Question 6

To derive the gradient of the regularized error function, we need to compute the derivative of E(w) with respect to w.

The regularized linear regression objective function is defined as follows:

$$E\left(w\right) \;=\; \frac{1}{2}\sum_{n=1}^{N}\left(t_i \;-\; y\left(x_n, w\right)\right)^2 \;+\; \frac{\lambda}{2}|w|^2$$

Which we will derivate to get:

$$\frac{\partial E\left(w\right)}{\partial w} = \frac{\partial\left(\frac{1}{2}\sum_{n=1}^{N}\left(t_i \;-\; y\left(x_n, w\right)\right)^2 \;+\; \frac{\lambda}{2}|w|^2\right)}{\partial w}$$

Using the chain rule, we can split this into two parts:

$$\frac{\partial E\left(w\right)}{\partial w} = \frac{\partial\left(\frac{1}{2}\sum_{i=1}^{N}\left(t_n \;-\; y\left(x_n, w\right)\right)^2\right)}{\partial w} + \frac{\partial\left(\frac{\lambda|w|^2}{2}\right)}{\partial w}$$

Derivative of the Mean Squared Error (MSE) term and Derivative of the L2 regularization term is and using the chain rule we have:

$$\frac{\partial E\left(w\right)}{\partial w} = \frac{2\sum_{i=1}^{N}x_i\left(t_n \;-\; y\left(x_n, w\right)\right)}{2} + \frac{2\lambda w}{2}$$

The derivative of the L2 regularization term is straightforward:

$$\frac{\partial E\left(w\right)}{\partial w} = -\sum_{i=1}^{N}\Phi(t_n \;-\; \Phi) \;+\; \lambda w$$

Let

$$\Phi = y\left(x_n, w\right)$$

And to find the ideal weights, let

$$\frac{\partial E\left(w\right)}{\partial w} = 0$$

so by rearranging the terms

$$w \;=\; \left(\lambda I \;+\; \Phi^T\Phi\right)\Phi^T t$$

In [1]:
```python
import numpy as np
from sklearn.model_selection import train_test_split
```

In [2]:
```python
def f(x):
    return np.sin(5*np.pi*x)/(1+2*x)
```

```python
def make_additive_noise_data(n, f, a, b, noise=0.1**0.5, random_state=None):
    RNG = np.random.default_rng(random_state)
    x = RNG.uniform(a, b, size=(n, 1))
    y = f(x) + RNG.normal(0, noise, size=(n, 1))
    return x, y
```

In [3]:
```python
X_train,y_train = make_additive_noise_data(100, f, -0.3, 0.3, random_state=None)
```

In [4]:
```python
X_test,y_test = make_additive_noise_data(10000, f, -0.3, 0.3, random_state=None)
```

In [13]:
```python
class DirectRidgeRegressor:

    def __init__(self,lambd = .1):
        self.lambd = lambd

    def fit(self, x, y):
        I = np.identity(x.shape[1]) #defining the identity matrix
        rhs = x.T.dot(y)
        lhs = x.T.dot(x) + self.lambd * I    #adding the regularization parameter
        self.coef_ = np.linalg.solve(lhs, rhs)  # find optimal weights w (coefficien
        return self

    def predict(self, x):
        return x.dot(self.coef_)
```

In [14]:
```python
import numpy as np
from sklearn.pipeline import make_pipeline
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import Ridge
from sklearn.metrics import mean_squared_error

lambda_values = np.geomspace(10**-10, 0.1, 101, endpoint=True)  #generating the lamb

results = []
for lambda_param in lambda_values:  #looping through all the lambda values
    model = make_pipeline(        # Creating a pipeline with polynomial feature trans
        PolynomialFeatures(degree=5),
        DirectRidgeRegressor(lambda_param)
    )

    model.fit(X_train, y_train)  #Fitting the data with the training data
    y_pred = model.predict(X_test)  #making the predictions on the test data
    mse = mean_squared_error(y_test, y_pred)  #calculating the mse for the lambda va
    results.append({'lambda': lambda_param, 'mse': mse})

for result in results:
    print(f"Lambda: {result['lambda']}, MSE: {result['mse']}")
```

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```

In [15]:
```python
import numpy as np
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import PolynomialFeatures
from sklearn.metrics import mean_squared_error

lambda_values = np.geomspace(10**-10, 0.1, 101, endpoint=True)  #generating the lamb
results = []

num_repetitions = 10

for _ in range(num_repetitions):
    random_indices = np.random.choice(len(X_train), size=20, replace=False)  # selec
    X_train_repeated = X_train[random_indices]
    y_train_repeated = y_train[random_indices]

    for lambda_param in lambda_values:  #looping through all the lambda values
        model = make_pipeline(
            (PolynomialFeatures(degree=5)),  #creating a pipeline with polynomial fe
            (DirectRidgeRegressor(lambda_param))
        )
        model.fit(X_train_repeated, y_train_repeated) #fitting the model
        y_pred = model.predict(X_test)  #predicting the test data
        mse = mean_squared_error(y_test, y_pred)  # calculating the mse for each lam
        results.append({'lambda': lambda_param, 'mse': mse})

for result in results:
    print(f"Lambda: {result['lambda']}, MSE: {result['mse']}")
```

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Lambda: 0.04365158322401665, MSE: 0.87413139861324
Lambda: 0.053703179637025325, MSE: 0.8832440156435565
Lambda: 0.06606934480075964, MSE: 0.8918181449260917
Lambda: 0.0812830516164099, MSE: 0.8997087695522783
Lambda: 0.1, MSE: 0.9068539085076845
```

In [16]:
```python
import matplotlib.pyplot as plt

lambda_values = np.geomspace(10**-10, 0.1, 101, endpoint=True)

training_mses = []  #initializing a list
testing_mses = []  #initializing a list

reps = 10

for iii in range(reps):
    idx = np.random.choice(len(X_train), size=20, replace=False)
    X_train_new = X_train[idx]
    y_train_new = y_train[idx]

    training_mses_rep = [] #initializing a list
    testing_mses_rep = [] #initializing a list

    for lambda_param in lambda_values:
        model = make_pipeline(          # Creating a pipeline
            (PolynomialFeatures(degree=5)),
            (DirectRidgeRegressor(lambda_param))
        )
        model.fit(X_train_new, y_train_new)  #fitting the data
        y_train_pred = model.predict(X_train_new)    #making training predictions
        training_mse = mean_squared_error(y_train_new, y_train_pred) #calculating ms
        training_mses_rep.append(training_mse)
        y_test_pred = model.predict(X_test)  #making testing predictions
        testing_mse = mean_squared_error(y_test, y_test_pred)  #calculatng mse for t
        testing_mses_rep.append(testing_mse)
    training_mses.append(training_mses_rep)
    testing_mses.append(testing_mses_rep)

training_mses = np.array(training_mses) #changinf to numpy array
testing_mses = np.array(testing_mses)
mean_tr_mses = np.mean(training_mses, axis=0)
mean_tst_mses = np.mean(testing_mses, axis=0)

plt.figure(figsize=(16, 8))  #plotting plot
plt.plot(np.log(lambda_values), np.log(mean_tr_mses), label='Training MSE', color='b
plt.plot(np.log(lambda_values), np.log(mean_tst_mses), label='Testing MSE', color='r
plt.xlabel('Log Lambda')
plt.ylabel('Log Mean Squared Error')
```
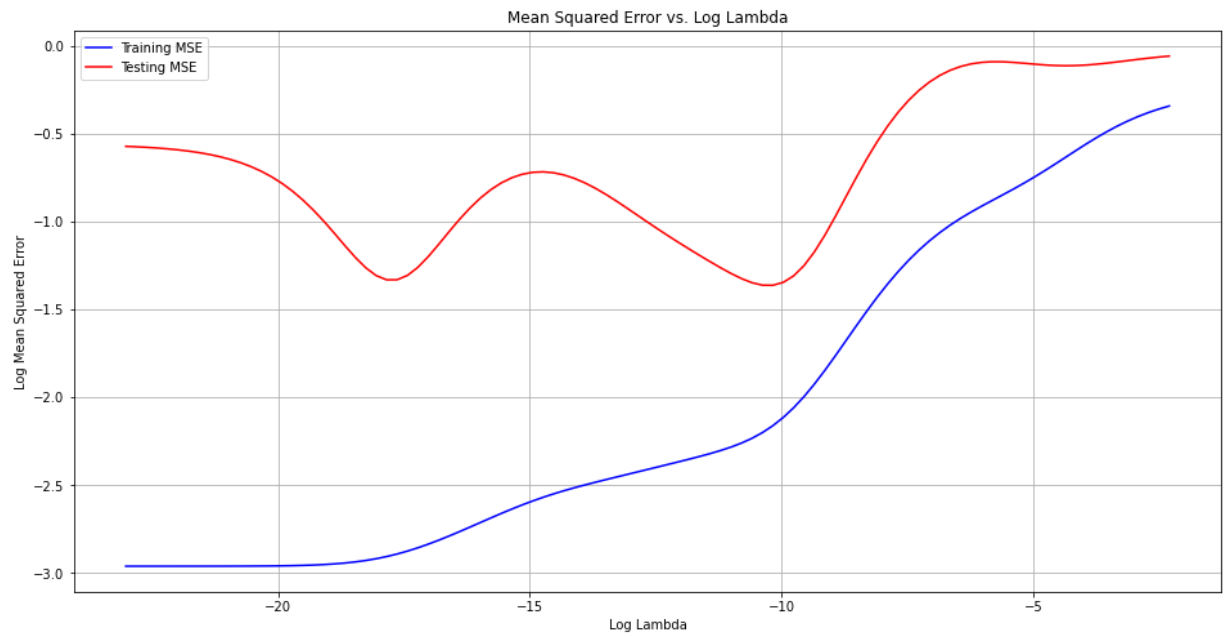
```python
plt.title('Mean Squared Error vs. Log Lambda')
plt.legend()
plt.grid(True)
plt.show()
```


Mean Squared Error vs. Log Lambda

In [ ]: