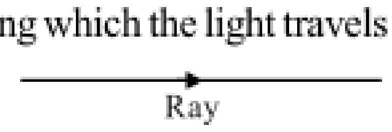
Reflection of light at plane surface

Some Basic terms

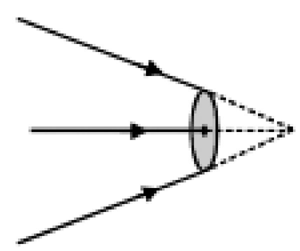
Ray : The straight line path along which the light travels in a homogeneous medium is called a ray.



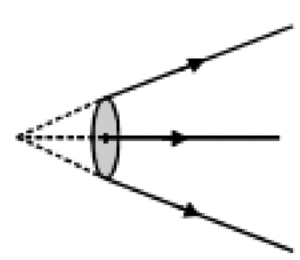
Ray

Beam of light : A bundle or bunch of rays is called a beam. It is of following three types :

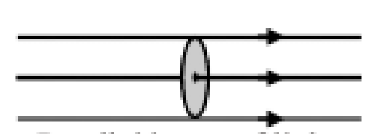
(a) Convergent beam : In this case diameter of beam decreases in the direction of ray.



(b) Divergent beam : It is a beam in which all the rays meet at a point when produced backward and the diameter of beam goes on increasing as the rays proceed forward.



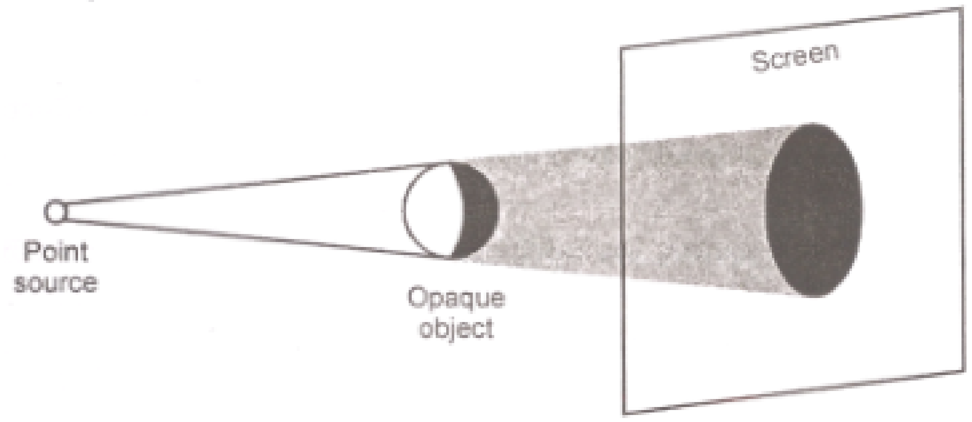
(c) Parallel beam : It is a beam in which all the rays constituting the beam move parallel to each other and diameter of beam remains same.



Parallel beam of light

Shadow formation

Shadow formation is explained by the law of rectilinear propagation of light which state that in a homogeneous medium light travels along straight paths. Thus, an opaque object placed between a point source of light and screen will cast a shadow with a sharply defined boundary.



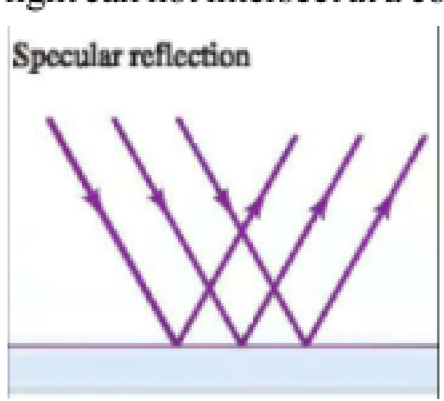
Reflection of light

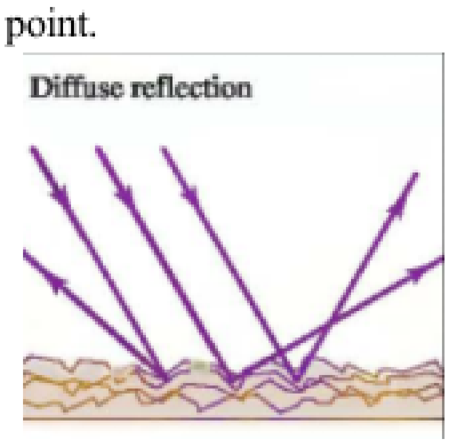
When a beam of light is incident on a boundary separating two media then some part of it may be transmitted and some is turned back in the medium from which it became incident (reflection of light).

Specular and diffuse reflection

The type of reflection which is usually invoked in a discussion of reflection at plane and spherical mirrors is known as specular or regular reflection. An incident parallel beam of light is reflected as a parallel beam in figure. The energy in the incident light is confined to one direction only on reflection.

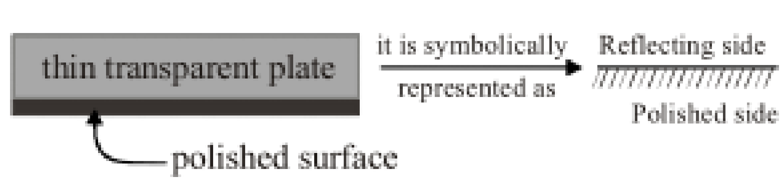
Diffuse or irregular reflection is the most common type of reflection and no image formation takes place as the reflected light can not intersect at a common point.



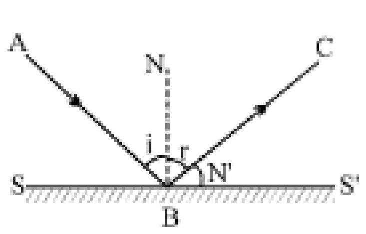


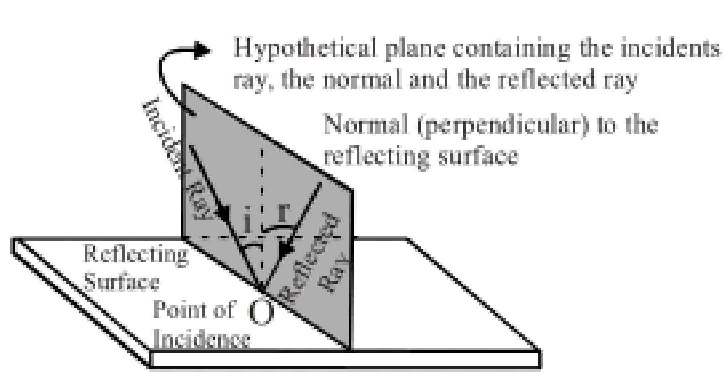
Plane Mirror

A highly polished smooth surface is a mirror. To form a good mirror a thin layer of silver is chemically deposited on a glass surface for high reflectivity.



Laws of Reflection





Reflection from a plane surface

(i) The incident ray (AB), the reflected ray (BC) and normal (NN') to the surface (SC') of reflection at the point of incidence (B) lie in the same plane. This plane is called the plane of incidence (also plane of reflection).

(ii) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal

$$\angle i = \angle r$$

Laws of Reflection in vector form

Let $\hat{e}_1$ = unit vector along incident ray

$\hat{n}$ = unit vector along normal

$\hat{e}_2$ = unit vector along reflected ray

Now $\vec{e}_{\parallel}$ = component of $\hat{e}$ parallel to mirror

$$= \hat{e}_1 - (\hat{e}_1 \cdot \hat{n})\hat{n}$$

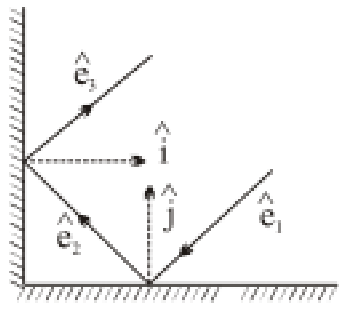
and $\vec{e}_{\perp}$ = component of $\hat{e}_1$ perpendicular to mirror

$$= (\hat{e}_1 \cdot \hat{n})\hat{n}$$

Hence $$\hat{e}_2 = \vec{e}_{\parallel} - \vec{e}_{\perp} = \hat{e}_1 - 2\hat{n}\,(\hat{e}_1 \cdot \hat{n})$$

Note :

Whenever reflection takes place, the component of incident ray parallel to reflecting surface remains unchanged, while component perpendicular to reflecting surface (i.e., along normal) reverses in direction.

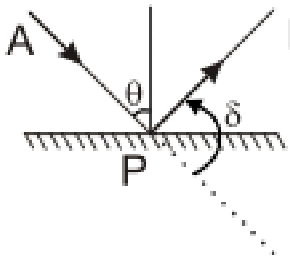


Consider incident ray along unit vector $\hat{e}_1$ given $\hat{e}_1 = -x\hat{i} - y\hat{j}$ unit vector along reflected ray will be given by $\hat{e}_2 = -x\hat{i} + y\hat{j}$ similarly $\hat{e}_3 = x\hat{i} + y\hat{j}$ diverge.

Principle of reversibility of light :

According to this principle if the path of light is reversed then it will retrace its path. i.e., if BP would incident ray then PA would be corresponding reflected ray.

Deviation produced by plane mirror :



$$\delta = 180^o - 2\theta$$

Angle of deviation produced by a single surface depends on the angle of incidence, Greater the angle of incidence lower will be the deviation and vice–versa.

Rotation of Mirror and Incident ray :

(i) If incident ray is rotated (Mirror is kept fixed) in the plane of incidence by angle θ then reflected ray rotates by the same angle in the same angle in plane of incidence but in opposite sense.

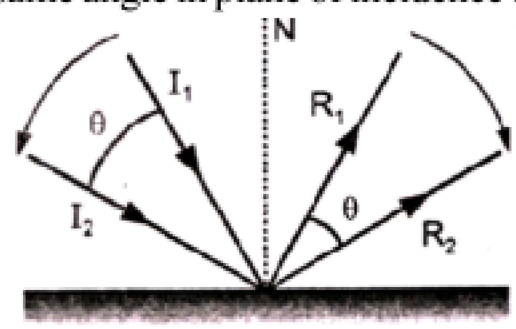


Fig. Rotation of incident ray

(ii) If mirror is rotated (taking position of incident ray same) by angle θ such that normal at the point of incidence rotates in the plane of incidence then reflected ray rotates by 2θ and in same sense.

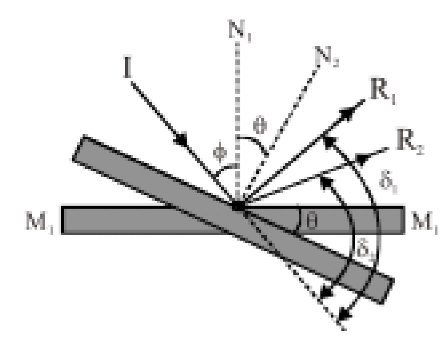
δ_1 = Deviation in first position of mirror

$= \pi - 2\phi$

δ_2 = Deviation in second position of mirror

$= \pi - 2\ (\theta + \phi)$

$\therefore \quad \delta_1 - \delta_2 = \pi - 2\phi\ \{\pi - 2\ (\theta + \phi)\} = 2\theta$

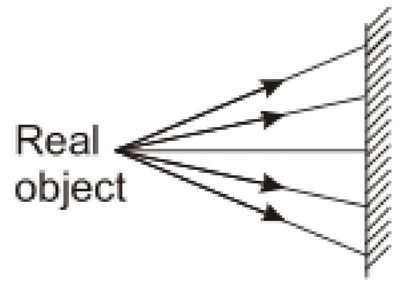


Rotation of plane mirror

Image formed by Plane mirror

Object : The point of intersection of incident beam is called point object.

Real object : If the incident beam is diverging then its intersection point is called real object . It can be seen by human eye and can be photographed by camera.



Virtual object point :

If the incident beam is converging then its intersection point is called virtual object. It cannot be seen by human eye and photographed by a camera.

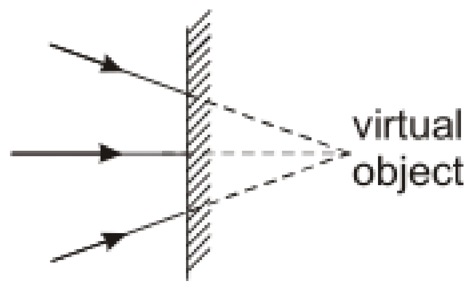
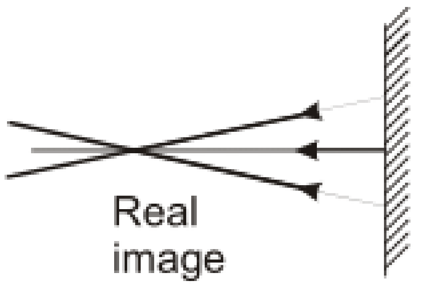


Image : The point of intersection of reflected or refracted beam is called image.

Real image: If the reflected or refracted beam is converging then its intersection point is called real image. It can be seen by eye, photographed by a camera and can be taken on screen.



Virtual image: If the reflected or refracted beam is diverging then its intersection point is called virtual image. It can be seen by eye, photographed by a camera but can't be taken on screen.

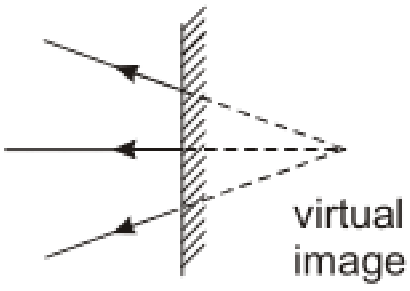
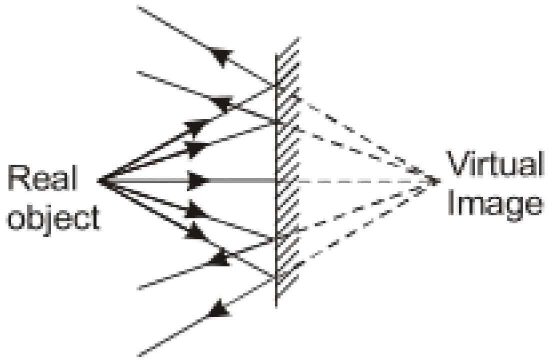
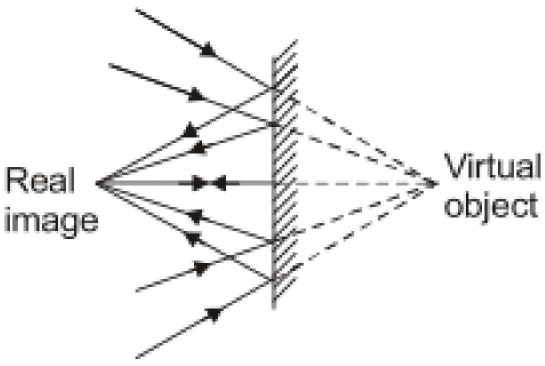


Image of a point object formed by plane mirror

Case : I For real object



Case : II For virtual object



Features :

(i) Distance of object from mirror = Distance of image from the mirror.

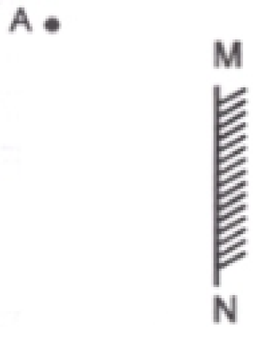
(ii) The line joining a point object and its image is normal to the reflecting surface.

(iii) The size of the image is the same as that of the object.

(iv) For a real object the image is virtual and for a virtual object the image is real.

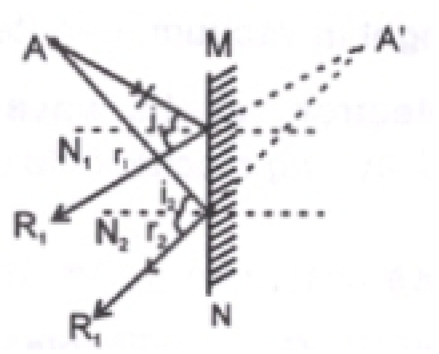
Illustration :

Figure shows a point object A and a plane mirror MN. Find the position of image of object A, in mirror MN, by drawing ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (This region is called field of view.)



Sol. *See figure, consider any two rays emanating from the object N_1 and N_2 are normals ;*

$i_1 = r_1$ and $i_2 = r_2$



The meeting point of reflected rays R_1 and R_2 is image A′. Though only two rays are considered it must be understood that all rays from A reflect from A reflect from mirror such that their meeting point is A′. To obtain the region in which reflected rays are present, join A′ with the ends of mirror and extend. The following figure shows this region as shaded. In figure there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q cannot see the image because they do not receive any reflected ray.

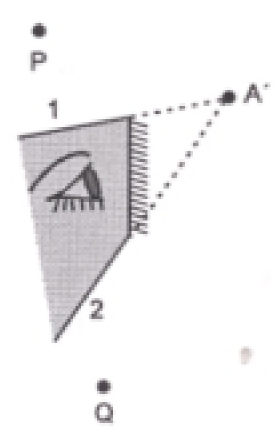
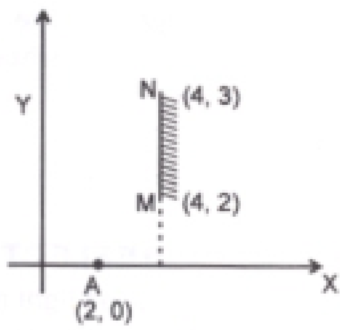


Illustration :

Find the region on Y axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown.

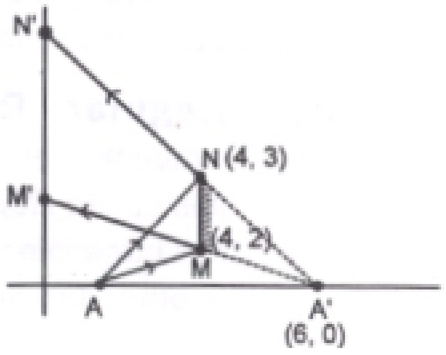


Sol. *The image of point A, in the mirror is at A′ (6, 0). Join A′M and extend to cut Y axis at M′ (Ray originating from A which strikes the mirror at M gets reflected as the ray MM′ which appears to come from A′). Join A′N and extend to cut Y axis at N′ (Ray originating from A which strikes the mirror at N get reflected as the ray NN′ which appears to come from A′).*

From Geometry.

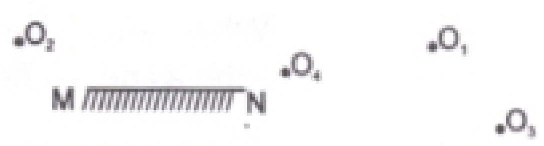
$M' \equiv (0, 6)$

$N' \equiv (0, 9)$. *M′N′ is the region on Y axis in which reflected rays are present.*



Practice Exercise

Q.1 See the following figure. Which of the object(s) shown in figure will not from its image in the mirror.



Answers

Q.1 O_3.

Motion of object and mirror :

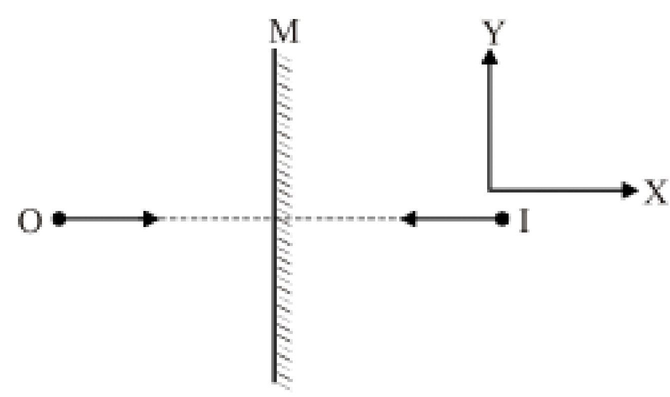
Let, $X_{O/m}$ = X co-ordinate of object w.r.t. mirror

$X_{I/m}$ = X co-ordinate of image w.r.t. mirror

$Y_{O/m}$ = Y co-ordinate of object w.r.t. mirror

$Y_{I/m}$ = Y co-ordinte of image w.r.t. mirror

For plane mirror



$$X_{o/m} = -X_{I/m}$$

Differentiating both sides w.r.t. 't'

$$\frac{d}{dt}(X_{o/m}) = \frac{-d}{dt}(X_{I/m})$$

$$[\vec{V}_{o/m}]_x = -[\vec{V}_{IX} - \vec{V}_{mX}]$$

$$\therefore \quad \vec{V}_{IX} = 2\vec{V}_{mX} - \vec{V}_{oX}$$

Similarly, $Y_{I/m} = Y_{o/m}$

Differentiating both side w.r.t. 't' we get

$$(\vec{V}_{I/m})_Y = (\vec{V}_{o/m})_Y$$

In nutshell, for sloving numerical problems involving calculation of velocity of image of object with respect to any observer, always calculate velocity of image first with respect to mirror using following points.

$$(\vec{V}_{I/M})_{||} = (\vec{V}_{o/m})_{||}$$

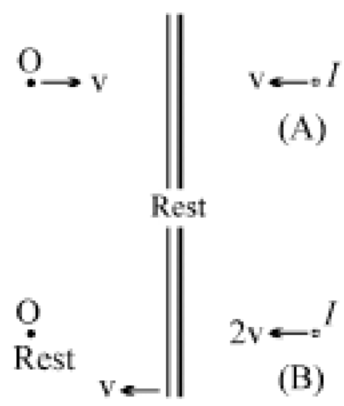
$$(\vec{V}_{I/M})_{\perp} = (\vec{V}_{O/M})_{\perp}$$

$$\vec{V}_{I/M} = (\vec{V}_{I/M})_{||} + (\vec{V}_{I/M})_{\perp}$$

Velocity of image with respect to required observer is then calculated using basic equation for relative motion.

$$\vec{V}_{A/B} = \text{Velocity of A with respect B}$$
$$= \vec{V}_A - \vec{V}_B$$

(i) If an object moves towards (or away from) a plane mirror at speed v, the image will also approach (or recede) at the same speed v, and the relative velocity of image with respect to object will be 2v, as shown in figure.



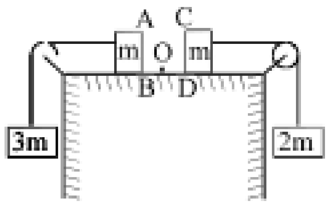
The object and its image move towards each other with same speed v.

The image of a stationary object moves with 2v if the mirror moves toward it with v

(ii) If the mirror is moved toward (or away from) the object with speed v, the image will also move toward (or away from) the object with a speed 2v, as shown in figure.

Illustration :

Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the figure. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB & CD of the two blocks are made reflecting. The acceleration of two images formed in those two reflecting surfaces w.r.t. each other is:



(A) $5g/6$ (B) $5g/3$ (C) $g/3$ (D) $17g/6$

Sol. *We know that*

$$V_I = 2V_m + V_0$$

differentiating $a_I = 2a_m + a_0$

$$a_0 = 0$$
$$a_I = 2a_m$$

$$a_A = \frac{3}{4}g \qquad a_C = \frac{2g}{3}$$

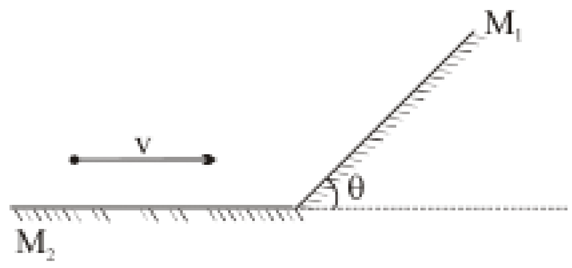
accelerate of image in AB $= 2a_A = \dfrac{3g}{2}$

accelerate of image in CD $= 2a_C = \dfrac{4g}{3}$

acceleration of image in AB w.r.t. that CD $= \dfrac{3g}{2} + \dfrac{4g}{3} = \dfrac{17g}{6}\ m/s^2$

Practice Exercise

Q.1 A point object is moving with a speed v before an arrangement of two mirrors as shown in figure. Find the velocity of image in mirror M_1 with respect to image in mirror M_2.



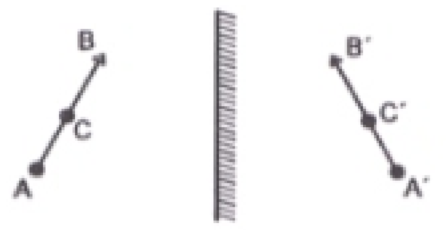
Q.2 A person walks at a velocity v in a straight line forming an angle α with the plane of a mirror. What is the velocity v_{rel} at which he approaches his image assuming that the object and its image are symmetric relative to the plane of the mirror?

Answers

Q.1 $2v\sin\theta$ Q.2 $2v\sin\alpha$

Image of extended object :

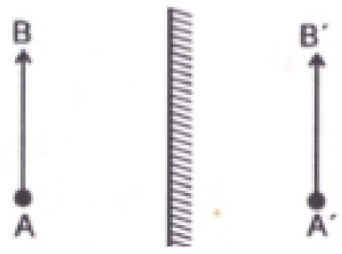
An extended object like AB shown in figure is combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed A′ will be images of A, C′ will be image of C, B′ will be image of B etc. All point images together form extended image. Thus extended image is formed of an extended object.



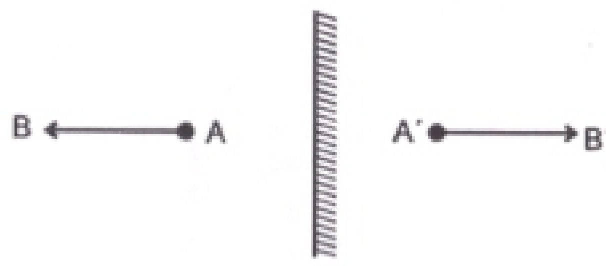
Properties of image of an extended object, formed by a plane mirror :

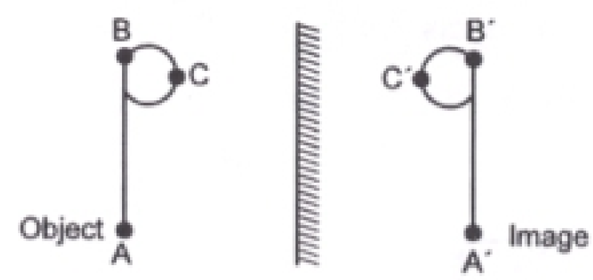
(1) Size of extended object = size of extended image.

(2) The image of erect, if the extended object is placed parallel to the mirror.

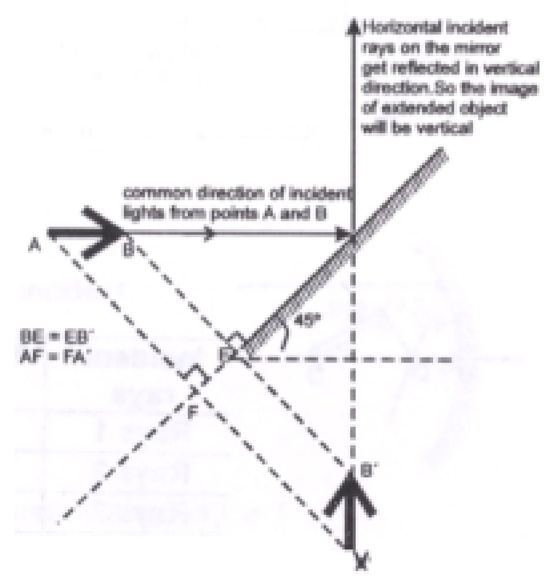


(3) The image is inverted if the extended object lies perpendicular the plane mirror.



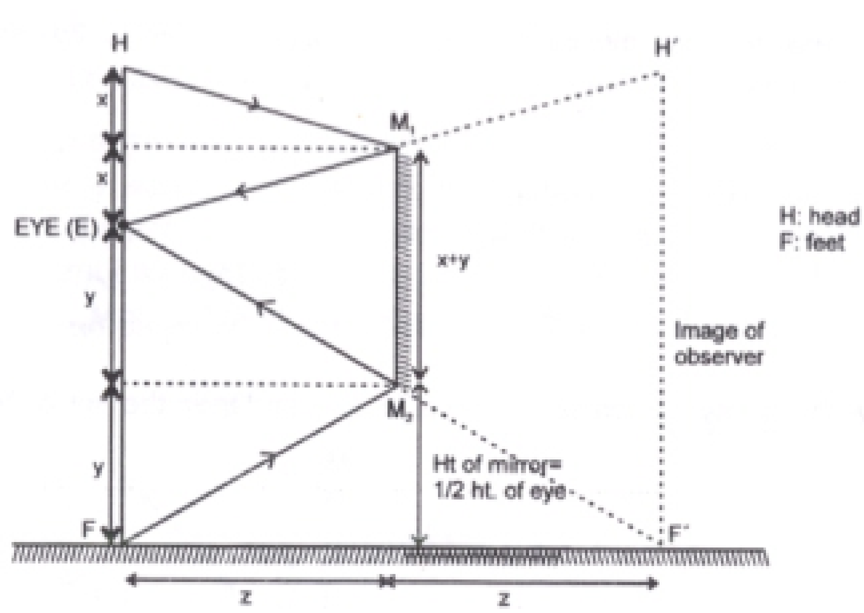


Note: If an extended horizontal object is placed infront of a mirror inclined 45° with the horizontal, the image formed will be vertical. See fig.



Calculation of minimum height of mirror :

(i) Minimum height of a single mirror required for a man to see its complete image



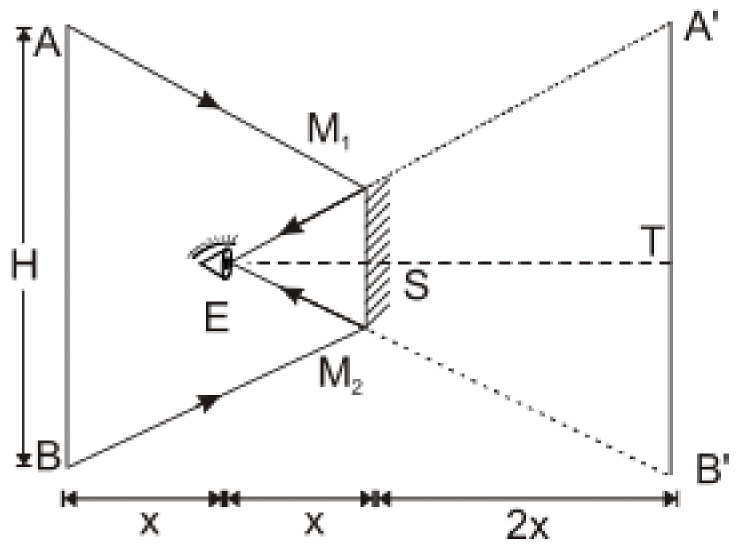
$\Delta E\, M_1, M_2$ and $\Delta E\, H'F'$ are similar

$$\therefore \quad \frac{M_1M_2}{H'F'} = \frac{z}{2z}$$

or $\quad M_1M_2 = H'F' / 2 = HF / 2$

$\therefore$ the minimum size of a plane mirror, required to see the full image of an observer is half the size of that observer.

(ii) Minimum height of a mirror required to see top as well as bottom of wall when man is mid of wall and mirror.



$\Delta E\, M_1, M_2$ and $\Delta E\, H'F'$ are similar

$\therefore \quad \dfrac{M_1M_2}{A'B'} = \dfrac{x}{3x}$

or $\quad M_1M_2 = \dfrac{A'B'}{3} = \dfrac{AB}{3} = \dfrac{H}{3}$

$\therefore$ the minimum size of a plane mirror, required to see the full image of an observer is one third the size of wall if observer is standing extactly between wall and mirror.

Reflection at two mirrors

Calculation of deviation :

In this case net derviation surffered by incident ray is algebraic sum of deviation due to individeula reflection.

$$\delta_{net} = \Sigma\delta_i$$

where δ_i = Devidation due to single reflection

Note : While summing up. sense of rotation is taken inot account

Illustration :

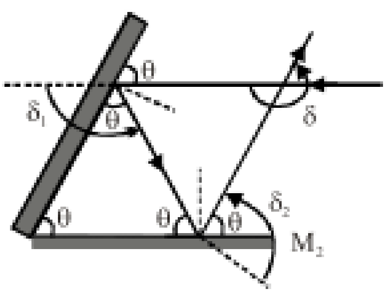
Two plane mirrors are inclined to each othere such that a ray of light incident on the first mirror and parallel to the second is reflected from the second mirror parallel to the first mirror. Determine the angle between the two mirror. Also determine the totaldeviation produced in the incidnet ray due to the two reflections.

Sol. *Form figure* $3\theta = 180^o$

$\theta = 60^o$

$\delta_1 = 180^o - 2 \times 30^o$

$= 120^o$



$\delta_2 = 180^o - 2 \times 30^o = 120^o$

$\therefore \quad$ *Total deviation* $= \delta_1 + \delta_2$

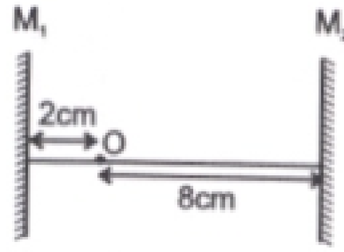
$= 240^o$

Formation of Multiple image by two parallel mirrors:

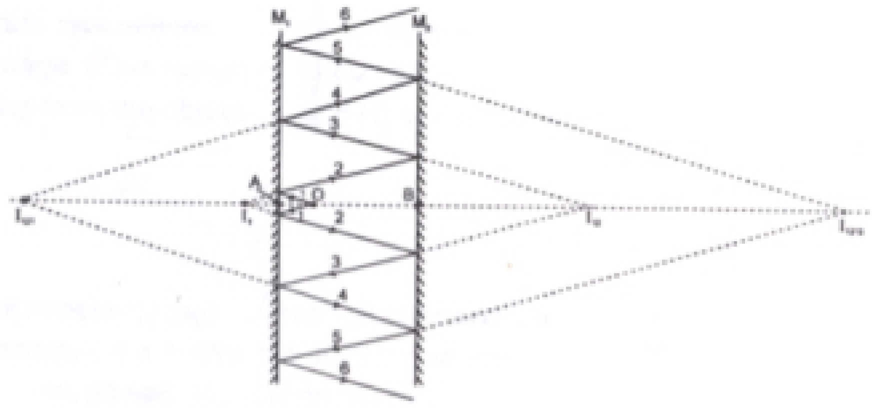
If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

Illustration :

Figure shows a point object placed between two parallel mirrors. Its distance from M_1 is 2cm and that from M_2 is 8 cm. Find the distance of image from the two mirrors considering reflection on mirror M_1 first.



Sol. *Let us start forming image from M_1. O is an object for M_1 which from image I_1 behind it. Now I_1 act an object for M_2 which from image I_{12} behind it. Again I_{12} act as object for M_1 and M_1 form image I_{121} behind it and so on. Here we get a series of image.*



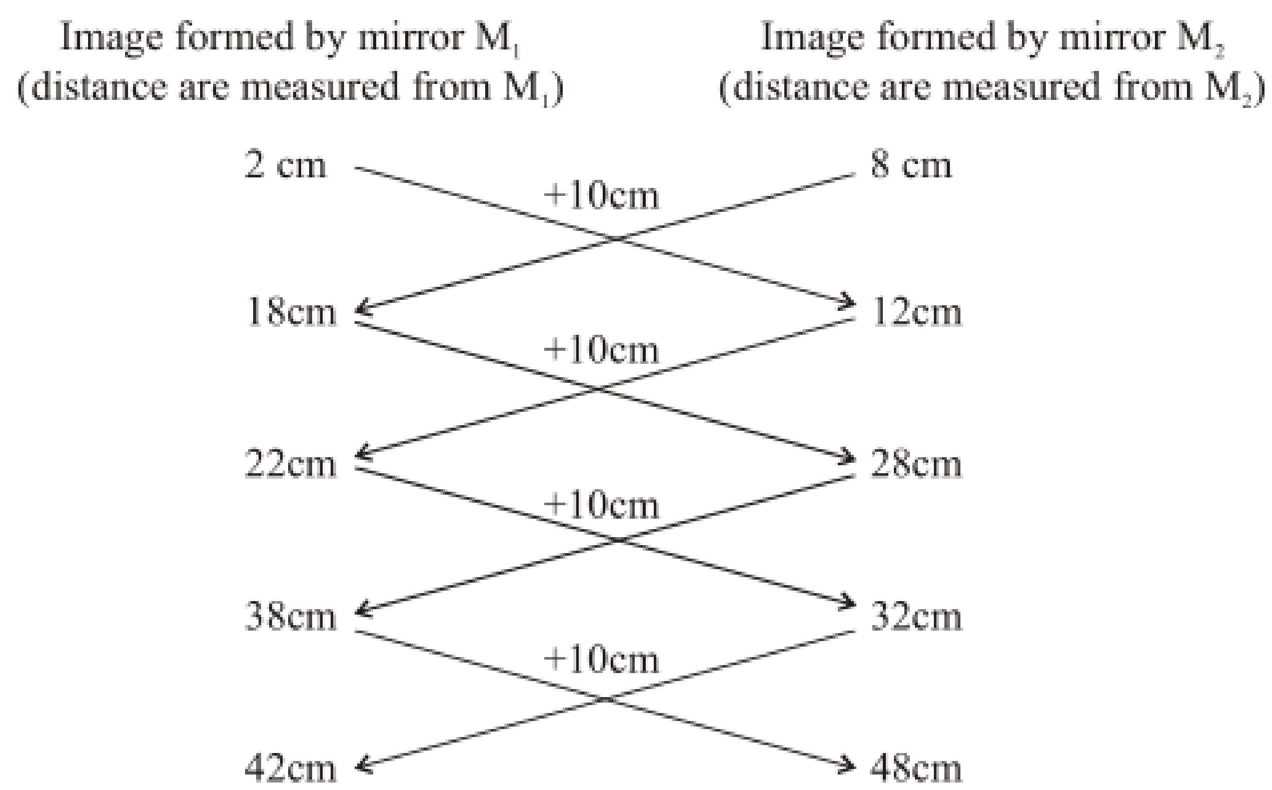
Incident Ray	Reflected by	Reflected ray	Object	Image	Object Distance	Image Distance
Ray 1	M_1	Ray 2	O	I_1	$AO = 2cm$	$AI_1 = 2cm$
Ray 2	M_2	Ray 3	I_1	I_{12}	$BI_1 = 12cm$	$BI_{12} = 12cm$
Ray 3	M_1	Ray 4	I_{12}	I_{121}	$AI_{12} = 22cm$	$AI_{121} = 22cm$
Ray 4	M_2	Ray 5	I_{121}	I_{1212}	$BI_{121} = 32cm$	$BI_{1212} = 32cm$

And so on ....

Similarly a series of images will be formed by the rays striking mirror M_2 first. Total number of image = ∞.

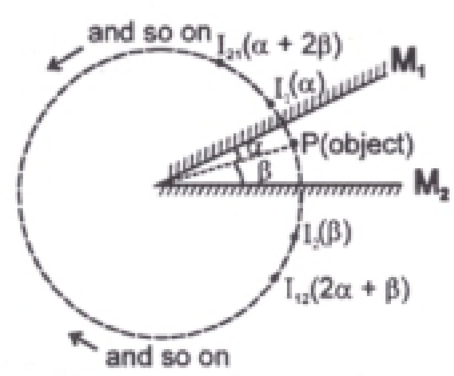
Incident Ray	Reflected by	Reflected ray	Object	Image	Object Distance	Image Distance
Ray 1	M_2	Ray 2	O	I_2	$BO = 8cm$	$BI_2 = 8cm$
Ray 2	M_1	Ray 3	I_2	I_{21}	$AI_2 = 18cm$	$AI_{21} = 18cm$
Ray 3	M_2	Ray 4	I_{21}	I_{212}	$BI_{21} = 28cm$	$BI_{212} = 28cm$
Ray 4	M_1	Ray 5	I_{212}	I_{2121}	$AI_{212} = 38cm$	$AI_{2121} = 38cm$

A scheme is given in which both series of images are covered.



Locating all the images formed by two inclined plane Mirrors

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in figure.



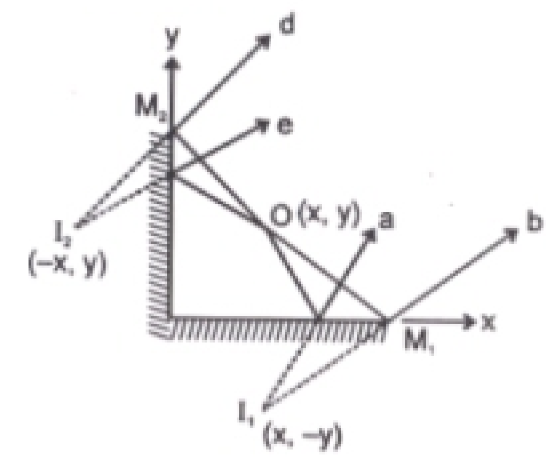
Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I1. Similarly image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I2.

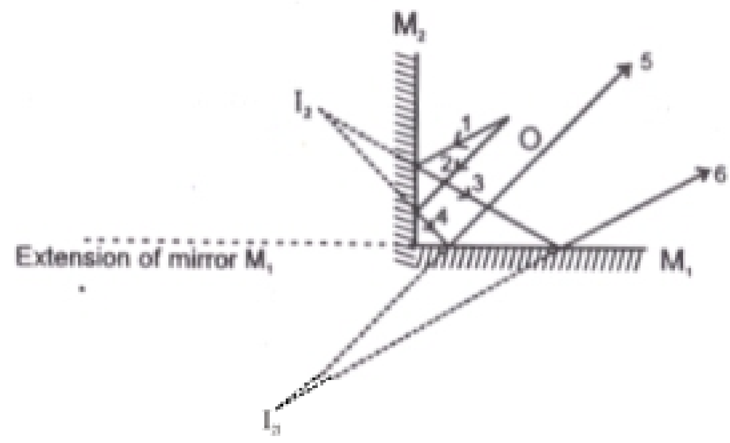
Now I_2 will act as an obect for M_1 which is at an angle $(\alpha + 2\beta)$ from M_1. Its image will be formed at $(\alpha + 2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21}, and so on. Think when hsi will process stop. Hint : The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

Illustration :

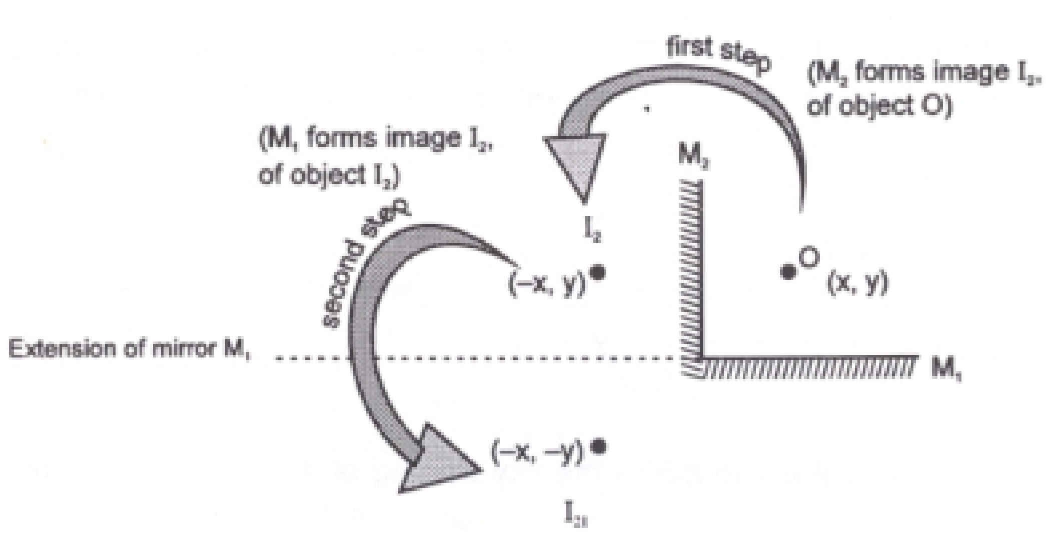
Consider two perpendicular mirrors. M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinate of object as (x, y), find the position and number of images.

Sol. *Rays 'a' and 'b' strike mirror M_1 only and these rays will from image I_1 at $(x, -y)$, such that O and I_1 are equidistant from mirror M_1. These rays do not form further image because they do not strike any mirror again. Similarly rays 'd' and 'e' strike mirror M_2 only and these rays will form image I_2 at $(-x, y)$, such that O and I_2 are equidistant from mirror M_2. Now consider those rays which strike mirror M_2 first and then the mirror M_1.*





For incident rays 1, 2 object is O, and reflected rays 3, 4 form image I_2.
Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.
I_2 and I_{21}, are equidistant from M_1. To summarize see the following figure.
Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5,6 do not strike any mirror, so image formation stops.



For rays reflecting first from M_1 and then from M_2, first image I_1 (at $(x, -y)$) will be formed and this will function as object for mirror M_2 and then its image I_{12} (at $(-x, -y)$) will be formed.

I_{12} and I_{21} coincide.

$\therefore$ *Three images are formed*

Shortcut :

When 360º is exactly divisible by θ.

Here two cases may arise

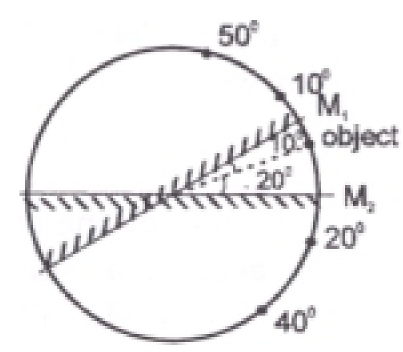
(a) If $\frac{360^o}{\theta}$ is even integer then number of images = $\frac{360^o}{\theta}-1$. What ever the location of object (symmetric or unsymmetric)

(b) If $\frac{360^o}{\theta}$ is odd integer then number of images = $\frac{360^o}{\theta}$ for unsymmetric placement

$=\frac{360^o}{\theta}-1$ for symmetric placement.

Illustration:

Two mirrors are inclined by an angle 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.



Sol. Figure is self explanatory

Number of images

(i) Using direct formula ; $\frac{360°}{30°}$ = 12 (even number)

∴ number of images = 12 – 1 = 11

(ii) By counting. See the following table

Image formed by Mirror M_1 (angles are measured from the mirror M_1.)	Image formed by Mirror M_2 (angles are measured from the mirror M_2.)
10°	20°
50°	40°
70°	80°
110°	100°
130°	140°
170°	160°
Stop because next angle will be more than 180°	Stop because next angle will be more than 180°

+30° +30° +30° +30° +30°

To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide. Here last angles made by the mirrors + the angle between the mirrors = 160° + 170° + 30° = 360°. Therefore in this case the last images coincide. Therefore the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 -1 (as the last images coincide)= 6 + 6 –1 = 11.

Practice Exercise

Q.1 Two plane mirrors forms an angle of 120°. The distance between the two image of a point source formed in them is 20 cm and the point source lies on the bisector of the angle formed by the mirrors. What is the distance of the light source from the point where the mirrors touch?

Q.2 To get three images of a single object, what should be the angle between two plane mirrors?

Q.3 Two plane mirrors are inclined at an angle of 60° to each other. If an object is placed between them,find the number of images produced.

Answers

Q.1 11.55 cm   Q.2 90°   Q.3 5

Reflection at spherical mirror

Aperture : The edge of a spherical mirror is a circle. Part of the plane of circle, enclosed by the circle is called its aperture.

Paraxial Ray: A light ray incident on the mirror at very small angle then the ray is called paraxial ray.

Marginal Ray: A light ray incident on the mirror at finite angle then the ray is called marginal ray.

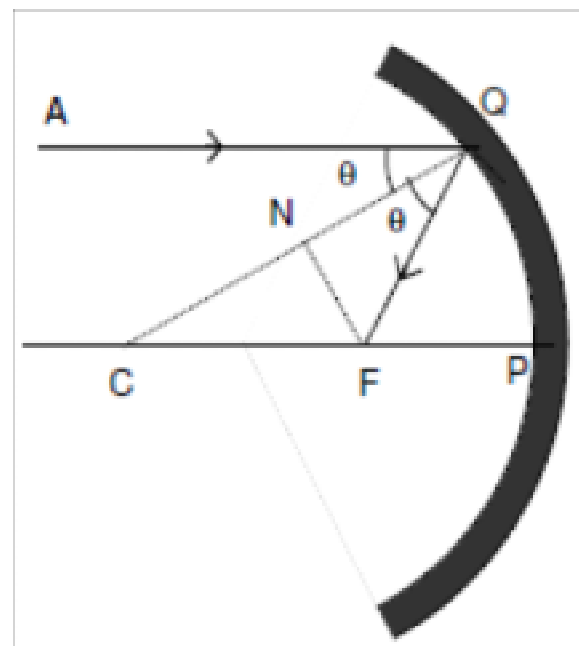
focus : Suppose a light ray AQ parallel to x axis become incident on a concave mirror at angle of incidence θ (fig). After reflection we have reflected ray QF at angle of reflection θ which intersects x axis at F. We want to calculate PF

In triangle CFQ

$$\angle QCF = \angle AQC = \theta \qquad \text{(alternate angle)}$$

$\Rightarrow$ triangle CFQ is an isoscless triangle (CF=QF).

$\Rightarrow$ CN=QN=CQ/2=R/2



In triangle NFQ

$$\text{COS}\theta = \frac{\text{QN}}{\text{QF}} = \frac{\text{R}/2}{\text{QF}}$$

$$\Rightarrow \qquad \text{QF} = \frac{\text{R}}{2\text{COS}\theta}$$

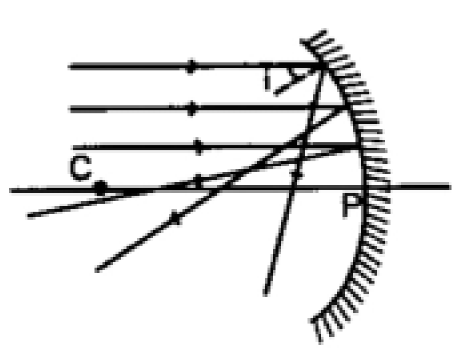
$$\Rightarrow \qquad \text{CF} = \text{QF} = \frac{\text{R}}{2\text{COS}\theta}$$

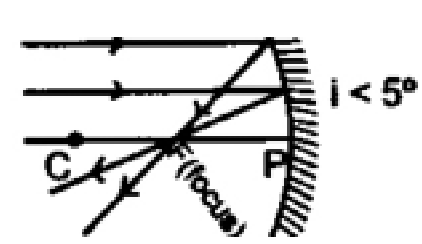
$$\therefore \qquad \text{PF} = \text{PC} - \text{CF} = \text{R} - \text{CF} = \text{R} - \frac{\text{R}}{2\text{COS}\theta}$$

For marginal rays θ is not small. Hence different light rays intersects x-axis at different points. But if we consider paraxial beam $(\theta \to 0)$

$$\Rightarrow \qquad \text{PF} = \frac{\text{R}}{2} \qquad (\text{As } \theta \to 0 \Rightarrow \cos\theta \to 1)$$

i.e all the light rays intersects x-axis at single point. This single point is called focus of the spherical mirror





As i increases cos i decreases.

Hence CQ increases

If i is a small angle cos i ≈ 1

$\therefore$ CQ = R/2

Principal axis : A line passing through focus and centre of curvature.

Pole : Point of intersection of principal axis and mirror.

Focal length (f) : The distance between focus and pole is called focal length .

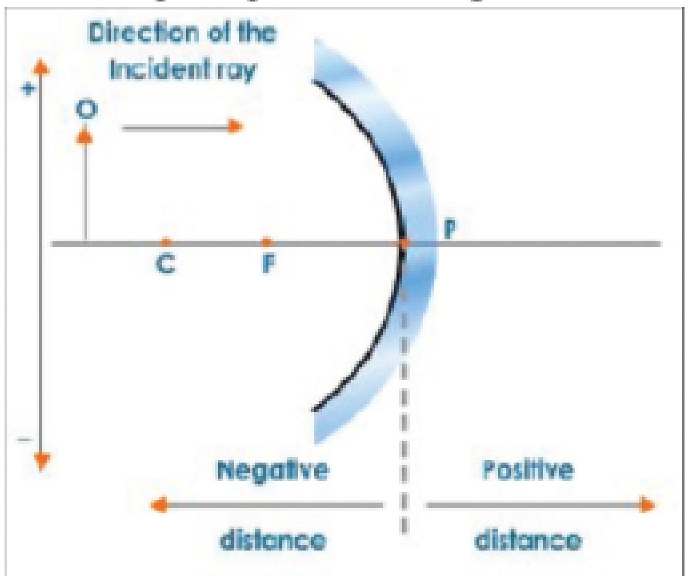
Sign Convention :

The following sign convention is used for measuring various distances in the ray diagrams of spherical mirrors:

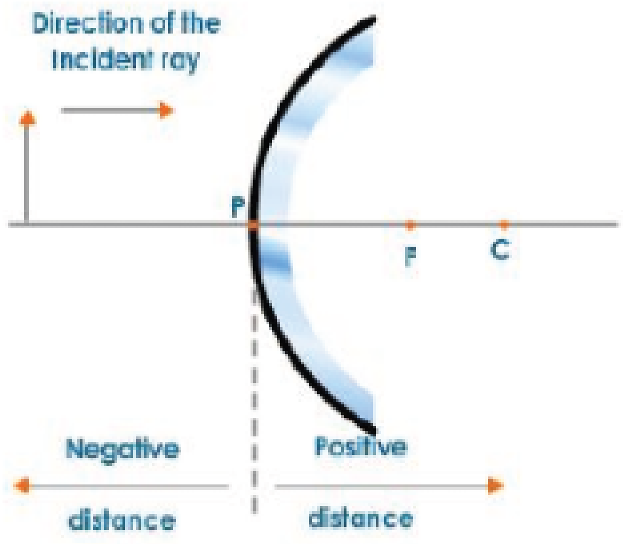
All distances are measured from the pole of the mirror.

Distances measured in the direction of the incident ray are positive and the distances measured in the direction opposite to that of the incident rays are negative.

Distances measured along y-axis above the principal axis are positive and that measured along y-axis below the principal axis are negative.



Concave Mirror

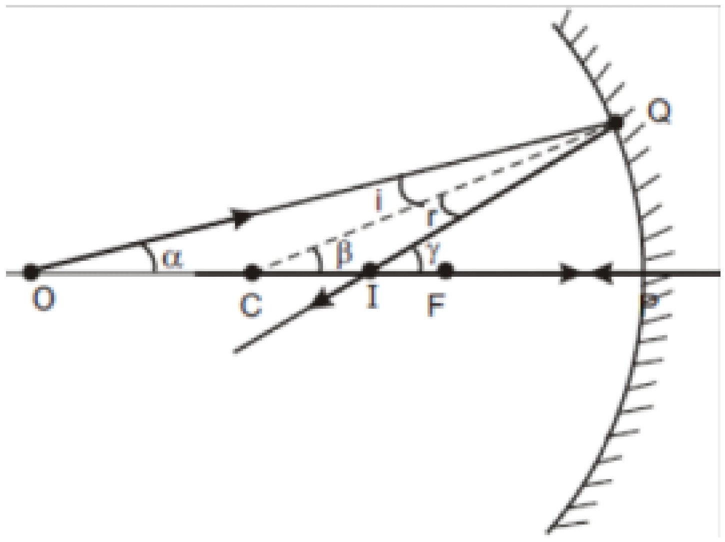


Convex Mirror

	Spherical Mirrors	Lenses
Focal Length	+ for concave mirrors	+ for a converging lens
	- for convex mirrors	- for a diverging lens
Object Distance	+ if object is in front of the mirror (real object)	+ if the object is to the left of the lens (real object)
	- if object is behind the mirror (virtual object)*	- if the object is to the right of the lens (virtual object)*
Image Distance	+ if the image is in front of the mirror (real image)	+ for an image (real) formed to the right of the lens by a real object
	- if the image is behind the mirror (virtual image)	- for an image (virtual) formed to the left of the lens by a real object
Magnification	+ for an image that is upright with respect to the object	+ for an image that is upright with respect to the object
	- for an image that is inverted with respect to the object	-for an image that is inverted with respect to the object.

Mirror formula

In this section we describe quantitatively where images are formed when light rays are reflected at spherical mirror. Consider two transparent media having indices a spherical mirror of radius R (Fig.). We assume that the object at O. Let us consider the paraxial rays leaving O. As we shall see, all such rays are reflected at the spherical surface and focus at a single point I, the image point.Figure shows a single ray leaving point O and reflecting to point I.



Now we use the fact that an exterior angle of any triangle equals the sum of the two opposite interior angles. Applying this rule to triangles OQC and CQI in Figure gives

$$i=\beta-\alpha \qquad\qquad r=\gamma-\beta$$

From law of reflection

$$i = r$$

$$\Rightarrow \quad \beta - \alpha = \gamma - \beta$$

$$\Rightarrow \quad \alpha + \gamma = 2\beta$$

$$\Rightarrow \quad \frac{QP}{OP}+\frac{QP}{IP}=\frac{QP}{CP} \qquad \text{(paraxial ray approximation)}$$

Taking sign convention

$$u = -OP \qquad\qquad v = -IP \qquad\qquad R = -CP$$

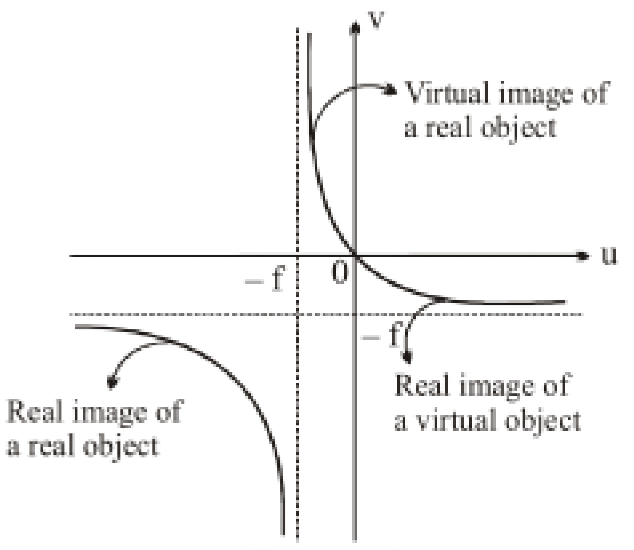
we get

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

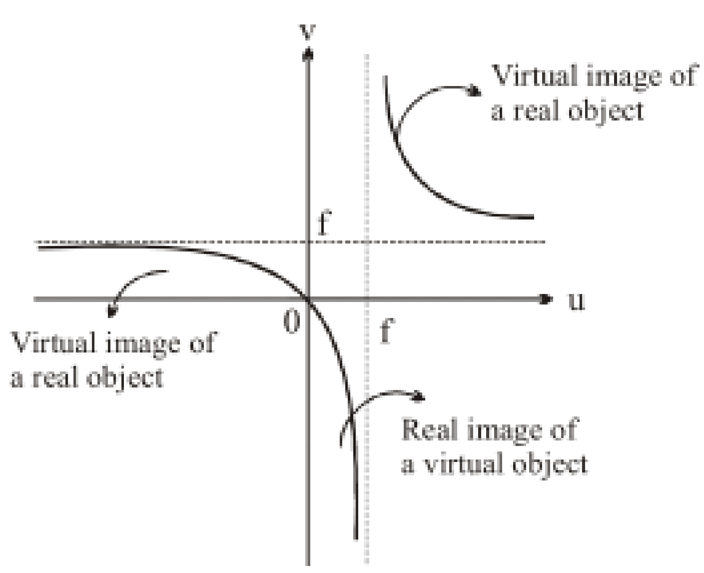
Note : This relation is same in all cases no matter object or image point is real or virtual, mirror is concave or convex.

Graph : v vs u :

(a) For concave mirror **(b) For convex mirror**



Concave mirror



Concave mirror

Image Tracing for Transverse Extended Object

In tracing image of a transverse extended object we should keep in mind following :

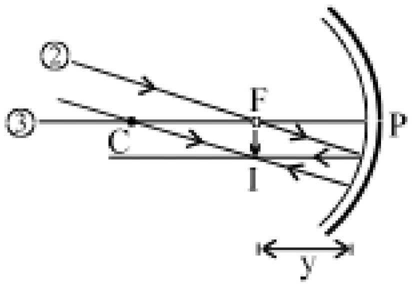
(1) A ray parallel to principal axis after reflection from the mirror passes or appears to pass through its focus

(2) A ray passing through or directed towards centre of curvature, after reflection from the mirror, retraces its path (as for it $\theta_1 = 0$ and so $\theta_2 = 0$).

(3) Ray drawn from the top of the object toward the focal point on the back side of the mirror and is reflected parallel to the principal axis.

Image Tracing in some cases

(i) When the object is placed at infinity, a real, inverted and very small image is formed at the focus.

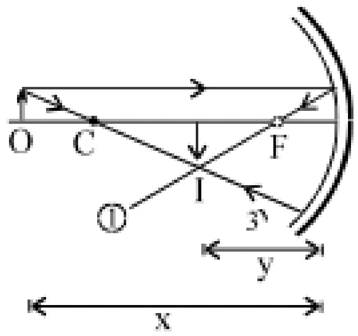


For a distance object image is formed at the focus

$x = \infty$

$v = -y$	where	$y = f_0$
$m = -\delta$	where	$\delta << 1$

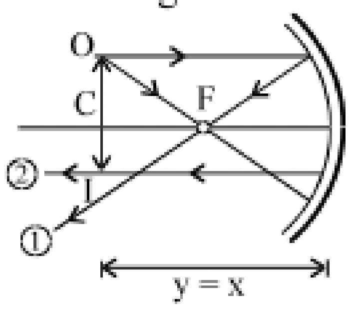
(ii) When the object is placed beyond C ($2f_0 < x < \infty$), a real, inverted and diminished image is formed between F acid C.



For an object placed beyond C, image is formed between C and F

$v = -y$	where	$f_0 < y < 2f_0$
$m = -\delta$	where	$0 < \delta < 1$

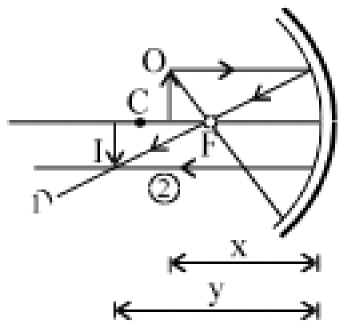
(iii) When the object is placed at C.
($x = 2f_0$), a real, inverted and equal size image is formed at C.



At C both object and image coincide

$v = -y$	where	$y = 2f_0$
$m = -\delta$	where	$\delta = 1$

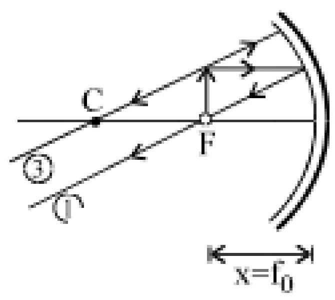
(iv) When the object is placed between F and C ($f_0 < x < 2f_0$), a real, inverted and large image is formed beyond C.



For an object placed between F and C image is formed beyond C.

$v = -y$	where	$y < 2f_0$
$m = -\delta$	where	$\delta > 1$

(v) When the object is placed at focus F, a real, inverted and very large image is formed at infinity.

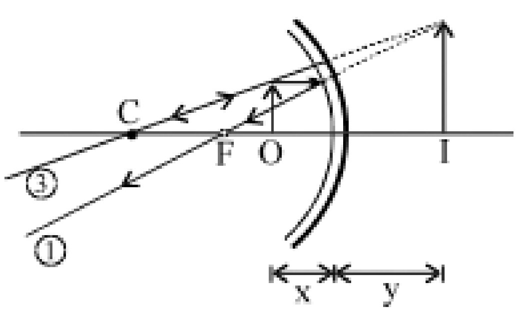


For an object placed at focus, image is formed at infinity

$v = -y$	where	$y = \infty$
$m = -\delta$	where	$\delta >> 1$

Note virtual object

When the object is placed between F and P, a virtual, erect and enlarged image is formed behind the mirror.



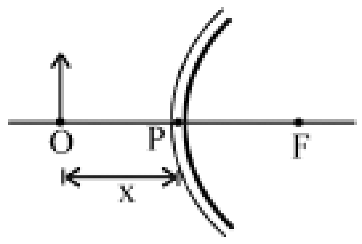
A virtual image is formed for an object placed within focus

$v = +y$

$m = +\delta$ where $\delta > 1$

Convex mirror

The fig. shows a convex mirror of focal length f_0 in front of which an object O is placed at a distance x from the pole P.



An object O placed in front of a convex mirror

According to Cartesian sign convention, the formulae may be modified as

$$u = -x \quad \text{and} \quad f = +f_0$$

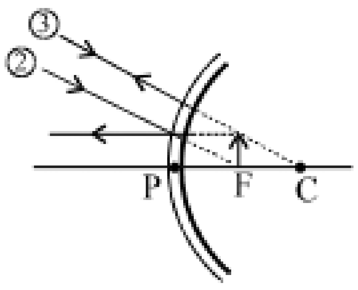
Thus $\quad v = \dfrac{xf_0}{f_0 + x}$

The above expression shows that whatever may be the value of x, v is always positive and its value is always less than or equal to f_0.

The magnification formula may be modified as

$$m = \frac{f_0}{f_0 + x}$$

When the object is placed at infinity, a virtual, erect and very diminished image is formed at the focus.



For a distance object image is formed at the focus

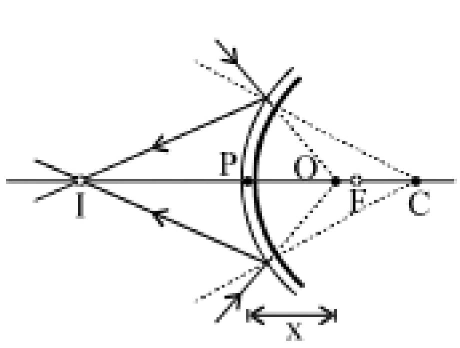
$x = \infty \qquad v = f_0 \qquad m << 1$

Illustration :

Can a convex mirror form real images ?

Sol. *Yes, only when the object is virtual and is placed between F and P.*

The fig. shows a convex mirror exposed to a converging beam which converges to a point that lies between F and P.



A real image formed by convex mirror

$$v = \frac{-xf_0}{f_0 - x}$$

v becomes negative (real image) only when $x < f_0$.

FOR CONCAVE MIRROR

Position of object (real)	Position of image	Characteristics of image
At infinity	At. F	Real, inverted, highly diminished
Between infinity and C	Between C and F	Real, inverted, diminished
At C	At C	Real, inverted, same size as that of object
Between C and F	Between infinity and C	Real, inverted, magnified
At F	At infinity	Real, inverted, highly magnified
within F	Behind the mirror	Virtual, erect, magnified

FOR CONVEX MIRROR

Position of object (real)	Position of image	Characteristics of image
At infinity	At F	Virtual, erect, highly diminished
At a finite distance	Between P and F	Virtual, erect, diminished

Transverse or Lateral or linear magnification

It is defined as

$$m_T = \frac{\text{height of image}}{\text{height of object}} = \frac{h_I}{h_O}$$

After using geometry we get

$$m_T = -\frac{v}{u} = \frac{f}{f-u}$$

Note :

Sign of m_T states orientation of image w.r. to object and its magnitude compares size of image with size of object

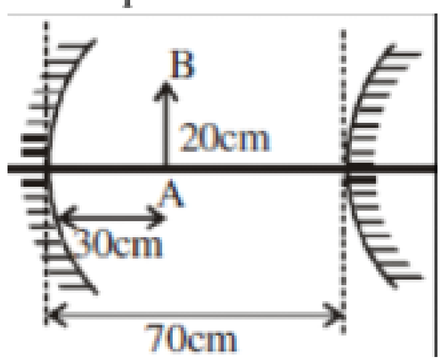
Newtonian formula for mirrors

here distances are measured from focus

$$uv = f^2 \quad \& \qquad m_T = -\frac{f}{u}$$

Illustration :

A concave and convex mirror of focal length 10 cm and 15 cm are placed at distance 70 cm. An object AB of height 2 cm is placed at distance 30 cm from concave mirror. First ray is incident on concave mirror then on convex mirror. Find size position and nature of image.



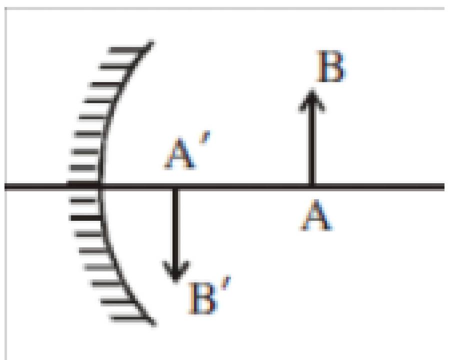
Sol. For concave mirror,

$$u=-30\text{cm},\quad f=-10\text{cm}$$

Using

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{30}=\frac{-1}{10} \Rightarrow \qquad v=-15\,cm$$



Now,

$$\frac{A'B'}{AB}=\frac{-V}{u}=\frac{(-15)}{(-30)} \quad \Rightarrow \quad A'B'=-1\,cm$$

Image formed by first reflection will be real inverted and diminished

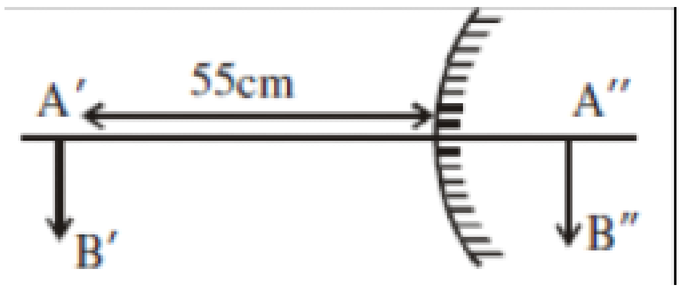
For convex mirror

$$u'=-55\text{cm},\quad f'=+15\text{cm}$$

Using

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v'}-\frac{1}{55}=\frac{1}{15} \quad \Rightarrow \quad v'=\frac{165}{14}\,cm$$



Now,

$$\frac{A''B''}{A'B'}=-\frac{V'}{u'}=-\frac{\left(+\frac{165}{14}\right)}{(-55)} \Rightarrow \qquad A''B''=\left(-\frac{3}{14}\right)(-1)=-0.2\ \ cm$$

Image for Longitudinal Object

If an object is placed along principal axis then it is called longitudinal object.

If object is of very small size then longitudinal magnification is defined as.

$$m_L=\frac{length\ of\ image}{length\ of\ image}=\frac{dv}{du}$$

Now using

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

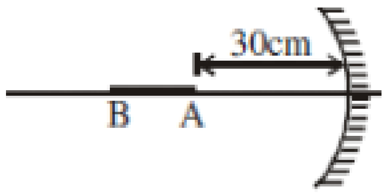
Differentiating w.r.to. u we get

$$\frac{dv}{du}=-\frac{v^2}{u^2}=-m_T^2$$

$$\Rightarrow \quad m_L=-m_T^2$$

Illustration :

*An object AB is placed on the axis of concave mirror of focal length 10 cm end A of the object is at 30 cm from mirror. Find the length of the image **(a)** If length of object is 5 cm **(b)** If length of object is 1 mm*



Sol. *For point A,* $u=-30\text{cm}, \quad f=-10\text{cm}$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v'}-\frac{1}{30}=-\frac{1}{10} \Rightarrow \quad v=-15\text{cm}$$

Similarly for point B $u=-35\text{cm}$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

we will get $v'=-14\text{cm}$

Now size of image $|A'B'|=|v-v'|=|(-15)-(-14)|=1\text{cm}$

(b) *here* $u=-30\text{cm}, \ f=-10\text{cm}$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v'}-\frac{1}{30}=-\frac{1}{10} \Rightarrow \quad v=-15\text{cm}$$

Now, $\frac{dv}{du}=-\frac{(-15)^2}{(-30)^2} \Rightarrow \quad |dv|=\frac{(15)^2}{(30)^2}|du|$

$$\Rightarrow \quad |dv|=\left(\frac{225}{900}\right)(10^{-3})=2.5\times10^{-4}\ m$$

i.e.the length of the image is 2.5×10^{-4} m.

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Relation between velocity of object and image if object is moving on principal axis:

we can write

velocity of object relative to mirror= $v_{O/M} = \dfrac{du}{dt}$

velocity of image relative to mirror= $v_{I/M} = \dfrac{dv}{dt}$

Now using

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Differentiating w.r.to. t we get

$$\frac{dv}{dt}=-\frac{v^2}{u^2}\frac{du}{dt}$$

$$\Rightarrow \quad v_{I/M}=m_L v_{O/M}$$

Relation between velocity of object and image if object is moving perpendicular to principal axis:

we can write

velocity of object relative to mirror = $v_{O/Mirror} = \dfrac{dh_O}{dt}$

velocity of image relative to mirror = $v_{I/Mirror} = \dfrac{dh_I}{dt}$

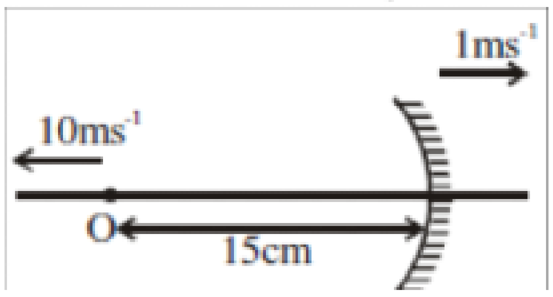
dividing we get

$$\frac{v_{I/Mirror}}{v_{O/Mirror}}=\frac{dh_I}{dh_O}=\frac{h_I}{h_O}=m_T$$

$$\Rightarrow \quad v_{I/Mirror}=m_T v_{O/Mirror}$$

Illustration :

A mirror of radius of curvature 20 cm and an object which is placed at distance 15 cm both are moving with velocity 1 ms^{-1} and 10 ms^{-1} as shown in figure. Find the velocity of image.



Sol. Using

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{15}=-\frac{1}{10} \quad \Rightarrow \quad v=-30\text{cm}$$

Now, using

$$V_{im}=-\frac{v^2}{u^2}V_{OM}$$

$$\Rightarrow \quad (V_i - V_m) = -\frac{v^2}{u^2}(V_0 - V_m)$$

$$\Rightarrow \quad V_i - (1) = -\frac{(-30)^2}{(-15)^2}[(-10)-(1)]$$

$$\Rightarrow \quad V_i = 45\,\text{cm/s}$$

Practice Exercise

Q.1 A beam of light converges towards a point O, behind a convex mirror of focal length 20 cm. Find the nature and position of image if the point O is (a) 10 cm behind the mirror (b) 30 cm behind the mirror.

Q.2 A plane mirror is placed 22.5 cm in front of a concave mirror of focal length 10 cm. Find where an object can be placed between the two mirrors, so that the first image in both the mirror coincides.

Q.3 A concave mirror of focal length f produces a real image n times the size of the object. What is the distance of the object from the mirror?

Q.4 The focal length of a concave mirror is 30 cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

Q.5 A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole. What is the size of the image?

Q.6 If a luminous point is moving at a speed V_0 towards a spherical mirror, along its axis, show that the speed at which the image of this object is moving will be given by :

$$V_i = \left[\frac{f}{u-f}\right]^2 V_0$$

Answers

Q.1 (a) –20 (b) +60 Q.2 15 cm from the concave mirror Q.3 (n+1)f/n

Q.4 Case I If the image is inverted (i.e., real) : 40 cm in front of the mirror.
Case II If the image is erect (i.e., virtual) : 20 cm in front of the mirror

Q.5 $b\left[\frac{f}{(u-f)}\right]^2$

Refraction of Light At plane Surface

Refractive Index

The refractive index (denoted by μ or n) of a material for a given colour at light is defined as

$$\mu = \frac{c}{v}$$

Where c = speed of light in vacuum and

v = speed of light in the medium for that colour of light.

Note :

(i) $\mu_{vac} = 1$

(ii) We can also say refractive index as absolute refractive index.

(iii) The refractive index of a material depends on wavelength of light and given by (Cauchy's equation)

$$\mu_\lambda^2 = \mu_0 + \frac{A}{\lambda_0^2}$$

Where μ_λ = refractive index of a material for light ray of wave length λ.

μ_0 = constant depending on nature of material.

A = constant depending on nature of material.

It is well known that in visible range the wavelength violet light is minimum of red light is maximum. So using Cauchy's equation we can say that in visible range the refractive index of a material is maximum for violet and minimum for red and hence (i) speed of light in a material is maximum for red and minimum for violet in visible range (ii) deviation of violet ray is maximum and red colour of light ray is minimum in visible range for same material and same angle incidence.

(iv) When light passes from one medium to another then speed and wavelength changes such that frequency remains unchanged (hence colour also).

$$\lambda = \frac{\lambda_0}{\mu}$$

(v) **Relative Refractive Index :**

The ratio μ_2/μ_1 is called refractive index of second medium relative to first and represented by μ_{21} or ${}_1\mu_2$.

${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$ (for given colour of light and different two media). Note : ${}_1\mu_2 = \frac{1}{{}_2\mu_1}$

Illustration:

How long will the light take in travelling a distance of 500 metre in water? Given that μ for water is 4/3 and the velocity of light in vacuum is 3×10^{10} cm/sec.

Sol. *We know that*

$$\mu = \frac{\text{velocity of light in vaccum}}{\text{velocity of light in water}}$$

$$\frac{4}{3} = \frac{3\times10^{10}}{\text{velocity of light in water}}$$

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or velocity of light in water $= 2.25 \times 10^{10}$ cm/sec.

Time taken $= \dfrac{500 \times 100}{2.25 \times 10^{10}} = 2.22 \times 10^{-6}$ sec.

Illustration :

(a) Find the speed of light of wave length $\lambda = 780$ nm (in air) in a medium of refractive index $\mu = 1.55$

(b) What is the wavelength of this light in the given medium ?

Sol. (a) $v = \dfrac{c}{\mu} = \dfrac{3.0 \times 10^8}{1.55} = 1.94 \times 10^8$ m/s (b) $\lambda_{medium} = \dfrac{\lambda_{air}}{\mu} = \dfrac{780}{1.55} = 503$ nm

Practice Exercise

Q.1 Light of wavelength 6000Å enters from air into water [a medium of index of refraction(4/3)]. What is the (a) speed (b) wavelength (c) frequency and (d) colour of light in water? [Speed of light in free space is 3×10^8 m/s]

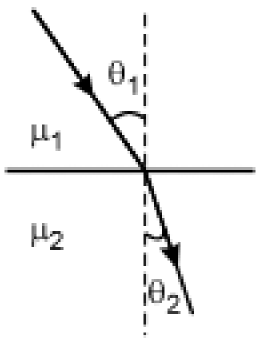
Q.2 If the wavelength of light in vacuum is 5800 Å, then calculate the wavelength in glass [${}_a\mu_g = 3/2$]

Answers

Q.1 (a) 2.25×10^8 m/s, (b) 4500Å (c) 5×10^{14} Hz and (d) yellow Q.2 3866A

Refraction of Light

If a light ray passes from one transparent medium to another medium but having oblique incidence then it deviates from its path (either towards normal or away from the normal) This bending of light ray due to change of medium is called refraction of light.



Laws of Refraction

Ist Law : Incident ray, refracted ray and normal at the point of incidence are coplanar i.e. refracted ray must lie in the plane of incidence.

IInd Law (Snell's law) : The ratio of sine of angle of incidence with sine of angle of refraction is constant for given two media and given colour of light which is equal to refractive index of second medium relative to first for that colour of light.

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

In vector form :

$$\mu_1(\hat{u}_i \times \hat{u}_n) = \mu_2(\hat{u}_r \times \hat{u}_n)$$

Deductions :

(1) (a) If there is no change of medium light ray passes undeviated.

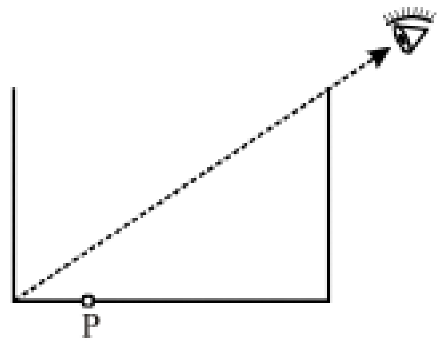
(b) If light ray incident normally then transmitted light ray is also normal.

(c) If light ray passes from rarer to denser then it bends towards the normal.

(d) If light passes from denser to rarer then it bends away from the normal.

Illustration :

A cylindrical vessel, whose diameter and height both are equal to 30cm is placed on a horizontal surface and a small particle P is placed in it at a distance of 5.0 cm from the centre. An eye is placed at a position such that the edge of the bottom is just visible. The particle P is in the plane of drawing. Up to what minimum height should water be poured in the vessel to maks the particle P visible ? Given : refractive index of water = 4/3.



Sol. *From figure*

$$x = 30 - h$$

$$PA = 20 - x$$

$$= 20 - (30 - h) = h - 10$$

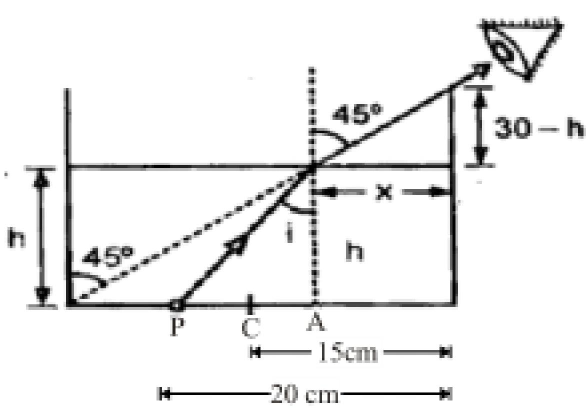
Using Snell's law we get

$$i \times \sin r = \frac{4}{3} \sin i$$

$$1 \times \sin 45^o = \frac{4}{3} \times \frac{h-10}{\sqrt{h^2 + (h-10)^2}}$$

$$23\,(h - 10)^2 = 9h^2$$

$$h = 26.7\ cm$$

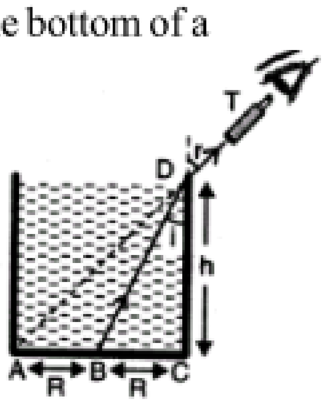


Practice Exercise

Q.1 If one face of a prism of prism angle 30° and $\mu = \sqrt{2}$ is silvered, the incident ray retraces its initial path. What is the angle of incidence?

Q.2 A ray of light falls on a transparent glass slab with refractive index (relative to air) μ find the angle of incidence for which the reflected and refracted rays are mutually perpendicular.

Q.3 A person looking through the telescope T just sees the point A on the rim at the bottom of a cylindrical vessel when the vessel is empty. When the vessel is completely filled with a liquid of refractive index 1.5, he observes a mark at the centre B of the bottom, without moving the telescope or the vessel. What is the height of the vessel if the diameter of its cross–section is 10 cm?



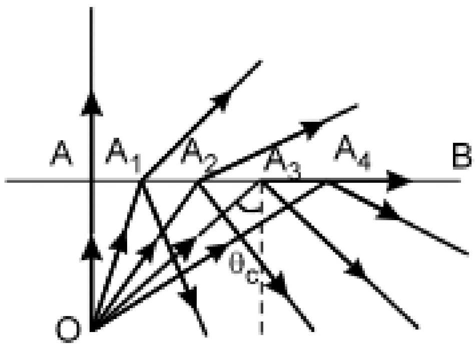
Q.4 A pole standing in a clear water pond stands 1 m above the water surface, the pond is 2 m deep. What are the lengths of the shadows thrown by the pole on the surface and bottom of the pond if the sun is 30^o over the horizon?[Refractive index of water is 4/3]

Answers

Q.1	45^o	Q.2	$\tan^{-1}\mu$	Q.3	8.45 cm	Q.4	$2\sqrt{3}$ m

Critical Angle of incidence

Here AB is refracting surface, O is a point source of light, placed in denser medium. A number of rays become incident from O to the surface AB. The ray OA_1 being normally incident, travels undeviated. As we consider the rays having larger angle the rays will be deviated more and more. For a particular value of angle of incidence (called critical angle of incidence) light ray grazes in the surface.



For $\theta_1 = \theta_c \quad \Rightarrow \quad \theta_2 = 90^0$

Using snell's law,

$$\mu_{denser} \sin\theta_c = \mu_{rarer} \sin 90^0$$

$$\Rightarrow \sin\theta_c = \frac{\mu_{rarer}}{\mu_{denser}}$$

If the light ray is incident having angle of incidence greater than critical angle of incidence then the light is totally reflected and obeys the laws of reflection. This phenomenon is called total internal reflection.

Illustration :

A ray of light traveling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. Find the refractive index of the medium

Sol. *Here* $45^o > \theta_c$

$\sin 45^o > \sin \theta_c$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{1}{n}$$

$$\Rightarrow \quad n > \sqrt{2}$$

Illustration :

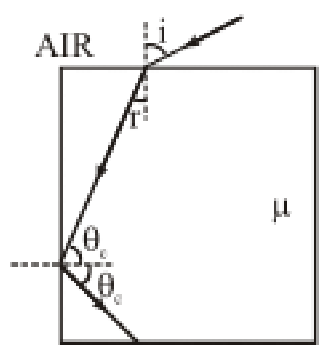
In the figure shown, for an angle of incidence i at the top surface, what is the minimum refractive index needed for total internal reflection at the vertical face?

Sol. *Applying Snell's law at the top surface*

$$\mu \sin r = \sin i \qquad ...(i)$$

For total internal reflection the vertical face

$$\mu \sin \theta_c = 1$$



Using geometry, $\theta_c = 90^o - r$

$\therefore \quad \mu \sin(90 - r) = 1$

or $\quad \mu \cos r = 1 \qquad ...(ii)$

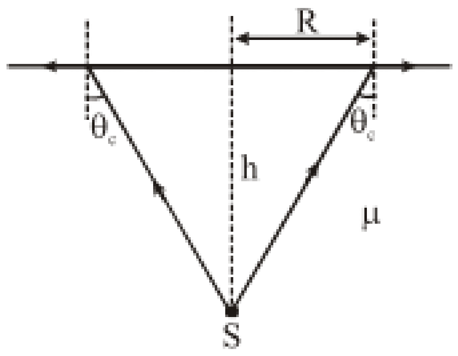
On squaring and adding equation (i) and (ii), we get

$$\mu^2 \sin^2 r + \mu^2 \cos^2 r = 1 + \sin^2 i$$

or $\quad \mu = \sqrt{1 + \sin^2 i}\,.$

Illustration :

A point source of light is placed at the bottom of a tank containing a liquid (refractive index = μ) upto a depth h. A bright circular spot is seen on the surface of the liquid. Find the radius of this bright spot.



Sol. *Rays coming out of the source and incident at an angle greater than* θ_c *will be reflected back into the liquid therefore, the corresponding region on the surface will appear dark. As is obvious from the figure, the radius of the bright spot is given by*

$$R = h \tan \theta_c = \frac{h \sin \theta_c}{\cos \theta_c} \qquad or \qquad R = \frac{h \sin \theta}{\sqrt{1 - \sin^2 \theta_c}}$$

Since $\quad \sin \theta_c = \dfrac{1}{\mu}$

$\therefore \quad R = \dfrac{h}{\sqrt{\mu^2 - 1}}\,.$

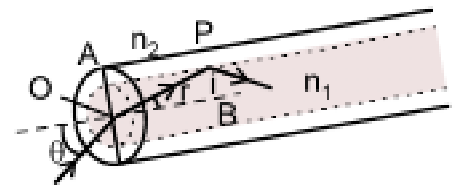
Practice Exercise

Q.1 A ray of light from a denser medium strike a rarer medium at an angle of incidence i. If the reflected and refracted rays are mutually perpendicular to each other, what is the value of critical angle?

Q.2 A beam of parallel rays of width 20 cm propagates in glass at an angle 60^o to its plane face. Find the beam width after it goes over to air through this face. The refractive index of glass is 1.8.

Q.3 A ray of light travelling in glass ($\mu = 3/2$) is incident on a horizontal glass–air surface at the critical angle θ_c. If a thin layer of water ($\mu = 4/3$) is now poured on the glass–air surface, at what angle will the ray of light emerge into air at the water–air surface?

Q.4 A particular optical fibre consists of a glass core (index of refraction n_1) surrounded by a cladding (index of refraction $n_2 < n_1$). Suppose a beam of light enters the fibre from air at an angle θ with the fibre axis as shown in **figure**. Show that the greatest possible value of θ for which a ray can be propagated down in fibre is given by $\theta = \sin^{-1}[\sqrt{n_1^2 - n_2^2}]$

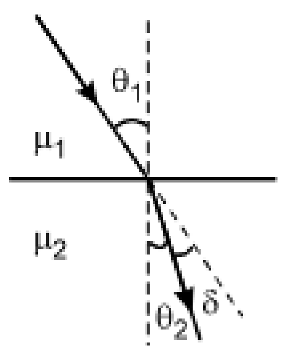


Answers

Q.1 $\sin^{-1}(\tan i)$ Q.2 20 cm Q.3 90^o

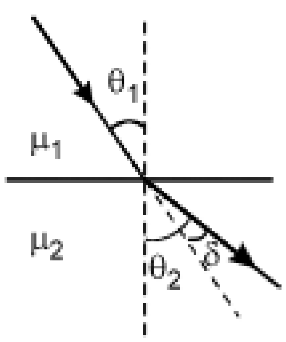
(3) **Calculation of angle of deviation :**

When $\mu_1 < \mu_2$



$\delta = \theta_1 - \theta_2$

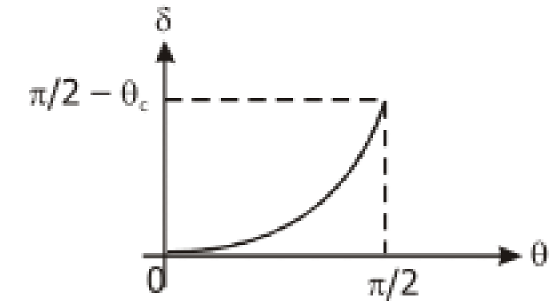
When $\mu_1 > \mu_2$



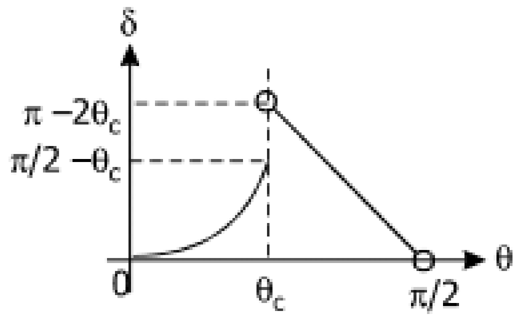
$\delta = \theta_2 - \theta_1$

Plot of deviation vs angle of incidence

light travells from rarer to denser



light travells from rarer to denser



Practice Exercise

Q.1 A ray of light is incident on the surface of a spherical glass paper weight making an angle α with the normal and is refracted in the medium at an angle β. Calculate the deviation.

Answers

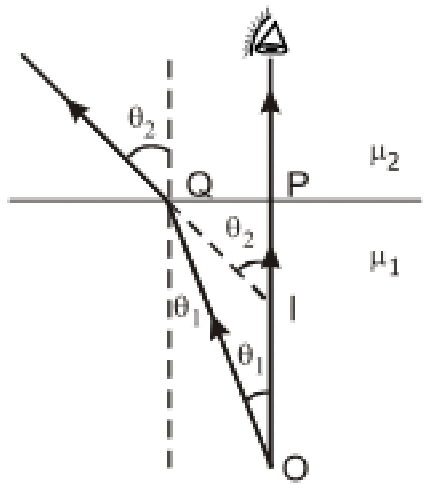
Q.1 $2(\alpha - \beta)$

Shifting of image :

Here we will encounter two cases

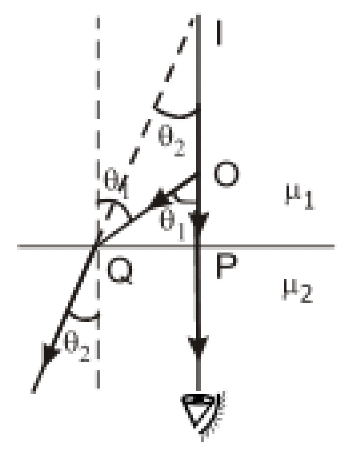
(a) When object is in denser medium with respect to observer

Here $\mu_1 > \mu_2$



(b) when object is in rarer medium with respect to observer.

Here $\mu_1 < \mu_2$



From Snell's law

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

for paraxial rays $\theta \to 0$

$$\Rightarrow \quad \mu_1\theta_1 = \mu_2\theta_2$$

$$\Rightarrow \quad \mu_1 \tan\theta_1 = \mu_2 \tan\theta_2$$

$$\Rightarrow \quad \mu_1 \frac{PQ}{OP} = \mu_2 \frac{PQ}{IP}$$

$$\Rightarrow \quad \frac{IP}{OP} = \frac{\mu_2}{\mu_1}$$

Relation between the velocities of object and image if object is moving perpendicular to the surface:

$$v_{O/surface} = \frac{d}{dt}(OP) \qquad \text{and} \qquad v_{I/surface} = \frac{d}{dt}(IP)$$

but

$$\frac{IP}{OP} = \frac{\mu_2}{\mu_1} \qquad \Rightarrow \qquad IP = \frac{\mu_2}{\mu_1} OP$$

differentiating both sides with respect to time we get

$$\frac{d}{dt} IP = \frac{\mu_2}{\mu_1} \frac{d}{dt} OP$$

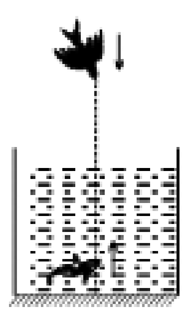
$$\Rightarrow \quad v_{I/surface} = \frac{\mu_2}{\mu_1} v_{O/surface}$$

Practice Exercise

Q.1 A fish rising vertically to the surface of water in a lake uniformly at the rate of 3 m/s observes a king–fisher (bird) diving vertically towards the water at a rate of 9 m/s vertically above it. If the refractive index of water is (4/3), find the actual velocity of the dive of the bird.

Q.2 A concave mirror of radius of curvature one metre is place at the bottom of a tank of water. The mirror forms an image of the sun when it is directly overhead. Calculate the distance of the image from the mirror for (a) 80 cm and (b) 40 cm of water in the tank.

Q.3 A bird in air is diving vertically over a tank with speed 6 cm/s. Base of the tank is silvered. A fish in the tank is rising upward along the same line with speed 4 cm/s. [Take: $\mu_{water} = 4/3$]



(A) Speed of the image of fish as seen by the bird directly

(B) Speed of image of bird relative to the fish looking upwards

Answers

Q.1 4.5 m/s

Q.2 (a) 50cm from the mirror (b) 47.5 cm

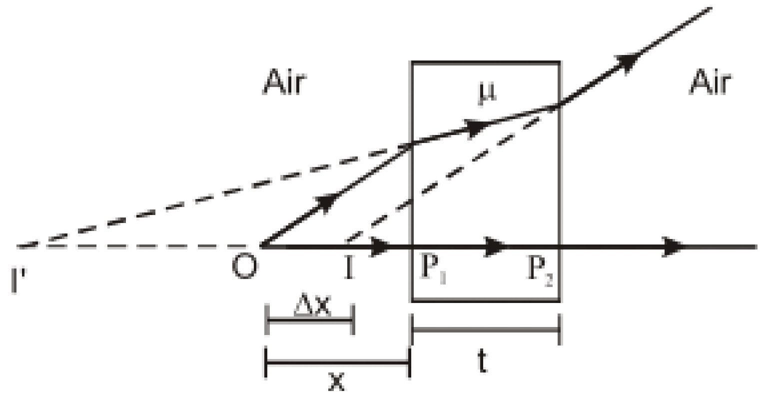
Q.3 (A) Velocity of fish in air $= 4 \times \frac{3}{4} = 3 \uparrow$, Velocity of fish w.r.t bird $= 3 + 6 = 9 \uparrow$

(B) Velocity of bird in water $= 6 \times \frac{4}{3} = 8 \downarrow$, w.r.t fish $= 8 + 4 = 12 \downarrow$

Refraction Through Slab

Shifting of image :

Let an object O be placed at one of the slab (thickness–t, R. I.–μ) and the distance between the surface closer to the object and the object be x.



At first surface

$$\frac{I'P_1}{OP_1} = \frac{\mu}{1} \quad \Rightarrow \quad I'P_1 = \mu OP_1 = \mu x$$

At second surface

$$\frac{IP_2}{I'P_2} = \frac{1}{\mu} \quad \Rightarrow \quad \frac{IP_2}{\mu x + t} = \frac{1}{\mu} \quad \Rightarrow \quad IP_2 = x + \frac{t}{\mu}$$

Now, shifting will be given by

$$\Delta x = OP_2 - IP_2 = (x + t) - \left(x + \frac{t}{\mu}\right)$$

$$\Rightarrow \quad \Delta x = t\left(1 - \frac{1}{\mu}\right)$$

Note that shifting occurs in the direction of propagation of light

Practice Exercise

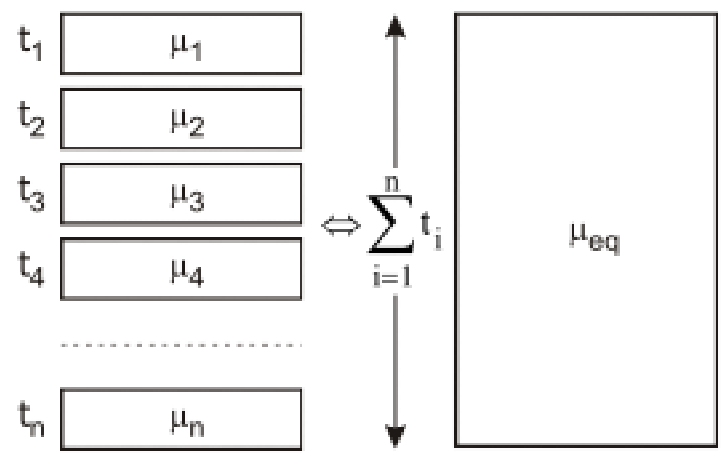
Q.1 An object is placed 21 cm infront of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror. Find the position of the final image formed. (You may take the distance of the near surface of the slab from the mirror to be 1 cm).

Q.2 A 20 cm thick glass slab of refractive index 1.5 is kept in front of a plane mirror. Find the position of the image (relative to mirror) as seen by an observer through the glass when a point object is kept in air at a distance of 40 cm from the mirror.

Answers

Q.1 7.67 cm in front of mirror Q.2 80/3 cm

Combination of slabs :



At first surface

$$\frac{I'P_1}{OP_1} = \frac{\mu}{1} \qquad \Rightarrow \qquad I'P_1 = \mu OP_1 = \mu x$$

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Now, shifting will be given by

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Answers

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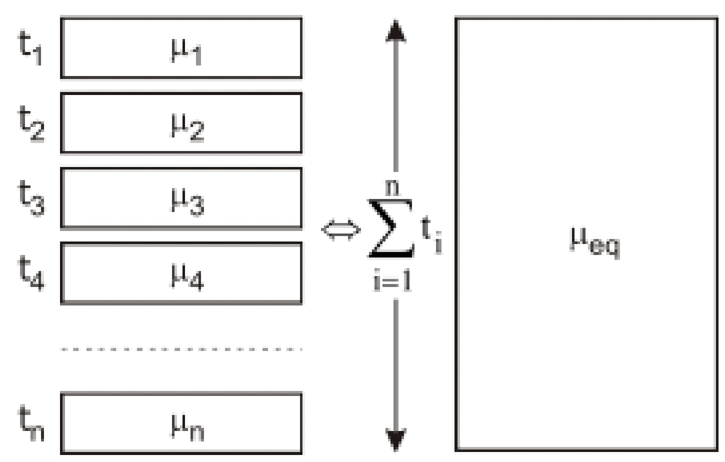


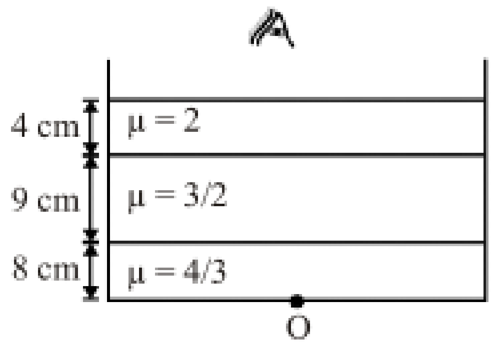
Net shift will be

$$(t_1 + t_2 + t_3 + ...+ t_n)\left[1-\frac{1}{\mu_{eq}}\right] = t_1\left[1-\frac{1}{\mu_1}\right] + t_2\left[1-\frac{1}{\mu_2}\right] + t_3\left[1-\frac{1}{\mu_3}\right]$$

$$\Rightarrow \quad \frac{t_1 + t_2 + t_3 + ...t_n}{\mu_{eq}} = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \frac{t_3}{\mu_3} + ...\frac{t_n}{\mu_n}$$

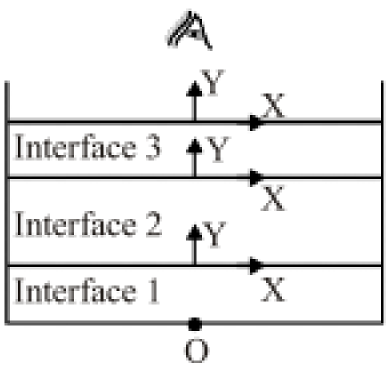
Illustration :

A tank contains three layers of immiscible liquids (figure). The first layer is of water with refractive index 4/3 and thickness 8 cm. The second layer is an oil with refractive index 3/2 and thickness 9cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.



Sol. I : Method of interfaces :

A ray of light from the object undergoes refraction at three interfaces. (1) Water-oil, (2) Oil-glycerine (3) Glycerine-air. The coordinate system for each of the interfaces is shown in figure.



Water-Oil Interface :

$d_1 = -8cm,\ \mu_1 = 4/3,\ \mu_2 = 1.5$

As $\quad \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \quad$ *or* $\quad d_2 = \frac{\mu_2}{\mu_1} d_1$ *we get* $d_2 = -9cm$

Oil-Glycerine Interface :

$d_1 = -(9 + 9) = -18cm,\ \mu_1 = 1.5,\ \mu_2 = 2$

As $\quad \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \quad$ *or* $\quad d_2 = \frac{\mu_2}{\mu_1} d_1$ *we get* $d_2 = -24\ cm$

Glycerine-Air Interface :

$d_1 = -(4 + 24) = -28\ cm,\ \mu_1 = 2,\ \mu_2 = 1$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ *or* $d_2 = \frac{\mu_2}{\mu_1} d_1$ *we get* $d_2 = -14$ *cm*

Thus the final image is 14cm below the glycerine - air interface.

Sol II : Method of elements :

The system now comprises three slabs - one of water, one of oil and one of glycerine. As discussed in this Section and given by equation, the net shift of the system in the sum of the individual shifts each of the slabs asuming they were in air.

Therefore,

Net shift $$s = t_1\left[1-\frac{1}{\mu_1}\right] + t_2\left[1-\frac{1}{\mu_2}\right] + t_3\left[1-\frac{1}{\mu_3}\right]$$

$$s = 8\left[1-\frac{3}{4}\right] + 9\left[1-\frac{2}{3}\right] + 4\left[1-\frac{1}{2}\right]$$

$$= 7cm.$$

The direction of the shift is in the direction of the incident rays which is upwards. Therefore, final position of the object is = (4 + 9 + 8) – 7 = 14cm below the glycerine-air interface.

Illustration :

In a tank a 4 cm thick layer of water $\left(\mu = \frac{4}{3}\right)$ *floats on a 6 cm thick layer of an organic liquid (μ = 1.48). Wiewing at normal incidence how far below the water surface does the battom of tank appear to be ?*

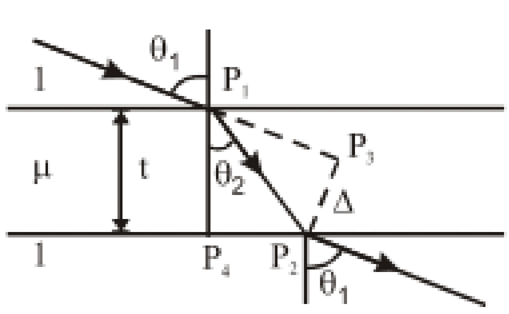
Sol. $$d_{Ap} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2}$$

$$= \frac{6}{1.48} + \frac{4}{4/3}$$

$$d_{Ap} = 7.05 \text{ cm}$$

Sifting in path :

When a ray passes through a slab placed in a medium then after refraction the emergent ray is parallel to the incident ray but it seems that it has translated some distance (Δd, called shifting in path)



In triangle $P_1 P_2 P_4$

$$P_1 P_2 = \frac{t}{\cos\theta_2}$$

In triangle $P_1 P_2 P_3$

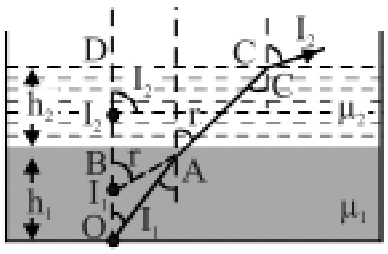
$$\Delta = P_2 P_3 = P_1 P_2 \sin(\theta_1 - \theta_2) = \frac{t}{\cos\theta_2} \sin(\theta_1 - \theta_2)$$

Elemenating θ_2 (using Snell's law), we get

$$\Delta = \frac{t}{\cos\theta_2} \sin(\theta_1 - \theta_2) = t\sin\theta_1 \left[1 - \frac{\mu_1 \cos\theta_1}{\sqrt{\mu_2^2 - \mu_1^2 \sin^2\theta_1}}\right]$$

Practice Exercise

Q.1 An object O is located at the bottom of a tank containing two immiscible liquids and is seen vertically from above. The lower and upper liquids are of depths h_1 and h_2 respectively and of refractive indices μ_1 and μ_2 respectively. Locate the position of the image of the object O from the surface.



Q.2 A light is incident at 40° on a glass plate of refractive index $\mu = 1.3$ and width $h = 1$ cm, and emerges from the other side of it. Find the linear displacement of the light ray caused by refraction.

Answers

Q.1 $\frac{h_2}{\mu_2} + \frac{h_1}{\mu_1}$ Q.2 $x = 0.58$ cm

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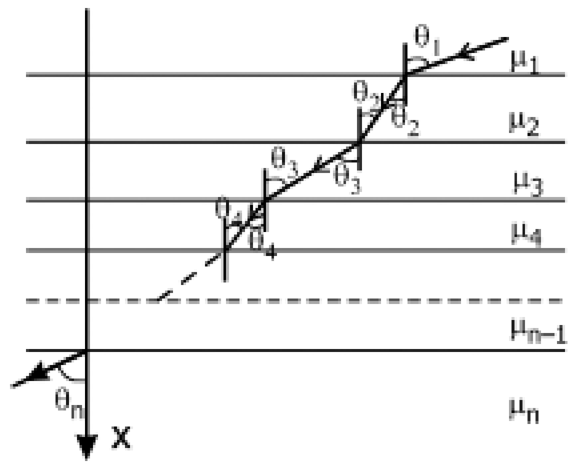
Snell's law for a number of parallel surface

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 = \ldots\ldots\ldots\ldots = \mu_n \sin\theta_n$$



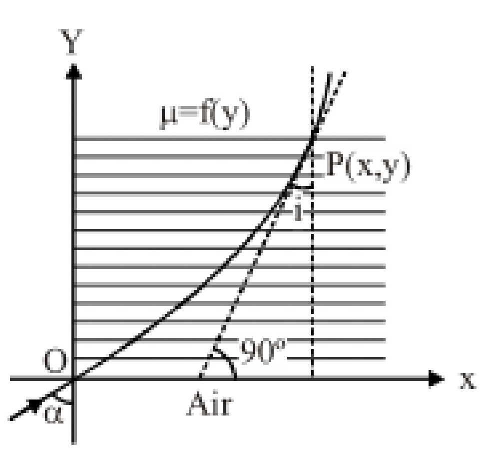
$$\mu_x \sin\theta_x = \text{const.}$$

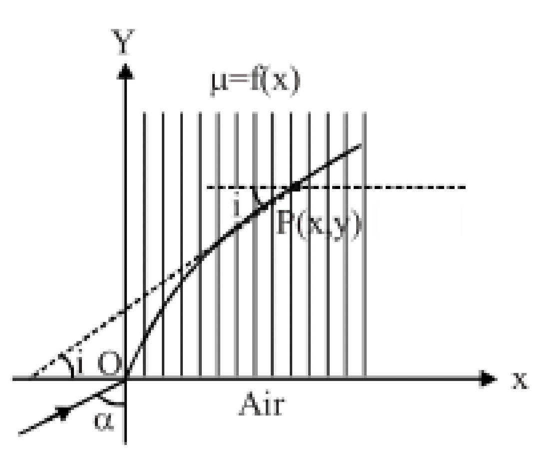
i.e., the product of refractive index of a medium and sine of angle made by light ray in that medium with normal is constant for refraction through many parallel surfaces.

If $\mu_1 = \mu_n \Rightarrow \theta_1 = \theta_n$

i.e., the ray in 1st medium is parallel to the ray in the nth medium.

Continuous variation of refractive





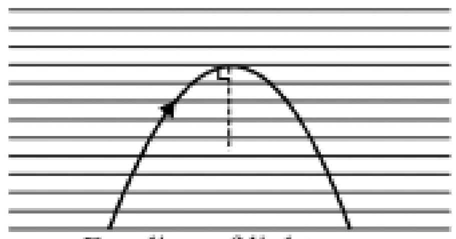
$$\mu \sin\theta = \text{constant}$$

$$\frac{dy}{dx} = \text{slope of tangent}$$

$$= \cot i, \text{ if } \quad \mu = f(y)$$

$$= \tan i, \quad \text{if } \quad \mu = f(x)$$

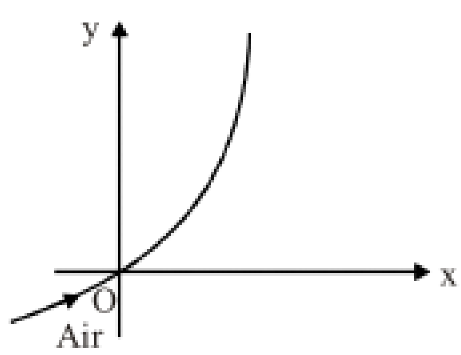
In situation where there is continuous varation of refractive index, critical angle is approximately 90° and bending of ray takes place if angle of incidence approaches 90° while travelling successively from denser to rarer layers.



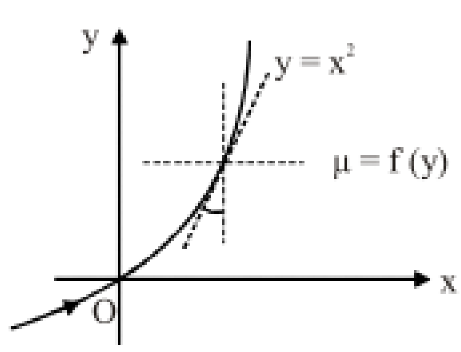
Bending of light ray

Illustration :

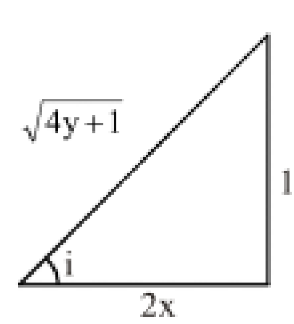
Find the variation of R. I. assuming it to be function of y such that a ray entering at origin at grazing incidence follows a parabolic path $y = x^2$ as shown in figure.



Sol. $\dfrac{dy}{dx} = \tan(90 - i) = \cos i = 2x$



Using Snell's law :



$$1 \cdot \sin\frac{\pi}{2} = \mu \sin i$$

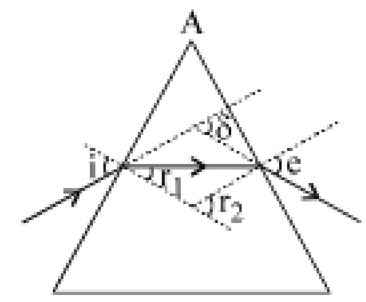
or $$\sin i = \frac{1}{\mu}$$

or $$\mu = \text{coses } i = \sqrt{4x^2 + 1}$$

or $$\mu = \sqrt{4y + 1}$$

REFRACTION THROUGH PRISM

Prism is a transparent medium whose refracting surfaces are not parallel but are inclined to each other.



A prism deviates a light ray

1. Basic Terms

(i) Angle of prism or reflecting angle (A)

The angle between the faces on which light is incident and from which it emerges.

(ii) Angle of deviation (δ)

It is the angle between the emergent and the incident ray. It other words, it is the angle through which incident ray turns in passing through a prism.

$$\delta = (i - r_1) + (e - r_2)$$

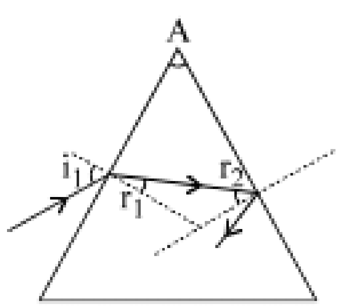
or $$\delta = i + e - (r_1 + r_2)$$

or $$\delta = i + e - A$$

2. Condition of no emergence

A ray of light incident on a prism of angle A and refractive index μ will not emerge out of a prism (whatever may be the angle of incidence) if $A > 2\theta_c$, where θ_c is the critical angle.

i.e. $$\mu > \frac{1}{\sin\ (A/2)}$$

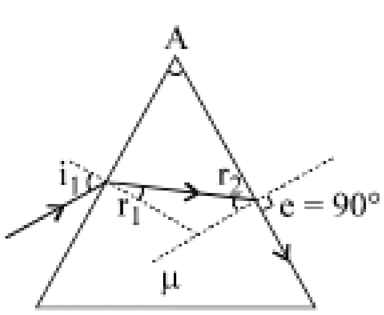


Condition of no emergence

3. Condition of grazing emergence

By the condition of grazing emergence we mean the angle of incidence i at which the angle of emergence becomes e = 90°.

$$i = \sin^{-1}\left[\sqrt{\mu^2 - 1}\ \sin A\ -\ \cos\ A\right]$$



Grazing emerging ray in a prism

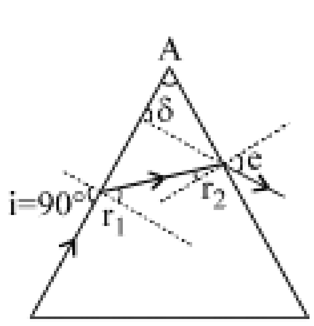
Note : That the light will emerge out of a given prism only if the angle of incidence is greater than the condition of grazing emergence.

4. Condition of maximum deviation

Maximum deviation occurs when the angle of incidence is 90°.

$$\delta_{max} = 90° + e - A$$

where $e = \sin^{-1}[\mu \sin(A - \theta_c)]$



Condition of maximum derivation

5. Condition of minimum deviation

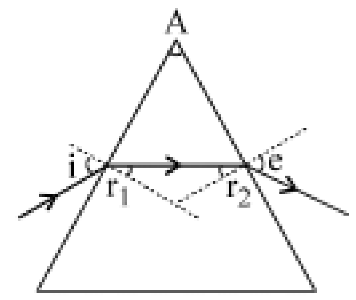
The minimum deviation occurs when the angle of incidence is equal to the angle of emergence, i.e

$i = e$

$\delta_{min} = 2i - A$

Using Snell's law

$$\mu = \frac{\sin\left[\frac{\delta_{min} + A}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$



Light ray passes through a prism symmetrically in the condition of the minimum deviation

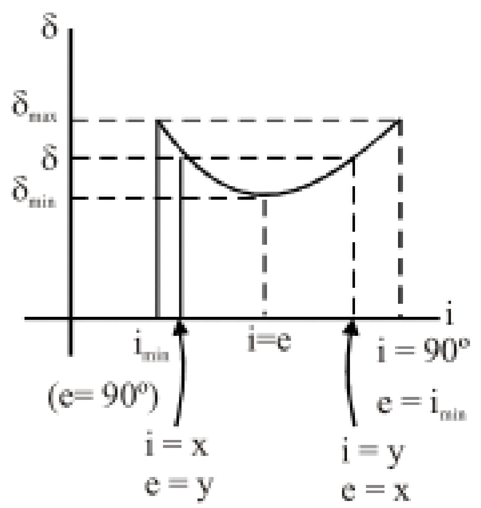
Note : That in the condition of minimum deviation the light ray passes through the prism symmetrically, i.e. the light ray in the prism becomes parallel to its base.

Characteristic of a prism

(a) Variation of δ versus i (shown in diagram).

For one δ (except δ min) there are two values of angle of incidence.

If i and e are interchanged then we get the same value of δ because of reversibility principle of light.



(b) There is one and only one angle of incidence for which the angle of deviation is minimum.

(c) When $\delta = \delta_{min}$, the angle of minimum deviation, then $i = e$ and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces. We can show by simple calculation that $\delta_{min} = 2i_{min} - A$ where i_{min} = angle of incidence for minimum deviation, and $r = A/2$

$$\therefore \quad n_{rel} = \frac{\sin\left[\frac{A+\delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \text{ where } n_{rel} = \frac{n_{prism}}{n_{surroundings}}$$

Also $\delta_{min} = (n - 1)A$ (for small values of $\angle A$)

Illustration :

A prism with angle A = 60° produces a minimum deviation of 30°. Find the refractive index of the material.

Sol. *We know that*

$$\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Here $A = 60°,\ \delta_{min} = 30°$

$$\therefore \qquad \mu = \frac{\sin\left(\frac{90+30}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

Thin prisms

In thin prisms the distance between the refracting surfaces is negligible and the angle of prism (A) is very small.

Since $A = r_1 = r_2$, therefore, if A is small then both r_1 and r_2 are also small, and the same is true for i_1 and i_2.

According to Snell's law $\sin i_1 = \mu \sin r_1$ or $i_1 = \mu r_1$

$\sin i_2 = \mu \sin r_2$ oqr $i_2 = \mu r_2$

Therefore, deviation $\delta = (i_1 - r_1) + (i_2 - r_2)$

$\delta = (r_1 + r_2) + (i_2 - r_2)$

$\delta = A(\mu - 1)$

Note : That the deviation for a small angled prism is independent of the angle of incidence.

Illustration :

A thin prism of angle A = 6° produces a deviation δ = 3°. Find the refractive index of the material of prism.

Sol. *We know that* $\delta = A(\mu - 1)$

or $\mu = 1 + \frac{\delta}{A}$

Here $A = 6°,\ \delta = 3°$, *therefore*

$$\mu = 1 + \frac{3}{6} = 1.5$$

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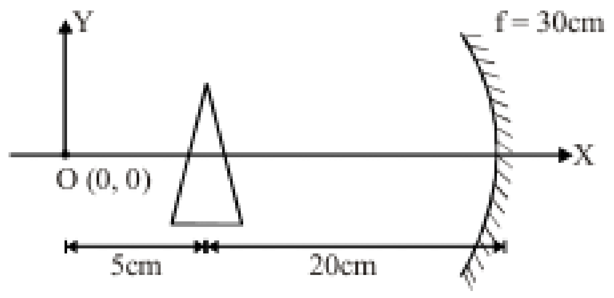
Illustration :

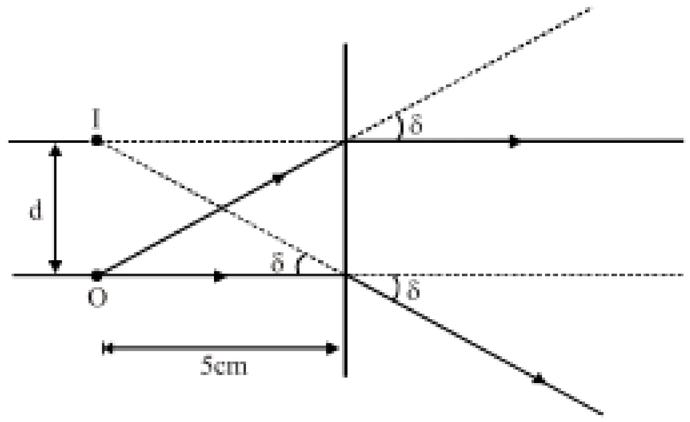
Find the co-ordinates of image of the point object 'O' formed after reflection from concave mirror as shown in figure assumming prism to be thin and small in size of prism angle 2°. Refractive index of prism materal is 3/2.



Sol. *Consider image formation through prism. All incident rays will be deviated by*

$$\delta = (\mu - 1)A = \left(\frac{3}{2} - 1\right)2° = 1° = \frac{\pi}{180} \text{ rad}$$

Now as prism is thin so object and imae will be in same plane as shown in figure.



It is clear $\frac{d}{5} = \tan\delta \approx \delta$ $(\therefore\ \delta \text{ is very small})$

or $d = \frac{\pi}{36}$ cm

Now this image will act as an object for concave mirror.

$u = -25$ cm, $f = -30$ cm

$$\therefore \quad v = \frac{uf}{u - f} = 150 \text{ cm}$$

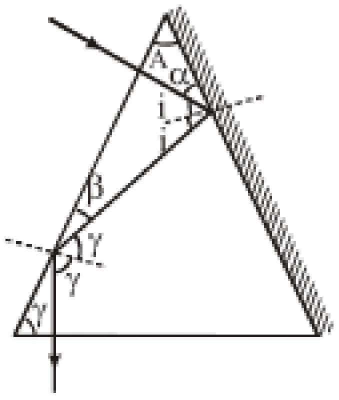
$$\text{Also,} \quad m = \frac{-v}{u} = +6$$

$\therefore$ *Distance of image from principal axis* $= \frac{\pi}{36} \times 6 = \frac{\pi}{6}$ cm

Hence, co-ordinates of image formed after reflection from concave mirror are $\left(175\text{cm}, \frac{\pi}{6}\text{cm}\right)$

Illustration :

The cross-section of the glass prism has the form of an isosceles triangle. One of the equal faces is silvered. A ray of light incident normally on the other equal face and after being reflected twice, emerges through the base of prism along the normal. Find the angle of the prism.



Sol. *From the figure,*

$$\alpha = 90^\circ - A$$

$$i = 90^\circ - \alpha = A \qquad \ldots(i)$$

Also $\quad \beta = 90^\circ - 2i = 90^\circ - 2A$

and $\quad \gamma = 90^\circ - \beta = 2A$

Thus, $\quad \gamma = r = 2A$

From geometry,

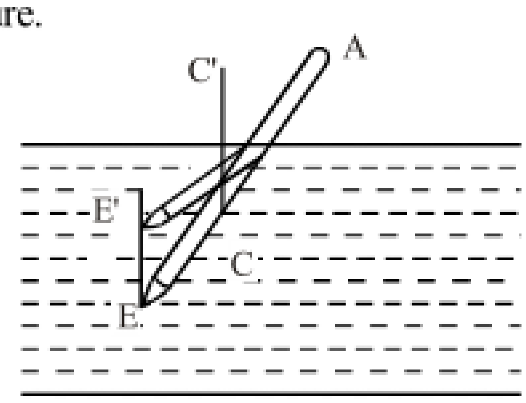
$$A + \gamma + \gamma = 180^\circ.$$

or $\quad A = \dfrac{180}{5} = 36^\circ.$

Some interesting Facts related to refraction and total internal reflection

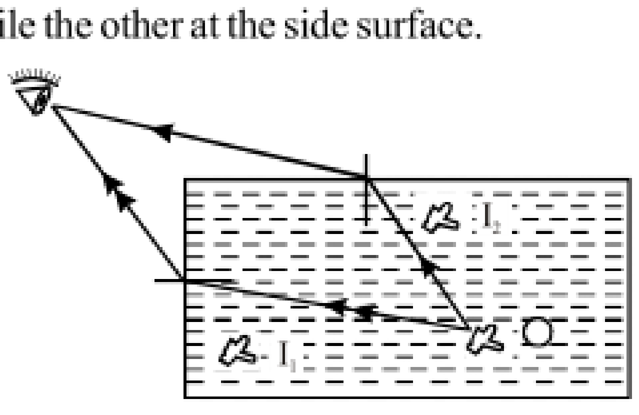
Bending of an object

When a point object in a denser medium is seen from a rarer medium it appears to be at a depth (d/μ). so if a linear object is dipped inclined to the surface of a liquid, (say water) actual depth will be different for its different points and so apparent depth. Due to this the object appears to be inclined from its actual position or BE as shown in figure.



Visibility of two images of an object

When an object is in a glass container and is seen from a level higher than that of liquied in the container as shown in figure , two images I_1 and I_2 of object O can be seen simultaneously-one due to refraction at the upper surface while the other at the side surface.



The sun is oval shaped at the time of rising and setting.

In the moring or eveing, the sunin at the horzon and refactive index in the atmosphere of the earth decreases with height. Due to this, light reaching earth's atmophere from different parts of vertical diameter of the sun enters at different heights in earth's atmophere and so travels in media of different refractive indices at the same instant and hence bends unequally.

Due to this unequal bending of light from vertical diameter, the image of the sun gest distorted and it apperars oval and larger. Howver at noon when the sun is overhead, then due to normal incidence there will be no bending and the sun will appear circular.

Similarly you can explain **Sun rises before it actually rises and sets after it actually sets.**

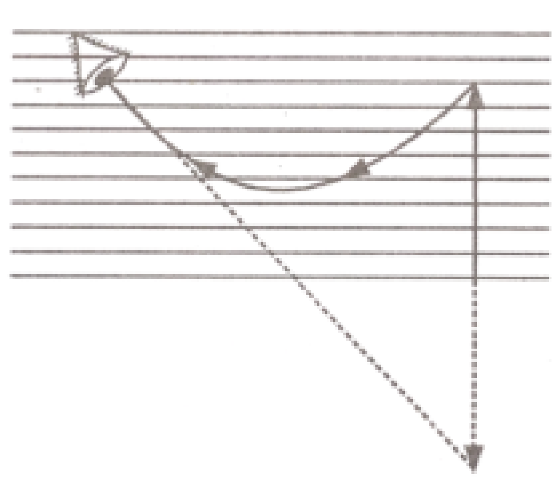
Stars twinkle.

Stars are self - luminous distant object, so only a few rays of light reach the eye through the atmosphere. However, due to fluctuations in refractive index of atmosphere the refraction becomes irregular and the light sometimes reaches the eye and sometimes it does not. This gives rise to twinkling of stars. If from moon or free space we look at a star this effect will not take place and star light will reach the eye continuously.

Tree appear inverted in deserts (mirage)

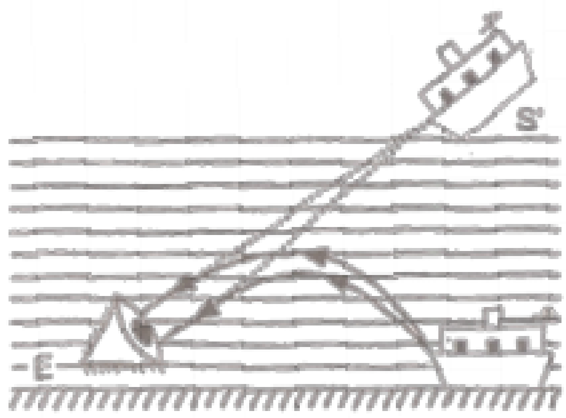
It is an optical illusion created due to the phenomenon of total internal reflection. This is seen in hot region. In hot areas like deserts surface of earth is very hot. So, air in the lower regions of atmosphere is hot as compared to that in higher regions. This result in variation of density with height and it increases as we go up. In this situation atmosphere can be assumed to be made of large number of thin layers of air.

A beam of light starting from an object say a tree and travelling downward finds itself going from denser to rater medium. Therefore, its angle of incidence at consecutive layers goes on increasing gradually till it surpasses the critical value and is reflected back due to total-internal reflection. A virtual image of the object is seen by eye at E. Due to the disturbance of air, the mirage is wavy in nature, thus giving an illusion for the presence of water which is actually not there. This effect is also called inferior mirage.



Ships appear above in the air in cold countries (looming)

This effect occurs when the density of air decreases much more rapidly with increasing height than it does under normal conditions. This situation sometimes happens in cold region particularly in the vicinity of the cold surface of sea or of a lake. Light rays starting from an object S (say a ship) are curved downward and on entering the eye the rays appear to come from S′, thus giving an impression that the ship is floating in air. This effect is also called superior mirage.

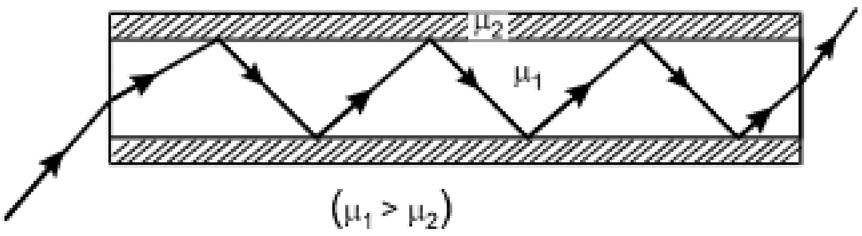


An eye placed inside water sees the external world within a cone and rest surface of water appears as a vast sheet of mirror. Radius is $r = \dfrac{h}{\sqrt{\mu^2 - 1}}$

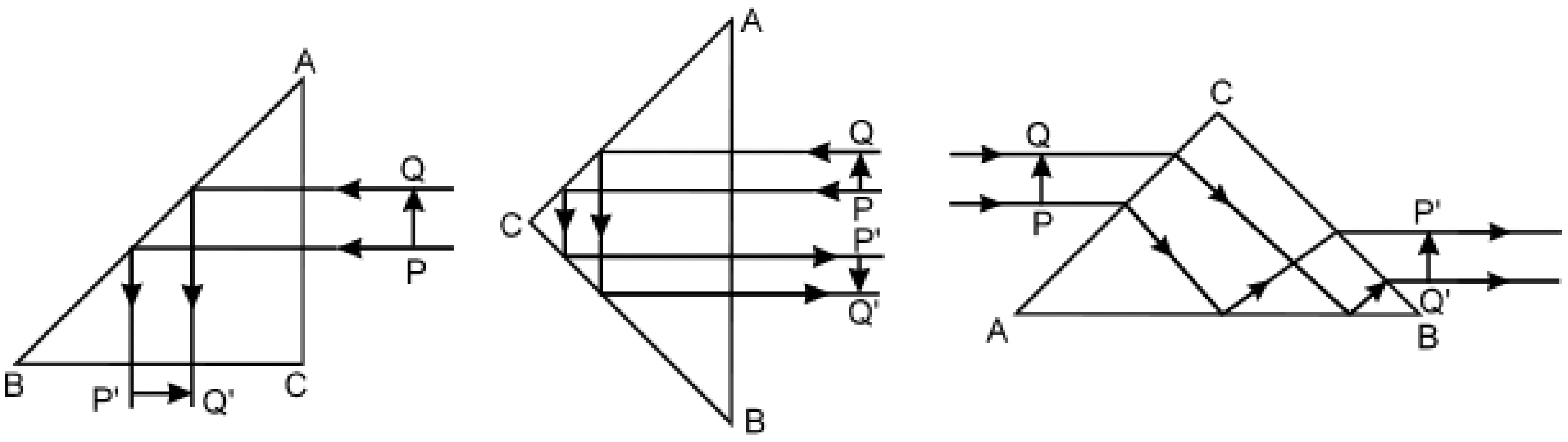
Diamond and glass both shine if cut to the special shape, but diamond shine more than glass piece cut to same shape.

Optical fibre

In optical fibre a fine material of high refractive index is coated with a material of relatively low refractive index. When a light is incident in it, it suffers a no. of total internal reflection and comes out of it. It is used as light pipe.



reflecting prisms.



Dispersion of Light

When a beam of light (containing several wavelengths) falls on one face of a prism, it splits into its constituent colours. This phenomenon of splitting of light into its constituent colours is called dispersion of light and the band of colours obtained on a screen is called spectrum. The cause of dispersion is variation of refractive index with wavelength of light. An approximate empirical relation as proposed by Cauchy is given by

$$\mu(\lambda) = A + \frac{B}{\lambda^2}$$

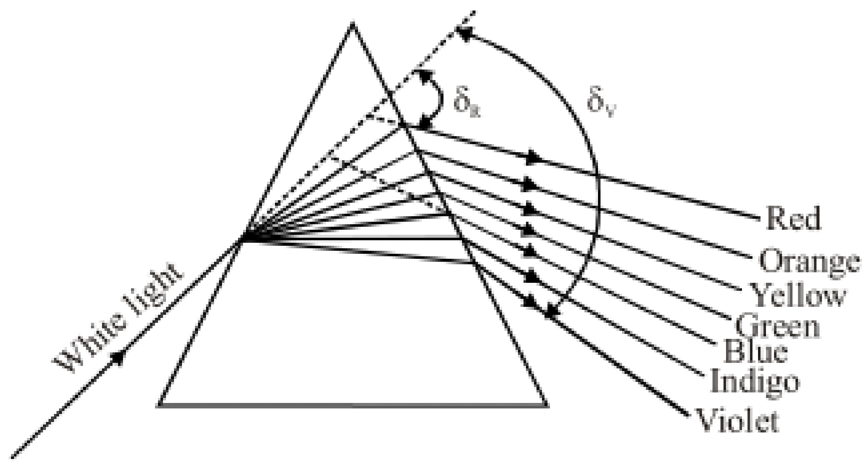
where A and B are known as Cauchy's constant. The value of A and B depends on material of prism.

We know that $\lambda_{Red} > \lambda_{Violet}$

$\therefore \quad \mu_{Red} < \mu_{Violet}$

Hence, $\quad \delta_{Red} < \delta_{Violet}$

The difference in the deviations suffered by two colours in passing through a prism gives the angular dispersion for these colours. The angle between the violet and red colours is known as angular dispersion. We know that for small angle of prism, deviation is given by



$$\delta = (\mu - 1)A$$

$\therefore \quad \delta_V$ = Deviation in violet colour $= (\mu_V - 1)A$

δ_R = Deviation in red colour $= (\mu_R - 1)A$

Hence, Angular Dispersion (A.D) $= \delta_V - \delta_R$

$= (\mu_V - \mu_R)A$

It is clear from above relation that angular dispersion depends upon (i) the nature of material of the prism and (ii) the angle of the prism. This is also defined as the rate of change of angle of deviation with wavelngth i.e., A.D. $= \dfrac{d\delta}{d\lambda}$.

Dispersive power of a prism is defined as the ratio between angular dispersion to mean deviation produced by the prism.

ω = Dispersive power

$$= \frac{\delta_V - \delta_R}{\delta_Y} = \frac{\mu_V - \mu_R}{\delta_Y - 1} = \frac{d\mu}{\mu_Y - 1}$$

Where $d\mu$ denotes the difference between the refractive indices of material of prism for violet and red light. It is also defined as dispersion per unit deviation. Yellow colour is taken as mean colour.

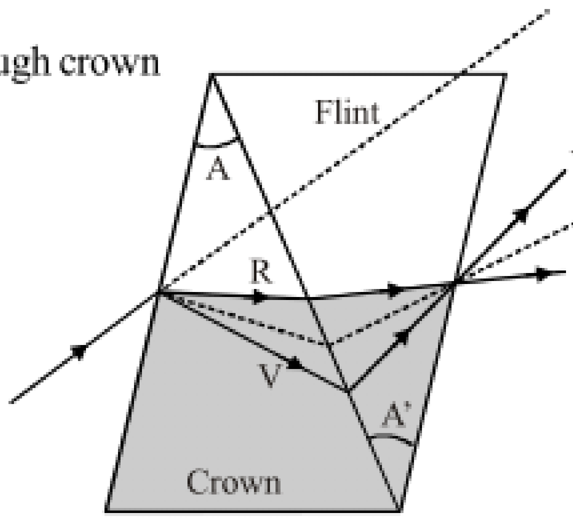
Also, $\quad \mu_Y = \dfrac{\mu_V + \mu_R}{2}$ or $\dfrac{\mu_B + \mu_R}{2}$

Dispersion without Deviation

Let us consider a crown glass prism combined with a flint glass prism in position as shown in figure. Let A and A' be the angles of crown glass prism and flint glass prism respectively. Let μ_0, μ and μ_r be the refractive indices of the crown glass for violet, yellow and red colours respectively.

Let μ_0', μ' and μ_r' be the corresponding values for the flint glass prism.

Let δ and δ' be the deviations suffered by yellow light through crown glass prism and flint glass prism respectively.



If the combination does not produce any deviation, then

$$\delta + \delta' = 0$$

or $\quad (\mu - 1)A + (\mu' - 1)A' = 0$

or $\quad (\mu' - 1)A' = -(\mu - 1)A$

or $\quad A' = -\left(\dfrac{\mu - 1}{\mu' - 1}\right)A$

This is the condition for no devition. The negative sign indicates that the two prisms are to be placed in position as shown in figure.

Net angular dispersion $= [(d_v - d_r) + (d'_v - d'_r)] = (\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A'$

$$= A\left[(\mu_v - \mu_r) + (\mu_v' - \mu_r')\frac{A'}{A}\right]$$

$$= A\left[(\mu_v - \mu_r) + \left(\frac{\mu - 1}{\mu' - 1}\right)(\mu_v' - \mu_r')\right]$$

$$= (\mu - 1)A\left[\frac{\mu_v - \mu_r}{\mu - 1} - \frac{\mu_v' - \mu_r'}{\mu' - 1}\right] = \delta(\omega - \omega')$$

Here ω and ω' are the dispersive powers of crown glass and flint glass respectively. As the dispersive powers ω and ω' are not equal, in such a combination there will be resultant dispersion and the final dispersed beam is parallel to the incident beam.

Since $\omega' > \omega$ therefore the net angular dispersion is negative. This explains why the order of colours in the spectrum due to combination is opposite to that in the crown glass prism.

Deviation without dispersion :

For the combination of prism shown in figure, if there is to be no angular dispersion, then

$$(\delta_v - \delta_r) + (\delta_v' - \delta_r') = 0$$

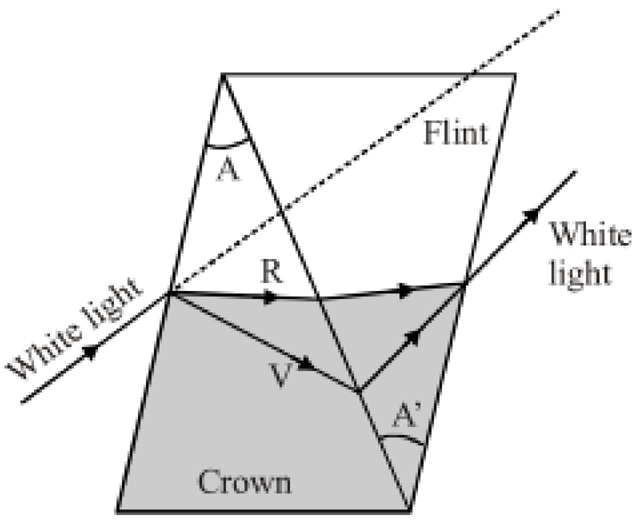
or $\quad (\mu_v - \mu_r)A + (\mu_v' - \mu_r')A' = 0$

or $\quad (\mu_v' - \mu_r')A' = -(\mu_v - \mu_r)A$

or $\quad A' = -\left(\dfrac{\mu_v - \mu_r}{\mu_v' - \mu_r'}\right)A$

This is the condition for achromatism i.e., the condition for no dispersion. This condition can be written in another form as given below.

From equation (1),



$$(\mu-1)A\frac{\mu_v-\mu_r}{\mu-1}+(\mu'-1)A\frac{\mu_v'-\mu_r'}{\mu'-1}=0$$

or $$\delta\omega+\delta'\omega'=0$$

or $$\frac{\omega}{\omega'}=-\frac{\delta'}{\delta}$$

Since, $\omega'>\omega$, $\quad\therefore\ \delta>\delta'$

or $\quad(\mu-1)A>(\mu'-1)A'$

But, $\quad(\mu-1)<(\mu'-1)\qquad\therefore\ A>A'$

So, the crown glass prism should have a larger angle than the flint glass prism.

Net deviation $=\delta+\delta'$

$$=(\mu-1)A+(\mu'-1)A'=(\mu-1)A\left[1+\frac{(\mu'-1)A'}{(\mu-1)A}\right]$$

$$=(\mu-1)A\left[1-\frac{\mu'-1}{\mu-1}\frac{\mu_v-\mu_r}{\mu_v'-\mu_r'}\right]\qquad\text{(Using equation (2))}$$

$$=(\mu-1)A\left[1-\frac{\mu_v-\mu_r}{\mu-1}\times\frac{\mu_v-\mu_r}{\mu-1}\times\frac{\mu'-1}{\mu_v'-\mu_r'}\right]=\delta\left(1-\frac{\omega}{\omega'}\right)$$

Since $\omega'>\omega$, therefore the net deviation is in the direction of the deviation produced by crown glass prism.

Remark :

A parallel sided glass slab can be looked upon as the combination of two prisms producing no deviation and no dispersion.

Practice Exercise

Q.1 A ray of light undergoes deviation of 30^o when incident on an equilateral prism of refractive index $\sqrt{2}$. What is the angle subtended by the ray inside the prism with the base of the prism?

Q.2 A thin prism P_1 with angle 4^o and made from glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. What is the angle of the prism P_2?

Q.3 A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism.

Q.4 A glass prism of angle 72° and index of refraction 1.66 is immersed in a liquid of refractive index 1.33. Find the angle of minimum deviation for a parallel beam of incident light passing through the prism.

Answers

Q.1 0 Q.2 3° Q.3 $\sqrt{3}$ Q.4 22.37°

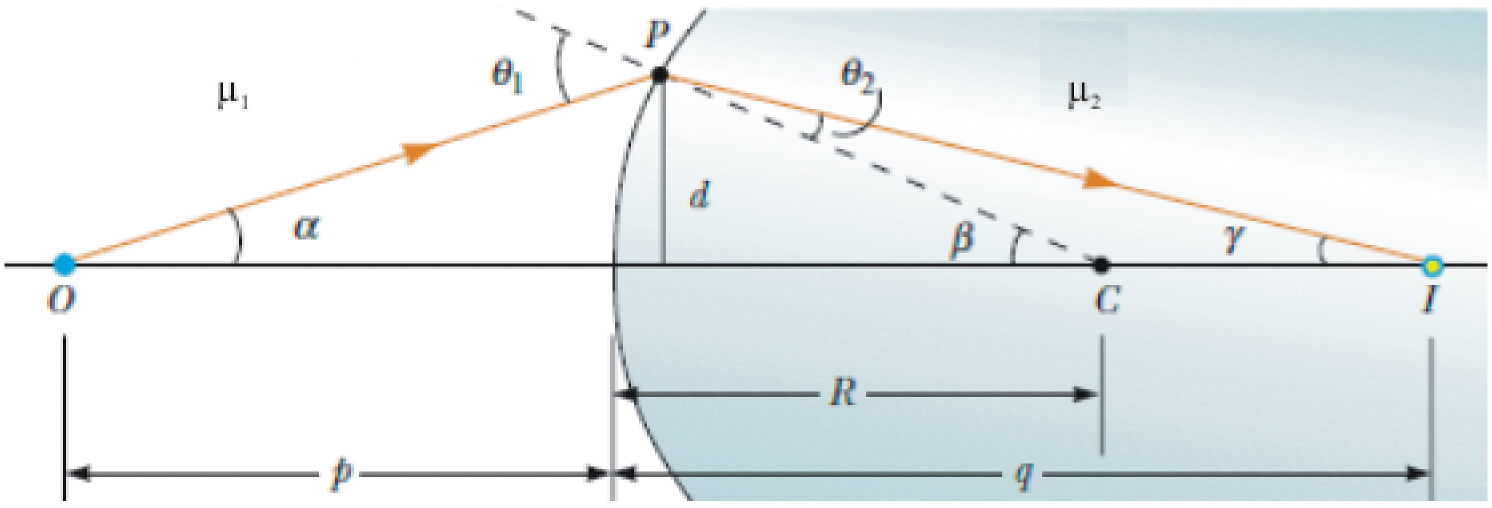
formation of image by a Spherical refracting Surface

Pole (vertex) : Point of intersection of principal axis and the refracting surface.
Focal Point (F) : It is an axial point having the property that any incident ray traveling parallel to the axis will after refraction, proceed toward, or appear to come from this point.
Focal length (f) : The distance of the focus from the vertex of the refracting surface is called focal length. There is a great significance of the sign of focal length as it is able to state whether the spherical refracting surface is converging or diverging as in the chart.

Sign of f	Nature of the system
+ ve	converging
– ve	diverging

Relation between position of object distance, image distance and radius of the spherical refracting surface :

In this section we describe how images are formed when light rays are refracted at the boundary between two transparent materials. Consider two transparent media having indices of refraction μ_1 and μ_2, where the boundary between the two media is a spherical surface of radius R (Fig.). We assume that the object at O is in the medium for which the index of refraction is μ_1. Let us consider the paraxial rays leaving O. As we shall see, all such rays are refracted at the spherical surface and focus at a single point I, the image point.Figure shows a single ray leaving point O and refracting to point I.



Snell's law of refraction applied to this ray gives

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

Because θ_1 and θ_2 are assumed to be small, we can use the small-angle approximation $\sin\theta \approx \theta$ (with angles in radians) and say that

$$\mu_1\theta_1 = \mu_2\theta_2$$

Now we use the fact that an exterior angle of any triangle equals the sum of the two opposite interior

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angles. Applying this rule to triangles OPC and PIC in Figure gives

$\theta_1 = \alpha + \beta$

$\beta = \theta_2 + \gamma$

If we combine all three expressions and eliminate θ_1 and θ_2, we find that

$\mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta$

From Figure, we see three right triangles that have a common vertical leg of length d. For paraxial rays (unlike the relatively large-angle ray shown in Fig.), the horizontal legs of these triangles are approximately p for the triangle containing angle α, R for the triangle containing angle β, and q for the triangle containing angle γ). In the small-angle approximation, $\tan\alpha \approx \alpha$, so we can write the approximate relationships from these triangles as follows:

$$\tan\alpha \approx \alpha = \frac{d}{p} \qquad \tan\beta \approx \beta = \frac{d}{R} \qquad \tan\gamma \approx \gamma = \frac{d}{q}$$

We substitute these expressions into above Equation and divide through by d to give

$$\frac{\mu_2}{p} + \frac{\mu_1}{q} = \frac{\mu_2 - \mu_1}{R}$$

Let

u= object distance (with sign convention)

v= image distance (with sign convention)

R= Radius of curvature (with sign convention)

then

$p = -u \qquad q = +v \qquad R = +R$

putting the values of p, q and R in above equation we get

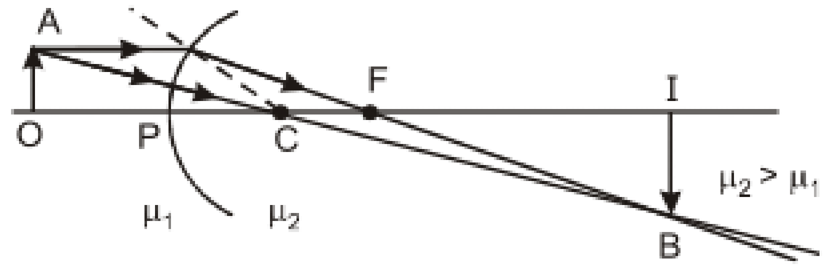
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Rules for image tracing for a linear, transverse extended object :

The basic rule is same as that in mirrors. Briefly,

(i) Draw a ray parallel to the principal axis which after refraction will be along the line passing through F.

(ii) Draw a ray incident along the line through centre it will pass undeviated.



Transverse Magnification :

Snell's law of refraction applied to this ray gives

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

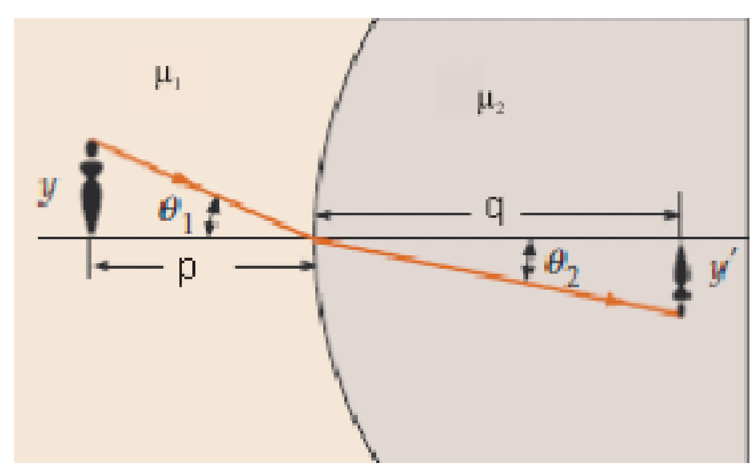
Because θ_1 and θ_2 are assumed to be small, we can say that

$$\sin\theta_1 \approx \theta_1 \approx \tan\theta_1 = \frac{y}{p}$$

$$\sin\theta_2 \approx \theta_2 \approx \tan\theta_2 = \frac{-y'}{q}$$

From the above equations we get

$$\mu_1 \frac{y}{p} = \mu_2 \frac{-y'}{q}$$



$$\Rightarrow \qquad \frac{y'}{y} = -\frac{\mu_1}{\mu_2}\frac{q}{p}$$

Transverse magnification is given by

$$m_T = \frac{y'}{y} = -\frac{\mu_1}{\mu_2}\frac{q}{p}$$

Let

u= object distance (with sign convention)

v= image distance (with sign convention)

then

p=-u q=+v R=+R

putting the values of p and q in above equation we get

$$m_T = \frac{\mu_1 v}{\mu_2 u}$$

Longitudinal Magnification

for small longitudinal object,

$$m_L = \frac{dv}{du} = \frac{\mu_1}{\mu_2}\frac{v^2}{u^2}$$

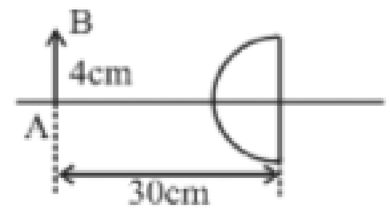
Optic Power :

It represents the conversing ability of an element. For a single spherical refracting surface it is defined as

$$P = \frac{\mu_2 - \mu_1}{R}$$

Illustration :

A linear object of length 4 cm is placed at 30 cm from the plane surface of hemispherical glass of radius 10 cm. The hemispherical glass is surrounded by water. Find the final position and size of the image.



Sol. *For 1st surface*

$\mu_1 = \frac{4}{3},\ \mu_2 = \frac{3}{2}, u = -20$ **cm**, and $R = +10$ cm,

using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

$\Rightarrow \frac{\left(3/2\right)}{v} - \frac{\left(4/3\right)}{(-20)} = \frac{\left(3/2 - 4/3\right)}{10}$

$\Rightarrow v = -30$ cm

Using $\frac{A'B'}{AB} = \frac{\mu_1 v}{\mu_2 u} \Rightarrow \frac{A'B'}{(4\text{cm})} = \frac{\left(4/3\right)(-30)}{\left(3/2\right)(-20)}$

$\Rightarrow A'B' = 5.3$ cm.

$A'B'$ behaves as the object for plane surface

$\mu_1' = \frac{3}{2},\ \mu_2' = \frac{4}{3}$ and $R = \infty,\ u' = -40$

$\Rightarrow \frac{\mu_2'}{v'} = \frac{\mu_1'}{u'} \Rightarrow \frac{\left(4/3\right)}{v'} = \frac{\left(3/2\right)}{(-40)}$

Solving it we will get, $v' = -25.4$ cm

Now using, $\frac{A''B''}{A'B'} = \frac{(\mu_1' v)}{(\mu_2' u)}$

$\Rightarrow \frac{A''B''}{(5.3)} = \frac{\left(3/2\right)(-35.4)}{\left(4/3\right)(-40)} \Rightarrow A''B'' = 5.3$

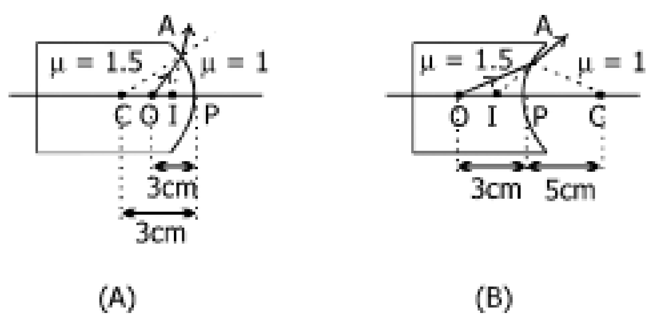
The final images in all the above cases are shown in figure.

Practice Exercise

Q.1 A parallel beam of light travelling in water ($\mu = 4/3$) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial (a) Find the position of the image due to refraction at the first surface and the position of final image. (b) Draw a ray diagram showing the positions of both the images.

Q.2 A point source of light is placed in air at a distance 2R from the centre of a glass sphere of radius of curvature R and refractive index 1.5. Obtain the position of the intermediate and final images.

Q.3 An air bubble in glass ($\mu = 1.5$) is situated at a distance 3 cm from a spherical surface of diameter 10 cm as shown in **figure**. At what distance from the surface will the bubble appear if the surface is (a) convex (b) concave.



Q.4 If a mark of size 0.2 cm on the surface of a glass sphere of diameter 10 cm and $\mu = 1.5$ is viewed through the diametrically opposite point, where will the image be seen and of what is size?

Answers

Q.1	(a) 0.6 cm and 0.1 cm left of first surface	Q.2	∞ and 2R
Q.3	(a) –2.5 cm (b) –1.66 cm	Q.4	0.6 cm

Refraction through lens

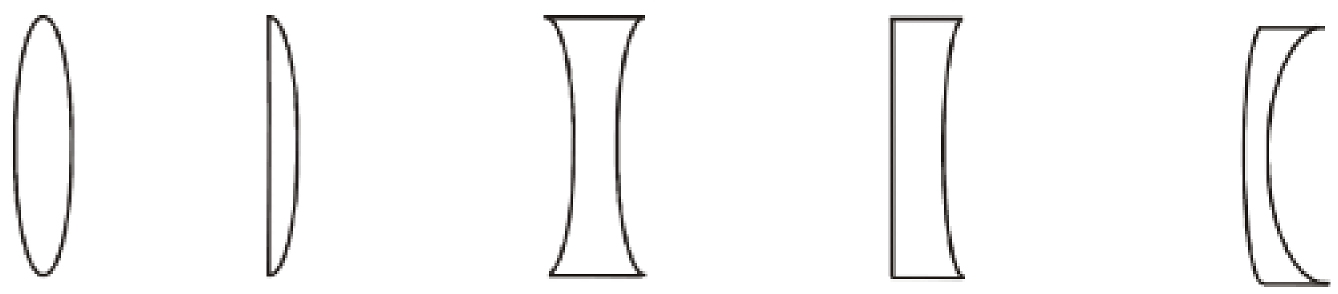
Lens :

A lens is an optical system bounded by two or more than two refracting surfaces having common axis.

Refraction through a simple thin lens :

In simple lens if the surfaces are very close to each other then the lens is called simple thin lens if they are separated by non negligible distance then called simple thick lens.

Thin lens is classified (Geometrically) to following categories.



Biconvex **Plane convex** **Biconcave** **Plano concave** **Convexo concave**

But according to its action a lens may be of two types.

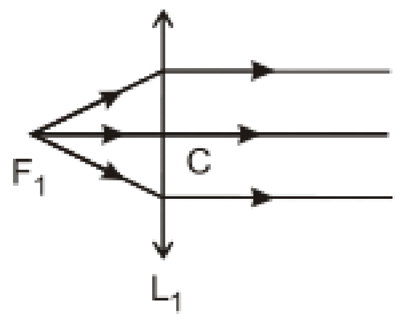
Simple Thin Spherical lens

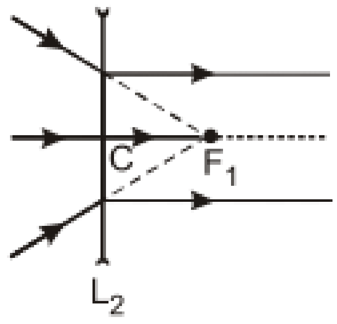
Converging lens or convex lens	**Diverging lens or concave lens**
(If it converges Parallel beam)	(If it diverges parallel beam)

Some basic terms related to thin spherical lens :

Optic Centre (C) : If a light ray is incident on lens is such that after refraction from lens the emergent ray is parallel to the incident ray then the point at which refracted ray intersects the principal axis is called optic centre of that lens.

Primary Focal Point (F_1) : The position of object if image is at infinity is called primary focal point

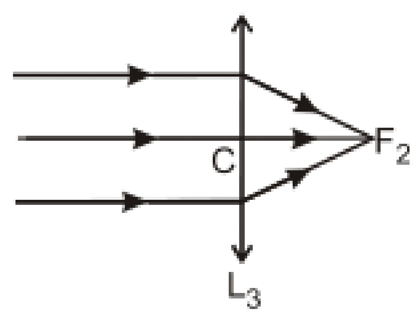


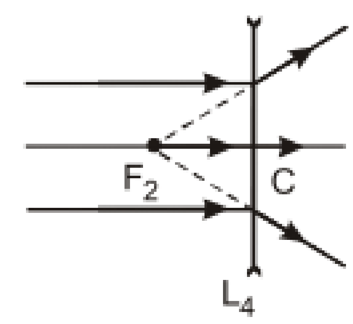


Where L_1 = converging lens

L_2 = diverging lens

Secondary focal point (F_2) : The position of image if object is at infinity is called secondary focal point





Where L_3 = converging lens

L_4 = diverging lens

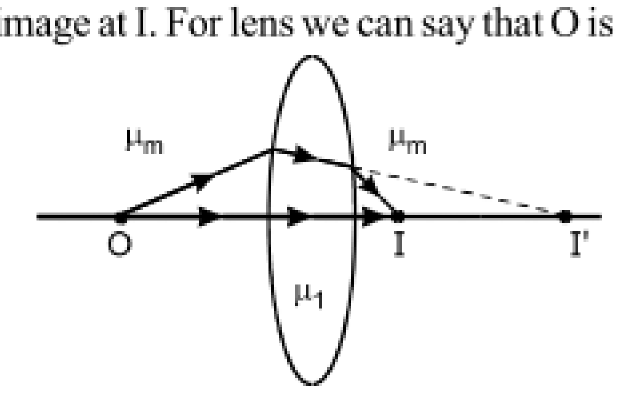
focal length f : The distance between optical centre and secondary focal point is termed as focal length.

Note :	**Sign of f**	**Nature of the lens**
	+ ve	converging
	– ve	diverging

Quantitative Discussion of a thin spherical lens placed in a Medium for an axial point object :

Consider a thin spherical lens ($m = m_l$) is placed in a medium ($m = m_m$) as shown in figure.

Let a point object O is placed on the principal axis of the lens. For the first refracting surface O is an object point and the corresponding image point it I_1. Now I_1 will act as object point for second surface which again form an image at I. For lens we can say that O is object point and I is image point.



At first surface, $$\frac{\mu_l}{v'} - \frac{\mu_m}{u} = \frac{\mu_l - \mu_m}{R_1} \qquad \text{... (i)}$$

At second surface, $$\frac{\mu_m}{v} - \frac{\mu_l}{v'} = \frac{\mu_m - \mu_l}{R_2} \qquad \text{... (ii)}$$

Combining (i) and (ii) we get.

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_1}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{... (iii)}$$

Calculation of f : If $u = \infty$ $\Rightarrow v = f$

$$\frac{1}{f} = \left(\frac{\mu_1}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{... (iv)}$$

This formula is known as **lens maker formula.**

from (iii) and (iv)

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

This formula is known as **lens formula**

Power of a thin spherical lens :

It is defined as sum of the power of individual surfaces

$$P=P_1+P_2=\frac{\mu_l-\mu_m}{R_1}+\frac{\mu_m-\mu_l}{R_2}$$

$$P=(\mu_l-\mu_m)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{\mu_m}{f}$$

Note : The sign of P and f are same.

Mathematical Analysis to state whether the lens is converging or diverging :

Biconvex :

$$\frac{1}{f}=\left(\frac{\mu_l}{\mu_m}-1\right)\left(\frac{1}{+R_1}-\frac{1}{-R_2}\right)=\left(\frac{\mu_l}{\mu_m}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

If	$\mu_l > \mu_m$	$\mu_l < \mu_m$	$\mu_l = \mu_m$
f =	+ve	− ve	∞
P =	+ ve	− ve	0
Nature→	Converging	diverging	neither converging nor diverging

Plano convex :

$$\frac{1}{f}=\left(\frac{\mu_l}{\mu_m}-1\right)\left(\frac{1}{\pm\infty}-\frac{1}{-R}\right)=\left(\frac{\mu_l}{\mu_m}-1\right)\frac{1}{R}$$

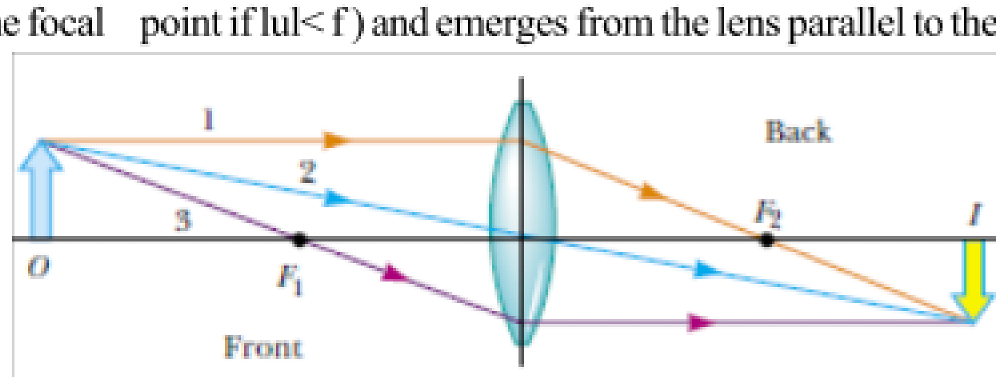
If	$\mu_l > \mu_m$	$\mu_l < \mu_m$	$\mu_l = \mu_m$
f =	+ve	− ve	∞
P =	+ ve	− ve	0
Nature→	Converging	diverging	neither converging nor diverging

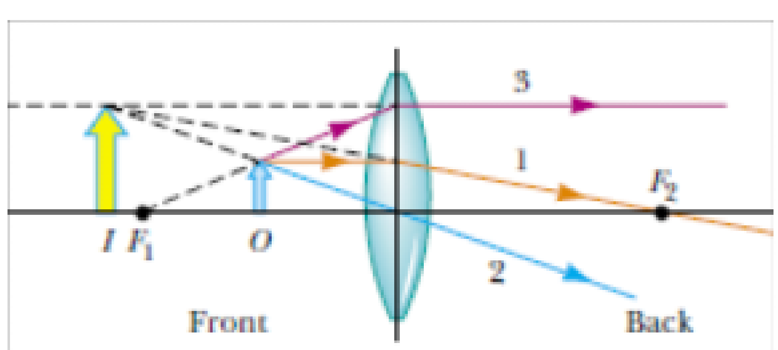
Extended objects :

Way of image tracing

To locate the image of a converging lens (Fig.), the following three rays are drawn from the top of the object:

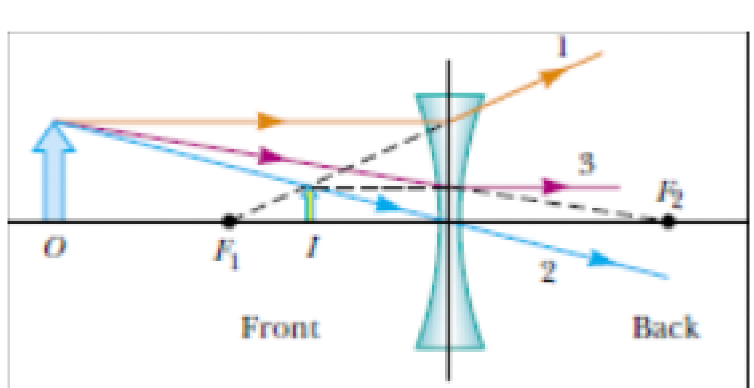
- Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through the focal point on the back side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn through the focal point on the front side of the lens (or as if coming from the focal point if lul< f) and emerges from the lens parallel to the principal axis.





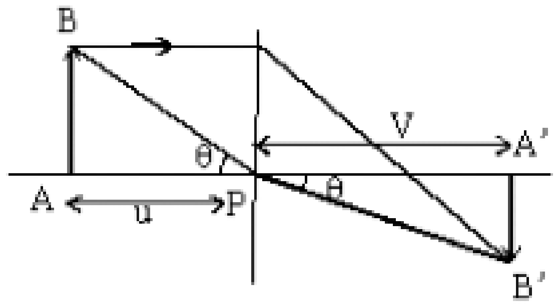
To locate the image of a diverging lens (Fig.), the following three rays are drawn from the top of the object:

- Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray emerges directed away from the focal point on the front side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis.



Magnification :

Transverse magnification :



From $\Delta 'S \;\; PAB$ and

$$\frac{AB}{AP} = \frac{A'B'}{A'P} \quad \Rightarrow \quad \frac{+h_{object}}{-u} = \frac{-h_{image}}{+v}$$

$$m_T = \frac{h_{image}}{h_{object}} = \frac{v}{u} = \frac{f}{f+u}$$

Longitudinal magnification : For small linear longitudinal object

Using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Differentiating w.r.to. u we get

$$\frac{dv}{du} = \frac{v^2}{u^2} = m_T^2$$

$$m_L = m_T^2$$

Illustration :

What if the object moves right up to the lens surface, so that $u \to 0$? Where is the image ?

Sol. *In this case because u << R, where R is either of the radii of the surface of the lens, the curvature of the lens can be ignored and it should appear to have the same effect as a plane piece of material. This would suggest that the image is just on the front side of the lens, at v = 0. We can verify this mathematically by rearranging the thin lens equation.*

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

If we let $u \to 0$, the second term on the right become verty large compared to the first and we can neglect 1/f. The equation becomes

$$\frac{1}{v} = -\frac{1}{u}$$

$$v = -u = 0$$

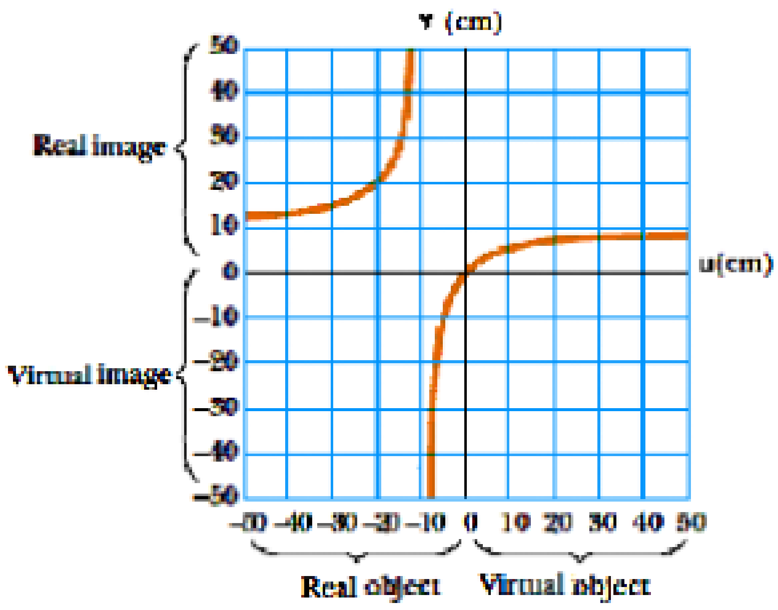
Thus v is on the front side of the lens (because it has the oposite sign as v), and just at the lens surface.

Illustration

Plot graphs of image distance as a function of object distance for a lens for which the focal length is 10 cm if the lens is

(a) convex (b) concave

Sol. *(a)*



(b)

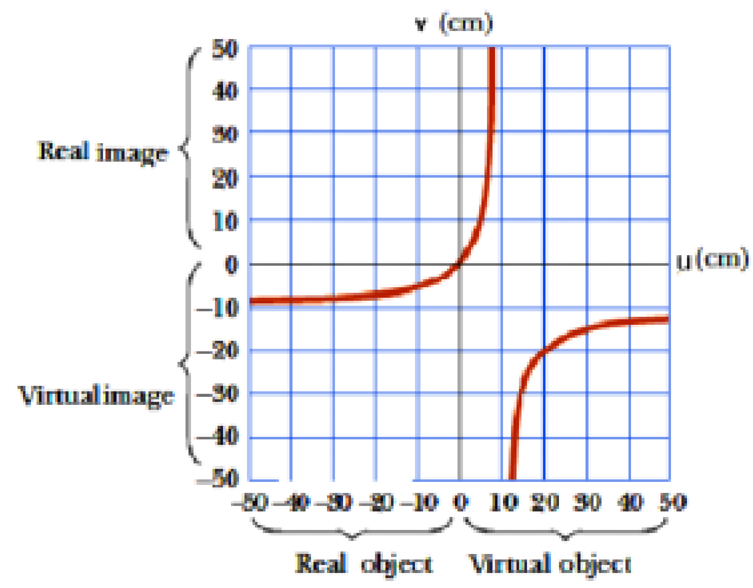


Illustration :

A thin equiconvex lens of refractive index 3/2 and radius of curvature 30 m is put in water (refractive index $= \frac{4}{3}$). Find its focal length.

Sol.
$$\frac{1}{f} = \left(\frac{\mu_1}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{0.3} + \frac{1}{0.3}\right)$$

or
$$\frac{1}{f} = \left(\frac{9}{8} - 1\right)\left(\frac{2}{0.3}\right)$$

or
$$\frac{1}{f} = \frac{1}{8} \times \frac{2}{0.3}$$

or $f = 1.20\,\text{m}$.

Illustration :

A lens of focal length f projects m times magnified image of an object on a screen. Find the distance of the screen from the lens.

Sol. *Image will be real.*

We know that

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{v}{f} = 1 - \frac{v}{u}$$

$$\Rightarrow \quad \frac{v}{f} = 1 + m \qquad [\because \text{ u is negative}]$$

$$\Rightarrow \quad v = f(m+1).$$

Practice Exercise

Q.1 A glass convex lens of refractive index (3/2) has got a focal length equal to 0.3 m. Find the focal length of the lens if it is immersed in water of refractive index (4/3).

Q.2 A projector lens marks an image of an object on a screen 6 m from the lens. If the magnification is 24, what is the focal length of the lens?

Q.3 A thin glass (refractive index 1.5) lens has optical power of -5 D in air. Find optical power in a liquid medium with refractive index 1.6 .

Q.4 Find the relation between n_1 and n_2 if the behaviour of a light ray is as shown in the figure aside .

Q.5 A point object is located at a distance of 15 cm from the front surface thick bi–convex lens. The lens is 10 cm thick and radii of its front and back surfaces are 10 cm and 25 cm respectively. How far beyond the back surface of this lens ($m = 1.5$) is the image formed?

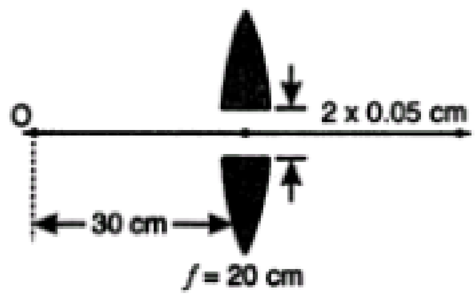
Q.6 An equiconvex lens of refractive index (3/2) and focal length 10 cm in air is held with its axis vertical and its lower surface immersed in water ($m = 4/3$), the upper surface being in air. At what distance from the lens, will a vertical beam of parallel light incident on the lens be focused?

Q.7 A magnifying lens has a focal length of 10 cm. (a) Where should the object be placed if the image is to be 30 cm from the lens? (b) What will be the magnification?

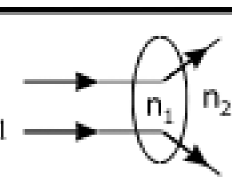
Q.8 A converging beam of light forms a sharp image on a screen. A lens is placed in the path of the beam, the lens being 10 cm from the screen. It is found that the screen has to be moved 8 cm further away from the lens to obtain a sharp image. Find the focal length and nature of lens.

Q.9 An object 25 cm high is placed in front of a convex lens of focal length 30 cm. If the height of image formed is 50 cm, find the distance between the object and the image?

Q.10 A point object O is placed at a distance of 0.3 m from a convex lens (focal length 0.2 m) cut into two halves each of which is displaced by 0.0005 m as shown in figure. Find the position of the image. If more than one image is formed, find their number and distance between them.



Answers

Q.1 1.2 m   Q.2 0.24 metre   Q.3 1 D   Q.4 $n_2 > n_1$ 

Q.5 200cm   Q.6 20 cm   Q.7 (a) –7.5 cm (b) 4   Q.8 –22.5 cm

Q.9 Case 1 (If the image is inverted i.e., real) $\rightarrow$ 90 cm ; Case 2 (If the image is erect i.e., virtual) $\rightarrow$ 15 cm

Q.10 0.3 cm

Note:

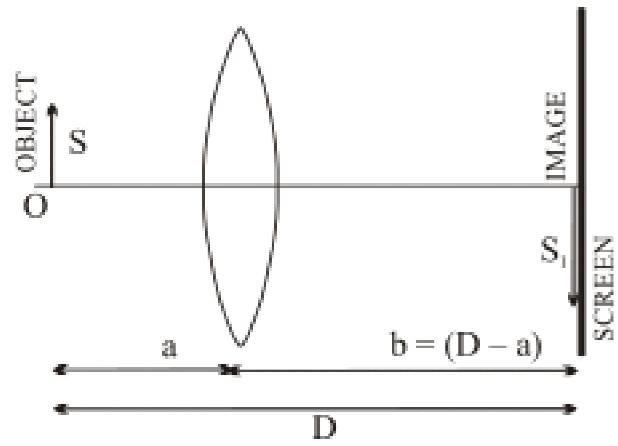
All the formula derived earlier is valid only for thin lens and applicable when lens is thin as well as medium on both sides of the lens is same. If the lens is thick or medium on both sides of the lens is different then we have to work with each surface step by step.

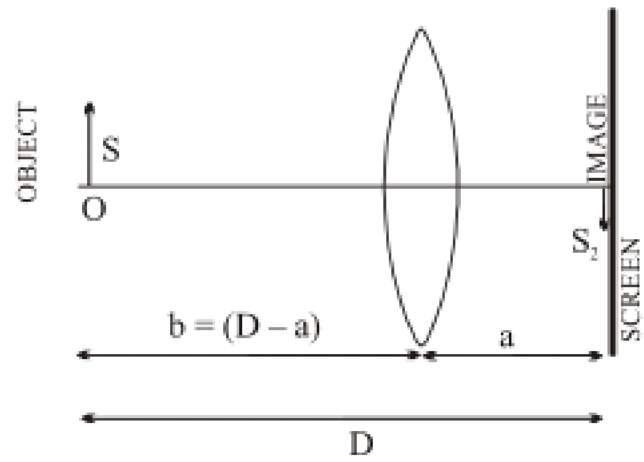
Displacement method :

This is a laboratory method to find the focal length of convex lens. In displacement method, in two different situation real image of a real object are formed on the screen for given position of object and screen by displacing the lens.

If a thin converging lens of focal length 'f' is placed between an object and a screen fixed at a distance D apart and if D > 4f, then there are two positions of the lens at which a sharp image of the object is formed on the screen.

If the object is at a distance 'u' from the lens, the distance of image from the lens v = (D – u). So from lens formula,





$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we have, $$\frac{1}{(D-a)} - \frac{1}{-a} = \frac{1}{f}$$

i.e. $$a^2 - Da + Df = 0$$

So that $$a = \frac{1}{2}\left[D \pm \sqrt{D(D-4f)}\,\right] \qquad \text{... (i)}$$

Now there are three possibilities

(a) If D < 4f : a will be imaginary, so physically no position of lens is possible.

(b) If D = 4f : In this situation a = D/2 = 2f. So only one position is possible and in this situation,

$b = D - a = 4f - 2f = 2f\ (= a)$

(c) If D > 4f : In this situation both the roots of equation (i) will be real.

i.e. $a_1 = a = \frac{1}{2}\left[D - \sqrt{D(D-4f)}\right]$

and $a_2 = D - a = \frac{1}{2}\left[D + \sqrt{D(D-4f)}\right]$ ... (ii)

So if $d > 4f$, there are two positions of lens at distance a_1 and a_2 from the object for which real image is formed on the screen.

Let us assume $a > b$

$\Rightarrow$ $a = \frac{1}{2}\left[D - \sqrt{D(D-4f)}\right]$

and $b = \frac{1}{2}\left[D + \sqrt{D(D-4f)}\right]$

Calculation of focal length of lens :

The displacement of the lens will be

$$x = b - a = \sqrt{D^2 - 4Df}$$

$$\Rightarrow f = \frac{D^2 - x^2}{4D}$$

Note : (a) $m_1 m_2 = 1 \quad \Rightarrow \quad \left(\frac{-S_1}{S_0}\right)\left(\frac{-S_2}{S_0}\right) = 1 \qquad \Rightarrow S_o = \sqrt{S_1 S_2}$

(b) $m_2 - m_1 = \frac{v'}{u'} - \frac{v}{u} = \frac{+a}{-b} - \frac{+b}{-a} = \frac{b}{a} - \frac{a}{b} = \frac{b^2 - a^2}{ab} = \frac{(a+b)(b-a)}{ab} = \frac{Dx}{Df} = \frac{x}{f}$

Illustration :

For two positions of a converging lens between an object and a screen which are 96 cm apart, two real images are formed. The ratio of the lengths of the two images is 4.84. Calculate the focal length of the lens.

Sol. $a+b=96$ ---------- (i)

Here $\frac{m_1}{m_2} = 4.84$

$\Rightarrow \quad m_1^2 = 4.84 \quad \Rightarrow \quad m_1 = -2.2 \quad \Rightarrow \quad \frac{+b}{-a} = -2.2$

$\Rightarrow \quad b = 2.2a$ ... (ii)

Solviing we get $a = 20$ cm gives $d = 76$ cm $\Rightarrow$ x=b-a=56cm

$\therefore \quad \Rightarrow f = \frac{D^2 - x^2}{4D} = 20.625$ cm .

Practice Exercise

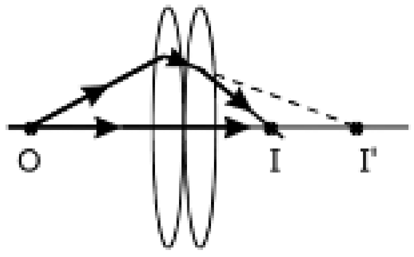
Q.1 A source and a screen are fixed in a place a distaqnce 'l' apart. A thin lens is placed betwen them at a position such that the source is focussed on the screen. For what ranges of lens focal lengths are there for (a) two (b) one (c) no such positions?

Q.2 An object is placed at a distance of 75 cm from a screen. Where should a convex lens of focal length 12 cm be placed so as to obtain a real image of the object?

Answers

Q.1 (a) f< l/4 (b) f= l/4 (c) f> l/4 Q.2 15 cm or 60 cm

Combination of two thin spherical lenses in contact



For first lens :

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}$$

For second lens :

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2}$$

Combining, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{i.e., } P_{eq} = P_1 + P_2$$

Similarly for n thin lenses in contact

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots \frac{1}{f_n}$$

$$\Rightarrow \quad P_{eq} = P_1 + P_2 + P_3 + \ldots\ldots + P_n$$

Note :

(1) If two thin lenses of equal focal but of opposite nature (i.e. one convergent and other divergent) are put in contact, the resultant focal length of the combination will be.

$$\frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{-f} = 0$$

i.e., $f_{eq} = \infty$ and $P_{2eq} = 0$

i.e., the system will be have as a plane glass plate.

(2) If two lenses of same nature are put in contact, then as (f_1 and f_2 are magnitude of focal lengths)

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

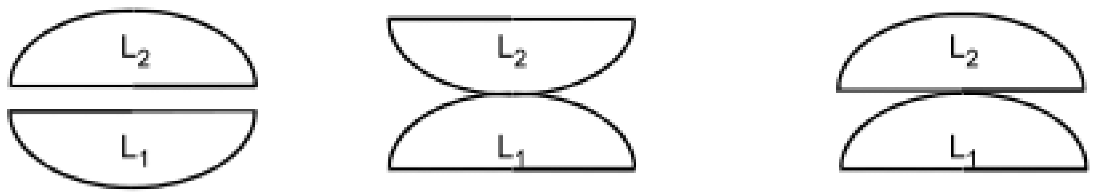
$$\frac{1}{f_{eq}} > \frac{1}{f_1} \text{ and } \frac{1}{f_{eq}} > \frac{1}{f_2}$$

The resultant focal length will be lesser than individual.

(3) If two thin lenses of opposite nature with different focal lengths are put in contact, the resultant focal length will be of same nature as that of the lens of shorter focal length but its magnitude will be more than that of shorter focal length.

(4) If a lens of focal length f is divided into two equal parts as shown in figure and each part has a focal length f' then each part will have focal length 2 times.

Now if these part are put in contact as in figure (A), (B) or (C) the resultant focal length of the combination will be equal to initial value.



(a) (b) (c)

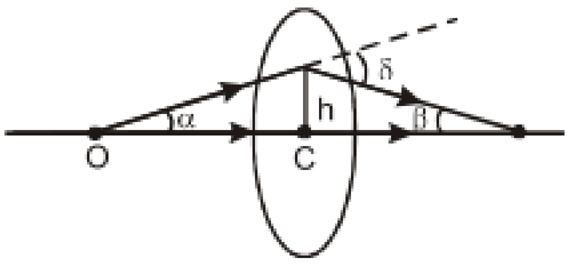
Practice Exercise

Q.1 Two plano–concave lenses of glass of refractive index 1.5 have radii of curvature of 20 and 30 cm. They are placed in contact with the curved surfaces towards each other and the space between them is filled with a liquid of refractive index (4/3). Find the focal length of the system.

Answers

Q.1 72 cm

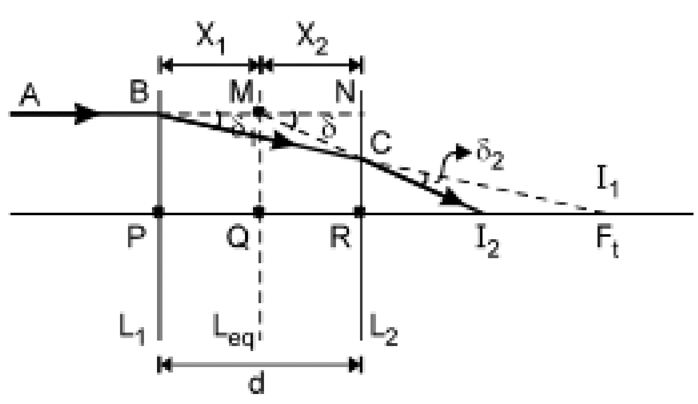
Deviation produced by thin lens.



$$\delta = \alpha + \beta = \frac{h}{-u} + \frac{h}{v} = h\left(\frac{1}{v} - \frac{1}{u}\right) = \frac{h}{f}$$

Combination of two thin spherical lenses separated by some distance

Let L_1 and L_2 are two thin lenses separated at distance 'd' from each other and focal lengths of the lenses are f_1 and f_2 respectively. We have to calculate position, focal length, power of the equivalent lens. The deviation produced by the Leq is the sum of deviations produced by L_1 & L_2.



Let a ray parallel to principal axis of the lens is coming and it is deviated by the first lens by angle d_1 and would from image at F_1 or say l_1 is the absence of L_2 but L_2 deviates it again by angle d_2 before forming image at F_1.

Produce AB by a dotted line and also l_2C. They intersect each other at a point M. Again produce BM such that it intersects L_2 at a point say N.

Draw BP, MQ and NR perpendicular to the principal axis.

Let $BP = h_1$ & $CR = h_2$

$= NR$

$= MQ$

Using $d = d_1 + d_2$ and geometry we get

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \text{i.e.} \quad P_{eq} = P_1 + P_2 - dP_1P_2$$

and position of the equivalent lens :

$$\frac{df_{eq}}{f_1} \text{ left of the second lens.}$$

Note :

The above formula is applicable for only parallel incident rays (object is situated at ∞).

Illustration :

A convergent lens of 6 diopters is combined with a diverging lens of –2 diopter. Find the power and focal length of the combination.

Sol. *Here* $P_1 = 6$ *diopter,* $P_2 = -2$ *diopter*

Power of the combination is given by

Using the formula $P = P_1 + P_2 = 6 - 2 = 4$ *diopters*

$$f = \frac{1}{P} = \frac{1}{4} = 0.25\,\text{m} = 25\,\text{cm}$$

Practice Exercise

Q.1 A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. What is the value of d if a parallel beam of light incident on A leaves B as a parallel beam?

Answers

Q.1 15 cm

Lens mirror combination

Concept of image forming at object itself :

In some unique situations the image is formed at the same point as the object. For a lens, if we apply the len formula we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

or $$\frac{1}{u}-\frac{1}{u}=\frac{1}{f}$$

which is not possible.

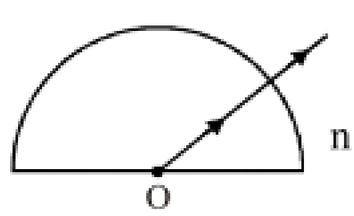
On the other hand, for the case of refraction across a single curved surface, the formula is

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$$

which reduces to

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$$

The solution for which is u = R. Therefore the object and image coincide when the object is placed at the center of curvature (figure.)



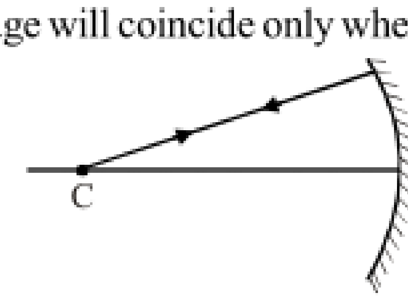
Similarly, in the case of mirrors, the formula is

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

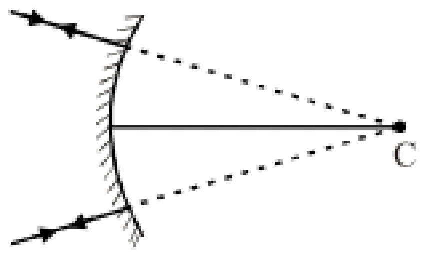
which simplifies to

$$\frac{1}{u}+\frac{1}{u}=\frac{2}{R}$$

and the solution once again is u = R. Therefore for mirrors and refraction across a single curved surface, we can say that object and image will coincide only when the object is kept at the center of curvature.



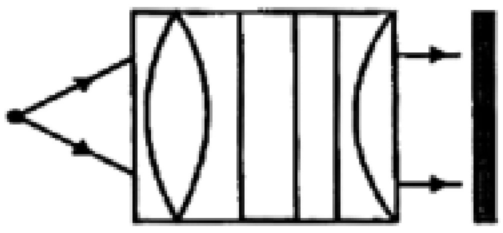
In problems in optics, we will usually have a train of optical elements with the stipulation that the image is formed on the object itself. In such cases, there will have to be a mirror at the end of the optical train and the rays have to be incident normally on the mirror in order to retrace their paths. Three kinds of mirrors are possible.



Case-1 :

A plane mirror at the end of the optical train

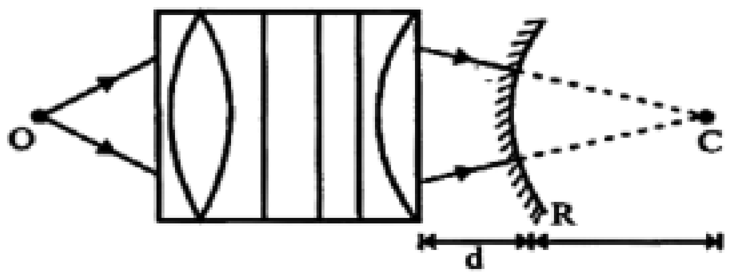
In this situation, the ray of light emerging from the system just before it impinges on the mirror has to be parallel so as to strike the mirror normally. Thus, the image after the last lens must be formed at infinity.



Case-2:

A concvex mirror of radius of curvature R

Here, the bundle of rays must convege on to the centre of curvature which is in front of the concave mirror. If the distance of the last lens from the mirror is d, we can say that the image formed from the last lens must be at a distance d + R from the lens (figure).



Case-3:

A concave mirror of radius of curvature R

Here the bundle of rays must converge on to the center of curvature of the convex of the convex mirror. If the distance of the last lens from the mirror isd, we can say that the image formed from the last lens must be at a distance (d – R) from the lens (figure)

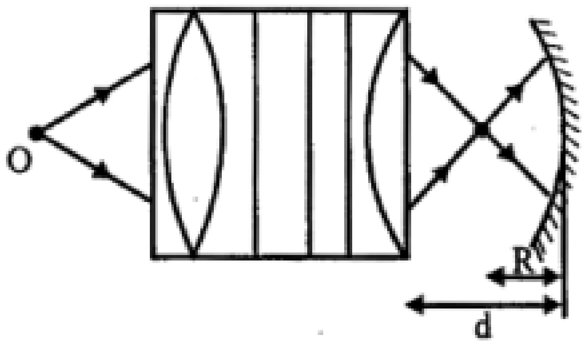


Illustration :

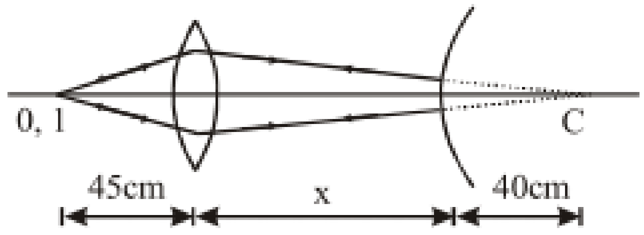
An object is placed at a distance of 45 cm from a converging lens of focal length 30 cm. A mirror of radius of curvature 40 cm is to be placed on the other side of lens so that the object coincides with its image.

Find the position of the mirror if it is

(a) convex ?

(b) concave ?

Sol. *(a) The object and image will coincide only if the light ray retraces its path and it will occur only when the ray normally strike at the mirror. In other words, the centre of curvature of the mirror and the rays incident on the mirror are collinear.*



The rays after refraction from lens must be directed towards the centre of curvature of mirror at C. If x is the separation, then for the lens

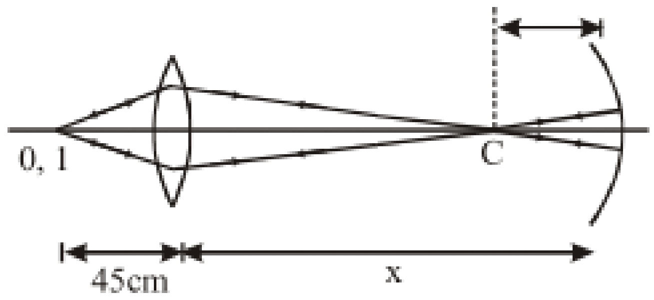
$$u = -45\text{cm}, v = x + 40, f = 30\,\text{cm}$$

Using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or $\frac{1}{x+40} - \frac{1}{-45} = \frac{1}{30}$

or $x = \frac{45(30)}{45-30} - 40 = 50\,\text{cm}$

(b)



In case of concave mirror, the refracted rays through the lens meets at C, the centre of curvature (C) of the mirror.

Using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$u = -45\text{ cm}, v = x - 40, f = 30\text{ cm}$

$$\frac{1}{x-10} - \frac{1}{-45} = \frac{1}{30}$$

or $x - 10 = \frac{(45)\times 30}{45-30}$

or $x = 90 + 40 = 130\,\text{cm}$.

Practice Exercise

Q.1 A concave lens of focal length 20 cm is placed 15 cm in front of a concave mirror of radius of curvature 26 cm and further 10 cm away from the lens an object is placed . The principal axis of the lens and the mirror are coincident and the object is on this axis. Find the position and nature of the image.

Q.2 A point object is placed at a distance of 12 cm on the axis of a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at a distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of convex mirror?

Q.3 An object 2 cm high is placed in front of a double convex lens of focal length 12.5 cm. On the other side of the lens a concave mirror of focal length 10 cm is placed at a distance of 45 cm. If the separation between the object and the mirror is 70 cm, calculate the location, nature and magnification of the image.

Answers

Q.1 Final image will be inverted, real and 8 times of the object   Q.2 25 cm

Q.3 25 cm, inverted, real and of same size as object, –1

Effect of Silvering A surface of a Thin spherical Lens

If a surface of a thin spherical lens is silvered it behaves like a mirror and we can calculate is focal length. Let a thin spherical lens is polished at the right face. The radii of curvature of the left and right faces are R_1 and R_2.

When a ray of light becomes incident on this silvered lens it will be first refracted by the lens, then reflected from mirror and again refracted by the lens .

Hence power of equivalent mirror can be written as

$$P_{eq} = P_{lens} + P_{mirror} + P_{lens}$$

$$\Rightarrow \quad P_{eq} = 2P_{lens} + P_{mirror}$$

$$\Rightarrow \quad \left(-\frac{1}{f_{eq}}\right) = 2\left(\frac{1}{f_{lens}}\right) + \left(-\frac{1}{f_{mirror}}\right)$$

$$\Rightarrow \quad \frac{1}{f_{eq}} = -\frac{2}{f_{lens}} + \frac{1}{f_{mirror}}$$

Illustration :

One face of an equiconvex lens of focal length 60 cm made of glass $(\mu = 1.5)$ *is silvered. Does it behave like a concave mirror or convex mirror?*

Sol. *here* $f_l = +60$ *cm (converging lens)*

$f_m = -30$ *(converging mirror)*

$$\Rightarrow \quad \frac{1}{f_{eq}} = -\frac{2}{f_{lens}} + \frac{1}{f_{mirror}} = -\frac{2}{+60} + \frac{1}{-30}$$

$$\therefore \quad f_{eq} = -15$$

The positive sign indicates that the resulting mirror is converging or concave.

Practice Exercise

Q.1 A thin hollow equiconvex lens, silvered at the back, converges a parallel beam of light at a distance of 0.2 m in front of it. Where will it converge the same light if filled with water having m = 4/3 ?

Q.2 A plano convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object ?

Q.3 A pin is placed 10 cm infront of a convex lens of focal length 20 cm, made of material having refractive index 1.5. The surface of the lens further away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. Is the image real or virtual?

Answers

Q.1 –12 cm Q.2 20 cm Q.3 –11 cm and real

Optical defects of mirrors & lenses

In the formation of image we have considered :

(i) Incident rays are paraxial

(ii) Incident rays are monochromatic

But practically all these points are not perfectly correct hence image is defected. The defects (aberrations) are classified broadly in two parts.

Aberrations

Chromatic aberration

When the rays used in the formation of image having more than one wavelength then the defect in image only due to presence of more than one wavelength.

Monochromatic aberration

When the rays taken in image formation are monochromatic, then the defect produced is called monochromatic aberration. This defect is also called seidel aberration.

Spherical aberration

The defect in image produced in the formation of image of an axial point object (of monochromatic light) by a spherical mirror or lens is called spherical aberration.The image of an object in point object formed by a spherical mirror or by a spherical lens is usually blurred. This defect of image is called spherical aberration

Methods to reduce spherical aberration :

A. For mirrors : By using a proper surface e.g., paraboloidal surface for parallel incident beam.

B. For lenses : In lenses spherical aberration cannot be completely vanished. It can be minimized only.

(i) By using stops.

(ii) By using crossed lens.

Note : for minimum spherical aberration.

$$\frac{R_1}{R_2} = \frac{2\mu^2 - \mu - 4}{\mu(2\mu + 1)}$$

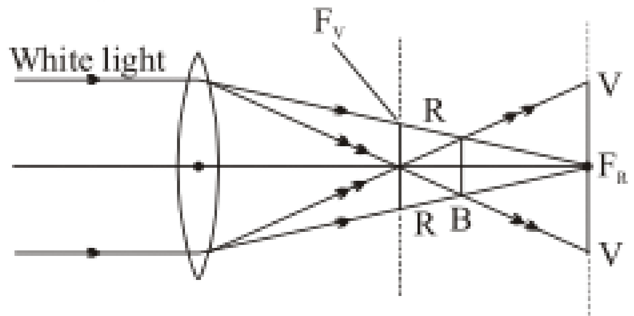
(iii) By using combination of lenses, $d = f_1 - f_2$.

Chromatic aberration

The image of an object in white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colours. For a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

and an μ of lens in maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it as shown in figure.



As a result of this in case of convergent lens, at F_V centre of image will be violet and focused while sides red and blurred while at F_R reverse is the case, i.e., centre will be red and focused while sides violet and blurred. The difference between f_V and f_R is a measure of longitudinal chromatic aberration, i.e.,

$$\text{L.C.A.} = f_R - f_V = -df$$

with $df = f_V - f_R$ ... (1)

However, as for a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{... (2)}$$

i.e.,
$$-\frac{df}{f^2} = d\mu\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{... (3)}$$

So dividing equation (3) by (2)

$$-\frac{df}{f} = \frac{d\mu}{(\mu-1)} = \omega\left[\text{as } \omega\ \frac{d\mu}{(\mu-1)}\right] \quad \text{... (4)}$$

And hence, from equation (1) and (4),

$$\text{L.C.A.} = -df = \omega f \quad \text{... (5)}$$

Now, as for a single lens neither f nor ω can be zero, we cannot zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism

In case of two thin lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{i.e.,} \qquad -\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if $dF = 0$

i.e.,
$$\frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

which in the light of equation (5) reduces to

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0$$

i.e.,
$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad \text{... (6)}$$

This condition is called condition of achromatism (for two thin lenses in contact) and the lens combination which satisfies this condition achromatic lens. From this condition, i.e., from equation (6) it is clear that in case of achromatic doublet :

(1) The two lenses must be of different materials.

Since, if $\omega_1 = \omega_2$, $\frac{1}{f_1} + \frac{1}{f_2} = 0$, i.e., $\frac{1}{F} = 0$ or $F = \infty$

i.e., combination will not behave as a lens, but as a plane glass plate.

(2) As ω_1 and ω_2 are positive quantities, for eq. (6) to hold, f_1 and f_2 must be of opposite nature, i.e. if one of the lenses is convex the other must be concave.

(3) If the achromatic combination is convergent,

$f_C < f_D$ and as $-\frac{f_C}{f_D} = \frac{\omega_C}{\omega_D}$, $\omega_C < \omega_D$

i.e., a convergent achromatic doublet, convex lens has lesser focal length and dispersive power than divergent one.

(ii) For lenses separated by a distance : $d = \frac{\omega_2 f_1 + \omega_1 f_2}{\omega_1 + \omega_2}$

Practice Exercise

Q.1 An optical doublet is formed from two lenses A and B made of glass of different refractive indices μ_A, μ_B respectively. Lens A has two convex sides of radius of curvature R and lens B has one flat side and one concave side of radius of curvature R. What is the power of the doublet? For red, yellow and blue wavelengths, the refractive index μ_A is 1.50, 1.51 and 1.52 respectively whereas μ_B is 1.60, 1.62 and 1.64 respectively. What is the difference in power of the doublet for these three wavelengths?

Q.2 Two lenses, one made of crown glass and the other of flint glass, are to be combined so that the combination is achromatic for the blue and red light and acts as a convex lens of focal length 35 cm. Calculate the focal length of the components if for –

Crown glass	$\mu_Y = 1.5175$ and $(\mu_B - \mu_R) = 0.00856$
Flint glass	$\mu_Y = 1.6214$ and $(\mu_B - \mu_R) = 0.01722$

Q.3 An equiconvex lens of crown glass and an equiconcave lens of flint glass make an achromatic system. The radius of curvature of convex lens is 0.54 m and the refractive indices for the crown glass are $\mu_R = 1.53$ and $\mu_V = 1.55$, find the dispersive power of flint glass.

Q.4 A telescope objective of focal length 60 cm is made of two thin lenses, one of crown glass of refractive index 1.52 and other of flint glass refractive index 1.66. One surface of the flint glass is plane. Calculate the radii of curvature of both the lenses which form the achromatic doublet if dispersive powers of crown and flint glass are 0.0151 and 0.0302 respectively.

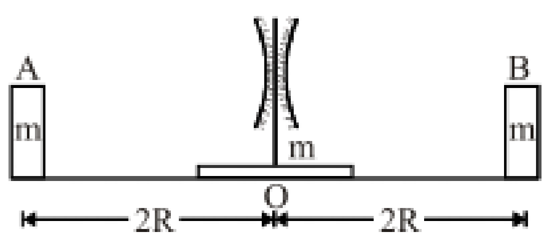
Answers

Q.1 $\frac{1}{R}[2\mu_A - \mu_B - 1]$, 0 Q.2 $f_C = 14.17$ cm & $f_D = 23.8$ cm Q.3 0.055

Q.4 The radii of curvature of convex lens (of crown glass) are 25.74 cm and 39.6 cm respectively while of concave lens (of flint glass) are 39.6 cm and ∞ respectively

Solved Example

Q.1 Two concave mirrors of equal radii of curvature R are fixed on a stand facing opposite directions. The whole system has a mass m and is kept on a frictionless horizontal table (figure).



Two block A and B, each of mass m, are placed on the two sides of the stand. At t = 0, the separation between A and the mirror is 2R and also the separation between B and the mirror is 2R. The block B moves towards the mirror at a speed v. All collisions which take place are elastic. Taking the original position of the mirrors standard system to be x = 0 and x-axis along AB, find the position of the images of A and B at

(a) $t = \frac{R}{v}$, (b) $t = \frac{3R}{v}$ (c) $t = \frac{5R}{v}$

Sol. (a) At $t = \frac{R}{v}$

For block A, $u = -2R$

$\therefore \quad \frac{1}{v} + \frac{1}{-2R} = \frac{2}{-R}$

or $\quad v = \frac{-2R}{3}$

For block B : The distance travels by block B in time $\frac{R}{v}$ is R

Thus $u = -R$

$$\frac{1}{v} + \frac{1}{-R} = \frac{2}{-R}$$

or $\quad v = R$

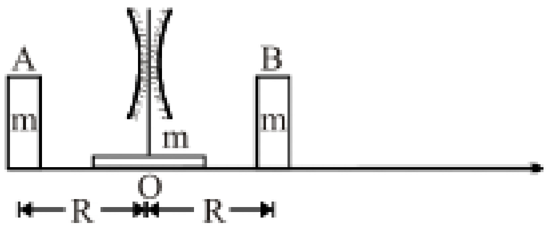
The x-coordinate of the image of the block with respect to the mirror will be +R.

(b) At $\quad t = \frac{3R}{v}$

The block B will collide with the stand after time $\frac{2R}{v}$

After collision block B becomes at est and mirror starts moving with the same velocity v. In the remaining time R/v, the distance moved by the mirror is R.

The position of blocks and mirror are shown in figure.



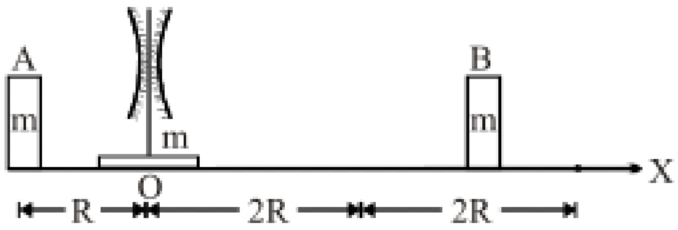
At this time the blocks lie at the centre of curvature of the respective mirrors. Their images will form at the centres of cuvature. So their co-ordinates are :

For block A, $x = -R$

For block B, $x = +R$

(c) At $t = \frac{5R}{v}$.

The block B will collide to the mirror after a time $\frac{2R}{v}$. Thereafter mirror starts moving towards block **A** with velocity v. At $t = \frac{4R}{v}$, the mirror will collide with block A and stops after collision. The positions of blocks and mirror are shown in figure.



For block A : Its image will form on the same place. Therefore the positions of the blocks are

$$x_A = -3R$$

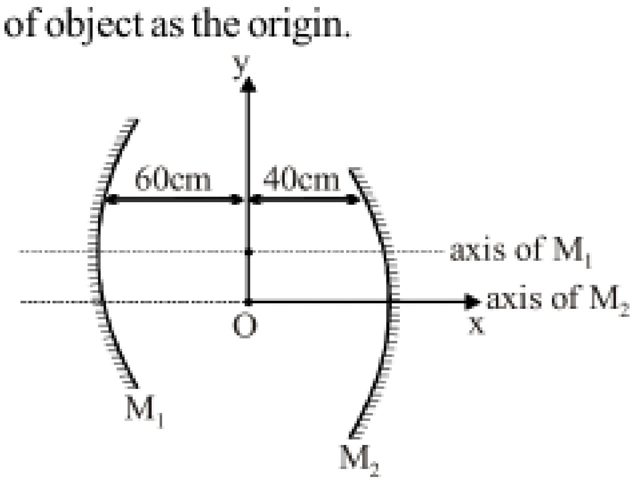
For block B : $u = -2R$

$$\frac{1}{v} + \frac{1}{-2R} = \frac{2}{-R}$$

$$v = -\frac{2R}{3}$$

The co-ordinates of B are $-\left(2R - \frac{2R}{3}\right) = \frac{-4R}{3}$

Q.2 Two concave mirrors each of radius of curvature 40 cm are placed such that their principal axes are parallel to each other and at a distance of 1cm to each other. Both the mirrors are at a distance of 100 cm to each other. Consider first reflection at M_1 and then at M_2, find the coordinates of the image thus formed. Take locatioin of object as the origin.



Sol. Using mirror formula for first reflection :

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}\Rightarrow\frac{1}{-20}=\frac{1}{v}+\frac{1}{-70}$$

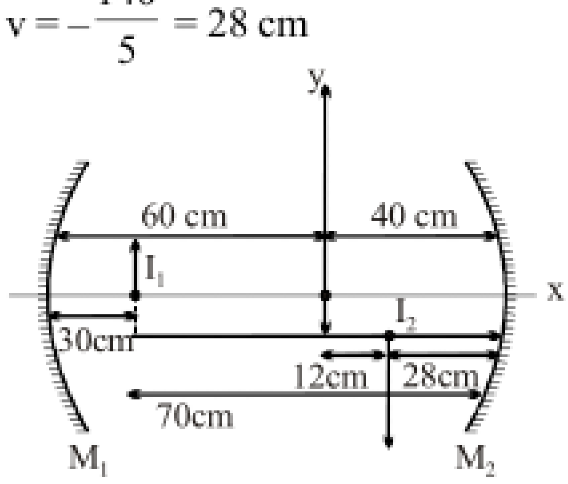
$$\Rightarrow \quad \frac{1}{v}=\frac{1}{60}-\frac{1}{20}\Rightarrow v=-30\text{ cm}$$

Using mirror formula for second reflection

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}\Rightarrow\frac{1}{-20}=\frac{1}{v}+\frac{1}{-70}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{70}\quad\frac{1}{20}=\frac{2-7}{140}$$

$$\Rightarrow \quad v=-\frac{140}{5}=28\text{ cm}$$



Height of $I_2\Rightarrow m=\left(\frac{-30}{-60}\right)=\frac{I_1}{-1}$

$\Rightarrow I_1=\frac{1}{2}$ cm

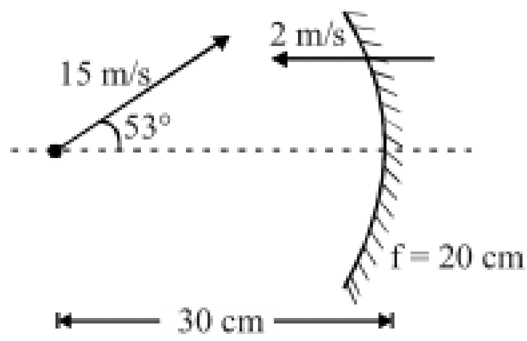
Height of first image from x-axes $=1+\frac{1}{2}=\frac{3}{2}$ cm

Height of $I_2\Rightarrow m=\left(\frac{-28}{-70}\right)=\frac{2I_2}{3}$

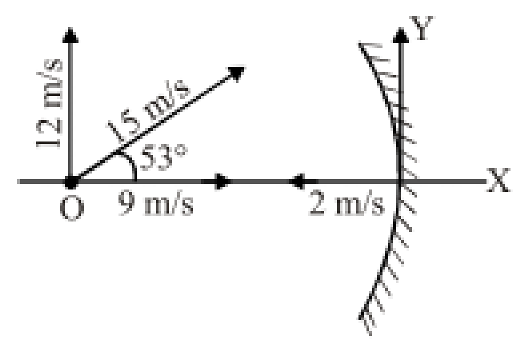
$\Rightarrow I_2=\frac{3\times 28}{2\times 70}$ $I2=0.6$ cm

Co-ordinate of $I_2=(12-0.6)$

Q.3 Find the velocity of image in situation as shown in figure.



Sol. $\vec{V}_O$ = Velocity of obeject = $(9\hat{i}+2\hat{j})\,m/s$



$\vec{V}_m$ = Velocity of mirror $= -2\hat{i}$ m/s

$$m = \frac{f}{f-u} = \frac{-20}{-20-(-30)} = -2$$

For velocity component parallel to optical axis

$$(\vec{V}_{I/m})_{||} = -m^2\,(\vec{V}_{O/m})_{||}$$

$$(\vec{V}_{I/m})_{||} = (-2)^2\,11\,\hat{i} = -44\,\hat{i}\ \text{m/s}$$

For velocity component perpendicular to optical axis

$$(\vec{V}_{I/m})_{\perp} = (\vec{V}_{O/m})_{\perp}$$

$$= (-2)\ 12\,\hat{j} = -24\,\hat{j}\ \text{m/s}$$

$\therefore$ $\vec{V}_{I/m}$ = Velocity of image w.r.t. mirror

$$= (\vec{V}_{I/m})_{||} + (\vec{V}_{I/m})_{\perp}$$

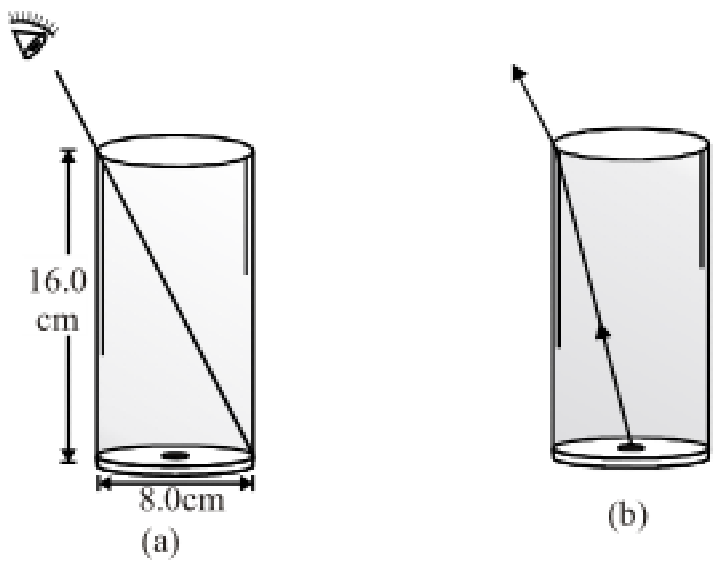
$$= (-44\,\hat{i} - 24\,\hat{j})\,m/s$$

Also, $\vec{V}_{I/m} = \vec{V}_I - \vec{V}_m$

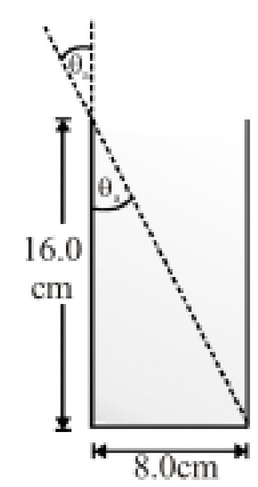
or $\vec{V}_I = (-44\hat{i} - 24\hat{j}) - 2\hat{i}$

$$= (-46\hat{i} - 24\hat{j})\,m/s$$

Q.4 You sight along the rim of a glass with vertical sides so that the top rim is lined up with the opposite edge of the bottom. The glass is a thin-walled, hollow cylinder 16.0 cm high with a top and bottom of the glass diameter of 8.0 cm. While you keep your eye in the same position, a friend fills the glass with a transparent liquid, and you then see a dime that is lying at the center of the bottom of the glass. What is the index of refraction of the liquid ?



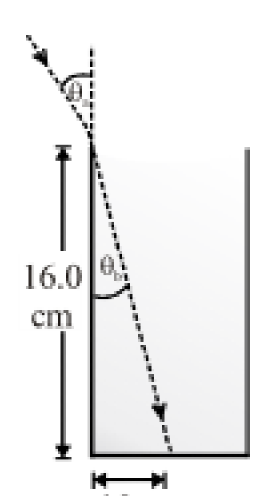
Sol. Use geometry of find the angles of incidence and refraction. Before the liquid is poured in the ray along your line of sight has the path shown in figure.



$$\tan \theta_a = \frac{8.0\text{cm}}{16.0\text{cm}} = 0.500$$

$$\theta_a = 26.57^\circ$$

After the liquid is poured in, θ_a is the same and the refracted ray passes throught the center of the bottom of the glass as shown in figure.



$$\tan \theta_b = \frac{4.0\text{cm}}{16.0\text{cm}} = 0.250$$

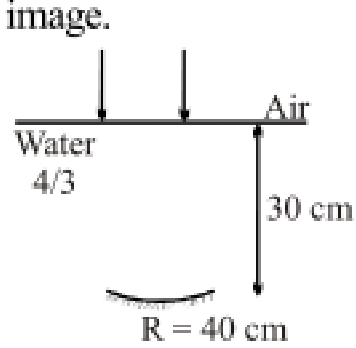
$$\theta_b = 14.04^\circ$$

Use Snell's law to find n_b the refractive index of the liquid:

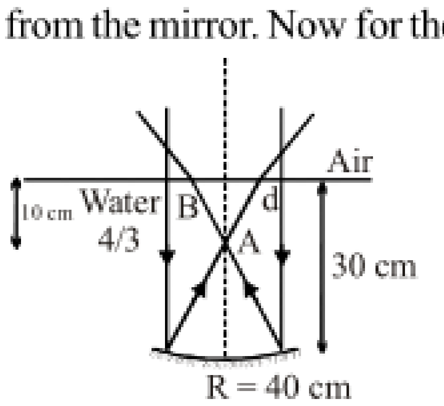
$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$n_b = \frac{n_a \sin\theta_a}{\sin\theta_b} = \frac{(1.00)(\sin 26.57^\circ)}{\sin 14.04} = 1.84$$

Q.5 A concave mirror is placed inside water with its shining surface upwards and principal axis of concave mirror. Find the position of final image.

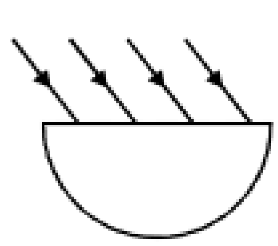


Sol. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore for the mirror, object is at ∞. Its image A (in figure) will be formed at focus which is 20 cm from the mirror. Now for the interface between water and air, d = 10 cm.



$$\therefore \qquad d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5 \text{ cm}$$

Q.6 Rays of light fall on the plane surface of a semicylinder of refractive index $n = \sqrt{2}$, at angle 45° in the plane normal to the axis of cylinder. Discuss the condition that the rays do not suffer total internal reflection.

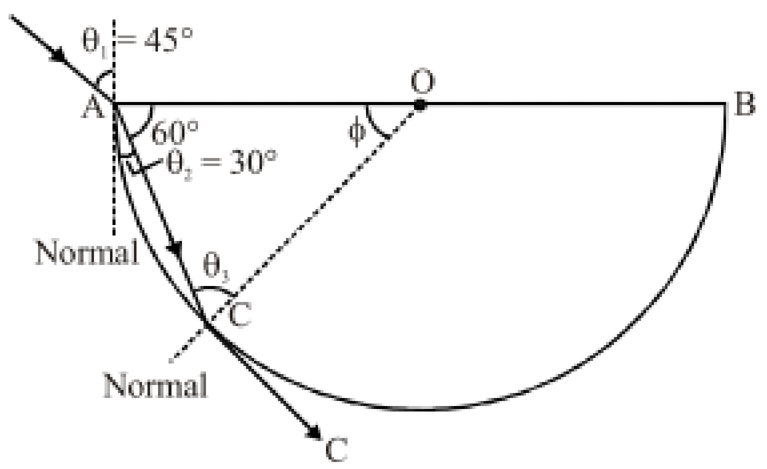


Sol. First we consider a ray incident at A. From Snell's law,

$$1 \sin 45^\circ = \sqrt{2} \sin\theta_2$$

$$\sin\theta_2 = \frac{1}{2}$$

$$\theta_2 = 30^\circ$$



Let the angle $\phi = \angle AOC$ denote the position of the point C on the curved surface.

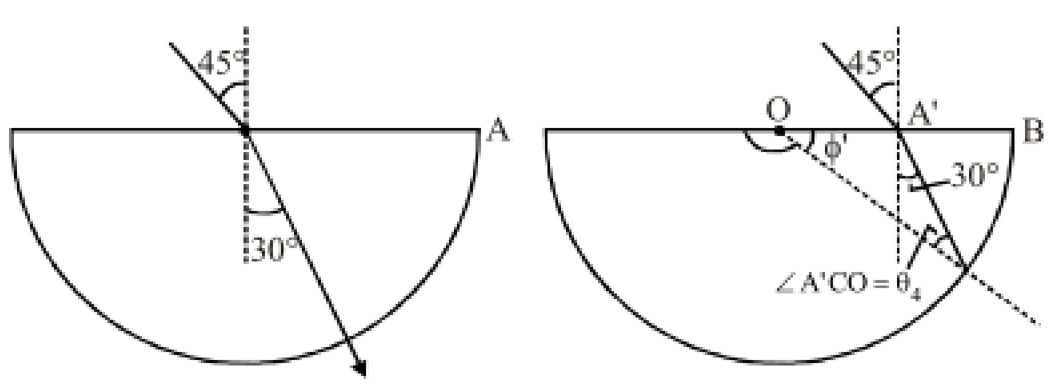
$$\angle CAO = 60°$$

The critical angle for glass to air interface can be determined from Snell's law.

$$n \sin C = 1 \sin 90°$$

$$\sin C = \frac{1}{n} = \frac{1}{\sqrt{2}}$$

$$C = 45°$$



If total internal reflection has to take place at the curved surface, angle θ_3 must be greater than the critical angle, $C = 45°$.

As $\theta_3 = 180° - \phi - 60°$, therefore, for no total internal reflection,

$$180° - \phi - 60° < 45°$$

or $\quad \phi > 75°$

When the ray falls at O, the refracted ray will move radially out, without deviation. The normal rays do not suffer deviation. Next we consider a ray to the right of O.

For no total internal reflection,

$$\angle A'CO < 45°$$

In $\Delta OA'C$, $\angle OAA' + \angle A'CO + \angle COA' = 180°$

$120° + \theta_4 + (180 + \phi) = 180°$

$$\theta_4 = \phi - 120°$$

Thus, $\quad \phi - 120° < 45°$

$$\phi < 165°$$

Hence for rays to transmit through curved surface,

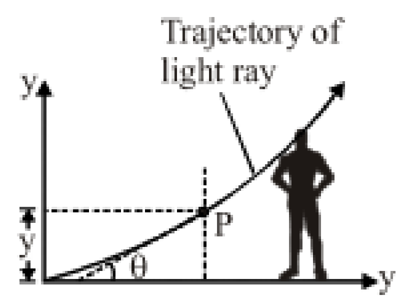
$$75° < \phi < 165°$$

Q.7 Due to a vertical temperature gradient in the atmosphere the index of refraction varies. Suppose index of refraction varies as $n = n_0\sqrt{1+ay}$ where n_0 is the index of refraction at the surface and $a = 2.0 \times 10^{-6}\ m^{-1}$. A person of height $h = 2.0$ m stands on a level surface. Beyond what distance he cannot see the runway ?

Sol. Let O be the distant object just visible to the man. Let P be a point on the trajectory of the ray. From figure, $\theta = 90 - i$.

The slope of tangent at point P is $\tan\theta = dy/dx = \cot i$. From Snell's law, $n \sin i$ = constant

At the surface $n = n_0$ and $i = 90°$



$$n_0 \sin 90° = n \sin i = (n_0\sqrt{1+ay})\ \sin i$$

$$\sin i = \frac{1}{\sqrt{1+ay}}$$

$$\cot i = \frac{dy}{dx} = \sqrt{ay}$$

$$\int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx$$

$$x = 2\sqrt{\frac{y}{a}}$$

On substituting $y = 2.0$ m and $a = 2 \times 10^{-6}\ m^{-1}$, we have

$$x_{max} = 2\sqrt{\frac{2}{2\times10^{-6}}} = 2000\ m$$

Q.8 A ray of light passes through an equilateral prism such that the angle of incidence and the angle of emergence are both equal to 3/4th of the angle of prism. Find the angle of minimum deviation .

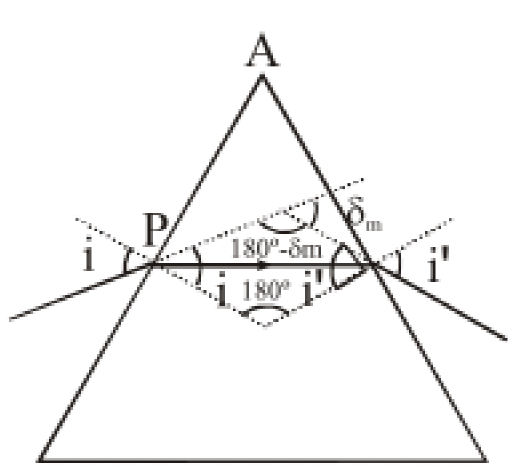
Sol. Givne $A = 60°$

$$i = i' = \frac{3}{4}A = 45°$$

$\because$ $i + i' = A + \delta$

or $90° = 60° + \delta$

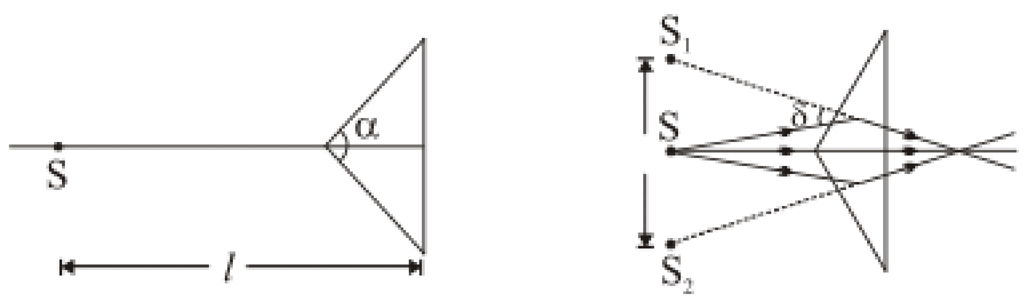
$\therefore$ $\delta = 30°$



Note that $i = i'$ is the condition for minimum deviation.

Hence $\delta = 30° = \delta_{min}$.

Q.9 A thin biprism (figure) of obtuse angle $\alpha = 178°$ is placed at a distance $l = 20$ cm from a slit. How many images are formed and what is the separation between them ? Refractive index of he material $\mu = 1.6$.



Sol. Two image are formed by the two thin prisms-one above the axis and the oher below the axis by the same distance. The fracting angle of each thin prism $= \dfrac{\pi - \alpha}{2} = \dfrac{1}{2}(\pi - \alpha)$ where α is the obtuse angle in radian.

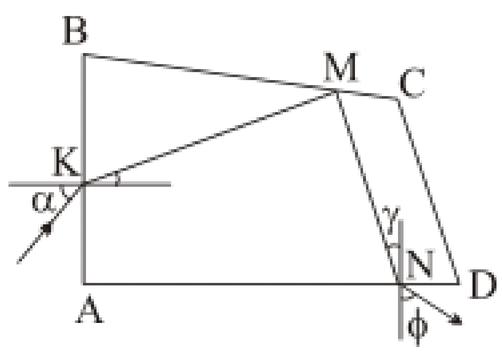
Then δ (deviation of a ray) $= (\mu - 1)\dfrac{1}{2}(\pi - \alpha)$

$\therefore \dfrac{d}{2} = l\delta$

or $\quad d = 2l(\mu - 1)\dfrac{1}{2}(\pi - \alpha)$

Here $\quad d = (1.6 - 1) \times 0.20\left(\pi - 178 \times \dfrac{\pi}{180}\right) = 0.6 \times 0.20 \times \pi \times \dfrac{1}{90} = 0.004 \text{ m} = 4 \text{ mm}$

Q.10 The faces of prism ABCD made of glass with a refraction index n form dihedral angle: $\angle A = 90°$, $\angle B = 75°$, $\angle C = 135°$ and $\angle D = 60°$ (the Abbe prism). A beam of light falls on face AB and after complete internal reflection from face BC escapes through face AD. Find the angle of incidence α of the beam onto face AB if a beam that has passed through the prism is perpendicular to the incident beam.



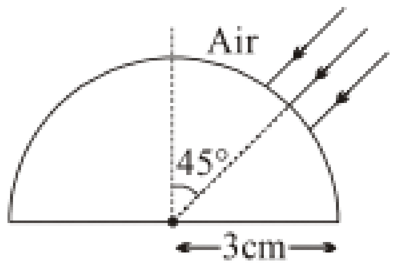
Sol. According to the initial condition, the incident beam and the beam that has passed through the prism are mutually perpendicular. Therefore, $\angle\phi = \angle\alpha$ and also $\angle\gamma = \angle\beta$ (figure). The sum os the angles of the quadrangle AKMN is 360°. Therefore, $\angle KMN = 90°$ and beam KM is incident on to face BC at an angle of 45°. If we know the angles of triangle KBM, it is easy to find that $b = 30°$. In confomity with the

law of refraction, $\dfrac{\sin\alpha}{\sin\beta} = n$

Hence, $\quad \sin\alpha = 0.5$ n and arc sin 0.5 n

Since full internal reflection at an angle of 45° is observed only when $n \geq \sqrt{2}$, the angle a is within $45° \leq a \leq 90°$.

Q.11 Shows a transparent hemisphere of radius 3.0 cm made of ia material of refractie index 2.0 :



(a) A narrow beam of parallel rays is incident ofthe hemisphere as shown in figure. Are the rays totally reflected at plane surface ?

(b) Find the image formed by refraction at the first surface.

(c) Find the image formed by the reflection or by refraction at the plane surface.

Sol. (a) The critical angle for material air interface

$$\sin C = \frac{1}{\mu} = \frac{1}{2}$$

$\therefore\ C = 30°$

The rays are incident normally on the spherical surface, so they pass undeviated and then incident of plane face at an angle 45°. As the angle of incidence is greater than critical angle (30°), so rays get totally reflected.

(b) For spherical surface :

$$u = \infty$$

We have $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

or $\dfrac{2}{v} - \dfrac{1}{\infty} = \dfrac{2-1}{R}$

$\therefore\quad v = 2R$

Thus the image will form on diametrically opposite point.

(c) Some of the rays get totally reflected and so they will form the image at I_2.

Q.12 (a) A ray of light suffers an internal reflection inside a water drop. Find the condition for minimum deviation, the angle of incidence at minimum deviation and the value of minimum deviajtion.

(b) A source and a screen are held fixed at a distance l from each other] A thin lens is placed between them such that the source is foucused on the screen. For, what values of focal length of the lens there are one, two or no positions for the lens ?

Sol. D(deviation) $= (i - \theta) + (\pi - 2\theta) + (i - \theta) = \pi + 2i - 4\theta$

$$\frac{dD}{di} = 2 - 4\frac{d\theta}{di}$$

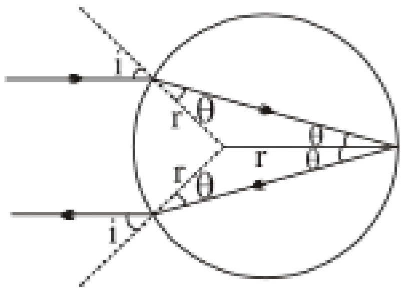
By Snell's law, $\mu \sin\theta = \sin I$

Differentiating w.r.t. 'i' we get,

$$\mu \cos\theta \frac{d\theta}{di} = \cos i$$

$$\Rightarrow \quad \frac{dD}{di} = 2 - 4\left[\frac{\cos i}{\mu\cos\theta}\right]$$

When D is minimum $\frac{dD}{di} = 0$



$$\Rightarrow \quad 2 - \frac{4\cos i}{\mu\cos\theta} = 0$$

$$2 = \frac{4\cos i}{\mu\cos\theta}$$

$$\mu\cos\theta = 2\cos i$$

$$\mu \bullet \sqrt{1-\sin^2\theta} = 2\cos i$$

$$\mu \bullet \sqrt{1-\frac{\sin^2 i}{\mu^2}} = 2\cos i$$

$$\Rightarrow \quad \mu^2 - \sin^2 i = 4\cos^2 i = 4[1 - \sin^2 i]$$

$$\Rightarrow \quad \mu^2 = 4 - 3\sin^2 i$$

$$\sin i = \sqrt{\frac{4-\mu^2}{3}}$$

$$\Rightarrow \quad i = \sin^{-1}\sqrt{\frac{4-\mu^2}{3}}$$

$$D_{min} = \pi + 2\sin^{-1}\sqrt{\frac{4-\mu^2}{3}} - \sin^{-1}\frac{1}{\mu}\sqrt{\frac{4-\mu^2}{3}}$$

Q.13 A lens has a power of +5 diopter in air. What will be its power if completely immersed in water ? Given

$\mu_g = \frac{3}{2}$; $\mu_w = \frac{4}{3}$

Sol. Let f_a and f_w be the focal lengths of the lens in air water respectively, then

$$P_a = \frac{1}{f_a} \quad \text{and} \quad P_w = \frac{\mu_w}{f_w}$$

$$f_a = 0.2 \text{ m} = 20 \text{ cm}$$

Using lensmaker's formula

$$P_a = \frac{1}{f_a} = (\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...(i)}$$

$$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

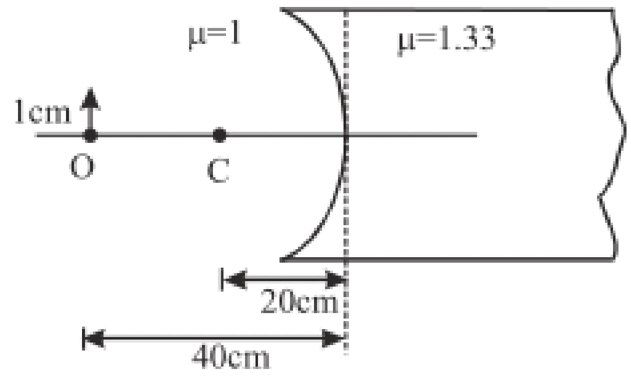
$$\Rightarrow \quad P_w = \frac{\mu_w}{f_w} = (\mu_g - \mu_w)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...(ii)}$$

Dividing equation (ii) by equation (i), we get,

$$\frac{P_w}{P_a} = \frac{(\mu_g - \mu_w)}{(\mu_g - 1)} = \frac{1}{3}$$

or $$P_w = \frac{P_a}{3} + \frac{+5}{3} D$$

Q.14 For the optical arrangement shown in the figure.



Sol. *According to Cartesian sign convention*

$u = -40\ cm, \quad R = -20\ cm$

$\mu = 1,\ \mu_2 = 1.33$

Applying equation for refraction through spherical surface, we get

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.33}{v} - \frac{1}{-40} = \frac{1.33-1}{-20}$$

After solving, $v = -32\ cm.$

The magnification is $m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u}$

$$\therefore \quad \frac{h_2}{1} = -\frac{1(32)}{1.33(-40)} \quad or \quad h_2 = 0.6\,\text{cm}$$

The positive sign shows that the image is erect.

Q.15 A glass slab of thickness 3cm and refractive index 1.5 is placed in front of a concave mirror of focal length 20 cm. Where should a point object be placed if it is to image on to itself? The glass slab and the concave mirror are shown in figure.

Sol. Let the distance of the object from the mirror be x. We know that the slab simply shifts the object. The shift being equal to

$$s = t\left[1 - \frac{1}{\mu}\right] = 1\text{cm}$$

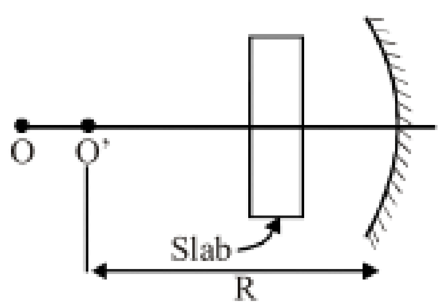
The direction of shift is towards the concave mirror.

$\therefore$ the apparent distance of the object from the mirror is $x - 1$.

If the rays are to retrace their paths, the object should appear to be at the center of curvature of the mirror.

$$\therefore x - 1 = 2f = 40 \text{ cm}$$

or $\quad x = 41$ cm from the mirror.



Q.16 An thin equiconvex lens of glass ($\mu = 1.5$) having a focal length of 30 cm in air is placed at a distance of

10 cm from a plane mirror, which in turn, is placed with its plane perpendicular to the optic axis of the lens. Water ($\mu = 4/3$) fills the space between the lens and the mirror. A parallel beam of light is incident on to the lens parallel to the principal axis.

(a) Find the position of the final image w.r.t. the optical centre of the lens.

(b) If the mirror is rotated by 1°, as shown in the figure, find the displacement of the image.

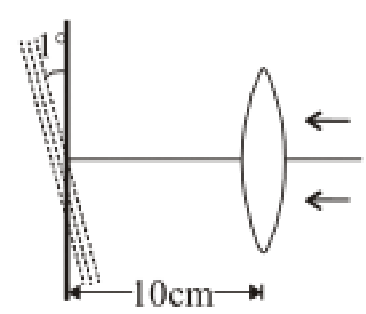
Sol. (a) The focal length of the glasslens is 30 cm

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2(\mu - 1)}{R}$$

The radius of curvature, $R = 30$ cm

The object distance, $m = \infty$.

Now we apply Gauss's law at surface S_1 ans S_2.



$$\frac{3/2}{v_1} - \frac{1}{u} = \frac{\frac{3}{2} - 1}{30} \quad \text{.... (1)}$$

$$\frac{\frac{4}{3}}{v} - \frac{\frac{3}{2}}{v_1} = \frac{-1/6}{-30} \quad \text{... (2)}$$

$\Rightarrow v = 60$ cm

After reflection from the mirror, the light rays appear to converge to a point 40 cm to the right of the convex lens. This serves as a virtual object for the lens : $u = +40$ cm

$$\frac{\frac{3}{2}}{v_1} - \frac{\frac{4}{3}}{+40} = \frac{\frac{1}{6}}{30}$$

$$\frac{1}{v} - \frac{\frac{3}{2}}{v_1} = \frac{-\frac{1}{2}}{-30}$$

$\Rightarrow v = 18$ cm to the right of the convex lens.

(b) If the mirror is rotated by 1°, the reflected ray rotaties by 2°. The virtual object for the lens formed by the reflection from the mirror is displaced by:

$$\Delta y_1 = 50 \times \frac{2\pi}{180}\text{cm}$$

The magnification due to the refraction at the two surfaces of the lens is

$$m = m_1 m_2 = \left(\frac{v}{\mu_3}\Big/\frac{v_1}{\mu_2}\right) \times \left(\frac{v_1}{\mu_2}\Big/\frac{u}{\mu_1}\right) = \left(\frac{v}{\mu_3}\Big/\frac{u}{\mu_1}\right) = \frac{18}{1}\Big/\frac{40}{(4/3)} = \frac{18}{30}$$

The displacement of the final image is

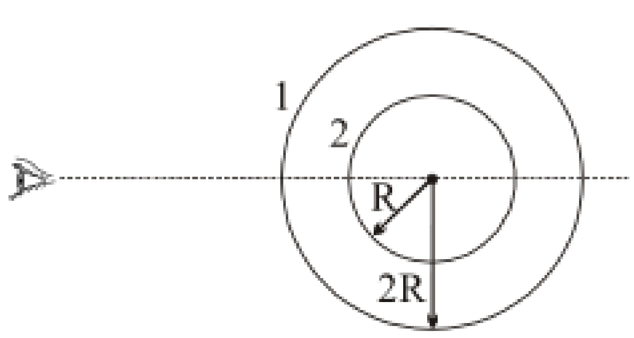
$$\frac{18}{30} \times 50 \times \frac{2\pi}{180}\text{cm} = \frac{\pi}{3}\text{cm}$$

Q.18 A hollow sphere of glass of inner and outer radii R and 2R respectively has a small mark on its inner surface. This mark is observed from a point outside the sphere such that the centre of the sphere lies in between. Prove that the mark will appear nearer than it really is, by a distance $\frac{(\mu-1)R}{(3\mu-1)}$, where R is the radius of the inner surface.

Sol. Refraction at surface 2,

$$\frac{\mu}{v} + \frac{1}{2R} = \frac{\mu-1}{-R} \qquad \text{or} \qquad \frac{\mu}{v} = -\frac{1}{R}\left[(\mu-1)+\frac{1}{2}\right] = \frac{1}{R}\left[\frac{2\mu-1}{2}\right] = \frac{-(2\mu-1)}{2R}$$

or $$v = -\left[\frac{2\mu R}{2\mu-1}\right]$$



For surface 1

$$u = -\left(R + \frac{2\mu R}{2\mu-1}\right) = -\left(\frac{4\mu-1}{2\mu-1}\right)R$$

$$\frac{1}{v} + \frac{\mu(2\mu-1)}{(4\mu-1)R} = \frac{1-\mu}{-2R}$$

$$\frac{1}{v} = -\frac{1}{R}\left[\frac{1-\mu}{2} + \frac{\mu(2\mu-1)}{(4\mu-1)}\right] = -\frac{1}{R}\left[\frac{4\mu-1-4\mu^2+\mu+4\mu^2-2\mu}{2(4\mu-1)}\right] = -\frac{1}{R}\left[\frac{3\mu-1}{2(4\mu-1)}\right]$$

or $\quad v = -\dfrac{2R(4\mu-1)}{(3\mu-1)}$

$\therefore$ Distance between the final image and object,

$$3R - \frac{2R(4\mu-1)}{(3\mu-1)} = R\left[\frac{9\mu-3-8\mu+2}{3\mu-1}\right] = \frac{(\mu-1)R}{(3\mu-1)}$$