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Batch Gradient Descent [n dimensions]

- We have already learned about the Batch Gradient Descent but for 2 dimensional data. But actually in real time we generally got the data which is of "n" dimensional.

- So to handle the "n" dimensional data we have to calculate "n" slope values and "n" intercept values.

- Understanding this, with taking a 4 Dimensional Data.

So, this is our data. Shape ⇒ (3,4)
⇒ 3 rows, 3+1 → Output Column, → Input Columns

cgpa (X_1)	iq (X_2)	gender (X_3)	package(lpa) (Y_1 or Y)
9.8	50	M	5.56
6.3	90	F	4.62
7.5	70	M	5.39

X → cgpa, iq, gender; → Y package. Data

$$\boxed{\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3}$$

For "N" input Columns..

$$\boxed{\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \dots \beta_n X_n}$$ General Formula...

$$\boxed{\text{Loss Function} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}$$

where ⇒ n = number of rows.
Y_i = Y_actual value.
$\hat{Y}_i$ = Y_Predicted value.

$$\Rightarrow \boxed{\frac{1}{n}\sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)}$$

Q Now, How to Calculate this β_0, β_1 and $\beta_2 \dots \beta_n$..??

Ans β_0 ⇒ Intercept term..

$\left.\begin{matrix}\beta_1 \\ \vdots \\ \beta_n\end{matrix}\right\}$ ⇒ Slopes or Coefficient terms..

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Calculating the β_0 Intercept..

$$\frac{\partial L}{\partial \beta_0} = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)^2$$

$\Downarrow$ As per the Data given

$$\frac{\partial L}{\partial \beta_0} = \left[\sum_{i=1}^{3} (Y_{11} - \beta_0 - \beta_1 X_{11} - \beta_2 X_{12} - \beta_3 X_{13})^2 + (Y_{21} - \beta_0 - \beta_1 X_{21} - \beta_2 X_{22} - \beta_3 X_{23})^2 + (Y_{31} - \beta_0 - \beta_1 X_{31} - \beta_2 X_{32} - \beta_3 X_{33})^2\right]$$

$$\frac{\partial L}{\partial \beta_0} = \frac{-2}{3} \sum_{i=1}^{3} (Y_i - \hat{Y}_i) \Rightarrow \boxed{\frac{\partial L}{\partial \beta_0} = \frac{-2}{n} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)} \rightarrow 1$$

Calculating the β_1 Slope..

$$\frac{\partial L}{\partial \beta_1} = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)^2$$

$\Downarrow$ As per the Data given

$$\frac{\partial L}{\partial \beta_1} = \left[\sum_{i=1}^{3} (Y_{11} - \beta_0 - \beta_1 X_{11} - \beta_2 X_{12} - \beta_3 X_{13})^2 + (Y_{21} - \beta_0 - \beta_1 X_{21} - \beta_2 X_{22} - \beta_3 X_{23})^2 + (Y_{31} - \beta_0 - \beta_1 X_{31} - \beta_2 X_{32} - \beta_3 X_{33})^2\right]$$

$$\frac{\partial L}{\partial \beta_1} = \frac{-2}{3} (Y_i - \hat{Y}_i) X_{i1} \Rightarrow \boxed{\frac{\partial L}{\partial \beta_1} = \frac{-2}{n} \sum_{i=1}^{n} (Y_i - \hat{Y}_i) X_{i1}} \rightarrow 2$$

Where $\Rightarrow$ n = number of rows

$X_{i1} \Rightarrow$ All the values of First Column

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Calculating the β_2 Slope ..

So, $\frac{\partial L}{\partial \beta_2} = \frac{-2}{3} \sum_{i=1}^{n} (Y_i - \hat{Y}_i) X_{i2} \Rightarrow \boxed{\frac{-2}{n} \sum_{i=1}^{n} (Y_i - \hat{Y}_i) X_{i2}} \rightarrow (3)$

where :-

$X_{i2} \Rightarrow$ All the values of Second Column.

$n \Rightarrow$ Number of rows.

So from (1), (2) and (3)

[For $\beta = \beta_0, \beta_1, \beta_2 \ldots \beta_m$]

$$\boxed{\beta_m = \frac{-2}{n} \sum_{i=1}^{n} (Y_i - \hat{Y}_i) X_{im}}$$

For "m" columns...

where :-

m = number of columns

n = number of rows

X_{im} = At every row it's column values at every iteration.

$$\boxed{\beta_m = \frac{-2}{n} \sum_{i=1}^{n} (Y_i - \hat{Y}_i) X_{im}}$$

—X—X—