Measuring Electoral Bias: Australia, 1949–93

SIMON JACKMAN*

Electoral systems translate citizens' votes into seats in the legislature, and are thus critical components of democracies. But electoral systems can be unfair, insulating incumbents from adverse electoral trends, or biasing the mapping of votes to seats in favour of one party. I assess methods for measuring bias and responsiveness in electoral systems, highlighting the limitations of the popular 'multi-year' and 'uniform swing' methods. I advocate an approach that incorporates constituency-level *and* jurisdiction-wide variation in party's vote shares. I show how this method can be used to elaborate both the extent and consequences of malapportionment. I then present election-by-election estimates of partisan bias and responsiveness for ninety-three state and federal elections in Australia since 1949. The empirical results reported show that the coalition parties have generally 'out-biased' the Australian Labor party, despite some notable pro-ALP biases. The overall extent of partisan bias in Australian electoral systems, however, has generally diminished in magnitude over time.

Electoral systems are key components of democracies, translating citizens' votes into seats in the legislature. Legislatures in turn determine cabinet composition and government policy, and so for this reason an understanding of electoral systems is central to an understanding of democratic politics. The importance of electoral systems is lost on neither students nor practitioners of democratic politics. Political scientists have variously concerned themselves with understanding electoral systems as mechanisms for collective choice,[1]

* Woodrow Wilson School of Public and International Affairs, Princeton University (visiting), and Department of Political Science, University of Rochester.

[1] See Kenneth J. Arrow, *Social Choice and Individual Values*, 2nd edn (New Haven, Conn.: Yale University Press, 1963); Dennis C. Mueller, *Public Choice* (Cambridge: Cambridge University Press, 1979); William H. Riker, *Liberalism Against Populism: A Confrontation Between the Theory of Democracy and the Theory of Social Choice* (Prospect Heights, Ill.: Waveland Press, 1982).

with the political implications of adopting particular electoral systems,[2] and with the need to specify the properties of real electoral systems.[3] Practitioners of democratic politics, on the other hand, have provided many instances of manipulation of electoral systems – both in terms of the choice of voting rules,[4] and, in the case of constituency-based electoral systems, in terms of the drawing of electoral boundaries.

I focus here on this latter type of manipulation – which is apparently as old as electoral politics itself.[5] Australia's long-standing susceptibility to gerrymandering provides a rich and convenient context in which to measure electoral manipulation and elaborate its consequences. Queensland's experiences with the zonal systems of both Labor (1949–57) and the coalition parties (1960–89) are the stuff of political legend.[6] The same can be said for the so-called 'Playmander', the malapportionment that helped ensure conservative domination of South Australian politics from 1932 to 1965.[7] Even a brief international survey of electoral manipulation does not fail to mention the various systems of 'rural weighting' that have operated in Queensland and Western Australia, and in elections for the House of Representatives (the lower house of the federal parliament).[8] Since most Australian parliaments have three-year terms, good constituency-level data are available for a large number of elections during the period studied here (roughly 1949 to the present).

Rather than present a detailed analysis of any one electoral redistricting or of bias in a particular jurisdiction, my approach is deliberately broad and comparative, part methodological and part substantive. I first provide a brief summary of the salient features of Australia's political system, and the methods

---

[2] Douglas Rae, *The Political Consequences of Electoral Laws* (New Haven, Conn.: Yale University Press, 1967); Bernard Grofman and Arend Lijphart, eds, *Electoral Laws and their Political Consequences* (New York: Agathon Press, 1986).

[3] For instance, Edward R. Tufte, 'The Relationship between Seats and Votes in Two-Party Systems', *American Political Science Review*, 67 (1973), 540–54; Gary King and Robert X. Browning, 'Democratic Representation and Partisan Bias in Congressional Elections', *American Political Science Review*, 81 (1987), 1251–73; Rein Taagepera and Matthew Soberg Shugart, *Seats and Votes: The Effects and Determinants of Electoral Systems* (New Haven, Conn.: Yale University Press, 1989).

[4] I have in mind here choices among the following features of electoral systems: single-member versus multi-member constituencies, proportional representation with or without gatekeeping, transferable preferences (the 'alternative vote'), majority versus plurality rule, etc.

[5] See Elmer C. Griffith, *The Rise and Development of the Gerrymander* (New York: Scott, Forseman, 1907).

[6] Useful summaries include Peter Coaldrake, *Working the System: Government in Queensland 1983–1989* (St Lucia: University of Queensland Press, 1989); Ross Fitzgerald, *From 1915 to the Early 1980s: A History of Queensland* (St Lucia: University of Queensland Press, 1984); Hugh Lunn, *Joh: The Life and Political Adventures of Sir Johannes Bjelke-Petersen*, 2nd edn (St Lucia: University of Queensland Press, 1984).

[7] Dean Jaensch, 'A Functional "Gerrymander" – South Australia, 1944–1970', *Australian Quarterly*, 42, No. 4 (1970), 96–101.

[8] David Butler and Bruce Cain, *Congressional Redistricting: Comparative and Theoretical Perspectives* (New York: Macmillan, 1992), pp. 122–3.

used to elect legislatures in the Australian jurisdictions considered here. I then survey some of the popular methods currently used for measuring the properties of electoral systems. I indicate the shortcomings of these methods and advocate a model proposed by Gelman and King.[9] With a generalized version of the Gelman and King model, I generate estimates of 'bias' and 'responsiveness' in ninety-three Australian elections. I compare these estimates and offer some generalizations about electoral manipulation in Australia since 1949.

Debates as to 'who are the biggest cheats in Australian politics' will continue to enthral students of Australian politics (and practitioners, no doubt) for generations to come. I do not presume to adjudicate those debates here. More modestly, I present methods and evidence which may improve the quality of those debates.

AUSTRALIAN JURISDICTIONS: ELECTORAL SYSTEMS AND CONSTITUTIONAL SETTING[10]

Australia is a federation of six former British colonies – Queensland, New South Wales, Victoria, Tasmania, South Australia and Western Australia – and two territories, the Australian Capital Territory and the Northern Territory. The colonies became states when the Commonwealth of Australia came into being on 1 January 1901.[11] The British origins of the Australian constitution mean that parliaments in Australian jurisdictions (the states, territories and the Commonwealth) retain many of the features of Westminster.[12] Bicameralism is the norm, though upper houses largely function as chambers of review, and cabinets are generally drawn from the majority party (or coalition) of the lower houses, usually elected under single-member constituency electoral systems.

Australian state parliaments pre-date the federal parliament, having been established as colonial parliaments in the 1800s. Like Westminster, these

[9] Andrew Gelman and Gary King, 'Estimating the Electoral Consequences of Legislative Redistricting', *Journal of the American Statistical Association*, 85 (1990), 274–82.

[10] In this brief summary I do not provide a comprehensive survey of the complexities of Australian constitutionalism, federalism and its political and electoral history. My aim here is merely to introduce enough detail so as to render my substantive focus on electoral manipulation accessible to readers unfamiliar with the Australian political system. Standard textbook introductions to Australian politics include Don Aitkin, Brian Jinks and John Warhurst, *Australian Political Institutions*, 4th edn (Melbourne: Pitman, 1989); Henry Mayer and Helen Nelson, eds, *Australian Politics: A Fifth Reader* (Melbourne: Longman Cheshire, 1980); and L. F. Crisp, *Australian National Government*, 5th edn (Melbourne: Longman Cheshire, 1983).

[11] The territories are subsequent creations of the federal government. Constitutional amendments and federal legislation since 1973 have gradually granted the territories more representation in the federal parliament, though the Northern Territory has had an elected member of the House of Representatives since 1922.

[12] However, as I point out below, Australia's federal system creates many important exceptions to this general principle. Some observers find the term 'Washminster' more appropriate in describing Australia's mix of Westminster and federalist political institutions. See Dean Jaensch, *Getting Our Houses in Order* (Ringwood, Victoria: Penguin, 1986).

parliaments are 'sovereign', subject to the constraints of common law, West-minster conventions, and the Commonwealth Constitution.[13] While each state has a Constitution Act, these statutes can be amended as the government of the day sees fit.[14] For current purposes it is important to note that electoral redistricting traditionally occurs at the government's pleasure, with no consti-tutional fetters on the properties of the resulting electoral system.

The Commonwealth Constitution literally enumerates the powers of the federal parliament and the states are presumed to retain powers not otherwise handed over to the Commonwealth. The High Court of Australia, the ultimate arbiter of Australian constitutional matters, is generally reluctant to interpret the Constitution so as to limit the powers of the state parliaments to regulate their own affairs[15] and no federal statute or constitutional principle has been successfully used to object to a state's electoral arrangements.[16] Like the state parliaments, the House of Representatives (the lower house of the federal parliament) regulates its own redistricting.[17]

Over the last twenty years Australian parliaments have gradually ceded much of their involvement in redistricting to independent electoral commis-sions. The commissions review electoral boundaries frequently and their decisions are not subject to parliamentary review.[18] The legislative remit of the Australian Electoral Commission (AEC) is far reaching. It aims to achieve equality among House of Representatives constituencies, within a tolerance of 2 per cent, some three and a half years *after* each redistricting – which requires close and frequent consultation between the AEC and the Australian Bureau of Statistics so as to obtain demographic forecasts.[19] While these recent self-

[13] See James A. Thomson, 'State Constitutions and Institutional Systems', in Brian Galligan, ed., *Australian State Politics* (Longman Cheshire: Melbourne, 1986), pp. 177–93.

[14] Common law and Westminster conventions fill some of the constitutional breach. Grey areas persist, particularly over the status of the remaining legal and constitutional ties to the British monarchy, still the head-of-state in Australian jurisdictions: a 'Governor' for each state, and a 'Governor-General' for the Commonwealth. In November 1975, after failing to pass its budget through the opposition-controlled Senate, a federal Labor government was removed from office and parliament dissolved by the then Governor-General.

[15] For example, *Melbourne Corp* v. *Commonwealth* (1947) 74 CLR 31; *Victoria* v. *Common-wealth* (1971) 122 CLR 353; *Queensland Electricity Commission* v. *Commonwealth* (1985) 159 CLR 192. See also the discussion in R. D. Lumb, *The Constitution of the Commonwealth of Australia Annotated*, 4th edn (Sydney: Butterworth, 1986).

[16] This principle was affirmed as recently as 1992 in *Australian Capital Television Pty. Ltd. and others* v. *Commonwealth (No. 2)* 108 ALR 577.

[17] For example, sections 24, 27, 29 of the Commonwealth Constitution. Contrast the United States, where the state legislatures are responsible for redistricting of their respective congressional districts.

[18] See, for example, the 1980s and 1990s amendments to the Commonwealth Electoral Act 1918; the Queensland Electoral Act 1992; and the 1991 amendments to the South Australian Constitution Act 1934 (see also fn. 60 below).

[19] Redistricting of a state's House of Representatives constituencies begins 'automatically' when for a period of three months more than a one-third of the state's constituencies deviate from the average constituency enrolment by more than 10 per cent, or if more than seven years has passed

imposed reforms are a marked change from previous practices of 'stacking' electoral commissions,[20] they fall short of a constitutional guarantee of electoral fairness. A proposed 1988 amendment to the Commonwealth Constitution providing for 'fair and democratic' (one vote, one value) elections failed at a referendum, winning the support of only 37.4 per cent of voters.[21]

Three other features of Australia's electoral systems deserve elaboration.[22] First, virtually all lower house elections in Australian jurisdictions take place using the alternative vote: voters rank the candidates in order of preference, and counting proceeds by eliminating the candidate with the fewest first preferences, and distributing second preferences from that candidate among the remaining candidates until one candidate has a majority. Secondly, the electoral system for the Senate (the federal upper house) and the Tasmanian lower house combine the alternative vote with proportional representation (the Hare-Clark method). Thirdly, electoral enrolment and voting is compulsory in Australian jurisdictions for all eligible citizens.[23] Accordingly, turnout is consistently high in Australian elections, normally in excess of 90 per cent.

Studying Australian electoral systems is both complicated and simplified by these features. One complication is how to determine a party's electoral strength under the alternative vote, and how 'fairly' it is being represented in the legislature given that electoral strength. The number of first preference votes a party receives is often a misleading indicator in this respect, since seats are decided after distributing preferences.[24] For this reason students of

Australian elections focus on 'two-party preferred' (2PP) vote shares,[25] which assess the strengths of the two major political groupings that have by and large dominated electoral politics in Australia, at least since 1949: the Australian Labor Party (a broad-based social democratic party); and the coalition between the conservative parties, the Liberal Party of Australia and the National Party (formerly known as the Country Party, reflecting its rural base). Two-party preferred vote shares are calculated by inspecting (or estimating) preference distributions from minor parties to either of the major parties, and between the two conservative coalition partners. In this way 2PP provides convenient summaries of the electoral strengths of the major partisan groupings. The use of 2PP vote shares here allows the analysis to focus directly on partisan bias from the manipulation of electoral boundaries, without there being any undue worry about the effects of the alternative vote.[26] Likewise, compulsory voting ensures that differential rates of turnout will not confound the empirical results.

The analysis here focuses exclusively on single-member constituency electoral systems for which 2PP data are available. Federal upper house (the Senate) and Tasmanian lower house elections are excluded because they use the (proportional) Hare-Clark system. State upper-house elections are also excluded because, until fairly recently, these were appointed bodies. These exclusions mean that constituency-level 2PP data are available for ninety-three elections – encompassing every lower-house election in the mainland states and for the federal House of Representatives from 1949 to mid-1993.[27]

With these features of Australia's electoral systems in mind I now turn to my main substantive concern: the measurement of 'bias' in Australian electoral systems.

MEASURES OF ELECTORAL MANIPULATION: BIAS AND RESPONSIVENESS

Research on the manipulation of electoral boundaries was invigorated by the ruling of the United States Supreme Court in *Baker v. Carr*[28] which made manipulations of electoral boundaries justiciable. The Civil Rights Act (1964) and the Voting Rights Act (1965) institutionalized challenges to electoral redistricting in the United States under the principle of 'one person, one vote'.[29] This

---

[25] Malcolm Mackerras is generally attributed with introducing two-party preferred vote totals to the study of Australian elections. For a review see Joan Rydon, 'Two-Party Preferred: The Analysis of Voting Figures under Preferential Voting', *Politics*, 21 (1986), 68–74.

[26] Contrast G. Gudgin and P. Taylor, *Seats, Votes, and the Spatial Organisation of Elections* (London: Pion, 1979), pp. 165–9.

[27] Except the 1983 elections for the Western Australian Legislative Assembly for which I was unable to obtain 2PP data.

[28] 369 U.S. 186 (1962).

[29] Robert B. McKay, *Reapportionment: The Law and Politics of Equal Representation* (New York: Clarion, 1965); Nelson Polsby, *Reapportionment in the 1970s* (Berkeley: University of California Press, 1971).

body of law has grown significantly since the 1960s, ensuring a steady demand for measures of properties of given sets of electoral boundaries.

Attempts to generate such measures frequently rely on a model of single-member constituency electoral systems derived from the 'cube law', one of the first relationships posited between seats and votes in single-member constituencies: 'if the votes are divided between the parties in the ratio A : B then the seats will be divided in the ratio $A^3 : B^3$, and the winning majority of votes will be magnified in the proportion of seats won'.[30] This strict relationship between parties' vote proportions and seat proportions was thought to express the tendency of single-member constituency electoral systems to 'reward' winning parties, and 'punish' losing parties.[31] Investigations of real-world single-member constituency electoral systems showed the 'cube law' neither to accurately approximate observed relationships between seats and votes, nor to provide any compelling normative justification of how vote shares ought to translate into seats shares.[32] A generalization of the model underlying the 'cube law' was proposed:

$$\frac{s}{1 - s} = \left(\frac{v}{1 - v}\right)^{\rho} \tag{1}$$

where s is a party's proportion of legislative seats won, v is that party's proportion of the two-party vote, and ρ is a parameter measuring the 'responsiveness' of the electoral system: i.e., 'how much does a party's seat share change for a given change in its vote share?' The 'cube law' arbitrarily set ρ equal to 3, but this generalization allows it to vary. For instance, $\rho = 1$ characterizes a system of pure proportional representation while $\rho = \infty$ characterizes a knife-edge 'winner-take-all' system, the difference between winning all legislative seats and none being one vote at 50 per cent of the votes. Figure 1 plots seats–votes curves with different values of ρ. Higher values of ρ indicate more responsive electoral systems; lower values indicate less responsive electoral systems.

[30] M. G. Kendall and A. Stuart, 'The Law of Cubic Proportion in Election Results', *British Journal of Sociology*, 1 (1950), 183–97. An early statement of the 'cube law' is in F. Y. Edgeworth's paper presented to the Royal Statistical Society in 1898, 'Miscellaneous Application of the Calculus of Probabilities', *Journal of the Royal Statistical Society*, 61 (1898), 534–42. The Right Hon. J. Parker Smith, a Fellow of the Royal Society from 1891 and a four-term member of the House of Commons, cited Edgeworth's work and presented applications of the 'cube law' for British elections between 1885 and 1906 to a Royal Commission on Systems of Election (*House of Commons Sessional Papers*, 26 (1910), 79–81). The Commission heard evidence on many different electoral systems operating in Europe and the Australian states at the time and makes fascinating reading.

[31] Examples include Dean McHenry, 'The Australian General Election of 1954', *Australian Quarterly*, 27, No.1 (1955), 14–23; Joan Rydon, 'The Relation of Votes to Seats in Elections for the Australian House of Representatives, 1949–1954', *Political Science*, 9, No.2 (1957), 49–61; John Warhurst, 'State Elections: Queensland and South Australia', *Politics*, 13 (1978), 121–30.

[32] Tufte, 'The Relationship between Seats and Votes in Two-Party Systems'; Phillip A. Schrodt, 'A Statistical Study of the Cube Law in Five Electoral Systems', *Political Methodology*, 7 (1981), 31–53; King and Browning, 'Democratic Representation and Partisan Bias in Congressional Elections'; cf. Gudgin and Taylor, *Seats, Votes, and the Spatial Organisation of Elections*.
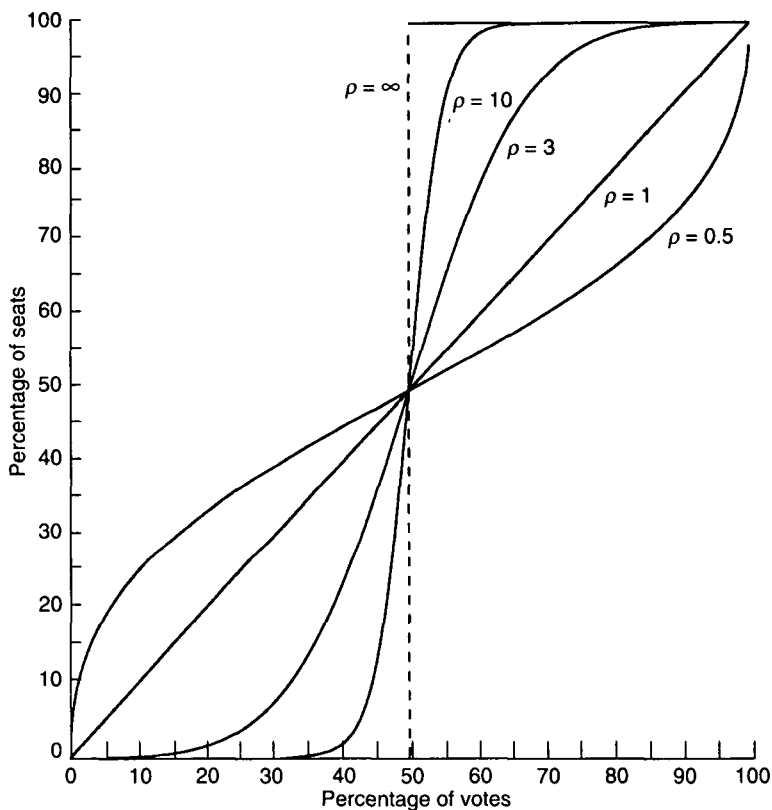
Fig. 1. Hypothetical seats–votes curves

With data on s and v from real-world electoral systems ρ can be estimated statistically. Taking natural logs of (1) constrains predicted seat proportions to their (natural) [0,1] interval, though there are substantive arguments for the widespread use of the logistic functional form in this setting.[33] Adding an error-term[34] yields:

$$\ln\left(\frac{s}{1-s}\right) = \rho\ln\left(\frac{v}{1-v}\right) + e, \tag{2}$$

which can be estimated using least squares regression. While this approach permits estimation of the responsiveness of two-party electoral systems it assumes 'partisan symmetry': i.e., changes in votes shares yield equal changes

[33] Gudgin and Taylor, *Seats, Votes, and the Spatial Organisation of Elections*, Appendix One.
[34] Debates as to the functional form of the error term are reviewed in William J. Linehan and Phillip A. Schrodt, 'A New Test of the Cube Law', *Political Methodology*, 4 (1978), 353–67, and King and Browning, 'Democratic Representation and Partisan Bias in Congressional Elections.'

in seats shares for both parties. For instance, party A's share may fall from 50 per cent of the vote to 40 per cent and thus fall from 50 per cent of the seats to 30 per cent. Equations (1) and (2) imply that as a corollary party B goes from 50 per cent of the votes to 60 per cent and from 50 per cent of the seats to 70 per cent. In fact, Equations (1) and (2) constrain the seats–votes curve to pass through the (50%,50%) point, and to be symmetric about that point. Observations of electoral systems suggest that this is unrealistic – manipulations of electoral systems are designed to ensure secure majorities of legislative seats with insecure majorities of the vote. The point where a party obtains 50 per cent or more of the seats may well lie at some point below (or above) 50 per cent of the votes. In this way electoral systems are said to be 'biased' in favour of one party or another. To measure partisan bias in an electoral system we can modify Equation (1), so that

$$\frac{s}{1-s} = \beta\left(\frac{v}{1-v}\right)^{\rho}. \tag{3}$$

This in turn implies a modified version of Equation (2),

$$\ln\left(\frac{s}{1-s}\right) = \ln \beta + \rho \ln\left(\frac{v}{1-v}\right) + e \tag{4}$$

where $\ln \beta$ measures the bias of an electoral system. Here I define s and v in terms of the Australian Labor Party's (ALP) share of seats and two party preferred (2PP) vote share, respectively, and thus partisan bias is defined with respect to the ALP. Substituting $v = 0.5$ and re-arranging Equation (4) yields the following expression for the seat share Labor obtains with 50 per cent of the votes:

$$s^* = E(s \mid v = 0.5) = \frac{\exp(\widehat{\ln \beta})}{1 + \exp(\widehat{\ln \beta})} \tag{5}$$

Similarly, the Labor vote share required for 50 per cent of the seats is

$$v^* = \frac{\exp(-\widehat{\ln \beta/\rho})}{1 + \exp(-\widehat{\ln \beta/\rho})}. \tag{6}$$

Details as to the derivation of the sampling distributions of these quantities are provided in the Appendix.

The 'Multi-Year' Method

This method of estimating seats–votes curves is sometimes referred to as the 'multi-year' method,[35] since it pools aggregate seat and vote shares over time (or space). This method can also be extended to the case where the observations

---

[35] Variously attributed to Richard Niemi and Patrick Fett, 'The Swing Ratio: An Explanation and an Assessment', *Legislative Studies Quarterly*, 11 (1986), 75–90.
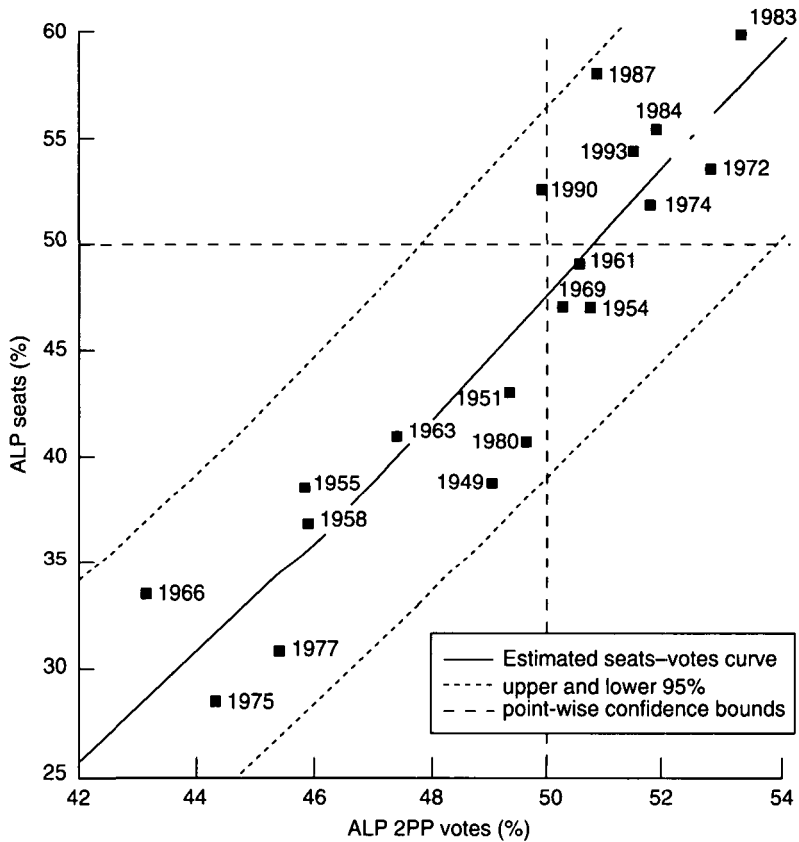
Fig. 2. Australian House of Representatives, 1949–93

are drawn from different jurisdictions, and allocated into sub-samples depending upon which parties controlled the electoral redistricting.[36] In this way the estimates of bias and responsiveness can be compared across sub-samples, and inferences drawn about one party 'out-manipulating' the others.

While pooling electoral data permits us to obtain fairly precise estimates of bias and responsiveness, these estimates are drawn over such a broad selection of elections as to limit the political meaning we might like to attach to those estimates. Thus the 'multi-year' method poses a trade-off to the analyst: precision at the cost of political relevance.

To demonstrate this trade-off I estimated Equation (4) with data from elections for the Australian House of Representatives. These data are plotted in

[36] Richard Niemi and Simon Jackman, 'Bias and Responsiveness in State Legislative Redistricting', *Legislative Studies Quarterly*, 16 (1991), 183–202.

TABLE 1   *Australian House of Representatives, 1949–93*

Parameter estimate	All elections	Labor 'control' (1949–54, 1984–93)	(1984–93)	Non-Labor 'control' (1955–83)
Bias				
$\widehat{\ln \beta}$	− 0.08	− 0.10	0.17	− 0.12
Standard error	(0.04)	(0.08)	(0.09)	(0.05)
Responsiveness				
$\hat{\rho}$	3.04	5.54	0.99	2.82
Standard error	(0.33)	(1.79)	(1.83)	(0.31)
ALP seats if ALP obtains 50% 2PP votes				
s*	47.9%	47.6%	54.2%	47.1%
prob [s* > 50%]	0.02	0.09	0.96	0.001
ALP 2PP votes necessary for 50% ALP seats				
v*	50.7%	50.4%	45.7%	51.0%
prob [v* > 50%]	0.98	0.88	0.26	0.99
adj. r^2	0.83	0.59	− 0.31	0.88
n	19	7	4	12

Figure 2, and Table 1 reports the estimates of Equation (4) obtained with various sub-sets of these data.

The entire 1949–93 set of elections reveals a moderate anti-Labor bias, with Labor obtaining about 48 per cent of the seats with 50 per cent of the 2PP vote. Co-incidently, the responsiveness estimate is very close to three, which is the prediction of the 'cube law'. Elections held under Labor incumbency (Table 1, column two) display the same amount of bias as for the whole set of elections, but the estimate is only as large as its standard error. The responsiveness estimate is very high, the result of fitting a single seats–vote curve to two, small, disparate clusters of elections: (1) the 1980s elections, in which Labor efficiently translated votes into seats; and (2) the elections of 1949, 1951 and 1954, when despite Labor having 'controlled' the drawing of electoral boundaries in 1948, Labor's vote share was either low, or relatively inefficiently translated into seats (see Figure 2). Electoral systems engineered under non-Labor incumbency display a sizeable and statistically significant anti-Labor bias and are relatively unresponsive (Table 1, column four). Fifty per cent of the vote typically yields Labor 47 per cent of the seats when the coalition 'controls' electoral redistricting.

I subset further to cover the four elections held since the 1984 Labor-initiated reforms of the electoral system, resulting *inter alia* in an increase in

the size of the House of Representatives from 125 to 148 members. Results of this analysis appear in column three of Table 1. There appears to be a substantial pro-Labor bias in the 1984–93 electoral systems, worth (on average), about 54 per cent of the seats for 50 per cent of the votes. However, these estimates are only on the threshold of statistical significance (at conventional levels). Note further that the responsiveness estimate is very small (0.99), and roughly half the size of its standard error. This point estimate of responsiveness describes a relatively unresponsive electoral system. Note also the extremely poor fit of the model to the data, as indicated by the large negative adjusted r^2.

Such a seats–vote curve is so at odds with results for other sub-samples of the House of Representatives data and analyses of other single-member electoral systems in the literature that I reject it out of hand. The unresponsive electoral system implied by these results almost certainly does not correspond with Australian electoral systems of 1984–93. The odd estimates of bias and unresponsiveness I obtain are almost certainly caused by fitting two parameters to four data points: a highly unrecommended state of affairs, but unavoidable when we attempt to extract specific political content using the 'multi-year' method. In assessing the political consequences of the 1984 redistricting, application of the 'multi-year' method has pushed us towards an absurd answer.

Shortcomings of the Multi-Year Method

As the above analyses demonstrate, estimating bias and responsiveness is subject to the limitations of any statistical enterprise: less data yields less precise estimates of parameters of interest, and the influence of each data point increases as sample size decreases. To obtain a reasonable number of data points I pool across many elections, but at the expense of substantive clarity. For instance, pooling across forty-five years of Australian elections yields measures of bias and responsiveness that are precise, but devoid of political import, since this bundles together periods of both Labor and non-Labor control of electoral redistricting. Estimating seats–votes curves with progressively fewer observations brings more political content to the enterprise, since we are able to estimate bias and responsiveness for periods where one party controlled or instigated electoral redistricting. But as the analyses above indicate, there are simply insufficient data to permit the kinds of politically meaningful comparisons we desire.

To push this argument further, the political content of bias and responsiveness measures would be greatly increased if we were able to derive bias and responsiveness estimates for a single redistricting, say over two or three elections. But at this point the 'multi-year' estimation procedure breaks down for want of degrees of freedom: there are close to as many parameters to estimate as there are observations (elections). In any event, long before running out of degrees of freedom the estimates are swamped by their standard errors, or yield manifestly absurd measures of bias and responsiveness, as the analyses above indicate. In short, it is easy to push the trade-off between political meaning and

precision to the point where imprecision becomes so large as to render the bias and responsiveness measures meaningless (politically, or otherwise), or even mathematically undefined.

Even if bias and responsiveness could be estimated for a small group of elections, there is still a deeper problem to be considered. Bias and responsiveness arise *not* just via the machinations of politicians. Voters move between elections, thereby changing the partisan composition of constituencies. For a given set of electoral boundaries and a given aggregate level of partisan support, different distributions of partisans through the constituencies will produce different electoral outcomes. What we observe in a specific election, or specific group of elections, is not just the result of partisan manipulation of the electoral system, but also the way in which partisans are distributed throughout constituencies. The result of a single election is just the aggregation of a particular spatial distribution of vote choices through a set of electoral boundaries. Thus when we estimate bias and responsiveness we measure manipulation of the electoral system *conditional* on a spatial distribution of partisan support. As the spatial distribution changes, so too will the bias and responsiveness of the electoral system, even though the electoral boundaries stay the same.

Further variation in bias and responsiveness stems from variation in the spatial distribution of other characteristics germane to vote choices. For instance, popular candidates or a set of economic conditions particular to a constituency or variation in turnout[37] may also cause variation in bias and responsiveness – even while electoral boundaries are constant.

The preceding discussion makes clear why we would like election-by-election measures of bias and responsiveness. But it is also clear that the multi-year method is totally incapable of generating election-by-election measures. What methods can provide election-by-election measures of bias and responsiveness?

Uniform Partisan Swing

One method, attributed to Butler, 'builds' a seats–votes curve from constituency-level data from a single election.[38] The method starts with the distribution of constituency-level results observed in a single election, sometimes referred to as the 'constituency proportion distribution' (CPD).[39] This distribution is then 'shifted' such that the shifted mean vote proportion corresponds to various points in the (0,1) interval. For all 'shifts' of the CPD the relative pattern of constituency-level results is held constant: shifting the location of the CPD changes all constituency results uniformly. For each 'shift' the

---

[37] Compulsory voting ensures that this particular confounding variable (differential turnout) is far less an issue in Australia than in the United States or the United Kingdom.

[38] David E. Butler, 'Appendix III: The Relation of Seats to Votes', in R. B. McCallum and Alison Readman, *The British General Election of 1945* (London: Oxford University Press, 1947).

[39] Gudgin and Taylor, *Seats, Votes, and the Spatial Organisation of Elections*, pp. 14–20.
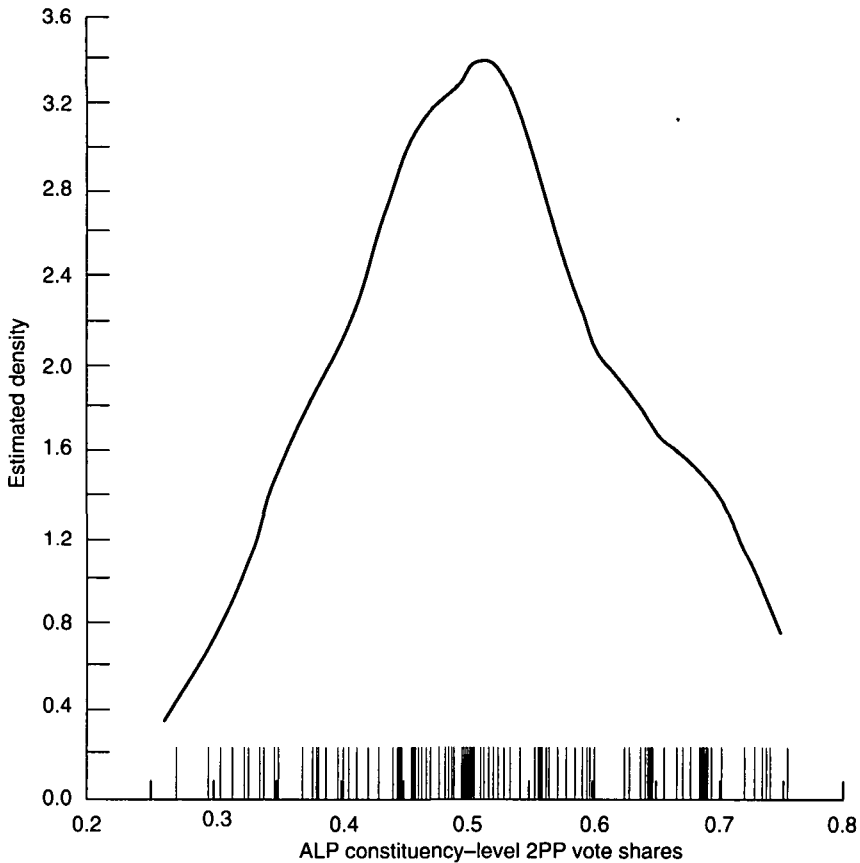
Fig. 3. Distribution of constituency-level ALP 2PP vote shares, House of Representatives, 1993

proportion of constituency results above 50 per cent is recorded. The resulting series of 'shifted means' and seat shares is then plotted, and the bias and responsiveness measures obtained via inspection. Figure 3 presents an estimate of the CPD for the 1993 House of Representatives election, and Figure 4 presents the seats–votes curve obtained assuming uniform swing, suggesting a moderate bias against Labor: assuming uniform swing, 50 per cent of the 2PP would yield Labor only 46.9 per cent of the seats. Contrast the seats–votes curve this method generates for the 1950 election for the South Australian House of Assembly (Figure 5). The small number of seats then in the South Australian Assembly (thirty-nine) produces a jagged step function, and the bias resulting from the anti-Labor malapportionment is clear (Labor obtained 30.8 per cent of the seats with 48.7 per cent of the 2PP vote).

   This method assumes 'uniform partisan swing': that is, by shifting the CPD all constituencies change by an equal amount. For some elections, the assump-
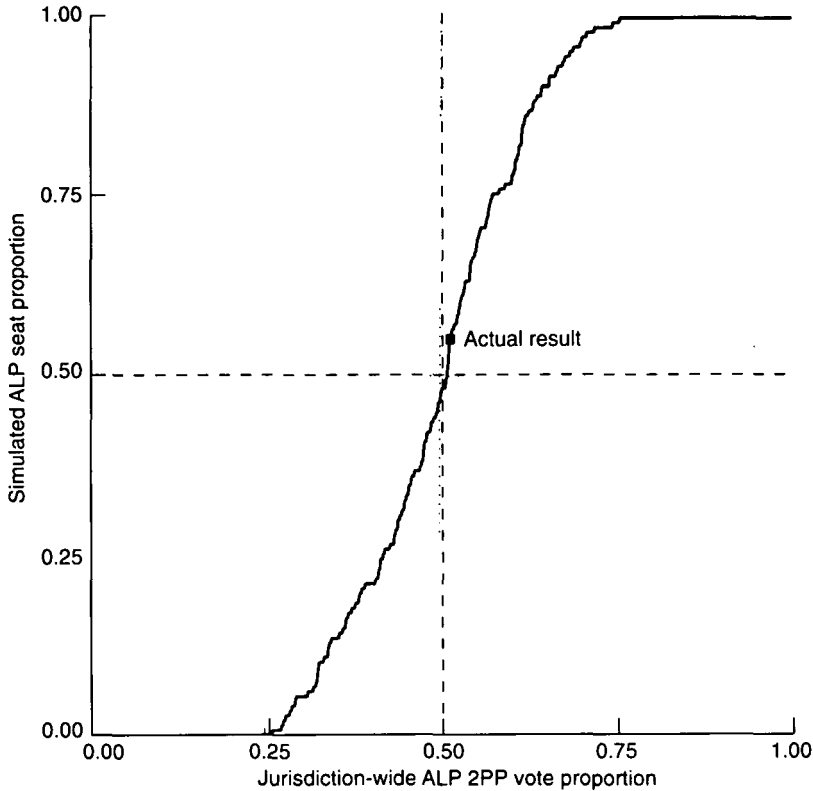
Fig. 4. Seats–votes curve (uniform swing), House of Representatives, 1993

tion of 'uniform swing' is not far from reality but, as a theoretical matter, it is an overly restrictive assumption.[40] None the less, assumptions of 'uniform swing' figure prominently (and not uncontroversially) in analyses of Australian elections, particularly in the generation of electoral 'pendulums'.[41] The

[40] See Gudgin and Taylor, *Seats, Votes, and the Spatial Organisation of Elections*, p. 16.

[41] The literature on this question is mammoth. Malcolm Mackerras has made the case for uniform swing in various articles, replies and rejoinders: see 'Uniform Swing: Analysis of the 1975 Election', *Politics*, 11 (1976), 41–6; 'No Change: Analysis of the 1977 Election', *Politics*, 13 (1978), 131–8; 'Rejoinder to Campbell Sharman', *Politics*, 13 (1978), 339–42. Clive Bean and David Butler are also impressed with the uniformity of swings in Australian elections – see their 'Uniformity in Australian Electoral Patterns: The 1990 Federal Election in Perspective', *Australian Journal of Political Science*, 26 (1991), 127–36. For contrary arguments and evidence, see Joan Rydon, 'Swings and Predictions: The Analysis of Australian Electoral Statistics', in Henry Mayer, ed., *Labor to Power* (Sydney: Angus and Robertson, 1973); Campbell Sharman, 'Swing and the Two-Party Preferred Vote: A Comment on Malcolm Mackerras', *Politics*, 13 (1978), 336–9; Brian Austen, *Uniformity and Variation in Australian Electoral Behavior: State Voting Patterns in House*
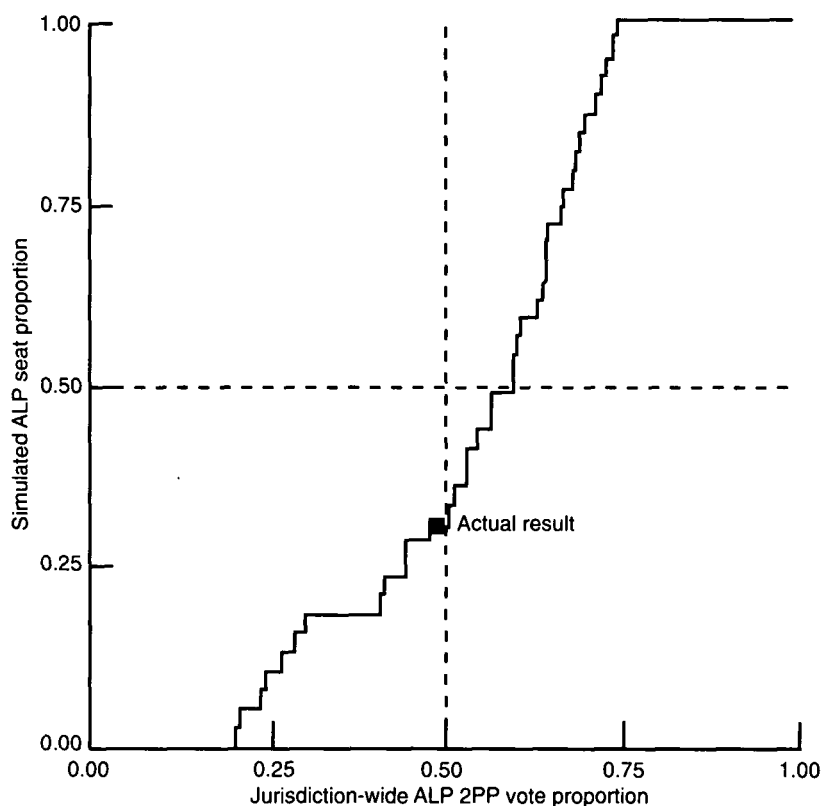
Fig. 5. Seats–votes curve (uniform swing), South Australian Legislative Assembly, 1950

controversy surrounding the uniform swing assumption in the context of predicting electoral outcomes underlies its shortcomings in the present context of measuring electoral manipulation, though seldom do critics of uniform swing make this connection explicit.[42] In both contexts I find the assumption

---

(F'note continued)

of *Representatives Elections 1946–1975*, Occasional Monograph No. 1 (Hobart: Department of Political Science, University of Tasmania, 1977); Brian Austen, 'A Comment on Malcolm Mackerras', *Politics*, 13 (1978), 342-4; Owen Hughes, 'Uniform Swing Revisited: Further Comments on Mackerras', *Politics*, 19 (1984), 111–18; D. Peetz, 'Donkeys, Deserters and Targets: Causes of Swing in Electorates in the 1987 Federal Election', *Australian Quarterly*, 61 (1989), 468–80; Alistair Fischer, 'Swings and Gerrymanders', *Electoral Studies*, 10 (1991), 299–312; Christian Leithner, 'The Geographic Configuration of the Vote: New Results and Interpretations from Australian Data' (paper presented to the Annual Meetings of the Australasian Political Studies Association, Australian National University, Canberra, 1992).

[42] Fischer's 'Swings and Gerrymanders' is a notable exception among the 'uniform swing' literature cited in the previous footnote.

of uniform swing neither theoretically nor empirically satisfying. Since uniform swing has so pre-occupied many students of electoral politics, I pause briefly here to elaborate its shortcomings before presenting my preferred model.

One major shortcoming of using uniform swing to measure bias and responsiveness is that constituency-specific variation about the jurisdiction-wide 'uniform swing' is set to zero. This is convenient, but patently unrealistic. As a consequence, changes over time in the measures of bias and responsiveness generated under the assumption of 'uniform swing' are confounded with changes in the spatial distribution of voters and effects specific to each constituency. At best, measuring bias and responsiveness in this way can only fit one election at a time.

To appreciate the extent that electoral systems do indeed deviate from the assumption of uniform swing I plotted results for the 1983 election for the Australian House of Representatives against those for the 1980 election (Figure 6). If the assumption of uniform swing held, then the data points would lie on a 45° line, with the intercept on the vertical axis indicating the size of the uniform swing. The line in Figure 6 has these properties, while simultaneously minimizing the sum of the squared errors. Clearly the data are scattered all about this 'best-fitting' uniform-swing line, a pattern I found in many other similar scatterplots of Australian electoral data.

The prediction errors inherent to uniform swing are substantial, and politically consequential. Applying uniform swing to the 1980 and 1983 House of Representatives elections I find a root mean square error of just over 4 per cent – many elections are decided by smaller margins. Relying on uniform swing as a predictor of election results is a luxury few real-world politicians could afford.[43]

None the less, one argument for uniform swing is that in the aggregate it makes few prediction errors in terms of seat tallies.[44] But the aggregate prediction of seat tallies is an exercise political scientists are seldom interested in, save for the occasional appearance on election night television.[45] More often than not, we are interested in how aggregate seat tallies come about, which inevitably directs attention below the aggregate level to state-level, regional-level and, ultimately, individual-level explanations.

This focus on sub-aggregate relationships also applies to efforts to measure bias and responsiveness in electoral systems. A given aggregate seat and vote combination comes about for a variety of reasons. Which marginal seats fell to

[43] The uniform swing line in Figure 6 was estimated using restricted least squares, the restriction being that the slope parameter equal one. The unrestricted intercept parameter indicates the size and direction of the fitted uniform swing. The unrestricted slope estimate is 0.91 (s.e. = 0.03), and the root mean square error of the unrestricted model is marginally under 4 per cent, suggesting a slight improvement over the uniform swing model.

[44] Bean and Butler, 'Uniformity in Australian Electoral Patterns', p. 133.

[45] Though even in this application uniform swing is of dubious value. See the interesting review in Michael Maley and Rodney Medew, 'Some Approaches to Election Night Forecasting in Australia', *Australian Journal of Political Science*, 26 (1991), 51–62.
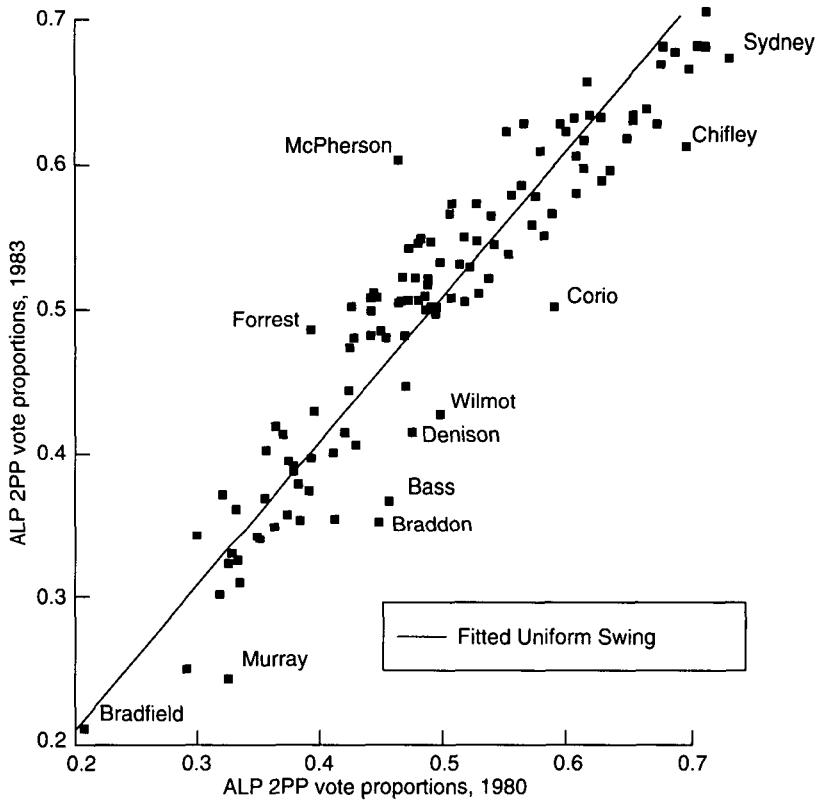
Fig. 6. *Electoral swing, Australian House of Representatives, 1980–83*

which party? How different would the configuration of influences on the vote have to be to in order to produce a different aggregate outcome? Uniform swing struggles with these questions. So too do aggregate measures of 'bias' in electoral systems such as Gini coefficients, David–Eisenberg or Dauer–Kelsay indices, which enjoyed a long history in the study of malapportionment.[46] None of these single aggregate measures are concerned with how aggregate outcomes arise, nor with the implications should a different sub-aggregate configuration of influences on voting arise. Just as we do not blind ourselves to these factors in explaining election outcomes, nor should we when characterizing electoral systems.

Specifically, uniform swing is premised with a number of deterministic

[46] For examples, see Joan Rydon, ' "Malapportionment" – Australian Style', *Politics*, 3 (1968), 133–47; James Kelly, 'Vote Weightage and Quota Gerrymanders in Queensland, 1931–1971', *Australian Quarterly*, 43 (1971), 39–54; and Colin A. Hughes, *A Handbook of Australian Government and Politics, 1975–1984* (Sydney: Australian National University Press, 1986), Appendix A.

assumptions about the generation of patterns of constituency vote shares. Uniform swing assumes there is no variation in constituencies' vote proportions apart from that induced by uniform swing, and that the particular distribution of constituency-level vote proportions observed are the only data that could be observed under a given electoral system. Zero probability is attached to observing a CPD other than the one obtained in a given election. Implicit here is the rejection of any notion of constituency-level election results being generated randomly. As a result, uniform swing provides no measures of confidence (e.g., standard errors) in the measures of bias and responsiveness it generates,[47] nor in forecasts about election results.

Finally, uniform swing is merely a special (though uninteresting) case of the model I elaborate below. By embedding uniform swing in a more general model of vote shares the interminable debate as to the utility of the 'uniform swing' assumption is made finally, and thankfully, redundant. In short, there is no *a priori* reason to constrain constituency-specific effects to be zero when we have the means to estimate them statistically and incorporate these effects into models of electoral systems, and of electoral processes more generally.

BEYOND 'UNIFORM SWING': INCORPORATING CONSTITUENCY-LEVEL VARIABILITY

A better method would be one that did not deterministically fix the distribution of constituency-level vote shares. Such a method would involve estimating or simulating constituency-level effects on vote shares.[48]

Such a method has been proposed by Gelman and King.[49] In their model the unit of analysis is the constituency. Each constituency's result is modelled as a (stochastic) function of two variables: a jurisdiction-wide swing (affecting all constituencies equally), and an effect specific to each constituency. In this model simulations are conducted not only over the range of possible jurisdiction-wide (and hence, uniform) swings, but also over a plausible range of constituency-specific deviations about the jurisdiction-wide swing. Like the

---

[47] Strictly speaking, uniform swing does indeed yield standard errors, though they are always constrained to zero, implying an infinite level of confidence in the bias and responsiveness estimates. Contrast James C. Garand and T. Wayne Parent, 'Representation, Swing, and Bias in US Presidential Elections, 1872–1988', *American Journal of Political Science*, 35 (1991), 1011–31, where Equation (4) is fit to the step-functions generated by uniform swing (e.g., Figures 4 and 5).

[48] One particular source of constituency-specific variation that has been modelled in this way is 'incumbency advantage'. For example, see Gary King, 'Representation through Legislative Redistricting: A Stochastic Model', *American Journal of Political Science*, 33 (1989), 787–824; Gary King and Andrew Gelman, 'Systemic Consequences of Incumbency Advantage in the US House', *American Journal of Political Science*, 35 (1991), 110–38; Andrew Gelman and Gary King, 'A General Method for Evaluating Electoral Systems and Redistricting Plans', *American Journal of Political Science*, 38 (1994), forthcoming.

[49] Gelman and King, 'Estimating the Electoral Consequences of Legislative Redistricting.'

'uniform swing' method, measures of bias and responsiveness emerge from summaries of these simulations.

To outline this model I introduce the following notation and definitions. Define $v_{it} \in (0,1)$ as the Labor party 2PP vote proportion in constituency i for election t, where i indexes $(1, \ldots, n)$, the set of constituencies comprising the jurisdiction under consideration. Let $u_{it} = \text{logit}(v_{it}) \equiv \ln[v_{it}/(1 - v_{it})]$. The model of constituency level results is

$$u_{it} \sim N(\alpha_{it}, \sigma^2), \; \alpha_{it} = \gamma_i + \delta_t, \tag{7}$$

where γ_i is a constituency-specific effect, δ_t is a jurisdiction-wide effect, and these effects are independent. Less formally, a single constituency's Labor vote share is a draw from a normal distribution with a mean given by two components: a jurisdiction-wide component (δ_t), common to all constituencies (note there is no i subscript on δ_t); and a constituency-specific component (γ_i). The variance of this normal distribution is a scalar (σ^2), and therefore common to all constituencies. It is assumed that constituency-specific departures from the jurisdiction-level average vote share are spatially independent across constituencies: for instance, the extent to which a given constituency departs from the jurisdiction-level vote share (and hence jurisdiction-level swing) is assumed to be unrelated to other constituencies' departures from the jurisdiction average.

This assumption is possibly controversial. One can imagine instances in the Australian context where constituency-specific departures from a jurisdiction average might be correlated: e.g., recession in rural constituencies; dissatisfaction with a particular state government producing a state-specific swing in a federal election at odds with the national swing; and so on. With constituency-level data one could model constituency-level variation in vote shares. Such data might include census data organized by (federal or state) constituencies, federal vote results broken down into state constituencies, incumbency status, or campaign expenditures. However, in the absence of constituency-level information as to what drives the constituency-specific component of Equation (7) (i.e., γ_i), it is difficult to relax the assumption of spatial independence and keep the model tractable.[50]

It bears repeating that this model assumes the presence of *both* constituency-level and jurisdiction-wide effects on a constituency's vote share. The possibly restrictive assumption here is that constituency-level swings (in so far as they depart from the jurisdiction-wide swing) are independent across constituencies. Uniform swings are a special case of this model, where $\sigma^2 = 0$.

---

[50] In this article I have opted for a broad approach, analysing as many state and federal Australian elections as practical, rather than a detailed analysis of a smaller group of elections. Gathering constituency-level covariates for the large group of elections I analyse here is beyond my scope in this article. If such data are available I recommend use of an alternative model proposed by Gelman and King ('A General Method for Evaluating Electoral Systems and Redistricting Plans'), which can easily be implemented with their free computer program 'JudgeIt'.

Since the jurisdiction-wide swing, δ_t, is by definition constant across constituencies for a given election, an electoral system can be characterized by the vector of constituency-specific effects, $\gamma = (\gamma_i, \ldots, \gamma_n)$ and the variance parameter σ^2. Without loss of generality, in estimating the model for a single election, δ_t is set to zero, since adding an arbitrary constant to all of the constituency-level Labor vote shares would not change the results of the model. In later stages of estimation varying δ_t is important, since variation in δ_t corresponds to different hypothetical levels of jurisdiction-wide swing. Conditional on estimates of constituency-specific effects, and of other parameters in the model, I eventually let the common jurisdiction-level effect (the δ_t) vary between negative and positive infinity on the logit scale (0 and 1 on the 2PP vote share scale) so as to generate seats–votes curves and the measures of bias and responsiveness for a single election.

Learning about bias and responsiveness thus requires first learning about γ, the unknown constituency-specific effects. This task is made harder by the absence of constituency-specific covariates, and so I employ a fairly involved statistical procedure which is detailed in the Appendix. Here I simply outline the model.

Because I lack data on the determinants of constituency-level effects on the constituency-level vote shares, I resort to simulation. First, I use data from a sequence of elections held under the same electoral boundaries to estimate the variability of constituency-level results in a particular jurisdiction. This estimate informs the simulation of constituency-level effects by providing a lower bound on their variability. Secondly, after inspecting many plots of distributions of constituency-level vote shares in my data (e.g., Figure 3) I make 'educated guesses' as to the values of the parameters of the Gelman and King model of constituency-level effects. These guesses are the same for all ninety-three elections I analyse, symmetric with respect to the parties, and are not stringent.[51]

An iterative process is used to find estimates of the parameters of the model. These estimates are optimal in the sense that they maximize the probability of observing the distribution of constituency-level results for a given election, given the model I assume to underlie this distribution, and my initial educated guesses as to what the true parameter values are. Convergence of this iterative procedure occurs relatively quickly and I sample from the distribution of constituency-level effects characterized by the parameters obtained from the iterative procedure. I then add hypothetical levels of jurisdiction-wide swing (δ_t) to these constituency-specific effects to trace out a seats–vote curve over the (0,1) interval in which average ALP vote shares lie. I repeat the entire process fifty times and note the 'average' seats–vote curve, and average levels of bias and responsiveness.

---

[51] By making 'guesses' about variance parameters deliberately high I offset any undue 'over-confidence' in my guesses about means. The data overwhelmingly dominate the distribution of constituency-level influences on vote shares I ultimately settle on for each election.

CONTROLLING FOR MALAPPORTIONMENT

Both the 'uniform swing' method and the Gelman and King model generate seats–votes curves with jurisdiction-level summaries of simulations of the parties' vote shares. More formally, a seats–votes curve generated by either of these methods is an interpolation of a series of ordered (v_t, s_t) pairs, each ordered pair being the aggregate-level summary of simulation t. The s_t term is simply the expected proportion of the legislative seats won by a party should it obtain proportion v_t of the 2PP vote.

But note that the v_t term is the average of the simulated *constituency-specific* vote shares, which may or may not correspond to the *jurisdiction-level* vote share. In talking about political parties it is more intuitive to talk about a 'party's strength' in terms of its 'overall vote share' than in terms of its average constituency-level strength. For this reason I define the votes–seats curves implied by my simulations in terms of jurisdiction-wide vote share.

In addition to this appeal to intuition, there are other reasons why we ought to define the domain of the seats–votes curve in terms of a jurisdiction-level vote share. Using the average constituency-specific vote share in place of overall vote share blinds us to a very common form of electoral manipulation: malapportionment. In fact, it is this difference between the average constituency vote share (the 'effective vote') and the jurisdiction-level vote share that Soper and Rydon use as a measure of malapportionment in elections for the Australian House of Representatives from 1949 to 1954.[52] Using the notation introduced earlier, 'effective vote' corresponds to

$$\frac{1}{n} \Sigma^n_{i=1} v_i,$$

while the jurisdiction-level vote share is

$$V = \frac{\Sigma^n_{i=1} w_i v_i}{n}, \tag{8}$$

where w_i is a weighting term, equal to the size of constituency i relative to the average constituency, such that $\bar{w} = 1$, and $\Sigma^n_{i=1} w_i = n$. V is thus a weighted

[52] C. S. Soper and Joan Rydon, 'Under-Representation and Electoral Prediction', *Australian Journal of Politics and History*, 4 (1958), 94–106. Subsequent applications include Joan Rydon, 'A Note on Voting and ALP Under-Representation in the 1958 Federal Elections', *Australian Journal of Politics and History*, 5 (1959), 84–6; Joan Rydon, 'Some Aspects of Voting in the 1961 Elections', *Australian Journal of Politics and History*, 8 (1962), 98–101; Joan Rydon, 'The South Australian "Gerrymander" ', *Australian Journal of Politics and History*, 9 (1963), 86–7; Jaensch, 'A Functional "Gerrymander" – South Australia, 1944–1970'; Gudgin and Taylor, *Seats, Votes, and the Spatial Organisation of Elections*, pp. 57–8. Contrast uses of Gini coefficients and differences between median and mean constituency-level vote shares. See Colin A. Hughes, 'Fair and Equal Constituencies: Australia, Jamaica and the United Kingdom', *Journal of Commonwealth and Comparative Politics*, 16 (1978), 256–71, and 'A Close-Run Thing', in Howard E. Penniman, ed., *Australia at the Polls: The National Elections of 1980 and 1983* (American Enterprise Institute, Washington, DC: George Allen & Unwin, 1983).

average of the constituency-level vote shares, v_i. Clearly, V will equal the constituency-averaged vote share when $w_i = 1$ for all constituencies: malapportionment is impossible in this pathological case of equally sized constituencies.

To demonstrate how malapportionment works, I take expectations of Equation (8), noting that the expectation of the product of two variables is equal to the product of the expectations plus their covariance, and that the summation operator and the n term in the denominator cancel:

$$E(V) = E(w_i)\, E(v_i) + \text{cov}(w_i, v_i). \tag{9}$$

By definition, $E(w_i) = 1$, yielding

$$E(V) - E(v_i) = \text{cov}(w_i, v_i). \tag{10}$$

The left-hand side of this equation is simply Soper and Rydon's measure of malapportionment: the difference between 'effective' and overall vote. Equation (10) makes clear that malapportionment is a function of the covariance between vote shares and the number of voters in a constituency, or more precisely, the relative number of voters in a constituency. As the correlation between constituency size and vote shares increases in magnitude, so too will the difference between overall vote share and the 'effective vote'. A necessary condition for malapportionment is that constituencies be unequally sized.[53] But unequal numbers of voters in constituencies does not constitute malapportionment *unless* constituency sizes are related to the parties' (likely) vote shares. In the notation I use here, malapportionment is defined by a relationship between the weights (w_i) defining a constituency's size and the parties' vote shares (v_i).

Figure 7 displays the relationship in Equation (10) graphically. The difference between jurisdiction-level ALP 2PP vote shares (V) and average constituency-level ALP 2PP vote shares ($\bar{v}$) is regressed on the product of the correlation between constituency size[54] and ALP 2PP vote share and the Gini coefficient measuring inequality in constituency size. For a given election, the Gini coefficient takes high values when constituencies are increasingly unequal in size, and, by definition, takes values in the unit interval.[55] The relationship depicted in Figure 7 is striking: positive correlations between constituency size and ALP 2PP vote share, in the presence of inequalities in constituency size,

[53] In the pathological case of equal numbers of voters in all constituencies ($w_i = 1$, for all i), the covariance term in Equation (10) is zero because w_i is invariant: i.e., no malapportionment is possible.

[54] I use the sum of the Labor and non-Labor 2PP votes as a proxy for constituency size, since compulsory voting helps ensure that voter turnout is consistently high in Australian elections. Any biases introduced by this proxy are unlikely to be substantial nor bear heavily on my later conclusions.

[55] Taking the product of the correlation between votes and constituency size and Gini coefficient results in weighting down elections with strong correlations but relatively equally sized constituencies (e.g., NSW 1988, VIC 1985). This product is a close approximation to the covariance term in Equation (10), and has the advantage that both correlation and Gini coefficients have well-defined and easily interpreted metrics.
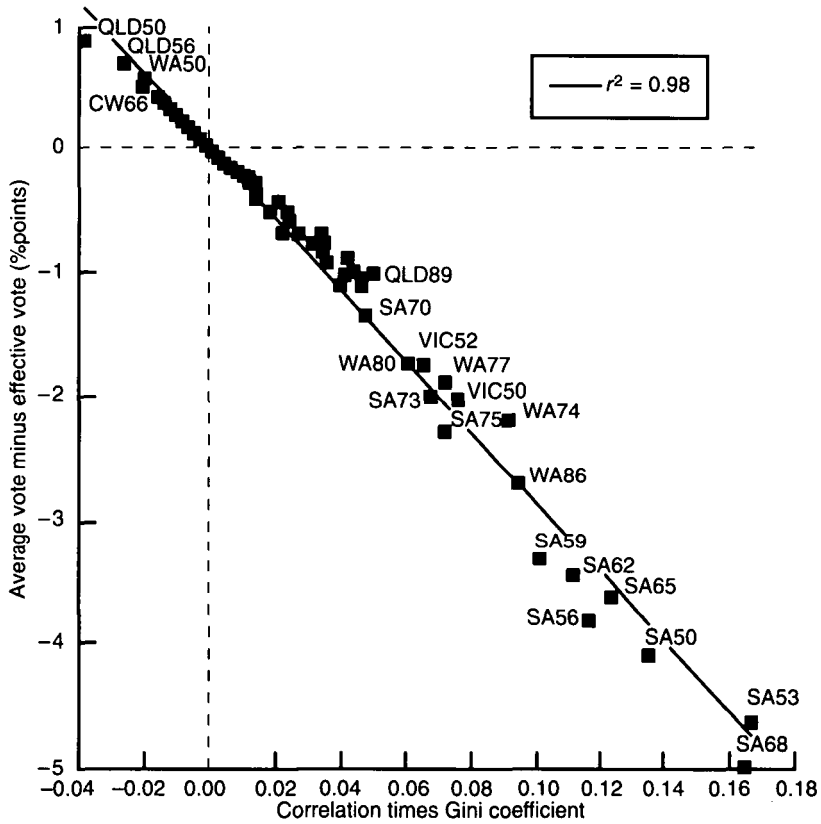
Fig. 7. Components of malapportionment

are strongly associated with electoral disadvantage for the ALP. When ALP 2PP vote shares are strongly and positively correlated with constituency size, *and* constituency sizes vary substantially, we have evidence of anti-Labor malapportionment. Specifically, ALP support is being 'wasted' in large constituencies, while non-Labor support is being more efficiently spread through smaller constituencies. As a result, Labor's 'effective vote' is less than Labor's overall vote share.

In particular, Figure 7 clearly shows the results of South Australia's infamous malapportionments. The seven most extreme cases of anti-Labor malapportionment are the electoral systems of South Australia from 1950 (the first South Australian election in my analysis) to 1968. In only a few cases do I find evidence of substantial electoral disadvantages to the coalition parties arising from malapportionment. The malapportionments of the Hanlon Labor government in Queensland are the most prominent cases of pro-Labor

malapportionment in my data, but amount to modest abuses compared with the consistency and severity of the South Australian malapportionments.

While the depiction of Australian malapportionments in Figure 7 is striking, my main point here is to note the difference between 'effective' vote and average vote. Tracing out seats–votes curves using the effective vote ignores the 'partisan weighting' introduced through malapportionment. In effect, such a method statistically controls away malapportionment. The bias and responsiveness estimates obtained in this way measure electoral manipulation *over and above* malapportionment. But if malapportionment is strongly suspected to be the villain, as is the case in many Australian jurisdictions, then such a procedure is likely to blind us to the very thing we are looking for!

The solution I propose to this shortcoming is exceedingly simple. Recall what malapportionment is: the 'weighting' of votes on partisan criteria. Successful malapportionment involves 'weighting up' one's partisans (by having them in numerically smaller, and thus many, constituencies) relative to one's opponents' partisans. A party's overall vote share is simply the 'weighted' average of the constituency-specific vote shares, where the 'weights' are the relative numeric sizes of the constituencies. I take these 'weights' into account when I calculate the aggregate-level summary of a given set of constituency-level simulations. Each simulated constituency-level ALP vote share is multiplied by the relative size of each constituency. In this way the domain of each seats–votes curve I generate is not the 'effective vote', but overall vote. Accordingly, the resulting estimates of bias and responsiveness are indeed sensitive to malapportionment as a tool of electoral manipulation.

RESULTS

I summarize the results of the simulations with a plot of how the bias and responsiveness estimates change over time, for each state. Figures 8, 9 and 10 contain these plots for Queensland, South Australia and the Commonwealth, respectively. Each plotted point is a pair of bias and responsiveness estimates, where I have defined bias in terms of the expected seat share above or below 50 per cent won by the ALP when it obtains 50 per cent of the jurisdiction-wide 2PP vote. Dotted lines join elections with redistricting intervening, and a solid square indicates the first election in my data for a particular jurisdiction, or the first election after redistricting. The reference lines indicate the average standard deviations for the bias and responsiveness estimates, helping to convey what might pass for a statistically significant change in either of these quantities.

The Queensland results (Figure 8) show the large pro-ALP bias in the 1950s due to the malapportionment introduced by the Hanlon government in 1949. After losing office in the 'split' election of 1957[56] Queensland's electoral

---

[56] No 2PP data is available for the 1957 Queensland election, held under plurality voting at the height of the ALP 'split'.
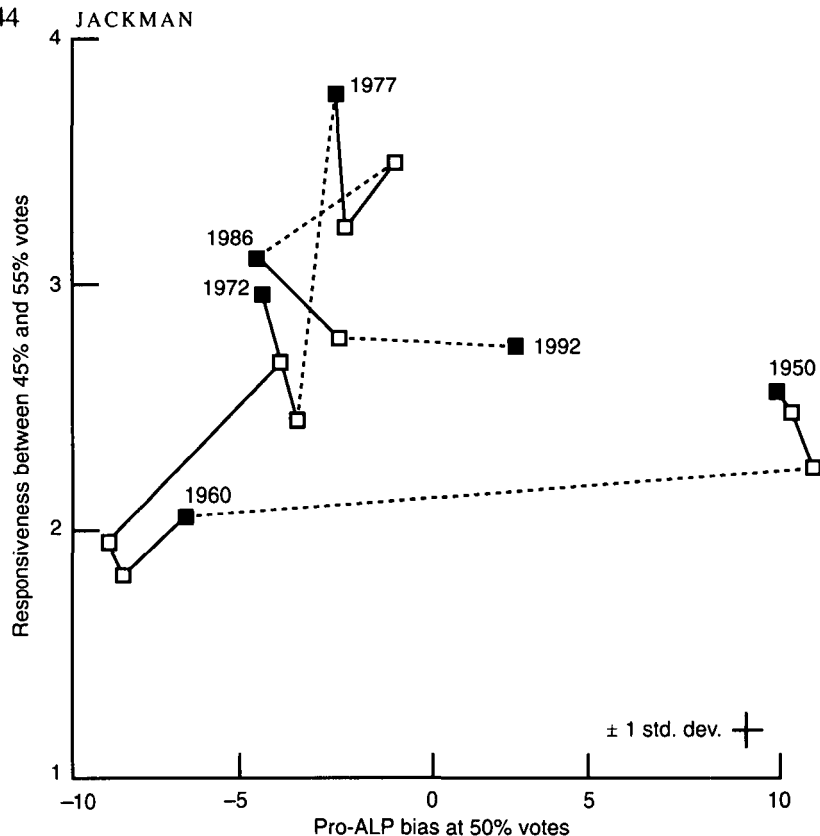
Fig. 8. Queensland Legislative Assembly, 1950–92, bias and responsiveness

system underwent a dramatic shift, reflecting both a shift in partisan loyalties and the weight those voters were given under the coalition's version of the zonal system of malapportionment. Labor suffered through most of the 1960s under pro-coalition biases almost as large as the pro-Labor biases it instituted in the 1950s. Bias was generally low in Queensland throughout the 1970s and responsiveness quite high. Under these conditions Labor could have won government with only a fraction over 50 per cent of the 2PP vote. Using Equation (6) I estimate that after 1969 Labor never required more than 51 per cent of the 2PP vote to win a majority of seats (on average). In fact, my estimate of bias for 1983 is indistinguishable from zero. My results are consistent with Mackerras's self-styled 'revisionist' conclusion that Labor's failure to win office was as much due to Labor's failure to win the necessary votes as it was the result of the coalition's malapportionments.[57] The 1985 redistricting moved the Queensland electoral system away from being almost unbiased in 1983, as

---

[57] Malcolm Mackerras, 'A Revisionist Interpretation of the Impact of Queensland's Electoral Scheme', *Australian Journal of Political Science*, 25 (1990), 339–49.
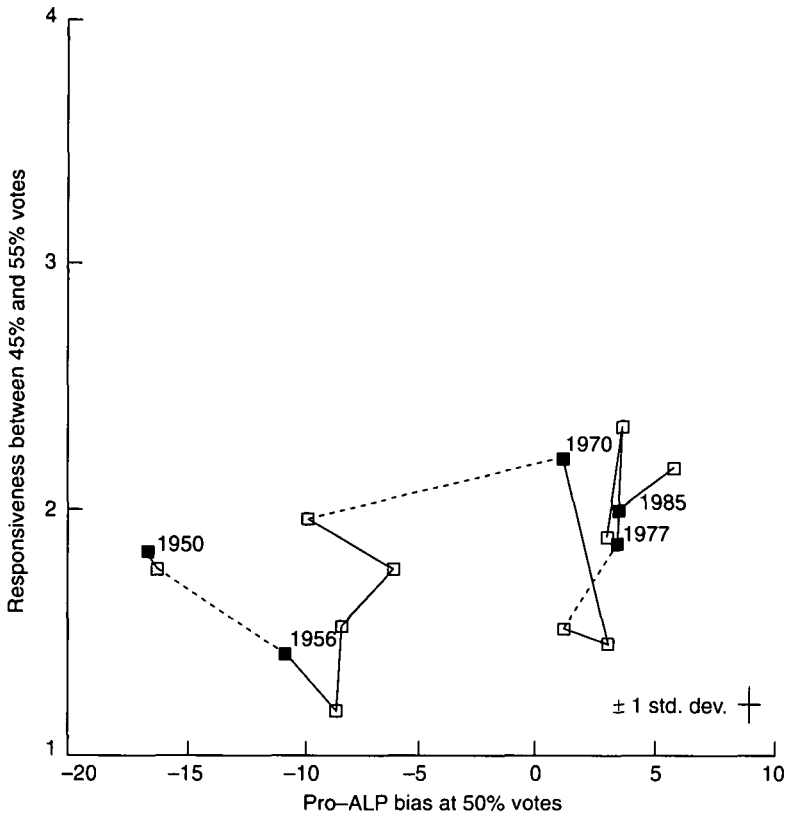
Fig. 9. South Australian House of Assembly, 1950–89, bias and responsiveness

well as decreasing responsiveness, thereby creating fewer marginal seats, and helping to insulate incumbent legislators. The ALP came to power in Queensland in 1989 after thirty-two years in opposition and campaigned for reform in government. Under Labor an independent electoral commission was created and the zonal systems responsible for malapportionment in Queensland since 1949 were eliminated. The new electoral arrangements certainly eliminated any anti-Labor bias in Queensland: I find a small pro-Labor bias in the 1992 Queensland electoral system. I estimate that Labor could have won roughly 52.6 per cent of the seats with 50 per cent of the 2PP vote in 1993.

South Australia's electoral system exhibits very high anti-Labor biases during the 1950s and 1960s, so high that I have had to use a different scale on the horizontal bias axis (Figure 9). Like Queensland, these biases are largely attributable to malapportionment, though operating exclusively to the coalition's advantage. Further, the South Australian electoral systems of this period are quite unresponsive, indicating a low proportion of marginal seats. Swings
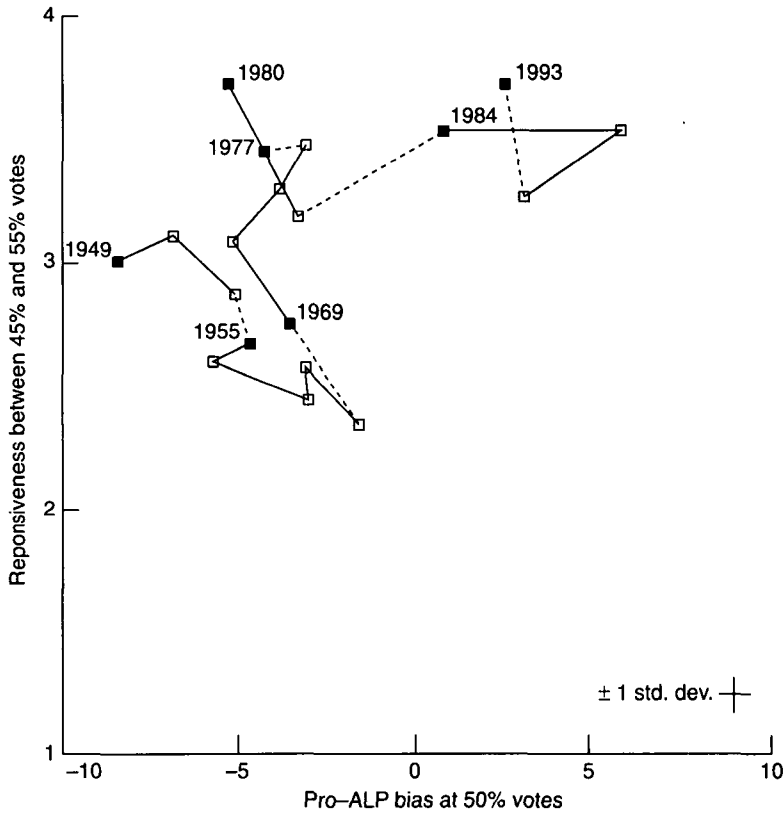
Fig. 10. House of Representatives, 1949–93, bias and responsiveness

to Labor through this period would have yielded unfairly small seat turnover. South Australia's electoral arrangements through this period thoroughly insulated coalition incumbents from adverse electoral swings. Redistricting in time for the 1970 election ended the era of mammoth anti-Labor biases, and subsequent elections have displayed small to mild pro-Labor bias. The 1989 election displays the largest pro-Labor bias seen in South Australia (worth about 56 per cent of the seats for 50 per cent of the 2PP vote). The Liberal party's failure to win office with 50 per cent of the popular vote in South Australia prompted amendments to the South Australian Constitution requiring unbiased electoral redistricting.[58]

Electoral systems for the Commonwealth display none of the extreme biases witnessed in Queensland and South Australia (Figure 10). At no time has bias

[58] See the *Report of the Electoral Districts Boundaries Commission* (Adelaide: Government Printer, 1991), p. 2, and note B.2 below.

exceeded 10 per cent in magnitude, though for a long time it hovered around 2 to 3 per cent in favour of the coalition. Electoral systems of the 1950s and 1960s gravitated towards neutrality, after initially displaying large anti-Labor biases. The 1968 redistricting moved the House of Representatives electoral system away from being almost unbiased in 1966 to the region of pro-coalition bias where it remained until the 1984 redistricting. Throughout this time responsiveness was relatively high, and Labor's victories at various elections throughout that period suggest that while significant, the pro-coalition bias in the electoral system was not a grave setback to Labor's chances of winning office. Labor was a clear beneficiary from the 1984 redistricting, which transformed a small pro-coalition bias in 1983 into a small pro-Labor bias in 1984, and into a substantial pro-Labor bias in 1987. The swing against Labor in the 1990 election slightly reduced this advantage. Redistricting before the 1993 election appears to have even further eroded Labor's electoral advantage. For 1993 I find a small though not insignificant anti-Labor bias: Labor would have obtained just 47.3 per cent of the seats had it won 50 per cent of the 2PP vote. Furthermore, electoral systems for the House of Representatives have been generally quite responsive, and exhibit a fairly steady increase in responsiveness from 1966 to 1980. This indicates an increase in the proportion of marginal seats over the period, possibly stemming from a decline of partisan loyalties and a growing concentration of swinging voters in outer metropolitan seats since the 1960s.

Further information on bias and responsiveness is contained in Tables 2 and 3. In general, manipulations of electoral systems in Australia have been to the disadvantage of Labor parties, despite some notable pro-Labor biases in Queensland through the early 1950s. The largest biases I find are pro-coalition biases, from South Australia, and like the largest pro-Labor biases, also from the 1950s. Figure 11 shows that since the 1950s average electoral bias has become progressively less anti-Labor.[59] Averaged over the jurisdictions in my analysis, electoral biases in Australia required Labor to win around 55 per cent of the 2PP vote in order to form governments in 1950. This average figure has fallen slowly but fairly steadily since 1950. The average bias of Australian electoral systems in 1992 was indistinguishable from zero, despite some notable individual exceptions in the 1980s and 1990s.

Figure 12 shows that large biases are not confined to the decade of the 1950s. While bias seems to generally diminish in magnitude over time (perhaps largely through the increasing independence of the various state and federal electoral commissions), there is some evidence to suggest a slight increase in the magnitude of biases through the mid and late 1980s. New South Wales exhibits large pro-Labor biases in 1984 and 1988, as does Western Australia in

---

[59] The 'loess' curves in Figures 11 and 12 are locally weighted regression fits to the data, as described in William S. Cleveland, Eric Grosse and William M. Shyu, 'Local Regression Models', in John M. Chambers and Trevor J. Hastie, eds, *Statistical Models in S* (Pacific Grove, Calif.: Wadsworth and Brooks, 1992).
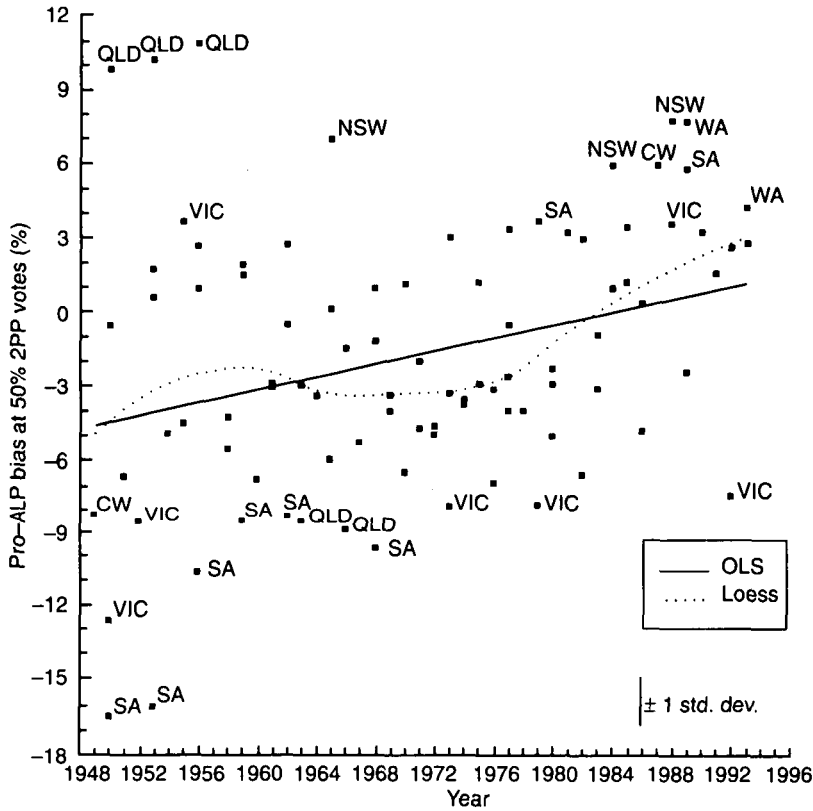
Fig. 11. Partisan biases in Australian electoral systems, 1949–93

1989, and the House of Representatives in 1987. The large swing against Labor in Victoria in 1992 helps account for the sizeable anti-Labor bias associated with that election.

CONCLUSION

Measuring properties of sets of electoral boundaries can be done in several different ways. Here I have presented three different methods: 'multi-year' estimates, estimates based on uniform swing, and estimates allowing for constituency-specific deviations from uniform swing. I argue for the latter method. 'Multi-year' estimates work best when making broad historical characterizations about electoral systems, but fail miserably when we attempt to extract substantive content about a small set of elections, and yield no information at all about bias and responsiveness in any one election. Uniform swing permits election-by-election estimates of bias and responsiveness but is based on a
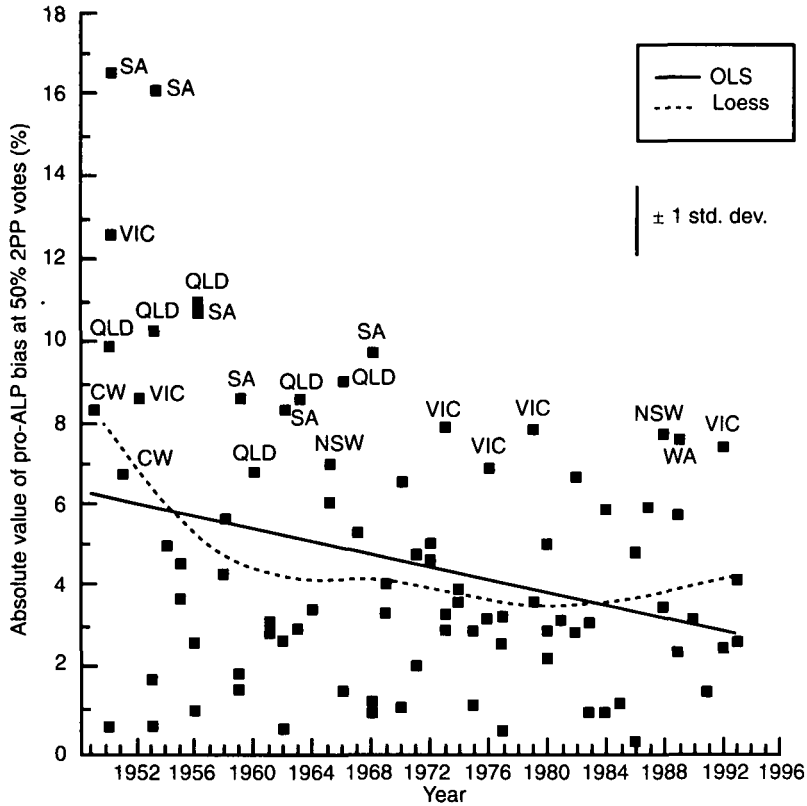
Fig. 12. Magnitude of biases in Australian electoral systems, 1949–93

patently unrealistic assumption. Constituencies vary about a jurisdiction-wide swing, and the model I employ captures this fact about real-world electoral systems.

To the extent that we have better data and models with which to model constituency-specific swings, then we can have more confidence in the resulting characterizations of electoral systems. The specific form of the model I employ here is complicated by the absence of constituency-level covariates: with data on incumbency, local economic conditions, characteristics of the candidates, or the social-structural composition of constituencies, a better model of constituency-level swings could lead to improved measures of bias and responsiveness. Gathering and using these contextual data is standard fare for students of politics in the United Kingdom or the United States, but rare in the study of Australian politics.

Yet even without these data for the Australian case I draw some distinct substantive conclusions. By and large, since 1949, the non-Labor parties have

TABLE 2   *Ten Largest Bias Estimates (Standard Deviations)*

	Anti-ALP				Pro-ALP		
SA	1950	− 16.51	(1.24)	QLD	1956	10.91	(0.96)
SA	1953	− 16.12	(1.13)	QLD	1953	10.22	(1.12)
VIC	1950	− 12.64	(1.23)	QLD	1950	9.84	(0.96)
SA	1956	− 10.68	(1.27)	NSW	1988	7.75	(0.38)
SA	1968	− 9.74	(1.75)	WA	1989	7.66	(1.23)
QLD	1966	− 9.01	(0.90)	NSW	1965	7.01	(0.60)
VIC	1952	− 8.58	(0.83)	CW	1987	5.97	(0.43)
QLD	1963	− 8.56	(0.78)	NSW	1984	5.92	(0.72)
SA	1959	− 8.55	(0.92)	SA	1989	5.79	(1.13)
SA	1962	− 8.32	(1.26)	WA	1993	4.25	(0.63)

benefited from manipulating electoral boundaries more than Labor. Both parties have instituted gross electoral manipulations: the malapportionments of South Australia (anti-Labor) and Queensland (pro-Labor) in the 1950s and 1960s are blights on the democratic process unlikely to be seen again. Recent biases, such as the pro-Labor biases in New South Wales and Western Australia, and to a lesser extent in Commonwealth elections, pale by comparison. None the less, they remind us that biases in electoral systems persist and matter in close elections, despite increasingly stringent legal requirements for unbiased electoral distributions[60] and the best efforts and intentions of the

TABLE 3   *Responsiveness Estimates (Standard Deviations)*

	Ten smallest				Ten largest		
SA	1959	1.18	(0.17)	VIC	1973	3.96	(0.10)
WA	1953	1.36	(0.14)	QLD	1977	3.78	(0.32)
NSW	1991	1.40	(0.11)	VIC	1970	3.73	(0.11)
SA	1956	1.41	(0.23)	CW	1980	3.72	(0.09)
NSW	1978	1.45	(0.09)	WA	1993	3.71	(0.10)
SA	1973	1.45	(0.13)	VIC	1982	3.59	(0.10)
SA	1975	1.50	(0.23)	VIC	1979	3.56	(0.09)
SA	1962	1.52	(0.17)	CW	1984	3.51	(0.10)
WA	1950	1.55	(0.18)	CW	1987	3.51	(0.10)
WA	1965	1.61	(0.18)	QLD	1983	3.51	(0.12)

[60] Amendments to the South Australian Constitution (Constitution Act 1991) provide, inter alia that '83. (1) ... the Commission must ensure, as far as practicable, that ... if candidates of a particular group attract more than 50 per cent of the popular vote (determined by aggregating votes

relatively independent electoral commissions charged with redistricting Australian jurisdictions.

As I argued at the outset, terms such as the 'Playmander', the 'Bjelkemander', and even the more technical 'gerrymander' are familiar not only to professional observers of Australian politics, but are well and truly part of Australian political folklore. These terms are recognized by many as symbols of 'shady dealings', sharp practice, or even benign 'larrikinism' by their politicians.

In addition to what they signify about their politicians, these terms also embody the systematic deprivation of many Australians of their electoral 'power', and, arguably, their rights as citizens. When unchecked, electoral manipulations cement incumbents in office, de-legitimize opposition and induce stasis in public policy.[61] As the findings of the Fitzgerald Inquiry suggest, corruption within government is the handmaid of electoral abuse.[62] These and other characteristics of one-party government were clearly elaborated in V. O. Key's seminal work on politics in the Southern United States[63] – a work that with only a little imagination accurately describes Australian jurisdictions afflicted with long histories of electoral manipulation.

While the details may not be understood by lay people, many do understand that manipulation has long been part of the cut and thrust of electoral politics. To the extent that relatively technical properties of electoral systems figure in popular (mis-)understandings of politics and in appraisals of the legitimacy of governments, it is incumbent on social scientists to understand and elaborate them.

To this end, the analysis presented here provides a set of concepts and methodological tools for measuring electoral manipulation and drawing

(F'note continued)

cast throughout the State and allocating preferences to the necessary extent), they will be elected in sufficient numbers to enable a government to be formed.' Such requirements obviously require a consideration of many of the issues I raise above. Further, South Australia's '50 per cent rule' comes close to a constitutional mandate for the very measures of electoral bias I employ here.

[61] 'One wonders whether this [malapportionment] might be best for the party itself; without vigorous and effective criticism from outside can it maintain its record of efficiency?' These reflections refer, not a little ironically, to the ALP in Queensland during the 1950s. See A. A. Morrison, 'The Queensland Electoral System', *Australian Quarterly*, 28 (1956), 80–5.

[62] Formally known as the 'Commission of Inquiry Into Possible Illegal Activities and Associated Police Misconduct' (1987–89), the Fitzgerald Inquiry had far-reaching political consequences in Queensland. Labor resumed office in the 1989 election after thirty-two years in opposition, in the wake of the Commission's findings of high-level corruption in Queensland public life. Among Fitzgerald's recommendations was the creation of an independent electoral commission in Queensland. See the *Report of the Commission* and discussions in Coaldrake's *Working the System: Government in Queensland*, and Evan Whitton, *The Hillbilly Dictator: Australia's Police State* (Crows Nest: ABC Enterprises, 1989). Contrast Mackerras, 'A Revisionist Interpretation of the Impact of Queensland's Electoral Scheme', p. 347.

[63] V. O. Key, *Southern Politics in State and Nation* (New York: Knopf, 1949; 2nd reprinting, Knoxville: University of Tennessee, 1986), chap. 14.

substantive conclusions about electoral manipulations and its consequences. Only with such tools can we systematically draw conclusions about the 'fairness' of a set of electoral arrangements, and in turn, the legitimacy of elections and governments themselves.

APPENDIX A: SAMPLING DISTRIBUTIONS

The quantities in Equations (5) and (6) involve taking exponents and dividing least squares regression estimates. The sampling distributions of the estimates themselves are asymptotically normal, and testing hypotheses about their sampling distributions is straightforward. However, the sampling distributions of the quantities in Equations (5) and (6) are not known to mathematical statistics. I build these sampling distributions 'from scratch', using computer-intensive techniques. This is a special case of what is called the 'bootstrap' in the statistics literature.[64]

In the case of Equation (6) I sample the duple ($\overline{\ln \beta, \hat{\rho}}$) from a bivariate normal distribution with means, variances and covariances equal to the least squares estimates of $\ln \beta$ and ρ and the estimated asymptotic variances and covariances of these estimates, respectively. In the case of Equation (5) I sample from a univariate normal distribution. I then substitute these sampled values into Equations (5) and (6) and store the resulting quantities. One thousand repetitions of this process yields a precise approximation of the asymptotic sampling distribution of these quantities. I then simply note the proportion of sampled values that lie above 50 per cent and report this as the 'p-value' in the tables in the text.

APPENDIX B: ESTIMATION OF SEATS–VOTE CURVES

This discussion closely follows that of Gelman and King.[65] Since that discussion is rather compact, this appendix may provide more practical details as to estimation. None the less, interested readers are strongly advised to consult the original Gelman and King article.

Summary

Estimates of bias and responsiveness are obtained in four stages. First, constituency-level data from multiple elections are pooled to estimate σ^2, the variance of constituency-level results (on the logit scale). Secondly, using data from just one election, I use Bayesian methods to derive the posterior distribution of γ, the constituency-specific effects. Thirdly, averaging over the variation in γ allows estimation of the posterior seats–votes curve, $E(S_t \mid V_t, u_0)$. For a given estimate of γ, V_t is allowed to range from 0 to 1 by allowing δ_t (the jurisdiction-wide swing) to range from $-\infty$ to ∞ on the logit scale. Last, summaries of the seats–votes curve are estimated, yielding a measure of bias and responsiveness for each election under consideration.

Hyperparameters. The first task is to estimate σ^2. Gelman and King suggest the following conceptual model of variation in constituency-level results:

(variance between two elections, Y years apart)
= (variance due to randomness in individual elections) + (variance due to changes in underlying electoral system)

---

[64] For an introduction see Robert Stine, 'An Introduction to Bootstrap Methods: Examples and Ideas', in John Fox and J. Scott Long, eds, *Modern Methods of Data Analysis* (Newbury Park, California: Sage, 1990).

[65] Gelman and King, 'Estimating the Electoral Consequences of Legislative Redistricting'.

Note that the difference between u_{it_1} and u_{it_2} has variance $2\sigma^2$ if, as assumed in Equation (7), their two normal distributions are independent. Note also that σ^2 is constant across elections. Gelman and King suggest the second quantity on the right side of the above expression to be roughly proportional to Y. This being the case, $2\sigma^2$ is estimated as η_0 in the following regression equation:

$$s_{t_1 t_2}^2 = \eta_0 + \eta_1(t_2 - t_1) + \omega \tag{B1}$$

where

$$s_{t_1 t_2}^2 = \frac{1}{n_{t_1 t_2}} \sum_i [u_{it_1} - u_{it_2} - (\bar{u}_{t_1} - \bar{u}_{t_2})]^2, \tag{B2}$$

and $n_{t_1 t_2}$ is the number of constituencies in both elections t_1 and $t_2, \forall t_1 < t_2$ (this quantity is not estimated where a redistricting intervenes between t_1 and t_2), so at least three elections are required to estimate this quantity.

Where redistricting intervenes before three elections transpire I set σ equal to the value estimated from the previous redistricting, or to 0.15, unless otherwise noted. Western Australia redistricts frequently and no estimate of σ could be obtained. New South Wales also redistricts frequently, and I was only able to obtain one estimate of σ, 0.1566 (s.e. = 0.0246) for the 1973 to 1978 set of elections. I obtained thirteen estimates of various σ in all, ranging from 0.0733 (s.e. = 0.0009) for House of Representatives elections from 1949 to 1954, to 0.2319 (s.e. = 0.1064) for South Australia, 1977 to 1982, with an average around 0.15.

In general, these estimates indicate fairly stable electoral politics, compared with the variation observed in the American states. Gelman and King report estimates of around 0.20 in their analyses of redistricting in Ohio, Connecticut and Wisconsin. The smaller variations in Australian electoral systems come as no surprise: where parties and party loyalties are stronger, prior expectations are that swings will be more uniform.[66] While these estimates of σ show Australian swings to be smaller and less variable across constituencies than those in the American states, they are a far cry from uniform swing. Even the smallest variation in constituency-level swings I find corresponds to about a 95 per cent confidence interval of ± 3.6 per cent if we were to predict a constituency-level result of 50 per cent, using uniform swing. The corresponding confidence bound for the largest estimate of variability in swing (South Australia, 1977–82) is ± 11.2 per cent.[67]

The constituency-level effects, γ, are assumed to be drawn from a mixture of three normal distributions: one distribution corresponding to Labor strongholds, another corresponding to competitive constituencies, and the third corresponding to non-Labor strongholds. In the absence of constituency-level covariates, imposing this weak structure on the data is an appropriate way to model uncertainty about the pattern of variation in constituency-level swings about the jurisdiction-wide swing. Eight parameters characterize this normal mixture distribution: three means, three variances and two mixture proportions (since the constraint that the mixture proportions must sum to one exactly identifies the third mixture proportion). Formally, define the vector of parameters as $\theta = (\mu_j, \rho_j^2 - \sigma^2, \lambda_j; j = 1, 2, 3)$ with the constraint $\Sigma_j \lambda_j = 1$, where j indexes the three constituency types defined above. Directly estimating these parameters via maximum likelihood is impossible, since there exist an infinite set of maxima of the likelihood function where the variances of any of the j normal distributions is zero.[68] To avoid this problem with direct ML estimation Gelman and King recommend positing prior distributions over the parameters and estimating them with Bayesian methods.

[66] See Hughes, 'Fair and Equal Constituencies', p. 262.

[67] This estimate is obtained by substituting the estimate of σ in the following expression:

$$100\left(0.5 - \frac{\exp(1.96\sigma)}{(1 + \exp(1.96\sigma))}\right),$$

recalling that σ is on the logit scale, and that logit (0.5) = 0.

[68] Gary King, *Unifying Political Methodology* (New York: Cambridge University Press, 1989), p. 82.

Prior distributions for θ conjugate with a $N(\gamma_i, \sigma^2)$ distribution that are mathematically convenient and substantively adequate are:[69]

$$\mu_j \sim N(\mu_{\mu_j}, \sigma^2_{\mu_j}), \; j = 1, 2, 3$$
$$\rho_j^{-2} \sim \Gamma(\tfrac{1}{2}\alpha_{\rho_j}, \tfrac{1}{2}\beta_{\rho_j}), \; j = 1, 2, 3$$
$$(\lambda_1, \lambda_2, \lambda_3) \sim \text{Dirichlet} \; (a_{\lambda_1}, a_{\lambda_2}, a_{\lambda_3}). \tag{B3}$$

After inspecting many plots of the distribution of Labor vote proportions from state and federal elections (for example, Figure 3), and appraisal of work in other two-party single-member constituency systems, I assign the following values to the parameters characterizing my prior distributions over θ:

j	1	2	3
μ_j	-0.5	0	0.5
$\sigma^2_{\mu_j}$	0.5	3	0.5
α_{ρ_j}	4	4	4
β_{ρ_j}	0.16	0.64	0.16
a_{λ_j}	20	10	20

These are quite diffuse priors, consistent with the notion of imposing a weak structure on the distribution of constituency-level deviations from the jurisdiction-wide component. Note also that since $j = 1$ corresponds to non-Labor strongholds, and $j = 3$ corresponds to Labor strongholds, these priors are substantively identical with respect to both Labor and non-Labor. In this way there is no *a priori* tendency in the structure of this model to find a preponderance of constituency-level vote swings towards one party or the other.

Estimation for a single election. I first estimate $P(\theta \mid u_0)$. Maximum likelihood estimation of θ is impossible, but the problem becomes tractable with recourse to Tanner and Wong's 'data augmentation' algorithm,[70] noting that $P(\theta \mid u_0) = \int (\theta \mid \tau, u_0) P(\tau \mid \theta, u_0) \, d\tau$ where τ is a matrix of unobserved indicators corresponding to the j normal mixtures, and where the integration

---

[69] For details on the properties of these distributions see Norman L. Johnson and Samuel Kotz, *Distributions in Statistics: Continuous Univariate Distributions I and II* (New York: Wiley, 1970). Algorithms for generating random samples from these distributions can be found in Luc Devroye, *Non-Uniform Random Variate Generation* (New York: Springer-Verlag, 1986) and A. Narayanan, 'Computer Generation of Dirichlet Random Vectors', *Journal of Statistical Computation and Simulation*, 36 (1990), 19–30.

[70] Martin A. Tanner and Wing Hung Wong, 'The Calculation of Posterior Distributions by Data Augmentation', *Journal of the American Statistical Association*, 82 (1987), 528–40. This method is a special case of what is known as 'Gibbs sampling' in the statistics literature. See Martin A. Tanner, *Tools for Statistical Inference: Observed Data and Data Augmentation Methods* (New York: Springer Verlag, 1991), the review in Andrew Gelman and Donald B. Rubin, 'Inference from Iterative Simulation Using Multiple Sequences', *Statistical Science*, 7 (1992), 457–511, and the essays in J. M. Bernardo, J. O. Berger, A. P. Dawid and A. F. M. Smith, eds, *Bayesian Statistics 4* (Oxford: Oxford University Press, 1992).

is over the parameter space of τ. The second term of the right-hand side of this expression is a normal density, conditional on the unobserved indicators. Methods exist for estimating the parameters of normal mixtures factored in this way, involving either the EM algorithm for missing data or methods derived around it.[71] With imputed values $\tau*$ for the unobserved indicators derived from these EM-like procedures, sampling from $P(\theta \mid \tau*, u_0)$ is done via Bayes theorem and the prior distributions defined earlier.

The process starts with an application of the EM algorithm to obtain $P(\theta \mid u_0)$, treating τ as missing data. Convergence of this application of the EM algorithm is generally slow since the entire n by 3 matrix of indicators, τ, is regarded as missing information. The values obtained from the EM algorithm are considered 'starting values' and are 'updated' using the prior distributions above. This 'updating' step closely resembles the EM algorithm again. The τ are 'data' manufactured by the analyst, and assign the 'real' data into the J sub-samples, characterized by θ. The aim is to sample from the distribution of θ conditional on the 'real data', but this is simply intractable as stated, since we need to know something about how the 'real data' are distributed through the J mixtures. Using the 'starting values' for θ, denoted as $\theta*$, I sample the indicators $\tau*$ from $P(\tau \mid \theta*, u_0)$, noting that

$$(\tau_{i1}, \tau_{i2}, \tau_{i3} \mid \theta, u_0) \sim \text{multinomial } (\lambda_{i1}^*, \lambda_{i2}^*, \lambda_{i3}^*; 1), \quad i = 1, \ldots, n, \tag{B4}$$

where

$$\lambda_{ij}^* = \frac{\lambda_j}{\rho_j} \phi \left(\frac{u_i - \mu_j}{\rho_j} \right), \tag{B5}$$

and ϕ is the standard normal density function. With the current sample of indicators, $\tau*$, I then sample the elements of $\theta*$ from the following distributions, each defined using Bayes' Rule and the prior distributions defined earlier:

$$(\rho_j^{-2} \mid \tau, u_0) \sim \Gamma(\frac{1}{2}(\alpha_{\rho j} + n_j), \frac{1}{2}(\beta_{\rho j} + SS_j)), \tag{B6}$$

$$(\mu_j \mid \rho_j^2, \tau, u_0) \sim N(\mu_j^*, \rho_j^{*2}), \tag{B7}$$

$$(\lambda_1, \lambda_2, \lambda_3 \mid \tau, u_0) \sim \text{Dirichlet}(a_{ij} + n_j), \tag{B8}$$

where

$$n_j = \sum_i \tau_{ij}, \tag{B9}$$

$$\mu_j^* = \frac{\sigma_{\mu j}^2 n_j \bar{u}_j + \rho_j^2 \mu_{\mu j}}{\sigma_{\mu j}^2 n_j + \rho_j^2}, \tag{B10}$$

$$SS_j = \sum_i \tau_{ij}(u_{i0} - \bar{u}_j)^2, \tag{B11}$$

$$\bar{u}_j = \frac{1}{n_j} \sum_i \tau_{ij} u_{i0}, \tag{B12}$$

$$\rho_j^{*2} = \frac{\sigma_{\mu j}^2 \rho_j^2}{n_j \sigma_{\mu j}^2 + \rho_j^2}, \tag{B13}$$

where $j = 1,2,3$. The sampled values of ρ_j^2 are constrained to be no less than σ^2 simply by re-sampling until appropriate values are found. This constraint follows from Equation (7), which shows that the variance of the constituency-specific effects is a component of the total variation in constituency results. Since the total variation in constituency-level outcomes is σ^2,

[71] See A. P. Dempster, N. M. Laird and D. B. Rubin, 'Maximum Likelihood from Incomplete Data via the EM Algorithm', *Journal of the Royal Statistical Association*, Series B, 39, No.1 (1977), 1–22; Murray Aitkin and Granville T. Wilson, 'Mixture Models, Outliers, and the EM Algorithm', *Technometrics*, 22 (1980), 325–31; D. B. Rubin and R. Little, *Statistical Analysis with Missing Data* (New York: Wiley, 1987); and Geoffrey J. McLachlan and Kaye E. Basford, *Mixture Models: Inference and Applications to Clustering* (New York: Marcel Dekker, 1988).

and ρ_j^2 is the variance of the jth mixture, the variance of the constituency-specific effects is $\rho_j^2 - \sigma^2 > 0$ implying the constraint $\rho_j^2 > \sigma^2$, recalling that constituency-specific and jurisdiction-wide effects are assumed to have zero covariance.

Ten iterations of this second-step 'EM-like' algorithm produce reasonably stable estimates of θ. Each repetition of the two-step process yields an approximately independent sample from the posterior distribution of θ.

With each sample from the posterior of θ I then sample from $P(\gamma \mid \theta, u_0)$ to obtain estimates of constituency-level effects, γ. This is accomplished by factoring the conditional posterior of γ as follows

$$P(\gamma \mid \theta, u_0) = \prod_i P(\gamma_i \mid \theta, u_0)$$
$$\propto \prod_i P(u_{i0} \mid \gamma_i, \theta) P(\gamma_i \mid \theta).$$

Each γ_i is sampled from the mixture of the j normal distributions, with means μ_{ij}^*, variances ρ_{ij}^{2*} and mixture proportions λ_{ij}^* conditional on the data for each district. Formally,

$$\mu_{ij}^* = \frac{\sigma^2 \mu_j + \rho_j^2 u_{i0}}{\sigma^2 + \rho_j^2},$$

$$\rho_{ij}^{2*} = \frac{\sigma^2 \rho_j^2}{\sigma^2 + \rho_j^2},$$

and

$$\lambda_{ij}^* = \frac{\lambda_j \exp\left(-\frac{1}{2} \frac{(u_{i0} - \mu_j)^2}{\sigma^2 + \rho_j^2} \right)}{\Sigma_{k=1}^3 \lambda_k \exp\left(-\frac{1}{2} \frac{(u_{i0} - \mu_k)^2}{\sigma^2 + \rho_k^2} \right)}$$

for $j = 1,2,3$.

Simulating the Seats–Votes Curve

With each sample of γ I then estimate the seats-votes curve by adding simulated levels of jurisdiction-wide partisan swing, δ_t to form

$$E(u_{it}) = \alpha_{it} = \gamma_i + \delta_t \tag{B14}$$

as per Equation (7). I chose about fifty unequally spaced values of δ_t between about -3 and 3 (on the logit of votes scale) so as to give a reasonably fine-grained approximation of the effects of a wide range of national (or state) partisan swings. For each sample of γ_i, and for each level of δ_t, α_{it} is sampled from a (α_{it}, σ^2) normal distribution. Averaging across fifty samples gives a relatively precise estimate of u_{it}, for each of the fifty levels of δ_t I chose.

Since $E(u_{it} \mid \alpha_{it}, \sigma^2)$ is on the logit scale, transformations are required to have the results of the simulations interpretable in terms of vote and seat proportions, rather than logits of these proportions. The following expectations are required to trace out a seats–vote curve conditional on α_{it} and σ^2:

$$E(V_{it} \mid \alpha_{it}) = \int_{-\infty}^{\infty} \mathrm{logit}^{-1} u \frac{1}{\sigma} \phi \left(\frac{u - \alpha_{it}}{\sigma} \right) du$$

$$= \int_{-\infty}^{\infty} \frac{e^u}{1 + e^u} \frac{1}{\sigma} \phi \left(\frac{u - \alpha_{it}}{\sigma} \right) du \tag{B15}$$

$$E(S_{it} \mid \alpha_{it}) = p[(V_{it}/\alpha_{it}) > 0.5]$$

$$= \int_0^\infty \frac{1}{\sigma} \phi\left(\frac{u - \alpha_{it}}{\sigma}\right) du$$

$$= \Phi\left(\frac{\alpha_{it}}{\sigma}\right) \tag{B16}$$

where ϕ and Φ are the standard normal probability density and the cumulative normal density function, respectively. Evaluating these conditional expectations is made tractable by replacing the inverse logit term with a third-degree Taylor series expansion, which is a reasonable approximation over the ranges of α_{it} in my data.

These constituency-specific results are averaged to the jurisdiction level. In the case of overall vote, this is done using the weighting procedure to account for malapportionment, as described on p. 343. In the case of seat shares, I simply average the constituency-specific probabilities of the ALP winning that seat, given in Equation (B16).

Bias and responsiveness are 'read off' the curves implied by these averages. Each simulation produces a seats–vote curve, and a set of bias and responsiveness estimates. The bias and responsiveness estimates for a single election are just the average of the fifty bias and responsiveness estimates: one pair for each simulation, where responsiveness is defined as the slope of the (linearized) seats–votes curve between $V_{it} = 0.45$ and $V_{it} = 0.55$, and bias is the height of the seats–vote curve above $S_{it} = 0.5$ where $V_{it} = 0.5$.