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Assigment-6 Parameter Estimation

1) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$x_1, x_2, --- x_n \rightarrow$ Sample of size n

$L(x_1, x_2, x_3, --- x_n) = f(x_1) \cdot f(x_2) \cdot f(x_3) --- f(x_n)$

$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) ----$

Taking ln both side

$\ln(L) = \frac{-n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)$ — (i)

Taking derivative w.r.t μ of eq (i)

$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} \frac{-(2(x_i-\mu))}{2\sigma^2}$

$= \sum_{i=1}^{n} (x_i-\mu) = 0$

$= n\bar{x} - n\mu = 0$

$\bar{x} = \mu$ Ans

Hence $\theta_1 = \mu$ is the sample mean

Taking derivative w.r.t σ^2 of eq (i)

$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2(\sigma^2)^2} = 0$

$-n + \sum_{i=1}^{n} -\frac{(x_i-\mu)}{\sigma^2} = 0$

$n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}$

$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$

Hence $\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$ Ans

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(2) Binomial distribution $\rightarrow {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

log on both side

$$\log L = \sum_{i=1}^{n} \left(\log ({}^{n}C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

Differentiate w.r.t to θ

$$\frac{d(\log(L))}{d\theta} = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta}$$

$$\frac{\sum x_i}{\theta} = n^2$$

$$\boxed{\theta = \frac{\sum x_i}{n^2}}$$ Ans