1. Prove or disprove the following inference rules for functional dependencies. A proof can be made either by a proof argument or by using inference rules IR1 through IR3. A disproof should be done by demonstrating a relation instance that satisfies the conditions and functional dependencies in the left hand side of the inference rule but do not satisfy the conditions or dependencies in the right hand side.

a. {W → Y, X → Z} |= {WX → Y}

1. W --> Y, given
2. WX --> YX (Using IR2 to augment 1 with X)
3. YX --> Y, (Using IR1, Y subset of YX)
4. WX --> Y, (Using IR3 on 2 and 3)

b. {X → Y} and Z subset-of Y |= {X →Z }

1. X --> Y , given
2. Y→ Z (Using IR1, Z subset of Y)
3. X --> Z, (Using IR3 on 1 and 2)

C. {X → Y, X → W, WY → Z} |= {X → Z}

1. X → Y, GIVEN
2. X → W, GIVEN
3. WY → Z, GIVEN
4. X → YX (Using IR2 to augment 1 with X)
5. XY → WY (Using IR2 to augment 2 with Y)
6. X → WY (Using IR3 on 4 and 5)
7. X → Z (Using IR3 on 3 and 6)

D. {XY → Z, Y → W} |= {XW → Z}

1. XY → Z (Given)
2. Y → W (Given)
3. XY → XW (Using IR2 to augment 2 with X)

Tuple	X	Y	Z	W
1.	X1	Y1	Z1	W1
2.	X1	Y1	Z2	W1

Since in tuple 1 and 2 we have same value of XW but different value of Z: So, XW does not functionally determine Z.

E. {X → Z, Y → Z} |= {X → Y}

1. X→ Z (Given)
2. Y → Z (Given)

Tuple	X	Y	Z
1.	X1	Y1	Z1
2.	X1	Y2	Z1

The above two tuples satisfy X→ Z and Y→ Z but do not satisfy X → Y. So, X → Y.

F. $\{X \to Y, XY \to Z\} \models \{X \to Z\}$

1. $X \to Y$ (Given)
2. $XY \to Z$ (Given)
3. $X \to XY$ (Using IR2 to augment 1 with X)
4. $X \to Z$ (Using IR3 on 2 and 3)

G. $\{X \to Y, Z \to W\} \models \{XZ \to YW\}$

1. $X \to Y$ (Given)
2. $Z \to W$ (Given)
3. $XZ \to YZ$ (Using IR2 to augment 1 with Z)
4. $YZ \to YW$ (Using IR2 to augment 2 with Y)
5. $XZ \to YW$ (Using IR3 on 3 and 4)

H. $\{XY \to Z, Z \to X\} \models \{Z \to Y\}$

Tuple	X	Y	Z
1.	X1	Y1	Z1
2.	X1	Y2	Z1

Since in tuple 1 and 2 we have same Z value but different Y value: Z !→ y (Z does not functionally determine Y)

I. $\{X \to Y, Y \to Z\} \models \{X \to YZ\}$

1. $X \to Y$ (Given)
2. $Y \to Z$ (Given)
3. $Y \to YZ$ (Using IR2 to augment 2 with Y)
4. $X \to YZ$ (Using IR3 on 1 and 3)

J. $\{XY \to Z, Z \to W\} \models \{X \to W\}$

1. $XY \to Z$
2. $Z \to W$

Tuple	X	Y	Z	W
1.	X1	Y1	Z1	W1
2.	X1	Y2	Z2	W2

Since in tuple 1 and 2 we have same X value for different W value. So,
X does not functionally determine W.

2. Consider the following two sets of functional dependencies
$F = \{A \to C, AC \to D, E \to AD, E \to H\}$ and $G = \{A \to CD, E \to AH\}$. Check whether or not they are equivalent.

Solution,
Given that

F: $A \to C$	G: $A \to CD$
$AC \to D$	$E \to AH$
$E \to AD$	
$E \to H$	

→ Finding closure set of F using functional dependencies in G
$[A]^+ = ACD$, which makes $A \to C$ true
$[AC]^+ = ACD$, which makes $AC \to D$ true
$[E]^+ = ACDH$, which makes $E \to AD$ and $E \to H$ true
All the functional dependencies in F holds true for G as well.
We can conclude that F is subset of G.

→ Finding closure set of G using functional dependencies in F.
$[A]^+ = ACD$, which makes $A \to CD$ true
$[E]^+ = ACDEH$, which makes $E \to AH$ true
All the functional dependencies in G holds true for F as well.
We can conclude that G is subset of F.

Since both F and G are subset of each other, they are equivalent.