AI Project I Report

<u>Instructions on how to run the program:</u>

To run our program, navigate to the working directory and run "python test.py" on the command line. You'll be asked to provide the file paths of the input and output file (you want to save to) and the value of k (penalty) for the step cost function. To visualize the solution path, remove the first four lines of text from your output file and run "python vis.py [name of output file]" on the command line to see an illustration of the path.

```
<u>Test.py source code:</u>
import heapq
import math
import sys
# List of all possible directions the robot could move in the grid
directions = [(0, 1), (1, 1), (1, 0), (1, -1), (0, -1), (-1, -1), (-1, 0), (-1, 1)]
# Used to make the origin as the top-left corner initially to make it easier for us to work on the
grid
def flipGrid(grid):
  return grid[::-1]
# Check if input grid is valid as per the instructions of input format
def checkInputGrid(start, goal, grid):
  # Number of rows = 30
  # Number of cols = 50
  # Start position depicted by '2' in grid
  # End position depicted by '5' in grid
  if len(grid) == 30 and len(grid[0]) == 50 and grid[goal[1]][goal[0]] == 5 and
grid[start[1]][start[0]] == 2:
     return True
```

return False

```
# Read input file returns start, goal, and grid
def read input(file path):
  with open(file path, 'r') as file:
     # Read start and goal positions
     start goal line = file.readline().strip()
     start i, start j, goal i, goal j = map(int, start goal line.split())
     start = (start i, start j)
     goal = (goal i, goal i)
     # Read the workspace grid
     grid = []
     for in range(30): # 30 rows as specified
       row = list(map(int, file.readline().strip().split()))
       grid.append(row)
  grid = flipGrid(grid)
  res = checkInputGrid(start, goal, grid)
  if not res:
     print('Inavlid input file. Please check specifications.')
     sys.exit(0)
  return start, goal, grid, res
def heuristicFunction(currPos, goalPos):
  # Euclidean distance as a heuristic, ensuring non-negative values
  return math.sqrt((goalPos[0] - currPos[0]) ** 2 + (goalPos[1] - currPos[1]) ** 2)
# Create a list of all valid neighbours
def get neighbors(position, grid):
  neighbors = []
```

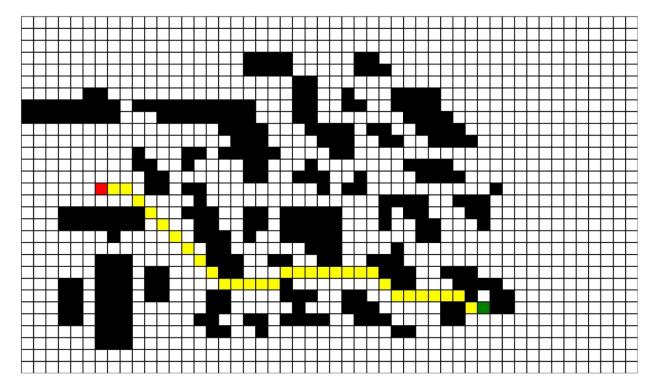
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# Use directions list to compute all possible neighbour indices in grid
  for direction in directions:
     new i = position[0] + direction[0]
     new j = position[1] + direction[1]
     # Do not include neighbours if out of bounds or black cells
     if 0 \le \text{new i} \le \text{len}(\text{grid}[0]) and 0 \le \text{new j} \le \text{len}(\text{grid}) and \text{grid}[\text{new i}][\text{new i}] != 1:
        neighbors.append((new i, new j))
  return neighbors
# Calculate stepcost = angular + distance cost
def stepCost(k, start, currPos, nextPos):
  direction = (nextPos[0] - currPos[0], nextPos[1] - currPos[1])
  # Ensure distCost calculation only involves real numbers
  distCost = math.sqrt(direction[0] ** 2 + direction[1] ** 2)
  if distCost == 1.0 or distCost == math.sqrt(2):
     # Handle angleCost calculation
     if currPos == start:
        angleCost = 0 # Angle cost is 0 for start as mentioned in question
     else:
        # Calculate difftheta
        diffTheta = abs(math.atan2(direction[1], direction[0]) * 180 / math.pi)
        # Adjust difftheta if more than 180
       if diffTheta > 180:
          diffTheta = 360 - diffTheta
        angleCost = k * (diffTheta / 180)
     return distCost + angleCost
  else:
     print(direction)
     print(distCost)
```

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print('Invalid distance cost while calculating step cost for neighbour.')
     sys.exit(0)
# Run the A* search algorithm
def a star search(start, goal, grid, k):
  # Open list is the Frontier (Sorted based on f(n) values of nodes)
  open list = []
  heapq.heappush(open list, (heuristicFunction(start, goal), start))
  # Came from keeps track of the parent node
  came from = {start: None}
  # g score / f score dictionaries keep track of the scores
  g score = \{start: 0\}
  f score = {start: heuristicFunction(start, goal)}
  # While frontier not empty
  while open list:
     f score curr, current = heapq.heappop(open list)
     # If goal found in frontier return solution
     if current == goal:
       path = []
       while current:
          path.append(current)
          current = came from[current]
       # Reverse path (Start goes to begining and Goal to end)
       path.reverse()
       return path, f score, len(came from)
     # If goal not found as current node explore neighbours to add to tree
     for neighbor in get neighbors(current, grid):
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step cost = stepCost(k, start, current, neighbor)
       # Update g score of neighbours
       tentative g score = g score [current] + step cost
       tentative f score = tentative g score + heuristicFunction(neighbor, goal)
       # If lower f score found for neighbor update dictioaries
       if neighbor not in g score or tentative f score < f score[neighbor]:
          came from[neighbor] = current
          g score[neighbor] = tentative g score
          f score[neighbor] = tentative f score
         # Push new values to heap for resorting for next iteration
         if isinstance(f score[neighbor], float) and not isinstance(f score[neighbor], complex):
            heapq.heappush(open list, (f score[neighbor], neighbor))
  # If no solution found
  return None, float('inf'), len(came from)
def write output(file path, path, f score, total nodes, grid):
  with open(file path, 'w') as file:
     # Depth of goal node = len(path) [LINE 1]
     file.write(f"{len(path)}\n")
     # len(came from) = total nodes [LINE 2]
     file.write(f"{total nodes}\n")
     if path:
       moves = []
       f score print list = [f score[path[0]]]
       for i in range(1, len(path)):
         # Print index of appropriate direction
         di = path[i][1] - path[i - 1][1]
         di = path[i][0] - path[i - 1][0]
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moves.append(directions.index((di, dj)))
          # Print f score of each node in solution path
          f score print list.append(f score[path[i]])
          # Number of f scores should be one more than number of directions
          if len(f score print list) != (len(moves) + 1):
            print('Error in output format creation.')
            sys.exit(0)
          # Replace path grid indices with '4' instead of '0'
          if i < len(path) - 1:
             grid[path[i][1]][path[i][0]] = 4
       file.write(" ".join(map(str, moves)) + "\n") # [LINE 3]
       file.write(" ".join(map(str, f score print list)) + "\n") # [LINE 4]
     # Updated grid
     grid = flipGrid(grid)
     for i in range(len(grid)):
       line = " ".join(map(str, grid[i]))
       file.write(line + "\n") # [LINE 5-34]
# MAIN
file path = input('Enter filepath / filename: ')
start, goal, grid, res = read input(file path)
k = int(input('Enter value for k: '))
path, f_score, total_nodes = a star search(start, goal, grid, k)
write output('Output1.txt', path, f score, total nodes, grid)
Output of Input1.txt [k = 0]:
32
194
```

$0\ 0\ 7\ 7\ 7\ 7\ 7\ 7\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 7\ 7\ 0\ 0\ 0\ 0\ 0\ 7\ 0$

32.57299494980466 32.622776601683796 32.67572330035593 32.82509590207858 32.988682805403634 33.1684647227918 33.366774513857266 33.586369156128 33.830516434096076 34.103098247786185 34.40873160871375 34.413459741226546 34.41868167352756 34.4244787752596 34.43095126760845 35.006742657457565 35.02498060213621 35.045743124634214 35.06958612548419 35.097238938210836 35.12967631234924 35.16822857026841 35.214755041863 35.388346874966274 35.627416997969526 35.63911171640227 35.65536869969684 35.67945481172171 35.71862684627243 35.792417163603844 35.97056274847714 35.97056274847714 $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 4\ 4\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 4\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ 

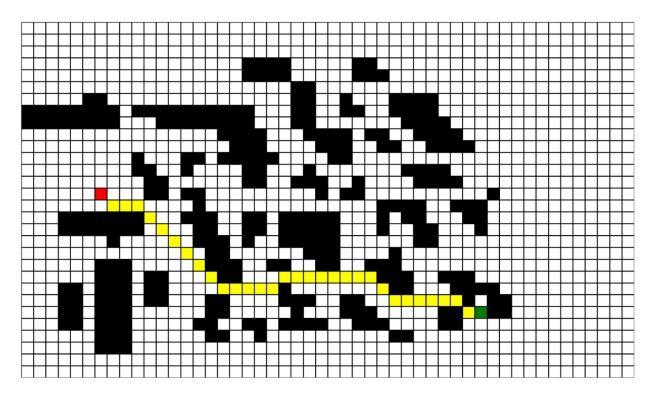
Output of Input1.txt [k = 2]:

32

282

700777777700001000000770000070

 $32.57299494980466\ 32.73513308910474\ 32.778666463751044\ 32.82509590207858\\ 33.488682805403634\ 34.1684647227918\ 34.866774513857266\ 35.586369156128\\ 36.330516434096076\ 37.103098247786185\ 37.90873160871375\ 37.913459741226546\\ 37.91868167352756\ 37.9244787752596\ 37.93095126760845\ 39.006742657457565\\ 39.02498060213621\ 39.045743124634214\ 39.06958612548419\ 39.097238938210836\\ 39.12967631234924\ 39.16822857026841\ 39.214755041863\ 39.888346874966274$

40.627416997969526 40.63911171640227 40.65536869969684 40.67945481172171 40.71862684627243 40.792417163603844 41.47056274847714 41.47056274847714 $0\ 0\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 4\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 4\ 4\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 4\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 4\ 4\ 4\ 4\ 4\ 4\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 4\ 5\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ 

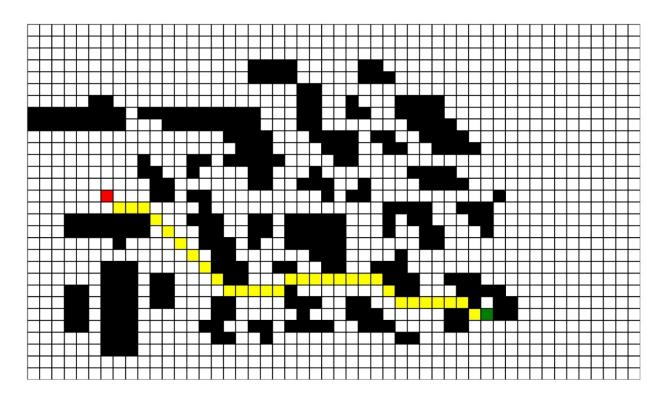
Output for Input1.txt [k = 4]:

32

326

 $7\ 0\ 0\ 7\ 7\ 7\ 7\ 7\ 7\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 7\ 7\ 0\ 0\ 0\ 0\ 0\ 7\ 0$

 $0\ 0\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 4\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 4\ 4\ 4\ 4\ 4\ 4\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\,0\,0\,1\,1\,0\,1\,1\,1\,0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,1\,1\,0\,0\,0\,0\,0\,1\,1\,4\,5\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0$



Output of Input2.txt [k = 0]:

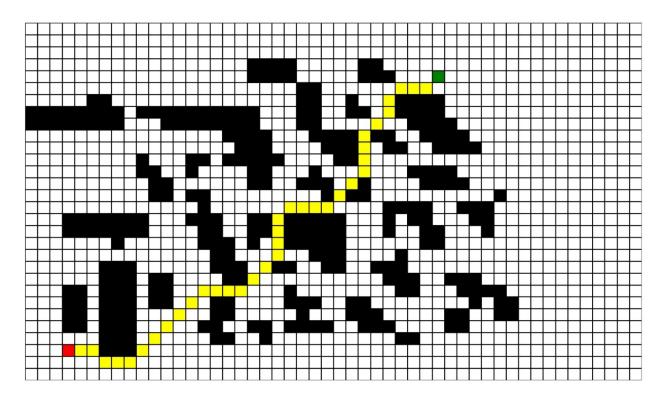
38

380

$0\ 0\ 7\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 2\ 2\ 2\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 2\ 2\ 2\ 1\ 1\ 2\ 1\ 0\ 0\ 1$

 $37.8021163428716\ 38.013511046643494\ 38.235341863986875\ 39.538997298749976$ $39.797825588281356\ 40.06966046470001\ 40.069967401935514\ 40.07030161279838$ $40.07066690098348\ 40.071067811865476\ 40.071509822506016\ 40.07199959321647$ $40.35533905932738\ 40.65833174289156\ 40.98268409419626\ 40.98527659649403$ $40.98821368682716\ 40.991568865010166\ 41.24710879827376\ 41.52691193458119$ $41.83394163668508\ 41.83516978220376\ 42.112698372208094\ 42.42241793342256$ $42.76901958965595\ 42.77681122334285\ 42.78653056184162\ 42.798989873223334$ $43.01853433051622\ 43.2842712474619\ 43.60923954912999\ 43.616327673029275$ $43.62741699796952\ 44.0995529529691\ 44.20390822051099\ 44.2776985378424$ 44.4558441227157

 $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 4\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 4\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 4\ 4\ 4\ 4\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 4\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 4\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$



Output of Input2.txt [k = 2]:

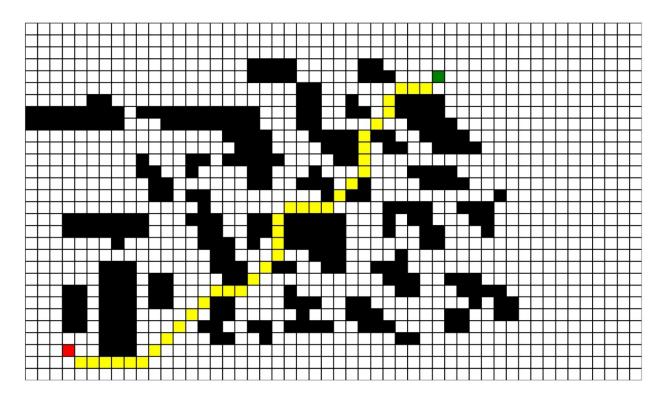
38

599

7000001111111100111222100011112221121001

37.8021163428716 39.05727401181051 39.29239139154464 39.538997298749976 39.797825588281356 40.06966046470001 40.35533905932737 40.85533905932738 41.35533905932738 41.85533905932738 42.85533905932738 42.85533905932738 43.35533905932738 43.65833174289156 43.98268409419626 44.48527659649403 44.98821368682716 45.491568865010166 46.74710879827376 48.02691193458119 49.33394163668508 49.83516978220376 50.112698372208094 50.42241793342256 50.76901958965595 51.27681122334284 51.786530561841616 52.29898987322333 53.51853433051622 54.784271247461895 56.10923954912998 56.61632767302927 57.12741699796951 58.59955295296909 59.20390822051098 59.2776985378424 59.455844122715696 59.955844122715696

 $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 4\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 4\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 4\ 4\ 4\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 4\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 4\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$



Output for Input2.txt [k = 4]:

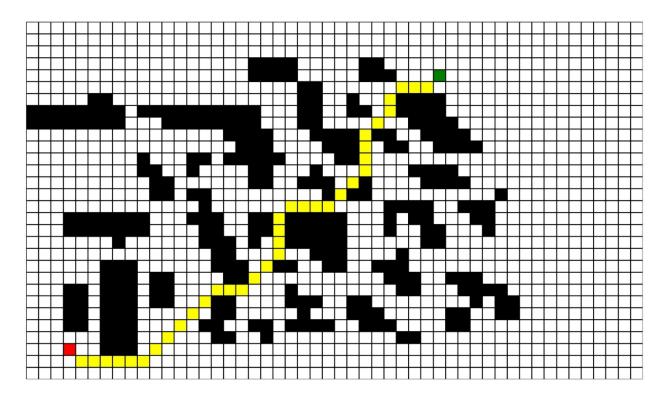
38

780

7000001111111100111222100011112221121001

 $37.8021163428716 \ 39.05727401181051 \ 39.29239139154464 \ 39.538997298749976 \ 39.797825588281356 \ 40.06966046470001 \ 40.35533905932737 \ 41.35533905932738 \ 42.35533905932738 \ 43.35533905932738 \ 44.35533905932738 \ 45.35533905932738 \ 46.35533905932738 \ 46.65833174289156 \ 46.98268409419626 \ 47.98527659649403 \ 48.98821368682716 \ 49.991568865010166 \ 52.24710879827375 \ 54.526911934581186 \ 56.83394163668508 \ 57.83516978220375 \ 58.11269837220809 \ 58.42241793342255 \ 58.76901958965594 \ 59.77681122334283 \ 60.78653056184161 \ 61.79898987322332 \ 64.0185343305162 \ 66.28427124746189 \ 68.60923954912997 \ 69.61632767302926 \ 70.6274169979695 \ 73.09955295296908 \ 74.20390822051098 \ 74.27769853784238 \ 74.45584412271569 \ 75.45584412271569$

 $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 4\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 4\ 4\ 4\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 4\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 4\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$



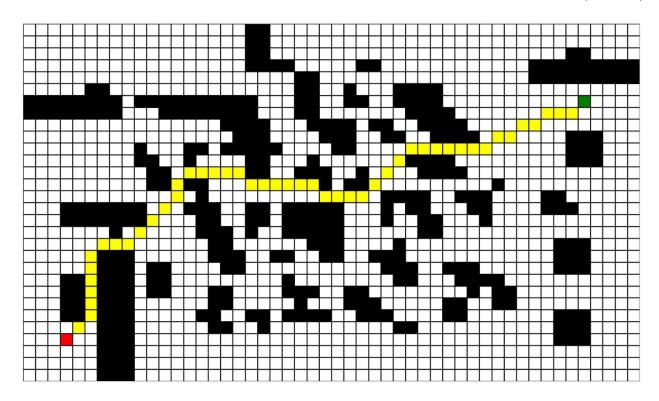
Output of Input3.txt [k=0]:

49

395

 $46.51881339845203 \ 46.602707673153105 \ 46.69185152366881 \ 47.29105474894765 \\ 47.90974558182222 \ 48.548445851333845 \ 49.207667325580374 \ 49.88790881437237 \\ 50.047052213325614 \ 50.092358377461835 \ 50.13994136467273 \ 50.299914698929804 \\ 50.47161725826807 \ 50.656292164376524 \ 50.85534862931096 \ 51.633750523477666 \\ 51.87134969118418 \ 51.88901530667865 \ 51.90782558054147 \ 51.92789428890646 \\ 51.94935062553747 \ 52.620573423320565 \ 52.65374609703038 \ 52.689432032852196 \\ 52.72792206135786 \ 52.76955262170047 \ 52.81471482258825 \ 53.614340677975186 \\ 53.68279485226897 \ 53.75766375181925 \ 53.83985122732316 \ 53.94112549695428 \\ 54.0585702391122 \ 54.196164611692126 \ 54.359209651784354 \ 54.400460381958766 \\ 54.44810051389684 \ 54.503689783942946 \ 54.56931948749233 \ 54.64784767501942 \\ 54.74326178322248 \ 54.82897654146032 \ 54.921407368095785 \ 55.012581805104006 \\ 55.099552952969084 \ 55.203908220510975 \ 55.27769853784238 \ 55.45584412271569 \\ 55.45584412271569$

 $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 4\ 4\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 4\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 4\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 4\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 4\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$



Output for Input3.txt [k=2]:

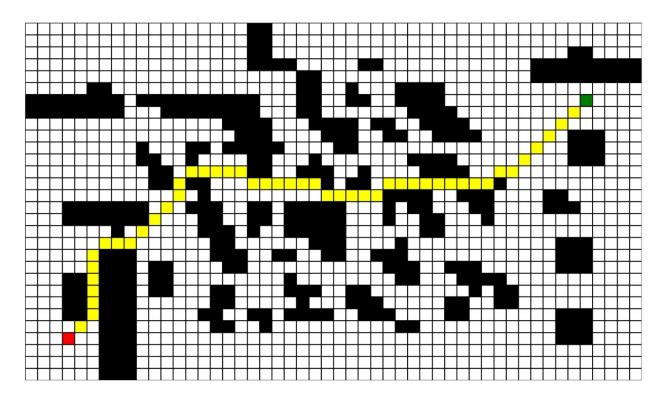
49

571

112222210011112100007000070000100000000101111111

 $46.51881339845203 \ 46.602707673153105 \ 47.19185152366881 \ 48.79105474894765 \\ 50.40974558182222 \ 52.048445851333845 \ 53.707667325580374 \ 55.38790881437237 \\ 56.047052213325614 \ 56.092358377461835 \ 56.13994136467273 \ 56.7999146989298 \\ 57.47161725826807 \ 58.156292164376524 \ 58.85534862931096 \ 60.633750523477666 \\ 61.37134969118418 \ 61.38901530667865 \ 61.40782558054147 \ 61.42789428890646 \\ 61.44935062553748 \ 62.620573423320565 \ 62.65374609703038 \ 62.689432032852196 \\ 62.72792206135786 \ 62.76955262170047 \ 62.81471482258825 \ 64.11434067797518 \\ 64.18279485226897 \ 64.25766375181925 \ 64.33985122732315 \ 64.43042985178688 \\ 65.02059838267702 \ 65.1092945433509 \ 65.20882502860258 \ 65.32117224633744 \\ 65.44879317555385 \ 65.59475399650934 \ 65.76290480183775 \ 65.95810343709542 \\ 66.1864949988387 \ 66.69010720577002 \ 66.9558441227157 \ 67.4558441227157 \\ 67.9558441227157 \ 68.4558441227157 \ 68.95584412271569 \ 69.45584412271569 \\ 69.95584412271569$

 $0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 4\ 4\ 4\ 4\ 4\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 4\ 4\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 4\ 4\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 4\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 4\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$



Output for Input3.txt [k=4]:

49

678

112222210011112100007000070000100000000101111111

46.51881339845203 46.602707673153105 47.69185152366881 50.29105474894765 52.90974558182222 55.548445851333845 58.207667325580374 60.88790881437237 62.047052213325614 62.092358377461835 62.13994136467274 63.299914698929804 64.47161725826807 65.65629216437652 66.85534862931095 69.63375052347766 70.87134969118418 70.88901530667864 70.90782558054147 70.92789428890646 70.94935062553746 72.62057342332056 72.65374609703036 72.6894320328522 72.72792206135784 72.76955262170047 72.81471482258824 74.61434067797518 74.68279485226896 74.75766375181925 74.83985122732315 74.93042985178688 76.02059838267701 76.10929454335088 76.20882502860256 76.32117224633744 76.44879317555385 76.59475399650934 76.76290480183773 76.9581034370954 77.18649499883868 78.19010720577 78.45584412271569 79.45584412271569 80.45584412271567 81.45584412271567

 $0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 4\ 4\ 4\ 4\ 4\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 4\ 4\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 4\ 4\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 4\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 4\ 4\ 4\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 0\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 1\ 1\ 4\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$ $0\ 0\ 0\ 0\ 4\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0$

