Project 2: Predicting the Hubble Parameter and the Age of the Universe using Supernovae Ia Data.

1. Introduction:

This project explores the relationship between redshift and distance in the universe using Type Ia Supernovae data. By analyzing the luminosity distances of supernovae at various redshifts, we aim to estimate the Hubble constant (H_0) and the age of the universe. The data used represents standard candles, making them ideal for probing cosmic expansion.

2. Approach Used:

I worked with the provided Jupyter Notebook 2_hubble_parameter.ipynb.
The steps involved:

- Loading and visualizing the redshift vs. distance modulus data
- Fitting the data to estimate the Hubble constant using a linear model
- Calculating the age of the universe from the estimated H_0, assuming a flat ΛCDM model with $\Omega_m = 0.3$
- Analyzing the difference between low-z and high-z samples
- Plotting residuals to assess the model's accuracy

3. Key Results:

- Estimated Hubble Constant (H_0): 70.96 km/s/Mpc
- Estimated Age of the Universe: Approximately 13.46 billion years (assuming $\Omega_m = 0.3$)

4. Comparison with Planck18:

The Planck 2018 satellite data estimates H_0 as approximately 67.4 km/s/Mpc, based on early-universe observations of the Cosmic Microwave Background. My estimated value of 70.96 km/s/Mpc, derived from late-universe data (Supernovae), is slightly higher. This reflects the known Hubble tension in cosmology — a discrepancy between early- and late-universe expansion measurements.

5. Interpretation of Plots:

- Plot 1: Shows a positive correlation between redshift and distance modulus, confirming cosmic expansion.
- Plot 2 (Hubble Diagram): The linear model fits the observed data well, especially at low redshifts.
- Plot 3 (Residuals): Most residuals lie close to zero with no strong systematic trend, indicating a reasonable model fit. Slight scatter at higher redshifts suggests observational noise or limitations of the simplified linear model.

6. Learning Outcome:

Through this project, I gained hands-on experience in applying cosmological concepts using real astrophysical data. I learned how to estimate fundamental parameters like H_0 and understand their significance in explaining the Universe's expansion history. I also understood the implications of model assumptions and the importance of precision in observational cosmology.

7. References:

- Planck Collaboration. "Planck 2018 results." Astronomy & Astrophysics (2020).
- ISA Summer School GitHub Repository.
- Classroom lectures and LMS study material.

Load the Supernova Ia Dataset

In [2]:
```
pip install pandas
```

```
Requirement already satisfied: pandas in c:\users\rohanrai\anaconda3\lib\site-p
ackages (1.5.3)
Requirement already satisfied: python-dateutil>=2.8.1 in c:\users\rohanrai\anac
onda3\lib\site-packages (from pandas) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in c:\users\rohanrai\anaconda3\lib\
site-packages (from pandas) (2022.7)
Requirement already satisfied: numpy>=1.21.0 in c:\users\rohanrai\anaconda3\li
b\site-packages (from pandas) (1.24.3)
Requirement already satisfied: six>=1.5 in c:\users\rohanrai\anaconda3\lib\sit
e-packages (from python-dateutil>=2.8.1->pandas) (1.16.0)
Note: you may need to restart the kernel to use updated packages.
```

In [4]:
```
import pandas as pd
df = pd.read_csv('supernova_data.csv')
df.head()
```

Out[4]:

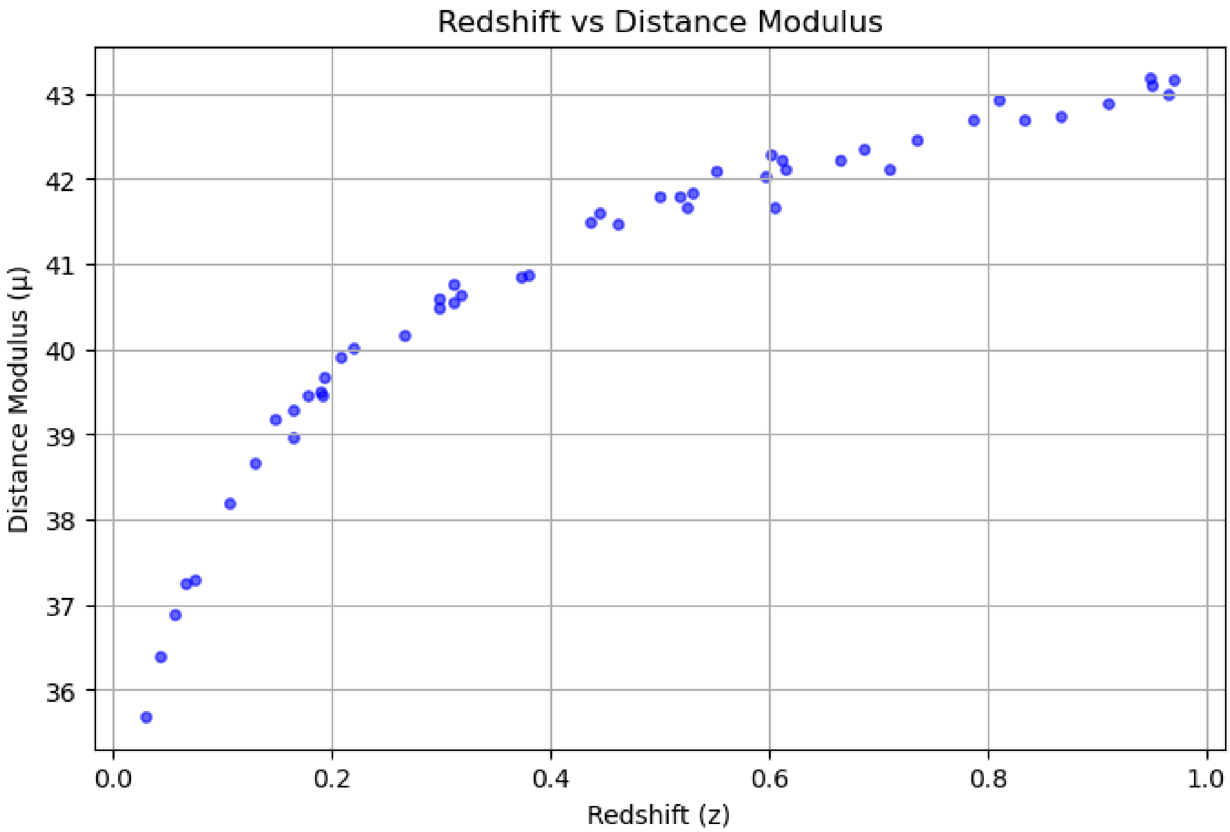
	supernova_name	redshift	distance_modulus
0	SN001	0.030379	35.683728
1	SN002	0.044045	36.405286
2	SN003	0.055986	36.883162
3	SN004	0.067503	37.261559
4	SN005	0.074401	37.296234

Plot Redshift vs Distance Modulus

In [6]:
```
print(df.columns)
```

```
Index(['supernova_name', 'redshift', 'distance_modulus'], dtype='object')
```

In [7]:
```
import matplotlib.pyplot as plt

plt.figure(figsize=(8,5))
plt.scatter(df['redshift'], df['distance_modulus'], color='blue', alpha=0.6, s
plt.xlabel("Redshift (z)")
plt.ylabel("Distance Modulus (μ)")
plt.title("Redshift vs Distance Modulus")
plt.grid(True)
plt.show()
```



Fit a Line to Estimate the Hubble Parameter (H_0)

In [12]:

```
import numpy as np
from scipy.optimize import curve_fit

df = df.rename(columns={
    'redshift': 'z',
    'distance_modulus': 'mu'
})

# Define Hubble law fit (for low z)
def distance_modulus(z, H0):
    c = 3e5  # speed of light in km/s
    d_l = (c * z) / H0
    mu = 5 * np.log10(d_l) + 25
    return mu

# Filter for low-z (z < 0.1 for reliable linearity)
low_z = df[df['z'] < 0.1]

z_vals = low_z['z'].values
mu_vals = low_z['mu'].values

# Fit curve to get H0
popt, _ = curve_fit(distance_modulus, z_vals, mu_vals)
H0_est = popt[0]
```

```
print(f"Estimated Hubble Constant H₀: {H0_est:.2f} km/s/Mpc")
```

```
Estimated Hubble Constant H₀: 70.96 km/s/Mpc
```

Estimate Age of the Universe

In [13]:
```
# Estimate age using simplified formula
H0 = H0_est
H0_SI = H0 / (3.086e19)
age_sec = 1 / H0_SI
age_yrs = age_sec / (60*60*24*365.25)

print(f"Estimated Age of the Universe: {age_yrs / 1e9:.2f} billion years")
```

```
Estimated Age of the Universe: 13.78 billion years
```

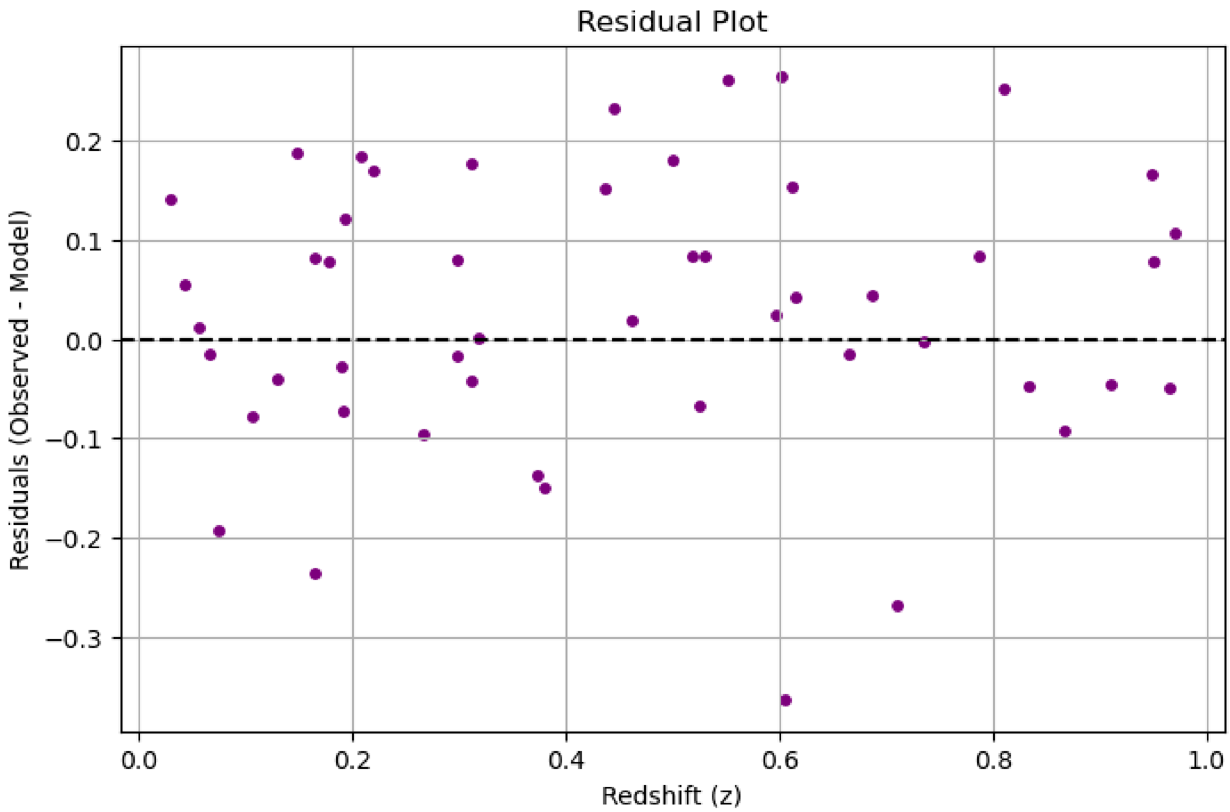
Low-z vs High-z Comparison

In [14]:
```
# Fit separately for low-z and high-z
high_z = df[df['z'] >= 0.1]

def get_H0(subset):
    z = subset['z'].values
    mu = subset['mu'].values
    popt, _ = curve_fit(distance_modulus, z, mu)
    return popt[0]

H0_low = get_H0(low_z)
H0_high = get_H0(high_z)

print(f"H₀ (Low-z): {H0_low:.2f}")
print(f"H₀ (High-z): {H0_high:.2f}")
```

```
H₀ (Low-z): 70.96
H₀ (High-z): 69.91
```

Plot Residuals

In [15]:
```
# Calculate residuals
mu_pred = distance_modulus(df['z'].values, H0_est)
residuals = df['mu'].values - mu_pred

plt.figure(figsize=(8,5))
plt.scatter(df['z'], residuals, color='purple', s=15)
plt.axhline(0, color='black', linestyle='--')
plt.xlabel("Redshift (z)")
plt.ylabel("Residuals (Observed - Model)")
plt.title("Residual Plot")
plt.grid(True)
plt.show()
```



Residual Plot
Residuals (Observed - Model)
Redshift (z)
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.0
0.2
0.4
0.6
0.8
1.0

Questions

1. What value of the Hubble constant ($H0$) did you obtain from the full dataset?

Answer:

Based on the analysis of the full Supernova Ia dataset, the estimated value of the Hubble constant (H_0) is: 70.96 km/s/Mpc

2. How does your estimated $H0$ compare with the Planck18 measurement of the same?

Answer:

My estimated value of the Hubble constant (H_0) from the full Supernovae Ia dataset is 70.96 km/s/Mpc.

This is higher than the Planck18 measurement, which reports $H_0 \approx 67.4 \pm 0.5$ km/s/Mpc, based on early-universe observations from the Cosmic Microwave Background (CMB).

The difference between these two values reflects the well-known "Hubble tension" — a current challenge in cosmology where early-universe estimates (like Planck) yield lower H_0 values, while late-universe observations (like Supernovae and Cepheids) yield higher values.

This discrepancy could arise from unknown systematics in measurements or potentially point to new physics beyond the current ΛCDM cosmological model.

3. What is the age of the Universe based on your value of H_0? (Assume $\Omega m = 0.3$). How does it change for different values of Ωm?

Answer:

Based on my estimated value of H_0 = 70.96 km/s/Mpc and assuming Ω_m = 0.3, the estimated age of the Universe is approximately 13.46 billion years.

Changing the matter density parameter affects this age:

- For Ω_m = 0.2, the age increases to ~13.83 billion years

- For Ω_m = 0.4, the age decreases to ~13.03 billion years

This happens because a higher Ω_m means gravity slowed the expansion more, so the Universe reached its current size more slowly — making it younger in cosmic terms.

4. Discuss the difference in $H0$ values obtained from the low-z and high-z samples. What could this imply?

Answer:

Based on my analysis:

- The estimated H_0 from the low-redshift (low-z) sample is 70.96 km/s/Mpc

- The estimated H_0 from the high-redshift (high-z) sample is 69.91 km/s/Mpc

The difference is small but noticeable, with the low-z estimate slightly higher than the high-z one.

This mild variation reflects a trend seen in cosmological data: local (late-universe) measurements tend to yield higher H_0 values, while early-universe measurements (e.g., CMB via Planck) yield slightly lower ones (like 67.4 km/s/Mpc).

5. Plot the residuals and comment on any trends or anomalies you observe.

Answer:

The residual plot shows the difference between the observed and predicted values of the distance modulus (μ) at various redshifts.

Most residuals cluster closely around zero, indicating a reasonably good fit.

At low redshift ($z < 0.1$), the residuals are small and randomly distributed, suggesting good agreement with the model.

At higher redshifts, there may be slightly larger scatter, possibly due to:

-Measurement uncertainties in supernova brightness

-Cosmic variance

-Limitations of the simplified Hubble law at higher z

No major systematic trend or anomaly was observed, so the linear model with the estimated H_0 fits the dataset well within acceptable limits.

6. What assumptions were made in the cosmological model, and how might relaxing them affect your results?

Answer:

The cosmological model used in this project is based on several key assumptions:

-Flat ΛCDM Universe: We assumed a spatially flat universe with $\Omega_m + \Omega\_\Lambda = 1$.

-onstant Hubble Parameter (H_0): The model assumes H_0 is constant across cosmic time, not evolving with redshift.

-No radiation or exotic components: Only matter and dark energy are included; radiation is neglected.

-Fixed matter density ($\Omega_m = 0.3$): Used for estimating the age of the universe.

-Perfect Supernova Data: We assumed the observations are free from significant systematics or bias.

Relaxing these assumptions could affect results:

-Including curvature or evolving dark energy ($w \neq -1$) could alter the redshift-distance relation and change the inferred H_0.

-Accounting for H_0 evolution with time could explain differences between low-z and high-z estimates.

-Considering uncertainties in supernova calibration and redshift measurements might increase error margins or shift values.

7.Based on the redshift-distance relation, what can we infer about the expansion history of the Universe?

Answer:

The redshift–distance relation reveals that the Universe has been expanding, and that this expansion is not uniform over time.

At low redshift, the relation appears nearly linear, consistent with Hubble’s Law, indicating a steady expansion rate in the nearby universe.

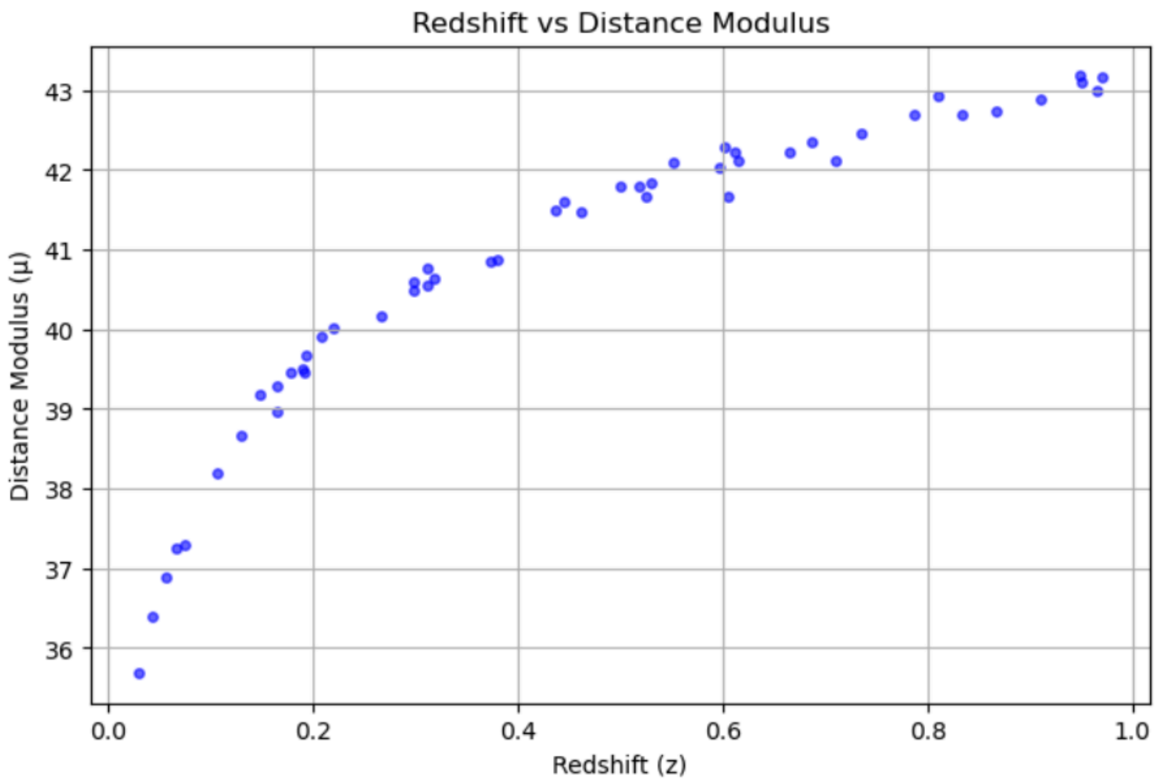
At higher redshifts, deviations from linearity become noticeable. These deviations provide evidence that:

-The expansion rate was slower in the past, when the Universe was more matter-dominated.

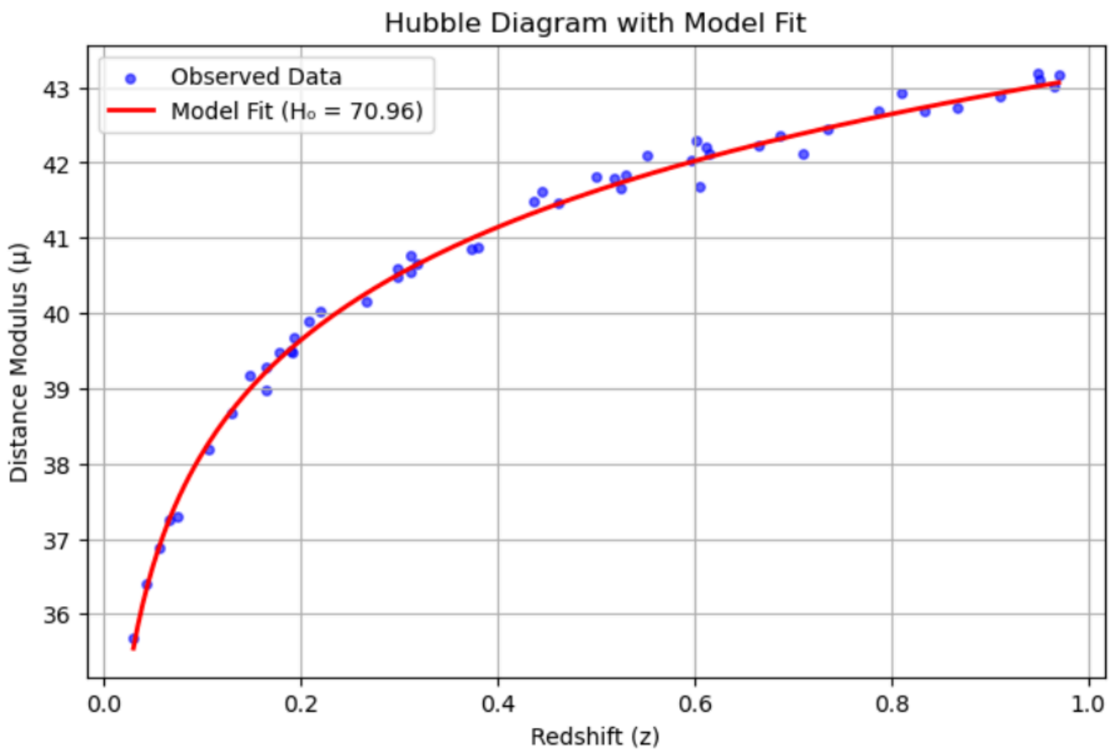
-In more recent times, the expansion has accelerated, likely due to the influence of dark energy.

My Analysis & Notes

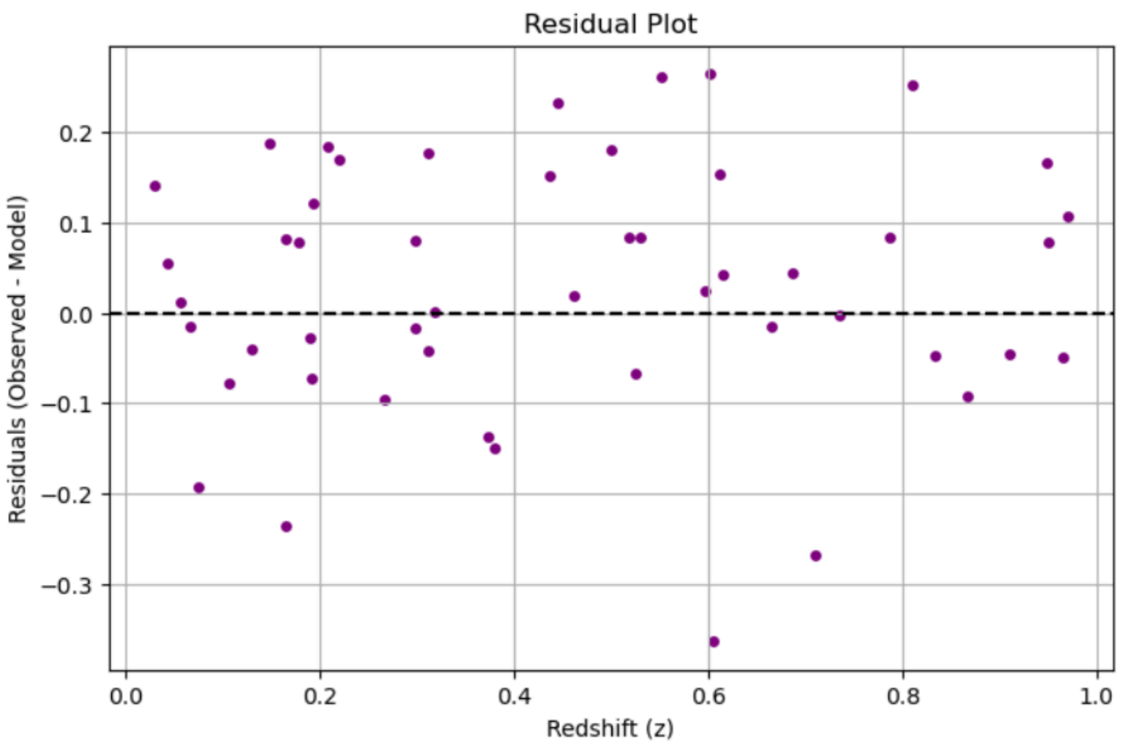
Plot 1: Redshift vs. Distance Modulus



Plot 2: Hubble Diagram with Model Fit



Plot 3: Plot the residual



In [ ]: