22 SPRING CSCE 629 600: ANALYSIS OF ALGORITHMS - Homework 6

Name: Rohan Chaudhury ¶

UIN: 432001358

Question (1) Textbook page 879, Exercise 29.3-6.

Solve the following linear program using SIMPLEX:

$$\begin{array}{ll} maximize & 5x_1 - 3x_2 \\ subject\ to & x_1 - x_2 \leq 1 \\ & 2x_1 + x_2 \leq 2 \\ & x_1, x_2 \geq 0\ . \end{array}$$

Answer:

Given:

$$x_1 - x_2 \leq 1$$
$$2x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0 \ .$$

Required:

$$maximize \qquad 5x_1 - 3x_2$$

In order to solve the linear program we have to first convert this linear program into it's slack form. The slack form is given by:

$$z = 5x_1 - 3x_2$$
$$x_3 = 1 - x_1 + x_2$$
$$x_4 = 2 - 2x_1 - x_2$$

The Basic solution is:

$$x_1 = 0, x_2 = 0, x_3 = 1, x_4 = 2$$

and the objective value is = 0

$x_1, and\ x_2$ are the non-basic variables.

We can see that only x_1 (non-basic variable) has a positive co-efficient in the above objective function whereas x_2 has a negative co-efficient. So we pick x_1 and increase its value in the slack-form LP.

In the equation $x_3 = 1 - x_1 + x_2$, we can increase the value of x_1 upto a value of 1 so that x_3 doesn't become negative and in the equation $x_4 = 2 - 2x_1 - x_2$, we can increase the value of x_1 upto a value of 1 so that x_4 doesn't become negative. So $x_1 = 1$ is the highest that we can increase the value of x_1 here. Now we rewrite the equation $x_3 = 1 - x_1 + x_2$ as $x_1 = 1 - x_3 + x_2$ and replace this value in the above slack-form LP to get the new slack-form LP which is given as:

$$z = 5(1 - x_3 + x_2) - 3x_2 = 5 - 5x_3 + 2x_2$$
$$x_1 = 1 - x_3 + x_2$$
$$x_4 = 2 - 2(1 - x_3 + x_2) - x_2 = 2x_3 - 2x_2 \ .$$

Now the Basic solution is:

$$x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 0$$

and the objective value is = 5

We can see now that x_2 (non-basic variable) has a positive co-efficient in the above objective function. So we pick x_2 and increase its value in this slack-form LP.

In the equation $z = 5 - 5x_3 + 2x_2$, we can increase the value of x_2 with no restrictions as value of z increases with the increase in the value of x_2 in this equation but in the equation $x_4 = 2x_3 - 2x_2$ we can increase the value of x_2 only upto 0 so that x_4 doesn't become negative. So $x_2 = 0$ is the highest that we can increase the value of x_2 here. Now we rewrite the equation $x_4 = 2x_3 - 2x_2$ as $x_2 = \frac{2}{3}x_3 - \frac{1}{3}x_4$ and replace this value in the above slack-form LP to get the new slack-form LP which is given as:

$$z = 5 - 5x_3 + 2(\frac{2}{3}x_3 - \frac{1}{3}x_4) = 5 - \frac{11}{3}x_3 - \frac{2}{3}x_4$$

$$x_1 = 1 - x_3 + (\frac{2}{3}x_3 - \frac{1}{3}x_4) = 1 - \frac{1}{3}x_3 - \frac{1}{3}x_4$$

$$x_2 = \frac{2}{3}x_3 - \frac{1}{3}x_4$$

Now the Basic solution is:

$$x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 0$$

and the objective value is = 5

Now from the equations we can see that both the non-basic variables $x_1\ and\ x_2$ has negative co-efficient in the objective function. So our final solution is the basic solution:

$$x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 0$$

and the objective value with this solution is = 5