

COORDINATE SYSTEMS IN ASTRONOMY

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WHY DO WE NEED COORDINATES

- In astronomy, a celestial coordinate system is a system for specifying positions of celestial objects: satellites, planets, stars, galaxies, and so on. Coordinate systems can specify a position in 3-dimensional space, or merely the direction of the object on the celestial sphere, if its distance is not known or not important.
- For communication of object and also some what for predictions

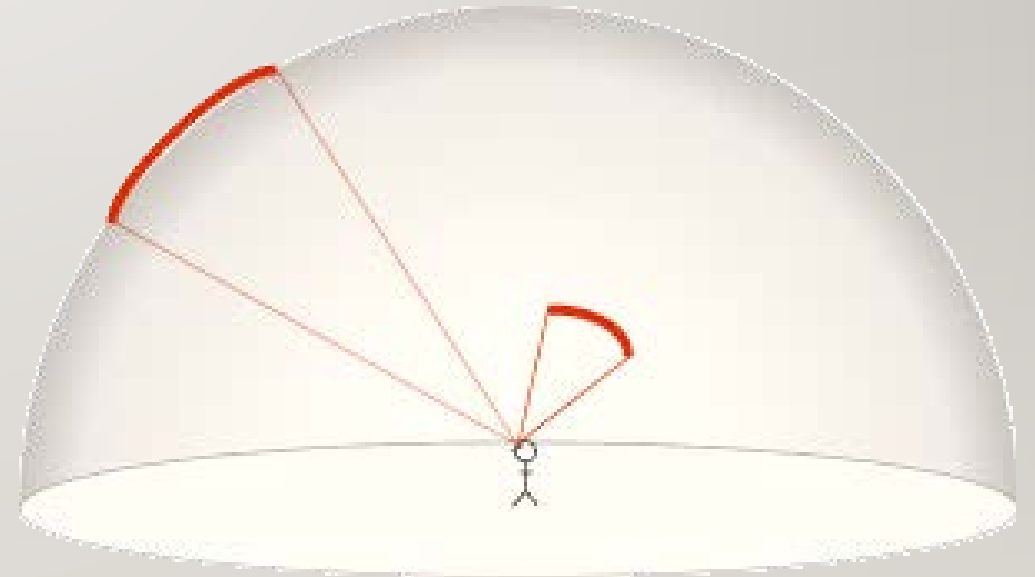
SOME TERMS TO NOTE

You would learn them in upcoming slides

- Ecliptic
- Celestial Equator
- Zenith
- Nadir
- Equinoxes
- Solstices

MEASURING DISTANCE IN ASTRONOMY

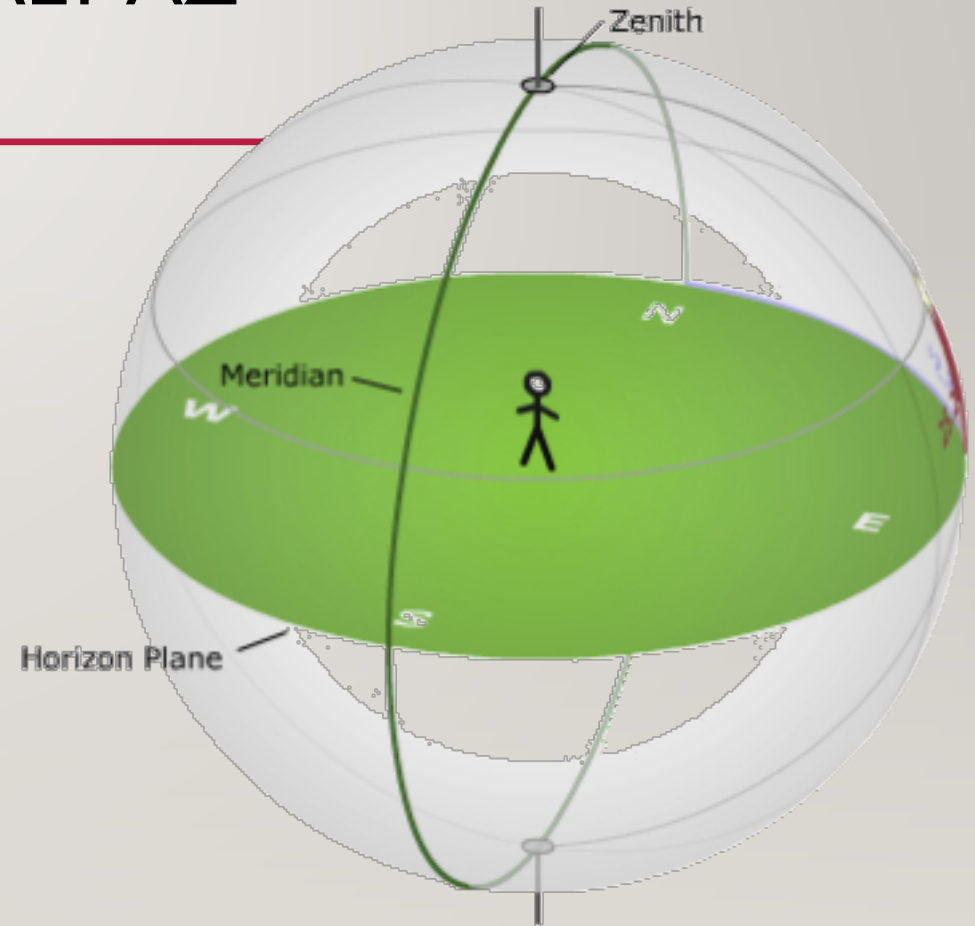
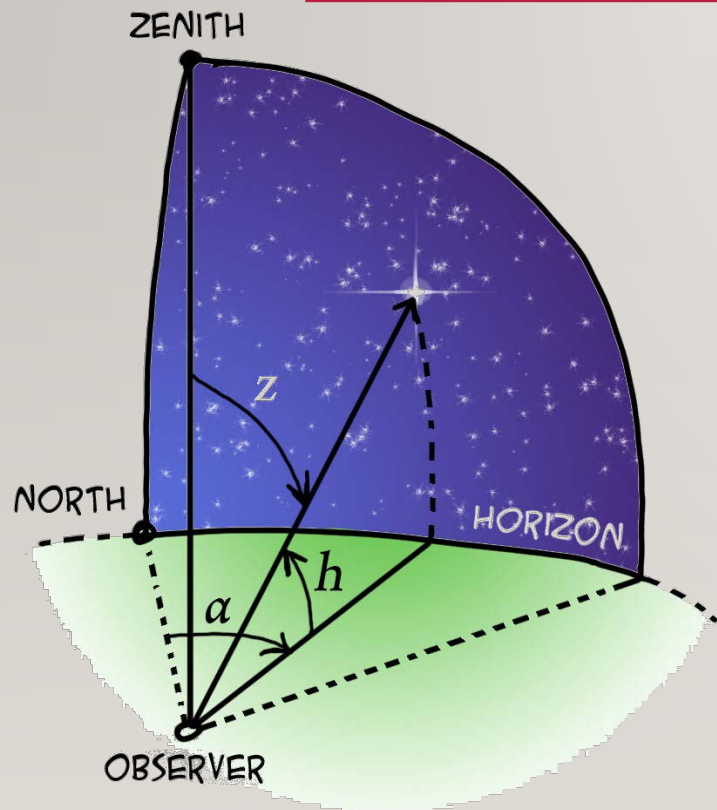
- When we are measuring distances in observational astronomy. The actual distance from earth doesn't matter to us.
- We measure the distance in angles



HOW DO WE DEFINE COORDINATES ON A SPHERE

- In spherical coordinates we need (r, θ, ϕ) , Since we are neglecting distances so we need just 2 angles to define any coordinate.
- Have you seen any example earlier for like this sort. Where we have defined 2D coordinates on a surface of sphere
- Earth's latitude and longitude.

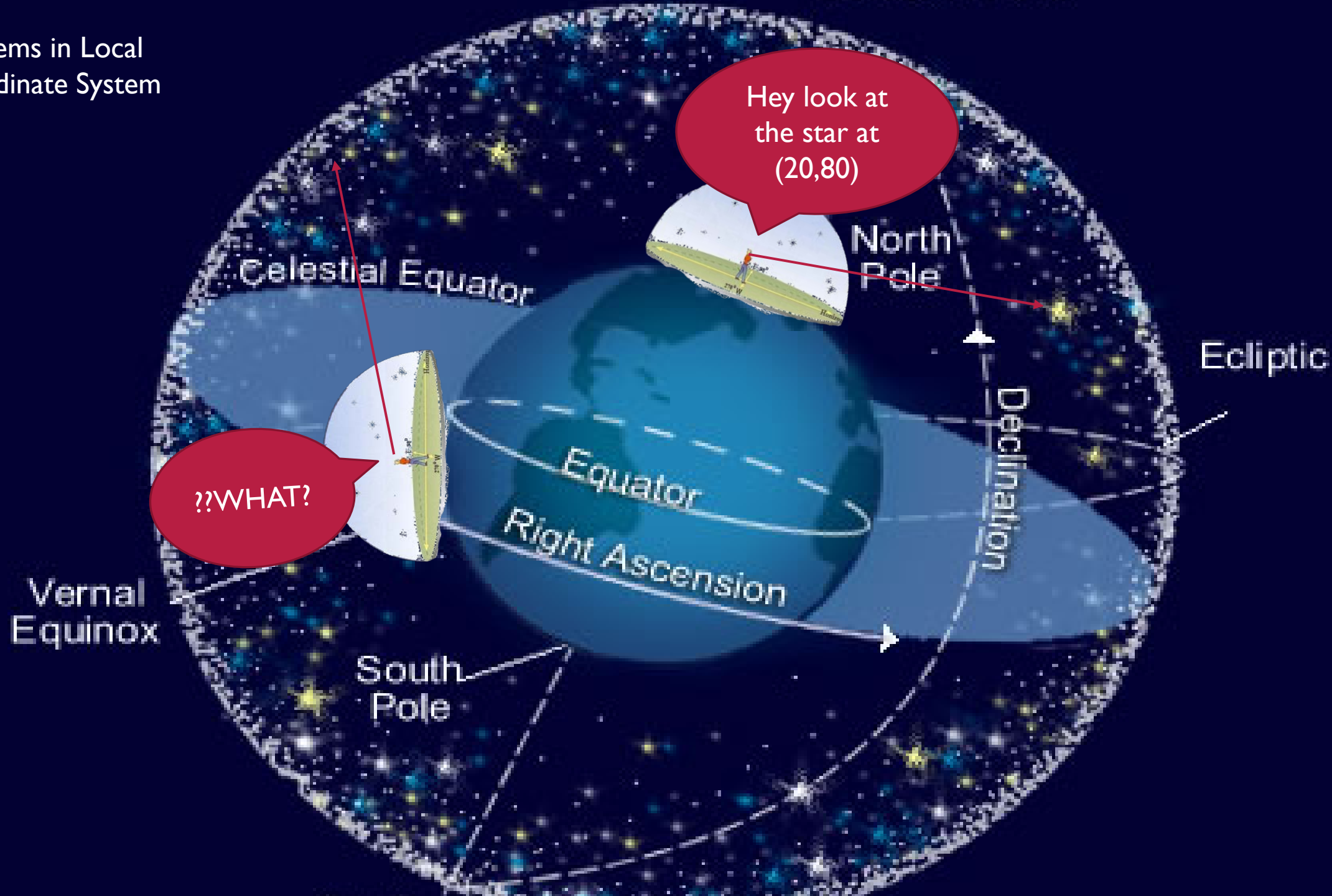
LOCAL COORDINATE SYSTEM ALT AZ



LOCAL COORDINATE SYSTEM ALT AZ

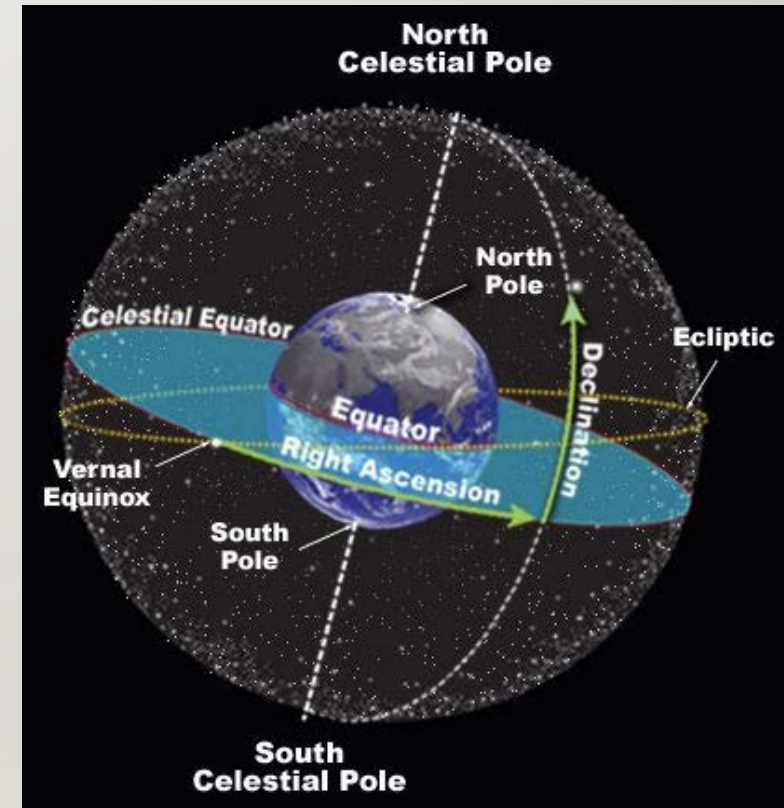
- It is local coordinate system. Because if you change your latitude your position of star changes
- **Altitude** is measured in degrees above the horizon
- **Azimuth** is measured on a parallel to the horizon, in degrees East from North
- Expressed as (Alt,Az). Unlike all other are represented first horizontal and then vertical componenet
- Polaris in Mumbai is at $(19.1^{\circ}, 0^{\circ})$

Problems in Local Coordinate System



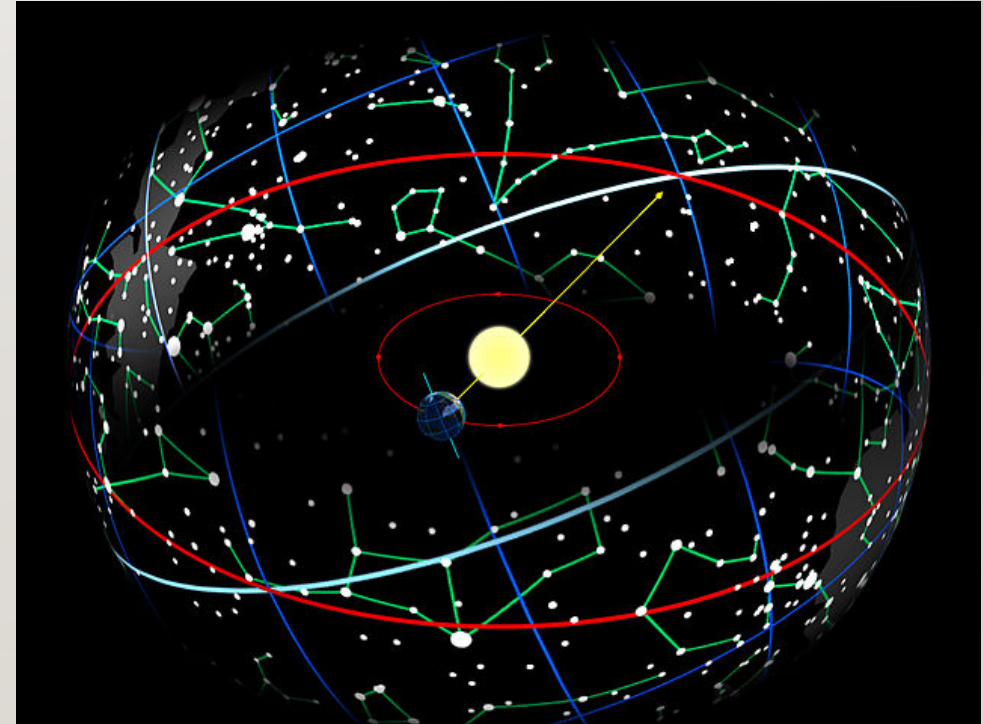
NEW SYSTEM OF COORDINATES

- **Celestial Equator** Is the projection of earth's equator in the space
- This gives rise to **North Celestial Pole** and **South Celestial Pole**

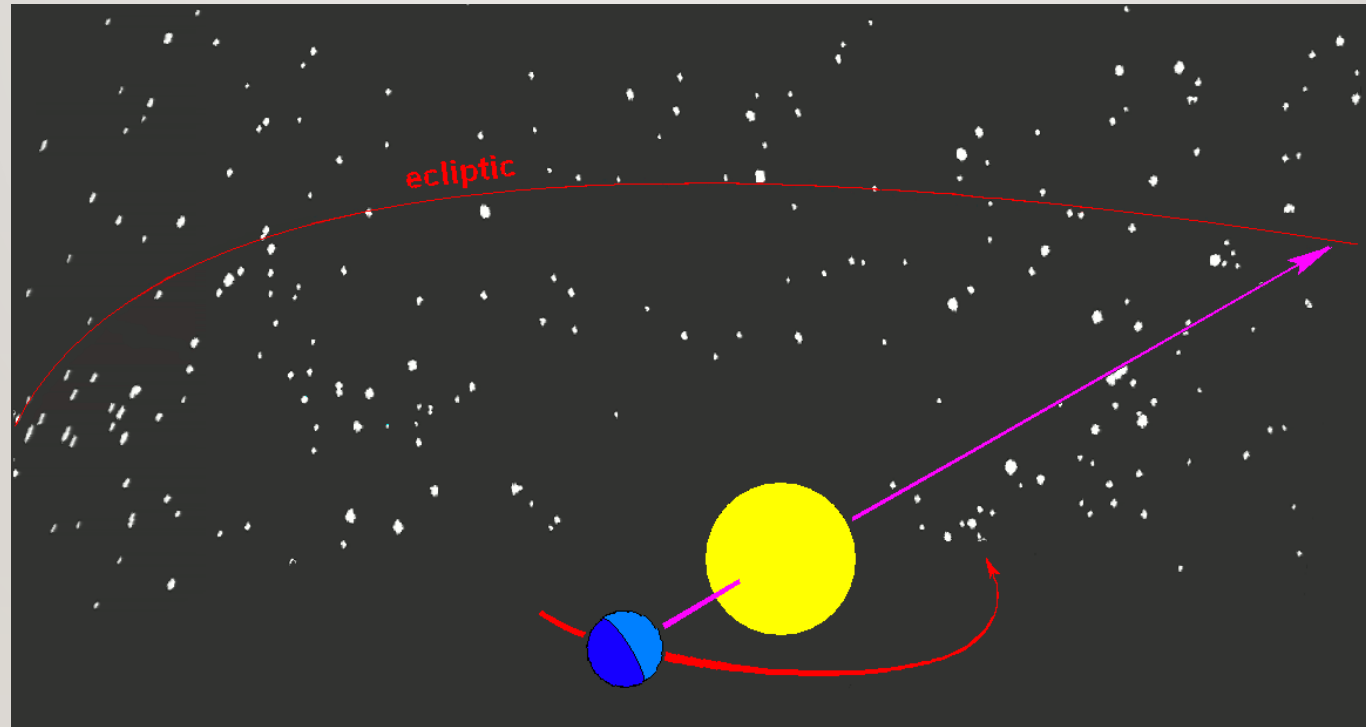


NEW SYSTEM OF COORDINATES

- **Ecliptic** is the imaginary path of the sun in the celestial sphere. Sun passes through zodiacal constellation that are Aries, Taurus, Gemini, Pisces
- **Equinoxes** are formed when celestial equator and ecliptic meet in celestial sphere. **Vernal equinox** lies in **Pisces** and **Autumnal Equinox** lies in **Virgo**



ECLIPTIC



EQUATORIAL COORDINATES SYSTEM

- This coordinates define two angles Declination(Dec) and Rights Ascension (RA)
- **Declination** (Dec) As on the Earth, North-South positions are measured from the celestial equator, with the vertex at the Earth's center. Like latitude, it is measured in degrees, with negative values for positions South of the celestial equator. The equator is 0° Dec, and the South celestial pole is at -90° Dec.
- The East-West celestial coordinate is called **Right Ascension** (RA). Like longitude, East-West positions are measured on a parallel, with the vertex on the Earth's axis, from an arbitrary zero-point meridian on the fixed sky. This prime meridian is set at the Vernal Equinox. It is expressed in hour, minutes and seconds $24^h=360^\circ$. And 0^h is set for Vernal Equinox. It is expected that zodiacal constellation changes after every 2h RA
- Order of giving coordinates is RA, Dec. Coordinates of Vega are $18^h 36^m 56^s$ and $38^\circ 47' 01''$

ECLIPTIC COORDINATE SYSTEM

- The **ecliptic coordinate system** is a celestial coordinate system commonly used for representing the positions and orbits of Solar System objects. Because solar system is planar
- It consists of Latitude and Longitude. Same as earth's system Vernal equinox is the centre

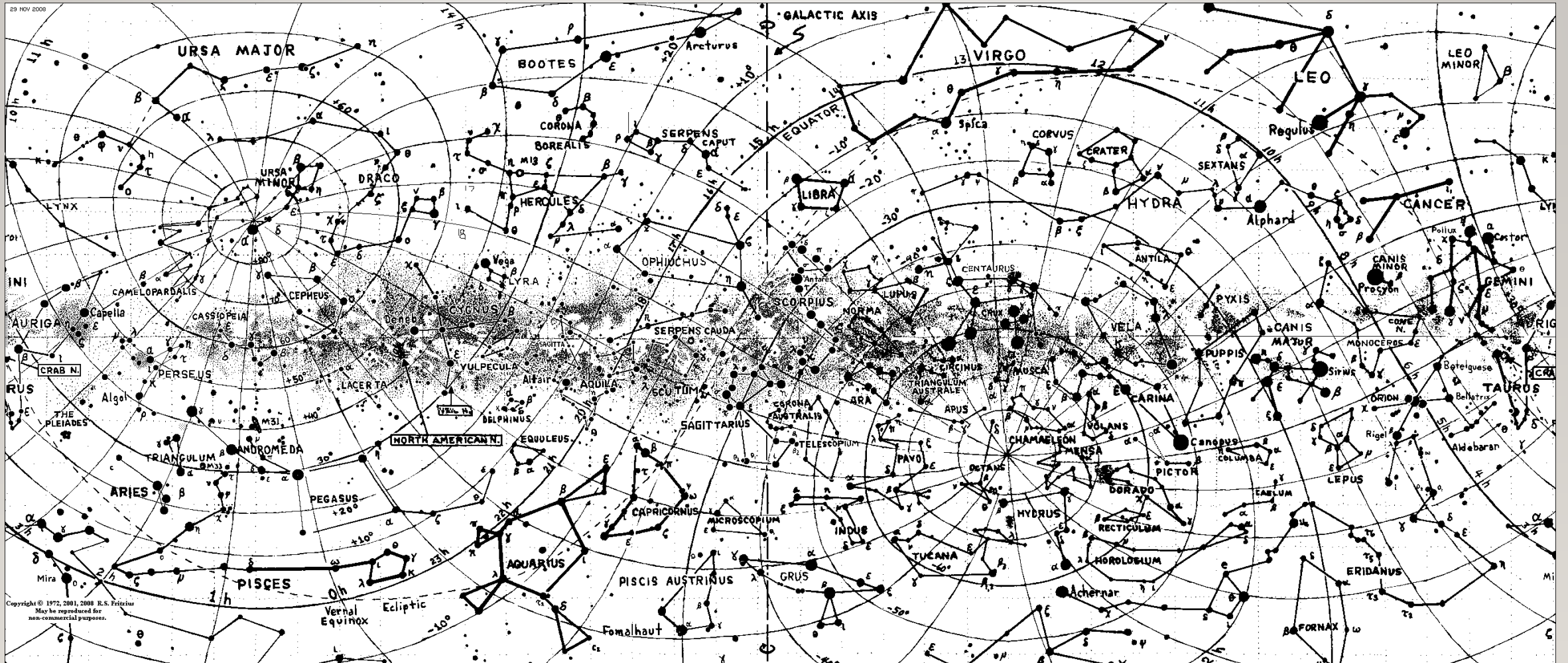
GALACTIC COORDINATE SYSTEM



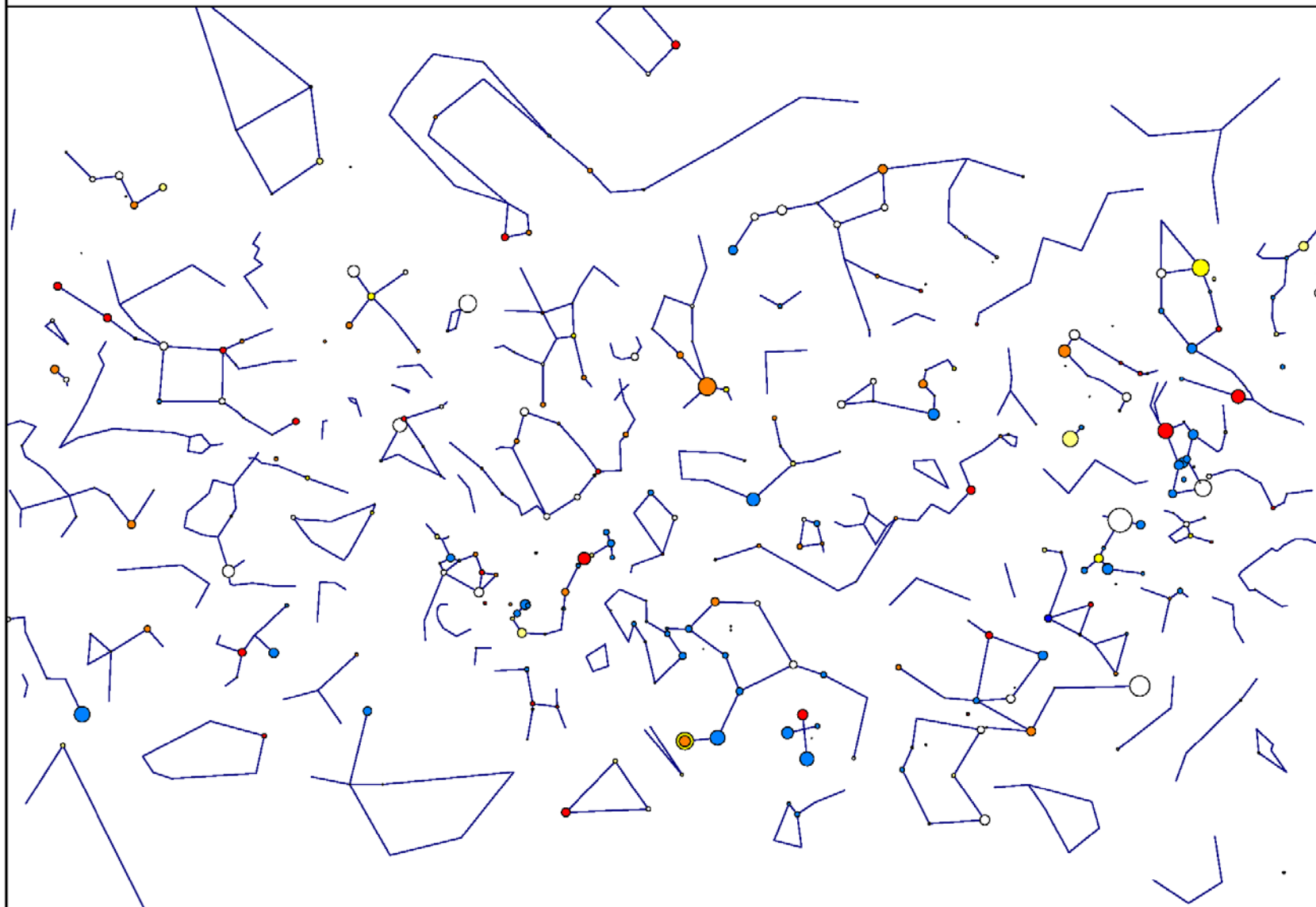
GALACTIC COORDINATE SYSTEM

- Centre of galaxy is taken as (0,0), And plane of galaxy as horizontal plane
- Longitude (symbol l) measures the angular distance of an object eastward along the galactic equator from the galactic center. Analogous to terrestrial longitude, galactic longitude is usually measured in degrees ($^{\circ}$).
- Latitude (symbol b) measures the angular distance of an object perpendicular to the galactic equator, positive to the north, negative to the south. For example, the north galactic pole has a latitude of $+90^{\circ}$. Analogous to terrestrial latitude, galactic latitude is usually measured in degrees ($^{\circ}$).

GALACTIC CENTRED MAP



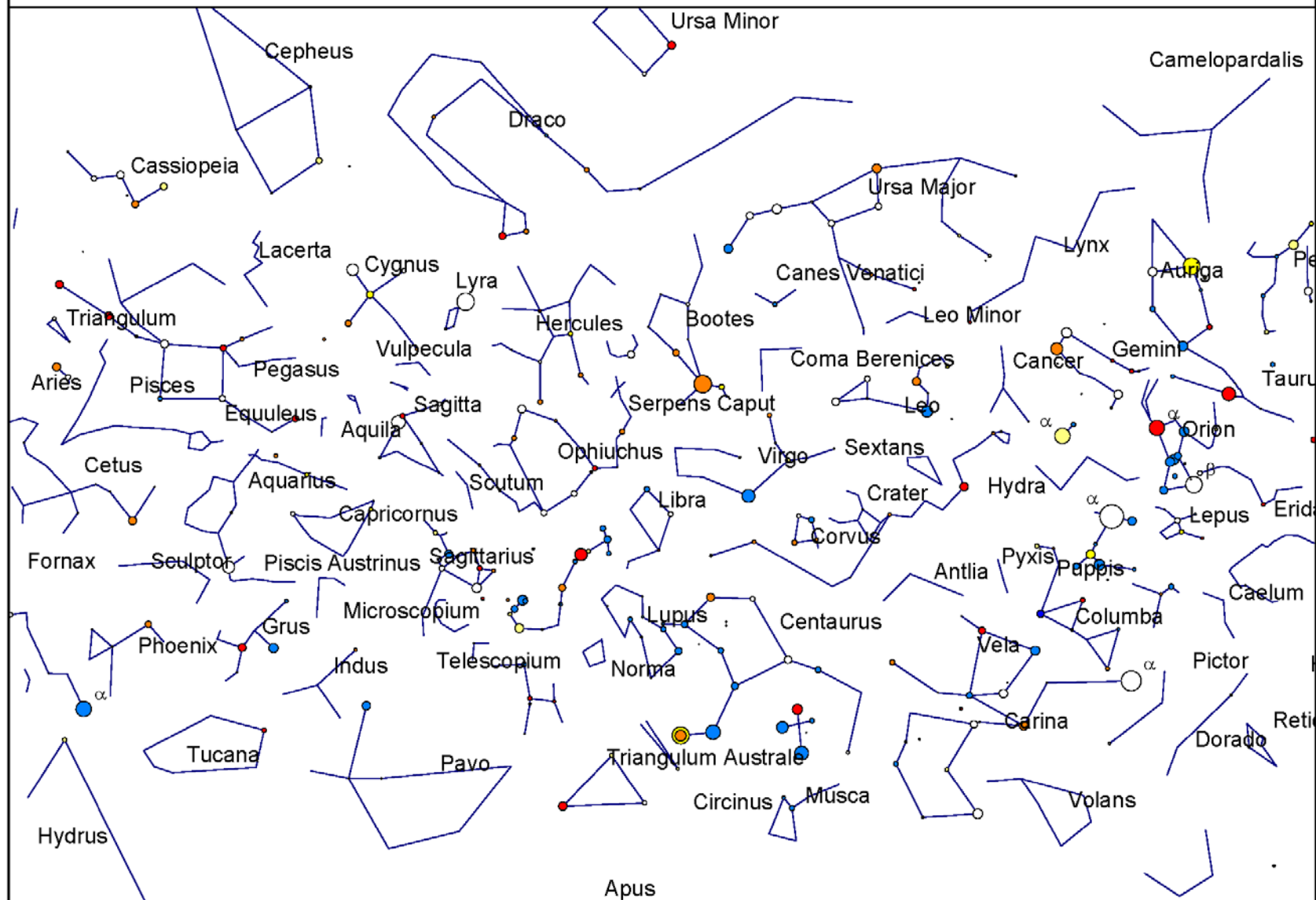
Sky Charts, Observatory Geneva, Center RA: 15h01m DEC:-00°00'



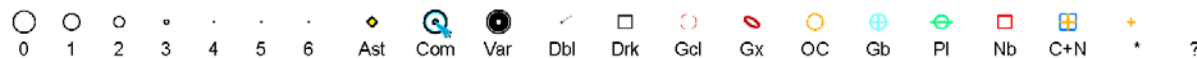
Equatorial coord. MER, Apparent, 2016-09-17, 17h43m40s (CEST), Mag:3.6/6.0,60.0', FOV:+360°00'00"

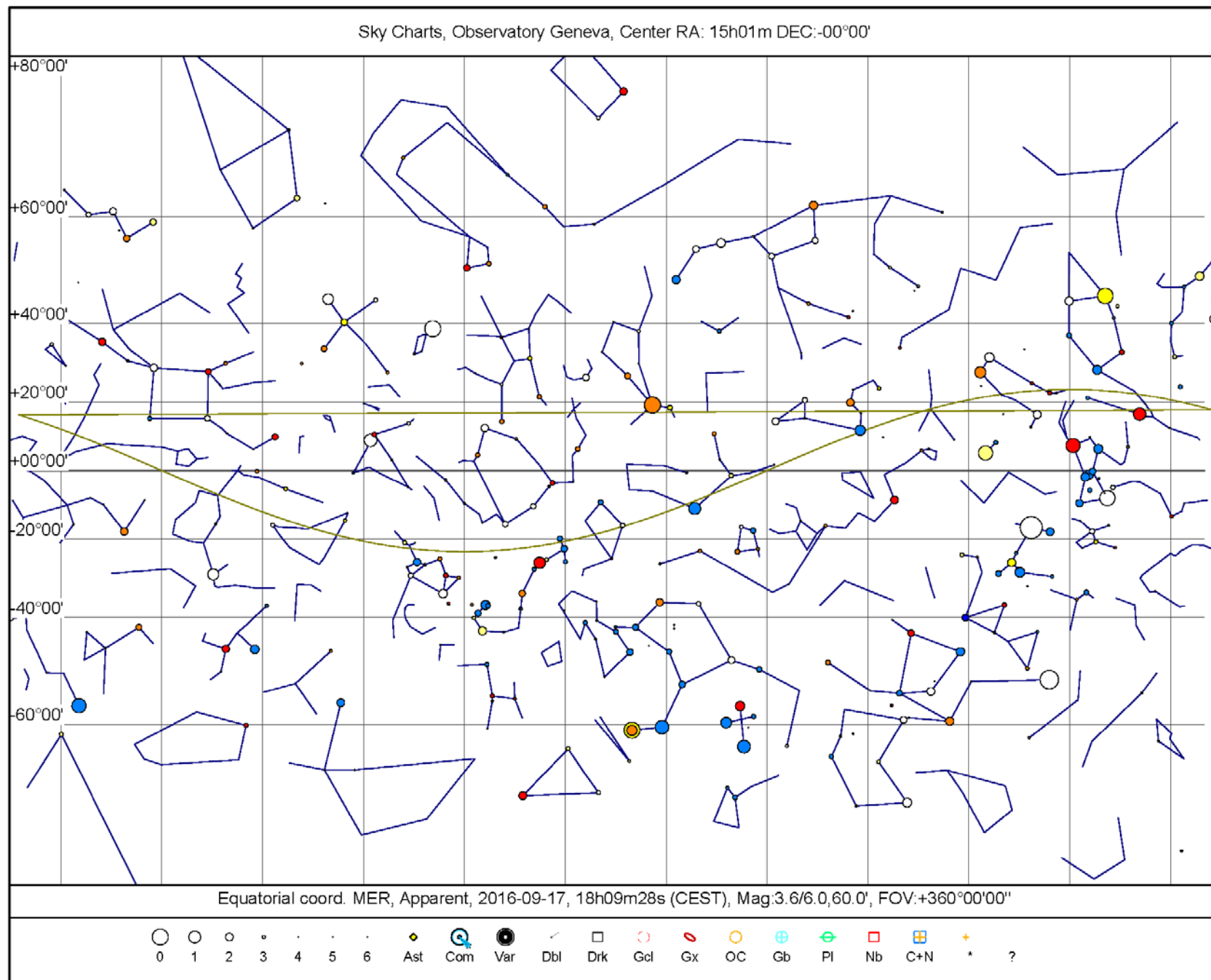
0 1 2 3 4 5 6 Ast Com Var Dbl Drk Gcl Gx OC Gb PI Nb C+N + ?

Sky Charts, Observatory Geneva, Center RA: 15h01m DEC:-00°00'



Equatorial coord. MER, Apparent, 2016-09-17, 17h42m23s (CEST), Mag:3.6/6.0,60.0', FOV:+360°00'00"





POSITIONAL ASTRONOMY

Celestial Coordinates in Action



WHAT TO SEEK IN POSITIONAL ASTRONOMY

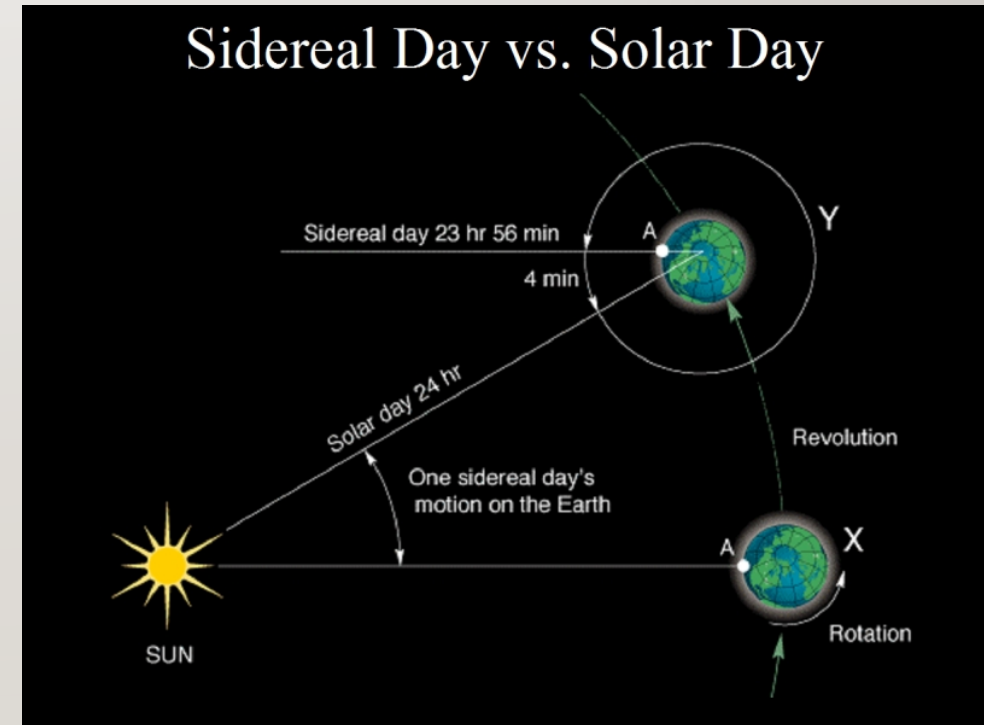
- We know that locally it is better to use Alt-Az coordinate system. Whereas for commuting data or remembering it is better to use Equatorial coordinates system.
- So what are the methods to convert one system to another.
- How to predict rise of stars, sun etc

YOU LEARNT ABOUT THEM

- Earlier Mentioned
 - Ecliptic
 - Celestial Equator
 - Zenith
 - Nadir
 - Equinoxes
 - Solstices

SOME IMPORTANT DEFINITION

- Day
 - **Solar Day** is the time between 2 successive culmination of sun ($t=24\text{h}$)
 - **Sidereal Day** is the time between 2 successive culmination of distant star ($t=23\text{h } 56\text{m } 4.093\text{s}$)



SOME IMPORTANT DEFINITION

- Year
 - **Tropical Year** is the time between 2 successive passage of sun through vernal equinox ($t=365.242199$ days)
 - **Sidereal Year** is the time taken by earth for one revolution around the sun w.r.t. distant star ($t=365.2564$ days)
 - **Anomalistic Year** is the time taken by earth between 2 successive passages of earth through its perihelion ($t=365.2596$ days)

SOME IMPORTANT DEFINITION

- Period (Relative between bodies)
 - **Synodic Period** is the time between 2 successive (same type of conjunctions of solar system objects

$$\frac{1}{T_{synodic}} = \frac{1}{T_{earth}} - \frac{1}{T_{object}}$$

- **Draconic Period** is the time between two successive passages through ascending node.

CALENDARS

- **Lunar** based on phases of moon old Jewish/Islamic
- **Roman (Lunar) Calendar**
 - 12 months total 355 days
 - Intercalary months once in 2-3 years decided by local priest
- **Julian Calendar (46 BC)**
 - Year had 365 days. Extra leap day every 4 year. Avg. length of the year 365.25
 - Error: 1 day in 128 years
- **Gregorian Calendar**
 - In 1582, the Julian calendar accumulated 10 days. Vernal Equinox occurred on 11 Mar
 - That year many countries after 4th October had 15th October directly
 - This calendar added the rule that Centuries are leap year only when they are divisible by 400
 - Error: 1 day in 3300 years

ADAPTION OF GREGORIAN CALENDAR AND NEW TIME

- Gregorian Calendar changes
 - **16th to 18th Century** changes implemented in Catholics but not in protestants or orthodox
 - **1700:** Prussia
 - **1752:** India, America, England, British colonies subtracted 11 days from calendar.
 - **1918:** Russia They subtracted 13 days from month of October. That's why they celebrate October revolution in November.
- **Herschel's Calendar**
 - He added rule that years divisible by 4000 are not leap year. Error: 1 day in 20,000 years

JULIAN DATES

- For calculation we use Julian Dates
- Continuous day counting. Reference point 1.1.4713 BC
- Day change at noon in UT (Astronomer's Night!)
- Simple subtraction is used to find no. of days between events

1/Jan/1980 00:00 UT = JD 2,444,239.5

12/Jan/2011 13:30 IST = JD 2,455,936.833

SOME IMPORTANT TERMS

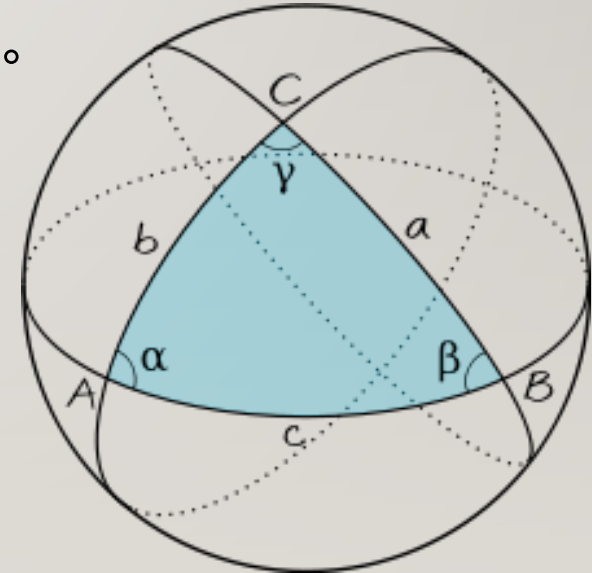
- Zenith
- Nadir
- North Celestial Pole (NCP) , SCP, NEP, SEP
- Meridian
- **Transit** When an object crosses Meridian
- **Hour Angle** It is the time since last transit

CIRCLES

- **Great Circle** Any circle on the surface of the sphere such that its centre coincides with the centre of sphere e.g. Equator, Longitudes
- **Small Circle** All other circles on surface of sphere e.g. Latitudes
- **Spherical Angle** is the Angle between the plane of any 2 great circle.

SPHERICAL TRIANGLE

- All 3 sides are arcs of great circles (namely a, b, c)
- Sum of any 2 sides is greater than 3rd side
- Sum of 3 spherical angles namely (A, B, C) is greater than 180°
- Each spherical angle is less than 180°



FORMULAS OF SPHERICAL TRIANGLE

- Some formulas
- Sine Rule

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

- Cosine Rule

$$\cos a = \cos b \cos c + \sin b \sin c \sin A$$

- Analogue of Cosine Rule

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$

- Four Parts Formula

$$\cos a \cos C = \sin a \cot b - \sin C \cot B$$

CONVERSION OF COORDINATES (DEFINING VARIABLES)

- **Horizontal Coordinates**

- Alt \rightarrow a , Azimuth \rightarrow A
- **Zenith Distance** = $90^\circ - a$

- **Equatorial Coordinates**

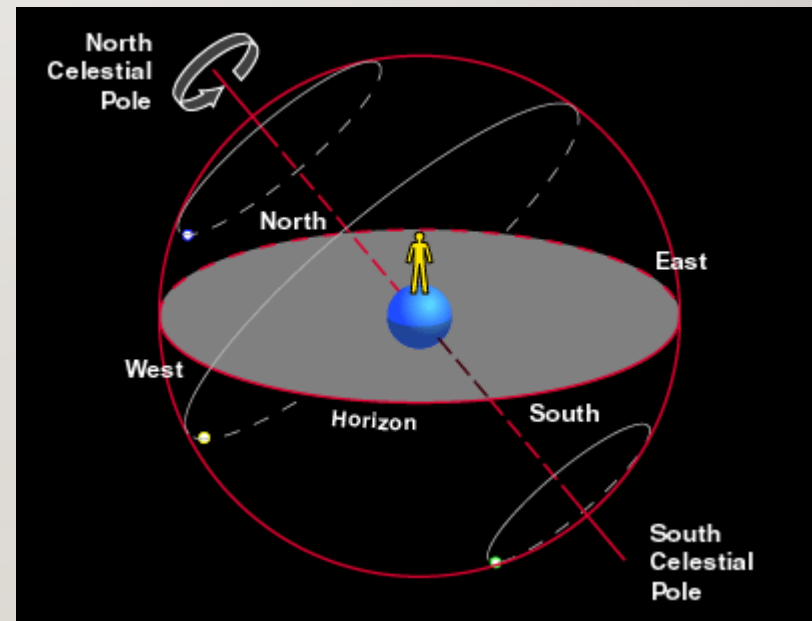
- Declination \rightarrow δ , Latitude on Earth \rightarrow ϕ , hour angle \rightarrow h

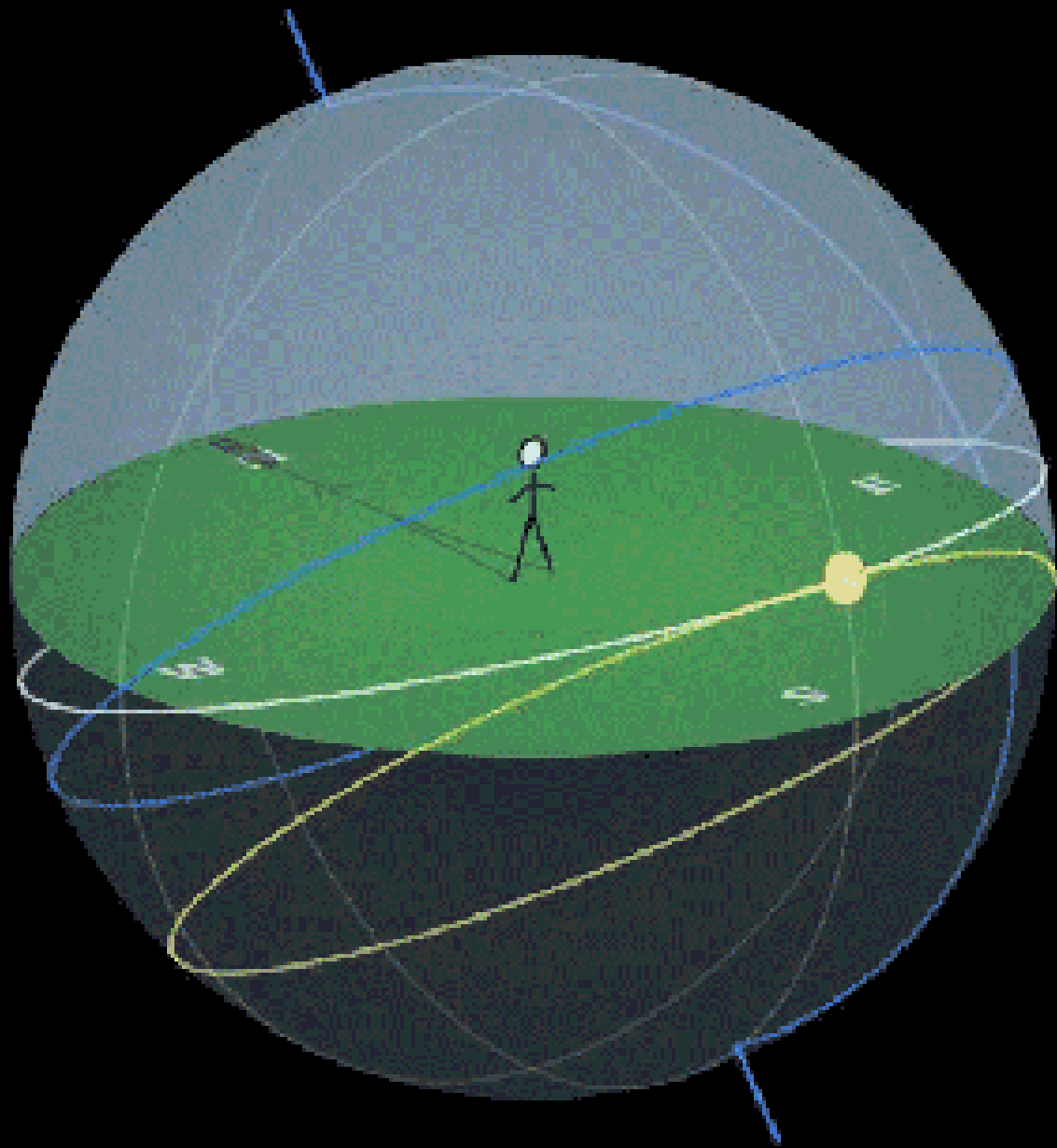
- **Ecliptic**

- Latitude \rightarrow β , Longitude \rightarrow α

SOME ADDITIONAL INFORMATION

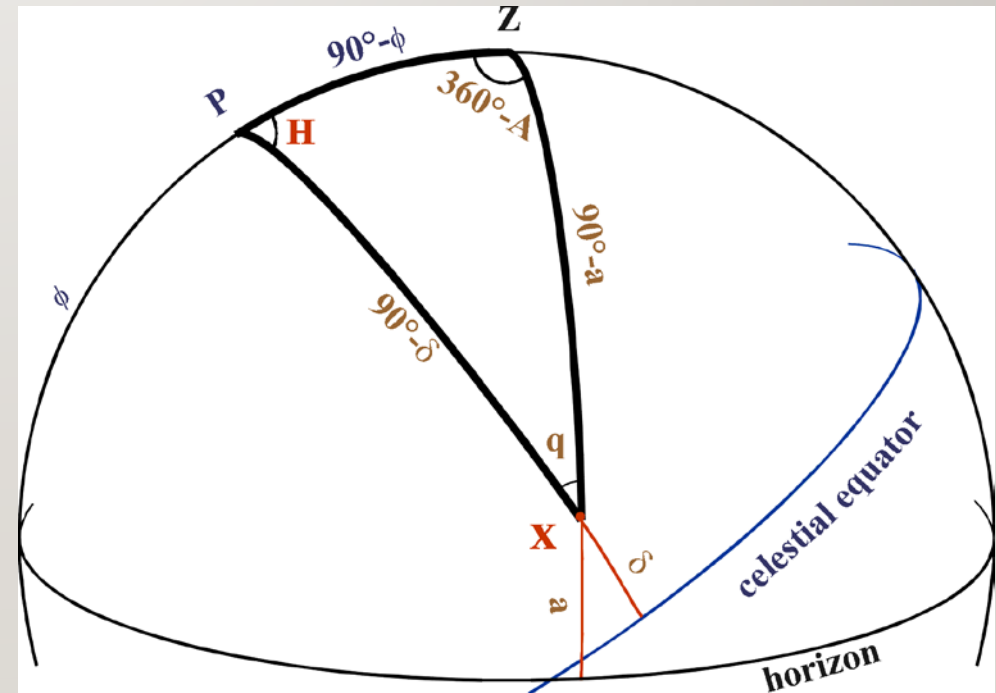
- Stars are Circumpolar if
$$\delta \geq 90 - \phi$$
- Stars are Never Rising if
$$\delta < \phi - 90$$





HORIZONTAL TO EQUATORIAL

- We use a spherical triangle often called Spherical Triangle: XPZ, where Z is the zenith, P is the North Celestial Pole, and X is the object.
- The sides of the triangle:
 - PZ is the observer's co-latitude = $90^\circ - \phi$.
 - ZX is the zenith distance of X = $90^\circ - a$.
 - PX is the North Polar Distance of X = $90^\circ - \delta$.
- The angles of the triangle:
 - The angle at P is H, the local Hour Angle of X.
 - The angle at Z is $360^\circ - A$, where A is the azimuth of X.
 - The angle at X is q, the **parallactic angle**.
- **Local Sidereal Time** is b



HORIZONTAL TO EQUITORIAL

- $\sin(\delta) = \sin(a)\sin(\phi) + \cos(a) \cos(\phi) \cos(A)$

- We can now use the sine rule to get H, using the same formula as above:

$$\sin(H) = - \sin(A) \cos(a) / \cos(\delta)$$

Or use the cosine rule instead:

$$\sin(a) = \sin(\delta)\sin(\phi) + \cos(\delta) \cos(\phi) \cos(H)$$

and rearrange to find H:

$$\cos(H) = \{ \sin(a) - \sin(\delta) \sin(\phi) \} / \cos(\delta) \cos(\phi)$$

- Having calculated H, ascertain the Local Sidereal Time t.

Then the R.A. follows from

$$\mathbf{R.A. = t - H .}$$

EQUATORIAL TO HORIZONTAL

- Local Hour Angle $H = \text{LST} - \text{RA}$, in hours;
convert H to degrees (multiply by 15).
Given H and δ , we require azimuth A and altitude a .
- By the cosine rule: $\cos(90^\circ - a) = \cos(90^\circ - \delta) \cos(90^\circ - \phi) + \sin(90^\circ - \delta) \sin(90^\circ - \phi) \cos(H)$
which simplifies to:
 $\sin(a) = \sin(\delta) \sin(\phi) + \cos(\delta) \cos(\phi) \cos(H)$
This gives us the altitude a .
- By the sine rule:
 $\sin(360^\circ - A) / \sin(90^\circ - \delta) = \sin(H) / \sin(90^\circ - a)$
which simplifies to:
 $-\sin(A) / \cos(\delta) = \sin(H) / \cos(a)$
i.e. **$\sin(A) = -\sin(H) \cos(\delta) / \cos(a)$**
which gives us the azimuth A .
- Alternatively, use the cosine rule again: which simplifies to
 $\sin(\delta) = \sin(\phi) \sin(a) + \cos(\phi) \cos(a) \cos(A)$
Rearrange to find A :
 $\cos(A) = \{ \sin(\delta) - \sin(\phi) \sin(a) \} / \cos(\phi) \cos(a)$
which again gives us the azimuth A

HORIZONTAL AND EQUATORIAL

HORIZONTAL TO EQUATORIAL

- $\sin \delta = \sin a \sin \varphi + \cos a \cos \varphi \cos A$
- $\sin(H) = -\frac{\sin(A) \cos(a)}{\cos(\delta)}$
- $\cos(H) = \frac{\{\sin(a) - \sin(\delta) \sin(\varphi)\}}{\cos(\delta) \cos(\varphi)}$
- $RA = t - H$

EQUATORIAL TO HORIZONTAL

- $H = t - RA$
- $\sin(a) = \sin \delta \sin \varphi + \cos \delta \cos \varphi \cos H$
- $\sin(A) = -\frac{\sin(H) \cos(\delta)}{\cos(a)}$
- $\cos(A) = \frac{\{\sin(\delta) - \sin(\varphi) \sin(a)\}}{\cos(\varphi) \cos(a)}$

FOR RISE OF SUN OR ANY OTHER BODY

$$\cos H = -\tan \delta \tan \phi$$

- Used to find out equinox day length
- Equinox day length
- Maximum minimum length at a latitude
- Calculate rise time at Mumbai on Winter Solstice

EXAMPLE (ASTRO GC 2016)

The most well known scientist Dr. Cloe Bhatore deploys her personal droid N1T-FAT3HPKR to monitor the sun. Though it normally gives delayed results it is known for its excellent performance when it comes to imaging. Prof Chloe would like to image the sun right from the beginning of sunrise to the end of sunset. Towards the end of summer in North pole, she sets up the droid in the south pole, gives the command to start as soon as sun begins to rise and comes back her home in North pole. After 6 months when she comes back to the south pole to retrieve the images, she finds that there are no images from the beginning of sunrise to the end of sunrise. This was because of the inherent delay in the working of droid.

Find this inherent delay in the working of N1T-FAT3HPKR. Given sun extends an angle of 13.71° at the earth (That's not exact it's hypothetical.)

Hint: The sunrise and sunset at the poles happen due to the motion of the sun along the ecliptic (The path along which the sun travels in the background of stars). Assume circular orbit of earth and that the sun moves 1° along the ecliptic everyday. Assume plane trigonometry. (Don't go into spherical trigonometry).

QUESTION I

- Question requires us to calculate rise time of sun on poles. How to do it. Given diameter of sun = 13.712°

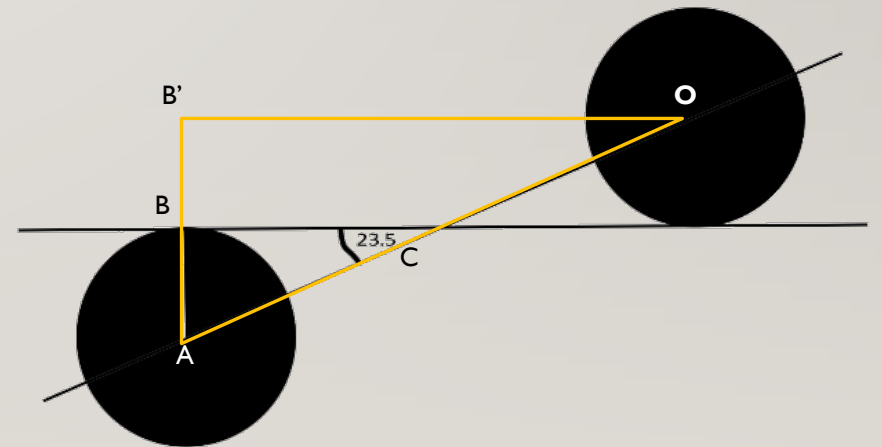
$$\frac{\sin \angle B'OA}{\sin \angle AB'O} = \frac{\sin \angle OB'A}{\sin \angle AOB'}$$

$$\angle B'OA = 23.5^\circ, \angle B'OA = 90^\circ, AB' = 13.712$$

$$\Rightarrow AO = \sin^{-1} \left(\frac{\sin 13.712}{\sin 23.5^\circ} \right) = 17.4199$$

Sun moves along ecliptic at a pace of $\frac{360}{365.25} = .9856^\circ \text{ day}^{-1}$

No. of days required for sunrise = 35.9501 days



EQUATORIAL AND ECLIPTIC

EQUATORIAL TO ECLIPTIC

- $\tan \alpha = \frac{\sin(RA) \cos i + \tan \delta \sin i}{\cos(RA)}$
- $\sin \beta = \sin \delta \cos i - \cos \delta \sin i \sin(RA)$

ECLIPTIC TO EQUATORIAL

- $\tan(RA) = \frac{\sin \alpha \cos i - \tan \beta \sin i}{\cos \alpha}$
- $\sin \delta = \sin \beta \cos i + \cos \beta \sin i \sin \alpha$
- $i = 23.5^\circ$ (*ecliptic obliquity*)

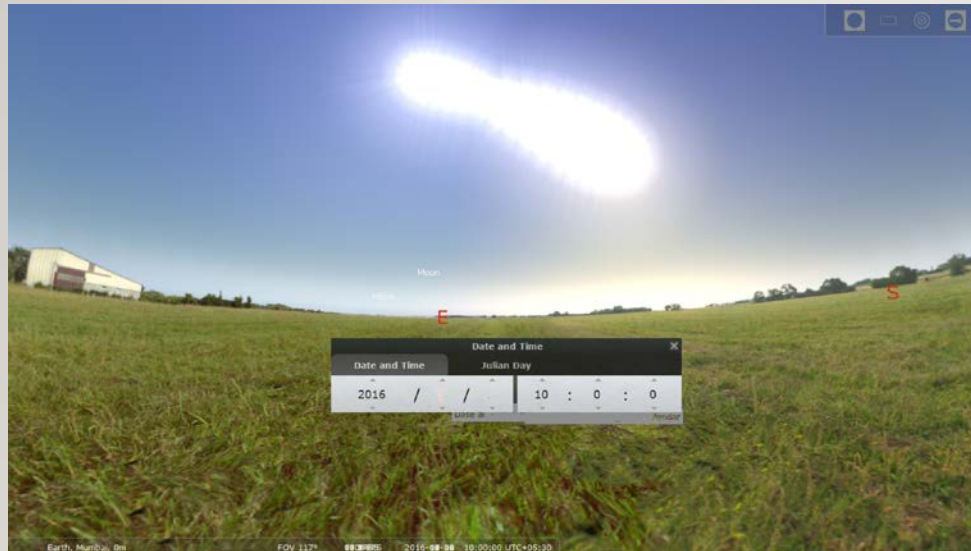
TYPES OF SUN

- **True Sun** is defined by Hour angle +12 h of sun, It is the time shown by sundial
- **Mean Sun** is defined as hour angle of sun assuming it moves with constant angular velocity
- Difference in mean sun and true sun can be up to 16 mins. This is due to
 - Orbit is elliptical
 - Obliquities of ecliptic
 - Equation of time = (True – Mean) Solar Time
- One of the visible effect is that Sunrise in early January to a nearly fixed time yet day length is increasing

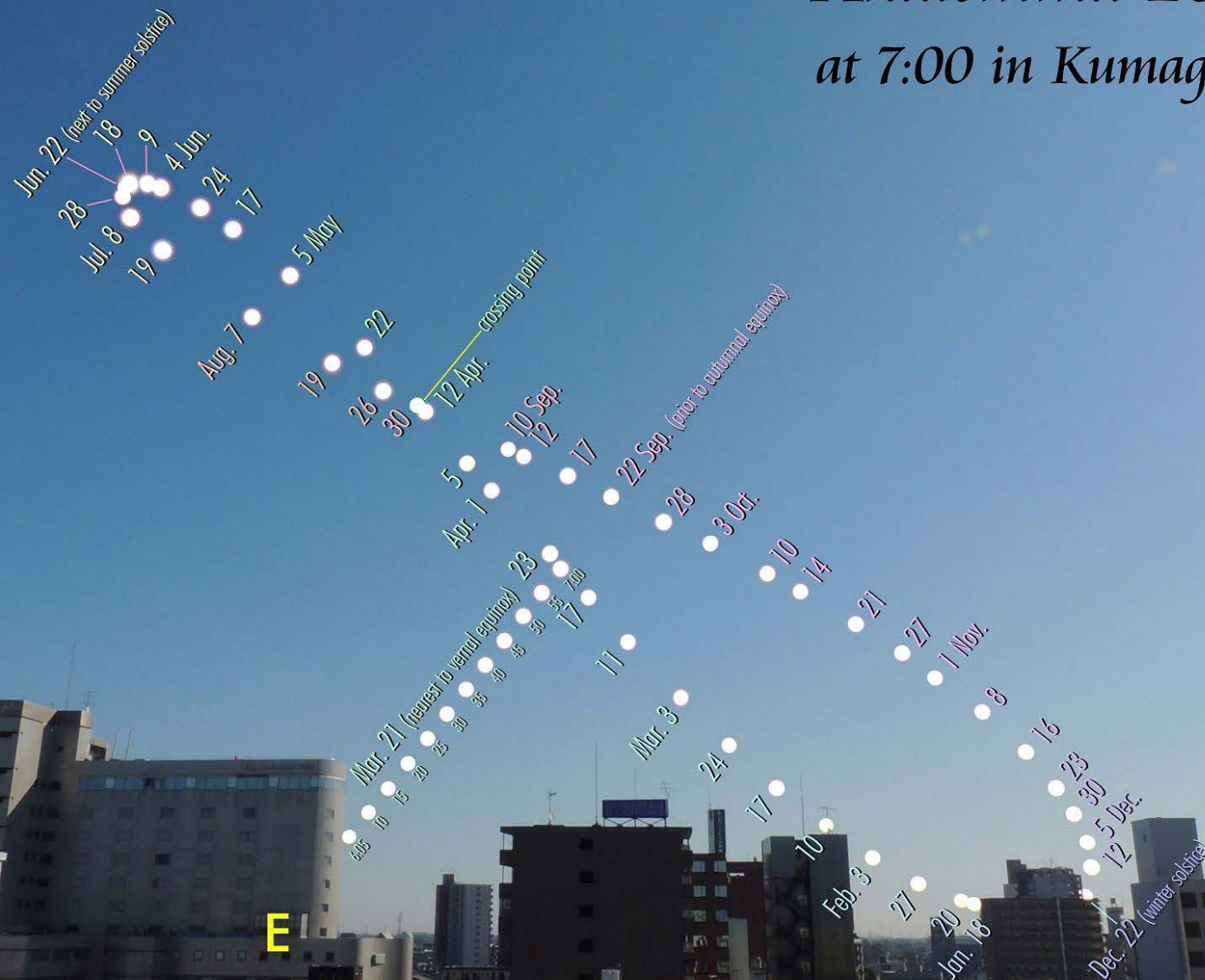
SUN'S ANALEMMA

- If you daily picture sun at same time everyday you would get a 8 like figure

SUN'S ANALEMMA



Analemma 2013
at 7:00 in Kumagaya



THANK YOU

