

### Assignment-6 Parameter Estimation

$$1) f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$x_1, x_2, \dots, x_n \rightarrow$  Sample of size  $n$

$$\begin{aligned} L(x_1, x_2, x_3, \dots, x_n) &= f(x_1) \cdot f(x_2) \cdot f(x_3) \dots f(x_n) \\ &= \left( \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \right) \cdot \left( \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}} \right) \dots \end{aligned}$$

Taking ln both side

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^n \left( -\frac{(x_i - \mu)^2}{2\sigma^2} \right) \quad \text{--- (1)}$$

Taking derivative w.r.t  $\mu$  of eq (1)

$$\begin{aligned} \frac{\partial \ln(L)}{\partial \mu} &= 0 + \sum_{i=1}^n -\frac{(x_i - \mu)}{\sigma^2} \\ &= \sum_{i=1}^n (x_i - \mu) = 0 \end{aligned}$$

$$= n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu \quad \text{Ans}$$

Hence  $\theta_1 = \mu$  is the sample mean

Taking derivative w.r.t  $\sigma^2$  of eq (1)

$$\begin{aligned} \frac{\partial \ln(L)}{\partial \sigma^2} &= -\frac{n}{2\sigma^2} + \sum_{i=1}^n -\frac{(x_i - \mu)^2}{2(\sigma^2)^2} = 0 \\ -\frac{n}{2\sigma^2} + \sum_{i=1}^n -\frac{(x_i - \mu)^2}{2\sigma^4} &= 0 \end{aligned}$$

$$n = \sum_{i=1}^n \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$

$$\text{Hence } \theta_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2 \quad \text{Ans}$$