LAWS OF MOTION

We described motion in terms of position, velocity, and acceleration without considering what might cause that motion. Now we consider what might cause one object to remain at rest and another object to accelerate ? The two main factors we need to consider are the forces acting on an object and the mass of the object. We discuss the three basic laws of motion, which deal with forces and masses and were formulated more than three centuries ago by Isaac Newton. Once we understand these laws, we can answer such questions as "What mechanism changes motion ? And "Why do some objects accelerate more than other ?'

Force

Everyone has a basic understanding of the concept of force. In a vague language, **Force** is push or pull which is an attempt to change the state of rest or motion of an object, or merely deform it. The effect of force on the state of motion of body was first understood by Isaac Newton (1642 – 1727)

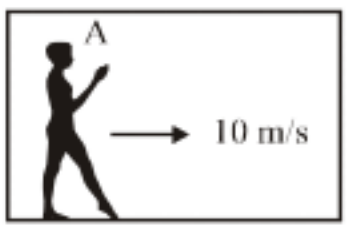
Newtonian Mehanics

The study of impact of force on the motion of a body as Newton presented it, is called Newtonian mechanics. Newtonian mechanics does not apply to all situations. If the speed of the interacting bodies is very large, an appreciable fraction of the speed of light, we must replace Newtonian mechanics with Einstein's special theory of relativity. If the interacting bodies are on the scale of atomic structure for example, they might be electrons in an atom, We must replace newtonian mechanics with quantum mechanics. Newtonian mechanics is a special case of these two theories, it applies to the motion of objects ranging in size from very small (macroscopic) to astronomical which move with speed neglible w.r.t. speed of light.

Newton's First Law

When net force acting on a body is zero, then we can always find frames of reference in which acceleration of the body is zero. Such frames are called **inertial reference frames.**

A frame moving with constant velocity w.r.t. inertial reference frame is also an inertial reference frame. Thus in an inertial reference frame, if no net force acts on a body, it's velocity connot change. Let us take an example, A block is kept on the floor, such that net force acting on it is zero, and it is at rest



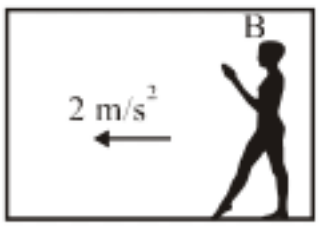
(block is at rest on the floor)

Frame A moving with constant velocity 10 m/s

In the frame A : Observer A observes the block, to be moving with same velocity in opposite direction.

the velocity of block remains 10 m/s towards left. Thus the block has zero acceleration w.r.t. A, Therefore A is an inertial reference frame.

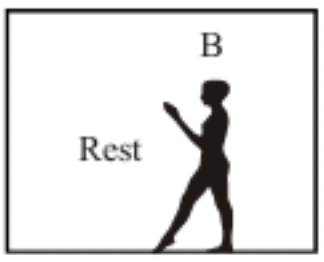
(block is at rest on the floor)



(Frame B moving with an acceleration 2 m/s².)

In the frame B : observer B observes the block to be accelerated in opposite direction. Thus velocity of block is seen to be changing (w.r.t. B) with time, although no force acts on it here, therefore B is not an inertial reference frame and these type of reference frames are known as non inertial reference frame.

A frame of reference accelerated w.r.t. inertial frame of reference is a non-inertial reference frame.

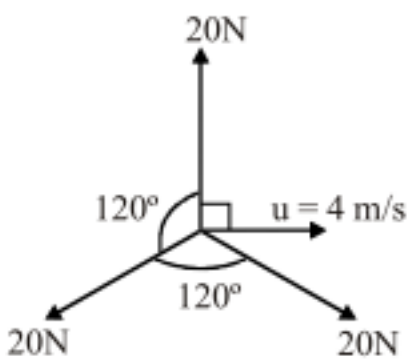


Inertial frame is an ideal concept because only an isolated object can have zero acceleration. However we can assume earth to be an inertial reference frame, if we neglect its rotation & revolution. This can be done because Earth's acceleration due to revolution and rotation is much smaller than the acceleration we notice in daily life i.e acceleration due to gravity ($g = 9.8$ m/s²).

Illustration :

An object is acted upon by three forces, 20N each as shown in fig. If its initial velocity is 4 m/s in the direction shown in figure. Find its displacement in first 4 sec.

Sol.



As net force (vector sum of all forces) acting on the object is zero, therefore according to Newton's first law,

$$\vec{a} = 0$$

applying, $$S = ut + \frac{1}{2}at^2$$

$$S = 4 \times 4 = 16 \text{ m/s}$$

Mass

Mass is that property of a substance which specifies how much resistance an object exhibits to changes in its velocity i.e. it gives an idea of inertia of that body

Suppose a ball is moving in a straight line and a train is moving in a straight line; both with same constant speed. Which one is more difficult to stop? Of course the train.

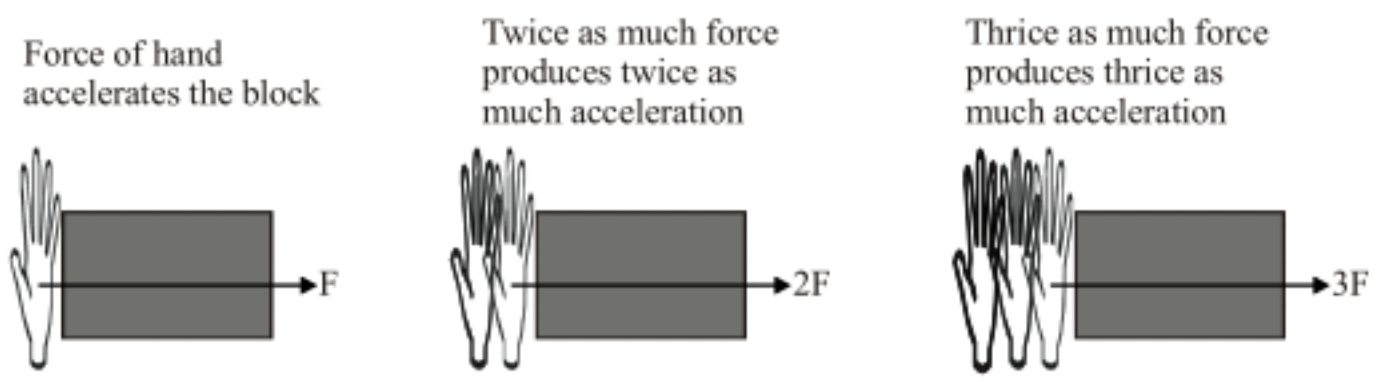
Mass is an inherent property of an object and of the method used to measure it. The mass of a closed system of bodies is independent of the processes going on in the system, no matter what kind these processes are. Also, mass is a scalar quantity and thus obeys the rules of ordinary arithmetic.

Mass should not be confused with weight. Mass and weight are two different quantities. Weight is the force with which Earth attracts the object and is dependent on mass.

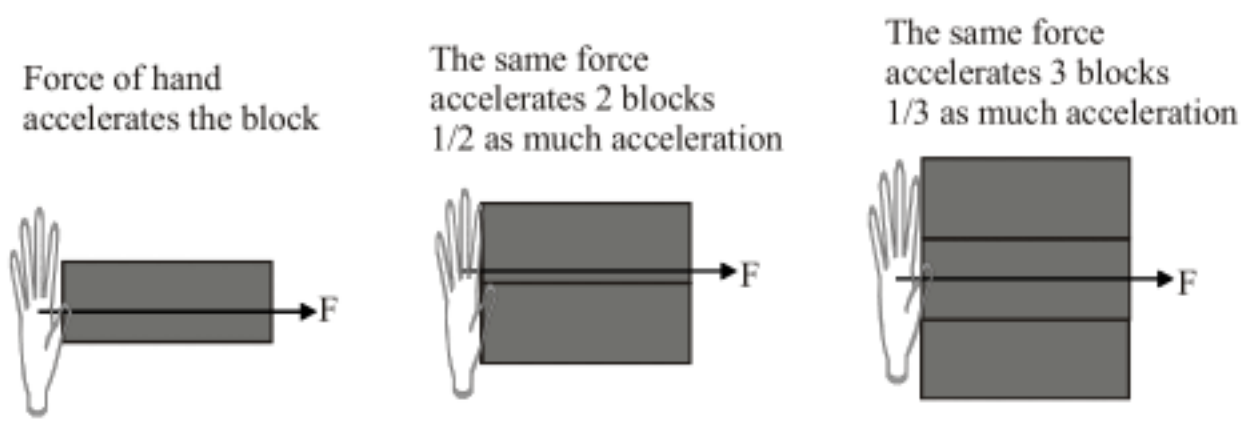
Newton's Second law

Newton's first law explains what happens to an object when no net force acts on it. It either remains at rest or moves in straight line with constant speed. Newton's second law answers the question, what happens to an object that has a nonzero resultant force acting on it.

When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.



$$\textbf{acceleration} \propto \textbf{force}$$



$$\textbf{acceleration} \propto \frac{\textbf{1}}{\textbf{mass}}$$

Thus, we can relate mass, acceleration, and force through the following mathematical statement of Newton's second law, by choosing proper units so that constant of proportionality is 1 :

$$\sum \vec{F} = m\vec{a} \qquad \text{.....(i)}$$

we have indicated that the acceleration is due to the net force acting on an object. The net force on an

object is the vector sum of all forces acting on the object. In solving a problem using Newton's second law, it is important to determine the correct net force on an object.

What happens when several forces act simultaneously on an object ?

In this case, the object accelerates only if the net force acting on it is not equal to zero.

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$: vector sum of all forces. .....(ii)

There may be many forces acting on an object, but there is only one acceleration.

Note that equation 2 is vector expression and hence is equivalent to three component equations

$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$

When net force is zero, then the object is said to be in equilibrium. And if net force is zero, acceleration will also be zero, therefore velocity will be constant. **When the velocity of an object is constant (including when object is permanently at rest), the object is said to be in equilibrium.**

Force is the cause of change in motion :

Force does not cause motion. We can have motion in the absence of force, as described in Newton's first law. Force is the cause of change in motion as measured by acceleration.

"ma" is not a force :

Equation (i) does not say that the product ma is a force. All forces on object are added vectorialy as in equation (ii) to get the net force on the left side of the equation. This net force is then equated to the product of the mass of object and the acceleration that results from the net force.

Do not include an "ma force" in your analysis of the forces on an object.

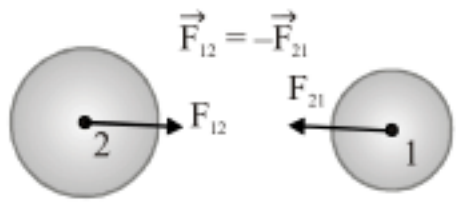
Definition of the newton

The SI unit of force is the newton, which is defined as the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s^2. From the definition and Newton's second law, we see that the newton can be expressed in terms of the following fundamental units of mass, length, and time

$1\ N \equiv 1\ kg.\ m/s^2$

Newton's Third law

Every action has equal and opposite reaction, and the action reaction act on the different bodies, simultaneously These action reaction pair must be of same type, viz. friction for friction, gravitation for gravitation etc.

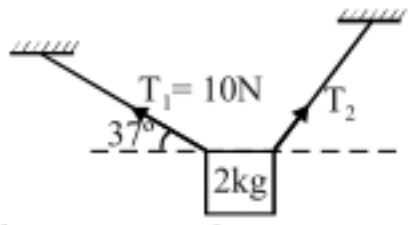


Caution

(i) Action and reaction forces act on different objects. Two forces acting on the same object, even if they are equal in magnitude and opposite in direction, cannot be an action-reaction pair.

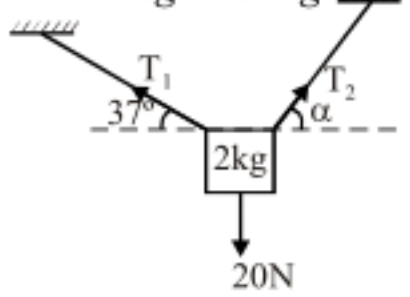
(ii) In an action reaction pair, both forces act simultaneously i.e. we can not say one as action and other reaction.

Illustration :



As in figure, A block of 2 kg hangs by two massless string. The tension in left string is 10N and makes 37° from horizontal. Find tension in right string. Also show action reaction pairs.

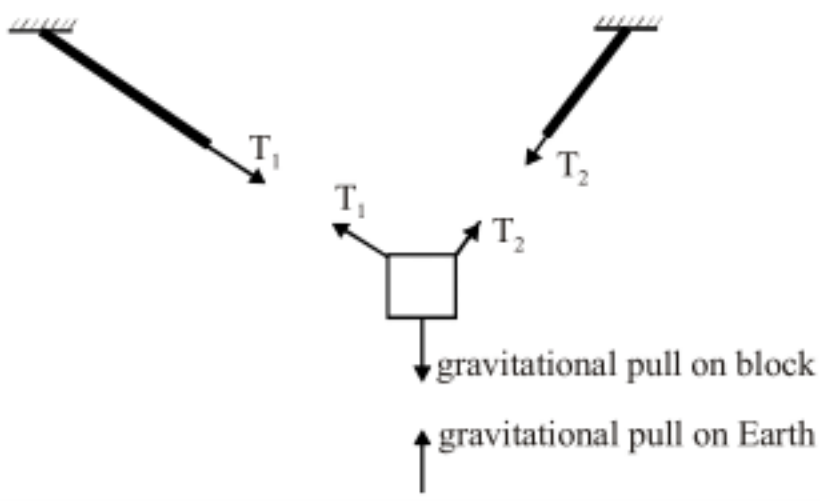
Sol.



block is in equilibrium i.e. rest so the net force on the block must be zero accounding to newton's first law as observed from an inertial reference frame

$\vec{T}_1 + \vec{T}_2 + m\vec{g} = \vec{0} \quad \Rightarrow \quad (-8\hat{i} + 6\hat{j}) + \vec{T}_2 - 20\hat{j} = \vec{0}$

$\vec{T}_2 = 8\hat{i} + 14\hat{j}$, *i.e. tension in right string* $T_2 = \sqrt{8^2 + 14^2} = 16.12\text{N}$ *and string makes angle* $\alpha = \tan^{-1}\left(\frac{7}{4}\right)$ *with the horizontal*



Practice Exercise

Q.1 An object experiences no acceleration. Which of the following cannot be true for the object ?

(A) A single force acts on the object.

(B) No force acts on the object.

(C) Forces act on the object, but the forces cancel.

Q.2 An object experiences net force and exhibits an acceleration in response. Which of the following statements is always true ?

(A) the object moves in the direction of the force.

(B) The acceleration is in the same direction as the velocity.

(C) The acceleration is in the same direction as the net force.

(D) The velocity of the object increases.

Q.3 If a fly collides with the windshield of a fast-moving bus, which object experiences an impact force with a larger magnitude ?

(A) the fly (B) the bus (C) the same force is experienced by both.

Q.4 If a fly collides with the windshield of a fast-moving bus, which object experiences the greater acceleration:

(A) The fly (B) the bus (C) the same acceleration is experienced by both.

Answers

Q.1	A	Q.2	C	Q.3	C	Q.4	A

Free body diagram (FBD)

In this diagram the object of interest is isolated from its surroundings and the interactions between the object and the surroundings are represented in terms of forces.

Remember not to show " M $\vec{a}$ " as force in FBD. $\vec{a}$ is the effect of net force acting on an object.

System

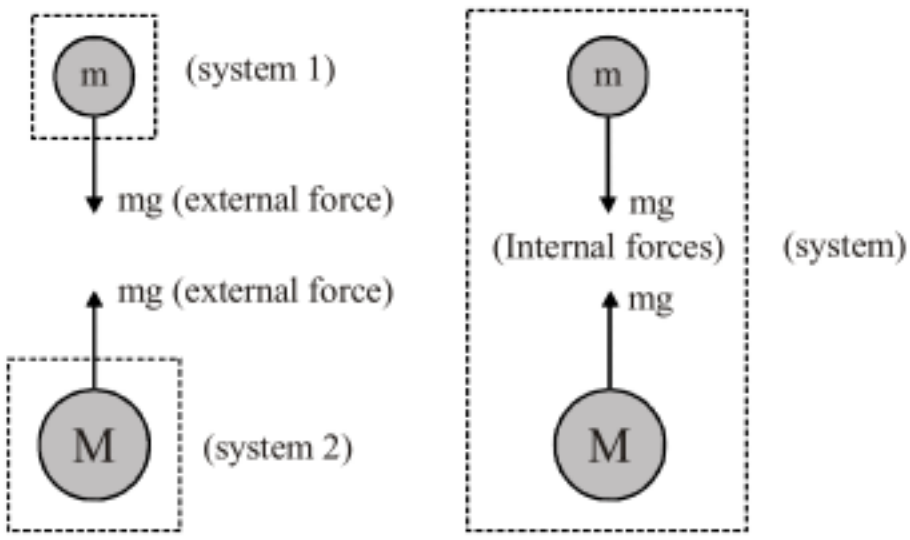
The system is what you choose to analyse motion.

1. According to system chosen, forces can be divided into two categories :

(i) Internal force (ii) External force

Whether a force is external or internal will depend on the system chosen. For example, earth is pulling a mass by applying a force mg on it. The mass is also applying a force mg on earth.

If we chose earth-mass as system, then mg becomes an internal force as it is within the system. But if choose only mass m as system; then mg acting on the mass is an external force.



We do not show internal force in FBD while solving a problem.

2. According to the origin of force, we can classify it into two types :

(i) Field forces

(ii) Contact forces

Field forces

The force experienced by an object without physical contact is known as field force.

e.g. Gravitational force, Electromagnetic force.

Gravitational force :

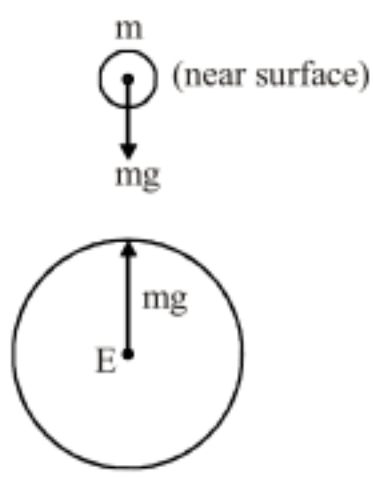
The force by which two bodies attract each other by virtue of their masses.

If two particles haivng masses m_1 & m_2 are seperated by a distance r, then the magnitude of force is given by $F = \frac{Gm_1m_2}{r^2}$.

This is called Newton's law of Gravitation

The force acts along the line joining the particles, this force is always attractive in nature.

For earth mass system, earth applies a force mg on mass m, which always acts towards the center of earth downwards. mg is called weight of the body.



Electromagnetic force :

If two particles having charges q_1 and q_2 are separated by a distance r, then the magnitude of force is given by : $F = \frac{Kq_1q_2}{r^2}$. This is called Coulomb's law and it acts along the line joining the particles. For like charges, the force is repulsive, and for unlike charges, the force is attractive.

Contact Force

The force experienced by an object by physical contact is known as contact force.

e.g. Frictional force (which will be dealt in detail later), Normal force, tension.

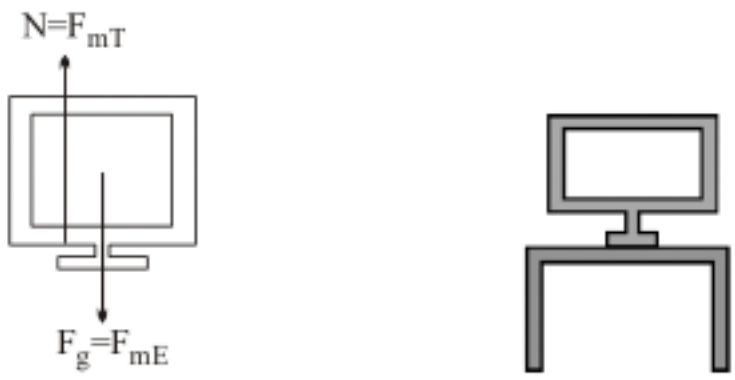
Normal force :

A contact force perpendicular (normal means perpendicular) to the contact surface that prevents two objects from passing through one another is called the normal contact force.

In the situation shown, LCD is kept on the table. Weight mg acts on it downwards still it is at rest. i.e. some force acts in the upward direction (Normal to the table) which balances the weight. This force is the normal force by the table on the LCD.

$F_{mT} \Rightarrow$ F on mass by table

$F_{me} \Rightarrow$ F on mass by Earth



(b)

Here $N = mg$ (not an action reaction pair)

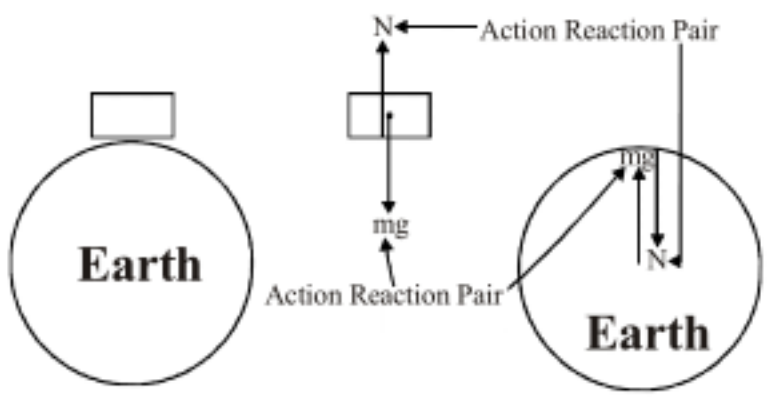
N Does not always equal mg

In the situation shown in figure and in many others, we find that the normal force has the same magnitude as the gravitational force. However, this is not always true. If an object is on an incline, if there are applied forces with vertical components, or if there is a vertical acceleration of the system, then $N \neq mg$. Always apply Newton's second law to find the relationship between N and mg.

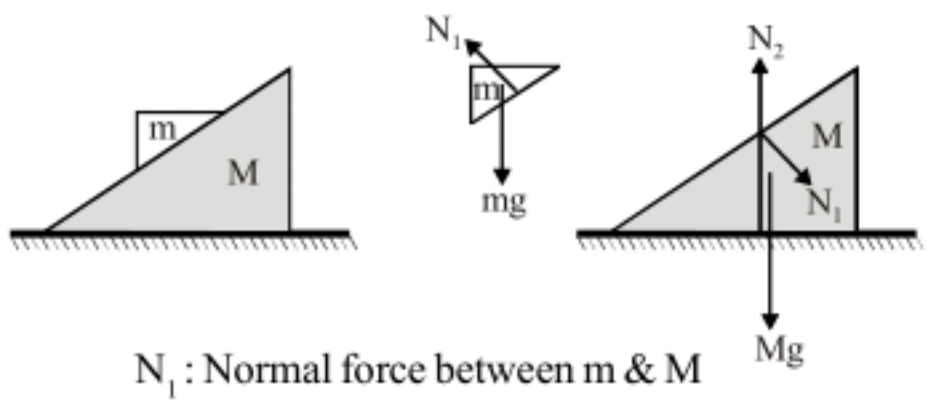
Note : Normal force is always a pushing force.

Normal force acts along the common normal

e.g. (i)



e.g. (ii)



N_1 : Normal force between m & M

N_2 : Normal force between M & floor

Normal force acts perpendicular to the surface in contact.

If one surface is not well defined, then normal force acts perpendicular to the surface of second object.

Here we can not specity the perpenducular to the rod at the point of contact. Thus normal acts perpenducular to the floor and the wall respectively.

e.g.

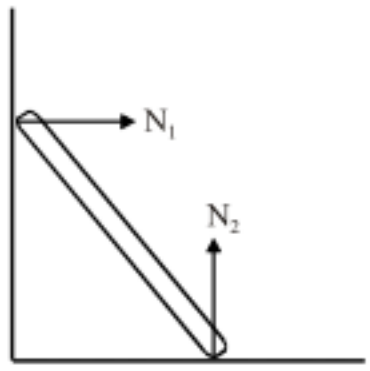
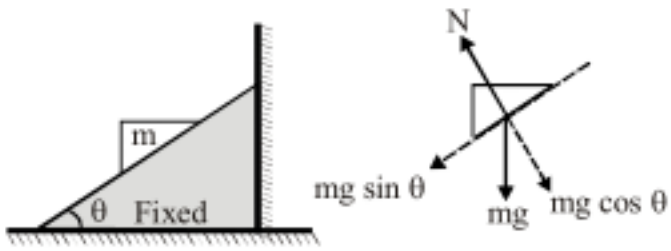


Illustration :

Find normal force at all contact points.

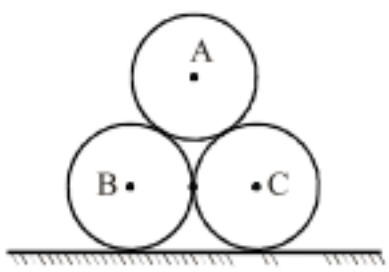
(i)



Force along the perpendicular to the inclined must be zero, as m has no acceleration in this direction

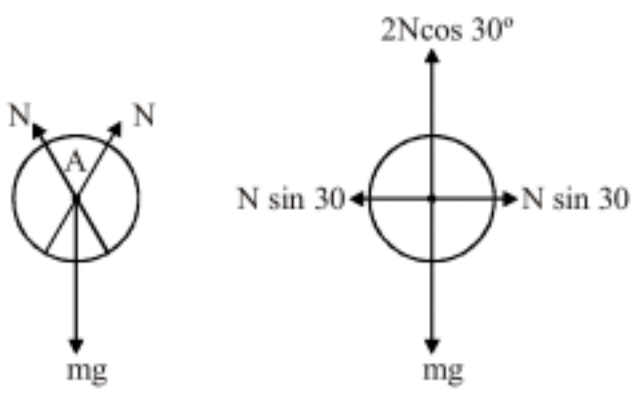
$\therefore \quad N = mg\cos\theta$

(ii)



Cylinders B & C are fixed and each cylinder is of mass m.

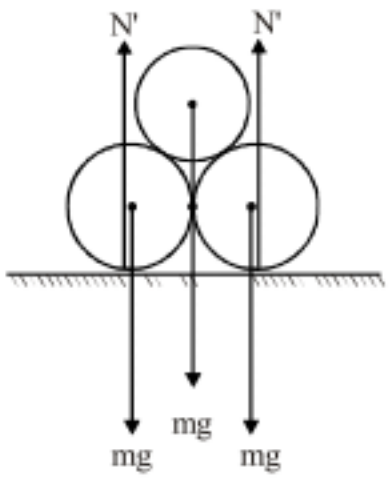
FBD of A



$$2\,N\cos 30^\circ = mg$$

$$N = \frac{mg}{\sqrt{3}}$$

Considering all the three together as system,

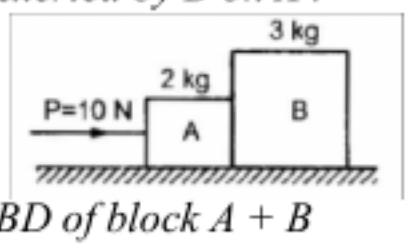


$$2N' = 3mg$$

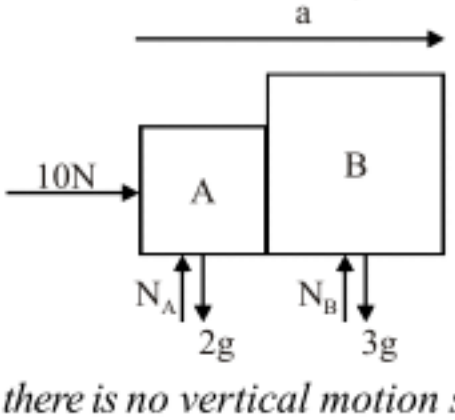
$$N' = \frac{3mg}{2}$$

Illustration :

Blocks A and B have masses of 2kg and 3kg respectively. The ground is smooth. P isan external force of 10 N. Find the force exerted by B on A .



Sol. *Block A and B as a system, FBD of block A + B*

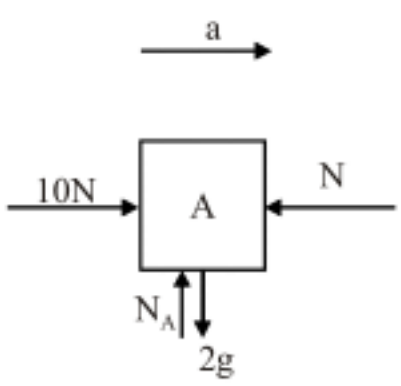


(here normal force between A & B is internal so we can not show it in FBD)

there is no vertical motion so the net force on system in vertical direction $\left(\Sigma\vec{f}_y = \vec{0}\right)$. *In horizotnal direction there is an external force 10N so the system will accelelate with acceleration a.*

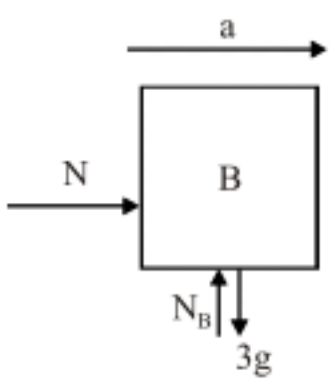
$$a = \frac{10}{5} = 2\,m/s^2$$

FBD of block A



$10 - N = 2 \times 2 \Rightarrow N = 6\ N$

FBD of block B



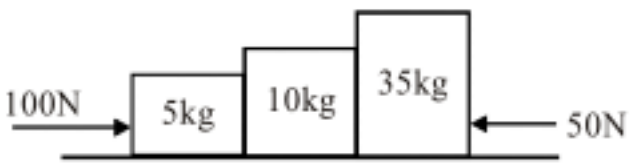
$N = 3 \times 2 = 6\ N$

(Here block A & block B are the seperate systems so the normal force N is external, normal force N must be shown in each FBD)

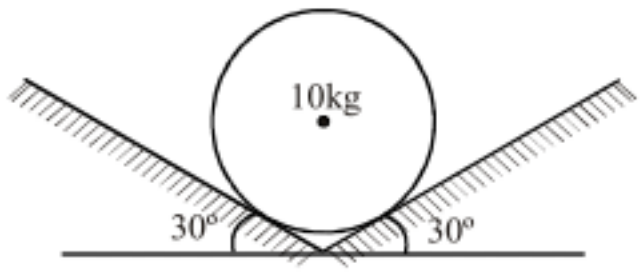
Practice Exercise

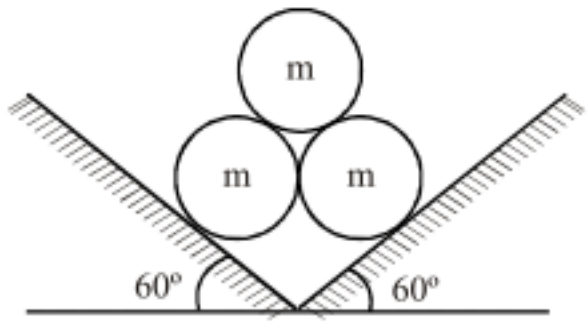
Q.1 Find normal forces at all the contact points.

(i)



(ii)





Q.2 Find normal contact force between sphere and inclind plane.

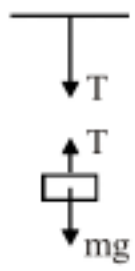
Answers

Q.1 (i) Normal contact force between 5 kg and 10 kg is 95 N. while 10 kg and 35 kg is 85 N.

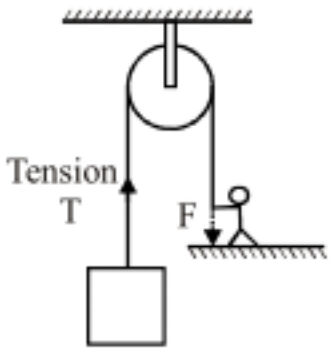
(ii) $\frac{100}{\sqrt{3}}$ N

Q.2 3 mg

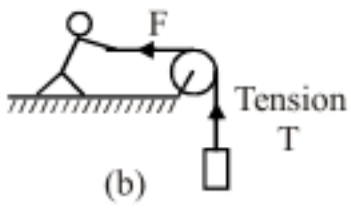
Tension force :



- It is self adjusting pulling force, which is electromagnetic in nature.
- We assume string is massless, Unless specified.
- Strings will be tight only when the ends are pulled apart.
- **Tension is a pulling force, acts away from the object along the string**
- Tension throughout the massless string is same usually.
- A pulley can change the direction of the force exerted by a cord.



(a)

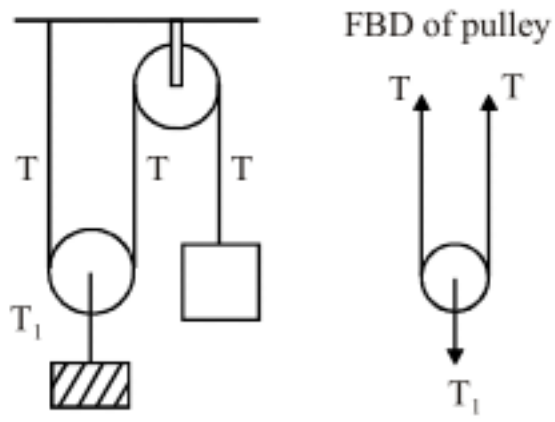


(b)

Illustration :

In the situation shown in figure draw the FBD of the moving pulley assume that the pulley is massless. Also relate tension in both strings.

Sol.



According to Newton's second law :

$T_1 - 2T = m_{pulley}\, a$ ($\because$ mass of pulley is negligible i.e $m_{pulley} \to 0$)

$T_1 - 2T = 0 \Rightarrow T_1 = 2T$

Illustration :

When two unequal masses are hung vertically over a frictionless pulley of negligible mass the arrangement is called atwood machine. The device is sometimes used in the laboratory to measure the gravitational field strength. Determine the magnitude of the acceleration of the two masses and the tension in the string. Consider $m_2 > m_1$.

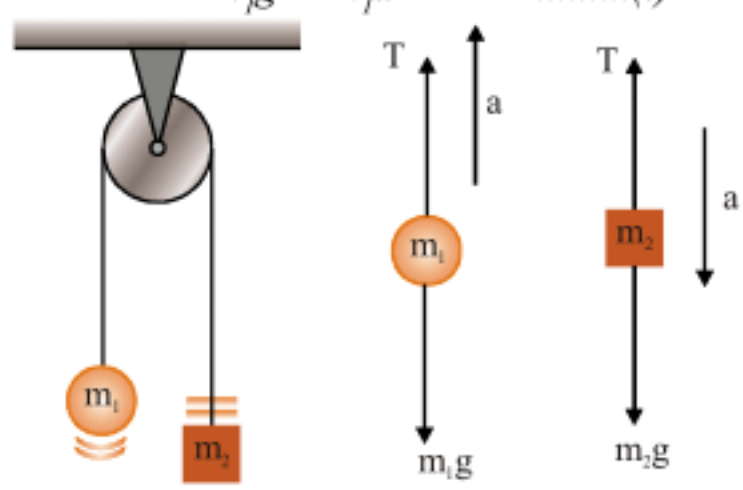
Sol. *The free body diagrams for the two masses are shown in figure. Two forces act on each block: the upward force exerted by the string. T, and the downward force of gravity.*

The magnitude of the net force exrted on m_1 is $T - m_1g$, the magnitude of the net force exerted on m_2 is $m_2g - T$.

Because the blocks are connected by a string, their accelerations must be equal in magnitude. m_1 must accelerate upward, while m_2 must accelerate downward.

When Newton's second law is applied to m_1, with acceleration upward for this mass we find (taking upward to be the positive y direction)

$$T - m_1g = m_1a \qquad .....(i)$$



Similarly, for m_2 we find

$$m_2g - T = m_2a \qquad .....(ii)$$

Adding (i) & (ii)

$$m_2g - T + T - m_1g = m_1a + m_2a$$

or

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g \qquad .....(iii)$$

When (iii) substituted into (i), we get

$$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g \qquad .....(iv)$$

The result for the acceleration, in equation (iii), can be interpreted as the ratio of the unbalanced force on the system to the total mass of the system.

Special cases

When $m_1 = m_2$, $a = 0$ *and* $T = mg = m_1g = m_2g$

If $m_2 >> m_1$, $a \approx g$ *(a freely falling body) and* $T \approx 2m_1g$.

Basic steps for applying Newton's Laws

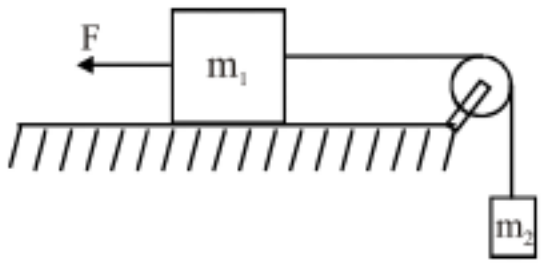
The following procedure is recommended when dealing with problems involving Newton's laws:

- Draw a simple, neat diagram of the system to help conceptualize the problem.
- If any acceleration component is zero, the particle is in equilibrium in this direction and $\Sigma F = 0$ in this direction. If not the particle is undergoing an acceleration, the problem is one of non-equilibrium in this direction and $\Sigma F = ma$.
- Isolate the object whose motion is being analyzed. Draw a free-body diagram for this object. For systems containing more than one object, draw separate free-body diagrams for each object.

  Do not include in the free-body diagram forces exerted by the object on its surroundings.
- Establish convenient coordinate axes for each object and find the components of the forces along these axes. Apply Newton's second law, $\Sigma F = ma$, in component form. Check your dimensions to make sure that all terms have units of force.
- Solve the component equations for the unknowns. Remember that you must have as many independent equations as you have unknowns to obtain a complete solution.
- Make sure, your results are consistent with the free-body diagram. Also check the predictions of your solutions for extreme value of the variables. By doing so you can often detect errors in your results.

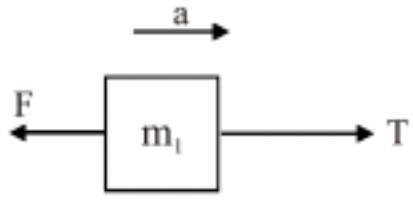
Illustration :

A constant force $F = m_2g/2$ *is applied on the block of mass* m_1 *as shown in figure. The string and the pulley are light and the surface of the table is smooth. Find the acceleration of* m_1.



Sol. *When Newton's second law is applied to m_1, with acceleration rightward for this mass (because $m_2g > F$).*

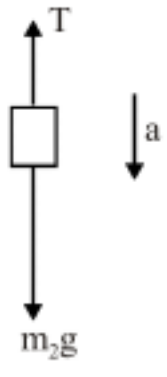
FBD of m_1



$$T - F = m_1 a \qquad \text{.....(i)}$$

similarly for m_2

FBD of m_2



$$m_2 g - T = m_2 a \qquad \text{.....(ii)}$$

Adding (i) & (ii)

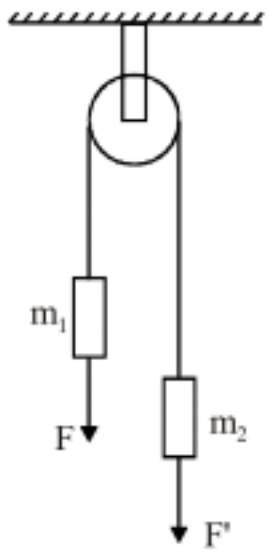
$$m_2 g - T + T - F = (m_1 + m_2)\, a$$

$$a = \frac{m_2 g - F}{m_1 + m_2} \qquad \left(F = \frac{m_2 g}{2}\right)$$

$$a = \frac{m_2 g}{2(m_1 + m_2)}$$

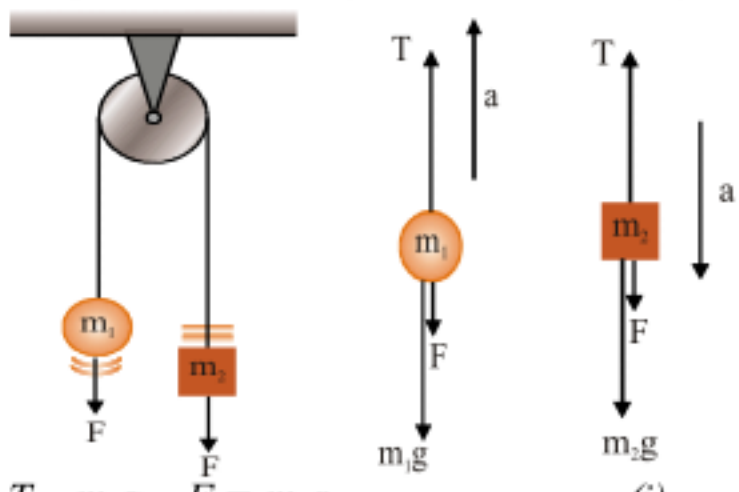
Illustration :

In figure m_1 = 2 kg, m_2 = 5 kg and F = 1 N. Find the acceleration of blocks.



Sol. *The free body diagrams for the two masses are shown in figure. Three forces act on each block: the upward force exerted by the string. T, the downward force of gravity and the downward force F. Thus, the magnitude of the net force ecerted on m_1 is $T - m_1g - F$, while the magnitude of the net force exerted on m_2 is $m_2g + F - T$. Because the blocks are connected by a string, their accelerations must be equal in magnitude. It is given that $m_2 > m_1$, then m_1 must accelerate upward, while m_2 must accelerate downward.*

When Newton's second law is applied to m_1, with a (acceleration) upward for this mass (because $m_2 > m_1$), we find (taking upward to be the positive y direction)



$$T - m_1g - F = m_1a \qquad .....(i)$$

Similarly, for m_2 we find

$$m_2g + F - T = m_2a \qquad .....(ii)$$

Adding (i) & (ii)

$$m_2g + F - T + T - m_1g - F = m_1a + m_2a$$

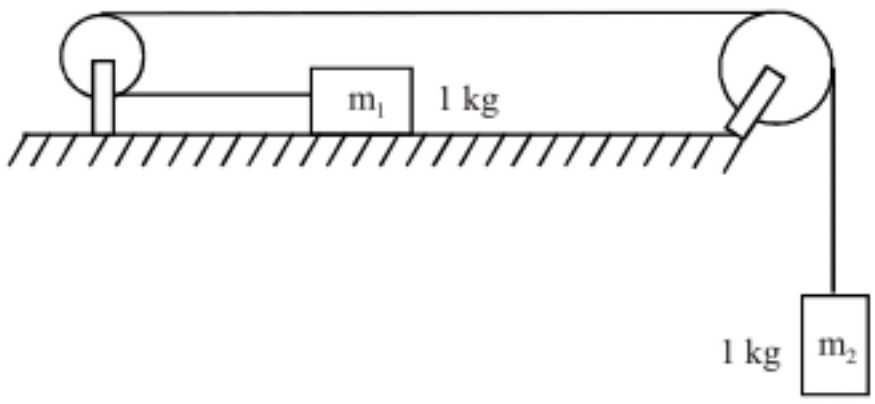
$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g \qquad .....(iii)$$

Given that $m_1 = 2$ kg, $m_2 = 5$ kg and $F = 1$ N

therefore $$a = \left(\frac{5-2}{5+2}\right)g = \frac{3g}{7}$$

Illustration :

Calculate the acceleration of the two blocks (m_1 and m_2) and tension in the string shown in figure. The pulley and the string are light and all the surfaces are frictionless. Take $g = 10$ m/s^2.



Sol. *The free body diagrams for the two masses are shown in figure. Two forces act on m_2: the upward force exerted by the string T, and the downward force of gravity. Thus, the magnitude of the net force exerted on m_2 is $m_2g - T$. Because the blocks are connected by a string, their accelerations must be equal in magnitude.*

When Newton's second law is applied to m_2, with a (acceleration) downward for this mass.

$$m_2 g - T = m_2 a \qquad .....(i)$$

similarly for m_1 $\qquad T = m_1 a \qquad .....(ii)$

Adding (i) & (ii)

$$m_2 g - T + T = (m_1 + m_2) a$$

$$a = \frac{m_2 g}{m_1 + m_2}$$

Given that $m_1 = m_2 = 1$, *therefore*

$$a = \frac{g}{2}$$

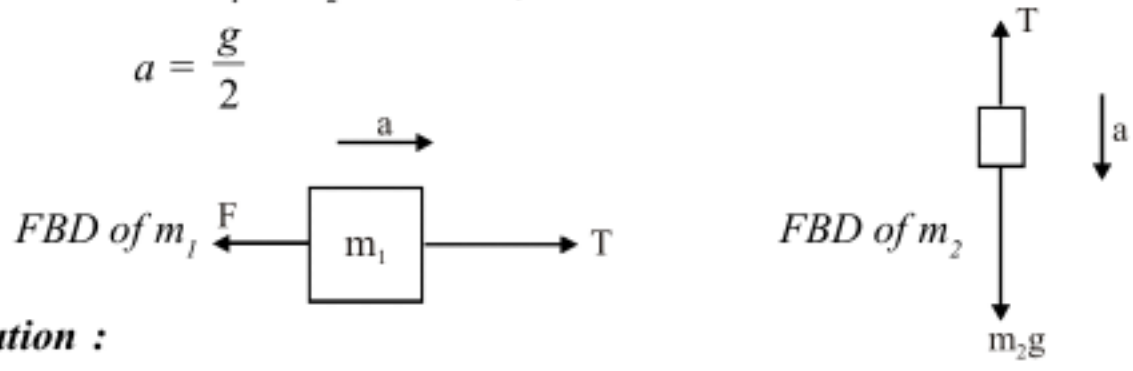
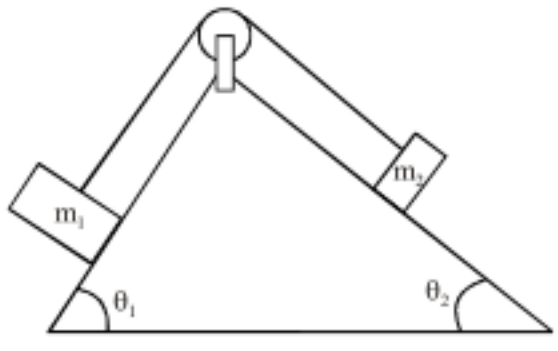
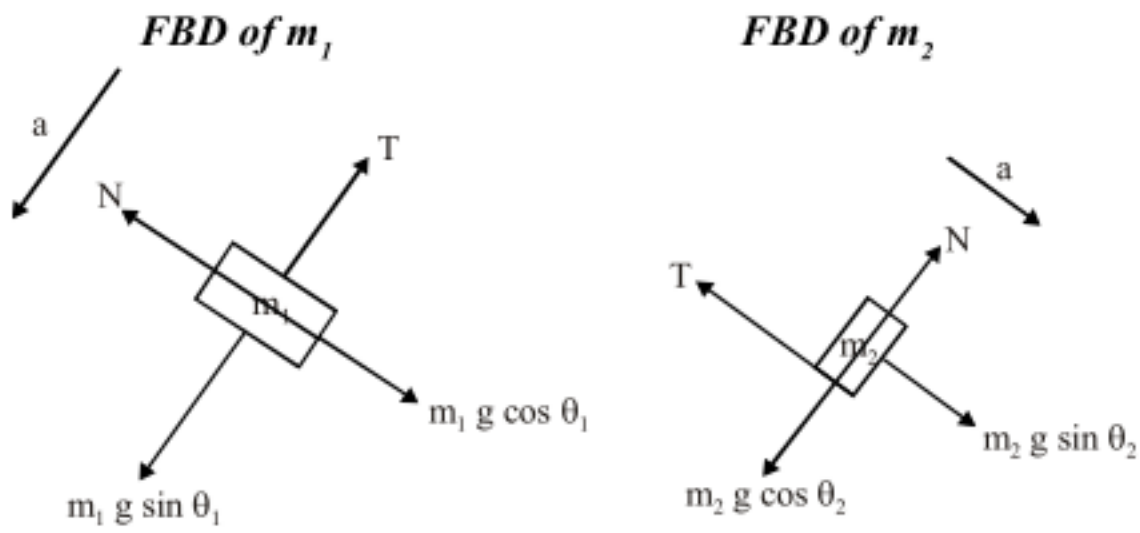


Illustration :

Consider the situation shown in figure. All the surface are frictionless and the string and the pulley are light. Find the magnitude of the acceleration of the two blcoks.



Sol. *The free body diagrams for the two masses are shown in figure. Two forces act on each block: the upward force along the inclined plane exerted by the string T, the downward force of gravity and normal force (N_1 & N_2). Thus, the magnitude of the net force exerted on m_1 along the inclined is $m_1 g \sin\theta_1 - T$, while the magnitude of the net force exerted on m_2 along the inclined is $T - m_2 g \sin\theta_2$. Because the blocks are connected by a string, their accelerations must be equal in magnitude. If we assume that $m_2 > m_1$, then m_1 must accelerate down the inclined, while m_2 must accelerate up the inclined.*



When Newton's second law is applied to m_1, with a (acceleration) down the inclined for this mass

(because $m_1 g \sin\theta_1 > m_2 g \sin\theta_2$)

$$m_1 g \sin\theta_1 - T = m_1 a \qquad .....(i)$$

Similarly, for m_2 we find

$$T - m_2 g \sin\theta_2 = m_2 a \qquad .....(ii)$$

Adding (i) & (ii)

$$m_1 g \sin\theta_1 - T + T - m_2 g \sin\theta_2 = m_1 a + m_2 a$$

or

$$a = \left(\frac{m_1 g \sin\theta_1 - m_2 g \sin\theta_2}{m_1 + m_2}\right) .....(iii)$$

Illustration :

A ball of mass 'm' falls from rest under gravity in a resistive medium. The resistive force is given by f = – kv. Find the velocity of ball as a function of time. Plot a v-t graph for it.

Sol. $m\dfrac{dv}{dt} = mg - kv$

$$\frac{m}{k}\int\frac{dv}{v} = -\int dt \qquad \frac{m}{k}\frac{dv}{dt} = \frac{mg}{k} - v$$

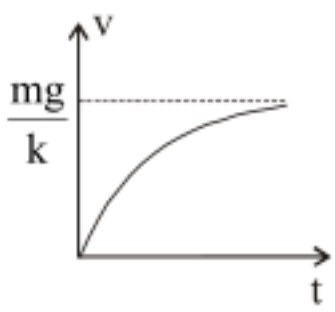
$$\int_0^v \frac{dv}{\frac{mg}{k} - v} = \frac{k}{m}\int_0^t dt$$

$$\left[\frac{\ell n\left(\frac{mg}{k} - v\right)}{-1}\right]_0^v = \frac{k}{m}[t]_0^t$$

$$\ell n\,\frac{mg - kv}{mg} = -\frac{k}{m}t$$

$$1 - \frac{kv}{mg} = e^{-\frac{kt}{m}}$$

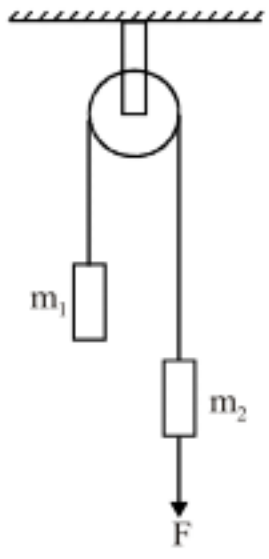
$$v = \frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right)$$



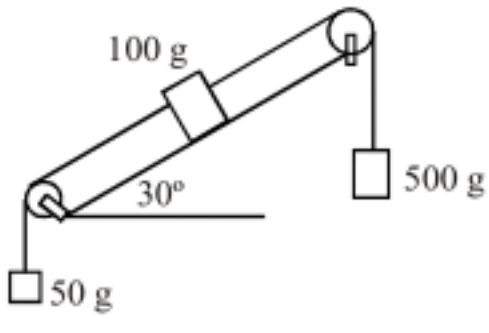
Practice Exercise

Q.1 Find the accelerations of blocks A, B & C and tentions in the strings. All the blocks are of equal mass 10 kg and F = 60 N

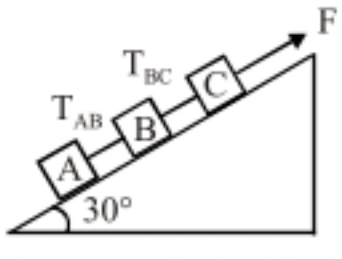
Q.2 In figure $m_1 = 2$ kg, $m_2 = 5$ kg and F = 21N. Find the acceleration of blocks.



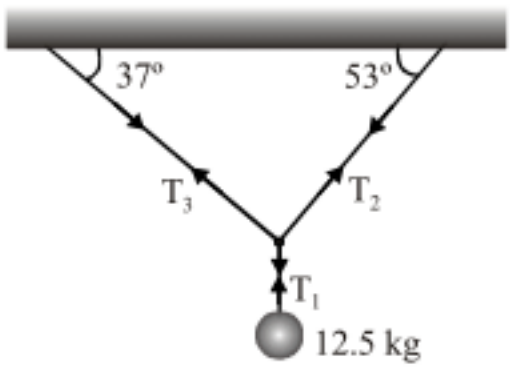
Q.3 Find the acceleration of the 500 g block in figure.



Q.4 Find the accelerations of blocks A, B & C and tensions in the strings. All the blocks are of equal mass 10 kg and F = 300 N



Q.5 A body of mass 12.5 kg is suspended with the help of light inextensible strings as shwon in figure. Find tension in three strings. Strings are light [$g = 10\ ms^{-2}$]



Q.6 Which of the following is the reaction force to the gravitational force acting on the body as you sit in your desk chair ?

(a) The normal force exerted by the chair

(b) The force you exert downward on the seat of the chair

(c) Neither of these forces.

Answers

Q.1	$a_A = a_B = a_C = 2\ m/s^2$, $T_{AB} = 40\ N$, $T_{BC} = 20\ N$			Q.2	$a = 7 m/s^2$
Q.3	$a = 80/13\ m/s^2$	Q.4	$a = 5\ m/s^2$	Q.5	$T_1 = 125\ N$, $T_2 = 100\ N$, $T_3 = 75\ N$
Q.6	C				

Spring Force

We know that the more force we apply to a spring, the more it stretches. For a spring that obeys Hooke's low, the extension of the spring is proportional to the applied force. If we stretch spring by a distance x from its equilibrium position, it applies a restoring force F, towards its equilbrium position, which is proportional to x, given by

$$F = kx$$

Here k is a proportionality constant, known as spring constant or stiffness. A spring has tendecy of restoring its natural length state, thus whether we stretch it or compress, it always opposes the external force in the direction towards its equilibrium position.

One more point is to be noted is that, a spring applies restoring force equally at both of its ends, doesn't matter whether an end is fixed or not.

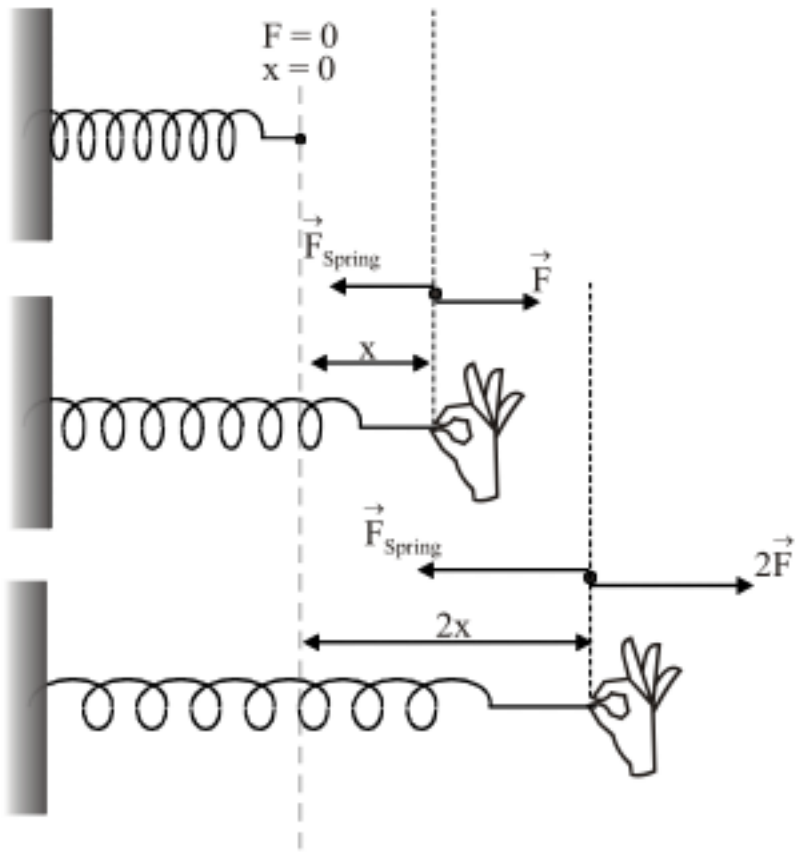
If we look at FBD of spring we will note that equal force on spring must act from both ends.

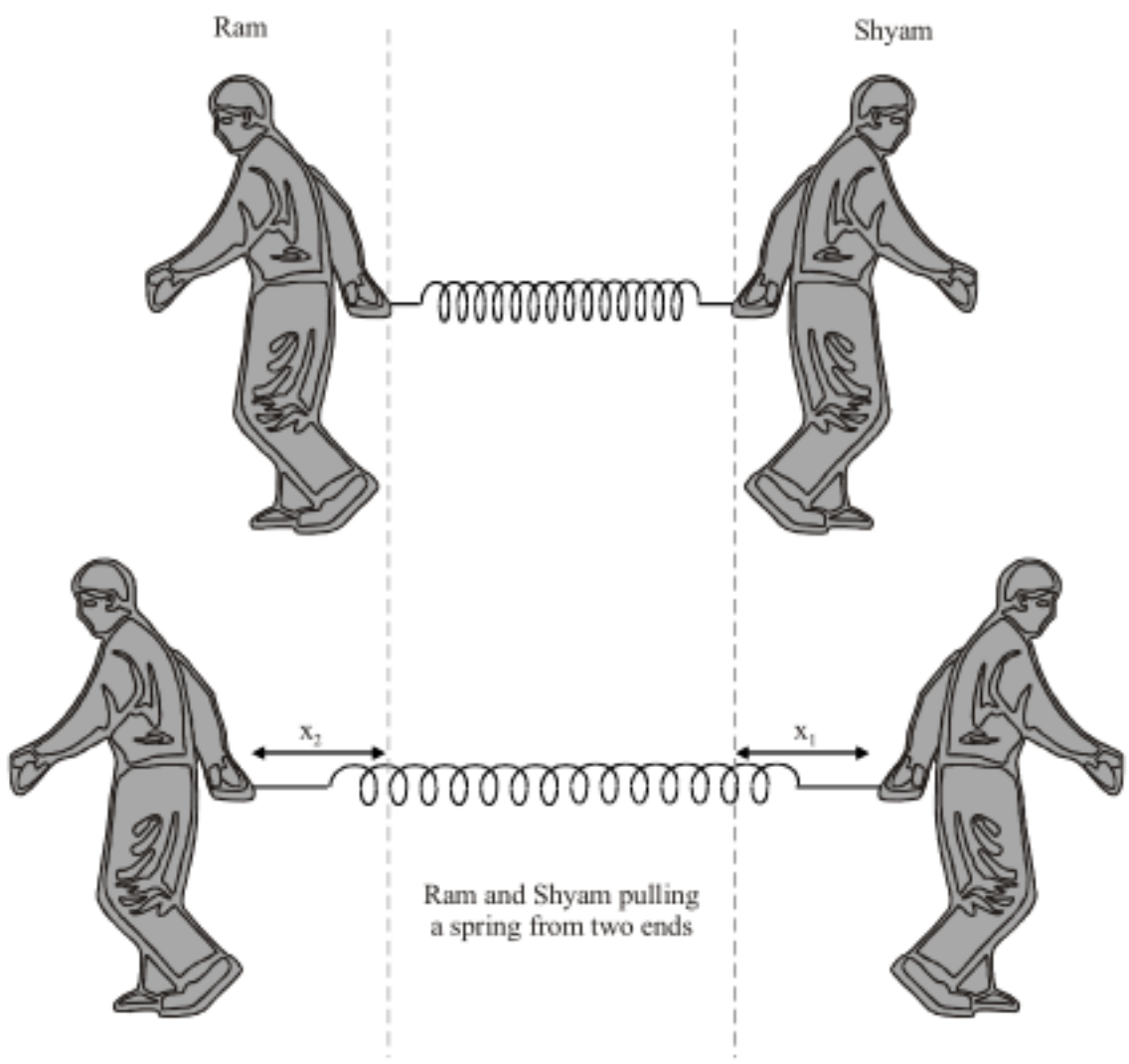
$$F - F' = m_s \times a \qquad \text{(spring is mass less, } m_s \to 0)$$

therefore $\quad F = F'$

In figure an end of the spring is fixed to wall and other is pulled by applying a force. As the restoring force is directly proportional to the deformation in it, for stretching it by x, we apply a force F on it and for stretching it to double the length (2x), we have to apply a force double of the previous value i.e. 2F.

Let's say Ram & Shyam are pulling a spring from two ends as shown in figure Ram moves x_2 and Shyam moves x_1.

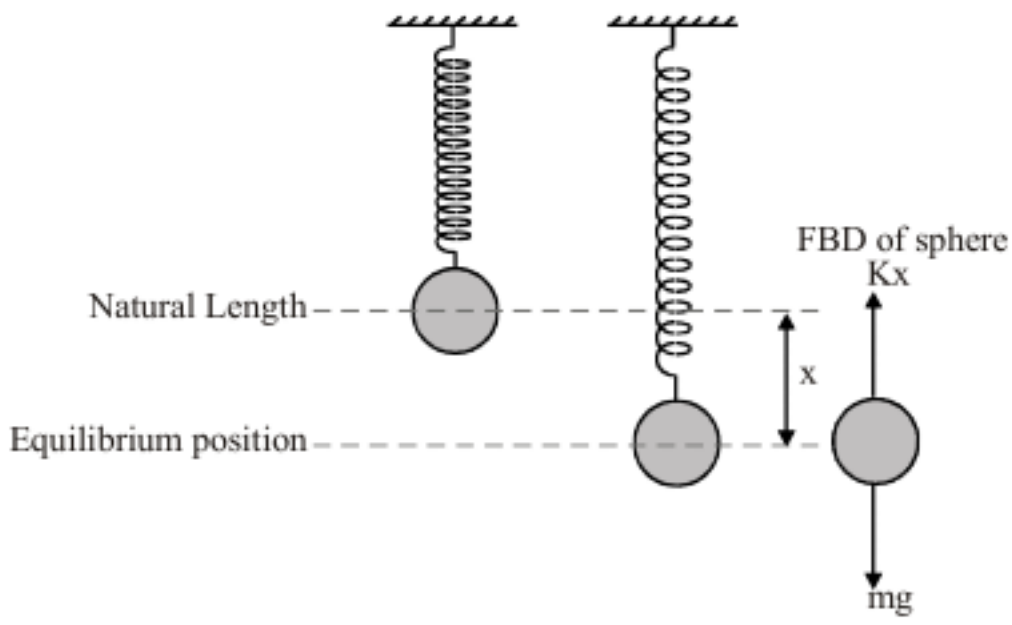




The force acting on Ram and Shyam is $k(x_1 + x_2)$, not kx_2 on Ram and kx_1 on Shyam. Force due to spring is kx where x is defined as $|l - l_0|$, where l is present length and l_0 is natural length,

Illustration :

The spring is hanging vertically, and a sphere of mass m is attached to its lower end. Under the pull of the weight, the spring stretches a distance x from its natural length. If a spring is stretched 2.0 cm by a suspended object having a mass of 0.5 kg, what is the force constant of the spring ?



Sol. *Free-body diagrams for sphere in Shown in figure. Two force are acting on sphere : the upward force exerted by the spring (T = kx) and downward force of gravity (mg). As the sphere is in equilibrium the two forces will be equal in magnitude.Equilibrium is the condition when the acceleration of the block is zero)*

$$kx = mg \qquad \Rightarrow \qquad k = \frac{mg}{x} = \frac{0.5 \times 10}{2.0 \times 10^{-2}} = 250 N/m$$

Equivalent spring constant

(In all the discussions we will ignore mass and and damping of spring)

When two or more springs are connected in some manner then the combination can be replaced by a single equivalent spring such that it produces same elongation for same applied force.

Parallel combination

When springs are connected in parallel, then we can replace them by single spring of spring constant k_{eq} where $k_{eq} = k_1 + k_2$. This situation is shown in figure. Here we present the proof.

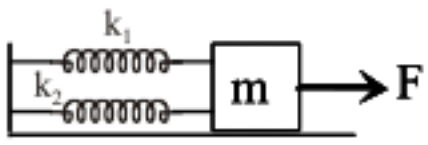
If the force F pulls the mass m by y, the stretch in each spring will be same.

$$y_1 = y_2 = y$$

Now for an equivalent spring $F = k_{eq} \times y$ and as spring constants are not equal so $F_1 \neq F_2$.

For equivalence,

$$F = F_1 + F_2 \Rightarrow K_{eq}\, y = k_1 y + k_2 y$$



this reduces to

$$k_{eq} = k_1 + k_2$$

For more springs $k = k_1 + k_2 + k_3 + ...$

Series combination

When spring are connected in series then we can replace them by a single spring of spring constant k_{eq} where $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$. This situation is shown in figure.

Here we present the proof. As the spring are massless, so force in the spring will be the same

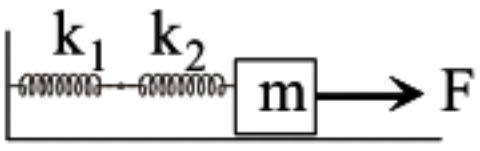
$$F_1 = F_2 = F$$

Now for equivalent spring $F = k_{eq}\, y$, as spring constants are not equal so extensions will not be equal, but total extension y can be written as sum of two extensions $y = y_1 + y_2$

or $$\frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2} \qquad \left[as\ for\ F = ky, y = \frac{F}{k} \right]$$

For more than two springs Connected in series:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$$



Parts of a spring :

If a spring of force constant k of length l is cut in two parts say of l_1 and l_2, let us assume that new force constants are k_1 and k_2 for the two parts. If we connect these two parts in series, the equivalent force constant must be initial k. Thus we have

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

According to the molecular properties of a spring, the force constant of a part of the spring is inversely proportional to its length, which gives us

$$k_1 = \frac{c}{l_1} \qquad \text{and} \qquad k_2 = \frac{c}{l_2}$$

Where c is a positive constant depending upon the material of spring. Substituting the above values of new force constants k_1 and k_2 in equation, we get

$$\frac{1}{k_{eq}} = \frac{l_1}{c} + \frac{l_2}{c}$$

or $$c = \frac{l}{k_{eq}}$$

Using value of c in equation, we have

$$k_1 = \frac{k_{eq} l}{l_1} \qquad \text{and} \qquad k_2 = \frac{k_{eq} l}{l_2}$$

Illustration :

A spring of constant K is cut into two parts in ratio 1:3. Find the spring constant of individual springs.

Sol. *Let us suppose natural length of spring is l*

Now, we have to find spring constant of both the parts (of length l/4 and 3l/4)

lets say spring constants of the parts are k_1 and k_2

we know $k \propto \frac{1}{l}$ *or* kl = *constant i.e.* $k_1 l_1 = k_2 l_2 = kl$

$$k_1 \times \frac{l}{4} = k_2 \times \frac{3l}{4} = kl$$

$$\therefore \quad k_1 = 4k \ \& \ k_2 = \frac{4k}{3}$$

Illustration :

A mass m is connected to a spring of spring constant K as shown. Find the extension of the spring when the block is at its equilibrium position.

Sol. *The free body diagrams for the sphere is shown in figure. Two forces act on sphere: the upward force exerted by the spring (T=kx), and the downward force of gravity (mg).*

Given that sphere is in equilibrimum therefore these two forces should be equal in magnitude.

$$kx = mg \Rightarrow x = \frac{mg}{k}$$

FBD of sphere

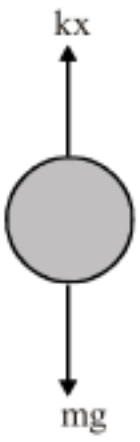
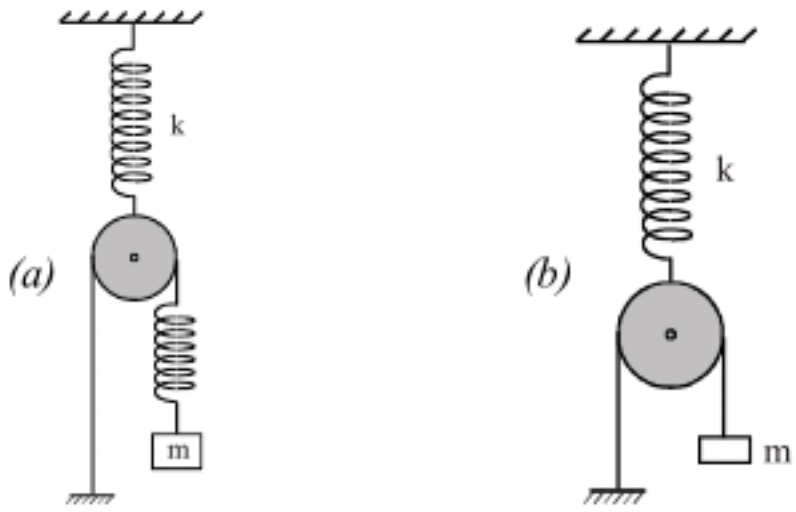


Illustration :

Find extension of spring in equilibrium condition, all the blocks are of mass m and all springs are of spring constant k.

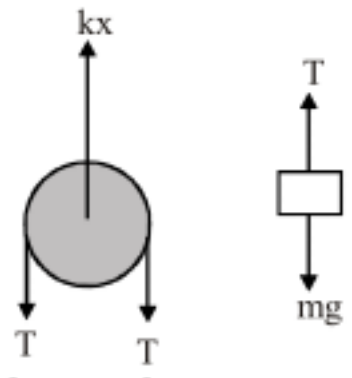


Sol. *(a) The free body diagrams for the block and pulley are shown in figure. Three forces act on pulley Two forces act on block: the upward force exerted by the string (T), and the downward force of gravity (mg). Given that block is in equilibrium therefore these two forces should be equal in magnitude.*

kx = 2T (force on pulley should be zero because it is massless)

$$\frac{kx}{2} = mg \Rightarrow x = \frac{mg}{2k}$$

FBD of pulley & block



(b) *Here again mass M is in equilibrium, thus tension in the string connected to it must be equal to Mg and hence the restoring force in the lower spring will also be Mg. If x_1 be the extension in this spring. we have*

$$k_1x_1 = Mg$$

$$x_1 = \frac{Mg}{k_1}$$

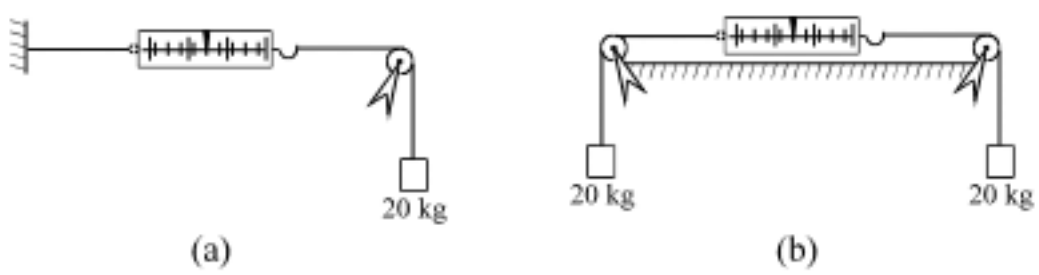
As pulley is light, the tension in upper string must be twice that of the lower string, 2Mg which is equalt to the restioring force in the upper spring. If x_2 is the extention in the upper spring, we have

$$k_2x_2 = 2Mg$$

or $$x_2 = \frac{2Mg}{k_2}$$

Illustration :

Figures show a light spring balance connected in two different arrangements. The graduations in the balance measure the tension in the spring.



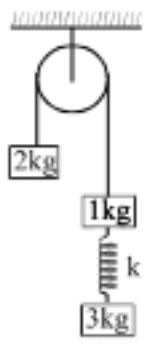
The ratio of reading of balances (T_a/T_b) is _____?

Sol. *Spring balnce gives tension in the string*
Because tension at both side of the spring balance will be same in case (a) & in case (b), $T_a = T_b = 20$ g

$$\therefore \quad \frac{T_a}{T_b} = 1$$

Illustration :

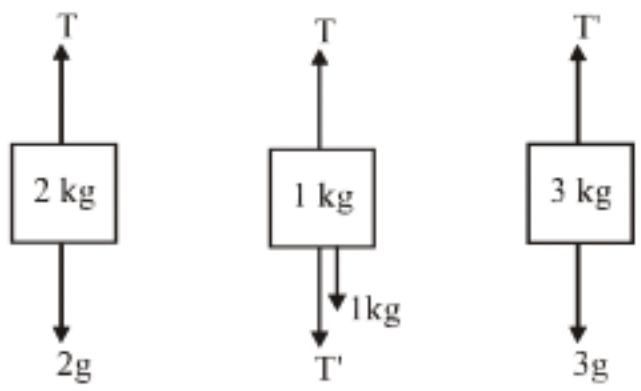
From the fixed pulley, masses 2 kg, 1 kg and 3 kg are suspended as shown in the figure. Find the extension in the spring if k = 100 N/m. (assume 1 kg and 3 kg block move with same acceleration)



Sol. *The FBD for the three block are shown in figure :*

Two force act on 2 kg block upward force exerted by the string T & downward force of gravity 2g thus, the magnitude of net force on 2 kg block is T – 2g.

FBD of blocks



Applying newton's II law on 2 kg block;

$$T - 2g = 2a \qquad \text{.....(i)}$$

Similarly for 1 kg & 3 kg block;

$$T' + 1g - T = 1a \qquad \text{.....(ii)}$$

$$3g - T' = 3a \qquad \text{.....(iii)}$$

Adding (i), (ii) & (iii)

we get $\quad 2g = 6a$

$$a = \frac{g}{3}$$

$\therefore$ *substituting value of a in equation (iii); tension in spring* $T' = 3g - 3a \quad (T' = kx)$

$$T' = 2g$$

$$\therefore x = \frac{2g}{k} = \frac{g}{50} = 0.2\ m$$

Cutting of spring and string :

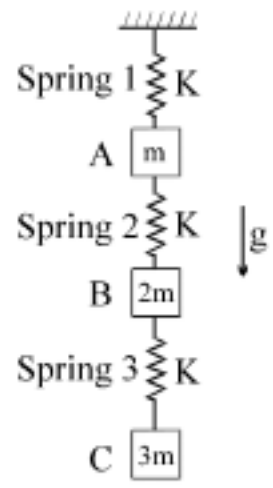
Spring force doesn't change instantaneously, whereas, the tension in the string changes instantaneously.

If a spring is cut, the tension in that spring becomes zero instantaneously.

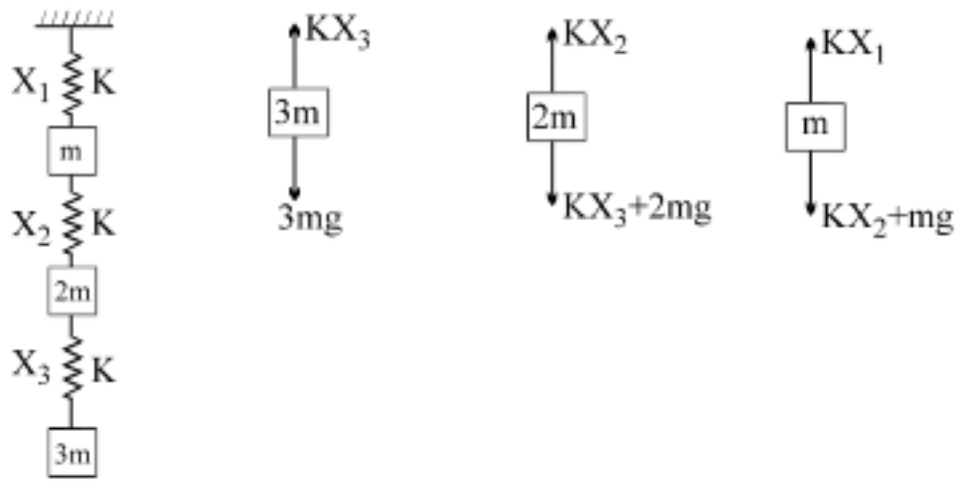
Illustration :

The system shown in the figure is in equilibrium. Find the initial acceleration

of A, B and C just after the spring-2 is cut.



Sol. *FBD of blocks before cutting the spring*



$$3mg = KX_3 \qquad ....(1)$$

$$2mg + KX_3 = KX_2$$

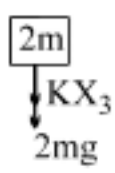
$$\therefore \quad 2mg + 3mg = KX_2 \Rightarrow 5\,mg = KX_2 \qquad ....(2)$$

$$KX_1 = 6\,mg \qquad ....(3)$$

when spring 2 is cut

spring force in other two spring remain becomes zero, while unchanged.

$$KX_1 - mg = ma_3 \Rightarrow a_3 = 5g \uparrow$$



$$KX_3 + 2mg = 2ma_2 \Rightarrow a_2 = \frac{5g}{2} \downarrow$$

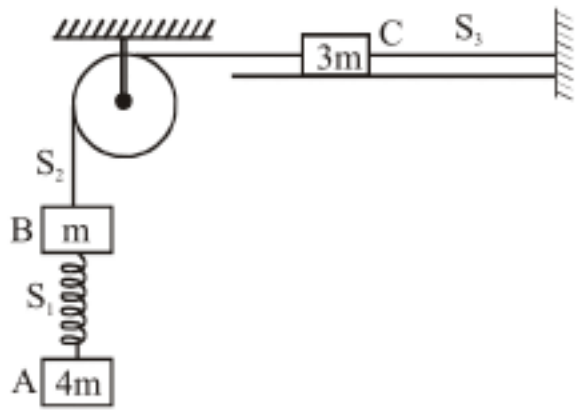
acceleration of 3 m will be zero.

Conclusion :

It is important to remember that ropes can change tension instantaneously while spring need to move to change tension, so in this example tension in spring is not changing instantaneously

Illustration :

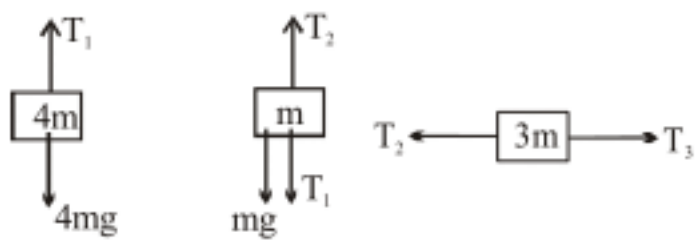
In following setup pulley strings and spring are light. Initially all masses are in equilibrium and at rest.



(a) Find tension in spring and tension in ropes

(b) Find accleration of masses immediately after the string S_3 is cut.

Sol. (a)



applying Newton's 2nd law to block A

$$4mg - T_1 = 0$$

applying Newton's 2nd law to block B

$$mg + T_1 - T_2 = 0$$

applying Newton's 2nd law to block C

$$T_3 - T_2 = 0$$

Solving $T_1 = 4mg$

$T_2 = 5mg$

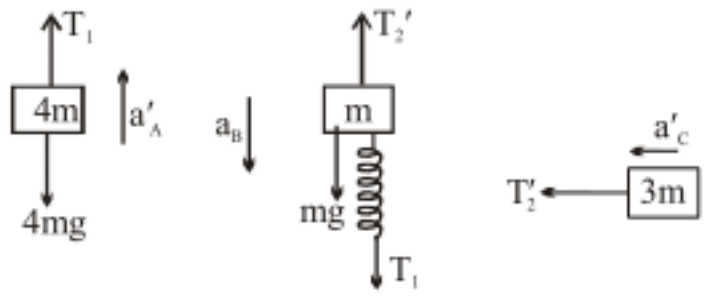
$T_3 = 5mg$

Here spring is behaving same as string except that it is streched while string can not strech.

(b) *The most important point in this problem is that any object of finite mass can not change its position instantaneously, as this require infinite velocity.*

Thus immediately after cutting the string S_3 all masses will remain at same position and force due to spring will not change. As force of spring is k x and x is $l - l_0$. At the same time we would like to emphasize that tension in string S_2 will change instantaneously (Tension is a self adjusting Force). To maintain constraint relation between blocks B & C have same since they magnitude of accleration. We can identify all the forces acting on all objects. Only tension T in string S_2 is unknown force all other forces are known.

Considering FBD



We know from part (a) that tension in spring is

T_1 *and* $T_1 = 4\,m\,g$

Writing Newton's Second Law for A

$$4\,mg - T_1 = 4m\,a_A'$$

Writing Newton's Second Law for B

$$T_1 + mg - T_2' = ma_B'$$

Writing Newton's Second Law for C

$$T_2' = 3ma_C'$$

Quantities which may have different value from part (a)are represented using symbol ', for eg a tenstion in string s_2 is T_2', others which have same value as part (a) have been retained with same symbol.

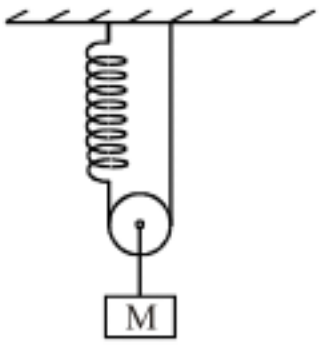
as $a_B' = a_C'$

solving we get $a_A' = 0 \quad a_B' = a_C' = \frac{5}{4}g$

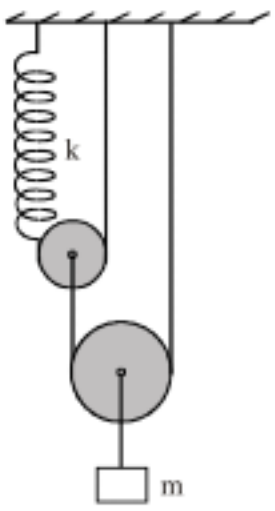
Practice Exercise

Q.1 Find extension of spring in equilibrimum condition, all the blocks are of mass m and all springs are of spring constant k.

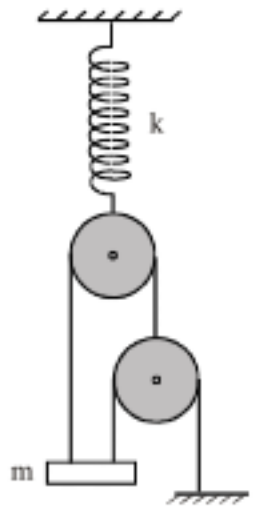
(i)



(ii)



(iii)



Q.2 Two blocks A and B of mass M and 2M respectively, are hanging from a ceiling by means of a light spring and a light string as shown. If the string between the blocks is suddenly cut, tinα, the accelerations of the block A and B.

Answers

Q.1 (i) $\frac{Mg}{2k}$; (ii) $\frac{mg}{4k}$; (iii) $\frac{4mg}{3k}$

Q.2 $a_A = 2g$; $a_B = g$

Pseudo force

Motion in Accelerated Frames :

Till now we have restricted ourselves to apply Newton's laws of motion, only to describe observations that are made in an inertial frame of reference. In this part, we learn how Newton's laws can be applied by an observer in a noninertial reference frame. For example,consider a block kept on smooth surface of a compartment of train.

If the train acclerates, the block accelerates toward the back of the train. When observed from the train we may conclude based on Newton's second law F= ma that a force is acting the block to cause it to accelerate, but the Newton's second law is not applicable from this non- inertial frame. So we can not relate observed accleration with the Force acting on the block.

If we still want to use Newton's second law we need to apply a psuedo force, acting in backward direction, ie opposite to the aceleration of noninertial reference frame. This force explains the motion of block towards the back of train. The fictious force is equal to $-\overrightarrow{ma}$, where $\vec{a}$ is the acceleration of the non inertial reference frame. Fictitious force appears to act on an object in the same way as a real force, but real forces are always interactions between two objects, on the other hand there is no second object for a fictitious force.

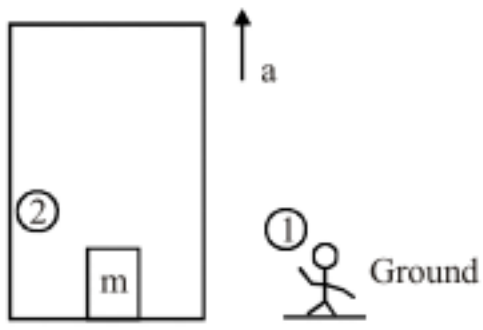
pseudo force, $\vec{F}_P = -m\vec{a}_0$

where a_0 is acceleration of non inertial reference frame

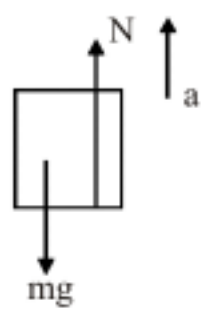
Thus, we may conclude that pseudo force is not a real force. When we draw the free body diagram of a mass, with respect to an inertial frame of reference we apply only the real forces (forces which are actually acting on the mass), but when the free body diagram is drown from a non-inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation

$\vec{F} = m\vec{a}_0$, valid in this frame also.

Suppose a block A of mass m is placed on a lift ascending with an acceleration a_0. Let N be the normal reaction between the block and the floor of the lift Free body diagram of A is shown in figure.

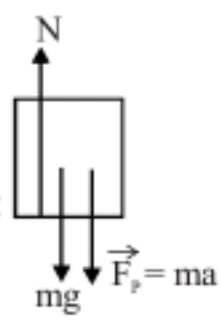


for observer 1 (Inertial reference frame):



$N - mg = ma$

For observer 2 (Non inertial reference frame):



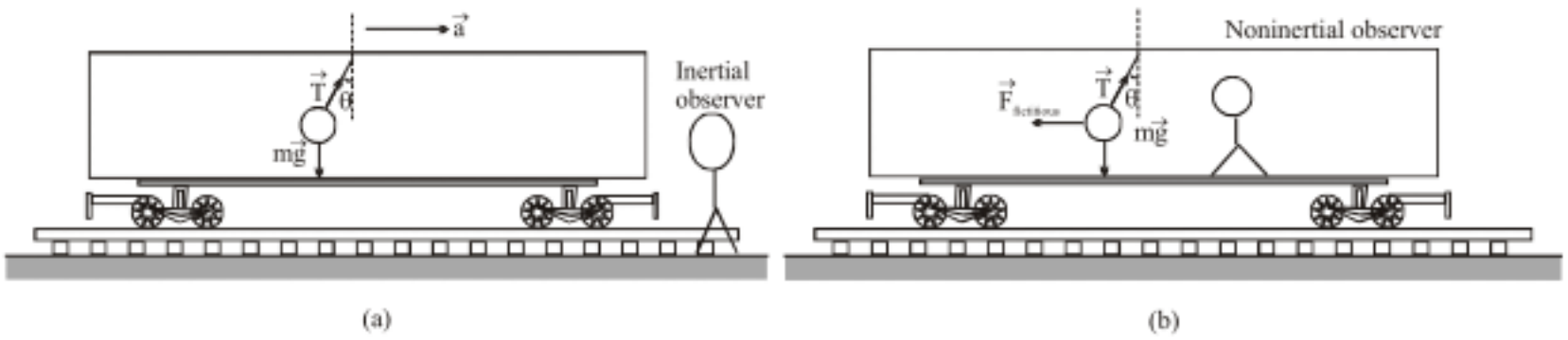
$a = 0$

$\therefore \quad N = mg + ma$

Illustration :

A small ball of mass m hangs by a cord from the ceiling of a compartment of a train that is accelerating to the right as shown in Figure. Analyze the situaions for two observers A & B.

Sol. *The observer A on the ground, is inertial Frame. He sees the compartment is accelerating and knows that the horizontal comp. of tension in the cord provides the ball, required horizontal force. The noninertial observer on the compartment, can not see the car's motion, he is not aware of its acceleration. He will say that Newton's second law is not valid as the object has net horizontal force (the horizontal component of tension) but no horizontal acceleration.*



For the inertial observer, ball has a net force in the horizontal direction and is in equilibrium in the vertical direction. For the noninertial observer, we apply fictitious force towards left and consider it to be in equilibirium.

According to the inertial observer A, the ball experience two forces, T exerted by the cord and the weight.

Applying, Newton's second law in in horizontal and vertical direction we get

Inertial observer $T\sin\theta - mg = 0$ .....(i)

$T\cos\theta = ma$ .....(ii)

According to the noninertial observer B riding in the car (Fig. b), the ball is always at rest and so its acceleration is zero. The noninertial observer applies a fictitious force in the horizontal direction of magnitude ma towards left. This fictious force balances the horizontal component of T and thus the net force on the ball is zero.

Apply Newton's second law in horizontal and vertical direction we get

Noninertial observer $T\sin\theta - mg = 0$

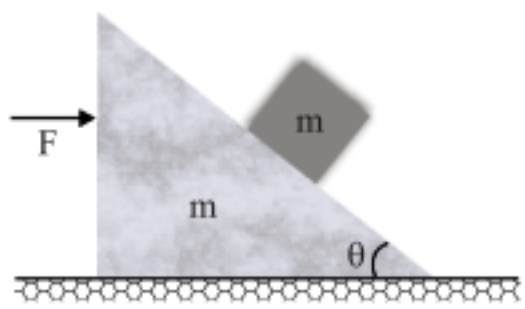
$T\cos\theta - ma = 0$

These expressions are equivalent to Equations (1) and (2).

The noninertial observer B obtains the same equations as the inertial observer. The physical explanation of the cord's deflection, however, differs in the two frames of reference.

Illustration :

All surfaces are smooth in following figure. Find F, such that blcok remains stationary with respect to wedge.



Sol. *Acceleration of (block + wedge)* $a = \dfrac{F}{(M+m)}$

Let us solve the problem by both the methods.

From inertal frame of reference (Guound)

FBD of block w.r.t. ground (Apply real forces):

With respect to ground block is moving with an acceleration 'a'.

$\therefore \quad \Sigma F_y = 0 \quad \Rightarrow \quad N\cos\theta = mg$ .....(i)

and $\quad \Sigma F_x = ma \quad \Rightarrow \quad N\sin\theta = ma$ .....(ii)

From Equation (i) and (ii)

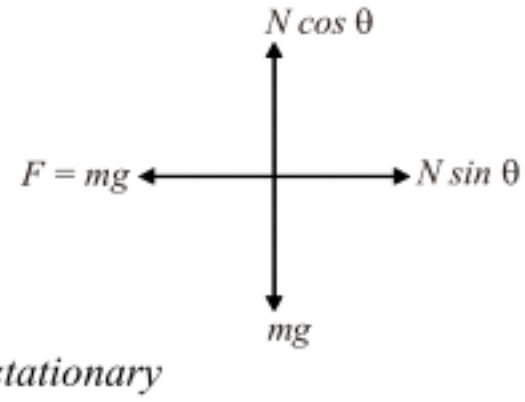
$$a = g\tan\theta$$

$$F = (M+m)a$$

$$= (M+m)g\tan\theta$$

From non-inertial frame of reference (Wedge)

FBD of block w.r.t. wedge (real force + pseudo force)



w.r.t. wedge block is stationary

$\therefore \quad \Sigma F_y = 0 \quad \Rightarrow \quad N\cos\theta = mg$ .....(iii)

and $\quad \Sigma F_x = ma \quad \Rightarrow \quad N\sin\theta = ma$ .....(iv)

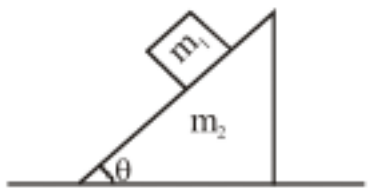
From equation (iii) and (iv), we will get the same result

$$F = (M+m)g\tan\theta$$

Illustration:

There is no friction at any contact. Wedge is free to move

Find force acting on wedge due to block. Also find acceleration of wedge.



Sol. *You may want to directly reach to conclusion that answer is* $m_1 g\cos\theta$. *but it is being solved in refrence frame of wedge which may be acclerating. Horizontal component of normal contact force applied by block on wedge will accelerate the wedge. Thus refrence frame attached to wedge is non-inertial refrence frame.*

Acceration vector of block in ground frame is sum of acceleration of wedge and acceleration of block w.r.t. wedge $(\vec{a}_{b/w})$

$$\vec{a}_b = \vec{a}_{b/w} + \vec{a}_w$$

$$\vec{a}_b \equiv$$

Consider F.B.D of wedge. Take horizontal component of normal contact force and apply Newton's 2nd Law

$$N \sin\theta = m_2\, a_w$$

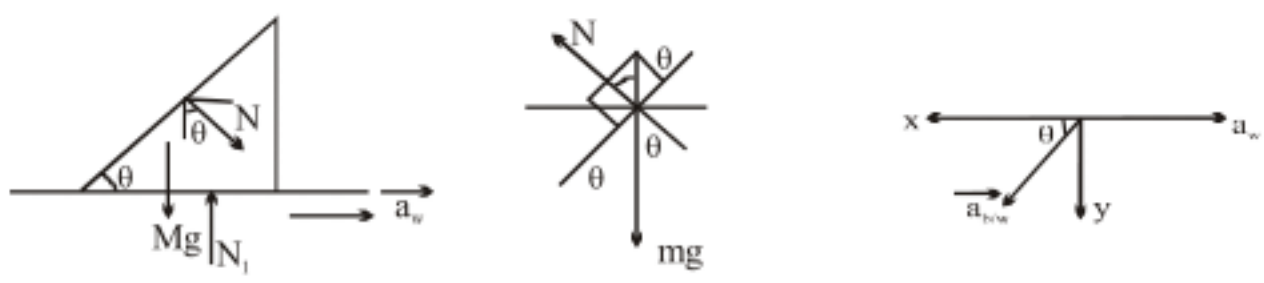
Consider F.B.D. of block and acceleration vector of block, Take horizontal and vertical component of forces and acceleration and apply Newton's second law.

$$a_x = a_{b/w} \cos\theta - a_w$$

$$a_y = a_{b/w} \sin\theta$$

$$N \sin\theta = m_1 (a_{b/w} \cos\theta - a_w)$$

$$m_1 g - N \cos\theta = m_1 (a_{b/w} \sin\theta)$$



Solving we get

$$N = \frac{m_1 m_2 g \cos\theta}{(m_2 + m_1 \sin^2\theta)}$$

$$a_w = \frac{m_1 g \cos\theta \sin\theta}{(m_2 + m_1 \sin^2\theta)}\ ,\quad a_{b/w} = \frac{(m_1 + m_2) g \sin\theta}{(m_1 \sin^2\theta + m_2)}$$

Friction

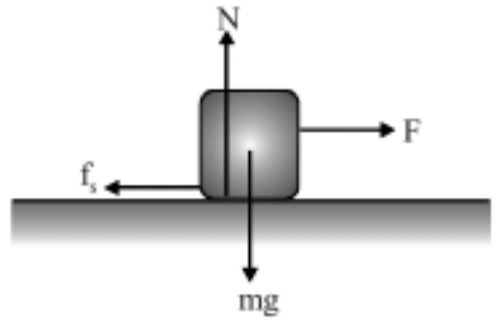
Frictional forces are unavoidable in our daily lives. If we were not able to counteract them, they would stop every moving object and bring to a halt every rotating shaft. About 20% of the gasoline used in an automobile is needed to counteract friction in the engine and in the drive train on the other hand, if friction were totally absent, we could not get an automobile to go anywhere, and we could not walk or ride a bicycle. We could no hold a pencil, and, if we could, it would not write. Nails and screws would be useless, woven cloth would fall apart, and knots would untie.

When surfaces slide or tend to slide over one another, a force of friction acts. Friction is caused by the irregularities in the surfaces in mutual contact, and it depends on the kinds of material and how much they are pressed together. Even surfaces that appear to be very smooth have microscopic irregularities that obstructs motion. Atoms cling together at many points of contact. When one object slides against another, it must either rise over the irregular bumps or else scrape atoms off. Either way requires force. Although the details of friction are quite complex at atomic level, it ultimately involves the elctromagnetic force between atoms & molecules.

The direction of the friction force is always in a direction opposing relative motion. An object sliding down an incline experiences friction directed up the incline; an object that slides to the right experiences friction toward the left. Thus, if an object is to move at constant velocity, a force equal to the opposing force of friction must be applied so that the two forces exactly cancel each other. The zero net force then results in zero acceleration and constant velocity.

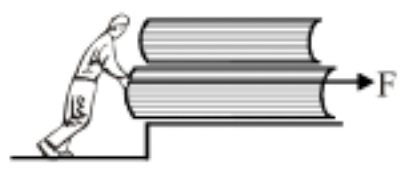
Static Friction

Static friction acts when two contact surfaces are not moving relative to each other. For example, consider a block on a horizontal table, as in figure. If we apply an external horizontal force F to the block, acting to the right, the block remains stationary if F is not too large. The force that counteracts F and keeps the block from moving acts to the left and is the frictional force f. As long as the block is not movng, f = F. Since the block is stationary, we call this frictional force the force of static friction, f_s.



F = f_s, As long as the block is not moving

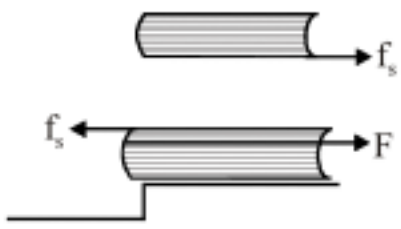
If we keep two books one on top the other and now we slowly push the lower book, both the books move together, the force moving the upper book is friction. Since the books are moving with respect to ground but they are not moving with respect to each other, this force of friction between two books is of static nature.



Two books kept on top of each other & the lower book being pushed slowly

Similarly when we walk on ground the friction force acting on the foot is of static nature. You may be suprised but the foot in contact with ground is not moving while

the other foot is moving. During this time friction acting between foot and ground is static friction.



Free body diagram of two books

If we are increasing our speed then static friction is acting in forward direction. This happens because we try to pull our leg backward while walking in forward direction. This means foot is trying to move in backward direction with respect to ground and ground applies friction in forward direction

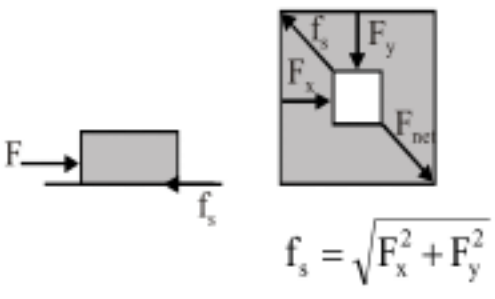
Man is moving but the point in contact of the foot is not moving i.e. static friction is acting on the man

Person is walking due to static friction

Static friction is self-adjusting and it opposes the tendency of relative motion

In fact, in the previous case of books also, we can see that static friction acting in forward direction on upper book has given it some motion so it can move with the lower book. Another point to be noted here is that even by third law, pair of static friction is opposing relative motion by trying to slow down the lower book (which is being accelerated externally).

for example in figure we assume that the block is stationary and we can see that static friction is acting in such a direction so as to oppose the relative motion. F represents external force and f_s represents friction.



Block is stationary due to static friction

The magnitude of the static friction between any two surfaces in contact can have the values

$$f_s \le \mu F_N$$

where the dimensionless constant μ_s is known as the coefficient of static friction and F_N is the magnitude of the normal contact force exerted by one surface on the other. The equality in equation holds when the surfaces are on the verge of slipping, that is when $f_s = f_{s,max} = \mu_s F_N$, This maximum value of f_s is called limiting friction. This situation is called impending motion. The inequality hold when the surfaces are not on the verge of sliping. Maximum strength of the joints formed is directly proportional to the normal contact force, that is $f_{s,max} \propto F_N$.

Maximum strength also depends on the roughness of contact surface $f_{s,max}$ (also called $f_{limiting}$) = $\mu_s N$. Magnitude of static friction is self-adjusting such that relative motion does not start (but still it has maximum value).

Let us say we are applying force F on a block kept on horizontal rough surface with coefficient of static friction $\mu_s = 0.1$ & mass of block is 5 kg. When applied F is less than 5 N the value of static friction is

equal to the applied force, not 5 N. It is the maximum value of friction. But when applied force F is equal to 5 N, the value of static friction is 5 N

Illustration :

A block lying on a horizontal surface is pulled by a force of 0.1 N but the block does not move i.e., remains at rest. Find friction force

Sol. *To analyse the frictional force on the block we proceed as follows.*

As the block remains at rest, the force of static friction is balancing the 0.1 N force (the applied force). So the frictional force - 0.1 N

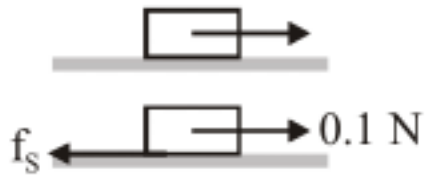
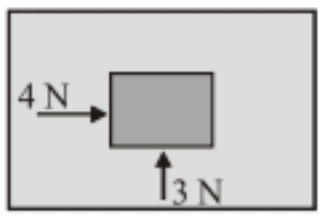


Illustration :

A block lying on a horizontal surface is pushed by forces as shown in figure but the block does not move i.e., it remains at rest.

Find the frictional force on the block.



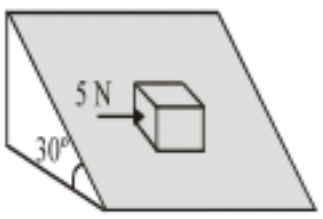
Sol. *Net force acting on the block is* $\sqrt{3^2+4^2} = 5N$

As the block remains at rest, the force of static friction is balancing the applied force (5 N)

Illustration :

A block lying on an inclined surface at an angle 30° with the horizontal is pushed by force as shown in figure but the block does not move i.e., it remains at rest.

Find the frictional force on the block.



Sol. *Two forces are acting on the block along the surface of inclined, first one is mg sin 30° down the inclined and second 5 N as shown in figure*

So the net force acting on the block along the surface of inclined is $\sqrt{12^2+5^2} = 13N$

As the block remains at rest, the force of static friction is balancing the applied force (13 N)

Illustration :

A block of weight 100 N lying on a horizontal surface just begins to move when a horizontal force of 25 N acts on it. Determine the coefficient of static friction.

Sol. *As the 25 N force brings the block to the point of sliding i.e., limiting friction is 25 N.*

limiting frictional force $= \mu_s N$.

From force diagram : $N = 100\ N$

$\mu_s N = 25 \quad \Rightarrow \quad \mu_s = 0.25.$

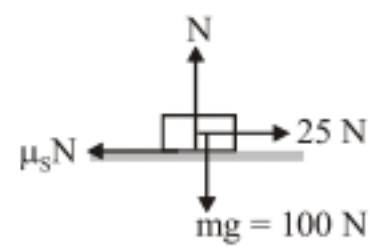


Illustration :

A block of weight 100 N lying on a horizontal surface is pushed by a force F acting at an angle 30° with horizontal. For what value of F will the block begin to move if $\mu_s = 0.25$ *?*

Sol. *Consider the force diagram of the block at the moment when it is just to start moving.*

Balancing force :

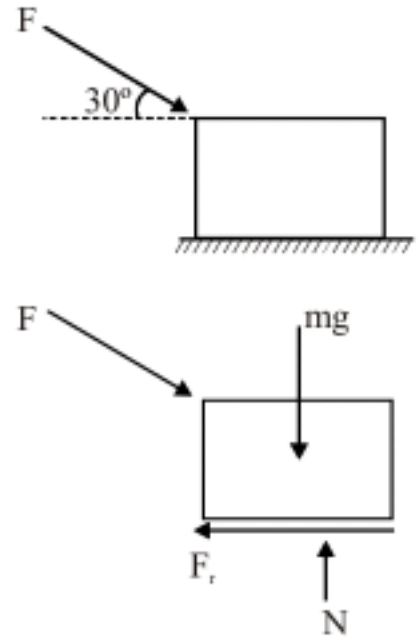
$N = mg + F \sin 30^o$

$F = \cos 30^o = \mu_s N$

$F = \cos 30^o = \mu_s(mg + F \sin 30^o)$

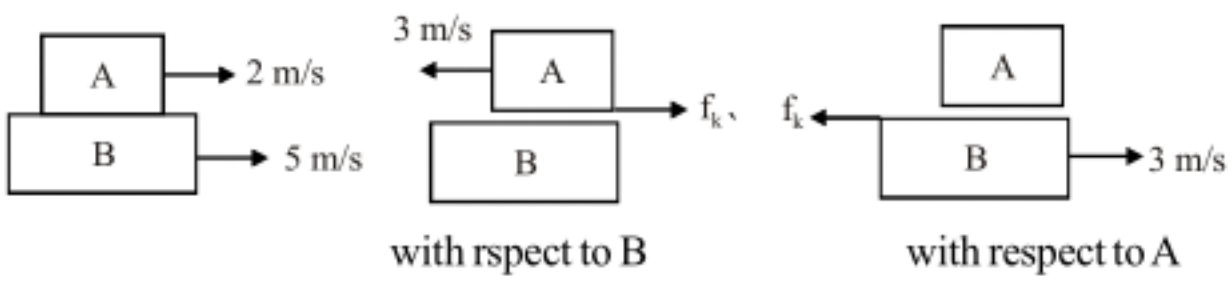
$$F = \frac{\mu_s mg}{\cos 30^o - \mu_s \sin 30^o} = \frac{0.25(100)2}{\sqrt{3} - 0.25}$$

$F = 33.74\ N$



Kinetic friction

Kinetic friction acts when there is relative motion between two surfaces in contact. It acts always opposite to the relative velocity as we can see in figure. The magnitude is not self-adjusting as in static friction, it is always is equal to $\mu_k F_N$.



Experimentally, we find that, to a good approximation, both $f_{s,max}$ and f_k are proportional to the magnitude of the normal force. The following empirical laws of friction summarize the experimental observations :

1. The magnitude of the force of kinetic friction acting between two surfaces is

   $f_k = \mu_k N$

   where μ_k is the coefficient of kinetic friction. Although the coefficient of kinetic friction can vary with speed, we shall usually neglect any such variations in problems.

2. The value of μ_k and μ_s depend on the nature of the surfaces.

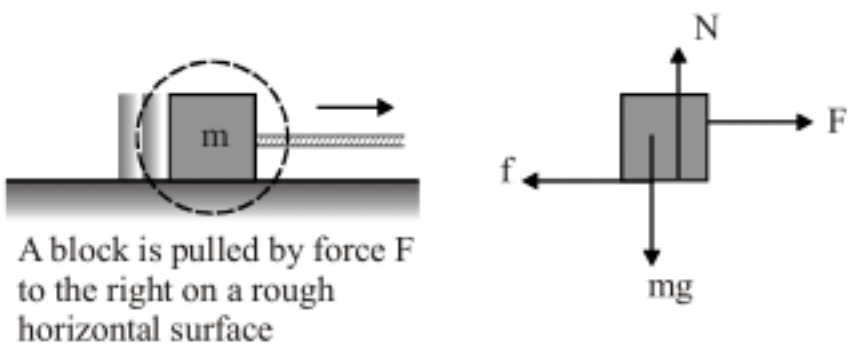
3. μ_k is generally less than μ_s.

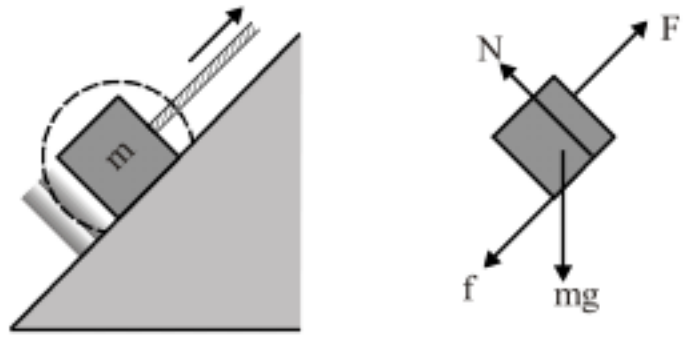
The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary. Dust impurities, surface oxidation etc. have a great role in determining the friction coefficient. Suppose we take two blocks of pure copper, clean them carefully to remove any oxide or dust layer at the surfaces, heat them to push out any dissolved gases and keep them in contact with each other in an evacuated chamber at a very low pressure of air. The blocks stick to each other and a large force is needed to slide on over the other. The friction coefficient as defined above, becomes much larger than one this is called cold welding. If a small amount of air is allowed to go into the chamber so that some oxidation takes place at the surface, the friction coefficient reduces to usual values.

4. The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the motion (kinetic friction) or the tendency of motion (static friction) of the object relative to the surface.

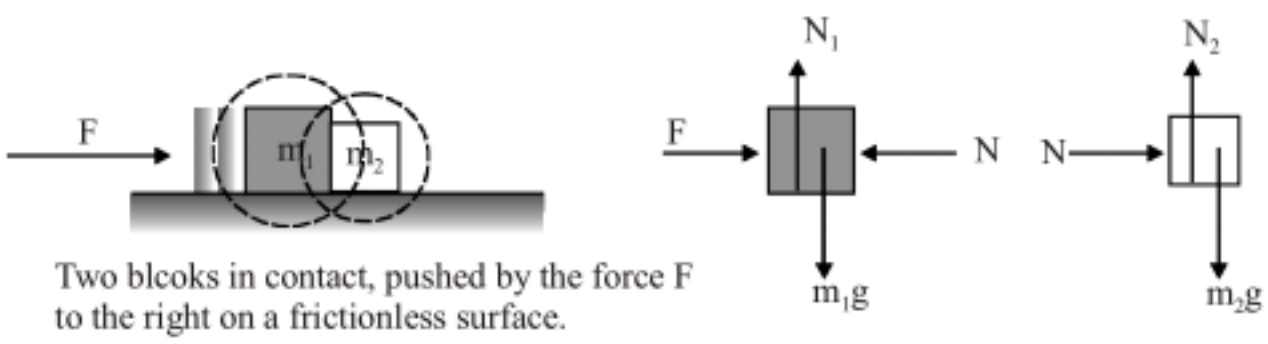
5. The coefficients of friction are nearly independent of the area of contact between the surfaces. We might expect that placing an object on the side having the more area might increase the friction force. While this provides more points in contact, the weight of the object is spread out over a larger area, so that the individual points are not pressed so tightly together. These effects approximately compensate for each other, so that the friction force is independent of the area. So those extra wide tires you see on some cars provide no more friction than narrower tires. The wider tire simply spreads the weight of the car over more surface area to reduce heating and wear. Similarly, the friction between a truck and the ground is the same whether the truck has four tires or eighteen! More tires spread the load over more ground area and reduces the pressure per tire. Interestingly, stopping distance when brakes are applied is not affected by the number of tires.But the wear that tires experience, very much depends on the number of tires.

Various mechanical configurations (left) and the corresponding free-body diagrams (right). The term rough here means only that the surface is not frictionless.

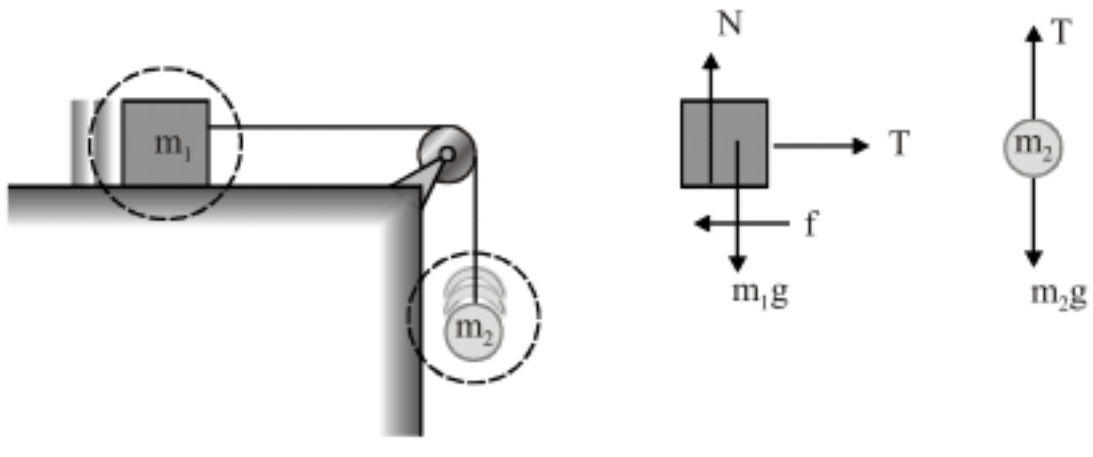




A block is pulled up the rough incline



Two blcoks in contact, pushed by the force F
to the right on a frictionless surface.



Two masses connected by a light cord.
The surface is rough and the pulley is frictionless

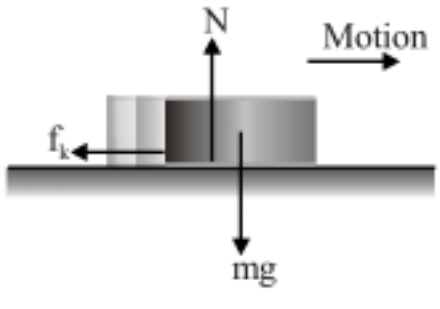
Illustration:

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

Sol. *Imagine that the puck in figure slides to the right and eventually comes to rest. The forces acting on the puck after it is in motion are shown in figure first, we find the acceleration in terms of the coefficient of kinetic friction, using Newton's second law. Knowing the acceleration of the puck and the distance it travels, we can then use the equation of knematics to find the numerical value*

of the coefficient of kinetic friction.

Defining rightward and upward as our positive directions, we apply Newton's second law in component form to the puck and obtain



$$\Sigma F_x = -f_k = ma_x \qquad \text{.....(1)}$$

$$\Sigma F_y = N - mg = 0 \qquad (a_y = 0) \qquad \text{.....(2)}$$

But $f_k = \mu_k N$, and from (2) we see that $N = mg$. Therefore,

(1) becomes

$$-\mu_k N = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

The negative sign means the acceleration is to the left in figure because the velocity of the puck is to the right, this means that the puck is slowing down. The acceleration is independent of the mass of the puck and is constant because we assume that μ_k remains constant.

Because the acceleration is constant, we can use Equation $v^2 = u^2 + 2as$

$$0 = u^2 + 2a\,S = u^2 - 2\mu_k gS \qquad v = 0$$

$$\mu_k = \frac{u^2}{2gS}$$

$$\mu_k = \frac{\left(20.0m/s^2\right)^2}{2(9.80m/s^2)(115m)} = 0.136$$

Illustration :

A 5 kg block slides down a plane inclined at 30° to the horizontal. Find

(a) The acceleration of the block if the plane is frictionless.

(b) the acceleration if the coefficient of kinetic friction is 0.2.

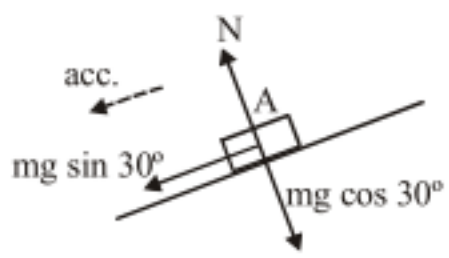
Sol. *(i)* $N = mg \cos 30^o$

$mg \sin 30^o = ma$

$a = g \sin 30^o$,

down the plane if plane is smooth.

$a = g/2 = 4.9\ m/s^2$



(ii) $N = mg \cos 30^o$

$mg \sin 30^o - \mu_k N = ma$

$a = g \sin 30^o - \mu_k g \cos 30^o$

$a = 3.20\ m/s^2$

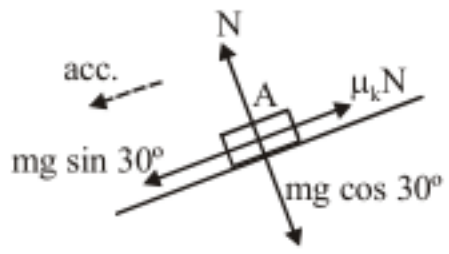


Illustration :

5 kg block projected upwards with an initial speed of 10 m/s from the bottom of a plane inclined at 30° with horizontal. The coefficient of kinetic friction between the block and the plane is 0.2.

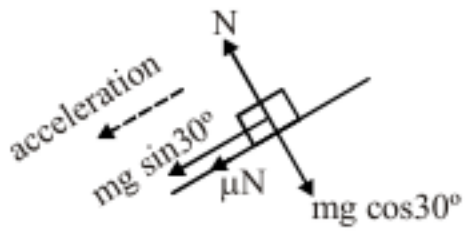
(a) How far does the block move up the plane ?

(b) How long it move up the plane ?

Sol *While the block is moving up the frictional forc acts downward.*

As the block is slowing downs, the velocity and acceleration must be in opposite direction.

Velocity in this case is upwards, so acceleration is in downward direction.



the megnitude of acceleration $= \dfrac{mg \sin 30^o + \mu\, mg \cos 30^o}{m} = g\,(\sin 30^o + \mu\, mg \cos 30^o)$

$\Rightarrow \quad a = -g\,(\sin 30^o + \mu\, mg \cos 30^o) = -\,6.6\ m/s^2$

For the motion of block from the bottom to up the plane :

$u = +\,10\ m/s$ $\qquad v^2 = u^2 + 2as$, we get

$0^2 = 10^2 + 2\,(-6.6)\,(s) \quad \Rightarrow \quad s = 7.58\ m$

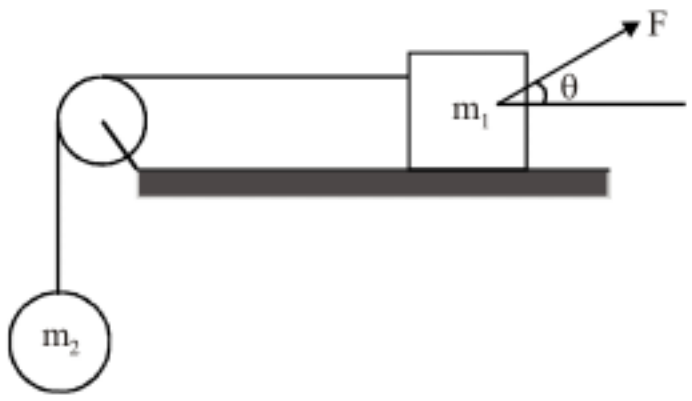
$v = u + at$

$0 = 10 - 6.6 \times t \quad \Rightarrow \quad t = 1.5$ seconds

Hence the block moves up the plane for 1.5 sec covering 7.58 m.

Illustration :

A block of mass m_1 on a rough, horizontal surface is connected to a ball of mass m_2 by a lightweight cord over a lightweight, frictionless pulley, as shown in figure. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown. The coefficient of kinetic friction be tween the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.



Sol. *The block will side to the right and the ball will rise. We can identify forces and we want an acceleration, so we categorize this as a Newton's second law problem, one that includes the friction force. To analyze the problem, we begin by drawing free-body diagrams for the two objects ,as shown in figure and Next, we apply Newton's second law in component from to each object and use Equation* $f_k = \mu_k n$. *Then we can above for the acceleration in terms of the parameters given.*

This applied force F has x and y components F cos θ and F sin θ, respectively. Applying Newton's second law to both objects and assuming the motion of the block is to the right, we obtain.

Motion of block : $\Sigma F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$ .....(1)

$\Sigma F_y = N + F\sin\theta - m_1 g = m_1 a_y = 0$ .....(2)

Motion of ball : $\Sigma F_x = m_2 a_x = 0$

$\Sigma F_y = T - m_2 g = m_2 a_y = m_2 a$ .....(3)

Because the two objects are connected, we can equate the magnitudes of the x component of the acceleration of the block and the y component of the acceleration of the ball. From Equation we know that $f_k = \mu_k N$, *and form (2) we know that* $N = m_1 g - F\sin\theta$ *(in this case n is not equal to* $m_1 g$*) therefore,*

$$f_k = \mu_k(m_1 g - F\sin\theta) \qquad .....(4)$$

That is, the friction force is reduced because of the positive y component of F. Substituting (4) and the value of T from (3) into (1) gives

$$F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2(a + g) = M_1 a$$

Solving for a, we obtain

$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - g(m_2 + \mu m_1)}{m_1 + m_2} \qquad .....(5)$$

The acceleration of the block can be either to the right or to the left,[5] *depending on the sign of the numerator in (5). If the motion is to the left, then we must reverse the sign of* f_k *in (1) because the force of kinetic friction must oppose the motion of the block relative to the surface.*

Illustration :

A horse pulls a sled along a level, snow-covered road, causing the sled to accelerate, as shown in figure. Newton's third law state that the sled exerts a force of equal magnitude and opposite direction on the horse. In view of this, how can the sled accelerate – don't the forces cencel ? Under what condition does the system (horse plus sled) move with constant velocity ?

Sol. *Remember that the forces described in Newton's third law act on different objects the horse exerts a force on the sled, and the sled exerts an equal magnitude and oppositely directed force on the horse. Because we are interested only in the motion of the sled, we do not consider the forces it exerts on the horse. When determining the motion of an object, you must add only the forces on that object. (This is the principle behind drawing a free-body diagram.) The horizontal forces exerted on the sled are the forward force T exerted by the horse and the backward force of friction* f_{sled} *between sled and snow. When the forward force on the sled exceeds the backward force, the sled accelerates to the right.*

The horizontal forces exerted on the horse are the forward force f_{horse} exerted by the earth and the backward tension force T exerted by the sled. The resultant of these two forces causes the horse to accelerate.

The force that accelerates the system (horse plus sled) is the net force $f_{horse} - f_{sled}$. When f_{horse} balances f_{sled}, the system moves with constant velocity.

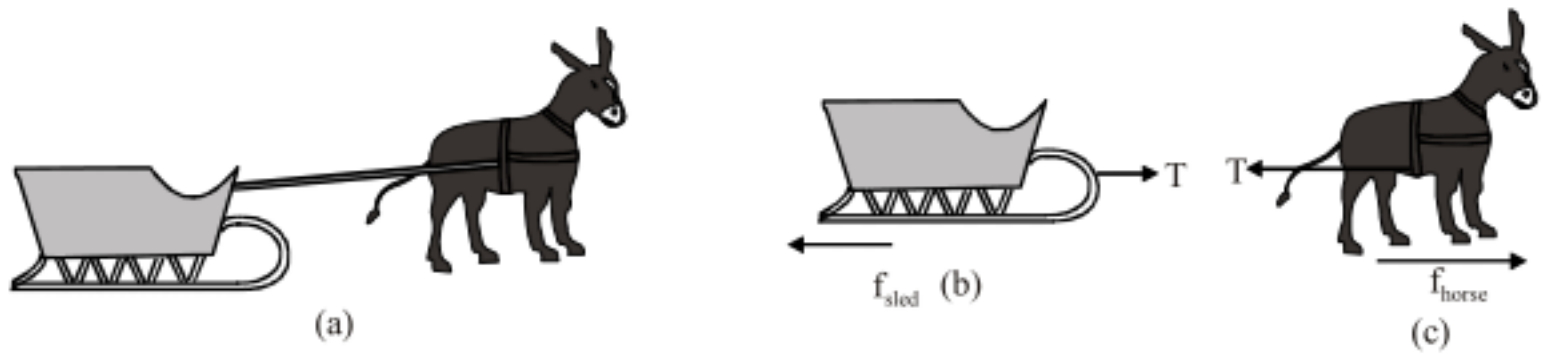
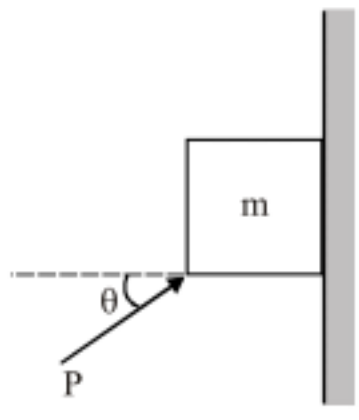


Illustration :

A block of mass 'm' is supported on a rough wall by applying a force P as shown in figure Coefficient of static friction between block and wall is μ_s. For what range of values of P, the block remains in static equlilibrium ?



Sol. *Impending state of motion is a critical border line between static and dynamic states of body. The block under the influence of P sin θ (Component of P) may have a tendency to move upward or it may be assumed that P sinθ just prevents downward fall of the block. Therefore there are two possibilties:*

Case (i) *Impending motion upwards : In this case force of friction is downward.*

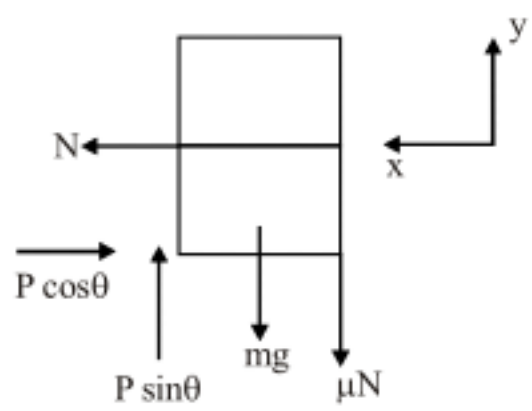
from conditions of equilibrium

$$\Sigma F_x = N - P\cos\theta = 0$$

or $$N = P\cos\theta$$

$$\Sigma F_y = P\sin\theta - \mu N - mg = 0$$

or $$P\sin\theta - \mu P\cos\theta - mg = 0$$



or $$P_{max} = \frac{mg}{\sin\theta - \mu\cos\theta}$$

Case (ii) *Impending motion downward : In this case friction force acts upward.*

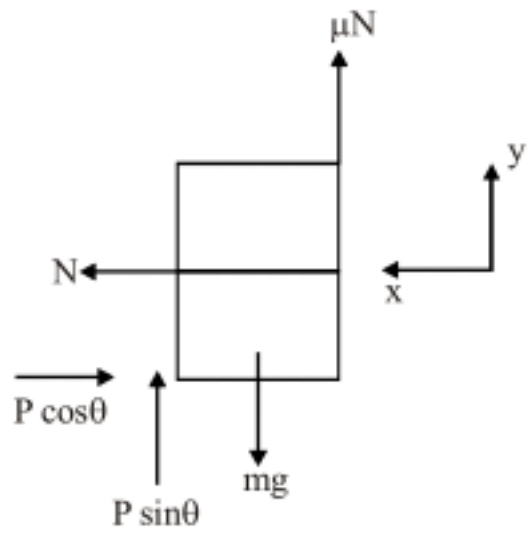
$$\Sigma F_x = N - P\cos\theta = 0$$

or $$N = P\cos\theta$$

$$\Sigma F_y = P\sin\theta + \mu N - mg = 0$$

or $$P\sin\theta + \mu P\cos\theta - mg = 0$$

or $$P_{min.} = \frac{mg}{\sin\theta + \mu\cos\theta}$$



Therefore the block will be in static equilibrium for

$$\frac{mg}{\sin\theta + \mu\cos\theta} \le P \le \frac{mg}{\sin\theta - \mu\cos\theta}$$

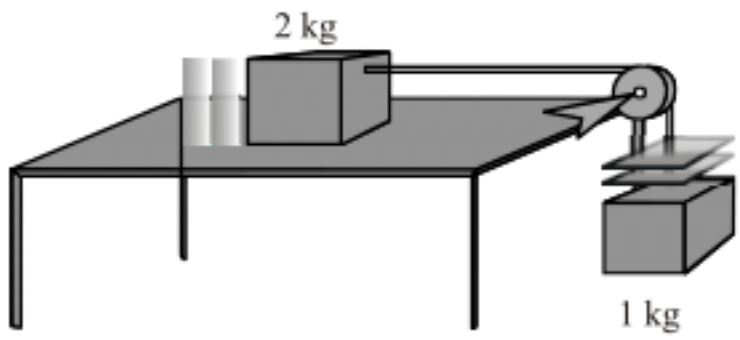
Practice Exercise

Q.1 You press your physics textbook flat against a vertical wall with your hand. What is the direction of the friction force exerted by the wall on the book?

Q.2 A crate is located in a truck. The truck accelerates to the east, and the crate moves with it, without sliding. What is the direction of the friction force exerted by the crate on the truck ?

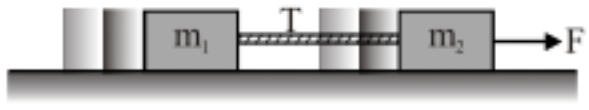
Q.3 A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor ? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it ? (c) If the maximum value of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N ? If it is 12 N ? (d) What is the magnitude of the frictional force in part (c)?

Q.4 A 1 kg hanging block is connected by a string over a pully to a 2 kg block sliding on a flat table. If the coefficient of sliding friction is 0.20, find the tension in the string.



Q.5 A 25 kg block is initially at rest on a horizontal surface. A horizontal force of 75 N is required to set the block in motion. After it is in motion, a horizontal force of 60 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.

Q.6 Two blocks connected by a massless rope are being dragged by a horizontal force F. Suppose that F = 60 N, m_1 = 12 kg, m_2 = 18 kg, and coefficient of kinetic friction between each block and the surface is 0.10. (a) Find the magnitude of the acceleration of the system.



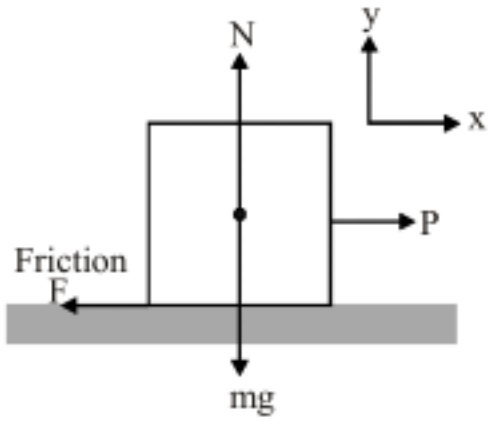
Answers

Q.1	upward	Q.2	to the west	Q.3	(a)0; (b)5; (c) No, yes (d) 8,10
Q.4	T = 8N, a = 2m/s²	Q.5	$\mu_s = 0.3$, $\mu_k = 0.24$	Q.6	a = 1 m/s²

Graph of frictional force versus applied force

Figure shows a block of weight mg resting on a rough surface. A horizontal force is applied to the block. Force P is gradually increased from zero.

When force P is very small, the block does not move form condition of equilibrium,



$$\Sigma F_x = P - F_{friction} = 0;\ F_{friction} = P$$

$$\Sigma F_y = N - mg = 0;\ N = mg$$

Friction force counter balance external force, till the block is static. This friction force is static friction. As external force is increased, static friction also increases.

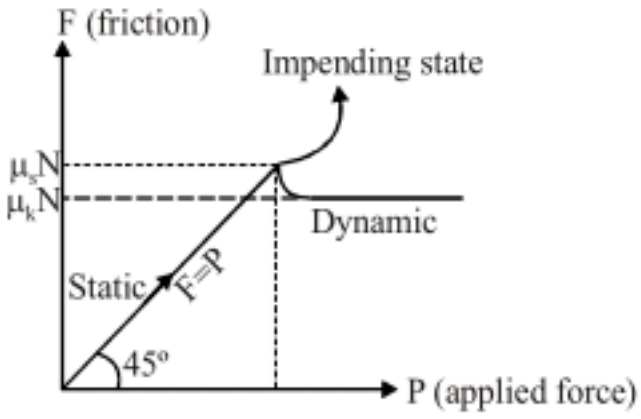
As the force P is gradually increased, a limiting point is reached where friction force F is not sufficient to prevent onset of motion. When the block is about to move, the state of motion is called impending state of motion. At this point friction force has maximum value and given by the equation

$$F_{max} = \mu_s N$$

where μ_s is coefficient of static friction, N is normal reaction.

If force P is greater than F_{max}, the block will have a resultant froce $P - F_{max}$ on it. The block will accelerate in the direction of resultant force when sliding motion starts. At this moment friction force is given by

$$F = \mu_k N$$



Where μ_k is coefficient of kinetic friction. The figure shows variation of friction force versus external force graph.

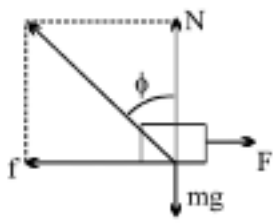
Angle of friction (ϕ)

Mathematically, the angle of friction (ϕ) may be defined as the angle between the normal reaction N and the resultant of the maximum friction force f and the normal reaction.

Thus $\tan\phi = \frac{f}{N}$

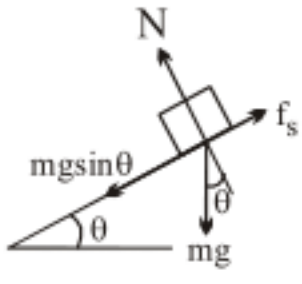
Since $f = \mu N$, therefore,

$$\tan\phi = \mu$$



Angle of Repose

Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in figure. The incline angle is increased until the block starts to move. The angle θ for which slipping just occurs is **angle of repose**.



The only forces acting on the block are the gravitational force mg, the normal force N, and the force of static friction f_s. These forces balance when the block is not moving. When we choose x-axis to be parallel to the plane and y perpendicular to it, Newton's second law applied to the block for this balanced situation gives

$$\Sigma F_x = mg\sin\theta - f_s = 0 \qquad \text{.....(1)}$$

$$\Sigma F_y = N - mg\cos\theta = 0 \qquad \text{.....(2)}$$

We can eliminate mg by substituting $mg = N\cos\theta$ from (2) into (1) to find

$$f_s = mg\sin\theta = \left(\frac{N}{\cos\theta}\right)\sin\theta = N\tan\theta \qquad \text{.....(3)}$$

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value $\mu_s N$. The angle θ in this situation is the critical angle θ_c and (3) become

$$\mu_s N = N\tan\theta_c$$

$$\mu_s = \tan\theta_c$$

this θ_c called **angle of repose**

At this angle inclined we feel net force on the block is zero and it will move with constant velocity but, if pushed. If block is pushed slightly then it moves with acceleration because frictional force on the block will decrease. ($\mu_k < \mu_s$)

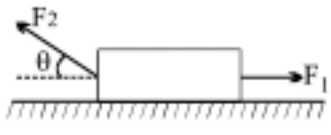
Application Automobile Antilock braking system (ABS)

(Topic for interest)

If an automobile tire is rolling and not slipping on a road surface, then the maximum friction force that the road can exert on the tire is the force of static friction μ_s. One must use static friction in this situation because at the point of contact between the tire and the road, no sliding should occur. However, if the tire starts to skid , the friction force exerted on it is reduced to the force of kinetic friction μ_k. Thus, to maximize the friction force and minize stopping distance, the wheels must maintain pure rolling motion and not skid. In directional control situations drivers typically press the brake very hard, "locking the brakes." This stops the wheels from rotating, ensuring a skid and reducing the friction force from the static to the kinetic value. To address this problem, automotive engineer have developed antilock braking systems (ABS). The purpose of the ABS is to help maintain control of automobiles and minimize stopping distance. The system briefly releases the brakes when a wheel is just about to stop turning. This maintains rolling contact between the tire and the pavement. When the brakes are released momentarily, the stopping distance is greater than it would be, if the brakes were being applied continuously.

Practice Exercise

Q.1 A block of mass m = 3 kg is experiencing two forces acting on it as shown.



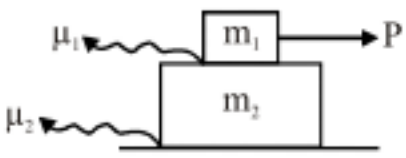
If $F_2 = 20$ N and $\theta = 60°$, determine the minimum and maximum values of F_1 so that the block remains at rest. (Take: $\mu_s = 1/\sqrt{3}$ and $\mu_k = 1/3$, $g = 10\,m/s^2$)

Q.2 A block, of mass m slips on a rough horizontal table under the action of a horizontal force applied to it. The coefficient of friction between the block and the table is μ. The table does not move on the floor. Find the total frictional force applied by the floor the legs of the table.

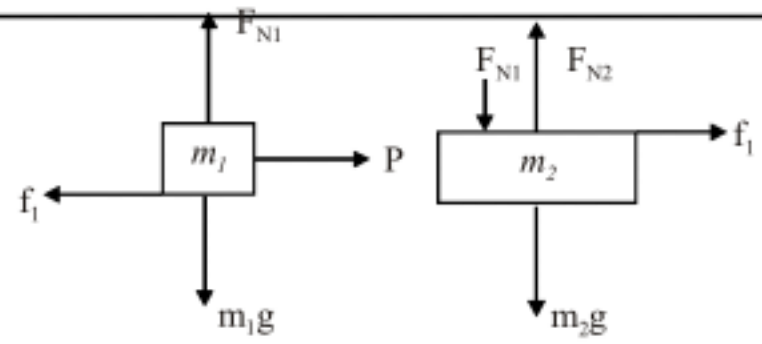
Answers

Q.1 $10\sqrt{3}$ N , $20 - 10\sqrt{3}$ N, 2N, left Q.2 μmg

Block over block problems



Consider two block kept one over the other as shown in Figure where m_1 is kept on top of m_2. Let's say coefficient of friction between m_1 and



m_2 is μ_1 and between m_2 and ground is $\mu_2 = 0$.

To find acceleration of m_1 and m_2 we make their free-body diagrams and write Newton's second law

Along y axis

$$F_{N_1} - m_1g = 0 \text{ and } F_{N_1} + m_2g - F_{N_2} = 0$$

Solving the two gives,

$$F_{N_1} - m_1g = 0 \text{ and } F_{N_2} = (m_1 + m_2)g$$

Along x axis

$$P - f_1 = m_1a_1 \text{ and } f_1 = m_2a_2$$

Also we know $\quad f_1 < \mu_1 F_{N_1}$

We now have two equation and three unknowns in equation along x direction. This problem cannot be solved unless we make some assumptions.

Case I : If we assume both blocks are moving together then friction between them need not be equal to maximum value. Thus, taking $a_1 = a_2$, we are down to two unknowns and we can solve the equations, but we must verify our answer by checking $f_1 \le \mu_1 F_{N_1}$. If this check fails we go to next possibility.

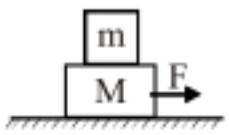
Case II : If we assume blocks are moving relative to each other, then friction between them must have reached maximum value (Kinetic friction)

$$a_1 \ne a_2 \text{but } f_1 = \mu_1 F_{N_1}$$

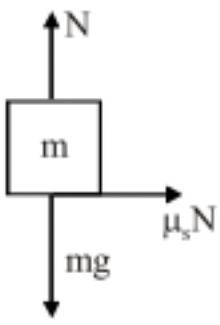
Again we get two unknowns and two equation. We can solve it, But we must verify our answer. We will check the answer by verifying that friction is opposite to the direction relative motion.

Illustration :

A block of mass m is placed on another block of mass M lying on a smooth horizontal surface. The confficient of static friction between m and M is μ_s. What is the maximum force that be applied to M so that the blocks remains at rest relative to each other ?



Sol. *Draw the force diagrams of blocks at the moment when F is at its maximum value and m is about to slide relative to it.*



Frictional force between m and M = $\mu_s N$

(N ; normal reaction between the block)

Due to the friction, M will try to drag m towrds right and hence frictional force will act on m towards right.

Let a = acceleration each block

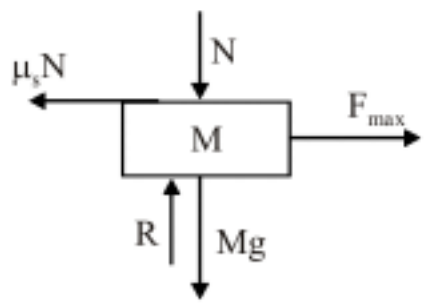
R = normal reaction between M and the surface.

Frome force diagram of m :

$N = mg$

$\mu_s N = ma$

From force diagram of M:



$N + Mg = R$

$F_{max}\ \mu_s N = Ma$

combining these two equation, we get

$F_{max} = \mu_s (m + M) g$

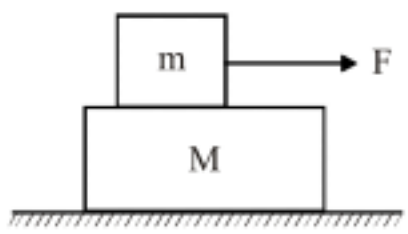
Hence $\mu_s (m + M)$ is the critical value of force F.

If F is greater than this critical value, m begins to slip relative to M and their acceleration will be different.

If F is smaller than this critical value, m and M move together without any relative motion.

Illustration :

A block of mass m is placed on another block of mass M lying on a smooth horizontal surface. The coefficient of static friction between m and M is μ_s. What is the maximum force that can be applied to m so that blocks remains at rest relative to each other ?



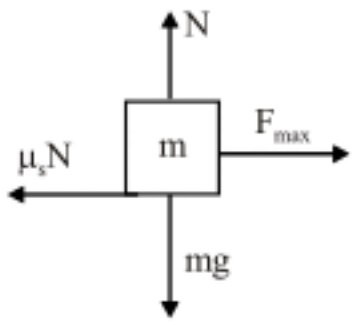
Sol. *Imagine the situation when F is at its maximum value so that m is about to start slipping relative to M.*

The mass m tries to drag M toward right due to frction.

From forces on m :

$F - \mu_s N = ma$

$N = mg$



From forces on M :

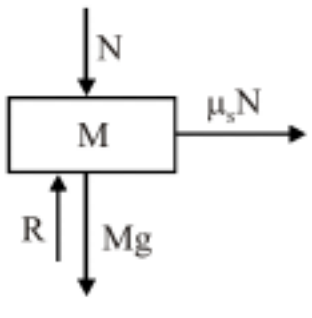
$$\mu_s N = ma$$

$$F_{max} = m_s (m + M)g$$

Solving these equations, we get :

Hence frictional force on M exerted by m will be towards right

Let a = magnitude of acceleation of blocks towards right.



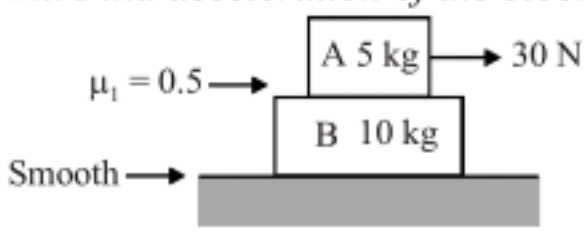
$$F_{max} = \frac{\mu_s (m+M)mg}{M}$$

If F is less than this critical value, the blocks stick together without any relative motion.

If F is greater than this critical value, the blocks slide relative to each and their acceleration are different.

Illustration :

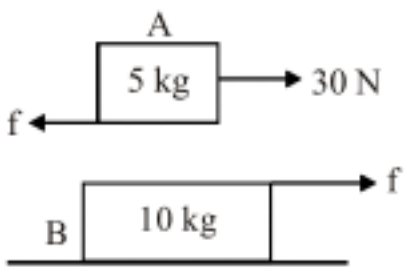
Friction coefficient between the blocks is 0.5 and ground is smooth if force of 30 N is applied on the upper block as shown. Find acceleration of the blocks



Sol. *Maximum friction force between blocks is 25 N ($f_{max} = \mu N = \mu mg = 0.5 \times 5 \times 10$) and ground is smooth. Block B must move because some force on upper surface will act on it.*
Block B can either move with same velocity and acceleration as block A or it can move relative to A.

Let's assume block B moves with block A and solve.

Making free-body diagram as shown in figure and applying Newton's second law, we get



$$30 - f = 5a \qquad \text{(for A)}$$
$$f = 10a \qquad \text{(for B)}$$

Solving we get,

$$a = 2\ m/s^2 \text{ and } f = 20\ N$$

Now check if this acceleration is possible by verifying $f \le f_{max\ A}$

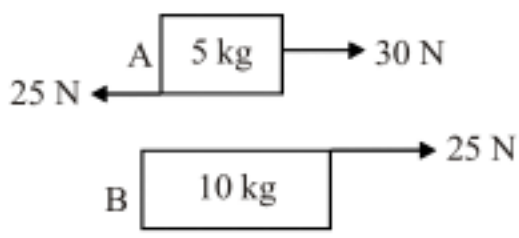
$$f = 20 < 25$$

Hence our assumption is true.
We can try by assuming they are not moving together and solve the equations to get an absurd result.

If we assume they are moving separately, friction attains maximum value.
Again making free-body diagram

$$a_A = \frac{30-25}{5} = 1\ m/s^2$$

$$a_B = \frac{25}{10} = 2.5\ m/s^2$$



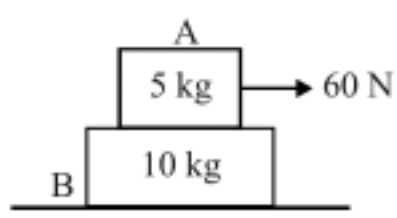
If we look at block A from reference from attached to block B we get

$$a_{A/B} = a_A - a_B = 1.5\ m/s^2$$

that is block A will be moving towards left and friction on it also acting in the same direction. This is not possible as friction must oppose relative motion.

Illustration :

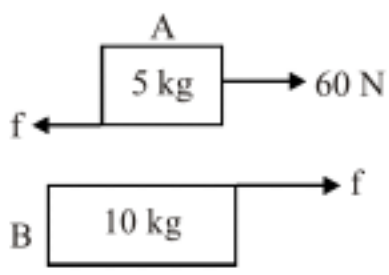
Friction coefficient between the block is 0.5 and ground is smooth. If force of 60 N is applied on the upper block as shown in figure find acceleration of the blocks.



$$f_{max} = 0.5 \times 5\ g = 25\ N$$

Let's assume they have same acceleration. The value of acceleration is 60/(10+5) = 4 m/s².

Making free-body diagram



Writing Newton's second law,

$$60 - f = 5 \times 4 \qquad \text{(for A)}$$

$$f = 10 \times 4 = 40\ N \qquad \text{(for B)}$$

Solving we get, $a = 4\ m/s^2$ and $f = 40\ N$ while limiting value of friction is 25 N, hence our assumption is wrong.

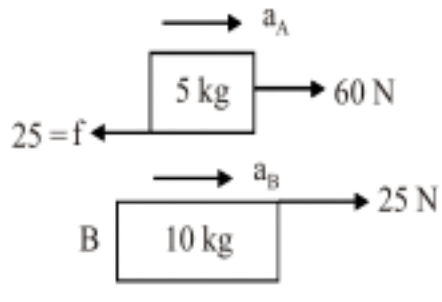
Let's say they are not moving together. Then the free-body diagram

Writing Newton's second law,

$$60 - 25 = 5\ a_A \qquad (f_K = 25)$$

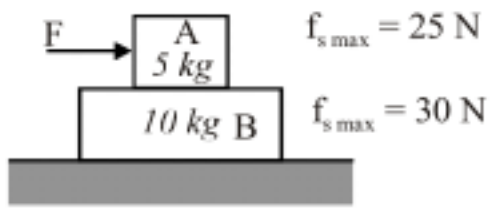
$$25 = 10\ a_B$$

$a_A = 7\ m/s^2,\ a_B = 2.5\ m/s^2$



If we look at block A from frame attached to block B we will find it is moving toward right and friction is acting towards left. This is satisfying friction's tendency to oppose relative motion.

Illustration :



Friction coefficient between block is 0.5 and between ground and 10 kg block is 0.2. Find acceleration of blocks if force F = 40 N is applied on 5 kg block as shown in figure

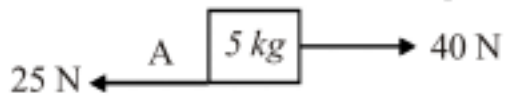
Sol. *First we find the values of limiting friction at all contact surfaces ($f_{s,max}$)*

f_{max} *between blocks* $f_1 = 0.5 \times 5 \times 10 = 25\ N$

f_{max} *between 10 kg block & ground,* $f_2 = 0.2 \times 15 \times 10 = 30\ N$

The only driving force that the block B can experience is the one applied by the lower surface of block A on block B.

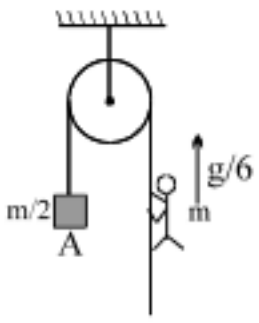
The maximum value of this force is 25N. This is lower than the minimum force required to move with respect to ground $f_{required} = 0.2 \times 15 \times 10 = 30\ N$ *Hence only the block A will move*



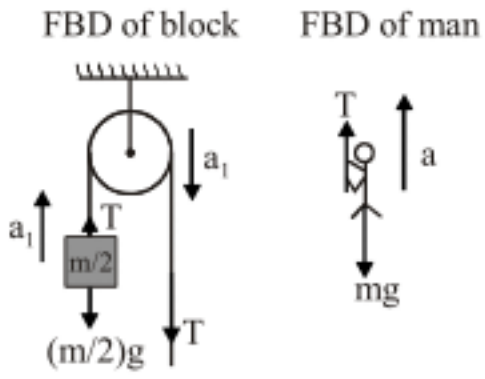
$$a_A = \frac{40-25}{5};\ a_A = 3\ m/s^2,\ a_B = 0\ m/s^2$$

Solved Example

Q.1 Block A of mass m/2 is connected to one end of light rope which passes over a pulley as shown in the Fig. Man of mass m climbs the other end of rope with a relative acceleration of g/6 with respect to rope. Find acceleration of block A and tension in the rope.



Sol.



a and a_1 are w.r.t ground.

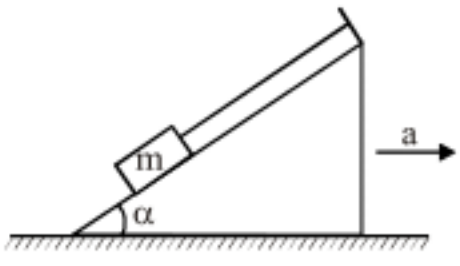
$a + a_1 = g/6$

$$T - \frac{m}{2}g = \frac{m}{2}a_1$$

$T - mg = ma$ on solving above equations we get $T = \frac{13}{18}mg$ and $a_1 = \frac{4}{9}g$

Paragraph for question nos. 2 to 4

A body of mass m = 1.8 kg is placed on an inclined plane, the angle of inclination is $\alpha = 37°$, and is attached to the top end of the slope with a thread which is parallel to the slop. Then the slope is moved with a horizontal acceleration of a. Friction is negligible.

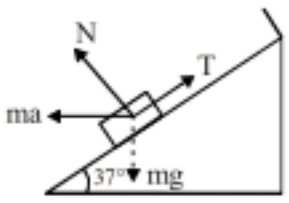


Q.2 Find the acceleration if the body pushes the slope with a force of $\frac{3}{4}mg$?

Sol. $N = mg\cos 37° - ma\sin 37° = \frac{3}{4}mg$

$a = \frac{5}{6}\ m/s^2$

FBD of block w.r.t wedge



Q.3 Find the tension in thread ?:

Sol. $T = mg\sin 37° + ma\cos 37°$

$T = 12$ N

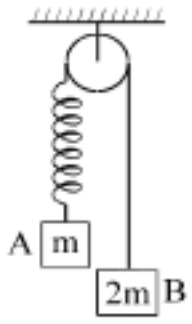
Q.4 At what acceleration will the body lose contact with plane?

Sol. $N = mg\cos 37° - ma\sin 37°$ (for lose contact $N = 0$)

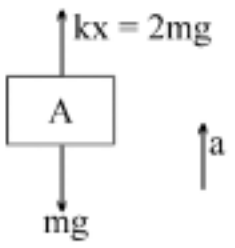
$mg\cos 37° = ma\sin 37°$

$$a = \frac{40}{3}\ m/s^2$$

Q.5 In the figure, block 'A' of mass 'm' is attached to one end of a light spring and the other end of the spring is connected to another block 'B' of mass 2m through a light string. 'A' is held and B is at rest in equilibrium. Now A is released. The acceleration of A just after that instant is 'a'. The same thing is repeated for 'B'. In that case the acceleration of 'B' is 'b', then value of a/b is.....?



Sol. When A is held, B will be at rest in equilibrium, if $kx = T = 2mg$
Now, just after A is released

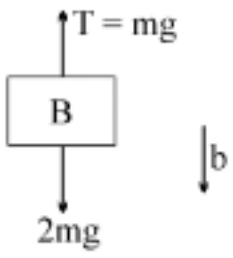


$$a = \frac{2mg - mg}{m} = g$$

When B held and A in equilibrium,

$T = kx = mg$

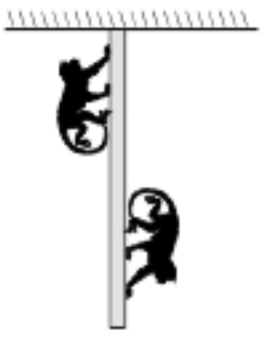
After B is released,



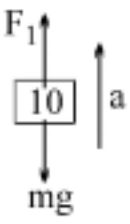
$$b = \frac{2mg - mg}{2m} = g/2$$

$a/b = \mathbf{2}$

Q.6 Two monkeys of masses 10 and 8 kg are moving along a vertical rope, the former climbing up with an acceleration of $2 m/s^2$ while the latter coming down with a uniform velocity of 2m/s. Find the tension in the rope at the fixed support.



Sol. FBD of monkey (1),



$F_1 - m_1g = m_1a$ $\Rightarrow$ $F_1 = m_1(g + a) = 10(10 + 2) = 120$ N

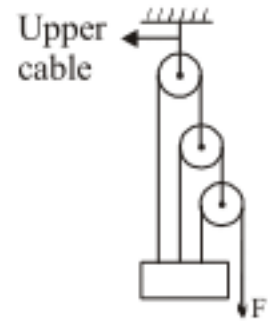
FBD of monkey (2),

$F_2 = m_2g = 10 \times 8 = 80$ N

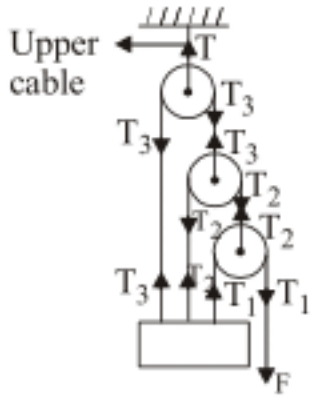
FBD of rope

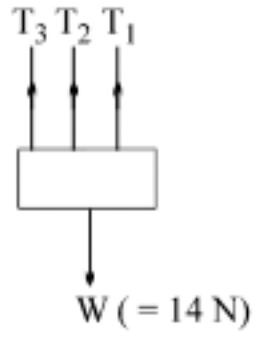
Tension $= F_1 + F_2 = 120 + 80 = 200$ N

Q.7 The pull F is just sufficient to keep the 14 N block in equilibrium as shown. Pulleys are ideal. Find the tension (in N) in the upper cable.



Sol. $T_1 = F$





$T_2 = 2T_1 = 2F$

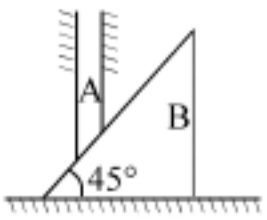
$T_3 = 2T_2 = 4F$

$T = 2T_3 = 8F$

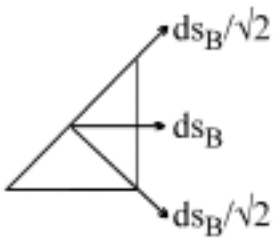
For equilibrium of block $T_1 + T_2 + T_3 = 14 \quad \Rightarrow \quad 7F = 14 \quad \Rightarrow \quad F = 2$ N

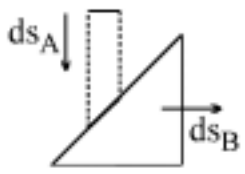
$\therefore$ T = 8F = 16 N

Q.9 Find the acceleration of rod A and wedge B in the arrangement shown in fig. if the mass of rod equal that of the wedge and the friction between all contact surfaces is negligible. Take angle of wedge as 45°.



Sol.





Perpendicular to the plane of contact displacement must be same.

$$\frac{ds_B}{\sqrt{2}} = \frac{ds_A}{\sqrt{2}}$$

$$ds_B = ds_A$$

Differentiating, $a_B = a_A = a$ (Let)

$mg - N/\sqrt{2} = ma$ ...(1)

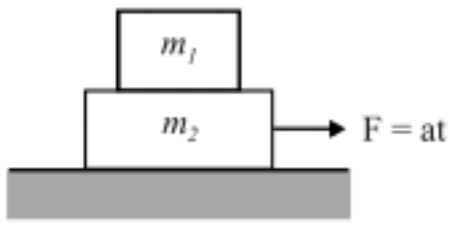
$N/\sqrt{2} = ma$ ...(2)

Equation (1) + (2)

$\Rightarrow \quad mg = 2ma$

$\Rightarrow \quad a = g/2$

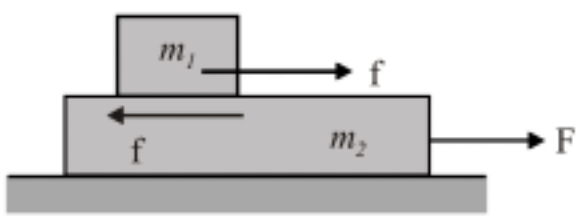
Q.10 A bar of mass m_1 is placed on a plank of mass m_2. which rests on a smooth horizontal plane. The coefficient of friction between the surfaces of the bar and the plank is equal to μ. The plank is subjected to the horizontal force F depending on time t as F = at (a is a constant).



Find (a) the moment of time t_0 at which the plank starts sliding from under the bar and (b) the acceleration of the bar a_1 and that of plank a_2 during motion.

Sol. As the force F grows, so does the static friction force f_s. However, the friction force f has the limiting value $f_{limit} = \mu m_1 g$. Unless this value is reached, both bodies move together with equal accelerations. But as soon as the force f reaches this limit, mass m_2 starts sliding under mass m_1.

Let us make free-body diagram



Writing Newton's second law for the plank and the bar, having taken the positive direction of the x axis

$f = m_1 a_1$, $F - f = m_2 a_2$

Acceleration of m_2 must be always greater than or equal to the acceleration of m_1 that is $a_2 \geq a_1$ where the equality corresponds to the moment $t = t_0$. Hence, when $f = \mu m_1 g$, then sliding begins. Putting $f = \mu m_1 g$.

$$t_0 = (m_1 + m_2)\frac{\mu g}{a}$$

When $t \leq t_0$, then

$$a_1 = a_2 = \frac{at}{(m_1 + m_2)}$$

and when $t > t_0$ then they seperate. Only force acting on m_1 is friction, whose value is constant.

Thus $a_1 = \mu g = \text{constant}$

Now m_2 is experiencing force F and constant friction

thus, $at - \mu m_1 g = m_2 a_2$

Solving we get

$$a_2 = \frac{(at - \mu m_1 g)}{m_2}$$

You may have been tempted to think that when external force F (=at) is equal to $\mu m_1 g$, slipping will being. You can check that at this instant t_2 they are moving with same acceleration.

$$at_2 = \mu m_1 g$$

$$t_2 = \frac{\mu m_1 g}{a}$$

Let's find force of friction between the blocks at this instant.

$$\mu m_1 g - f = m_2 a$$
$$f = m_1 a$$

Solving for friction

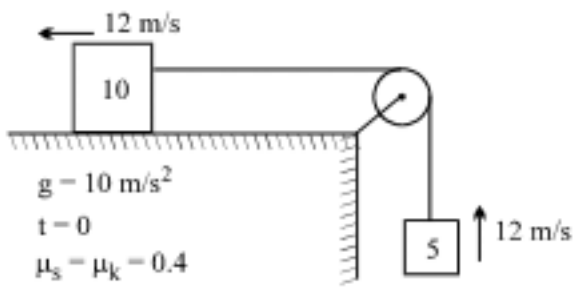
$$f = \frac{\mu m_1 g}{\left(1 + \frac{m_2}{m_1}\right)}$$

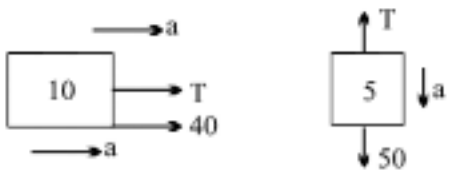
Since friction is less than the limiting value, slipping has not yet begun.

Q.11 Blocks are given velocities as shown at t = 0, find velocity and position of 10 kg block.

(A) at t = 1 sec

(B) at t = 4 sec.



Sol. making F.B.D. 

]

(A) $40 + T = 10a$; $50 - T = 5a$; $a = 6\ m/s^2$

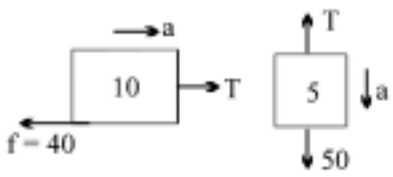
$u = 12$; $a = -6$

$v = 12 - 6 \times 1 = 6\ m/s$; $s = 12 \times 1 - 3 \times 1 = 9\ m$

(B) Let them solve it wrong. Then explain that since velocity has changed the direction during motion friction would also have changed thus direction and acceleration will change.

$u = 12$; $a = -6$ (till velocity becomes zero)

$v = 0 \Rightarrow v = 2$ sec ; $s = 2 \times 2 - 3 \times 4 = 12\ m$

now FBD 

$50 - T = 5a$

$T - 40 = 10a$

$a = 2/3\ m/s^2$

$u = 0, a = 2/3, t = 2, v = 4/3$

$s = \frac{1}{2} \times \frac{2}{3} \times 4 = \frac{4}{3}$; total displacement $12 - \frac{4}{3} = \frac{32}{3}$ m