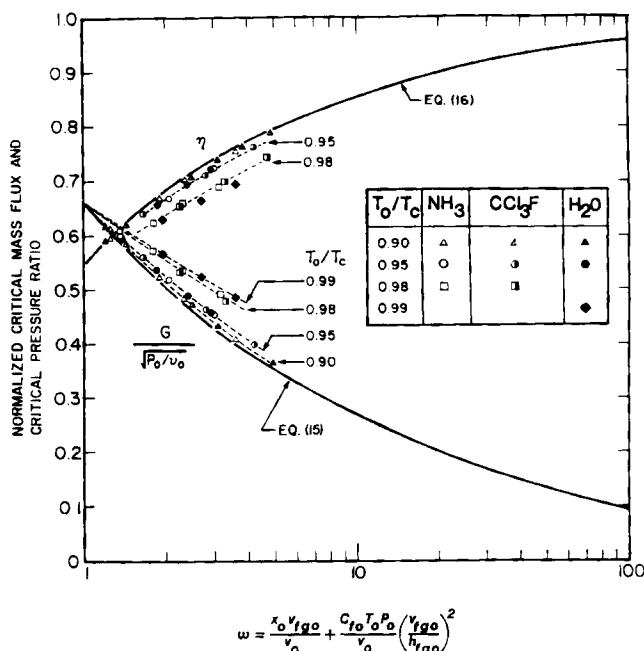
Conclusions

The validity of the present correlation for calculating critical flow parameters has been demonstrated for eleven fluids with widely differing properties. Since only stagnation properties are required, it is extremely easy to apply in most engineering calculations and it will be most useful for those cases where extensive property data are not readily available. The correlation can further provide quick bounding estimates on flow rates due to uncertainties in physical properties and/or stagnation conditions.

Acknowledgments

Notation

C_f = liquid-phase specific heat at constant pressure
$\overline{C_f}$ = average C_f between T_o and T, Eq. 11
G = mass velocity or flux
G_L = limiting flow, Eq. 8
h = specific enthalpy
P = pressure
s = specific entropy
T = temperature
v = specific volume
x = static quality or vapor mass fraction
ω = correlating parameter, Eq. 13
η = critical pressure ratio

Subscripts

c = thermodynamic critical point
f = liquid phase
g = vapor phase
fg = difference between vapor and liquid phase property
o = stagnation condition

Superscripts

$\bar{}$ = average property value between T_o and T, Eqs. 11 and 12

Literature cited

[DuPont], "Thermodynamic Properties of Freon-11 Refrigerant (Trichlorofluoromethane)," Tech. Bull. No. T-11, E. I. DuPont de Nemours & Company, Inc. (1965).

Epstein, M., R. E. Henry, W. Midvidy, and R. Pauls, "One-Dimensional Modeling of Two-Phase Jet Expansion and Impingement," *Thermal-Hydraulics of Nuclear Reactors* II, (2nd Int. Topical Meet. Nuclear Reactor Thermal-Hydraulics, Santa Barbara, CA (Jan., 1983).

Fauske, H. K., "Flashing Flows—Some Practical Guidelines for Emergency Releases," *Plant/Operations Prog.*, 4(3), 132 (1985).

Grolmes, M. A., and J. C. Leung, "Scaling Considerations for Two-Phase Critical Flow," T. N. Veziroglu and A. E. Bergles, eds., *Multi-Phase Flow and Heat Transfer* III, A: *Fundamentals*, 549, Elsevier, Amsterdam (1984).

Hall, D. G., and L. S. Czapary, "Tables of Homogeneous Equilibrium Critical Flow Parameters for Water in SI Units," Report No. EGG-2056, EG&G Idaho, Inc. (Sept., 1980).

Lahey, R. T., and F. J. Moody, *The Thermal-Hydraulics of a Boiling Water Nuclear Reactor*, Am. Nuclear Soc., Hinsdale, IL (1977).

Moody, F. J., "Maximum Discharge Rate of Liquid Vapor Mixtures from Vessels," *Non-Equilibrium Two-Phase Flows, ASME Symp. Vol.*, Am. Soc. Mech. Eng. (1975).

Reynolds, W. C., *Thermodynamic Properties in SI; Graphs, Tables, and Computational Equations for Forty Substances*, Dept. Mech. Eng., Stanford Univ. (1979).

Yaws, C. L., *Physical Properties: A Guide to the Physical, Thermodynamic and Transport Properties of Industrially Important Chemical Compounds*, McGraw-Hill, New York (1977).