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# Bayes Theorem

- Bayes Theorem is one of the important theorem for Naive Bayes Algorithm.
- According to Bayes' Theorem ⇒ $\boxed{\underbrace{P(A|B)}_{\text{Posterior}} = \frac{\overbrace{P(B|A)}^{\text{Livelihood}} * \overbrace{P(A)}^{\text{Prior}}}{\underbrace{P(B)}_{\text{Evidence}}}}$

Proving this theorem:-

As, we Know:- $P(A|B) = \frac{P(A \cap B)}{P(B)}$ → ①

$P(B|A) = \frac{P(B \cap A)}{P(A)}$ and $\{P(A \cap B) = P(B \cap A)\}$

$P(A) * P(B|A) = P(A \cap B)$ → ②

Putting the value of $P(A \cap B)$ ② in ①

$$\boxed{P(A|B) = \frac{P(B|A) * P(A)}{P(B)}}$$

∴ Hence Proved...

Q Sample Problem using Bayes Theorem.

Marker Factory

| $M_1$ | $M_2$ | $M_3$ | |
|---|---|---|---|
| (20) | (30) | (50) | ← Manifacture Percentage.. |
| 5% | 3% | 1% | ← Defect Percentage.. |

(i) Randomly One Marker is Picked from Bag and it is Defective too. What is the Probability that it belongs to $M_3$.

Sol: What all things we have given in this Question, we will write them all :-

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Given :- $P(M_1) = \frac{20}{100} \Rightarrow \frac{1}{5}$ , $P(M_2) = \frac{30}{100}$ , $P(M_3) = \frac{5\cancel{0}}{10\cancel{0}} = \frac{1}{2}$

$P(D|M_1) = \frac{1}{20}$ , $P(D|M_2) = \frac{3}{100}$ , $P(D|M_3) = \frac{1}{100}$

To find :- $P(M_3|D) = ?$

Sol. As, we Know :- According to Bayes theorem $\Rightarrow P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$

So :- $P(M_3|D) = \frac{P(D|M_3) * P(M_3)}{P(D)}$

$P(D|M_3) = \frac{1}{100}$ , $P(M_3) = \frac{1}{2}$

To Calculate $P(D) = ?$

$P(D) = P(D \cap M_1) + P(D \cap M_2) + P(D \cap M_3)$

$P(D) = [P(D|M_1) * P(M_1)] + [P(D|M_2) * P(M_2)] + [P(D|M_3) * P(M_3)]$

As, we Know :- $P(D \cap M_1) = P(D|M_1) * P(M_1)$

$\Rightarrow \left[\frac{1}{20} \times \frac{1}{5}\right] + \left[\frac{3}{100} \times \frac{30}{100}\right] + \left[\frac{1}{100} \times \frac{1}{2}\right]$

as :- $P(D|M_1) = \frac{P(D|M_1)}{P(M_1)}$

$\Rightarrow \frac{1}{100} + \frac{9\cancel{0}}{1000\cancel{0}} + \frac{1}{200} \Rightarrow 0.01 + 0.009 + 0.005 \Rightarrow \boxed{0.024}$ Answer :-

$P(D|M_2) = \frac{P(D \cap M_2)}{P(M_2)}$

$\Rightarrow \frac{0.010 + 0.5}{0.024} \Rightarrow \frac{0.510}{0.024}$ Answer $\Rightarrow \boxed{21.250}$

$P(D|M_3) = \frac{P(D \cap M_3)}{P(M_3)}$

(ii) Randomly One Marker is picked from the bag and it is ~~def~~ defective. then what is the Probability that it belongs to $M_2$.

solution. Given $\Rightarrow P(M_1) \Rightarrow \frac{20}{100} = \frac{1}{5}$ , $P(M_2) = \frac{30}{100} = \frac{3}{10}$ , $P(M_3) = \frac{5\cancel{0}}{10\cancel{0}} = \frac{1}{2}$

$P(D|M_1) = \frac{5}{100} \Rightarrow \frac{1}{20}$ , $P(D|M_2) = \frac{3}{100}$ , $P(D|M_3) = \frac{1}{100}$

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To find $\Rightarrow P(M_2|D) = ?$

As per the Bayse theorem.

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

So:- $P(M_2|D) = \frac{P(D|M_2) * P(M_2)}{P(D)}$

$P(D|M_2) = \frac{3}{100}$ , $P(M_2) = \frac{3}{10}$

Calculating the value of $P(D) = P(D \cap M_1) + P(D \cap M_2) + P(D \cap M_3)$

$P(D) \Rightarrow P(D|M_1) = \frac{P(D \cap M_1)}{P(M_1)} + P(D|M_2) = \frac{P(D \cap M_2)}{P(M_2)} + P(D|M_3) = \frac{P(D \cap M_3)}{P(M_3)}$

$\Rightarrow [P(D|M_1) * P(M_1)] + [P(D|M_2) * P(M_2)] + [P(D|M_3) * P(M_3)]$

$\Rightarrow \left[\frac{1}{20} \times \frac{1}{5}\right] + \left[\frac{3}{100} \times \frac{3}{10}\right] + \left[\frac{1}{100} \times \frac{1}{2}\right]$

$\Rightarrow \frac{1}{100} + \frac{9}{1000} + \frac{1}{200} \Rightarrow 0.010 + 0.009 + 0.005$

$\Rightarrow \boxed{0.024}$

$P(D) = 0.024$ , Now $P(M_2|D) = \frac{\frac{3}{100} \times \frac{3}{10}}{0.024} \Rightarrow \frac{0.03 * 0.3}{0.024}$

So:- $\boxed{0.375}$

$\Rightarrow \frac{0.009}{0.024} \Rightarrow \boxed{0.375}$

Ans Therefore the Probability of Picking a Marker which is Defective and come from $M_3$ is 0.375 respectively...