IEEE 1999 International Conference on Power Electronics and Drive Systems, PEDS'99, July 1999, Hong Kong.

Analysis of Single-Phase Boost Power Factor Correction (PFC) Converter

Liu Ping , Meng Yu , Kang Yong , Zhang Hui and Chen Jian , *Senior Member, IEEE*

Department of Electrical Engineering

Huazhong University of Science and Technology

Wuhan,Hubei, P.R.C (430074)

Abstract – In this paper, the single-phase boost power factor correction converter is discussed in detail. With the Fourier series analysis of the general model , the low frequency model and high frequency model are proposed. From the low frequency model, the expression of the duty cycle, the boundary between continuous current mode(CCM) and discontinuous current mode(DCM), the study of the cusp distortion and the ripple output voltage are developed. While the high frequency components of the input current are described basing on the high frequency model. Theoretical results are demonstrated by the simulation and the experiment.

I . INTRODUCTION

Harmonic pollution in power systems caused by power converters has been taken a great consideration. Conventional off-line switch-mode power supplies draw pulsating AC line current, resulting in low power factor, high THD (Total Harmonic Distortion) and high RMS line current. Switching power utilities with diodes bridge rectifiers have a power factor less than 0.7. The harmonic current generated by converter will distort the AC line voltage and cause power disturbance. The high RMS line current also places a high stress on the switching devices. To solve this problem, several methods for power factor correction (PFC) converters have been proposed in recent years.

Because the boost converter (Fig.1) has many advantages such as : the continuous current and that the current waveform can be shaped to trace the desired waveform. So it is widely used in the field of PFC. During the past few years, several technical papers have been published on this subject. Most of these papers focus on circuit topologies[1,3,4] and control methods[2,5,6,7,8] .

Ignoring various control strategies, there are some inherent characteristics existing in the converter. The characteristics are very important to design the converter. This is why this paper is presented. In this paper, the general model of the boost PFC circuit is described. Based on the Fourier analysis, the low frequency model and the high frequency model are obtained. From the former, the expression of the duty cycle, the boundary between continuous current mode(CCM)and discontinuous current mode(DCM), the depiction of the cusp distortion are presented. From the later, the high frequency harmonic components are analyzed.

0-7803-5769-8/99/$10.00©1999IEEE

II . MODEL OF THE BOOST PFC CONVERTER

The circuit will be analyzed under the following assumptions:

(1)The AC line voltage is the sinusoidal voltage $U_m \sin\omega_1 t$. U_m is the amplitude and ω_1 is the line angular frequency.

(2) The switch "S" operates at the constant frequency

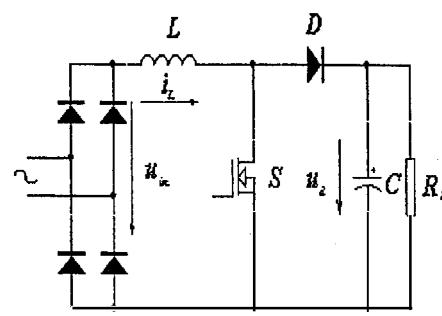

Fig.1 Boost PFC Converter

f_S. f_S is much higher than the line frequency f_1 and f_S / f_1 is an integer.

(3) The inductor L is linear. Saturation is not considered.

(4) The conduction loss is neglected. The average input power is equal to the average output power.

Therefore, the general model can be expressed as follows:

$$\left.\begin{aligned} L\frac{di_L}{dt} &= u_{in} - D_{off}u_d \\ C\frac{du_d}{dt} &= D_{off}i_L - \frac{u_d}{R_S} \end{aligned}\right\} \qquad (1)$$

Where $u_{in} = U_m|\sin\omega_1 t|$, D_{off} is the switching function of "S". $D_{off} = 1$ when "S" is closed, otherwise $D_{off} = 0$ while "S" is opened. The relationship between D_{off} and the duty cycle D_{on} is $D_{on} = 1 - D_{off}$. But this model just gives a segmented solution. In order to obtain continuous equations to describe the converter, Fourier analysis is employed in analyzing the model.

The Fourier series within one period of the variables in the model are

$$D_{off}(\omega_S t) = D_{off(l)} + \sum_{n=1}^{\infty} D_{off(n)} \sin(n\omega_S t + \alpha_n)$$
$$= D_{off(l)} + D_{off(h)} \tag{2}$$

$$i_L(\omega_S t) = i_{L(l)} + \sum_{n=1}^{\infty} i_{L(n)} \sin(n\omega_S t + \alpha_n)$$
$$= i_{L(l)} + i_{L(h)} \tag{3}$$

$$u_d(\omega_S t) = u_{d(l)} + \sum_{n=1}^{\infty} u_{d(n)} \sin(n\omega_S t + \alpha_n)$$
$$= u_{d(l)} + u_{d(h)} \tag{4}$$

where $\omega_s = 2\pi f_s$

In the subscripts, "(l)" expresses the low frequency component, "(h)"expresses the high frequency component. In the equations, the first items are the average values of the variables in a switching period. They indicate the low frequency components of the variables. The second items are the high frequency components of the variables.

Thus, the model can be divided into two models. One is the low frequency model (5), the other is the high frequency model (6).

$$\left.\begin{aligned} L\frac{di_{L(l)}}{dt} &= u_{in} - D_{off(l)}u_{d(l)} \\ C\frac{du_{d(l)}}{dt} &= D_{off(l)}i_{L(l)} - u_{d(l)}/R_S \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} L\frac{di_{L(h)}}{dt} &= -D_{off(h)}u_{d(l)} - D_{off(h)}u_{d(h)} - D_{off(l)}u_{d(h)} \\ C\frac{du_{d(h)}}{dt} &= D_{off(h)}\left[i_{L(l)} + i_{L(h)}\right] - u_{d(h)}/R_S \end{aligned}\right\} \tag{6}$$

III. THE EXPRESSION OF THE DUTY CYCLE

Assuming that the average input current has traced the desired sinusoidal waveform $I_m \sin(\omega_1 t)$. So $i_{L(l)} = I_m|\sin(\omega_1 t)|$ in a half sinusoidal period which begins on $t = 0$.

$$i_{L(l)} = I_m \sin\omega_1 t \qquad \omega_1 t \in [0,\pi] \tag{7}$$

So (5) can be rewritten as:

$$\left.\begin{aligned} L\frac{dI_m \sin\omega_1 t}{dt} &= -D_{off(l)}u_{d(l)} + u_{in} \\ C\frac{du_{d(l)}}{dt} &= D_{off(l)}I_m \sin\omega_1 t - u_{d(l)}/R_S \end{aligned}\right\} \tag{8}$$

$$D_{off(l)} = \frac{u_{in} - I_m L\omega_1 \cos\omega_1 t}{u_{d(l)}} \tag{9}$$

The Fourier series of $u_{d(l)}$ is $u_{d(0)} + \sum_{n=1}^{f_S/f_1-1} U_{d(n)} \sin n\omega_1 t$.

Generally, the output filter capacitor is large enough to guarantee that the peak value of the ripple voltage is much less than the DC output voltage. Approximately

$$D_{off(l)} = \frac{\sqrt{U_m^2 + I_m^2 L^2 \omega_1^2}}{u_{d(0)}} \sin(\omega_1 t - \theta)$$
$$\theta = arctg\frac{I_m L\omega_1}{U_m} \tag{10}$$

From the equations ,the output voltage can be obtained as:

$$u_d = \sqrt{\frac{U_m I_m R_S}{2} + \frac{\sqrt{(U_m I_m)^2 + (I_m^2 L\omega_1)^2}}{4u_{d(0)}\omega_1 C}\sin(2\omega_1 t + \pi - arctg\frac{I_m L\omega_1}{U_m})} \tag{11}$$

It is composed of DC component $u_{d(0)} = \sqrt{\frac{U_m I_m R_S}{2}}$ and twice line frequency component $\frac{\sqrt{(U_m I_m)^2 + (I_m^2 L\omega_1)^2}}{4u_{d(0)}\omega_1 C}$.

Note that $0 \le D_{off(l)} \le 1$, the expression of $D_{off(l)}$ is

$$D_{off(l)} = \begin{cases} 0 & 0 \le \omega_1 t \le \varphi \\ \frac{\sqrt{U_m^2 + I_m^2 L^2 \omega_1^2}}{u_{d(0)}}\sin[\omega_1 t - \theta] & \varphi \le \omega_1 t \le \pi \end{cases}$$
$$\varphi = 2\theta \tag{12}$$

The expression of φ is acquired in section V

Then $D_{on(l)}$ can be obtained as :

$$D_{on(l)} = \begin{cases} 1 & 0 \le \omega_1 t \le \varphi \\ 1 - \frac{\sqrt{U_m^2 + I_m^2 L^2 \omega_1^2}}{u_{d(0)}}\sin[\omega_1 t - \theta] & \varphi \le \omega_1 t \le \pi \end{cases} \tag{13}$$

IV THE BOUNDARY BETWEEN CCM(CONTINUOUS CURRENT MODE) AND DCM (DISCONTINUOUS CURRENT MODE)

As we know, the boost converter has two operation modes: CCM(Continuous current mode) and DCM(Discontinuous Current mode). Some important characteristics of boost converter depend on the operation mode of the converter. The boundary between CCM and DCM is based on the output power, the switching frequency and the value of the inductor.

In one switching period, if the value of the current fluctuation is higher than twice of the average current value($\Delta I > 2I_{avg}$) , the converter is operating on DCM (As fig.2).

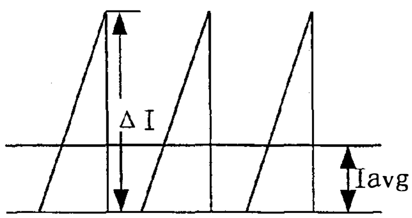

Fig.2 The current waveform under DCM

· According to the expression of the duty cycle, the value of the current fluctuation within one switching period $[t, t+T_S]$ is

$$\frac{U_m \sin\omega_1 t}{L}\left[1-\frac{\sqrt{U_m^2+I_m^2L^2\omega_1^2}}{u_{d(t)}}\sin(\omega_1 t-\theta)\right]\cdot T_S \qquad (14)$$

T_S is the switching period.

It describes that if the current is continuous in a switching period, then:

$$\frac{U_m \sin\omega_1 t}{L}\left[1-\frac{\sqrt{U_m^2+I_m^2L^2\omega_1^2}}{u_{d(t)}}\sin(\omega_1 t-\theta)\right]\cdot T_S \le 2I_m \sin\omega_1 t \quad (15)$$

$$\frac{2LI_m}{U_m}\cdot f_S \ge 1-\frac{\sqrt{U_m^2+I^2L^2\omega_1^2}}{u_{d(t)}}\sin(\omega_1 t-\theta) \qquad (16)$$

If the current is continuous in the whole half line period,. It must be satisfied that:

$$L \ge \frac{U_m}{2I_m f_S}. \qquad (17)$$

$\frac{U_m}{2I_m f_S}$ is the critical value of the inductor.

V. THE CUSP DISTORTION

In a half line period, the boost PFC converter can be equivalent to a simplified circuit as Fig.3 . As shown in the simplified circuit, if i_L is the sinusoidal waveform in phase with U_{in}, the fundamental waveform of U_T is a sinusoidal wave whose phase lags U_{in} or i_L (Fig.4).

But in the field of $\omega_1 t \in \left[0, arctg\frac{I_m L\omega_1}{U_m}\right]$, if waveform of the inductor current is maintained sinusoid, the fundamental component of U_T must be negative (As Fig.4).

Since U_T can't be negative., the inductor current can't trace the sinusoid waveform in the domain of $\omega_1 t \in [0,\varphi]$. There is a cusp distortion after the AC line input has crossed zero volts. In this domain, the switch "S" is always opened or $D_{on} \equiv 1$ to force the current to trace the sinusoid waveform.

Based on the analysis above, the expression of the average inductor current is :

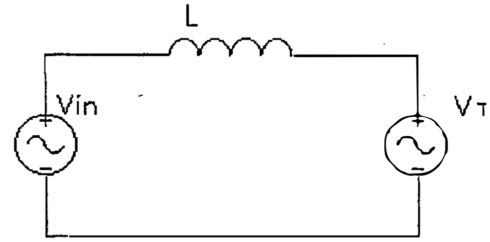

Fig.3 Simplified circuit of the converter

$$i_L = \begin{cases} \int_0^t U_m \sin\omega_1 t dt / L & 0 \le \omega_1 t \le \varphi \\ I_m \sin\omega_1 t & \varphi \le \omega_1 t \le \pi \end{cases} \qquad (18)$$

Because the average inductor current is continuous, so

$$\frac{U_m(1-\cos\varphi)}{L\omega_1} = I_m \sin\varphi \qquad (19)$$

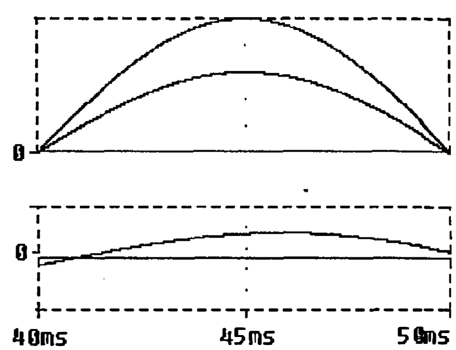

Fig.4 U_{in}, i_L (upper traces) and U_T (lower trace)

φ can be obtained as: $\varphi = 2arctg\frac{I_m L\omega_1}{U_m} = 2\theta$

VI. THE OUTPUT VOLTAGE HARMONICS

The output voltage is composed of three parts—the DC component , the AC components with the frequency range lower than the switching frequency and the AC components with the frequency range equal to or higher than the switching frequency. Generally, the output filter capacitor is large enough to make the third part of the output voltage as former mentioned to be negligible. So the output voltage harmonics are approximately concentrated in the second part .

From (11),the output voltage harmonics is combined of

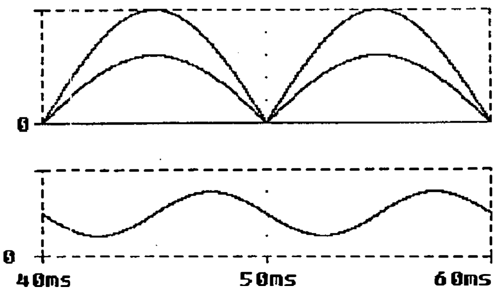

Fig 5 The twice line frequency harmonicof the output voltage(lower trace) , U_{in}, i_L (upper traces)

high frequency harmonics and a AC component with twice the line frequency , the later is the effect of the instantaneous input power which has a component with twice the line frequency(Fig.5).

Generally , the output filter capacitor is large enough to make the high frequency harmonics to be negligible .The ratio between the maximum ripple voltage value and the DC voltage is

$$\frac{\Delta u_{\max}}{u_{d(0)}} = \frac{\sqrt{(U_m I_m)^2 + (I_m^2 L\omega_1)^2}}{2U_m I_m R_S \omega_1 C} \quad (20)$$

From the expression, the ratio is proportional to the output power and inverse proportional to the value of the output filter capacitor.

VII.THE HIGH FREQUENCY CURRENT HARMONICS

The analysis of the high frequency current harmonics includes the high frequency current harmonics and the high frequency voltage harmonics. With aforementioned reasons, the high frequency voltage harmonics are neglected. The output voltage is considered constant in one switching period.

The high frequency model can be simplified as:

$$L\frac{di_{L(h)}}{dt} = -D_{off(h)} u_{d(l)} \quad (21)$$

Assuming that the pulse-width modulation waveform is symmetrical in a switching period.

$$D_{off(h)} = \sum_{n=1}^{\infty} (-1)^n \frac{2}{n\pi} \sin(nd_{off(l)}\pi) \cos n\omega_S t \quad (22)$$

The high frequency harmonics is

$$i_{L(h)} = \sum_{n=1}^{\infty} 2\frac{(-1)^{n+1} \sin(nd_{off(l)}\pi) u_{d(l)}}{Ln^2 \pi\omega_S} \sin(n\omega_s t) \quad (23)$$

For the amplitude of the n th harmonic is proportional to $1/n^2$, the harmonic current expression can be approximated by the first component.

$$i_{L(h)} = 2\frac{\sin(d_{off(l)}\pi) u_{d(l)}}{L\pi\omega_S} \sin(\omega_s t) \quad (24)$$

VIII. SIMULATION AND EXPERIMENT

The computer simulation with PSPICE and the prototype converter based on the PFC integrate circuit UC3854BN are accomplished to demonstrate the principles and expressions.. In all the simulation and experiment results given, the line voltage is 220V,the output DC voltage is 400V, the output power is 2KW, the switching frequency is 100K.So the critical value of the inductor is 0.13mH.

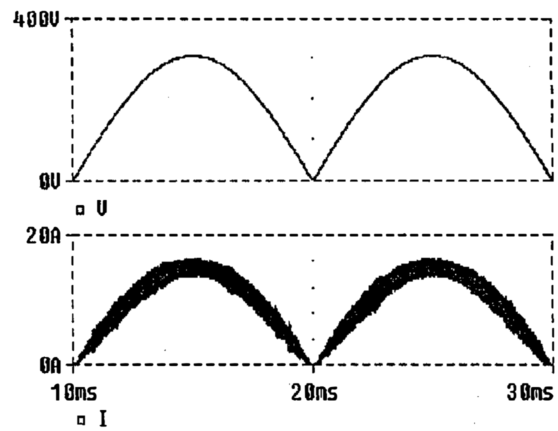

Fig.6 The simulation waveforms of U_T (upper trace) and i_L (nether trace)

In fig.6 the waveforms are U_T and i_L, the value of the inductor is 0.35mH . The current is continuous. The power factor is over 0.997. The current is discontinuous. In fig 7, the value of the inductor is 0.09mH. The current is discontinuous In Fig.8 the

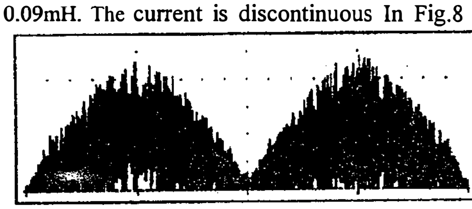

Fig.7 The simulation waveforms of i_L

value of the inductor is 1mH. The current is continuous but the cusp distortion is heavy. The waveforms in fig.9 and fig.10 show U_{in}, i_L,the input voltage and the input current in the experiment. The value of the inductor in the prototype

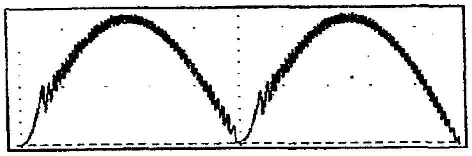

Fig.8 The simulation waveforms of i_L

equipment is 0.35mH. The power factor is approximately 0.999. The results accords with the principles and expressions.

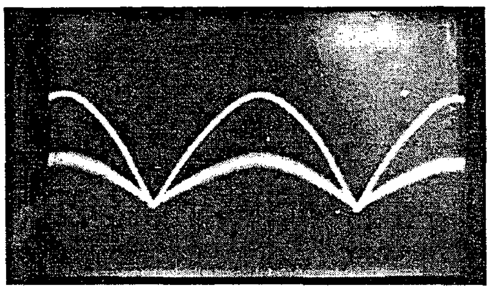

Fig.9 U_{in} and i_L

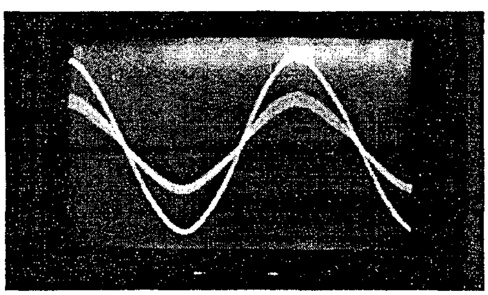

Fig.10 The input voltage and input current

Ⅸ. CONCLUSION

Just the same as the boost converter which works in DC/DC mode , the expression of the duty cycle is the key to analyze the converter working in AC/DC mode .In this paper, the expression of the duty cycle is derived from the low frequency model of the converter. Based on the expression , the boundary between the two operation mode of the converter ,the cusp distortion , the harmonics of the output voltage and the harmonic input current are obtained . With the analysis and deeper acknowledgment of the converter , the optimization of the parameters' design can be reached.

References

(1) John C. salmon "Circuit Topologies for Single-Phase Votage -Doubler Boost Rectifiers" IEEE TRANS. on POWER ELECTRONOCS VOL.8. NO.4. OCTOBER 1993 521~529.

(2)John C .Salmon "Techniques for Minimizing the Input Current Distortion of Current-Controlled Single-Phase Rectifiers" IEEE TRANS. on POWER ELECTRONOCS VOL.8. NO.4. OCTOBER 1993 509~520

(3) Roger J. King " Analysis and Design of an Unusual Unity-Power-Factor Rectifier" IEEE TRANS. on INDUSTRIAL ELECTRONICS VOL.38. NO.2. APRIL 1991 126~134

(4) Eric X.Yang , Yimin Jiang , Guichao Hua , and Fred C. Lee "Isolated Boost Circuit for Power Factor Correction" IEEE PESC'93 Conf. Proc .196~203

(5) Chen Zhou , Raymond B . Ridley ,and Fred C. Lee "DESIGN AND ANALYSIS OF A HYSTERETIC BOOST POWER FACTOR CORRECTIONCIRCUIT" IEEE PESC'90 Conf. Proc .800~807

(6) Seshadri Sivakumar, K.Natarajan, Rajmund Gudelewicz "Control of Power Factor Correcting Boost Converter Without Instantaneous Measurement of Input Current" IEEE TRANS. on POWER ELECTRONOICS. VOL. 10 NO.4 JULY 1995 435~445

(7) Ahmed H . Mitwalli, Steven B .Leeb , George C. Verghese and V. Joseph "An Adaptive Digital Controller for a Unity Power Factor Converter" IEEE TRANS. on POWER ELECTRONOCS VOL.11. NO.2. MARCH 1996 374~382

(8) Mohamed O. Eissa,Steven B Leeb, George C Verghese and Aleksandar M. Stankovic "Fast Controller for a Unity-Power-Factor PWM Rectifier" IEEE TRANS. on POWER ELECTRONOCS VOL.11 NO.1. JANUARY 1996 1~6