Heat Transfer

Heat may be transported from one point to another by any of three possible mechanisms : conduction, convection, and radiation. We study the rate of energy transfer between bodies due to temperature difference between them.

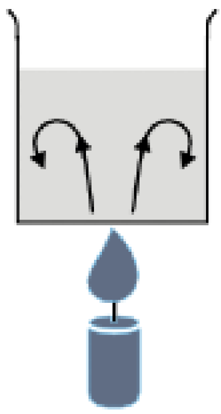
Convection

Convection is the process in which heat is carried from place to place by the bulk movement of a fluid. In liquid and gases, the atoms or molecules can move from point to point. The transfer of heat that accompanies mass transport is called convection.

In forced convection, a fan or pump sets up fluid currents. For examples, a fan blows air, or a pump circulates water in a hot-water heating system in a house.

In free convection, it occurs because the density of a fluid varies with its temperature.

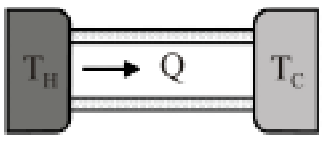
An example of convection currents in a pan of water being heated on a gas burner. The currents distribute the heat from the burning gas to all parts of the water. The direction of convection current is opposite to acceleration due to gravity as shown in figure.



In convection, heat transfer accompanies the movement of a fluid

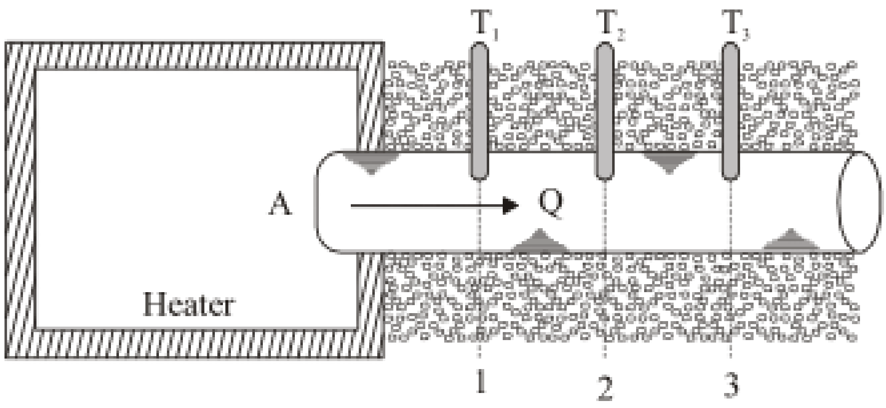
Conduction

A rod whose ends are in thermal contact with a hot reservoir at temperature T_H and a cold reservoir at temperature T_C. The sides of the rod are covered with insulation meterial, so the transport of heat is along the rod, not through the sides. The molecules at the hot reservoir have greater vibrational energy. This energy is trransferred by collisions to the atoms at the end face of the rod. These atoms in turn transfer enery to their neighbors futher along the rod. Such transfer of heat through a substance is called conduction as shown in figure.

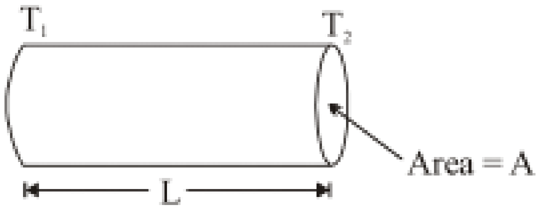


Heat is conducted through an insulated bar whose ends are in thermal contact with two reservoirs

Steady and Transient State :



Consider a metal rod AB, with one end A inserted into a chamber containing a heater with other end B left free and exposed to the surrounding as shown in figure. The rod is thermally insulated sideways with some bad conductor of heat say cotton. Three thermometers are installed in the rod at three distinct sections numbered (1), (2) and (3). Initially, the enitre system is at the room temperature and the three thermometers show the same room temperature. The heater is then switched on. The end A first gets heated up and simultaneously heat is conducted to the adjacent sections towards end B. Due to heat absorption at each sections. The corresponding temperatures start rising with $T_1 > T_2 > T_3$. Such a state, encountered initially is known as a transient state. In this state, the heat coming through end A, is continuously absorbed at each sections with a temperature rise as time elapses. After some time when the temperature of end B becomes equal to that of surrounding and thus becomes constant. Similarly, the temperature of each of the sections of the rod (for example 1, 2, 3) becomes constant or steady. But these steady values at different sections are different.



Consider a portion of the rod of cross sectional area A as shown in figure. Let the temperatures of the two sections separated by a length L be T_1 and T_2 respectively (with $T_1 > T_2$).

Temperature gradient (fall in temperature per unit length) along the length of the rod will be $\frac{T_1 - T_2}{L}$.

Experiments show that the conduction rate (energy transferred per unit time) is given by: Fourier's Law of Heat Conduction

$$H = \frac{\partial Q}{\partial t} = KA\frac{d(-T)}{dx}$$

(Where K : Thermal conductivity of material

H : Thermal current

$\frac{dT}{dx}$: Temperature gradient

A : cross-sectional area of heat path)

The reciprocal of thermal conductivity (K) is called thermal resistivity or thermal specific resistance. Substances having high values of K are good conductors of heat.

Temperature distribution along a conductor :

In order to study conduction in more detail consider figure (i), which shows a metal bar AB whose ends have been soldered into the walls of two metal tanks H and C. Tanks H contains boiling water and C contains ice-water. Heat flows along the bar from A to B and when conditions are steady the temperature θ of the bar is measured at points along its length.

The curve in the upper part of the figure shows how the temperature falls along the bar, less and less steeply from the hot end to the cold. So the temperature gradient decreases from the hot end to the cold.

The figure (ii) shows how the temperature varies along the bar, if the bar is well lagged with a bad conductor, such as asbestos or wool. It now falls uniformly from the hot to the cold end, so the temperature gradient along the bar is constant.

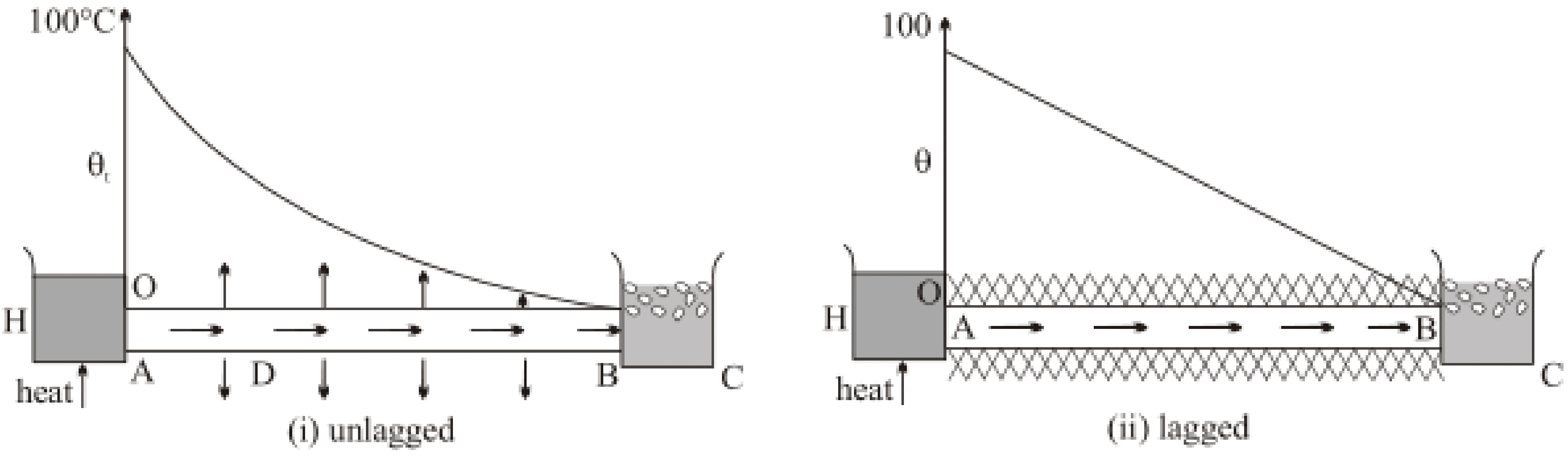
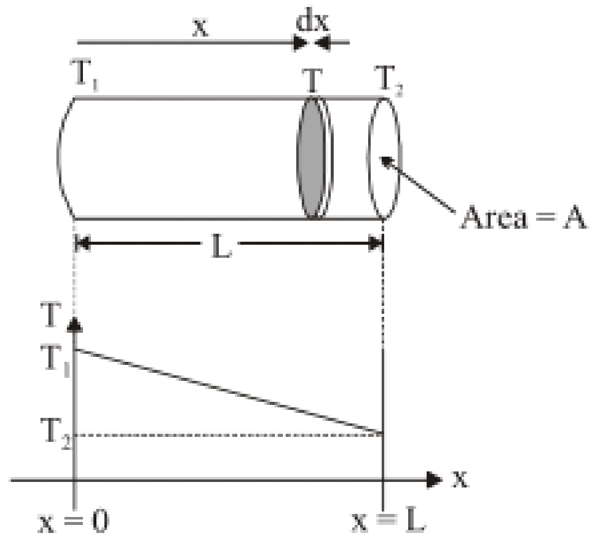


Figure : Temperature fall along lagged and unlagged bars

The difference between the temperature distributions is due to the fact that, when the bar is unlagged, heat escapes from its sides, by convection in the surrounding air, figure (i). The arrows in the figure represent the heat escaping per second from the surface of the bar, and the heat flowing per second along its length. The heat flowing per second along the length decreases from the hot end to the cold. But when the bar is lagged, the heat escaping from its sides is negligible, and the flow per second is now constant along the length of the bar, figure (ii).

Steady State Heat Conduction :



Temperature variation along length of rod

At steady state, energy transferred through one cross-section of the rod during a certain time interval is equal to the energy transferred by at the other cross-section of the rod during the same time interval.

$$H = \frac{\Delta Q}{\Delta t} = KA\left(\frac{\Delta T}{\Delta x}\right) = KA\left(\frac{T_1 - T_2}{L}\right)$$

Temperature distribution across the rod :

Let at distance x we take element of length dx having a cross-sectional area A and temperature T (As shown in figure). In steady state, rate of heat flow H remains constant

$$H = -KA\frac{dT}{dx}$$

$$\int_{T_1}^{T} dT = -\int_{0}^{x}\frac{H}{KA}dx$$

$$T - T_1 = -\frac{Hx}{KA} \qquad \left(\because \frac{H}{KA} = \frac{T_1 - T_2}{L}\right)$$

$$T = T_1 - \frac{x}{L}(T_1 - T_2)$$

The variation has been plotted above.

Thermal Resistance :

The heat transfer by conduction due to temperature difference has an analogy with flow of electric current through a wire when a potential difference is applied. In that case, electrical resistance is defined as

$$R = \frac{V}{i}$$

Similarly, thermal resistance is defined as

$$R = \frac{(T_1 - T_2)}{H}$$

For a rod having length L , area of cross-section A and thermal conductivity K,

$$R = \frac{(T_1 - T_2)}{H}$$

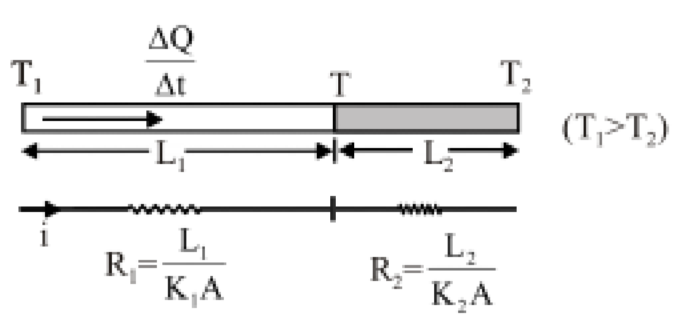
$$= \frac{(T_1 - T_2)}{KA(T_1 - T_2)/L}$$

$$R = \frac{L}{KA}$$

Having calculated the thermal resistance, we can now apply the results of series combination and parallel combination of resistors. It has been explained below.

Composite Rods :

Series Connection : If same heat current are flowing both the rods in steady state, they are said to be in series.



$$(\because R_{eq} = R_1 + R_2 = \Sigma R)$$

Where A - cross-section area of rods

T - Temperature at the juction or Interface temperature

K_1 & K_2 - Themal conductivities of rods having lenghts L_1 and L_2 respectively.

In steady state, heat current is constant throughout the rods.

$$i = \frac{\Delta Q}{\Delta t} = \frac{T_1 - T}{R_1} = \frac{T - T_2}{R_2}$$

$$\therefore T_1 - T = iR_1 \qquad ...(i)$$

$$T - T_2 = iR_2 \qquad ...(ii)$$

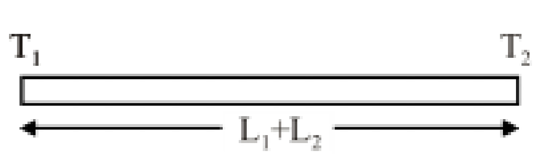
From (i) & (ii)

$$\frac{T_1 - T_2}{R_1 + R_2} = i \quad \text{and} \quad T = \frac{(T_1 R_2 + T_2 R_1)}{R_1 + R_2}$$

$$i = \frac{\Delta T}{R_{eq}}, \text{ in series } R_{eq} = R_1 + R_2$$

Equivalent conductivity of composite Rods (K_{eq}) :

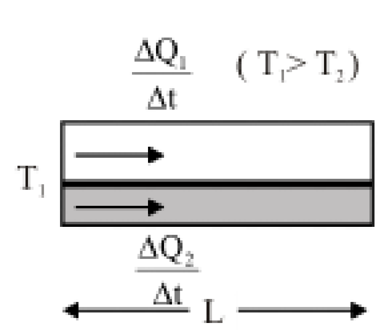
If this rod is replaced by a single rod, then $i = (T_1 - T_2)/R_{eq}$



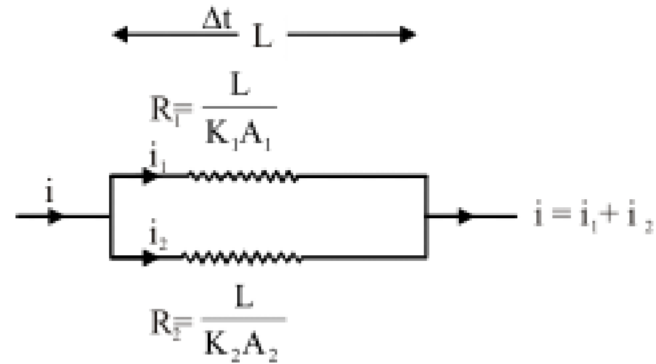
$$\therefore \qquad R_{eq} = R_1 + R_2 = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} = \frac{L_1 + L_2}{K_{eq} A}$$

$$K_{eq} = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}}$$

Parallel Connection : If the tow rods have the same temperature difference across it, they are said to be in parallel.



$$\frac{\Delta Q}{\Delta t} = \frac{\Delta Q_1}{\Delta t} + \frac{\Delta Q_2}{\Delta t}$$



$$\left(\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \Sigma \frac{1}{R}\right)$$

$$i_1 = \frac{T_1 - T_2}{R_1}, i_2 = \frac{T_1 - T_2}{R_2}$$

$$\therefore\ i = i_1 + i_2 = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

In parallel, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

If the two rods are repleaced by a single rod, then K_{eq} will be

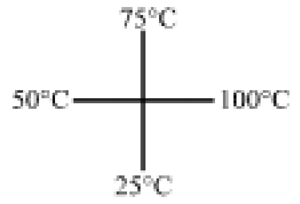
$$K_{eq} = \frac{L}{R_{eq}(A_1 + A_2)} \text{ and } i = \frac{T_1 - T_2}{R_{eq}}$$

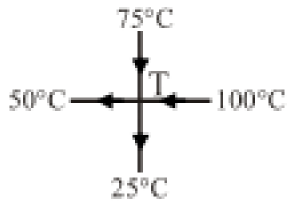
Thus, the heat current in thermal resistances in terms of total thermal current is given by :

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) \times i \qquad \text{and} \qquad i_2 = \left(\frac{R_1}{R_1 + R_2}\right) \times i$$

Illustration :

Two identical rods are joined at their middle points. The ends are maintained at constant temperatures as indicated. The temperature of the junction is _______?





Sol. *Let junction temperature be T*

Accourding to kirchouff's junction law,

Net input thermal current is equal to net output thermal current on a junction. i.e.

$$\sum\left(\frac{\Delta Q}{\Delta t}\right)_{in} = \sum\left(\frac{\Delta Q}{\Delta t}\right)_{out}$$

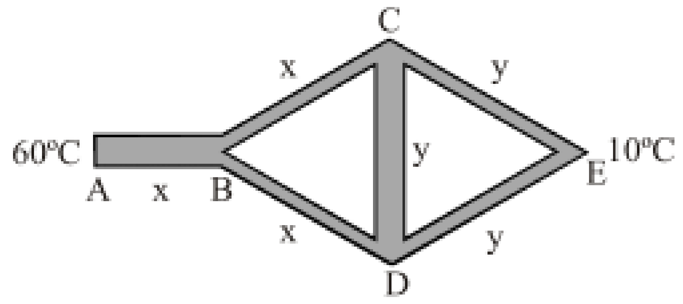
$$\frac{(100 - T)}{(R/2)} + \frac{(75 - T)}{(R/2)} = \frac{(T - 50)}{(R/2)} + \frac{(T - 25)}{(R/2)}$$

$175 + 75 = 4T$

$T = 62.5°C$

Illustration :

Three rods of material x and three of material y are connected as shown in figure. All the rods are identical in length and cross-sectional area. If the end A is maintained at 60ºC and the junction E at 10ºC, calculate the temperature of the junction B. The thermal conductivity of x is 800 W/n – ºC and that of y is 400W/m –ºC.



Sol. *It is clear from the symmetry of the figure that the points C and D are equivalent in all respect and hence, they are at the same temperature, say T. No heat will flow through the rod CD. We can, therefore neglect this rod in further analysis. (Treated as balance wheat stone bridge)*

Let L and A be the length and the area of cross-section of each rod. The thermal resistnces of AB, BC and BD are equal. Each has a value

$$R_1 = \frac{1}{K_x}\frac{L}{A} \qquad ...(i)$$

Similarly, thermal resistances of CE and DE are equal, each having a value

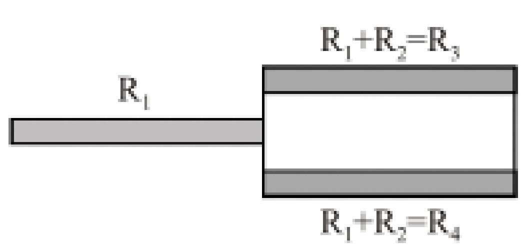
$$R_2 = \frac{1}{K_y}\frac{L}{A} \qquad ...(ii)$$

As the rod CD has no effect, we can say that the rods BC and CE are joined in series. Their equivalent thermal resistance is

$$R_3 = R_{BC} + R_{CE} = R_1 + R_2.$$

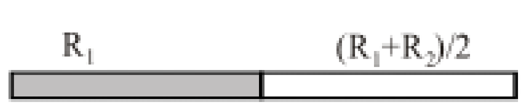
Also, the rods BD ad DE together have an equivalent thermal resitance

$$R_4 = R_{BD} + R_{DE} = R_1 + R_2.$$



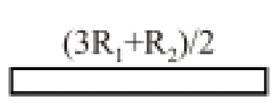
The resistances R_3 and R_4 are joined in parallel and hence their equivalent thermal resistance is given by

$$\frac{1}{R_5} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{2}{R_3} \qquad or \qquad R_5 = \frac{R_3}{2} = \frac{R_1 + R_2}{2}$$



This resistance R_5 is connected in series with AB. Thus, the total arrangement is equivalent to a thermal resistance.

$$R = R_{AB} + R_5 = R_1 + \frac{R_1 + R_2}{2} = \frac{3R_1 + R_2}{2}$$



The heat current through A is

$$i = \frac{T_A - T_E}{R} = \frac{2(T_A - T_E)}{3R_1 + R_2} \qquad ...(i)$$

This current passes through the rod AB. We have

$$i = \frac{T_A - T_B}{R_{AB}} \qquad ...(ii)$$

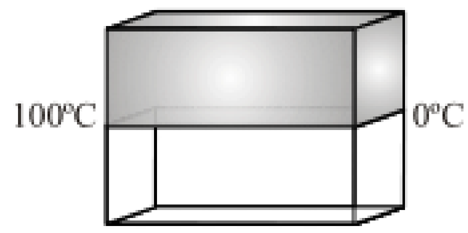
by using (i) and (ii) we get

$$T_A - T_B = \frac{2K_y(T_A - T_E)}{K_x + 3K_y} = \frac{2 \times 400}{800 + 3 \times 400} = 20^\circ C$$

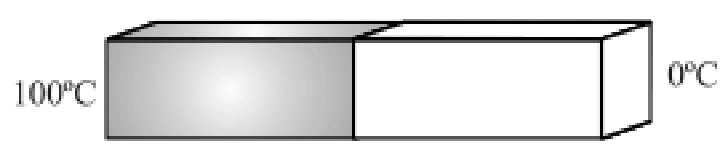
$$T_B = T_A - 20^\circ C = 40^\circ C$$

Illustration :

Two identical rectangular rods of metal are welded as shown in figure (1) and 20 J of heat flows through the rods in 1 min. How long would it take for 20 J heat to flow through the rods if they are welded as shown in figure (2).



(Figure - 1)



(Figure - 2)

Sol. *Let R be the thermal resistance of each rod.*

$\therefore$ *In first case* $\frac{1}{R_1} = \frac{1}{R} + \frac{1}{R}$ *or* $R_1 = \frac{R}{2}$

So the rate of flow of heat in this situation will be

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta T}{R_1} = \frac{100 - 0}{R/2} = \frac{20}{60}$$

$R = 600\ ^\circ C/W$

Now for case (2)

$R_2 = R + R = 600 + 600 = 1200\ ^\circ C/W$

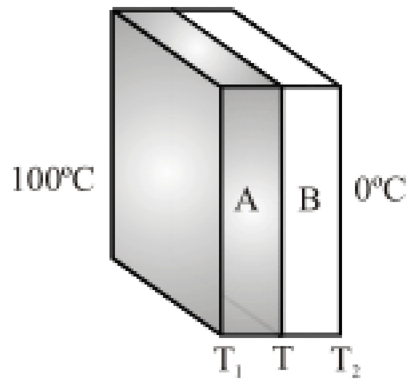
$$\therefore \quad \frac{\Delta Q}{\Delta t} = \frac{\Delta T}{R_2}$$

$$\frac{20}{t} = \frac{100}{1200}$$

$t = 240$ *sec.*

Illustration :

Two parallel plates A and B are joined together to form a compound plate (in figure). The thicknesses of the plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 100 cm² for each plate. The thermal conductivities are $K_A = 200$ W/m-° for the plate A and $K_B = 400$ W/m-°C for the plate B. The outer surface of the plate A is maintained at 100°C and the outer surface of the plate B is maintained at 0°C. Find (a) the rate of heat flow through any cross-section, (b) the temperature at the interface and (c) the equivalent thermal conductivity of the compound plate.



Sol. (a) *Let the temperature of the interface be T.*

The area of cross-section of each pate is $A = 100\ cm^2 = 0.01 m^2$. The thicknesses are $x_A = 0.04$ m and $x_B = 0.025$ m

The thermal resistance of the plate A is

$$R_A = \frac{x_A}{K_A A}$$

and that of the plate B is

$$R_B = \frac{x_B}{K_B A}$$

The equivalent thermal resistance is

$$R_{eq} = R_A + R_B = \frac{1}{A}\left(\frac{x_A}{K_A} + \frac{x_B}{K_B}\right) \qquad \text{... (i)}$$

Thus, $$\frac{\Delta Q}{\Delta t} = \frac{T_1 - T_2}{R_{eq}} = \frac{A(T_1 - T_2)}{x_A/K_A + x_B/K_B}$$

$$= \frac{(0.01\,m^2)\,(100\,°C)}{(0.04\,m)/(200\,W/m-°C) + (0.025\,m)/(400\,W/m-°C)} = 3810\ W.$$

(b) *We have* $$\frac{\Delta Q}{\Delta t} = \frac{A(T - T_2)}{x_B/K_B}$$

or, $$3810\ W = \frac{(0.01\,m^2)\,(T - 0°C)}{(0.025\,m)\,/\,(400\,W/m-°C)}$$

or, $T = 25°C$

(c) *If K is the equivalent thermal conductivity of the compound plate, its thermal resistance is*

$$R_{eq} = \frac{1}{A}\;\frac{x_A + x_B}{K_{eq}}$$

Comparing with (i),

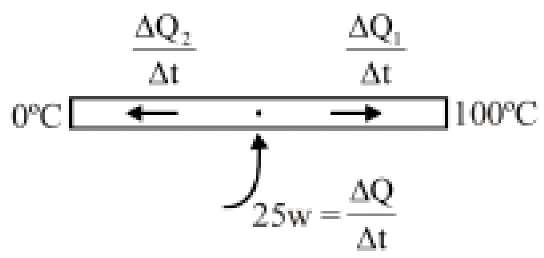
$$\frac{x_A + x_B}{K_{eq}} = \frac{x_A}{K_A} + \frac{x_B}{K_B}$$

or, $$K_{eq} = \frac{x_A + x_B}{x_A / K_A + x_B / K_B} = 248\ W/m\text{–}°C$$

Illustration :

The ends of copper rod of length 1 m and area of cross section 1 cm² are maintained at 0°C and 100°C. At the centre of the rod there is a source of heat of power 25 W. Thermal conductivity of copper is 400 W/m-K. In steady state, the temperature at the section on rod at which source is supplying heat, will be__________?

Sol.



Net thermal current supplied by source $\left(\frac{\Delta Q}{\Delta t}\right)$ *then*

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta Q_1}{\Delta t} + \frac{\Delta Q_2}{\Delta t}$$

$$25 = \frac{kA}{0.5}(T-100) + \frac{kA}{0.5}(T-0)$$

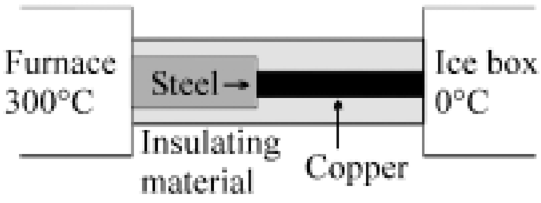
$$\frac{25 \times 0.5}{400 \times 1 \times 10^{-4}} = 2T - 100$$

$$\therefore\ 2T = \frac{1250}{4} + 100$$

$$T = 206.25\ °C$$

Illustration :

What is the temperature of the steel-copper junction in the steady state of the system shown in the figure. Length of the steel rod = 25 cm, length of the copper rod = 50 cm, temperature of the furnace = 300 °C, temperature of the other end = 0°C. The area of cross section of the steel rod is twice that of the copper rod. (Thermal conductivity of steel = 50 J s⁻¹ m⁻¹ K⁻¹ and of copper = 400 J s⁻¹ m⁻¹ K⁻¹)



Sol. *Let temperature be T, in steady state series connection heat transfered* $\left(\frac{\Delta Q}{\Delta t}\right)$ *through each rod is same.*

$$\frac{\Delta Q}{\Delta t} = \frac{k_1 A_1 (T_1 - T)}{L_1} = \frac{k_2 A_2 (T - T_2)}{L_2}$$

$$300 - T = \left(\frac{L_1}{L_2}\right)\left(\frac{k_2}{k_1}\right)\left(\frac{A_2}{A_1}\right)(T - 0)$$

$$300 - T = 2T$$

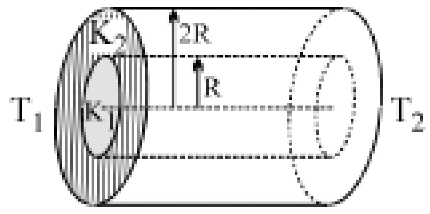
$$T = 100°C$$

Practice Exercise

Q.1 Three identical metal rods A, B and C are placed end to end and a temperature difference is maintained between the free ends of A and C. If the thermal conductivity of B (K_B) is twice that of C (K_C) and half that of A (K_A), (K_A = 49 w/mK)calculate the effective thermal conductivity of the system ?

Q.2 Two identical rectangular rods of metal are welded end to end in series between temperatures of 0° C and 100°C and 10J of heat is conducted (in a steady state process) through the rods in 2.0min. How long would it take for 10J to be conducted through the rods if they are welded together in parallel across the same temperatures?

Q.3 A composite cylinder is made of two materials having thermal conductivities K_1 and K_2 as shown. Temperature of the two flat faces of cylinder are maintained at T_1 and T_2. For what ratio K_1/K_2 the heat current through the two materials will be same. Assume steady state and the rod is lagged (insulated from the curved surface).



Answers

Q.1 21 w/mK Q.2 30 secs. Q.3 $\frac{K_1}{K_2} = 3$

Radiation

Radiation is the process in which energy is transferred by means of electromagnetic waves.

All bodies continuously radiate energy in the form of electromagnetic waves. It does not require a material medium. Electromagnetic waves from the sun, for example, travel through the void of space during their journey to earth. Even an ice cube radiates energy, although so little of it is in the form of visible light that an ice cube cannot be seen in the dark. The surface of an object plays a significant role in determining how much radiant energy the object will absorb or emit.

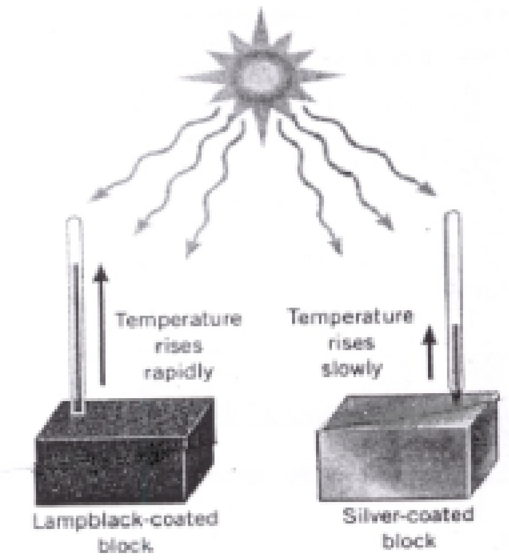


Figure : The temperature of the block coated with lampblack rises faster than the temperature of the block coated absorbs radiant energy from the sun at the greater rate

The two blocks in sunlight in figure, for example, are identical, except that one has a rough surface coated with lamblack (a fine black soot), while the other has a highly polished silver surface. As the thermometers indicate, the temperature of the black block rises at a much faster rate than that of the silver block. This is because lampblack absorbs about 97% of the incident radiant energy, while the silvery surface absorbs only about 10%. The remaining part of the incident energy is reflected in each case. We observe the lampblack as black in color because it reflects so little of the light falling on it, while the silvery surface looks like a mirror because it reflects so little of the light falling on it, while the silvery surface looks like a mirror because it reflects so much light. Since the color black is associated with nearly complete absorption of visible light, the term perfect blackbody or, simply, blackbody is used when referring to an object that absorbs all the electromagnetic waves falling on it.

Black body:

The experiments described before lead us to the idea of a perfectly black body, one which absorbs all the radiation that falls upon it, and reflects and transmits none. The experiments also lead us to suppose that such a body would be the best possible radiator.

Radiation

Radiation is the process in which energy is transferred by means of electromagnetic waves.

All bodies continuously radiate energy in the form of electromagnetic waves. It does not require a material medium. Electromagnetic waves from the sun, for example, travel through the void of space during their journey to earth. Even an ice cube radiates energy, although so little of it is in the form of visible light that an ice cube cannot be seen in the dark. The surface of an object plays a significant role in determining how much radiant energy the object will absorb or emit.

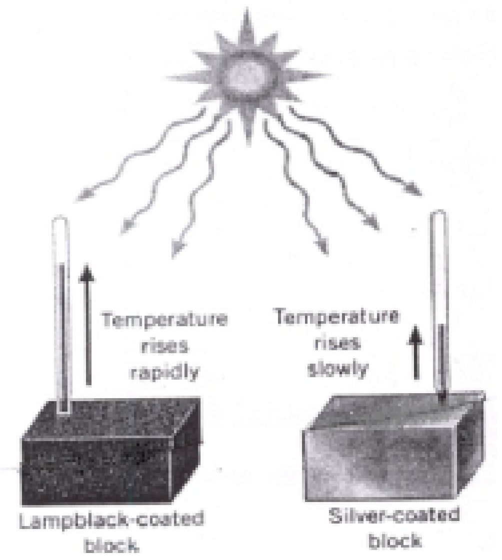


Figure : The temperature of the block coated with lampblack rises faster than the temperature of the block coated absorbs radiant energy from the sun at the greater rate

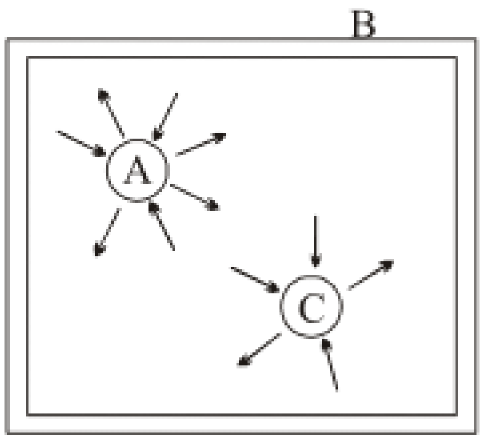
The two blocks in sunlight in figure, for example, are identical, except that one has a rough surface coated with lamblack (a fine black soot), while the other has a highly polished silver surface. As the thermometers indicate, the temperature of the black block rises at a much faster rate than that of the silver block. This is because lampblack absorbs about 97% of the incident radiant energy, while the silvery surface absorbs only about 10%. The remaining part of the incident energy is reflected in each case. We observe the lampblack as black in color because it reflects so little of the light falling on it, while the silvery surface looks like a mirror because it reflects so little of the light falling on it, while the silvery surface looks like a mirror because it reflects so much light. Since the color black is associated with nearly complete absorption of visible light, the term perfect blackbody or, simply, blackbody is used when referring to an object that absorbs all the electromagnetic waves falling on it.

Black body:

The experiments described before lead us to the idea of a perfectly black body, one which absorbs all the radiation that falls upon it, and reflects and transmits none. The experiments also lead us to suppose that such a body would be the best possible radiator.

Prevost's theory of exchange:

Any body having temperature greater then zero kelvin, must emit or aborb radiation.



A is placed in an evacuated enclosure B, at lower temperature than A, then A cools until it reaches the temperature of B. If a body C, cooler than B, is put in B, then C warms up to the temperature of B. We conclude that radiation from B falls on C, and therefore also on A, even through A is at a higher temperature. Thus A and C each come to equilibrium at the temperature of B when each is absorbing and emitting radiation at equal rates.

If Q is the total incident energy on a body, Q_1 is the part absorbed, Q_2 is the part reflected and Q_3 is the part transmitted then

$Q = Q_1 + Q_2 + Q_3$

Absorption coefficient or absorptive power $a = Q_1/Q$

Reflection coefficient $r = Q_2/Q$

Transmission coefficient $t = Q_3/Q$

Thus $a + r + t = 1$

If, for a body, $r = t = 0$ and $a = 1$, i.e. it absorbs all the energy falling on it, such bodies are known as black bodies.

Emissive Power:

Emissive power of a surface is the quantity of heat energy emitted per second, per unit area of surface through unit solid angle. It depends on the nature and the temperature of the surface.

Emissivity:

Emissivity of a surface is the ratio of the emissive power of that surface to the emissive power of a black body at the same temperature.

Kirchhoff's Law:

At a given temperature, the ratio of emissive power to absorptive power of any body is equal to the emissive power of a black body at that temperature. Thus,

$$\frac{E_1}{a_1} = \frac{E_2}{a_2} = E_{\text{Black body}}$$

From Kirchhoff's law, it can be deduced that good absorbers are also good emitters

Stefan's radiation law

An idealized body that absorbs all the radiation incident upon it is called a blackbody. A blackbody absorbs not only all visible light, but infrared, ultraviolet, and all other wavelengths of electromagnetic radiation. It turns out that a good absorber is also a good emitter of radiation. A blackbody emits more radiant ower per unit surface area than any real object at the same temperature. The rate at which a blackbody emits rdiation per unit surface area is proportional to the fourth power of the absolute temperature.

$$P = \frac{dQ}{dt} = \sigma A T^4 \qquad \text{(for a black body)}$$

In equation, A is the surface area and T is the surface temperature of the blackbody in kelvins. Since Stefan's law involves the absolute temperature and not a temperature difference, °C cannot be substituted. The universal constant σ (Greek letter sigma) is called Stefan's constant :

$$\sigma = 5.670 \times 10^{-8}\ \text{W/(m}^2\text{.K}^4\text{)}$$

The fourth-power temperature dependence implies that the power emitted is extremely sensitive to temperature changes. If the absolute temperature of a body doubles, the energy emitted increases by a factor of $2^4 = 16$.

Since real bodies are not perfect absorbers and therefore emit less than a blackbody, we define the emissivity (e) as the ratio of the emitted power of the body to that of a blackbody at the same temperature. Then Stefan's law becomes.

$$P = e\sigma A T^4 \qquad \text{(for a non-black body)}$$

The emissivity ranges from 0 to 1.

e = 1 for a perfect radiator and absorber (a blackbody).

e = 0 for a perfect reflector.

Hot object placed in isothermal enclosure:

Consider a body at a temperature of T_0 and T_e is the temperature of the room or enclosure containing the body. If A is the surface area of the body and emissivity (e).

Since the body is in temperature equilibrium, the energy per second it radiates must equal the energy per second it absorbs. then, from Stefan's law,

energy per second emitted (P_{emit}) $= e\sigma A T_0^4$

energy per second absorbed ($P_{absorbed}$) $= e\sigma A T_e^4$

$P_{emit} = P_{aborbed} \Rightarrow T_e = T_o$

Now suppose the body X is heated electrically by a heater of power W watts and finally reaches a constant temperature T. In this case, from Prevost's theory,

energy per second from heater, W = net energy per second radiated by X

The net energy per second radiated by X = $e\sigma A T^4 - e\sigma A T_0^4$. So

$$W = e\sigma A T^4 - e\sigma A T_0^4 = e\sigma A (T^4 - T_0^4)$$

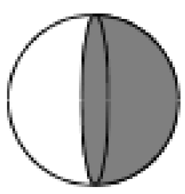
Illustration:

A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is mantained at a constant temperature of 500 K by heating it electrically. A total of 300 W of electric power is needed to do it. When half of the surface of the copper sphere is completely blackened, 600W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

Sol. *Applying Stefan's Law*

Initially $T = 300K$ and $T_s = 500K$

$$300 = \sigma e A [500^4 - 300^4] \quad ....(1)$$

afterwards  *half of the surface of sphere is completely blackened*

$$600 = \frac{\sigma e A}{2}[500^4 - 300^4] + \frac{\sigma A}{2}[500^4 - 300^4] \quad ....(2)$$

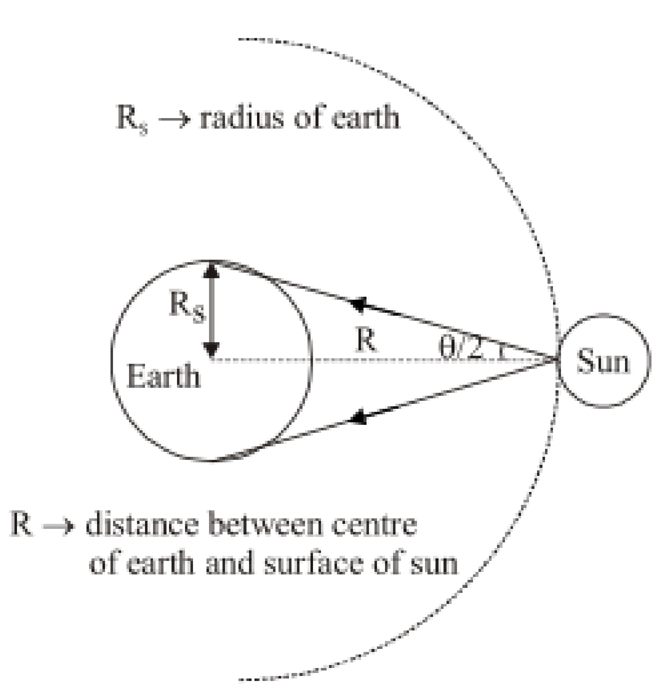
dividing (2) by (1)

$$2 = \frac{\left(\frac{e}{2}+\frac{1}{2}\right)}{e} \Rightarrow 2e = \frac{e}{2}+\frac{1}{2} \Rightarrow e = \frac{1}{3}$$

Illustration:

The solar constant for a planet is S. The surface temperature of the sun is T K. The sun subtends an angle θ at the planet. Find S.

Sol. *Solar constant (S) is defined the rate at which radiations are received from sun per unit surface area.*



$$\tan(\theta/2) = \frac{Rs}{R}$$

$$\tan(\theta/2) \simeq \theta/2 = \frac{Rs}{R}$$

$$\left[\frac{Rs}{R} = \theta/2\right]$$

$$\text{Solar constant } (S) = \frac{\text{Power received}}{\text{Surface Area}}$$

$$S = \frac{\sigma(4\pi Rs^2)T^4}{4\pi R^2} = \sigma\left(\frac{Rs}{R}\right)^2 T^4$$

Using small angle approximation we get

$$S = \sigma\left(\frac{\theta^2}{4}\right)T^4$$

$$S = \frac{\sigma T^4 \theta^2}{4}$$

Illustration:

A highly conducting solid sphere of radius R, density ρ and specific heat s is kept in an evacuated chamber. A parallel beam of electromagnetic radiation having uniform intensity I is incident on its surface. Assuming surface of the sphere to be perfectly black and its temperature at $t = 0$ to be equal to T_0, Calculate maximum attainable temperature of the sphere. (Stefan's constant = σ)

Sol. *At maximum temperature,*

heat received by solid sphere from electromagnetic radiation = heat radiated by solid sphere

$$I \times \pi R^2 = \sigma(4\pi R^2)\ (T_{max})^4.$$

($\because$ Power received per second (P_{abs}) = Intensity (I) × Projection area of sphere)

$$T_{max} = \left(\frac{I}{4\sigma}\right)^{1/4}$$

Illustration:

The distance of the Earth from the Sun is 4 times that of the planet Mercury from the Sun.The temperature of the Earth in radiative equilibrium with the Sun is 290 K. Find the radiative equilibrium temperature of the Mercury. Assume all three bodies to be black body.

Sol. $$P_{received} = \left(\pi R_p^2\right)\left(\frac{P_{sun}}{4\pi r_s^2}\right)$$

$$P_{emitted} = \sigma(e)4\pi R_p^2 T_p^4$$

In Thermal Equilibrium

$$P_{received} = P_{emitted}$$

$$\Rightarrow \quad (T_P)^2 \propto \frac{1}{r_s}$$

$$\frac{T_{earth}}{T_{mercury}} = \sqrt{\frac{r_{mercury}}{r_{earth}}}$$

$$T_{mercury} = (290K)\left(\frac{4}{1}\right)^{1/2}$$

$$= 580\ K$$

Illustration :

The tungsten filament of an electric lamp has a length of 0.5m and a diameter 6×10^{-5} m. The power rating of the lamp is 60W. Assuming the radiation from the filament is equivalent to 80% that of a perfect black body radiator at the same temperature, estimate the steady temperature of the filament. (Stefan constant = $5.7 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$)

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Sol. *When the temperature is steady,*

power radiated from filament = power received = 60W

$\therefore \quad 0.8 \times 5.7 \times 10^{-8} \times 2\pi \times 3 \times 10^{-5} \times 0.5 \times T^4 = 60$

since surface area of cylindrical wire is $2\pi rh$ *with the usual notation.*

$$\therefore \quad T = \left(\frac{60}{0.4 \times 5.7 \times 10^{-8} \times 2\pi \times 3 \times 10^{-5}} \right)^{1/4} = 1933\ K$$

Newton's law of cooling

For small temperature differences, the rate of cooling, due to conduction, convection, and radiation combined, is proportional to the difference in temperature. It is a valid approximation in the transfer of heat from a radiator to a room, the loss of heat through the wall of a room, or the cooling of a cup of tea on the table.

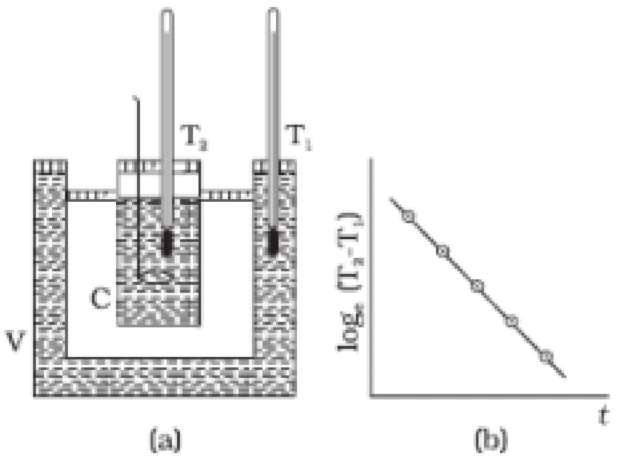


Figure : Verification of Newton's Law of cooling

Suppose, a body of surface area A at an absolute temperature T is kept in a surrounding having a lower temperature T_0. The net rate of loss of thermal energy from the body due to radiation is

$$\Delta u_1 = e\sigma A(T^4 - T_0^4)$$

If the temperature difference is small, we can write

$$T = T_0 + \Delta T$$

or, $$T^4 - T_0^4 = (T_0 + \Delta T)^4 - T_0^4$$

$$= T_0^4 \left(1 + \frac{\Delta T}{T_0}\right)^4 - T_0^4$$

$$= T_0^4 \left[1 + 4\frac{\Delta T}{T_0} + \text{higher powers of } \frac{\Delta T}{T_0}\right] - T_0^4$$

$$\approx 4T_0^3\, \Delta T = 4T_0^3\,(T - T_0)$$

Thus, $$\Delta u_1 = 4e\sigma A T_0^3\,(T - T_0)$$

$$= b_1 A\,(T - T_0)$$

The body may also lose thermal energy due to convection in the surrounding air. For small temperature difference, the rate of loss of heat due to convection is also proportional to the temperature difference and the area of the surface. This rate may, therefore, be written as

$$\Delta u_2 = b_2 A\,(T - T_0)$$

The net rate of loss of thermal energy due to convection and radiation is

$$\Delta u = \Delta u_1 + \Delta u_2 = (b_1 + b_2)\, A\, (T - T_0).$$

If s be the specific heat capacity of the body and m its mass, the rate of fall of temperature is

$$\frac{-dT}{dt} = \frac{\Delta u}{ms} = \frac{b_1 + b_2}{ms} A\,(T - T_0)$$
$$= bA\,(T - T_0)$$

Thus, for small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed. We can write

$$\frac{dT}{dt} = -\,bA\,(T - T_0)$$

Cooling curve:

The law holds good only for small difference of temperature. Also, the loss of heat by radiation depends upon the nature of the surface of the body and the area of the exposed surface. We can write

$$-\frac{dT}{dt} = k\,(T - T_s)$$

where k is a positive constant depending upon the area and nature of the surface of the body. Suppose a body of mass m and specific heat capacity s is at temperature T. Let T_s and T_o be the temperature of the surroundings and body respectively. If the temperature falls by a small amount dT in time dt, then the amount of heat lost is

$dQ = msdT$

∴ Rate of loss of heat is given by

$$\frac{dQ}{dt} = ms\,\frac{dT}{dt}$$

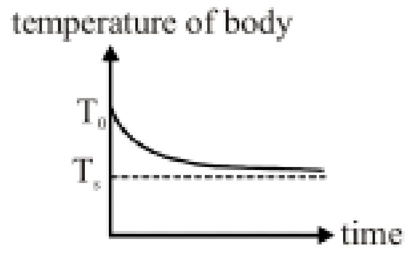
from equation

$$-\frac{dQ}{dt} = k\,(T - T_s) \text{ and } \frac{dQ}{dt} = ms\,\frac{dT}{dt}$$

we have $$-ms\frac{dT}{dt} = k(T - T_s)$$

$$\frac{dT}{T - T_s} = -\frac{k}{ms}\,dt = -\,Kdt \qquad \text{(where } K = k/ms)$$

On integrating,



$$ln\left(\frac{T - T_s}{T_0 - T_s}\right) = -\,kt$$

$$T = T_s + (T_0 - T_s)e^{-kt}$$

enables you to calculate the time of colling of a body through a particular range of temperature.

Illustration :

The temperature of a body falls from 52°C to 36°C in 10 minutes when placed in a surrounding of constant temperature 20°C. What will be the temperature of the body after another 10 min. (Use Newton's law of cooling)

Sol. *Applying newton's law of cooling*

$$\frac{\Delta T}{\Delta t} = -\frac{b}{mc}\left(T_{avg} - T_S\right)$$

$$\frac{52-36}{10\,\text{min}} = \frac{-b}{mc}\left[\frac{52+36}{2} - 20\right]$$

Let the tempature of the body after another 10 min be T

$$\frac{36-T}{10\,\text{min}} = \frac{-b}{mc}\left[\frac{36+T}{2} - 20\right]$$

solving we get T = 28°C

Illustration :

A metal block is placed in a room which is at 10°C. It is heated by an electric heater of power 500 W till its temperature becomes 50°C. Its initial rate of rise of temperature is 2.5°C/sec. The heater is switched off and now a heater of 100W is required to maintain the temperature of the block at 50°C.(Assume Newtons Law of cooling to be valid)

(i) *What is the heat capacity of the block?*

(ii) *What is the rate of cooling of block at 50°C if the 100W heater is also switched off?*

(iii) *What is the heat radiated per second when the block was 30°C?*

Sol.(i) $P_{heater} = P_{given\ to\ block} + P_{Loss\ to\ surroundings}$

Initial $P_{Loss} = 0$

$$\therefore \quad 500 = ms\frac{dT}{dt} + 0$$

$$500 = C\,2.5$$

$$\therefore \quad C = 200\ J/°C$$

(ii) *At 50°, power loss to surroundings = 100W*

$$0 = ms\frac{dT}{dt} + 100$$

$$\frac{dT}{dt} = -\frac{100}{200} = -0.5\ s/sec$$

(iii) *Given at 50°C : Newton's Law of cooling*

$$\text{Power Loss} = 100 = k(50-10) \quad \Rightarrow \quad k = \frac{10}{4} = \frac{5}{2}$$

$$\therefore \text{At } 30°C \qquad P_{Loss} = k(30-10) = \frac{5}{2}\times 20 = 50\ W$$

Practice Exercise

Q.1 A metal sphere with a black surface and radius 30 mm, is cooled to – 73 C (200 K) and placed inside an enclosure at a temperature of 27°C (300K). Calculate the initial rate of temperature rise of the sphere, assuming the sphere is a black body. (Assume density of metal = 8000 kg m^{-3} specific heat capacity of metal = 400 J kg^{-1} K^{-1}, and Stefan constant = 5.7×10^{-8} W m^{-2} K^{-4}).

Q.2 A pan filled with hot food cools from 94 °C to 86 °C in 2 minutes when the room temperature is at 20 °C. How long will it take to cool from 71 °C to 69 °C?

Q.3 A body cools from 50° C to 40°C in 5 minutes. The surrounding temperature is 20°C. What will be its temperature 5 minutes after reading 40°C? Use approximate method.

Answers

Q.1 0.012 K s^{-1} (approx). Q.2 42 sec Q.3 $T = \frac{100}{3}$ °C

Wien's Displacement Law

The wavelength corresponding to highest intensity m is inversely proportional to the absolute temperature. Thus

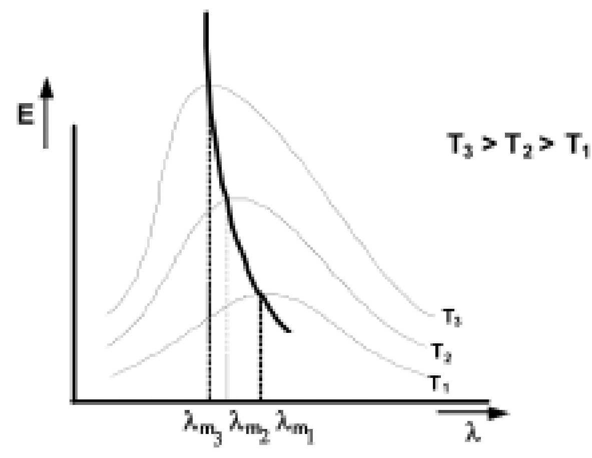
$$\lambda_m = \frac{b}{T}$$

where b (= 2.89×10^{-3} meter Kelvin) is known as the Wien's constant.

When the temperature of a black body is increased, the contribution of low wavelength radiation increases. This explains why a body on heating first appears red, then orange, then white and finally blue. This law also helps us in determining the temperatures of the stars.

Energy Distribution in Black Body Radiation:

The radiation emitted by a black body at any temperature is a mixture of all wavelengths. The graph shows qualitative variation in intensity wavelength, at different temperatures.



Spectral emmissive power:

To speak of the intensity of a single wavelength is meaningless. The slit of the spectrometer always gathers a band of wavelengths the narrower the slit the narrower the band –and we always speak of the intensity of a given band. We express it as follows :

energy radiated $m^{-2}\ s^{-1}$, in band λ to $\lambda + \Delta\lambda = E_\lambda\ \Delta\lambda$

The quantity E_λ is called emissive power of a black body for the wavelength λ and at the given temperature ; its definition follows from equation λ to $\lambda + \Delta\lambda = E_\lambda\ \Delta\lambda$:

$$E_\lambda = \frac{\text{energy radiated } m^{-2}\ s^{-1}, \text{ in band } \lambda \text{ to } \lambda + \Delta\lambda}{\text{band width}, \Delta\lambda}$$

$$E_\lambda = \frac{\text{power radiated } m^{-2} \text{ in band } \lambda \text{ to } \lambda + \Delta\lambda}{\Delta\lambda}$$

In the figure, E_λ is expressed in watts per m^2 per nanometre (10^{-9} m).

The quantity $E_\lambda\ \Delta\lambda$ in equation λ to $\lambda + \Delta\lambda = E_\lambda\ \Delta\lambda$ is the area beneath the radiation curve between the wavelength λ and $\Delta\lambda$ (figure). Thus the energy radiated per meter² per second between those wavelengths in proportional to that area.

Similarly, the total radiation emitted per metre² per second over all wavelengths is proportional to the area under the whole curve.

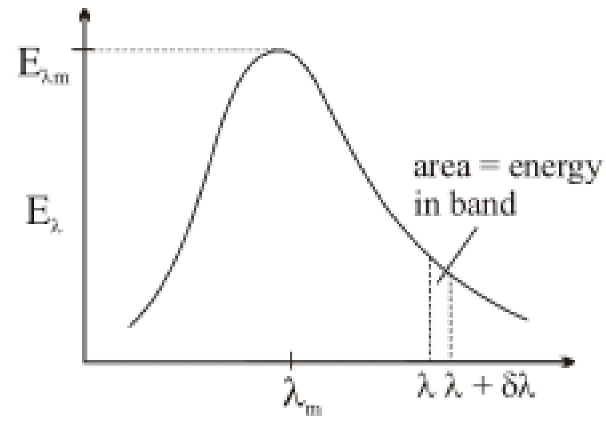


Figure : Definition of $E{\cdot}\lambda_m$ and $E_{\lambda m}$

Laws of black body radiation:

The curves of figure can be explained only Planck's quantum theory of radiation, which is outside our scope. Both theory and experiment lead to three generalisations, which together describe well the properties of black body radiation.

(i) If λ_m is the wavelength of the peak of the curve for T (in K), then

$$\lambda_m T = \text{constant} \qquad \text{... (2)}$$

The value of the constant is 2.9×10^{-3} m K. In figure the dotted line is the locus of the peaks of the curves for different temperatures.

The relationship in (2) is sometimes called Wien's displacement law.

(ii) If $E_{\lambda m}$ is the height of the peak of the curve for the temperature T, then

$$E_{\lambda m} \propto T^5 \qquad \text{... (3)}$$

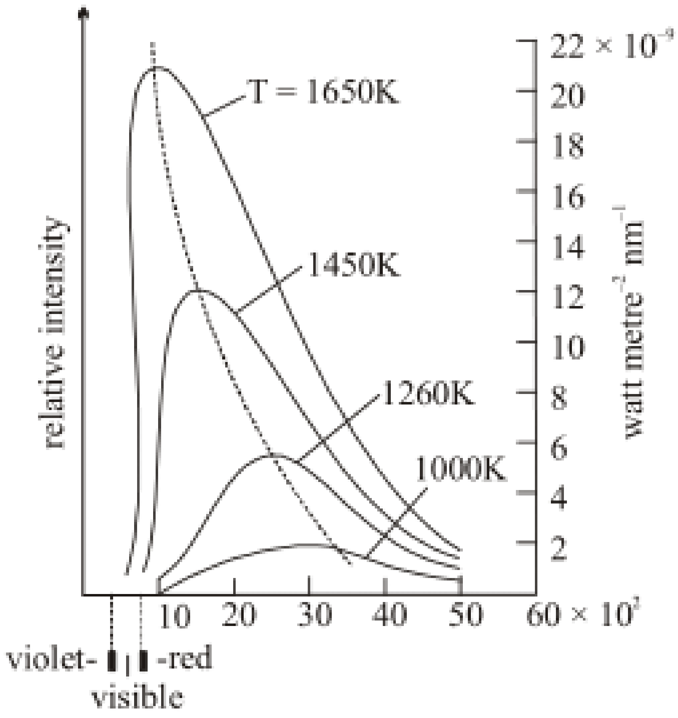


Figure : Distribution of intensity in black body radiation

(iii) If E is the total energy radiated per metre^2 per second at a temperature T, which is represented by the total area under the particular $E_\lambda - \lambda$ curve, then

$$E = \sigma T^4$$

So in figure, which shows four $E_\lambda - \lambda$ graphs at different temperatures T, the total area below the graphs should be proportional to the corresponding value of T^4.

Illustration:

Estimate the surface temperature of sun. Given for solar radiations, $\lambda_m = 4753$ Å.
($b = 2.89 \times 10^{-3}$ meter Kelvin)

Sol. *From Wien's displacement law*

$\lambda_m T = b$

$T = 6097$ K.

Illustration :

The energy radiated by a black body at 2300 K is found to have the maximum at a wavelength 1260 nm, its emissive power being 8000 Wm^{-2}. When the body is cooled to a temperature T K, the emissive power is found to decrease to 500 Wm^{-2}. Find:

(i) *the temperature T*

(ii) *the wave length at which intensity of emission in maximum at the temperature T.*

Sol. *(i)* $\lambda m_1 = 1260$ nm

$\lambda m_1 = 1260 \times 10^{-9} m,\ T_1 = 2300\ K,\ T_2 = T\ K$

$$\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$\left(\frac{E_1}{E_2}\right)^{1/4} = \frac{T_1}{T_2},\ T_2 = T_1 \times \left(\frac{E_2}{E_1}\right)^{1/4}$$

$$T_2 = T_1 \times \left(\frac{E_2}{E_1}\right)^{1/4}$$

$$T_2 = 2300 \times \left[\frac{500}{8000}\right]^{1/4} = 2300 \times \frac{1}{2} = 1150\ k$$

$T_2 = 1150\ k$ *Ans.*

(ii) *by using Wein's law*

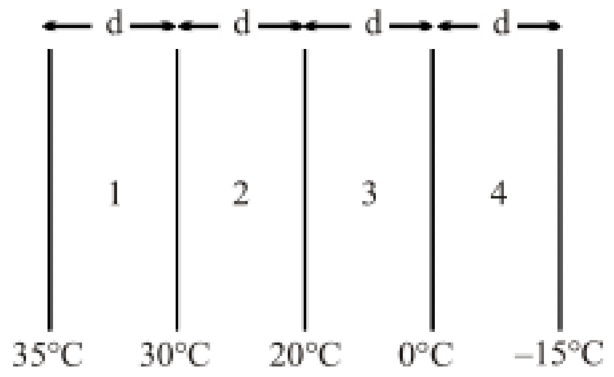
$$T_1\lambda_{m_1} = T_2\lambda_{m_2}\ ;\ \lambda_{m_2} = \left(\frac{T_1\lambda_{m_1}}{T_2}\right)$$

$$\lambda_{m_2} = \frac{2300 \times 1260 \times 10^{-9}}{1150} = 2520 \times 10^{-9}\ M$$

$$\lambda_{m_2} = 2520\ nm$$

Solved Example

Q.1 The diagram shows four slabs of different materials with equal thickness, placed side by side. Heat flows from left to right and the steady-state temperatures of the interfaces are given. Rank the materials according to their thermal conductivities, smallest to largest.



Sol In steady state heat transfered through each slab is same.

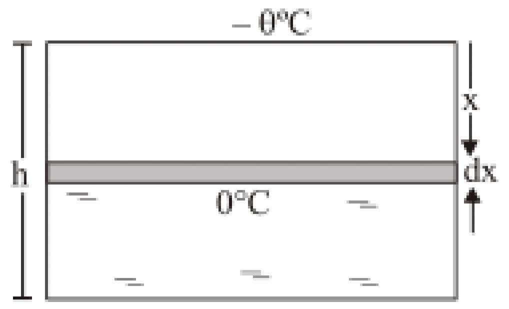
Than $\frac{\Delta Q}{\Delta t} = \frac{k_1 A \Delta T_1}{d} = \frac{k_2 A \Delta T_2}{d} = \frac{k_3 A \Delta T_3}{d} = \frac{k_4 A \Delta T_4}{d}$

$k \propto \frac{1}{\Delta T}$ thus 3, 4, 2, 1

Q.2 **(Growth of ice on Pond)**

On a cold winter day, the atmospheric temperature is – θ (on Celsius scale) which is below 0°C. A cylindrical drum of height h made of a bad conductor is completely filled with water at 0°C and is kept outside without any lid. Calculate the time taken for the whole mass of water to freeze. Thermal conductivity of ice is K and its latent heat of fusion is L. Neglect expansion of water on freezing.

Sol.



Suppose, the ice starts forming at time t = 0 and a thickness x is formed at time t. The amount of heat flown from the water to the surrounding in the time interval t to t = dt is

$$\Delta Q = \frac{KA\theta}{x}\, dt$$

The mass of the ice formed due to the loss of this amount of heat is

$$dm = \frac{\Delta Q}{L} = \frac{KA\theta}{x}\, dt.$$

The thickness dx of ice formed in time dt is

$$dx = \frac{dm}{A\rho} = \frac{K\theta}{\rho x L}\, dt$$

or, $$dt = \frac{\rho L}{K\theta}\, x\, dx$$

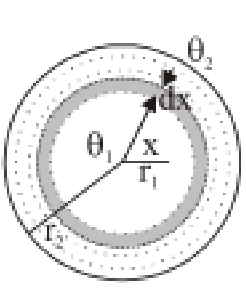
Thus, the time T taken for the whole mass of water to freeze is given by

$$\int_0^T dt = \frac{\rho L}{K\theta}\int_0^h x dx$$

$$T = \frac{\rho L h^2}{2K\theta}$$

Q.3 Two thin metallic spherical shells of radii r_1 and r_2 ($r_1 < r_2$) are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature θ_1 and the outer shell at temperature θ_2 ($\theta_1 < \theta_2$). Calculate the rate at which heat flows radially through the material.

Sol. Let us draw two spherical shells of radii x and x + dx concentric with the given system. Let the temperatures at these shells be θ and $\theta + d\theta$ respectively. The amount of heat flowing radially inward through the material between x and x + dx is



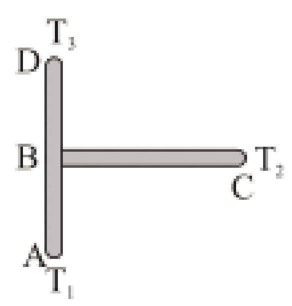
$$\frac{\Delta Q}{\Delta t} = \frac{K 4\pi x^2\, d\theta}{dx}$$

Thus, $$K4\pi \int_{\theta_1}^{\theta_2} d\theta = \frac{\Delta Q}{\Delta t}\int_{r_1}^{r_2}\frac{dx}{x^2}$$

$$K4\pi\,(\theta_2 - \theta_1) = \frac{\Delta Q}{\Delta t}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{\Delta Q}{\Delta t} = \frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}$$

Q.4 Three rods AB, BC and BD having thermal conductivities in the ratio 1 : 2 : 3 and lengths in the ratio 2 : 1 : 1 are joined as shown is figure. The ends A, C and D are at temperatures T_1, T_2 and T_3 respectively. Find the temperature of the junction B. Assume steady state.



Sol. Let the thermal conductivities of the rods AB, BC and BD be K, 2K and 3K respectively. Also, let their lengths be 2L, L and L.

If T be the required temperature of the junction B and assuming $T_1 > T > T_2 > T_3$, we have

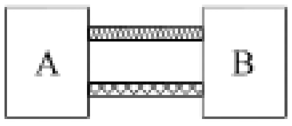
$$\left.\frac{\Delta Q}{\Delta t}\right]_{AB} = \left.\frac{\Delta Q}{\Delta t}\right]_{BC} + \left.\frac{\Delta Q}{\Delta t}\right]_{BD}$$

i.e. $$\frac{KA(T_1 - T)}{2L} = \frac{2KA(T - T_2)}{L} + \frac{3KA(T - T_3)}{L}$$

or $$\frac{T_1 - T}{2} = 2(T - T_2) + 3(T - T_3)$$

or $$T = \frac{1}{11}(T_1 + 4T_2 + 6T_3)$$

Q.5 The container A is constantly maintained at 100°C and insulated container B of the figure initially contains ice at 0°C. Different rods are used to connect them. For a rod made of copper, it takes 30 minutes for the ice to melt and for a rod of steel of same cross-section taken in different experiment it takes 60 minutes for ice to melt. When these rods are simultaneously connected in parallel. Find the time interval in which ice melts ?



Sol. $Q = it$ where i = heat flow rate $= \frac{\Delta T}{R} = \frac{100}{R}$

For copper rod :

$$Q = \left(\frac{100}{R_1}\right)(30) \quad \Rightarrow \quad R_1 = \left(\frac{100}{Q}\right) \times 30$$

Also for steel rod :

$$Q = \left(\frac{100}{R_2}\right) \times 60 \quad \Rightarrow \quad R_2 = \left(\frac{100}{Q}\right) \times 60$$

Now, $$Q = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) t_{reqd.}$$

$$\therefore \quad t_{reqd} = \frac{Q}{\left(\frac{Q}{100}\right) \times \frac{1}{30} + \left(\frac{Q}{100}\right) \times \frac{1}{60}} = 20 \text{ min}$$

Q.6 **(Temperature of Sun)**

Estimate the temperature T_e of the earth, assuming it is in radiative equilibrium with the sun. (Assume radius of sun, $r_s = 7 \times 10^8$ m, temperature of solar surface = 6000 K, distance of earth from sun, $R = 1.5 \times 10^{11}$ m)

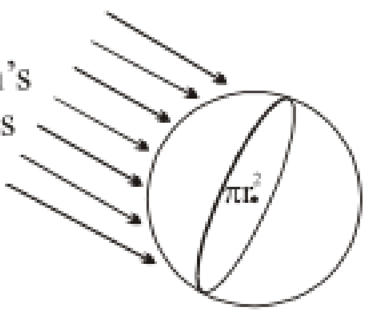
Sol. Power radiated from sun = $\sigma \times$ surface area $\times T^4$

$$= \sigma \times 4\pi r_s^2 \times T_s^4$$

$$\text{Power received by earth} = \frac{\pi r_e^2}{4\pi R^2} \times \text{power radiated by sun}$$

since πr_e^2 is the effective area of the earth on which the sun's radiation is incident normally. figure and $4\pi R^2$ is the total area on which the sun's radiation falls at a distance R from the sun where the earth is situated.

Now power radiated by earth = $\sigma \cdot 4\pi r_e^2 \cdot T_e^4$



Assuming radiative equilibrium

power radiated by earth = power received by earth

$$\therefore \quad \sigma \cdot 4\pi r_e^2 \cdot T_e^4 = \sigma \cdot 4\pi r_s^2 \cdot T_s^4 \times \frac{\pi r_e^2}{4\pi R^2}$$

Cancelling r_e^2 and simplifying, then

$$T_e^4 = T_s^4 \times \left(\frac{r_s^2}{4R^2}\right)$$

$$\therefore \quad T_e = T_s \times \left(\frac{7 \times 10^8}{2 \times 1.5 \times 10^{11}}\right)^{1/2} = 209 \text{ K}$$

Note that the calculation is approximate, for example, the earth and the sun are not perfect black body radiators and the earth receives heat from its interior.

Q.7 A wood-burning stove stands unused in a room where the temperature is 18°C (291 K). A fire is started inside the stove. Eventually, the temperature of the stove surface reaches a constant 198°C (471 K), and the room warms to a constant 29°C (302 K). The stove has an emissivity of 0.900 and a surface area of 3.50 m². Determine the net radiant power generated by the stove when the stove (a) is unheated and has a temperature equal to room temperature and (b) has a temperature of 198°C.

Sol. **Reasoning :** The stove emits more radiant power heated than when unheated. In both cases, however, the Stefan-Bolfzmann law can be used to determine the amount of power emitted. Power is energy per unit time or Q/t. But in this problem we need to find the net power produced by the stove. The net power is the power the stove emits minus the power the stove absorbs. Then power the stove absorbs comes from the wall, ceiling, and floor of the room, all of which emit radiation.

(a) Remembering that temperature must be expressed in kelvins when using the Stefan-Boltzmann law, we find that

$$\text{Power emitted by unheated} = \frac{Q}{t} = e\sigma T^4 A$$

$$= (0.900)\,[5.67 \times 10^{-8}\ \text{J/(s} \cdot \text{m}^2 \cdot \text{K}^4]\,(291\ \text{K})^4\,(3.50\ \text{m}^2) = 1280\ \text{W}$$

The fact that the unheated stove emits 1280 W of power and yet maintains a constant temperature means that the stove also absorbs 1280 W of radiant power from its surroundings. Thus, the net power generated by the unheated stove is zero.

$$\text{Net power generated by stove at } 18°\text{C} = \underbrace{1280\ \text{W}}_{\substack{\text{Power emitted}\\ \text{by stove at}\\ 18°\text{C}}} - \underbrace{1280\ \text{W}}_{\substack{\text{Power emitted by}\\ \text{room at } 18°\text{C and}\\ \text{absorbed by stove}}} = 0\ \text{W}$$

(b) The hot stove (198 °C) or 471 K) emits more radiant power than it absorbs from the cooler room. The radiant power the stove emits is

$$\text{Power emitted by stove at } 198°\text{C} = \frac{Q}{t} = e\sigma T^4 A$$

$$= (0.900)\,[5.67 \times 10^{-8}\ \text{J/(s} \cdot \text{m}^2 \cdot \text{K}^4)]\,(471\ \text{K})^4\,(3.50\ \text{m}^2) = 8790\ \text{W}$$

The radiant power the stove absorbs from the room is identical to the power that the stove would emit at the constant room temperature of 29°C (302 K). The reasoning here is exactly like that in part (a).

$$\begin{array}{l}\text{Power emitted by}\\ \text{room at } 29°\text{C and}\\ \text{absorbed by stove}\end{array} = \frac{Q}{t} = e\sigma T^4 A$$

$$= (0.900)\,[5.67 \times 10^{-8}\ \text{J/(s} \cdot \text{m}^2 \cdot \text{K}^4)]\,(302\ \text{K})^4\,(3.50\ \text{m}^2) = 1490\ \text{W}$$

The net radiant power the stove produces from the fuel it burn is

$$\begin{array}{l}\text{Net power}\\ \text{generated by}\\ \text{stove at } 198°\text{C}\end{array} = \underbrace{8790\ \text{W}}_{\substack{\text{Power emitted}\\ \text{by stove at}\\ 198°\text{C}}} - \underbrace{1490\ \text{W}}_{\substack{\text{Power emitted by}\\ \text{room at } 29°\text{C and}\\ \text{absorbed by stove}}} = 7300\ \text{W}$$

Q.8 The room heater can provide only 16°C in the room when the temperature outside is -20°C. It is not warm and comfortable, that is why the electric stove with power of 1 kW is also plugged in. Together these two devices maintain the room temperature of 22°C. Determine the thermal power of the heater.

Sol. Rate of heat loss with only room heater

$$P_h = \frac{\Delta Q}{\Delta t} = C\,(16 + 20) \qquad \text{[where C = constant]}$$

while both heater and stove it is

$$P_h + P_s = \left(\frac{\Delta Q}{\Delta t}\right)' = C\,(22 + 20)$$

$$\therefore \frac{P_h}{P_h + P_s} = \frac{36}{42} \Rightarrow 7P_h = 6\,P_h + 6P_s$$

$$\Rightarrow P_h = 6P_s = 6\text{ kW}$$

Q.9 A hot body palced in air is cooled down according to Newton's law of cooling, the rate of decrease of temperature being k times the temperature difference from the surrounding. Starting from $t = 0$, find the time in which the body will loose half the maximum heat it can lose.

Sol. We have,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

where θ_0 is the temperature of the surrounding and θ is the temperature of the body at time t. Suppose $\theta = \theta_1$ at $t = 0$

Then,

$$\int_{\theta_1}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k\int_0^t dt$$

or $$\text{In}\,\frac{\theta - \theta_0}{\theta_1 - \theta_0} = -kt$$

or, $$\theta - \theta_0 = (\theta_1 - \theta_0)\,e^{-kt} \qquad \text{...(i)}$$

The body continues to lose heat till its temperature becomes equal to that of the surrounding. The loss of heat in this entire period is

$$\Delta Q_m = ms\,(\theta_1 - \theta_0).$$

This is the maximum heat the body can looses. If the body loses half this heat, the decreases in its temperature will be,

$$\frac{\Delta Q_m}{2ms} = \frac{\theta_1 - \theta_0}{2}$$

If the body loses this heat in time t_1, the temperature at t_1 will be

$$\theta_1 - \frac{\theta_1 - \theta_0}{2} = \frac{\theta_1 + \theta_0}{2}$$

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Putting these values of time and temperature in (i),

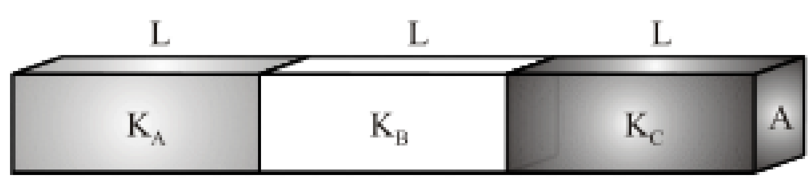
$$\frac{\theta_1 + \theta_0}{2} - \theta_0 = (\theta_1 - \theta_0)e^{-kt}$$

or, $$e^{-kt} = \frac{1}{2}$$

or, $$t_1 = \frac{\ln 2}{k}$$

Q.10 Three identical metal rods A, B and C are placed end to end and a temperature difference is maintained between the free ends of A and C. If the thermal conductivity of B (K_B) is twice that of C (K_C) and half that of A (K_A), (K_A = 49 w/mK)calculate the effective thermal conductivity of the system ?

Sol.



$$K_B = K_A/2$$
$$K_C = K_A/4$$

$$\frac{1}{R_{eq}} = \frac{L}{A}\left(\frac{1}{K_A} + \frac{1}{K_B} + \frac{1}{K_C}\right) = \frac{3L}{A}\left(\frac{1}{K_{eff}}\right)$$

$$K_{eff} = \frac{3K_A}{7} = 21 \text{ w/mK}$$