22 SPRING CSCE 629 600: ANALYSIS OF ALGORITHMS - Homework 8

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Question (1) Textbook page 1066, Exercise 34.2-6.

A hamiltonian path in a graph is a simple path that visits every vertex exactly once. Show that the language HAM-PATH = {<G, u, v> : there is a hamiltonian path from u to v in graph G} belongs to NP.

ANSWER:

It is required to show that the language HAM-PATH belongs to NP. This can be proved by showing that the language HAM-PATH can be verified in polynomial time.

Let us consider the following:

(a) The input x as <G, u, v> (G.V are the vertices of the graph and G.E are the edges of the graph)

(b) the certificate y as a sequence of vertices $\{v_1, v_2, v_3, \ldots, v_n\}$.

So now, we can write an algorithm T(x,y) which can verify HAM-PATH by using the following steps:

=> We check if size_of (G.V) == n ------------------ (i)

=> We check if v_1 == u and v_n == v ------------------ (ii)

=> We check if for all i ϵ {1, 2,…., n}, v_i belongs to G.V ------------------ (iii)

=> We check if for all i, j ϵ {1, 2, , n}, $v_i \neq v_j$ ------------------ (iv)

=> We check if for all i ϵ {1, 2, , n-1}, (v_i, v_{i+1}) belongs to G.E ------------------ (v)

=> If (any of the above steps fail): ------------------ (vi)

=> return False (i.e., the path is not a Hamiltonian path) ------------------ (vii)

=> Else: ------------------ (viii)

=> return True (i.e., the path is a Hamiltonian path) ------------------ (ix)

Time complexity of the algorithm: In the above algorithm:

Step (i) to count the number of vertices will take O(1) time

Step (ii) checks if $v_1 = u$ and $v_n = v$ and it will take O(1) time

Step (iii) which checks if all the vertices in the path belong to the graph will take O(V) time

Step (iv) which checks if there are any duplicate vertices in the path will take $O(V^2)$ time

Step (v) which checks if the edges between the consecutive vertices of the path belong to the graph will take O(E) time

So, this algorithm T(x,y) which can verify HAM-PATH runs in $O(V^2)$ time i.e., all the steps of the algorithm run in polynomial time. Hence, it proves that: HAM-PATH belongs to NP.

Question (2) Textbook page 1077, Exercise 34.3-2.

Show that the $\leq_P$ relation is a transitive relation on languages. That is, show that if L1 $\leq_P$ L2 and L2 $\leq_P$ L3, then L1 $\leq_P$ L3.

ANSWER:

We are given the following relationship between languages L1, L2, and L3: (a) L1 $\leq_P$ L2 and (b) L2 $\leq_P$ L3

We have to prove that L1 $\leq_P$ L3.

With the above provided relations (a) and (b), we can say that there exists two reduction functions (f1 : $\{0, 1\}^* \rightarrow \{0, 1\}^*$ and f2 : $\{0,1\}^* \rightarrow \{0, 1\}$) which are polynomial time computable and which satisfy the following:

1. $x \in L1 \leftrightarrow f1(x) \in L2$
2. $x \in L2 \leftrightarrow f2(x) \in L3$

Now, as f1 and f2 are polynomial time computable functions, we can say that f2(f1(x)) is also a polynomial time computable function which satisfies the following:

$\mathbf{x \in L_1 \leftrightarrow f2(f1(x)) \in L_3}$

Hence this shows that L1 $\leq_P$ L3. So, $\leq_P$ relation is a transitive relation on languages.