Problem Specification

- **Scheduling Strategy:** Open scheduling with shared OR capacity.
- **Planning Horizon & Decision Level:** Single-stage tactical assignment over a one-week horizon; strategic capacity fixed exogenously.
- **Uncertainty Treatment:** Deterministic model using predicted durations $\hat{d}_i$.
- **Decision Scope:** Assignment-only of cases to OR–day blocks; no sequencing.
- **Block Duration:** 8 hours (480 minutes) per OR–day block.
- **Block Capacity:** $Cap = 480$ minutes.
- **Cost Parameters:** $c^{OT} = 15$, $c^{IT} = 10$, $c^{REJ} = 20$.
- **Objective:** Minimize total idle, overtime, and rejection costs.

1 Deterministic formulation

Sets

- B – pool of OR–day blocks.
- I – pool of elective surgeries.

Parameters

- b_i – booked time of surgery i (minutes).
- $\hat{d}_i$ – predicted duration of surgery i (minutes).
- $Cap = 480$ – capacity of each OR block (minutes).
- $c^{OT} = 15,\ c^{IT} = 10$ – overtime/idle penalty per minute.
- $c^{REJ} = 20$ – rejection penalty per booked minute.

$$
\begin{aligned}
&X_{it} \in \{0,1\} && \forall i \in I,\ t \in B,\\
&R_i \in \{0,1\} && \forall i \in I,\\
&OT_t,\ IT_t \ge 0 && \forall t \in B.
\end{aligned}
$$

$$
\min \sum_{t \in B} \big(15\, OT_t + 10\, IT_t\big) \ +\ 20 \sum_{i \in I} b_i\, R_i
$$

$$
\sum_{t \in B} X_{it} + R_i = 1 \qquad \forall i \in I, \tag{1}
$$

$$
\sum_{i \in I} \hat{d}_i\, X_{it} - 480 = OT_t - IT_t \qquad \forall t \in B, \tag{2}
$$

$$
X_{it} \in \{0,1\},\ R_i \in \{0,1\},\ OT_t,\ IT_t \ge 0. \tag{3}
$$

2 Predict–then–optimize formulation

Sets

- B – pool of OR–day blocks.
- I – pool of elective surgeries.

Parameters

- d_i – random duration of surgery i (minutes).
- $\hat{d}_i$ – predicted duration of surgery i (minutes).
- b_i – booked time of surgery i (minutes).
- $Cap = 480$ – capacity of each OR block (minutes).
- $c^{OT} = 15,\ c^{IT} = 10$ – overtime/idle penalty per minute.
- $c^{REJ} = 20$ – rejection penalty per booked minute.

$$\begin{aligned} &X_{it} \in \{0,1\} && \forall i \in I,\ t \in B, \\ &R_i \in \{0,1\} && \forall i \in I, \\ &OT_t,\ IT_t \geq 0 && \forall t \in B. \end{aligned}$$

$$\min \sum_{t \in B} \big(15\, OT_t + 10\, IT_t\big) \ +\ 20 \sum_{i \in I} b_i\, R_i$$

$$\sum_{t \in B} X_{it} + R_i = 1 \qquad \forall i \in I, \tag{4}$$

$$\sum_{i \in I} \hat{d}_i\, X_{it} - 480 = OT_t - IT_t \qquad \forall t \in B, \tag{5}$$

$$X_{it} \in \{0,1\},\ R_i \in \{0,1\},\ OT_t,\ IT_t \geq 0. \tag{6}$$

3 Stochastic optimization (SAA) formulation

Additional notation

- Ω – set of scenarios.
- p_ω – probability of scenario ω.
- d_i^ω – realized duration of surgery i in scenario ω.
- OT_t^ω, IT_t^ω – overtime/idle time in block t under scenario ω.

First-stage (here-and-now) decisions

$$X_{it},\ R_i \quad \forall i \in I,\ t \in B.$$

Second-stage (recourse) decisions

$$OT_t^\omega,\ IT_t^\omega \quad \forall t \in B,\ \omega \in \Omega.$$

Two-stage formulation

$$\min_{X,R} 20 \sum_{i \in I} b_i R_i + \mathbb{E}_\omega\big[Q(X,\omega)\big], \tag{7}$$

$$Q(X,\omega) = \min_{OT^\omega, IT^\omega} \sum_{t \in B} \big(15\, OT_t^\omega + 10\, IT_t^\omega\big) \tag{8}$$

$$\text{s.t.} \sum_{i \in I} d_i^\omega\, X_{it} - 480 = OT_t^\omega - IT_t^\omega \qquad \forall t \in B,\ \omega \in \Omega, \tag{9}$$

$$OT_t^\omega,\ IT_t^\omega \geq 0 \qquad \forall t \in B,\ \omega \in \Omega. \tag{10}$$

Sample Average Approximation (SAA) Given a sample $\hat{\Omega}$ of size N,

$$\mathbb{E}_\omega\big[Q(X,\omega)\big] \;\approx\; \frac{1}{N} \sum_{\omega \in \hat{\Omega}} Q(X,\omega).$$