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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

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MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **October/November 2024**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

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1 At a college, the students choose exactly one of tennis, hockey or netball to play. The table shows the numbers of students in Year 1 and Year 2 at the college playing each of these sports.

	Tennis	Hockey	Netball
Year 1	16	22	12
Year 2	24	18	28

One student is chosen at random from the 120 students. Events X and N are defined as follows:

X: the student is in Year 1

N: the student plays netball.

$P(X|N) = P(X)$

$\frac{P(X \cap N)}{P(N)} = P(X)$

(a) Find $P(X|N)$. [1]

$\frac{P(X \text{ and } N)}{P(N)} = \frac{12/120}{40/120} = \frac{12}{40}$

(b) Find $P(N|X)$. [1]

$\frac{12}{50}$

(c) Determine whether or not X and N are independent events. [1]

→ $P(X \text{ and } N) = P(X) \cdot P(N)$ → for independent events

→ $12/120 \neq \frac{50}{120} \cdot \frac{40}{120}$ ⇒ Dependent events.

One of the students who plays netball takes 8 shots at goal. On each shot, the probability that she will succeed is 0.15, independently of all other shots.

(d) Find the probability that she succeeds on fewer than 3 of these shots. [3]

→ X: "No. of shots taken at goal"

→ $X \sim B(8, 0.15)$

→ $P(X < 3) = P(X = 0, 1, 2)$

$= \binom{8}{0}(0.15)^0(0.85)^8 + \binom{8}{1}(0.15)^1(0.85)^7 + \binom{8}{2}(0.15)^2(0.85)^6$

$= 0.895$

2 **(a)** Find the number of different arrangements of the 9 letters in the word ALGEBRAIC. [1]

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(b) Find the number of different arrangements of the 9 letters in the word ALGEBRAIC in which there are no more than two letters between the two As. [3]

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3 A fair coin and an ordinary fair six-sided dice are thrown at the same time. The random variable X is defined as follows.

- If the coin shows a tail, X is twice the score on the dice.
- If the coin shows a head, X is the score on the dice if the score is even and X is 0 otherwise.

(a) Draw up the probability distribution table for X. [3]

	1	2	3	4	5	6
H	0	2	0	4	0	6
T	2	4	6	8	10	12

x	0	2	4	6	8	10	12
$P(X=x)$	$3/12$	$2/12$	$2/12$	$2/12$	$1/12$	$1/12$	$1/12$

(b) Find $\text{Var}(X)$. [3]

$\rightarrow E(X) = 4/12 + 8/12 + 12/12 + 8/12 + 10/12 + \frac{12}{12} = \frac{9}{2}$

$\rightarrow \text{Var}(X) = 8/12 + 32/12 + 72/12 + 64/12 + 100/12 + 144/12 - (9/2)^2$

$\therefore \text{Var}(X) = 14.75$

4 The heights, in metres, of white pine trees are normally distributed with mean 19.8 and standard deviation 2.4 .

In a certain forest there are 450 white pine trees.

(a) How many of these trees would you expect to have height less than 18.2 metres? [4]

X: "Height of randomly chosen white pine trees"

$X \sim N(19.8, 2.4^2)$

$\rightarrow P(X < 18.2) = P(Z < -0.667)$

$= 1 - P(Z < 0.667)$

$= 1 - 0.7477$

$= 0.252$

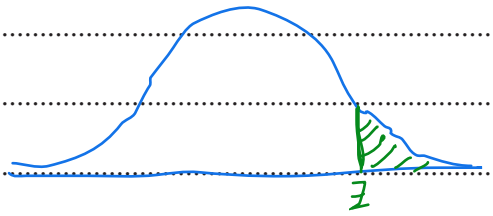
$\rightarrow E(X) = 450 \times 0.252$

The heights, in metres, of red pine trees are normally distributed with mean 23.4 and standard deviation σ. It is known that 26% of red pine trees have height greater than 25.5 metres.

(b) Find the value of σ. [3]

R: "Height of randomly chosen red pine tree"

$R \sim N(23.4, \sigma^2)$



$\rightarrow P(R > 25.5) = 0.26$

$\rightarrow Z = 0.6432$

$\rightarrow 0.6432 = \frac{25.5 - 23.4}{\sigma}$

$\therefore \sigma = 3.26$

5 In a class of 21 students, there are 10 violinists, 6 guitarists and 5 pianists. A group of 7 is to be chosen from these 21 students. The group will consist of 4 violinists, 2 guitarists and 1 pianist.

(a) In how many ways can the group of 7 be chosen? [2]

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On another occasion a group of 5 will be chosen from the 21 students. The group must contain at least 2 violinists, at least 1 guitarist and at most 1 pianist.

(b) In how many ways can the group of 5 be chosen? [4]

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6 Teams of 15 runners took part in a charity run last Saturday. The times taken, in minutes, to complete the course by the runners from the Falcons and the runners from the Kites are shown in the table.

Falcons	38	39	42	44	46	48	50	51	52	56	58	59	64	69	76
Kites	32	40	40	45	47	48	52	54	58	59	59	60	61	63	65

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with the Falcons on the left-hand side. [4]

(b) Find the median and the interquartile range of the times for the Falcons. [3]

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Let x and y denote the times, in minutes, of a runner from the Falcons and a runner from the Kites respectively.

It is given that

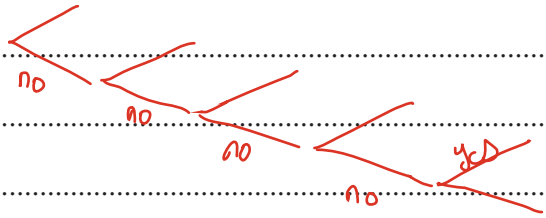
$$\Sigma x = 792, \qquad \Sigma x^2 = 43\,504, \qquad \Sigma y = 783, \qquad \Sigma y^2 = 42\,223.$$

(c) Find the mean and the standard deviation of the times taken by all 30 runners from the two teams. [3]

7 In a game, players attempt to score a goal by kicking a ball into a net. The probability that Leno scores a goal is 0.4 on any attempt, independently of all other attempts. The random variable X denotes the number of attempts that it takes Leno to score a goal.

(a) Find $P(X = 5)$. [1]

$(0.6)^4(0.4)$



(b) Find $P(3 \leqslant X \leqslant 7)$. [2]

(c) Find the probability that Leno scores his second goal on or before his 5th attempt. [3]

Leno has 75 attempts to score a goal.

(d) Use a suitable approximation to find the probability that Leno scores more than 28 goals but fewer than 35 goals. [5]

→ $n = 75$, $p = 0.4$, $q = 0.6$

→ np and nq ≥ 5 ⇒ Approx as ND.

→ $X \sim N(30, 18)$

→ $P(28 < X < 35) = P(28.5 < x < 34.5)$

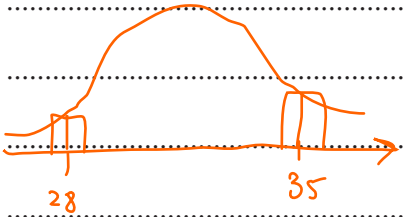
$= P(X < 34.5) - P(X < 28.5)$

$= P(Z < 1.06) - P(Z < -0.353)$

$= P(Z < 1.06) - [1 - P(Z < 0.353)]$

$= 0.8554 - 1 + 0.6379$

$= 0.493$



Additional page

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