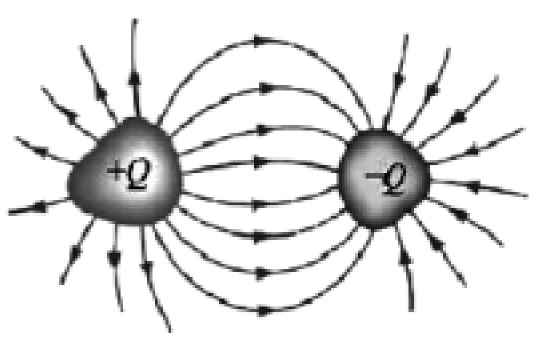
Capacitance

Introduction

Capacitor is an arrangement of two conductors generally carrying charges of equal magnitudes and opposite sign and separated by an insulating medium . A capacitor is a device which stores electric charge. Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges (Figure). Capacitors have many important applications in electronics. Some examples include storing electric potential energy, delaying voltage changes when coupled with resistors, filtering out unwanted frequency signals, forming resonant circuits and making frequency-dependent and independent voltage dividers when combined with resistors. Some of these applications will be discussed in latter chapters



When charges are pulled apart, energy is associated with the pulling apart of charges, just like energy is involved in stretching a spring. Thus, some energy is stored in capacitors.

In the *uncharged* state, the charge on either one of the conductors in the capacitor is zero. During the charging process, a charge Q is moved from one conductor to the other one, giving one conductor a charge $+Q$, and the other one a charge $-Q$. A potential difference ΔV is created, with the positively charged conductor at a higher potential than the negatively charged conductor. Note that whether charged or uncharged, the net charge on the capacitor as a whole is zero.

Note :

1. The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge Q, we mean that the positively charged conductor has charge +Q and negatively charged conductor has a charge –Q.

2. In a circuit, a capacitor is represented by the symbol : 

Limitations on charging a conductor

How much electric charge can be placed on a conductor?
As more air is pumped into the tank, the pressure opposing the flow of additional air becomes greater so it becomes further difficult to pump more air. . Similarly, as more charge Q is transferred to the conductor, the potential V of the conductor becomes higher, making it increasingly difficult to transfer more charge. Suppose we try to place an indefinite quantity of charge Q on a spherical conductor of radius r. The air surrounding the conductor is an insulator, sometimes called a dielectric, which contains few charges free to move. The electric field intensity E and the potential V at the surface of the sphere are given by

$$E = \frac{kQ}{r^2} \quad \text{and} \quad V = \frac{kQ}{r}$$

Since the radius r is constant, both the field intensity and the potential at the surface of the sphere increase in direct proportion to the charge Q. There is a limit, however, to the field intensity that can exist on a conductor without ionizing the surrounding air. When this occurs, the air essentially becomes a conductor, and any additional charge placed on the sphere will "leak off" to the air. This limiting value of electric field intensity for which a material loses its insulation properties is called the dielectric strength of that material.

The dielectric strength for a given material is that electric field intensity for which the material ceases to be an insulator and becomes a conductor.

The dielectric strength for dry air at 1 atm pressure is around 3MN/C. Since the dielectric strength of a material varies considerably with environmental conditions, such as pressure and humidity, it is difficult to compute accurate values.

Note that the amount of charge that can be placed on a spherical conductor decreases with the radius of the sphere. Thus, smaller conductors can usually hold less charge. But the shape of a conductor also influences its ability to retain charge. Consider the charged conductors. If these conductor are tested with an electroscope, it will be discovered that the charge on the surface of a conductor is concentrated at points of greatest curvature. Because of the greater charge density in these regions, the electric field intensity is also greater in regions of higher curvature. If the surface is reshaped to a sharp point, the field intensity may become great enough to ionize the surrounding air. A show leakage of charge sometimes occurs at these locations, producing a corona discharge, which is often observed as a faint violet glow in the vicinity of the sharply pointed conductor. It is important to remove all sharp edges from electrical equipment to minimize this leakage of charge.

Practice Exercise

Q.1 What is the maximum charge that may be placed on a spherical conductor 1m in diameter? Assume it is surrounded by air. Assume the dielectric strength for dry air at 1 atm pressure is around 3MN/C.

Answers

Q.1 $\frac{1}{12} \times 10^{-3}\,C$

Capacitance

We can say that the increase in potential V is directly proportional to the charge Q placed on the conductor. Symbolically: $V \propto Q$

Therefore, the ratio of the quantity of charge Q to the potential V produced will be a constant for a given conductor. This ratio reflects the anility of a conductor to store charge and is called its capacitance C.

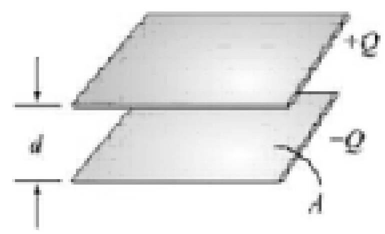
$$C = \frac{Q}{V}$$

The unit of capacitance is the coulomb per volt, which is redefined as a farad (F). Thus, if a conductor has a capacitance of 1 farad, a transfer of 1 coulomb of charge to the conductor will raise its potential by 1 volt.

The value of C for a given conductor is not a function of either the charge placed on a conductor or the potential produced. In principle, the ratio Q/V will remain constant as charge is added indefinitely, but the capacitance depends on the size and shape of a conductor as well as on the nature of the surrounding medium.

The capacitor

The simplest example of a capacitor consists of two conducting plates of area A, which are parallel to each other, and separated by a distance d, as shown in Figure.



A parallel-plate capacitor

Experiments show that the amount of charge Q stored in a capacitor is linearly proportional to $|\Delta V|$, the electric potential difference between the plates. Thus, we may write

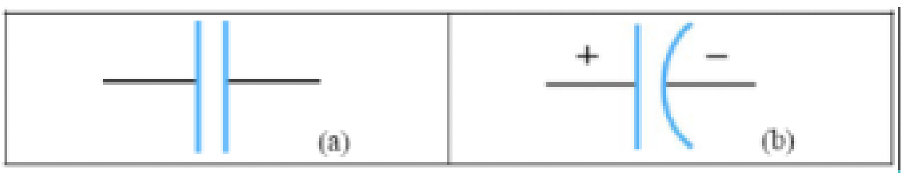
$$Q = C\,|\Delta V|$$

where C is a positive proportionality constant called *capacitance*. Physically, capacitance is a measure of the capacity of storing electric charge for a given potential difference ΔV. The SI unit of capacitance is the *farad* (F) :

$$1\text{ F} = 1\text{ farad} = 1\text{ coulomb / volt} = 1\text{C/V}$$

A typical capacitance is in the picofarad ($1\text{pF} = 10^{-12}$ F) to millifarad range, ($1\text{ mF} = 10^{-3}\text{ F} = 1000\ \mu\text{F}$; $1\mu\text{F} = 10^{-6}$ F).

Figure (a) shows the symbol which is used to represent capacitors in circuits. For a polarized fixed capacitor which has a definite polarity, Figure (b) is sometimes used.



Capacitor symbols.

Practice Exercise

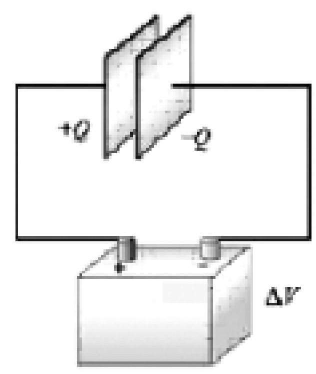
Q.1 A capacitor having a capacitance of 4μF is connected to a 60V battery. What is the charge on the capacitor?

Answers

Q.1 240 μC

Capacitors in Electric Circuits

A capacitor can be charged by connecting the plates to the terminals of a battery, which are maintained at a potential difference ΔV called the ***terminal voltage.***

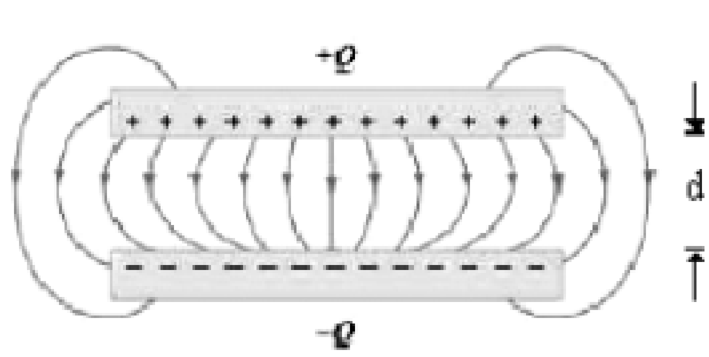


Charging a capacitor.

The connection results in sharing the charges between the terminals and the plates. For example, the plate that is connected to the (positive) negative terminal will acquire some (positive) negative charge. The sharing causes a momentary reduction of charges on the terminals, and a decrease in the terminal voltage. Chemical reactions are then triggered to transfer more charge from one terminal to the other to compensate for the loss of charge to the capacitor plates, and maintain the terminal voltage at its initial level. The battery could thus be thought of as a charge pump that brings a charge Q from one plate to the other.

Parallel-Plate Capacitor

Consider two metallic plates of equal area A separated by a distance d, as shown in Figure below. The top plate carries a charge $+Q$ while the bottom plate carries a charge $-Q$. The charging of the plates can be accomplished by means of a battery which produces a potential difference. Find the capacitance of the system.



The electric field between the plates of a parallel-plate capacitor

To find the capacitance C, we first need to know the electric field between the plates. A real capacitor is finite in size. Thus, the electric field lines at the edge of the plates are not straight lines, and the field is not contained entirely between the plates. This is known as *edge effects*, and the non-uniform fields near the edge are called the *fringing fields*. In Figure the field lines are drawn by taking into consideration edge effects. However, in what follows, we shall ignore such effects and assume an idealized situation, where field lines between the plates are straight lines.

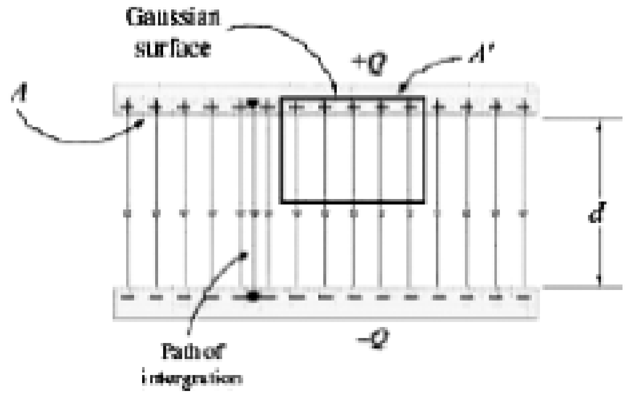
In the limit where the plates are infinitely large, the system has planar symmetry and we can calculate the electric field everywhere using Gauss's law given in Eq. :

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

By choosing a Gaussian "pillbox" with cap area A' to enclose the charge on the positive plate (see Figure), the electric field in the region between the plates is

$$EA' = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma A'}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

The same result has also been obtained using superposition principle.



Gaussian surface for calculating the electric field between the plates.

The potential difference between the plates is

$$\Delta V = V_- - V_+ = -\int_+^- \vec{E} \cdot d\vec{s} = -Ed$$

where we have taken the path of integration to be a straight line from the positive plate to the negative plate following the field lines. Since the electric field lines are always directed from higher potential to lower potential, $V_- < V_+$. However, in computing the capacitance C, the relevant quantity is the magnitude of the potential difference:

$$|\Delta V| = Ed$$

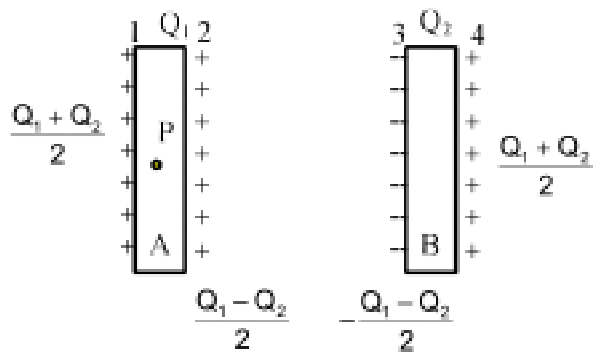
and its sign is immaterial. From the definition of capacitance, we have

$$C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d} \text{ (parallel plate)}$$

Note that C depends only on the geometric factors A and d. The capacitance C increases linearly with the area A since for a given potential difference ΔV, a bigger plate can hold more charge. On the other hand, C is inversely proportional to d, the distance of separation because the smaller the value of d, the smaller the potential difference $|\Delta V|$ for a fixed Q.

Plates of a Parallel Plate Capacitor carrying Different Charges

Two identical plates of parallel plate capacitor are given unequal charges Q_1 and Q_2. The charges appearing on the inner surface be $+\frac{Q_1 - Q_2}{2}$ and $-\frac{Q_1 - Q_2}{2}$ and the charges appearing on outer surfaces are $\frac{Q_1 + Q_2}{2}$ (as shown in the figure). Here the potential difference between the plates is



$$V = \frac{\left(\frac{Q_1 - Q_2}{2}\right)}{\left(\frac{\varepsilon_0 A}{d}\right)}$$

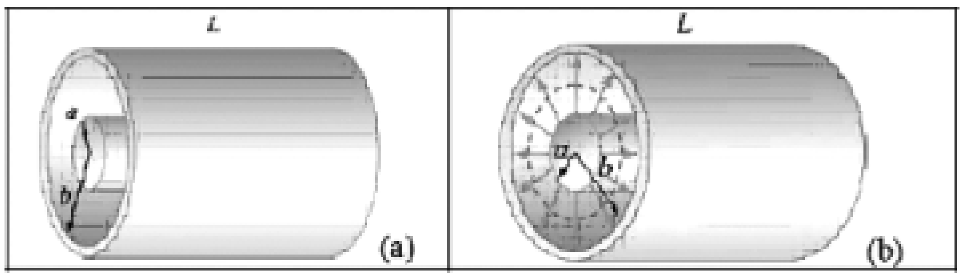
This means

charge of the capacitor is $q = \frac{Q_1 - Q_2}{2}$

capacitance is still $C = \frac{\varepsilon_0 A}{d}$

Cylindrical Capacitor

Consider next a solid cylindrical conductor of radius a surrounded by a coaxial cylindrical shell of inner radius b, as shown in Figure. The length of both cylinders is L and we take this length to be much larger than $b - a$, the separation of the cylinders, so that edge effects can be neglected. The capacitor is charged so that the inner cylinder has charge $+Q$ while the outer shell has a charge $-Q$. What is the capacitance?



(a) A cylindrical capacitor. (b) End view of the capacitor. The electric field is non-vanishing only in the region $a < r < b$.

The potential difference is given by

$$\Delta V = V_b - V_a = -\int_a^b E_r\,dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

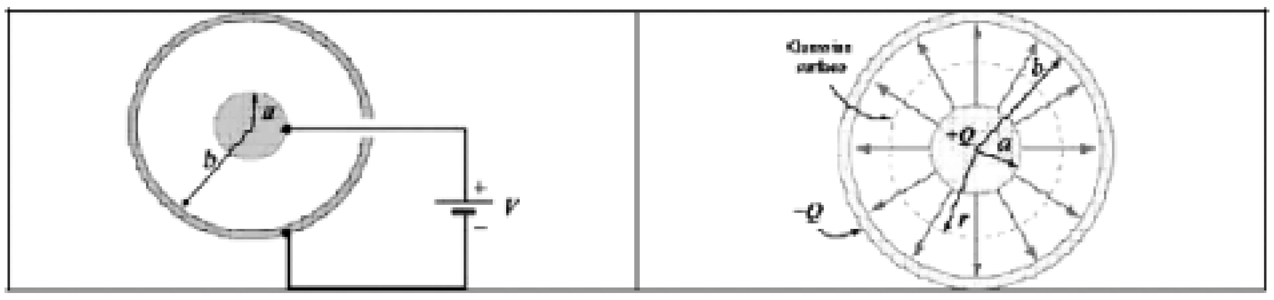
where we have chosen the integration path to be along the direction of the electric field lines. As expected, the outer conductor with negative charge has a lower potential. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{\lambda \ln(b/a)/r\pi\varepsilon_0} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$$

Once again, we see that the capacitance *C* depends only on the geometrical factors, *L*, *a* and *b*.

Spherical Capacitor

As a third example, let's consider a spherical capacitor which consists of two concentric spherical shells of radii *a* and *b*, as shown in Figure. The inner shell has a charge $+Q$ uniformly distributed over its surface, and the outer shell an equal but opposite charge $-Q$. What is the capacitance of this configuration?



(a) spherical capacitor with two concentric spherical shells of radii a and b.
(b) Gaussian surface for calculating the electric field.

The potential difference between the two conducting shells is :

$$\Delta V = V_b - V_a = -\int_a^b E_r\,dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

which yields

$$C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

Again, the capacitance C depends only on the physical dimensions, a and b.

An "isolated" conductor (with the second conductor placed at infinity) also has a capacitance. In the limit where $b \to \infty$, the above equation becomes.

$$\lim_{b\to\infty} C = \lim_{b\to\infty} 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \lim_{b\to\infty} 4\pi\varepsilon_0 \frac{a}{\left(1 - \frac{a}{b}\right)} = 4\pi\varepsilon_0 a$$

Thus, for a single isolated spherical conductor of radius R, the capacitance is

$$C = 4\pi\varepsilon_0 R$$

The above expression can also be obtained by noting that a conducting sphere of radius R with a charge Q uniformly distributed over its surface has $V = Q/4\pi\varepsilon_0 R$, using infinity as the reference point having zero potential, $V(\infty) = 0$. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{Q/4\pi\varepsilon_0 R} = 4\pi\varepsilon_0 R$$

As expected, the capacitance of an isolated charged sphere only depends on its geometry, namely, the radius R.

Illustration:

A parallel plate air capacitor is made using two plates 0.2 m square, spaced 1 cm apart. It is connected to a 50 V battery.

(a) *what is the capacitance ?*

(b) *what is the charge on each plate ?*

(c) *what is the electric field between the plates ?*

(d) *if the battery is disconnected and then the plates are pulled apart to a separation of 2 cm, what are the Answers to the above parts ?*

Sol. *(a)* $$C_0 = \frac{\varepsilon_0 A}{d_0} = \frac{8.85\times10^{-12}\times0.2\times0.2}{0.01}$$

$$C_0 = 3.54\times10^{-5}\ \mu F$$

(b) $Q_0 = C_0 V_0 = \left(3.54\times10^{-5}\times50\right)\mu C \quad = 1.77\times10^{-3}\ \mu C$

(c) $$E_0 = \frac{V_0}{d_0} = \frac{50}{0.01} = 5000\ V/m.$$

(d) *If the battery is disconnected, the charge on the capacitor plates remains constant while the potential difference between plates can change.*

$$C = \frac{A\,\epsilon_0}{2d} = 1.77\times10^{-5}\ \mu F$$

$$Q = Q_0 = 1.77\times10^{-3}\ \mu C$$

$$V = \frac{Q}{C} = \frac{Q_0}{C_0/2} = 2V_0 = 100 \text{ volts.}$$

$$E = \frac{V}{c}\frac{2V_0}{2d_0} = E_0 = 5000\ V/m.$$

Practice Exercise

Q.1 A capacitor having plate area A, separation between plates d is connected to a battery having potential difference across it as V. Find what happens to its provided the battery remains connected

(a) Capacitance (c) P.d, across capacitor plates

(b) Charge (d) Field between the plates

when its area is doubled

Q.2 A capacitor having plate area A, separation between plates d is connected to a battery having potential difference across it as V. Find what happens to its provided the battery is disconnected

(a) Capacitance (c) P.d, across capacitor plates

(b) Charge (d) Field between the plates

when its area is doubled

Q.3 Two identical metal plates are given positive charges Q_1 and Q_2 ($<Q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is :

Answers

Q.1 (a) doubled (b) doubled (c) remain same (d) remain same

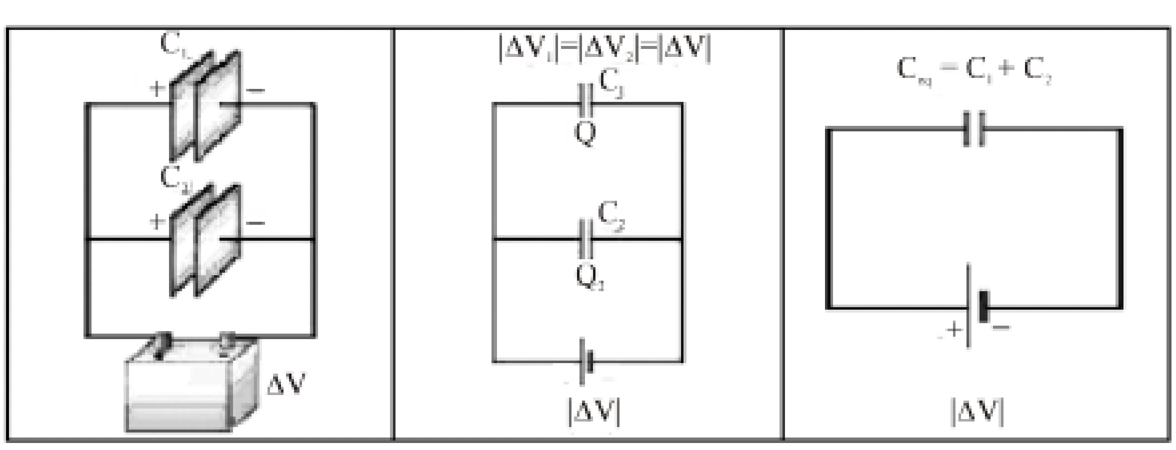
Q.2 (a) doubled (b) remain same (c) halved (d) halved

Q.3 $\dfrac{Q_1 - Q_2}{2C}$

Grouping of capacitors

Parallel Connection :

Suppose we have two capacitors C_1 and C_2 that are connected in parallel, as shown in Figure.



Capacitors in parallel and an equivalent capacitor.

The left plates of both capacitors C_1 and C_2 are connected to the positive terminal of the battery and have the same electric potential as the positive terminal. Similarly, both right plates are negatively charged and have the same potential as the negative terminal. Thus, the potential difference $|\Delta V|$ is the same across each capacitor. This gives

$$C_1 = \frac{Q_1}{|\Delta V|},\ C_2 = \frac{Q_2}{|\Delta V|}$$

These two capacitors can be replaced by a single equivalent capacitor with a total charge Q supplied by the battery. However, since Q is shared by the two capacitors, we must have

$$Q = Q_1 + Q_2 = C_1\,|\Delta V| + C_2\,|\Delta V| = (C_1 + C_2)\,|\Delta V|$$

The equivalent capacitance is then seen to be given by

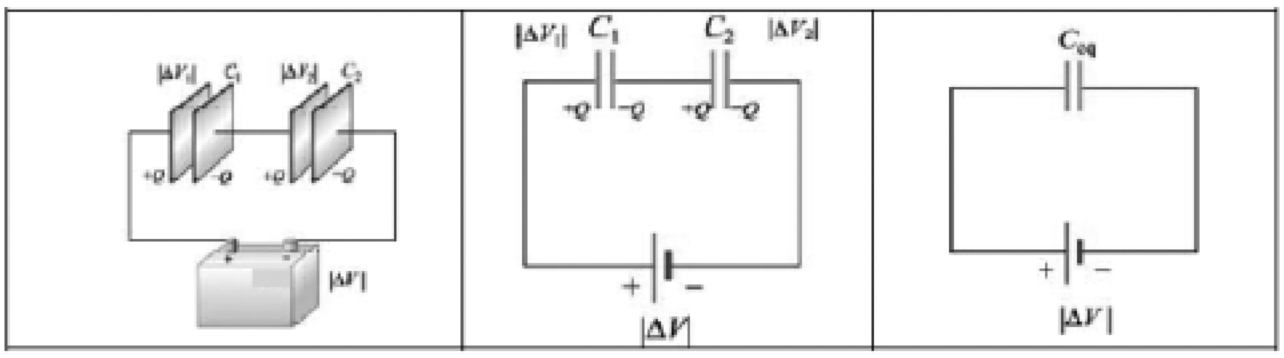
$$C_{eq} = \frac{Q}{|\Delta V|} = C_1 + C_2$$

Thus, capacitors that are connected in parallel add. The generalization to any number of capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + ... + C_N = \sum_{i=1}^{N} C_i \quad \text{(parallel)}$$

Series Connection :

Suppose two initially uncharged capacitors C_1 and C_2 are connected in series, as shown in Figure. A potential difference $|\Delta V|$ is then applied across both capacitors. The left plate of capacitor 1 is connected to the positive terminal of the battery and becomes positively charged with a charge $+Q$, while the right plate of capacitor 2 is connected to the negative terminal and becomes negatively charged with charge $-Q$ as electrons flow in. What about the inner plates? They were initially uncharged; now the outside plates each attract an equal and opposite charge. So the right plate of capacitor 1 will acquire a charge $-Q$ and the left plate of capacitor 2 will acquire a charge of + Q.



Capacitors in series and an equivalent capacitor

The potential differences across capacitors C_1 and C_2 are

$$|\Delta V_1| = \frac{Q}{C_1},\ |\Delta V_2| = \frac{Q}{C_2}$$

respectively. From Figure, we see that the total potential difference is simply the sum of the two individual potential differences:

$$|\Delta V| = |\Delta V_1| + |\Delta V_2|$$

In fact, the total potential difference across any number of capacitors in series connection is equal to the sum of potential differences across the individual capacitors. These two capacitors can be replaced by a single equivalent capacitor $C_{eq} = \dfrac{Q}{|\Delta V|}$. Using the fact that the potentials add in series,

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

and so the equivalent capacitance for two capacitors in series becomes

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Also

$$|\Delta V_1| = \frac{Q}{C_1} = \frac{C_2 V}{C_1 + C_2} \qquad |\Delta V_2| = \frac{Q}{C_2} = \frac{C_1}{C_1 + C_2} V$$

The generalization to any number of capacitors connected in series is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_N} = \sum_{i=1}^{N} \frac{1}{C_i} \text{ (series)}$$

Illustration:

Two capacitors of capacitance $C_1 = 6\ \mu F$ and $C_2 = 3\ \mu F$ are connected in series across a cell of emf 18 V.

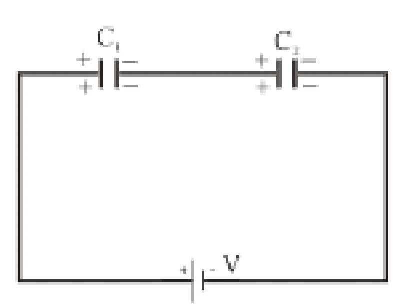
Calculate :

(a) the equivalent capacitance

(b) the potential difference across each capacitor

(c) the charge on each capacitor.

Sol. (a) $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$\Rightarrow \quad C = \frac{C_1 C_2}{C_1 + C_2} = \frac{6 \times 3}{6 + 3} = 2\mu F.$$

(b) $$V_1 = \frac{C_2}{C_1 + C_2} V = \frac{3}{6+3} \times 18 = 6 \text{ volts}$$

$$V_2 = \frac{C_1}{C_1 + C_2} V = \frac{6}{6+3} \times 18 = 12 \text{ volts}$$

Note that the smaller capacitor C_2 has a larger potential difference across it.

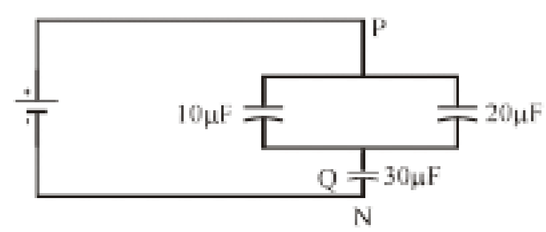
(c) $$Q_1 = Q_2 = C_1 V_1 = C_2 V_2 = CV$$

charge on each capacitor $= C_{eq} V$

$$= 2\mu F \times 18 \text{ volts} = 36\mu C$$

Illustration:

Find the equivalent capacitance of the combination shown in figure between the points P and N.

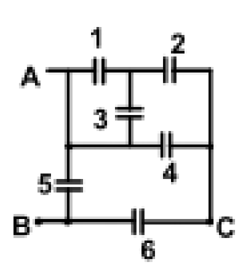


Sol. *The* $10\ \mu F$ *and* $20\ \mu F$ *capacitors are connected in parallel. Their equivalent capacitance is* $10\,\mu F + 20\,\mu F = 30\ \mu F$. *We can replace the* $10\ \mu F$ *and the* $20\ \mu F$ *capacitors by a single capacitor of capacitance* $30\,\mu F$ *between P and Q. This is connected in series with the given* $30\ \mu F$ *capacitor. The equivalent capacitance C of this combination is given by*

$$\frac{1}{C} = \frac{1}{30\mu F} + \frac{1}{30\mu F} \quad or, \quad C = 15\ \mu F.$$

Illustration:

Find the equivalent capacitance between points A and B capacitance of each capacitor is 2 μF.



Sol. *The circuit can be redrawn as*

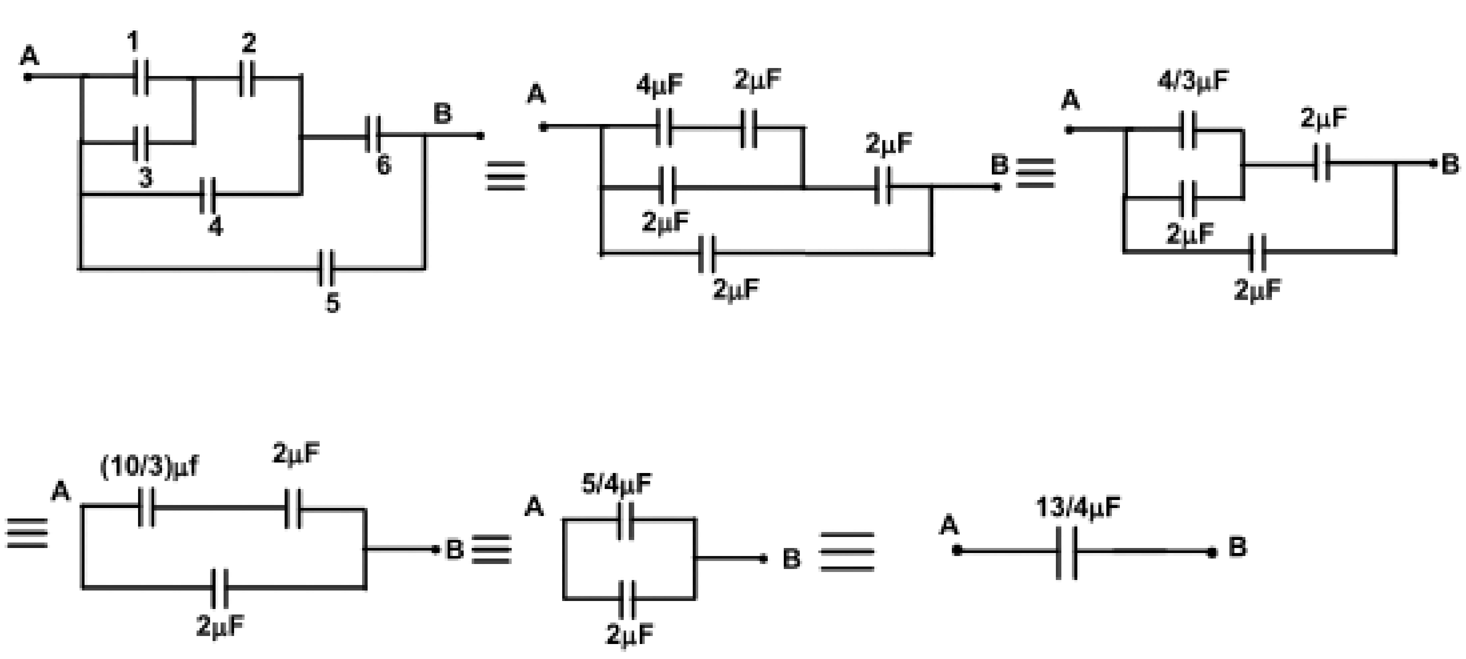
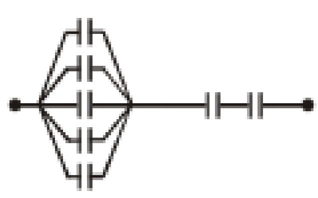
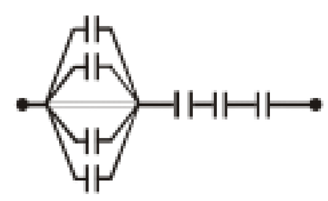
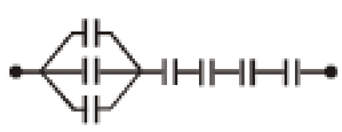


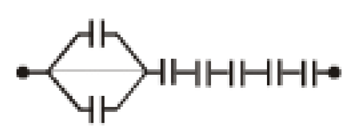
Illustration :

Seven capacitors each of capacitance $2\mu F$ *are to be connected in a configuration to obtain an effective capacitance of* $(10/11)\mu F$. *Which of the combination(s), shown in figure below, will achieve the desired result?*

(a) 

(b) 

(c) 

(d) 

Sol.

(a) $\frac{1}{C} = \frac{1}{5 \times 2} + \frac{2}{2} = \frac{11}{10}$ or $C = \frac{10}{11}\mu F$

(b) $\frac{1}{C} = \frac{1}{4 \times 2} + \frac{3}{2} = \frac{13}{8}$ or $C = \frac{8}{13}\mu F$

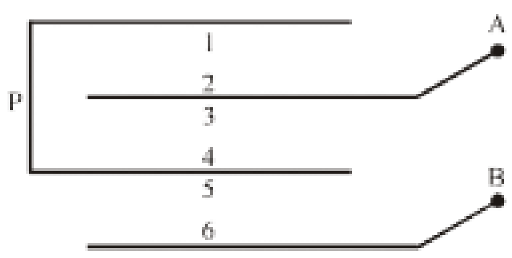
(c) $\frac{1}{C} = \frac{1}{3 \times 2} + \frac{4}{2} = \frac{13}{6}$ or $C = \frac{6}{13}\mu F$

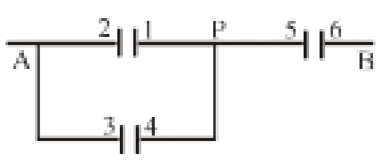
(d) $\frac{1}{C} = \frac{1}{2 \times 2} + \frac{5}{2} = \frac{11}{4}$ or $C = \frac{4}{11}\mu F$.

Illustration :

Four identical metal plates are located in air at equal separations d as shown. The area of each plate is A. Calculate the effective capacitance of the arrangement across A and B.

Sol. *Let us call the isolated plate as P. A capacitor is formed by a pair of parallel plates facing each other. Hence we have three capacitor formed by the pairs (1, 2), (3, 4) and (5, 6). The surface 2 and 3 are at same potential as that of A. The arrangement can be redrawn as a network of three capacitors.*

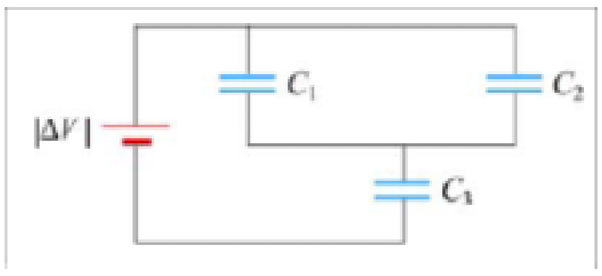




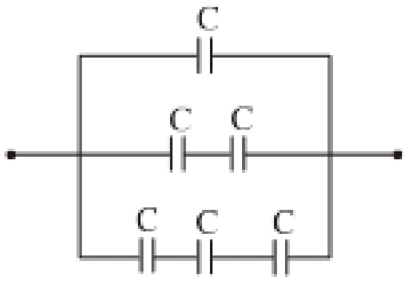
$$C_{AB} = \frac{2C.C}{2C + C} = \frac{2C}{3} = \frac{2}{3}\frac{\varepsilon_0 A}{d}$$

Practice Exercise

Q.1 Find the equivalent capacitance for the combination of capacitors shown in Figure.



Q.2 Find the equivalent capacitance, assuming that all the capacitors have the same capacitance C.



Q.3 Four identical metal plates are located in air at equal distances d from one another. The area of each plate is equal to A. Find the capacitance of the system between points A and B if the plates are interconnected as shown (a) in Fig. (a) (b) in Fig. (b)

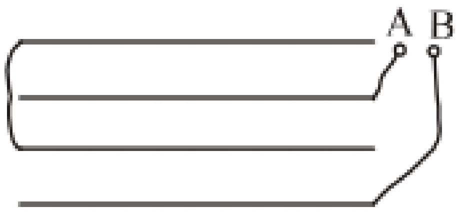


Fig. : (a)

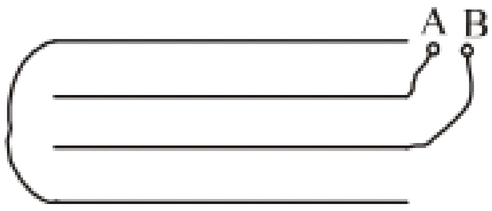
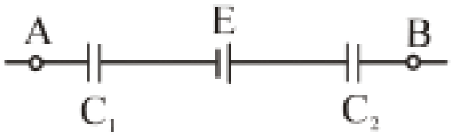


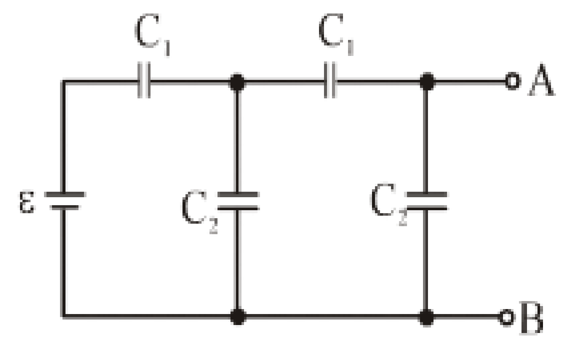
Fig. (b)

Q.4 A capacitor of capacitance $C_1 = 1.0\ \mu F$ with stands the maximum voltage $V_1 = 6.0$ kV while a capacitor of capacitance $C_2 = 2.0\ \mu F$, the maximum voltage $V_2 = 4.0$ kV. What voltage will the system of these two capacitors withstand if they are connected in series?

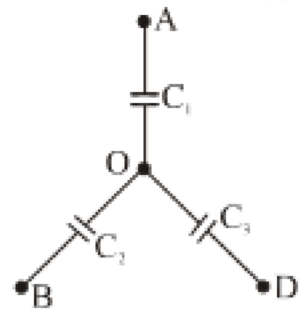
Q.5 A circuit has a section AB shown in Fig. The emf of the source equals E = 10V, the capacitor capacitances are equal to $C_1 = 1.0\ \mu F$ and $C_2 = 2.0\ \mu F$, and the potential difference $V_A - V_B = 5.0$ V. Find the voltage across each capacitor.



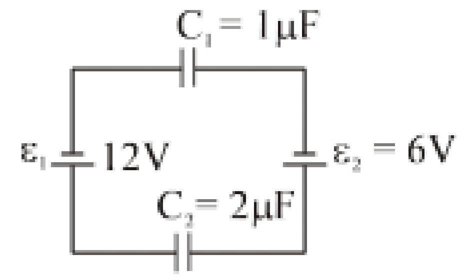
Q.6 Find the potential difference between points A and B of the system shown in Fig. If the emf is equal to $\xi = 110$ V and the capacitance ratio $C_2/C_1 = 2.0$.



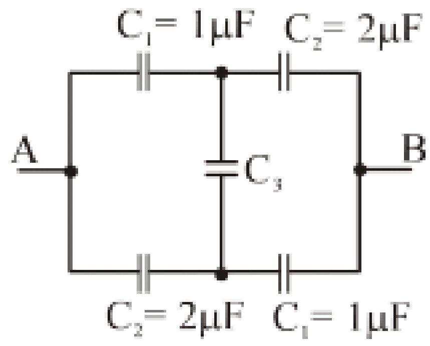
Q.7 Three uncharged capacitors of capacitance C_1, C_2 and C_3 are connected as shown in figure to one another and to points A, B and D at potentials V_A, V_B and V_D. Determine the potential V_0 at point O.



Q.8 In a circuit shown in Fig. Find the potential difference between the left and right plates of each capacitor.

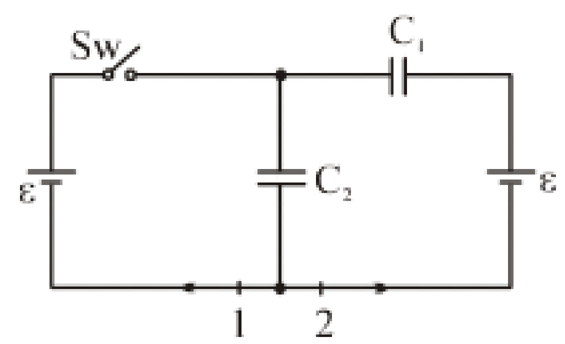


Q.9 Find the capacitance of the circuit shown in Fig. between points A and B.



Q.10 A capacitor of capacitance $C_1 = 1.0$ μF charged up to a voltage $V = 110$ V is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing the capacitances $C_2 = 2.0$ μF and $C_3 = 3.0$ μF. What charge will flow through the connecting wires?

Q.11 What charges will flow after the shorting of the switch Sw in the circuit illustrated in Fig. through sections 1 and 2 in the directions indicated by the arrows?



Practice Exercise

Q.1 $\dfrac{(C_1+C_2)C_3}{C_1+C_2+C_3}$

Q.2 $\dfrac{11}{6}$ C

Q.3 (a) $C = 2\varepsilon_0 A / 3d$; (b) $C = 3\varepsilon_0 A/2d$

Q.4 $V \le V_1 (1 + C_2 / C_2) = 9$ kV

Q.5 $V_1 = q / C_1 = 10V$, $V_2 = q / C_2 = 5V$, where $q = (V_A - V_B + E)\, C_1C_2 / (C_1 + C_2)$

Q.6 V= 10V

Q.7 $\dfrac{V_A C_1 + V_B C_2 + V_D C_2}{C_1 + C_2 + C_3}$

Q.8 $V_1 = -4V$, $V_2 = 2V$

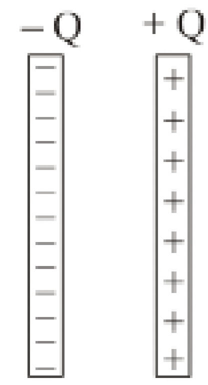
Q.9 $\dfrac{7}{5}$ μF

Q.10 0.06 mC

Q.11 $q_1 = \xi C_2$, $q_2 = -\xi C_1C_2 / (C_1 + C_2)$

Force on the Plates of a Capacitor

The plates of a parallel-plate capacitor have area A and carry total charge $\pm Q$ (see Figure). Electric field due to negative plate at the location of positive plate



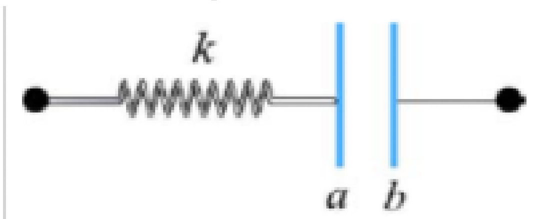
$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0 A}$$

force on the positive plate

$$F = q\,E_{ext} = (Q)\left(\frac{Q}{2\epsilon_0 A}\right) = \frac{Q^2}{2\epsilon_0 A} \text{ (attracting)}$$

Illustration:

Consider an air-filled parallel-plate capacitor with one plate connected to a spring having a force constant k, and another plate held fixed. The system rests on a table top as shown in Figure



Sol. For equilibrium

$$\frac{Q^2}{2A\epsilon_0} = kx \quad \Rightarrow \quad x = \frac{Q^2}{2kA\varepsilon_0}$$

Practice Exercise

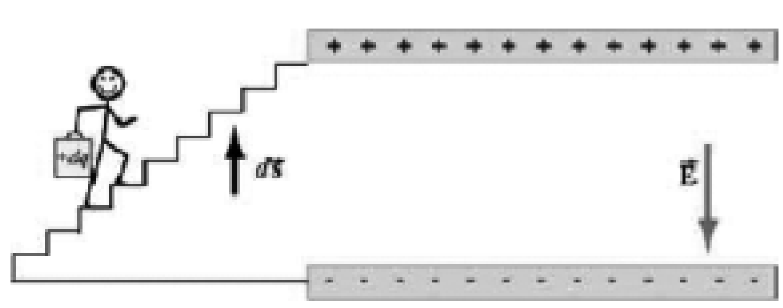
Q.1 Plates of a parallel plate of area A and separation between the plates d. Is charged to a potential difference of V. Find the attraction force between plates.

Answers

Q.1 $\dfrac{\epsilon_0 AV^2}{2d^2}$

Storing Energy in a Capacitor :

As discussed in the introduction, capacitors can be used to stored electrical energy. The amount of energy stored is equal to the work done to charge it. During the charging process, the battery does work to remove charges from one plate and deposit them onto the other.



Work is done by an external agent in bringing $+dq$ from the negative plate and depositing the charge on the positive plate.

Let the capacitor be initially uncharged. In each plate of the capacitor, there are many negative and positive charges, but the number of negative charges balances the number of positive charges, so that there is no net charge, and therefore no electric field between the plates. We have a magic bucket and a set of stairs from the bottom plate to the top plate (Fig.).

We start out at the bottom plate, fill our magic bucket with a charge +dq, carry the bucket up the stairs and dump the contents of the bucket on the top plate, charging it up positive to charge +dq. However, in doing so, the bottom plate is now charged to $-dq$. Having emptied the bucket of charge, we now descend the stairs, get another bucketful of charge $+dq$, go back up the stairs and dump that charge on the top plate. We then repeat this process over and over. In this way we build up charge on the capacitor, and create electric field where there was none initially.

Suppose the amount of charge on the top plate at some instant is $+q$, and the potential difference between the two plates is $|\Delta V| = q/C$. To dump another bucket of charge $+dq$ on the top plate, the amount of work done to overcome electrical repulsion is $dW = |\Delta V|\, dq$. If at the end of the charging process, the charge on the top plate is $+Q$, then the total amount of work done in this process is

$$W = \int_0^Q dq\, |\Delta V| = \int_0^Q dq \frac{q}{C} = \frac{1}{2}\frac{Q^2}{C}$$

This is equal to the electrical potential energy U_E of the system:

$$U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} Q\, |\Delta V| = \frac{1}{2} C\, |\Delta V|^2$$

Energy Density of the Electric Field :

One can think of the energy stored in the capacitor as being stored in the electric field itself. In the case of a parallel-plate capacitor, with $C = \varepsilon_0 A / d$ and $|\Delta V| = Ed$, we have

$$U_E = \frac{1}{2} C|\Delta V|^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2 = \frac{1}{2} e_0 E^2\,(Ad)$$

Since the quantity Ad represents the volume between the plates, we can define the electric energy density as

$$u_E = \frac{U_E}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

Note that u_E is proportional to the square of the electric field. Alternatively, one may obtain the energy stored in the capacitor from the point of view of external work. Since the plates are oppositely charged, force must be applied to maintain a constant separation between them. From Eq., we see that a small patch of charge $\Delta q = \sigma(\Delta A)$ experiences an attractive force $\Delta F = \sigma^2(\Delta A)/2\varepsilon_0$. . If the total area of the plate is A, then an external agent must exert a force $F_{ext} = \sigma^2 A/2\varepsilon_0$ to pull the two plates apart. Since the electric field strength in the region between the plates is given by $E = \sigma/\varepsilon_0$, the external force can be rewritten as

$$F_{ext} = \frac{\varepsilon_0}{2} E^2 A$$

Note that F_{ext} is independent of d. The total amount of work done externally to separate the plates by a distance d is then

$$W_{ext} = \int \vec{F}_{ext} . d\vec{s} = F_{ext} d = \left(\frac{\varepsilon_0 E^2 A}{2}\right) d$$

consistent with Eq. Since the potential energy of the system is equal to the work done by the external agent, we have $u_E = W_{ext} / Ad = \varepsilon_0 E^2 / 2$. The electric energy density u_E can also be interpreted as electrostatic pressure P.

First law of thermodynamics in Capacitors:

If heat liberated by system = $Q_{liberated}$

$\Rightarrow \quad \Delta Q = -Q_{liberated}$

Work done by the system

$$\Delta W = -\Delta W_{battery}$$

Now using

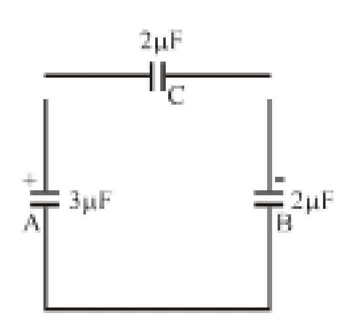
$$\Delta Q = \Delta U + \Delta W$$

$\Rightarrow \quad -Q_{liberated} = \Delta U_{capacitor} - \Delta W_{battery}$

$\therefore \quad \Delta W_{battery} = \Delta U_{capacitor} + Q_{liberated}$

Therefore, as mentioned in the beginning of chapter, the work done by battery goes in storing energy in capacitor and rest goes as heat loss in resistor.

Illustration :



Two capacitors A and B with capacities 3 μF and 2μF are charged to a potential difference of 100V and 180V respectively. The plates of the capacitors are connected as shown in the figure with one wire of each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate :

(i) the final charge on the three capacitors, and

(ii) the amount of electrostatic energy stored in the system before and after the completion of the circuit.

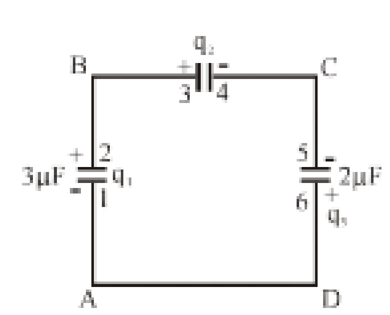
Sol. *Charge on capacitor A, before joining with an uncharged capacitor,*

$$q_A = CV = (100) \times 3\ \mu c = 300\ \mu c$$

similarly charge on capacitor B,

$$q_B = 180 \times 2\ \mu c$$

$$= 360\ \mu c$$



Let q_1, q_2 and q_3 be the charges on the three capacitors after joining them as shown in fig.

From conservation of charge,

Net charge on plates 2 and 3 before joining

= *Net charge after joining*

$\therefore \quad 300 = q_1 + q_2 \qquad \ldots (1)$

Similarly, net charge on plates 4 and 5 before joining

= *Net charge after joining*

$$-360 = -q_2 - q_3$$

$$360 = q_2 + q_3 \qquad \text{.... (2)}$$

applying Kirchoff's 2nd law in loop ABCDA,

$$\frac{q_1}{3} - \frac{q_2}{2} + \frac{q_3}{2} = 0$$

$$2q_1 - 3q_2 + 3q_3 = 0 \qquad \text{.... (3)}$$

From equations (1), (2) and (3),

$$q_1 = 90\ \mu c,\ q_2 = 90\ \mu c \text{ and } q_3 = 150\ \mu c$$

(ii) (a) Electrostatic energy stored before completing the circuit,

$$U_i = \frac{1}{2}(3 \times 10^{-6})\,(100)^2 + \frac{1}{2}(2 \times 10^{-6})(180)^2 \qquad \left(U = \frac{1}{2}CV^2\right)$$

$$= 4.74 \times 10^{-2}\ J$$

$$= 47.4\ mJ.$$

(b) Electrostatic energy stored after completing the circuit,

$$U_f = \frac{1}{2}(90\times10^{-6})^2.\frac{1}{3\times10^{-6}} + \frac{1}{2}(90\times10^{-6})^2.\frac{1}{2\times10^{-6}} + \frac{1}{2}(150+10^{-6})^2.\frac{1}{2\times10^{-6}} \qquad \left(U = \frac{1}{2}\frac{q}{c^2}\right)$$

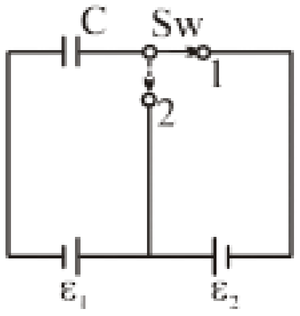
$$= 90\times10^{-4}\ J$$

$$= 9\ mJ.$$

Practice Exercise

Q.1 A capacitor of capacitance $C_1 = 1.0\ \mu F$ carrying initially a voltage $V = 300$ V is connected in parallel with an uncharged capacitor of capacitance $C_2 = 2.0\ \mu F$. Find the loss of the electric energy of this system by the moment equilibrium is reached. Explain the result obtained.

Q.2 What amount of heat will be generated in the circuit shown in Fig. after the switch Sw is shifted from position 1 to position 2?



Q.3 Each plate of a parallel-plate air capacitor has an area A. What amount of work has to be performed to slowly increase the distance between the plates from x_1 to x_2 if
(a) the charge on the capacitor, which is equal to q, or (b) the voltage across the capacitor, which is equal to V, is kept constant in the process?

Answers

Q.1 $\Delta W = -1/2\ V^2C_1C_2 / (C_1 + C_2)$ 0.03 mJ Q.2 $Q = 1/2C\,\xi_2^2$

Q.3 (a) $W = q^2 (x_2 - x_1) / 2\varepsilon_0 A$; (b) $W = \varepsilon_0 AV^2 (x_2 - x_1) / 2x_1x_2$

Dielectrics

Dielectric is any insulating substance (insulator). It can be rubber, plastic wood, oil etc. In contrast to conductors, the electrons in dielectrics are attached to specific atoms or molecules, so they are not allowed from moving randomly at will. They are in tight leash; all they can do is move a bit within the atom or a molecule.

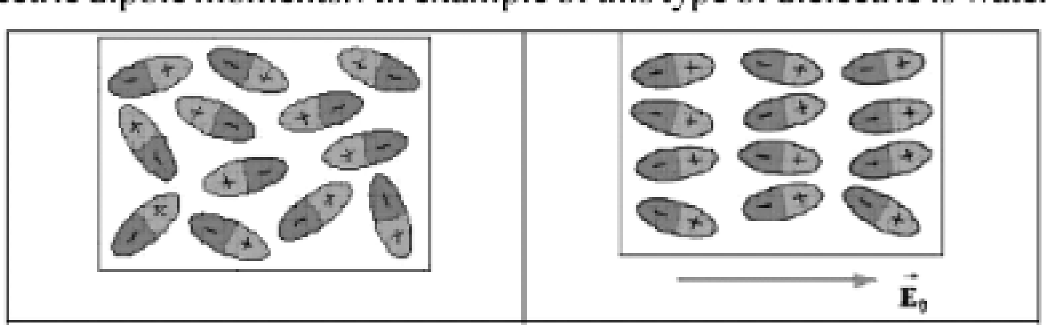
Experimentally it was found that capacitance *C* increases when the space between the conductors is filled with dielectrics. To see how this happens, suppose a capacitor has a capacitance when there is no material between the plates. When a dielectric material is inserted to completely fill the space between the plates, the capacitance increases to K_e times i.e.

$$C = K_e C_0$$

where K_e is called the dielectric constant or relative permittivity. In the Table below, we show some dielectric materials with their dielectric constant. Experiments indicate that all dielectric materials have $K_e > 1$.

The fact that capacitance increases in the presence of a dielectric can be explained from a molecular point of view. We shall show that K_e is a measure of the dielectric response to an external electric field.

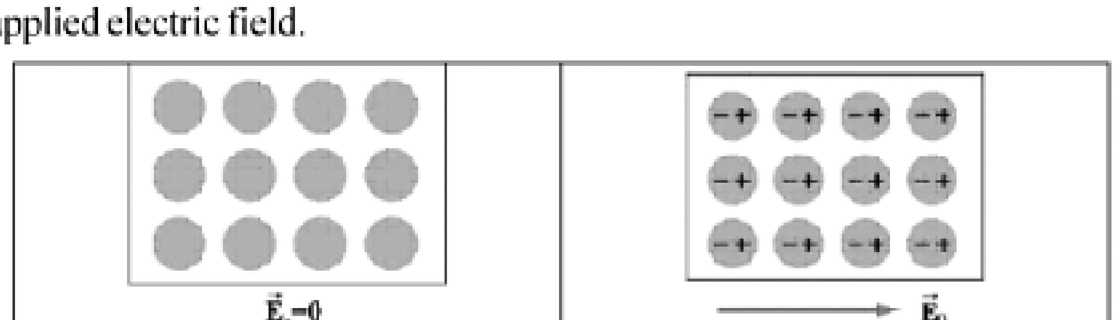
There are two types of dielectrics. The first type is polar dielectrics, which are dielectrics that have permanent electric dipole moments. An example of this type of dielectric is water.



Orientations of polar molecules when (a) $\vec{E}_0 = \vec{0}$ and (b) $\vec{E}_0 \neq 0$

As depicted in Figure, the orientation of polar molecules is random in the absence of an external field. When an external electric field $\vec{E}_0$ is present, a torque is set up and causes the molecules to align with $\vec{E}_0$. However, the alignment is not complete due to random thermal motion. The aligned molecules then generate an electric field that is opposite to the applied field but smaller in magnitude.

The second type of dielectrics is the non-polar dielectrics, which are dielectrics that do not possess permanent electric dipole moment. Electric dipole moments can be induced by placing the materials in an externally applied electric field.



Orientations of non-polar molecules when (a) $\vec{E}_0 = \vec{0}$ and (b) $\vec{E}_0 \neq \vec{0}$

Figure illustrates the orientation of non-polar molecules with and without an external field $\vec{E}_0$. The induced surface charges on the faces produces an electric field $\vec{E}_P$ in the direction opposite to $\vec{E}_0$, leading to $\vec{E} = \vec{E}_0 + \vec{E}_P$, with $|\vec{E}_P| < |\vec{E}_0|$. Below we show how the induced electric field $\vec{E}_P$ is calculated.

Let us now examine the effects of introducing dielectric material into a system. We shall first assume that the atoms or molecules comprising the dielectric material have a *permanent* electric dipole moment. If left to themselves, these permanent electric dipoles in a dielectric material never line up spontaneously, so that in the absence of any applied external electric field $\vec{p} = \vec{0}$ due to the random alignment of dipoles, and the average electric field $\vec{E}_P$ is zero as well. However, when we place the dielectric material in an external field $\vec{E}_0$, the dipoles will experience a torque $\vec{\tau} = \vec{p} \times \vec{E}_0$ that tends to align the dipole vectors $\vec{p}$ with $\vec{E}_0$. The effect is a net polarization $\vec{p}$ parallel to $\vec{E}_0$, and therefore an average electric field of the dipoles $\vec{E}_P$ *anti-parallel* to $\vec{E}_0$, i.e., that will tend to *reduce* the total electric field strength below $\vec{E}_0$. The total electric field $\vec{E}$ is the sum of these two fields:

$$\vec{E} = \vec{E}_0 + \vec{E}_P$$

Note that every dielectric material has a characteristic dielectric strength which is the maximum value of electric field before breakdown occurs and charges begin to flow.

Material	κ_e	Dielectric strength $(10^6\,\mathrm{V/m})$
Air	1.00059	3
Paper	3.7	16
Glass	4–6	9
Water	80	–

Revised energy density becomes: $u = \frac{1}{2}K\varepsilon_0 E^2$

Calculation of induced (polarised) charge on dielectric :

However, we have just seen that the effect of the dielectric is to weaken the original field E_0 by a factor K_e. Therefore,

$$E = \frac{E_0}{K_e} = \frac{Q}{K_e \varepsilon_0 A} = \frac{Q - Q_P}{\varepsilon_0 A}$$

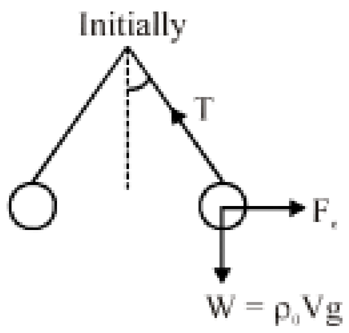
from which the induced charge Q_P can be obtained as

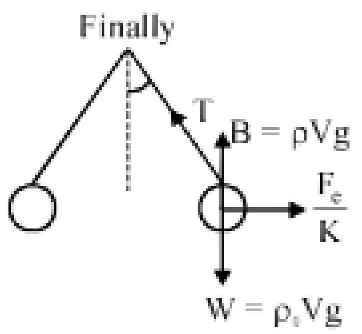
$$Q_P = Q\left(1 - \frac{1}{K_e}\right)$$

Illustration :

Two small identical balls carrying the charges of the same sign are suspended from the same point by insulating threads of equal length. When the surrounding space was filled with kerosene (of density ρ_0*, dielectric constant K) the divergence angle between the threads remained constant. What is the density of the material of which the balls are made?*

Sol.





When dielectric is filled, force on charge is reduced by K factor

According to question

$$\theta = \theta'$$

$$\Rightarrow \quad \tan\theta = \tan\theta' \Rightarrow \quad \frac{F}{\rho_0 Vg} = \frac{\frac{F}{K}}{(\rho - \rho_0)Vg} \quad \Rightarrow \quad \rho = \frac{K}{K-1}\rho_0$$

Practice Exercise

Q.1 The charges on the plates of a parallel-plate capacitor are of opposite sign, and they attract each other. To increase the plate separation, is the external work done positive or negative? What happens to the external work done in this process?

Q.2 How does the stored energy change if the potential difference across a capacitor is tripled?

Q.3 Does the presence of a dielectric increase or decrease the maximum operating voltage of a capacitor? Explain.

Answers

Q.1 +ve Q.2 9 times Q.3 Depends on the dielectric strength

Capacitor Containing Dielectrics without Battery :

As shown in Figure, a battery with a potential difference $|\Delta V_0|$ .across its terminals is first connected to a capacitor C_0, which holds a charge $Q_0 = C_0|\Delta V_0|$. We then disconnect the battery, leaving Q_0 = const.

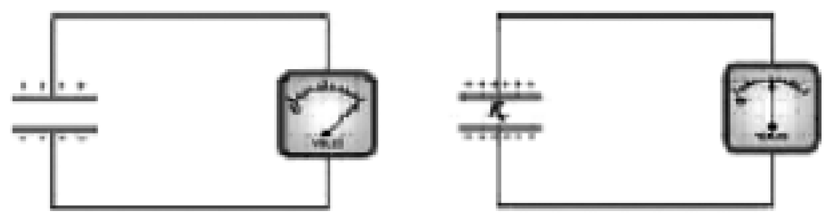


Fig.: 44 - Inserting a dielectric material between the capacitor plates while keeping the charge Q_0 constant
If we then insert a dielectric between the plates, while keeping the charge constant, experimentally it is found that the potential difference decreases by a factor of *Ke*.

$$|\Delta V| = \frac{|\Delta V_0|}{K_e}$$

This implies that the capacitance is changed to

$$C = \frac{Q}{|\Delta V|} = \frac{Q_0}{|\Delta V_0|/K_e} = K_e \frac{Q_0}{|\Delta V_0|} = K_e C_0$$

Thus, we see that the capacitance has increased by a factor of *Ke. The* electric field within the dielectric is now

$$E = \frac{|\Delta V|}{d} = \frac{|\Delta V_0|/K_e}{d} = \frac{1}{K_e}\left(\frac{|\Delta V_0|}{d}\right) = \frac{E_0}{K_e}$$

We see that in the presence of a dielectric, the electric field decreases by a factor of K_e.

Capacitor Containing Dielectrics with Battery :

Consider a second case where a battery supplying a potential difference remains connected as the dielectric is inserted. Experimentally, it is found (first by Faraday) that the charge on the plates is increased by a factor K_e.

$$Q = K_e Q_0$$

where Q_0 is the charge on the plates in the absence of any dielectric.

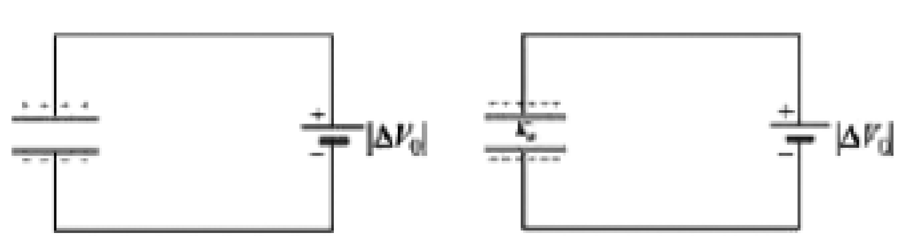


Fig. : 45 - (a) Fig. : 45 - (b)

Figure : Inserting a dielectric material between the capacitor plates while maintaining a constant potential difference $|V_0|$

The capacitance becomes

$$C = \frac{Q}{|\Delta V_0|} = \frac{K_e Q_0}{|\Delta V_0|} = K_e C_0$$

which is the same as the first case where the charge Q_0 is kept constant, but now the charge has increased.

Illustration:

A parallel plate capacitor has plates of area 4 m^2 separated by a distance of 0.5 mm. The capacitor is connected across a cell of emf 100 volts. Find the capacitance, charge & energy stored in the capacitor if a dielectric slab of dielectric constant k=3 and thickness 0.5 mm is inserted inside this capacitor after it has been disconnected from the cell.

Sol.

when the capacitor is without dielectric

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 4}{0.5 \times 10^{-3}}$$

$$C_0 = 7.08 \times 10^{-2}\ \mu F.$$

$$Q_0 = C_0 V_0$$

$$= (7.08 \times 10^{-2} \times 100)\mu C = 7.08\,\mu C$$

$$U_0 = \frac{1}{2} C_0 V_0^2 = 354 \times 10^{-6}\ J.$$

as the cell has been disconnected, charge on the capacitor remain constant

$$C = \frac{k\varepsilon_0 A}{d} = kC_0 = 0.2124\,\mu F$$

$$V = \frac{Q}{C} = \frac{Q_0}{kC_0} \frac{V_0}{k} = \frac{100}{3}\ \text{volts.}$$

$$U = \frac{1}{2}\frac{Q_0}{C} = \frac{1}{2}\frac{Q_0^2}{kC_0} = \frac{U_0}{k} = 118 \times 10^{-6}\ J.$$

Electric field inside the plates $= E = \frac{V}{d} = \frac{V_0}{kD} = \frac{E_0}{k}$

Note that the field becomes 1/k times by insertion of dielectric.

Illustration :

A 6 × 10^{-9} F parallel plate capacitor is connected to a 500 V battery. When air is replaced by another dielectric material. 7.5 × 10^{-6} C charge flows into the capacitor. Find the dielectric constant of the material

Sol. $Q = CV$

$$Q_1 = 6 \times 10^{-9} \times 500$$
$$= 3 \times 10^{-6}\ C$$

After insertion of dielectric

$$Q'_1 = (3+7.5) \times 10^{-6}\ C$$
$$= 10.5 \times 10^{-6}\ C$$
$$Q'_1 = CVK$$
$$10.5 \times 10^{-6} = 6 \times 10^{-9} \times 500\, K$$
$$K = 3.5$$

Capacitance of capacitor filled partially with dielectric :

A non-conducting slab of thickness t, area A and dielectric constant K_e is inserted into the space between the plates of a parallel-plate capacitor with spacing d, charge Q and area A, as shown in Figure(a). The slab is not necessarily halfway between the capacitor plates. What is the capacitance of the system?

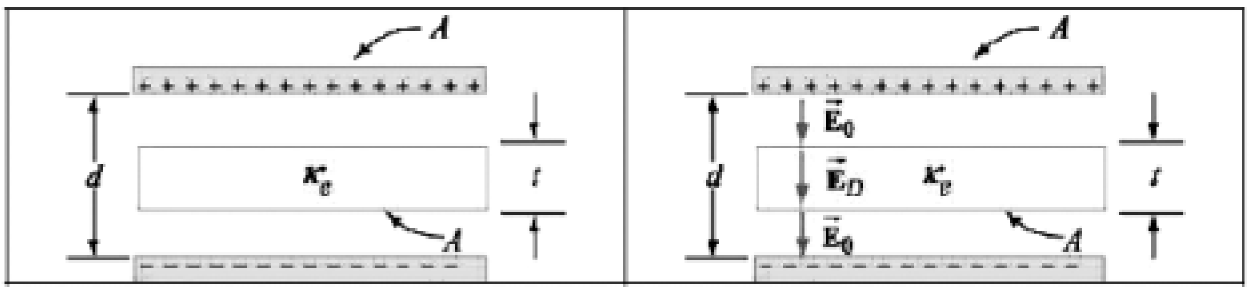


Fig. : 46 (a) Capacitor with a dielectric. (b) Electric field between the plates.

To find the capacitance C, we first calculate the potential difference ΔV. We have already seen that in the absence of a dielectric, the electric field between the plates is given by $E_0 = Q/\varepsilon_0 A$ and $E_D = E_0 / K_e$ when a dielectric of dielectric constant K_e is present, as shown in Figure (b). The potential can be found by integrating the electric field along a straight line from the top to the bottom plates:

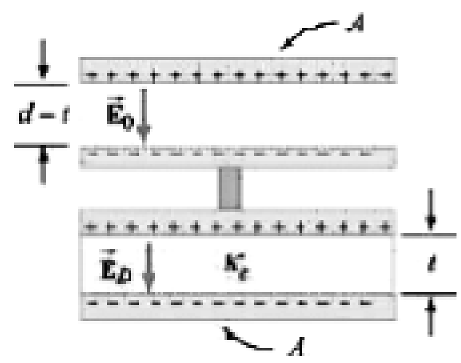
$$\Delta V = -\int_{+}^{-} E\,d\ell = -\Delta V_0 - \Delta V_D = -E_0(d-t) - E_D t = -\frac{Q}{A\varepsilon_0}(d-t) - \frac{Q}{A\varepsilon_0 K_e}t$$

$$= -\frac{Q}{A\varepsilon_0}\left[d - t\left(1 - \frac{1}{K_e}\right)\right]$$

where $\Delta V_D = E_D t$ is the potential difference between the two faces of the dielectric. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K_e}\right)}$$

We also comment that the configuration is equivalent to two capacitors connected in series, as shown in Figure.



Using Eq. for capacitors connected in series, the equivalent capacitance is

$$\frac{1}{C}=\frac{d-t}{\varepsilon_0 A}+\frac{t}{K_e \varepsilon_0 A}$$

It is useful to check the following limits :

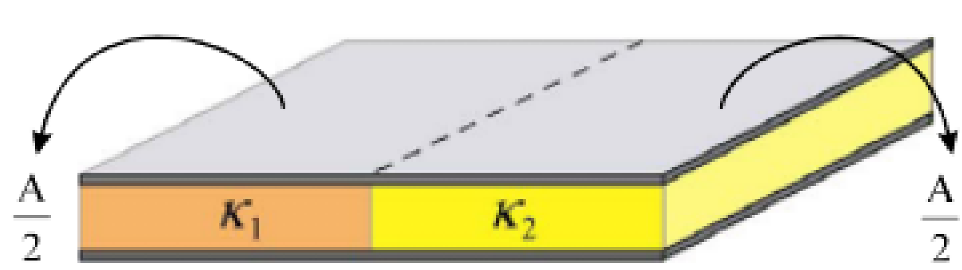
(i) *As $t \to 0$ i.e., the thickness of the dielectric approaches zero, we have $C = \varepsilon_0 A/d = C_0$, which is the expected result for no dielectric.*

(ii) *As, $K_e \to 1$, we again have $C \to \varepsilon_0 A/d = C_0$, and the situation also correspond to the case where the dielectric is absent.*

(iii) *In the limit where $t \to d$, the space is filled with dielectric, we have. $C \to K_e \varepsilon_0 A/d = K_e C_0$,*

Capacitor filled with two different dielectrics

Two dielectric with dielectric constant K_1 and K_2 each fill half the space between the plates of a parallel-plate capacitor as shown in figure



Capacitor filled with two different dielectrics

Each plate has an area A and the plates are separated by a distance d. Compute the capacitance of the system.

$$C_1 = \frac{K_1 \epsilon_0 (A/2)}{d} = \frac{K_1 \epsilon_0 A}{2d}, \qquad C_2 = \frac{K_2 \epsilon_0 A}{2d}$$

Since the two capacitors are in parallel

$$\Rightarrow \quad C = C_1 + C_2 = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

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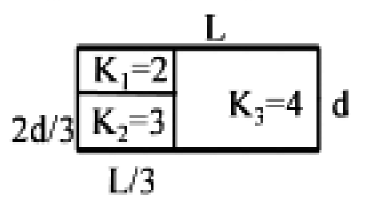
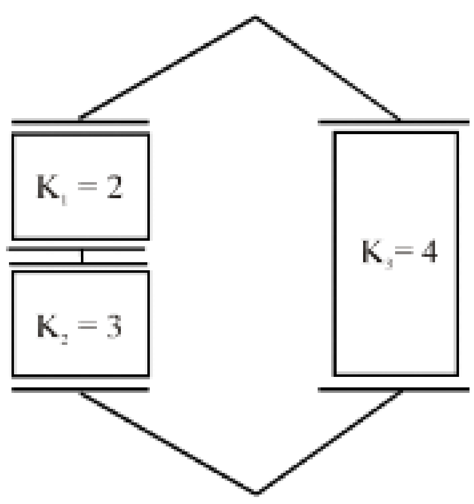
Illustration :

Find the equivalent capacitance of the system shown (assume square plates).



Sol. *The system can be represent as*



Taking $K_1 = 2$ to be series in $K_2 = 3$

$$\Rightarrow \quad \frac{1}{C_{\text{left}}} = \frac{1}{\dfrac{(2)\epsilon_0\left\{(L)\left(\frac{L}{3}\right)\right\}}{\left(\frac{d}{3}\right)}} + \frac{1}{\dfrac{(3)\epsilon_0\left\{(L)\left(\frac{L}{3}\right)\right\}}{\left(\frac{2d}{3}\right)}} \quad \Rightarrow \quad C_{\text{left}} = \frac{6\epsilon_0 L^2}{7d}$$

Now

$$C_{\text{right}} = \frac{(4)\epsilon_0\left\{(L)\left(\frac{2L}{3}\right)\right\}}{d} = \frac{8\epsilon_0 L^2}{3d}$$

Now $\quad C_{left}$ *and* C_{right} *are in parallel*

$$\Rightarrow \quad C_{eq} = C_{left} + C_{right} = \frac{6\epsilon_0 L^2}{7d} + \frac{8\epsilon_0 L^2}{3d} = \frac{74\epsilon_0 L^2}{21d}$$

Practice Exercise

Q.1 A parallel plate capacitor has a capacitance of 112 pF, a plate area of 96.5 cm², and a mica dielectric ($k_e = 5.40$). At a 55 V potential difference, calculate

(a) The electric field strength in a the mica.

(b) The magnitude of the free charge on the plates.

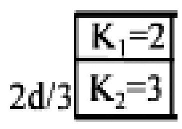
(c) The magnitude of the induced surface charge.

Q.2 Two parallel-plate air capacitors, each of capacitance C, were connected in series to a battery with emf ξ. Then one of the capacitors was filled up with slab of dielectric constant k.

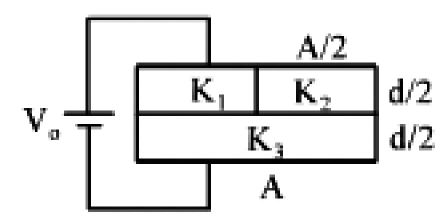
(a) What amount of charge flows through the battery?

(b) Find the factor by which electric field in each capacitor charges during the process. $\left(\text{i.e. } \frac{E_{after}}{E_{before}}\right)$

Q.3 Find the equivalent capacitance of the system shown (assume square plates each of area A).



Q.4 An ideal parallel plate capacitor of area A is filled with three dielectric slabs having dielectric constants $K_1 = 3.0$, $K_2 = 5.0$ and $K_3 = 2.0$ as shown. If a single dielectric material is to be used to have the same capacitance as this capacitor, then find its dielectric constant



Answers

Q.1 (a) 13.4 kV/m, (b) 6.16 nC, (c) 5.02 nC

Q.2 The strength decreased 1/2 (ε + 1) times ; (a) q = 1/2 C ξ (ε –1) / (ε + 1)]

Q.3 $\frac{18 \epsilon_0 A}{5d}$ Q.4 8/3

Energy Related discussion for dielectric capacitor

Illustration :

In the figure shown, a parallel plate capacitor is connected across a source of emf ε. The plates are square shaped with edge 'ℓ' and separated by a distance d. A dielectric slab of dielectric constant k and thickness d is inserted between the plates with constant speed v. Find the current in the connecting wires [ignore the resistance of connecting wires].

Sol. *Consider that length x of the dielectric is inside the capacitor. The capacitance of the system is*

$$C = \frac{\varepsilon_0 k x \ell}{d} + \frac{\varepsilon_0 \ell(\ell - x)}{d} = \frac{\varepsilon_0 \ell}{d}[(k-1)x + \ell]$$

Charge on the capacitor,

$$q = \frac{\varepsilon_0 \ell}{d}[(k-1)x + \ell]\varepsilon$$

$$\Rightarrow \quad I = \frac{dq}{dt} = \frac{\varepsilon_0 \ell}{d}(k-1)\varepsilon\frac{dx}{dt} = \frac{\varepsilon_0 \ell \varepsilon v(k-1)}{d}$$

Illustration :

A dielectric completely fills the gap between the plates of a parallel-plate capacitor whose capacitance is equal to C_0 when the dielectric is absent. Find the mechanical work which must be done against electric forces for extracting the dielectric out of the capacitor if

(i) Voltage (V) of the capacitor is maintained constant.

(ii) Charge (Q) of the capacitor is maintained constant.

Neglect the resistance of the circuit (If any)

Sol. *(i) T maintain constant voltage we have to used ideal battery*

change in capacitance

$$\Delta C = C_0 - KC_0 = -(K-1)\,C_0$$

Charge supplied by battery

$$\Delta q = V\Delta C$$

Work done by battery

$$W_b = V\Delta q = V^2\Delta C$$

Change in energy of capacitor

$$\Delta U = \frac{1}{2}V^2\Delta C$$

Now using work energy theorem

$$W_{mechanical} + W_b = \Delta K + \Delta U$$

$$\Rightarrow \quad W_{mechanical} = \Delta K + \Delta U - W_b = 0 + \frac{1}{2}V^2\Delta C - V^2\Delta C = -\frac{1}{2}V^2\Delta C = \frac{1}{2}(K-1)C_0V^2$$

(ii) To maintain constant charge capacitor should not be connected with the battery or any thing else]

$$\Delta q = 0$$

$$\Delta U = \frac{Q^2}{2C_0} - \frac{Q^2}{2KC_0} = \frac{Q^2}{2C_0}\left(1-\frac{1}{K}\right)$$

$$W_{mechanical} = \Delta K + \Delta U - W_b = 0 + \frac{Q^2}{2C_0}\left(1-\frac{1}{K}\right) - 0 = \frac{Q^2}{2C_0}\left(1-\frac{1}{K}\right)$$

Practice Exercise

Q.1 Between the plates of a parallel-plate capacitor there is a metallic plate whose thickness takes up $\eta = 0.60$ of the capacitor gap. When that plate is absent the capacitor has a capacity $C = 20$ nF. The capacitor is connected to a dc voltage source $V = 100$ V. The metallic plate is slowly extracted from the gap. Find: (a) the change in the energy of the capacitor; (b) the mechanical work performed in the process of plate extraction.

Answers

Q.1 (a) $\Delta U = -1/2\, CV^2\eta / (1-\eta) = -0.15$ mJ ; (b) $W = 1/2CV^2\eta / (1-\eta) = 0.15$ mJ

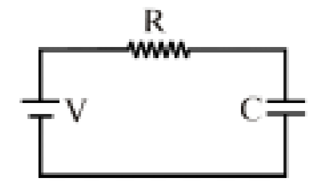
RC Circuit

Charging a capacitor :

Let the capacitor be initially uncharged. As soon as the circuit completes, the charge begins to flow. Let 'q' be the charge on the capacitor at certain instance & i be the current in the circuit. Then,

$$iR + \frac{q}{C} = V \qquad \& \qquad i = \frac{dq}{dt}$$

$$\Rightarrow \qquad R\frac{dq}{dt} = \frac{CV - q}{C}$$

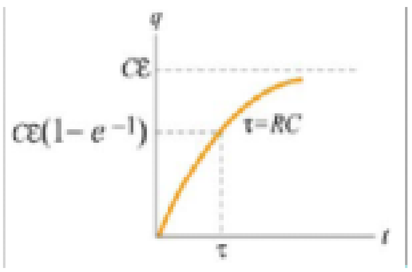


$$\Rightarrow \qquad \int_0^q \frac{-dq}{CV - q} = \int_0^t \frac{-dt}{CR}$$

$$\therefore \qquad \ell n\frac{CV - q}{CV} = \frac{-t}{CR}$$

$$\therefore \qquad q = q_0\,(1 - e^{-t/CR})$$

where, $q_0 = CV$ = maximum amount of charge stored on the plates



Now, $\frac{dq}{dt} = i = \frac{V}{R}e^{-t/CR} = \frac{V}{R}e^{-t/\tau}$

Once we know the charge on the capacitor we also can determine the voltage across the capacitor,

$$V_C(t) = \frac{q(t)}{C} = \varepsilon\,(1 - e^{-t/RC})$$

The graph of voltage as a function of time has the same form as figure. From the figure, we see that after a sufficiently long time the charge on the capacitor approaches the value.

$$q(1-\infty) = C\varepsilon = Q$$

At that time, the voltage across the capacitor is equal to the applied voltage source and the charging process effectively ends,

$$V_C = \frac{q(t=\infty)}{CQ} = \frac{Q}{C} = \varepsilon$$

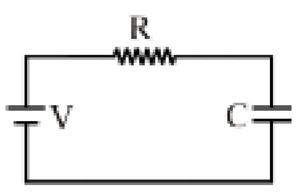
For current a **capacitor acts as :**

- Short-circuit just after closing the switch.
- Open circuit a long time after closing the switch.

Discharging a Capacitor

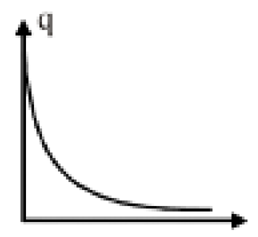
Suppose initially the capacitor has been charged to some value Q. For $t < 0$, the switch is open and the potential difference across the capacitor is given by $V_C = Q/C$. On the other hand, the potential difference across the resistor is zero because there is no current flow, that is, $I = 0$. Now suppose at $t = 0$ the switch is closed (Figure). The capacitor will begin to discharge.

$$iR - \frac{q}{C} = 0 \qquad \& \qquad i = -\frac{dq}{dt}$$



$$\Rightarrow \qquad R\frac{dq}{dt} = \frac{CV - q}{C}$$

$$\Rightarrow \qquad \int_{q_0}^{q} \frac{dq}{q} = \int_0^t \frac{-dt}{CR}$$



$$\therefore \qquad \ell n\frac{q}{q_0} = \frac{-t}{CR}$$

$$\therefore \qquad q = q_0\, e^{-t/CR} = CV_0\, e^{-t/CR}$$

Now, $i = \dfrac{dq}{dt} = \dfrac{V_0}{R}e^{-t/CR} = \dfrac{V_0}{R}e^{-t/\tau}$

Illustration :

In the circuit shown in Fig. the sources have emf's $\xi_1 = 1.0\ V$ and $\xi_2 = 2.5\ V$ and the resistances have the values $R_1 = 10\Omega$ and $R_2 = 20\ \Omega$. The internal resistances of the sources are negligible. Find the potential difference between the plates A and B of the capacitor C.

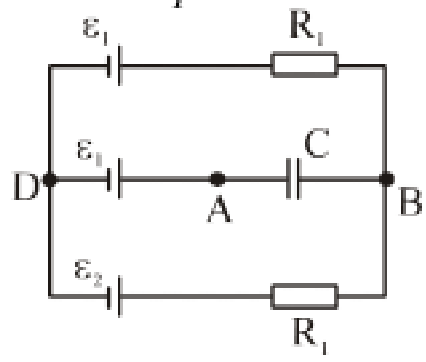


Fig. : 63

Sol. *At steady state there is no current through capacitor i.e. current exist in bigger loop anticlockwise which will be*

$$I = \frac{2.5 - 1.0}{10 + 20} = \frac{1}{20}\,A$$

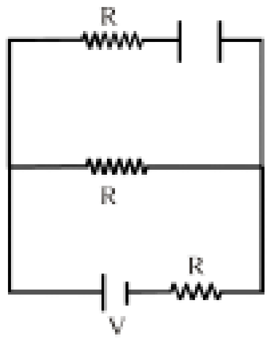
Now

$$V_{BD} = V_{BA} + V_{AD}$$

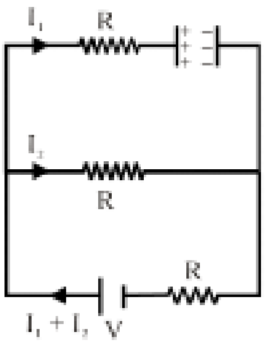
$$\Rightarrow \qquad I + \frac{1}{20}\times 10 = V_{BA} + 1 \qquad \Rightarrow \qquad V_{BA} = 0.5\ V$$

Illustration :

In the figure shown find (i) The charge of the capacitor as a function of time (ii) equivalent time constant.



Sol. $I_1 = \frac{dq}{dt}$ ...(i)



$I_1 R + \frac{q}{C} + (I_1 + I_2)R = V$

$I_1 R + \frac{q}{C} - I_2 R = 0$

$2I_1 + I_2 + \frac{q}{RC} = \frac{V}{R}$ ...(ii)

$I_2 = I_1 + \frac{q}{RC}$ ...(iii)

From (ii) and (iii)

$2I_1 + \left(I_1 + \frac{q}{RC}\right) + \frac{q}{RC} = \frac{V}{R}$

$3I_1 + \frac{2q}{RC} = \frac{V}{R}$ ...(iv)

from (i) and (iv)

$\frac{3dq}{dt} + \frac{2q}{RC} = \frac{V}{R}$

$3RC\frac{dq}{dt} + 2q = CV$

$3RC\frac{dq}{dt} = CV - 2q$

$\int_0^q \frac{dq}{CV - 2q} = \frac{1}{3RC}\int_0^t dt$

$$\left[\frac{CV-2q}{-q}\right]_0^q = \frac{t}{3RC}$$

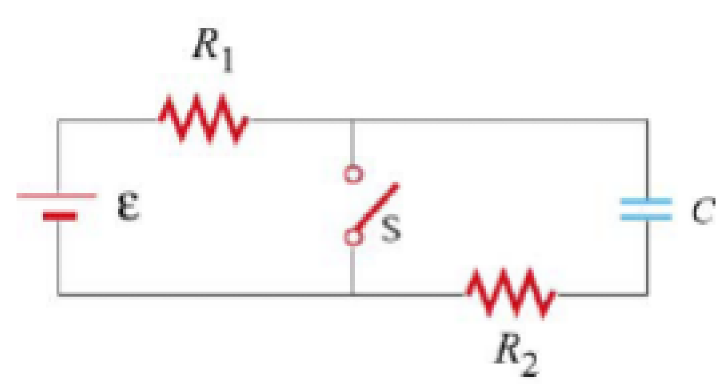
$$in\left(1-\frac{2q}{CV}\right) = \frac{-2t}{3RC}$$

$$\Rightarrow q = \frac{CV}{2}\left(1-e^{-\frac{2t}{3RC}}\right)$$

$$obviously\ \tau_{eq} = \frac{3RC}{2}$$

Practice Exercise

Q.1 In the circuit in figure, suppose the switch has been open for a very long time. At time t = 0, it is suddenly closed.

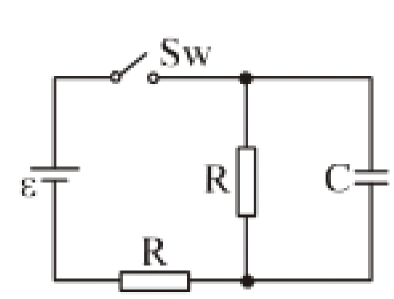


(a) What is the time constant before the switch is closed?

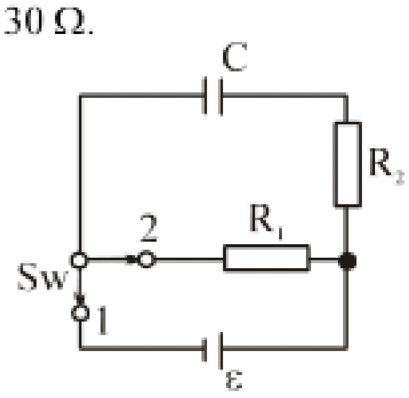
(b) What is the time constant after the switch is closed?

(c) Find the current through the switch as a function of time after the switch is closed.

Q.2 Find how the voltage across the capacitor C varies with time t (Fig.) after the shorting of the switch Sw at the moment t = 0.



Q.3 A capacitor of capacitance C = 5.00 μF is connected to a source of constant emf ξ = 200 V (Fig.). Then the switch Sw was thrown over from contact 1 to contact 2. Find the amount of heat generated in a resistance R_1 = 500 Ω if R_2 = 330 Ω.



Answers

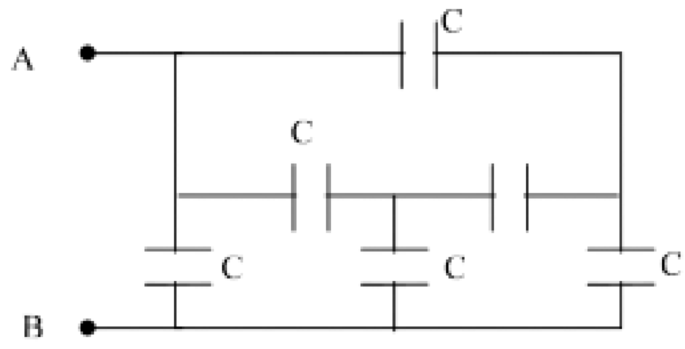
Q.1 (a) $(R_1 + R_2)\,C$ (b) $R_2\,C$ (c) $I(t) = I_1 + I'(t) = \frac{\varepsilon}{R_1} + \left(\frac{\varepsilon}{R_2}\right)e^{-t/R_2C}$

Q.2 $V = 1/2\,\xi\ (1 - e^{-2t/RC})$

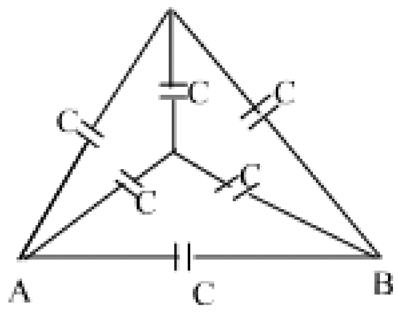
Q.3 $Q = 1/2C\,\xi^2 R_1/(R_1 + R_2) = 60$ mJ

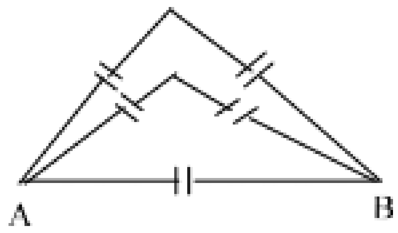
Solved Example

Q.1 In the given figure, what is the equivalent capacitance between points A and B?



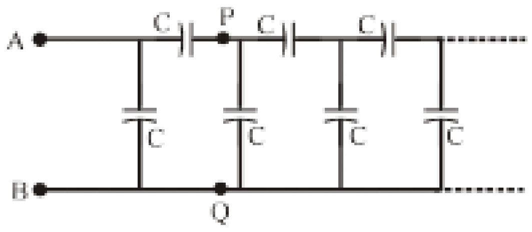
Sol. Circuit can be redrawn as.



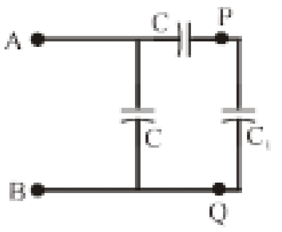


Hence, equivalent capacitance = 2C

Q.2 Find the capacitance of the infinite ladder shown in figure.



Sol. As the ladder is infinitely long, the capacitance of the ladder to the right of the points P, Q is the same as that of the ladder to the right of the points A, B. If the equivalent capacitance of the ladder is C_1, the given ladder may be replaced by the connections shown in figure.



The equivalent capacitance between A and B is easily found to be $C + \frac{CC_1}{C + C_1}$. But being equivalent to the original ladder, the equivalent capacitance is also C_1.

Thus, $C_1 = C + \frac{CC_1}{C + C_1}$

Or, $\quad C_1C + C_1^2 = C^2 + 2CC_1$

Or, $\quad C_1^2 - CC_1 - C^2 = 0$

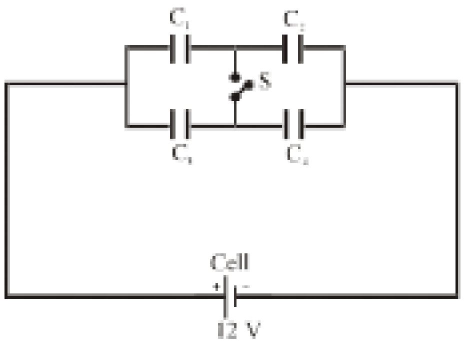
Giving $C_1 = \dfrac{C + \sqrt{C^2 + 4C^2}}{2} = \dfrac{1+\sqrt{5}}{2}C$.

Negative value of C_1 is rejected.

Q.3 The emf of the cell in the circuit is 12 volts and the capacitors are : $C_1 = 1\ \mu f, C_2 = 3\ \mu f, C_3 = 2\ \mu f, C_4 = 4\ \mu f$. Calculate the charge on each capacitor and the total charge drawn from the cell when

(a) the switch s is closed

(b) the switch s is open.



Sol. (a) Switch S is closed :

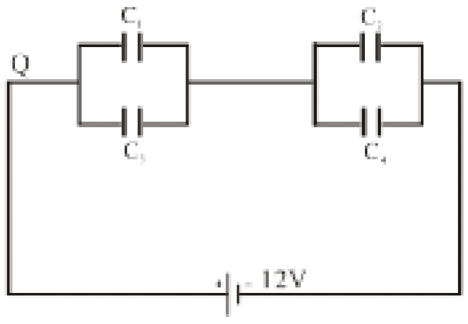
$$C = \frac{(C_1 + C_3)(C_2 + C_4)}{(C_1 + C_3) + (C_2 + C_4)}$$

$$C = \frac{3 \times 7}{3 + 7} = 2.1\mu F$$

total charge drawn from the cell is :

$$Q = C\ V = 2.1\ \mu F \times 12 \text{ volts} = 25.2\ \mu C$$

C_1, C_3 are in parallel and C_2, C_4 are in parallel.



Charge on C_1

$$Q_1 = \frac{C_1}{C_1 + C_3}Q = \frac{1}{1+2} \times 25.2\mu C = 8.4\mu C.$$

Charge on C_3

$$Q_3 = \frac{C_3}{C_1 + C_3}Q = \frac{2}{1+2} \times 25.2\mu C = 16.8\mu C.$$

Charge on C_2

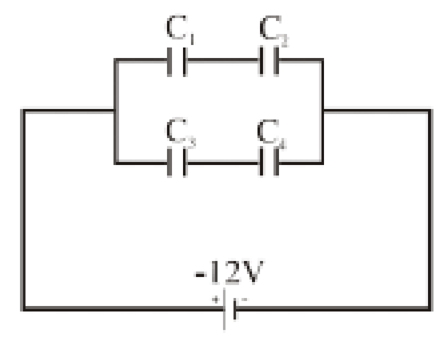
$$Q_2 = \frac{C_2}{C_2 + C_4}Q = \frac{3}{3+4} \times 25.2\mu C = 10.8\mu C.$$

Charge on C_4

$$Q_4 = \frac{C_4}{C_2 + C_4}Q = \frac{4}{3+4} \times 25.2\mu C = 14.4\mu C.$$

(b) Switch S is open :

$$C = \frac{C_1C_2}{C_1 + C_2} + \frac{C_3C_4}{C_3 + C_4}$$



$$C = \frac{1 \times 3}{1 + 3} + \frac{2 \times 4}{2 + 4} = \frac{25}{12}\,\mu F$$

total charge drawn from battery is :

$$Q = CV = \frac{25}{12} \times 12 = 25\mu C$$

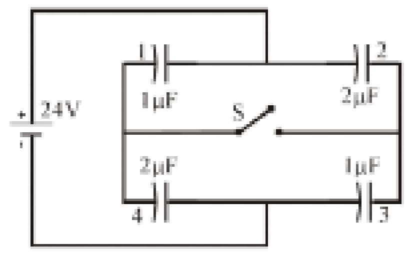
C_1 & C_2 are in series and the potential difference across combination is 12 volts.
charge on C_1 = charge on C_2

$$= \left(\frac{C_1C_2}{C_1 + C_2}\right)V = \frac{3}{4} \times 12 = 9\mu C.$$

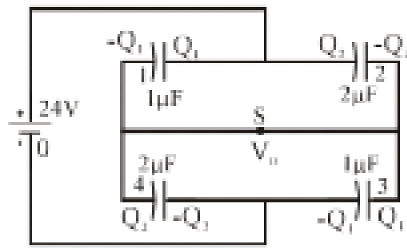
C_3 & C_4 are in series and the potential difference across combination is 12 volts.
charge on C_3 = charge on C_4

$$= \left(\frac{C_3C_4}{C_3 + C_4}\right)V = \frac{8}{6} \times 12 = 16\mu C.$$

Q.4 The connections shown in figure are established with the switch S open. How much charge will flow through the switch if it is closed?



(a)



(b)

Sol. When the switch is open, capacitors (2) and (3) are in series. Their equivalent capacitance is $\frac{2}{3}\,\mu F$. The charge appearing on each of these capacitors is, therefore, $24\,V \times \frac{2}{3}\,\mu F = 16\mu C$.

The equivalent capacitance of (1) and (4), which are also connected in series, is also $\frac{2}{3}\mu F$ and the charge on each of these capacitors is also $16\mu C$. The total charge on the two plates of (1) and (4) connected to the switch is, therefore, zero.

The situation when the switch S is closed is shown in figure. Let the charges be distributed as shown in the figure. Q_1 and Q_2 are arbitrarily chosen for the positive plates of (1) and (2). The same magnitude of charges will appear at the negative plates (3) and (4).

Take the potential at the negative terminal to the zero and at the switch to be V_0.

Writing equations for the capacitors (1), (2), (3) and (4).

$$Q_1 = (24V - V_0) \times 1\ \mu F \qquad ...(i)$$

$$Q_2 = (24V - V_0) \times 2\mu F \qquad ...(ii)$$

$$Q_1 = V_0 \times 1\ \mu F \qquad ...(iii)$$

$$Q_2 = V_0 \times 2\mu F \qquad ...(iv)$$

From (i) and (iii), $V_0 = 12V$.

Thus, from (iii) and (iv),

$$Q_1 = 12\mu C \text{ and } Q_2 = 24\ \mu C.$$

The charge on the two plates of (1) and (4) which are connected to the switch is, therefore $Q_2 - Q_1 = 12\mu C$.

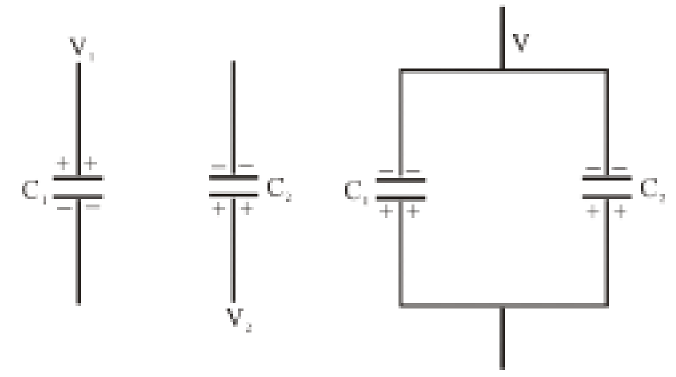
When the switch was open, this charge was zero. Thus, $12\mu C$ of charge has passed through the switch after it was closed.

Q.5 Two capacitors $C_1 = 1\mu F$ and $C_2 = 4\mu F$ are charged to a potential difference of 100 volts and 200 volts respectively. The charged capacitors are now connected to each other with terminals of opposite sign connected together. What is the

(a) final charge on each capacitor in steady state ?

(b) decrease in the energy of the system ?

Sol.



Initial charge on $C_1 = C_1V_1 = 100\mu C$

Initial charge on $C_2 = C_2V_2 = 800\mu C$

$$C_1V_1 < C_2V_2$$

when the terminals of opposite polarity are connected together, the magnitude of net charge finally is equal to the difference of magnitude of charges before connection.

(charge on $C_2)_i$ - (charge on $C_1)_i$

= (charge on $C_2)_f$ - (charge on $C_1)_f$

Let V be the final common potential difference across each.

The charges will be redistributed and the system attains a steady state when potential difference across each capacitor becomes same.

$$C_2V_2 - C_1V_1 = C_2V + C_1V$$

$$V = \frac{C_1V_2 - C_1V_1}{C_2 + C_1} = \frac{800 - 100}{5} = 140 \text{ volts}$$

Note that because $C_1V_1 < C_2V_2$, the final charge polarities are same as that of C_2 before connection.

Final charge on $C_1 = C_1 V = 140\ \mu C$

Final charge on $C_2 = C_2 V = 560\ \mu C$

Loss of energy $= U_i - U_f$

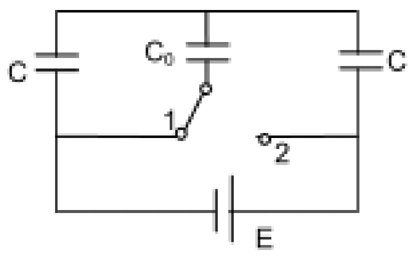
$$\text{Loss of energy} = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}C_1V^2 - \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}1(100)^2 + \frac{1}{2}4(200)^2 - \frac{1}{2}(1+4)(140)^2$$

$$= 36000\mu J = 0.036 J$$

Note: The energy is lost as heat in the connected wires due to the temporary currents that flow while the charge is being redistributed.

Q.6 Find the amount of heat generated in the circuit shown in the figure after the switch is shifted from position 1 to position 2.



Sol. When the switch is in position 1, the combination has C and C_0 in parallel and C in series for which the equivalent capacitance is

$$C_{eq} = \frac{C(C + C_0)}{2C + C_0}$$

The total charge on the combination is

$$Q = EC_{eq} = \frac{EC(C + C_0)}{2C + C_0}$$

The total charge on the three capacitors can be obtained as

$$q_3 = EC_{eq} = \frac{EC(C + C_0)}{2C + C_0}$$

$$q_2 = \frac{EC(C + C_0)C_0}{(2C + C_0)(C + C_0)} = \frac{ECC_0}{2C + C_0}$$

$$q_1 = \frac{EC(C + C_0)C}{(2C + C_0)(C + C_0)} = \frac{EC^2}{2C + C_0}$$

When the switch is in position 2, the charge distribution on the three capacitors is obtained as

$$q_3' = \frac{EC^2}{2C + C_0}, \quad q_2' = q_2 \quad \text{and} \quad q_1' = \frac{EC(C + C_0)}{2C + C_0}$$

Now, heat produced = (loss in stored electrical energy) + (extra energy drawn from the battery).

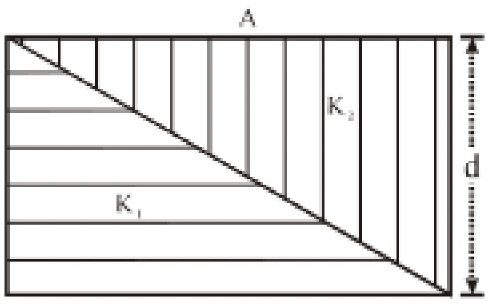
Since the equivalent capacitance C_{eq} remains unchanged in both the positions of the key, the loss in stored energy is zero. Hence,

$$\text{Heat produced} = \text{energy drawn from the battery}$$

$$= E\Delta q = E\,(q'_1 - q_1) = E(q_3 - q'_3)$$

$$= E\left[\frac{EC(C + C_0)}{2C + C_0} - \frac{EC^2}{2C + C_0}\right] = \frac{E^2CC_0}{2C + C_0}$$

Q.7 The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2 respectively (Fig.). Find the capacitance of the resulting capacitor.



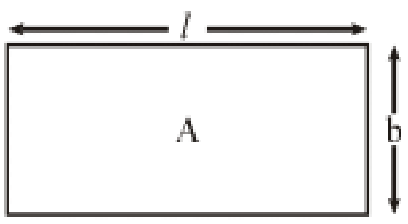
Sol. Let length and breadth of the capacitor be l and b respectively and d be the distance between the plates as shown in fig. Then consider a strip at a distance x of width dx.

Now QR = x tan θ

and PQ = d – x tan θ

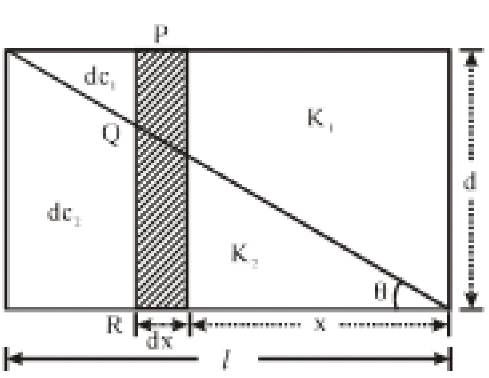
Where tan θ = d/l,

Capacitance of PQ



$$dC_1 = \frac{k_1\varepsilon_0(b\,dx)}{d - x\tan\theta} = \frac{k_1\varepsilon_0(b\,dx)}{d - \dfrac{xd}{l}}$$

$$dC_1 = \frac{k_1\varepsilon_0 bl\,dx}{d(l - x)} = \frac{k_1\varepsilon_0 A(dx)}{d(l - x)}$$



and dC_2 = capacitance of QR

$$dC_2 = \frac{k_2\varepsilon_0 b(dx)}{d\tan\theta}$$

$$dC_2 = \frac{k_2\varepsilon_0 A(dx)}{x\,d} \quad \ldots\ldots \quad \left\{\because \tan\theta = \frac{d}{l}\right\}$$

Now dC_1 and dC_2 are in series. Therefore, their resultant capacity dC will be given by

$$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2}$$

then $$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2}$$

$$= \frac{d(l - x)}{K_1\varepsilon_0 A(dx)} + \frac{x.d}{K_2\varepsilon_0 A(dx)}$$

$$\frac{1}{dC}=\frac{d}{\varepsilon_0 A(dx)}\left(\frac{l-x}{K_1}+\frac{x}{K_2}\right) \quad = \quad \frac{d[K_2(l-x)+K_1x]}{\varepsilon_0 AK_1K_2(dx)}$$

$$dC=\frac{\varepsilon_0 AK_1K_2}{d[K_2(l-x)+K_1x]}dx \quad , \quad dC=\frac{\varepsilon_0 AK_1K_2}{d[K_2 l+(K_1-K_1)x]}dx$$

All such elemental capacitor representing DC are connected in parallel.

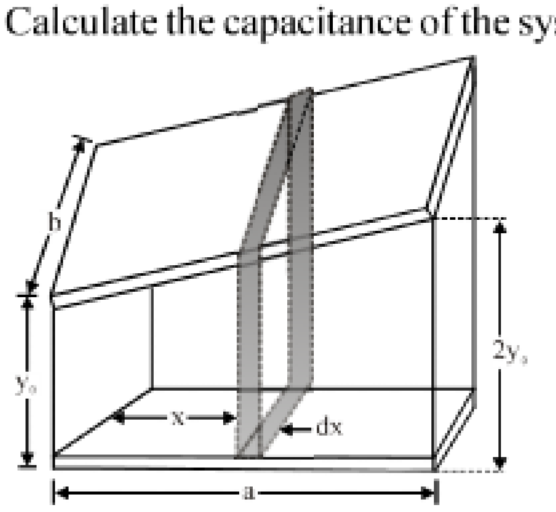
Now the capacitance of the given parallel plate capacitor is obtained by adding such infinitesimal capacitors parallel from x = 0 to x = l.

i.e. $$C=\int_{x=0}^{x=l} dC$$

$$=\int_0^l \frac{\varepsilon_0 AK_1K_2}{d[K_2 l+(K_1-K_2)x]}dx$$

$$C=\frac{K_1K_2\varepsilon_0 A}{(K_1-K_2)d}\ln\frac{K_2}{K_1}.$$

Q.8 A capacitor has rectangular plates of length a and width b. The top plate is inclined at a small angle as shown in figure The plate separation varies from $d = y_0$ at the left to $d = 2y_0$ at the right where y_0 is much less than a or b. Calculate the capacitance of the system.



Sol. We consider a differential strip of width dx and length b to approximate a differential capacitor of area b dx and separation $d = y_0 + \left(\frac{y_0}{a}\right)x$. All such differential capacitor are in parallel arrangement.

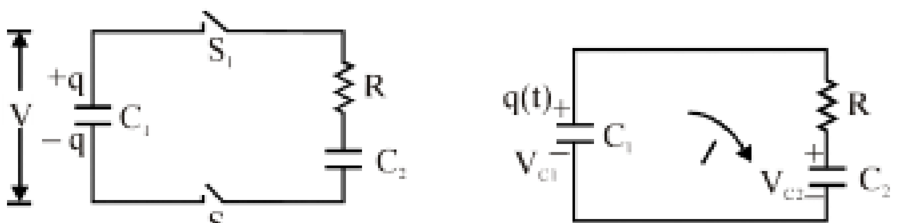
$$dC=\frac{\varepsilon_0(bdx)}{y_0+\left(\frac{y_0}{a}\right)x}$$

$$C=\varepsilon_0 b\int_0^a \frac{dx}{\left(y_0+\frac{y_0}{a}x\right)}$$

$$= \frac{\varepsilon_0 b}{(y_0 / a)}\left[\text{In}\left(\frac{y_0 + \frac{y_0}{a} \times a}{y_0}\right)\right]$$

$$= \frac{\varepsilon_0 ab}{y_0}\text{ in } 2$$

Q.9 The capacitor C_1 figure initially carries a charge q_0. When the switches S_1 and S_2 are shut, capacitor C_1 is connected in series to resistor R and a second capacitor C_2, which initially does not carry any charge.
(i) Find the charge deposited on the capacitor and the current through R as a function of time.
(ii) What is the heat lost in the resistor after a long time of closing the switch ?



(a) (b)

Sol. (i) Suppose at a moment 't' the charge deposited on C_1 is q (t).

$$\therefore \quad V_{C_1} = \frac{q(t)}{C_1}$$

and $$V_{C_2} = \frac{q_0 - q(t)}{C_2}$$

$$V_R = IR$$

and $$I = \frac{dq}{dt}$$

Applying KVL,

$$\frac{q}{C_1} - \frac{(q_0 - q)}{C_2} = IR = -R\frac{dq}{dt}$$

$$\therefore \quad R\frac{dq}{dt} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)q = \frac{q_0}{C_2}$$

Put $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore \quad q(t) = \left(1 - \frac{C}{C_2}\right)q_0\, e^{-t/RC} + \frac{C}{C_2}q_0$$

or $\quad q(t) = \frac{C}{C_1} q_0 e^{-t/RC} + \frac{C}{C_2} q_0$

$\therefore \quad I(t) = -\frac{dq}{dt} = \frac{q_0}{RC_1} e^{-t/RC}$

Charge C_2,

$$q_0 - q(t) = q_0 \frac{C}{C_1} (1 - e^{-t/RC})$$

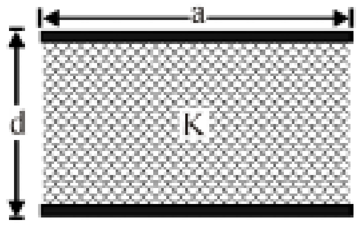
(ii) Electrostatic energy at t = 0 is

$$U(0) = \frac{q_0^2}{2C_1}$$

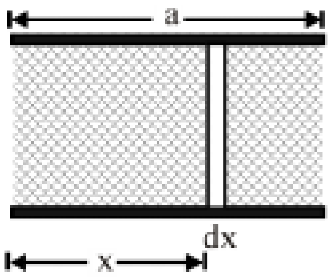
Final energy = $U(\infty) = \frac{q_0^2}{2(C_1 + C_2)}$

$$\Delta U = U(0) - U(\infty) = \frac{q_0^2 C_2}{2C_1(C_1 + C_2)}$$

Q.10 Shows a parallel-plate capacitor having square plates of edge a and plate-separation d. The gap between the plates is filled with a dielectric of dielectric constant K which varies parallel to an edge as



(a)



(b)

Where K and α are constants and x is the distance from the left ends. Calculate the capacitance.

Sol. Consider a small strip of width dx at separation x from the left end. This strip forms small capacitor of plate area adx. Its capacitance is

$$dC = \frac{(K_0 + ax)\varepsilon_0 a dx}{d}$$

The given capacitor may be divided into such strips with x varying from 0 to a. All these strips are connected in parallel. The capacitance of the given capacitor is,

$$C = \int_0^a \frac{(K_0 + ax)\varepsilon_0 a dx}{d}$$

$$= \frac{\varepsilon_0 a^2}{d}\left(K_0 + \frac{a\alpha}{2}\right)$$

Q.11 A capacitor of capacitance C is charged by connecting it to battery of emf ε. The capacitor is now disconnected and reconnected to the battery with the polarity reversed. Calculate the heat developed in the connecting wires.

Sol. When the capacitor is connected to the battery, a charge. When the polarity is reversed, a charge - Q on the other. When the polarity is reversed, a charge $-Q$ appears on the first plate and $+Q$ on the second. A charge 2Q, therefore passes through the battery from the negative to the positive terminal. The battery does a work

$$W = (2Q)\varepsilon = 2C\varepsilon^2$$

in the process. The energy stored in the capacitor is the same in the two cases. Thus, the work done by the battery appears as heat in the connecting wires. The heat produced is, therefore, $2C\varepsilon^2$.

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