PHYSICS

1. λ_m T = const.

$\ell n\, \lambda_m + \ell n T = C$

$$\frac{d\lambda_m}{\lambda_m} + \frac{dT}{T} = 0 \qquad \therefore \frac{d\lambda_m}{\lambda_m} = -\frac{dT}{T}$$

Now $\frac{d\lambda_m}{\lambda_m}$ = – 1 % = $-\frac{1}{100}$ (–ve sign indicates decrease)

dT = 1 (given)

$\therefore$ T = 100 K.

2. As dQ = msdT

$$\frac{dQ}{dt} = ms\frac{dT}{dt}$$

From question : $S \propto T$

or $S = K_1T$. (K_1 being propertionality constant)

Also, $\frac{d\theta}{dt}$ = constant = K_2 (say) $\Rightarrow ms\frac{dT}{dt} = K_2 \Rightarrow m(K_1T)\frac{dT}{dt} = K_2$

$$\Rightarrow \left(m\frac{K_1}{K_2}\right)\frac{T^2}{2} = t \Rightarrow T \propto \sqrt{t}$$

3. Rate of heat produced

$$\frac{dQ}{dt} = \frac{v^2}{R} = \frac{v^2}{R_0(1+\alpha(T-0))} = \frac{v^2}{R_0(1+\alpha T)} \qquad \text{and } \frac{dQ}{dt} = ms\frac{dT}{dt}$$

$$\Rightarrow ms\frac{dT}{dt} = \frac{v^2}{R_0(1+\alpha T)}$$

$$\int_{T=0}^{T=T}(1+\alpha T)dT = \frac{v^2}{R_0 ms}\int_{t=0}^{t=t}dt$$

$$T + \frac{\alpha T^2}{2} = \frac{v^2}{R_0 ms}t, \qquad t = \frac{R_0 ms}{v^2}\left(T + \frac{\alpha T^2}{2}\right).$$

4. For sphere :

$$\sigma T^4 S = m_1C.\left(\frac{-d\theta}{dt}\right)_{sphere}$$

For cube :

$$\sigma T^4.\, S = m_2.C\left(\frac{-d\theta}{dt}\right)_{cube}$$

$$\therefore \frac{\left(\frac{-d\theta}{dt}\right)_{sphere}}{\left(-\frac{d\theta}{dt}\right)_{cube}} = \frac{m_2}{m_1} = \frac{V_2}{V_1} \qquad [S = 6a^2 = (4\pi r^2)] = \sqrt{\frac{\pi}{6}}$$

5. Loss in heat from calorimeter + water as temperture changes from 10°C to 0°C

$= m_1C_1 10 + m_2C_2 10 \quad = 1 \times 1 \times 10 + 1 \times 0.1 \times 10 = 11$ kcal

Gain in heat of ice as its temperature changes from –11°C to 0°C

$= m_3C_3 \times 11 = 2 \times 0.5 \times 11 = 11$ kcal

Hence ice and water will coexist at 0°C without any phase change.

6. Clock is designed to indicate correct time at 20°C at height 'h'. It will indicate correct time at 30°C on the ground if in this case the time period is same as the earlier.

$$\therefore 2\pi\sqrt{\frac{L}{g_h}} = 2\pi\sqrt{\frac{L'}{g_s}}$$

here $L' = L(1 + \alpha\, 10)$, $\quad g_s = \dfrac{GM}{R^2}$ and $g_h = \dfrac{GM}{(R+h)^2} \Rightarrow \dfrac{L}{g_n} = \dfrac{L'}{g_s}$

$\Rightarrow L(R + h)^2 = L(1 + \alpha\, 10)\, R^2 \qquad \Rightarrow \qquad 1 + \dfrac{h}{R} = (1 + \alpha\, 10)^{1/2}$

$\Rightarrow 1 + \dfrac{h}{R} = 1 + 5\alpha$ (by binomial expansion) $\qquad \Rightarrow \qquad \alpha = \dfrac{h}{5R}$

7. Since $\vec{F} \perp \vec{V}$, the particle will move along a circle.

$\therefore \qquad F = \dfrac{mv^2}{R} \qquad \& \;\; \theta = \dfrac{S}{R} \qquad \Rightarrow \qquad \theta = \dfrac{FS}{mv^2}$

8. By symmetry

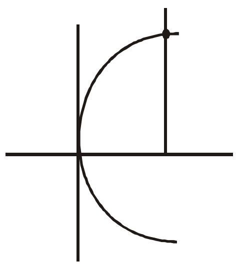
$I_{AB} = I_{BC}$ & $I_{AD} = I_{DC}$

$\therefore$ No current in BO and OD

$\therefore$ $T_B = T_O = T_D$

9. $x = 4y^2$

$\dfrac{dx}{dt} = 8y\,\dfrac{dy}{dt}$



$V_x = 8y\, V_y$

$V_x = 4$

$a_x = 0$

$0 = a_x = 8[y.a_y + V_y^2]$

$-y\, a_y = V_y^2$

$|a_y| = \dfrac{v_y^2}{y}$

$|a_y| = \dfrac{v_x^2}{64\, y^3} = \dfrac{16}{64 \times y^3}$

at $y = 1 \Rightarrow \quad |a_y| = \dfrac{1}{4}$

10. Applying Newton's Law on water calorimeter :

$$(m_1s_1 + m_2s_2)\ \frac{dT}{dt} = kA\ (T - T_0)$$

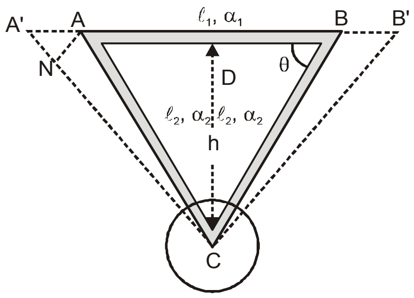
$$[(\rho v1) + v)]\left(\frac{50-40}{100}\right) = kA\ (45 - T_0)$$

$$[(0.8\rho\ )vS + v)\left[\frac{50-40}{74}\right] = kA\ (45 - T_0)$$

using ρ = 1 gm/cm^3, by solving

S = 0.6 cal/gm°C

11.



According to condition of the problem, height of the isosceles triangle ABC is unchanged. The dotted lines show configuration after a temperature rise. Increase in length of rod AB,

$$\Delta\ell_1 = \ell_1\alpha_1\Delta T$$

Thus $\quad AA' = 1/2\ \ell_1\alpha_1\Delta T$

We draw a normal from A to A'C (the final length of AC). Increase in length of AC is A'N

$$A'N = \ell_2\alpha_2\Delta T$$

Considering increase in angle θ to be very small.

$$A'N \simeq AA' \cos\theta$$

Where $\cos\theta = \dfrac{\ell_1}{2\ell_2}$

Thus, we have $\ell_2\alpha_2\Delta T = \left(\dfrac{1}{2}\ell_1\alpha_1\Delta T\right)\left(\dfrac{\ell_1}{2\ell_2}\right)$

Hence $\dfrac{\ell_1}{\ell_2} = 2\sqrt{\dfrac{\alpha_2}{\alpha_1}}$

12. Stress = 2R $[\alpha_{Al} - \alpha_{st}]\ \Delta\theta Y$ If aluminium ring is alowed to expand freely

so stress < 2R $(\alpha_{Al})\ \Delta\theta Y$

13. Power radiated $P = 4\pi r^2\sigma T^4 = -\left(ms\dfrac{dT}{dt}\right)$

$$-\frac{dT}{T^4} = \frac{4\pi r^2\sigma dt}{m} = c\ dt$$

$$-\int_{T_1}^{T_2}\frac{dT}{T^4} = ct \qquad \Rightarrow \qquad t = K\left[\frac{1}{T_2^{\,3}} - \frac{1}{T_1^{\,3}}\right].$$

14. $\Delta Q = \dfrac{kA(100-0)}{L} \,.\, T$ ....(i)

In second case :

$\Delta Q' = 2.\ \dfrac{kA(100-0)}{L/2} \,.\, T'$

since $\Delta Q = \Delta Q'$

$\therefore \quad T = 4T' \quad T' = \dfrac{T}{4}$

15. $i = -\dfrac{k_0 x^2}{L^2} A \dfrac{dT}{dx} \quad \Rightarrow \quad \int_{T_H}^{T} dT = -\dfrac{iL^2}{k_0 A} \int_1^x \dfrac{dx}{x^2}$

$T - T_H = \dfrac{iL^2}{k_0 A} \left[\dfrac{1}{x}\right]_1^x$

17. Heat obviously flows from higher temperature to lower temperature in steady state. $\Rightarrow$ A is true.

Temperature gradient $\propto \dfrac{1}{\text{cross section area}}$ in steady state. $\Rightarrow$ B is false.

Thermal current through each cross section area is same. $\Rightarrow$ C is true.

Temperature decreases along the length of the rod from higher temperature end to lower temperature end. $\Rightarrow$ D is false.

18. For steady state

$\left(\dfrac{dQ}{dt}\right)_{in} = \left(\dfrac{dQ}{dt}\right)_{out}$

$(V)\,(i_{55}) = 45(T-20)$

$(500)\,(4.5) = 45(T-20)$

$T_{55} = 70°C.$

Resistance at 20ºC is $R = \dfrac{v}{i} = \dfrac{500}{5}$

$R_{20} = 100\ \Omega$

Resistance at 70ºC is $R = \dfrac{v}{i} = \dfrac{500}{4.5} \simeq 111\ \Omega$

$R_f = R_0(1 + \alpha\Delta T)$

$111 = 100(1 + \alpha(50))$

$\alpha = \dfrac{0.11}{50} \simeq 2.2 \times 10^{-3}\ /°C.$

19. Let at any instant temperature of water be T, then heat current

$i = \dfrac{kA}{x}\,.\,(T-0)$ —— (1)

where $A = 6\,a^2 = 6\ m^2$; x = thickness = 1 mm = 10^{-3} m

Rate of heat lost from water, $\dfrac{dQ}{dt} = +\,m\,s\,\dfrac{dT}{dt}$ —— (2)

So, we get from (1) & (2), $-\,m\,s\,\dfrac{dT}{dt} = \dfrac{kAT}{x} \quad \Rightarrow \quad -\int_{50°}^{25°} \dfrac{dT}{T} = \dfrac{kA}{mSx} \int_0^{10\ell n2} dt$

$\Rightarrow \ \ell n\,(2) = \dfrac{kA}{mSx}\,.\,10\,(\ell n\ 2) \qquad$ So, $\dfrac{kA\,(10)}{mSx} = 1$

Putting values $\Rightarrow$ $k = \frac{mSx}{10A} = \frac{(10^3\ kg)\ (4.2 \times 10^3\ J/kg°C)\ 10^{-3}\ m}{10 \times (6\ m^3)}$ = k = 70 J/m ºC

$\Rightarrow$ Total heat transferred will be = total heat

$Q = \int dQ = \int \frac{kA}{x} T\,dt$ Lost by water.

$Q = m S \Delta T = 10^3 \times 4200 \times 25$ J $= (10^6$ gm) (1 cal) (25) $= m_{ice} L$

Giving $m_{ice} = \left(\frac{10^6 \times 25}{80}\right)$ gm $= \frac{25000}{80}$ kg = 312.5 kg

mass of ice melted = 312.5 kg

20. $\frac{\Delta A}{A} \times 100 = 2\left(\frac{\Delta \ell}{A}\right) \times 100$

$\Rightarrow$ % increase in Area = 2 × 0.2 = 0.4

$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6$ %

Since $\Delta \ell = \ell \alpha \Delta T$

$\frac{\Delta \ell}{\ell} \times 100 = \alpha \Delta T \times 100 = 0.2$

$\Rightarrow$ $\alpha = 0.25 \times 10^{-4}$ / ºC

21. Since , e = a = 0.2 (Since, a = (1 – r – t) = 0.2 for the body B)
E = (100) (0.2) = 20 W/m^2
Power emitted = e.A = 20 × 10 = 200 Watt

22. $\sigma(T^4 - T_S^4) . (6a^2)\, t = (d . a^3)\, s.\Delta T$

$\Rightarrow$ $t = \frac{d a s \Delta T}{6.\sigma(T^4 - T_s^4)} = \frac{4.8 \times 10^3 \times 0.9 \times 2.0 \times 10^3 \times 5}{6 \times 6 \times 10^{-8} \times (400^4 - 200^4)}$ = 5000 s.

X = 5

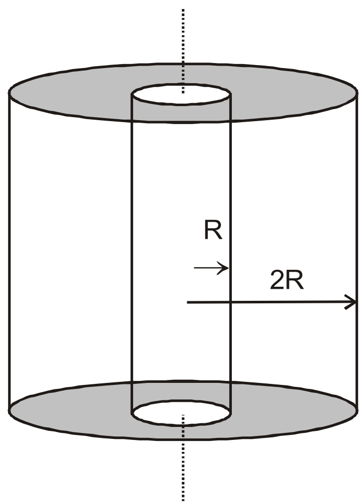
23. In equilibrium, power released= power absorbed

or $4\pi(2R)^2(1)\sigma T^4 = \frac{1}{32} 4\pi(R)^2(\frac{1}{2})\sigma 200^4$

24. $H = -K.\ 2\pi r l \frac{dT}{dr}$

$\int_{R_1}^{R_2} \frac{H\,dr}{2\pi r l} = -K \int_{T_1}^{T_2} dT$

$H = \frac{2\pi l k(T_1 - T_2)}{ln\frac{R_2}{R_1}}$ $H_i = H_f$ $\therefore$ **Ans.** n = 4

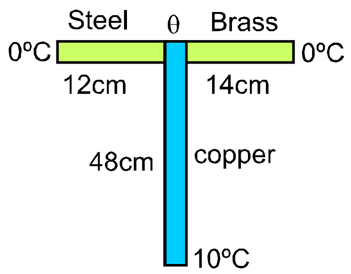


25. $i = i_1 + i_2$

$$\frac{0.96 \times 4 \times (10 - \theta)}{48} = \frac{0.28 \times 4(\theta - 0)}{14} + \frac{0.12 \times 4 \times (\theta - 0)}{12}$$

$0.02(10 - \theta) = 0.02\theta + 0.01\theta$

$\theta = 4°C.$



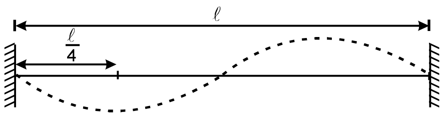
26. The mechanical strain $= \frac{\Delta \ell}{\ell} = \alpha\, \Delta T = 1.21 \times 10^{-5} \times 20 = 2.42 \times 10^{-5}$

The tension in wire $= T = Y \frac{\Delta \ell}{\ell} A = 2 \times 10^{11} \times 2.42 \times 10^{-5} \times 10^{-6} = 48.4$ N

$\therefore$ speed of wave in wire

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48.4}{0.1}} = 22 \text{ m/s}$$

Since the wire is plucked at $\frac{\ell}{4}$ from one end



The wire shall oscillate in 1st overtone (for minimum number of loops)

$\lambda = \ell = 1\text{m}$

Now $\quad V = f\lambda \quad$ or $\quad f = \frac{V}{\lambda} = 22$ Hz.

27. Rate of cooling

$$\frac{\Delta T}{\Delta t} = K(T - T_0)$$

For cooling from 60°C to 40°C

$$\Rightarrow \frac{60 - 40}{7} = K\left(\frac{60 + 40}{2} - 10\right)$$

$$\Rightarrow K = \frac{20}{7 \times 40} = \frac{1}{14}$$

For cooling from 40°C to T

$$\frac{40 - T}{7} = K\left(\frac{40 + T}{2} - 10\right) \Rightarrow \frac{40 - T}{7} = \frac{1}{14}\left(\frac{40 + T - 20}{2}\right)$$

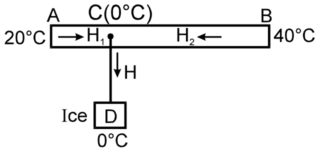
$\Rightarrow 160 - 4T = 20 + T$

$\Rightarrow 140 = 5T$

$$\Rightarrow T = \frac{140}{5} = 28°C$$

$\Rightarrow \quad T = 28°C$

28.



Thermal resistance of AC $\left(=\dfrac{L}{KA}\right) = \dfrac{0.1}{336 \times 1 \times 10^{-4}} = \dfrac{10^3}{336}$ = R (suppose)

thermal resistance of BC = $\dfrac{0.2}{336 \times 10^{-4}}$ = 2R

temperature of C = 0°C

$\therefore \quad H_1 = \dfrac{20}{R} \quad ; \; H_2 = \dfrac{40}{2R} = \dfrac{20}{R}$

$\therefore \quad H = H_1 + H_2 = \dfrac{40}{R} = \dfrac{40 \times 336}{10^3} = \dfrac{13440}{10^3}$ = 13.44 watt

Rate of melting of ice= $\dfrac{H}{L_f} = \dfrac{13.44/4.2}{80}$ g/s = 40 mg/s

29. $L = L_0 (1 - \alpha_s \Delta t)$

$1 = 1.015 - 1.015\, \alpha \Delta T$

$0.015 = 1.015 \times 10^{-3} \times \Delta T$

$\Delta T = \dfrac{15}{1.015} = 15.$

30. In steady state $\left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer 4}}$

$\Rightarrow \dfrac{0.06 \times A \times (30 - 25)}{1.5 \times 10^{-2}} = \dfrac{0.10 \times A \times \Delta T}{3.5 \times 10^{-2}}$ $\qquad \Rightarrow$ DT =7°C

T_3 = (– 10 + 7)°C = – 3°C

31. $\left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer 3}}$

$\Rightarrow \dfrac{0.06 \times A \times 5}{1.5 \times 10^{-2}} = \dfrac{0.04 \times A \times \Delta T}{2.8 \times 10^{-2}}$ $\qquad \Rightarrow$ DT = 14°C

T_3 = (– 3 + 14)°C = 11°C

32. $\left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\dfrac{\Delta Q}{\Delta t}\right|_{\text{layer 2}}$

$\Rightarrow \dfrac{0.06 \times A \times 5}{1.5 \times 10^{-2}} = \dfrac{K_2 \times A \times 14}{1.4 \times 10^{-2}}$ $\quad \Rightarrow K_2$ = 0.02 W/mK

33 to 35

For spherical surface, at steady state

$P_{Heater} + 50 \text{ W} = 120 \text{ W}$

$\Rightarrow \quad P_{Heater} = 70 \text{ W}$

At steady state $P_{Heater} = \dfrac{(100 - t_{out})\, kA}{\ell} = 70 = k'\,(t_{out} - 20)$ ....(i)

From the given observation

$50 = k'\,(40 - 20)$ ...(ii)

from equation (ii) $\quad k' = \dfrac{5}{2}$

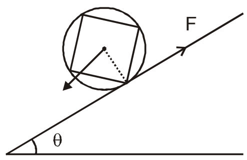
from equation (i)

$t_{out} = 48°$

$\ell = 0.52$ mm

36 to 38 $I = \left[\dfrac{M(R\sqrt{2})^2}{12} + M\left(\dfrac{R}{\sqrt{2}}\right)^2\right] \times 4 + mR^2$

$= 20 \text{ kgm}^2.$



$(4M + m)g \sin\theta - F = (4M + m)a.$

$F.R. = I\left(\dfrac{a}{R}\right)$

Solving

$a = \dfrac{7g}{24}$

$F = 20a \le \mu\ (4M + m)g \cos 30$

$\mu \ \ge \ \dfrac{5}{12\sqrt{3}}$

$\therefore \qquad \mu_{min} = \dfrac{5}{12\sqrt{3}}$

39. We have $\theta - \theta_s = (\theta_0 - \theta_s)\, e^{-kt}$

where θ_0 = Initial temperature of body = 40°C

θ = temperature of body after time t.

Since body cools from 40 to 38 in 10min, we have

$38 - 30 = (40 - 30)\, e^{-k\,10}$ .... (1)

Let after 10 min, The body temp. be θ

$\theta - 30 = (38 - 30)\, e^{-k\,10}$ .... (2)

$\dfrac{(1)}{(2)}$ gives $\qquad \dfrac{8}{\theta - 30} = \dfrac{10}{8}$, $\ \theta - 30 = 6.4 \qquad \Rightarrow \ \theta = 36.4$ °C

40. Self Explanatory

41. During heating process from 38 to 40 in 10 min. The body will lose heat in the surrounding which will be exactly equal to the heat lost when it cooled from 40 to 38 in 10 min, which is equal to ms $\Delta\theta = 2 \times 2 = 4$ J.

During heating process heat required by the body = m s $\Delta\theta$ = 4 J.

$\therefore$ Total heat required = 8 J.

42. (P) Total energy, total linear momentum, total angular momentum is conserved but mass is converted into energy.

(Q) Total energy, total linear momentum, total angular momentum and total mass is conserved

(R) Total energy, total linear momentum, total angular momentum and total mass is conserved

(S) Total energy, total linear momentum, total angular momentum is conserved but mass is converted into energy.