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1 At a college, the students choose exactly one of tennis, hockey or netball to play. The table shows the numbers of students in Year 1 and Year 2 at the college playing each of these sports.

	Tennis	Hockey	Netball
Year 1	16	22	12
Year 2	24	18	28

One student is chosen at random from the 120 students. Events X and N are defined as follows:

X: the student is in Year 1

N: the student plays netball.

$P(X|N) = P(X)$

$\frac{P(X \cap N)}{P(N)} = P(X)$

(a) Find $P(X|N)$. [1]

$\frac{P(X \text{ and } N)}{P(N)} = \frac{12/120}{40/120} = \frac{12}{40}$

(b) Find $P(N|X)$. [1]

$\frac{12}{50}$

(c) Determine whether or not X and N are independent events. [1]

→ $P(X \text{ and } N) = P(X) \cdot P(N)$ → for independent events

→ $12/120 \neq \frac{50}{120} \cdot \frac{40}{120}$ ⇒ Dependent events.

One of the students who plays netball takes 8 shots at goal. On each shot, the probability that she will succeed is 0.15, independently of all other shots.

(d) Find the probability that she succeeds on fewer than 3 of these shots. [3]

→ X: "No. of shots taken at goal"

→ $X \sim B(8, 0.15)$

→ $P(X<3) = P(X=0,1,2)$

$= \binom{8}{0}(0.15)^0(0.85)^8 + \binom{8}{1}(0.15)(0.85)^7 + \binom{8}{2}(0.15)^2(0.85)^6$

$= 0.895$

3 A fair coin and an ordinary fair six-sided dice are thrown at the same time. The random variable X is defined as follows.

- If the coin shows a tail, X is twice the score on the dice.
- If the coin shows a head, X is the score on the dice if the score is even and X is 0 otherwise.

(a) Draw up the probability distribution table for X. [3]

	1	2	3	4	5	6
H	0	2	0	4	0	6
T	2	4	6	8	10	12

x	0	2	4	6	8	10	12
$P(X=x)$	$3/12$	$2/12$	$2/12$	$\frac{2}{12}$	$1/12$	$1/12$	$1/12$

(b) Find $\text{Var}(X)$. [3]

$\rightarrow E(X) = 4/12 + 8/12 + 12/12 + 8/12 + 10/12 + \frac{12}{12} = \frac{9}{2}$

$\rightarrow Var(X) = 8/12 + 32/12 + 72/12 + 64/12 + 100/12 + 144/12 - (9/2)^2$

$\therefore Var(X) = 14.75$

4 The heights, in metres, of white pine trees are normally distributed with mean 19.8 and standard deviation 2.4 .

In a certain forest there are 450 white pine trees.

(a) How many of these trees would you expect to have height less than 18.2 metres? [4]

X: " Height of randomly choosen white pine trees"

$X \sim N(19.8, 2.4^2)$

$\rightarrow P(X < 18.2) = P(Z < -0.667)$

$= 1 - P(Z < 0.667)$

$= 1 - 0.7477$

$= 0.252$

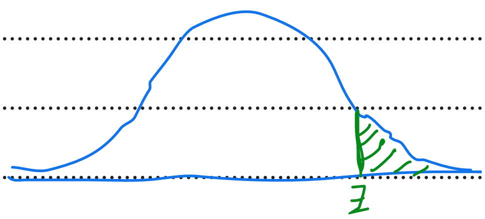
$\rightarrow E(X) = 450 \times 0.252$

The heights, in metres, of red pine trees are normally distributed with mean 23.4 and standard deviation σ. It is known that 26% of red pine trees have height greater than 25.5 metres.

(b) Find the value of σ. [3]

R: " Height of randomly choosen red pine tree"

$R \sim N(23.4, \sigma^2)$



$\rightarrow P(R > 25.5) = 0.26$

$\rightarrow Z = 0.6432$

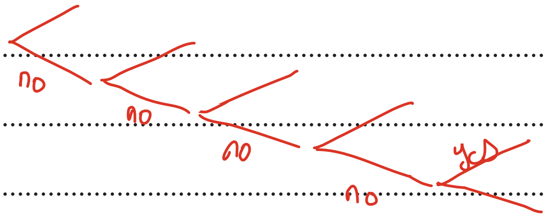
$\rightarrow 0.6432 = \dfrac{25.5 - 23.4}{\sigma}$

$\therefore \sigma = \underline{3.26}$

7 In a game, players attempt to score a goal by kicking a ball into a net. The probability that Leno scores a goal is 0.4 on any attempt, independently of all other attempts. The random variable X denotes the number of attempts that it takes Leno to score a goal.

(a) Find $P(X = 5)$. [1]

$\rightarrow (0.6)^4(0.4)$



(b) Find $P(3 \leqslant X \leqslant 7)$. [2]

(c) Find the probability that Leno scores his second goal on or before his 5th attempt. [3]

Leno has 75 attempts to score a goal.

(d) Use a suitable approximation to find the probability that Leno scores more than 28 goals but fewer than 35 goals. [5]

→ $n = 75$, $p = 0.4$, $q = 0.6$

→ np and $nq \geqslant 5$ ⇒ Approx as ND.

→ $X \sim N(30, 18)$

→ $P(28 < X < 35) = P(28.5 < X < 34.5)$

$= P(X < 34.5) - P(X < 28.5)$

$= P(Z < 1.06) - P(Z < -0.353)$

$= P(Z < 1.06) - [1 - P(Z < 0.353)]$

$= 0.8554 - 1 + 0.6379$

$= 0.493$