# Question 4

In [17]:
```
import numpy as np

def fruit_experiment(num):
    boxs = ['red', 'blue', 'yellow']
    box_probs = [1/3, 1/3, 1/3]
    contents = {
        'red': ['apple', 'apple', 'apple', 'orange', 'orange', 'orange', 'orange', '
        'blue': ['apple', 'apple', 'apple', 'apple', 'orange', 'orange', 'orange', '
        'yellow': ['apple', 'orange']
    }

    results_box = np.random.choice(boxs, size=num, p=box_probs)
    results_fruit = [np.random.choice(contents[box]) for box in results_box]

    return np.array(results_box), np.array(results_fruit)

result1, result2 = fruit_experiment(4)
result1 = np.asarray(result1)
print(result1)
print(result2)
```

```
['blue' 'blue' 'blue' 'red']
['apple' 'apple' 'apple' 'apple']
```

If A is an event that the yellow box is picked

And B is the event that apple is the fruit picked

To Find the probability that the fruit from yellow box if it is apple $P\left(A|B\right)$,

As there are 3 boxes with uniform probability for selecting each, so the probability of picking a yellow box at is

$$P(A) = \frac{1}{3}$$

And as the yellow box only has one apple and one orange hence the probability of picking an apple from the yellow box is:

$$P(B|A) = \frac{1}{2}$$

The probability is the sum of probability it being from yellow box and picking apple from box which is not yellow

$$P(B) = \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{3} \cdot \left(\frac{3}{8} + \frac{1}{2}\right)$$
$$P(B) = \frac{1}{6} + \frac{7}{24}$$
$$\text{And taking 24 as lcm and multiplying 4 to} \frac{1}{6}$$
$$P(B) = \frac{4}{24} + \frac{7}{24}$$
$$P(B) = \frac{11}{24}$$

Then using Bayes theorm we get

$$
\begin{aligned}
P(A|B) &= \frac{P(B|A) \cdot P(A)}{P(B)} \\
&= \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{11}{24}} \\
&= \frac{\frac{1}{6}}{\frac{11}{24}} \\
&= \frac{4}{11}
\end{aligned}
$$

So the probablity is $\frac{4}{11}$

# Question 5

In [7]:

```
import random
import numpy as np
import scipy.stats as stats

def die_experiment(repetitions):
    scores = []

    for _ in range(repetitions):
        X = random.randint(1, 6)
        score = 0
        for a in range(X):
            x1 = random.randint(1, 6)
            score = score + x1
        scores.append(score)

    return scores

print(die_experiment(10))
```

```
[8, 5, 5, 12, 5, 7, 4, 29, 14, 3]
```

In [3]:

```
repetitions = 10000
scores = die_experiment(repetitions)

avg = np.mean(scores)

std_error = np.std(scores) / np.sqrt(repetitions)

confidence_interval = stats.norm.interval(0.95, loc=avg, scale=std_error)

print(f"Estimated Average Score (E[Z]): {avg:.2f}")
print(f"95% Confidence Interval: ({confidence_interval[0]:.2f}, {confidence_interval
```

```
Estimated Average Score (E[Z]): 12.27
95% Confidence Interval: (12.14, 12.41)
```

Step 1: Find $E[Z|X=x]\}$

Let's consider a specific value of $x$ for $X$. We want to find the expected score $Z$ given that $X = x$. In this case, we have $x$ subsequent dice rolls $(Y_1, Y_2, \ldots, Y_x)$, and we need to find the expected sum of these rolls.

$$\begin{aligned} E[Z|X=x] &= E[Y_1 + Y_2 + \ldots + Y_x|X=x] \\ &= E[Y_1|X=x] + E[Y_2|X=x] + \ldots + E[Y_x|X=x] \end{aligned}$$

Each $E[Y_i|X=x]$ is simply the expected value of a single die roll, which is $(1/6)$ times the sum of the possible outcomes $(1, 2, 3, 4, 5, 6)$:

$$\begin{aligned} E[Y_i|X=x] &= \frac{1}{6} \cdot (1+2+3+4+5+6) \\ &= \frac{1}{6} \cdot 21 \\ &= \frac{7}{2} \end{aligned}$$

So, for a specific value of $x$, $E[Z|X=x] = \frac{7}{2} \cdot x$.

Now Using the Law of Total Expectation:

Now, we want to find the marginal expectation $E[Z]$, which is the expected score regardless of the value of $X$. We can use the law of total expectation to find this:

$$\begin{aligned} E[Z] &= E[E[Z|X]] \\ &= \sum_{x=1}^{6} E[Z|X=x] \cdot P(X=x) \\ &= \frac{1}{6}\left(\frac{7}{2} \cdot 1 + \frac{7}{2} \cdot 2 + \frac{7}{2} \cdot 3 + \frac{7}{2} \cdot 4 + \frac{7}{2} \cdot 5 + \frac{7}{2} \cdot 6\right) \\ &= \frac{1}{6} \cdot \frac{7}{2} \cdot (1+2+3+4+5+6) \\ &= \frac{1}{6} \cdot (73.5) \\ &= 12.25 \end{aligned}$$

So, analytically, the expected player score $E[Z]$ is $\frac{49}{4}$ or $12.25$.

In [ ]: