AI1103 Assignment-2

Rohan Atkurkar
CS20BTECH11041

Download all latex codes from

https://github.com/Rohan673/AI1103/blob/main/AI1103_Assignment2/Assignment2.tex

Problem

Gate EC: Q.69

Let X and Y be continuous random variables with joint probability density function

$$f(x,y) = \begin{cases} ae^{-2y} & 0 < x < y < \infty \\ 0 & otherwise \end{cases}$$

The value of a is

(A) 4
(B) 2
(C) 1
(D) 0.5

Solution

Let X and Y be continuous random variables as mentioned in the question above.

For a joint PDF we know that,

$$\int_{x=-\infty}^{x=\infty} f_X(x,y)\,\mathrm{d}x = 1$$

Similarly,

$$\int_{y=-\infty}^{y=\infty}\int_{x=-\infty}^{x=y} f_{XY}(x,y)\,\mathrm{d}x\,\mathrm{d}y = 1$$

Now,

$$\int_{y=-\infty}^{y=0}\int_{x=-\infty}^{x=y} f_{XY}(x,y)\,\mathrm{d}x\,\mathrm{d}y + \int_{y=0}^{y=\infty}\int_{x=0}^{x=y} a\times e^{-2y}\,\mathrm{d}x\,\mathrm{d}y = 1 \tag{1}$$

$$0 + \int_{y=0}^{y=\infty}\left(\int_{x=0}^{x=y} a\times e^{-2y}\,\mathrm{d}x\right)\mathrm{d}y = 1 \tag{2}$$

$$\int_{y=0}^{y=\infty}\left(a\times y\times e^{-2y}\right)\mathrm{d}y = 1 \tag{3}$$

$$a\times\left(\frac{-y}{2}\times e^{-2y} - \frac{e^{-2y}}{4}\right)_0^\infty = 1 \tag{4}$$

$$a\times\frac{1}{4} = 1 \tag{5}$$

$$a = 4 \tag{6}$$

Therefore, the answer is (A).

Marginal density of X is,

$$f_X(X) = \int_{y=-\infty}^{y=\infty} f_{XY}(x,y)\,\mathrm{d}y \tag{7}$$

$$= \int_{y=-\infty}^{y=0} f_{XY}(x,y)\,\mathrm{d}y + \int_{y=0}^{y=\infty} 4\times e^{-2y}\,\mathrm{d}y \tag{8}$$

$$= 0 + 2 \tag{9}$$

$$= 2 \tag{10}$$

Marginal density of Y is,

$$f_Y(Y) = \int\limits_{x=-\infty}^{x=\infty} f_{XY}(x,y)\,\mathrm{d}x \tag{11}$$

$$= \int\limits_{x=-\infty}^{x=0} f_{XY}(x,y)\,\mathrm{d}x + \int\limits_{x=0}^{x=\infty} 4 \times e^{-2y}\,\mathrm{d}x \tag{12}$$

$$= 0 + \left(4y \times e^{-2y}\right)_0^{\infty} \tag{13}$$

$$= 4ye^{-2y} \qquad \forall y \epsilon\,(0,\infty) \tag{14}$$

$$\tag{15}$$