Current Electricity

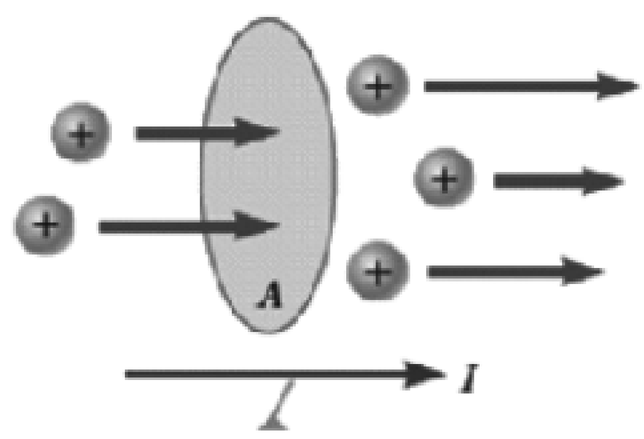
Introduction

In the last chapter we discussed electrostatics-the physics of stationary charges. In this chapter, we discuss the physics of electric currents-that is, charges in motion.

Examples of electric currents abound and involve many professions. Meteorologists are concerned with lightning and with the less dramatic slow flow of charge through the atmosphere. Biologists, physiologists, and engineers working in medical technology are concerned with the nerve currents that control muscles and especially with how those currents can be reestablished after spinal cord injuries. Electrical engineers are concerned with countless electrical systems, such as power systems, lightning protection systems, information storage systems, and music systems. Space engineers monitor and study the flow of charged particles from our Sun because that flow can wipe out telecommunication systems in orbit and even power transmission systems on the ground.

In this chapter we discuss the basic physics of electric currents and why they can be established in some materials but not in others. We begin with the meaning of electric current.

Electric Current



The time rate of flow of charge through any cross-section is called current.

$$i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

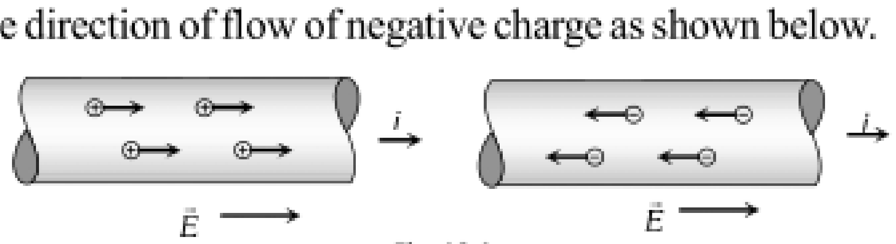
If flow is uniform then $i = \frac{Q}{t}$.

Current is a scalar quantity. It's S.I. unit is ampere (A) and C.G.S. unit is emu and is called biot (Bi), or ab ampere. 1A = (1/10) Bi (ab amp.)

Note :

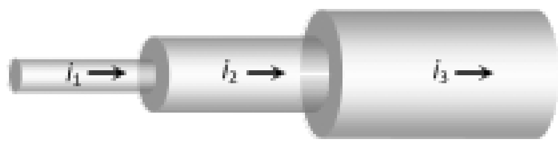
(1) Ampere of current means the flow of 6.25×10^{18} electrons/sec through any cross-section of the conductor.

(2) The conventional direction of current is taken to be the direction of flow of positive charge, i.e. field and is opposite to the direction of flow of negative charge as shown below.

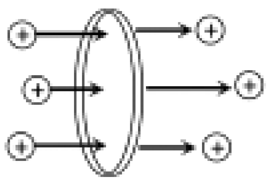


(3) The net charge in a current carrying conductor is zero.

(4) For a given conductor current does not change with change in cross-sectional area. In the following figure $i_1 = i_2 = i_3$



(5) Current due to translatory motion of charge : If n particle each having a charge q, pass through a given area in time t then $i = \frac{nq}{t}$

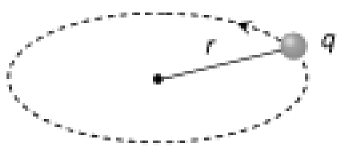


If n particles each having a charge q pass per second per unit area, the current associated with cross-sectional area A is $\mathbf{i = nqA}$

If there are n particle per unit volume each having a charge q and moving with velocity v, the current thorough, cross section A is $\mathbf{i = nq\upsilon A}$

(6) Current due to rotatory motion of charge : If a point charge q is moving in a circle of radius r with speed v (frequency ν, angular speed ω and time period T) then corresponding current

$$i = q\nu = \frac{q}{T} = \frac{q\upsilon}{2\pi r} = \frac{q\omega}{2\pi}$$



(7) **Current carriers :** The charged particles whose flow in a definite direction constitutes the electric current are called current carriers. In different situation current carriers are different.

(i) **Solids :** In solid conductors like metals current carriers are free electrons.

(ii) **Liquids :** In liquids current carriers are positive and negative ions.

(iii) **Gases :** In gases current carriers are positive ions and free electrons.

(iv) **Semi conductor :** In semi conductors current carriers are holes and free electrons.

(8) Current, as defined by above Equation, is a scalar because both charge and time in that equation are scalars. Yet, as in Figure (a), we often represent a current with an arrow to indicate that charge is moving. Such arrows are not vectors, however, and they do not require vector addition. Figure shows a conductor with current i_0 splitting at a junction into two branches. Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, so that

$$\mathbf{i_0 = i_1 + i_2}$$

(a) (b)

As Figure (b) suggests, bending or reorienting the wires in space does not change the validity of above Equation. Current arrows show only a direction (or sense) of flow along a conductor, not a direction in space.

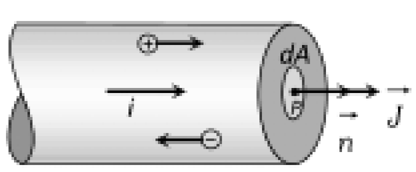
The relation $i_0 = i_1 + i_2$
is true at junction a no matter what the orientation in space of the three wires. Currents are scalars, not vectors.

9. Total charge flown through a cross section of conductor whoes current (i) is given will be $q = \int i\ dt$, we integrate with in prescribed limits to time

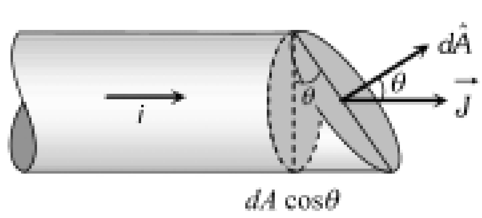
Current Density ($\vec{J}$)

Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point.

Current density at point P is given by $\vec{J} = \frac{di}{dA}\vec{n}$



If the cross-sectional area is not normal to the current, but makes an angle to θ with the direction of current then



$$J = \frac{di}{dA\cos\theta} \Rightarrow di = JdA\cos\theta = = \vec{J}.\vec{dA} = i = \int \vec{J}\cdot\vec{dA}$$

Note :

(1) Direction of $\vec{J}$ coincides with the direction of current flow at that point. So it is a vector quntity whose direction is defined with the electric field at that point.

(2) If current density $\vec{J}$ is uniform for a normal cross-section $\vec{A}$ then $J = \frac{i}{A}$

(3) Current density $\vec{J}$ is a vector quantity. It's direction is same as that of $\vec{E}$. It's S.I. unit is amp/m^2 and dimension [$L^{-2}A$].

(4) In case of uniform flow of charge through a cross-section normal to it as $i = nqvA$

so $J = \frac{i}{A} = nqv$

(5) Current density relates with electric field as $\vec{J} = \sigma\vec{E} = \frac{\vec{E}}{\rho}$; where σ = conductivity and ρ = resistivity or specific resistance of substance.

Illustration :

A copper wire of diameter 1.02 mm carries a current of 1.7 amp. Find the drift velocity (v_d) of electrons in the wire. Given n, number density of electrons in copper $= 8.5 \times 10^{28}\ /m^3$.

Sol. $I = 1.7\ A$

J = *current density*

$$= \frac{1}{\pi r^2} = \frac{1.7}{\pi \times (0.51 \times 10^{-3})^2}$$

$= nev_d$

$= 8.5 \times 10^{28} \times (1.6 \times 10^{-19}) \times v_d$

$$\therefore \quad v_d = \frac{1.7}{\pi \times (0.51 \times 10^{-3})^2 \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$= 1.5 \times 10^{-3}$ *m/sec.* $= 1.5$ *mm/sec.*

Illustration :

A solution of NaCl discharges 6.5×10^{16} Na^+ ions and 4.2×10^{16} Cl^- ions in 1 sec. Find the total current passing through the solution.

Sol. *The total current through a solution (conductor) is due to all the charge carriers (moving in opposite directions if they are oppositely charged.*

$$I_{tot} = \frac{6.5 \times 10^{16} + 4.2 \times 10^{16}}{1\,\text{sec}} \times e$$

$= 10.7 \times 10^{16}\ \ 1.6 \times 10^{-19}$ *coulomb/sec.*

$= 1.7 \times 10^{-4}\ A$

Illustration :

The magnitude J of the current density in a certain lab wire with a circular cross section of radius R = 2.00 mm is given by $J = (3.00 \times 10^8) r^2$, with j in amperes per square meter and radial distance r in meters. What is the current through the outer section bounded by r = 0.900R and r = R ?

Sol. *Assuming J is directed along the wire (with no radial flow) we integrate, starting*

$$i = \int |\vec{J}|\,dA = \int_{9R/10}^{R} (kr^2) 2\pi r dr = \frac{1}{2} k\pi (R^4 - 0.656R^4)$$

Where $k = 3.0 \times 10^8$ and SI units are understood. Therefore if R = 0.00200m. We obtain $i = 2.59 \times 10^{-3}$ A.

Illustration :

What is the current in a wire of radius R = 3.40 mm if the magnitude of the current density is given by (a) $J_a = J_0/R$ and (b) $J_b = J_0 (1 - r/R)$, in which r is the radial distance and $J_0 = 5.50 \times 10^4\ A/m^2$? (c) Which function maximizes the current density near the wire's surface ?

Sol. *(a) The current resulting from this nonuniform current density is*

$$i = \int_{cylinder} J_a\, dA = \frac{J_0}{R}\int_0^R r\cdot 2\pi r dr = \frac{2}{3}\pi R^2 J_0 = \frac{2}{3}\pi(3.40\times 10^{-3} m)(5.50\times 10^4 A/m^2)$$

$= 1.33\ A.$

(b) *In this case*

$$i = \int_{cylinder} J_a dA = \frac{J_0}{R}\int_0^R r\cdot 2\pi r dr = \frac{2}{3}\pi R^2 J_0 = \frac{1}{3}\pi R^2 J_0 = \frac{1}{2}\pi(3.40\times 10^{-3} m)^2(5.50\times 10^4 A/m^2)$$

$= 0.666\ A$

(c) *The result is different from that in part (a) because J_b is higher near the center of the cylinder (where the area is smaller for the same radial interval) and lower outward, resulting in lower average current density over the cross section and consequently a lower current than that in part (a). So J_a has its maximum value near the surface of the wire.*

Practice Exercise

Q.1 A steady current passes through a cylindrical conductor. Is there an electric field inside the conductor?

Q.2 If 0.6 mol of electrons flow through a wire in 45 min what are (a) the total charge that passes through the wire, and (b) the magnitude of the current.

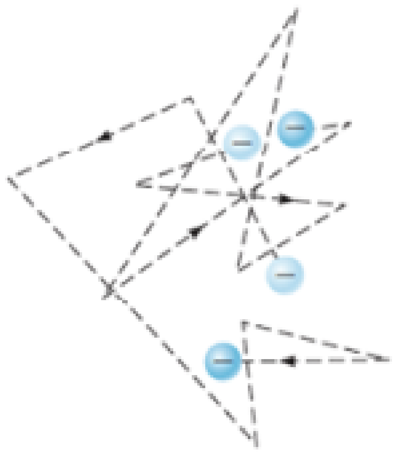
Answers

Q.1 Yes Q.2 (a) 5.7×10^4 C (b) 21.41 Amp

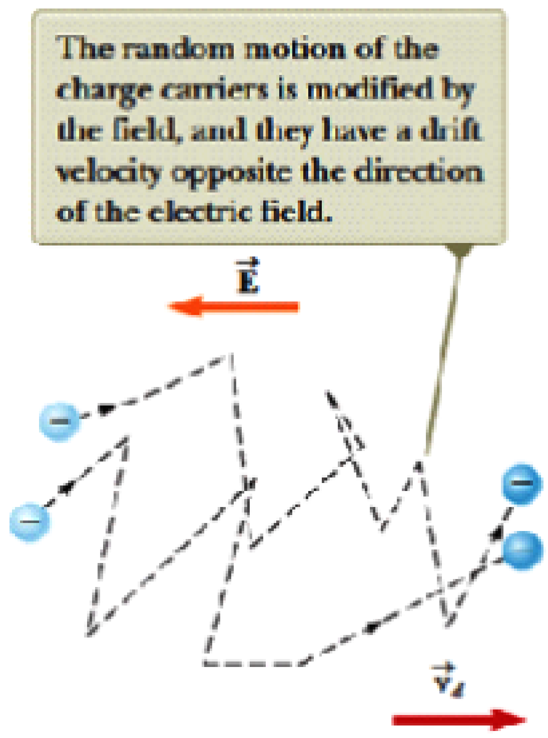
Model for Electric Conduction

We describe a classical model of electrical conduction in metals that was first proposed by Paul Drude (1863-1906) in 1900.

Consider a conductor as a regular array of atoms plus a collection of free electrons, which are sometimes called conduction electrons. The conduction electrons, although bound to their respective atoms when the atoms are not part of a solid, become free when the atoms condense into a solid. In the absence of an electric field, the conduction electrons move in random directions through the conductor Fig below. The situation is similar to the motion of gas molecules confined in a vessel. In fact, some scientists refer to conduction electrons in a metal as an electron gas.



When an electric field is applied, the free electrons drift slowly in a direction opposite that of the electric field (Figure Below), with an average drift speed v_d that is much smaller (typically 10^{-4}m/s) than their average speed between collisions (typically 10^6m/s).



In our model, we make the following assumptions:

1. The electron's motion after a collision is independent of its motion before the collision.
2. The excess energy acquired by the electrons in the electric field is transferred to the atoms of the conductor when the electrons and atoms collide.

We are now in a position to derive an expression for the drift velocity. When a free electron of mass m_e and charge q (= – e) is subjected to an electric field $\vec{E}$, it experiences a force $\vec{F} = q\vec{E}$. The electron is a particle under a net force, and its acceleration can be found from Newton's second law, $\sum \vec{F} = m\vec{a}$:

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{q\vec{E}}{m_e}$$

Because the electric field is uniform, the electron's acceleration is constant, so the electron can be modeled as a particle under constant acceleration. If $\vec{v}_i$ is the electron's initial velocity the instant after a collision (which occurs at a time defined as t = 0), the velocity of the electron at a very short time t later (immediately before the next collision occurs) is, from equation

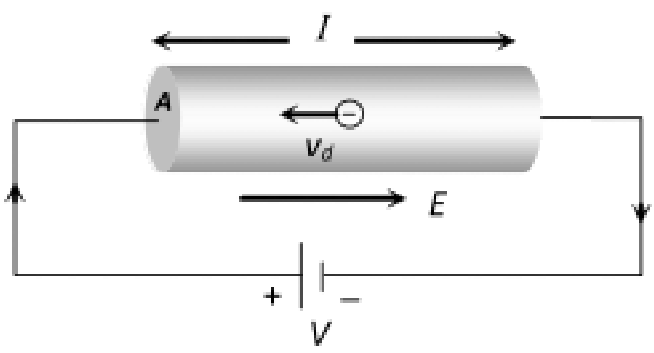
$$\vec{v}_f = \vec{v}_i + \vec{a}t = \vec{v}_i + \frac{q\vec{E}}{m_e}t$$

Let's now take the average value of $\vec{v}_f$ for all the electrons in the wire over all psoible collision times t and all possible value of $\vec{v}_i$. Assuming the initial velocities are randomly distributed over all possible directions, the average value of $\vec{v}_i$ is zero. The average value of the second terms of equation is $(q\vec{E}/m_\ell)\,\tau$, where τ is the average time interal between successive collisions. Because the average value of $\vec{v}_f$ is equal to the drift velocity.

$$\vec{v}_{f,avg} = \vec{v}_d = \frac{q\vec{E}}{m_e}\tau$$

Drift Velocity

Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it. Drift velocity is very small it is of the order of 10^{-4} m/s as compared to thermal speed ($\simeq 10^5$ m/s) of electrons at room temperature.



If suppose for a conductor
n = Number of electron per unit volume of the conductor
A = Area of cross-section
V = potential difference across the conductor
E = electric field inside the conductor

i = current, J = current density, ρ = specific resitance, σ = conductivity $\left(\sigma = \frac{1}{\rho}\right)$

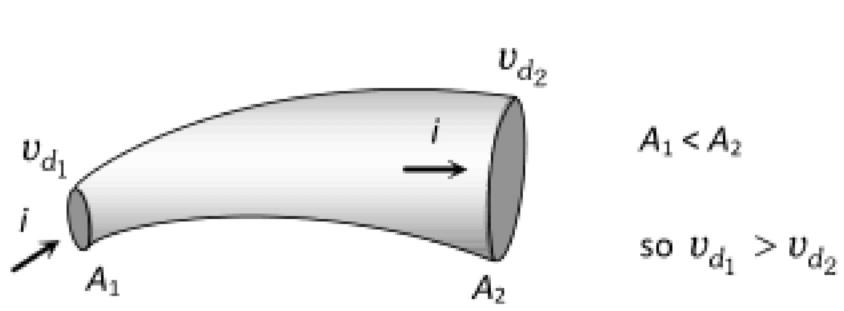
then current relates with drift velocity as $i = neAv_d$ ***we can also write***

$$v_d = \frac{i}{neA} = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne} = \frac{V}{\rho l n e}$$

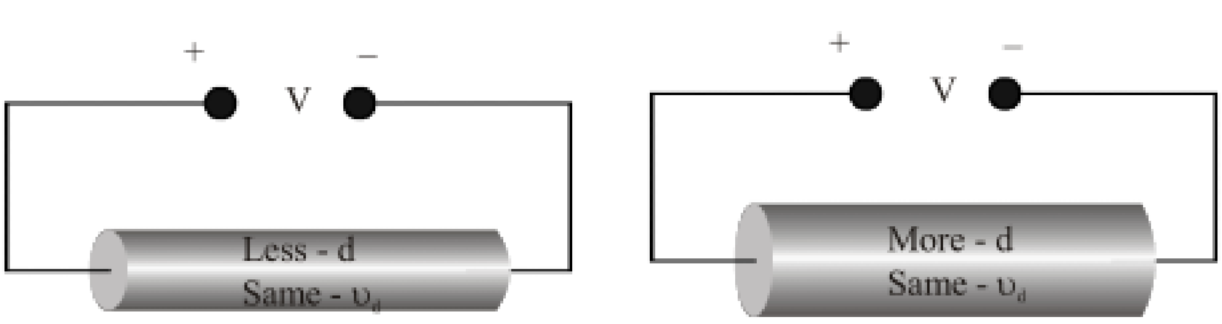
(1) The direction of drift velocity for electron in a metal is opposite to that of applied electric field (i.e. current density $\vec{j}$).

$v_d \propto E$ i.e. greater the electric field, larger will be the drift velocity

(2) When a steady current flows through a conductor of non-uniform cross-section drift velocity varies inversely with area of cross-section $\left(v_d \propto \frac{1}{A}\right)$



(3) If diameter (d) of a conductor is doubled, then drift velocity of electrons inside it will not change.



(4) **Relaxation time (τ) :** The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time $\tau = \frac{\text{mean free path}}{\text{r.m.s. velocity of electrons}} = \frac{\lambda}{v_{rms}}$. With rise in temperature υ_{rms} increases consequently τ decreases.

(5) **Mobility :** Drift velocity per unit electric field is called mobility of electron i.e. $\mu = \frac{\upsilon_d}{E}$. It's unit is $\frac{m^2}{\text{volt} - \text{sec}}$

Illustration :

Find the electric current in a conductor (copper) of cross-section $A = 1nm^2$, conduction electron density $n = 8.69 \times 10^{28}/m^2$ and drift speed $v_d = 1\ cm/s$.

Sol. $i = nev_d A$

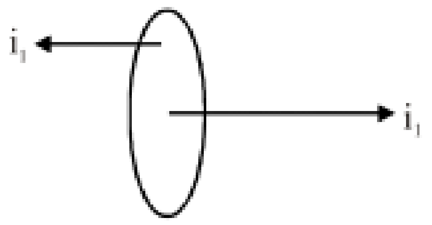
$= 8.69 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-2} \times 1 \times 10^{-4}$

$= 8.69 \times 1.6 \times 10^{5}$ *amp*

Illustration :

n_1 electron/s passes through a given cross-section towards right with velocity v_1 and n_2 proton/s passes through the same cross-section with velocity v_2 in the same direction. Find the current through a given cross-sectional. Put $n_1 = 1.5 \times 10^{10}$ and $n_2 = 10^{10}$.

Sol. $i_1 = \frac{\Delta q}{\Delta r} = \frac{\Delta N_1 q_1}{\Delta r} = \frac{dN_1}{dt} q_1$



$i_2 = \frac{dN_2}{dt} q_2$

$i = i_1 + i_2$

$= \left(\frac{dN_1}{dt}\right)(-e) + \left(\frac{dN_2}{dt}\right)e$

$i = (n_2 - n_1)\, e$

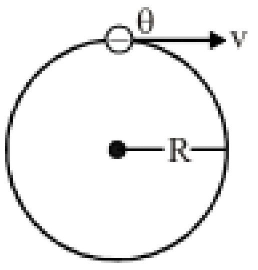
$= (1.5 \times 10^{10} - 1 \times 10^{10})\, 1.6 \times 10^{-19} = 0.5 \times 10^{-9}$ *amp*

Illustration :

Find the current associated with an electron revolving with a speed $v = 10^6$ m/s in an orbit of radius $R = 1Å$.

Sol. *The charge $\Delta q\ (= -e)$ flows (passes) through a fixed point during a time $\Delta t = T$.*

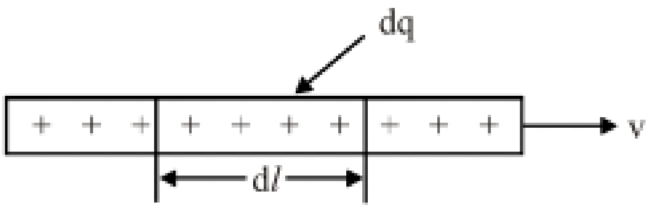
Then, $i = \frac{\Delta q}{\Delta t} = \frac{e}{T}$, *where* $T = \frac{2\pi R}{v}$



or, $i = \frac{ev}{2\pi R} = \frac{(1.6 \times 10^{-19})(10^6)}{2 \times \frac{22}{7} \times (10^{-10})} \approx 0.26 \times 10^{-3}\ A$

Illustration :

Find the current associated with a moving straight wire of linear charge density $\lambda = 2\ \mu C/m$ and of cross-section $A = 2\ mm^2$, when the wire is pulled with a speed $v = 2\ m/s$.



Sol. *Let $dq\ (= \lambda\ dl)$ passes through a given vertical plane in time dt.*

Then, $i = \frac{dq}{dt}$

$= \frac{\lambda dl}{dt}$

$= lv \qquad \left(\because v = \frac{dl}{dt}\right)$

$= 2 \times 10^{-3} \times 2 = 4\ mA$

Illustration :

A homogeneous beam of proton accelerated through a potential difference $V = 500\ KV$ has a circular cross-section of radius $R = 4\ mm$. Assuming beam current $i = 32 \times 10^{-3}\ A$. Find the
(i) number of protons passing through a cross-section per second.
(ii) electric field at the surface of the beam.
(iii) potential difference between the surface and axis of the beam.

Sol. *(i) The numer of protons/second*

$$= \frac{i}{e} = \frac{32 \times 10^{-3}}{1.6 \times 10^{-19}} = 2 \times 10^{16}$$

(ii) $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, *where* $l = \frac{i}{v}$

or, $$E = \frac{i}{2\pi\varepsilon_0 rv} \qquad ...(i)$$

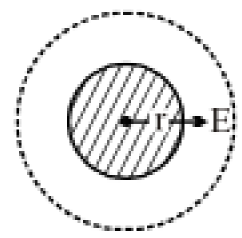
since $\frac{1}{2}mv^2 = eV$, *substituting* $v = \sqrt{\frac{2eV}{m}}$ *in eq. (i).*

$$E = \frac{1}{2\pi\varepsilon_0 R}\sqrt{\frac{m}{2eV}}$$

$$= \frac{2 \times 9 \times 10^9 \times 32 \times 10^{-3}}{4 \times 10^{-3}}\sqrt{\frac{1.6 \times 10^{-27}}{2 \times 1.6 \times 10^{-19} \times 500 \times 10^3}} \qquad \left(\because \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\right)$$

$= 144 \times 10^9 \times 10^{-7}\ V/m$
$= 14.4\ KV/m$

(iii) Applying Gauss Law.



$$E.\ 2\pi rl = \left(\frac{Q_0}{Q\pi R^2 l}\right)\left(\frac{Q\pi r^2 l}{\varepsilon_0}\right)$$

or, $$E = \frac{Qr}{2\pi\varepsilon_0 R^2 l} = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$$

Then, $$\Delta V = \int_0^R E\ dr = \frac{\pi}{2\pi\varepsilon_0 R^2}\int_0^R r\ dr = \frac{\lambda}{4\pi\varepsilon_0},$$ where $\lambda = i\sqrt{\frac{m}{2eV}}$

or, $$\Delta V = \frac{i}{4\pi\varepsilon_0}\sqrt{\frac{m}{2eV}} = \frac{ER}{2} = \frac{14.4\times10^3\times4\times10^{-3}}{2} = 28.8\ V$$

Practice Exercise

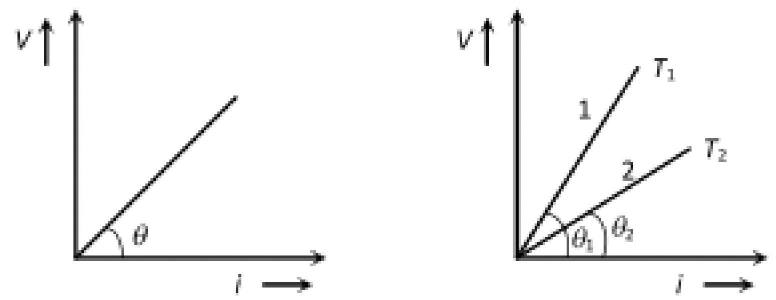
Q.1 A beam of fast moving electrons having cross-sectional area $A = 1\ cm^2$ falls normally on a flat surface. The electrons are absorbed by the surface and the average pressure exerted by the electrons on this surface is found to be $P = 9.1$ Pa. If the electrons are moving with a speed $v = 8\times10^7$ m/s, then find the effective current (in A) through any cross-section of the electron beam.
(mass of electron $= 9.1\times10^{-31}$ kg)

Answers

Q.1 0002 A

Ohm's Law

If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remains same, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends i.e. $i \propto V \Rightarrow V = iR$ where R is a proportionality constant, known as electric resistance.
(1) Ohm's law is not a universal law, the substances, which obey ohm's law are known as ohmic substance.
(2) Graph between V and i for a metallic conductor is a straight line as shown. At different temperatures V-i curves are different.



(A) Slope of the line $= \tan\theta = \frac{V}{i} = R$

(B) Here $\tan\theta_1 > \tan\theta_2$
So $R_1 > R_2$
i.e. $T_1 > T_2$

Resistance

(1) The property of substance by virtue of which it opposes the flow of current through it, is known as the resistance.

(2) Formula of resistance : For a conductor if l = length of a conductor A = Area of cross-section of conductor, n = No. of free electrons per unit volume in conductor, τ = relaxation time then resistance of conductor $R = \rho \frac{l}{A} = \frac{m}{ne^2\tau} \cdot \frac{l}{A}$; where ρ = resistiveity of the material of conductor

(3) **Unit and dimension :** It's S.I. unit is Volt/Amp. or Ohm (Ω). Also 1 ohm = $= \frac{1\,\text{volt}}{1\,\text{Amp}} = \frac{10^8\,\text{emu of potential}}{10^{-1}\,\text{emu of current}}$ = 10^9 emu of resistance. It's dimension is $[ML^2T^{-3}A^{-2}]$

(4) **Dependence of resistance :** Resistance of a conductor depends upon the following factors.

(i) Length of the conductor : Resistance of a conductor is directly proportional to it's length i.e. $R \propto l$ and and inversely proportional to it's area of cross-section i.e. $R \propto \frac{1}{A}$

(ii) Temperature : For a conductor

Resistance $\propto$ temperature.

If R_0 = resistance of conductor at 0°C

R_t = resistance of conductor at t°C

and α, β = temperature co-efficient of resistance then $R_t = R_0\,(1+\alpha t+\beta t^2)$ for t > 300°C and $R_t = R_0$ $(1+\alpha t)$ for $t \leq 300$°C or $\alpha = \frac{R_t - R_0}{R_0 \times t}$

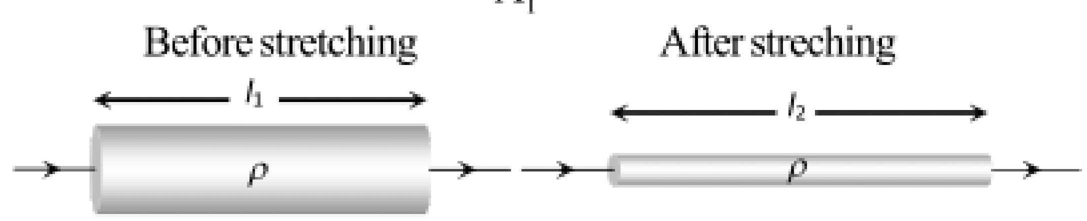
If R_1 and R_2 are the resistance at t_1°C and t_2°C respectively then $\frac{R_1}{R_2} = \frac{1+\alpha t_1}{t+\alpha t_2}$

The value of α is different at different temperature rage t_1°C to t_2°C is given by $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$ which given $R_2 = R_1\,[1+\alpha(t_2 - t_1)]$. This formula gives an approximate value.

(5) **Stretching of wire**

If a conducting wire stretches, it's length increases, area of cross-section decreases so resistance increases but volume remain constant.

Suppose for a conducting wire before stretching it's length = l_1, area of cross-section = A_1 radius = r_1 diameter = d_1, and resistance $R_1 = \rho \frac{l_1}{A_1}$



Volume remains constant *i.e.* $A_1 l_1 =$

After streching length = l_2 area of cross-section = A_2 radius = r_2 diameter = d_2 and resistance = $R_2 = \rho \frac{l_2}{A}$

Ratio of resistance before and after streching

$$\frac{R_1}{R_2}=\frac{l_1}{l_2}\times\frac{A_2}{A_1}=\left(\frac{l_1}{l_2}\right)^2=\left(\frac{A_2}{A_1}\right)^2=\left(\frac{r_2}{r_1}\right)^4=\left(\frac{d_2}{d_1}\right)^4$$

(i) If length is given then $R \propto l^2 \Rightarrow \frac{R_1}{R_2}=\left(\frac{l_1}{l_2}\right)^2$

(ii) If radius is given then $R \propto \frac{1}{r^4} \Rightarrow \frac{R_1}{R_2}=\left(\frac{r_2}{r_1}\right)^4$

Resistivity (ρ), Conductivity (σ) and Conductance (C)

(1) **Resistivity :** From $R=\rho\frac{l}{A}$; If l = 1m, A = 1 m² then R = ρ i.e. resistivity is numerically equal to the resistance of a substance having unit area of cross-section and unit length.

(i) Unit and dimension : It's S.I. unit ohm × m and dimension is $[ML^3T^{-3}A^{-2}]$

(ii) It's formula : $\rho=\frac{m}{ne^2\tau}$

(iii) Resistivity is the intrinsic property of the substance. It is independent of shape and size of the body (i.e. l and A).

(v) Resistivity depends on the temperature. For metals $\rho_t=\rho_0\,(1+\alpha\Delta t)$ i.e. resitivity increases with temperature.

(vi) Resistivity increases with impurity and mechanical stress.

(vii) Magnetic field increases the resistivity of all metals except iron, cobalt and nickel.

(viii) Resistivity of certain substances like selenium, cadmium, sulphides is inversely proportional to intensity of light falling upon them.

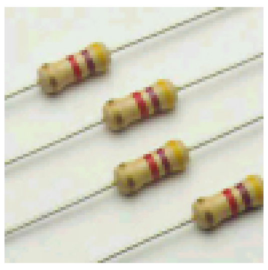
(2) **Conductivity :** Reciprocal of resistivity is called conductivity i.e. $s=\frac{1}{\rho}$ with unit mho/m and dimensions $[M^{-1}L^3T^{-3}A^2]$

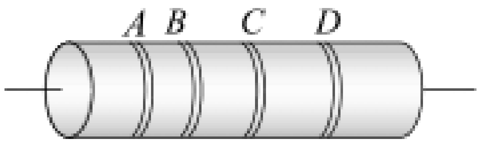
(3) **Conductance:** Reciprocal of resistance is known as conductance. $C=\frac{1}{R}$. It's unit is $\frac{1}{\Omega}$ or Ω^{-1} or "siemen".

Colour Coding of Resistance

To know the value of resistance colour code is used. These code are printed in form of set of rings or strips. By reading the values of colour bands, we can estimate the value of resistance.

The carbon resistance has normally four coloured rings or bands say A, B, C and D as shown in following figure.





Colour band A and B : Indicate the first two significant figures of resistance in ohm.

Band C : Indicates the decimal multiplier i.e. the number of zeros that follows the two significant figures A and B.

Band D : Indicates the tolerance in percent about the indicated value or in other words it represents the percentage accuracy of the indicated value.

The tolerance in the case of gold is ± 5% and in silver is ± 10%. If only three bands are marked on carbon resistance, then it indicate a tolerance of 20%.

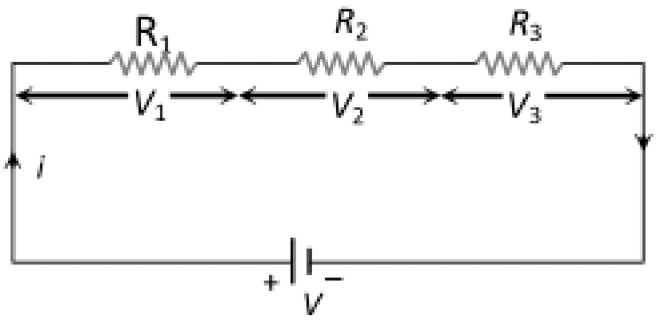
Table : Colour code for carbon resistance

Letters as an aid to memory	Colour	Figure (A, B)	Multiplier (C)
B	Black	0	10^o
B	Brown	1	10^1
R	Red	2	10^2
O	Orange	3	10^3
Y	Yellow	4	10^4
G	Green	5	10^5
B	Blue	6	10^6
V	Violet	7	10^7
G	Grey	8	10^8
W	White	9	10^9

Grouping of Resistance

(1) Series grouping

(i) Same current flows through each resistance but potential difference distributes in the ratio of resistance i.e. $V \propto R$

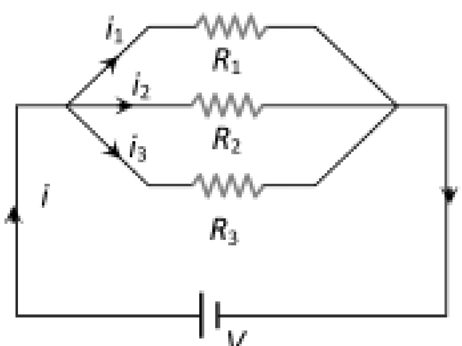


(ii) $R_{eq} = R_1 + R_2 + R_3$ equivalent resistance is greater than the maximum value of resistance in the combination.

(iii) If n identical resistance are connected in series $R_{eq} = nR$ and potential different across and resistance $V' = \frac{V}{n}$

(2) Parallel grouping

(i) Same potential difference appeared across each resistance but current distrbutes in the reverse ratio of their resistance i.e. $i \propto \frac{1}{R}$



(ii) Equivalent resistance is given by $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ or $R_{eq} = (R_1^{-1} + R_2^{-1} + R_3^{-1})^{-1}$ or

$$R_{eq} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_2R_1}$$

Equivalent resistance is smaller than the minimum value of resistance in the combination.

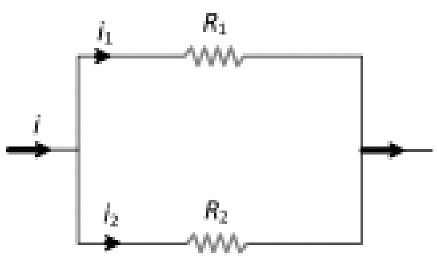
(iv) If two resistance in parallel $R_{eq} = \frac{R_1R_2}{R_1 + R_2}$

(v) Current through any resistance $i' = i \times \left[\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right]$

Where i' = required current (branch current), i = main current

$$i_1 = i\left(\frac{R_2}{R_1 + R_2}\right)$$

and $$i_2 = i\left(\frac{R_1}{R_1 + R_2}\right)$$



(vi) In n identical resistance are connected in parallel $R_{eq} = \frac{R}{n}$ and current through each resistance $i' = \frac{i}{n}$

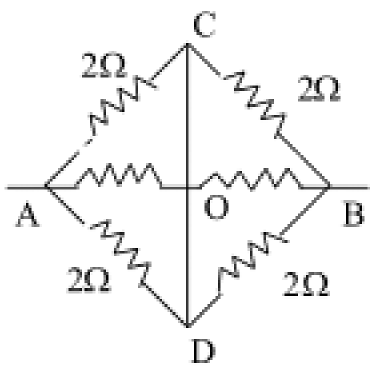
Note :

Rules for finding Req complicated resistance circuit:

- We can join any number of points in a circuit that are connected by a simple conducting wire as they will be at same potential.
- We can join any number of points in a circuit that are lying on plane of symmetry.
- We can break a single point in multiple points if after breaking new points formed are lying on plane of symmetry.

Illustration :

Find the equivalent resistance between A and B in the circuit shown here. Every resistance shown here is of 2Ω.



Sol. *Points C, O & D are at the same potential. Therefore, resistances AO, AC and AD are in parallel . Similarly BC, BO and BD are in parallel.*

$\therefore \quad R_{AB} = \frac{1}{3} \times (2\Omega) + \frac{1}{3} \times (2\Omega)$ *Plane of symmetrg passes through c,o and D*

$= \frac{4}{3}\Omega = 1.33\Omega$

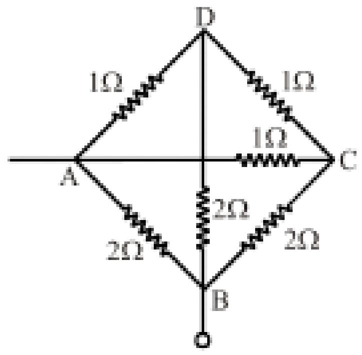
Illustration :

It is desired to make a 20 Ω coil of wire which has a zero thermal coefficient of resistance. To do this, a carbon resistor of resistance R_1, is placed in series with an iron resistor of resistance R_2. The proportions of iron and carbon are so chosen that $R_1 + R_2 = 20\ \Omega$ for all temperatures near 20°C, how large are R_1 and R_2 ? $(\alpha_c = -0.5\times10^{-3}, \alpha_{Fe} = 5\times10^{-3})$

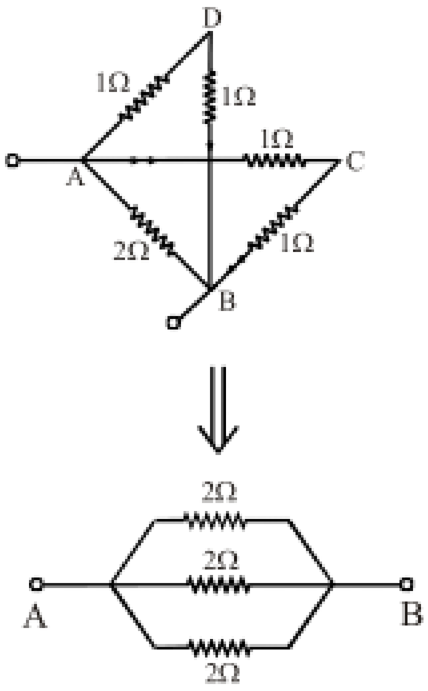
Sol. *We need $R_1(1 + \alpha_1\Delta t) + R_2(1 + \alpha_2\Delta T) = 20$ because $R_1 + R_2 = 20$ where $\Delta t = 0$,*
We must have $R_1\alpha_1 = -R_1\alpha_2$ with $\alpha_1 = -0.5 \times 10^{-3}$ solving the two equation $R_1+R_2 = 20$ and $R_1 = 18.18\ \Omega$ and $R_2 = 1.82\ \Omega$.

Illustration :

Six resistors form a pyramid. Find the effective resistance between A and B.



Sol. *The branches ADB and ACB are symmetrical relative tot the terminals A and B. Hence, the points D and C are equipotential. Since, $R_{DC} \neq 0$ $i_{DC} = 0$. Then remove the branch DC and then the circuit is reduced to a simpler one as shown in the figure.*

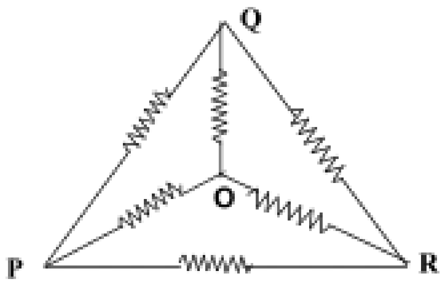


Then $$\frac{1}{R_{AB}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

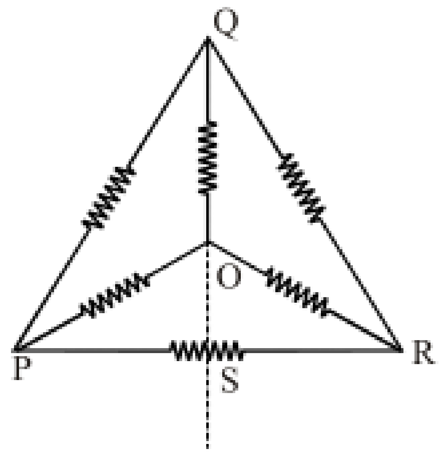
or, $$R_{AB} = \frac{2}{3}\Omega$$

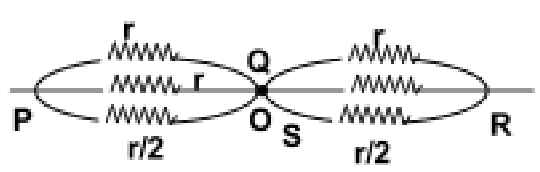
Illustration :

Six equal resistances each of resistance 4Ω are connected to form the following figure. What is the resistance between any two corners.



Sol. *There is symmetry about the line passing through QO and mid point of PR.*

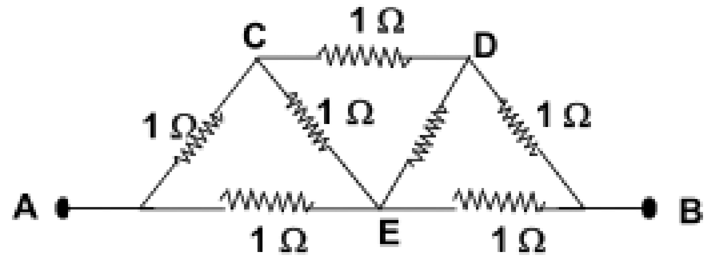




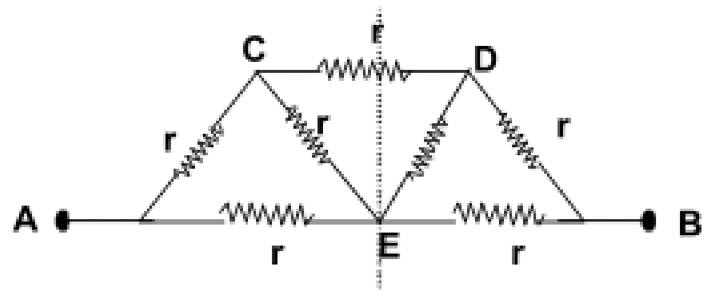
$$= \frac{r}{2} = 2\Omega$$

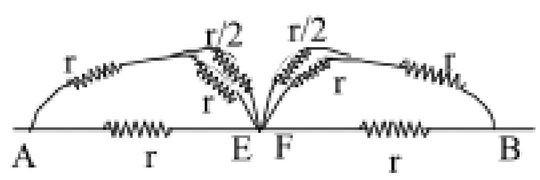
Illustration :

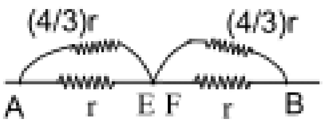
In the network shown in figure, each resistance is 1 Ω. What is the effective resistance between A and B

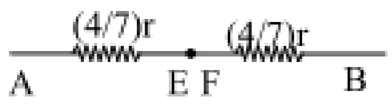


Sol. *There is a symmetry about line passing through E and mid point of CD.*





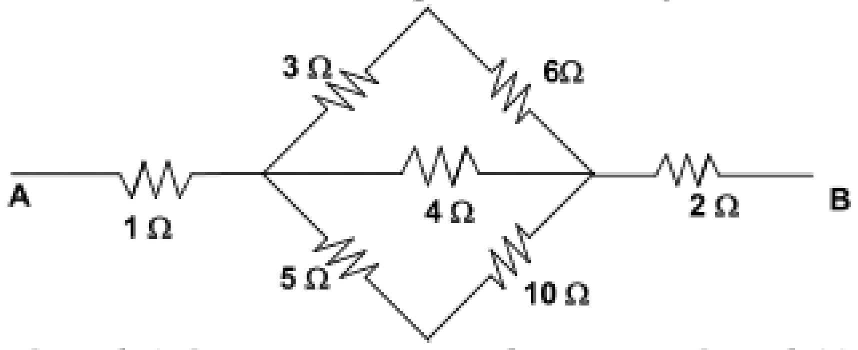




$= \frac{8}{7} r = \frac{8}{7} \Omega$

Illustration :

Find the equivalent resistance between points A & B of the network shown in the given diagram

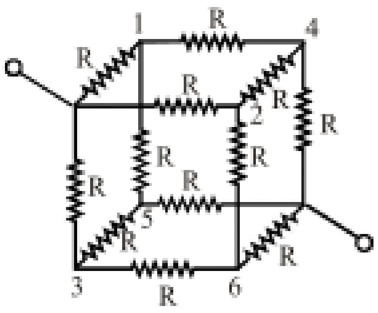


Sol. *The resistors 3 Ω and 6 Ω are in series and so are 5 Ω and 10 Ω resistors. These two series equivalents are in parallel to each other and also to the 4 Ω resistors. Hence the network reduces to the one given below :*

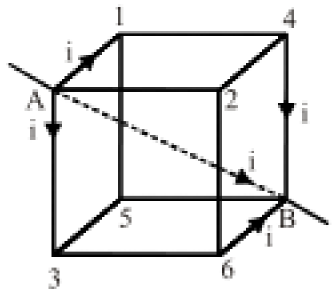
$R_{eq} = 5.34\ \Omega$

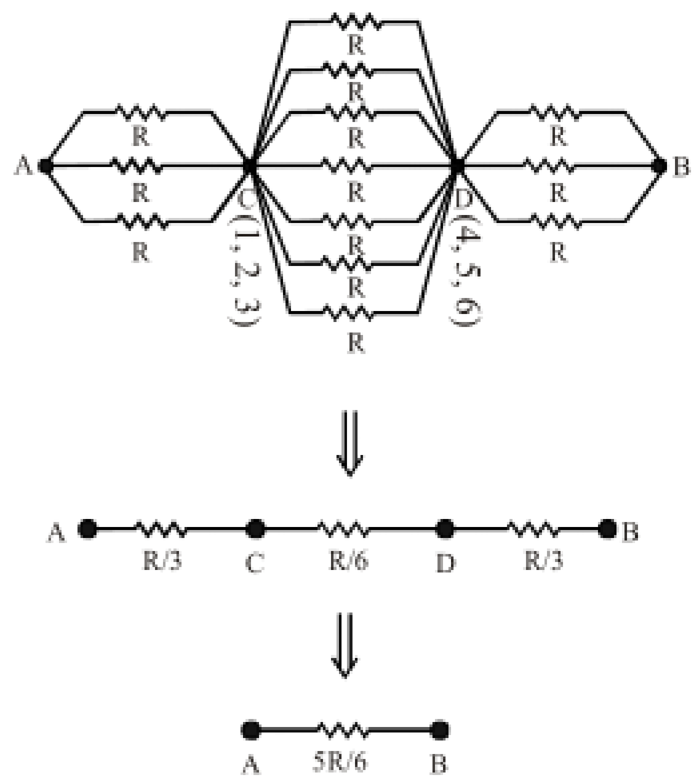
Illustration :

Find R_{AB} in the cubic network of twelve resistors each of resistance R.



Sol. *The network is symmetrical about the body diagonal AB. Since equal currents flow in the branches between A and (1, 2 and 3), the points 1, 2 and 3 are equipotential,. Similarly, the point 4,5,6 are equipotential. Let us now superimpose the points 1,2 and 3 at C and 4, 5, and 6 at D. You can now see that there are 3 resistors between A and C, six resistors between C and D and three resistors between D and B.*





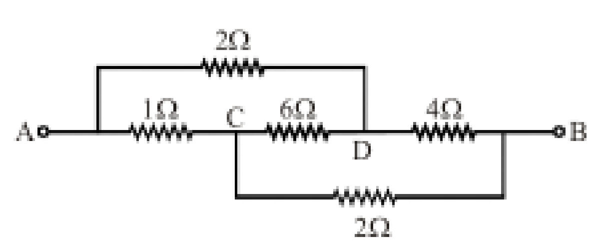
Then, $R_{AB} = R_{AC} + R_{CD} + R_{DB}$

$$= \frac{R}{3} + \frac{R}{6} + \frac{R}{3}$$

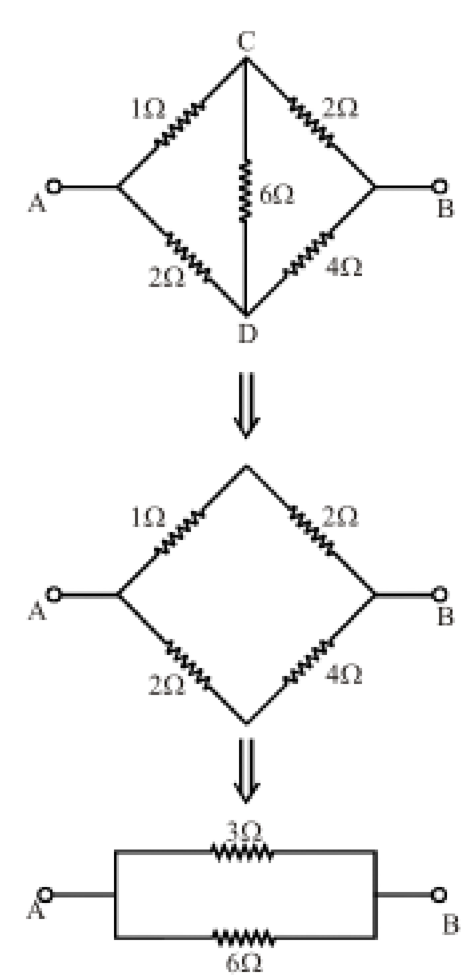
$$= \frac{5R}{6}$$

Illustration :

Find R_{AB} in the network



Sol. *The given network is a Wheatstone bridge as shown in the figure.*



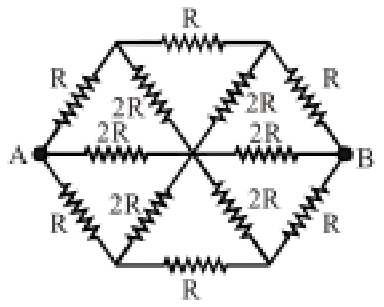
Since, $\frac{R_{AC}}{R_{AD}} = \frac{R_{CB}}{R_{DB}} = \frac{1}{2}$

The remove the branch CD to obtain a simple circuit.

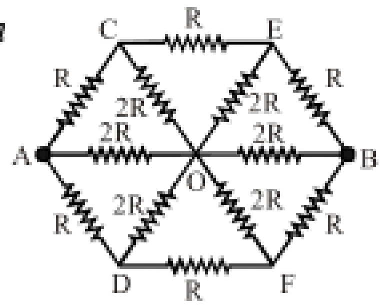
Hence $R_{AB} = \frac{3 \times 6}{3 + 6} = 2\Omega$

Illustration :

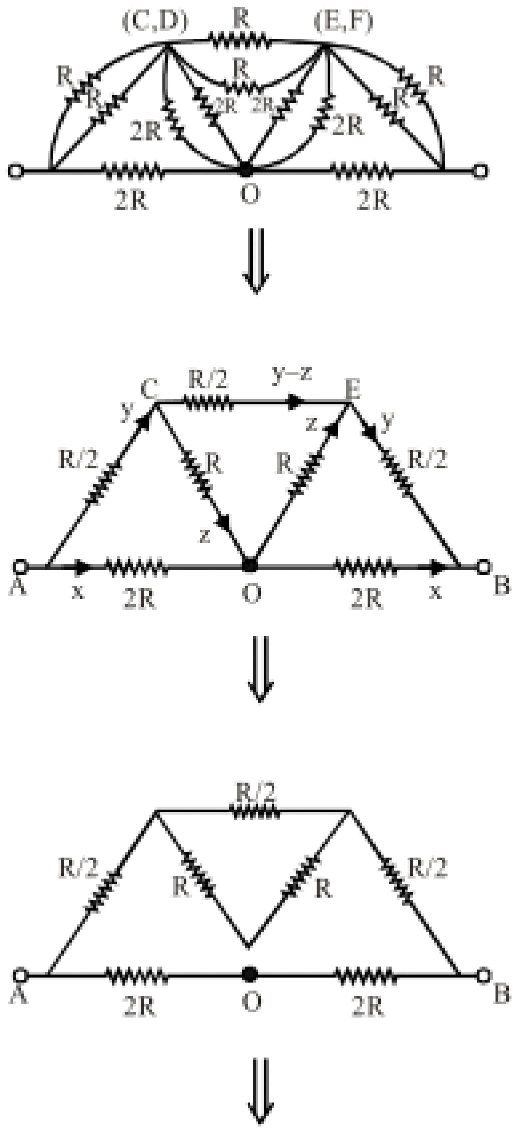
Find R_{AB}



Sol. *By inspection we can say that lower half and upper half of the given circuit is symmetrical about AB. Then, C and D are equipotential; E and F are equipotential. Superimposing D with C and F with E we have the following circuite. You can see that (AC and AD), (CE and DF), (EB and FB), (CO and DO) and (EO and FO) are superimposed.*



By current distribution following KCL, we understand that equal current passes through the branches CO and OE. Then, you can separate the branch COE, from AOB as shown in the figure and solve it by the processes of series and parallel combination.



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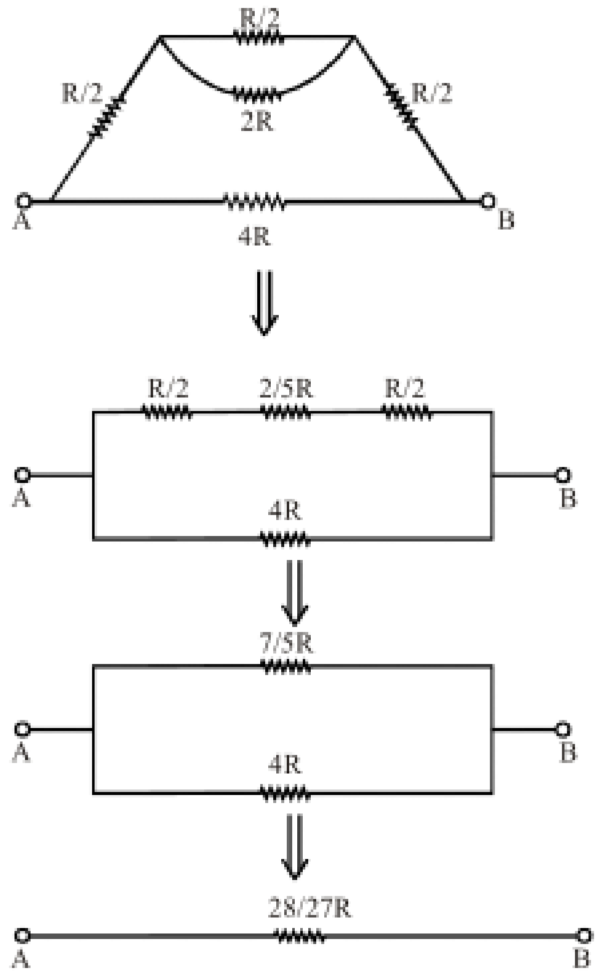
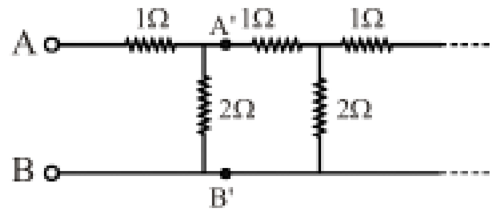


$$R_{AB}=\frac{\left(\frac{7}{5}R\right)(4R)}{\frac{7}{5}R+4R}=\frac{28}{27}R$$

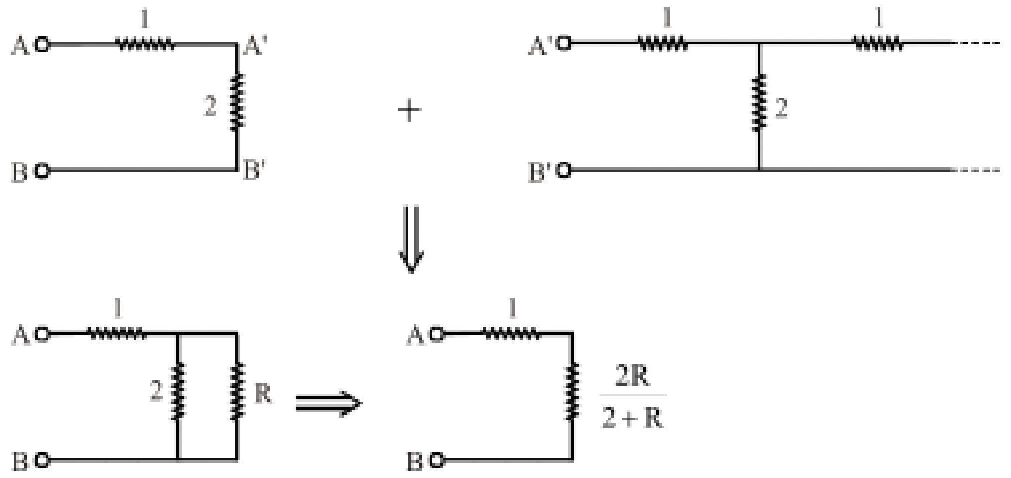
Illustration :

Find R_{AB}.



Sol. *Let* $R_{AB} = R$ *since infinite minus something is infinite, if you cut one well* $R_{A'B'} = R_{AB} = R$.

Hence $$\frac{(R_{A'B'})(2)}{2+R_{A'B'}}+1$$



Then, $R_{AB} = 1 + \dfrac{2R}{2+R}$

$$= \frac{2+3R}{2+R}$$

Putting $R_{AB} = R$, *we have*

$$R = \frac{2+3R}{2+R}$$

or, $R^2 - R - 2 = 0$

or, $R = \dfrac{1 \pm \sqrt{1+8}}{2}$

It gives $R = 2\Omega$

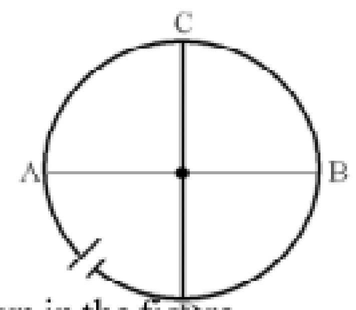
Practice Exercise

Q.1 A square pyramid is formed by joining 8 equal resistances R across the edges. The square base of the pyramid has the corner at A, B, C, D. The vertex is at M. Calculate the

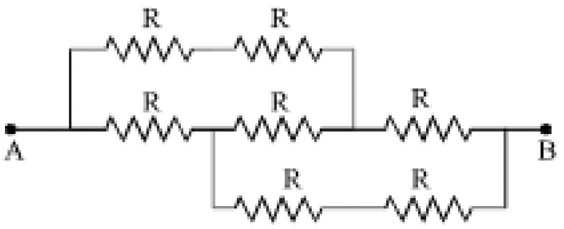
(a) current in the edge MC if an ideal cell of emf E is connected across the adjacent corners A and B.

(b) current in the edge MA if an ideal cell of emf E is connected across the opposite corners A and C.

Q.2 Calculate the equivalent resistance between the terminals of the cell shown in figure. The resistance of each quadrant is 1 ohm and the intersecting diameters have resistance 2 ohm each.



Q.3 Find the equivalent resistance of the configuration of equal valued resistors shown in the figure.



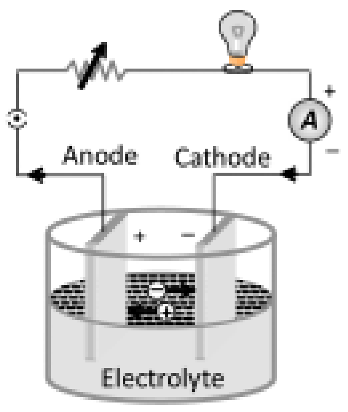
Q.4 Two conducting plates each of area A are separated by a distance d and they are parallel to each other. A conducting medium of varying conductivity fills the space between them. The conductivity varies linearly from σ and 2σ as you move from one plate to the other plate. Find the resistance of the medium between the conducting plates.

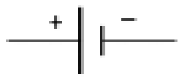
Answers

Q.1 (a) E/8R, (b) E/2R   Q.2 $\dfrac{15}{7}\Omega$ Q.3 7/5 R   Q.4 $\left[\dfrac{d}{\sigma A}\right]\ln(2)$

Electrice cell or Battery :

The device which converts chemical energy into electrical energy is known as electric cell. Cell is a source of constant emf but not constant current.





Symbol of cell

(1) **Emf of cell (E) :** The potential difference across the terminals of a cell when it is not supplying any current is called it's emf.

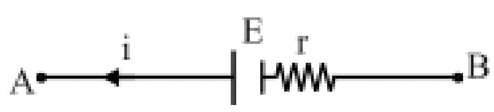
(2) **Potential difference (V) :** The voltage across the terminals of a cell when it is supplying current to external resistance is called potential difference or terminal voltage.

(3) Internal resistance (r) : In case of a cell the opposition of electrolyte to the flow of current through it is called internal resistance of the cell. The internal resistance of a cell depends on the distance between electrodes ($r \propto d$). area of electrodes [$r \propto (1/A)$] and nature, concentration ($r \propto C$) and temperture of elctrolyte [$r \propto (1/temp)$]

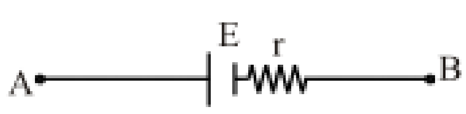
A cell is said to be ideal, if it has zero internal resistance.

Note : (i) During charging 

$$V_A - V_B = E + ir$$

During discharging 

$$V_A - V_B = E - ir$$

If no current is drawn 

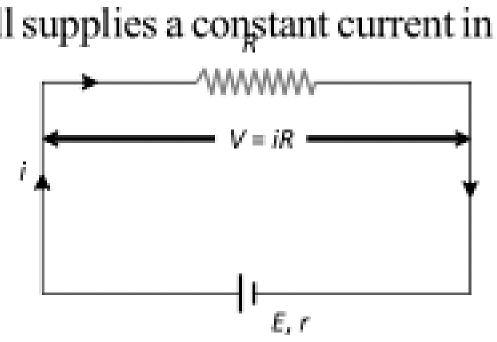
$$V_A - V_B = E$$

(ii) Inside a battery during discharging, charge is taken from -ve terminal (lower Potential) to +ve terminal (higher potential) by battery mechanism.

(iii) Work done by a battery during discharging = charge flown from +ve to -ve in outer circuit × emf of battery.

Cell in various Positions

(1) **Closed circuit :** Cell supplies a constant current in the circuit.



(i) Current given by the cell $i = \frac{E}{R+r}$

(ii) Potential difference across the resistance $V = iR$

(iii) Potential drop inside the cell $= ir$

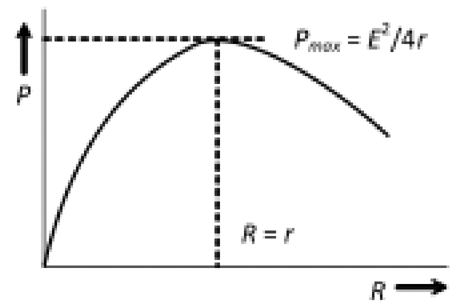
(iv) Equation of cell $E = V + ir\ (E > V)$

(v) Internal resistance of the cell $r = \left(\frac{E}{V} - 1\right) \cdot R$

(vi) Power dissipated in external resistance (load) $P = Vi = i^2R = \frac{V^2}{R} = \left(\frac{E}{R+r}\right)^2 .R$

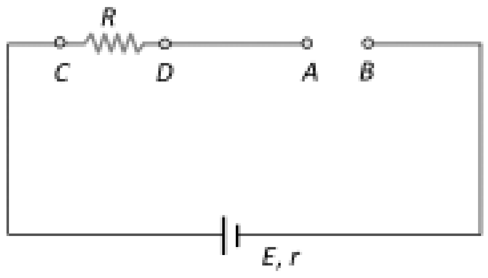
Power delivered will be maximum when $R = r$ so $P_{max} = \frac{E^2}{4r}$

This statement in generalised from is called "maximum power transfer theorem".



(vii) When the cell is being charged i.e. current is given to the cell then $E = V - ir$ and $E < V$.

(2) Open circuit : When no current is taken from the cell it is said to be in open circuit

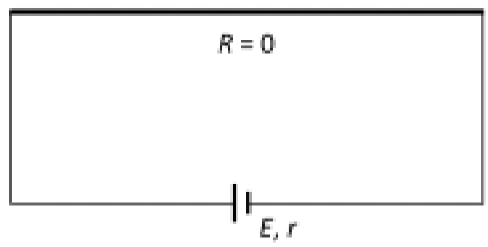


(i) Current through the circuit $i = 0$

(ii) Potential difference between A and B, $V_{AB} = E$

(iii) Potential difference between C and D, $V_{CD} = 0$

(3) **Short circuit :** If two terminals of cell are join together by a thick conducting wire



(i) Maximum current (called short circuit current) flows momentarily $i_{sc} = \frac{E}{r}$

(ii) Potential difference $V = 0$

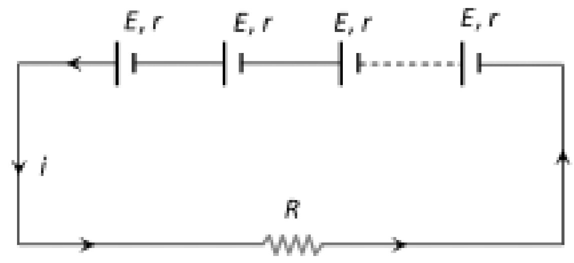
Grouping of cells

In series grouping of cell's their emf's are additive or subtractive while their internal resistances are always additive. If dissimilar plates of cells are connected together their emf's are added to each other while if their similar plates are connected together their emf's are subtractive.

$E_{eq} = E_1 + E_2$ $\qquad$ $E_{eq} = E_1 - E_2 \ (E_1 > E_2)$

$r_{eq} = r_1 + r_2$ $\qquad$ $r_{eq} = r_1 + r_2$

(1) **Series grouping :** In series grouping anode of one cell is connected to cathode of other cell and so on. If n identical cells are connected in series



(i) Equivalent emf of the combination $E_{eq} = nE$

(ii) Equivalent internal resistance $r_{eq} = nr$

(iii) Main current = Current from each cell $= i = \dfrac{nE}{R + nr}$

(iv) Potential difference across external resistance $V = iR$

(v) Potential difference across each cell $V' = \dfrac{V}{n}$

(vi) Power dissipated in the external circuit $= \left(\dfrac{nE}{R + nr}\right)^2 . R$

(vii) Condition for maximum power $R = nr$ and $P_{max} = n\left(\dfrac{E^2}{4r}\right)$

(viii) This type of combination is used when $nr << R$.

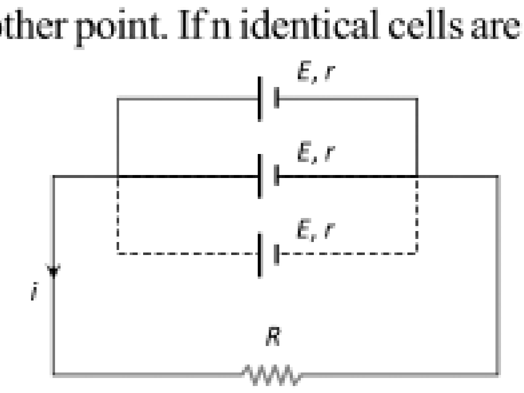
Note :

If Batteries are different

$E_{eq} = E_1 + E_2 + ...E_n$ if they are connected in same sense

$r_{eq} = r_1 + r_2 + ... + r_n$

(2) **Parallel grouping :** In parallel grouping all anodes are connected at one point and all cathode are connected together at other point. If n identical cells are connected in parallel



(i) Equivalent emf $E_{ef} = E$

(ii) Equivalent internal resistance $R_{eq} = r/n$

(iii) Main current $i = \dfrac{E}{R + r/n}$

(iv) potential difference across external reistance = p.d. across each cell $V = iR$

(v) Current from each cell $i' = \dfrac{i}{n}$

(vi) Power dissipated in the circuit $P = \left(\dfrac{E}{R + r/n}\right)^2 . R$

(vii) Condition for max. power is $R = r/n$ and $P_{max} = n\left(\dfrac{E^2}{4r}\right)$

(viii) This type of combination is used when $nr >> R$

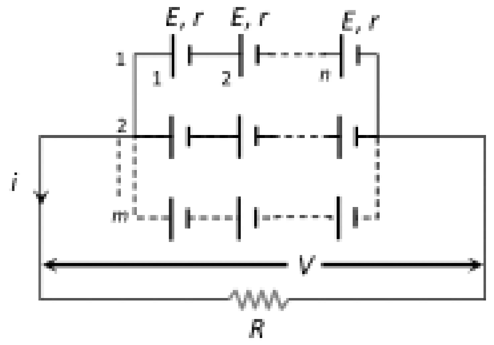
Note :

If Batteries are different

$$E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + ...}{\frac{1}{r_1} + \frac{1}{r_2} + ...}$$ If they are connected in same sense

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ...\frac{1}{r_n}$$

(3) **Mixed Grouping :** If n identical cell's are connected in a row and such m row's are connected in parallel as shown.



(i) Equivalent emf of the combination $E_{eq} = nE$

(ii) Equivalent internal resistance of the combination $r_{eq} = \dfrac{nr}{m}$

(iii) Main current flowing through the load $i = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{mnE}{mR + nr}$

(iv) Potential difference across load $V = iR$

(v) Potential difference across each cell $V' = \dfrac{V}{n}$

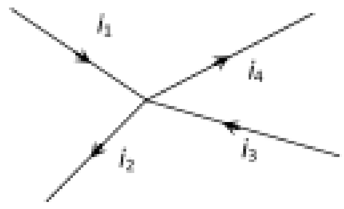
(vi) Current from each cell $i' = \dfrac{i}{n}$

(vii) Condition for maximum power $R = \frac{nr}{m}$ and $P_{max} = (mn)\frac{E^2}{4r}$

(viii) Total number of cell = mn

Kirchoff's Laws

Kirchoff's first law : This law is also known as junction rule or current law (KCL). According to it the algebraic sum of currents meeting at a junction is zero i.e.$\Sigma i = 0$



In a circuit, at any junction the sum of the currents entering the junction must equal the sum of the currents leaving the junction. $i_1 + i_3 = i_2 + i_4$

This law is simply a statement of ***"conservation of charge".***

Kirchoff's second law : This law is also known as loop rule or voltage law (KVL) and according to it "the algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero", i.e. $\Sigma V = 0$

This law represents ***"conservation of energy".***

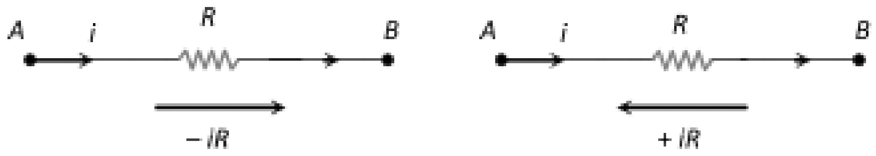
If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be (n - 1).

Note :

Sign convention for the application of Kirchoff's law :

For the application of Kirchoff's laws following sign convention are to be considered

(i) The change in potential in traversing a resistance in the direction of current is - iR while in the opposite direction +iR



(ii) The change in potential in traversing an emf source from negative to positive terminal is +E while in the opposite direction - E irrespective of the direction of current in the circuit.

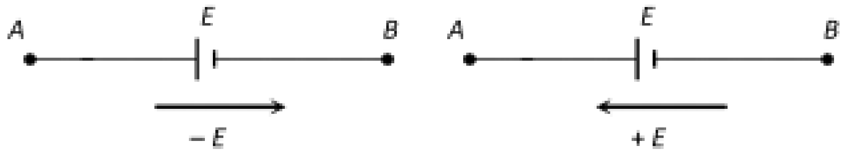


Illustration :

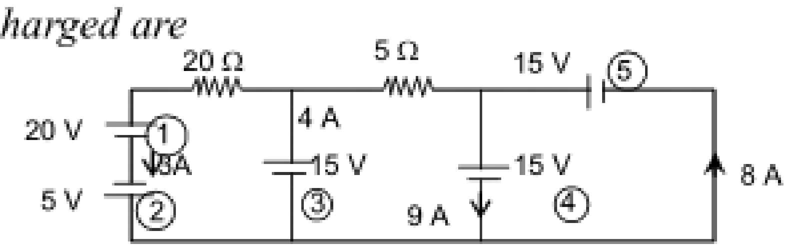
In the given network, the batteries getting charged are

(A) 1 and 3

(B) 1, 3 and 5

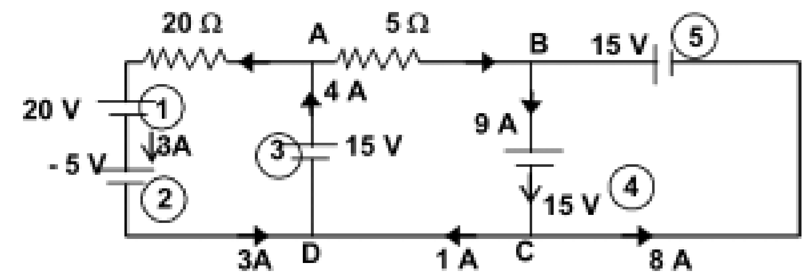
(C) 1 and 4

(D) 1, 2 and 5



Sol. *Applying Kirchhoff law at A, C and D, the direction of the currents in each branch will be as shown in the figure. It is clear from the figure that the batteries 1 and 4 are being charged.*

$\therefore$ *Hence (C) is correct*



Circuit solving Techniques

Case - (I)

Circuits having single Battery :

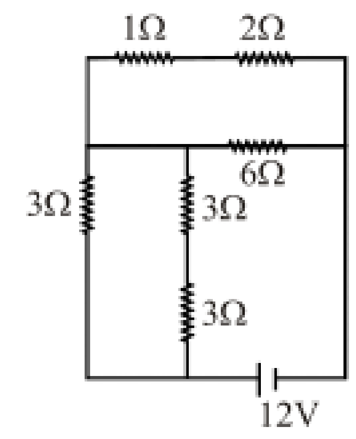
Step 1 - Remove Battery and find R_{eq} across the terminals of Battery

Step 2 -Total current through Battery $I_{total} = \dfrac{\text{Emf of battery}}{R_{eq}}$

Step 3 - Now divide the current as series- parallel combination.

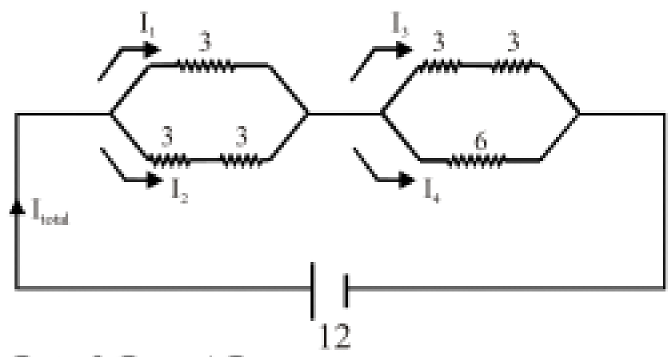
i.e. In series branches current remains same and in parallel current divides in inverse proportion of resistance.

Illustration :



Find current through each ressistance.

Sol.



$$R_{eq} = 2\Omega + 2\Omega = 4\Omega$$

$$I_{total} = \frac{12}{4} = 3 \ \ Amp$$

$I_1 + I_2 = 3\ Amp$ and $\dfrac{I_1}{I_2} = \dfrac{2}{1}$ hence $I_1 = 2A$

$I_2 = 1A$

Similarly $I_3 = 2A$

$I_4 = 1A$

Case - (II)

Circuits having many Batteries (can be reduced to single battery using Battery combination)

Step - 1

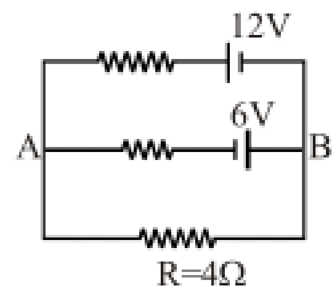
Apply Battery combination formula to reduced multiple batteries in single battery.

Step - 2

Solve as pervious case (I).

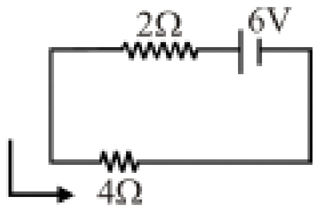
Illustration :

Find current through
$R = 4\ \Omega$
Also find $V_A - V_B$



Sol. *Applying parallel combination of Batteries,*

$$E_{eq} = \frac{\frac{+12}{3} - \frac{6}{6}}{\frac{1}{3} + \frac{1}{6}} = \frac{4-1}{1/2} = +6\text{V}$$



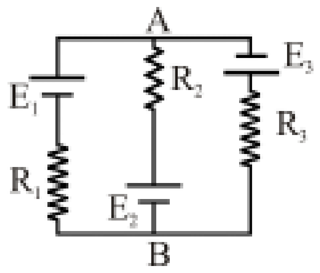
$V_{eq} = 2\Omega$

$$i = \frac{6}{2+4} = 1Amp$$

$V_A - V_B = iR = 2\ volt$

Illustration :

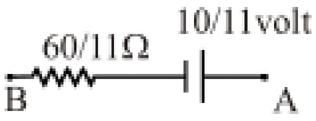
Find Potential difference $(V_A - V_B)$ *in the circuit*
Shown $E_1 = 1.5\ V, E_2 = 2.0\ V$
$E_3 = 2V, R_1 = 10\ \Omega,$
$R_2 = 20\ \Omega, R_3 = 30\ \Omega$



Sol. *We can reduced the wholde circuit into one Battery and on resistance.*

$$E_{eq} = \frac{\frac{1.5}{10} + \frac{2.0}{20} - \frac{2.5}{30}}{\frac{1}{10} + \frac{1}{20} + \frac{1}{30}} = \frac{\frac{1}{6}}{\frac{11}{60}} = \frac{10}{11}\text{volt}$$

$$R_{eq} = \frac{60}{11}\Omega$$



$$V_A - V_B = \frac{10}{11}volt$$

Case - (III)

Circuits having many Batteries.

(Using loop rule)

Step - I

Assume current in each Independent loop.

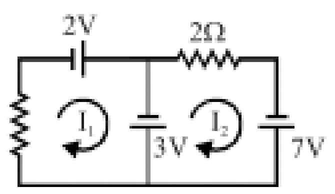
Step - II

Apply kirchoff's voltage law in each independent mesh (loop).

Illustration :

Find the current through each resistance

Sol. Let us assume currents I_1 and I_2 in the directions shown.



Using KVL, $-2-3+10\,I_1=0$ ...(i)

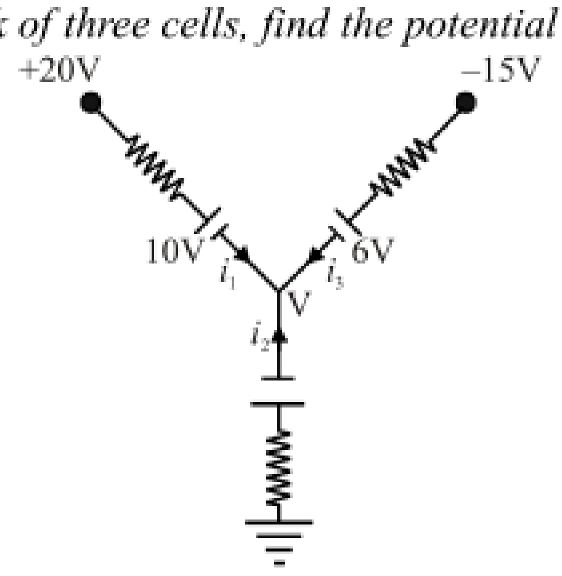
and $+3+2I_2-7=0$ ...(ii)

from (i) & (ii) $I_1=0.5$ A

$I_2=2$A

Illustration :

In the network of three cells, find the potential V of theri function.



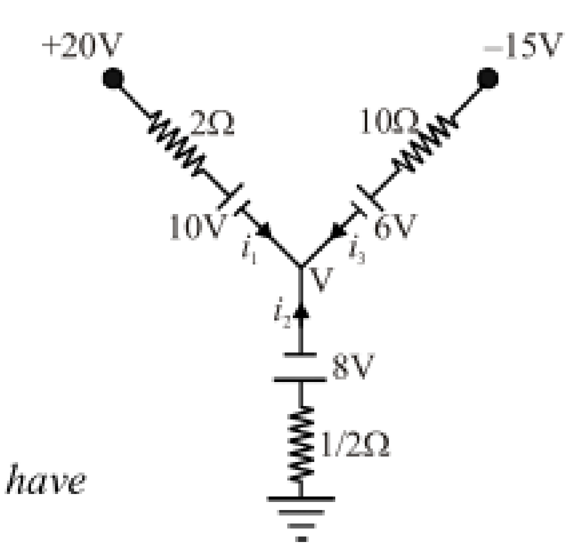
Sol. *Applying KCL for the individual branches,*

$$20-i_1(2)+10=V \quad ...(i)$$

$$0-i_2\left(\frac{1}{2}\right)-8=V \quad ...(ii)$$

$$-15-i_3(1)-6=V \quad ...(iii)$$

$$i_1+i_2+i_3=0 \quad ...(iv)$$



Putting i_1, i_2 and i_3 from eqs. (i), (ii) and (iii) in eq. (iv) we have

$$\frac{30-V}{2}+\frac{V+8}{-1/2}+\frac{V+21}{-1}=0$$

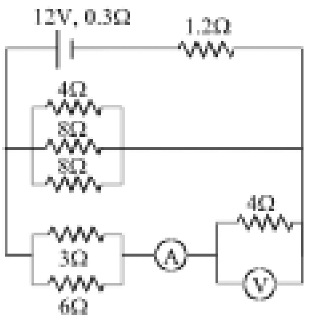
or, $V=-\dfrac{44}{7}$ *volt*

Practice Exercise

Q.1 For the circuit shown in the figure, find

(i) the equivalent external resistance of the circuit

(ii) the reading in ammeter (A) and voltmeter (V)

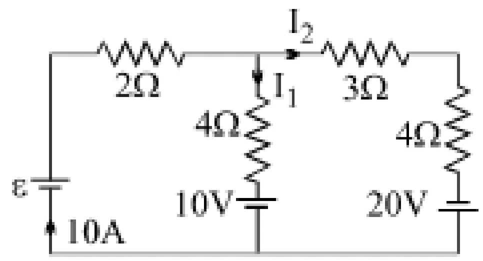


Q.2 For the circuit shown in the figure,

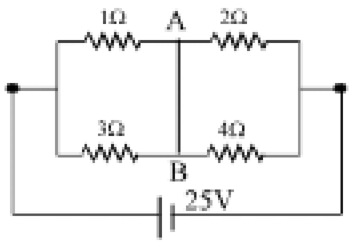
(i) find the currents I_1 and I_2, and the emf ε of the battery.

(ii) which batteries are supplying energy and at what rate to the circuit? Which batteries are absorbing energy and at what rate?

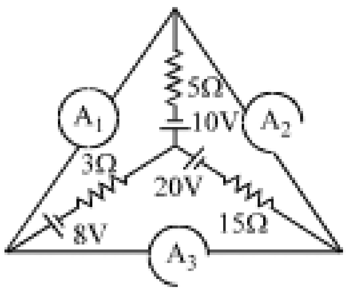
(iii) is total energy conserved? Justify.



Q.3 Find the current flowing through the segment AB of the circuit shown in figure.



Q.4 In the given circuit the ammeter A_1 and A_2 are ideal and the ammeter A_3 has a resistance of $1.9 \times 10^{-3}\Omega$. Find the readings of all the three meters.



Answers

Q.1 (i) 2.7 Ω, (ii) 1 A, 4 Volts

Q.2 (i) $\frac{40}{11}$ A, $\frac{70}{11}$ A (ii) $\frac{400}{11}$ W, $\frac{1400}{11}$ W, $\frac{4900}{11}$ W (iii) $\therefore$ energy is conserved

Q.3 1 A from A to B

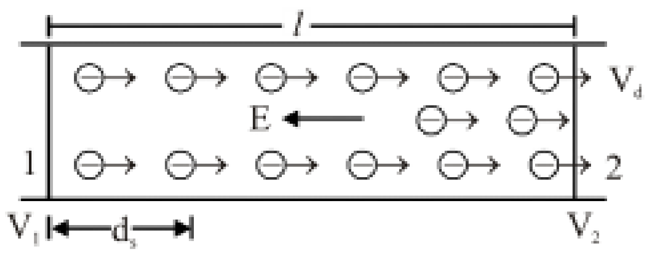
Q.4 $\frac{82}{27}$ A, $\frac{34}{27}$ A, 0

Energy conversion and electrical power

Input electrical energy

Let us consider a length l of the straight conductor of uniform cross-section A and conduction electron density n. Then the total number of conduction electrons in the considered segment is

$$N = nAl$$



The electric field does a positive work in pushing the conduction electron in opposite direction of the field $\vec{E}$

Since, the uniform electric field E pushes each electron with a constant drift speed v_d against the resistance (offered by the fixed atoms in the lattice), the total work done by the field during a time dt in shifting the electrons by a distance ds is

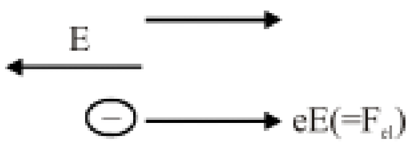
dW = (work done each electron) × (no. of electrons present in the segment)

$= (F_{el} \,.\, ds)\,(N)$

$= (eEds)\,(nAl) \qquad (\because\ F_{el} = eE)$

or, $\quad dW = (eEv_d\,dt)\,(nAl) \qquad (\because\ ds = v_d\,dt)$

$= (nev_dA)\,(El)\,dt$



Each electron experiences a force $\vec{F}_{el} = -e\,\vec{E}$, opposite to the applied field $\vec{E}$

By putting $nev_dA = i$ and $El = V_1 - V_2\,(=V)$, we have

Then, the total work done by the electric field on the assumed portion of theconductor during a time t is

$$W = \int_0^t iV\,dt$$

Where V = potential fifferece between the terminals 1 and 2 of the given portion ofthe conductor.

Input Electron Power

The electrical power of a voltage V while sending a current i can be give as rate of electrical work done.

or, $$P_{el} = \frac{dW}{dt}$$

or, $$P_{el} = iV$$

Heat Disslpated

As the electrons travel from lower potential V_1 to higher potenial V_2 they must lose their electrostatic potential energy or excess kinetic energy while accelerating in the applied electric field. This appears in the form of heat, light and sound etc., due to the resistance offered by the coductor. Hence, the amount of eat liberated in the considered portion of the conductor is

$$Q = \int_0^t iV\,dt$$

$$= \int_0^t i^2 R\,dt \qquad (\because V = iR)$$

$$= \int_0^t \frac{V^2}{R}\,dt \qquad \left(\because i = \frac{V}{R}\right)$$

Thermal Power

The rate of heat is liberated, that is power loss in the resistaor is called Ohmic heating, or Joule heating or Copper-loss or thermal power or $i^2 R$ loss which can be given as

$$\frac{dQ}{dt}(= P_R) = iV$$

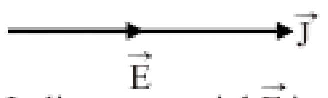
$$= i^2 R$$

$$= \frac{V^2}{R}$$

We can use thermal energy in room heater, toaster, electric iron etc. and in other electric circuits (power distribution and transmission) power lost cannot be used.

Joule - Lenz Law

The above expression is called mocroscopic form of Joule-Lenz law.



In linear material $\vec{E}$ is parallel to $\vec{J}$ at any point

Substing $i = JA$, $R = \rho\dfrac{l}{A}$ in the formula $\dfrac{dQ}{dt} = i^2 R$,

we have $\dfrac{dQ}{dt} = (JA)^2\left(\rho\dfrac{l}{A}\right)$

$= \rho J^2 (Al)$, where $Al = V$ (volume of the segment)

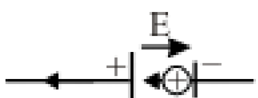
Then, the power loss (rate of heat generated) per unit volume is

$$\frac{dQ}{dt}\Big/V = Q_V = \rho J^2 = J.E = E^2/\rho \qquad (\because J = \rho E)$$

This expression is valid for any point of the conductor. Hence, we call i "point (or differenial) from" of Joule-Lenz law.

Micro-interportation of Heat Dissipation

The emf (battery) sets on electric field which pushes the electrons in the conductor. As a result, the electrons gain kinetic energy or loses electrostatic potential energy. The gain in K.E. is lost due to their repeated collision with the site atoms of the lattice. the exchange in kinetic energy and momenta of the electrons cause the lattics atoms to vibrate with more amplitudes. The vibrating metallic kernels of the lattice radiate electromagnetic energy in the form of heat, light etc., obeying the principle of electromagnetic radiation.



A +ve charge is brought from –ve to +ve electrode of the battery by its emf against the electric field $\vec{E}$ inside the battery

The excess K.E. of the elctrons received from the electric field (ultimately from the battery) is spent in exciting the atoms of the lattice which in turn radiate electromagnetic energy in the form of heat and light.

Power of an EMF

A pattery is ultimately respeonsible for setting electric field inside and outside of the conducting wires. Hence, the battery does work in circulating the charges. The rate of work done by a seat of emf (battery) to establish a current is defined as electrical power of a battery.

$$P_{el} = \frac{dW_b}{dt}$$

As discussed earlier, the work is done by a battery to push the conventinal +ve charge dq from its – ve terminal to +ve termial against the electrostatic force can be given as

$$dW_b = \varepsilon\, dq$$

Then, the power delivered by the batter in setting a current i is

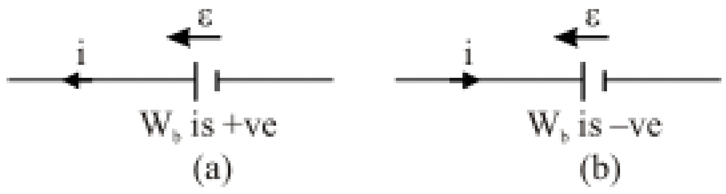
$$P_{el} = \frac{dW_b}{dt}$$

$$= \varepsilon \frac{dq}{dt}$$

$$= \varepsilon i$$

or, $$P_{el} = \varepsilon i$$

If current (or dq) flows in the direction of the emf, work done and power deliverd by the battery is +ve and vice-versa.



Illustraion :

Two bulb's of powers P_1 and P_2 are connected in series. If the supply voltage is equal to the rated voltage, find the power of the combination.

Sol. *Let their resistance be R_1 and R_2 respectively.*

For a rated voltage V, the power of the combination is

$$P = \frac{V^2}{R_1 + R_2} \quad (\because \text{ the resistance are connected in series})$$

Putting $R_1 = \frac{V^2}{P_1}$ *and* $R_2 = \frac{V^2}{P_2}$ *we obtain*

$$P = \frac{P_1 P_2}{P_1 + P_2}$$

Illustration :

A 1000 watt heater coil can be cut into two parts and when each part is used in the rated supply voltage, it gives more power as $P \propto \frac{1}{R}$, but we do not recommed this , explain.

Sol. *Since, the power dissipate in the coil is*

$$P = \frac{V^2}{R}$$

and R decreases by two fold if we cut it into two equal halves (say), power dissipation will be doubled. The heat liberation will be doubled which in turn, damages the coil by heating it or reduces its life.

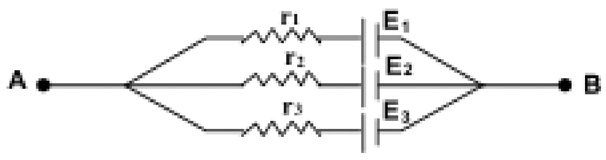
Illustration :

In the circuit shown in figure,

$E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V and $r_1 = r_2 = r_3 = 1$ ohm.

(a) Find the potential difference between the points A and B and the currents through each branch.

(b) If r_2 is short circuited and the point A is connected to point B through a resistance R, find the currents through E_1, E_2, E_3 and the resistor R.



Sol. *(a) Applying Kirchoff's loop law to mesh PLMQP and PLMQONP in the figure shown below, we have*

Shown below, we have

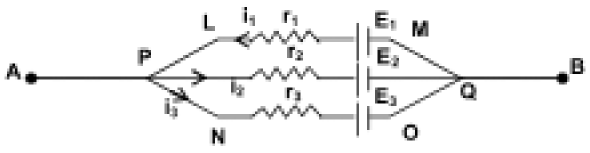
$i_1 r_1 + i_2 r_2 = E_1 - E_2$ or $i_1 + i_2 = 1$ . . . (i)

$i_1 r_1 + i_3 r_3 = E_1 - E_3$ or $i_1 + i_3 = 2$ . . . (ii)

At P, $i_2 + i_3 = i_1$ . . . (iii)

On solving (i), (ii) and (iii)

$i_1 = 1$ amp, $i_2 = 0$ amp, $i_3 = 1$ amp.



Since no current is drown along the brance AP

$\therefore \quad V_{AB} = V_{PQ}$

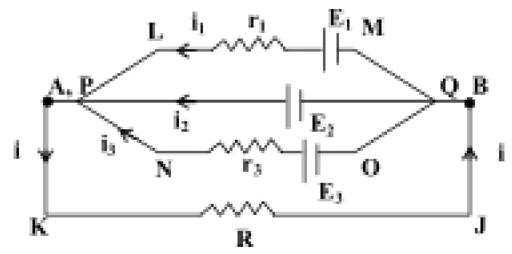
Potential difference across PQ,

$V_{PQ} = E_1 - r_1 r_1 = 2$ volt

(b) *The figure shows the circuit when point A is connected to point B and r_2 is short-circuited.*

Applying Kirchoff's junction rule at P, we get

$i = i_1 + i_2 + i_3$ . . (iv)



Applying Kirchoff's law to mesh ABMLA

$i_1 r_1 = E_1 - E_2$ or $i_1 = 1$ amp.

Applying Kirchoff's law to mesh ANOQML

$i_1 r_1 - i_3 r_3 = E_1 - E_3$ or $i_1 - i_3 = 2$ . . . (v)

From above equations

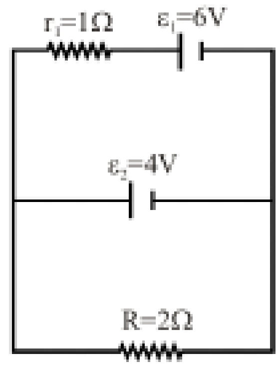
$i_1 = 1$ amp, $i_2 = 2$ amp, $i_3 = 1$ amp.

(direction of current is opposite)

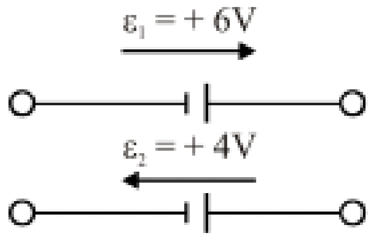
So, current through resistor R will be $I = I_1 + I_2 + I_3 = 2$ amp.

Illustration :

Two cells are connected to an external load of resistance R = 2 W. Find the current in the resistor.



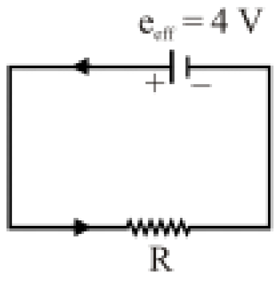
Sol. $\varepsilon_{eff} = \dfrac{\dfrac{\varepsilon_1}{r_1} + \dfrac{\varepsilon_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}}$



$= \dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$

$= \dfrac{6(0) + (-4)(1)}{1}$

$= -4\ V$



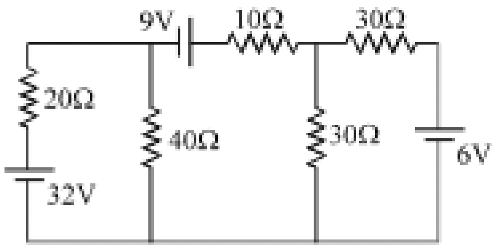
$r_{eff} = \dfrac{r_1 r_2}{r_1 + r_2} = \dfrac{(0)(1)}{0+1} = 0$

$i = \dfrac{e_{eff}}{R} = \dfrac{4}{2} = 2A$

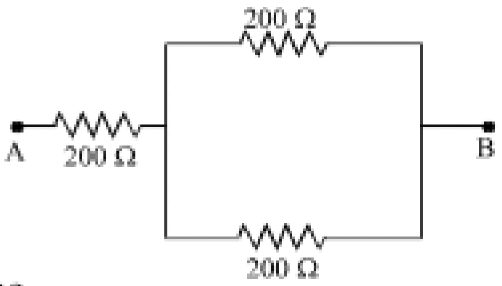
Practice Exercise

Q.1 In a house there are 3 lamps of 40W each, 8 lamps of 60W each, a radio of 40W and a TV of 160W. The lamps are in operation, on an average, for 2hrs a day, the radio for 4hrs a say and the TV for an hr a day. On Sundays an electric iron of 750W is used for an hour and the TV for an extra 3 hrs. Calculate the electricity bill for the month of February of a leap year at the rate of 45 paise per unit. The first Sunday falls on 3rd February.

Q.2 Obtain the power imparted to the 10 Ω resistor in the shown network.



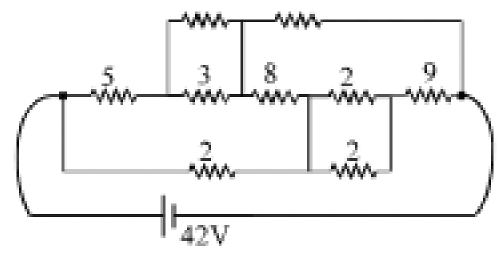
Q.3 Three 200 Ω resistors are connected as shown in figure. The maximum power that can be dissipated in any one of the resistor is 50 W. Find:



(i) the maximum voltage that can be applied to the terminals A and B.

(ii) the total power dissipated in the circuit for maximum voltage across the terminals A and B.

Q.4 Find the power dissipated in 5Ω and 8Ω resistors.



Answers

Q.1	22.05	Q.2	5.1 W	Q.3	(a) 150 V, (b) 75 W	Q.4	5W in 5Ω, 0 in 8Ω

Different Measuring Instruments

(1) **Galvanometer :**

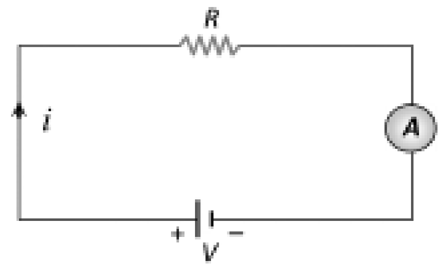
It is an instrument used to detect small current passing through it by showing deflection. Galvanometers are of different types e.g. moving coil galvanometer, moving magnet galvanometer, hot wire galvanometer. In dc circuit usually moving coil galvanometer are used.

(i) It's symbol : ——(G)——; where G is the total internal resistance of the galvanometer.

(ii) Full scale deflection current : The current required for full scale deflection in a galvanometer is called full scale deflection current and is represented by i_g.

(iii) Shunt : The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer is known as shunt.

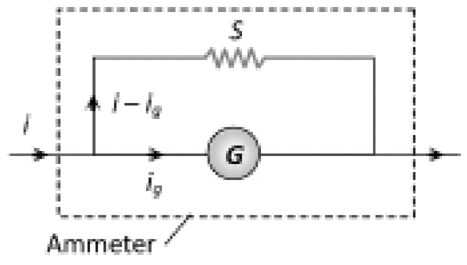
(2) **Ammeter :**



(i) The reading of an ammeter is always lesser than actual current in the circuit.

(ii) Smaller the resistance of an ammeter more accurate will be its reading. An ammeter is said to be ideal if its resistance r is zero.

(iii) **Conversion of galvanometer into ammeter :** A galvanometer may be converted into an ammeter by connecting a low resistance (called shunt S) in parallel to the galvanometer G as shown in figure.



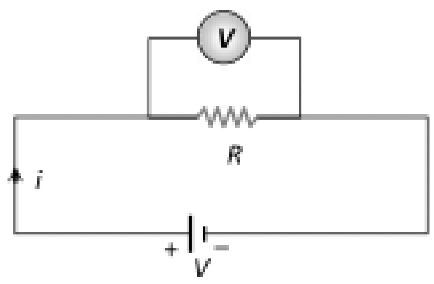
(a) Equivalent resistance of the combination $= \dfrac{GS}{G+S}$

(b) G and S are parallel to each other hence both will have equal potential difference i.e. $i_g G = (i - i_g)S$;

which gives Required shunt $S = \dfrac{i_g}{(i - i_g)} G$

(c) To pass nth part of main current (i.e. $i_g = \dfrac{i}{n}$) through the galvanometer, required shunt $S = \dfrac{G}{(n-1)}$

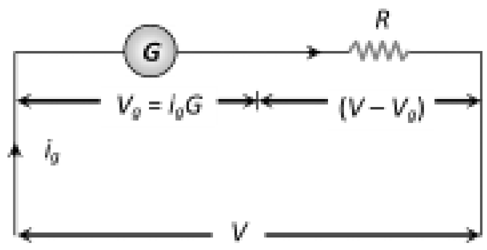
(3) **Voltmeter :** It is a device used to measure potential difference and is always put in parallel with the 'circuit element' across which potential difference is to be measured.



(i) The reading of a voltmeter is always lesser than true value.

(ii) Greater the resistance of voltmeter, more accurate will be its reading. A voltmeter is said to be ideal if its resistance is infinite, i.e., it draws no current from the circuit element for its operation.

(iii) **Conversion of galvanometer into voltmeter :** A galvanometer may be converted into a voltmeter by connecting a large resistance R in series with the galvanometer as shown in the figure.



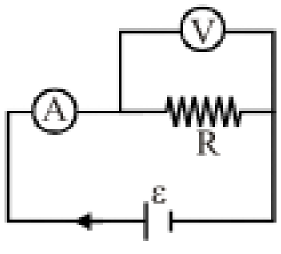
(a) Equivalent resistance of the combination $= G + R$

(b) According to ohm's law $v = i_g (G + R)$; which gives required series resistance $R = \dfrac{V}{i_g} - G = \left(\dfrac{V}{V_g} - 1\right)G$

(c) If n^{th} part of applied voltage appeared across galvanometer (i.e. $v_g = \dfrac{V}{n}$) then required series resistance $R = (n-1)G$.

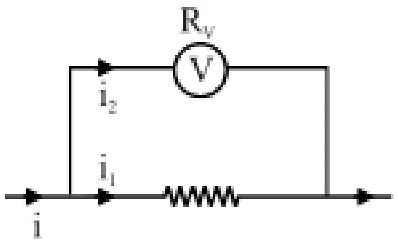
Illustration :

To measure the value of the resistance R, we have connected the voltmeter and ammeter as shown in the figure. Can the ratio of voltmeter and ammeter reading $\dfrac{V}{i}$ *given the correct value of R ? Discuss.*



Sol. *Let* $\frac{\text{Voltmeter reading}}{\text{Ammeter reading}} = R_m$

Where, R_m *= meter reading of resistance*



or, $R_m = \frac{V}{i}$ ...(i)

Since, R_V *and R are parallel,*

$i_2 R_V = i_1 R$ ...(ii)

According to KCL (1st law),

$i = i_1 + i_2$ ...(iii)

Using these three equation, we have

$$\frac{1}{R} = \frac{1}{R_m} - \frac{1}{R_V}$$

If $R_V \to \infty$, $R \to R_m$

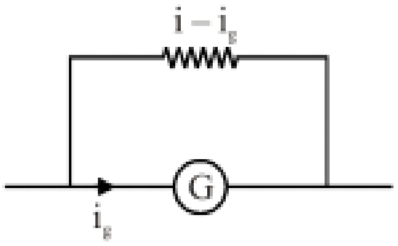
Hence, the ration of voltmeter and ammeter reading cannot give the exact volue of the resistance R.

Illustration :

The deflection of a moving coil galvanometer falls from 60 divisions to 12 divisions when a shunt of 12 Ω is connected. What is the resistance of the galvanometer ?

Sol. *The current i in the galvanometer is directly proportional to the angle of deflection* $(i \propto \theta)$

Then, $\frac{i_g}{i} = \frac{12}{60} = \frac{1}{5}$



or, $i_g = \frac{i}{5}$ ...(i)

For shunted galvanometer,

$(i - i_g)\, S = i_g\, G$

$G = (i - i_g)\frac{S}{i_g}$ ...(ii)

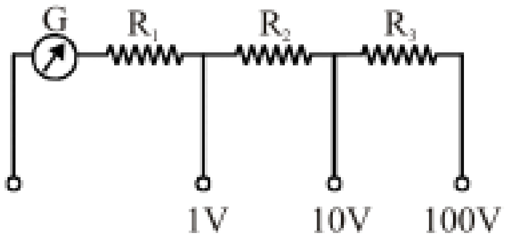
Putting i_g *from eq. (i) in eq. (ii) and S = 12 ohm.*

$G = 48$ *ohm*

Illustration :

The galvanometer G has internal resistance G = 50 Ω and full scale deflection occurs at i = 1 mA. Find the series resistors R_1, R_2 *and* R_3 *needed to use the arrangement as a voltmeter with different rages as shown in the figure.*

Sol. *For the range of* $V_1 = 1$ *volt,*



$$i_g = \frac{V_1}{G + R_1}$$

or, $10^{-3} = \frac{1}{50 + R_1}$

or, $R_1 = 950$ *ohm*

For the range of $V_2 = 10$ *volt*

$$i_g = \frac{V_2}{G+R_1+R_2}$$

or, $$10^{-3} = \frac{10}{50+950+R_2}$$

or, $R_2 = 9\times10^3$ *ohm*

For the range of $V_3 = 100$ *volt*

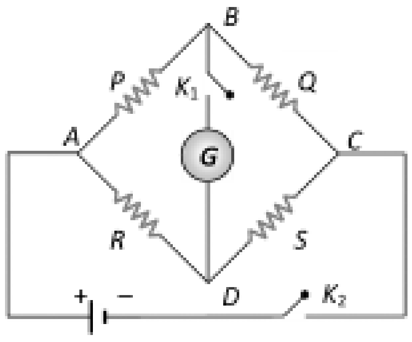
$$i_g = \frac{V_3}{G+R_1+R_2+R_3}$$

or, $$10^{-3} = \frac{100}{50+950+9000+R_3}$$

or, $R_3 = 90 \times 10^3$ *ohm*

Wheatstone bridge :

Wheatstone bridge is an arrangement of four resistance which can be used to measure one of them in terms of rest. Here arms AB and BC are called ratio arm and arms AC and BD are called conjugate arms



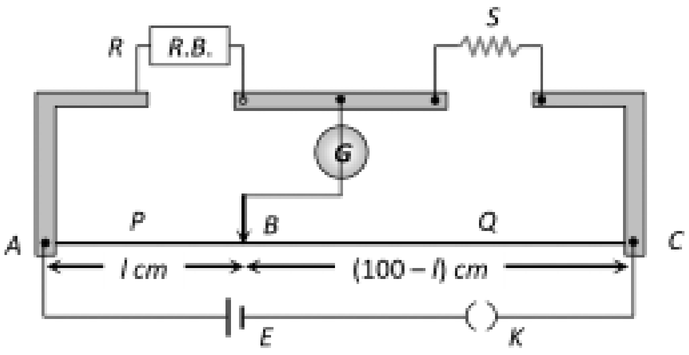
(i) Balanced bridge : The bridge is said to be balanced when deflection in galvanometer is zero i.e. no current flows through the galvanometer or in other words $V_B = V_D$. In the balanced condition $\frac{P}{Q} = \frac{R}{S}$, on mutually changing the position of cell and galvanometer this condition will not change.

(ii) Unbalanced bridge : If the bridge is not balanced current will flow from D to B if VD > VB i.e. $(V_A - V_D) < (V_A - V_B)$ which gives PS > RQ.

Applications of wheatstone bridge :

Meter bridge, post office box and Carey Foster bridge are instruments based on the principle of wheatstone bridge and are used to measure unknown resistance.

(4) Meter bridge : In case of meter bridge, the resistance wire AC is 100 cm long. Varying the position of tapping point B, bridge is balanced.



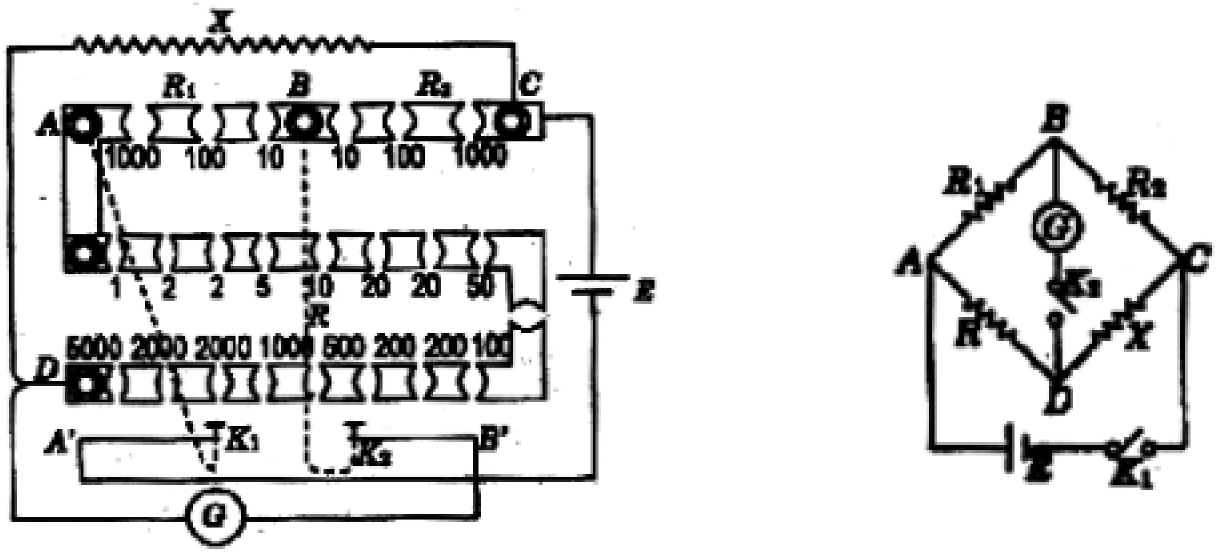
If in balanced position of bridge AB = ℓ , BC (100 - l)

so that $\dfrac{Q}{P} = \dfrac{(100-\ell)}{\ell}$ Also $\dfrac{P}{Q} = \dfrac{R}{S} \Rightarrow S = \dfrac{(100-\ell)}{\ell}R$

Note that :

* The balance-point is obtained by trial and error–not by scraping the jockey along the wire.
* The value of R in the resistance box should be chosen so that the balance point comes near to the center of the wire, i.e. from 40 cm to 60 cm from the end A.
* If the length either ℓ_1 or ℓ_2 is small, then the resistance of its end connections AA' and BB' will not be negligible in comparison with R_{AB} or R_{CB}. Then, the equation will not valid.
* The end resistance error can be minimized by interchanging R and X, and balancing again. The average values of ℓ_1 and ℓ_2 are taken to calculate the value of X.
* Since galvanometer is a sensitive instrument, therefore, a high resistance is sometimes connected in series with it until a near balance point is obtained. Then the high resistor is shunted or removed and the final balance point is obtained.
* The lowest resistance that can be measured with this bridge is about 1Ω.

(5) The post office Box



(a) The post office Bax (b)

It is a compact form of the Wheatstone bridge. It consists of compact resistance so arranged that different desired values of resistances may be selected in the three arms of Wheatstone bridge, as shown in figure.

Each of the arms AB and BC contains three resistances of 10, 10^2 and 10^3 Ω, respectively. These are called the ratio arms. Using these resistances the ratio $\dfrac{R_2}{R_1}$ can b made to have any of the following values : 100 : 1, 10 : 1, 1 : 1, 1 : 10 or 1 : 100.

The arm AD is a complete resistance box containing resistances from 1 to 5000 Ω. The tap keys K_2 and K_2 are are also provided in the post office box. The key K_1 is internally connected to the point A and the key K_2 to the point B (as shown by dotted line in the figure). The unknown resistance X is connected between C and D, the battery between C and the key K_1 and the galvanometer between D and the key K_2. The circuit shown in figure (A) is exactly the same as that of the Wheatstone bridge shown in figure. Hence, the value of the unknown resistance is given by

$$X = R\left(\frac{R_2}{R_1}\right)$$

Note that :

* The accuracy of the post office box depends on the choice of ratio arm $\frac{R_2}{R_1}$.
* If $R_2 : R_1$ is 1 : 1, then the value of the unknown resistance is obtained within $\pm 1\Omega$.
* If the ratio $R_2 : R_1$ is selected as 1 : 10, then the unknown resistance $X = R\left(\frac{1}{10}\right)$ is accurately measured upto $\pm 0.1\Omega$.
* If the ratio $R_2 : R_1$ is adjusted to 1 : 100, then the value of unknown resistance $X = R\left(\frac{1}{100}\right)$ is obtained to an accuracy of $\pm 0.01\ \Omega$.

Illustration :

The value of an unknown resistance is obtained by using a post office box. Two consecutive readings of R are observed at which the galvanometer deflects in the opposite directions for three different value of R_1. These two values are recorded under the column-I and II in the following observation table.

S.No.	R_1 (Ω)	R_2 (Ω)	R lies in-between		X = R (R_2/R_1)	
			I (Ω)	II (Ω)	I (Ω)	II (Ω)
1	10	10	16	17		
2	100	10	163	164		
3	1000	10	1638	1639		

Determine the value of the unknown resistance.

Sol. *The observation table may be complete as follows :*

S.No.	R_1 (Ω)	R_2 (Ω)	R lies in-between		X = R (R_2/R_1)	
			I (Ω)	II (Ω)	I (Ω)	II (Ω)
1	10	10	16	17	16.0	17.0
2	100	10	163	164	16.3	16.4
3	1000	10	1638	1639	16.38	16.39

The value of the unknown resistance lies in-between 16.38 Ω and 16.39 Ω.
The unknown value may be the average of the two

i.e. $X = \frac{16.38 + 16.39}{2}$

or $X = 16.385\ \Omega$

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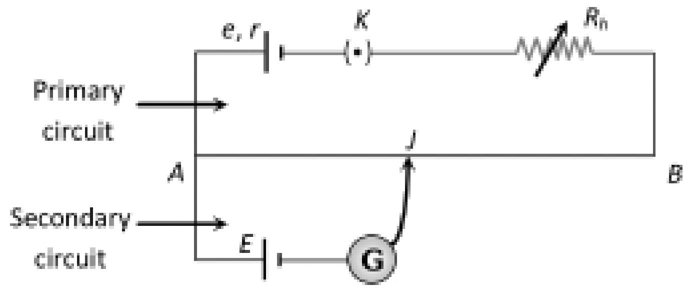
(6) Potentiometer

Potentiometer is a device mainly used to measure emf of a given cell and to compare emf's of cells. It is also used to measure internal resistance of a given cell.

Circuit diagram :

Potentiometer consists of a long resistive wire AB of length L (about 6m to 10 m long) made up of mangnine or constantan and a battery of known voltage e and internal resistance r called supplier battery or driver cell. Connection of these two forms primary circuit.

One terminal of another cell (whose emf E is to be measured) is connected at one end of the main circuit and the other terminal at any point on the resistive wire through a galvanometer G. This forms the secondary circuit. Other details are as follows



J = Jockey

K = Key

R = Resistance of potentiometer wire,

ρ = Specific resistance of potentiometer wire.

R_h = Variable resistance which controls the current through the wire AB

(i) The specific resistance (ρ) of potentiometer wire must be high but its temperature coefficient of resistance (α) must be low.

(ii) All higher potential points (terminals) of primary and secondary circuits must be connected together at point A and all lower potential points must be connected to point B or jockey.

(iii) The value of known potential difference must be greater than the value of unknown potential difference to be measured.

(iv) The potential gradient must remain constant. For this the current in the primary circuit must remain constant and the jockey must not be slided in contact with the wire.

(v) The diameter of potentiometer wire must be uniform everywhere.

(vi) Potential gradient (x) : Potential difference (or fall in potential) per unit length of wire is called potential gradient i.e. $x = \frac{V}{L}\frac{\text{volt}}{\text{m}}$ where $V = iR = \left(\frac{e}{R + R_h + r}\right).R$.

So $x = \frac{V}{L} = \frac{iR}{L} = \frac{i\rho}{A} = \frac{e}{(R + R_h + r)}.\frac{R}{L}$

Potential gradient directly depends upon

(a) The resistance per unit length (R/L) of potentiometer wire.

(b) The radius of potentiometer wire (i.e. Area of cross-section)

(c) The specific resistance of the material of potentiometer wire (i.e.ρ)

(d) The current flowing through potentiometer wire (i)

(ii) potential gradient indirectly depends upon

(a) The emf of battery in the primary circuit (i.e. e)

(b) The resistance of rheostat in the primary circuit (i.e. R_h)

Working :

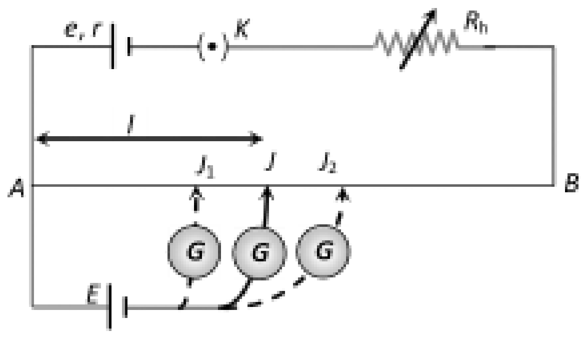
Suppose "jockey" is made to touch a point J on wire then potential difference between A and J will be

$V = xl$

At this length (l) two potential difference are obtained

(i) V due to battery e and

(ii) E due to unknown cell



If V > E then current will flow in galvanometer circuit in one direction

If V < E then current will flow in galvanometer circuit in opposite direction

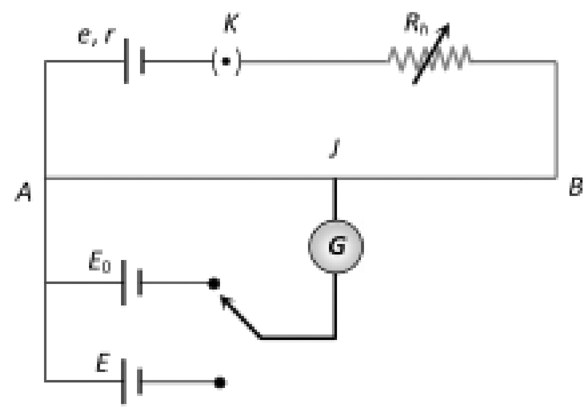
If V = E then no current will flow in galvanometer circuit this condition to known as null deflection position, length l is known as balancing length.

In balanced condition $E = xl$

or $$E = xl = \frac{V}{L}l = \frac{iR}{L}l = \left(\frac{e}{R + R_h + r}\right).\frac{R}{L} \times l$$

If V is constant then $L \propto l \Rightarrow \frac{x_1}{x_2} = \frac{L_1}{L_2} = \frac{l_1}{l_2}$

(vii) **Standardization of potentiometer :** The process of determining potential gradient experimentally is known as standardization of potentiometer.



Let the balancing length for the standard emf E_0 is l_0 then by the principle of potentiometer

$$E_0 = x l_0 \Rightarrow x = \frac{E_0}{l_0}$$

(viii) **Sensitivity of potentiometer :** A potentiometer is said to be more sensitive, if it measures a small potential difference more accurately.

(a) The sensitivity of potentiometer is assessed by its potential gradient. The sensitivity is inversely proportional to the potential gradient.

(b) In order to increase the sensitivity of potentiometer

(c) The resistance in primary circuit will have to be decreased.

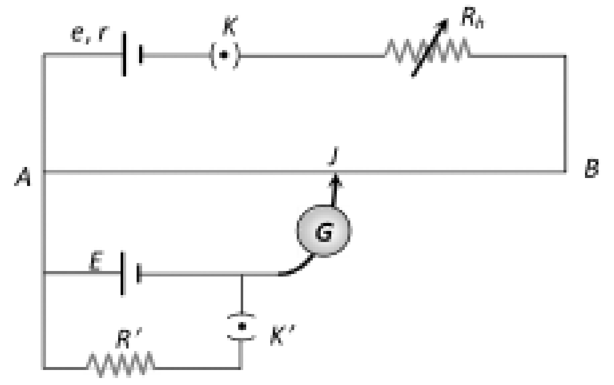
(d) The length of potentiometer wire will have to be increased so that the length may be measured more accuracy.

Difference between voltmeter and potentiometer

Voltmeter	Potentiometer
It's resistance is high but finite	Its resistance is infinite
It draws some current from source of emf	It does not draw any current from the source of unknown emf
The potential difference measured by it is lesser than the actual potential difference	The potential difference measured by it is equal to actual potential difference
Its sensitivity is low	Its sensitivity is high
It is a versatile instrument	It measures only emf or potential difference
It is based on deflection method	It is based on zero deflection method

Application of Potentiometer

(1) **To determine the internal resistance of a primary cell**



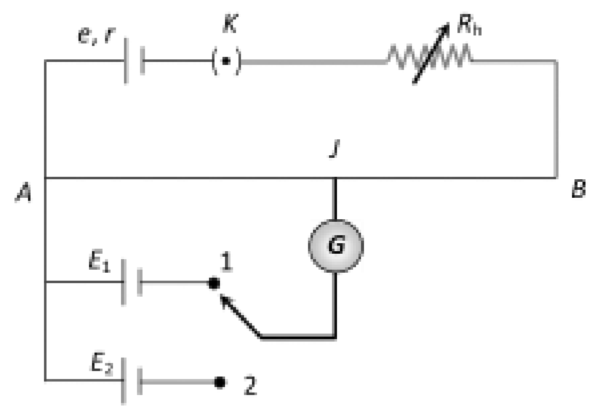
(a) Initially in secondary circuit key K' remains open and balancing length (l_1) is obtained. Since cell E is in open circuit so it's emf balances on length l_1 i.e. $E = x l_1$ .... (i)

(b) Now key K is closed so cell E comes in closed circuit. If the process of balancing repeated again then potential difference V balances on length l_2 i.e. $V = x l_2$ .... (ii)

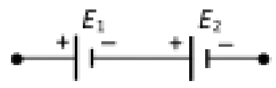
(c) By using formula internal resistance $r = \left(\frac{E}{V} - 1\right).R'$

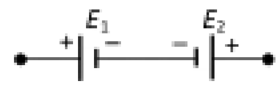
$$r = \left(\frac{l_1 - l_2}{l_2}\right).R'$$

(2) **Comparison of emf's of two cell :** Let l_1 and l_2 be the balancing lengths with the cells E_1 and E_2 respectively then $E_1 = xl_1$ and $E_2 = xl_2 \Rightarrow \frac{E_1}{E_2} = \frac{l_1}{l_2}$



Let $E_1 > E_2$ and both are connected in series. If balancing length is l_1 when cell assist each other and it is l_2 when they oppose each other as shown then :





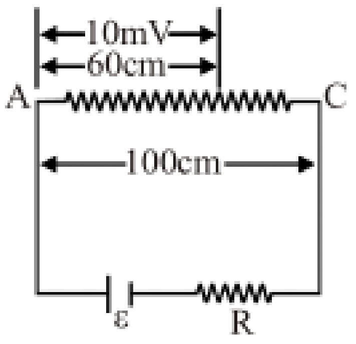
$$(E_1 + E_2) = xl_1 \qquad\qquad (E_1 - E_2) = xl_2$$

$$\Rightarrow \quad \frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2} \qquad\qquad \frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

Illustration :

A potentiometer wire of length 1 m has a resistance of 10 ohm. It is connected in series with a resistance R and a cell of emf 3 V and negligible internal resistance. A source of emf 10 mV is balanced against a length of 60 m of the potentiometer wire. Find the volue of R.

Sol. *Following the theory of potentiometer,*



$$V_{AB} = i\,R_{AB}$$

$$= \left(\frac{\varepsilon}{R + R_{AB}}\right) R_{AB}$$

$$\varepsilon = 3\ V,\ R_{AB} = 10\ \Omega,\ V_{AB} = 10 \times 10^{-3}\ V$$

and $$R_{AC} = \frac{AB}{AC} R_{AB} = \frac{60}{100} \times 10 = 6\ \Omega,$$

We have $$10 \times 10^{-3} = \left(\frac{3}{R + 10}\right) \times 6$$

or, $$R = 1790\ ohm$$

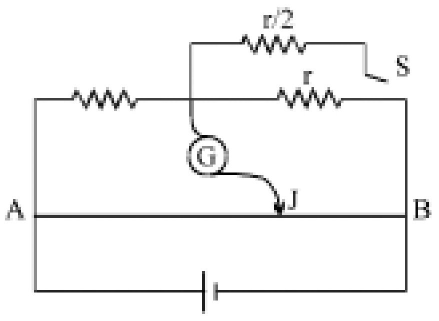
Practice Exercise

Q.1 A moving coil galvanometer of resistance 20Ω gives a full scale deflection when a current of 1mA is passed through it. It is to be converted into an ammeter reading 20A on full scale. But the shunt of 0.005Ω only is available. What resistance should be connected in series with the galvanometer coil ?

Q.2 In a potentiometer experiment it is found that no current passes through the galvanometer when the terminals of the cell are connected across 0.52 m of the potentiometer wire. If the cell is shunted by a resistance of 5Ω, a balance is obtained when the cell is connected across 0.4m of the wire. Find the internal resistance of the cell.

Q.3 There is a milliammeter each division of which reads 1mA. It has a resistance of 15Ω. How would you convert it into a voltmeter so that each division of its graduation would read 1 volt.

Q.4 The diagram shows a meter bridge with the wire AB having uniform resistance per unit length. When the switch S is open, AJ is the balance length and when the switch is closed, AJ' is the balance length. If AB = L and AJ = L/2 then what is the value of AJ'?



Q.5 How can the sensitivity of a potentiometer be increased?

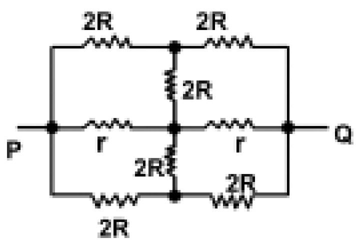
Q.6 An ammeter and a voltmeter are connected in series to a cell of e.m.f. 12 volts. When a certain resistance is connected in parallel with voltmeter the reading of voltmeter is reduced 3 times whereas the reading of ammeter increases 3 times. Find the voltmeter reading after the connection of resistance.

Answers

Q.1	79.995 Ω	Q.2	1.5Ω	Q.3	985 Ω in series	Q.4	$\frac{3l}{4}$
Q.5	Increasing rheostat in primary circuit it ↓ potential drop per unit length of wire					Q.6	3 volts

Solved Examples

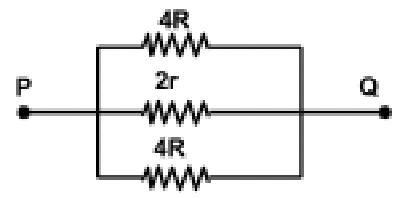
Q.1 The effective resistance between points P and Q of the electrical circuit shown in the figure is



(A) $\frac{2Rr}{R+r}$ (B) $\frac{8(R+r)}{3R+r}$

(C) $2r+4R$ (D) $\frac{5R}{2}+2r$

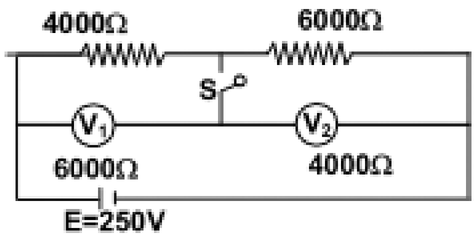
Sol. Sol. The circuit can be reduced to the one given alongside



$$R_e = \frac{2rR}{r+R}$$

Hence, (A) is correct

Q.2 In the circuit shown in the figure, V_1 and V_2 are two voltmeters having resistances 6000Ω and 4000Ω respectively emf of the battery is 250 volts, having negligible internal resistance. Two resistances R_1 and R_2 are 4000Ω and 6000Ω, respectively. Find the reading of the voltmeters V_1 and V_2 when



(i) switch S is open

(ii) switch S is closed

Sol.

(a) When switch S is open

R_1 and R_2 are in series. Let their equivalent resistance be R'

$R' = 4000 + 6000 = 10000$

The voltmeter are also in series. Let their resistance be R'', then

$R'' = 6000 + 4000 = 10000$

The resistance R' and R'' are connected in parallel. Their equivalent resistance is given by

$$R_{eq} = \frac{R' \times R''}{R' + R''} = \frac{10000 \times 10000}{20000} = 5000\ \Omega$$

$$\text{Current from battery} = \frac{E}{R_{eq}} = \frac{250}{5000} = \frac{1}{20}\text{A}$$

$$\text{Current } i_1 \text{ in the voltmeter branch} = \frac{1}{2} \times \frac{1}{20} = \frac{1}{40} \text{ amp}$$

$$\text{Potential difference across } V_1 = \frac{1}{40} \times 6000 = 150 \text{ volt}$$

$$\text{Potential difference across } V_2 = \frac{1}{40} \times 4000 = 100 \text{ volt}$$

(b) When switch S is closed. The circuit redrawn in this case is shown in figure. In this case V_1 and R_1 are in parallel. Similarly V_2 and R_2 are in parallel.

Equivalent resistance of V_1 and R_1

$$R' = \frac{6000 \times 4000}{6000 + 400} = 2400\ \Omega$$

Similarly for R_2 and V_2

$$R'' = \frac{6000 \times 4000}{6000 + 400} = 2400\ \Omega$$

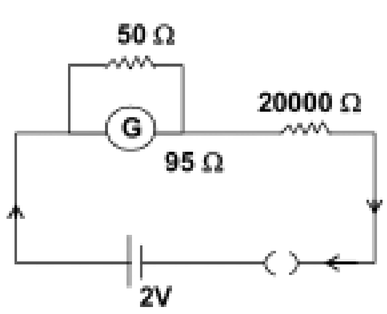
So, the two equal resistances are connected in series.

Hence reading of $V_1 = 125$ volt

And reading of $V_2 = 125$ volt

Q.3 A galvanometer of resistance 95Ω , shunted by a resistance of 50 Ω gives a deflection of 50 divisions when joined in series with a resistance of 20 kΩ and a 2 volt battery, what is the current sensitivity of galvanometer (in div/μA)?

Sol. Current in the circuit

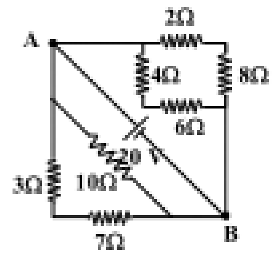


$$I = \frac{2}{20 \times 10^3} = 100\ \mu A$$

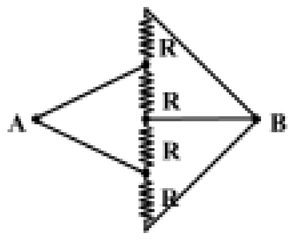
This current produces deflection of 50 div in the galvanometer

$$CS = \frac{\theta}{I} = \frac{50\,\text{Div}}{100\mu A} = \frac{1}{2}\frac{\text{Div}}{\mu A}$$

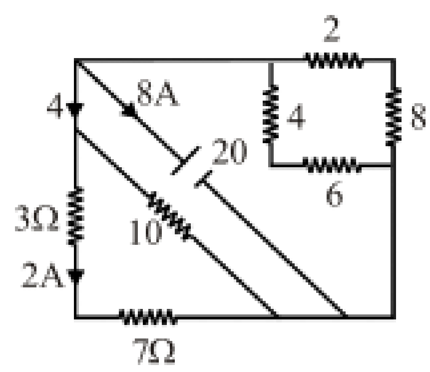
Q.5 (a) The potential difference across 7Ω resistor is equal to _______ and the current flowing through the battery is equal to _______.



(b) The equivalent resistance across A and B is equal to_____________.



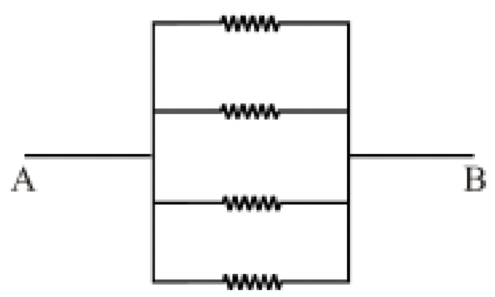
Sol. (a) $R_{AB} = \frac{5}{2}\Omega$, $i_{total} = \frac{20}{5/2} = 8A$



$$\Delta V_{7\Omega} = 7 \times 2 = 14\ \text{V}$$

14 volt, 8A

(b) The circuit can redrawn as,



$$R_{AB} = \frac{R}{4}$$

Q.6 Two resistors, 400 ohm and 800 ohm, are connected in series with a 6 V battery. It is desired to measure the current in the circuit. An ammeter of 10 ohm resistance is used for this purpose. What will be the reading in the ammeter? Similarly, if a voltmeter of 10,000 ohm resistance is used to measure the potential difference across 400 ohm, what will be the reading of the voltmeter?

Sol. Ammeter has law resistance and voltmeter has high resistance as compared with resistance of circuit hence

$$i = \frac{6}{400+800} = \frac{6}{1200} = 5\ \text{mA},\ V = 400 \times 5\text{mA} = 2\ \text{volt}$$

Q.7 Two cells, having emfs of 10 V and 8 V, respectively, are connected in series with a resistance of 24 Ω in the external circuit. If the internal resistances of each of these cells in ohm are 200% of the value of their emf's, respectively, find the terminal potential difference across 8 V battery.

Sol. We determine the internal resistance of each these cells :

$$r_1 = 2\Omega/\text{V} \times 10\text{V} = 20,$$
$$r_2 = 2\Omega/\text{V} \times 8\text{V} = 16\Omega$$

$\therefore$ Total resistance in circuit $= (24 + 16 + 20) = 60\Omega$

$$\therefore \quad \text{Current} = \frac{18\text{V}}{60\Omega} = 0.3\ \text{A}.$$

Thus terminal potential difference $V = E \quad ir = 8 - 0.3(16) = 3.2\ \text{V}$

Q.8 A galvanometer having 50 divisions provided with a variable shunt S is used to measure the current when connected in series with a resistance of 90Ω and a battery of internal resistance 10Ω. It is observed that when the shunt resistances are 10Ω and 50Ω, the deflections are, respectively, 9 and 30 divisions. What is the resistance of the galvanometer?

Sol. $$I = \frac{\varepsilon}{\left(90+10+\dfrac{SG}{S+G}\right)} = \frac{\varepsilon}{\left(100+\dfrac{SG}{S+G}\right)} \qquad \ldots\text{(i)}$$

Applying kirchhoff's of law

we get, $\qquad i_g = \dfrac{IS}{S+G}$

$$\Rightarrow \quad i_g = \frac{S}{S+G} \times \frac{\varepsilon}{\left(100 + \dfrac{SG}{S+G}\right)} \qquad \ldots\text{(ii)}$$

Let $i_g = i_1$ of $S = 10\Omega$ and $i_g = i_2$ for $S = 50\Omega$

$$\frac{i_1}{i_2} = \frac{\left(\dfrac{10}{10+G}\right) \times \left(\dfrac{\varepsilon}{100 + \dfrac{100G}{10+G}}\right)}{\left(\dfrac{50}{50+G}\right) \times \left(\dfrac{\varepsilon}{100 + \dfrac{50G}{50+G}}\right)}$$

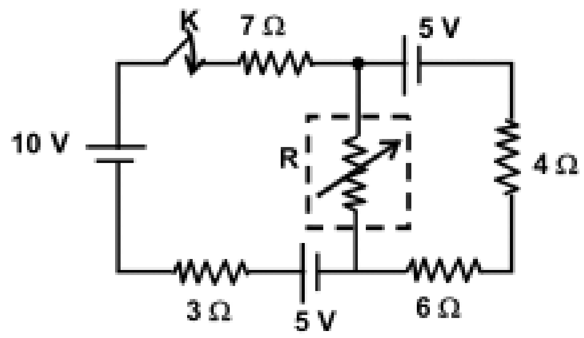
$$\frac{i_1}{i_2} = \frac{100+3G}{100+11G}$$

$\therefore$ Defletion is proportional to the current

$$\Rightarrow \quad \frac{9}{30} = \frac{100+3G}{100+11G}$$

Solving we get, $G = 233.3\ \Omega$

Q.9. In the circuit shown the resistance R is kept in a chamber whose temperature is 20°C which remains constant. The initial temperature and resistance of R is 50°C and 15Ω respectively. The rate of change of resistance R with temperature is $\frac{1}{2}\ \Omega/{}^\circ C$ and the rate of decrease of temperature of R is $\ln\left(\frac{3}{100}\right)$ times the temperature difference from the surrounding (Assume the resistance R loses heat only in accordance with Newton's law of cooling). If K is closed at $t = 0$, then find the



(a) value of R for which power dissipation in it is maximum.
(b) temperature of R when power dissipation is maximum.
(c) time after which the power dissipation will be maximum.

Sol.

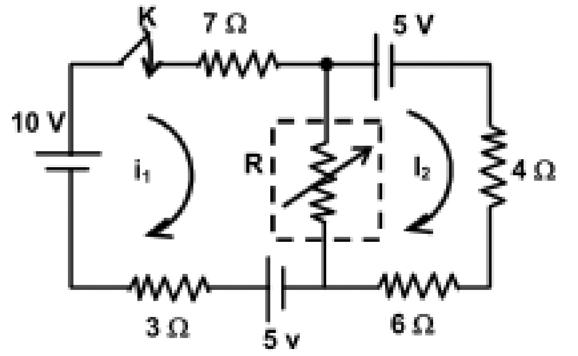
(a) Let i_1 and i_2 be the current in two loops respectively

$\therefore \quad (10 - 10)\, i_1 - R(i - i_2) + 5 = 0$ (for loop 1)

$\quad (10 + R)\, i_2 - Ri_1 = -5 \qquad$ (for loop 2)

Power dissipated in R,

$$P = (i_1 - i_2)^2 R = \frac{25}{(5+R)^2} \times R$$



$\Rightarrow$ For maximum power dissipation $\frac{dP}{dR} = 0$

$\Rightarrow$ $R = 5\Omega$

(b) $R = R_0 - \left(\frac{dR}{d\theta}\right)\Delta\theta$

$5 = 15 - \frac{1}{2}\Delta\theta$

$\Rightarrow \Delta\theta = 20^\circ C \Rightarrow$ temperature at that instant $= 30^\circ C$

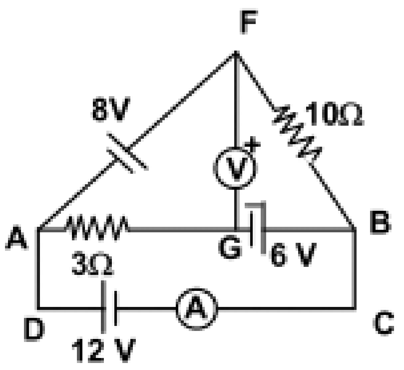
(c) According to Newton's law :

$$\frac{d\theta}{dt} = -k(\theta - 20^\circ)$$

$$\int_{50}^{20} \frac{d\theta}{\theta - 20} = -kt, \quad = \frac{-\ln 3}{100} t, = -\ln 3$$

$\therefore$ $t = 100$ sec.

Q.10 Find the reading of ammeter A and voltmeter V shown in the figure assuming the instruments to be ideal.

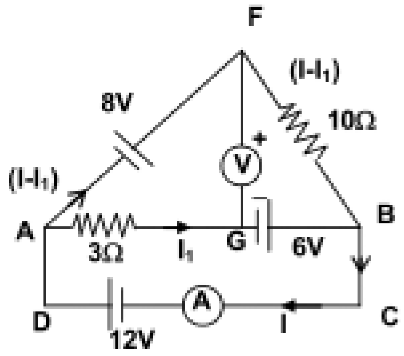


Sol. Distributing the currents in the circuit according to Kirchhoff's I law is shown in the figure. In ideal voltmeter current = 0. Applying Kirchhoff's law in mesh ABCDA

$-3I_1 + 6 + 12 = 0$

i.e. $I_1 = 6A$

Now apply Kirchoff's law in AFBA



$8 - (I - I_1) \times 10 - 6 + 3I_1 = 0$

i.e $10I - 13I_1 = 2$

or $I = \frac{2}{10} + \frac{13}{10} \times 6 = 8A$

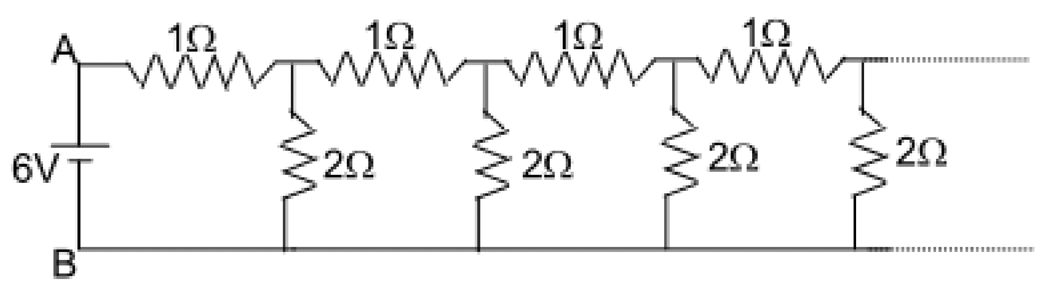
Hence reading of ammeter = 8A

Reading of the voltmeter $V = V_F - V_G$
applying Kirchhoff's law in mesh AFGA

$$8 - V + 6 \quad 3 = 0 \qquad \text{i.e. } V = 26\text{ V}$$

Hence reading of voltmeter $= 26$ V.

Q.11 An infinite ladder network of resistance is constructed with 1 and 2 resistance, as shown in fig. The 6V battery between A and B has negligible internal resistance.
(i) Show that the effective resistance between A and B is 2 .
(ii) What is the current that passes through 2 resistance nearest to the battery?



Sol.
(i) Since the network is an infinite ladder, we can assume that resistance across AB is equal to that of A' B'

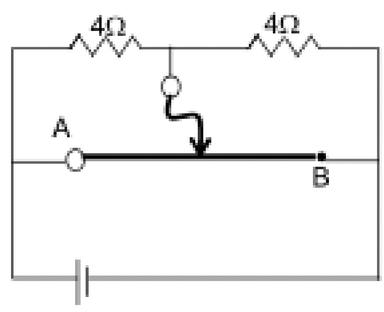
$$R = 1 + \frac{2R}{2+R}$$

$\Rightarrow \quad 2R + R^2 = 2 + R + 2R \qquad$ or $\qquad R = 2$ ohm.

(ii) $i = \dfrac{6}{2} = 3$ amp.

$i' = \dfrac{3}{2} = 1.5$ amp

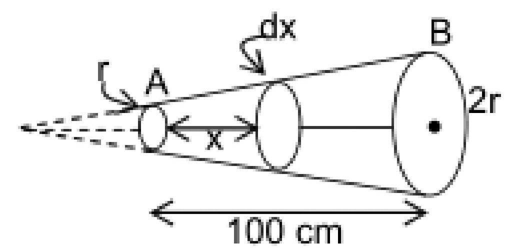
Q.12 The wire AB of a meter bridge continuously changes from radius r to $2r$ from left end to right end. Where should the free end of galvanometer be connected on AB so that the deflection in the galvanometer is zero?



Sol. Let the galvanometer be connected at a point $x = x_1$ from end A where $x = 0$.
Let R_1 = resistance of left part i.e. AX_1 and
R_2 = resistance of right part i.e. X_1B
Length = 100 cm = 1 m.
Consider an element of thickness dx at a distance x from end A and of radius r_x.

Thus, $r_x = \left(r + \dfrac{r}{1}x\right) = r(1 + x)$

Resistance of this element will be, $dRx = \dfrac{\rho dx}{\pi r_x^2}$



$$R_1 = \int_0^{x_1} \frac{\rho\, dx}{\pi(1+x)^2 r^2} = \frac{\rho}{\pi r^2}\left[1 - \frac{1}{x_1}\right]$$

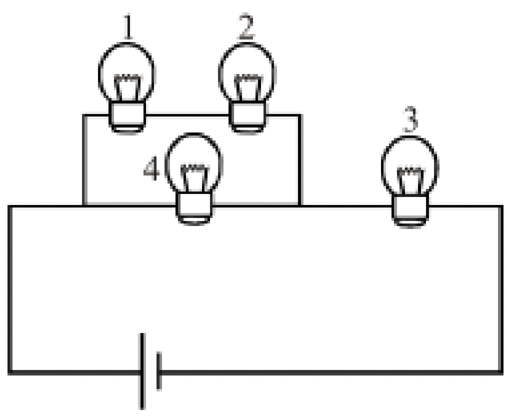
$$R_2 = \int_{x_1}^{4} \frac{\rho\, dx}{(1+x)^2 r^2} = \frac{\rho}{\pi r^2}\left[\frac{1}{1+x_1} - \frac{1}{1+1}\right]$$

For null point or zero deflection,

$$\frac{R_1}{R_2} = \frac{4}{4} \Rightarrow 1 - \frac{1}{1+x_1} = \frac{1}{1+x_1} - \frac{1}{1+1}$$

$$\Rightarrow \qquad x_1 = \frac{1}{3}\,\text{m} = 33.33\text{ cm}$$

Q.13 Four identical bulbs, each of same rating (100 W, 220 V) are connected across an ideal battery of emf 550 volts. Which of the 4 bulbs will have a voltage across it, which is greater than voltage rating. (i.e. which of them will fuse)



Sol. By voltage division
$v_3 = 330$ volts
$v_4 = 220$ volts
$v_1 = v_2 = 100$ volts
Ans. only bulb (3)

Q.14 What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil
(a) decreases down to zero uniformly during a time interval Δt
(b) decreases down to zero halving its value every Δt seconds?

Sol. (a) As current i is linear function of time, and at $t = 0$ and Δt, it equals i_0 and zero respectively, it may be represented as,

$$i = i_0\left(1 - \frac{t}{\Delta t}\right)$$

Thus $\quad q = \displaystyle\int_0^{\Delta t} i\,dt = \int_0^{\Delta t} i_0\left(1 - \frac{t}{\Delta t}\right)dt = \frac{i_0 \Delta t}{2}$

So, $\quad i_0 = \dfrac{2q}{\Delta t}$ $\qquad$ Hence $\quad i = \dfrac{2q}{\Delta t}\left(1 - \dfrac{t}{\Delta t}\right)$

The heat generated.

$$H=\int_0^{\Delta t} i^2 R dt=\int_0^{\Delta t}\left[\frac{2q}{\Delta t}\left(1-\frac{t}{\Delta t}\right)\right]^2 R dt=\frac{4q^2R}{3\Delta t}$$

(b) Obviously the current through the coil is given by

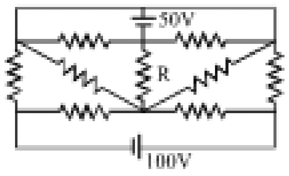
$$i=i_0\left(\frac{1}{2}\right)^{t/\Delta t}$$

Then charge $q=\int_0^{\infty} i dt=\int_0^{\infty} i_0 2^{-t/\Delta t} dt=\frac{i_0\Delta t}{\ln 2}$ So, $i_0=\frac{q\ln 2}{\Delta t}$

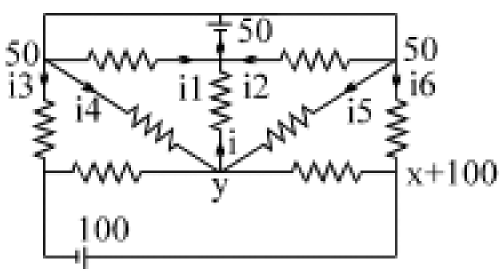
And hence, heat generated in the circuit in the time interval $t[0,\infty]$,

$$H=\int_0^{\infty} i^2 R dt=\int_0^{\infty}\left[\frac{q\ln 2}{\Delta t}2^{-t/\Delta t}\right]^2 R dt=-\frac{q^2\ln 2}{2\Delta t}R$$

Q.15 Find the current in the resistance R. Each resistance is of 2Ω.



Sol.



Nodal analysis $\frac{y-0}{2}+2.\frac{y-50}{2}+\frac{y-x}{2}+\frac{y-x-100}{2}=0$

$\Rightarrow$ $y+2y-100+(y-x)-100=0$

$5y-2x=200$ ... (1)

$i=i3+i4+i5+i6$

$$\frac{y-0}{2}=\frac{50-x}{2}+\frac{50-y}{2}+\frac{50-y}{2}+\frac{50-x-100}{2}$$

$y=150-x-y-50-x$

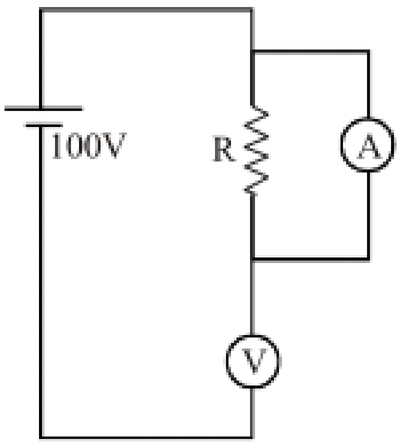
$2x+3y=100$ ... (2)

$-2x+5y=200$ ... (1)

$8y=300$

$\frac{y}{2}=\frac{300}{16}$ $=18.75$ A

Q.16 A voltmeter of resistance 995 Ω and an ammeter of resistance 10 Ω is connected as shown to calculate the unknown resistance R which is connected to the ideal battery. Voltmeter reading is 99.5 volts. The value of resistance R is calculated as $\frac{\text{Voltmeter reading}}{\text{Ammeter reading}}$ by student A.



(i) Find his answer.

(ii) Also find the actual value of resistance.

Sol. (i) Voltage across ammeter = 0.5 volts

Resistance = 10 Ω

Ammeter reading = 0.05 A

$$R = \frac{\text{Voltmeter reading}}{\text{Ammeter reading}} = \frac{99.5}{0.05} = 1990\ \Omega$$

(ii) Current across voltmeter $= \frac{99.5}{995} = 0.1$ **A**

and current through ammeter = 0.05 A

∴ Current through R = 0.05 A and voltage across R = 0.5 V

∴ $R = \frac{0.5}{0.05} = 10\ \Omega$