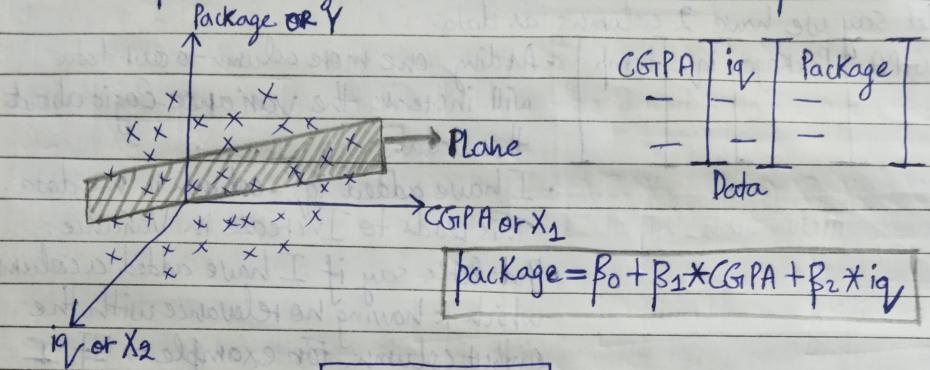


Multiple Linear Regression

- In Multiple Linear Regression there are more than one input columns...



- Equation of Plane $\Rightarrow mX_1 + hX_2 + b$ which is also standardly written as :-

$$\hat{Y} \Rightarrow \beta_0 + \beta_1 X_1 + \beta_2 X_2 \quad \text{where } \beta_0 = b$$

$$\beta_1 = m$$

$$\beta_2 = h$$

- This equation in General written as :- $\hat{Y} = \beta_0 + \sum_{i=1}^n \beta_i X_i$

Q How to calculate the $\beta_0, \beta_1, \beta_2, \dots, \beta_n$?

Ans

$$\begin{array}{c|ccc} \hat{Y}_1 & \beta_0 & \beta_1 X_{11} & \beta_2 X_{12} & \beta_3 X_{13} \\ \hat{Y}_2 & \beta_0 & \beta_1 X_{21} & \beta_2 X_{22} & \beta_3 X_{23} \\ \hat{Y}_3 & \beta_0 & \beta_1 X_{31} & \beta_2 X_{32} & \beta_3 X_{33} \\ \hat{Y}_4 & \beta_0 & \beta_1 X_{41} & \beta_2 X_{42} & \beta_3 X_{43} \\ \vdots & & \vdots & \vdots & \\ \hat{Y}_{100} & \beta_0 & \beta_1 X_{1001} & \beta_2 X_{1002} & \beta_3 X_{1003} \end{array}$$

$\hat{Y}_{\text{Predicted}}$
Matrix

Date.....

Assuming that now, we have columns = "m" and rows = "n".
In that case, the Matrix will be written as:-

$$\begin{array}{c|c}
 \begin{matrix} \hat{Y}_1 \\ Y_2 \\ Y_3 \\ \vdots \\ \vdots \\ \vdots \\ Y_n \end{matrix} & \Rightarrow \begin{matrix} B_0 & B_1 X_{11} & B_2 X_{12} & B_3 X_{13} & \dots & B_m X_{1m} \\ B_0 & B_1 X_{21} & B_2 X_{22} & B_3 X_{23} & \dots & B_m X_{2m} \\ B_0 & B_1 X_{31} & B_2 X_{32} & B_3 X_{33} & \dots & B_m X_{3m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ B_0 & B_1 X_{n1} & B_2 X_{n2} & B_3 X_{n3} & \dots & B_m X_{nm} \end{matrix} \\
 \text{Matrix_1} & \qquad \qquad \qquad \text{Matrix_2} \\
 \hline
 \begin{matrix} 1 & X_{11} & X_{12} & X_{13} & \dots & X_{1m} \\ 1 & X_{21} & X_{22} & X_{23} & \dots & X_{2m} \\ 1 & X_{31} & X_{32} & X_{33} & \dots & X_{3m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{n1} & X_{n2} & X_{n3} & \dots & X_{nm} \end{matrix} & \left[\begin{matrix} B_0 \\ B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_m \end{matrix} \right] \\
 \text{Matrix_a or } X & \qquad \qquad \qquad \text{Matrix_b or } B
 \end{array}$$

$$\therefore \boxed{\text{Matrix_a} * \text{Matrix_b} = \text{Matrix_2}} \Rightarrow \boxed{\hat{Y} = XB}$$

$$e = \begin{bmatrix} Y_1 - \hat{Y}_1 \\ Y_2 - \hat{Y}_2 \\ Y_3 - \hat{Y}_3 \\ \vdots \\ Y_n - \hat{Y}_n \end{bmatrix} \quad (n \times 1)$$

$$\text{As per the Loss Function} \Rightarrow \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad \text{"Mean Squared Error"}$$

This Loss Function can be rewritten as:-

$$\text{Loss Function} = \boxed{e^T e} \quad (n \times 1)$$

$$\boxed{[(Y_1 - \hat{Y}_1) \ (Y_2 - \hat{Y}_2) \ (Y_3 - \hat{Y}_3) \ \dots \ (Y_n - \hat{Y}_n)]^T = e^T} \quad (n \times 1)$$

$$\boxed{e^T \cdot e = [(Y_1 - \hat{Y}_1)^2 + (Y_2 - \hat{Y}_2)^2 + \dots + (Y_n - \hat{Y}_n)^2] \quad (1 \times 1)}$$

Date.....

So finally the "B" value or Formula is derived as :-

$$\beta = (X^T X)^{-1} \cdot X^T Y$$

where : $X = X_{\text{train}}$
 $Y = Y_{\text{train}}$

let say we have 3 dimensional data i.e.

CGPA	iq	package
X_1	X_2	Y

- For this data the β will provides you the 3 values :-

① β_0

Intercept

② $\beta_1 X_1$

Coefficient

③ $\beta_2 X_2$

Coefficient