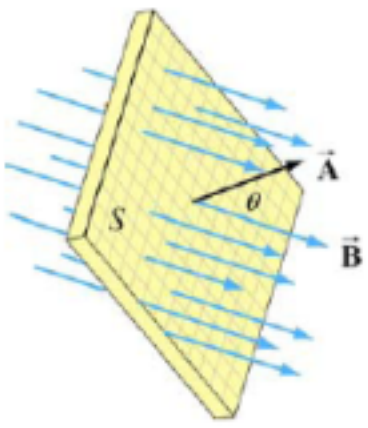
ELECTRO MAGNETIC INDUCTION

Magnetic Flux

Consider a uniform magnetic field passing through a surface *S*, as shown in the figure below:

Let the area vector be $\vec{A} = A\hat{n}$, where *A* is the area of the surface $\hat{n}$ and its unit normal. The magnetic flux through the surface is given by

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$



where θ is the angle between $\vec{B}$ and $\hat{n}$. If the field is non-uniform, Φ_B then becomes

$$\Phi_B = \int_S \vec{B} \cdot d\vec{A}$$

The SI unit of magnetic flux is the weber (Wb): $1\,Wb = 1\ T.m^2$

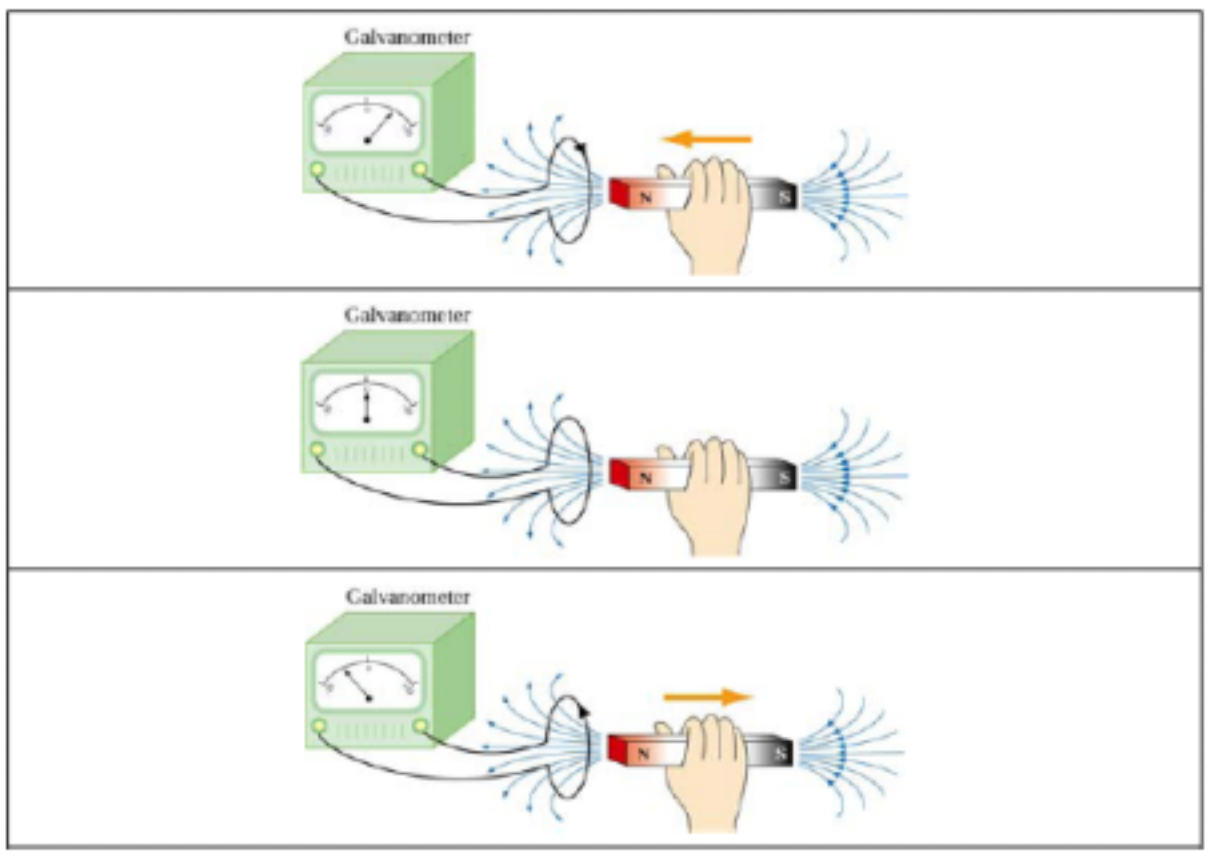
Electromagnetic induction

The electric fields and magnetic fields considered up to now have been produced by stationary charges and moving charges (currents), respectively.

It was observed that when a closed current carrying loop can set up a magnetic field . By nature of symmetry an obvious question comes in mind, can a current be generated with the help of magnetic field?

Answer to this care in 1831 when michacl faraday discovered that a change in magnetic filed can produce a current in a loop . The phenomenon is known as electromagnetic induction. Figure illustrates Faraday's experiments.

Faraday's Experiment



Faraday observed that no current is registered in the galvanometer when bar magnet is stationary with respect to the loop. However, a current is induced in the loop when a relative motion exists between the bar magnet and the loop. In particular, the galvanometer deflects in one direction as the magnet approaches the loop, and the opposite direction as it moves away.

Faraday concluded that whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop.

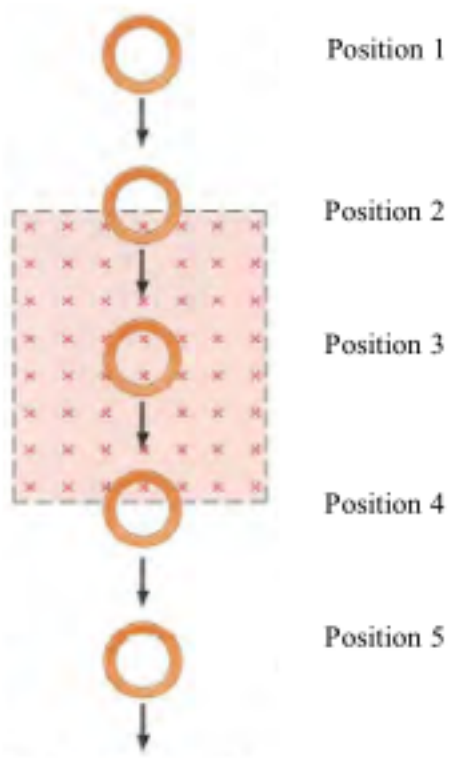
Lenz's Law (Deciding direction of inducted e.m.f.)

The induced emf resulting from a changing magnetic ftux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original ftux change.We shall discuss later on how it is in accordance with conservation of energy

Illustration

In Figure there is a constant magnetic field in a rectangular region of space. This field is directed perpendicularly into the page. Outside this region there is no magnetic field. A copper ring moves through the region from position 1 to position 5. Find the induced current in the ring as it passes through positions

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5



Sol *(a) Since the field is zero outside the rectangular region, no flux passes through the ring in position 1 , there is no change in the flux through the ring, and there is no induced emf or current in the ring*

(b) In position 2 the flux increases. According to Lenz's law, the induced current must create an induced magnetic field that opposes the increase. To oppose the increase, the induced field must point opposite to the external field and, therefore, must point out of the page. for which the induced current must be counterclockwise .

(c) Here the field is not zero, Hence nonzero flux passes through the ring in position 3 but the flux through the ring is constant, and there is no induced emf or current in the ring

(d) In position 4 the flux decreases. According to Lenz's law, the induced current must create an induced magnetic field that opposes the decrease. To oppose the decrease, the induced field must point in the direction of external field and, therefore, must point into the page. For which the induced current must be clockwise .

(e) Since the field is zero outside the rectangular region, no flux passes through the ring in positions 5, there is no change in the flux through the ring, and there is no induced emf or current in the ring

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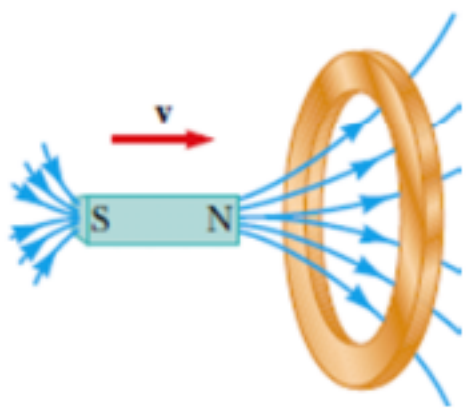
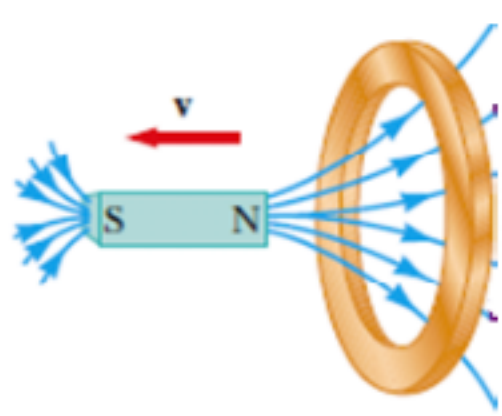
Practice Exercise

Q.1 Determine the direction of the induced current for the following situations.

(a)



(b)

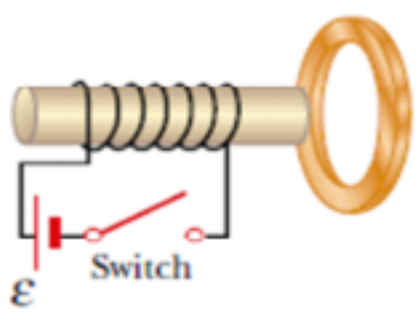


Q.2 A metal ring is placed near a solenoid, as shown in Figure. Find direction of induced current in the ring

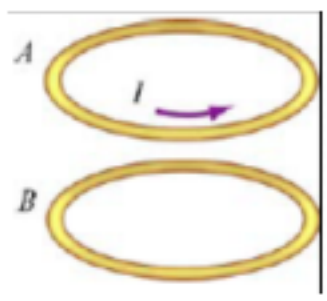
a. at the instant the switch in the circuit containing the solenoid is thrown closed,

b. after the switch has been closed for several seconds, and

c. at the instant the switch is thrown open.



Q.3 Two circular loops *A* and *B* have their planes parallel to each other, as shown in Figure.



Loop *A* has a current flowing in the counterclockwise direction, viewed from above.

(a) If the current in loop *A* decreases with time, what is the direction of the induced current in loop *B*? Will the two loops attract or repel each other?

(b) If the current in loop *A* increases with time, what is the direction of the induced current in loop *B*? Will the two loops attract or repel each other?

Q.4 Fig. illustrates plane figures made of thin conductors which are located in a uniform magnetic field directed away from a reader beyond the plane of the drawing. The magnetic induction starts diminishing. Find how the currents induced in these loops are directed.

(a) (b) (c) (d)

Answers

Q.1 (a) anticlockwise when seen from the side of magnet (b) clockwise when seen from the side of magnet

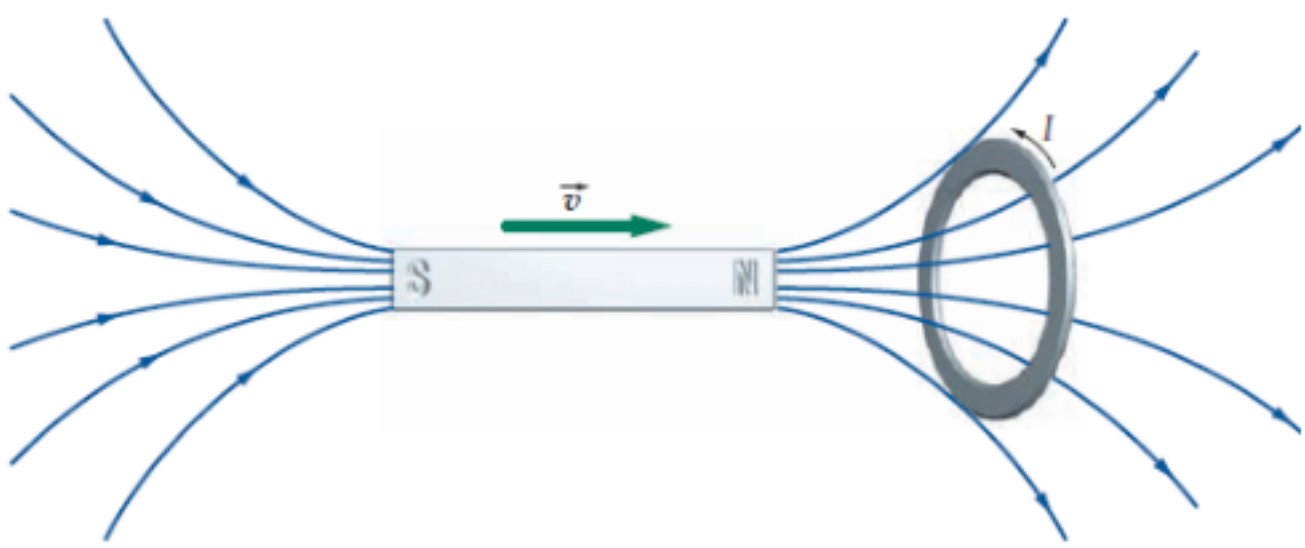
Q.2 (a) clockwise when seen from the side of solenoid (b) zero
(c) anticlockwise when seen from the side of solenoid

Q.3 (a) anticlockwise when seen from above ,attract (b) clockwise when seen from above ,repel

Q.4 (a) In the round conductor the current flows clockwise there is no current in the connector ; (b) in the outside conductor, clockwise ; (c) in both round conductors, clockwise ; no current in the connector, (d) in the left-hand side of the figure eight, clockwise

Lenz's law And conservation of energy

First figure (below) shows a bar magnet moving toward a conducting loop. It is the motion of the bar magnet to the right that induces an emf and current in the loop. Lenz's law tells us that this induced emf and current must be in a direction to oppose the motion of the bar magnet. That is, the current induced in the loop produces a magnetic field of its own, and this magnetic field must exert a force to the left on the approaching bar magnet.



Second figure shows the induced magnetic moment of the current loop when the magnet is moving toward it. The loop acts like a small magnet with its north pole to the left and its south pole to the right. Because like poles repel, the induced magnetic moment of the loop repels the bar magnet; that is, it opposes its motion toward the loop. This result means the direction of the induced current in the loop must be as shown in Second figure .

Suppose the induced current in the loop shown in Second figure was opposite to the direction shown. Then there would be a magnetic force toward the right on the approaching bar magnet, causing the bar magnet to gain speed. This gain in speed would cause an increase in the induced current, which in turn would cause the force on the bar magnet to increase, and so on. This result is too good to be true. Any time we nudge a bar magnet toward a conducting loop it would move toward the loop with ever increasing speed and with no significant effort on our part. Were this situation to occur, it would be a violation of energy conservation. The reality, however, is that energy is conserved, and Lenz's law is consistent with this reality.

Faraday's Law

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

$$: \qquad \varepsilon = -\frac{d\Phi_B}{dt}$$

Note:

(1) The -ve sign is according to Lenz's law (opposition)

(2) For a coil that consists of *N* loops, the induced emf would be:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

(3) Average emf is is given by

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t}$$

(4) We can see that an emf may be induced in the following ways:

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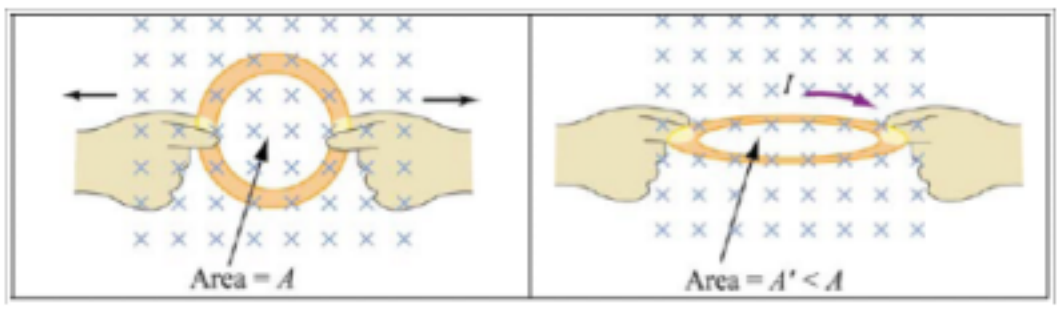
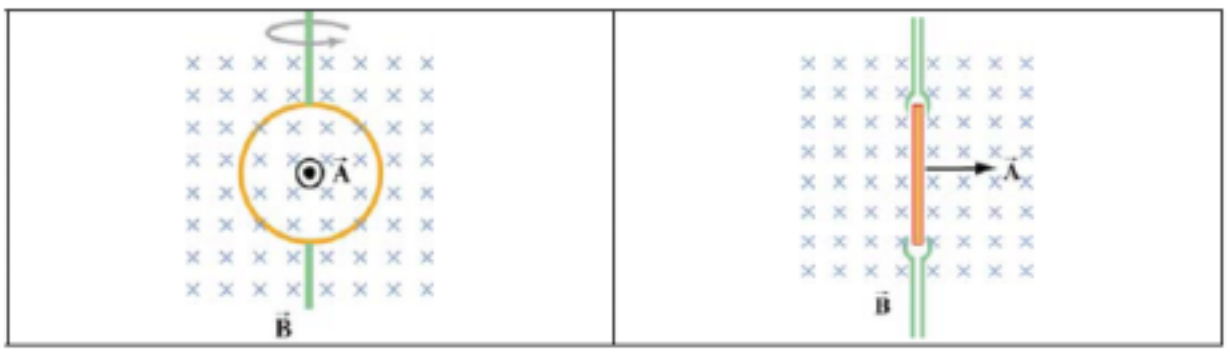
(i) by varying the magnitude of $\vec{B}$ with time (illustrated in Figure)

(ii) by varying the magnitude of $\vec{A}$, i.e., the area enclosed by the loop with time (illustrated in Figure)



(iii) varying the angle between $\vec{B}$ and the area vector $\vec{A}$ with time (illustrated in Figure)



(5) Induced current is given by

$$i = -\frac{1}{R}\frac{d\Phi_B}{dt}$$

(6) Charge flown through a coil

$$\Delta q = \int i\,dt = \int_{t_1}^{t_2} -\frac{1}{R}\frac{d\Phi}{dt}dt = \frac{1}{R}\int_{\Phi_1}^{\Phi_2} -d\Phi$$

$$\Delta q = \frac{\Phi_1 - \Phi_2}{R}$$

(7) Heat developed will be

$$H = \int i^2 R\,dt$$

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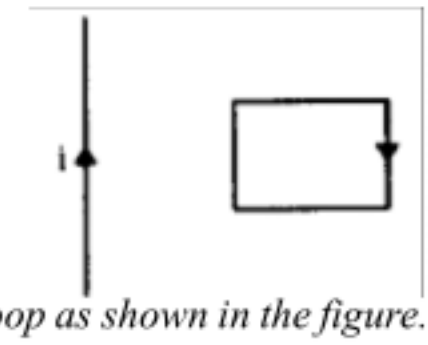
(8) You can also evaluate the direction of ξ from equation $\varepsilon = -\dfrac{d\Phi_B}{dt}$

The procedure to decide the direction is as follows:

Put an arrow on the loop to choose the positive sense of current. This choice is arbitrary. Using righthand thumb rule find the positive direction of the normal to the area bounded by the loop. If the fingers curl along the loop in the positive sense, the thumb represents the positive direction of the normal. Calculate the flux through the area bounded by the loop. If the flux increases with time, $\dfrac{d\Phi_B}{dt}$ is positive and ε is negative from equation $\varepsilon = -\dfrac{d\Phi_B}{dt}$. Correspondingly, the current is negative. It is, therefore, in the direction opposite to the arrow put on the loop. If the flux decreases with time, $\dfrac{d\Phi_B}{dt}$ is negative, ε is positive and the current is along the arrow.

Illustration :

Figure shows a conducting loop placed near a long, stright wire carrying a current i as shown. If the current increases continuously, find the direction of the induced current in the loop.



Sol. *Let us put an arrow on the loop as shown in the figure. The right-hand thumb rule shows that the positive normal to the loop is going into the plane of the diagram. Also, the same rule shows that the magnetic field at the site of the loop due to the current is also going into the plane of the diagram. Thus $\vec{B}$ and $\vec{A}$ are along the same direction everywhere so that the flux is positive. If i increases, the magnitude of Φ_B increases. Since Φ_B is positive and its magnitude increases, $\dfrac{d\Phi_B}{dt}$ is positive. Thus, ε is negative and hence , the current is negative. The current is, therefore, induced in the direction opposite to the arrow.*

Illustration :

The magnetic flux through each turn of a 100 turn coil is $(t^3 - 2t) \times 10^{-3}$ Wb, where t is in second.Find the induced emf at t = 2 s .

Sol.

$$\phi = (t^3 - 2t) \times 10^{-3}$$

$$\frac{d\phi}{dt} = (3t^2 - 2) \times 10^{-3}$$

$$\left.\frac{d\phi}{dt}\right|_{t=2} = (3\times 4 - 2) \times 10^{-3} \quad . \; Wb/s$$

$$= 10^{-2}\ \text{Wb/s} \qquad \Rightarrow \qquad e = -N\frac{d\phi}{dt}$$

$$= -100\times 10^{-2}\ \text{V} \qquad \Rightarrow \qquad = -1\ V.$$

Illustration :

A wire in the form of a circular loop of radius 10 cm lies in a plane normal to a magnetic field of 100 T. If this wire is pulled to take a square shape in the same plane in 0.1 s,Find average induced emf in the loop is:

Sol. *According to Faraday's law of electromagnetic induction,*

$$E_{induced} = \frac{-\Delta\phi}{\Delta t} = -\frac{B(A_f - A_i)}{\Delta t}$$

Let r be the radius of circle; then side of square formed $= \frac{2\pi r}{4} = \frac{\pi r}{2}$

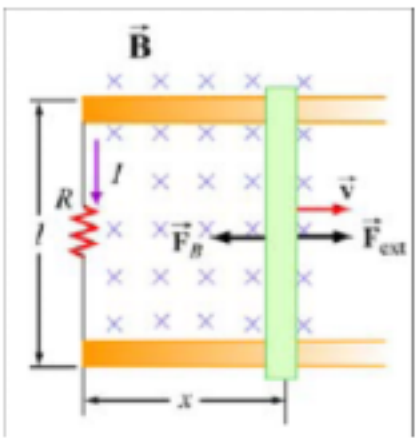
Change is area of loop $= A_i - A_f = \pi r^2 - \left(\frac{\pi r}{2}\right)^2 = \frac{\pi(4-\pi)r^2}{4}$

Hence average emf induced $= \frac{\pi(4-\pi)r^2}{4} \cdot \frac{B}{t}$

$$= \frac{\pi(4-\pi)\times(0.1)^2 \times 100}{4\times 0.1} \qquad = 6.75 \text{ volt.}$$

Illustration :

A conducting bar moves through a region of uniform magnetic field $\vec{B} = -B\hat{k}$ *(pointing into the page) by sliding along two frictionless conducting rails as shown in figure. An external force* $\vec{F}_{ext}$ *be applied so that the conductor moves to the right with a constant speed v. Find the*



a) emf induced in the coil.
b) curent through the resistor and its direction.
c) External force required to move the rod with constant speed.
d) power delivered by external force
e) power dissipated in resistor.

Sol

(a) *Let us take* $\vec{A}$ *out of the page.*

Now magnetic flux over the coil.will be

$$\Phi_B = -B\ell x$$

emf induced in the coil.will be

$$\varepsilon = -\frac{d\Phi_B}{dt} = B\ell\frac{dx}{dt} = B\ell v$$

(b) *current through the resistor will be*

$$i = \frac{\varepsilon}{R} = \frac{B\ell v}{R}$$

(c) *Magnetic force on slider will be*

$$\vec{F}_B = i\vec{\ell} \times \vec{B} = \frac{B^2\ell^2 v}{R} \text{ (left)}$$

External force required to move the rod with constant speed. will be

$$\vec{F}_{ext} = -\vec{F}_B = \frac{B^2\ell^2 v}{R} \text{ (right)}$$

d) *power delivered by external force*

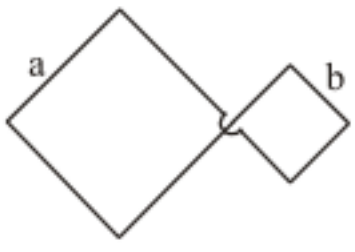
$$P_{ext} = F_{ext} v = \frac{B^2\ell^2 v^2}{R}$$

e) *power dissipated in resistor.*

$$P_R = i^2 R = \frac{B^2\ell^2 v^2}{R}$$

Illustration :

. *A plane loop shown in Fig. is shaped as two squares with sides a = 20 cm and b = 10 cm and is introduced into a uniform magnetic field at right angles to the loop's plane out of the page. The magnetic induction varies with time as $B = B_0 \sin \omega t$. Find the amplitude of the current induced in the loop if its resistance per unit length is equal to ρ. The inductance of the loop is to be neglected.*



Sol. *In both the loops e.m.f. are in opposing nature so we have to subtract in writing net flux*
Let us take $\vec{A}$ out of the page for bigger part.
Now magnetic flux over the coil.will be

$$\Phi_B = \left(a^2 - b^2\right) B_0 \cos \omega t$$

emf induced in the coil.will be

$$\varepsilon = -\frac{d\Phi_B}{dt} = \left(a^2 - b^2\right)\omega B_0 \sin \omega t$$

current through the resistor will be

$$i = \frac{\varepsilon}{R} = \frac{\left(a^2 - b^2\right)\omega B_0 \sin \omega t}{(\rho)\{4(a+b)\}} = \frac{(a-b)\omega B_0 \sin \omega t}{4\rho}$$

current amplitude through the resistor will be

$$i_{max} = \frac{(a-b)\omega B_0}{4\rho}$$

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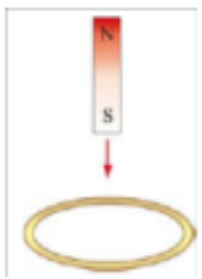
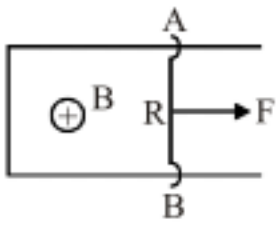
Practice Exercise

Q.1 A bar magnet falls through a circular loop, as shown in Figure.
(a) Describe qualitatively the change in magnetic flux through the loop when the bar magnet is above and below the loop.
(b) Make a qualitative sketch of the graph of the induced current in the loop as a function of time, choosing I to be positive when its direction is counterclockwise as viewed from above.



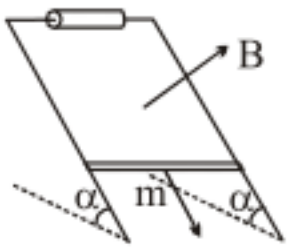
Q.2 A small coil is introduced between the poles of an electromagnet so that its axis coincides with the magnetic field direction. The cross-sectional area of the coil is equal to $S = 3.0\ mm^2$, the number of turns is $N = 60$. When the coil turns through 180° about its diameter, a ballistic galvanometer connected to the coil indicates a charge $q = 4.5\ \mu C$ flowing through it. Find the magnetic induction magnitude between the poles provided the total resistance of the electric circuit equals $R = 40\ \Omega$.

Q.3 A connector AB can slide without friction along a shaped conductor located in a horizontal plane. The connector has a length l, mass m, and resistance R. The whole system is located in a uniform magnetic field of induction B directed vertically. At the moment $t = 0$ a constant horizontal force F starts acting on the connector shifting it translationwise to the right. Find how the velocity of the connector varies with time t.



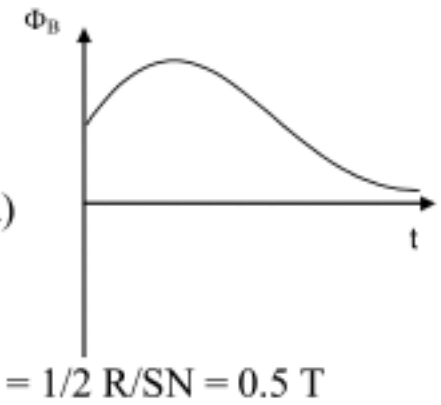
Q.4 In the previous question find the velocity of rod as a function of time, if initially it hasbeen given a velocity vo towards right and F is not acting.

Q.5 A copper connector of mass m slides down on two smooth copper bars, set at an angle α to the horizontal, due to gravity (Fig). At the top the bars are interconnected through a resistance R. The separation between the bars is equal to l. The system is located in a uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady-state velocity of the connector.

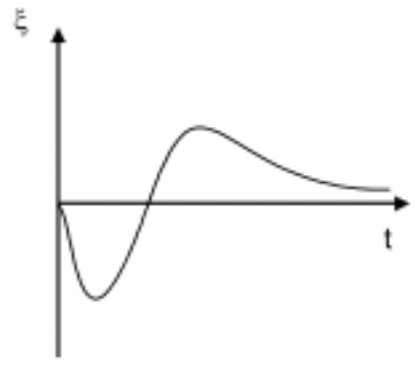


Answers

Q.1 (a)



(b)



Q.2 B = 1/2 R/SN = 0.5 T

Q.3 $v = \frac{FR}{B^2l^2}\left(1 - e^{-\frac{B^2l^2}{mR}t}\right)$

Q.4 $v = v_0 e^{-\frac{B^2l^2}{mR}t}$

Q.5 $v = \frac{mg\,R\sin\alpha}{B^2l^2}$

Generators :

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.

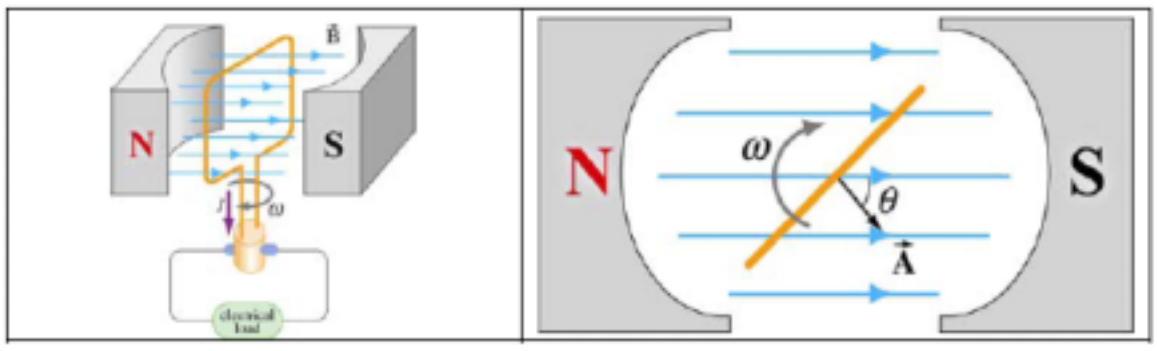


Figure : (a) A simple generator. (b) The rotating loop as seen from above.

Figure (a) is a simple illustration of a generator. It consists of an *N*-turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Figure (b), we see that the magnetic flux through the loop may be written as

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta = BA\cos\omega t$$

The rate of change of magnetic flux is $\frac{d\Phi_B}{dt} = -BA\,\omega\sin\omega t$

Since there are N turns in the loop, the total induced emf across the two ends of the loop is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = NB\,A\omega\sin\omega t$$

If we connect the generator to a circuit which has a resistance R, then the current generated in the circuit is given by

$$I = \frac{|\varepsilon|}{R} = \frac{NBA\omega}{R} \sin \omega t$$

The current is an alternating current which oscillates in sign and has an amplitude $I_0 = NBA\omega / R$. The power delivered to this circuit is $P = I|\varepsilon| = \frac{(NBA\omega)^2}{R} \sin^2 \omega t$

On the other hand, the torque exerted on the loop is

$$\tau = \mu B \sin \theta = \mu B \sin \omega t$$

Thus, the mechanical power supplied to rotate the loop is

$$P_m = \tau\omega = \mu B\, \omega \sin \omega t$$

Since the dipole moment for the N-turn current loop is

$$\mu = NIA = \frac{N^2 A^2 B\omega}{R} \sin \omega t$$

the above expression becomes

$$P_m = \left(\frac{N^2 A^2 B\omega}{R} \sin \omega t \right) B\omega \sin \omega t = \frac{(NAB\omega)^2}{R} \sin^2 \omega t$$

As expected, the mechanical power in put is equal to the electric power output.

Motional EMF

Consider a conducting bar of length *l* moving through a uniform magnetic field which points into the page, as shown in Figure. Particles with charge $q > 0$ inside experience a magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ which tends to push them upward, leaving negative charges on the lower end.

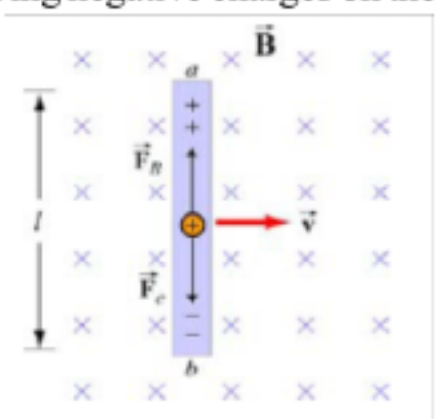


Figure : A conducting bar moving through a uniform magnetic field

The separation of charge gives rise to an electric field $\vec{E}$ inside the bar, which in turn produces a downward electric force $\vec{F}_e = q\vec{E}$. At equilibrium where the two forces cancel,

we have $qvB = qE$ or $E = vB$. Between the two ends of the conductor, there exists a potential difference given by

$$V_{ab} = V_a - V_b = \varepsilon = E\ell = B\ell v$$

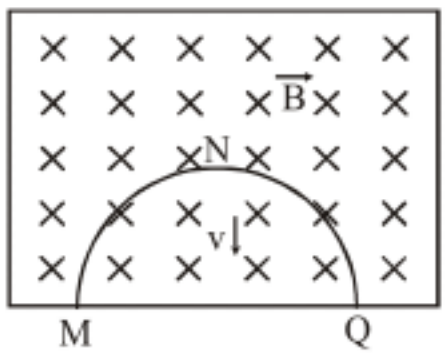
Since ε arises from the motion of the conductor, this potential difference is called the motional emf. In general, motional emf around a closed conducting loop can be written as

$$|\varepsilon| = \left|\oint(\vec{v}\times\vec{B}).d\vec{s}\right|$$

where $d\vec{s}$ is a differential length element.

Illustration :

A thin semi-circular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ (see figure). At the position MNQ the speed of the ring is v . Find the potential difference developed across the ring .



Sol. *The induced emf as given by Faraday's law of induction is*

$E = - B\, l\, v$

l = *projection of ring perpendicular to the direction of* $v = 2\,R$

$= B \times 2\,R \times v$

$= 2\,B\,v\,R.$

Illustration :

An air-plane with 20m wing spread is flying at 250 ms^{-1} straight south parallel to the earth's surface. The earth's magnetic field has a horizontal component of $2\times10^{-5}\,\mathrm{Wb\,m^{-2}}$ and the dip angle is 60°. Calculate the induced emf between the plane tips is:

Sol. *As the plane is flying horizontally it will cut the vertical component of earth's field B_V. So the emf induced between its tips,*

$e = B_V\, vl$

But as by difinition of angle of dip,

$$\tan\phi = \frac{B_V}{B_H} \quad i.e., \quad B_V = B_H \tan\phi$$

So $\quad e = (B_H \tan\phi)vl = 2\times10^{-5}\times\sqrt{3}\times250\times20$

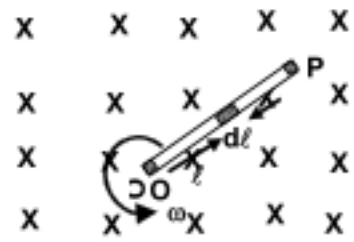
i.e., $\quad e = (\sqrt{3})\times10^{-1}\,V = 0.173\,V$.

Illustration :

A copper rod of length 'ℓ' rotates at an angular velocity ω in a uniform magnetic field B as shown in figure. What is the induced emf across its ends?

Solution:

The rod is supposed to be the combination of a number of infinitesimal elements. Speed of each element is different. Consider an element at a distance ℓ from O.



Speed of this element is ωℓ, and is perpendicular to its length

$\Rightarrow \varepsilon = \int vBd\ell$

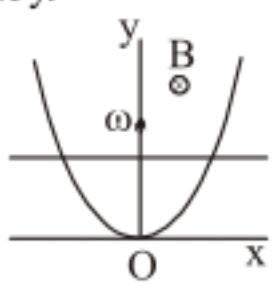
$$\text{As} \quad v = \omega\ell$$

$$\varepsilon = \omega B\int_0^L \ell d\ell = \frac{1}{2}\omega BL^2$$

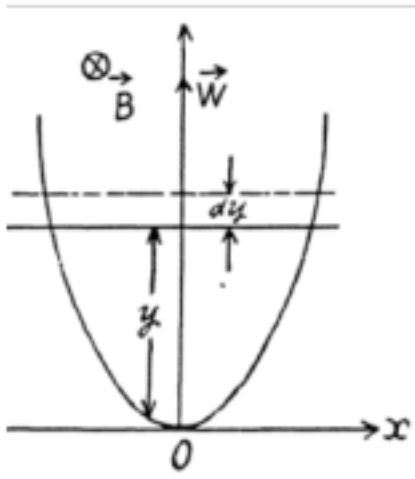
'O' turns out to be '+' and P the – ve terminal

Illustration :

A wire bent as a parabola $y = ax^2$ is located in a uniform magnetic field of induction B, the vector B being perpendicular to the x, y-plane. At the moment t = 0 a connector starts sliding translationwise from the parabola apex with a constant acceleration ω (Fig.). Find the emf of electromagnetic induction in the loop thus formed as a function of y.



Sol. Obviously, from Lenz's law, the induced current and hence the induced e.m.f. in the loop is anticlockwise.



From faraday's law of electromagnetic induction.

$$\xi_{in} = \left|\frac{d\phi}{dt}\right|$$

here, $d\phi = \vec{B}.d\vec{S} = -2B\,x\,dy$,

and from

$$y = ax^2,\ x = \sqrt{\frac{y}{a}}$$

Hence,

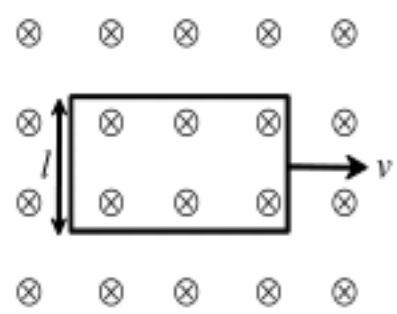
$$\xi_{in} = 2B\sqrt{\frac{y}{a}}\frac{dy}{dt} = By\sqrt{\frac{8\omega}{a}},\ \text{using}\ \frac{dy}{at} = \sqrt{2\omega y}$$

Alternative sol

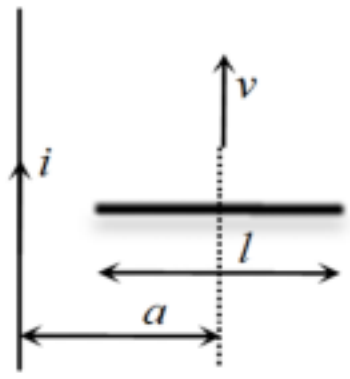
$$\varepsilon = vB\ell = vB(2x) = \left(\sqrt{2\omega y}\right)(B)(2x)$$

Practice Exercise

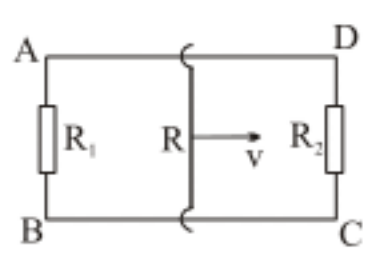
Q.1 For the situation shown, find the emf induced in the loop. Explain your answer by both flux theory and theory of motional emf.



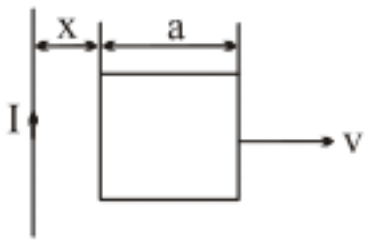
Q.2 Figure shows a straight, long wire carrying a current i and a rod of length l coplanar with the wire and perpendicular to it. The rod moves with a constant velocity v in a direction parallel to the wire. The distance of the wire from the near end of the rod is a. Find the motional emf induced in the rod.



Q.3 A rectangular loop with a sliding connector of length l is located in a uniform magnetic field perpendicular to the loop plane (Fig.). The magnetic induction is equal to B. The connector has an electric resistance R, the sides AB and CD have resistances R_1 and R_2 respectively. Neglecting the self-inductance of the loop, find the current flowing in the connector during its motion with a constant velocity v.

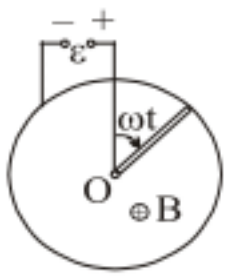


Q.4 A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in Fig. The frame translates to the right with a constant velocity v. Find the emf induced in the frame as a function of distance x.



Q.5 A spherical conducting shell is placed in a vertical time-varying uniform magnetic field. Is there an induced current along the equator?

Q.6 A metal rod of mass m can rotate about a horizontal axis O, sliding along a circular conductor of radius a (Fig.). The arrangement is located in a uniform magnetic field of induction B directed perpendicular to the ring plane. The axis and the ring are connected to an emf source to form a circuit of resistance R. Neglecting the friction, circuit inductance, and ring resistance, find the law according to which the source emf must vary to make the rod rotate with a constant angular velocity ω.



Answers

Q.1 Zero

Q.2 $\varepsilon = \frac{\mu_0 I v}{2\pi} \ln(1+\frac{\ell}{a})$

Q.3 $I = Bvl/(R+R_m)$, where $R_m = R_1R_2/(R_1+R_2)$

Q.4 $\xi_i = \frac{\mu_0}{4\pi} \frac{2Ia^2v}{x(x+a)}$

Q.5 Yes

Q.6 $\xi_i = \frac{\omega a^3B^3 + 2mg \sin \omega t}{2aB}$

Induced Electric Field

We have seen that the electric potential difference between two points *A* and *B* in an electric field $\vec{E}$ can be written as

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

When the electric field is conservative, as is the case of electrostatics, the line integral of it is path-independent, which implies $\oint \vec{E} \cdot d\vec{s} = 0$

Faraday's law shows that as magnetic flux changes with time, an induced current begins to flow. What

causes the charges to move? It is the induced emf which is the work done per unit charge. However, since magnetic field can do no work, as we have shown, the work done on the mobile charges must be electric, and the electric field in this situation cannot be conservative because the line integral of a conservative field must vanish. Therefore, we conclude that there is a non-conservative electric field $\vec{E}_{nc}$ associated with an induced emf : $\varepsilon = \oint \vec{E}_{nc} \cdot d\vec{s}$

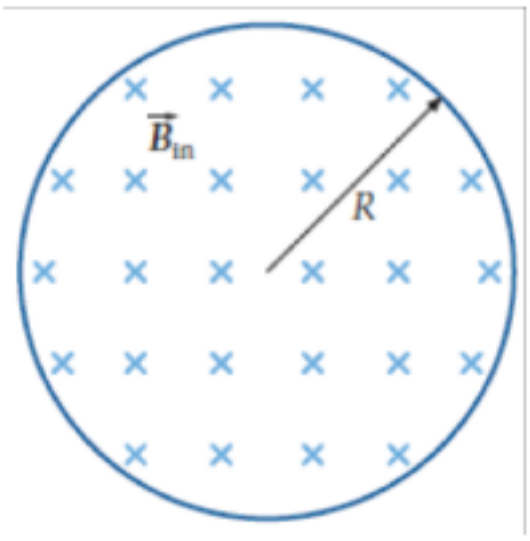
Combining with Faraday's law then yields :

$$\oint \vec{E}_{nc} . d\vec{s} = -\frac{d\Phi_B}{dt}$$

The above expression implies that a changing magnetic flux will induce a non-conservative electric field which can vary with time. It is important to distinguish between the induced, non-conservative electric field and the conservative electric field which arises from electric charges.

As an example, let's consider a uniform magnetic field which points *into* the page and is confined to a circular region with radius *R*, as shown in Figure. Suppose the magnitude of $\vec{B}$ increases with time, *i.e.*, $dB/dt > 0$. Let's find the induced electric field everywhere due to the changing magnetic field.

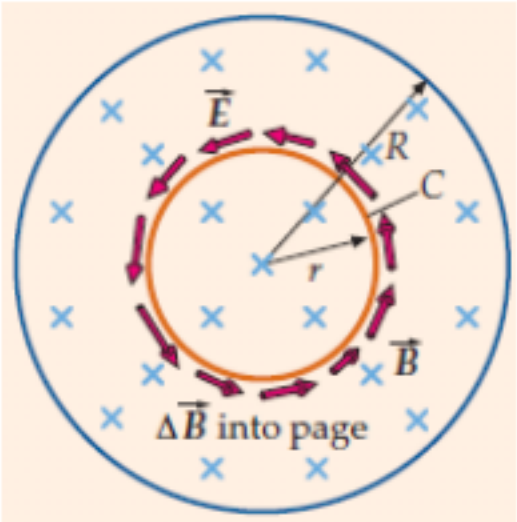
Since the magnetic field is confined to a circular region, from symmetry arguments we choose the integration path to be a circle of radius *r*. The magnitude of the induced field $\vec{E}_{nc}$ at all points on a circle is the same. According to Lenz's law, the direction $\vec{E}_{nc}$ of must be such that it would drive the induced current to produce a magnetic field opposing the change in magnetic flux. With the area vector $\vec{A}$ pointing *out* of the page, the magnetic flux is negative or inward. With $dB/dt > 0$, the inward magnetic flux is increasing. Therefore, to counteract this change the induced current must flow counterclockwise to produce more outward flux. The direction of $\vec{E}_{nc}$ is shown in Figure.



Let's proceed to find the magnitude of $\vec{E}_{nc}$.

In the region r >R

the rate of change of magnetic flux is :



$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(\vec{B}\,.\,\vec{A}) = \frac{d}{dt}(-BA) = -\left(\frac{dB}{dt}\right)\pi r^2$$

Using equation, we have

$$\oint \vec{E}_{nc}.d\vec{s} = E_{nc}(2\pi r) = -\frac{d\Phi_B}{dt} = \left(\frac{dB}{dt}\right)\pi r^2$$

which implies

$$E_{nc} = \frac{r}{2}\frac{dB}{dt}$$

Similarly, for r > R

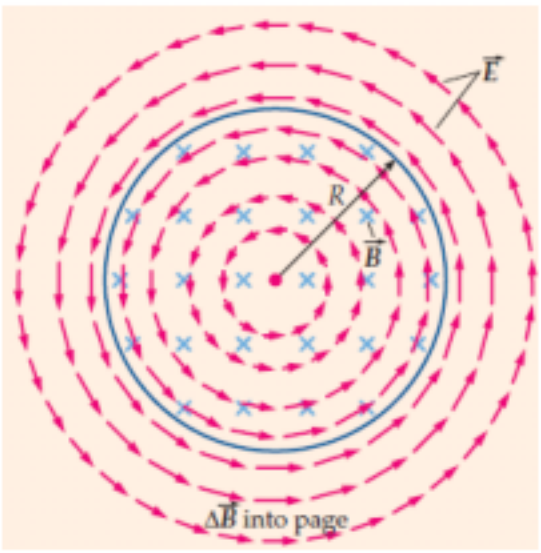
the induced electric field may be obtained as

$$E_{nc}\,(2\pi r) = -\frac{d\Phi_B}{dt} = \left(\frac{dB}{dt}\right)\pi R^2$$

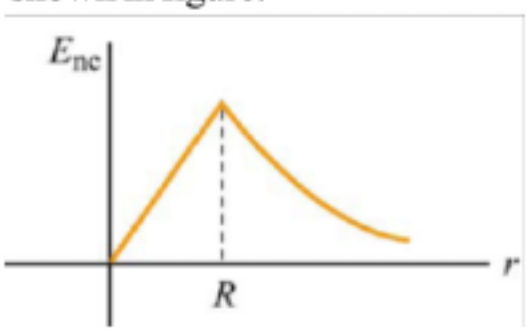
or

$$E_{nc} = \frac{R^2}{2r}\frac{dB}{dt}$$

The pattern of E_{nc} as a function of r is shown in figure.



A plot of E_{nc} as a function of r is shown in figure.



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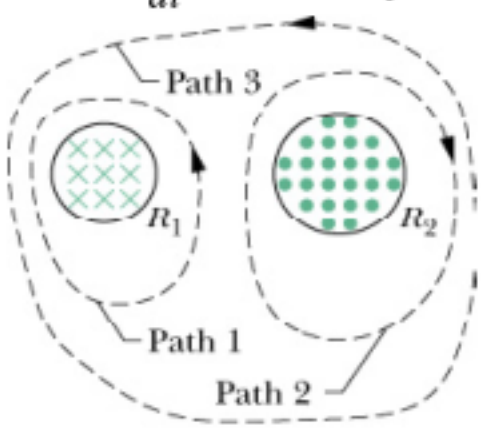
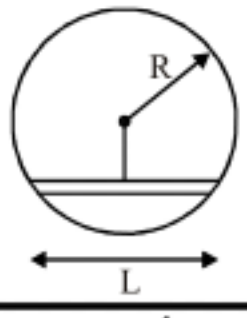
Practice Exercise

Q.1 Figure shows two circular regions of radii R_1 & R_2. (R_2 being the bigger circle). The magnetic field in both are decreasing at a rate of $\frac{dB}{dt}$. Calculate $\oint E.d\vec{l}$ for each of the three paths indicated.



Q.2 A uniform magnetic field fills a cylindrical volume of radius R. If the magnetic field is decreasing at a rate $\frac{dB}{dt}$, find induced emf at rod's end.



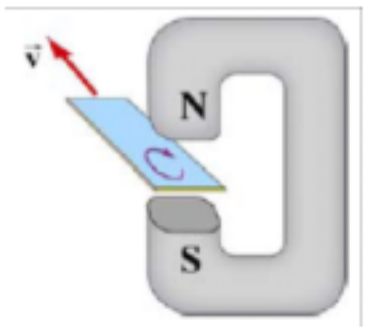
Answers

Q.1 $-\frac{dB}{dt}\pi R_1^2$ for path 1, $-\frac{dB}{dt}\pi R_2^2$ for path 2, $\frac{dB}{dt}\pi\left(R_2^2 - R_1^2\right)$ for path 3

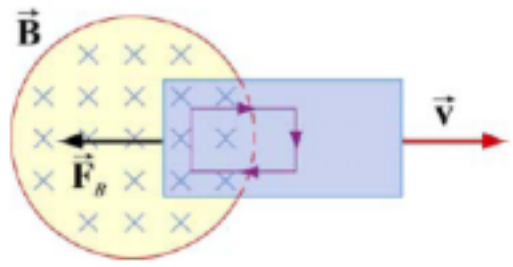
Q.2 $\frac{L}{2}\sqrt{R^2 - \frac{L^2}{4}}\frac{dB}{dt}$

Eddy Currents :

We have seen that when a conducting loop moves through a magnetic field, current is induced as the result of changing magnetic flux. If a solid conductor were used instead of a loop, as shown in Figure, currents can also be induced along any closed loop in the conductor. The induced current are called an *eddy current*.



The induced eddy currents also generate a magnetic force that opposes the motion, making it more difficult to move the conductor across the magnetic field (Figure).



Since the conductor has non-vanishing resistance *R*, Joule heating causes a loss of power by an amount $P = \varepsilon^2 / R$. Therefore, by increasing the value of R, power loss can be reduced. One way to increase *R* is to laminate the conducting slab, or construct the slab by using gluing together thin strips that are insulated from one another (see Figure-a). Another way is to make cuts in the slab, thereby disrupting the conducting path (Figure-b).

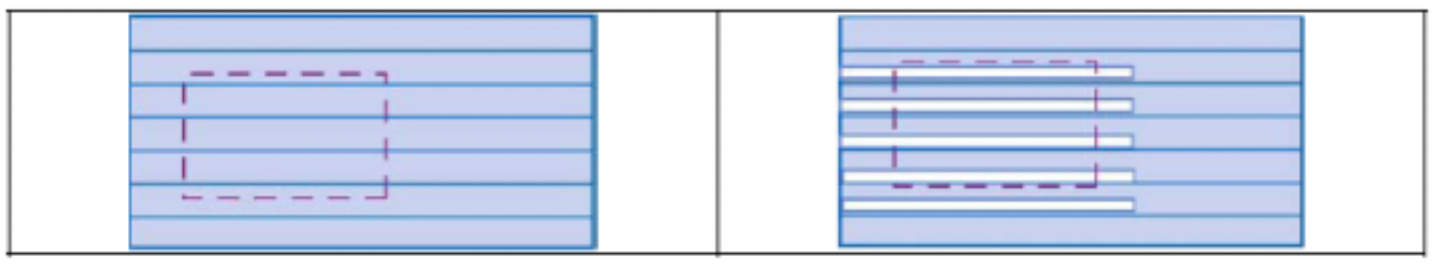
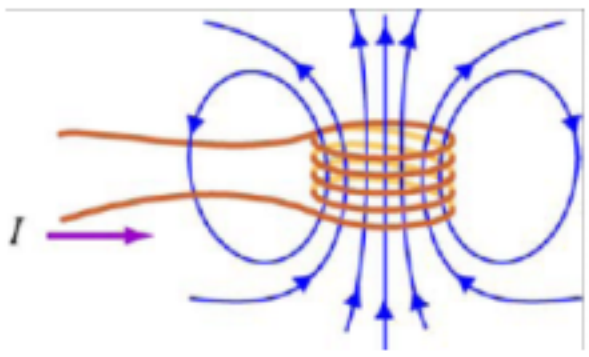


Figure (a) Figure (b)

There are important applications of eddy currents. For example, the currents can be used to suppress unwanted mechanical oscillations. Another application is the magnetic braking systems in high-speed transit cars.

Self-Inductance :

Consider again a coil consisting of N turns and carrying current I in the counterclockwise direction, as shown in Figure. If the current is steady, then the magnetic flux through the loop will remain constant. However, suppose the current I changes with time, then according to Faraday's law, an induced emf will arise to oppose the change.According to lenz law the induced emf will be clockwise if $\frac{dI}{dt} > 0$, and counterclockwise if $\frac{dI}{dt} < 0$. The property of the loop in which its own magnetic field opposes any change in current is called "self-inductance," and the emf generated is called the self-induced emf or back emf, which we denote as ε_L. All current-carrying loops exhibit this property. In particular, an inductor is a circuit element (symbol) which has a large self-inductance.



Mathematically, the self-induced emf can be written as

$$\varepsilon_L = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\int \vec{B}\,.\,d\vec{A}$$

and is related to the self-inductance L by

$$\varepsilon_L = -L\frac{dI}{dt}$$

The above two expressions can be combined to yield

$$L = \frac{N\Phi_B}{I}$$

Physically, the inductance L is a measure of an inductor's "opposition" to the change of current; larger the value of L, lower the rate of change of current.

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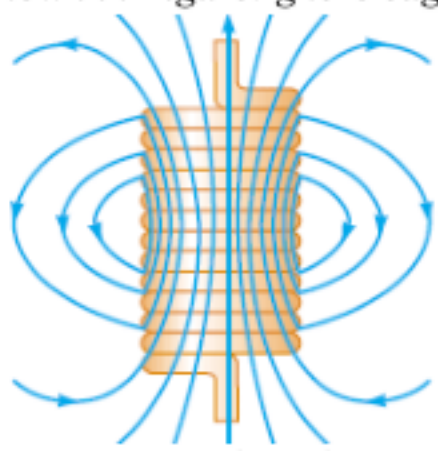
Illustration :

Compute the self-inductance of a solenoid with N turns, length ℓ, and radius R with a current I flowing through each turn, as shown in Figure.Ignore edge effects



Sol. *Ignoring edge effects and applying Ampere's law, the magnetic field inside a solenoid is given by Eq. :*

$$\vec{B} = \frac{\mu_0 NI}{\ell}\hat{k} = \mu_0 nI\ \hat{k}$$

where $n = N / \ell$ is the number of turns per unit length. The magnetic flux through each turn is

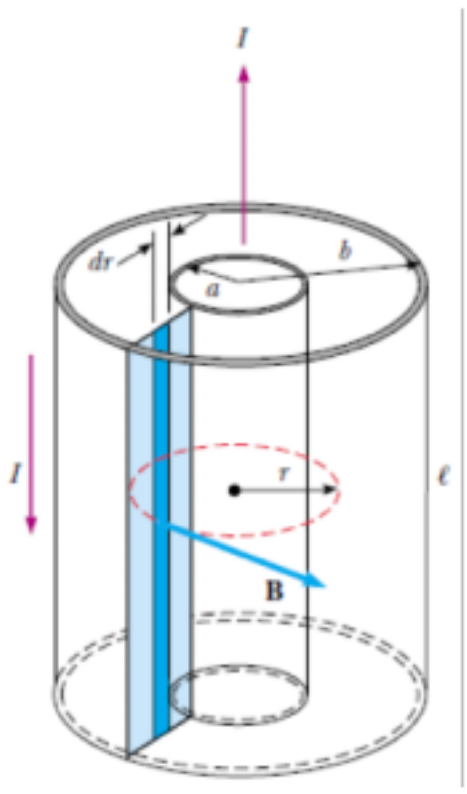
$$\Phi_B = BA = \mu_0 nI\ .\ (\pi R^2) = \mu_0 nI\pi R^2$$

Thus, the self-inductance is $L = \dfrac{N\Phi_B}{I} = \mu_0 n^2 \pi R^2 \ell$

Note We see that L depends only on the geometrical factors (n, R and ℓ) and is independent of the current I.

Illustration :

A long coaxial cable as consisting of two thin concentric cylindrical conducting shells of radii a and b and length l, . The conducting shells carry the same current I in opposite directions. Imagine that the inner conductor carries current to a device and that the outer one acts as a return path carrying the current back to the source. Calculate the self-inductance L of this cable.



Sol. *Imagine a thin radial slice of the coaxial cable, such as the shaded rectangle in Figure. The magnetic field is perpendicular to the rectangle of length l and width b-a, the cross section of interest. Divide this rectangle into strips of width dr. We see that the area of each strip is ldr and that the flux through each strip is*

$$B\,dA = Bldr.$$

Hence, we find the total flux through the entire cross section by integrating

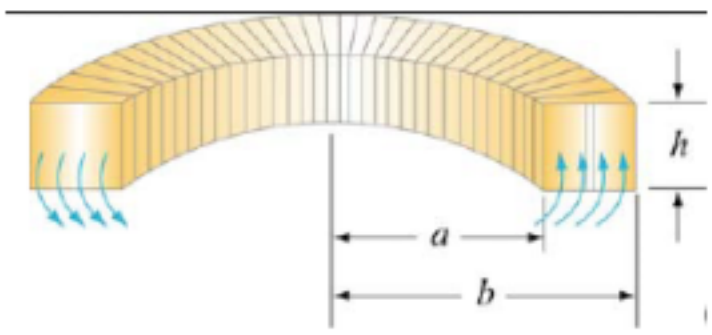
$$\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 I}{2\pi r}\,\ell\,dr = \frac{\mu_0 I \ell}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I \ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

Using this result, we find that the self-inductance of the cable is

$$L = \frac{\Phi_B}{I} = \frac{\mu_0 \ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

Practice Exercise

Q.1 Calculate the self-inductance of a toroid which consists of N turns and has a rectangular cross section, with inner radius a, outer radius b and height h, as shown in Figure.



Q.2 How many metres of a thin wire are required to manufacture a solenoid of length $l_0 = 100$ cm and inductance L = 1.0 mH if the solenoid's cross-sectional diameter is considerably less than its length? Ignore edge effects

Ans. $l = \sqrt{4\pi l_0 \mathrm{L}/\mu_0} = 100$ m

Practice Exercise

Q.1 $L = \dfrac{\mu_0 N^2}{\ell}\dfrac{h}{b-a}$

Q.2 $l = \sqrt{4\pi l_0 \mathrm{L}/\mu_0} = 100$ m

Energy Stored in Inductor

Since an inductor in a circuit serves to oppose any change in the current through it, work must be done by an external source such as a battery in order to establish a current in the inductor. From the work-energy theorem, we conclude that energy can be stored in an inductor.

The power, or rate at which an external emf ε_{ext} works to overcome the self-induced emf ε_L and pass current *I* in the inductor is

$$P_L = \frac{dW_{ext}}{dt} = I\varepsilon_{ext}$$

If only the external emf and the inductor are present, then $\varepsilon_{ext} = -\varepsilon_L$ which implies

$$P_L = \frac{dW_{ext}}{dt} = -I\varepsilon_L = +\,IL\,\frac{dI}{dt}$$

If the current is increasing with $dI / dt > 0$, then $P > 0$ which means that the external source is doing positive work to transfer energy to the inductor. Thus, the internal energy U_B of the inductor is increased. On the other hand, if the current is decreasing with $dI/dt < 0$, we then have $P < 0$. In this case, the external source takes energy away from the inductor, causing its internal energy to go down. The total work done by the external source to increase the current form zero to I is then :

$$W_{ext} = \int dW_{ext} \int_0^I LI'dI' = \frac{1}{2}LI^2$$

This is equal to the magnetic energy stored in the inductor :

$$U_B = \frac{1}{2}LI^2$$

We comment that from the energy perspective there is an important distinction between an inductor and a resistor. Whenever a current I goes through a resistor, energy flows into the resistor and dissipates in the form of heat regardless of whether I is steady or time-dependent (recall that power dissipated in a resistor is $P_R = IV_R = I^2R$). On the other hand, energy flows into an ideal inductor only when the current is varying with $dI / dt > 0$. The energy is not dissipated but stored there; it is released later when the current decreases with $dI/dt < 0$. If the current that passes through the inductor is steady, then there is no change in energy since $P_L = LI\,(dI / dt) = 0$. Also note that there is an important distinction between an inductor and a resistor. The potential difference across a resistor depends on I, while the potential difference across an inductor depends on dI / dt. The self-induced emf does not oppose the current itself, but the rate of change of current.

Energy density in a magnetic field

A long solenoid with length ℓ and a radius *R* consists of *N* turns of wire. A current *I* passes through the coil. We have to find the energy density in the magnetic field..

Energy of the solenoid is given by

$$U_B = \frac{1}{2}LI^2 = \frac{1}{2}\,\mu_0 n^2 I^2 \pi R^2 \ell$$

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The result can be expressed in terms of the magnetic field strength $B = \mu_0 nI$:

$$U_B = \frac{1}{2\mu_0}(\mu_0 nI)^2 (\pi R^2 \ell) = \frac{B^2}{2\mu_0}(\pi R^2 \ell)$$

Since $\pi R^2 \ell$ is the volume within the solenoid, and the magnetic field inside is uniform, the term

Hence magnetic energy density (i.e.the energy per unit volume) of the magnetic field will be given by

$$u_B = \frac{U_B}{\pi r^2 \ell} = \frac{B^2}{2\mu_0}$$

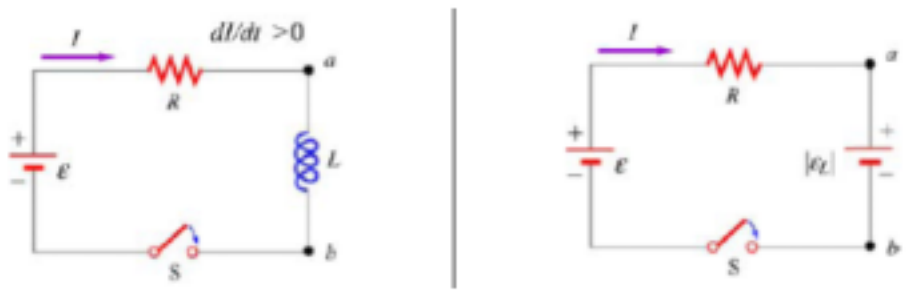
may be identified as the magnetic energy density, or the energy per unit volume of the magnetic field. The above expression holds true even when the magnetic field is non-uniform. The result can be compared with the energy density associated with an electric field:

$$u_E = \frac{\varepsilon_0 E^2}{2}$$

Growth and decay of current in L-R circuit

Growth of Current :

Consider the *RL* circuit shown in Figure. At $t = 0$ the switch is closed. We find that the current does not rise immediately to its maximum value ε / R. This is due to the presence of the self-induced emf in the inductor.



RL circuit is described by the following differential equation:

$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

The above equation can be rewritten as

$$\frac{dI}{I - \varepsilon / R} = -\frac{dt}{L/R} \quad \Rightarrow \quad \frac{dI}{I - \varepsilon / R} = -\frac{dt}{\tau}$$

where $\tau = \frac{L}{R}$

is the time constant of the RL circuit. The qualitative behaviour of the current as a function of time is depicted in figure.

Integrating over both sides with prper limits we get

$$\int_0^I \frac{dI}{I - \varepsilon / R} = -\int_0^t \frac{dt}{\tau}$$

$$I = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$$

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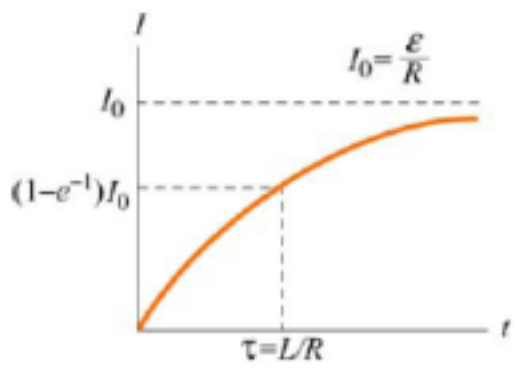
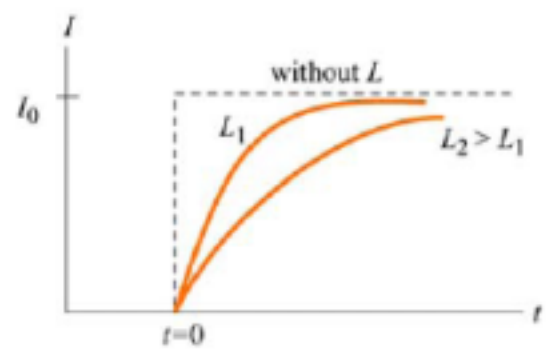




Note that after a sufficiently long time, the current reaches its equilibrium value ε/R. The time constant τ is a measure of how fast the equilibrium state is attained; the larger the value of *L*, the longer it takes to build up the current. A comparison of the behavior of current in a circuit with or without an inductor is shown in Figure .

Similarly, the magnitude of the self-induced emf can be obtained as

$$|\varepsilon_L| = \left| -L\frac{dI}{dt} \right| = \varepsilon e^{-t/\tau}$$

which is at a maximum when $t = 0$ and vanishes as t approaches infinity. This implies that a sufficiently long time after the switch is closed, self-induction disappears and the inductor simply acts as a conducting wire connecting two parts of the circuit.

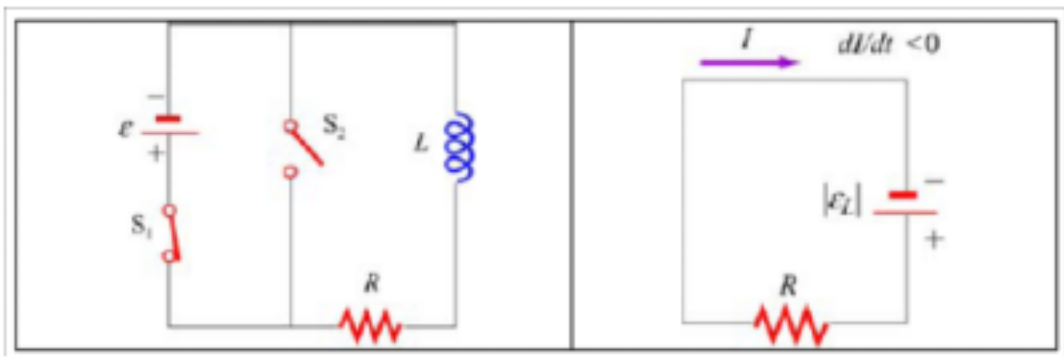
To see that energy is conserved in the circuit, we multiply Eq. by *I* and obtain

$$P = \quad I\varepsilon = I^2R + LI\frac{dI}{dt}$$

The left-hand side represents the rate at which the battery delivers energy to the circuit. On the other hand, the first term on the right-hand side is the power dissipated in the resistor in the form of heat, and the second term is the rate at which energy is stored in the inductor. While the energy dissipated through the resistor is irrecoverable, the magnetic energy stored in the inductor can be released later.

Decay of Current :

Next we consider the *RL* circuit shown in Figure. Suppose the switch S_1 has been closed for a long time so that the current is at its equilibrium value ε/*R*. What happens to the current when at $t = 0$ switches S_1 is opened and S_2 closed?



Writhing loop equation

$$-L\frac{dI}{dt} - IR = 0$$

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which can be rewritten as

$$\frac{dI}{I} = -\frac{dt}{L/R} \qquad \Rightarrow \qquad \frac{dI}{I} = -\frac{dt}{\tau}$$

Integrating over both sides with prper limits we get

$$\int_0^I \frac{dI}{I} = -\int_0^t \frac{dt}{\tau}$$

We get

$$I = \frac{\varepsilon}{R} e^{-t/\tau}$$

where $\tau = L/R$ is the same time constant as in the case of rising current. A plot of the current as a function of time is shown in Figure.

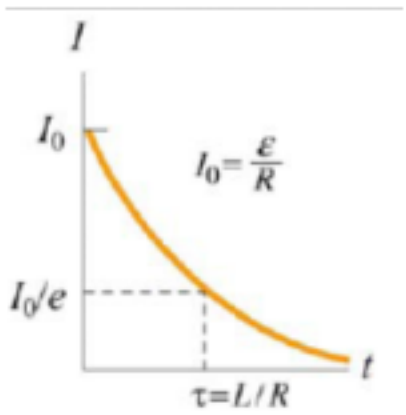


Illustration :

The inductance of a closed-packed coil of 400 turns is 8 mH. A current of 5 mA is passed through it.Find the magnetic flux through the coil .

Sol.

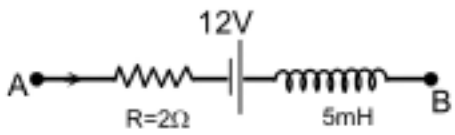
$$L = \frac{N\phi}{i}$$

$$\Rightarrow \qquad 8\times10^{-3} = \frac{400\times\phi}{5\times10^{-3}}$$

$$\Rightarrow \qquad \phi = \frac{40\times10^{-6}}{400}\,\text{Wb} = 10^{-7}\,\text{Wb}$$

Illustration :

A current of I = 10 A is passed through the part of a circuit shown in the figure. What will be the potential difference between A and B when I is decreased at constant rate of 10^2 amp/s, at the beginning?



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Sol. *Applying the law of potential between the points A and B we obtain,*

$$V_B - V_A = -IR + E - L\frac{di}{dt}$$

$$\Rightarrow \quad V_B - V_A = -10 \times 2 + 12 - 5 \times 10^{-3} \times (-10^2) = -20 + 12 + 0.5 = -7.5 \text{ volt.}$$

Illustration :

Two different coils have self inductances L_1 = 8 mH and L_2 = 2 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then find

(a) $\frac{i_1}{i_2}$; *(b)* $\frac{W_1}{W_2}$.

Sol. $e_1 = L_1\frac{di_1}{dt}$ and $e_2 = L_2\frac{di_2}{dt} \Rightarrow \frac{di_1}{dt} = \frac{di_2}{dt} \Rightarrow \frac{e_1}{e_2} = \frac{L_1}{L_2} = \frac{8}{2}$

$$\Rightarrow \frac{e_2}{e_1} = \frac{1}{4}.$$

Power given to the coils are same.

So $e_1 i_1 = e_2 i_2 \Rightarrow \frac{i_1}{i_2} = \frac{e_2}{e_1} = \frac{1}{4}$

$$Energy = \frac{1}{2}Li^2$$

$$\Rightarrow \frac{W_1}{W_2} = \frac{\frac{1}{2}L_1 i_1^2}{\frac{1}{2}L_2 i_2^2} = \frac{L_1}{L_2}\left(\frac{i_1}{i_2}\right)^2 = 4 \times \frac{1}{16} = \frac{1}{4}.$$

Illustration :

A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. Find the time after which current in the coil is approximately 1.0 A

Sol. *Current developed with time in a coil of inductance*

$$I = \frac{V}{R}(1 - e^{-t/\tau}) \text{ where } \tau = L/R$$

we have

$$\tau = \frac{8.4\,\text{mH}}{6\Omega} = 1.4\text{ms}$$

Hence

$$1\ A = \left(\frac{12\text{V}}{6\Omega}\right)(1 - e^{-t/1.4\text{ms}})$$

$$\Rightarrow \quad e^{-t/1.4ms} = 1 - \frac{1}{2} = \frac{1}{2} \quad \Rightarrow \quad -t/1.4ms = \ln\left(\frac{1}{2}\right) = -0.693$$

$$\Rightarrow \quad t = (1.4 \times 0.693)ms = 0.97ms \approx 1\ ms.$$

Illustration :

The current in an L – R circuit builds up to $3/4^{th}$ of its steady state value in 4 seconds. Find the time constant of this circuit .

Sol. $I = I_0(1 - e^{-t/\tau})$ *where* $\tau \rightarrow$ *time constant*

$$\therefore \quad \frac{3}{4} I_0 = I_0(1 - e^{-t/\tau})$$

$$\Rightarrow \quad \frac{3}{4} = 1 - e^{-t/\tau} \quad \Rightarrow \quad e^{-t/\tau} = \frac{1}{4} \quad \Rightarrow \quad \frac{-t}{\tau} \ln e = \ln \frac{1}{4}$$

$$\Rightarrow \quad \frac{-4}{\tau} = -2 \ln 2 \quad \Rightarrow \quad \tau = \frac{2}{\ln 2}.$$

Practice Exercise

Q.1 Consider the circuit shown in figure below :
Determine the current through each resistor
(a) immediately after the switch is closed.
(b) a long time after the switch is closed.
Suppose the switch is reopened a long time after it's been closed. What is each current
(c) immediately after it is opened?
(d) after a long time?

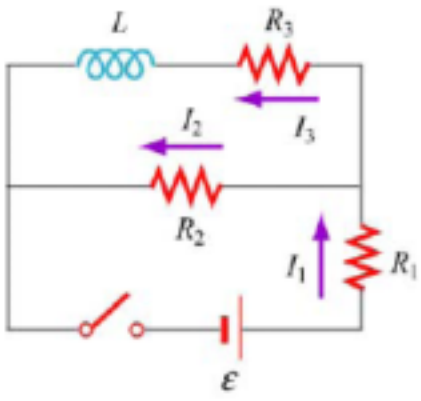
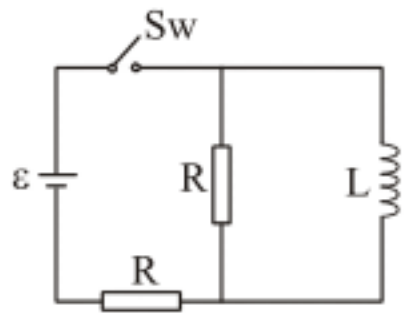


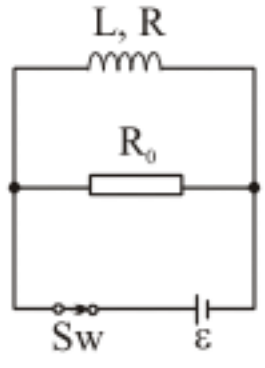
Figure *RL* circuit

Q.2 A closed circuit consists of a source of constant emt ξ and a choke coil of inductance L connected in series. The active resistance of the whole circuit is equal to R. At the moment t = 0 the choke coil inductance was decreased abruptly η times. Find the current in the circuit as a function of time t.
Instruction. During a stepwise change of inductance the total magnetic flux (flux linkage) remains constant.

Q.3 Find the time dependence of the current flowing through the inductance L of the circuit shown in Fig. after the switch Sw is shorted at the moment t = 0.



Q.4 A coil of inductance L = 2.0 μH and resistance R = 1.0 Ω is connected to a source of constant emf $\xi = 3.0$ V (Fig.). A resistance $R_0 = 2.0\,\Omega$ is connected in parallel with the coil. Find the amount of heat generated in the coil after the switch Sw is disconnected. The internal resistance of the source is negligible.



Practice Exercise

Q.1 (a) $I_1 = I_2 = \dfrac{\varepsilon}{R_1 + R_2}$

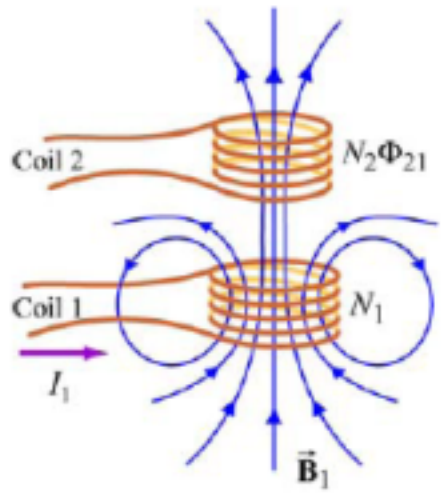
(b) $I_1 = \dfrac{(R_2 + R_3)\varepsilon}{R_1R_2 + R_1R_3 + R_2R_3}$, $I_2 = \dfrac{R_3\varepsilon}{R_1R_2 + R_1R_3 + R_2R_3}$, $I_3 = \dfrac{R_2\varepsilon}{R_1R_2 + R_1R_3 + R_2R_3}$

(c) $I_3 = -I_2 = \dfrac{R_2\varepsilon}{R_1R_2 + R_1R_3 + R_2R_3}$ (d) $I_1 = I_2 = I_3 = 0.$

Q.2 $I = \dfrac{\xi}{R}\,[1 + (\eta - 1)\,e^{-t\eta R/L}$ Q.3 $I = \dfrac{\xi}{R}\,(1 - e^{-tR/2L})$ Q.4 $Q = \dfrac{L\xi^2}{2R^2\,(1 + R_0/R)} = 3\mu J$

Mutual Inductance

Suppose two coils are placed near each other, as shown in Figure.



The first coil has N_1 turns and carries a current I_1 which gives rise to a magnetic field $\vec{B}_1$. Since the two coils are close to each other, some of the magnetic field lines through coil 1 will also pass through coil 2. Let Φ_{21} denote the magnetic flux through one turn of coil 2 due to I_1. Now, by varying I_1 with time, there will be an induced emf associated with the changing magnetic flux in the second coil :

$$e_{21} = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{N_2 d}{dt} \int_{\text{coil 2}} \vec{B}_1 . d\vec{A}_2$$

The time rate of change of magnetic flux Φ_{21} in coil 2 is proportional to the time rate of change of the current in coil 1 :

$$N_2 \frac{d\Phi_{21}}{dt} = M_{21} \frac{dI_1}{dt}$$

where the proportionality constant M_{21} is called the mutual inductance. It can also be written as

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$

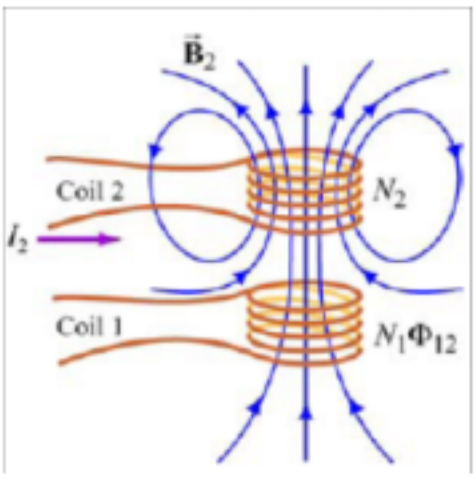
The SI unit for inductance is the henry (H) :

$$1 \text{ henry} = 1 \text{ H} = 1 \text{ T.m}^2/\text{A}$$

We shall see that the mutual inductance M_{21} depends only on the geometrical properties of the two coils such as the number of turns and the radii of the two coils.

In a similar manner, suppose instead there is a current I_2 in the second coil and it is varying with time (Figure). Then the induced emf in coil 1 becomes

$$\varepsilon_{12} = -N_1 \frac{d\Phi_{12}}{dt} = -\frac{N_1 d}{dt} \int_{\text{coil 1}} \vec{B}_2 . d\vec{A}_1$$



This changing flux in coil 1 is proportional to the changing current in coil 2,

$$N_1 \frac{d\Phi_{12}}{dt} = M_{12} \frac{dI_2}{dt}$$

where the proportionality constant M_{12} is another mutual inductance and can be written as

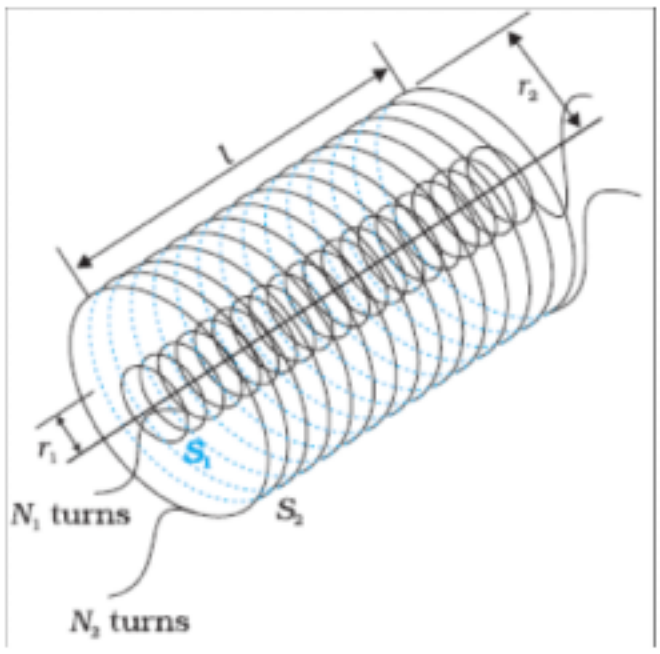
$$M_{12} = \frac{N_1 \Phi_{12}}{I_2}$$

According to *reciprocity theorem*

$$M_{12} = M_{21} \equiv M$$

Illustration :

Consider Fig. which shows two long co-axial solenoids each of length l. We denote the radius of the inner solenoid S_1 by r_1 and the number of turns per unit length by n_1. The corresponding quantities for the outer solenoid S_2 are r_2 and n_2, respectively. Let N_1 and N_2 be the total number of turns of coils S_1 and S_2, respectively. Calculate the mutual-inductance M between the solenoids. Neglect the edge effects



Sol *When a current I_2 is set up through S_2, it in turn sets up a magnetic flux through S_1.*
The magnetic field due to the current I_2 in S_2 is
$B=\mu_0 n_2 I_2$.
The resulting flux linkage with coil S_1 is,
$\Phi_1 = (n_1 l)(\pi r_1^2)(\mu_0 n_2 I_2)$
Hence the mutual-inductance M between the solenoids will be
$M=\Phi_1/I_2=\mu_0 \pi n_1 n_2 r_1^2 l$

Illustration :

The coefficient of mutual induction between the primary and secondary of a transformer is 5 H. Calculate the induced emf in the secondary when 3 ampere current in the primary is cut off in 2.5 10^{-4} second.

Sol. *Induced emf in the secondary* $\varepsilon_S = -M\frac{di_P}{dt} = -5\frac{3}{1/4000} = -6 \times 10^4\ V$

The negative sign merely indicates that the emf opposes the change.

Illustration :

A small square loop of wire of side l is placed inside a large square loop of wire of side L (L>>l). The loops are co-planar and their centers coincide. Find the mutual inductance of the system .

Sol. *Magnetic field produced by a current in a large square loop of wire at its center*

$$B = \frac{2\sqrt{2}\mu_0 i}{\pi L}$$

The magnetic flux ϕ_{12} *that links big loop with the small square loop of side l (l<<L) is*

$$\phi_{12} = B(l^2) = \frac{2\sqrt{2}\mu_0 i}{\pi}\left(\frac{l^2}{L}\right),$$

$\therefore$ *The mutual inductance*

$$M_{12} = \frac{\phi_{12}}{i} = \frac{2\sqrt{2}\mu_0 i}{\pi}\left(\frac{l^2}{L}\right)$$

Practice Exercise

Q.1 Calculate the mutual inductance of a long straight wire and a rectangular frame with sides a and b. The frame and the wire lie in the same plane, with the side b being closest to the wire, separated by a distance *l* from it and oriented parallel to it.

Q.2 Two thin concentric wires shaped as circles with radii a and b lie in the same plane. Allowing for a << b, find:
(a) their mutual inductance;
(b) the magnetic flux through the surface enclosed by the outside wire, when the inside wire carries a current I.

Q.3 There are two stationary loops with mutual inductance L_{12}.. The current in one of the loops starts to be vang as $I_1 = \alpha t$, where α is a constant, t is time. Find the time dependence I_2 (t) of the current in the other loop whose inductance is L_2 and resistance R.

Answers

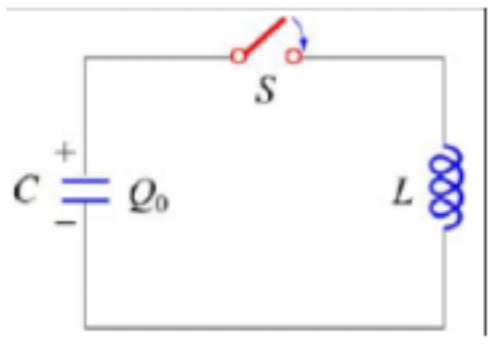
Q.1 $L_{12} = \frac{\mu_0 b}{2\pi}\, ln\left(1 + \frac{a}{l}\right)$

Q.2 (a) $L_{12} \approx \mu_0 \pi a^2 / 2b$; $\Phi_{21} = \mu_0 \pi a^2 I / 2b$

Q.3 $I_2 = \frac{\alpha L_{12}}{R}(1 - e^{-t/\tau})$ where $\tau = L_2/R$

LC Oscillations :

Consider an LC circuit in which a capacitor is connected to an inductor, as shown in Figure.



Suppose the capacitor initially has charge Q_0. When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.

The total energy in the LC circuit at some instant after closing the switch is

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

The fact that U remains constant implies that

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

where

$$I = -\frac{dQ}{dt}$$

and

$$\frac{dI}{dt} = -\frac{d^2Q}{dt^2}$$

Notice the sign convention we have adopted here. The negative sign implies that the current I is equal to the rate of decrease of charge in the capacitor plate immediately after the switch has been closed.

The general solution to equation is

$$Q = Q_0 \cos(\omega_0 t + \phi)$$

where Q_0 is the amplitude of the charge and ϕ is the phase. The angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The corresponding current in the inductor is

$$I = -\frac{dQ}{dt} = \omega_0 Q_0 \sin(\omega_0 t + \phi) = I_0 \sin(\omega_0 t + \phi)$$

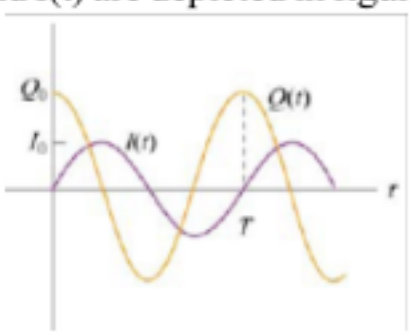
where $I_0 = \omega_0 Q_0$.

From the initial conditions $Q(\text{at } t = 0) = Q_0$ and $I(\text{at } t = 0) = 0$, the phase ϕ can be determined to $\phi = 0$. Thus, the solutions for the charge and the current in our LC circuit are

$$Q(t) = Q_0 \cos \omega_0 t$$

and $$I(t) = I_0 \sin \omega_0 t$$

The time dependence of Q(t) and I(t) are depicted in figure.

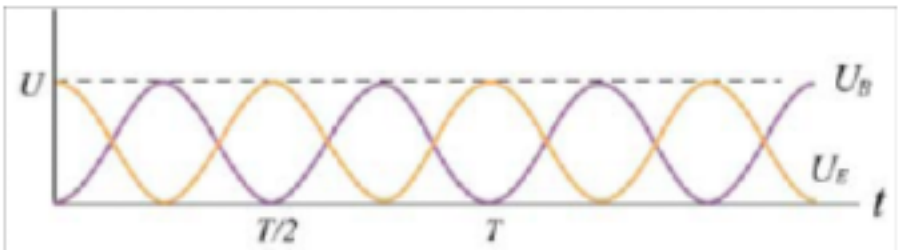


Using Eqs., we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E = \frac{Q^2(t)}{2C} = \left(\frac{Q_0^2}{2C}\right)\cos^2 \omega_0 t$$

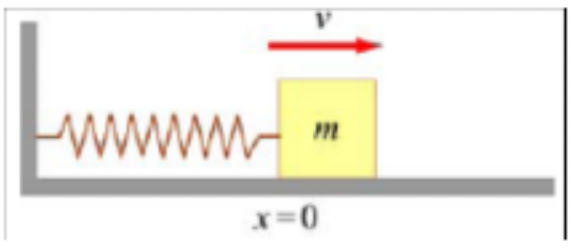
and $$U_B = \frac{1}{2} LI^2(t) = \frac{LI_0^2}{2} \sin^2 \omega t = \frac{L(-\omega_0 Q_0)^2}{2} \sin^2 \omega_0 t = \left(\frac{Q_0^2}{2C}\right)\sin^2 \omega_0 t$$

The electric and magnetic energy oscillation is illustrated in figure.



Mechanical analogy

The mechanical analog of the *LC* oscillations is the mass-spring system, shown in Figure.



If the mass is moving with a speed v and the spring having a spring constant k is displaced from its

equilibrium by x, then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic energy of the mass and the potential energy of the spring, respectively. In the absence of friction, U is conserved and we obtain

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

Using $\quad v = \frac{dx}{dt} \quad$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$,

the above equation may be rewritten as

$$m\frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is

$$x = x_0 \cos(\omega_0 t + \phi)$$

where $\quad \omega_0 = \sqrt{\frac{k}{m}}$

is the angular frequency and x_0 is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system may be written as

$$U = \frac{1}{2}mx_0^2\omega_0^2 \sin^2(\omega_0 t + \phi) + \frac{1}{2}kx_0^2\cos^2(\omega_0 t + \phi)$$

$$= \frac{1}{2}kx_0^2[\sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi) = \frac{1}{2}kx_0^2$$

In figure we illustrate the energy oscillations in the LC circuit and the mass spring system (harmonic oscillator).

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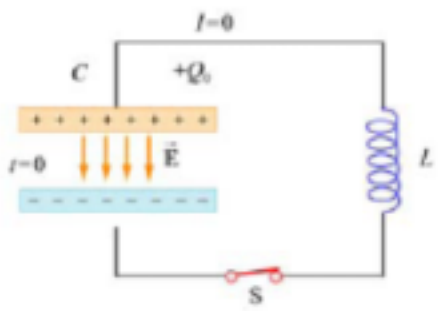
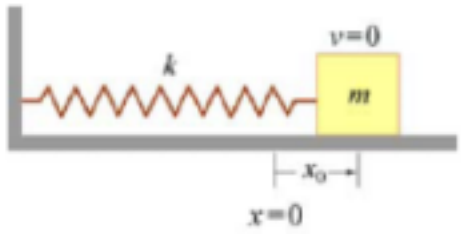
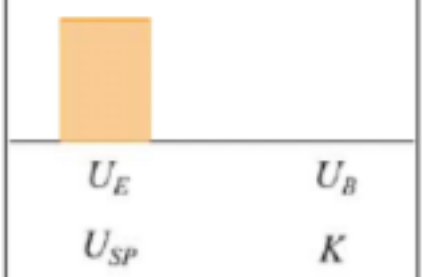
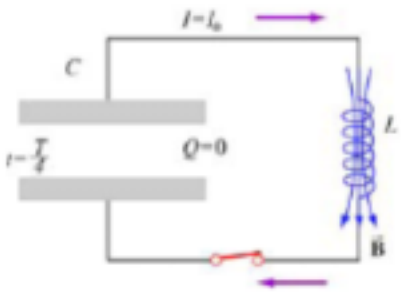
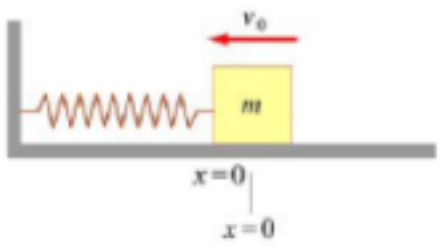
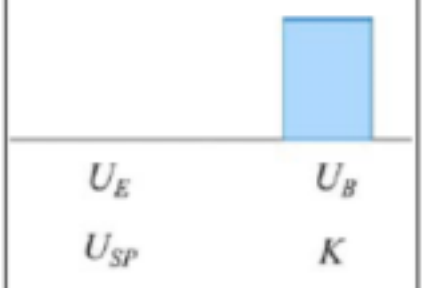
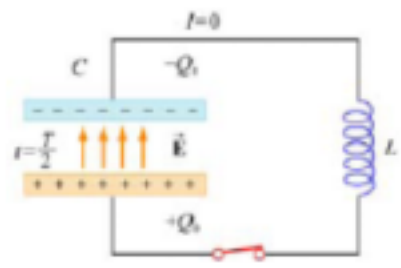
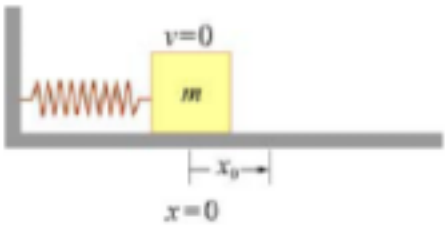
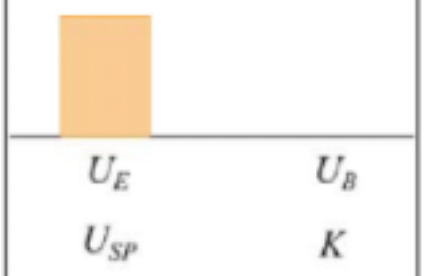
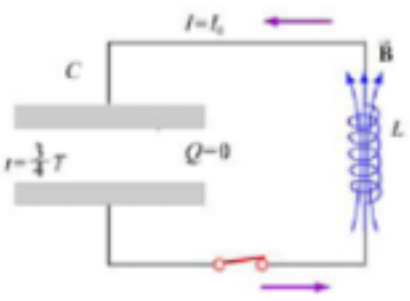
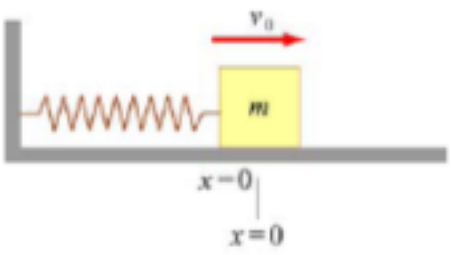
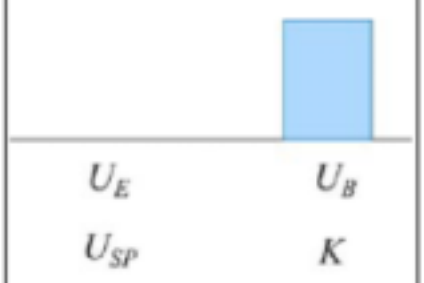
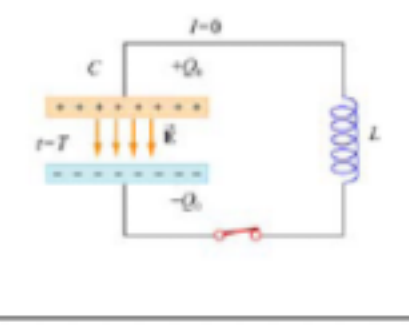
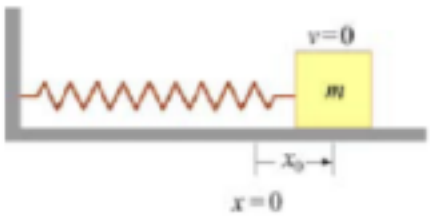
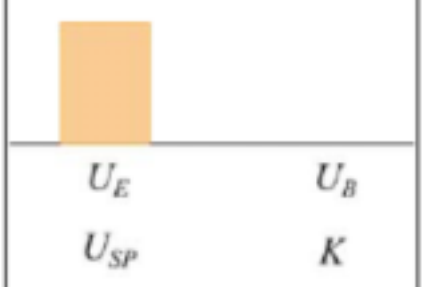
LC Circuit	Mass-spring System	Energy
		
		
		
		
		

Illustration :

An inductor of inductance 2 mH is connected across a charged capacitor of 5 μF. Let q denote the instantaneous charge on the capacitor, and i the current in the circuit. Maximum value of q is Q = 200 μC.

(a) When q = 100 μC, what is the value of $|di/dt|$?

(b) When q = 200 μC, what is the value of i?

Sol. *(a) Charge stored in the capacitor oscillates simple harmonically as,*

$$Q = Q_0 \sin(\omega t \pm \phi) \text{ and}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3}H)(5.0\times10^{-6}F)}} = 10^4 s^{-1}$$

At t = 0, if $Q = Q_0$, then

$$Q(t) = Q_0 \cos \omega t \text{ and if } Q = Q_0/2, \text{ then } \cos \omega t = ½$$

$$\frac{di(t)}{dt} = -Q_0\omega^2 \cos(\omega t) = (200\times10^{-6}C)(10^4 s^{-1})^2\left(\frac{1}{2}\right) = 10^4 \text{ A/s}$$

(b) When the energy of the capacitor is maximum, the energy stored in the inductor will be zero.

i.e. $\frac{1}{2}Li^2 = 0 \Rightarrow i = 0$

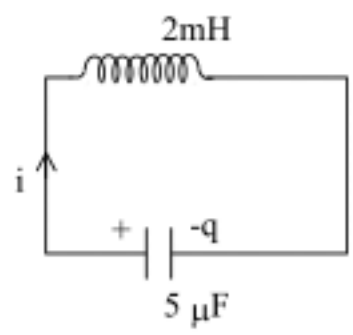


Illustration :

A capacitor of capacitance 2 μF is charged to a potential difference of 12 V. It is then connected across an inductor of inductance 0.6 mH. What is the current in the circuit at a time when the potential difference across the capacitor is 6.0 V?

Sol. *As the capacitor is charged to a potential difference of 12 V, the initial charge on the capacitor is*

$$q_0 = CV_0 = 2 \times 10^{-6} \times 12 \text{ C.} \qquad \ldots (1)$$

At any instant as the capacitor discharges through the inductor (LC circuit), the instantaneous charge on the capacitor is given by

$$q = q_0 \cos \omega t \qquad \ldots (2) \quad [\text{because at } t = 0, q = q_0]$$

$$\text{But } q = CV. \qquad \ldots (3)$$

where V is the potential difference at the instant 't'.

From (1) and (3) we obtain $\frac{q}{q_0} = \frac{V}{V_0}$

Putting the value of V and V_0 we obtain

$$\frac{q}{q_0} = \frac{1}{2} \quad \Rightarrow \quad \cos\omega t = \frac{1}{2} \quad \Rightarrow \quad \omega t = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \omega t = /3 \text{ rad} \qquad \ldots (4)$$

$$\text{Here } \omega = \frac{1}{\sqrt{LC}} = \frac{1}{[0.6\times10^{-3}\times2\times10^{-6}]^{\frac{1}{2}}}$$

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$$\Rightarrow \quad \omega = \frac{10^5}{2\sqrt{3}} \text{ rad/sec.} \qquad \ldots (5)$$

The current through the circuit at that instant is given by,

$$i = \frac{d}{dt}[q_0 \cos \omega t]$$

$$\Rightarrow \quad i = -q_0 \omega \sin \omega t$$

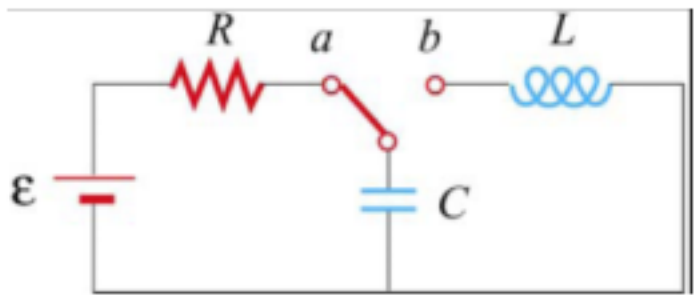
Putting the value of q_0 from (1), ω from (5) and ωt from (4) we obtain.

$$|i| = 2 \times 10^{-7} \times 12 \times \frac{10^5}{2\sqrt{3}} \sin(\pi/3)$$

$$= 0.6 \text{ A}$$

Practice Exercise

Q.1 Consider the circuit shown in Figure. Suppose the switch which has been connected to point *a* for a long time is suddenly thrown to *b* at $t = 0$.



Find the following quantities :

(a) the frequency of oscillation of the *LC* circuit.

(b) the maximum charge that appears on the capacitor.

(c) the maximum current in the inductor.

(d) the total energy the circuit possesses at any time *t*.

Answer

Q.1 (a) $f = \frac{1}{2\pi\sqrt{LC}}$ (b) $Q = C\varepsilon$ (c) $I_0 = \varepsilon\sqrt{\frac{C}{L}}$ (d) $\frac{1}{2}C\varepsilon^2$

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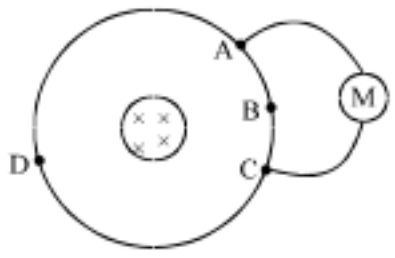
Solved Examples

Q.1 A variable magnetic field creates a constant emf E in a conductor ABCDA. The resistances of portion ABC, CDA and AMC are R_1, R_2 and R_3 respectively. What current will be shown by meter M? The magnetic field is concentrated near the axis of the circular conductor.

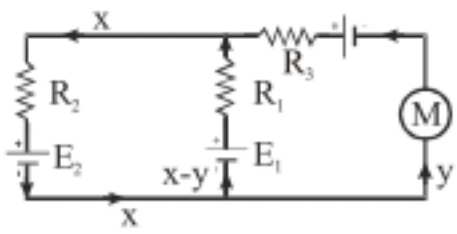


Sol. Let E_1 and E_2 be the emfs developed in ABC and CDA, respectively. Then $E_1 + E_2 = E$. There is no net emf in the loop AMCBA as it does not enclose the magnetic field. If E_3 is the emf in AMC then $E_1 - E_3 = 0$. The equivalent circuit and distribution of current is shown in figure.

By the loop rule

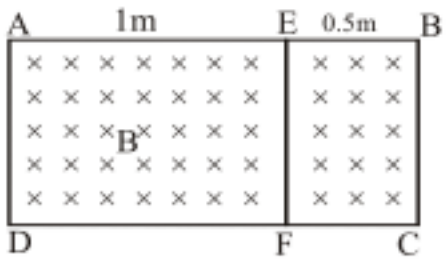
$$R_1(x-y) + R_2x = E_1 + E_2 = E$$

and $$R_3y - R_1(x-y) = E_3 - E_1 = 0$$



Solving for y, $y = \dfrac{ER_1}{R_1R_2 + R_2R_3 + R_3R_1}$.

Q.2 A rectangular frame ABCD made of a uniform metal wire has a straight connection between E & F made of the same wire as shown in the figure. AEFD is a square of side 1 m & EB = FC = 0.5 m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the place of the paper & normal to it. The rate of change of the magnetic field is 1 T/s, the resistance per unit length of the wire is 1 Ω/m. Find the current in segments AE, BE & EF.



Sol. Induced e.m.f.

$$= -\frac{d\Phi}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB}{dt}$$

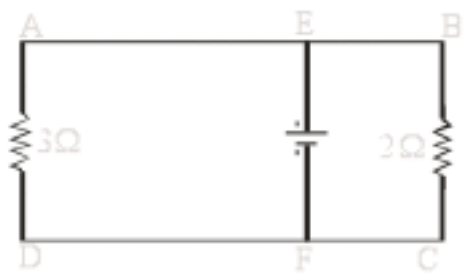
$\therefore$ Induced e.m.f. in AEFD = 1 × 1 = 1V (area = 1 m^2)

Induced e.m.f. in EBCF $= \frac{1}{2} \times 1 = 0.5V$ (area $= \frac{1}{2} m^2$)

Total induced e.m.f $= 1 + 0.5 = 1.5V$

Given that resistance per unit length of the wire is $1\Omega/m$. Hence the equivalent circuit take the form as shown in figure. The resistances 3Ω and 2Ω are in parallel. Hence their equivalent resistance would

$$\frac{(3\times 2)}{(3+2)}=\left(\frac{6}{5}\right)$$

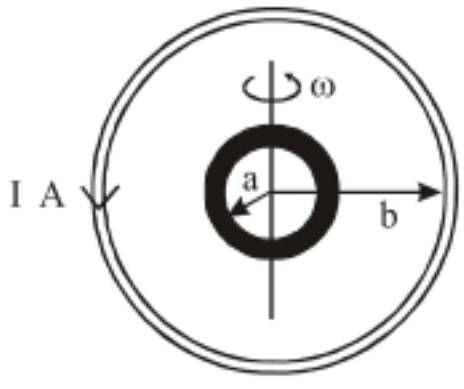


$\therefore$ Current from $EF=\frac{1.5}{(6/5)}=\frac{5}{4}$ amp.

Current in $AE=\frac{5}{4}\times\frac{2}{5}=\frac{1}{2}$ amp.

Current in $BE=\frac{5}{4}\times\frac{3}{5}=\frac{3}{4}$ amp.

Q.3 A very small circular loop of area $5\times10^{-4}\,m^2$, resistance 2 ohm and negligible inductance is initially coplanar and concentric with a much larger fixed circular loop of radius 0.1 m. A constant current of 1 Amp. is passed in the bigger loop and the smaller loop is rotated with angular velocity ω rad/s about a diameter. Calculate (a) the flux linked with the smaller loop (b) induced emf and induced current in the smaller loop as a function of time.



Sol. (a) The situation is shown in figure. The field at the center of larger loop,

$$B_1=\frac{\mu_0}{4\pi}\frac{2\pi I}{R}=10^{-7}\frac{2\pi\times1}{0.1}=2\pi\times10^{-6}\frac{Wb}{m^2}$$

is initially along the normal to the area of smaller loop. Now as the smaller loop (and hence normal to its plane) is rotating at angular velocity ω, so in time t it will turn by an angle $\theta=\omega t$ w.r. to $\vec{B}$ and hence the flux linked with the smaller loop at time t,

$$\phi_2=B_1S_2\cos\theta=(2\pi\times10^{-6})(5\times10^{-4})\cos\omega t$$

i.e., $\phi_2=\pi\times10^{-9}\cos\omega t$ Wb

(b) The induced emf in the smaller loop,

$$e_2=-\frac{d\phi_2}{dt}=-\frac{d}{dt}(\pi\times10^{-9}\cos\omega t)$$

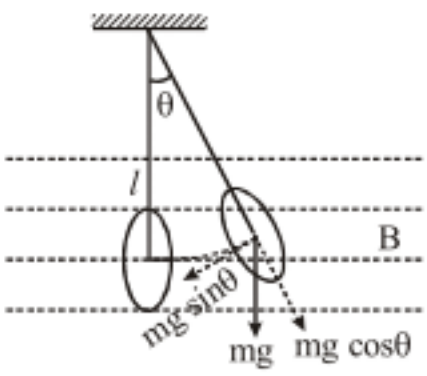
i.e., $e_2=\pi\times10^{-9}\omega\sin\omega t$ volt

(c) The induced current in the smaller loop,

$$I_2=\frac{e_2}{R}=\frac{1}{2}\pi\omega\times10^{-9}\sin\omega t$$

Q.4 A wire frame of area A and resistance R is suspended freely from a 0.392 m long thread. There is a uniform magnetic field of B tesla and the plane of wire-frame is perpendicular to the magnetic field. The frame is forcely made to oscillate using external force with small angular amplitude θ_0 along the direction of magnetic field according to the law $\theta = \theta_0 \sin \omega t$. The plane of the frame is always along the direction of thread and does not rotate about it. What is the inducted e.m.f. in wire-frame as a function of time? Also find the maximum current is the frame.

Sol. The situation is shown in figure.The instantaneous flux through the frame when displaced through an angle θ is given by



Equation of S.H.M.

$$\theta = \theta_0 \sin \omega t$$

Now

$$\Phi = BA \cos \theta$$

Instantaneous induced e.m.f.

$$e = -\frac{d\Phi}{dt} = BA \sin \theta \frac{d\theta}{dt} = BA\theta \frac{d\theta}{dt} \qquad (\because \sin \theta = \theta)$$

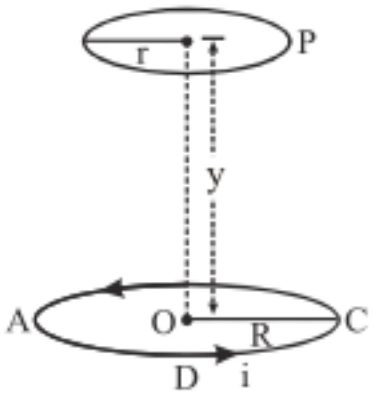
$$e = BA(\theta_0 \sin \omega t)\frac{d}{dt}(\theta_0 \sin \omega t)$$

$$= BA\theta_0 \sin \omega t \quad \theta_0 \omega \cos \omega t$$

$$= \frac{1}{2} BA\omega\theta_0^2 \sin 2\omega t$$

and $\quad i_{max} = BA\omega\theta_0^2 \sin 2\omega t / 2R$

Q.5 A coil A C D of radius R and number of turns n carries a current i amp. and is placed in the plane of paper. A small conducting ring P of radius r is placed at a distance y from the centre and above the coil A C D. Calculate the induced e.m.f. produced in the ring when the ring is allowed to fall freely. Express induced e.m.f. in terms of speed of the ring.



Sol. We know that magnetic induction at some point on the axis of a current carrying coil at a distance y from its centre is given by

$$B = \frac{\mu_0}{4\pi} \times \frac{2\pi n i R^2}{(R^2 + y^2)^{3/2}}$$

Now, the magnetic flux linked with ring P is

$$\Phi = BA = \frac{\mu_0}{4\pi} \times \frac{2\pi n i r^2 R^2}{(R^2 + y^2)^{3/2}} \times \pi r^2 = \frac{\mu_0}{2} \times \frac{\pi n i R^2 r^2}{(R^2 + y^2)^{3/2}}$$

Let the ring P falls with velocity v. Now y varies with time as

$$y = y_0 - vt$$

The induced e.m.f. in the ring

$$e = \frac{d\Phi}{dt} = \frac{\mu_0}{2} \times \pi n i R^2 r^2 \frac{d}{dt}(R^2 + y^2)^{-3/2} = -\frac{3}{2}\mu_0 \pi n i R^2 r^2 (R^2 + y^2)^{-5/2}.2y\left(\frac{dy}{dt}\right)$$

$$= -\frac{3}{2}\frac{\mu_0 \pi n i R^2 r^2}{(R^2 + y^2)^{5/2}} y(-v) = \frac{3}{2}\frac{\mu_0 \pi n i R^2 r^2 y v}{(R^2 + y^2)^{5/2}}.$$

Q.6 An infinitesimally small bar magnet of dipole moment $\overrightarrow{M}$ is pointing and moving with the speed v in the x-direction. A small closed circular conducting loop of radius a and of negligible self-inductance lies in the y-z plane with its center at x = 0, and its axis coinciding with the x-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the center of the loop is much greater than a.

Sol. Field due to the bar magnet at distance x (near the loop)

$$B = \frac{\mu_0}{4\pi}\frac{2M}{x^2}$$

$\Rightarrow$ Flux lined with the loop : $\phi = BA = \pi a^2 . \frac{\mu_0}{4\pi}\frac{2M}{x^3}$

emf induced in the loop : $e = -\frac{d\phi}{dt} = \frac{\mu_0}{4\pi}\frac{6\pi M a^2}{x^4}\frac{dx}{dt}$

$$= \frac{\mu_0}{4\pi}\frac{6\pi M a^2}{x^4} v.$$

$\Rightarrow$ Induced current : $i = \frac{e}{R} = \frac{\mu_0}{2\pi}\frac{3\pi M a^2}{R x^4}.v. = \frac{3\mu_0 M a^2}{R x^4}$

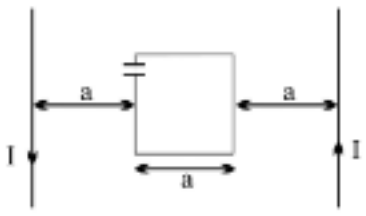
Let F = force opposing the motion of the magnet

Power due to the opposing force = Heat dissipated in the coil per second.

$\Rightarrow$ $Fv = i^2 R \Rightarrow F = \frac{i^2 R}{v} = \left(\frac{\mu_0}{4\pi}\right)^2 \times \left(\frac{6\pi M a^2}{R x^4}\right)^2 \times v^2 \times \frac{R}{v}$

$$F = \frac{9}{4}\frac{\mu_0^2 M^2 a^4 v}{R x^4}.$$

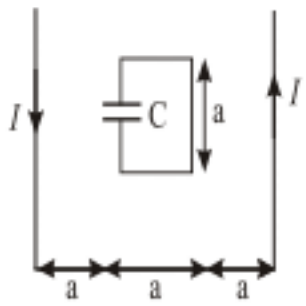
Q.7 A square loop of side 'a' with a capacitor of capacitance C is located between two current carrying long parallel wires as shown. The value of I in the wire is given as $I = I_0 \sin\omega t$.



(a) calculate maximum current in the square loop.

(b) Draw a graph between charge on the lower plate of the capacitor v/s time. [JEE 2003]

Sol. (a) $I = I_0 \sin \omega t$



Flux linked with square loop

$$\int d\phi = \int_a^{2a} \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{3a - x}\right] a\,dx$$

$$= \frac{\mu_0 Ia}{2\pi}[\ln x - \ln(3a - x)]_a^{2a}$$

$$= \frac{\mu_0 Ia}{2\pi}[\ln 2a - \ln a - \ln a + \ln 2a]$$

$$= \frac{\mu_0 Ia}{2\pi} 2\ln 2 = \frac{\mu_0 Ia}{\pi}\ln 2$$

Charge an capacitor $= \left|C\frac{d\phi}{dt}\right| = C\frac{\mu_0 a}{\pi}\ln 2\frac{dI}{dt}$

$$= \frac{\mu_0 Ca}{\pi}\ln 2(I_0\omega)\cos\omega t \quad \ldots(i)$$

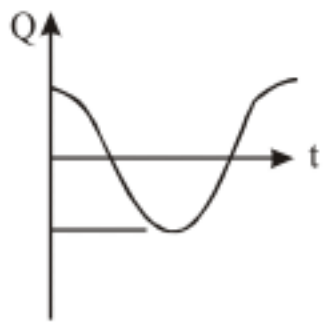
$$= \frac{\mu_0 C(I_0\omega)a}{\pi}\ln 2\cos\omega t \quad \ldots(ii)$$

Maximum current $I = \left|\frac{dq}{dt}\right|_{max} = \frac{\mu_0 CI_0\omega^2 a}{\pi}\ln 2$

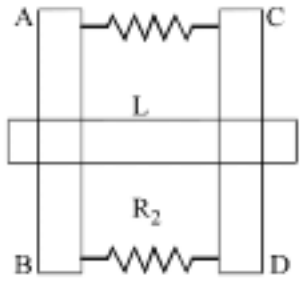
(b) The graph for charge and time can be drawn from equation (i) as shown in figure.

$\frac{\mu_0 CI_0\omega a}{\pi}\ln 2$

$-\frac{\mu_0 CI_0\omega a}{\pi}\ln 2$.



Q.8 Two parallel vertical metallic rails AB & CD are separated by 1 m. They are connected at the two ends by resistance R_1 & R_2 as shown in the figure. A horizontally metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6T perpendicular to the plane of the rails, it is observed that when the terminal velocity is attained, the power dissipated in R_1 & R_2 are 0.76 W & 1.2 W respectively. Find the terminal velocity of bar L & value R_1 & R_2.



Sol. At terminal velocity

$$mg = I\ell B \quad \Rightarrow \quad 0.2 \times 9.8 = I \times 1 \times 0.6 \quad \Rightarrow \quad I = \frac{1.96}{0.6} \text{ Amp.} \qquad \text{..... (1)}$$

$$Ie = (0.76 + 1.2) = 1.96 \qquad \text{..... (2)}$$

From (1) and (2)

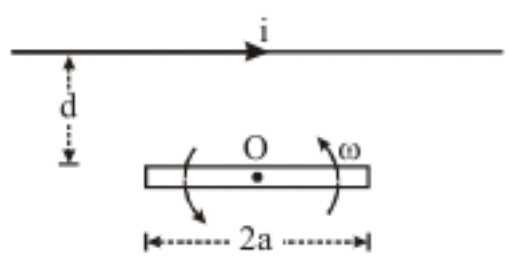
$$\frac{1.96}{0.6} e = 1.96 \quad \Rightarrow \quad e = 0.6 \text{ volt}$$

$$P_1 = \frac{e^2}{R_1} = 0.76 \quad \Rightarrow \quad R_1 = \frac{e^2}{0.76} = \frac{0.36}{0.76} = 0.47\ \Omega$$

$$P_2 = \frac{e^2}{R_2} = 1.2 \quad \Rightarrow \quad R_2 = \frac{e^2}{1.2} = 0.3\ \Omega$$

$$P = Fv \quad \Rightarrow \quad 1.56 = mg\,v \quad \Rightarrow \quad 1.96 = 1.96\,v \quad \Rightarrow \quad v = 1 \text{ m/s}$$

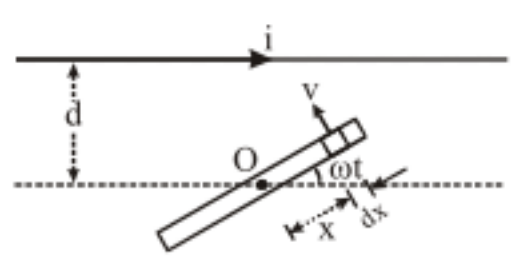
Q.9 A rod of length 2 a is free to rotate in a vertical plane, about a horizontal axis O passing through its midpoint. A long straight, horizontal wire is in the same plane and is carrying a constant current i as shown in figure. At initial moment of time the rod is horizontal and starts to rotate with constant angular velocity ω. Calculate e.m.f. induced in rod as a function of time.



Sol. The rotated position of the rod after a time t is shown in figure. Consider a small element of length dx of the rod at a distance x from the centre.

The velocity of the element $v = x\omega$ and its distance from the wire is $r = (d - x\sin\omega t)$. Magnetic induction at this position

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{2\pi(d - x\sin\omega t)}$$



The induced e.m.f in this element

$$de = Bv\ dx = \frac{\mu_0 i(x\omega)\,dx}{2\pi(d - x\sin\omega t)}$$

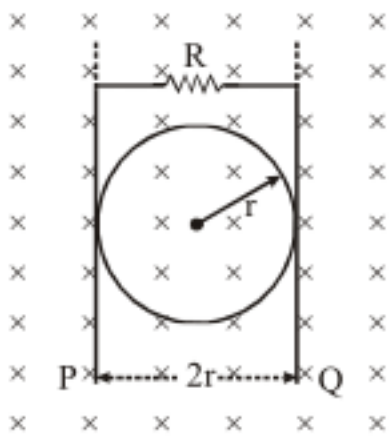
In order to obtain the resultant e.m.f., we integrate this expression from (–a) to +a. Hence

$$e = \frac{\mu_0 i\,\omega}{2\pi}\int_{-a}^{a}\frac{x\,dx}{(d - x\sin\omega t)}$$

$$= \frac{\mu_0 i\,\omega}{2\pi}\left(\frac{-1}{\sin^2\omega t}\right)\left[2a\sin\omega t + d\log\left(\frac{d - a\sin\omega t}{d + a\sin\omega t}\right)\right]$$

$$= \frac{\mu_0 i\omega}{2\pi\sin^2\omega t}\left[-2a\sin\omega t + d\log\left(\frac{d + a\sin\omega t}{d - a\sin\omega t}\right)\right].$$

Q.10 P and Q are two infinite conducting plates kept parallel to each other and separated by a distance 2r. A conducting ring of radius r falls vertically between the plates such that planes are always tangent to the ring. Both the planes are connected by a resistance R. There exists a uniform magnetic field of strength B perpendicular to the plane of ring. The arrangement is shown in figure. Plane Q is smooth and friction between the plane P and the ring is enough to prevent slipping. At t = 0, the planes and the ring. Find



(a) the current through R as a function of time

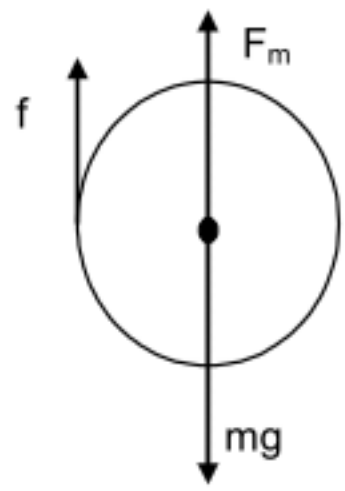
(b) terminal velocity of the ring (assume g to be constant).

Sol. Let v be the velocity of C.M. of the ring at any time t.

e.m.f. across the diameter of ring, e = 2 B v r.

$\therefore$ Current through R is I = (2 B v r/R) ...(i)

The different force on the ring are shown in figure.



For rotational motion

$$fr = I\alpha$$

$$f = \frac{I\alpha}{r} = \frac{Ia}{r^2} \qquad (\because\ \alpha = a/r) \qquad \text{...(ii)}$$

where a = acceleration of C.M.

Now $F_m = B\,i\,L = \frac{4B^2r^2}{R}v$ $(\because L = 2r)$ ...(iii)

For translational motion

$$mg - f - F_m = ma \qquad \text{...(iv)}$$

Substituting the value of f and F_m from eqs. (ii) and (iii) in equation (iv), we get

$$mg - \frac{Ia}{r^2} - \frac{4B^2r^2}{R}v = ma$$

$$mg - \frac{4B^2r^2}{R}v = ma + \frac{Ia}{r^2}$$

or $$mg - \frac{4B^2r^2}{R}v = a\left[m + \frac{I}{r^2}\right]$$

or $$\left(mg - \frac{4B^2r^2}{B}v\right) = \frac{dv}{dt}\left[m + \frac{I}{r^2}\right]$$

or $$\frac{dv}{\left[mg - \frac{4B^2r^2}{R}v\right]} \times \left(m + \frac{I}{r^2}\right) = dt$$

for a ring $I = mr^2$

$$\therefore \quad \frac{dv}{\left[mg - \frac{4B^2r^2}{R}v\right]}(2m) = dt \qquad \ldots(v)$$

Integrating equation (iv) with proper limits, we get

$$-\frac{mR}{2B^2r^2}\log_e\left[mg - \frac{4B^2r^2}{R}v\right]_0^v = t$$

or $$\frac{-mR}{2B^2r^2}\log_e\left[\frac{mg - \frac{4B^2r^2}{R}v}{mg}\right] = t$$

$$\therefore \quad v = \frac{mgR}{4B^2r^2}\left[1 - e^{-\frac{2B^2r^2}{mR}.t}\right] \qquad \ldots(vi)$$

For equation (i) and (vi), we get

$$i = \frac{2Bvr}{R} = \frac{mg}{2Br}\left[1 - e^{-\frac{2B^2r^2}{mR}.t}\right].$$

Q.11 A rectangular conducting loop in the vertical x-z plane has length L, width W, mass M and resistance R. It is dropped lengthwise from rest. At $t = 0$ the bottom of the loop is at a height h above the horizontal x-axis. There is a uniform magnetic field B perpendicular to the x-z plane, below the x-axis. The bottom and top of the loop cross this axis at $t = t_1$, and t_2 respectively. Obtain the expression for the velocity of the loop for the time $t_1 \le t \le t_2$

Sol. For time t_1, the loop is freely falling under gravity, so velocity attained by loop at

$$t = t_1$$

$$\upsilon_1 = gt_1 = \sqrt{2gh}$$

During the time $t_1 \le t \le t_2$, flux linked with the loop is changing, so induced emf

$$e = -\frac{d\phi}{dt} = -B\upsilon W$$

and Induced current

$$I = -\frac{B\upsilon W}{R} \text{ clockwise}$$

Magnetic Force

$$F = WIB = -\frac{B^2\upsilon W^2}{R}$$

So, $$m\frac{d\upsilon}{dt} = mg - \frac{B^2\upsilon W^2}{R}$$

$$dt = \frac{m d\upsilon}{\left[\left(mg - \frac{B^2W^2\upsilon}{R}\right)\right]}$$

Integrating,

$$t = -\frac{mR}{B^2W^2}\log_e\left[mg - \frac{B^2\upsilon W^2}{R}\right] + A$$

At $t = t_1, \upsilon = \upsilon_1 = gt_1$

$$\therefore \quad A = t_1 + \frac{mR}{B^2W^2}\log_e\left[mg - \frac{B^2\upsilon_1 W^2}{R}\right]$$

Substituting for A,

$$e^{-\frac{B^2W^2}{mR}(t-t_1)} = \log_e\left[\frac{mg - \frac{B^2\upsilon W^2}{R}}{mg - \frac{B^2\upsilon_1 W^2}{R}}\right]$$

Gives the velocity v of the loop in the interval $t_1 \le t \le t_2$.

Q.12 A coil of inductance L = 50 × 10^{-6} henry and resistance = 0.5 Ω is connected to a battery of emf = 5.0 V. A resistance of 10 Ω is connected parallel to the coil. Now at some instant the connection of the battery is switched off. Find the amount of heat generated in the coil after switching off the battery.

Sol Total energy stored in the inductor $= \frac{1}{2}Li_0^2$

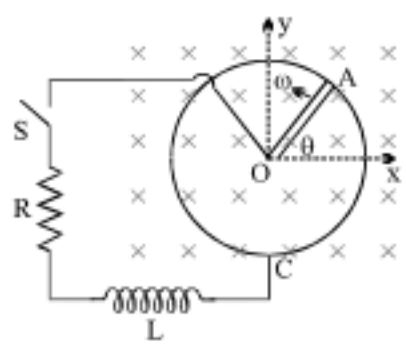
$$E_L = \frac{1}{2}L\left(\frac{V}{r}\right)^2$$

$\therefore$ Fraction of energy lost across inductor

$$= E_L \cdot \frac{r}{(R+r)}$$

$$= \frac{LV^2}{2r(R+r)} = \frac{50\times10^{-6}\times5^2}{2\times0.5(10+0.5)} = 1.19\times10^{-4}\,J$$

Q.13 A metal rod OA of mass m & length r is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform & constant magnetic induction $\vec{B}$ is applied perpendicular & into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O & a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.



(a) What is the induced emf across the terminals of the switch ?

(b) (i) Obtain an expression for the current as a function of time after switch S is closed.

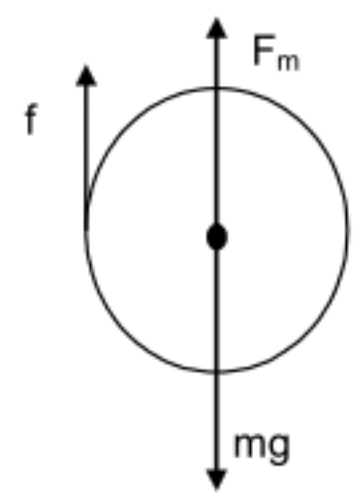
(ii) Obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at t = 0.

Sol. Let v be the velocity of C.M. of the ring at any time t.

e.m.f. across the diameter of ring, e = 2 B v r.

$\therefore$ Current through R is I = (2 B v r/R) ...(i)

The different force on the ring are shown in figure.



For rotational motion

$$fr = I\alpha$$

$$f = \frac{I\alpha}{r} = \frac{Ia}{r^2} \qquad (\because\ \alpha = a/r) \qquad ...(ii)$$

where a = acceleration of C.M.

Now $F_m = BiL = \frac{4B^2r^2}{R}v$ $(\because L = 2r)$ ...(iii)

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For translational motion

$$mg - f - F_m = ma \qquad \text{...(iv)}$$

Substituting the value of f and F_m from eqs. (ii) and (iii) in equation (iv), we get

$$mg - \frac{Ia}{r^2} - \frac{4B^2r^2}{R}v = ma$$

$$mg - \frac{4B^2r^2}{R}v = ma + \frac{Ia}{r^2}$$

or $$mg - \frac{4B^2r^2}{R}v = a\left[m + \frac{I}{r^2}\right]$$

or $$\left(mg - \frac{4B^2r^2}{B}v\right) = \frac{dv}{dt}\left[m + \frac{I}{r^2}\right]$$

or $$\frac{dv}{\left[mg - \frac{4B^2r^2}{R}v\right]} \times \left(m + \frac{I}{r^2}\right) = dt$$

for a ring $I = mr^2$

$$\therefore \quad \frac{dv}{\left[mg - \frac{4B^2r^2}{R}v\right]}(2m) = dt \qquad \text{...(v)}$$

Integrating equation (iv) with proper limits, we get

$$-\frac{mR}{2B^2r^2}\log_e\left[mg - \frac{4B^2r^2}{R}v\right]_0^v = t$$

or $$\frac{-mR}{2B^2r^2}\log_e\left[\frac{mg - \frac{4B^2r^2}{R}v}{mg}\right] = t$$

$$\therefore \quad v = \frac{mgR}{4B^2r^2}\left[1 - e^{-\frac{2B^2r^2}{mR}.t}\right] \qquad \text{...(vi)}$$

For equation (i) and (vi), we get

$$i = \frac{2Bvr}{R} = \frac{mg}{2Br}\left[1 - e^{-\frac{2B^2r^2}{mR}.t}\right].$$

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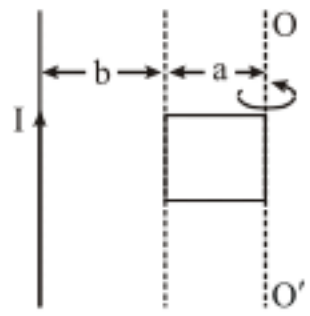
Q.14 A square loop of side a and a straight, infinite conductor are placed in the same plane with two sides of the square parallel to the conductor. The inductance and resistance are equal to L and R respectively. The frame is turned through 180° about the axis OO'. Find the electric charge that flows in the square loop.



Sol. By circuit equation $i = \left(\varepsilon - L\frac{di}{dt}\right)/R$ where $\varepsilon =$ induced emf and $L\frac{di}{dt} =$ self-induced emf

$$\Rightarrow \quad Ri = \varepsilon - L\frac{di}{dt} \quad \Rightarrow \quad \int Ri\,dt = \int \varepsilon dt - \int L\frac{di}{dt}dt$$

$$\Rightarrow \quad Rq = \int -\frac{d\varphi}{dt}dt - L[i]_i^f = \varphi_i - \varphi_f \quad (\because i_{initial} = 0, i_{final} = 0)$$

$$\Rightarrow \quad q = (\varphi_i - \varphi_f)/R$$

Consider a strip at a distance x in the initial position. Then $B = (\mu_0/4\pi)(2I/x)$ along the inward normal to the plane.

$$\therefore \quad d\varphi_i = (\mu_0 I/2\pi x)a\ dx\cos 0 = \frac{\mu_0 Ia}{2\pi}\frac{dx}{x}$$

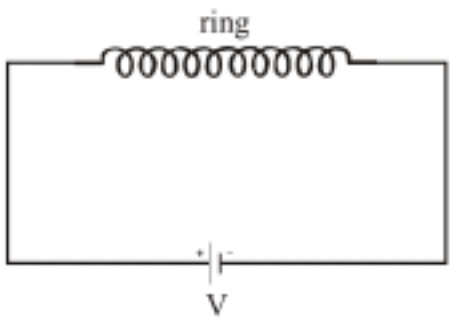
$$\Rightarrow \quad \varphi_i = \frac{\mu_0 Ia}{2\pi}\int_b^{a+b}\frac{dx}{x} = \frac{\mu_0 Ia}{2\pi}\ln\frac{a+b}{b}$$

Similarly $\varphi_f = \frac{\mu_0 Ia}{2\pi}\ln\frac{2a+b}{a+b}$

$$\therefore \quad |\varphi_i - \varphi_f| = \frac{\mu_0 Ia}{2\pi}\ln\frac{2a+b}{b}$$

$$\therefore \quad |q| = \frac{\mu_0 Ia}{2\pi R}\ln\frac{2a+b}{b}.$$

Q.15 A thin wire ring of radius a and resistance r is located inside a long solenoid so that their axes coincide. The length of the solenoid is equal to l its cross-sectional radius to b. At a certain moment, the solenoid was connected to a source of constant voltage V. The total resistance of the circuit is equal to R. Assuming the inductance of the ring to be negligible, find the maximum value of the radial force acting per unit length of the ring.



Sol. The inductance L of the solenoid

$$L = \mu_0 n^2 \pi b^2 l$$

The current through the solenoid varies as

$$I(t) = \frac{V}{R}(1 - e^{-tR/L})$$

B(t) = magnetic induction

$$= \mu_0 n\ I(t)$$

$$e = \frac{d\phi}{dt}\ \&\ \vec{B}.\vec{A}$$

flux through ring $= \mu_0 n.\pi a^2$

$$e = \frac{d\phi}{dt},\ \mu_0 n\pi a^2 \frac{dI}{dt}$$

where $\quad i(t)$ is $\dfrac{dI(t)}{dt}$

Force unit per length on the ring $= \dfrac{F}{\ell} = BI(t)$

$$\therefore \qquad F(t) = \mu_0 n\, I(t).\mu_0\, n\, \dot{I}(t)\, \pi a^2 / r$$

$$= \frac{\mu_0^2.\pi a^2}{r}.n^2 I(t)\dot{I}(t)$$

$$F(t) = \frac{\mu_0^2 \pi a^2}{r}.n^2\left[\frac{V}{R}(1-e^{-tR/L})\right].\left[\frac{V}{R}.\frac{Re^{-tR/L}}{L}\right]$$

$$= \frac{\mu_0^2 \pi a^2 V^2}{r}.e^{-t/RL}(1-e^{-t/RL})\left(\frac{n^2}{RL}\right)$$

Initial conditions are $F(t) = 0$ as $t = 0$

$$F(t) = 0 \text{ at } t = \infty$$

For finding maximum value of force, we differentiate F(t) w.r.t. t and put to zero. This gives the time at which this occurs.

$$e^{-t/RL} = \frac{1}{2}$$

$$\therefore \qquad F_{max} = \frac{\mu_0^2 \pi a^2 V^2}{4rR}\frac{n^2}{L} = \frac{\mu_0 a^2 v^2}{4\pi R/b^2}$$

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Alternating current

Introduction

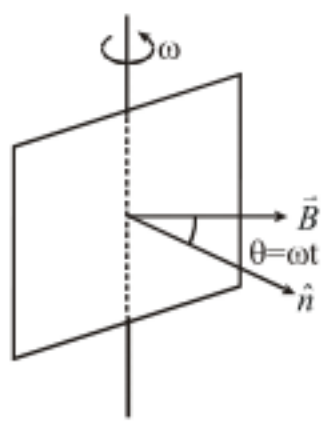
Voltages and currents that vary symmetrically in magnitude and direction with time are very common. The electric mains supply in our homes and offices is a voltage that varies like a sine function with time. Such a voltage is called alternating voltage (ac voltage) and the current driven through the appliances is called the alternating current (ac current)

Basic Principle of AC Generation:

Alternating voltage is generated by rotating a coil of conducting wire in a strong magnetic field. The magnetic flux linked with the coil changes with time and an alternating emf is thus induced. Instantaneous flux linked with coil is

$$\phi = (\vec{A}.\vec{B})n$$

$$= ABn\cos(\omega t + \theta_0)$$



where A = area of the coil (in m^2)

B = magnetic field (in tesla)

n = number of turns

ω = angular frequency $= \frac{2\pi}{T} = 2\pi f$ (in rad s^{-1})

f = frequency (in hertz)

θ_0 = initial phase angle.

With the change of time $\cos(\omega t + \theta_0)$ changes consequently an emf V is induced. According to Faraday's law

$$V = \frac{d\varphi}{dt}$$

$$= -\frac{d}{dt}[ABn\cos(\omega t + \theta_0)]$$

$$= ABn\ \omega\sin(\omega t + \theta_0)$$

$$V = V_m \sin(\omega t + \theta_0)$$

Here V_m = voltage amplitude of sinusoidal voltage or the peak value of ac voltage

where $V_m = ABn\omega$

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Average Values of ac Voltage and ac Current:

$$\textit{average volue of voltage (from } t = t_1 \textit{ to } t = t_2\textit{)} = \frac{\int_{t_1}^{t_2} V(t)\,dt}{t_2 - t_1}$$

AC voltage or currrents are commonly sinusoidal (sine or cosine function) and their mean values for complete cycle is zero.

(i) **Average value for positive half cycle (or rectified average value):**

$V = V_m \sin \omega t$

$$\therefore \quad (V)_{av} = \frac{\int_0^{T/2} V_0 \sin \omega t\, dt}{\frac{T}{2} - 0} = \frac{2}{T}\int_0^{T/2} V_0 \sin \omega t\ dt = \frac{2}{\pi} V_0 = 0.637 V_0.$$

This is also known as the **rectified average value of a sinusoidal voltage**

(ii) ***Root Mean Square Value*** (V_{rms} or I_{rms})**:**

$$V_{rms}\,(\text{from } t = t_1 \text{ to } t = t_2) = \sqrt{\int_{t_1}^{t_2} V^2(t)\,dt / t_2 - t_1}$$

rms value for a complete cycle of sinusoidal voltage

$\therefore \quad V = V_0 \sin \omega t$

$$V_{rms} = \sqrt{\frac{\int_0^T V_0^2 \sin^2 \omega t\, dt}{T}} = \frac{V_0}{\sqrt{2}}$$

rms value for a complete cycle of sinusoidal current

$I = I_0 \sin \omega t$

$$\text{and} \quad I_{rms} = \sqrt{\frac{\int_0^T I_0^2 \sin^2 \omega t\, dt}{T}} = \frac{I_0}{\sqrt{2}}$$

$$\text{or RMS value} = \frac{\text{Peak value}}{\sqrt{2}}$$

How do we measure sinusoidally varying voltage or current?

Moving coil galvanometer measure steady currents but if we pass sinusoidal current through them, the needle may wiggle at low fequency but its average deflection is zero. Hot wire instruments are commonly used to measure the rms values (also known as **virtual values**).

It must be emphasied here that meters used for ac voltage and current measurement are always calibrated to read rms values, not peak (or maximum) or rectified average values. The usual domestic power supply "220 - volt ac" has rms voltage of 220 V. The voltage amplitude or peak value is

$$V_0 = \sqrt{2}V_{rms} = \sqrt{2}(220V) = 311V.$$

Illustration:

If a domestic appliance draws 2.5 A from a 220-V, 60- Hz power supply, find

(a) The average current for full cycle

(b) The average of the square of the current

(c)The current amplitude

(d) The supply voltage amplitude.

Sol. *(a)* *The average of sinusoidal AC values over any whole number of cycles is zero.*

(b) *RMS value of current* $= I_{rms} = 2.5\ A$

$\therefore \quad (I^2)_{av} = (I_{rms})^2 = 6.25A^2$

(c) $I_{rms} = \dfrac{I_m}{\sqrt{2}}$

$\therefore$ *Current amplitude* $= \sqrt{2}I_{rms} = \sqrt{2}(2.5\ A)$

$= 3.5\ A$

(d) $V_{rms} = 220V = \dfrac{V_m}{\sqrt{2}}$

$\therefore$ *Supply voltage amplitude*

$V_m = \sqrt{2}(V_{rms}) = \sqrt{2}(220V)$

$= 311\ V.$

AC Voltage Applied to a Resistor :

A resistor connected to a source ε of ac voltage as shown in the ciruit digram . The symbol for an ac source in a circuit diagram is $\ominus$. For simplicity, we consider a source which produces sinusoidally varying potential difference across its terminals. Let this potential difference, also called ac voltage, be given by

$$V = V_0 \sin \omega t \qquad \ldots(i)$$

where V_0 is the amplitude of the sinusoidal voltage and ω is its angular frequency.

AC voltage applied to a resistor

The instantaneous potential drop across the resistor R is

$$V_0 \sin \omega t = IR$$

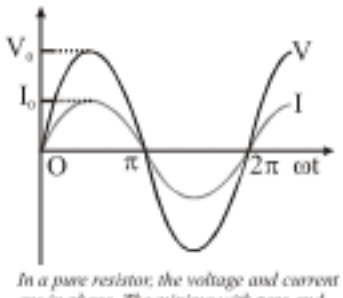
or $$I = \frac{V_0}{R} \sin \omega t$$

$$I = I_0 \sin \omega t \qquad \ldots(ii)$$

where I is the instantaneous current and the current amplitude I_0 is given by

$$I_0 = \frac{V_0}{R} \qquad \ldots(iii)$$

Equation (iii) is just Ohm's law which for resistors work equally well for both ac and dc voltages. The voltage across a pure resistor and the current through it, given by equation (i) and (ii) are plotted as a function of time in figure. Note, in particular that both V and I reach zero, minimum and maximum values at the same time. Clearly, the voltage and current are in phase for a circuit containing pure resistance.



In a pure resistor, the voltage and current are in phase. The minima with zero and maxima occur at the same respective times.

We see that, like the applied voltage, the current varies sinusoidally and has corresponding positive and negative values during each cycle. Thus, **the sum of the instantaneous current values over one complete cycle is zero, and the average current is zero**. The fact that the average current is zero, however, does not mean that the average power is zero and that there is no dissipation of electrical energy. As you know, joule heating is given by I^2R and depends on I^2 (which is always positive whether I is positive or negative) and not on I. Thus there is Joule heating and dissipation of electrical energy when an ac current passes through a resistor.

The instantaneous power dissipated in the resistor is

$$P = I^2 R = I_0^2 R \sin^2 \omega t \qquad \ldots[iv]$$

The average value of Power P over a cycle is

$$\overline{P} = \frac{1}{2} I_0^2 R = I^2_{rms} R \qquad \ldots[v]$$

Where the bar over a letter(here, P) denotes its average value.

To express ac power in the same form as dc power $(P = I^2R)$, as special value of current is used. It is called, root mean square (rms) or **effective current** and is denoted by I_{rms}.

Similarly, we define the rms voltage or **effective voltage**

From equation [iii], we have

$$V_0 = I_0 R \qquad ...[vi]$$

or $$\frac{V_0}{\sqrt{2}} = \frac{I_0}{\sqrt{2}} R \qquad ...[vii]$$

or $$V_{rms} = I_{rms} R \qquad ...[ix]$$

In terms of rms values, the equation for power and relation between current and voltage in ac circuits are essentially the same as those for the dc case.

In fact, the I_{rms} or rms current is the equivalent dc current that would produce the same average power loss as the alternating current. Equation [v] can also be written as

$$\overline{P} = V_{rms}^2 / R = I_{rms}^2 R \qquad (\text{since } V_{rms} = I_{rms} R) \qquad ...[ix]$$

Illustration:

A bulb is rated 60 W at 220 V/50 Hz. Find the maximum value of instantaneous current through the filament?

Sol. $V_{max} = 220\sqrt{2} = 311\,V$

$$R = \frac{220^2}{P} = \frac{220 \times 220}{60} = \frac{2420}{3} = 806.67\,\Omega$$

$$I = \frac{V_{max}}{R} = \frac{311}{806.67} = 0.39A$$

Illustration:

A light bulb is rated at 200 W for a 220 V supply. Find

(a) The resistance of the bulb;

(b) The peak voltage of the source; and

(c) The rms current through the bulb.

Sol. *(a) We are given P = 100 W and V = 220 V. The resistance of the bulb is*

$$R = \frac{V_{rms}^2}{P} = \frac{(220V)^2}{200W} = 242\Omega$$

(b) *The peak voltage of the source is*

$$V_m = \sqrt{2} V_{rms} = 311V$$

(c) *Since,* $P = I_{rms} V_{rms}$

$$I_{rms} = \frac{P}{V_{rms}} = \frac{200W}{220V} = 0.90A.$$

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Practice Exercise

Q.1 If E_0 represents the peak value of the voltage in an ac circuit. Find the rms voltage.

Q.2 In an ac circuit, the rms value of current I_{rms} is related to the peak current I_0 by the relation :

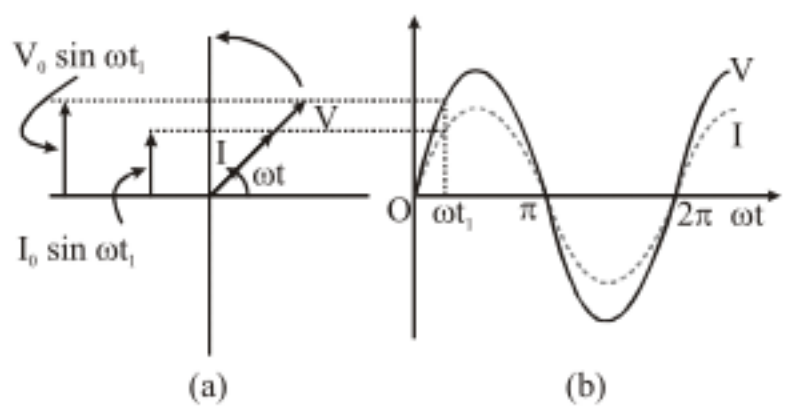
Q.3 The electric current in a circuit is given by $i = i_0\, t/\tau$ for some time. What is the rms current for the period $t = 0$ to $t = \tau$

Answers

Q.1 $E_0/\sqrt{2}$ Q.2 $I_{rms} = (I_0/\sqrt{2})$ Q.3 $i_0/\sqrt{3}$

Representation of AC Current and Voltage by Rotating Vectors – Phasors:

In the previous section, we saw that the current through a resistor is in phase with the ac voltage. But this is not so in the case of an inductor, a capacitor or a combination. In order to show phase relationship between voltage and current in an ac circuit, we use the motion of PHASORS. The analysis of an ac circuit is facilitated by the use of a phasor diagram. A phasor is a vector which rotates about the origin with angular speed ω, as shown in figure. The vertical components of phasors V and I represent the sinusoidally varying quantities V and I. The magnitudes of phasors V and I represent the amplitudes or the peak values V_0 and I_0 of these oscillating quantities. Figure (a) shows the voltage and their relationship at time t_1 i.e., corresponding to the circuit show in figure for the case of an ac source connected to a resistor. The projection of voltage and current phasors on vertical axis, i.e., $V_0 \sin \omega t$ and $I_0 \sin \omega t$, respectively represent the instaneous value of voltage and current at that instant. As they rotate with frequency ω, curves in figure(b) are generated which respresent the sinusoidal variation of voltage and current with time.



(a) A phasor diagram for the circuit in figure
(b) Graph of V and I versus ωt

From figure (a) we see that phasors V and I for the case of a resistor are in the same phase. This is so for all times. This means that the phase angle between the voltage and the current is zero.

AC Voltage Applied to an Inductor:

An ac source connected to an inductor as shown in the circuit below. Usually, inductors have appreciable resistance in their windings, but we shall assume that this is ideal inductor (having zero resistance). Thus, the circuit is a purely inductive ac circuit. Let the voltage across the source be $V = V_0 \sin \omega t$. Using the loop equation. $\sum \varepsilon(t) = 0$, and since there is no resistor in the circuit.

$$V - L\frac{dI}{dt} = 0 \qquad \ldots[x]$$

An AC source connected to an inductor

where the second term is the self-induced emf in the inductor; and L is the self-inductance of the coil. Combining equation [i] and [x], we have

$$\frac{dI}{dt} = \frac{V}{L} = \frac{V_0}{L}\sin \omega t \qquad \ldots[xi]$$

$$dI = \frac{V_0}{L}\sin \omega t \, dt$$

$$I = -\frac{V_0}{\omega L}\cos(\omega t)$$

Using $-\cos(\omega t) = \sin\left(\omega t - \frac{\pi}{2}\right)$, we have

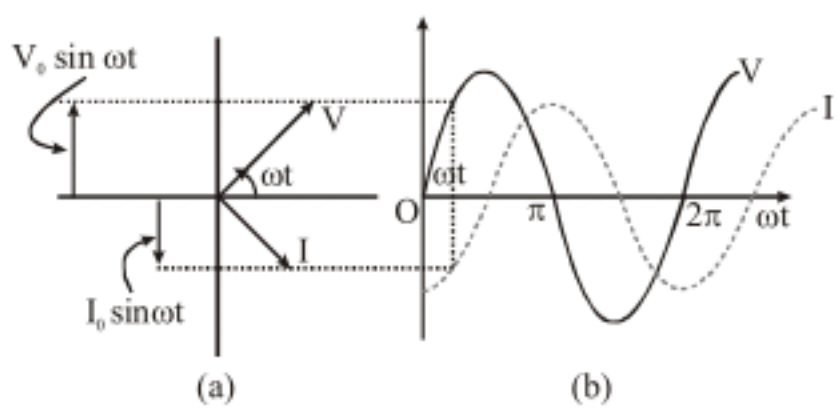
$$i = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \ldots(xii)$$

where $I_0 = \frac{V_0}{\omega L}$ is the amplitude of the current. The quantity ωL is analogous to the resistance and is called **inductive reactance**, denoted by $\mathbf{X_L}$:

$$X_L = \omega L = 2\pi f L \qquad \ldots(xiii)$$

The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a pure inductive circuit in the same way as does the resistance in a pure resistive circuit. The inductive reactance is directly proportional to the inductance frequency of the voltage source.

A comparison of equation (i) and (ii) for the source voltage and the current in an inductor shows that the current lags the voltage by $\frac{\pi}{2}$ or one-quarter $\left(\frac{1}{4}\right)$ cycle. Figure a shows the voltage and the current phasors in the present case at instant t. The current phasor I is $\frac{\pi}{2}$ behind the voltage phasor V. When rotated with frequency ω counter-clockwise, they generate the voltage and current given by equation [i] and [xii], respectively and as shown in figure (b)



(a) A phasor diagram for the circuit in figure
(b) Graph of V and I versus ωt.

We see that the current reaches its maximum value later than the voltage by one-fourth of a period $\left[\frac{T}{4} = \frac{\pi/2}{\omega}\right]$. You have seen that an inductor has reactance that limits current similar to resistance in a dc circuit. Does it also consume power like a resistance? Let us try to find out.

The instantaneous power supplied to the inductor is

$$P_L = IV = I_m \sin\left(\omega t - \frac{\pi}{2}\right) V_0 \sin(\omega t) = -I_0 V_0 \cos(\omega t).\ \sin(\omega t) = - -\frac{I_0 V_0}{2} \sin(2\omega t) \quad ...[xiv]$$

So, the average power over a complete cycle is zero

since the average of $\sin(2\omega t)$ over a complete cycle is zero.

Thus, the average power supplied to an inductor over one complete cycle is zero.

Physically, this result means the follows. During the first quarter of each current cycle, the flux through the inductor builds up and sets up a magnetic field and energy is stored in the inductor. In the next quarter of cycle, as the current decrease, the flux decreases and the stored energy is returned to the source. Thus, in each half cycle, the energy which is withdrawn from the source is returned to it without any dissipation of power.

Illustration:

A pure inductor of 50.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Sol. The inductive reactance.

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 50\times 10^{-3}\Omega$$

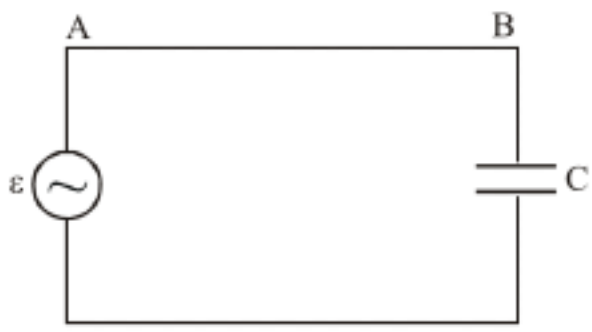
$$= 15.7\Omega$$

The rms current in the circuit is

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220V}{15.7\Omega} = 14.01A\ .$$

AC Voltage Applied to A Capacitor:

An ac source ε connected to a capacitor only, a purely capacitive ac circuit is as shown.



An ac source connected to a capacitor

When the capacitor is connected to an ac source, as in figure, it limits or regulates the current, but does not completely prevent the flow of charge. The capacitor is alternately charged and discharged as the current reverses each half cycle. Let q(t) be the charge on the capacitor at any time t. The instantaneous voltage V(t) across the capacitor is

$$V(t) = \frac{q(t)}{C} \qquad \ldots(xv)$$

From the Kirchhoff's loop rule, the voltage across the source and the capacitor are equal,

$$V_0 \sin\omega t = \frac{q}{C}$$

To find the current, we use the relation $I = \frac{dq}{dt}$

$$I = \frac{d}{dt}(V_0 C \sin\omega t) = \omega C V_0 \cos(\omega t)$$

Using the relation, $\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$, we have

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots(xvi)$$

where the amplitude of the oscillating current is

$$I_0 = \frac{V_0}{(1/\omega C)}$$

Comparing it to $I_0 = \frac{V_0}{R}$ for a purely resistive circuit, we find that $(1/\omega C)$ plays the role of resistance.

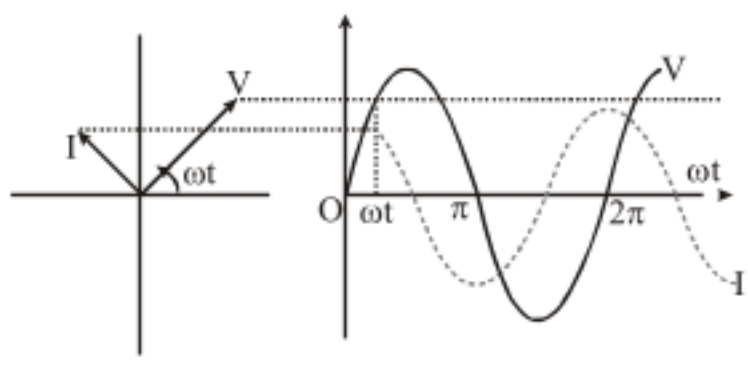
It is called **capacitive reactance** and is denoted by $\mathbf{X_C}$,

$$\mathbf{X_C = 1/\omega C = 1/2\pi fC} \qquad \ldots(xvii)$$

So that the amplitude of the current is

$$I_0 = \frac{V_0}{X_C} \qquad \ldots(xviii)$$

The dimension of capacitive reactance is the same as that of resistance and its SI unit is Ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as does the resistance in a purely resistive circuit. But it is inversely proportional to the frequency and the capacitance.



A comparison of equation (xvii) with the equation of source voltage equation (i) shows that the current in a capacitor leads the voltage by $\pi/2$. Figure shows the phasor diagram at an instant t. Here the current phasor I is $\frac{\pi}{2}$ rad ahead of the voltage phasor V as they rotate counter clockwise. Figure shows the variation of voltage and current with time. We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.

The instantaneous power supplied to the capacitor is

$$P_C = IV = I_0 \cos(\omega t).\, V_0 \sin(\omega t)$$
$$= I_0 V_0 \cos(\omega t) \sin(\omega t)$$
$$= \frac{I_0 V_0}{2} \sin(2\omega t) \qquad \ldots[xix]$$

So, as in the case of an inductor, the average power

Since average of sin 2ωt over a complete cycle is zero. As discussed in the case of an inductor, the energy stored by a capacitor in each quarter period is returned to the source in the next quarter period.

Thus, we see that in the case of an inductor. the current lags the voltage by 90° and in the case of a capacitor, the current leads the voltage by 90°.

Illustration:

30.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current.

Sol. *The capacitive reactance is*

$$X_C = \frac{1}{2\pi fC} = 106\Omega$$

The rms current is

$$I_{rms} = \frac{V_{rms}}{X_C} = 2.08A$$

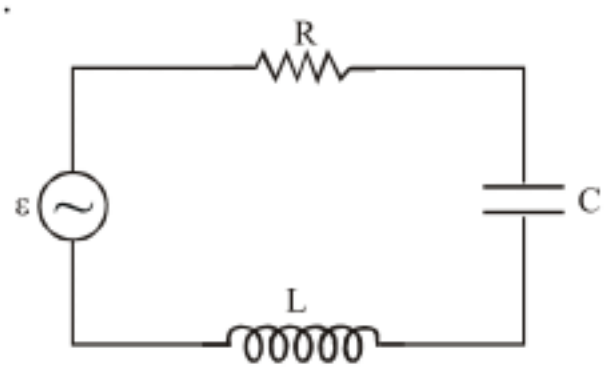
The peak current is

$$I_0 = \sqrt{2} I_{rms} = 2.96A$$

This current oscillates between 2.96A and – 2.96A and is ahead of the voltage by 90°.
If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

AC Voltage Applied to a Series LCR Circuit:

Figure shows a series LCR circuit connected to an ac source ε. As usual, we take the voltage of the source to be $V = V_0 \sin \omega t$.



A series LCR circuit connected to an ac source

If q is the charge on the capacitor and I the current, at time t, we have, from Kirchhoff's loop rule:

$$L\frac{dI}{dt} + IR + \frac{q}{C} = V \qquad \ldots(xx)$$

We want to determine the instantaneous current I and its phase relationship to the applied alternating voltage V. We shall use the technique of phasors to solve equation [xx] to obtain the time –dependence of I.

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Phasor-Diagram Solution:

From the circuit shown in figure we see that the resistor, inductor and capacitor are in series. therefore, the ac current in each element is the same, having the same amplitude and phase. Let it be

$$I = I_0 \sin(\omega t + \phi) \qquad \ldots \text{(xxi)}$$

where ϕ is the phase difference between the voltage across the source and the current in the circuit. On the basis of that we construct a phasor diagram for the present case.

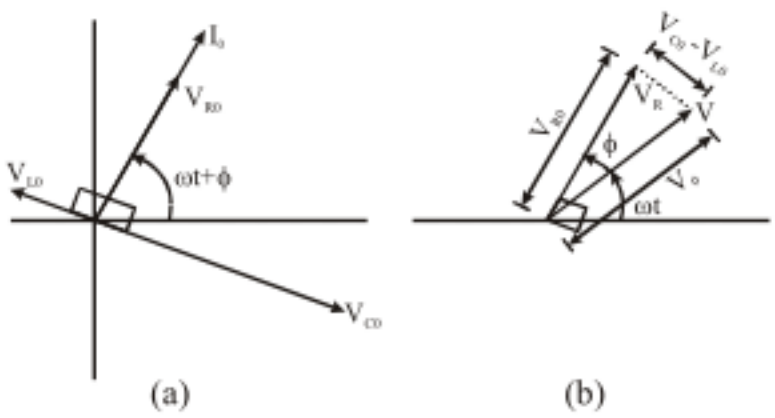
Let I be the phasor representing the current in the circuit as given by equation [xxi]. Further, let V_L, V_R, V_C, and V represent the voltage across the inductor, resistor, capacitor and the source, respectively. From previous section, we know that V_R is in phase with I, V_C is $\frac{\pi}{2}$ rad behind I and V_L is $\frac{\pi}{2}$ rad ahead of I. V_R, V_C and I are shown in figure (a) with appropriate phase-relations.

The length of these phasors or the amplitude of V_R, V_C and V_L are :

$$V_{R0} = I_0 R\,,\ V_{C0} = I_0 X_C,\ V_{L0} = I_0 X_L \qquad \ldots \text{(xxii)}$$

The voltage equation (xx) for the circuit can be written as

$$V = V_L + V_R + V_C \qquad \ldots \text{(xxiii)}$$



(a) (b)

(a) Relation between the phasors. (b) Relation between the phasors for the circuit in

This relation is represented in figure (b). Since $\vec{V}_{C0}$ and $\vec{V}_{L0}$ are always along the same line and in opposite direction, they can be combined vectorilly into a single phasor $(\vec{V}_C + \vec{V}_L)$ which has a magnitude $|\vec{V}_{C0} - \vec{V}_{L0}|$. Since, $\vec{V}_0$ is represented as the hypotenuse of a right- angle triangle

$$V_0^2 = V_{R0}^2 + (V_{C0} - V_{L0})^2 \qquad \ldots\text{[xxiv]}$$

Substituting the values of V_{R0}, V_{C0} and V_{L0} from equation [xxii] into the above equation, we have

$$V_0^2 = (I_0 R)^2 + (I_0 X_C - I_0 X_L)^2$$
$$= I_0^2 [R^2 + (X_C - X_L)^2]$$

or $$I_0 = \frac{V_0}{\sqrt{R^2 + (X_C - X_L)^2}} \qquad \ldots \text{(xxv (a))}$$

By analogy to the resistance in a circuit, we introduce the impedance Z in ac circuit :

$$I_0 = \frac{V_0}{Z} \qquad \ldots \text{(xxv (b))}$$

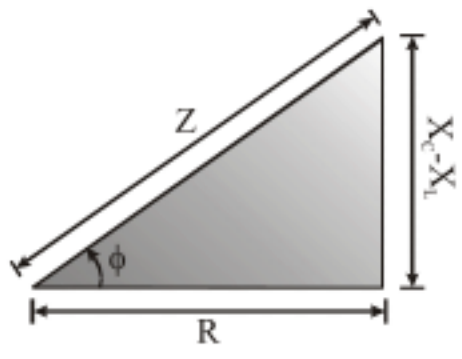
where $Z = \sqrt{R^2 + (X_C - X_L)^2}$ ... (xxvi)

Since phasor I is always parallel to phasor V_R, the phase angle ϕ is the angle between V_R and V and can be determined from figure:

$$\tan\phi = \frac{V_{C0} - V_{L0}}{V_{R0}}$$

from the impedance triangle

$$\tan\phi = \frac{X_C - X_L}{R} \qquad \ldots \text{(xxvii)}$$



impedance triangle

Equations (xxvi) and (xxvii) are shown in figure. This is called **Impedance triangle** which is a right triangle with Z as its hypotenuse.

Equation (xxv (a)) gives the amplitude of the current and figure gives the phase angle. With these, equation (xxi) is completely specified.

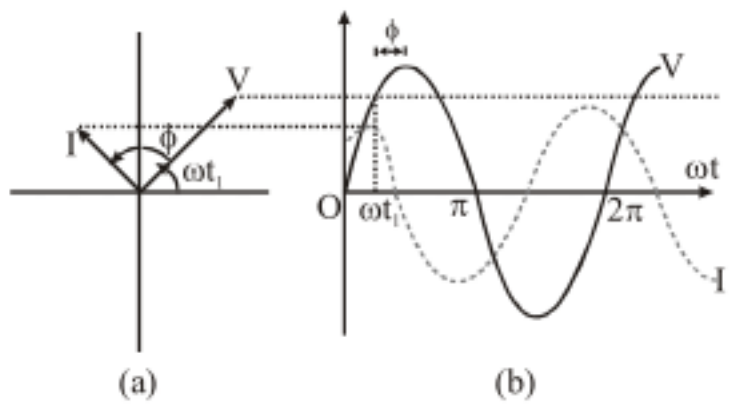
If $X_C > X_L, \phi$ is positive and the circuit is capacitive. Consequently, the voltage across the source lags the current.

If $X_C < X_L, \phi$ is negative and the circuit is inductive.

Consequently, the voltage across the source leads the current.

Figure shows the phasor diagram and variation of V and I with ωt for the case $X_C > X_L$.

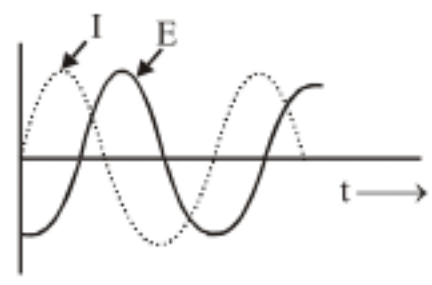
Thus, we have obtained the amplitude and phase of current for an LCR series circuit using the technique of phasors. But this method of analyzing ac circuits suffers from certain disadvantages. First, the phasor diagram states nothing about the initial condition. One can take any arbitrary value of t and draw different phasors which show the relative angle between different phasors. The solution so obtained is called the **steady-state solution.**



(a) Phasor diagram of V and I. (b) Graphs of V and I versus ωt for a series LCR circuit where $X_C > X_L$.

Illustration :

When an AC source of e.m.f. $E = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the e.m.f. E and the current I in the circuit is observed to be $\pi/4$, as shown in the figure. If the circuit consists possibly only of R-C in series. What will be the relation between the two elements of the circuit ?



Sol. *Given $E = E_0 \sin(100\,t)$. Comparing this with $E = E_0 \sin \omega t$, we have $\omega = 100\,\text{rad s}^{-1}$. It follows from the figure that the current leads the e.m.f. which is true only for R-C circuit, and not for R-L circuit. Hence the circuit does not contain an inductor. Thus choices (c) and (d) are not possible. For R-C circuit, the phase difference between E and I is given by*

$$\tan\phi = \frac{1}{\omega RC} \ldots\ldots$$

Given $\phi = \pi/4$. Also $\omega = 100\,\text{rad s}^{-1}$. Using these values in, we get

$$\tan\left(\frac{\pi}{4}\right) = \frac{1}{100\,RC} \text{ or } RC = \frac{1}{100}$$

Illustration :

An LCR series circuit with 100 Ω resistance is connected to an ac source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current in the LCR circuit.

Sol. *When capacitance is removed, the circuit becomes L-R with,*

$$\tan\phi = \frac{X_L}{R} \quad \text{i.e.,} \quad X_L = R\tan\phi = 100\sqrt{3}\ \Omega$$

and when inductance is removed the circuit becomes C-R with,

$$\tan\phi = \frac{X_C}{R} \quad \text{i.e.,} \quad X_C = R\tan\phi = 100\sqrt{3}\ \Omega$$

as here $X_L = X_C$ so the circuit is in series resonance and hence as $X = X_L - X_C = 0$,

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i.e., $Z = \sqrt{R^2 + X^2} = R$,

So, $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{200}{100} = 2\ A$

Resonance:

An interesting characteristic of the series RLC circuit is the phenomenon of resonance. The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the system's natural frequency. If such a system is driven by an energy source at a frequency that is near the natural frequency, the amplitude of oscillation is found to be large.

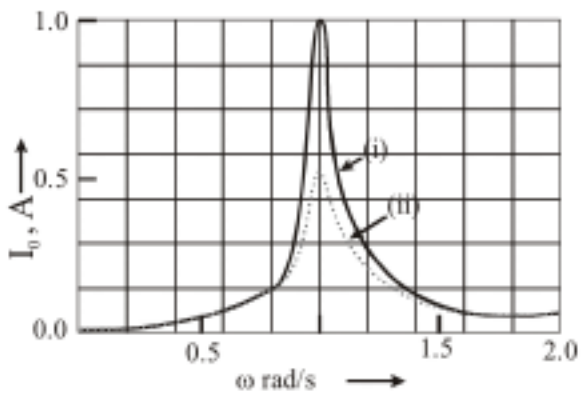
For an RLC circuit driven with voltage of amplitude V_0 and frequency ω, we found that the current amplitude is given by $I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_C - X_L)^2}}$

with $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$. So if ω is varied, then at a particular frequency $\omega_0, X_C = X_L$, the impendance is minimum. This frequency is called the **resonant frequency** :

then $\frac{1}{\omega_0 C} = \omega_0 L$ or $\omega_0 = \frac{1}{\sqrt{LC}}$ ...(xxxv)

At resonant frequency, the current amplitude is maximum; $I_0 = \frac{V_0}{R}$

The variation of I_0 with ω in a RLC series circuit with $L = 1.00$ mH, $C = 1.00$ nF for two values of R: (i) $R = 100\ \Omega$ and (ii) $R = 200\,\Omega$ are shown in the figure. For the source applied $V_0 = 100$ V. ω_0 for this case is $\left(\frac{1}{\sqrt{LC}}\right) = 1.00 \times 10^6$ rad/s.



Variation of I_0 with ω for two cases: (i) R = 100Ω, (ii)R = 200Ω, L = 1.00mH. C = 1.00nF and V_0 = 100V for both cases.

We see that the current amplitude is maximum at the resonant frequency.

Illustration :

Resonance frequency of a circuit is f. If the capacitance is made 4 times the initial value, then find the resonance frequency ?

Sol. $f = \frac{1}{2\pi\sqrt{LC}}$ i.e. $f \propto \frac{1}{\sqrt{C}} \rightarrow \frac{1}{\sqrt{4}} = \frac{1}{2}$ time

Power in AC Circuits: The Power Factor:

We have seen that a voltage $V = V_0 \sin \omega t$ applied to a series RLC circuit drives a current in the circuit given by $I = I_0 \sin(\omega t + \phi)$ where

$$I_0 = \frac{V_0}{Z} \text{ and } \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Therefore, the instantaneous power P supplied to the source is

$$P = VI = (V_0 \sin \omega t) \times [I_0 \sin(\omega t + \phi)]$$

$$= \frac{V_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)] \qquad \ldots(xxix)$$

The average power over a cycle is given by the average of the two terms in R.H.S. of equation [xxxvii]. Second term average is zero, therefore,

$$\overline{P} = \frac{V_0 I_0}{2} \cos\phi = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

$$= V_{rms} I_{rms} \cos\phi \qquad \ldots(xxx)$$

This can also be written as,

$$\overline{P} = I_{rms}^2 Z \cos\phi \qquad \ldots(xxxi)$$

So, the average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between them. The quantity $\cos\phi$ is called the **power factor**. Let us discuss the following cases: from impedence triangle $\cos\phi = \frac{R}{z}$

Case (I) Resistive circuit:

If the circuit contains only pure R, it is called resistive. In that case $\phi = 0$, $\cos\phi = 1$. There is maximum power dissipation.

Case (II) Purely Inductive or Capacitive Circuit :

If the circuit contains only an inductor or capacitor, we know that the phase difference between voltage and current is $\frac{\pi}{2}$. Therefore, $\cos\phi = 0$, and no power is dissipated even though a current is flowing in the circuit. This current is sometimes referred to as **wattless current**.

Case(III) LCR Series Circuit:

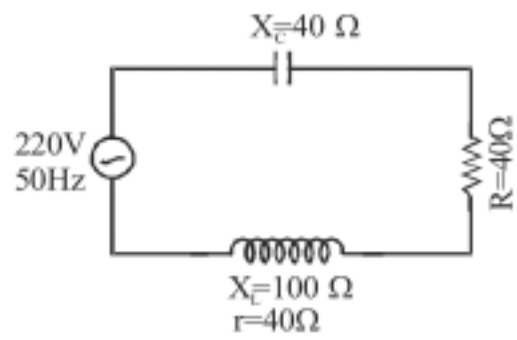
In an LCR series circuit, power dissipated is given by equation [xxxviii] where $\phi = \tan^{-1}\frac{X_C - X_L}{R}$. So, ϕ may be non-zero (except $\frac{\pi}{2}$) in a RL or RC or RCL circuit. Even in such cases, power is dissipated only in the resistor.

Case (IV) Power Dissipated at Resonance in LCR Circuit:

At resonance $X_C - X_L = 0$, and $\phi = 0$. Therefore, $\cos\phi = 1$, and $P = I_0^2 z = I_0^2 R$. That is, maximum power is dissipated in a circuit (through R) at resonance.

Illustration:

Find the power factor of the circuit shown in figure ?



Sol. $R = 40 + 40 = 80\,\Omega$,

$$X_L - X_C = 100 - 40 = 60\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{80^2 + 60^2} = 100$$

Power factor, cos $\phi = \frac{R}{Z} = \frac{80}{100} = 0.8$.

Illustration :

A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12 V, 50 rad/s ac source a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF *capacitor is connected in series with the coil.*

Sol. *In case of a coil as* $Z = \sqrt{R^2 + \omega^2 L^2}$

i.e., $$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

so when dc is applied

$$I = \frac{V}{R} \quad \text{i.e.,} \quad R = \frac{12}{4} = 3\,\Omega \qquad \text{... (i)}$$

and when ac is applied,

$$I = \frac{V}{Z} \quad \text{i.e.,} \quad Z = \left(\frac{V}{I}\right) = \left(\frac{12}{2.4}\right) = 5\,\Omega$$

or, $R^2 + X_L^2 = 5^2 \quad (\text{as } Z = \sqrt{R^2 + X_L^2})$

so, $X_L^2 = 5^2 - R^2 = 5^2 - 3^2 = 4^2 \quad$ i.e., $\quad X_L = 4\,\Omega$

but as $X_L = \omega L, \quad L = \frac{X_L}{\omega} = \frac{4}{50} = 0.08$

Now when the capacitor is connected to the above circuit in series,

As $$X_C = \frac{1}{\omega C} = \frac{1}{50 \times 2500 \times 10^{-6}} = \frac{10^3}{125} = 8\ \Omega$$

So, $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (4-8)^2} = 5\ \Omega$$

and hence $$I = \frac{V}{Z} = \frac{12}{5} = 2.4\ A$$

so, $$P_{av} = V_{rms} I_{rms} \cos\phi = (I_{rms} \times Z) \times I_{rms} \times \left(\frac{R}{Z}\right)$$

i.e., $$P_{av} = I_{rms}^2 R = (2.4)^2 \times 3 = 17.28\ W.$$

Illustration:

A 750 Hz., 20 V source is connected to a resistance of 100 ohm, an inductance of 0.1803 henry and a capacitance of 10 microfarad all in series. Calculate the time in which the resistance (thermal capacity 2 J/°C) will get heated by 10°C.

Sol. *As in this problem,*

$$X_L = \omega L = 2\pi f L = 2\pi \times 750 \times 0.1803 = 849.2\Omega$$

and, $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 750 \times 10^{-5}} = 21.2\Omega$$

so, $$X = X_L - X_C = 849.2 - 21.2 = 828\Omega$$

and hence, $$Z = \sqrt{R^2 + X^2} = \sqrt{(100)^2 + (828)^2} = 834\Omega$$

but as in case of ac,

$$P_{av} = V_{rms} I_{rms} \cos\phi = V_{rms} \times \frac{V_{rms}}{Z} \times \frac{R}{Z}$$

i.e., $$P_{av} = \left(\frac{V_{rms}}{Z}\right)^2 \times R = \left(\frac{20}{834}\right)^2 \times 100 = 0.0575W$$

and as, $U = P \times t = mc\Delta\theta = (TC)\Delta\theta$

$$t = \frac{(TC) \times \Delta\theta}{P} = \frac{2 \times 10}{0.0575} = 348s$$

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Practice Exercise

Q.1 An ac circuit having resistance (R) and inductance (L) connected in series with a source of ω angular frequency. Find the power factor.

Q.2 In an LCR series ac circuit, the voltage across each of the components, L, C and R is 50V. Find the voltage across the LC combination.

Q.3 If in a series L-C-R circuit, the voltage across R, L and C are V_R, V_L and V_C respectively. Find the voltage of applied AC source.

Q.4 In a L R circuit the A.C. source has voltage 220 V and the potential difference across the inductance is 176 V. Find the potential difference across the resistance.

Answers

Q.1 $R/(R^2+\omega^2L^2)^{1/2}$ Q.2 Zero Q.3 $\sqrt{[(V_R)^2+(V_L-V_C)^2]}$

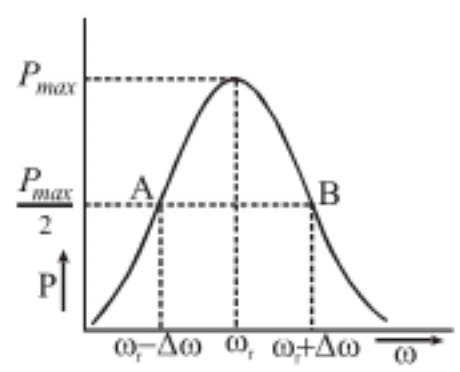
Q.4 132 V

Band Width and Q-Factor:

Angular frequency variation of power in LCR series circuit.

$$P = P_{max}\sqrt{\frac{R^2}{\left[R^2+\left(L\omega-\frac{1}{C\omega}\right)^2\right]}}$$

Graph between P & ω as shown in the figure.



$$\omega_1 = +\frac{R}{2L}+\left(\omega_r^2+\frac{R^2}{4L^2}\right)^{1/2} \text{ and } \omega_2 = -\frac{R}{2L}+\left(\omega_r^2+\frac{R^2}{4L^2}\right)^{1/2}$$

Now, $\omega_1-\omega_2=\frac{R}{L}$

or $(\omega_r+\Delta\omega)-(\omega_r-\Delta\omega)=\frac{R}{L}$ or $2\Delta\omega=\frac{R}{L}$.

The frequency interval between half maximum power points is known as **band width**.

The ratio of resonance frequency and band width is known as **quality factor (Q)**.

$\therefore$ $Q=\frac{\omega_r}{2\Delta\omega}=\frac{\omega_r L}{R}$.

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Q factor is a measure of the sharpness of resonance. Resonance will be sharp if the value of bandwidth $(2\Delta\omega)$ is small. This is of course possible only when the power-frequency curve fall steeply around $\omega = \omega_r$.

Illustration:

A resistance R and inductance L and a capacitor C all are connected in series with an AC supply. The resistance of R is 16 ohm and for a given frequency, the inductive reactance of L is 24 ohm and capacitive reactance of C is 12 ohm. If the current in the circuit is 5 amp., find

(a) The potential difference across R, L and C

(b) The impedance of the circuit

(c) The voltage of AC supply

(d) Phase difference between applied source voltage and current drawn

Sol. *(a) Potential difference across the resistance*

$$V_R = iR = 5\times16 = 80 \text{ Volt}$$

Potential difference across the inductor

$$V_L = iX_L = 5\times24 = 120 \text{ Volt}$$

Potential difference across the capacitor

$$V_C = iX_C = i \times (1/\omega C) = 5\times12 = 60 \text{ Volt}$$

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{[(16)^2 + (24-12)^2]} = 20 \text{ ohm}$$

(c) *The voltage of AC supply is given by*

$$E = IZ = 5 \times 20 = 100 \text{ volt}$$

(d) *Phase angle*

$$\phi = \tan^{-1}\left[\frac{\omega L - (1/\omega C)}{R}\right] = \tan^{-1}\left[\frac{24-12}{16}\right] = \tan^{-1}(0.75) = 36^\circ 46'.$$

Illustration:

Obtain the resonant freqency and Q-factor of a series LCR circuit with L = 3.0 H, C = 27μF and $R = 7.4\Omega$. It is desired to improve the sharpness of the resonance of the circuit by reducing its full width at half maximum by a factor of 2. Suggest a suitable way.

Sol. *L = 3 henry,* $C = 27\times10^{-6} F$, $R = 7.4\Omega$

Resonant angular frequency is given by $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3\times27\times10^{-6}}} rad\ s^{-1}$

$$= \frac{1}{9\times10^{-3}} rad\, s^{-1} = \frac{1000}{9} rad\, s^{-1} = 111\ rad\ s^{-1}$$

$$Q = \frac{\omega_0 L}{R} = \frac{111\times3}{7.4} = 45$$

To double Q without changing ω_0, *reduce R to* 3.7Ω.

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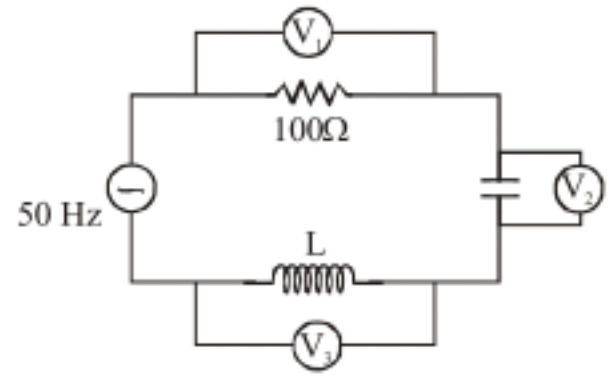
Illustration:

A series L-C-R circuit is connected to an AC source of 50 Hz. as shown in figure. If the readings of the three voltmeters V_1, V_2 and V_3 are 65 V, 415 V and 204 V respectively, calculate,



(i) The current in the circuit

(ii) The value of inductor

(iii) The value of the capacitor C and

(iv) The value of C (for the same L) required to produce resonance.

Sol. ***(i)*** *Here $V_R = i_{rms} R$,*

where i_{rms} is the rms value of current in the circuit.

$$\therefore \quad i_{rms} = \frac{V_R}{R} = \frac{65}{100} = 0.65 \text{ amp}$$

(ii) $V_L = i_{rms} \times X_L$ or $X_L = \frac{V_L}{i_{rms}}$ $\therefore$ $X_L = \frac{204}{0.65} = 313.85\Omega$

Now, $X_L = \omega L = 2\pi f L$ or $L = \frac{X_L}{2\pi f}$ $\therefore$ $L = \frac{313.85}{2\pi \times 50} = 1.0$ henry

(iii) $X_C = \frac{V_C}{i_{rms}} = \frac{415}{0.65} = 638.46\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \therefore C = \frac{1}{2\pi \times 50 \times 638.46} = 5 \times 10^{-6} = 5\mu F$$

(iv) *Let 'C' be the capacitance of capacitor that will produce resonance with inductor L = 1.0 henry. Then*

$$f = \frac{1}{2\pi\sqrt{(LC)}} \text{ or } C \frac{1}{4\pi^2 f^2 L}$$

$$= \frac{1}{4\pi^2 \times (50)^2 \times 1.0} = 10.1 \times 10^{-6} F = 10.1\mu F.$$

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Solved Examples

Q.1. A circuit consists of a series combination of a 50 mH inductor and a $20\mu F$ capacitor connected to 220 V, 50 Hz supply. The circuit has negligible ohmic resistance. (a) Find the amplitude and rms value of current. (b) Find the rms values of voltage across inductor and capacitor. (c) What is the average power transferred to (i) the inductor, and (ii) the capacitor ? (d) What is the total power absorbed by the circuit

Sol. Given L = 50 mH = 50 × 10^{-3} H, C = $20\mu F = 20\times10^{-6}$ F, $V_{rms} = 220V, V = 50Hz$ and

$\omega = 2\pi v = 100\pi \text{ rads}^{-1}$. Therefore, peak value of voltage is

$$V_0 = \sqrt{2}\, V_{rms} = \sqrt{2}\times 220 = 311V$$

The peak value of the current in a series LCR circuit is

$$I_0 = \frac{V_0}{\left[R^2\left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}} = \frac{V_0}{Z} \quad ..... (1)$$

Putting R = 0 we have

$$I_0 = \frac{V_0}{\omega L - \frac{1}{\omega C}} \quad ..... (2)$$

(a) Substituting the values of V_0, ω, L and C in equation (2) and solving we get $I_0 = -2.17A$. The amplitude of current is the magntidue of modulus of current. Therefore, amplitude of curernt is $|I_0| = 2.17A$.

$$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{2.17}{\sqrt{2}} = 1.53A$$

(b) Voltage drop across inductor is

$$(V_L)_{rms} = I_{rms}\times\omega L = 1.53\times100\pi\times50\times10^{-3} = 23.1V$$

Voltage drop across capictor is

$$(V_C)_{rms} = \frac{I_{rms}}{\omega L} = \frac{1.53}{100\pi\times20\times10^{-6}} = 243V$$

(c) Since in an inductor the voltage leads the current by a phase angle of 90°, the power transferred to it is 0. Thus $P_L = 0$. We know that in a capacitor the current leads the voltage by 90°. Hence power transfered to capcitor is 0. Thus $P_C = 0$.

(d) Total power absorbed by the circuit is $P = P_L + P_C = 0$

Q.2 A bulb is rated 55 W/110 V. It is to be connected to a 220 V/50 Hz with inductor in series. What should be the value of inductance so that bulb gets correct voltage.

Sol. We want $\frac{V_{app}R}{\sqrt{R^2+L^2\omega^2}} = 110$ or $\frac{220R}{\sqrt{R^2+L^2\omega}} = 110$

or $4R^2 = R^2 + L^2\omega^2$ or $L\omega = \sqrt{3}\,R$

$L(100\pi) = 220\sqrt{3}$

$L = \frac{2.2\sqrt{3}}{3.14} = \frac{2.2(1.732)}{3.14} = 1.2H$; $R = \frac{110\times110}{55} = 220\Omega$

Q.3 A circuit draws a power of 550 watt from a source of 220 volt, 50 Hz. The power factor of the circuit is 0.8 and the current lags in phase behind the potential difference. To make the power factor of the circuit as 1.0, what capacitance will have to be connected with it.

Sol. As the current lags behind the potential difference, the circuit contains resistance and inductance.

Power, $P = v_{rms} \times i_{rms} \times \cos\phi$

Here, $i_{rms} = \frac{V_{rms}}{Z}$, where $Z = \sqrt{[(R^2 + (\omega L)^2)]}$

$\therefore \quad P = \frac{V_{rms}^2 \times \cos\phi}{Z}$ or $Z = \frac{v_{rms}^2 \times \cos\phi}{P}$

So, $Z = \frac{(220)^2 \times 0.8}{550} = 70.4$ ohm

Now, power factor $\cos\phi = \frac{R}{Z}$ or $R = Z\cos\phi$

$\therefore \quad R = 70.4 \times 0.8 = 56.32$ ohm

Further, $Z^2 = R^2 + (\omega L)^2$ or $(\omega L) = \sqrt{(Z^2 - R^2)}$

Or $\omega L = \sqrt{(70.4)^2 - (56.32)^2} = 42.2$ ohm

When the capacitor is connected in the circuit,

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

and $\cos\phi = \frac{R}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$

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when $\cos\phi = 1, \omega L = \dfrac{1}{\omega C}$

$$\therefore \qquad C = \frac{1}{\omega(\omega L)} = \frac{1}{2\pi f(\omega L)}$$

$$= \frac{1}{(2\times 3.14\times 50)\times(42.2)}$$

$$= 75\times 10^{-6}\,F = 75\mu F.$$

Q.4 Find the average current in terms of I_0 for the waveform shown.

Sol. $I = 2I_0\dfrac{t}{T_0};\ 0 < t < \dfrac{T_0}{2}$;

$$I = 2I_0\left(\frac{t}{T_0} - 1\right); \frac{T_0}{2} < t < T_0$$

$$I_{av} = \frac{2}{T}\int_0^{T_0} I.dt = \frac{2}{T_0}\left[\int_0^{T/2}\frac{2I_0 t}{T_0}dt\right]$$

$$= \frac{2}{T_0^2}\left[\frac{2I_0T_0^2}{2\times 4}\right] = \frac{I_0}{2}$$

Q.5 A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\Omega$, $L = 25.48$ mH, and $C = 796\ \mu F$. Find (a) the impedance of the circuit; (b) the phase difference between the voltage across the source and the current; (c) the power dissipated in the circuit; and (d) the power factor.

Sol. (a) To find the impedance of the circuit, we first calculate X_L and X_C.

$$X_L = 2\pi fL$$

$$= 2\times 3.14\times 50\times 25.48\times 10^{-3}\Omega = 8\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 796\times 10^{-6}} = 4\Omega$$

Therefore,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = 5\Omega$$

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(b) Phase difference, $\phi = \tan^{-1}\dfrac{X_C - X_L}{R} = \tan^{-1}\left(\dfrac{4-8}{3}\right) = -53.1^\circ$

since ϕ is negative, the current in the circuit lags the voltage across the source.

(c) The power dissipated in the circuit is

$P = I_{rms}^2 R$

Now, $I_{rms} = \dfrac{I_m}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\left(\dfrac{283}{5}\right) = 40A$

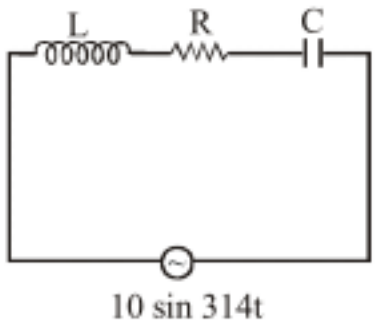
Therefore, $P = (40A)^2 \times 3\Omega = 4800W$

(d) Power factor $= \cos\phi = \cos 53.1^\circ = 0.6$.

Q.6 An inductor of 20×10^{-3} henry, a capacitor 100 μF and a resistor 50 Ω are connected in series across a source of emf V = 10 sin (314t). Find the energy dissipated in the circuit in 20 minutes. If resistance is removed from the circuit and the value of inductance is doubled, then find the variation of current with time in the new circuit.

Sol. Here, time of 1 cycle $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{314} = 1/50$ s. So, we have to calculate the average energy as time >> T.

Energy consumed in time $t = (V_{rms} I_{rms} \cos\phi)t$



$$= \frac{I_0}{\sqrt{2}} \cdot \frac{V_0}{\sqrt{2}} \cdot \frac{R}{Z} t$$

$$E = \frac{V_0^2 R}{2Z^2} t \qquad \left(\therefore I_0 = \frac{V_0}{Z}\right)$$

Now, $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega c}\right)^2}$

$$= \sqrt{(50)^2 + \left(314\times20\times10^{-3} - \frac{1}{314\times100\times10^{-6}}\right)^2}$$

$$= \sqrt{3153.6} \approx 56 \text{ ohm}$$

$\therefore$ Energy consumed $= \dfrac{10^2 \times 50 \times 20 \times 60}{2 \times 3153.6}$ joule

When resistance is removed,

$$\cos\phi = \frac{R}{Z'} = 0 \quad \text{or} \qquad \phi = \pi/2$$

$$Z' = \frac{1}{\omega c} - \omega L' = \frac{1}{314\times10^{-4}} - 314\times40\times10^{-3}\ \Omega$$

$$= 19.3\,\Omega$$

$$I = \frac{E_0}{Z'}\sin(\omega t + \phi)$$

$$= \frac{10}{19.3}\sin(314t + \pi/2)$$

$$= 0.52 \cos 314\, t \text{ Amp.}$$

Q.7 A resistor of 200Ω and a capacitor of $15.0\ \mu F$ are connected in series to a 220 V, 50 Hz ac source. (a) Calculate the current in the circuit; (b) Calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltage more than the source voltage? If yes, resolve the paradox.

Sol. Given

$$R = 200\Omega,\ C = 15.0\mu F = 15.0\times10^{-6}\,F$$

$$V_{rms} = 220\,V,\ f = 50Hz$$

(a) In order to calculate the current, we need the impedance of the circuit. It is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (2\pi f C)^{-2}}$$

$$= \sqrt{(200\Omega)^2 + (2\times3.14\times50\times15\times10^{-6}\,\Omega)^{-2}}$$

$$= \sqrt{(200\Omega)^2 + (212.3\Omega)^2}$$

$$= 291.5\Omega.$$

Therefore, the current in the circuit is

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{220V}{291.5\Omega} = 0.755A$$

(b) Since the current is the same throughout the circuit, we have

$$V_R = I_{rms}R = (0.755A)(200\Omega) = 151V$$

$$V_C = I_{rms}X_C = (0.755\,A)(212.3\Omega) = 160.3V$$

The algebraic sum of the two voltage, V_R and V_C is 311.3V which is more than the source voltage of 220 V. How to resolve this paradox? As you have learnt in the text, the two voltages are not in the same phase. Therefore, they cannot be added like ordinary numbers. The two voltages are out of phase by ninety degrees. Therefore, the total of these voltages must be obtained using the Pythagorean theorem:

$$V_{R+C} = \sqrt{V_R^2 + V_C^2} = 220\ V$$

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Thus, if the phase difference between two voltage is properly taken into account, the total voltage across the resistor and the capacitor is equal to the voltage of the source.

Q.8 An LCR circuit has L = 10 mH, R = 3 Ω and C = 1 μF connected in series to a source of 15 cos ωt V. Calculate the current amplitude and the average power dissipated per cycle at a frequency that is 10% lower than the resonance frequency.

Sol. As here resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 10^{-6}}} = 10^4 \frac{\text{rad}}{\text{s}}$$

so, $$\omega = \omega_0 - \frac{10}{100}\omega_0 = \frac{9}{10}\omega_0 = 9 \times 10^3 \frac{\text{rad}}{\text{s}}$$

and hence, $X_L = \omega L = 9 \times 10^3 \times 10^{-2} = 90\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{9 \times 10^3 \times 10^{-6}} = 111.11\Omega$$

so, $X = X_C - X_L = 111.11 - 90 = 21.11\Omega$

and hence, $Z = \sqrt{R^2 + X^2} = \sqrt{3^2 + (21.11)^2}$

i.e., $Z = \sqrt{9 + 445.63} = 21.32\Omega$

and as here E = 15 cos ωt , i.e., E_0 =15 V,

$$I_0 = \frac{E_0}{Z} = \frac{15}{21.32} = 0.704 \text{ A}$$

The average power dissipated,

$$P_{av} = V_{rms} I_{rms} \cos\phi = (I_{rms} \times Z) \times I_{rms} \times \frac{R}{Z}$$

i.e., $$P_{av} = I^2_{rms} R = \frac{1}{2} I_0^2 R \quad \left[\text{as } I_{rms} = \frac{I_0}{\sqrt{2}}\right]$$

so, $$P_{av} = \frac{1}{2} \times (0.704)^2 \times 3 = 0.74 \text{ W}$$

Now as $f = \frac{\omega}{2\pi} = \frac{9 \times 10^3}{2\pi} \frac{\text{cycle}}{\text{s}}$

So, $$\frac{P_{av}}{\text{cycle}} = \frac{0.74 \text{ J/s}}{(9 \times 10^3 / 2\pi) \text{ cycle/s}} = \frac{2\pi \times 0.74}{9 \times 10^3} \frac{\text{J}}{\text{cycle}}$$

i.e., $$\frac{P_{av}}{\text{cycle}} = 5.16 \times 10^{-4} \frac{\text{J}}{\text{cycle}}.$$

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Q.9 A series LCR circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 rad s^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what frequency the current in the circuit lags the voltage by 45° ?

Sol. At resonance as X = 0, $I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2}A$

and as $V_L = IX_L = I\omega L$ $\qquad L = \frac{V_L}{I\omega}$

so, $\qquad L = \frac{40}{(1/2)\times4\times10^5} = 0.2\,mH$

and as $\omega_0 = \frac{1}{\sqrt{LC}}$, $\qquad C = \frac{1}{L\omega_0^2}$

i.e., $\qquad C = \frac{1}{0.2\times10^{-3}\times(4\times10^5)^2} = \frac{1}{32}\mu F$

Now in case of series LCR circuit,

$$\tan\phi = \frac{X_L - X_C}{R}$$

so current will lag the applied voltage by 45° if,

$$\tan 45 = \frac{\omega L - \frac{1}{\omega C}}{R}$$

i.e., $\qquad 1\times120 = \omega\times2\times10^{-4} - \frac{1}{\omega(1/32)\times10^{-6}}$

i.e., $\qquad \omega^2 - 6\times10^5\omega - 16\times10^{10} = 0$

i.e., $\qquad \omega = \frac{6\times10^5 \pm \sqrt{(6\times10^5)^2 + 64\times10^{10}}}{2}$

i.e., $\qquad \omega = \frac{6\times10^5 + 10\times10^5}{2} = 8\times10^5\,\frac{rad}{s}$.

Q.10 In a series RC circuit $R = 500\Omega, C = 2\mu F, V = 282\sin(377t)$. The power consumed is

(A) 14100 W (B) 141 W (C) 10 W (D) 14.1 W

Sol. $$P = \frac{V_p^2 R}{2Z^2} = \frac{282\times282\times500}{2\times\left(\sqrt{(500)^2 + \left(\frac{10^6}{2\times377}\right)^2}\right)^2} = \frac{282\times282\times500}{2\times1410\times1410} = 10\,W$$

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Q.11 The value of L, C and R in an LCR series circuit are 4 mH, 40 pF and $100\,\Omega$ respectively. The quality factor of the circuit is

(A) 10,000 (B) 100 (C) 1000 (D) 10

Sol. $\omega = \frac{1}{\sqrt{LC}}$ $Q = \frac{L\omega}{R} = \frac{L}{R\sqrt{LC}} = \frac{4\times10^{-3}}{100\sqrt{4\times10^{-3}\times40\times10^{-12}}} = 100$

Q.12 In an LR series AC circuit the angular frequency of applied emf is $2\times10^{4}\,\text{rads}^{-1}$ and the value of resistance is $20\,\Omega$. The instant at which the value of emf is maximum E_0, the value of current is $\frac{i_{max}}{\sqrt{2}}$. The inductance in the circuit will be

(A) 1 mH (B) 40 mH (C) 8 mH (D) cannot be predicted

Sol. $\phi = \pi/2 - \pi/4 = \pi/4,\ \tan\pi/4 = 1 = \frac{L\omega}{R}$ $\therefore$ L = 1 m H

Q.13 The self inductance of the motor of an electric fan is 10 Henry. In order to impart maximum power at 50 Hz it should be connected to a capacitance of

(A) 3×10^{-6} Farad (B) 2×10^{-6} Farad (C) 10^{-6} Farad (D) 10^{-4} Farad

Sol. For maximum power to be transferred

$$X_L = X_C \text{ or } L\omega = \frac{1}{C\omega}$$

$$\text{or } C = \frac{1}{L\omega^2} = \frac{1}{10\times(100\pi)^2} = 10^{-6}\,F$$

Q.14 A coil of resistance 200 ohms and self inductance 1.0 henry has been connected to an a.c. source of frequency $200/\pi$ Hz. The phase difference between voltage and current is :

(A) 30° (B) 63° (C) 45° (D) 75°.

Sol. $\tan\phi = \frac{X_L}{R} = \frac{2\pi fL}{R} = \frac{2\pi(200/\pi)1}{200} = 2$

$\therefore \quad \phi = 63^{\circ}$.

Q.15 The p.d. across an instrument in an a.c. circuit of frequency f is V and the current flowing through it is I such that V = 5 cos π ft (B) volt and I = 2 sin (2 π ft) amp. The power dissipate in the instrument is :

(A) zero (B) 10 watt (C) 5 watt (D) 2.5 watt.

Sol. As V = 5 cos π ft (2) =5 sin (2 π ft + $\pi/2$)

And I = 2 sin (2 π ft)

$\therefore$ phase difference between V and I is $\phi = \frac{\pi}{2}$

Average power $P = V_{rms} I_{rms} \times \cos\phi = 0$

Q.16 In an L-R circuit, the value of L is $(0.4/\pi)$ henry and the value of R is 30 ohm. If in the circuit, an alternating emf of 200 volt at 50 cycles per second is connected, the impedance of the circuit and current will be :

(A) 11.4 ohm, 17.5 ampere (B) 30.7 ohm, 6.5 ampere

(C) 40.4 ohm, 5 ampere (D) 50 ohm, 4 ampere.

Sol. Here $X_L = \omega L = 2\pi f L$

$$= 2\pi \times 50 \times \frac{0.4}{\pi} = 40\Omega$$

$$R = 30\,\Omega$$

$$\therefore \quad Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{50} = 4A.$$

Q.17 In an a.c. circuit, V & I are given by

V = 100 sin (100 t) volt.

I = 100 sin (100 t + $\pi/3$) mA.

The power dissipated in the circuit is :

(A) 10^4 watt (B) 10 watt (C) 2.5 watt (D) 5 watt.

Sol. $P = V_{rms} I_{rms} \cos\phi$

$$= \frac{100}{\sqrt{2}} \cdot \left(\frac{100}{\sqrt{2}} \times 10^{-3}\right) \cos 60^\circ$$

$$= \frac{10}{2} \times \frac{1}{2} = 2.5 \text{ watt}.$$

Q.18 An alternating voltage (in volts) varies with time t (in seconds) as V = 200 sin (100 πt)

(A) The peak value of the voltage is 200 V

(B) The rms value of the voltage is 220 V

(C) The rms value of the voltage is $100\sqrt{2}$ V

(D) The frequency of the voltage is 50 Hz

Sol. V = 200 sin (100 πt)

Compare this equation with $V = V_0 \sin\omega t$

$$V_0 = 200 \text{ V}, \quad V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 100\sqrt{2}$$

$$\omega = 50$$

Q.19 A 50 Ω electric heater is connected to 100 V, 60 Hz ac supply.

(A) The peak value of the voltatge is 100 V

(B) The peak value of the current in the circuit is $2\sqrt{2}$ A

(C) The rms value of the voltage is 100 V

(D) The rms value of the current is 2 A

Sol. $V_{rms} = 100$ V. Peak value of voltage $= 100\sqrt{2}$V . Peak value of current $= \frac{100\sqrt{2}}{50} = 2\sqrt{2}$ A

$$I_{rms} = \frac{2\sqrt{2}}{\sqrt{2}} = 2A$$

Q.20 L, C and R respectively represent inductance, capacitance and resistance. Which of the following combinations have the dimensions of frequency ?

(A) R/L (B) 1/RC

(C) $R/\sqrt{LC}$ (D) $1/\sqrt{LC}$

Sol. The dimensions of ωL and $1/\omega C$ are the same as those of resistance, where $\omega = 2\pi v$.

Q.21 In a series LCR circuit

(A) the voltage V_L across the inductance leads the current in the circuit by a phase angle of $\pi/2$

(B) the voltage V_C across the capacitance lags behind the current by a phase angle of $\pi/2$

(C) the voltage V_R across the resistance is in phase with the current

(D) the votage across the series combination of L, C and R is $V_{max} = V_{L\,max} + V_{C\,max} + V_{R\,max}$.

Sol. The voltage across the combination is given by $V = \left[(V_R)^2 + (V_C - V_L)^2\right]^{1/2}$.

Q.22 If $V = V_P \sin(\omega t + \pi/3)$ when will the voltage be maximum for the first time ?

(A) T/6 (B) T/12 (C) T/3 (D) None of these

Sol. $\sin(\omega t + \pi/3) = 1$ or $\omega t = \pi/6$

$$\text{or } \frac{2\pi t}{T} = \pi/6 \text{ or } t = \frac{T}{12} \text{ seconds}$$

Q.23 If $i = t^2$, $0 < t < T$ then rms value of current is

(A) $\frac{T^2}{\sqrt{2}}$ (B) $\frac{T^2}{2}$ (C) $\frac{T^2}{\sqrt{5}}$ (D) None of these

Sol. $i_{rms}\sqrt{\frac{1}{T}\int_0^T i^2 dt} = \frac{T^2}{\sqrt{5}}$

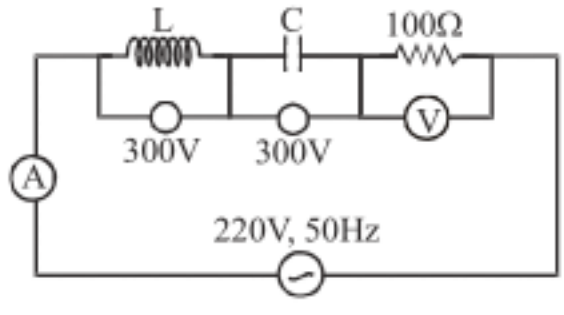
Q.24 In the circuit shown in figure, what will be the readings of voltmeter and ammeter ?

(A) 800 V, 2 A

(B) 220 V, 2.2 A

(C) 300 V, 2 A

(D) 100 V, 2 A.



Sol. As $V_L = V_C = 300\,V$,

and $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\therefore$ $V_R = V = 220\,V$

Also $I = \frac{V}{R} = \frac{220}{100} = 2.2$ A.

Q.25 An AC voltmeter in an LCR circuit reads 30 V across resistance 80 V across inductance and 40 V across capacitance. The value of applied voltage will be

(A) 25 V (B) 50 V (C) 70 V (D) 150 V

Sol. $V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{30^2 + 40^2} = 50V$

Q.26 In an AC circuit, a resistance of 3Ω, an inductance coil of 4Ω and a condenser of 8Ω are connected in series with an AC source of 50 V (rms). The average power loss in the circuit will be

(A) 300 W (B) 600 W

(C) 400 W (D) 500 W

Sol. $P = \frac{V_{rms}^2 \times R}{|Z|^2} = \frac{50^2 \times 3}{5^2} = 300W$

Q.27 The current flowing in a coil is 3 A and the power consumed is 108 W. If the a.c. source is of 120 V, 50 Hz, the resistance of the circuit is :

(A) 24Ω (B) 10Ω

(C) 12Ω (D) 6Ω.

Sol. From $P = I^2R, R = \frac{P}{I^2} = \frac{108}{3^2} = 12\Omega$

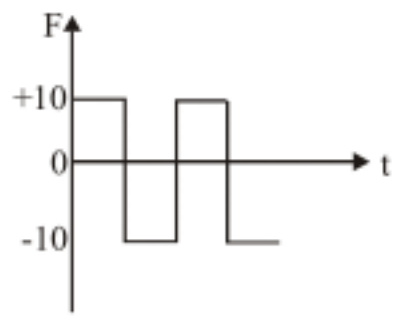
Q.28 If the phase difference between voltage and current is $\pi/6$ and the resistance in the circuit is $\sqrt{300}\,\Omega$, then the impedance of the circuit will be

(A) 40Ω (B) 20Ω

(C) 50Ω (D) 13Ω

Sol. $\cos\phi = \frac{R}{|Z|}$ or $\frac{\sqrt{3}}{2} = \frac{\sqrt{300}}{|Z|}$ or $Z = 20\ \Omega$

Q.29 The rms voltage of the wave form shown is



(A) 10 V (B) 7 V

(C) 6.37 V (D) None of these

Sol. $V_{rms} = \sqrt{\frac{1}{T}\int_0^T 10^2 dt} = 10V$

Question No. 30 to 32

A 100Ω resistance is connected in series with a 4H inductor. The voltage across the resistor is, $V_R = (2.0V)\sin(10^3 t)$.

Q.30 Find the expression of circuit current

(A) $(2\times10^{-2}A)\sin(10^3 t)$ (B) $(2\times10^{-3}A)\sin(10^2 t)$

(C) $(2\times10^{-3}A)\sin(10^3 t)$ (D) None of these

Q.31 Find the inductive reactance

(A) 2×10^3 ohm (B) 3×10^3 ohm

(C) 4×10^3 ohm (D) 5×10^3 ohm

Q.32 Find amplitude of the voltage across the inductor.

(A) 40 V (B) 60 V

(C) 80 V (D) 90 V

Sol. 30 to 32

30 $i = \frac{V_R}{R} = \frac{(2.0V)\sin(10^3 t)}{100} = (2.0\times10^{-2}A)\sin(10^3 t)$

31 $X_L = \omega L = (10^3)\times(4H) = 4.0\times10^3 \text{ ohm}$

32 Amplitude of voltage across inductor,

$V_0 = I_0 X_L = (2.0\times10^{-2}A)(4.0\times10^3 \text{ ohm}) = 80 \text{ volts}$

Question No. 33 to 35

If various elements, i.e., resistance, capcitance and inductance which are in series and having values 1000 Ω, 1μF and 2.0 H respectively. Given emf as, $V = 100\sqrt{2}\sin 1000t$ volts

Q.33 Voltage across the resistor is

(A) 70.7 Volts (B) 100 Volts

(C) 141.4 Volts (D) 270.7 Volts

Q.34 Voltage across the inductor is

(A) 70.7 Volts (B) 100 Volts

(C) 141.4 Volts (D) 270.7 Volts

Q.35 Voltage across the capacitor is

(A) 70.7 Volts (B) 100 volts

(C) 141.4 Volts (D) 270.7 Volts

Sol. 33 to 35

33 rms value of voltage across the source

$$V_{rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ Volt}$$

From question, $\omega = 1000 \text{ rad/s}$

$$i_{rms} = \frac{V_{rms}}{|Z|} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{rms}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{100}{\sqrt{(1000)^2 + \left(1000\times2 - \frac{1}{1000\times1\times10^{-6}}\right)^2}} = 0.0707 \text{ amp}$$

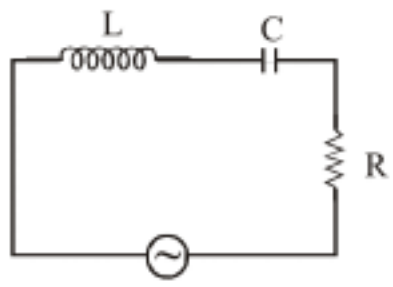
Since the curernt will be same every where in the circuit, therefore

P.D. across resistor $V_R = i_{rms}R = 0.0707\times1000 = 70.7$ volts

34 P.D. across inductor $V_L = i_{rms}XL = 0.0707 \times 1000 \times 2 = 141.4$ volt

35 P.D. across capalitor $V_C = i_{rms}X_C = 0.0707 \times \frac{1}{1 \times 1000 \times 10^{-6}} = 70.7$ volts

Q.36 Figure shows a series LCR circuit connected to a variable frequency 200 V source. L = 5 H, $C = 80\,\mu F$ and $R = 40\,\Omega$.



Column I	Column II
(A) The impedance of the circuit at resonance (in ohm)	(P) 1250
(B) The current amplitude at resonance (in A)	(Q) 200
(C) The rms potential drop across the inductor at resonance (in volt)	(R) 40
(D) The rms potential drop across the resistor at resonance (in V)	(S) $5\sqrt{2}$

Sol.

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

$\omega = \omega_r = 1/\sqrt{LC}$ (resonance)

$Z = R = 40\,\Omega$

$$I_0 = V_0 / Z = \frac{\sqrt{2}V_{rms}}{R} = 5\sqrt{2}\text{ A}$$

$$I_{rms} = \frac{V_{rms}}{R} = 5\text{ A}$$

The rms potential drop across L is 1250 V

The rms potential across R is $= I_{rms} \times R = 5 \times 40 = 200\text{V}$