AI1103 Assignment-2

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Download all latex codes from

https://github.com/Rohan673/AI1103/blob/main/AI1103_Assignment2/Assignment2.tex

PROBLEM

Gate EC: Q.69

Let X and Y be continuous random variables with joint probability density function

$$f\left(x,y\right)=\begin{cases}ae^{-2y} & 0<x<y<\infty\\ 0 & otherwise\end{cases}$$

The value of a is

(A) 4
(B) 2
(C) 1
(D) 0.5

SOLUTION

Let X and Y be continuous random variables as mentioned in the question above.

For a joint PDF we know that,

$$\int_{x=-\infty}^{x=\infty} f_X(x,y)\,\mathrm{d}x = 1$$

Similarly,

$$\int_{y=-\infty}^{y=\infty}\int_{x=-\infty}^{x=y} f_{XY}(x,y)\,\mathrm{d}x\,\mathrm{d}y = 1$$

Now,

$$\int_{y=-\infty}^{y=0}\int_{x=-\infty}^{x=y} f_{XY}(x,y)\,\mathrm{d}x\,\mathrm{d}y + \int_{y=0}^{y=\infty}\int_{x=0}^{x=y} a\times e^{-2y}\,\mathrm{d}x\,\mathrm{d}y = 1 \tag{1}$$

$$0+\int_{y=0}^{y=\infty}\left(\int_{x=0}^{x=y} a\times e^{-2y}\,\mathrm{d}x\right)\mathrm{d}y = 1 \tag{2}$$

$$\int_{y=0}^{y=\infty}\left(a\times y\times e^{-2y}\right)\mathrm{d}y = 1 \tag{3}$$

$$a\times\left(\frac{-y}{2}\times e^{-2y}-\frac{e^{-2y}}{4}\right)_0^{\infty} = 1 \tag{4}$$

$$a\times\frac{1}{4}=1 \tag{5}$$

$$a=4 \tag{6}$$

Therefore, the answer is (A).