NPGR026 Practical Exercise Assignment 1
Spectral Multiplication & Conversion to Colour Space

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March 27th, 2019

Goal:

Implement the functions needed to

1. calculate light-surface interactions in a spectral renderer, to
2. convert the result to a colour value and to
3. compare the performance of two approaches to representing/sampling spectra

Required results:

A working computer program that allows the user to read the spectral luminaire and reflectance data provided in appendix 1 of this exercise assignment, and perform a point-wise multiplication of a selected pair of luminaire and reflectance data using two techniques: random wavelength evaluation (both genuine single wavelength, and hero WL sampling), and fixed wavelength buckets. The result spectra from these multiplications then have to be converted to colour space, and displayed as RGB values. A comparison has to be made of the accuracy of these approaches. The techniques that the user can select should be:

1. Random wavelength sampling
   User input: number n of samples, with n usually being rather large (100, or more)
2. M samples that are equidistant across the visible range
   User input: number m of samples, with m usually in the range 8 to 45

The program should allow comparisons between using random wavelength sampling and fixed wavelength buckets, and the effect that the number of samples has on accuracy. To this effect, it should at least be capable of displaying the results of the three techniques side by side (random / hero / regular). Optionally, it could be capable of displaying the results in sequence for increasing sample numbers.

The computer program does *not* have to have a fancy user interface. Actually, a command-line program is completely sufficient, although you are of course free to do this in a GUI environment if you feel like it. Also, it does not necessarily have to offer the option to actually read reflectance and luminaire data from a file; you can hard-code the provided data into the program if you wish, and merely offer the user the option of selecting two of the in-built spectra for multiplication and conversion.

The focus of this exercise are three functions within this program:

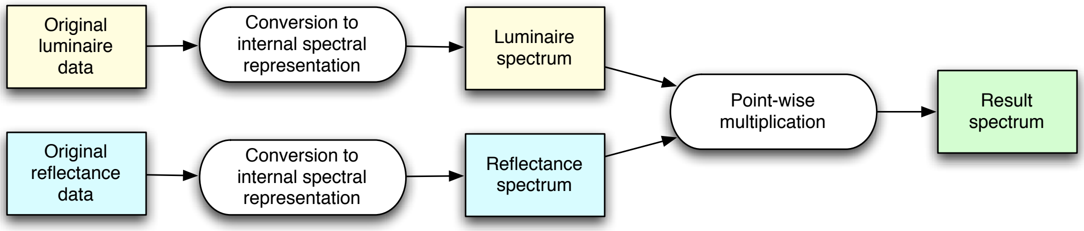
1. the conversion of input data to an internal representation of spectral data
2. the spectral multiplication routine, and the
3. conversion of spectral values to colour space.

Approximate expected results for the program are listed in appendix 2 for your reference, so you can verify that your program works correctly prior to handing it in.

Workflow of the program:

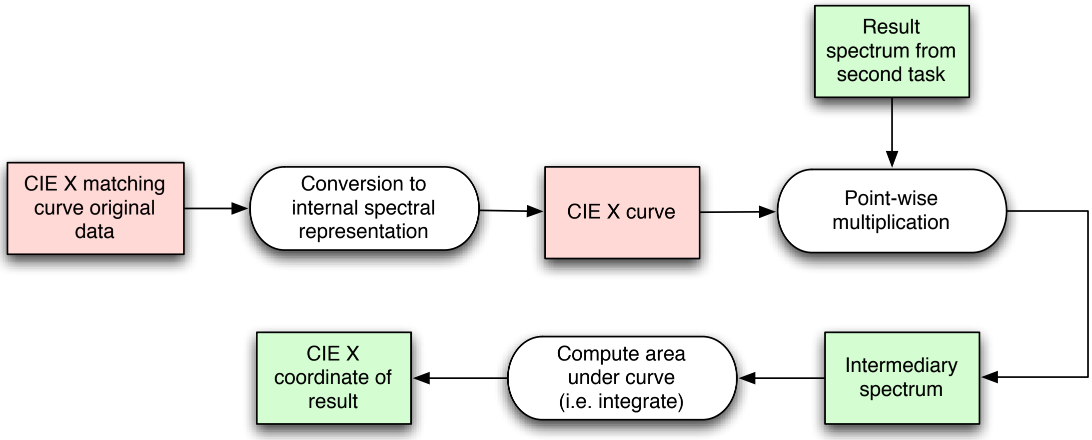
The following diagrams are intended to clarify the internal workings of the required program.

Workflow for the second sub-task — spectral multiplication:



Workflow for the third sub-task - conversion of the result to RGB:

Step 1 of *Spectrum* → *RGB*: the following has to be done for each of the three channels X, Y and Z to obtain the CIE XYZ colour space co-ordinates of the result:



Step 2 of *Spectrum* → *RGB*: the obtained XYZ triplet has then to be multiplied with the conversion matrix that is given in part D of the appendix.

Environment for doing this:

Any programming language of your choice, that can be run or compiled on the university computers. In the opinion of the lecturer, good candidates for such a simple test environment that just has to handle a few numerical operations would be the Python language, or perhaps Java. But within reason, do feel free to use any other system that you are familiar with instead.

Assignment examination details:

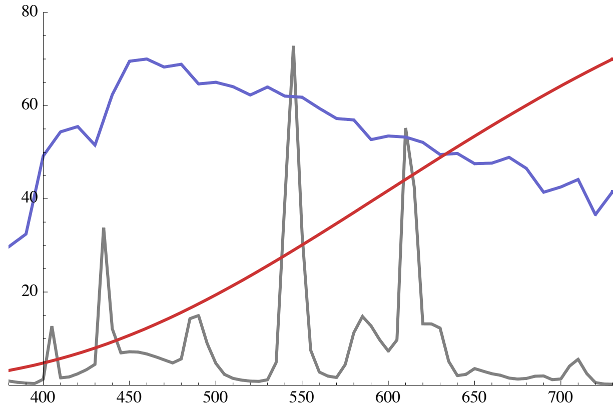
You are expected to mail the source code of your program to wilkie@cgg.mff.cuni.cz until April 25th, 2019; receipt of the source will be acknowledged via e-mail. Before handing in your solution, do make sure to compare the results your program computes with the figures listed in appendix 2! The functionality of this source code will later be tested in your presence on a university computer, and you are expected to exactly know the inner workings of your code. There will be a short practical examination and discussion about your results on a one-to-one basis.

Appendix 1 - Input Data for the exercise

This section contains spectral data for lights, reflectances, and the CIE matching functions (parts A, B and C), as well as the transformation matrix needed for conversion from CIE XYZ to sRGB (part D).

The data in the luminaire, reflectance and CIE XYZ matching function sections is provided in the curly brace notation typical for Mathematica® notebook files, and in a fairly small font size to save space. Each curly brace contains a pair of numbers - the first one is the wavelength of the sample in nanometers, and the second one the intensity (for luminaires, in arbitrary scale) or reflectance value (for surfaces, in values between 0 and 1). You can copy&paste the spectral data from this file, or, if you are having difficulties with this, the lecturer can also provide this data in an ASCII text file.

Part A - Luminaire data



Reference plot of the provided illuminants
(not to relative scale): Illuminant A (red), D65 (blue), F11 (grey)

CIE Illuminant A (incandescent light, i.e. a conventional light bulb)

```
{380, 0.045345}, {385, 0.050435}, {390, 0.055941}, {395, 0.061771}, {400, 0.068064}, {405, 0.074727},
{410, 0.081806}, {415, 0.089256}, {420, 0.097122}, {425, 0.105451}, {430, 0.114150}, {435, 0.123265},
{440, 0.132797}, {445, 0.142745}, {450, 0.153109}, {455, 0.163844}, {460, 0.174949}, {465, 0.186470},
{470, 0.198362}, {475, 0.210624}, {480, 0.223209}, {485, 0.236165}, {490, 0.249445}, {495, 0.263048},
{500, 0.276976}, {505, 0.291181}, {510, 0.305664}, {515, 0.320424}, {520, 0.335462}, {525, 0.350685},
{530, 0.366139}, {535, 0.381825}, {540, 0.397696}, {545, 0.413705}, {550, 0.429900}, {555, 0.446234},
{560, 0.462706}, {565, 0.479271}, {570, 0.495928}, {575, 0.512678}, {580, 0.529521}, {585, 0.546363},
{590, 0.563252}, {595, 0.580187}, {600, 0.597076}, {605, 0.614011}, {610, 0.630900}, {615, 0.647742},
{620, 0.664538}, {625, 0.681288}, {630, 0.697946}, {635, 0.714510}, {640, 0.730983}, {645, 0.747363},
{650, 0.763604}, {655, 0.779706}, {660, 0.795669}, {665, 0.811494}, {670, 0.827179}, {675, 0.842680},
{680, 0.857996}, {685, 0.873126}, {690, 0.888071}, {695, 0.902832}, {700, 0.917361}, {705, 0.931705},
{710, 0.945817}, {715, 0.959698}, {720, 0.973348}, {725, 0.986813}, {730, 1.000000}
```

CIE Illuminant D65 (daylight)

```
{380, 0.424448217}, {385, 0.443972835}, {390, 0.463497453}, {395, 0.583191851}, {400, 0.702886248},
{405, 0.739813243}, {410, 0.776740238}, {415, 0.784804754}, {420, 0.792869270}, {425, 0.764431239},
{430, 0.735993209}, {435, 0.813242784}, {440, 0.890492360}, {445, 0.941850594}, {450, 0.993208829},
{455, 0.996604414}, {460, 1.000000000}, {465, 0.987691002}, {470, 0.975382003}, {475, 0.979626486},
{480, 0.983870968}, {485, 0.953735144}, {490, 0.923599321}, {495, 0.926146010}, {500, 0.928692699},
{505, 0.921901528}, {510, 0.915110357}, {515, 0.902376910}, {520, 0.889643463}, {525, 0.901952462},
{530, 0.914261460}, {535, 0.900254669}, {540, 0.886247878}, {545, 0.884550085}, {550, 0.882852292},
{555, 0.865874363}, {560, 0.848896435}, {565, 0.833191851}, {570, 0.817487267}, {575, 0.815365025},
{580, 0.813242784}, {585, 0.783106961}, {590, 0.752971138}, {595, 0.758488964}, {600, 0.764006791},
{605, 0.762308998}, {610, 0.760611205}, {615, 0.752546689}, {620, 0.744482173}, {625, 0.725806452},
{630, 0.707130730}, {635, 0.708828523}, {640, 0.710526316}, {645, 0.694821732}, {650, 0.679117148},
{655, 0.679966044}, {660, 0.680814941}, {665, 0.689728353}, {670, 0.698641766}, {675, 0.681663837},
{680, 0.664685908}, {685, 0.628183362}, {690, 0.591680815}, {695, 0.599745331}, {700, 0.607809847},
{705, 0.619269949}, {710, 0.630730051}, {715, 0.576825127}, {720, 0.522920204}, {725, 0.558149406},
{730, 0.593378608}, {735, 0.593378608}, {740, 0.593378608}, {745, 0.593378608}, {750, 0.593378608},
{755, 0.593378608}, {760, 0.593378608}, {765, 0.593378608}, {770, 0.593378608}, {775, 0.593378608},
{780, 0.593378608}
```

F11 (a common type of fluorescent light)

```
{380, 0.91}, {385, 0.63}, {390, 0.46}, {395, 0.37}, {400, 1.29}, {405, 12.68}, {410, 1.59}, {415, 1.79},
{420, 2.46}, {425, 3.33}, {430, 4.49}, {435, 33.94}, {440, 12.13}, {445, 6.95}, {450, 7.19}, {455, 7.12},
{460, 6.72}, {465, 6.13}, {470, 5.46}, {475, 4.79}, {480, 5.66}, {485, 14.29}, {490, 14.96}, {495, 8.97},
{500, 4.72}, {505, 2.33}, {510, 1.47}, {515, 1.1}, {520, 0.89}, {525, 0.83}, {530, 1.18}, {535, 4.9},
{540, 39.59}, {545, 72.84}, {550, 32.61}, {555, 7.52}, {560, 2.83}, {565, 1.96}, {570, 1.67}, {575, 4.43},
{580, 11.28}, {585, 14.76}, {590, 12.73}, {595, 9.74}, {600, 7.33}, {605, 9.72}, {610, 55.27}, {615, 42.58},
{620, 13.18}, {625, 13.16}, {630, 12.26}, {635, 5.11}, {640, 2.07}, {645, 2.34}, {650, 3.58}, {655, 3.01},
{660, 2.48}, {665, 2.14}, {670, 1.54}, {675, 1.33}, {680, 1.46}, {685, 1.94}, {690, 2}, {695, 1.2}, {700, 1.35},
{705, 4.1}, {710, 5.58}, {715, 2.51}, {720, 0.57}, {725, 0.27}, {730, 0.23}, {735, 0.21}, {740, 0.24},
{745, 0.24}, {750, 0.2}, {755, 0.24}, {760, 0.32}, {765, 0.26}, {770, 0.16}, {775, 0.12}, {780, 0.09}
```

Part B- Reflectance Values

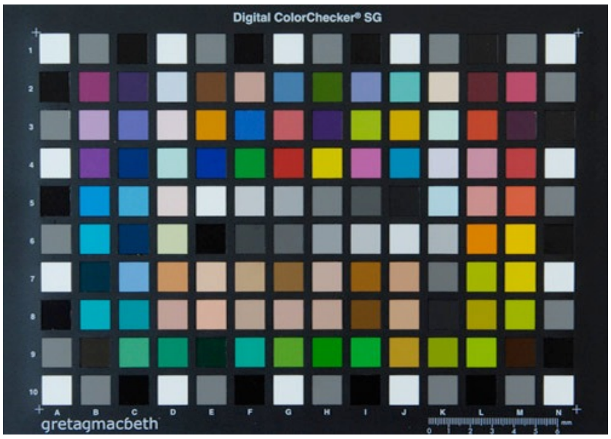
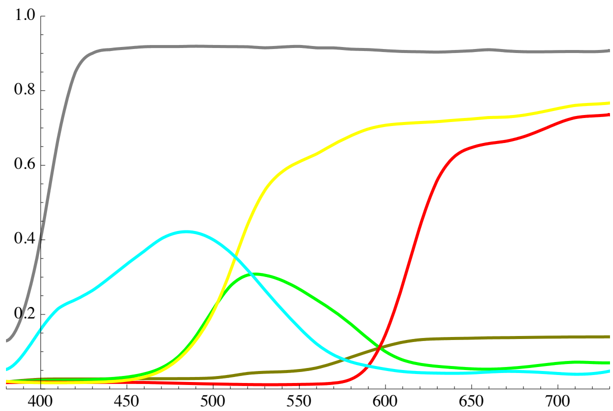


Photo of an XRite Color Checker® SG
The columns are labelled from left to right with the letters A to N,
while, starting from the top, the rows are labelled 1 to 10.



Reference plot of the provided XRite Colour Checker® reflectance data
E2 (olive), F4 (green), G4 (red), H4 (yellow), J4 (cyan), A1 (grey)

XRite Colour Checker SG Patch E2 (dark skin)

```
{380, 0.0191}, {390, 0.0231}, {400, 0.0261}, {410, 0.0268}, {420, 0.0269}, {430, 0.0269}, {440, 0.0270},
{450, 0.0272}, {460, 0.0275}, {470, 0.0275}, {480, 0.0277}, {490, 0.0281}, {500, 0.0296}, {510, 0.0347},
{520, 0.0417}, {530, 0.0448}, {540, 0.0462}, {550, 0.0494}, {560, 0.0565}, {570, 0.0693}, {580, 0.0852},
{590, 0.1006}, {600, 0.1144}, {610, 0.1255}, {620, 0.1320}, {630, 0.1344}, {640, 0.1354}, {650, 0.1362},
{660, 0.1372}, {670, 0.1376}, {680, 0.1381}, {690, 0.1386}, {700, 0.1391}, {710, 0.1395}, {720, 0.1396},
{730, 0.1399}
```

XRite Colour Checker SG Patch F4 (green)

```
{380, 0.0194}, {390, 0.0211}, {400, 0.0225}, {410, 0.0232}, {420, 0.0238}, {430, 0.0252}, {440, 0.0276},
{450, 0.0320}, {460, 0.0405}, {470, 0.0566}, {480, 0.0876}, {490, 0.1415}, {500, 0.2134}, {510, 0.2762},
{520, 0.3054}, {530, 0.3051}, {540, 0.2909}, {550, 0.2678}, {560, 0.2390}, {570, 0.2090}, {580, 0.1735},
{590, 0.1346}, {600, 0.1001}, {610, 0.0778}, {620, 0.0665}, {630, 0.0607}, {640, 0.0570}, {650, 0.0540},
{660, 0.0531}, {670, 0.0549}, {680, 0.0587}, {690, 0.0637}, {700, 0.0689}, {710, 0.0719}, {720, 0.0708},
{730, 0.0701}
```

XRite Colour Checker SG Patch G4 (red)

```
{380, 0.0170}, {390, 0.0180}, {400, 0.0186}, {410, 0.0182}, {420, 0.0178}, {430, 0.0177}, {440, 0.0176},
{450, 0.0176}, {460, 0.0174}, {470, 0.0164}, {480, 0.0153}, {490, 0.0143}, {500, 0.0134}, {510, 0.0126},
{520, 0.0119}, {530, 0.0115}, {540, 0.0116}, {550, 0.0122}, {560, 0.0132}, {570, 0.0159}, {580, 0.0263},
{590, 0.0621}, {600, 0.1462}, {610, 0.2796}, {620, 0.4361}, {630, 0.5597}, {640, 0.6234}, {650, 0.6474},
{660, 0.6583}, {670, 0.6646}, {680, 0.6763}, {690, 0.6935}, {700, 0.7129}, {710, 0.7276}, {720, 0.7322},
{730, 0.7357}
```

XRite Colour Checker SG Patch H4 (yellow)

```
{380, 0.0205}, {390, 0.0179}, {400, 0.0169}, {410, 0.0167}, {420, 0.0170}, {430, 0.0179}, {440, 0.0197},
{450, 0.0236}, {460, 0.0321}, {470, 0.0490}, {480, 0.0805}, {490, 0.1309}, {500, 0.2059}, {510, 0.3155},
{520, 0.4407}, {530, 0.5329}, {540, 0.5824}, {550, 0.6094}, {560, 0.6303}, {570, 0.6565}, {580, 0.6792},
{590, 0.6973}, {600, 0.7072}, {610, 0.7117}, {620, 0.7145}, {630, 0.7170}, {640, 0.7209}, {650, 0.7240},
{660, 0.7280}, {670, 0.7293}, {680, 0.7342}, {690, 0.7424}, {700, 0.7521}, {710, 0.7605}, {720, 0.7637},
{730, 0.7669}
```

XRite Colour Checker SG Patch J4 (cyan)

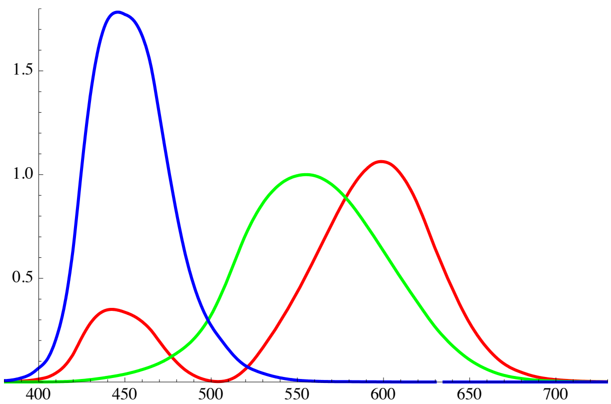
```
{380, 0.0523}, {390, 0.0932}, {400, 0.1601}, {410, 0.2149}, {420, 0.2388}, {430, 0.2642}, {440, 0.2983},
{450, 0.3351}, {460, 0.3695}, {470, 0.4030}, {480, 0.4200}, {490, 0.4191}, {500, 0.4004}, {510, 0.3663},
{520, 0.3196}, {530, 0.2653}, {540, 0.2128}, {550, 0.1640}, {560, 0.1211}, {570, 0.0911}, {580, 0.0726},
{590, 0.0615}, {600, 0.0531}, {610, 0.0468}, {620, 0.0438}, {630, 0.0425}, {640, 0.0420}, {650, 0.0430},
{660, 0.0455}, {670, 0.0472}, {680, 0.0465}, {690, 0.0445}, {700, 0.0416}, {710, 0.0395}, {720, 0.0412},
{730, 0.0479}
```

XRite Colour Checker SG Patch A1 (white)

```
{380, 0.1291}, {390, 0.2090}, {400, 0.4016}, {410, 0.6692}, {420, 0.8488}, {430, 0.9002}, {440, 0.9100},
{450, 0.9142}, {460, 0.9178}, {470, 0.9184}, {480, 0.9184}, {490, 0.9192}, {500, 0.9186}, {510, 0.9181},
{520, 0.9177}, {530, 0.9150}, {540, 0.9171}, {550, 0.9188}, {560, 0.9149}, {570, 0.9147}, {580, 0.9113},
{590, 0.9101}, {600, 0.9074}, {610, 0.9052}, {620, 0.9044}, {630, 0.9035}, {640, 0.9050}, {650, 0.9070},
{660, 0.9099}, {670, 0.9066}, {680, 0.9045}, {690, 0.9043}, {700, 0.9047}, {710, 0.9051}, {720, 0.9046},
{730, 0.9074}
```

Part C - CIE Matching Function Data

As discussed in the course, this data is needed for the conversion of spectral data to colour space. It can be readily found on the web, but for convenience‘s sake, it is also repeated here.



Reference plot of the CIE XYZ matching functions

CIE X

```
{380, 0.0014}, {385, 0.0022}, {390, 0.0042}, {395, 0.0077}, {400, 0.0143}, {405, 0.0232}, {410, 0.0435},
{415, 0.0776}, {420, 0.1344}, {425, 0.2148}, {430, 0.2839}, {435, 0.3285}, {440, 0.3483}, {445, 0.3481},
{450, 0.3362}, {455, 0.3187}, {460, 0.2908}, {465, 0.2511}, {470, 0.1954}, {475, 0.1421}, {480, 0.0956},
{485, 0.0580}, {490, 0.0320}, {495, 0.0147}, {500, 0.0049}, {505, 0.0024}, {510, 0.0093}, {515, 0.0291},
{520, 0.0633}, {525, 0.1096}, {530, 0.1655}, {535, 0.2257}, {540, 0.2904}, {545, 0.3597}, {550, 0.4334},
{555, 0.5121}, {560, 0.5945}, {565, 0.6784}, {570, 0.7621}, {575, 0.8425}, {580, 0.9163}, {585, 0.9786},
{590, 1.0263}, {595, 1.0567}, {600, 1.0622}, {605, 1.0456}, {610, 1.0026}, {615, 0.9384}, {620, 0.8544},
{625, 0.7514}, {630, 0.6424}, {635, 0.5419}, {640, 0.4479}, {645, 0.3608}, {650, 0.2835}, {655, 0.2187},
{660, 0.1649}, {665, 0.1212}, {670, 0.0874}, {675, 0.0636}, {680, 0.0468}, {685, 0.0329}, {690, 0.0227},
{695, 0.0158}, {700, 0.0114}, {705, 0.0081}, {710, 0.0058}, {715, 0.0041}, {720, 0.0029}, {725, 0.0020},
{730, 0.0014}, {735, 0.0010}, {740, 0.0007}, {745, 0.0005}, {750, 0.0003}, {755, 0.0002}, {760, 0.0002},
{765, 0.0001}, {770, 0.0001}, {775, 0.0001}, {780, 0.0000}
```

CIE Y

```
{380, 0.0000}, {385, 0.0001}, {390, 0.0001}, {395, 0.0002}, {400, 0.0004}, {405, 0.0006}, {410, 0.0012},
{415, 0.0022}, {420, 0.0040}, {425, 0.0073}, {430, 0.0116}, {435, 0.0168}, {440, 0.0230}, {445, 0.0298},
{450, 0.0380}, {455, 0.0480}, {460, 0.0600}, {465, 0.0739}, {470, 0.0910}, {475, 0.1126}, {480, 0.1390},
{485, 0.1693}, {490, 0.2080}, {495, 0.2586}, {500, 0.3230}, {505, 0.4073}, {510, 0.5030}, {515, 0.6082},
{520, 0.7100}, {525, 0.7932}, {530, 0.8620}, {535, 0.9149}, {540, 0.9540}, {545, 0.9803}, {550, 0.9950},
{555, 1.0000}, {560, 0.9950}, {565, 0.9786}, {570, 0.9520}, {575, 0.9154}, {580, 0.8700}, {585, 0.8163},
{590, 0.7570}, {595, 0.6949}, {600, 0.6310}, {605, 0.5668}, {610, 0.5030}, {615, 0.4412}, {620, 0.3810},
{625, 0.3210}, {630, 0.2650}, {635, 0.2170}, {640, 0.1750}, {645, 0.1382}, {650, 0.1070}, {655, 0.0816},
{660, 0.0610}, {665, 0.0446}, {670, 0.0320}, {675, 0.0232}, {680, 0.0170}, {685, 0.0119}, {690, 0.0082},
{695, 0.0057}, {700, 0.0041}, {705, 0.0029}, {710, 0.0021}, {715, 0.0015}, {720, 0.0010}, {725, 0.0007},
{730, 0.0005}, {735, 0.0004}, {740, 0.0002}, {745, 0.0002}, {750, 0.0001}, {755, 0.0001}, {760, 0.0001},
{765, 0.0000}, {770, 0.0000}, {775, 0.0000}, {780, 0.0000}
```

CIE Z

```
{380, 0.0065}, {385, 0.0105}, {390, 0.0201}, {395, 0.0362}, {400, 0.0679}, {405, 0.1102}, {410, 0.2074},
{415, 0.3713}, {420, 0.6456}, {425, 1.0391}, {430, 1.3856}, {435, 1.6230}, {440, 1.7471}, {445, 1.7826},
{450, 1.7721}, {455, 1.7441}, {460, 1.6692}, {465, 1.5281}, {470, 1.2876}, {475, 1.0419}, {480, 0.8130},
{485, 0.6162}, {490, 0.4652}, {495, 0.3533}, {500, 0.2720}, {505, 0.2123}, {510, 0.1582}, {515, 0.1117},
{520, 0.0782}, {525, 0.0573}, {530, 0.0422}, {535, 0.0298}, {540, 0.0203}, {545, 0.0134}, {550, 0.0087},
{555, 0.0057}, {560, 0.0039}, {565, 0.0027}, {570, 0.0021}, {575, 0.0018}, {580, 0.0017}, {585, 0.0014},
{590, 0.0011}, {595, 0.0010}, {600, 0.0008}, {605, 0.0006}, {610, 0.0003}, {615, 0.0002}, {620, 0.0002},
{625, 0.0001}, {630, 0.0000}, {635, 0.0000}, {640, 0.0000}, {645, 0.0000}, {650, 0.0000}, {655, 0.0000},
{660, 0.0000}, {665, 0.0000}, {670, 0.0000}, {675, 0.0000}, {680, 0.0000}, {685, 0.0000}, {690, 0.0000},
{695, 0.0000}, {700, 0.0000}, {705, 0.0000}, {710, 0.0000}, {715, 0.0000}, {720, 0.0000}, {725, 0.0000},
{730, 0.0000}, {735, 0.0000}, {740, 0.0000}, {745, 0.0000}, {750, 0.0000}, {755, 0.0000}, {760, 0.0000},
{765, 0.0000}, {770, 0.0000}, {775, 0.0000}, {780, 0.0000}
```

Part D - Transformation matrix for CIE XYZ → sRGB

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.2410 & -1.5374 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0570 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Appendix 2 - Results

The results obtained with your program will in all likelihood ***not*** be identical to the results listed in this appendix. However, a reasonably close match should be possible. The reason for this are the inaccuracies that are introduced by the internal representation your program uses; more samples usually mean higher accuracy, but require more memory and longer execution times.

The largest variance in results is to be expected for Illuminant F11; the spiky nature of this light source makes most practical sampling approaches problematic. The reference results were obtained from direct convolution and integration of a combination of cubic (reflectance) and linear (luminaires, to preserve the spikes in F11) interpolations of the sampled spectra in Mathematica®.

	A	D65	F11
E2	11.99, 2.01, 0.032	13.03, 4.42, 2.02	274.45, 66.78, 13.33
F4	6.33, 11.02, -0.18	2.76, 24.67, 2.61	118.61, 378.68, -0.67
G4	34.51, -1.38, -0.22	35.52, -0.60, 1.09	680.00, -3.56, 3.49
H4	69.19, 22.74, -2.91	73.73, 48.95, -2.29	1520.02, 812.03, -87.00
J4	1.12, 8.60, 5.43	-5.53, 20.42, 32.49	30.39, 260.78, 290.40
A1	83.35, 37.84, 10.54	80.79, 82.44, 80.84	1867.8, 1243.61, 704.05

Three important issues that are specific to spectral rendering are evident from these numbers.

1. Note the huge difference between the F11 numbers, and the ones for A and D65! The reason for this discrepancy is that the illuminant data for F11 is given to a different scale than the other two; a rendering system has to compensate for such things by normalising all input spectra to equal radiant power before use.
2. Some sRGB triplets have negative coordinates - some patches, such as G4/red, for all three illuminants, and some only under one. An example of this is J4/cyan, which is logical, since D65 is the bluest of the three illuminants, and hence the one that is most likely to drive an already blue patch out of gamut. One or more negative co-ordinates mean that the colour stimulus caused by the input spectrum is outside the range of colours that sRGB can represent; for all such colours, some form of gamut mapping has to be employed in a real renderer.
3. The white patch A1 has approximately equal values for all three channels only when illuminated by D65/daylight, which is the reference white for this particular type of object (the colour checker). Due to its higher accuracy, the problem area of white balance is more of a concern for spectral rendering than for normal RGB rendering.