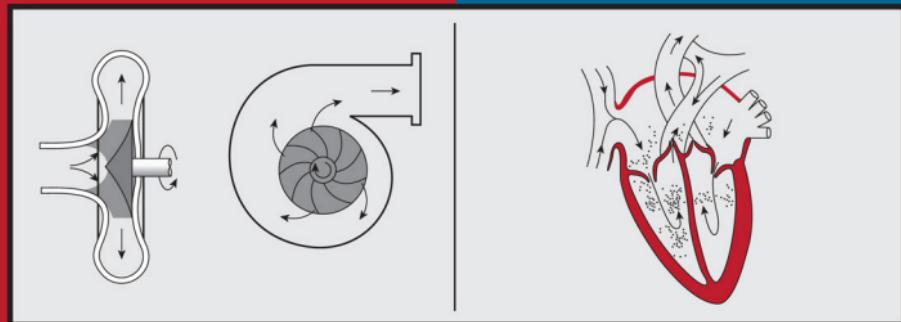


# Fundamentals of Momentum, Heat, and Mass Transfer



**Welty | Wicks | Wilson | Rorrer**

**SOLUTION MANUAL**

— Fifth Edition —

# CHAPTER 1

1.1  $n = 4 \times 10^{20}$  MOLECULES/m<sup>3</sup>

$$\bar{v} = \sqrt{k_B T} = 1.32 \times 10^4 \text{ m/s}$$

$$A = \pi/4 (10^{-3} \text{ m})^2$$

$$NA = \frac{1}{4} n \bar{v} A = 1.04 \times 10^{18} \text{ m/s}$$

## 1.2 FLOW PROPERTIES:

VELOCITY

PRESSURE GRADIENT

STRESS

## FLUID PROPERTIES:

PRESSURE

TEMPERATURE

DENSITY

SPEED OF SOUND

SPECIFIC HEAT

1.3 MASS OF SOLID =  $\rho_s V_s$

" " FLUID =  $\rho_f V_f$

$$x = \frac{\rho_s V_s}{\rho_s V_s + \rho_f V_f}$$

$$\Rightarrow \frac{V_f}{V_s} = \frac{1-x}{x} \frac{\rho_s}{\rho_f}$$

$$\rho_{mix} = \frac{\rho_s V_s + \rho_f V_f}{V_s + V_f} = \frac{\rho_s + \rho_f (V_f/V_s)}{1 + V_f/V_s}$$

$$= \frac{x \rho_s + (1-x) \rho_f}{x \rho_f + (1-x) \rho_f}$$

1.4 GIVEN  $\frac{P+B}{P_1+B} = \left(\frac{\rho}{\rho_1}\right)^7$

$$\text{For } P_1 = 1 \text{ atm} \quad \frac{\rho}{\rho_1} = 1.01$$

$$P = 3001 (1.01)^7 - 3000 \\ = 21 \text{ atm}$$

## 1.5 AT CONSTANT TEMPERATURE

$$P/\rho_T = \text{CONST.} \Rightarrow P/\rho = \text{CONST.}$$

FOR 10% INCREASE IN  $\rho$

P MUST ALSO INCREASE BY 10%

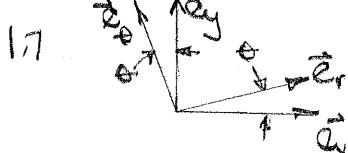
## 1.6 SINCE DENSITY VARIES AS

$$\rho = kP$$

$$\rho_{250,000 \text{ FT}} = \rho_{s.l.} \frac{P_{250,000 \text{ FT}}}{P_{s.l.}}$$

$$\therefore \rho = nM \quad (M = \text{MOLECULAR WT})$$

$$\therefore n_{250,000} = n_{s.l.} \left[ \frac{1.5 \times 10^{-7}}{2.378 \times 10^{-3}} \right] \\ = 4 \times 10^{20} \left[ \cancel{x} \right] = 2.5 \times 10^{16}$$



$$\vec{r} = |\vec{r}|_x \vec{e}_x + |\vec{r}|_y \vec{e}_y$$

$$= \cos \theta \vec{e}_x + \sin \theta \vec{e}_y$$

$$\vec{e}_\theta = |\vec{e}_\theta|_x \vec{e}_x + |\vec{e}_\theta|_y \vec{e}_y$$

$$= -\sin \theta \vec{e}_x + \cos \theta \vec{e}_y$$

Q.E.D.

$$1.8 \quad \frac{d\vec{e}_r}{d\theta} = -\sin\theta \vec{e}_x + \cos\theta \vec{e}_y \\ = \vec{e}_\theta$$

$$\frac{d\vec{e}_\theta}{dt} = -\omega_r \theta \vec{e}_x - \sin\theta \vec{e}_y \\ = -\vec{e}_r$$

Q.E.D.

### 1.9 TRANSFORMATION FROM $(x,y)$ TO $(r,\theta)$

$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x} \frac{\partial}{\partial r} + \frac{\partial \theta}{\partial x} \frac{\partial}{\partial \theta}$$

$$\frac{\partial}{\partial y} = \frac{\partial r}{\partial y} \frac{\partial}{\partial r} + \frac{\partial \theta}{\partial y} \frac{\partial}{\partial \theta}$$

$$r^2 = x^2 + y^2 \quad \theta = \tan^{-1} y/x$$

$$\text{so: } \frac{\partial r}{\partial x} = \frac{x}{(x^2+y^2)^{1/2}} = \frac{r \cos\theta}{r} = \cos\theta$$

$$\frac{\partial \theta}{\partial x} = -\frac{y}{x^2+y^2} = -\frac{r \sin\theta}{r^2} = -\frac{\sin\theta}{r}$$

$$\frac{\partial r}{\partial y} = \sin\theta \quad \frac{\partial \theta}{\partial y} = \frac{\cos\theta}{r}$$

$$\Rightarrow \frac{\partial}{\partial x} = \cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial \theta}$$

$$\frac{\partial}{\partial y} = \sin\theta \frac{\partial}{\partial r} + \frac{\cos\theta}{r} \frac{\partial}{\partial \theta}$$

$$1.10 \quad \nabla = \frac{\partial}{\partial x} \vec{e}_x + \frac{\partial}{\partial y} \vec{e}_y + \frac{\partial}{\partial z} \vec{e}_z \\ = \left( \cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial \theta} \right) \vec{e}_x \\ + \left( \sin\theta \frac{\partial}{\partial r} + \frac{\cos\theta}{r} \frac{\partial}{\partial \theta} \right) \vec{e}_y \\ + \frac{\partial}{\partial z} \vec{e}_z$$

1.10 (CONTINUED) --

$$= (\vec{e}_x \cos\theta + \vec{e}_y \sin\theta) \frac{\partial}{\partial r} \\ + \frac{1}{r} (-\vec{e}_x \sin\theta + \vec{e}_y \cos\theta) \frac{\partial}{\partial \theta} \\ + \vec{e}_z \frac{\partial}{\partial z}$$

thus:

$$\nabla = \vec{e}_r \frac{\partial}{\partial r} + \frac{1}{r} \vec{e}_\theta \frac{\partial}{\partial \theta} + \vec{e}_z \frac{\partial}{\partial z}$$

$$1.11 \quad \nabla P = \frac{\partial P}{\partial x} \vec{e}_x + \frac{\partial P}{\partial y} \vec{e}_y$$

$$\nabla P(a,b) = S_0 V_m^2 \left[ \left[ \frac{1}{a} \cos 1 \sin 1 + 2 \right] \vec{e}_x \right.$$

$$\left. + \frac{1}{b} (\sin 1 \cos 1) \vec{e}_y \right]$$

$$= S_0 V_m^2 \left[ \left[ \frac{1}{a} \sin \frac{2}{2} + 2 \right] \vec{e}_x \right.$$

$$\left. + \frac{1}{b} (\sin \frac{2}{2}) \vec{e}_y \right]$$

$$1.12 \quad \nabla T(x,y) = T_0 \bar{e}^{1/4} \left[ \left[ \frac{1}{a} \left( \frac{y-a}{a} \right) \left( \frac{y-b}{b} \right) \vec{e}_x \right. \right. \\ \left. \left. + \frac{1}{b} \left( \sin \frac{x}{a} \sin \frac{y}{b} \right) \vec{e}_y \right] \right]$$

$$\nabla T(a,b) = T_0 \bar{e}^{1/4} \left[ \frac{1}{a} (\cos 1 \sinh 1) \vec{e}_x \right. \\ \left. + \frac{1}{b} (\sin 1 \sinh 1) \vec{e}_y \right]$$

$$= T_0 \bar{e}^{1/4} \left[ \frac{\cos 1 (e + \bar{e}^1)}{2a} \vec{e}_x \right. \\ \left. + \frac{\sin 1 (e - \bar{e}^1)}{2b} \vec{e}_y \right]$$

$$= T_0 \frac{\bar{e}^{5/4}}{2} \left[ \frac{\cos 1 (1 + \bar{e}^2)}{a} \vec{e}_x \right. \\ \left. + \frac{\sin 1 (1 - \bar{e}^2)}{b} \vec{e}_y \right]$$

1.13 In prob 1.12  $T(x,y)$  is dimensionally homogeneous (D.H.)

$P(x,y)$  in prob 1.11 will be D.H. if

$$P_P \sim \frac{1}{V_P^2} L_B s^2 / \pi^4$$

or using the conversion factor  $g_C$

$$1.14 \quad \phi = 3x^2y + 4y^2$$

$$\text{a) } \nabla \phi = (6xy)\vec{e}_x + (3x^2 + 8y)\vec{e}_y$$

$$\nabla \phi(3,5) = 90\vec{e}_x + 67\vec{e}_y$$

$$\text{b) } \nabla \phi \cdot \vec{e}_S = \left[ 6xy\vec{e}_x + (3x^2 + 8y)\vec{e}_y \right] \cdot \left[ \cos\theta\vec{e}_x + \sin\theta\vec{e}_y \right]$$

AT POINT (3,5)

$$\begin{aligned} \nabla \phi \cdot \vec{e}_S &= (90\vec{e}_x + 67\vec{e}_y) \cdot (\cos(-60)\vec{e}_x + \sin(-60)\vec{e}_y) \\ &= 45 - 58.02 = -13.02 \end{aligned}$$

1.15 For an IDEAL GAS

$$P = \frac{g RT}{M}$$

$$\text{from Prob. 1.3 } g = \frac{g_m(1-x)}{1 - \frac{g_m}{g_s} x}$$

$$\therefore P = \frac{g_m(1-x)}{1 - \frac{g_m}{g_s} x} \frac{RT}{M}$$

$$1.16 \quad \psi = Ar \sin\theta \left( 1 - \frac{a^2}{r^2} \right)$$

$$\begin{aligned} \text{a) } \nabla \psi &= \frac{\partial \psi}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial \psi}{\partial \theta} \vec{e}_\theta \\ &= A \sin\theta \left( 1 - \frac{a^2}{r^2} \right) \vec{e}_\theta \end{aligned}$$

$$\text{b) } |\nabla \psi| = A \left[ \sin^2\theta \left( 1 + \frac{a^2}{r^2} \right)^2 + \cos^2\theta \left( 1 - \frac{a^2}{r^2} \right)^2 \right]^{1/2}$$

$|\nabla \psi|_{\max}$  is given by  $\partial |\nabla \psi| / \partial r = 0$

$$\text{or } \frac{\partial}{\partial r} |\nabla \psi| dr + \frac{\partial}{\partial \theta} |\nabla \psi| d\theta = 0$$

$$\text{REQUIRMENT } \frac{\partial}{\partial r} |\nabla \psi| = \frac{\partial}{\partial \theta} |\nabla \psi| = 0$$

for  $\frac{\partial}{\partial r} |\nabla \psi| = 0$ :

$$-\sin^2\theta \left( 1 + \frac{a^2}{r^2} \right) + \cos^2\theta \left( 1 - \frac{a^2}{r^2} \right) = 0 \quad (1)$$

$$\frac{1}{2} \text{ from } \frac{\partial}{\partial \theta} |\nabla \psi| = 0$$

$$\sin\theta \cos\theta \left[ \left( 1 + \frac{a^2}{r^2} \right)^2 - \left( 1 - \frac{a^2}{r^2} \right)^2 \right] = 0 \quad (2)$$

$$\text{From Eq 2: } \sin\theta \cos\theta 4\frac{a^2}{r^2} = 0$$

IF  $a \neq 0, r \neq 0$  THEN  $\sin\theta \cos\theta = 0$

FOR WHICH  $\theta = 0, \pi/2$  (3)

SUBST INTO EQ. 1  $\theta = 0, 1 - \frac{a^2}{r^2} = 0$

GIVEN  $a = r$

FOR  $\theta = \pi/2 \quad 1 + \frac{a^2}{r^2} = 0$  IMPOSSIBLE

THUS CONDITIONS FOR  $|\nabla \psi|_{\max}$  ARE

$$\theta = 0 \quad r = a$$

$$1.17 P = P_0 + \frac{1}{2} \rho U_\infty^2 \left[ \frac{2xz}{L^3} + 3\left(\frac{x}{L}\right)^2 + \frac{U_\infty t}{L} \right]$$

$$\frac{\partial P}{\partial x} \vec{e}_x = \frac{1}{2} \rho U_\infty^2 \left[ \frac{2z}{L^3} + \frac{6x}{L^2} \right] \vec{e}_x$$

$$\frac{\partial P}{\partial y} \vec{e}_y = \frac{1}{2} \rho U_\infty^2 \left[ \frac{2xz}{L^3} \right] \vec{e}_y$$

$$\frac{\partial P}{\partial z} \vec{e}_z = \frac{1}{2} \rho U_\infty^2 \left[ \frac{2xy}{L^3} \right] \vec{e}_y$$

$$\nabla P = \frac{1}{2} \rho U_\infty^2 \left[ \left( \frac{2xz}{L^3} + \frac{6x}{L^2} \right) \vec{e}_x + \frac{2xz}{L^3} \vec{e}_y + \frac{2xy}{L^3} \vec{e}_z \right]$$

$$1.18 \text{ VERTICAL CYLINDER } d = 10 \text{ m} \quad h = 6 \text{ m}$$

$$V = \frac{\pi}{4} (10 \text{ m})^2 (6 \text{ m}) = 471.2 \text{ m}^3$$

$$@ 20^\circ \text{C } S_w = 998.2 \text{ kg/m}^3$$

$$m = S_w V = (998.2)(471.2) = 470350 \text{ kg}$$

$$@ 80^\circ \text{C } S_w = 971.8 \text{ kg/m}^3$$

$$m = (971.8)(471.2) = 457910 \text{ kg}$$

$$\Delta m = 12440 \text{ kg}$$

$$1.19 \text{ LIQUID } - V = 1200 \text{ cm}^3 @ 1.25 \text{ MPa}$$

$$V = 1188 \text{ cm}^3 @ 2.5 \text{ MPa}$$

$$\beta = -V \left( \frac{\partial P}{\partial V} \right)_T \approx -V \frac{\Delta P}{\Delta V}$$

$$V = 1194 \text{ cm}^3 = 1.194 \times 10^{-3} \text{ m}^3$$

$$\Delta V = -12 \text{ cm}^3 = -1.2 \times 10^{-7} \text{ m}^3$$

$$\beta = -1.194 \times 10^{-3} \left[ \frac{1.25 \text{ MPa}}{-1.2 \times 10^{-7}} \right]$$

$$= +12440 \text{ MPa} = +12.44 \text{ MPa}$$

$$1.20 \quad \beta = -V \left( \frac{\partial P}{\partial V} \right)_T \approx -V \frac{\Delta P}{\Delta V}$$

$$V = 0.25 \text{ m}^3$$

$$\Delta V = -0.005 \text{ m}^3$$

$$\Delta P = 10 \text{ MPa}$$

$$\beta = -0.25 \left[ \frac{10}{-0.005} \right] = 500 \text{ MPa}$$

$$1.21 \text{ for H}_2\text{O} \quad \beta = 2.205 \text{ GPa}$$

$$\frac{\Delta V}{V} = -0.0075$$

$$\beta \approx -V \frac{\Delta P}{\Delta V} \quad \text{or} \quad \Delta P = \beta \frac{\Delta V}{V}$$

$$\Delta P = (2.205 \text{ GPa})(0.0075)$$

$$= 0.0165 \text{ GPa} = 16.5 \text{ MPa}$$

$$1.22 \text{ H}_2\text{O: } P_1 = 100 \text{ kPa} \quad P_2 = 120 \text{ MPa}$$

$$\beta = 2.205 \text{ GPa}$$

$$\beta \approx -V \frac{\Delta P}{\Delta V} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{\Delta P}{\beta}$$

$$\frac{\Delta V}{V} = \frac{\Delta P}{\beta} = \frac{(120000 - 100) \text{ kPa}}{120 \times 10^6 \text{ kPa}}$$

$$= 0.999 \times 10^{-3}$$

$$= 0.0999 \text{ PERCENT}$$

1.23  $H_2O @ 68^\circ C$  ( $341 K$ )

$$\sigma = 0.123 \left[ 1 - 0.00139(341) \right]$$

$$= 0.0647 \text{ N/m}$$

IN A CLEAN TUBE -  $\theta = 0^\circ$

$$h = \frac{2\sigma \cos \theta}{8gr}$$

$$= \frac{2(0.0647)}{979(9.81)(0.2875 \times 10^{-2}/2)}$$

$$= 9.37 \times 10^{-3} \text{ m} = 9.37 \text{ mm}$$

1.24 PARALLEL GLASS PLATES

$$- Gap = 1.625 \text{ mm}$$

$$\sigma = 0.0735 \text{ N/m}$$

FOR A UNIT DEPTH:

$$\text{SURFACE TENSION FORCE} = 2(1)\sigma \cos \theta$$

$$\text{WEIGHT OF } H_2O = 8gh(1)(1.625 \times 10^{-3})$$

FOR CUBAN GLASS  $\cos \theta = 1$

EQUATING FORCES:

$$2(1)(\sigma) = 8gh(1)(1.625 \times 10^{-3})$$

$$h = \frac{2(0.0735)}{(1000)(9.81)(1.625 \times 10^{-3})}$$

$$= 0.00922 \text{ m} = 9.22 \text{ mm}$$

ANSWER: 9.22 mm

1.25 GLASS TUBE -  $d_i = 0.25 \text{ mm}$

$$d_o = 0.35 \text{ mm}$$

$$\theta = 130^\circ$$

SURFACE TENSION FORCE -

$$\text{INSIDE} = 2\pi r_i \sigma \cos \theta$$

$$\text{OUTSIDE} = 2\pi r_o \sigma \cos \theta$$

TOTAL UPWARD FORCE -

$$F = 2\pi \sigma \cos \theta (r_i + r_o)$$

$$= 2\pi (0.144)(\cos 130^\circ) \left( \frac{0.25 + 0.35}{2} \times 10^{-3} \right)$$

$$= 5.33 \times 10^{-4} \text{ N}$$

1.26  $H_2O$ -AIR-GLASS INTERFACE @  $40^\circ C$

TUBE RADIUS = 1 mm

$$h = \frac{2\sigma \cos \theta}{8gr}$$

$$\sigma = 0.123 \left[ 1 - 0.00139(313) \right] = 0.0695 \text{ N/m}$$

$$h = \frac{2(0.0695)}{(993)(9.81)(1 \times 10^{-3})}$$

$$= 0.0143 \text{ m} (1.43 \text{ cm})$$

1.27 SOAP BUBBLE -  $T = 20^\circ C$   $d = 4 \text{ mm}$

$$\sigma = 0.025 \text{ N/m} (\text{TABLE 1.2})$$

FORCE BALANCE FOR BUBBLE:

$$\pi r^2 \Delta P = 2\pi r \sigma$$

$$\Delta P = \frac{2\sigma}{r} = \frac{2(0.025)}{2 \times 10^{-3}}$$

$$= 25 \text{ N/m}^2 \sim 25 \text{ Pa}$$

1.28 GLASS TUBE IN Hg ( $\sigma_{Ag} = 0.44 \text{ N/m}$ )

For Hg / glass -  $\sigma = 0.44 \text{ N/m}$

$$\theta = 130^\circ$$

$$h = \frac{2\sigma}{8gr} \quad r = 3 \text{ mm}$$

$$= \frac{2(0.44)}{13.6(1000)(18 \times 10^{-3})}$$

$$= -0.0277 \text{ m}$$

= 2.77 cm DEPRESSION

1.29 @  $60^\circ\text{C}$   $\sigma_{H_2O} = 0.0662 \text{ N/m}$

$$\sigma_{Ag} = 0.44$$

TUBE DIAMETER = 0.55 mm

$$h = \frac{2\sigma \cos \theta}{8gr}$$

for  $H_2O$ :

$$h = \frac{2(0.0662)(981)}{983(9.81)(0.55 \times 10^{-3}/2)}$$

$$= 0.0499 \text{ m (4.99 cm rise)}$$

for Ag:

$$h = \frac{2(0.44)(\cos 130^\circ)}{13.6(983)(9.81)(0.55 \times 10^{-3}/2)}$$

$$= -0.0157 \text{ m}$$

(1.57 cm Depression)

1.30  $H_2O$  / GLASS INTERFACE

$$T = 30^\circ\text{C}$$

$$\sigma = 0.123 [1 - 0.00139(303)]$$

$$= 0.0712 \text{ N/m}$$

$$\rho = 996 \text{ kg/m}^3$$

$$h \leq 1 \text{ mm}$$

$$h = \frac{2\sigma \cos \theta}{8gr}$$

$$r = 2\sigma / \rho gh$$

$$= \frac{2(0.0712)}{996(9.81)(1 \times 10^{-3})}$$

$$= 0.0146 \text{ m (1.46 cm)}$$

$$d = 2r = 2.915 \text{ cm}$$

## CHAPTER 2

2.1 Assume Local Gas Behavior.

$$\frac{dp}{dy} = -\rho g = -\frac{\rho g}{RT}$$

for  $T = a + by$

$$\Rightarrow T = 530 - 24y/h$$

$$\frac{dp}{P} = -\frac{g}{R} \frac{by}{530 - 24(y/h)}$$

$$\int_{P_0}^P \frac{dp}{P} = gh \int_0^1 \frac{-24}{530 - 24(y/h)} dy$$

$$\ln \frac{P}{P_0} = gh \ln \frac{530}{530 - 24h}$$

WITH  $P = 10.6 \text{ psia}$ ,  $P_0 = 30.1 \text{ inHg}$

$$h = 9192 \text{ FT}$$

2.2



$$\sum F_y = 0 \text{ on TANK}$$

$$\frac{P\pi d^2}{4} - \frac{P_{atm}\pi d^2}{4} - 250 = 0 \quad (1)$$

AT H<sub>2</sub>O LEVEL IN TANK:

$$P = P_{atm} + \rho_w g(h-y) \quad (2)$$

$$\text{from (1) } \frac{1}{2}(2) \quad h-y = 1,275 \text{ FT} \quad (3)$$

for Isothermal compression of air

$$P_{atm}V_{TANK} = P(V_{air})$$

$$P = \frac{3}{3-y} P_{atm} \quad (4)$$

Combining (1) & (4)

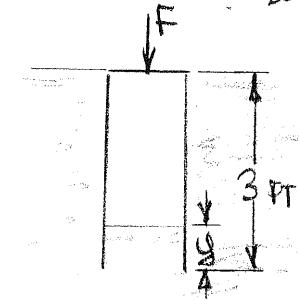
$$y = 0.12 \text{ FT}$$

$$h = 1,395 \text{ FT.}$$

2.2 CONT.

FOR TOP OF TANK FLUSH WITH H<sub>2</sub>O LEVEL

$$\sum F_y = 0$$



$$P = P_{atm} + \frac{250 + F}{\pi d^2/4}$$

AT H<sub>2</sub>O LEVEL IN TANK:

$$P = P_{atm} + \rho_w g(3-y)$$

COMBINING EQUATIONS:

$$F = 196(3-y) - 250$$

FOR ISOTHERMAL COMPRESSION OF AIR  
(AS IN PART (1))

$$3-y = 2.8 \text{ FT}$$

$$\Rightarrow F = 196(2.8) - 250 = 293.6 \text{ lbf}$$

2.3 WHEN NET FORCE ON TANK = 0

$$WT = \text{BOYANT FORCE} = 250 \text{ lbf}$$

$$V_w \text{ DISPLACED} = 250 / \rho_w g = 4.01 \text{ FT}^3$$

ASSUMING ISOTHERMAL COMPRESSION

$$P_{atm} + (3FT) = P(4.01 \text{ FT}^3)$$

$$= (P_{atm} + \rho g y)(4.01)$$

$$y = 45.88 \text{ FT}$$

$$\text{TOP IS AT LEVEL: } y = \frac{4.01}{\pi d^2/4}$$

OR AT 44.6 FT BELOW SURFACE

2.4.

$$\frac{dp}{dy} = \rho g = \rho_0 e^{\frac{\Delta P}{\beta}} \quad (1)$$

$$\int_0^{\Delta P} \frac{-\Delta P/\beta}{e^{\Delta P/\beta}} dy = \int_0^y -\frac{\rho_0 g \Delta y}{\beta} \quad (2)$$

$$e^{-\Delta P/\beta} = 1 - \frac{\rho_0 g y}{\beta} \quad (3)$$

$$\Delta P = -\beta \ln\left(1 - \frac{\rho_0 y}{\beta}\right) \quad (4)$$

$$= 300,000 \ln(1 - 0.0462) \quad (5)$$

$$= 14190 \text{ Pa} \quad (6)$$

DENSITY RATIO:

$$\frac{\rho}{\rho_0} = e^{-\Delta P/\beta} = 1.0484$$

$$\text{so } \rho = 1.0484 \rho_0$$

2.5. BUOYANT FORCE:

$$F_B = \rho V = \frac{\rho V}{RT}$$

FOR CONSTANT VOLUME:

F VARIES INVERSELY WITH T

2.6 SEA  $H_2O$ :  $\rho_0 = 1.025$ AT DEPTH  $y = 185 \text{ m}$ 

$$\begin{aligned} p_g &= 1.025 \rho_0 g y \\ &= 1.025 (1000)(9.81)(185) \\ &= 1.86 \times 10^6 \text{ Pa} \\ &= 1.86 \text{ MPa} \end{aligned}$$

2.7.

r measured from Earth's Surface

R = RADIUS OF EARTH

$$\frac{dp}{dr} = \rho g = \rho_0 \frac{g}{R} r$$

$$P - P_{\text{atm}} = \frac{\rho_0 r^2}{2R}$$

AT CENTER OF EARTH -  $r = R$ 

$$P_{\text{CE}} - P_{\text{atm}} = \frac{\rho_0 R}{2}$$

SINCE  $P_{\text{CE}} \gg P_{\text{atm}}$ 

$$\begin{aligned} P_{\text{CE}} &\approx \frac{\rho_0 R}{2} = \frac{(5.67)(1000)(9.81)(6330 \times 10^3)}{2} \\ &= 176 \times 10^9 \text{ Pa} \\ &= 176 \text{ MPa} \end{aligned}$$

2.8

$$\frac{dp}{dy} = -\rho g$$

$$\int_{P_{\text{atm}}}^p \frac{dp}{dy} dy = -\rho g \int_0^{-h} dy$$

$$P - P_{\text{atm}} = \rho g (-h)$$

$$= (1050)(9.81)(11034)$$

$$= 113.7 \text{ MPa}$$

$$\approx 1122 \text{ ATMOSPHERES}$$

2.9 AS IN PREVIOUS PROBLEM

$$P - P_{\text{atm}} = \rho g h$$

$$\text{for } P - P_{\text{atm}} = 101.33 \text{ kPa}$$

$$h = 101.33 / \rho g$$

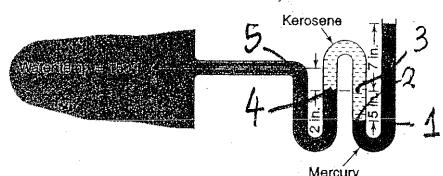
2.9 (CONT.)

$$\text{for } H_2O: h = \frac{10133}{(1000)(9.81)} = 10.33 \text{ m}$$

$$\text{SEA } H_2O: h = \frac{10133}{(1.025)(1000)(9.81)} = 10.08 \text{ m}$$

$$Hg \quad h = \frac{10133}{13.6(1000)(9.81)} = 0.80 \text{ m}$$

2.10



$$P_1 = P_{atm} + \rho_{Hg} g (12") \quad P_1 = P_2$$

$$P_2 = P_3 + \rho_k g (5") \quad P_3 = P_4$$

$$P_4 = P_A + \rho_w g (2") \quad P_4 = P_5$$

$$P_{atm} + \rho_{Hg} g (12) = P_A + \rho_w g (2) + \rho_k g (5)$$

$$P_A = P_{atm} + \rho_w g [(13.6)(12) - 2 - 0.75(5)]$$

$$= P_{atm} + 5.81 \text{ psi} = 5.81 \text{ psi}$$

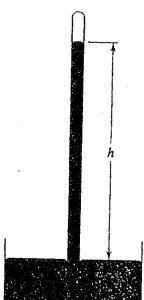
2.11 Force Balance on LIQUID COLUMN:

$A$  = AREA OF TUBE

$$-3A + 14.7A - \rho g h A = 0$$

$$h = \frac{11.7(144)}{62.4(12.2)}$$

$$= 26.6 \text{ in.}$$



2.12

$$P_A = P_B - \rho_0 g (10 \text{ ft})$$

$$P_C = P_B + \rho_w g (5 \text{ ft})$$

$$P_D = P_C - \rho_{Hg} g (1 \text{ ft})$$

$$P_A - P_D = \rho_{Hg} g (1) - \rho_w g (5) - \rho_0 g (10)$$

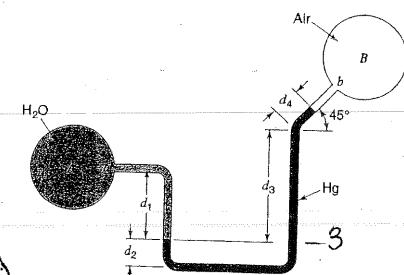
$$P_A - P_{atm} = \rho_w g (13.6 \times 1 - 5 - 0.8 \times 10 \times 1)$$

$$= 37.4 \text{ lb}_f/\text{ft}^2$$

2.13

$$P_3 = P_A - \rho_0 g \rho_w$$

$$= P_B + (\rho_{Hg} g) \times \\ \times (d_3 + d_4 \sin 45^\circ)$$



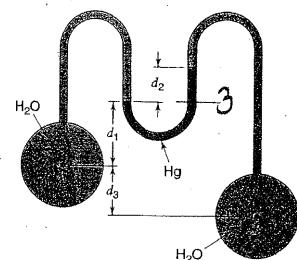
$$P_A - P_B = \frac{(62.4)(32.2)}{32.2} \left[ (12.4 + 4 \sin 45^\circ) / 13.6 - 2 \right]$$

$$= 245 \text{ lb}_f/\text{ft}^2 = 1.70 \text{ psi}$$

2.14

$$P_3 = P_A - \rho_w g d_1$$

$$P_3 = P_B - \rho_w g (d_1 + d_2 + d_3) \\ + \rho_{Hg} g d_2$$



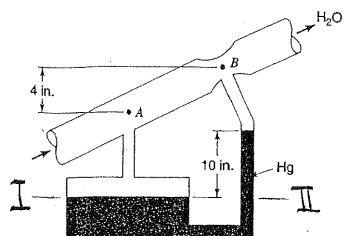
EQUATING:

$$P_A - P_B = \rho_{Hg} g d_2 - \rho_w g (d_2 + d_3)$$

$$= \rho_w g \left[ (13.6)(1/12) - 7.3/12 \right]$$

$$= 32.8 \text{ lb}_f/\text{ft}^2 = 0.227 \text{ psi}$$

2.15



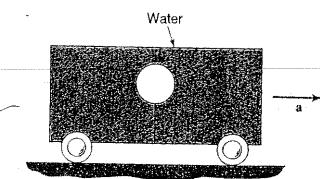
$$P_I = P_A + \rho_{Hg} g (10")$$

$$P_{II} = P_B + \rho_{Hg} g (4") + \rho_{Hg} g (10")$$

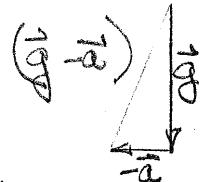
$$P_I = P_{II}$$

$$P_A - P_B = \rho_{Hg} [-6 + 13.6(10)] \\ = 56.3 \text{ psi}$$

2.16



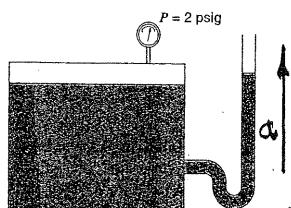
PRESSURE GRADIENT IS IN  
DIRECTION OF  $\vec{g} - \vec{a}$   
ISOBARS ARE PERPENDICULAR TO  $(\vec{g} - \vec{a})$



STRING WILL ASSUME THE  
 $(\vec{g} - \vec{a})$  DIRECTION & BALLOON  
WILL MOVE FORWARD.

2.17

AT REST:  
 $P = \rho g y_0$



ACCELERATION:

$$P = \rho |(\vec{g} - \vec{a})| = \rho (g + a) y_a$$

$$\text{EQUATION: } y_a = \frac{g}{g+a} \text{ WHICH } < y_0$$

LEVEL GOES DOWN

$$2.18 \quad F = P_{6.6} A - P_{Atm} A = \rho g h (\pi r^2)$$

$$h = 2 \text{ m} \quad r = 0.3 \text{ m}$$

$$F = 5546 \text{ N}$$

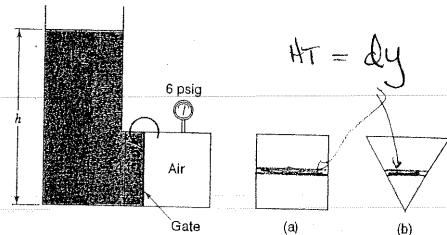
$$y_{c.p.} = \bar{y} + I_{bb}/A \bar{y}$$

$$\text{For A Circle: } I_{bb} = \pi r^4 / 4$$

$$y_{c.p.} = 2 \text{ m} + \frac{\pi (0.3 \text{ m})^4}{4 \pi (0.3 \text{ m})^2 (2 \text{ m})}$$

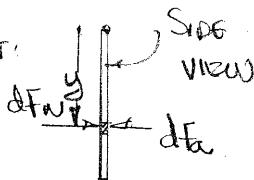
$$= 2.1011 \text{ m}$$

2.19



HEIGHT OF  $H_2O$  COLUMN  
ABOVE DIFFERENTIAL ELEMENT:

$$= h - 4 + y$$



For (a) - RECTANGULAR GATE -  $\Delta A = 4 \text{ dy}$

$$\Delta F_W = [\rho_w g (h - 4 + y) + P_{Atm}] \Delta A$$

$$\Delta F_A = [P_{Atm} + (6 \text{ psig})(144)] \Delta A$$

$$\sum M_o = \int_A y (\Delta F_W - \Delta F_A) = 0$$

$$\int_0^4 y [\rho g (h - 4 + y) - 864] (4 \text{ dy}) = 0$$

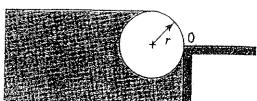
$$h = 15.18 \text{ FT}$$

For (b):  $\Delta A = (4 - y) \Delta y$

$$\int_0^4 y [\rho g (h - 4 + y) - 864] (4 - y) \Delta y = 0$$

$$h = 15.85 \text{ FT}$$

2.20



PER UNIT DEPTH:

$$\sum F_y = 0$$

$$F_{y\text{up}} = S_w g \pi r^2 / 2 \quad \{ \text{Buoyancy} \}$$

$$F_{y\text{down}} = S_w g \pi r^2 + S_w g (r^2 - \pi r^2 / 4)$$

FRONT SIDE:

$$\frac{S_w g \pi r^2}{2} = S_w g \pi r^2 + S_w g r^2 \left(1 - \frac{\pi}{4}\right)$$

$$S_w = S_w \left(\frac{\pi}{2} - 1 + \frac{\pi}{4}\right) / \pi$$

$$= S_w \left(\frac{3}{4} - \frac{1}{\pi}\right) = 0.432 S_w$$

$$= 432 \text{ kg/m}^3$$



a) To LIFT block from bottom

$$F = \{ \text{WT OF CONCRETE} \} + \{ \text{WT OF H2O} \}$$

$$= S_c g V + [S_w g (22.75') + P_{atm}] A$$

$$= (150) g (3 \times 3 \times 0.5)$$

$$+ [62.4 g (22.75) + 14.7 (144)] \times (3 \times 3)$$

$$= 675 + 31828 = \underline{32503 \text{ Lbf}}$$

2.21 (CONT)

b) TO MAINTAIN BLOCK IN FREE POSITION

$$F = \{ \text{WT OF CONCRETE} \} - \{ \text{BUOYANT FORCE} \} \text{ OF A}_{20}$$

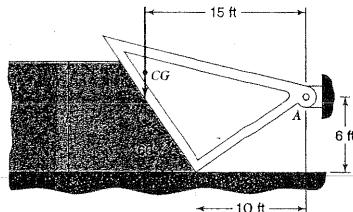
$$= 675 - S_w g V$$

$$= 675 - [62.4 g (3 \times 3 \times 0.5)]$$

$$= 675 - 281 = \underline{394 \text{ Lbf}}$$

2.22.

DISTANCE Z  
MEASURED ALONG  
GATE SURFACE  
FROM BOTTOM



$$\Sigma M_A = 500(15) - \int_{0}^{h/\sin 60} z S_w g (h - z \sin 60) dz = 0$$

$$S_w g \int_{0}^{h/\sin 60} (zh - z^2 \sin 60) dz = 7500$$

$$S_w g \left[ \frac{h^2}{2} - \frac{z^3}{3} \sin 60 \right]_0^{h/\sin 60} = 7500$$

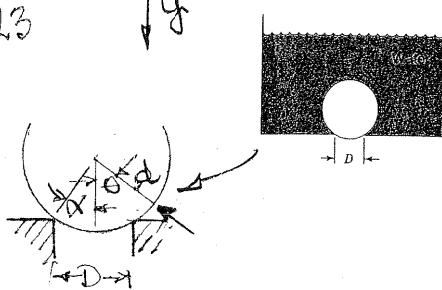
$$(62.4) g \left[ \frac{h^3}{(\sin 60)^2} \left( \frac{1}{2} - \frac{1}{3} \right) \right] = 7500$$

$$h^3 = \frac{7500 (6)(\sin 60)^2}{62.4 g} = 541$$

$$h = 8.15 \text{ FT}$$

2.23

↓ y



USING SPHERICAL COORDINATES FOR A PLATE  
AT  $y = \text{CONSTANT}$ :

$$dA = 2\pi r^2 \sin\theta d\theta$$

$$P = \rho g [h - r \cos\theta + r \cos\theta]$$

$$df_y = dP \cos\theta$$

$$F_y = \int \rho g (h - r \cos\theta + r \cos\theta) x \left( 2\pi r^2 \sin\theta d\theta \right)$$

$$= \cancel{2\pi} \rho g r^2 \int_{0}^{\pi} (h - r \cos\theta + r \cos\theta) \sin\theta \cos\theta d\theta$$

$$= C \left[ \int_{0}^{\pi} (h - r \cos\theta) \sin\theta \cos\theta d\theta + r \int_{0}^{\pi} \sin\theta \cos^2\theta d\theta \right]$$

$$= C \left[ (h - r \cos\theta) \sin^2\theta \Big|_0^\pi + r \left( -\frac{1}{3} \cos^3\theta \right) \Big|_0^\pi \right]$$

$$= C \left[ (h - r \cos\theta) (1 - \sin^2\theta) - \frac{r}{3} (0 - \cos^3\theta) \right]$$

Now - for  $F_y = 0$

$$\sin\theta = \frac{D}{r} \quad \text{and} \quad \left[ 1 - \left( \frac{D}{r} \right)^2 \right]^{1/2}$$

$$\therefore r = \frac{D}{2}$$

2.23 (cont)

$$0 = \left( h - \frac{D}{2} \cos\theta \right) (1 - \sin^2\theta)^{1/2} + \frac{D}{6} (1 - \cos^3\theta)$$

$$= h - \frac{D}{2} \cos\theta + \frac{D}{6} \cos\theta$$

$$\text{GIVEN} \quad h = \frac{D}{3} \cos\theta$$

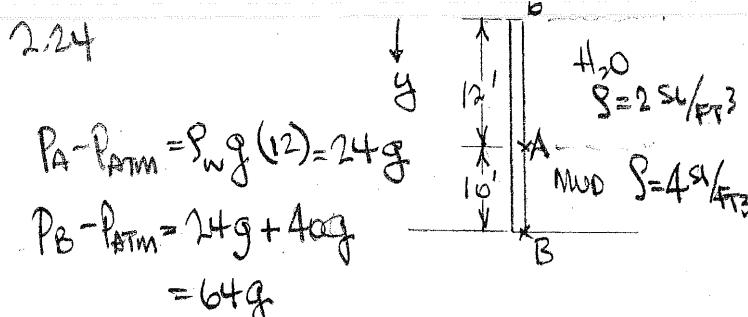
$$\frac{h}{D} = \frac{\cos\theta}{3} = \frac{1}{3} \left[ 1 - \left( \frac{D}{2} \right)^2 \right]^{1/2}$$

$$\text{For } D = 0.6 \text{ m}$$

$$h = \frac{0.6}{3} \left[ 1 - \left( \frac{50}{3} \right)^2 \right]^{1/2}$$

$$= \frac{1}{5} \left[ 1 - \left( \frac{50}{3} \right)^2 \right]^{1/2}$$

2.24



$$\text{BETWEEN } O \text{ & } A: \quad P - P_{\text{atm}} = \rho_w g y$$

$$\text{" } \quad \text{A & B} \quad P = \rho_w g (12) + \rho_m g (y - 12)$$

FOR UNIT DEPTH:

$$F = \int (P - P_{\text{atm}}) dA$$

$$= \int_0^{12} \rho_w g y dy + \int_{12}^{22} [\rho_w g (12) + \rho_m g (y - 12)] dy$$

$$= \rho_w g (192) + \rho_m g (50)$$

$$= \underline{18790 \text{ N}}$$

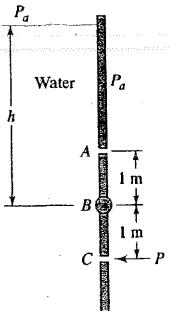
2.24 (cont.)

Force Location:

$$\begin{aligned}
 Fx\bar{y} &= \int_0^{22} y(P - P_{atm}) dA \\
 &= \int_0^{12} \rho_w g y^2 dy + \int_{12}^{22} \rho_w g / 2 y dy \\
 &\quad + \int_{12}^{22} \rho_m g (y^2 - 12y) dy \\
 &= \rho_w g (576 + 2040) \\
 &\quad + \rho_m g (2973 - 2040) \\
 &= 288400 \text{ ft lb}_f \\
 \bar{y} &= \frac{288400}{18790} = 15.35 \text{ ft}
 \end{aligned}$$

2.25

$$\begin{aligned}
 \text{Force on gate} &= \rho g \bar{y} A \\
 &= (1000)(9.81)(12) \frac{\pi}{4} (2)^2 \\
 &= 369.8 \text{ kN}
 \end{aligned}$$

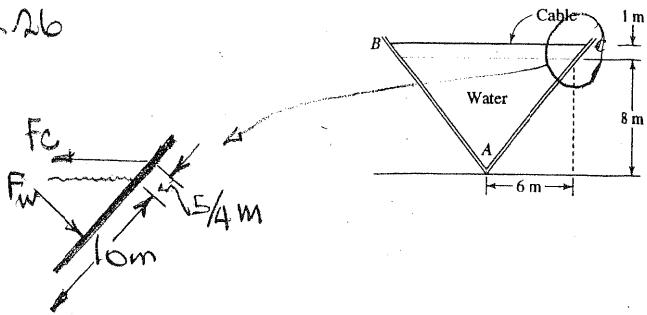


$$y_{cg} = \frac{I_{eo}}{\bar{y} A} = \frac{(\pi/4)(1)^4}{(12)(\pi/4)(2)^2} = 0.0208 \text{ m} \quad (\text{BELOW Axis B})$$

$$\sum M_B = 0$$

$$\begin{aligned}
 P(1) &= (369.8 \times 10^3)(0.0208) \\
 &= 7.70 \text{ kN}
 \end{aligned}$$

2.26



$$F_w = \rho g \bar{y} A$$

$$= (1000)(9.81)(4)(10)(1) = 392 \text{ kN}$$

$y_{cg}$  IS  $\frac{1}{3}$  DISTANCE FROM WATER LINE TO A

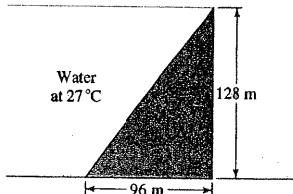
~ 6.66 m DOWN from  $H_2O$  LINE  
3.33 m UP from A

$$\sum M_A = F_c(9) = 392(3.33)$$

$$F_c = 145.2 \text{ kN}$$

2.27 WIDTH = 100m

$$H_2O @ 27^\circ C \quad \rho = 997 \text{ kg/m}^3$$



$$\begin{aligned}
 F &= \rho g \bar{y} A \\
 &= (997)(9.81)(64) \\
 &\quad \times (160)(100)
 \end{aligned}$$

$$= 10.016 \times 10^9 \text{ N} = 10.02 \times 10^3 \text{ MN}$$

FOR A FREE  $H_2O$  SURFACE

$$y_{cg} = \frac{2}{3}(128 \text{ m}) = 85.3 \text{ m} \quad \left. \begin{array}{l} \text{BELOW} \\ H_2O \\ \text{SURFACE} \end{array} \right\}$$

$$= 106.7 \text{ m } \left. \begin{array}{l} \text{MEASURED FRONT} \\ \text{DAM SURFACE} \end{array} \right\}$$

## 2.28 SPHERICAL FLOAT

UPWARD FORCES ~  $F + F_{BOOYANT}$   
 DOWNWARD " ~  $WT$

$$W = \rho g V = \rho g \left( \frac{4}{3} \pi R^3 \right)$$

$$F_b = \rho_w g V z = \rho_w g \left( \frac{4}{3} \pi R^3 \right) z$$

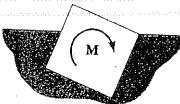
$z$  = FRACTION SUBMERGED

$$F = \rho g \left( \frac{4}{3} \pi r^3 \right) - \rho_w g z \left( \frac{4}{3} \pi r^3 \right)$$

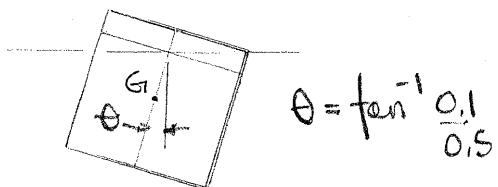
$$z = \frac{\rho g \left( \frac{4}{3} \pi r^3 \right) - F}{\rho_w g \left( \frac{4}{3} \pi r^3 \right)}$$

## 2.29

CUBE -



LENGTH OF SIDE =  $L$



$$\theta = \tan^{-1} \frac{0.1}{0.5}$$

G IS CENTER OF MASS OF SOLID

$$\begin{aligned} \sum M_G &= 2 \left[ \frac{1}{2} \left( \frac{L}{2} \right) (0.1L)(L) \rho g \left( \frac{2}{3} \frac{L}{2} \sin \theta \right) \right. \\ &\quad \left. - (0.9L)(L)(L) \rho g (0.05L \sin \theta) \right] \\ &+ M \end{aligned}$$

{ PART OF ORIGINAL SUBMERGED  
 VOLUME IS NOW OUT OF H2O - }  
 { PART THAT WAS ORIGINALLY OUT  
 IS NOW SUBMERGED }

## 2.29 (CONT.)

$$\begin{aligned} M &= \rho g L^4 \sin \theta \left[ -\frac{1}{60} + 0.045 \right] \\ &= \rho g L^4 \sin \theta (0.02833) \\ &= 0.00556 \rho g L^4 \end{aligned}$$

## CHAPTER 4

4.1  $\vec{v} = 10\vec{e}_x + 7x\vec{e}_y$   
 AT (2,2)  $\vec{v} = 10\vec{e}_x + 14\vec{e}_y$

AT  $-30^\circ$  from x axis:

UNIT VECTOR:  $\vec{e} = \frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y$

ALONG THIS DIRECTION THE  
COMPONENT IS  $\vec{e} \cdot \vec{v}$ :

$$\vec{e} \cdot \vec{v} = \left(\frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y\right) \cdot (10\vec{e}_x + 14\vec{e}_y)$$

$$= 5\sqrt{3} - 7 = 1.66 \text{ m/s}$$

4.2  $\vec{v} = 10\vec{e}_x + 7x\vec{e}_y$

$$\frac{dy}{dx} = \frac{v_y}{v_x} = \frac{7x}{10}$$

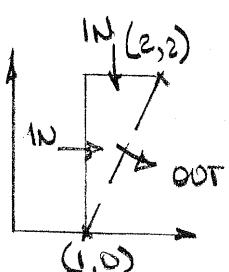
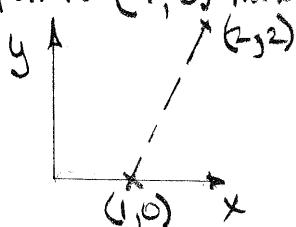
$$10y = 7x^2 + C$$

AT (2,1)  $C = 10 - 14 = -4$

EQN IS:  $\frac{7x^2}{2} - 10y + C = 0$

OR  $x^2 - \frac{10}{7}y - 8/7 = 0 \quad (\text{a})$

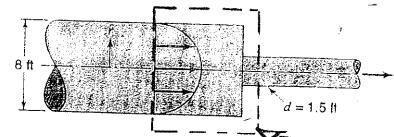
ACROSS THE SURFACE CONNECTING  
POINTS (1,0) AND (2,2):



## 4.2 (CONT)

$$\begin{aligned} \vec{v} &= \int_0^2 \vec{v} \cdot \vec{e}_x dy + \int_1^2 \vec{v} \cdot \vec{e}_y dx \\ &= 10y \Big|_0^2 + 7x^2 \Big|_1^2 \\ &= 20 + \frac{1}{2}(3) = 20 + 10.5 \\ &= 30.5 \text{ m}^3/\text{s} \end{aligned} \quad (\text{b})$$

4.3



$$\iint_{\text{C.S.}} S(\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{\text{C.V.}} S dV = 0$$

$$\iint_{\text{C.S.}} S(\vec{v} \cdot \vec{n}) dA = 8v_{2\text{avg}} A_2$$

$$- \int_0^R S g \left(1 - \frac{r^2}{16}\right) 2\pi r dr = 0$$

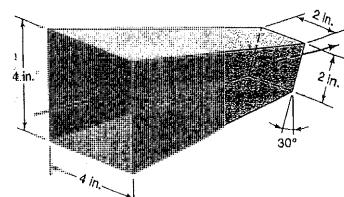
$$v_{2\text{avg}} A_2 = 18\pi \int_0^4 \left(r - \frac{r^3}{16}\right) dr$$

$$= 18\pi \left[\frac{r^2}{2} - \frac{r^4}{64}\right]_0^4 = 72\pi$$

$$v_{2\text{avg}} = \frac{72\pi}{\pi (3/4)^2} = 128 \text{ ft/s}$$

4.4

$$\iint_{\text{C.S.}} S(\vec{v} \cdot \vec{n}) dA = 0$$



$$\iint_{A_0} S v \cos 30 dA - \iint_{A_1} S v dA = 0$$

AA - CONTINUED

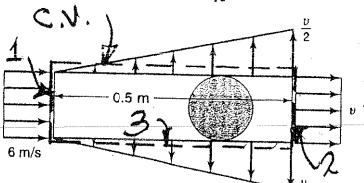
$$S_{IN} = S_{OUT} \quad A_{IN} = 4A_{OUT}$$

$$V_{OUT} = \frac{A_i V_i}{A_0 \cos 30^\circ}$$

$$= \frac{4(10)}{\cos 30^\circ} = 46.2 \text{ FT/s}$$

$$\dot{V} = A_i V = 10 \left( \frac{1}{3} \times \frac{1}{3} \right) \\ = 1.11 \text{ FT}^3/\text{s}$$

4.5 Steady Flow:  $\oint \vec{S}(\vec{v} \cdot \vec{n}) dA = 0$   
c.s.



$$-\rho g A_1 + \rho g A_2 + \int_A \vec{S}(\vec{v} \cdot \vec{n}) dA = 0$$

$\rho = \text{CONST.}$

$$V_1 A_1 = V_2 A_2 + \int_0^L \frac{V_2}{2} \pi D x dx$$

$$= V_2 A_2 + \frac{V_2}{2} \pi D \frac{x^2}{2} \Big|_0^L$$

$$= V_2 \frac{\pi D^2}{4} + \frac{V_2 \pi D L}{2}$$

$$= \frac{V_2 \pi D}{4} (D + L)$$

$$6 \frac{\pi}{4} (0.2)^2 = V_2 \frac{\pi}{4} [0.2(0.2 + 0.5)]$$

$$V_2 = 6(0.04) \\ (0.2)(0.7)$$

$$= 1.71 \text{ m/s}$$

4.6 FOR STEADY, INCOMPRESSIBLE FLOW:

$$\dot{V} = A_i V_{AVG} = \sum_i V_i \Delta A_i$$

From GIVEN DATA SET:

DIST FROM CENTER IN	$V_i$ FT/s	$\Delta A_i$ IN <sup>2</sup>	$V_i \Delta A_i$ FT <sup>3</sup> /s
0	7.5	7.844	0.4084
3.16	7.10	37.64	1.856
4.45	6.75	31.96	1.498
5.48	6.42	32.10	1.431
6.33	6.15	31.48	1.344
7.07	5.81	29.85	1.204
7.75	5.47	33.22	1.262
8.37	5.10	33.22	1.176
8.94	4.50	31.44	0.982
9.49	3.82	31.57	0.838
10.-	2.40	15.82	0.264

$$\sum \rightarrow 316.14 \text{ FT}^3/\text{s}$$

$$\sum \Delta A_i = 316.14 \text{ IN}^2 \quad \left\{ \begin{array}{l} \text{EXACT AREA} \\ = 314.16 \text{ IN}^2 \end{array} \right\} \\ = 2.195 \text{ FT}^2$$

$$\dot{V} = \sum V_i \Delta A_i = 18.26 \text{ FT}^3/\text{s}$$

$$V_{AVG} = \frac{\dot{V}}{A} = \frac{18.26}{2.195} = 8.32 \text{ FT/s}$$

4.7 INFLOW:  $\dot{V} = 2 \text{ gal/m} = 19.2 \text{ lb/m}$

OUTFLOW:  $\dot{V} = 19.2 \text{ lb}_m/m$

~ STEADY FLOW ~

for TOTAL FLOW:  $\oint \vec{S}(\vec{v} \cdot \vec{n}) dA = \dot{m}_{OUT} - \dot{m}_{IN} = 0$

TOTAL MASS IN TANK = M

MASS OF SALT IN TANK = S

for SALT:  $\dot{m}_{OUT} = 19.2 (S/M) \text{ lb}_m/m$

$\dot{m}_{IN} = 19.2 (1.92) \text{ "}$

## 4.7 - CONTINUED

for SAET: LENS. OF MASS

$$\frac{19.2S}{M} - 3.84 + \frac{dS}{dt} = 0$$

$$\frac{dS}{dt} = 3.84 - \frac{19.2S}{M}$$

$$= A - BS$$

$$A = 3.84$$

$$B = \frac{19.2}{M} = \frac{(19.2)(7.48)}{(100)(62.4)}$$

$$= 0.0230$$

$$\int_0^S \frac{dS}{A - BS} = \int_0^t dt$$

$$-\frac{1}{B} \ln A \frac{-BS}{A} = t$$

$$\ln \left[ 1 - \frac{BS}{A} \right] = -Bt$$

$$\text{or } S = \frac{A}{B} \left[ 1 - e^{-Bt} \right]$$

for  $t = 100 \text{ m}$ 

$$S = \frac{3.84}{0.0230} \left( 1 - e^{-230} \right)$$

$$\approx \underline{\underline{150 \text{ lb/m}}}$$
(a)

$$\text{for } t = \infty \quad S = \underline{\underline{167 \text{ lb/m}}} \quad (\text{b})$$

for  $S = 100 \text{ lb/m}$ 

$$t = \frac{1}{0.023} \ln \left[ 1 - \frac{0.023}{3.84} (100) \right]$$

## 4.7 - CONTINUED

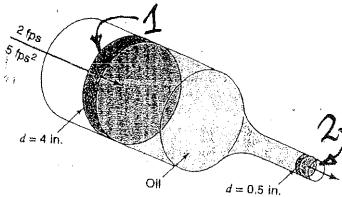
$$t = 39.8 \text{ m} \quad \text{for } S = 100 \text{ lb/m}$$

$$\text{from (a) } t = 100 \text{ m} \text{ for } S \approx 150 \text{ lb/m}$$

for  $S$  from 100 to 150

$$\Delta t = 100 - 39.8 = \underline{\underline{60.2 \text{ m}}} \quad (\text{c})$$

## 4.8



for piston &amp; cylinder shown:

$$\text{at 1} \quad V = V_1 = 2 \text{ ft}^3/\text{s} \quad a = a_1 = 5 \text{ ft/s}^2$$

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1}{A_2} V_1 = \left( \frac{d_1}{d_2} \right)^2 V_1$$

$$= \left( \frac{4}{0.5} \right)^2 (2) = \underline{\underline{128 \text{ ft/s}}}$$

$$a_2 = a_1 \left( \frac{d_1}{d_2} \right)^2 = 5 \left( \frac{4}{0.5} \right)^2 = \underline{\underline{320 \text{ ft/s}^2}}$$

4.9 for steady flow:

$$\iint_S S(\vec{v} \cdot \vec{n}) dA = 0$$

a.s.

$$\text{or } S v A = \text{constant}$$

$$\frac{d(S v A)}{S v A} = \frac{dS}{S} + \frac{dv}{v} + \frac{dA}{A} = 0$$

Q.E.D.

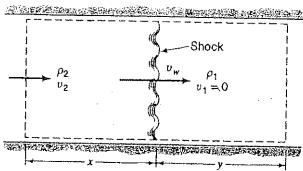
$$4.10 \quad \iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA = \iint_{CS} dm$$

$$\iint \rho (\vec{v} \cdot \vec{n}) dA = dm$$

$$\frac{\partial}{\partial t} \iint_{C.V.} \rho dV = \frac{\partial}{\partial t} M$$

$$\therefore \frac{\partial M}{\partial t} + \iint_{CS} dm = 0 \quad Q.E.D.$$

4.11



FOR THE C.V. SHOWN:

$$\iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iint_{C.V.} \rho dV = 0$$

C.V.  
O - STF FLOW

C.V. MOVES TO RIGHT WITH  $V = V_w$   
THUS:

$$-\dot{S}_1 A V_w + \dot{S}_2 A (V_w - V_2) = 0$$

$$V_2 = V_w (1 - \frac{\dot{S}_1}{\dot{S}_2})$$

4.12

$$V_{AVG} = \frac{1}{A} \int_A v dA$$

$$= \frac{V_{MAX}}{\pi R^2} \int_0^R 2\pi r \left[1 - \frac{r}{R}\right]^{\frac{1}{4}} dr$$

$$\text{FOR } Z = r/R \quad dz = dr/R$$

$$V_{AVG} = 2V_{MAX} \int_0^1 z (1-z)^{\frac{1}{4}} dz$$

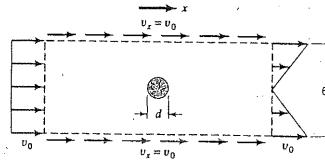
$$\text{FOR } \xi = 1-z \quad d\xi = -dz$$

$$V_{AVG} = -2V_{MAX} \int_1^0 (1-\xi) \xi^{\frac{1}{4}} d\xi$$

4.12 - CONTINUED

$$V_{AVG} = \frac{49}{60} V_{MAX} = 0.817 V_{MAX}$$

4.13



$$\iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iint_{C.V.} \rho dV = 0$$

O - STF FLOW

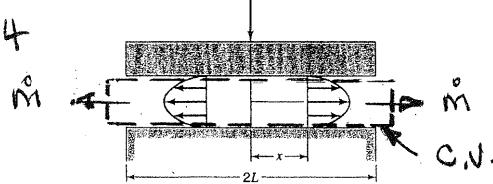
$$\iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA = - \dot{S} V_0 (bd) + \dot{m}_{\text{horiz}}$$

$$+ 2 \int_0^{3d} \rho \frac{V_0}{3d} y dy$$

$$\dot{m}_{\text{horiz}} = \dot{S} V_0 (bd) - \dot{S} V_0 (3d)$$

$$= \dot{S} V_0 (3d)$$

4.14



$$\frac{\partial m}{\partial t} + \int dm = 0$$

$$M = \dot{S}(2L)(b)(1) \quad \frac{\partial m}{\partial t} = 2SLb$$

$$\text{WHERE } b = -V$$

$$\int dm = 2 \dot{m}_{side} = 2 \int_0^b \dot{S} V(1) dy$$

$$\text{GIVEN: } -2SLV + 2S \int_0^b V dy = 0$$

$$\text{OR} \quad LV = \int_0^b V dy$$

## 4.14 - (CONTINUED)

(a) For  $V = V_{AVG}$  (A constant)

$$LV = V_{AVG} b$$

$$\underline{V_{AVG} = \frac{L}{b} V}$$

(b)  $V = ay + by^2$ WITH  $V(b) = 0$   $V\left(\frac{b}{2}\right) = V_{MAX}$ 

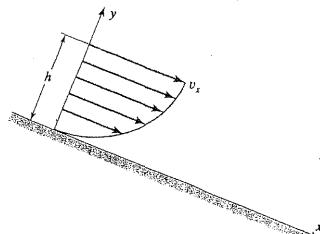
$$V = 4V_{MAX} \left[ \frac{y}{b} - \left( \frac{y}{b} \right)^2 \right]$$

$$LV = 4V_{MAX} \int \left[ \frac{y}{b} - \left( \frac{y}{b} \right)^2 \right] dy$$

{}

$$\underline{V_{MAX} = \frac{3}{2} \frac{LV}{b}}$$

## 4.15



$$\dot{V} = \int_A v_x dA$$

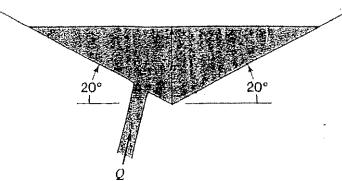
$$= W \int_0^h V_0 \left( 2 \frac{y}{h} - \frac{y^2}{h^2} \right) dy$$

$$= \frac{2}{3} W V_0 h$$

$$2000 \frac{\text{cm}^3}{\text{m}} = \frac{2}{3} (10) V_0 (2)$$

$$\underline{V_0 = 150 \text{ cm/m} = 2.5 \text{ cm/s}}$$

## 4.16



$$V = WA = Wh^2 \cot 20$$

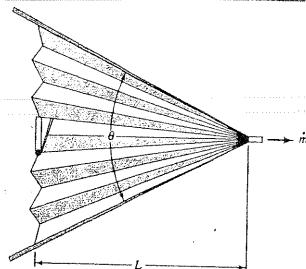
$$\dot{V} = w \cot 20 \cdot \frac{h^2}{2} / dt$$

$$\int_{h_1}^{h_2} dh^2 = \frac{\dot{V} \tan 20}{w} \int_0^t dt$$

$$h^2 \Big|_{h_1}^{h_2} = \frac{\dot{V} \tan 20}{w} t$$

$$\underline{t = (h_2^2 - h_1^2) \left[ \frac{w}{\dot{V}} \cot 20 \right]}$$

## 4.17



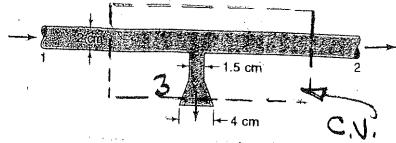
$$V = WA = WL^2 \tan \theta / 2$$

$$\begin{aligned} \dot{m} &= \rho \dot{V} = \rho WL^2 \frac{d}{dt} \left( \tan \frac{\theta}{2} \right) \\ &= \rho WL^2 \sec^2 \frac{\theta}{2} \frac{\dot{\theta}}{2} \end{aligned}$$

$$\underline{\omega^2 \frac{\theta}{2} = \frac{1 + \cos \theta}{2}}$$

$$\underline{\dot{m} = \frac{\rho WL^2 \dot{\theta}}{1 + \cos \theta} = \frac{\rho WL^2 \dot{\theta}}{2 \cos^2 \frac{\theta}{2}}}$$

4.18



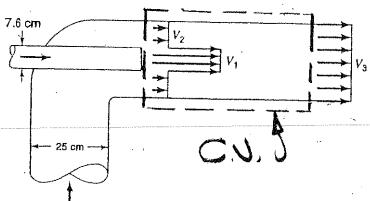
STEADY FLOW -  $\oint \dot{S}(\vec{G} \cdot \vec{n})dA = 0$   
C.S.

$$\oint \dot{V}_1 = 8A_1 V_2 + 8A_3 V_3$$

$$1.3 \times 10^{-3} = \frac{\pi}{4} (0.02)^2 (2) + (100) \frac{\pi}{4} (10^{-3})^2 V_3$$

$$V_3 = \underline{\underline{8.15 \text{ m/s}}}$$

4.19



STEADY FLOW -  $\oint \dot{S}(\vec{G} \cdot \vec{n})dA = 0$   
C.S.

$$\oint A_3 V_3 - \oint A_1 V_1 - \oint A_2 V_2 = 0$$

$$V_3 = \frac{A_1 V_1 + A_2 V_2}{A_3}$$

$$= \frac{\left[ \frac{\pi}{4} (0.076)^2 (40) + \frac{\pi}{4} (0.25^2 - 0.076^2) (3) \right]}{\frac{\pi}{4} (0.25^2)}$$

$$V_3 = \underline{\underline{5.15 \text{ m/s}}}$$

$$V_3 = \underline{\underline{5.15 \text{ m/s}}}$$

4.20

VOLUME DISPLACED BY PLUNGER

$$\dot{V} = A_p V_p = \frac{\pi}{4} d_p^2 V$$

VOLUME OF H<sub>2</sub>O MOVING PAST P:

$$\dot{V} = (A - A_p) V = \frac{\pi}{4} (D^2 - d_p^2) V$$

IN STEADY STATE OPERATION THESE  
MUST BE EQUAL:

$$\frac{\pi}{4} d_p^2 V = \frac{\pi}{4} (D^2 - d_p^2) V$$

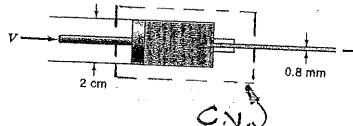
$$V = \sqrt{\frac{d_p^2}{D^2 - d_p^2}} \quad (a)$$

RELATIVE TO PLUNGER -

$$V_R = V + V$$

$$= V \left[ \frac{d_p^2}{D^2 - d_p^2} + 1 \right] \quad (b)$$

4.21

CONS. OF MASS - CONSTANT  $\rho$ 

$$\dot{V}_{\text{OUT}} = 6 \text{ cm}^3/\text{s} - \text{CONSTANT}$$

FOR NO LEAKAGE:

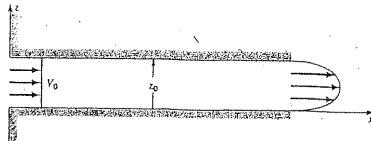
$$\dot{V} = A_p V = \frac{\pi}{4} (2)^2 V = 6$$

$$V = \underline{\underline{1.91 \text{ cm/s}}}$$

FOR LEAKAGE -  $\dot{V} = 6 + 0.6$ 

$$\dot{V} = \frac{6.6}{\pi/4 (2^2)} = \underline{\underline{2.1 \text{ cm/s}}}$$

4.22



PARALLEL PLATES -

INCOMPRESSIBLE, STEADY FLOW -

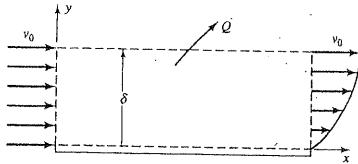
 $\dot{V}$  = CONSTANT

$$\begin{aligned} V_0 z_0 &= \int V \, dA \\ &= a \int_0^{z_0} z (z_0 - z) \, dz \\ &= a \frac{z_0^3}{6} \Rightarrow a = \frac{6V_0}{z_0^2} \end{aligned}$$

V IS MAX AT  $z = z_0/2$ 

$$\begin{aligned} V &= 6 \frac{V_0}{z_0^2} \left[ z_0 - \frac{z_0}{2} \right] \frac{z_0}{2} \\ &= 6V_0 / 4 = \underline{\underline{12 \text{ cm/s}}} \end{aligned}$$

4.23



FOR STEADY INCOMPRESSIBLE FLOW:

$$\dot{V}_{\text{OUT}} - \dot{V}_N = 0$$

$$Q + b \int_0^h V_0 \left( \frac{3y - y^3}{2} \right) dy = V_0 \delta b$$

$$\begin{aligned} \int_0^h \frac{3y - y^3}{2} dy &= \frac{\delta}{2} \int_0^1 \frac{3y - y^3}{2} dy \\ &= 5/8 \delta \end{aligned}$$

$$\underline{\underline{Q = V_0 \delta b \left( 1 - 5/8 \right) = \frac{3}{8} V_0 b \delta}}$$

4.24

SEE SKETCH FOR PROB 4.14

PLATES ARE CIRCULAR

$$\frac{\partial M}{\partial t} + \int \Delta m = 0$$

$$M = 8b\pi L^2$$

$$\frac{\partial M}{\partial t} = 8\pi L^2 b = -8\pi L^2 V$$

$$\dot{m}_{\text{exit}} = 8 \cdot 2\pi L b V_{\text{exit}} = 8\pi L^2 V$$

$$\Rightarrow \underline{\underline{V_{\text{exit}} = L V / 2b}} \quad (a)$$

AS IN PROB 4.14 - PARABOLIC  
EXIT PROFILE

$$V_{\text{exit}} = ay + by^2$$

$$= 4V_{\text{max}} \left[ \frac{y}{b} - \left( \frac{y}{b} \right)^2 \right]$$

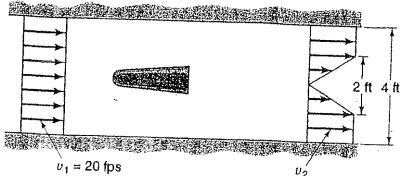
$$\dot{m}_{\text{exit}} = 8 \int_0^b V_{\text{exit}} 2\pi L dy$$

$$= 8 \frac{4}{3} \pi L b V_{\text{max}}$$

$$\therefore \underline{\underline{V_{\text{max}} = \frac{3}{4} \frac{L}{b} V}}$$

CHAPTER 5

5.1



Laws of Mass:  $\iint_{\text{CS}} S(\vec{v} \cdot \hat{n}) dA = 0$

$$\delta v_1 A_1 = 2 \left[ \int_0^1 8v_2 y dy + \int_1^2 8v_2 dy \right]$$

$$4v_1 = 2v_2 \left[ \int_0^1 y^2 dy + \int_1^2 y^2 dy \right] \\ = 3v_2$$

$$v_2 = \frac{4}{3} v_1 = 26.7 \text{ ft/s}$$

5.2 System Shown in prob 5.1

$$\sum f_x = \iint_{\text{CS}} v_2 S(\vec{v} \cdot \hat{n}) dA$$

- ASSUMING UNIT DEPTH -

$$F_x + (P_1 - P_2)A$$

$$= 2S \left[ \int_0^2 (v_2 y)^2 dy + \int_1^2 v_2^2 dy - v_1^2 \int_0^2 dy \right]$$

$$= 2S \left[ v_2^2 \frac{y^3}{3} \Big|_0^1 + v_2^2 y \Big|_1^2 - v_1^2 y \Big|_0^2 \right]$$

$$= 2S v_2^2 \left[ \left( \frac{1}{3} + 1 \right) \right] - 4S v_1^2$$

From 5.1 -  $v_2 = \frac{4}{3} v_1$

5.2 - CONTINUED -

$$F_x + (P_1 - P_2)A = \frac{20}{27} 8v_1^2$$

$$F_x = -800 \text{ N/m} = 52.8 \text{ lbf/ft}$$

$$P_1 - P_2 = \frac{1}{4} \left[ \frac{20}{27} 8v_1^2 + 52.8 \right]$$

$$= 157 \text{ lbf/ft}^2$$

$$\approx 7500 \text{ Pa} = 7.5 \text{ kPa}$$

5.3 Same General Configuration

EXCEPT EXIT VELOCITY DISTRIBUTION

$$\text{is } v = v_2 \left( 1 - \cos \frac{\pi y}{4} \right)$$

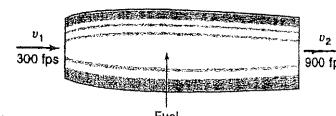
AS IN 5.1 THE EXPRESSION TO BE USED IS:

$$v_1 A_1 = 2 \left[ \int_0^2 v_2 \left( 1 - \cos \frac{\pi y}{4} \right) dy \right]$$

$$4v_1 = 2v_2 \left[ 2 - \frac{4}{\pi} \right]$$

$$v_2 = \frac{2v_1}{2 - \frac{4}{\pi}} = \frac{v_1}{1 - \frac{2}{\pi}} \\ = 55 \text{ ft/s}$$

5.4



STEADY FLOW

$$F_x = \iint_{\text{CS}} v_x S(\vec{v} \cdot \hat{n}) dA$$

$$= S_2 v_2^2 A_2 - S_1 v_1^2 A_1$$

$$= \dot{m}_2 v_2 - \dot{m}_1 v_1$$

$$= \dot{m}_1 (1.02 v_2 - v_1)$$

5.4 - CONTINUOUS -

$$\begin{aligned}\dot{m} &= \rho A_1 V_1 \\ &= \left(0.0805 \frac{\text{lb}_m}{\text{ft}^3}\right) (10.8 \text{ ft}^2) (300 \text{ ft/s}) \\ &= 260.8 \text{ lb}_m/\text{s} \\ F_x &= (260.8 \frac{\text{lb}_m}{\text{s}}) \left[ 1.02 (900 \text{ ft/s}) \right. \\ &\quad \left. - 300 \text{ ft/s} \right] \\ &= \underline{5005 \text{ lb}_f}\end{aligned}$$

5.5



STADY INCOMPRESSIBLE FLOW:

$$F_x = \iint_{c.s.} v_x s (\vec{v} \cdot \vec{n}) dA$$

$$F_y = \iint_{c.s.} v_y s (\vec{v} \cdot \vec{n}) dA$$

IN X-DIRECTION:

$$\begin{aligned}F_x &= (\dot{m} V A) [V_2 [v_x - 30^\circ] - (-V_1)] \\ &= \dot{m} [V (0.866) + 1]\end{aligned}$$

$$\dot{m} = \rho A_1 V = 62.4 \frac{\text{lb}_m}{\text{ft}^3} (3 \text{ ft}^3/\text{s})$$

$$= 187.2 \text{ lb}_m/\text{s}$$

$$\begin{aligned}F_x &= \frac{(187.2 \text{ lb}_m/\text{s})(25 \text{ ft/s})(1.866)}{32.2 \text{ lb}_m \text{ ft/s}^2 \text{ lb}_f} \\ &= \underline{271.2 \text{ lb}_f} \quad \text{FORCE ON BLADE IS IN } (-x)\end{aligned}$$

5.5 - CONTINUOUS

$$\begin{aligned}F_y &= \dot{m} V_2 \sin(-30) \\ &= \frac{(187.2)(25)(-0.5)}{32.2} \\ &= -72.7 \text{ lb}_f \quad \left. \begin{array}{l} \text{FORCE ON} \\ \text{BLADE IS IN} \\ +y \text{ DIRECTION} \end{array} \right\}\end{aligned}$$

PART (b) - BLADE MOVES TO RIGHT AT 15 FT/S

RELATIVE TO BLADE:  $V_1 = -40 \text{ FT/s}$

$$V_2 = 40 \text{ FT/s} @ -30^\circ$$

ABSOLUTE VELOCITY OF LEAVING JET

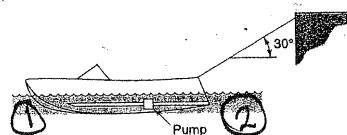
$$= \vec{V}_{\text{RELATIVE}} + \vec{V}_{\text{BLADE}}$$

$$= (34.64 \vec{e}_x - 20 \vec{e}_y) + 40 \vec{e}_x$$

$$= \underline{74.64 \vec{e}_x - 20 \vec{e}_y}$$

$$|V_{\text{exit}}| \approx \underline{77.3 \text{ FT/s}}$$

5.6



C.V. AROUND BOAT - STEADY INCOMPRESSIBLE FLOW

$$\begin{aligned}\sum F_x &= \iint_{c.s.} v_x s (\vec{v} \cdot \vec{n}) dA \\ &= \dot{m} (V_2 - V_1)\end{aligned}$$

5.6 - (CONTINUED)

$$F_x = \dot{m} \ddot{V} \left( \frac{1}{A_2} - \frac{1}{A_1} \right)$$

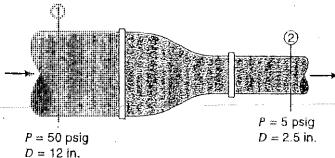
$$= \frac{(62.4)(6)^2 \left( \frac{1}{0.15} - \frac{1}{0.25} \right)}{32.2}$$

$$= 186 \text{ lbf}$$

$$\text{Tension in rope} = F_x / \cos 30^\circ$$

$$= 245 \text{ lbf}$$

5.7



Flow is steady, incompressible

$$\sum F_x = \iint_{C.S.} V_x S (\vec{V} \cdot \vec{n}) dA$$

$$= \dot{m} (V_2 - V_1)$$

$$\dot{m} = \rho \ddot{V} = 0.8 (62.4) (3 \text{ ft}^3/\text{s})$$

$$= 149.8 \text{ lbm/s}$$

$$\sum F_x = F_x + P_1 A_1 - P_2 A_2$$

{ + Atmospheric Pressure  
Cancels }

EQUATIONS:

$$F_x + P_1 \frac{\pi D_1^2}{4} - P_2 \frac{\pi D_2^2}{4} = \dot{m} \ddot{V} \left( \frac{1}{A_2} - \frac{1}{A_1} \right)$$

$$F_x = \frac{\pi}{4} (P_2 D_2^2 - P_1 D_1^2) + \dot{m} \ddot{V} \frac{4}{\pi} \left( \frac{1}{D_2^2} - \frac{1}{D_1^2} \right)$$

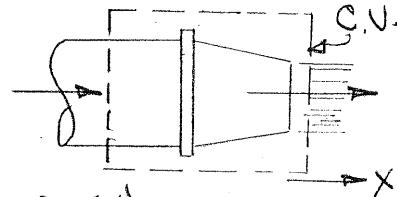
5.7 - (CONTINUED)

$$P_1 = 50 \text{ psig} \quad P_2 = 5 \text{ psia}$$

$$F_x = -5630 + 392$$

$$= -5238 \text{ lbf}$$

5.8



fluid is  $H_2O$

$$P_1 = 60 \text{ psig} \quad P_2 = 14.7 \text{ psia}$$

$$D_1 = 0.25 \text{ ft} \quad D_2 = \frac{1.5}{12} \text{ ft}$$

$$\ddot{V} = 400 \text{ gal/m} = 0.892 \text{ ft}^3/\text{s}$$

STEADY, INCOMPRESSIBLE FLOW

$$\iint_{C.S.} V_x S (\vec{V} \cdot \vec{n}) dA = 0$$

$$V_1 = \frac{\ddot{V}}{A_1} = \frac{0.892}{\frac{\pi}{4}(0.25)^2} = 18.17 \text{ ft/s}$$

$$V_2 = \frac{V_1 D_1^2}{D_2^2} = 18.17 \left( \frac{0.25(12)}{1.5} \right)^2 = 72.7 \text{ ft/s}$$

$$F_x + P_1 A_1 - P_2 A_2 = \dot{m} (V_2 - V_1)$$

$$F_x = \dot{m} (V_2 - V_1) - P_1 A_1 + P_2 A_2$$

$$= [62.4 (0.892) (72.7 - 18.17)] / 32.2$$

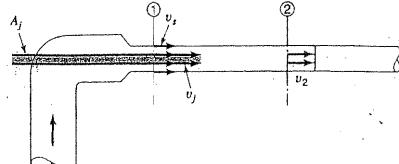
$$- (60 + 14.7) (144) \frac{\pi}{4} (0.25)^2$$

$$+ (14.7) (144) \frac{\pi}{4} (1.5)^2$$

$$= 94.3 - 528.0 + 25.4$$

$$= -408 \text{ lbf}$$

5.9



$H_2O$ -flow is STEADY, INCOMPRESSIBLE

$$\sum F_x = \iint_{CS} \rho v_x (\vec{v} \cdot \vec{n}) dA$$

$$\sum F_x = P_1 A_1 - P_2 A_2$$

$$\iint_{CS} \rho = \rho A_2 v_2^2 - \rho (A_s v_s^2 + A_j v_j^2)$$

EQUATION:

$$P_1 - P_2 = \rho \left[ v_2^2 - \frac{A_s}{A} v_s^2 - \frac{A_j}{A} v_j^2 \right]$$

By CONSERVATION OF MASS:

$$\iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA = 0$$

$$A_2 v_2 - A_s v_s - A_j v_j = 0$$

$$v_2 = \frac{A_s}{A_2} v_s + \frac{A_j}{A_2} v_j$$

$$= \frac{0.54}{0.60} (10) + \frac{0.06}{0.60} (90)$$

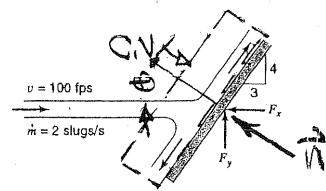
$$= 18 \text{ FT/S} \quad (a)$$

$$P_1 - P_2 = \frac{62.4}{32.2} \left[ (18)^2 - \frac{0.54}{0.6} (10)^2 - \frac{0.06}{0.6} (90)^2 \right]$$

$$= -1116 \text{ lbf/ft}^2 = -7.75 \text{ psi}$$

$$P_2 - P_1 = 7.75 \text{ psi}$$

5.10



flow is STEADY, INCOMPRESSIBLE, FRICTIONLESS

for FRICTIONLESS FLOW -

NO DRAG ON PLATE -

$$\sum F_n = \iint_{CS} v_n \rho (\vec{v} \cdot \vec{n}) dA = 0$$

$$F_n = - v_j \rho (v \cos \theta) A_j$$

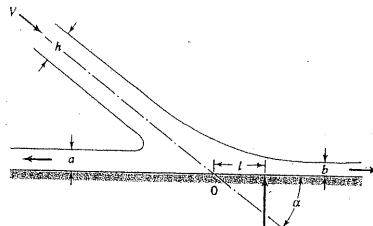
$$= - m v \cos \theta$$

$$= - 2 (100) (4/5) = -160$$

$$F_x = (-160)(4/5) = -128 \text{ lbf}$$

$$f_y = (-160)(3/5) = -96 \text{ lbf}$$

5.11



STEADY, INCOMPRESSIBLE FRICTIONLESS FLOW

IN X-DIRECTION:

$$\sum F_x = \iint_{CS} v_x \rho (\vec{v} \cdot \vec{n}) dA$$

$$= 8v^2 b - 8v^2 a - 8v^2 h \cos \alpha = 0$$

~~

$$\text{CONS. OF MASS: } 8vh = 8v(a+b)$$

$$h = a + b$$

$$a = \frac{h}{2} (1 - \cos \alpha) \quad b = \frac{h}{2} (1 + \cos \alpha)$$

~~

## 5.11 - CONTINUED

$$\begin{aligned}\sum F_y &= \iint_{C.S.} \delta v_y (\vec{v} \cdot \vec{n}) dA \\ &= \delta v^2 h \sin x \quad (a)\end{aligned}$$

PART (b):

$$\sum M_z = f_y l = \iint_{C.S.} (\vec{r} \times \vec{v})_z \delta (\vec{v}, \vec{n}) dA$$

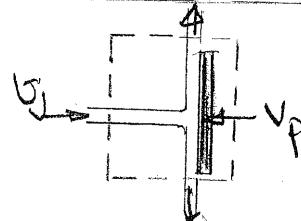
$$f_y l = \frac{a}{2} \delta (3va) - \frac{b}{2} \delta (3vb)$$

$$\delta v^2 h (\sin x) l = \frac{a^2 \delta v^2}{2} - \frac{b^2 \delta v^2}{2}$$

$$l = \frac{a^2 - b^2}{2h \sin x}$$

$$= \frac{h^2 \cos x}{2h \sin x} = \frac{h \cot x}{2}$$

5.12



Flow is steady, incompressible,  
frictionless -

Atmospheric pressure (analog)

CV moves to left with velocity,  $V_p$

$$\sum F_x = \iint_{C.S.} v_x \delta (\vec{v} \cdot \vec{n}) dA$$

$$\begin{aligned}F_x &= \delta A_o (V_o + V_p)^2 \\ &= \frac{(62.4)}{32.2} \frac{3}{30} (5 + 30)^2 \\ &= 237.4 \text{ lb}_F\end{aligned}$$

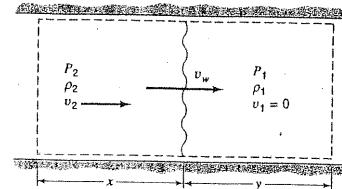
For moving plate

## 5.12 - CONTINUED

For  $V_p = 0$ 

$$\begin{aligned}F_x &= \frac{62.4}{32.2} \frac{3}{30} (30)^2 \\ &= 174.4 \text{ lb}_F\end{aligned}$$

5.13



cons. of mass: for unit gross section

$$\frac{\partial m}{\partial t} + \iint_{C.S.} dm = 0$$

$$M = S_2 x + S_1 y \quad S dm = -S_2 V_2$$

$$S_2 \dot{x} + S_1 \dot{y} - S_2 V_2 = 0$$

$$\text{since } \dot{x} = V_w \quad \dot{y} = -V_w$$

$$S_2 (V_w - V_2) - S_1 V_w = 0 \quad (1)$$

X-MOMENTUM:

$$\sum F_x = \iint_{C.S.} v_x \delta (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{C.V.} v_x \delta dv$$

$$\begin{aligned}P_2 - P_1 &= -V_2 S_2 V_2 + \frac{\partial}{\partial t} S_2 V_2 x \\ &= -S_2 V_2^2 + S_2 V_2 V_w \\ &= S_2 V_2 [V_w - V_2]\end{aligned}$$

$$\text{from (1): } S_2 V_2 (V_w - V_2) = S_1 V_2 V_w$$

$$\text{GIVEN: } P_2 - P_1 = S_1 V_2 V_w$$

Q.E.D.

5.14 FOR SITUATION CONSIDERED  
IN PROB 5.13

(a) AIR       $V_w = 1130 \text{ FT/s}$   
 $S = 0.00238 \text{ SWG/ft}^3$

$$P_2 - P_1 = S_1 V_2 S_w$$

$$= (0.00238)(10)(1130)$$

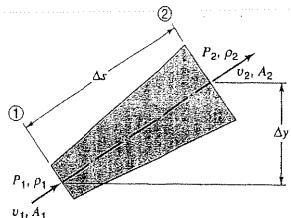
$$= 269 \text{ PSF} = 0.187 \text{ PSI}$$

(b)  $H_2O$        $V_w = 4700 \text{ FT/s}$   
 $S = 1.938 \text{ SWG/ft}^3$

$$\Delta P = (1.938)(10)(4700)$$

$$= 91,080 \text{ PSF} = 633 \text{ PSI}$$

5.15.



CONS. OF MASS:

TECHNIQUE IS TO LET

$$P_2 = P_1 + \frac{\partial P}{\partial S} AS$$

$$V_2 = V_1 + \frac{\partial V}{\partial S} AS$$

ETC

By CONSERVATION OF MASS -  $\{ \Delta S = \Delta y \neq 0 \}$

$$\frac{\partial}{\partial S} (S + V) = 0$$

By MOMENTUM THEOREM, USING  
CONS. OF MASS RESULT!

$$dP + SV dV + g dy = 0$$

- MESSY -

5.16

$D_1 = 0.3 \text{ m}$        $D_2 = 0.38 \text{ m}$   
 $V_1 = 12 \text{ m/s}$        $P_2 = 145 \text{ kPa}$   
 $P_1 = 128 \text{ kPa}$        $V_2 = 7.48 \text{ m/s}$   
 $A_1 = \frac{\pi}{4}(0.3)^2 = 0.0707 \text{ m}^2$        $A_2 = \pi = 0.1134 \text{ m}^2$   
 $\dot{V} = A_1 V_1 = (0.0707)(12) = 0.8484 \text{ m}^3/\text{s}$

IN X DIRECTION:  $\sum F_x = \iint_{C.S.} V_x S (\vec{v} \cdot \vec{n}) dA$

$$F_x + PA_1 - P_2 A_2 \cos 30^\circ = \dot{m} (V_2 \cos 30^\circ - V_1)$$

$$F_x = (1000)(0.8484) [7.48(\cos 30^\circ) - 12]$$

$$- (1000) [(128)(0.0707) + (145)(0.1134) \cos 30^\circ]$$

$$= -505.5 \text{ N}$$

IN Y DIRECTION:  $\sum F_y = \iint_{C.S.} V_y S (\vec{v} \cdot \vec{n}) dA$

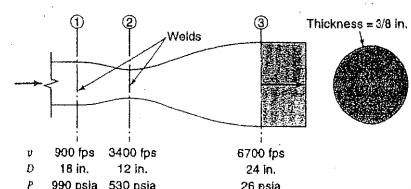
$$F_y - P_2 A_2 \sin 30^\circ = \dot{m} (V_2 \sin 30^\circ)$$

$$F_y = (1000)(0.8484)(7.48 \sin 30^\circ)$$

$$+ (1000)(145)(0.1134)(\sin 30^\circ)$$

$$= 11395 \text{ N} = 11,395 \text{ kN}$$

5.17



STEADY INCOMPRESSIBLE FLOW:

$$\sum F_x = \iint_{C.S.} V_x S (\vec{v} \cdot \vec{n}) dA$$

5.17 - (CONTINUED)

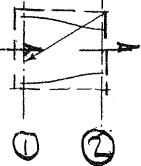
$$A_1 = \frac{\pi}{4}(1.5)^2 = 1.767 \text{ ft}^2$$

$$A_2 = \frac{\pi}{4}(1)^2 = 0.785 \text{ "}$$

$$A_3 = \frac{\pi}{4}(2)^2 = 3.142 \text{ "}$$

FOR C.N. BETWEEN

① & ② :



$$\sum F_x = \iint_{C.S.} \rho v_x (\vec{v} \cdot \vec{n}) dA$$

$$F_x + F_2 + P_1 A_1 - P_2 A_2 = \dot{m} (v_2 - v_1)$$

$$F_x = \frac{770(3400 - 900)}{32.2}$$

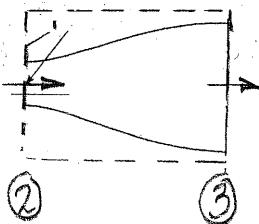
$$+ (530 - 14.7)(144)(0.785)$$

$$- (990 - 14.7)(144)(1.767) - f_2$$

$$= -130,000 \text{ lbf} - f_2$$

FOR C.N. BETWEEN

② & ③ :



$$F_x + P_2 A_2 - P_3 A_3 = \dot{m} (v_3 - v_2)$$

$$F_x = \frac{770}{32.2} (6700 - 3400)$$

$$+ (26 - 14.7)(144)(3.14)$$

$$- (530 - 14.7)(144)(0.785)$$

$$= 25777 \text{ lbf}$$

5.17 - (CONTINUED)

STRESS AT 2:

$$\sigma = \frac{F}{A} = \frac{25777 \text{ lbf}}{\pi (12)(3/8)} = 1823 \text{ psi}$$

(COMPRESSION)

AT 1

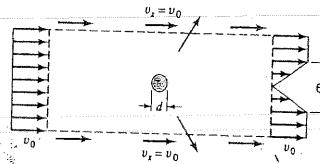
$$F_1 = -130,000 + 25777$$

$$= -104220 \text{ lbf}$$

$$\sigma = \frac{104220}{\pi (18)(3/8)} = 4915 \text{ psi}$$

TENSION

5.18



FLOW IS STEADY & INCOMPRESSIBLE  
NO NET PRESSURE FORCE

$$\sum F_x = \iint_{C.S.} \rho v_x s(\vec{v} \cdot \vec{n}) dA$$

$$F_x = \iint_{\text{OUT}} \rho v_x^2 dA - \rho v_0^2 A_{in}$$

$$= 2 \int_0^{3d} \rho v_0^2 \left( \frac{y}{3d} \right)^2 dy$$

$$+ \rho v_0^2 (3d) - \rho v_0^2 (6d)$$

MOMENTUM OUT TOP & BOTTOM

$$F_x = 2\rho v_0^2 \left[ \frac{y^3}{3} \right]_0^{3d} + \rho v_0^2 (3d - 6d)$$

$$= - \frac{\rho v_0^2 d}{2}$$

FORCE ON CYLINDER =  $\rho v_0^2 d$

5.19 FLUID IS  $H_2O$

$$V_{sonic} = 1433 \text{ m/s}$$

THIS IS JUST LIKE PROB 5.13

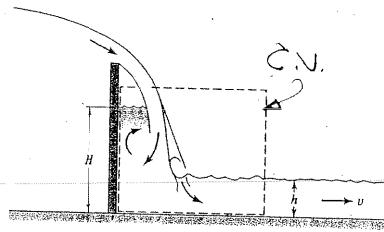
FOR AN OBSERVER MOVING WITH  
 $H_2O$  STREAM:  $V = 3 \text{ m/s}$

$$\text{THEN } \Delta P = \frac{1}{2} \rho A V^2$$

$$= (1000)(1433^2)(3) \times 10^{-4}$$

$$= 4287 \text{ kPa}$$

5.20



STATIC PRESSURE OF  $H_2O$ :

$$\text{ON LEFT} - P = \frac{\rho g h}{2}$$

$$\text{ON RIGHT} - P = \frac{\rho g h}{2}$$

$$\sum F_x = \iint_{CS} v_x \rho (\vec{v} \cdot \vec{n}) dA$$

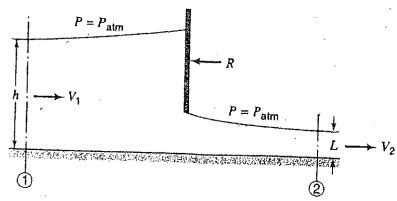
$$\frac{\rho g H^2}{2} - \frac{\rho g h^2}{2} = (\rho v h) v$$

$$H^2 = \frac{2}{\rho g} \left[ \rho h v^2 + \frac{\rho g h^2}{2} \right]$$

$$= \frac{2 h v^2}{g} + h^2$$

$$H = \left[ \frac{2 h v^2}{g} + h^2 \right]^{1/2}$$

5.21



CONSERVATION OF MASS:

$$\iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA = 0$$

$$-\rho h V_1 + \rho L V_2 = 0$$

$$V_2 = \frac{h V_1}{L} \quad (a)$$

X-MOMENTUM:

$$\sum F_x = \iint_{CS} v_x \rho (\vec{v} \cdot \vec{n}) dA$$

$$P_1 A_1 - P_2 A_2 + F_x = \dot{m} (V_2 - V_1)$$

$$F_x = \dot{m} (V_2 - V_1) + P_2 A_2 - P_1 A_1$$

$$= \rho V_1 h (V_2 - V_1) + \frac{\rho g L^2}{2} - \frac{\rho g h^2}{2}$$

$$= \rho V_1^2 h (h_L - 1) + \frac{\rho g}{2} (L^2 - h^2) \quad (b)$$

5.22.



CONSERVATION OF MASS:

$$V_1 h_1 = V_2 h_2$$

MOMENTUM THEM:

$$\sum F_x = \iint_{CS} \rho v_x (\vec{v} \cdot \vec{n}) dA$$

$$P_1 h_1 - P_2 h_2 = \dot{m} (V_2 - V_1)$$

$$P_1 = \frac{\rho g h_1}{2} \quad P_2 = \frac{\rho g h_2}{2}$$

## 5.22 - CONTINUED

$$\frac{8gh_1^2}{2} - \frac{8gh_2^2}{2} = 8V_1h_1(V_2 - V_1)$$

from Cons. of MASS:  $V_2 = V_1 h_1 / h_2$

$$\frac{g}{2}(h_1^2 - h_2^2) = V_1^2 h_1 \frac{h_1 - h_2}{h_2}$$

FACTORIZING  $\frac{g}{2}$  GIVES US  $h_1 - h_2$

$$\frac{gh_2}{2}(h_1 + h_2) = V_1^2 h_1$$

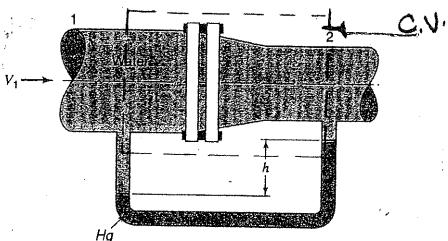
$$h_2^2 + h_1 h_2 - \frac{2V_1^2 h_1}{g} = 0$$

$$h_2 = \frac{h_1}{2} \left[ \sqrt{1 + \frac{8V_1^2}{gh_1}} - 1 \right]$$

$\frac{1}{2}$  from CONTINUITY

$$V_2 = \frac{gh_1}{\Delta A_1} \left[ 1 + \sqrt{1 + \frac{8V_1^2}{gh_1}} \right]$$

5.23



$$D_1 = 8 \text{ cm}$$

$$D_2 = 5 \text{ cm}$$

$$V_1 = 5 \text{ m/s}$$

$$P_2 = 1 \text{ atm}$$

$$h = 58 \text{ cm}$$

$$A_1 = \frac{\pi}{4} (8 \text{ cm})^2 = 50.3 \text{ cm}^2$$

$$A_2 = \frac{\pi}{4} (5 \text{ cm})^2 = 19.6$$

$$V_2 = 5 \text{ m/s} \left( \frac{50.3}{19.6} \right) = 12.83 \text{ m/s}$$

## 5.23 - CONTINUED

$$X\text{-MOMENTUM: } \sum F_x = \iint_{c.s.} v_x S (\vec{v} \cdot \vec{n}) dA$$

$$F_x + P_1 A_1 - P_2 A_2 = gV (V_2 - V_1)$$

$$P_1 - P_2 = \rho_w g h [13.6 - 1]$$

$$= (1000)(9.81)(0.58)(12.6) \\ = 71.69 \text{ kPa}$$

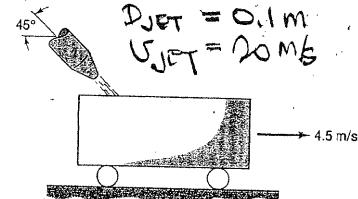
SINCE  $P_2 = 1 \text{ atm}$

$$P_1 = 71.69 \text{ kPa} \text{ ABS}$$

$$F_x + P_1 A_1 = (1000)(50.3 \times 10^{-4})(5)(12.83 - 5)$$

$$F_x = 197 - 71.69(50.3 \times 10^{-4})(1000) \\ = -163.7 \text{ N}$$

5.24



X MOMENTUM'

$$\sum F_x = \iint_{c.s.} v_x S (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint v_x S dV$$

$$F_x = -\rho_w A_J V_J (V_J \cos \theta) + \rho_w A_J V_J V_c$$

$$= \rho_w A_J V_J (V_c - V_J \cos \theta)$$

$$= 1000 \left( \frac{\pi}{4} \right) (0.1)^2 (20) [4.5 - 20 \cos 45^\circ]$$

$$= -1515 \text{ N}$$

Force on Car by Jet:  $F_x = 1515 \text{ N}$

5.24 - (CONTINUOUS) -

$\Sigma$  Momentum:

$$\begin{aligned}\sum F_y &= \iint_{C.S.} v_y S(\vec{v} \cdot \vec{n}) dA \\ &\quad + \frac{\partial}{\partial t} \iiint_{C.V.} v_y S dV\end{aligned}$$

$$\begin{aligned}F_y &= -v_j \sin \theta S(v_j A_j) + 0 \\ &= -(20 \sin 45^\circ)(1000)(-20) \\ &\quad \times \frac{\pi}{4}(0.1)^2 \\ &= +2220 \text{ N}\end{aligned}$$

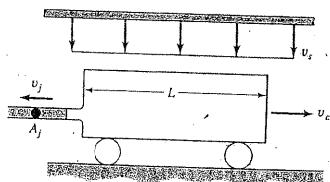
force EXERTED BY  $A_{20}$ :

$$F_y = -2220 \text{ N}$$

TOTAL Forces -

$$\vec{F} = 1515 \hat{e}_x - 2220 \hat{e}_y \text{ N}$$

5.25



COORDINATES FIXED TO GROUND  
~ MOVING TO RIGHT w/ TURB.

MOMENTUM THM IN X-DIRECTION

$$\begin{aligned}\sum F_x &= SA_j v_j (-v_j) \\ &\quad - SA_s v_s (-v_c)\end{aligned}$$

$$F_x = SA_s v_s v_c - SA_j v_j^2$$

IN y-DIRECTION

$$\begin{aligned}F_y &= SA_j v_j (0) - SA_s v_s (-v_c) \\ &= SA_s v_s^2\end{aligned}$$

FORCE OF FLUID ON CAR IS NEGATIVE  
OF MASS.

5.26

for C.V. Shown:

$$\int d\dot{m} + \frac{\partial}{\partial t} M = 0$$

$$M = SAh$$

$$d\dot{m} = SA h^\circ$$

$$\sum F_y = \iint_{C.S.} v_y S(\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{C.V.} v_y dV$$

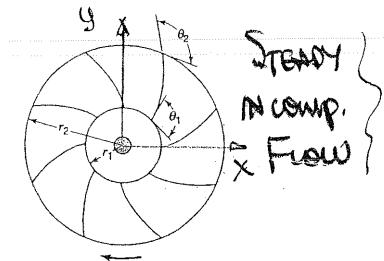
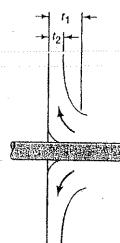
$$-8ghA = +8A h^\circ + 8A \frac{\partial}{\partial t} (hh^\circ)$$

$$-gh = -h^2 + \frac{\partial}{\partial t} (hh^\circ)$$

$$\Rightarrow \underline{\underline{h}} = -g$$

5.27

$$\begin{aligned}\omega &= 1180 \text{ rpm} \quad r_2 = 0.6 \text{ in.} \\ r_1 &= 2 \text{ in.} \quad \theta_2 = 135^\circ \\ r_2 &= 8 \text{ in.} \quad r_1 = 0.8 \text{ in.}\end{aligned}$$



ROTATION IS ABOUT Z-AXIS -

$$\sum M_z = \iint_{C.S.} (\vec{r} \times \vec{v})_z S(\vec{v} \cdot \vec{n}) dA$$

$$\begin{aligned}&= \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} M_{out} \\ &= (r_x v_y - r_y v_x) \dot{m}_{out}\end{aligned}$$

$$= (r_x v_y - r_y v_x) \dot{m}_{out}$$

AT POSITION ON X-AXIS -  $r_x = r_2$

$$r_y = 0$$

$$\dot{v} = 800 \left( \frac{1}{7.48} \right) \left( \frac{1}{60} \right) = 1.783 \text{ ft}^3/\text{s}$$

$$v_x = \frac{1.783}{\pi (8/12)(2)(0.6/12)} = 8.51 \text{ ft/s}$$

5.27 - CONTINUED

AT THIS LOCATION -

$$V_{tan} = V_y = V_x \\ = 8.51 \text{ ft/s}$$

ABS. VELOCITY @  $r_2$

$$V_y = -rw + V_{tan} \\ = -(8/12) \left( \frac{1180 \times 2\pi}{60} \right) + V_tan \\ = -82.38 + 8.51 = -73.87 \text{ ft/s}$$

Now - IN MOMENTUM EXPRESSION:

$$M_2 = (r_2 V_y) S \dot{V} \\ = \frac{8}{12} \left( -73.87 \right) (64) (1.783) \\ = 174 \text{ FT Lbf}$$

$$\text{POWER} = M_2 \omega$$

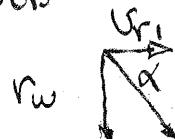
$$= 174 \left( \frac{1180 \times 2\pi}{60} \right) \left( \frac{1}{550} \right) \\ = \underline{\underline{39.1 \text{ HP}}}$$

5.28 FOR CONFIGURATION OF PROB 5.28

$$\dot{V} = 1.783 \text{ ft}^3/\text{s}$$

$$\text{AT INLET} - V_r = \frac{\dot{V}}{2\pi r_1 t_1} \\ = \frac{1.783}{2\pi (2/12)(0.8/12)} = 25.54 \text{ ft/s}$$

5.28 - CONTINUED



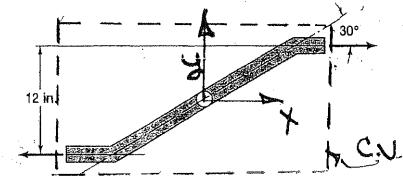
$$rw = -r_1 \omega = -\left(\frac{2}{12}\right) \left(\frac{1180 \times 2\pi}{60}\right) \\ = -20.6 \text{ ft/s}$$

$$\alpha = \tan^{-1} \frac{rw}{V_{r1}} = \tan^{-1} \frac{20.6}{25.54} = \underline{\underline{38.9^\circ}}$$

$$\text{PART (B): } \sum F_x = \iint_{C.S.} V_x S (\vec{r} \cdot \vec{n}) dA$$

$$F_x = -S V_z (-V_z) A, \\ = \dot{m} V_2 = S \dot{V} \frac{V}{A_1} \\ = \frac{(64)(1.783)}{\pi (2/12)^2 (32.2)} \\ = \underline{\underline{70.6 \text{ lbf}}}$$

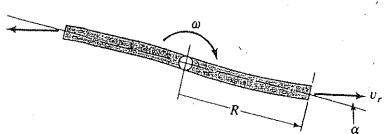
5.29



$$\text{FOR C.N. SHOWN: } \sum M_2 = \iint_{C.S.} (\vec{r} \times \vec{v})_z S (\vec{r} \cdot \vec{n}) dA$$

$$M_2 = \begin{vmatrix} \hat{e}_x \hat{e}_y \hat{e}_z \\ r_x r_y r_z \\ V_x V_y V_z \end{vmatrix} \dot{m} \\ = 2 \dot{m} (V_x r_y) \\ = \frac{2 (62.4) (\pi/4) \left(\frac{0.5}{12}\right)^2 (20) \left(\frac{6}{12}\right)}{32.2} \\ = \underline{\underline{1.057 \text{ FT LBF}}}$$

5.30



$$\sum M_A = \iint_{C.S.} (\vec{r} \times \vec{v})_z \delta(\vec{r} \cdot \vec{n}) dA$$

$$= \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} / (m)$$

$$M_A = 2(-r_y v_x) m$$

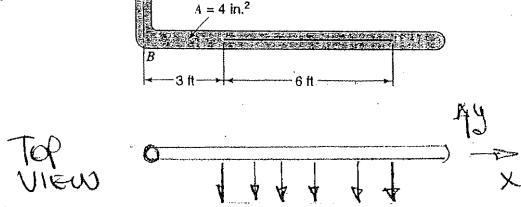
$$r_y = R \sin \alpha$$

$$v_x = v_r \sin \alpha - R \omega$$

$$M_A = 2m \left[ -R \sin \alpha (v_r \sin \alpha - R \omega) \right]$$

$$\omega = \frac{M_A}{2m r^2 \sin \alpha} + \frac{v_r \sin \alpha}{r}$$

5.31



$$\sum M_A = \iint_{C.S.} (\vec{r} \times \vec{v})_z \delta(\vec{v} \cdot \vec{n}) dA$$

$$= \int_3^9 \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} \delta v_y t dx$$

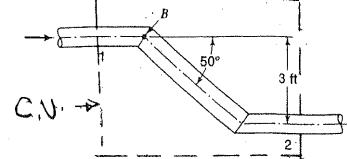
$$= \int_3^9 -3t v^2 x dx = -9v^2 t \left( \frac{x^2}{2} \right) \Big|_3^9$$

$$v = \frac{8}{6(0.25/12)} = 64 \text{ FT/s}$$

$$M = -\frac{624(64)^2 (0.25)(81-9)}{32,2}$$

$$= -5950 \text{ FT LBF}$$

5.32



$$\sum M_B = \iint_{C.S.} (\vec{r} \times \vec{v})_z \delta(\vec{v} \cdot \vec{n}) dA$$

$$M_B = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} m + r_2 p_2 A_2$$

$$= (r_x v_y - r_y v_x) m + r_2 p_2 A_2$$

$$\dot{v} = \left( 30 \frac{\text{gal}}{\text{m}} \right) \left( \frac{1}{7.48 \times 60} \right) = 0.0668 \text{ ft}^3/\text{s}$$

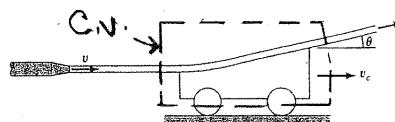
$$v_2 = \frac{\dot{v}}{A_2} = \frac{0.0668}{\frac{\pi}{4} \left( \frac{0.75}{12} \right)^2} = 21.79 \text{ FT/s}$$

$$M_B = + \frac{3(21.79)(62.4)(0.0668)}{32,2}$$

$$+ (3)(24)(\frac{\pi}{4})(0.75)^2$$

$$= 8.46 + 31.81 = \underline{40.3 \text{ FT LBF}}$$

5.33



LINEAR MOMENTUM! COORDINATE SYSTEM MOVES WITH CART

$$\sum F_x = \iint_{C.S.} v_x \delta(\vec{v} \cdot \vec{n}) dA$$

$$F_x = SA \left[ (v - v_c) \cos \theta \right] (v - v_c) + SA(v - v_c)^2$$

$$P = v_c F_x = SA(v - v_c)^2 [ \cos \theta - 1 ] v_c$$

$$\text{FOR } m = \frac{v_c}{v} - P = SA \left[ \frac{v}{v_c} \right] v^3 m (1 - m)^2$$

5.33 - (CONTINUED)

$$\text{For } P = P_{\max} \quad \frac{dP}{dm} = 0$$

$$\therefore \frac{dP}{dm} = 8A \left[ \frac{v^3}{m} \right] \frac{d}{dm} (m - 2m^2 + m^3)$$

$$= \left\{ \left. \left( 1 - 4m + 3m^2 \right) \right\} = 0$$

$$m = 1, \frac{1}{3}$$

$m = 1$  — MINIMUM

$m = \frac{1}{3}$  MAXIMUM

$$\therefore m = \frac{v_c}{v} = \frac{1}{3}$$

PART (b) ROTATION ABOUT Z-AXIS -

$$M_2 = \iint_{C.S.} (\vec{r} \times \vec{v})_z S(\vec{v} \cdot \vec{n}) dA$$

$$= \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} \begin{matrix} \overset{\circ}{m} & \text{OUT} \\ z \end{matrix}$$

$$- \begin{vmatrix} \overset{\circ}{m} & \text{IN} \\ z \end{vmatrix}$$

$$= rm \left[ (v - v_c) \cos \theta + v_c - v \right]$$

$$= m \overset{\circ}{(}\cos \theta - 1)(v - v_c)$$

$$P = M_2 \omega = M_2 \frac{v_c}{r}$$

$$= \left[ m \overset{\circ}{(}\cos \theta - 1) \right] v_c (v - v_c)$$

$$\text{for } m = \frac{v_c}{v}$$

5.33 - (CONTINUED)

$$P = \left[ \right] m v^2 (1 - m)$$

$$\frac{dP}{dm} = \left[ \right] v^2 (1 - 2m) = 0$$

or  $P_{\max}$  occurs when

$$m = \frac{v_c}{v} = \frac{1}{2}$$

## CHAPTER 6

6.1. For  $V = A + Br$

$$V(r_0) = 0 = A + B r_0$$

$$V(r_i) = \frac{w_d}{r_i} = A + B r_i$$

$$A = -Br_0 = \frac{\omega_d}{2} - Bri$$

$$B(r_0 - r_i) = -\frac{wd}{a}$$

$$B = -\frac{1}{r_0 - r_i} \frac{\omega d}{2}$$

$$A = \frac{r_o - r_i}{2} \frac{\omega d}{2}$$

$$V = \frac{r_0 - r}{r_0 - r_i} \frac{\omega d}{2}$$

$$6.2 \quad \text{STEADY FLOW: } \frac{dQ}{dt} = \frac{dW_h}{dt} = 0$$

$$-\frac{\partial \mathbf{w}}{\partial t} = \iint_{\text{c.s.}} (\mathbf{e} + \frac{1}{3}) \mathbf{g} (\vec{v} \cdot \vec{n}) dA$$

$$-\frac{d\mathbf{u}_2}{dt} = m \left( \mathbf{u}_2 - \mathbf{u}_1 \right) + \frac{\mathbf{P}_2 - \mathbf{P}_1}{s} + \frac{\mathbf{U}_2^2 - \mathbf{U}_1^2}{2} + \mathbf{g}(\mathbf{z}_2 - \mathbf{z}_1)$$

$$\dot{m} = \dot{V} \rho = 1025(2) = 215,25 \text{ kg/s}$$

$$U_1 = \frac{V}{A} = 4,278 \text{ m/s}$$

$$U_2 = \frac{g}{A_2} = 11,573 \text{ "}$$

SINCE  $\Delta T = 0$        $u_2 - u_1 = 0$

$$z_2 - z_1 = 18 \text{ m}$$

$$P_2 = 175 \text{ kPa}$$

$$P_1 = -0.15 \text{ mHg}$$

$$= -19.9 \text{ kPa}$$

## 6.2 - CONTINUOUS -

$$\frac{P_2 - P_1}{\rho} = \frac{175 + 19,9}{1,025} = 190 \text{ m}^2/\text{s}^2$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{(4.28)^2 - (1.57)^2}{2} = -57.7$$

$$g(z_2 - z_1) = g_{\text{SI}}(18) = 17.7 \text{ "}$$

$$-W = (190 - 57.7 + 17.7)(215,3)$$

$$= 32,295 \text{ W} = 32,3 \text{ kW}$$

$$6.3 \quad \frac{\partial \vec{S}}{\partial t} - \frac{\partial \vec{S}_{NS}}{\partial t} - \frac{\partial \vec{S}_{NP}}{\partial t} = \iint_{CS} (\vec{e} + \vec{P}) \cdot \vec{S} (\vec{r}, \vec{n}) dA$$

$$-\ddot{m} \left[ h_1 + \frac{v^2}{2} + g z_1 \right] + \frac{\partial}{\partial t} \left[ m u \right] = 0$$

NE(1)

$$SVC_V \frac{dT}{dt} = SAV \left( \frac{V^2}{2} \right)$$

$$\frac{dT}{dt} = \frac{AV}{Vc_p} \frac{U^2}{2}$$

$$\frac{\pi}{4} \left(\frac{8}{12}\right)^2 FT^2 \left( \frac{110FT}{5} \right)^3$$

$$2 \left( 10^3 \text{ ft}^3 \right) \left( 0.17 \frac{\text{Btu}}{\text{lbm F}} \right) \left( \frac{778 \text{ ft lb}}{\text{Btu}} \right) \left( \frac{32.2 \text{ lb ft}}{\text{s}^2 \text{ Btu}} \right)$$

$$= \underline{21.8 \text{ F/s}}$$

6.4 Energy Equation Reduces To

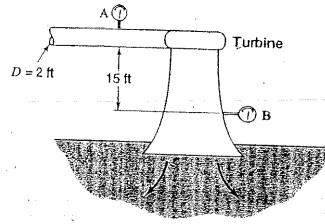
$$\iint_{C,S} (\epsilon + \frac{P}{\rho}) \delta(\vec{v} \cdot \vec{n}) dA = 0$$

$$U_2 - U_1 + P_B - P_A + \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$\Delta U = C \Delta T = \frac{P_1 - P_2}{S}$$

$$\Delta T = \frac{\Delta P}{C_S} = \frac{10(144)}{(1)(62.4)(778)} = \underline{0.0297 \text{ F}}$$

6.5.



Between A & B:

$$\dot{S}_{WS} = \iint_{C,S} (\epsilon + \frac{P}{\rho}) \delta(\vec{v} \cdot \vec{n}) dA$$

$$\dot{S}_{WS} = \frac{600(550)}{0.82} = 402000 \text{ ft lbf/s}$$

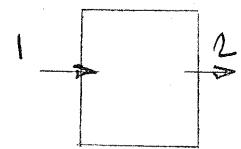
$$\begin{aligned} \dot{S}_{WS} &= m \left[ \frac{V_B^2 - V_A^2}{2} + P_B - P_A \right. \\ &\quad \left. + g(y_B - y_A) + \frac{\Delta U}{S} \right] \end{aligned}$$

$$\frac{P_B - P_A}{S} = \left\{ -\frac{\dot{S}_{WS}}{dt} + g(y_A - y_B) + \frac{V_A^2}{2} \right\}$$

$$P_B = P_A + S \{ \}$$

$$= \underline{465 \text{ PSIA}}$$

6.6.



$$\dot{V} = 4 \text{ ft}^3/\text{s}$$

$$f_i = -6 \text{ PSIA} \quad P_2 = 40 \text{ PSIA}$$

$$V_1 = \frac{4}{\pi \frac{(1)^2}{4}} = 510 \text{ ft/s} \quad V_2 = \frac{4}{\pi \frac{(10)^2}{4}} = 733 \text{ ft/s}$$

$$y_2 - y_1 = 5 \text{ ft}$$

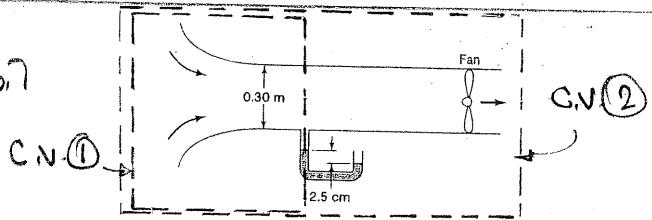
Energy Eqn. Reduces to:

$$\begin{aligned} - \dot{S}_{WS} \frac{d}{dt} &= m \left[ \frac{\Delta P}{S} + \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) \right] \\ &= mg \left[ \frac{\Delta P}{Sg} + \frac{V_2^2 - V_1^2}{2g} + y_2 - y_1 \right] \end{aligned}$$

$$= 27850 \text{ ft lbf/s}$$

$$= \underline{50.6 \text{ Hp}}$$

6.7



for C.N. ① - Energy Eqn. Reduces to

$$0 = \frac{V_2^2 - V_1^2}{2} + \frac{P_2 - P_1}{S} + g(z_2 - z_1)$$

$$\frac{V_2^2}{2} = \frac{P_1 - P_2}{S} = \frac{25 \text{ cm H}_2\text{O}}{8}$$

$$V_2 = \left[ 2 \frac{\Delta P}{S} \right]^{\frac{1}{2}} = 20 \text{ m/s}$$

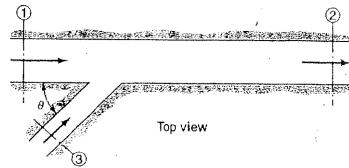
$$\dot{V} = f V_2 = \frac{\pi}{4} (0.3)^2 (20) = \underline{1.47 \text{ m}^3/\text{s}}$$

## 6.7 - CONTINUED

for CN, ② ENERGY EQN IS:

$$\begin{aligned} -\frac{dU_3}{dt} &= \dot{m} \left[ A_1 u + \frac{V_2^2}{2} A_1 + \frac{P_3}{g} A_1 + g A_1 \right] \\ &= \dot{m} V \frac{V_2^2}{2} \\ &= (1.2)(1.417)(20)^2 / 2 \\ &= \underline{\underline{346 \text{ W}}} \end{aligned}$$

6.8.



STEADY FLOW ENERGY EQUATION:

$$\begin{aligned} \dot{m}_1 \left( u_1 + \frac{V_1^2}{2} + \frac{P_1}{g} \right) + \dot{m}_3 \left( u_3 + \frac{V_3^2}{2} + \frac{P_3}{g} \right) \\ = \dot{m}_2 \left( u_2 + \frac{V_2^2}{2} + \frac{P_2}{g} \right) \end{aligned}$$

CONS. OF MASS:

$$u_1 A_1 + u_3 A_3 = u_2 A_2 \quad \textcircled{1}$$

ENERGY EQUATION CAN BE WRITTEN

$$\begin{aligned} A_1 u_1 \left[ C_v T_1 + \frac{V_1^2}{2} + \frac{P_1}{g} \right] \\ + A_3 u_3 \left[ C_v T_3 + \frac{V_3^2}{2} + \frac{P_3}{g} \right] \\ = A_2 u_2 \left[ C_v T_2 + \frac{V_2^2}{2} + \frac{P_2}{g} \right] \quad \textcircled{2} \end{aligned}$$

MOMENTUM:

$$\begin{aligned} (P_1 - P_2) A_1 &= S V_2^2 A_1 - S V_1^2 A_1 \\ &\quad - S V_3^2 A_3 u_3 \quad \textcircled{3} \end{aligned}$$

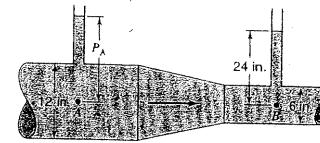
## 6.8 - CONTINUED

for  $u = C_v T$ ,  $T_1 = T_3$ ,  $P_1 = P_3$   
↑ LOTS OF ALGEBRA

$$C_v(T_2 - T_1) = \frac{V^2}{2} \left[ 1 + \left( \frac{A_3 V_3}{A_1 V_1} \right)^2 - 1 \right] \times$$

$$\times \left[ \frac{1 + 2 \frac{A_3 V_3}{A_1 V_1}}{1 + \frac{A_3 V_3}{A_1 V_1}} \right] + \frac{V^2}{2} \left[ \frac{A_3 V_3}{A_1 V_1} - \frac{2 A_3 V_3}{A_1 V_1} \right]$$

6.9



$$\dot{V} = 3 \text{ ft}^3/\text{s}$$

BETWEEN A &amp; B - ENERGY EQUATION:

$$\int_{C,S} \dot{S} (e + \frac{P}{g}) S (\vec{V} \cdot \vec{n}) dA = 0$$

$$\frac{V_B^2 - V_A^2}{2} + u_B - u_A + \frac{P_B - P_A}{g} = 0$$

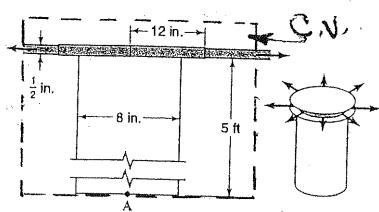
$$V_A = \frac{3}{(\pi/4)(1)^2} = 3.82 \text{ ft/s}$$

$$V_B = \frac{3}{\pi/4 (1/2)^2} = 15.28 \text{ ft/s}$$

$$\frac{P_A - P_B}{g} = \frac{(15.28)^2 - (3.82)^2}{2g} + 0.45 \\ = 2.15 \text{ ft of H}_2\text{O}$$

$$P_A = 2.15 + 2 = 4.15 \text{ ft of H}_2\text{O}$$

6.10



$$P_A = 10 \text{ psig}$$

$$\sum F_y = \iint_{c,s} v_y g (\vec{v} \cdot \vec{n}) dA$$

Flow RATE MUST BE DETERMINED

ENERGY EQUATION FOR C.N. SHOWN:

$$\frac{P}{\rho} + \frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g \Delta z = 0$$

$$\frac{\Delta P}{\rho} = \frac{10(44)(32.2)}{62.4} = 743 \text{ ft}^2/\text{s}^2$$

$$V_A = \frac{\dot{V}}{\pi/4 (2)^2} = 2.865 \text{ ft/s}$$

$$V_B = \frac{\dot{V}}{\pi/4 (0.5)^2} = 3.82 \text{ ft/s}$$

$$\frac{\Delta V^2}{2} = \left(2.865^2 - 3.82^2\right) \frac{\dot{V}}{2} = -3.19 \dot{V}^2$$

$$g \Delta y = 32.2(-5) = -61 \text{ ft}^2/\text{s}^2$$

$$743 - 3.19 \dot{V}^2 - 61 = 0$$

$$\dot{V}^2 = \frac{682}{3.19} \quad \dot{V} = 14.6 \text{ ft}^3/\text{s}$$

$$V_A = 41.9 \text{ ft/s} \quad V_B = 55.9 \text{ ft/s}$$

$$F_y + P_A A_A - \rho g V = \dot{m} (-V_A)$$

$$P_A A_A = 10 \left(\frac{\pi}{4}\right) (8)^2 = 502 \text{ lb}_f$$

$$\begin{aligned} \rho g V &= (62.4)(32.2)\left(\frac{\pi}{4}\right)\left(\frac{8}{12}\right)^2(5) \\ (\text{WT}) &= 109 \text{ lb}_f \end{aligned}$$

6.10 - CON THUONG

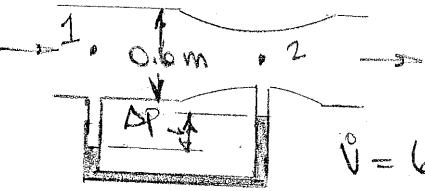
$$\dot{m} (-V_A) = \frac{-62.4(14.6)(41.9)}{32.2} = -1185 \text{ lb}_f$$

$$F_y = -502 + 109 - 1185$$

$$= -1578 \text{ lb}_f$$

Force on LD is 1578 lb\_f ↑

6.11



$$\dot{V} = 6 \text{ m}^3/\text{s}$$

$$\Delta P = 0.10 \text{ m of water} \quad (S_{1,2} = 0.6)$$

$$= 0.08 \text{ m H}_2\text{O} = 785 \text{ Pa}$$

$$A_1 = \frac{\pi}{4} (0.6)^2 = 0.283 \text{ m}^2$$

$$V_1 = \frac{\dot{V}}{A_1} = \frac{6}{0.283} = 21.2 \text{ m/s}$$

ENERGY EQUATION REDUCES TO:

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g \Delta y = 0$$

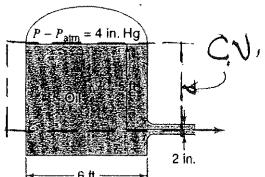
$$\begin{aligned} \frac{P_1 - P_2}{\rho} &= \frac{V_2^2 - V_1^2}{2} = \frac{\dot{V}^2}{2} \left[ \frac{1}{A_2^2} - \frac{1}{A_1^2} \right] \\ &= \frac{\dot{V}^2}{2} \left[ \left( \frac{A_1}{A_2} \right)^2 - 1 \right] \end{aligned}$$

$$\frac{P_1 - P_2}{\rho} = \frac{785}{11226} = 640 \text{ m}^2/\text{s}^2$$

$$640 = (21.2)^2 \left[ \left( \frac{A_1}{A_2} \right)^2 - 1 \right]$$

$$A_2 = 0.144 \text{ m}^2 \quad D_2 = 0.428 \text{ m}$$

6.12



ENERGY EQUATION REDUCES TO:

$$\Delta U + \frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g\Delta y = 0$$

FOR  $\Delta U = V_1 = 0$ 

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2}{2} + g(y_2 - y_1) = 0$$

$$V_2 = \left[ 2 \left( \frac{P_1 - P_2}{\rho} \right) + g\Delta y \right]^{1/2}$$

BY CONSERVATION OF MASS:

$$A_{TANK} \left( -\frac{dy}{dt} \right) = A_{VENT} V_2$$

$$-\frac{A_t}{A_v} \frac{dy}{dt} = [ ]^{1/2}$$

$$-\frac{A_t}{A_v} \int_{y_0}^{y_0-2} \frac{dy}{(K_1 + K_2 y)^{1/2}} = \int_0^t dt$$

$$K_1 = 2 \frac{\Delta P}{g} \quad K_2 = 2g$$

$$t = - \frac{A_t}{A_v} \left[ \frac{2}{K_2} (K_1 + K_2 y)^{1/2} \right]_{y_0}^{y_0-2}$$

$$K_1 = 344 \text{ FT}^2/\text{s}^2$$

$$\left[ K_1 + K_2 (y_0 - 2) \right]^{1/2} = 23.2 \text{ FT/s}$$

$$\left[ K_1 + K_2 (y_0) \right]^{1/2} = 25.8 \text{ "}$$

$$\underline{\underline{t = 105 \text{ s}}}$$

6.13. ENERGY EQUATION REDUCES TO

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$

$$P_1 = P_{atm} = 29 \text{ in. Hg} \left( \frac{14.7}{29.92} \right) = 14.25 \text{ psi}$$

$$P_2 = ?$$

$$V_1^2 = \left[ \left( 85 \text{ mi/hr} \right) \left( \frac{5280}{3600} \right) \right]^2 = (124.7)^2$$

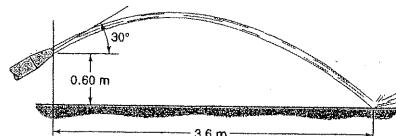
$$V_2^2 = 120^2$$

$$P_2 = 14.25 + \frac{g}{2} \left[ (124.7)^2 - (120)^2 \right]$$

$$\gamma = P/RT = \frac{14.25(144)}{53.3(500)} = 0.0710 \text{ lbm/ft}^3$$

$$\underline{\underline{P_2 = 14.25 + 1.37 = 15.6 \text{ psi}}}$$

6.14



$$\text{ENERGY EQUATION: } \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$\text{IN X-DIRECTION: } V_0 \cos \theta = V_x = \frac{dx}{dt}$$

$$\text{IN Y-DIRECTION: } V_0 \sin \theta - gt = \frac{dy}{dt}$$

$$x = (V_0 \cos \theta) t$$

$$y = (V_0 \sin \theta) t - \frac{gt^2}{2}$$

$$\text{COMBINING: } y = x \tan \theta - \frac{g}{2} \frac{x^2}{(V_0 \cos \theta)^2}$$

6.14 - CONTINUED

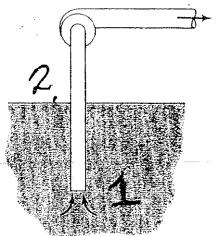
$$y = 0.6 \text{ m} \quad \tan \theta = 0.577 \\ x = 3.6 \text{ m} \quad \cos \theta = 0.816$$

$$0.6 = 3.6(0.577) - \frac{9.81}{2} \frac{3.6^2}{(0.816)^2}$$

$$V_0 = 7.57 \text{ ft/s}$$

$$\text{TOTAL HEAD} = 0.6 + \frac{V^2}{2g} \\ = 3.52 \text{ m}$$

6.15



$$\dot{V} = 550 \text{ g/m} = 1.225 \text{ ft}^3/\text{s}$$

$$V = \frac{\dot{V}}{A} = \frac{1.225}{\pi/4 (5.95/12)^2} = 6.35 \text{ ft/s}$$

ENERGY EQUATION: 2 IS AT H<sub>2</sub>O LEVEL  
OUTSIDE PIPE

$$\frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} + g(y_1 - y_2) = 0$$

$$\frac{P_1}{\rho} = -\frac{V_1^2}{2} - gy_1 \\ = -\frac{(6.35)^2}{2} - 32.2(6)$$

$$= -(20.16 + 193.2)$$

$$= -213.4 \text{ ft}^2/\text{s}^2$$

$$P_1 = -\frac{(6.35)(213.4)}{(144)32.2} = -2.87 \text{ psia}$$

40

6.16 WITH Reference To Prob 6.15

BETWEEN H<sub>2</sub>O SURFACE & PUMP INLET  
ENERGY EQUATION IS

$$\frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} + g(y_1 - y_2) = h_L$$

$$\frac{P_{atm} - P_2}{\rho g} - \frac{V_2^2}{2g} + y_1 - y_2 = h_L$$

$$\frac{V_2^2}{2g} = \frac{P_{atm} - P_2}{\rho g} - (y_2 - y_1) - h_L$$

$$= \frac{(14.7 - 0.47)(144)(32.2)}{62.4(32.2)} - 4-4$$

$$= 25.35 \text{ FT}$$

$$V_2 = \left[ 2(32.2)(25.35) \right]^{1/2} = 40.4 \text{ ft/s}$$

$$\dot{V} = A V_2 = \frac{\pi}{4} \left( \frac{5.95}{12} \right)^2 (40.4) = 7.8 \text{ ft}^3/\text{s}$$

6.17 From Prob 5.27

$$\Delta V_F = 10.22 \text{ ft/s}$$

$$\omega r_2 = 82.2 \text{ ft/s}$$

$$V_F = 10.22 \text{ ft/s}$$

$$\text{AT } r_2 - V_F = 82.2 - 10.22$$

$$V_F = 10.22$$

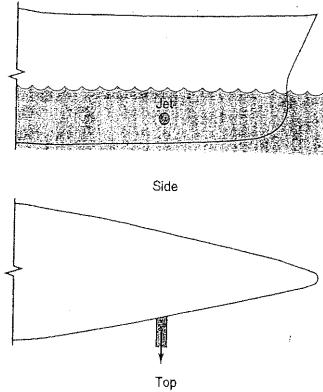
$$V = \sqrt{V_F^2 + V_y^2} = 82.9 \text{ ft/s}$$

$$\text{HEAD} = \frac{V^2}{2g} = 106.7 \text{ FT}$$

$$\Delta P = \rho \frac{V^2}{2} = 62.4 \left( \frac{82.9}{32.2} \right)^2$$

$$= 6660 \text{ lb}_F/\text{ft}^2 = 46.2 \text{ psi}$$

6.18



FOR THE SITUATION SHOWN -

$$\text{THRUST} = F = \rho V \sigma$$

$$\text{Power} = -\frac{\delta m}{dt} = \rho V \frac{\sigma^2}{2}$$

$$\frac{\text{Power}}{\text{THRUST}} \sim \frac{\rho V \sigma^2 / 2}{\rho V \sigma} \approx \sigma$$

$$\frac{\text{THRUST}}{\text{Power}} \sim \frac{1}{\sigma} \sim \frac{1}{m^{1/2}}$$

FANDRABIT CHOICE: { High Volume  
Low Pressure }

6.19 From Prob 5.7 :

$$P_1 = 50 \text{ psia} \quad P_2 = 5 \text{ psia}$$

$$D_1 = 12 \text{ in} \quad D_2 = 1.5 \text{ in}$$

$$\dot{V} = 3 \text{ ft}^3/\text{s} \quad S.G. = 0.8$$

$$h_L = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g}$$

$$V_1 = \frac{3}{4}\pi(1)^2 = 3.82 \text{ ft/s}$$

$$V_2 = \frac{3}{4}\pi(1.5)^2 = 88 \text{ ft/s}$$

6.19 - CONTINUED

$$h_L = \frac{(50-5)144}{0.8(62.4)} + \frac{3.82^2 - 88^2}{2(32.2)} = \underline{\underline{9.79 \text{ FT}}}$$

6.20 For A C.V. Enclosure Take focus:

$$\Delta u + \frac{\Delta p}{\rho} + \frac{\Delta V^2}{2} + g \Delta y = 0$$

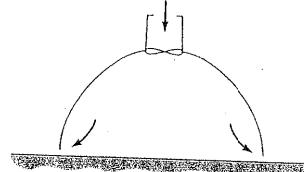
$$\Delta u = g \Delta y = 9.81(16.5) = 1620 \text{ m}^2/\text{s}^2$$

$$= 1620 \text{ m}^2/\text{s}^2 \left( 1 \frac{\text{J}}{\text{kg m}} \right) = 1620 \text{ J/kg}$$

$$\text{for } H_2O: C_p = 4184 \text{ J/kg K}$$

$$\Delta T = \frac{\Delta u}{C_p} = \frac{1620}{4184} \approx \underline{\underline{0.39^\circ\text{C}}}$$

6.21



ASSUME VERTICAL FORCES DO NOT INCLUDE MOMENTUM OF INCOMING AIR -

$$(P - P_{ATM})A = Mg \quad \left\{ \begin{array}{l} \text{PRESSURE} \\ \text{FORCE} \end{array} \right\} = WT$$

ENERGY EQUATION BECOMES BERNoulli EQUATION BETWEEN INSIDE & EXIT -

$$\frac{P - P_{ATM}}{\rho g} = \frac{V^2}{2}$$

$$\text{or, } V^2 = 2 \frac{Mg}{\rho A}$$

6.21 (CONTINUED)

$$V^2 = 2 \frac{(8100 \text{ kg})(9.81 \text{ m/s}^2)}{(1205 \text{ kg/m}^3)(27 \text{ m}^2)}$$

$$\Rightarrow 4885 \text{ m}^2/\text{s}^2 \quad V = 69.9 \text{ m/s}$$

$$\dot{V} = 69.9(24)(0.03) \\ = 50.3 \text{ m}^3/\text{s}$$

$$\dot{m} = 60.6 \text{ kg/s}$$

ENERGY EQUATION:

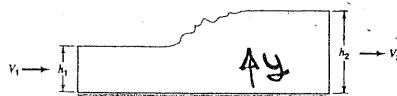
$$-\frac{\delta W_{\text{ext}}}{dt} = \dot{m} \left( \frac{\partial h}{\partial t} + \frac{\Delta V^2}{2} + \frac{\Delta P}{\rho g} + g \Delta y \right)$$

$$= \dot{m} \frac{\dot{V}^2}{2}$$

$$= 60.6 \frac{(69.9)^2}{2} = \underline{\underline{148 \text{ kW}}}$$

6.22 From PROB 5.22

$$h_2 = \frac{h_1}{2} \left[ \left( 1 + \frac{8V_1^2}{gh_1} \right)^{1/2} - 1 \right]$$



APPLIES TO  $\uparrow$

FOR BERNOULLI EQUATION TO BE

$$V_1 = 0 \quad h_L = 0$$

ENERGY EQUATION FOR THIS CASE IS

$$h_L = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} + y_1 - y_2$$

$$\frac{1}{2} \text{ SINCE } P = P_{\text{atm}} + \rho g(h - y)$$

6.22 - CONTINUED

$$h_L = \frac{V_1^2 - V_2^2}{2g} + h_1 - h_2$$

WRITING SAME TO PROB 5.22 AS

$$h_2 = \frac{h_1}{2} \left( \sqrt{1+B} - 1 \right) \quad \left\{ B = \frac{8V_1^2}{gh_1} \right\}$$

$\frac{1}{2}$  NOTE THAT - FOR  $h_2 > h_1$ ,  $B > 8$

BERNOULLI EQUATION APPLIES FOR  $B = 8$

$\frac{1}{2}$  OBVIOUSLY  $h_L > 0$  FOR  $B > 8$

6.23 ENERGY EQUATION APPLIES IN FORM:

$$-\frac{\delta W_{\text{ext}}}{dt} = \dot{m} \frac{P_{\text{ext}} - P_{\text{atm}}}{\rho g} = \dot{V} \Delta P$$

$$= \eta_P \eta_M (\text{Power})$$

$$\left\{ \begin{array}{l} \text{PER} \\ \text{PERSON} \end{array} \right\} \dot{V} = \frac{80}{(7.48)(24)(3600)} = 1.238 \times 10^{-4} \text{ ft}^3/\text{s}$$

$$P = \frac{\dot{V} \Delta P}{\eta_P \eta_M}$$

$$= \frac{(1.238 \times 10^{-4})(60)(144)}{1.075(0.9)}$$

$$= 1.584 \text{ ft lbf} \frac{1}{s}$$

$$= 2.148 \text{ W}$$

PER MONTH -

$$P = 2.148 \text{ W} (30)(24)$$

$$= 1547 \text{ Wh} = \underline{\underline{1.547 \text{ kWh}}}$$

6.24 Bernoulli Eqn  
BETWEEN FREE STREAM &  
A REFERENCE POINT (1) ON GM

$$\frac{P_{ATM}}{g} + \frac{(W+V)^2}{2g} = \frac{P_1}{g} + \frac{(V-W)^2}{2g}$$

$$\frac{P_1 - P_{ATM}}{g} = \frac{P_1 g}{g} = 2WV$$

$$\underline{P_1 g = 2WV}$$

6.25 Energy loss is

$$\frac{\delta Q}{dt} = \iint_{CS} (e + \frac{1}{2} V^2) \rho (\vec{V} \cdot \vec{n}) dA$$

$$\dot{Q} = \dot{m} \left[ (e_2 - e_1) + \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) \right]$$

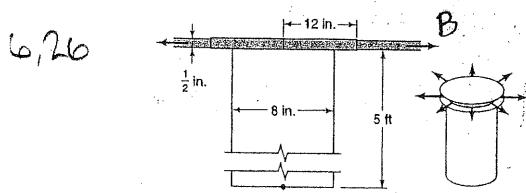
$$\Delta u = 200 \text{ kJ/kg}$$

$$\frac{\Delta P}{g} = \frac{340 \times 10^3}{1001} = 340 \text{ kJ/kg}$$

$$\frac{\Delta g^2}{2} = 0$$

$$g \Delta y = 9.81 (15) = 0.147 \text{ kJ/kg}$$

$$\dot{Q} = 200 + 340 + 0.15 = 540 \text{ kJ/kg}$$



$$\dot{V} = V_A \frac{\pi}{4} \left(\frac{8}{12}\right)^2 = V_B \left(2\pi\right)(1)\left(\frac{0.5}{12}\right)$$

6.26 - CONTINUED

$$V_A = 2.865 \text{ ft/s} \quad V_A^2 = 8.22 \text{ ft}^2/\text{s}$$

$$V_B = 3.82 \text{ ft/s} \quad V_B^2 = 14.6 \text{ ft}^2/\text{s}$$

for negligible friction -

Bernoulli Eqn applies

$$\frac{P_A - P_B}{g} + \frac{V_A^2 - V_B^2}{2} + g(y_2 - y_1) = 0$$

$$\frac{V_B^2 - V_A^2}{2} = \frac{V^2}{2} \frac{14.6 - 8.22}{2} = 3.32 \text{ ft}^2/\text{s}$$

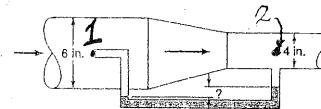
$$\frac{P_A - P_B}{g} = \frac{10(144)(32.2)}{62.4} = 743 \text{ ft}^2/\text{s}^2$$

$$g(y_A - y_B) = 32.2(-5) = -161$$

$$3.32 \text{ ft}^2/\text{s}^2 = 743 - 161 = 582$$

$$\underline{V = 13.2 \text{ ft}^3/\text{s}}$$

6.27



Bernoulli Eqn Between 1 & 2:

$$\frac{P_2 - P_1}{g} + \frac{V_2^2 - V_1^2}{2} = 0$$

$$V_1 = \frac{1}{\pi} \left(\frac{1}{2}\right)^2 = 5.09 \text{ ft/s}$$

$$V_2 = \frac{1}{\pi} \left(\frac{4}{12}\right)^2 = 11.5 \text{ ft/s}$$

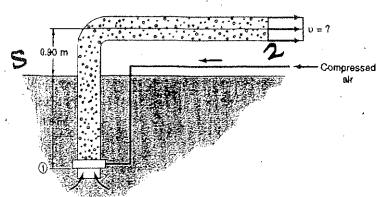
$$\frac{\Delta P}{g} = \frac{-(11.5^2 - 5.09^2)}{2} = -1.65 \text{ ft H}_2\text{O}$$

MANOMETER READING -

$$= -1.457 \text{ "Hg}$$

$$h \left[ 1 - \frac{1}{13.6} \right] = 1.457$$

6.28



For A CONTROL VOLUME BETWEEN  $1 \frac{1}{2}$   
(IN MIXTURE REGION)

$$\frac{P_1 - P_1}{\gamma_m} + \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$P_1 - P_{ATM} = \gamma_m g \Delta y_1 \quad (1)$$

MASS BALANCE AROUND MIXING CHAMBER.

$$\dot{m}_{A,air} + \dot{m}_w = \dot{m}_m$$

$$\text{AS GIVEN: } \gamma_m = \gamma_w / 2$$

$$\therefore V_m = 2V_w + 2 \frac{\gamma_A V_A}{\gamma_w V_w} \quad (2)$$

CONTROL VOLUME BETWEEN  $A_{2,0}$   
SURFACE  $\frac{1}{2}$  & 1 (H<sub>2</sub>O ONLY)

$$\frac{P_{ATM} - P_1}{\gamma_w} + \frac{0 - V_w^2}{2} + g \Delta y_2 = 0$$

$$P_1 - P_{ATM} = \gamma_w g \Delta y_2 - \gamma_w \frac{V_w^2}{2} \quad (3)$$

EQUATING (1) & (3):

$$\gamma_m g \Delta y_1 = \gamma_w \left( g \Delta y_2 - \frac{V_w^2}{2} \right)$$

$$\frac{V_w^2}{2} = g (\Delta y_2 - \Delta y_1 / 2)$$

$$V_w = \left[ 2g (0.45m) \right]^{1/2}$$

6.28 - CONTINUED

SUBSTITUTING EXPRESSION FOR  $V_w$   
INTO (2)

$$V_m = 2 \left[ g (0.9m) \right]^{1/2} + \frac{2 (g A) V_A}{\gamma_w V_m}$$

SINCE  $\frac{V_m}{V_A} \gg 1$  2ND TERM IS SMALL

$$\therefore V_m = 2 \left[ 9.81 (0.9) \right]^{1/2} = \underline{\underline{5.94 \text{ m/s}}}$$

6.29 for conditions of Prob 6.28

CONTROL VOLUME AROUND MIXING CHAMBER.

$$\sum F_y = \iint_{CS} V_y \rho (F \cdot n) dA$$

$$\Delta P A = \gamma_m A V_m \left[ V_m - \frac{\gamma_m}{\gamma_w} V_m \right]$$

THIS NEGLECTS MOMENTUM OF AIR

from PROB 6.28

ABOVE MIXER -  $P = P_{ATM} + \gamma_m g \Delta y_1$ Below "  $P = P_{ATM} + \gamma_w g \Delta y_2 - \gamma_w \frac{V_w^2}{2}$ 

$$\Delta P = \gamma_w g (\Delta y_2 - \Delta y_1) - \gamma_w \frac{V_w^2}{2}$$

EQUATING WITH MOMENTUM EXPRESSION

$$\Delta P = \gamma_m V_m^2 \left( 1 - \frac{\gamma_m}{\gamma_w} \right)$$

$$= \gamma_w \left[ g (\Delta y_2 - \Delta y_1) - \frac{V_w^2}{2} \right]$$

$$\frac{V_m^2}{4} = g (\Delta y_2 - \Delta y_1) - \frac{V_w^2}{2}$$

$$\text{For } V_w/V_m = 2 \quad \underline{\underline{V_m = 4.6 \text{ m/s}}}$$

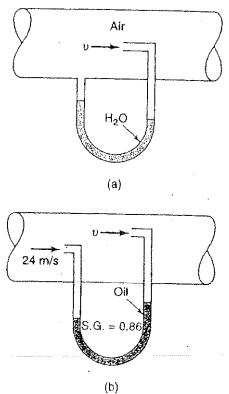
6.29 - (CONTINUED)

$$\Delta P = \rho_m v_m^2 (1 - \frac{1}{2})$$

$$= \frac{1}{2} (4.6)^2 (1000/2)$$

$$\approx \underline{5.3 \text{ kPa}}$$

6.30



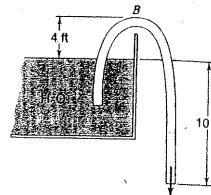
IN BOTH GASES - BEANOULLI EQUATION IS

$$\frac{V_2^2 - V_1^2}{2} + \frac{P_2 - P_1}{\rho g} = 0$$

(a)  $\Delta P = \frac{V^2}{2g} = \frac{15^2}{2(9.81)}$   
 $= 11.47 \text{ m AIR}$   
 $= \underline{1.39 \text{ cm H}_2\text{O}}$

(b)  $\Delta P = \frac{24^2 - 15^2}{2(9.81)}$   
 $= 17.9 \text{ m AIR}$   
 $= \underline{2.52 \text{ cm OIL}}$

6.31



BETWEEN LIQUID SURFACE & EXIT

BEANOULLI EQUATION:

$$\frac{V^2}{2} = g \Delta y$$

$$V = (2g \Delta y)^{1/2}$$

$$= 2(32.2)(10) = 2535 \text{ FT/S}$$

$$\dot{V} = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 (2535) = \underline{1.66 \text{ FT}^3/\text{s}}$$

BETWEEN POINT B & EXIT

$$\frac{P_B - P_{ATM}}{g} + g(y_B - y_{EXIT}) = 0$$

$$P_B = P_{ATM} - g(y_B - y_{EXIT})$$

$$= 14.7 \text{ psi} - \frac{62.4(32.2)(14)}{32.2(144)}$$

$$= \underline{8.63 \text{ psi}}$$

BY CONTINUITY -

$$\dot{V}_{TANK} = A_{TANK} \left(-\frac{dy}{dt}\right)$$

$$= A_{PIPE} \sqrt{2gy}$$

$$-\frac{dy}{dt} = \frac{A_{PIPE}}{A_{TANK}} \sqrt{2g y^{1/2}}$$

6.31 (CONTINUED)

$$\begin{aligned} -\int_{10}^7 y^{1/2} dy &= \frac{\Delta P}{A_t} \sqrt{2g} \int_0^t dt \\ 2y^{1/2} \Big|_{10}^7 &= \frac{\Delta P}{A_t} \sqrt{2g} t \\ 2 \left[ 10^{1/2} - 7^{1/2} \right] &= \frac{(1/12)^2}{10^2} \sqrt{2(32.2)} t \\ t &= 1854 \text{ s} = 0.515 \text{ h} \end{aligned}$$

6.32 ENERGY EQUATION FOR THIS

CASE:

$$\frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) + u_2 - u_1 = 0$$

$$V_1 = 0$$

$$y_2 - y_1 = -10 \text{ ft}$$

$$u_2 - u_1 = 32 \frac{V^2}{g}$$

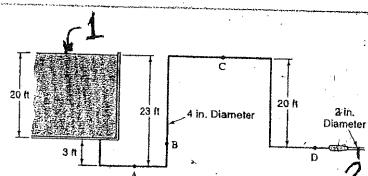
$$\frac{V_2^2}{2g} + (-10) + 32 \frac{V_2^2}{g} = 0$$

$$\frac{32V^2}{g} = 10$$

$$V = 10.03 \text{ ft/s}$$

$$V = \frac{\pi}{4} \left(\frac{1}{12}\right)^2 (10.03) = 0.0547 \text{ ft}^3/\text{s}$$

6.33



BETWEEN 1 & 2

$$\frac{V_2^2 - 0}{2} + g(y_2 - y_1) = 0$$

6.33 (CONTINUED)

$$V_2 = \left[ 2(32.2)(20) \right]^{1/2} = 35.9 \text{ ft/s}$$

$$V = AV = \frac{\pi}{4} \left(\frac{2}{12}\right)^2 (35.9) = 0.783 \text{ ft}^3/\text{s}$$

$$\text{IN 4" LINES} - V = \frac{V_2}{4} = 8.975 \text{ ft/s}$$

$$V^2 = 80.55 \text{ ft}^2/\text{s}^2$$

BETWEEN 1 & 4:

$$\frac{P_A - P_1}{g} + \frac{V_A^2 - V_1^2}{2} + g(y_A - y_1) = 0$$

$$\begin{aligned} P_A &= P_{\text{Atm}} + \rho \left(-\frac{V_A^2}{2}\right) + \rho g (y_1 - y_A) \\ &= P_{\text{Atm}} + \frac{62.4}{32.2} \left(-\frac{80.55}{2}\right) + 62.4(23) \end{aligned}$$

$$= P_{\text{Atm}} + 1356 \text{ lb/ft}^2 = \frac{3475 \text{ PSF}}{(24.12 \text{ psi})}$$

$$V_A = \underline{8.975 \text{ ft/s}}$$

BETWEEN A & B:

$$\frac{P_A - P_B}{g} + \frac{V_A^2 - V_B^2}{2} + g(y_A - y_B) = 0$$

$$P_B = P_A + \rho g (-3) = \frac{3290 \text{ PSF}}{(22.83 \text{ psi})}$$

$$V_B = \underline{8.975 \text{ ft/s}}$$

CONDITIONS AT D & B ARE EQUAL

$$\therefore P_D = \underline{3290 \text{ PSF}}$$

$$V_D = \underline{8.975 \text{ ft/s}}$$

6.33 (CONTINUED)

BETWEEN B &amp; C:

$$\frac{P_B - P_C}{g} + \frac{V_B^2 - V_C^2}{2} + g(y_B - y_C) = 0$$

$$P_C = P_B + g(y_B - y_C)$$

$$= P_B + 624(-20)$$

$$= \frac{2042 \text{ PSF}}{(14.18 \text{ psi})}$$

$$V_C = 8.975 \text{ ft/s}$$

6.34 BETWEEN bottom lower (1)  
& EXIT(2)

$$\frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$V_2 = \sqrt{2gA_y}$$

$$\dot{V} = \frac{\pi}{4} D_{\text{exit}}^2 [2gy]^{1/2}$$

H<sub>2</sub>O IN TANK:

$$\dot{V} = \frac{\pi}{4} D_{\text{TANK}}^2 \left( -\frac{dy}{dt} \right)$$

$$\frac{dy}{dt} = \left( \frac{D_{\text{exit}}}{D_{\text{TANK}}} \right)^2 \sqrt{2g} y^{1/2}$$

$$\int y^{1/2} dy = \left( \frac{D_e}{D_t} \right)^2 \sqrt{2g} \int dt$$

$$2y^{1/2} \Big|_4^{28} = \left( \frac{D_e}{D_t} \right)^2 \sqrt{2g} t$$

6.34 - (CONTINUED)

$$t = \frac{2(28^{1/2} - 4^{1/2})}{\left(\frac{2^{1/2}}{15}\right)^2 \left[ 2(32.2) \right]^{1/2}}$$

$$= \underline{6644 \text{ s}} = 1.846 \text{ hours}$$

6.35

From 1 TO 2

$$\frac{P_1 - P_2}{g_1} + \frac{V_2^2 - V_1^2}{2} + g(y_1 - y_2) = 0 \quad (1)$$

$$\frac{P_1 - P_2}{g_1} + \frac{V_2^2}{2} = 0 \quad (1)$$

From 3 TO 4:

$$\frac{P_3 - P_4}{g_2} + \frac{V_4^2 - V_3^2}{2} + g(y_3 - y_4) = 0$$

$$\frac{V_4^2}{2} = \frac{V_3^2}{2} + \frac{P_3 - P_4}{g_2} - gL \quad (2)$$

NOTE THAT  $P_4 + \rho_1 g L = P_1$ , GIVING

$$\frac{V_4^2}{2} = \frac{V_3^2}{2} + \frac{P_3 - P_1 + \rho_1 g L}{g_2} \quad (3)$$

From 2 TO 3

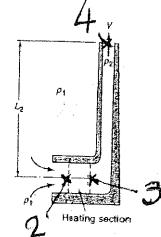
$$\frac{P_2}{g_1} - \frac{P_3}{g_2} + \frac{V_2^2 - V_3^2}{2} = 0$$

FOR  $V_2 \approx V_3$  NEGLIGIBLE.

$$P_2 = P_3$$

&amp; from (2)

$$\frac{V_2^2}{2} = -gL + \frac{\rho_1 g L}{g_2} = gL \left( \frac{\rho_1}{\rho_2} - 1 \right)$$



6.36 From Prob 6.28:

$$\frac{P_1 - P_2}{S_1} + \frac{V_2^2}{2} = 0$$

$$\frac{P_3 - P_1}{S_2} = \frac{V^2}{2} + gL \left( \frac{S_2 - S_1}{S_2} \right) - \frac{V_3^2}{2}$$

CONS. OF MASS:

$$S_1 V_2 = S_2 V_3 = S_2 V / R$$

$$\frac{\dot{A}_{ATR}}{\dot{A}_{STRA}} = R$$

$$\therefore V_2^2 = \left( \frac{S_2}{S_1} \right)^2 \left( \frac{V}{R} \right)^2 \quad V_3^2 = \frac{V^2}{R^2}$$

$$\text{GIVEN} \quad P_2 - P_1 = -\frac{S_1}{2} \left( \frac{S_2}{S_1} \right)^2 \frac{V^2}{R^2}$$

$$P_3 - P_1 = S_2 \frac{V^2}{2} + gL(S_2 - S_1) - \frac{S_2 V^2}{2R^2}$$

From Momentum Theorem:

$$\frac{V_2 - P_1}{P_2} \rightarrow V_3$$

$$F_x = \iint_{CS} V_x S(\vec{v}, \vec{n}) dA$$

$$(P_2 - P_3) A = S_1 V_2 A (V_3 - V_2)$$

$$P_2 - P_3 = \frac{S_2 V^2}{R^2} \left( 1 - \frac{S_2}{S_1} \right)$$

BERNOULLI EQU:

$$\frac{V^2}{R^2} \left( 1 - \frac{S_2}{S_1} \right) + \frac{S_1}{2 S_2} \left( \frac{P_2}{S_1} \right)^2 \frac{V^2}{R^2} + \frac{V^2}{2} + gL \left( 1 - \frac{S_1}{S_2} \right) - \frac{V^2}{2R^2} = 0$$

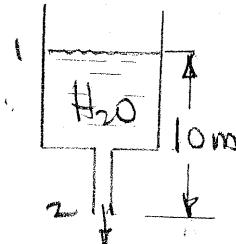
6.36 (CONTINUED)

DURING THE ALGEBRA:

$$V_2 = \frac{2gL \left( \frac{S_1}{S_2} - 1 \right)}{1 + \frac{1 - S_2/S_1}{R^2}}$$

6.37

FRICTIONLESS FLOW:



From Bernoulli

$$\frac{P_2 - P_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + y_2 - y_1 = 0$$

$$V_2^2 = 2g(y_1 - y_2)$$

$$V = \left[ 2(9.81)(10) \right]^{1/2} = 14 \text{ m/s}$$

$$\dot{m} = (1000) \left( \frac{\pi}{4} \right) (0.04)^2 (14) = 17.6 \text{ kg/s}$$

WITH NOZZLE -  $V = 14 \text{ m/s}$  {sinus}

$$\dot{m} = (1000) \left( \frac{\pi}{4} \right) (0.01)^2 (14) = 1.10 \text{ kg/s}$$

WITH  $u_2 - u_1 = 3V^2$

ENERGY EQU Reduces to

$$\frac{V_2^2}{2g} + \frac{3V^2}{g} = 2(y_1 - y_2)$$

$$V_2 = \left[ \frac{4}{7} (9.81) 10 \right]^{1/2} = 7.49 \text{ m/s}$$

$$\text{PIPE: } \dot{m} = 9.42 \text{ kg/s}$$

$$\text{NOZZLE: } \dot{m} = 0.589 \text{ "}$$

6.38 Same tank as in  
Prob 6.37 but 2 exit  
pipes -

Pipe 1:  $D = 0.04\text{m}$

$$\Delta y = 10\text{ m}$$

Pipe 2  $D = 0.04\text{m}$

$$\Delta y = 20\text{ m}$$

Frictionless flow!

Pipe 1 -

As in Prob 6.37

$$V = \sqrt{2g\Delta y} = 14\text{ m/s}$$

$$\dot{m} = \underline{17.6\text{ kg/s}}$$

Pipe 2: Also  $V = \sqrt{2g\Delta y}$

$$V = \left[ 2(9.81)(20) \right]^{1/2}$$

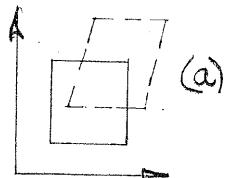
$$= \underline{19.81\text{ kg/s}}$$

$$\dot{m} = (1000)\left(\frac{\pi}{4}\right)(0.04)^2(19.81)$$

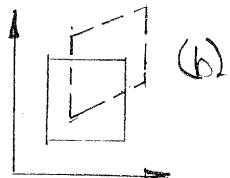
$$= \underline{24.9\text{ kg/s}}$$

## CHAPTER 7

7.1

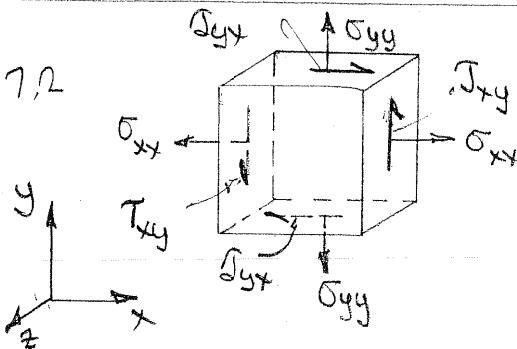


$$\frac{\partial u_x}{\partial y} \gg \frac{\partial u_y}{\partial x}$$



$$\frac{\partial u_y}{\partial x} \gg \frac{\partial u_x}{\partial y}$$

7.2



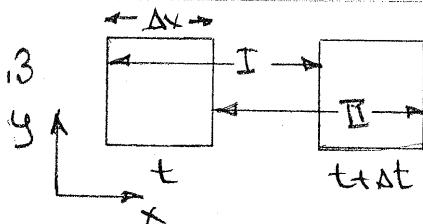
for 2-D flow - IN X, y

$$v_z = 0 \quad \sigma_{zz} = 0$$

$$\frac{\partial v_z}{\partial x} = 0 \quad \tau_{zx} = \tau_{xz} = 0$$

$$\frac{\partial v_z}{\partial y} = 0 \quad \tau_{zy} = \tau_{yz} = 0$$

7.3



$$I = v_x(x) \Delta t$$

$$II = v_x(x + \Delta x) \Delta t$$

## 7.3 - CONTINUED

AXIAL STRAIN RATE:

$$= \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta t \rightarrow 0}} \frac{v_x(x + \Delta x) \Delta t - v_x(x) \Delta t}{\Delta x \Delta t} = \frac{\partial v_x}{\partial t}$$

RATE OF VOLUME CHANGE:

$$= \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta t \rightarrow 0}} \frac{A \Delta x_{L+AT} - A \Delta x_L}{A \Delta x \Delta t}$$

$$= \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta t \rightarrow 0}} \frac{v_x(x + \Delta x) \Delta t - v_x(x) \Delta t}{A \Delta x \Delta t} = \frac{\partial v_x}{\partial x}$$

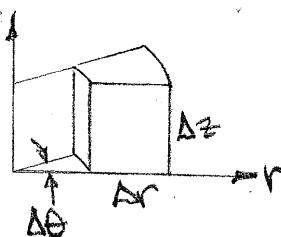
IN 3 DIMENSIONS

BOTH AXIAL STRAIN RATE AND

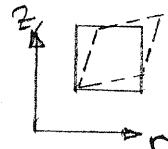
VOLUME CHANGE RATE ARE  
GIVEN BY

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

7.4



IN r-z PLANE:



$$-\frac{ds}{dt} = -\lim_{\Delta t \rightarrow 0} \frac{s_{t+\Delta t} - s_t}{\Delta t}$$

$$= -\lim_{\Delta t \rightarrow 0} \left[ \tan^{-1} \left( \frac{v_r|_{z+\Delta z} - v_r|_z}{\Delta z} \right) \Delta t \right]$$

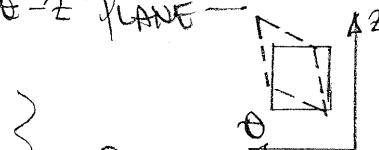
$$-\tan^{-1} \frac{v_z|r_{\Delta z} - v_z|r_z \Delta t + \bar{v}_z}{\Delta r} \Delta t$$

### 7.4 - (CONTINUED)

$$= \lim_{\Delta z \rightarrow 0} \left[ \frac{V_r|_{z+\Delta z} - V_r|_z}{\Delta z} + \frac{V_z|r_{z+\Delta z} - V_z|r_z}{\Delta r} \right]$$

$$\therefore \mathfrak{J}_{rz} = \mathfrak{J}_{zr} = \mu \left[ \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right]$$

IN THE  $\theta-z$  PLANE —

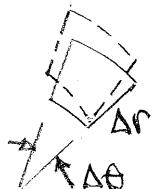


SAME PROCEDURE

$$\lim_{\Delta z \rightarrow 0} \left[ \frac{V_\theta|_{z+\Delta z} - V_\theta|_z}{\Delta z} + \frac{1}{r} \frac{V_z|_{z+\Delta z} - V_z|_z}{\Delta \theta} \right]$$

$$\mathfrak{J}_{\theta z} = \mathfrak{J}_{z\theta} = \mu \left[ \frac{\partial V_\theta}{\partial z} + \frac{1}{r} \frac{\partial V_z}{\partial \theta} \right]$$

1/ IN  $r-\theta$  PLANE



$$\lim_{\Delta r \rightarrow 0} \left[ \frac{V_r|_{\theta+\Delta \theta} - V_r|_\theta}{r \Delta \theta} + r \left( \frac{V_\theta}{r} \Big|_{\theta+\Delta \theta} - \frac{V_\theta}{r} \Big|_\theta \right) \right]$$

$$\mathfrak{J}_{r\theta} = \mathfrak{J}_{\theta r} = \mu \left( \frac{1}{r} \frac{\partial V_r}{\partial \theta} + r \frac{\partial}{\partial r} \left( \frac{V_\theta}{r} \right) \right)$$

### 7.5 NITROGEN @ 175 K

$$\mu = 2.6693 \times 10^6 \frac{\sqrt{MT}}{\sigma^2 \Omega \mu}$$

$$T = 175 \text{ K} \quad \sigma = 3.681 \text{ \AA}$$

$$M = 28 \quad \Omega \mu = 1.1942$$

$$E_A/k = 91.5$$

$$KT/e = 1.91$$

$$\mu \approx 11.55 \times 10^{-6} \text{ Pa.s.}$$

### 7.6 OXYGEN @ 350 K

$$\text{EQN. 7.10} \quad \mu = 2.6693 \times 10^6 \frac{\sqrt{MT}}{\sigma^2 \Omega \mu}$$

$$\frac{KT}{e} = \frac{T}{E/k} = \frac{350}{113} = 3.097$$

$$\text{YIELDING} \quad \Omega \mu = 1.03$$

$$M = 32 \quad \sigma = 3.433$$

$$\mu = 1.327 \times 10^{-5} \text{ Pa.s.}$$

$$\text{TABLE VALUE: } \mu = 1.318 \text{ Pa.s.}$$

### 7.7 for $H_2O$

$$\mu|_{60^\circ} = 0.76 \times 10^{-3} \text{ lbm/s.ft.}$$

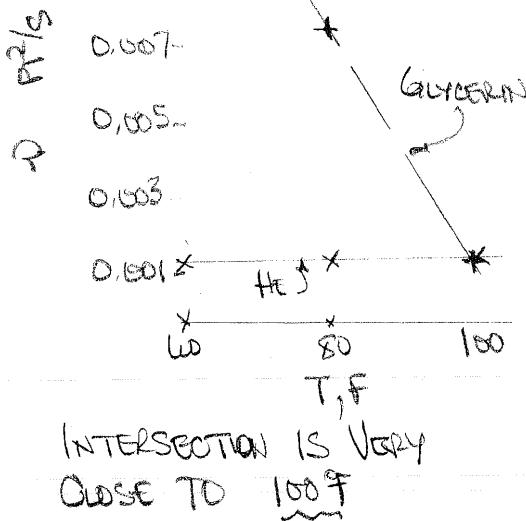
$$\mu|_{120^\circ} = 0.375 \times 10^{-3} \text{ "}$$

$$\text{PERCENT CHANGE} = \frac{0.76 - 0.375}{0.76} = 51\%$$

$$= \underline{0.51} \quad \text{OR} \quad \underline{51 \%}$$

### 7.8 PROPERTIES OF HELIUM, GLYCERIN FROM APPENDIX

T, F	$\rho, \text{HE}$	$\rho, \text{GLYCERIN}$
60	0.00125	0.017
80	0.00132	0.00762
100	0.00141	0.00128



### 7.9 FOR $H_2O$ $\dot{V} \sim 1/\mu$

$$\text{@ } 120^\circ\text{F} \quad \mu_w = 0.391 \times 10^{-3} \text{ lbf/s ft}$$

$$\text{@ } 32^\circ\text{F} \quad = 1.2 \times 10^{-3} \text{ "}$$

$$\frac{\dot{V}_{120}}{\dot{V}_{32}} = \frac{1.2 \times 10^{-3}}{0.391 \times 10^{-3}} = 3.07$$

FOR CENT CHANGE

$$= \frac{1.2 - 0.391}{0.391}$$

$$= 3.07 - 1 = 2.07$$

$$\text{OR } \underline{207 \%}$$

### 7.10 FOR AIR

$$\text{@ } 140^\circ\text{F} \quad \mu = 1.34 \times 10^{-5} \text{ lbf/s ft}$$

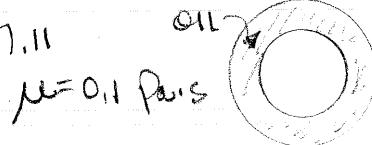
$$\text{@ } 32^\circ\text{F} \quad \mu = 1.15 \times 10^{-5} \text{ "}$$

$$\text{for } \dot{V} \sim 1/\mu$$

$$\frac{\dot{V}_{140}}{\dot{V}_{32}} = \frac{1.15}{1.34} = 0.852$$

$$\begin{aligned} \text{PER CENT CHANGE} &= \frac{1.15 - 1.34}{1.34} \\ &= 0.852 - 1 = -0.148 \\ &= -14.8\% \end{aligned}$$

### 7.11



$$R_o = 3.175 \text{ cm}$$

$$R_i = 3.183 \text{ "}$$

$$\begin{aligned} \text{1ST LAW: } \frac{dQ}{dt} - \frac{dW_s}{dt} - \frac{dW_u}{dt} &= 0 \\ Q &= \dot{W}_{\text{VISCOUS}} \quad \left. \begin{array}{l} \text{NO FLOW IN} \\ \text{OR OUT} \end{array} \right\} \\ &= \tau(A)U - \text{AT MOVING} \\ &\quad \text{BOUNDARY} \end{aligned}$$

$$\dot{Q} = \mu \frac{dW}{dr} \approx \mu \frac{rw}{t} \quad \left. \begin{array}{l} t = 6 \text{ sec} \\ \text{WIDTH} \end{array} \right\}$$

$$\dot{Q} = \mu \frac{rw}{t} (\pi D L)(rw)$$

$$= \frac{\mu (rw)^2 \pi D L}{t}$$

$$\omega = 1700 \left( \frac{2\pi}{60} \right) = 178 \text{ RAD/s}$$

$$\dot{Q} = \frac{(0.01) \left( 0.103175 \right)^2 \left( 178 \right)^2 \left( \pi \right) (0.03175) (0.028)}{4 \times 10^{-5}}$$

$$= \underline{5.58 \text{ W}}$$

7.12 REFER TO PROB 7.13

FOR  $\omega_2 = 2\omega_1$ ,

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2\omega_1)^2}{\omega_1^2} = 4$$

FOR CENT INCREASE

$$= \frac{\dot{Q}_2 - \dot{Q}_1}{\dot{Q}_1} = 4 - 1 = 3$$

$$= \underline{300\%}$$

7.13 SHIP 1  $\rightarrow V_1 = 4 \text{ m/s}$   
 SHIP 2  $\rightarrow V_2 = 3.1 \text{ m/s}$

CHOOSE CONTROL VOLUME  
 ATTACHED TO SHIP 1

$$\sum F_x = \iint_{C,S} V_x S (\vec{v} \cdot \vec{n}) dA \\ = \dot{m} V_{x1} - \dot{m} V_{x2}$$

RELATIVE TO MOVING SHIP

$$V_{x1} = 0 \quad V_{x2} = -0.9 \text{ m/s}$$

$$F_x = +\dot{m} V_{x2}$$

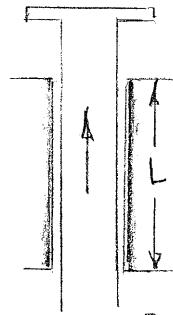
$$= 100 \text{ kg/s} (0.9 \text{ m/s})$$

$$= 90 \text{ N}$$

THIS IS FORCE APPLIED TO  
 MAINTAIN STATED CONDITIONS.

FORCE EXERTED BY FLUID  
 TRANSFER  $= -\underline{90 \text{ N}}$

7.14



$$t = GAP = \frac{D_{\text{OUTSIDE}} - D_{\text{INSIDE}}}{2}$$

$$= \frac{36.04 - 36.02}{2} = 0.01 \text{ cm}$$

$$f = \bar{J}A = \bar{J}\pi D L$$

$$\bar{J} = \mu \frac{du}{dy} = \mu \frac{du}{Ay} \quad \left. \begin{array}{l} \text{ASSUMES} \\ \text{LINEAR} \\ \text{PROFILE} \end{array} \right\}$$

$$\bar{J} = \mu \frac{V}{t}$$

$$F = \mu \frac{V}{t} \pi D L = g_D V \pi D L$$

$$= \frac{0.85(1000)(3.7 \times 10^{-4})(0.15) \times \pi (0.3602)(3.14)}{1 \times 10^{-4}}$$

$$= \underline{1676 \text{ N}}$$

7.15 REFER TO CONDITIONS OF PROB 7.14

LOAD ON RAM = 680 kg, L = 2.44 m

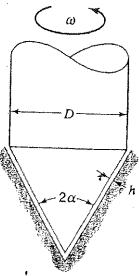
$$F = mg = \frac{g_D V \pi D L}{t}$$

$$V = \frac{mgt}{g_D \pi D L}$$

$$= \frac{680(9.81)(1 \times 10^{-4})}{0.85(1000)(3.7 \times 10^{-4})\pi(0.3602)(2.44)}$$

$$= \underline{0.768 \text{ m/s}}$$

7.16



$$M = \int r dF$$

$$dF = \tau dA$$

$dF$  IS ON THE CONICAL SURFACE  
 $= 2\pi r dL$

{  $dL$  IS A SLANTED SURF }  
 $dL = dr / \sin \alpha$

$$\text{SO: } dF = \tau dA$$

$$= \mu \frac{rw}{h} 2\pi r \frac{dr}{\sin \alpha}$$

$$\int_0^M dM = r dF \quad 0/2 \\ = \frac{2\pi \mu w}{h \sin \alpha} \int_0^r r^3 dr$$

$$M = \frac{\pi \mu w D^4}{32 h \sin \alpha}$$

$$7.17 \quad V_x = V_{max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \\ = 2V_{avg} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

$$\tau = \mu \frac{dv}{dr} \Big|_{r=R}$$

$$\frac{dv}{dr} = 2V_{avg} \left[ -\frac{2r}{R^2} \right]$$

7.17 - CONTINUED

$$\text{At } r=R \quad \frac{dv}{dr} = -4 \frac{V_{avg}}{R}$$

$$\tau = -\frac{4\mu V_{avg}}{R}$$

$$\mu_w = 0.76 \times 10^{-3} \text{ lbm/s.ft} @ 60^\circ F$$

$$\tau = -\frac{4(0.76 \times 10^{-3})(2)}{(0.05/12)(32.2)}$$

$$= -0.0453 \text{ lbf/ft}^2$$

7.17 For Conditions of Prob 7.16

$$\tau = -\frac{4\mu V_{avg}}{R}$$

$$F = \tau A = \tau \pi D L$$

$$= -\frac{4\mu V_{avg} (\pi D L)}{R}$$

$$=(-0.0453)(\pi)(0.1)(1)$$

$$= \underline{0.00119 \text{ lb}_f}$$

$$\Delta P = \frac{F}{\pi D^2/4}$$

$$= 0.00119 \text{ lb}_f \\ \pi (0.1/2)^2 / 4$$

$$= \underline{21.75 \text{ PSF}}$$

7.19 Shear Work Rate =  $\dot{\gamma} v$

$$\dot{\gamma} v = \mu v \frac{dv}{dr}$$

For Parabolic Profile -

$$v = v_{\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

$$\dot{\gamma} v = \mu v_{\max}^2 \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \left[ -\frac{2r}{R^2} \right]$$

$$= \mu v_{\max}^2 \left[ -\frac{2r}{R^2} + 2 \frac{r^3}{R^4} \right]$$

$$\frac{d}{dr}(\dot{\gamma} v) = \mu v_{\max}^2 \left[ -\frac{2}{R^2} + 6 \frac{r^2}{R^4} \right]$$

for  $\frac{d}{dr}(\dot{\gamma} v) = 0$

$$6 \frac{r^2}{R^4} = \frac{2}{R^2}$$

$$\underline{\underline{\frac{r}{R} = \frac{1}{\sqrt{3}}}}$$

## CHAPTER 8

### 8.1 HAGEN - POISEULLE EQU.

$$\frac{dp}{dx} = \frac{32 \mu V_{avg}}{D^2}$$

$$= \frac{32 \mu \dot{V}}{D^2 A} = \frac{32 \mu \dot{V}}{\pi/4 D^4}$$

For  $D = D_0$   $\dot{V}_0 = \left[ \left( -\frac{dp}{dx} \right) \frac{\pi/4}{32\mu} \right] D_0^4$

For  $D_1 = 2D_0$   $\dot{V}_1 = \left[ \left( 2D_0 \right)^4 \right] \dot{V}_0$   
 $\Rightarrow \dot{V}_1 = 16 \dot{V}_0$

PER CENT CHANGE

$$= \frac{\dot{V}_1 - \dot{V}_0}{\dot{V}_0} = \frac{\dot{V}_1}{\dot{V}_0} - 1 = 15$$

$$= \underline{1500 \%}$$

### 8.2 FOR SINGLE PIPE:

$$\Delta P_0 = \left[ \frac{32 \mu}{D^2 A} \right] \dot{V}_0 (40)$$

FOR SINGLE-PARALLEL COMBINATION

$$\Delta P_1 = \left[ \dot{V}_1 (22) - \left\{ \begin{array}{l} \text{SINGLE} \\ \text{BRANCH} \end{array} \right\} \right]$$

$$\Delta P_2 = \left[ \dot{V}_2 (18) \right] \left\{ \begin{array}{l} \text{PARALLEL} \\ \text{BRANCH} \end{array} \right\}$$

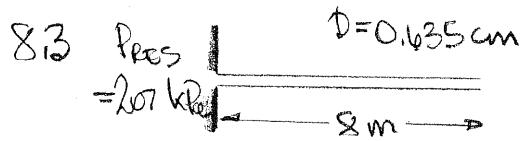
$$\Delta P_1 + \Delta P_2 = \Delta P_0 = 3.45 \times 10^6 \text{ Pa}$$

$$\dot{V}_1 = 2 \dot{V}_2$$

(CASE 2):  $\Delta P_1 + \Delta P_2 = \left[ \dot{V}_1 (22+4) \right] \dot{V}_1$

$$\dot{V}_1 = \frac{40}{31} \dot{V}_0 = \frac{40}{31} (4000)$$

$$= 5161 \text{ BBL/DAY}$$



for 1 — RESERVOIR

2 — PIPE ENTRANCE

3 — " EXIT

BETWEEN 1 & 2:  $\frac{P_1 - P_2}{8} + \frac{\dot{V}_1^2 - \dot{V}_2^2}{2} + g(y_1 - y_2) = 0$

$$\frac{P_1}{8} = \frac{P_2}{8} + \frac{\dot{V}_2^2}{2}$$

BETWEEN 2 & 3:

$$\frac{P_2 - P_3}{8} + \frac{\dot{V}_2^2 - \dot{V}_3^2}{2} + g(y_2 - y_3) - \Delta u = 0$$

$$\frac{P_2}{8} = \frac{P_{ATM}}{8} + \Delta u$$

FOR INVISCID FLOW —  $\Delta u = 0$

FOR LAMINAR, VISCOUS FLOW

$$\Delta u = \frac{\Delta P}{8} \Big|_{\text{FRICTION}} = \frac{32 \mu}{8 D^2} \dot{V}$$

INVISCID CASE:

$$\frac{\dot{V}^2}{2} = \frac{P_1 - P_{ATM}}{8} = \frac{P_{IG}}{8}$$

ASSUMING FLUID IS HYDRAULIC FLUID  
@ 60°F — 15.9 K

$$\rho = 849 \text{ kg/m}^3 \quad \mu = 0.0165 \text{ Pa.s}$$

$$\dot{V} = \left[ \frac{2 (207000)}{849} \right]^{\frac{1}{2}} = 22.08 \text{ m/s}$$

$$\dot{V} = \Delta u = \frac{\pi}{4} (0.00635^2) (22.08)$$

$$\approx 7 \times 10^{-4} \text{ m}^3/\text{s}$$

### 8.3 - CONTINUED

Viscous Case:

$$\frac{P_1 - P_{\text{ATM}}}{\rho} = \frac{V^2}{2} + \Delta u$$

$$\frac{P_{16}}{\rho} = \frac{V^2}{2} + \frac{32 \mu V}{\rho D^2}$$

$$V^2 + \frac{64 \mu V}{\rho D^2} - 2 \frac{P_{16}}{\rho} = 0$$

$$\frac{64 \mu}{\rho D^2} = \frac{64(0.0165)}{849(0.00635)^2}$$

$$= 30.85 \text{ m}^2/\text{s}^2$$

$$2 \frac{P_{16}}{\rho} = 487.6 \text{ "}$$

$$V^2 + 30.85 V - 487.6 = 0$$

$$V = \frac{-30.85 \pm \sqrt{(30.85)^2 + 4(487.6)}}{2}$$

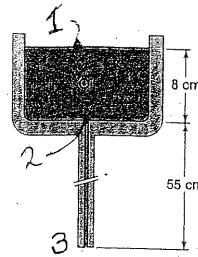
$$= 11.51 \text{ m/s}$$

$$V = \frac{\pi}{4} (0.00635)^2 (11.51)$$

$$= \underline{3.645 \times 10^{-4} \text{ m/s}}$$

$$\frac{V_{\text{inviscid}}}{V_{\text{viscous}}} = \frac{7}{3.645} = \underline{1.92}$$

### 8.4



from 1 to 2 (Bernoulli)

$$\frac{P_2 - P_{\text{ATM}}}{\rho} + \frac{V_2^2}{2} + g(y_2 - y_1) = 0$$

$$\frac{V_2^2}{2} = \frac{P_{\text{ATM}} - P_2}{\rho} + g(y_1 - y_2)$$

from 2 to 3

$$\frac{P_2 - P_{\text{ATM}}}{\rho} + \frac{V_2^2}{2} + g(y_2 - y_3) + \Delta u = 0$$

$$\Delta u = \frac{32 \mu V L}{\rho D^2} = \frac{32 V L}{D^2} \rightarrow$$

Combining Expressions:

$$\frac{32 V L}{D^2} \rightarrow = g(y_1 - y_3) - \frac{V^2}{2}$$

$$\rightarrow V = \frac{4V}{\pi D^2}$$

$$\rightarrow V = g A y \frac{\pi D^4}{128 L V} - \frac{g}{16 \pi L}$$

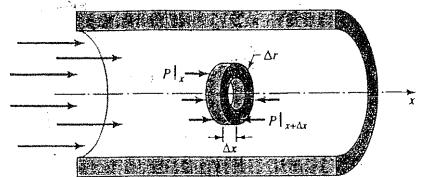
$$\frac{9.81(0.163)\pi(0.0018)^4}{128(0.55)(0.273 \times 10^{-4})} = 1.005 \times 10^{-5}$$

$$\frac{0.173 \times 10^{-4}}{16(\pi)(0.55)} = 0.000987 \times 10^{-5}$$

$$\rightarrow = (1.005 - 0.001) \times 10^{-5}$$

$$= \underline{1.0595 \times 10^{-5} \text{ m}^2/\text{s}}$$

8.5



USING THE SAME DEVELOPMENT AS IN SECTION 8.1:

$$\frac{d}{dr}(r\beta) = r \frac{df}{dx}$$

FOR AN ELEMENT OF LENGTH, L

$$\frac{d}{dr}(r\beta) \int_0^L dx = r \int_{P_1}^{P_2} df$$

WHICH BECOMES

$$\frac{d}{dr}(r\beta) = r \frac{\Delta P}{L}$$

INTEGRATING:

$$r\beta = \frac{\Delta P}{L} \frac{r^2}{2} + C_1$$

$$\beta = \frac{\Delta P}{L} \frac{r}{2} + C_1/r$$

FOR LAMINAR FLOW, NEWTONIAN

$$\beta = \mu \frac{dw}{dr}$$

$$\text{so } \mu \frac{dw}{dr} = \frac{\Delta P}{L} \frac{r}{2} + C_1/r$$

$$dw = \frac{\Delta P}{2\mu L} r dr + \frac{C_1}{\mu} \frac{dr}{r}$$

INTEGRATING:

$$w = \frac{\Delta P}{4\mu L} r^2 + \frac{C_1}{\mu} \ln r + C_2$$

BOUNDARY CONDITIONS:

$$w(r=R) = 0$$

$$w(r=kR) = 0 \quad k < 1$$

8.5 - (CONTINUED) -

CONSIDERABLE ALGEBRA YIELDS

$$C_1 = -\frac{\Delta P}{4\mu L} \frac{R^2(1-k^2)}{\ln^{1/k}}$$

$$C_2 = -\frac{\Delta P}{4\mu L} R^2 \left[ 1 - (1-k^2) \frac{\ln R}{\ln^{1/k}} \right]$$

WITH SUBSTITUTION & SIMPLIFICATION:

$$w = -\frac{\Delta P R^2}{4\mu L} \left[ 1 - \frac{r^2}{R^2} - \frac{1-k^2}{\ln^{1/k}} \frac{\ln R}{r} \right]$$

8.6 THIS IS SAME CONFIGURATION AS SHOWN IN PROB 8.5

$$\therefore \frac{d}{dr}(r\beta) - \frac{df}{dx} r = 0$$

$$\text{INTEGRATING: } Tr_x - \frac{df}{dx} \frac{r^2}{2} = \frac{C_1}{r}$$

FOR LAMINAR FLOW, NEWTONIAN FLUID:

$$Tr_x = \mu \frac{dw}{dr}$$

$$\frac{dw}{dr} - \frac{r}{2\mu} \frac{df}{dx} = \frac{C_1}{\mu r}$$

INTEGRATING:

$$w_x - \frac{1}{4\mu} \frac{df}{dx} r^2 = \frac{C_1}{\mu} \ln r + C_2$$

BOUNDARY CONDITIONS:

$$w(r=R/2) = 0$$

$$w(r=d/2) = v$$

MORE ALGEBRA!

$$C_1 = -\frac{\mu}{\ln d/R} \left[ v + \frac{1}{16\mu} \frac{df}{dx} (R^2 - d^2) \right]$$

## 8.6 - CONTINUED

$$C_2 = -\frac{1}{4\mu} \frac{dF}{dx} \frac{D^2}{4} - \frac{C_1}{\mu} \ln \frac{D}{2}$$

DRAG FORCE PER UNIT LENGTH

$$F = \bar{J}A = \bar{J}(\pi d)(1)$$

$$= \mu \frac{dV}{dr} \Big|_{r=d/2} (\pi d)$$

GIVEN:

$$\begin{aligned} F &= \pi d \mu \left[ \frac{C_1}{\mu r} + \frac{r}{2\mu} \frac{dF}{dx} \right]_{r=\frac{D}{2}} \\ &= \pi d \mu \left[ \frac{2C_1}{\mu d} + \frac{d}{4\mu} \frac{dF}{dx} \right] \end{aligned}$$

FOR THE CASE WITH  $\frac{dF}{dx} = 0$ 

$$F = -\frac{2\pi \mu V}{\ln D/d}$$

8.7

IN  $\theta$ -DIRECTION:

$$\begin{aligned} \sum F_\theta &= -r A \theta A Z \bar{J}_{r\theta} \Big|_r + r A \theta A Z \bar{J}_{r\theta} \Big|_r \\ &\quad + \underbrace{\Delta r A \theta Z \bar{J}_{r\theta} \Delta \theta}_{\theta \text{ component of force on } (\theta) \text{ face}} \end{aligned}$$

DIVIDE BY  $r A \theta A Z$  & TAKE LIMIT AS  $\Delta r \rightarrow 0$ :

$$\frac{d}{dr} (r \bar{J}_{r\theta}) + \bar{J}_{r\theta} = 0$$

$$r \frac{d\bar{J}}{dr} + 2\bar{J} = 0$$

## 8.7 - CONTINUED

$$\frac{d\bar{J}}{r} + 2 \frac{d\bar{J}}{r} = 0$$

$$\ln r + 2 \ln r = \ln (\text{constant})$$

$$\underline{r^2 \bar{J}} = \underline{\text{CONSTANT}}$$

$$\bar{J} = \mu r \frac{d}{dr} \left( \frac{V_\theta}{r} \right)$$

$$r^2 \bar{J} = \mu r^3 \frac{d}{dr} \left( \frac{V_\theta}{r} \right) = \text{constant} \quad (c)$$

$$d \left( \frac{V_\theta}{r} \right) = \frac{C_1}{\mu} \frac{dr}{r^3}$$

$$\frac{V_\theta}{r} = C_1 \left( -\frac{1}{r^2} \right) + C_2$$

$$V_\theta = -\frac{C_1}{r} + r C_2$$

Boundary Conditions:

$$V_\theta(R) = 0 \quad \Rightarrow \quad 0 = \frac{C_1}{R} + R C_2$$

$$V_\theta(KR) = V \quad \Rightarrow \quad V = -\frac{C_1}{KR} + KRC_2$$

ALGEBRA

$$V_\theta = \frac{VR}{K-1/K} \left( \frac{r}{R^2} - \frac{1}{r} \right)$$

IF PROFILE IS LINEAR:

$$V_\theta = ar + b$$

{}

$$V_\theta = \frac{V}{K-1} \left( \frac{r}{R} - 1 \right)$$

### 8.7 CONTINUED

$$\text{PERCENT ERROR} = \frac{\Delta V}{V}$$

$$\Delta V = V_{\text{ACTUAL}} - V_{\text{LINEAR}}$$

$$= \frac{VRk}{K^2-1} \left( \frac{r}{R^2} - \frac{1}{r} \right) - \frac{V}{R(K-1)} (r-R)$$

$$\frac{d}{dr} \Delta V = \frac{VRK}{K^2-1} \left( \frac{1}{R^2} + \frac{1}{r^2} \right)$$

$$- \frac{V}{R(K-1)}$$

$$= \frac{V}{K-1} \left[ \frac{RK}{K+1} \left( \frac{1}{R^2} + \frac{1}{r^2} \right) - \frac{1}{R} \right]$$

$$= 0$$

$$\Delta V_{\text{MAX}} \text{ occurs at } \frac{r}{R} = \sqrt{K}$$

$$\left| \frac{\Delta V}{V} \right|_{\text{MAX}} = 1 - \frac{(\sqrt{K}-1)(K+1)\sqrt{K}}{K(K-1)}$$

$$= 0.01$$

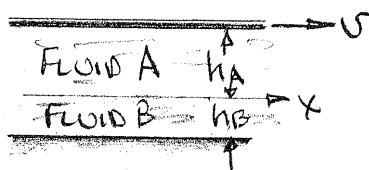
RESULTING IN

$$\frac{(\sqrt{K}-1)(K+1)}{\sqrt{K}(K-1)} = 0.99$$

}

$$\underline{\underline{K = 0.96}}$$

### 8.8 FOR FLOW BETWEEN 2 HORIZONTAL PLATES -



GOVERNING D.E. -

$$\frac{d}{dy} (T_{yx}) - \frac{dp}{dx} = 0$$

LAMINAR, STEADY NEWTONIAN

$$T_{yx} = \mu \frac{dv_x}{dy}$$

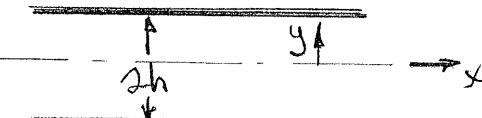
B.C. - AT INTERFACE ( $y=0$ )

$$(1) \quad V_{x,A} = V_{x,B}$$

$$(2) \quad T_{yx,A} = T_{yx,B}$$

$$(3) \quad V_x(-h_B) = V_x(h_A) = 0$$

### 8.9



FULLY DEVELOPED, STEADY, LAMINAR FLOW; NEWTONIAN FLUID -

$$\frac{d}{dy} T_{yx} - \frac{dp}{dx} = 0$$

$$T_{yx} = \mu \frac{dv_x}{dy}$$

B.C.  $V_x = 0$  for  $y = \pm h$

$$\mu e \frac{d^2 V}{dy^2} = \frac{dp}{dx}$$

$$\frac{dV}{dy} = \frac{1}{\mu e} \frac{dp}{dx} y + C_1$$

$$V = \frac{1}{2\mu e} \frac{dp}{dx} y^2 + C_1 y + C_2$$

## 8.9 - CONTINUED

Applying Boundary Conditions

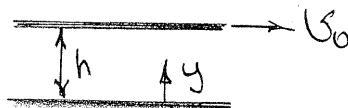
$$C_1 = 0$$

$$C_2 = -\frac{1}{2\mu} \frac{df}{dx} \frac{h^2}{2}$$

GIVEN:

$$U_x = \frac{1}{2\mu} \frac{df}{dx} \left( y^2 - \frac{h^2}{4} \right)$$

8.10



Boundary R.E. is

$$\frac{d}{dy} T_{yx} - \frac{df}{dx} = 0$$

INTEGRATING:  $T_{yx} - \frac{df}{dx} y = C_1$

LAMINAR FLOW, NEWTONIAN FLUID:

$$T_{yx} = \mu \frac{du}{dy}$$

$$\mu \frac{du}{dy} - \frac{df}{dx} y = C_1$$

FOR  $T_{yx}(0) = 0$   $C_1 = 0$

$$\frac{du}{dy} - \frac{1}{\mu} \frac{df}{dx} y = 0$$

$$U_x - \frac{1}{\mu} \frac{df}{dx} \frac{y^2}{2} = C_2$$

$$U_x @ y=h = U_o$$

$$C_2 = U_o - \frac{1}{\mu} \frac{df}{dx} \frac{h^2}{2}$$

## 8.10 - CONTINUED -

$$\text{Also } U_x @ y=0 = 0 \therefore C_2 = 0$$

$$\text{GIVEN: } \frac{df}{dx} = \frac{2\mu V_o}{h^2}$$

8.11 For Horizontal Pipe flow:

DEVELOPMENT IN SECTION 8.1

RESULTS IN

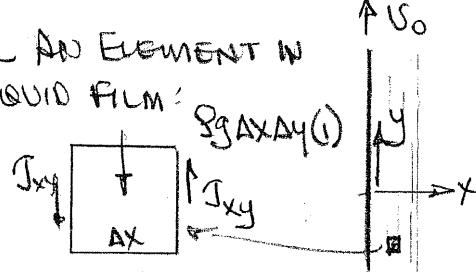
$$U_x = \left( \frac{df}{dx} \right) \frac{r^2}{4\mu} + C_2$$

FOR  $\mu = 0$   $\frac{df}{dx} \frac{r^2}{4}$  MUST  
= 0 FOR ALL  $r$

i.  $U_x = \text{CONSTANT} = V$

8.12

for an element in  
LIQUID FILM:



$$T_{xy}\Delta y|_{x+\Delta x} - T_{xy}\Delta y|_x - sgA_{xy}\Delta y = 0$$

$$\frac{T_{xy}|_{x+\Delta x} - T_{xy}|_x}{\Delta x} - sg = 0$$

IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{d}{dx} T_{xy} - sg = 0$$

8.12 - CONTINUOUS

$$\tau_{xy} = \mu \frac{dy}{dx}$$

$$\frac{dU_y^2}{dx^2} - \frac{8g}{\mu} = 0$$

$$\frac{dU}{dx} - \frac{8g}{\mu} x = C_1$$

$$U - \frac{8g}{2\mu} x^2 = C_1 x + C_2$$

BOUNDARY CONDITIONS:

$$U(0) = U_0 \quad \dot{U}(h) = 0$$

$$C_1 = -\frac{8g}{\mu} h \quad C_2 = U_0$$

GIVING

$$U = U_0 - \frac{8g}{2\mu} (2hx - x^2)$$

$$= U_0 - \frac{8gh^2}{2\mu} \left[ 2 \frac{x}{h} - \left( \frac{x}{h} \right)^2 \right]$$

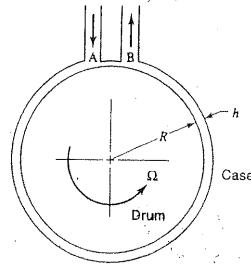
$$\dot{V} = \int_0^h U dy$$

$$= \int_0^h U_0 - \frac{8gh^2}{2\mu} \left[ 2 \frac{x}{h} - \left( \frac{x}{h} \right)^2 \right] dy$$

$$= U_0 h - \frac{8g}{2\mu} \left( h^3 - h^3/3 \right)$$

$$= U_0 h - \frac{8gh^3}{3\mu}$$

8.13



TREAT FLUID LAYER AS A THIN LINEAR LAYER:

$$\frac{\tau_{xy}}{\mu} = \frac{dy}{dx}$$

IN THE USUAL WAY:

$$\frac{dU}{dy} - \frac{df}{dx} = 0$$

TREAT  $\frac{df}{dx}$  CONSTANT  $\sim \frac{df}{dx} = \frac{\Delta P}{L}$

$$\therefore \tau_{xy} = \mu \frac{dy}{dx}$$

$$\text{GIVING } U = \frac{\Delta P}{L} \frac{y^2}{2\mu} + C_1 y + C_2$$

BOUNDARY CONDITIONS:

$$U(0) = R\Omega$$

$$U(h) = 0$$

$$\text{GIVING } R\Omega = C_2$$

$$0 = \frac{\Delta P}{L} \frac{h^2}{2\mu} + C_1 h + R\Omega$$

$$C_1 = -\frac{\Delta P}{L} \frac{h}{2\mu} - \frac{R\Omega}{h}$$

$$\therefore U = R\Omega \left( 1 - \frac{y}{h} \right) - \frac{\Delta P}{2\mu L} \left[ \left( \frac{y}{h} \right)^2 - \left( \frac{y}{h} \right) \right]$$

$$\text{Flow Rate} = \int_0^h U dy$$

## 8.13 - CONTINUOUS

$$\dot{V} = \int_0^h \text{Expression for } V^2 dy \\ = \frac{R \Omega h}{2} - \frac{\Delta P h^3}{12 \mu L}$$

GIVING:  $\Delta P = \frac{12 \mu L}{h^3} \left[ \frac{R \Omega h}{2} - \dot{V} \right]$

$$\begin{aligned} \text{EFFICIENCY} &= \frac{\text{POWER OUT}}{\text{POWER IN}} \\ &= \frac{\dot{V} R \Omega}{\dot{V} + \Delta P / \rho} \end{aligned}$$

$\dot{V}$  EVALUATED AT  $R(y=0)$

$$\dot{V} = \mu R \Omega \frac{h}{h} + \Delta P \frac{h}{2}$$

AFTER DOING THE ALGEBRA:

$$\eta = \frac{12 \dot{V}}{R \Omega h} \frac{R \Omega h / 2 - \dot{V}}{4 R \Omega h - 6 \dot{V}}$$

8.14



FLUID ENTERS AT  $y=0$   $\dot{V}$   
FLOWS EQUALLY IN  $+x$  &  $-x$   
DIRECTIONS EXITING AT  $x=L/2$   
WHERE  $P=P_{ATM}$ .

WORKING WITH THE R.H. SIDE  
(IN  $+x$  DIRECTION)

THE APPLICABLE DE. IS

$$\frac{dP}{dy} = \frac{dp}{dx}$$

## 8.14 - CONTINUOUS

$$\frac{dP}{dy} \text{ AS USUAL} = \frac{dP}{dy} = \mu \frac{dv}{dy}$$

$$\text{GIVEN: } \frac{d^2 V}{dy^2} = \frac{1}{\mu} \frac{dp}{dx}$$

$$\text{INTEGRATING: } \frac{dv}{dy} = \frac{1}{\mu} \frac{dp}{dx} y + C_1$$

$$\text{BOUNDARY COND: } \frac{dv}{dy}(0) = 0 \therefore C_1 = 0$$

$$\text{AGAIN } V = \frac{1}{2} \frac{dp}{dx} y^2 + C_2$$

$$\text{BOUNDARY LND: } V\left(\frac{L}{2}\right) = 0$$

$$\text{SO } C_2 = \left(-\frac{dp}{dx}\right) \frac{L^2}{8\mu}$$

VELOCITY EXPRESSION IS:

$$V = \frac{1}{2} \mu \left(-\frac{dp}{dx}\right) \left(\frac{L^2}{4} - y^2\right)$$

$$\dot{V} = 2 \int_0^{L/2} V dy \Big|_{L/2}$$

$$= \frac{1}{4} \left(-\frac{dp}{dx}\right) \int_0^{L/2} \left(\frac{L^2}{4} - y^2\right) dy$$

$$= \frac{1}{4} \left(-\frac{dp}{dx}\right) \frac{L^3}{12}$$

SO THE EXPRESSION FOR  $-\frac{dp}{dx}$  IS:

$$-\frac{dp}{dx} = \frac{12 \mu \dot{V}}{L^3}$$

$$\frac{1}{2} \int_{P_0}^{P_{ATM}} dp = \frac{12 \mu \dot{V}}{L^3} \int_0^{L/2} dx$$

$$P_0 - P_{ATM} = \frac{6 \mu \dot{V} L}{L^3}$$

$\frac{1}{2}$  FOR THE PLATE OF TOTAL LENGTH,  $L$ ,

$$F_y = (P_0 - P_{ATM}) 2L = \frac{12 \mu \dot{V} L^2}{L^3}$$

8.15 LIQUID FLOWING DOWN  
THE OUTSIDE OF A CYLINDER:

GOVERNING D.E. IS

$$\frac{1}{r} \frac{d}{dr}(r\dot{\theta}) + Sg = 0$$

$$\frac{1}{r}, \text{ AS USUAL } \dot{\theta} = \mu \frac{dw}{dr}$$

$$\mu \frac{d}{dr}\left(r \frac{dw}{dr}\right) + Sg = 0$$

$$r \frac{dw}{dr} + \frac{Sg}{\mu} \frac{r^2}{2} = C_1$$

$$\text{B.C. } \frac{dw}{dr} = 0 @ r = R+h$$

$$\frac{rdw}{dr} = \frac{Sg}{2\mu} \left[ (R+h)^2 - r^2 \right]$$

AND AGAIN:

$$v = \frac{Sg}{2\mu} \left[ (R+h)^2 \ln r - \frac{r^2}{2} \right] + C_2$$

$$\text{B.C. } v(R) = 0$$

GIVING:

$$\begin{aligned} S &= \frac{Sg}{2\mu} (R+h)^2 \ln \frac{r}{R} \\ &\quad + \frac{SgR^2}{4\mu} \left( 1 - \frac{r^2}{R^2} \right) \end{aligned}$$

8.16 FOR RESULT OF PROB. 8.15

$v_{\max}$  occurs WHERE  $\frac{dv}{dr} = 0$

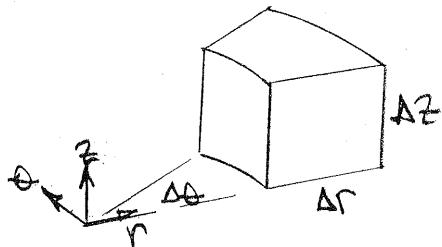
which is at  $r = R+h$

"

$$v_{\max} = \frac{SgR^2}{4\mu} \left[ 2\left(1 + \frac{h}{R}\right) \ln\left(1 + \frac{h}{R}\right) - \frac{h^2}{R^2} - \frac{2h}{R} \right]$$

# CHAPTER 9

9.1



$$\underset{\text{C.S.}}{\iint} \vec{v} \cdot \hat{n} dA + \frac{\partial}{\partial t} \underset{\text{C.V.}}{\iiint} \rho dV = 0$$

$$\underset{\text{C.S.}}{\iint} \vec{v} \cdot \hat{n} dA = \rho v_r \Delta x \Delta y \Big|_{r+\Delta r}$$

$$-\rho v_r \Delta z \Delta A \Big|_r + \rho v_\theta \Delta r \Delta z \Big|_{\theta+\Delta\theta}$$

$$-\rho v_\theta \Delta r \Delta z \Big|_\theta + \rho v_z \Delta \theta \Delta r \Big|_{z+\Delta z}$$

$$-\rho v_z \Delta r \Delta \theta \Big|_z$$

$$\frac{\partial}{\partial t} \underset{\text{C.V.}}{\iiint} \rho dV = \frac{\partial}{\partial t} \rho \Delta x \Delta y \Delta z$$

SUBSTITUTING INTO C.V. EQUATION &  
EVALUATING IN LIMIT AS  $\Delta r \rightarrow 0$

$$\cancel{1} \frac{\partial}{\partial r} (r v_r) + \cancel{1} \frac{\partial v_\theta}{\partial \theta} + \cancel{\frac{\partial v_z}{\partial z}} = 0$$

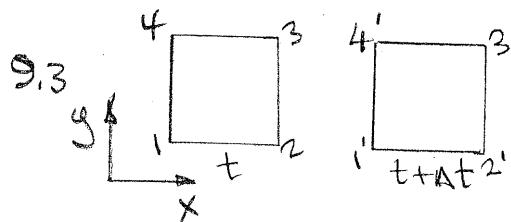
9.2  $\vec{v} = v_x \hat{e}_x + v_y \hat{e}_y + v_z \hat{e}_z$

$$\nabla = \frac{\partial}{\partial x} \hat{e}_x + \frac{\partial}{\partial y} \hat{e}_y + \frac{\partial}{\partial z} \hat{e}_z$$

$$\vec{v} \cdot \nabla = v_x \frac{\partial}{\partial x} (\hat{e}_x \cdot \hat{e}_x) + v_y \frac{\partial}{\partial y} (\hat{e}_y \cdot \hat{e}_y) + v_z \frac{\partial}{\partial z} (\hat{e}_z \cdot \hat{e}_z)$$

NOTE:  $\hat{e}_i \cdot \hat{e}_j = 1$  for  $j=i$   
 $= 0$  for  $j \neq i$

$$\therefore \vec{v} \cdot \nabla = v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$$



FOR 2-DIMENSIONAL FLOW:

$$\text{VOLUME CHANGE} = (1'2')(3'2') - (12)(32)$$

$$12 = \Delta x \quad 32 = \Delta y$$

$$12' = \Delta x + [v_x(x+\Delta x, y) - v_x(x, y)] \Delta t$$

$$3'2' = \Delta y + [v_y(x+\Delta x, y+\Delta y) - v_y(x+\Delta x, y)] \Delta t$$

$$(12)(32) = \Delta x \Delta y$$

$$(1'2')(3'2') = \Delta x \Delta y + [v_y(x+\Delta x, y+\Delta y) - v_y(x+\Delta x, y)] \Delta x \Delta t$$

$$+ [v_x(x+\Delta x, y) - v_x(x, y)] \Delta y \Delta t$$

$$+ [\quad] \Delta t^2$$

DIVIDING BY  $\Delta x \Delta y \Delta t$

EVALUATING IN LIMIT AS  $\Delta x, \Delta y, \Delta t \rightarrow 0$

$$\text{VOLUME CHANGE} = \frac{\partial v_y}{\partial y} + \frac{\partial v_x}{\partial x}$$

$$= \vec{v} \cdot \nabla$$

But, from CONTINUITY  $-\nabla \cdot \vec{v} = 0$

9.4  $\vec{v} = v_r \hat{e}_r + v_\theta \hat{e}_\theta$

$$d\vec{v} = \frac{\partial \vec{v}}{\partial r} dr + \frac{\partial \vec{v}}{\partial \theta} d\theta + \frac{\partial \vec{v}}{\partial t} dt$$

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial r} \frac{dr}{dt} + \frac{\partial \vec{v}}{\partial \theta} \frac{d\theta}{dt} + \frac{\partial \vec{v}}{\partial t}$$

### 9.4 CONTINUED -

$$\frac{\partial \vec{v}}{\partial r} = \frac{\partial v_r}{\partial r} \hat{e}_r + \frac{\partial v_\theta}{\partial \theta} \hat{e}_\theta + v_r \frac{\partial \hat{e}_r}{\partial r} + v_\theta \frac{\partial \hat{e}_\theta}{\partial r}$$

$$\frac{\partial \vec{v}}{\partial \theta} = \frac{\partial v_r}{\partial \theta} \hat{e}_r + \frac{\partial v_\theta}{\partial \theta} \hat{e}_\theta + v_r \frac{\partial \hat{e}_r}{\partial \theta} + v_\theta \frac{\partial \hat{e}_\theta}{\partial \theta}$$

$$\hat{e}_r = \hat{e}_x \cos \theta + \hat{e}_y \sin \theta$$

$$\hat{e}_\theta = -\hat{e}_x \sin \theta + \hat{e}_y \cos \theta$$

$$\frac{\partial \hat{e}_r}{\partial r} = \frac{\partial \hat{e}_r}{\partial \theta} \frac{\partial \theta}{\partial r} = \hat{e}_\theta \frac{\partial \theta}{\partial r} = 0$$

$$\frac{\partial \hat{e}_r}{\partial \theta} = -\hat{e}_x \sin \theta + \hat{e}_y \cos \theta = \hat{e}_\theta$$

IN SIMILAR FASHION

$$\frac{\partial \hat{e}_\theta}{\partial r} = 0 \quad \frac{\partial \hat{e}_\theta}{\partial \theta} = -\hat{e}_r$$

GIVEN:

$$\frac{\partial \vec{v}}{\partial r} = \frac{\partial v_r}{\partial r} \hat{e}_r + \frac{\partial v_\theta}{\partial \theta} \hat{e}_\theta$$

$$\begin{aligned} \frac{\partial \vec{v}}{\partial \theta} &= \left( \frac{\partial v_r}{\partial \theta} - v_\theta \right) \hat{e}_r \\ &\quad + \left( \frac{\partial v_\theta}{\partial \theta} + v_r \right) \hat{e}_\theta \end{aligned}$$

FOR  $\frac{d\vec{v}}{dt}$  TO BE  $\frac{D\vec{v}}{Dt}$   $\frac{dr}{dt} = v_r$

$$\frac{1}{r} \frac{d\theta}{dt} = \omega = \frac{v_\theta}{r}$$

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \left( v_r \frac{\partial v_r}{\partial r} + v_\theta \frac{\partial v_r}{\partial \theta} - \frac{v_r^2}{r} \right) \hat{e}_r$$

$$+ \left( v_r \frac{\partial v_\theta}{\partial r} + v_\theta \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} \right) \hat{e}_\theta$$

### 9.5 NAVIER-STOKES EQU - INCOMPRESSIBLE FORM:

$$\frac{D\vec{v}}{Dt} = \vec{g} - \frac{\nabla P}{\rho} + \nu \nabla^2 \vec{v}$$

a) FOR  $\vec{v}$  SMALL - ALL TERMS INVOLVING  $\vec{v}$  ( $\sim \frac{\partial \vec{v}}{\partial t} \& \nu \nabla^2 \vec{v}$ ) ARE SMALL RELATIVE TO OTHER TERMS.

b) FOR  $\nu$  SMALL  $\&$   $\vec{v}$  LARGE THE PRODUCT  $\nu \nabla^2 \vec{v}$  CANNOT BE CONSIDERED SMALL RELATIVE TO OTHER TERMS



INCOMPRESSIBLE N.S. EQU. IN X DIRECTION

$$\begin{aligned} \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \\ = g_x - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 v_x \end{aligned}$$

$$\Rightarrow \nabla^2 v_x = \frac{\partial^2 v_x}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial x}$$

$$\frac{\partial v_x}{\partial y} = \frac{1}{\mu} \frac{\partial p}{\partial x} y + C_1$$

$$v_x = \frac{1}{\mu} \frac{\partial p}{\partial x} \frac{y^2}{2} + C_1 y + C_2$$

B.C.  $v_x = 0$  @  $y = \pm L$

$$C_1 = 0 \quad C_2 = -\frac{1}{2\mu} \frac{\partial p}{\partial x} L^2$$

$$v_x = \frac{1}{2\mu} \frac{\partial p}{\partial x} (y^2 - L^2)$$

$$9.7 \quad \vec{v} = \frac{\omega R^2}{r} \hat{e}_\theta$$

$$\nabla \cdot \vec{v} = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \vec{v} = \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{\omega R^2}{r} \right)$$

$$= \frac{\omega R^2}{r} \frac{\partial}{\partial \theta} \left( \frac{1}{r} \right) = 0$$

AND CONTINUITY IS SATISFIED

$$9.8 \quad \frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \frac{\partial \vec{v}}{\partial \vec{r}} = -v_z \frac{\partial \vec{v}}{\partial y}$$

$$= -v_z \frac{\partial}{\partial y} \left[ \frac{-y/B}{B} \right] = \frac{8v_z y/B}{B}$$

$$\text{AT } y = 100,000 \text{ FT } \quad V = 20,000 \text{ FT/S}$$

$$\frac{D\vec{v}}{Dt} = \frac{20,000}{22,000} \text{ } S_0 \text{ } e^{-4.545}$$

$$= 0.0096 \text{ } S_0 \text{ } s^{-1}$$

$$9.9 \quad \nabla P = \rho \left( \vec{g} - \frac{D\vec{v}}{Dt} \right)$$

$$\vec{v} = 400 \left[ \left( \frac{y}{L} \right)^2 \hat{e}_x + \left( \frac{x}{L} \right)^2 \hat{e}_y \right]$$

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y}$$

$$= 400 \left( \frac{y}{L} \right)^2 800 \frac{x}{L^2} \hat{e}_y$$

$$+ 400 \left( \frac{x}{L} \right)^2 800 \frac{y}{L^2} \hat{e}_x$$

$$= \frac{32 \times 10^4}{L^4} \left[ x^2 y \hat{e}_x + x y^2 \hat{e}_y \right]$$

9.9 (CONTINUED) -

EVALUATED AT  $(L, 2L)$  WE GET

$$\nabla P = - \frac{128 \times 10^4}{L} \hat{e}_x$$

$$- \left[ 2g + \frac{256 \times 10^4}{L} \right] \hat{e}_y \quad \frac{LB^2/P^2}{ft}$$

9.10 IN X-DIRECTION:

$$\frac{Dv_x}{Dt} = \rho g_x - \frac{\partial P}{\partial x} - \frac{\partial}{\partial x} \left( \frac{2}{3} \mu \nabla \cdot \vec{v} \right)$$

$$+ \nabla \cdot \left( \mu \frac{\partial \vec{v}}{\partial x} \right) + \nabla \cdot \left( \mu \nabla v_x \right)$$

$$\rho \left[ \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right]$$

$$= \rho g_x - \frac{\partial P}{\partial x} - \frac{\partial}{\partial x} \left[ \frac{2}{3} \mu \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial y} + \frac{\partial v_x}{\partial z} \right) \right]$$

$$+ \frac{\partial}{\partial x} \left( \mu \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v_x}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial v_x}{\partial z} \right)$$

$$+ \frac{\partial}{\partial x} \left( \mu \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v_x}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial v_x}{\partial z} \right)$$

SIMILARLY IN  $y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z}$  -

A TOTAL OF 45 TERMS!

9.11 FOR  $\vec{v} = \vec{v}_0 + \vec{v}_r$

$0$  - OF COORDINATE ORIGIN

$r$  - RELATIVE TO COORD. ORIGIN

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}_0}{\partial t} + \frac{D \cdot \vec{v}_r}{Dt} = \vec{a}$$

1. N.S. EQU REDUCES TO

$$\rho \vec{a} = \rho \vec{g} - \nabla P$$

$$\therefore \nabla P = \rho (\vec{g} - \vec{a})$$

9.12 Given THAT

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0$$

a) for  $v_\theta = 0$   $\frac{\partial}{\partial r} (r v_r) = 0$

$$\therefore r v_r(\theta) = f(\theta)$$

$$\text{or } v_r = \underline{f(\theta)/r}$$

b) for  $v_r = 0$   $\frac{\partial v_\theta}{\partial \theta} = 0$

$$v_\theta = f(r)$$

9.13 N.S. for Incomp, Lam. flow

$$\frac{D\vec{v}}{Dt} = \vec{g} - \frac{\nabla p}{\rho} + \mu \nabla^2 \vec{v}$$

For  $\vec{g}$  NEGIGIBLE

a) VECTOR PROPERTIES  $\sim \vec{v} \nparallel \nabla p$   
ARE INDEPENDENT BY THEMSELVES  
BUT IN SAME RELATIONSHIP  
MUST LIE IN SAME PLANE.

b) IF VISCOSITY FORCES ARE NEGIGIBLE

$$\frac{D\vec{v}}{Dt} = - \frac{\nabla p}{\rho}$$

$\frac{D\vec{v}}{Dt}$  IS DETERMINED BY  $-\frac{\nabla p}{\rho}$

$\frac{1}{\rho}$  IS POSITIVE IN DIRECTION  
OF DECREASING PRESSURE.

c) IN SIMILAR FASHION, ANY  
FLUID - EITHER MOVING OR  
STATIC - WILL MOVE OR  
TEND TO MOVE IN  
DIRECTION OF DECREASING  $p$ .

9.14 For 1-D STEADY flow:

$$v_x = v_x(x) \quad v_y = v_z = 0$$

$$\rho v_x \frac{\partial v_x}{\partial x} = - \frac{dp}{dx} + \mu \left[ -\frac{2}{3} \left( \mu \frac{\partial v_x}{\partial x} \right) + \mu \frac{\partial v_x}{\partial x} \right]$$

$$\rho v_x \frac{\partial v_x}{\partial x} = - \frac{dp}{dx} + \frac{4}{3} \left( \mu \frac{\partial v_x}{\partial x} \right)$$

$$\underline{\frac{\partial}{\partial x} (\rho v_x) = 0}$$

9.15 CONTINUITY:  $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) = 0$

MOMENTUM:  $\rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} \right) = - \frac{\partial p}{\partial x}$

9.16 TAKING  $z$  AS POSITIVE DOWN

WITH  $v_r = v_z = 0 \quad \frac{1}{\rho} v_x = f(r)$

EQN. E.6. YIELDS

$z$  direction

$$\begin{aligned} \rho \left( \frac{\partial v_x}{\partial t} + v_r \frac{\partial v_x}{\partial r} + v_\theta \frac{\partial v_x}{\partial \theta} + v_z \frac{\partial v_x}{\partial z} \right) \\ = - \frac{\partial p}{\partial z} + \rho g_z + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_x}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_x}{\partial \theta^2} + \frac{\partial^2 v_x}{\partial z^2} \right] \end{aligned}$$

$\frac{1}{\rho}$  SINCE  $g_z = -g$

$$\frac{g}{\rho} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_x}{\partial r} \right)$$

PROCEED AS WAS DONE IN  
SOLNS TO PROBS 8.17 & 8.18

9.17 FOR INCOMPRESSIBLE,  
STEADY FLOW, WITH  $V_\theta = V_z = 0$   
EQUATION (E-4) HAS THE FORM

$r$  direction

$$\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_r v_\theta}{r} + v_z \frac{\partial v_r}{\partial z} \right) \quad \text{DUE TO CONTINUITY}$$

$$= -\frac{\partial P}{\partial r} + \rho g_r + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right]$$

↓ BECOMES

$$\frac{\partial}{\partial r} \left( P + \frac{\rho v_r^2}{2} \right) = \rho g_r$$

9.18 GOVERNING EQUATIONS ARE

$r$  direction

$$\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right)$$

$$= -\frac{\partial P}{\partial r} + \rho g_r + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right]$$

$\theta$  direction

$$\rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right)$$

$$= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right]$$

$z$  direction

$$\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right)$$

$$= -\frac{\partial P}{\partial z} + \rho g_z + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right]$$

WHEN  $V_\theta = f(r)$   $\downarrow$   $V_r = V_z = 0$   
THE ONLY NON-ZERO TERM ON  
THE LEFT-HAND SIDE OF ALL  
COMPONENT EQUATIONS,  
IS  $-V_\theta^2/r$

$$\therefore \frac{D \vec{V}}{Dt} = \frac{d \vec{V}}{dt} = - \frac{V_\theta^2}{r} \hat{e}_r$$

(Q.E.D.,

9.19 EQUATION (E-5) IS SIMPLIFIED  
FOR THIS CASE AS

$\theta$  direction

$$\rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + v_r v_\theta + v_z \frac{\partial v_\theta}{\partial z} \right)$$

$$= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right]$$

↓ IN THE ABSENCE OF GRAVITY  
WE HAVE

$$\frac{\partial v_\theta}{\partial t} = \nu \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right]$$

9.20 - FROM PROB. 9.19 ↓  
STEADY FLOW

$$\frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right] = 0$$

$$\text{GIVEN } \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) = C_1$$

INTEGRATING AGAIN

$$r v_\theta = C_1 \ln r + C_2$$

$$\text{B.C. } V_\theta(R_1) = R_1 \Omega_1$$

$$V_\theta(R_2) = R_2 \Omega_2$$

$$V_\theta = \frac{1}{r} \left[ R_1^2 \Omega_1 + \frac{(R_2^2 \Omega_2 - R_1^2 \Omega_1) \ln r / R_1}{\ln R_2 / R_1} \right]$$

## CHAPTER 10

$$10.1 \quad \nabla \times \vec{v} = \left( \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} \right) \times (v_r \vec{e}_r + v_\theta \vec{e}_\theta)$$

$$= \left( \vec{e}_r \times \vec{e}_r \right) \frac{\partial v_r}{\partial r} + v_r \vec{e}_r \times \frac{\partial \vec{e}_r}{\partial r}$$

$$+ \left( \vec{e}_r \times \vec{e}_\theta \right) \frac{\partial v_\theta}{\partial r} + v_\theta \vec{e}_r \times \frac{\partial \vec{e}_\theta}{\partial r}$$

$$+ \left( \vec{e}_r \times \vec{e}_r \right) \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_r}{r} \vec{e}_r \times \frac{\partial \vec{e}_r}{\partial \theta}$$

$$+ \left( \vec{e}_r \times \vec{e}_\theta \right) \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta}{r} \vec{e}_r \times \frac{\partial \vec{e}_\theta}{\partial \theta}$$

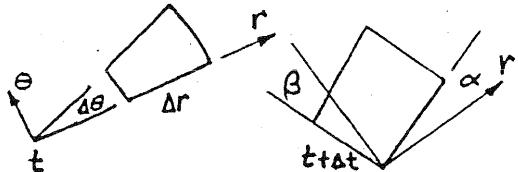
FOR REFERENCES SEE PROB 9.4

$$\frac{\partial \vec{e}_r}{\partial r} = 0, \frac{\partial \vec{e}_\theta}{\partial r} = 0, \frac{\partial \vec{e}_r}{\partial \theta} = \vec{e}_\theta \frac{\partial \vec{e}_\theta}{\partial \theta} = -\vec{e}_r$$

ALL REMAINING (NON-ZERO) TERMS GIVE:

$$\nabla \times \vec{v} = \left[ \frac{\partial v_\theta}{\partial r} + \frac{1}{r} \left( v_\theta - \frac{\partial v_r}{\partial \theta} \right) \right] \vec{e}_z$$

10.2



$$\omega_z = \frac{d}{dt} \left( \frac{\alpha + \beta}{2} \right)$$

$$= \lim_{\Delta t \rightarrow 0^2} \frac{1}{\Delta t} \left\{ \tan^{-1} \left( \frac{rv_\theta - r v_{\theta,r}}{r \Delta r} \right) \Delta r \right.$$

$$\left. + \frac{\tan^{-1} (v_r \alpha + v_\theta - v_{\theta,r}) \Delta t}{r \Delta \theta} \right]$$

10.2 (CONTINUED)

IN THE LIMIT: { NOTE THAT  $\tan z \approx z$ }

$$\omega_z = \lim_{\begin{cases} \Delta r \\ \Delta \theta \\ \Delta \theta \end{cases} \rightarrow 0} \left[ \frac{rv_\theta |_{r+\Delta r} - rv_{\theta,r}|_r}{r \Delta r} \right. \\ \left. - \frac{v_r |_{\theta+\Delta \theta} - v_{\theta,r}|_\theta}{r \Delta \theta} \right]$$

$$= \frac{1}{r} \frac{\partial}{\partial r} (rv_\theta) - \frac{1}{r} \frac{\partial v_r}{\partial \theta}$$

$$= \frac{\partial v_\theta}{\partial r} + \frac{1}{r} \left( v_\theta - \frac{\partial v_r}{\partial \theta} \right)$$

Q.E.D.

$$10.3 \quad d\psi = -v_y dx + v_x dy$$

$$= (-v_\theta \sin \alpha) dx + (v_\theta \cos \alpha) dy$$

$$\psi = -v_\theta (\sin \alpha) x + v_\theta (\cos \alpha) y + \psi_0$$

$$10.4 \quad \nabla \cdot \vec{v} = \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0$$

$$\text{Let } rv_r = \frac{\partial \psi(r, \theta)}{\partial \theta}$$

$$\text{Then } \nabla \cdot \vec{v} = \frac{1}{r} \left[ \frac{\partial}{\partial r} \frac{\partial \psi}{\partial \theta} + \frac{\partial v_\theta}{\partial \theta} \right] = 0$$

$$\frac{\partial}{\partial \theta} \left( \frac{\partial \psi}{\partial r} + v_\theta \right) = 0 \quad \therefore v_\theta = -\frac{\partial \psi}{\partial r}$$

$$\therefore v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad \therefore v_\theta = -\frac{\partial \psi}{\partial r}$$

Q.E.D.

$$10.5 \quad \phi = \frac{5}{3} x^3 - 5xy^2$$

$$\text{Since } \vec{V} = \nabla \phi$$

CONTINUITY CAN BE EXPRESSED AS  
 $\nabla \cdot \vec{V} = 0$  OR  $\nabla^2 \phi = 0$

$$\text{USING } \nabla^2 \phi = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$10x - 10x = 0$$

$$U_x = \frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y} = 5x^2$$

$$\text{OR } \psi = 5x^2 y$$

$$\sim \frac{\partial \phi}{\partial y} = U_y = -\frac{\partial \phi}{\partial x} \sim \underline{\text{CHECK}}$$

10.6 IN THE CORE: EULER'S EQU.

$$\frac{D\vec{V}}{Dt} = \vec{g} - \frac{\nabla P}{\rho}$$

$$= -\frac{U^2}{r} \hat{e}_r = \frac{\nabla P}{\rho} = \frac{\partial P}{\partial r} \hat{e}_r$$

$$\frac{dP}{dr} = -\frac{U^2}{r}$$

SINCE VELOCITY VARIATION IS LINEAR

$$U = U_{\max} r / R$$

$$\int_P^R \frac{dP}{dr} = -\frac{8U_{\max}^2}{R^2} \int_0^R r dr$$

$$P_R - P_0 = \frac{8U_{\max}^2}{2} \quad (1)$$

OUTSIDE THE CENTRAL CORE -  
 BERNOULLI EQU. APPLIES

10.6 - (CONTINUED)

$$\frac{P_\infty}{\rho} = P + \frac{U^2}{2}$$

$U$  VARIES INVERSELY WITH  $r$ :

$$U = U_{\max} \frac{R}{r}$$

$$\text{AT } r = R \quad P_\infty - P_r = \frac{8U_{\max}^2}{2} \quad (2)$$

ADDINB (1) & (2)

$$P_\infty - P_0 = \frac{8U_{\max}^2}{2}$$

$$U_{\max} = \left[ \frac{(58)(32.2)}{0.0766} \right]^{1/2} = \underline{126 \text{ FT/S}} \quad (a)$$

FOR  $P = -10 \text{ PSF}$

$$P_\infty - P = \frac{8U^2}{2}$$

$$U = \left[ \frac{(10)(32.2)}{0.0766} \right]^{1/2} = 91.7 \text{ FT/S}$$

$$r = \frac{U_{\max} R}{U} = \frac{126}{91.7} (100) = 138 \text{ FT}$$

PRESSURE WILL FALL FROM -10 TO -38 PSF  
 IN A DISTANCE OF 138 FT

AT 60 MPH = 88 FT/S

$$\text{TIME} = \frac{138}{88} = \underline{1.57 \text{ SECONDS}} \quad (b)$$

PRESSURE AT TORNADO CENTER = -38 PSF

$$\text{AT EDGE OF CORE: } = -\frac{8U_{\max}^2}{2} + P_\infty$$

FAR FROM CENTER  $P = P_{\text{ATM}}$

$$\text{TOTAL } \Delta P = \underline{38 \text{ PSF}} \quad (c)$$

$$10.7 \quad V_r = U_\infty \cos\theta (1 - a^2/r^2)$$

ALONG THE STAGNATION Streamline  
 $\theta = \pi$

$$V_r = -U_\infty (1 - a^2/r^2) \quad (a)$$

$$\frac{\partial V_r}{\partial r} = -\frac{2U_\infty a^2}{r^3}$$

$$\left. \frac{\partial V_r}{\partial r} \right|_{r=a} = -\frac{2U_\infty}{a} \quad (b)$$

10.8 From CONTINUITY

$$\frac{\partial}{\partial r}(rV_r) = -\frac{\partial V_\theta}{\partial \theta}$$

$$\therefore \left. \frac{\partial V_\theta}{\partial \theta} \right|_{r=a, \theta=\pi} = -\left. \frac{\partial}{\partial r}(rV_r) \right|_{r=a, \theta=\pi}$$

$$10.9 \quad P + \frac{V^2}{2} = \text{constant}$$

$$\text{For } P = P_\infty \quad V^2 = V_\infty^2$$

$$\therefore |V_\infty| = |V_\theta| = 2V_\infty \sin\theta$$

$$\sin\theta = 0.5$$

$$\therefore \theta = \pm 30^\circ, \pm 150^\circ$$

$$10.10 \text{ (a)} \quad \phi = U_\infty L \left[ \left( \frac{x}{L} \right)^3 - 3 \frac{x^2}{L^3} \right]$$

$$V = \nabla \phi = V_x \hat{i}_x + V_y \hat{i}_y$$

$$V_x = \frac{\partial \phi}{\partial x} = \frac{3U_\infty}{L^2} (x^2 - y^2) = \frac{\partial \phi}{\partial y}$$

$$V_y = \frac{\partial \phi}{\partial y} = -\frac{6U_\infty xy}{L^2} = -\frac{\partial \phi}{\partial x}$$

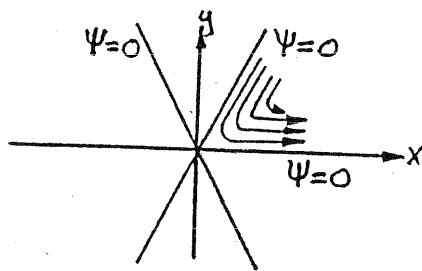
10.10 (CONTINUED)

$$\phi = \frac{3U_\infty}{L^2} \left( x^2 y - \frac{y^3}{3} \right) + f(y)$$

$$= \frac{3U_\infty x^2 y}{L^2} + g(y)$$

$$\text{So } \phi = \frac{U_\infty x^2 y}{L^2} (6x^2 - y^2)$$

Flow configuration is:



WHEN  $\psi = 0 \quad y = 0 \quad \text{or } \pm \sqrt{6}x$

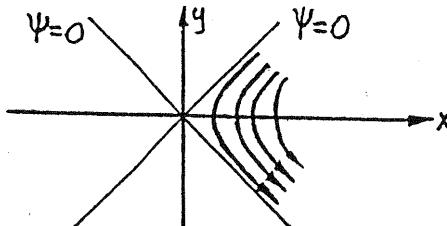
$$\text{b) } \phi = U_\infty xy$$

$$V_x = \frac{\partial \phi}{\partial x} = \frac{U_\infty y}{L} = \frac{\partial \phi}{\partial y}$$

$$V_y = \frac{\partial \phi}{\partial y} = \frac{U_\infty x}{L} = -\frac{\partial \phi}{\partial x}$$

$$\phi = \frac{U_\infty}{2L} y^2 + f(x); \quad \phi = -\frac{U_\infty}{2L} x^2 + g(y)$$

$$\phi = \frac{U_\infty}{2L} (y^2 - x^2)$$



WHEN  $\psi = 0 \quad y = \pm x$

10.10 CONTINUOUS

$$c) \phi = \frac{U_\infty L}{2} \ln(x^2 + y^2)$$

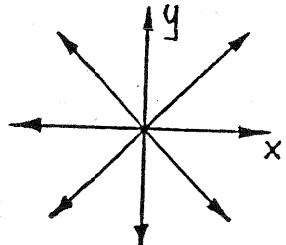
$$U_x = \frac{\partial \phi}{\partial x} = \frac{U_\infty L}{2} \frac{2x}{x^2 + y^2} = \frac{\partial \phi}{\partial y}$$

$$U_y = \frac{\partial \phi}{\partial y} = \frac{U_\infty L}{2} \frac{2y}{x^2 + y^2} = -\frac{\partial \phi}{\partial x}$$

$$\phi = \frac{U_\infty L}{2} x \tan^{-1}(y/x) + f(x)$$

$$\phi = \frac{U_\infty L}{2} x \tan^{-1}(y/x) + g(y)$$

$$\phi = U_\infty L \left[ \tan^{-1}(x/y) - \tan^{-1}(y/x) \right]$$



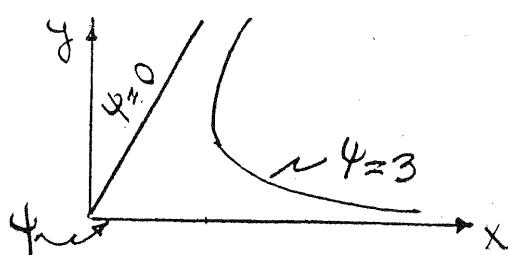
$$\text{WHEN } \phi = 0 \quad y = x$$

$$10.11 \quad \phi = 2r^3 \sin 3\theta \quad \text{FOR } \theta = 0, \frac{\pi}{3}$$

IN PICTURE - FOR  $\phi = 0$  - OR ANY NO.  
PICK 3 -

$$r = \left( \frac{3}{2 \sin 3\theta} \right)^{1/3}$$

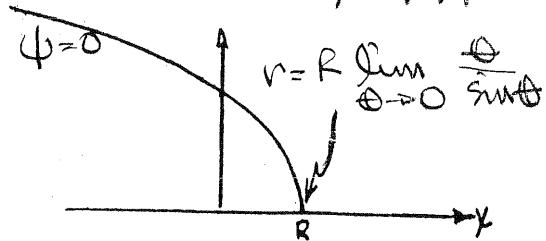
CHOOSE  $\theta$  - SOLVE FOR  $r$  -  
PLOT LOOKS LIKE:



$$10.12 \quad \phi = 0 = U_\infty r \sin \theta + \frac{Q}{2\pi} \theta$$

SINCE  $r > 0$  WHEN  $U_\infty > 0, \phi = 0$   
GIVES  $\theta = 0 \sim \text{THE + X AXIS}$

WHEN  $U_\infty < 0$  (FLOW IN -X DIRECTION)  
 $y = R\theta$  - WHERE  $R = Q / (2\pi |U_\infty|)$



$$10.13 \quad \text{FOR SOURCE AT ORIGIN } \phi = \frac{m \theta}{2\pi r}$$

$m$  = SOURCE STRENGTH

$$\text{FREE STREAM: } \phi = U_\infty y$$

$$\text{ADDING: } \phi = U_\infty y + \frac{m \theta}{2\pi r}$$

$$U_r = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = U_\infty \cos \theta + \frac{m}{2\pi r}$$

$$U_r = r \sin \theta$$

$$U_r = 0 @ \theta = \pi$$

$$\text{AT } \theta = \pi \quad r = \frac{m}{2\pi U_\infty} = \frac{Q}{2\pi U_\infty}$$

10.14

$$\nabla p = \rho \frac{D \vec{V}}{Dt}$$

$$= \rho \left[ \frac{\partial \vec{V}}{\partial t} + \nabla \left( \frac{U^2}{2} \right) - \vec{V} \times (\vec{V} \times \vec{V}) \right]$$

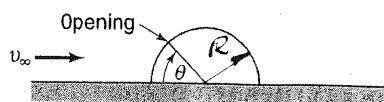
FOR STEADY, IRROTATIONAL FLOW

$$\nabla p = \rho \nabla \left( \frac{U^2}{2} \right)$$

@ stagnation point - WHERE  $V = 0$

$$\nabla p = 0$$

10.15



LIFT FORCE:

$$\begin{aligned} df_y &= df \sin \theta \\ &= (P_{in} - P_{out}) R \sin \theta d\theta \end{aligned}$$

$$f_y = \int_0^{\pi} AP R \sin^2 \theta d\theta$$

From BERNOULLI EQUATION:

$$P + \frac{1}{2} \rho V^2 = \text{CONST.}$$

$$P_{in} + \frac{1}{2} \rho V_{in}^2 = P + \frac{1}{2} \rho V^2$$

ON HUT:  $V = 2V_{in} \sin \theta$ 

$$\therefore P = P_{in} + \frac{1}{2} \rho V_{in}^2 [1 - 4 \sin^2 \theta]$$

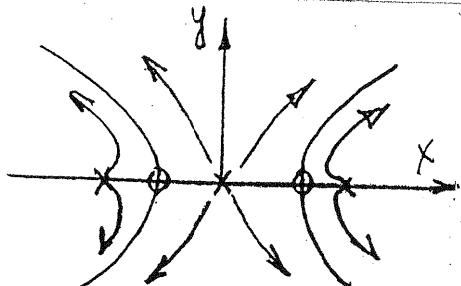
Subst. into Expression for  $f_y$ 

$$\begin{aligned} f_y &= \int_0^{\pi} \frac{1}{2} \rho V_{in}^2 [1 - 4 \sin^2 \theta - 1 + 4 \sin^2 \theta] R \sin \theta d\theta \\ &= 2 \rho V_{in}^2 R \int_0^{\pi} [\sin^3 \theta - \sin \theta \sin^2 \theta] d\theta \\ &= 2 \rho V_{in}^2 R \left[ \frac{4}{3} - 2 \sin^2 \theta \right] \end{aligned}$$

$$\text{for } f_y = 0 \quad \sin^2 \theta_0 = 2/3$$

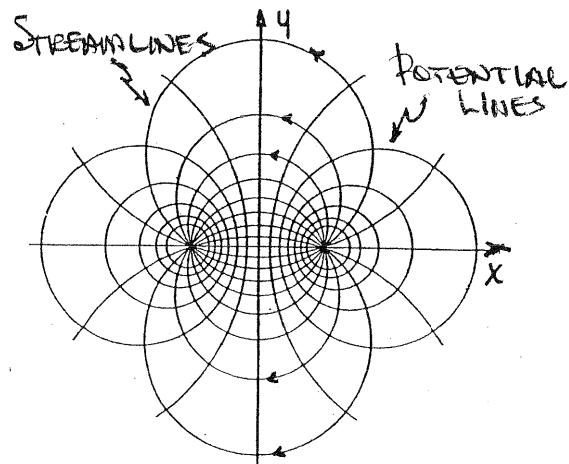
$$\theta_0 = 54.7^\circ$$

10.16

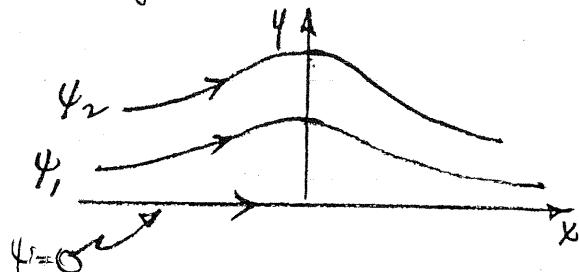


STAGNATION PTS AT CIRCLES

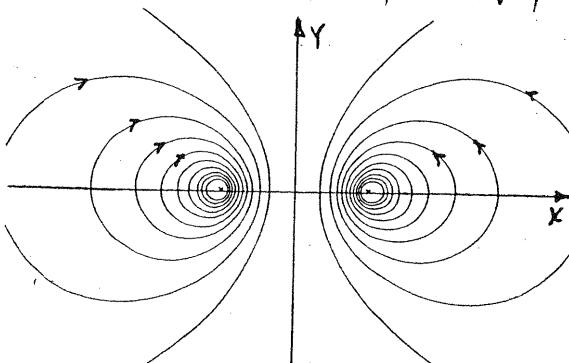
10.17

10.18 FOR THIS CASE -  $\nabla^2 \psi \neq 0$ 

$$y = \frac{\phi}{1+3x^2}$$

10.19  $\psi = -\frac{K}{2\pi} \ln r, V_\theta = \frac{K}{2\pi r}$ .

WHEN ORIGIN IS AT VORTEX

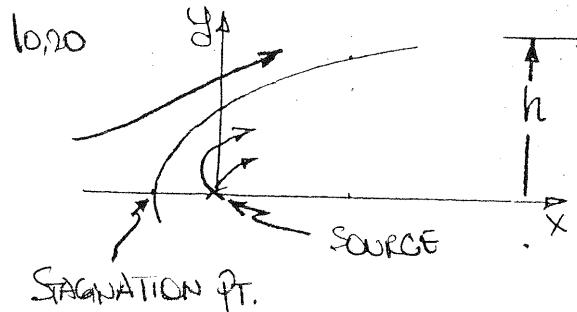


$$V_\theta(-a, 0) = \frac{K}{2\pi(2a)} = \frac{K}{4\pi a}$$

$$\vec{U}_p(-a, 0) = -K/4\pi a \hat{e}_y$$

$$\therefore \vec{U}(a, 0) = -K/4\pi a \hat{e}_y$$

$$\text{SINCE } \psi = \frac{K}{2\pi} \ln r$$



STAGNATION PT.

$$\psi = U_0 r \sin \theta + \frac{Q}{2} \theta$$

a) STAGNATION POINT

$$\vec{V} = 0 \text{ requires } V_r = V_\theta = 0$$

$$V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} = \frac{1}{r} \left( \frac{Q}{2\pi} + U_0 r \cos \theta \right)$$

$$U_\theta = - \frac{\partial \psi}{\partial r} = - U_0 \sin \theta$$

$$\therefore \theta = \pi, r \cos \pi = x = - \frac{Q}{2\pi U_0}$$

$\frac{1}{2}$  AT STAGNATION PT

$$x = - \frac{Q}{2\pi U_0} = - \frac{1.5}{2\pi(9)} = - \underline{0.0265 \text{ m}}$$

$$y = 0$$

b) BODY HEIGHT

STAGNATION STREAMLINE

$$\psi = U_0 r \sin \pi + \frac{Q\pi}{2\pi} = \frac{Q}{2}$$

THUS

$$\frac{Q}{2} = U_0 r \sin \theta + \frac{Q\theta}{2\pi}$$

SO WHEN  $\theta = \pi/2$

$$r \sin \theta = y = \frac{1}{U_0} \left( \frac{Q}{2} - \frac{Q\pi}{2\pi(2)} \right)$$

$$= \frac{Q}{4U} = \underline{0.0417 \text{ m}}$$

10.20 (CONTINUED)

c) FOR X LARGE - ALL FLOW IS AT  $U_0$   
 $\therefore Q = U_0 (2h)$

$$h = \frac{Q}{2U_0} = \frac{1.5}{2(9)} = \underline{0.0833 \text{ m}}$$

d) MAXIMUM SURFACE VELOCITY

$$U^2 = V_r^2 + V_\theta^2 \text{ IS ON S.L. } \phi = Q/2$$

$V_r \notin U_0$  DETERMINED IN PART (a)

$$U^2 = \frac{Q^2}{4\pi^2 r^2} + \frac{Q U_0 \cos \theta}{\pi r} + U_0^2$$

$$\text{ON SURFACE } \phi = \frac{Q}{2} = U_0 r \sin \theta + \frac{Q\theta}{2\pi}$$

$$\text{THUS } \frac{Q^2}{4\pi^2 r^2} = \frac{Q^2 U_0^2 \sin^2 \theta}{4\pi^2 Q^2 (1-\theta/\pi)^2}$$

$$\frac{Q U_0 \cos \theta}{\pi r} = \frac{U_0^2 Q \cos \theta \sin \theta}{\pi Q/2 (1-\theta/\pi)}$$

RESULTS IN

$$\frac{U^2}{U_0^2} = \frac{\sin^2 \theta}{\pi^2 (1-\theta/\pi)^2} + \frac{2 \sin \theta \cos \theta}{\pi (1-\theta/\pi)} + 1$$

$$\frac{U^2}{U_0^2} \text{ IS MAX AT } \theta \approx 63^\circ$$

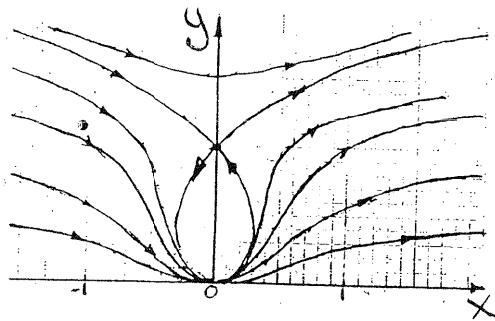
$$\text{SO } \frac{V_{MAX}}{U_0} \approx 1.26$$

10,21 IN THIS CASE -

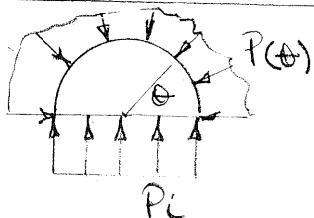
$$\phi = U_p r \sin\theta (1 + a^2/r^2)$$

STREAMLINES CAN BE  
PLOTTED FOR  $U_p = 1$ ,  $a = 1$

- IN UPPER HALF PLANE  
THEY APPEAR AS



10,22



$$\sum F_y = 0$$

$$P_i D - 12T - \frac{D}{2} \int_0^\pi P \sin\theta d\theta = 0$$

$$P = P_{atm} + \frac{8}{2} (U_p^2 - V^2)$$

$$V = -2U_p \sin\theta$$

$$12T = P_i D - P_{atm} + \frac{8}{2} U_p^3$$

$$+ 28 U_p^2 \frac{D}{2} \int_0^\pi \sin^3\theta d\theta$$

$$12T = (P_i - P_{atm})D + \frac{5}{6} 8U_p^2 D$$

$$\underline{\underline{T = 10.1 \text{ kN}}}$$

PER BOLT

## CHAPTER 11

### 11.1 VARIABLE DIMENSIONS

$D$	$L$
$H$	$M/L^3$
$\tau$	$L$
$\sigma$	$L/t^2$
$\epsilon$	$1/t$
$Q$	$L^3/t$
$P$	$M L^2/t^3$

$$i = 8 - 3 = 5$$

CHOOSE CORE AS  $S, D, W$

$$\Pi_1 = \eta \sim \text{ALREADY DIMENSIONLESS}$$

$$\Pi_2 = S^a D^b W^c H \sim = H/D$$

$$\Pi_3 = S^d D^e W^f \tau \sim = g/DW^2$$

$$\Pi_4 = S^g D^h W^i Q \sim = Q/D^3 W$$

$$\Pi_5 = S^j D^k W^l P \sim = P/S D^5 W^3$$

### 11.2 VARIABLE DIMENSIONS

$V$	$L/t$
$D$	$L$
$S$	$M/L^3$
$\mu$	$M/Lt$
$e$	$L$

$$i = 5 - 3 = 2$$

CHOOSE CORE AS  $D, V, \mu$

$$\Pi_1 = D^a V^b \mu^c \sim = \mu / DV^3 = 1/Re$$

$$\Pi_2 = D^d V^e \mu^f e \sim = e/D$$

$$\underline{f = f(Re, e/D)}$$

### 11.3 VARIABLE DIMENSIONS

$\Delta P$	$M/Lt^2$
$S$	$M/L^3$
$W$	$1/t$
$D$	$L$
$Q$	$L^3/t$
$P$	$M/Lt$

$$i = 6 - 3 = 3$$

CHOOSE CORE AS  $S, D, W$

$$\Pi_1 = S^a D^b W^c \Delta P \sim = \Delta P / S D^2 W^2$$

$$\Pi_2 = S^d D^e W^f Q \sim = Q / D^3 W$$

$$\Pi_3 = S^g D^h W^i P \sim = P / S D^2 W$$

### 11.4 VARIABLE DIMENSIONS

$C_{MAX}$	$ML^2/t^2$
$\alpha$	$M$
$\beta$	$L$
$\gamma$	$M/L^3$
$\delta$	$L/t^2$
$\epsilon$	$L$

$$i = 8 - 3 = 5$$

$$\Pi_1 = \alpha \sim \text{ALREADY DIMENSIONLESS}$$

$$\Pi_2 = \beta \sim " "$$

CHOOSE CORE AS  $M, L, \gamma$

$$\Pi_3 = M^a L^b \gamma^c C_{MAX} \sim = C_{MAX} / M \gamma$$

$$\Pi_4 = M^d L^e \gamma^f \sim = S L^3 / M$$

$$\Pi_5 = M^g L^h \gamma^i R \sim = R / L$$

## 11.5 VARIABLE DIMENSIONS

$R$	$L/t$
$D$	$L^2/t$
$d$	$L$
$w$	$1/t$
$\rho$	$M L^3$
$\mu$	$M/Lt$

$$l = 6 - 3 = 3$$

CHOOSE CORE AS  $d, w, \rho$

$$\Pi_1 = d^a w^b \rho^c k \sim = k/dw$$

$$\Pi_2 = d^d w^e \rho^f D \sim = D/d^2 w$$

$$\Pi_3 = d^g w^h \rho^i \mu \sim = \mu / g \rho^2 w$$

PLOT  $\Pi_1$  VS.  $\Pi_3$

OVER A RANGE IN VALUES OF  $\Pi_3$

## 11.6 VARIABLE DIMENSIONS

$Q$	$L^3/t$
$d$	$L$
$w$	$1/t$
$\mu$	$M/Lt$
$\sigma$	$M/t^2$
$\rho$	$M/L^3$

$$l = 6 - 3 = 3$$

CHOOSE CORE AS  $d, w, \rho$

$$\Pi_1 = d^a w^b \rho^c Q \sim = Q / \omega d^3$$

$$\Pi_2 = d^d w^e \rho^f \mu \sim = d^2 w \rho / \mu$$

$$\Pi_3 = d^g w^h \rho^i \sigma \sim = \sigma / g n^2 d^3$$

## 11.7 VARIABLE DIMENSIONS

$M$	$M$
$L$	$L$
$S$	$M/L^3$
$G$	$L/t^2$
$\sigma$	$M/t^2$

$$l = 5 - 3 = 2$$

CHOOSE CORE AS  $d, \rho, g$

$$\Pi_1 = d^a \rho^b g^c M \sim = M / d^3 g$$

$$\Pi_2 = d^b \rho^c g^f \sigma \sim = \sigma d^2 / g$$

## 11.8 VARIABLE DIMENSIONS

$n$	$1/t$
$L$	$L$
$d$	$L$
$\rho$	$M/L^3$
$T$	$ML/t^2$

$$l = 5 - 3 = 2$$

CHOOSE CORE AS  $n, d, \rho$

$$\Pi_1 = n^a d^b \rho^c L \sim = L/d$$

$$\Pi_2 = n^d d^e \rho^f T \sim = T / n^2 L \rho$$

## 11.9 VARIABLE DIMENSIONS

$P$	$ML^2/t^3$
$d$	$L$
$w$	$1/t$
$Q$	$L^3/t$
$S$	$M/L^3$
$\mu$	$M/Lt$

$$l = 6 - 3 = 3$$

## II.9 - CONTINUED

CHOOSE CORE AS  $\alpha, \beta, Q$

$$\begin{aligned}\pi_1 &= \alpha^a \beta^b Q^c w = \alpha^3 w / \alpha \\ \pi_2 &= \alpha^d \beta^e Q^f \mu = \alpha \mu / \beta Q \\ \pi_3 &= \alpha^g \beta^h Q^i P = \alpha^4 P / \beta Q^3\end{aligned}$$

$$\underline{\pi_3 = f(\pi_1, \pi_2)}$$

## II.10 VARIABLE DIMENSIONS

$r$	$L$
$t$	$t$
$g$	$m/L^3$
$E$	$ML^2/t^2$

$$L = 4 - 3 = 1$$

$$\pi_1 = t^a g^b E^c r = \frac{g^{1/5} r}{t^{2/5} E^{1/5}}$$

$$\text{OR } \pi_1 = \frac{g r^5}{t^2 E}$$

$$\text{So: } r^5 = C_1 \frac{Et^2}{g} \quad (1)$$

$$\text{SPEED OF WAVE FRONT} = \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{2}{5} C_2 t \frac{t}{r^4}$$

$$\text{From (1)} \quad t = C_3 r^{5/2}$$

$$\therefore \frac{dr}{dt} = C_4 / r^{3/2}$$

$\sim \frac{dr}{dt}$  DECREASES  $\rightarrow r$  INCREASES

## II.11 VARIABLE DIMENSIONS

$\alpha$	$L$
$\beta$	$L$
$\gamma$	$L/t$
$\delta$	$M/L^3$
$\epsilon$	$M/Lt$
$\zeta$	$M/t^2$

$$L = 6 - 3 = 3$$

CHOOSE CORE AS  $D, S, V$

$$\begin{aligned}\pi_1 &= D^a S^b V^c \alpha = \alpha / D \\ \pi_2 &= D^d S^e V^f \beta = \beta / DSV = 1/\rho_e \\ \pi_3 &= D^g S^h V^i \gamma = \frac{DS}{SV^2}\end{aligned}$$

$$\underline{\pi_4 = \pi_1(\pi_2, \pi_3)}$$

## II.12 VARIABLE DIMENSIONS

$\Delta P$	$M/Lt^2$
$Q$	$L^3/t$
$\gamma$	$L$
$\Delta$	$1/t$
$\mu$	$M/Lt$
$\zeta$	$L$
$R$	$L$

$$L = 7 - 3 = 4$$

CHOOSE CORE AS  $h, \Omega, \mu$

$$\begin{aligned}\pi_1 &= h^a \Omega^b \mu^c L = L/h \\ \pi_2 &= h^d \Omega^e \mu^f R = R/h \\ \pi_3 &= h^g \Omega^h \mu^i \Delta P = \Delta P / \Omega \mu \\ \pi_4 &= h^j \Omega^k \mu^l Q = Q / h^3 \Omega\end{aligned}$$

11.13  $Re = \frac{L U}{\nu}$  for air @  $20^\circ C$   
 $(0.931C)$   
 $\nu = 1.505 \times 10^{-5} m^2/s$

a) Based on  $L = 58m$

$$Re = \frac{(5.8)(22.2)}{1.505 \times 10^{-5}} = 8.55 \times 10^6$$

b) Based on  $D = 0.0004m$

$$Re = \frac{(0.0004)(22.2)}{1.505 \times 10^{-5}} = 9440$$

11.14  $\rho \frac{D\vec{U}}{Dt} = -\nabla P + \mu \nabla^2 \vec{U} + \rho g \left( \frac{T_A}{T_0} - 1 \right)$

TO PUT D.E. IN TO DIMENSIONLESS FORM - USE TEXT PROCEDURE -

$L$  = REFERENCE LENGTH

$U_0$  = " VELOCITY

THEN  $x^* = \frac{x}{L}, y^* = \frac{y}{L}, t^* = \frac{t}{U_0}$

$$\nabla^* = L \nabla \quad \nabla^* \cdot \nabla^* = L^2 \nabla^2$$

$$\rho \frac{D\vec{U}}{Dt} = \rho \frac{D\vec{U}^*}{Dt^*} \left( \frac{\partial \vec{U}}{\partial x^*} \right) \left( \frac{\partial t^*}{\partial t} \right)$$

$$= \frac{\rho U_0^2}{L} \frac{D\vec{U}^*}{Dt^*}$$

WHERE  $\frac{\rho U_0^2}{L}$  IS INERTIAL FORCE

WE COULD DO ALL OTHER TERMS IN A LIKE MANNER BUT PROBLEM STATEMENT ONLY ASKS FOR RATIO OF GRAVITY FORCES TO INERTIAL FORCES

11.14 - (CONTINUED) -

THE GRAVITATIONAL (BUOYANCY) FORCE IS  
 $\rho g \left( \frac{T_A}{T_0} - 1 \right)$

SO RATIO ASKED FOR IS

$$\frac{Lg}{U_0^2} \left[ \frac{T_A}{T_0} - 1 \right] \quad Q.E.D.$$

11.15	VARIABLE	MODEL	Prototype
	$D$	$D$	$6D$
	$U_0$	$U$	$20 \text{ ft}$
	$\rho$	$\rho$	$\rho$
	$\mu$	$\mu$	$\mu$
	$F$	$10^{12} F_p$	$F_p$
	$A$	$D^2$	$(6D)^2$

DYNAMIC SIMILARITY REQUIRES:

$$Re_m = Re_p$$

$$\frac{DU_0}{\mu L} = \frac{DU_p}{\mu L_p}$$

$$U_m = U_p \left[ \frac{D}{D_m} \frac{\rho_p}{\rho_m} \frac{\mu_m}{\mu_p} \right] = 6U_p$$

$$6 \quad 1 \quad 1 \quad 1 = 120 \text{ ft}$$

ALSO THAT  $E_{um} = E_{up}$

$$\frac{F/A}{8U^2} = \frac{F/A}{8U^2}_p$$

$$f_p = F_m \left[ \frac{A_p}{A_m} \left( \frac{\rho_p}{\rho_m} \right) \left( \frac{U_p}{U_m} \right)^2 \right]$$

$$36 \quad 1 \quad 1/36$$

$$= 10^{12} f_p$$

## 11.16 Similarity Features

$$Fr|_m = Fr|_p$$

$$\frac{V^2}{gL}|_m = \frac{V^2}{gL}|_p$$

MODEL PROTOTYPE

$$\begin{array}{lll} S & V_m & V_p \\ L & L/10 & L \end{array}$$

$$\left(\frac{V_m}{V_p}\right)^2 = \frac{L_m}{L_p} = 0.1$$

$$\frac{V_m}{V_p} = 0.316$$

## 11.17

MODEL PROTOTYPE

$$\begin{array}{ll} L = 3m & 4L \\ V_m & 16 \text{ m/s} \\ \rho_A & \rho_w \\ \nu_A & \nu_w \\ f_m & f_p \\ A_m & 16A_m \end{array}$$

For Air at  $20^\circ\text{C}$  (293K)

$$\nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{At } 6 \text{ atm} \quad \nu = 0.251 \times 10^{-5} \text{ "}$$

$$\text{For } H_2O \quad \nu = 0.995 \text{ "}$$

$$Re_m = Re_p$$

$$\frac{L|_S}{\nu}|_m = \frac{L|_U}{\nu}|_p$$

$$V_m = V_p \left( \frac{L|_U}{L|_m} \frac{\nu_m}{\nu_p} \right)$$

## 11.17 - CONTINUED

$$V_m = 16 \left[ (4) \frac{0.251 \times 10^{-5}}{0.995 \times 10^{-5}} (6) \right] = 969 \text{ m/s}$$

$$Eu_m = Eu_p$$

$$\frac{F/A}{gV^2}|_m = \frac{F/A}{gV^2}|_p$$

$$\begin{aligned} \frac{f_m}{f_p} &= \left( \frac{A_m}{A_p} \frac{\rho_m}{\rho_p} \frac{V_m^2}{V_p^2} \right) \\ &= \frac{1}{16} \left( \frac{7.229}{998.2} \right) \left( \frac{96.9}{16} \right)^2 \end{aligned}$$

$$\text{At } 20^\circ\text{C} \quad \rho_w = 998.2 \text{ kg/m}^3$$

$$\text{At } 20^\circ\text{C, } 6 \text{ atm} \quad \rho_A = 7.229 \text{ "}$$

$$\frac{f_m}{f_p} = 0.0166$$

RESULT IS  
QUITE  
TEMPERATURE  
SENSITIVE

## 11.18 Properties

$$\text{Air: } \rho = 5 \times 10^3 \frac{\text{slug}}{\text{ft}^3}, \quad \nu = 8 \times 10^{-5} \text{ ft}^2/\text{s}$$

$$H_2O: \rho = 194 \text{ "}, \quad \nu = 1 \times 10^{-5} \text{ m}$$

$$Re_m = Re_p$$

$$V_m = V_p \left[ \frac{L_p}{L_m} \frac{\nu_m}{\nu_p} \right]$$

$$= 60 \left[ \left( \frac{1}{2} \right) \left( \frac{8 \times 10^{-5}}{1 \times 10^{-5}} \right) \right]$$

$$= 240 \text{ MPH}$$

11.19

## VARIABLES Dimensions

$$\begin{array}{ll} L & L \\ a & L \\ \nu & LT \\ \tau & t \\ g & L/t^2 \end{array}$$

$$L = 5 - 2 = 3$$

{ NOTE -  $r = 2$  - NOT THE  
NO. OF FUNDAMENTAL DIM-  
ENSIONS - NO M }

CHOOSE CORE AS  $L, g$

$$\Pi_1 = L^a g^b \alpha \sim = \frac{\alpha}{L}$$

$$\Pi_2 = L^c g^d \nu \sim = \frac{\nu}{(Lg)^{1/2}}$$

$$\Pi_3 = L^e g^f \tau \sim = \sqrt{\frac{\nu}{L}}$$

a) FOR GEOMETRIC SIMILARITY

$$\Pi_1 |_m = \Pi_1 |_p$$

$$\alpha_m = \alpha_p \frac{L_m}{L_p} = 2 \left( \frac{1}{360} \right)$$

$$= 0.0056 m = \underline{5.6 \text{ mm}}$$

DYNAMIC SIMILARITY DICTATES

$$\Pi_2 |_m = \Pi_2 |_p$$

$$\nu_m = \nu_p \frac{(Lg)_m^{1/2}}{(Lg_p)^{1/2}}$$

$$= 8 \text{ m/s} \left( \frac{1}{360} \right)^{1/2}$$

$$= \underline{0.421 \text{ m/s}}$$

11.19 - CONTINUED

KINETIC SIMILARITY DICTATES

$$\Pi_3 |_m = \Pi_3 |_p$$

$$T_m = T_p \left[ \sqrt{\frac{\nu}{L}} \left( \frac{L}{\nu} \sqrt{\frac{L}{g}} \right) \right]$$

$$= 12 \pi R \left( \frac{1}{360} \right)^{1/2} = 0.632 \pi R \\ = \underline{37.9 \text{ min.}}$$

11.20 FOR EQUAL REYNOLDS NOS.:

$$Re |_m = Re |_p$$

$$\frac{L \nu S}{\mu} |_m = \frac{L \nu S}{\mu} |_p$$

$$S_m = S_p \left( \frac{L_p}{L_m} \right) \left( \frac{\nu_p}{\nu_m} \right) \left( \frac{\mu_m}{\mu_p} \right)$$

{ FIX IDEAL GAS BEHAVIOR -  $S = \frac{P}{RT}$

$$P_m = P_p \left( \frac{T_m}{T_p} \right) \left( \frac{L_p}{L_m} \right) \left( \frac{\nu_p}{\nu_m} \right) \left( \frac{\mu_m}{\mu_p} \right)$$

$$P_p = 287 \text{ Pa.}$$

$$T_p = 250.4 \text{ K} \quad T_m = 294 \text{ K}$$

$$\nu_m \approx 340.3 \text{ m/s} \quad \nu_p = 317.2 \text{ m/s}$$

$$\mu_m = 1.22 \times 10^{-5} \text{ lb m/s.FT}$$

$$\mu_p = 9.53 \times 10^{-6} \text{ "}$$

$$P_m = 287 \left( \frac{294}{250.4} \right) \left( \frac{1}{0.4} \right) \left( \frac{317.2}{340.3} \right) \left( \frac{1.22 \times 10^{-5}}{9.53 \times 10^{-6}} \right) \\ = 1000 \text{ Pa.} \sim \underline{1 \text{ kPa}}$$

11.20 - CONTINUED

DIMENSIONLESS TIME SCALE:

$$t^* = \frac{tU}{L}$$

$$\therefore \frac{tU}{L} |_m = \frac{tU}{L} |_p$$

$$\begin{aligned} \frac{t_m}{t_p} &= \left( \frac{U_p}{U_m} \right) \left( \frac{L_m}{L_p} \right) \\ &= \left( \frac{317.2}{340.3} \right) \left( \frac{0.41}{1} \right) \\ &= \underline{\underline{0.373}} \end{aligned}$$

11.21  $F_R = \frac{U^2}{gL}$  SPEED RATIO =  $\frac{U}{N_d}$

	MODEL	PROTOTYPE
L	0.41	1.45
U	2.58	U

EQUATING FRIEDE Numbers

$$\frac{U^2}{gL} |_m = \frac{U^2}{gL} |_p$$

$$\begin{aligned} U_p &= 2.58 \left( \frac{1.45}{0.41} \right)^{1/2} \\ &= \underline{\underline{6.31 \text{ m/s}}} \end{aligned}$$

EQUATING  $U/N_d$

$$\frac{U}{N_d} |_m = \frac{U}{N_d} |_p$$

11.21 CONTINUED

$$N_d |_p = N_d |_m \frac{U_p}{U_m}$$

$$\begin{aligned} N_p &= N_m \left( \frac{d_m}{d_p} \right) \left( \frac{U_p}{U_m} \right) \\ &= 450 \left( \frac{0.41}{2.45} \right) \left( \frac{6.31}{2.58} \right) \\ &= \underline{\underline{184 \text{ RPM}}} \end{aligned}$$

THRUST FORCE INVOLVES FULER No.

$$F_u |_m = F_u |_p$$

$$\begin{aligned} \frac{F/A}{S U^2/2} |_m &= \frac{F/A}{S U^2/2} |_p \\ F_p &= F_m \left( \frac{U_p}{U_m} \right)^2 \left( \frac{S_p}{S_m} \right) \left( \frac{L_p}{L_m} \right)^2 \\ &= 245 \left( \frac{6.31}{2.58} \right)^2 \left( \frac{2.45}{0.41} \right)^2 \\ &= \underline{\underline{52.3 \text{ kN}}} \end{aligned}$$

$$TORSION = FL$$

- FROM FULER \*

$$\frac{T/L}{S U^2/2} |_m = \frac{T/L}{S U^2/2} |_p$$

$$\begin{aligned} T_p &= T_m \left( \frac{L_p}{L_m} \right) \left( \frac{U_p}{U_m} \right)^2 \\ &= 70 \left( \frac{2.45}{0.41} \right) \left( \frac{6.31}{2.58} \right)^2 \\ &= \underline{\underline{715 \text{ N.m}}} \end{aligned}$$

## CHAPTER 12

12.1 AT TRANSITION  $Re_x = 2300$

$$Re = \frac{DV}{\nu} = 2300$$

$\tilde{\nu} = \text{AT } 20^\circ\text{C. : } \tilde{\nu} = 0.995 \times 10^{-6} \text{ m}^2/\text{s}$

$$V = \frac{2300(0.995 \times 10^{-6})}{0.038 \text{ m}}$$

$$= 0.060 \text{ m/s} \sim 6 \text{ cm/s}$$

12.2  $F_D = C_D A \frac{V^2}{2}$

For 35000 FT -  $\beta = 0.0237 \text{ lbm/ft}^3$   
S.L.  $\beta = 0.0766 \text{ lbm/ft}^3$

a) @ 35,000 FT  $500 \text{ mph} = 733 \text{ ft/s}$   
 $F_D = \frac{0.011(2400)(0.0237)(733)^2}{2(32.2)}$   
 $= 5220 \text{ lbf}$

b) @ SEA LEVEL  $700 \text{ mph} = 293 \text{ ft/s}$   
 $F_D = \frac{0.011(2400)(0.0766)(293)^2}{2(32.2)}$   
 $= 1700 \text{ lbf}$

12.3 AT TRANSITION  $Re_x = 2 \times 10^5$

for AIR @  $20^\circ\text{C}$ ,  $\tilde{\nu} = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$

$$Re_x = x \nu / \nu$$

$$x = (2 \times 10^5)(1.505 \times 10^{-5}) / 30 \text{ m/s}$$

$$= 0.100 \text{ m}$$

12.4  $\frac{U_x}{U_\infty} = C_1 + C_2 y + C_3 y^2 + C_4 y^3$

Boundary Conditions:

(1)  $U_x(0) = 0$

(2)  $U_x(\delta) = U_\infty$

(3)  $\frac{\partial U_x}{\partial y}(\delta) = 0$

ONE MORE B.C. IS NEEDED -

IF  $\frac{dP}{dx} = 0$  THE OTHER ONE IS  $\frac{\partial^2 U_x}{\partial y^2}(0) = 0$

BUT THIS ISN'T THE CASE CONSIDERED

THE GOVERNING EQUATION OF MOTION IS

$$\frac{U_x}{U_\infty} \frac{\partial U_x}{\partial x} + \frac{U_y}{U_\infty} \frac{\partial U_y}{\partial y} = - \frac{dP}{dx} + \mu \frac{\partial^2 U_x}{\partial y^2}$$

AT  $y=0$  -  $U_x = U_y = 0$

$\frac{dP}{dx}$  CAN BE RELATED TO  $U_\infty$  BY THE  
BERNOULLI EQUATION:  $P + \frac{U_\infty^2}{2} = \text{CONST}$

SO  $\frac{dP}{dx} = - \frac{U_\infty}{2} \frac{\partial U_\infty}{\partial x}$

i. @  $y=0$  THE EQUATION OF MOTION GIVES

$$(4) \quad \frac{\partial^2 U_x}{\partial y^2} = - \frac{1}{\mu} U_\infty \frac{\partial U_\infty}{\partial x}$$

THIS IS THE 4th B.C.

From (1):  $C_1 = 0$  THE REMAINING  
EXPRESSION FOR  $\frac{U_x}{U_\infty}$  WILL BE

$$\frac{U_x}{U_\infty} = C_2 \frac{y}{\delta} + C_3 \left(\frac{y}{\delta}\right)^2 + C_4 \left(\frac{y}{\delta}\right)^3$$

## 12.4 (CONTINUED) -

$$\text{from (2)} \quad 1 = C_2 + C_3 + C_4$$

$$(3) \quad 0 = C_2 + 2C_3 + 3C_4$$

$$(4) \quad -\frac{\delta^2}{\delta} \frac{dU_x}{dy} = 2C_3$$

SUBSTITUTION YIELDS:

$$\frac{U_x}{U_\infty} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3$$

$$+ \frac{\delta^2}{4\delta} \frac{dU_x}{dy} \left[ \frac{y}{\delta} - 2 \left( \frac{y}{\delta} \right)^2 + \left( \frac{y}{\delta} \right)^3 \right]$$

$$12.5 \quad \text{GIVEN } U_x = \alpha \sin \beta y$$

$$\text{FOR A LAMINAR B.L. } \frac{df}{dx} = 0$$

$$\text{B.C. (1)} \quad U_x(0) = 0$$

$$(2) \quad U_x(\delta) = U_\infty$$

$$(3) \quad \frac{dU_x}{dy}(\delta) = 0$$

$$\text{from (1)} \quad 0 = 0 \quad \rightarrow \text{NOT TRUE}$$

$$(2) \quad U_\infty = \alpha \sin \beta \delta$$

$$(3) \quad 0 = \alpha \beta U_\infty \beta \delta$$

$$\text{GIVEN } \beta \delta = \pi/2 \quad \beta = \pi/2\delta$$

$$\alpha = U_\infty$$

$$\text{SO PROFILE IS } U_x = U_\infty \sin \left( \frac{\pi y}{2\delta} \right)$$

VON KARMAN INTEGRAL EQN FOR B.L.

$$\frac{f}{f} = \frac{1}{2} \int_0^\delta U_x (U_\infty - U_x) dy$$

$$\frac{f}{f} = \frac{\mu}{f} \frac{dU_x}{dy} \Big|_0^\delta = \frac{\mu}{f} U_\infty \frac{\pi}{2\delta} \cos \frac{\pi y}{2\delta}$$

## 12.5 (CONTINUED) -

$$\text{TOPWATSON} \int_0^\delta U_x (U_\infty - U_x) dy$$

$$= U_\infty^2 \int_0^\delta \frac{U_x}{U_\infty} \left( 1 - \frac{U_x}{U_\infty} \right) dy$$

$$= U_\infty^2 \int_0^\delta \left[ \sin \frac{\pi y}{2\delta} - \sin^2 \frac{\pi y}{2\delta} \right] dy$$

$$= U_\infty^2 \left[ -\frac{2\delta}{\pi} \frac{\cos \pi y}{2\delta} - \frac{y}{2} + \frac{\delta}{2\pi} \sin \pi y \right]_0^\delta$$

$$= U_\infty^2 \left[ -2\frac{\delta}{\pi} - \frac{\delta}{2} \right] = U_\infty^2 \delta \left[ \frac{2}{\pi} - \frac{1}{2} \right]$$

$$\text{Now: } \frac{d}{dx} \left[ \int_0^\delta \sim \right] = \frac{d}{dx} \left\{ U_\infty^2 \delta \left[ \frac{2}{\pi} - \frac{1}{2} \right] \right\}$$

$$= \left( \frac{2}{\pi} - \frac{1}{2} \right) U_\infty^2 \frac{d\delta}{dx}$$

EQUATING BOTH PARTS:

$$\lambda \frac{dU_x}{dy} \frac{\pi}{2\delta} = \left( \frac{2}{\pi} - \frac{1}{2} \right) U_\infty^2 \frac{d\delta}{dx}$$

$$\delta d\delta = \frac{\lambda \pi^2}{U_\infty (4-\pi)} \int_0^\delta dx$$

$$\delta = \left[ \frac{\lambda x \pi^2}{U_\infty 4-\pi} \right]^{1/2}$$

$$\delta = 4.81 \sqrt{\frac{U_x}{U_\infty}} = \frac{4.81 x}{\sqrt{f_{ex}}}$$

$$C_{fx} = \frac{f}{8 U_\infty^2 / 2} = \frac{\mu U_\infty \pi / 2\delta}{8 U_\infty^2 / 2} = \frac{\mu \pi}{U_\infty \delta}$$

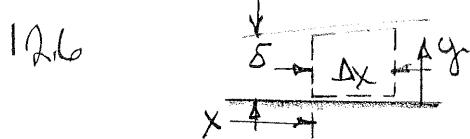
PUTTING IN OUR EXPRESSION FOR  $\delta$   
WE HAVE

$$C_{fx} = 0.653 f_{ex}^{-1/2}$$

## 12.5 - CONTINUED -

$$\begin{aligned}
 C_{FL} &= \frac{1}{L} \int_0^L C_{Fx} dx \\
 &= \frac{0.653}{L} \sqrt{\frac{D}{U_{\infty}}} \int_0^L x^{-\frac{1}{2}} dx \\
 &= \frac{0.653}{L} \sqrt{\frac{D}{U_{\infty}}} (2x^{\frac{1}{2}}) \Big|_0^L \\
 &= 1.306 \frac{D}{U_{\infty} L^{\frac{1}{2}}}
 \end{aligned}$$

	COMPARISON	APPROXIMATE	FACT
$\delta$	$4.81 \times Re_x^{-\frac{1}{2}}$	$50 \times Re_x^{-\frac{1}{2}}$	
$C_{Fx}$	$0.653 Re_x^{-\frac{1}{2}}$	$0.644 Re_x^{-\frac{1}{2}}$	
$C_{FL}$	$1.305 Re_x^{-\frac{1}{2}}$	$1.328 Re_x^{-\frac{1}{2}}$	



MOMENTUM THEOREM:

$$\begin{aligned}
 \sum F_x &= \iint_{CS} P(\vec{v} \cdot \vec{n}) dA + \frac{d}{dt} \cancel{10} \\
 \sum F_x &= PS|_x - PS|_{x+\Delta x} \\
 &\quad + P|_x + P|_{x+\Delta x} (\delta|_{x+\Delta x} - \delta|_x) \\
 &\quad - T_o \Delta x^2
 \end{aligned}$$

$$\begin{aligned}
 \iint_{CS} U_x P(\vec{v} \cdot \vec{n}) dA &= \int_0^{\delta} \delta U_x^2 dy \Big|_{x+\Delta x} \\
 &\quad - \int_0^{\delta} \delta U_x^2 dy \Big|_x - U_x \left[ \int_0^{\delta} \delta U_x dy \right] \Big|_{x+\Delta x} \\
 &\quad - \left[ \int_0^{\delta} \delta U_x dy \right] \Big|_x - U_{\infty} \Delta x
 \end{aligned}$$

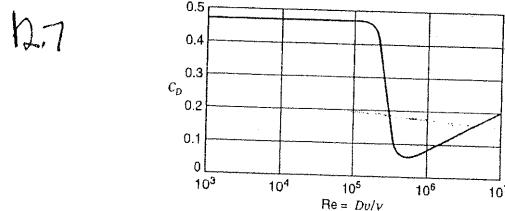
## 12.6 - CONTINUED -

REARRANGING, DIVIDING BY  $Ax$ , EVALUATING IN THE LIMIT AS  $\Delta x \rightarrow 0$ :

$$\begin{aligned}
 -S \frac{dP}{dx} &= T_o + U_{\infty} U_{\infty} + \frac{d}{dx} \int_0^{\delta} \delta U_x^2 dy \\
 &\quad - U_{\infty} \frac{d}{dx} \int_0^{\delta} \delta U_x dy
 \end{aligned}$$

NOTING THAT BERNOULLI'S EQU. APPLIES OUTSIDE THE B.L., WE CAN WRITE (SEE CHAPTER)

$$\begin{aligned}
 S \frac{dP}{dx} &= \frac{d}{dx} (SU_x^2) - U_{\infty} \frac{d}{dx} (SU_x) \\
 \frac{1}{S} \frac{dP}{dx} &= \frac{d}{dx} (U_x - U_{\infty}) dy \\
 \frac{T_o}{S} + \frac{U_{\infty} U_{\infty}}{S} &= \frac{dU_x}{dx} \int_0^{\delta} (U_x - U_{\infty}) dy \\
 &\quad + \frac{d}{dx} \int_0^{\delta} U_x (U_{\infty} - U_x) dy
 \end{aligned}$$



FOR A SMOOTH SPHERE - FIG. ABOVE

$$Re_{cr} \approx 2 \times 10^5$$

FOR AIR @ 20°C  $D = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$ 

$$\begin{aligned}
 U_{cr} &= \frac{D}{\rho} Re_{cr} \\
 &= \frac{(1.505 \times 10^{-5})(2 \times 10^5)}{0.042} \\
 &= 71.7 \text{ m/s}
 \end{aligned}$$

FOR SUCH A SPHERE (GOLF BALL SIZE)  
A VELOCITY GREATER THAN THIS  
WILL REDUCE DRAG & BALL WILL  
TRAVEL FURTHER

12.8 For Air @ 80°F

$$\rho = 0.169 \times 10^{-3} \text{ lbm/ft}^3$$

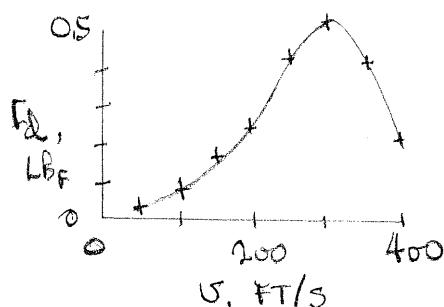
$$g = 0.0735 \text{ lbm/lb ft}^3$$

$$Re = \frac{Dv}{\nu} = \frac{(1.65/12)v}{0.169 \times 10^{-3}}$$

$$F_D = A \frac{v^2}{2} C_D$$

$$= \frac{\pi}{4} \left(\frac{1.65}{12}\right)^2 \left(\frac{0.0735}{32.2}\right) \frac{v^2}{2} C_D$$

$v, \text{ft/s}$	$C_D$	$Re$	$F_D, \text{lb}_F$
50	0.47	$4.07 \times 10^4$	0.0199
100	0.47	8.14	0.0797
150	0.46	12.2	0.175
200	0.45	16.3	0.305
250	0.41	20.3	0.434
300	0.35	24.4	0.534
350	0.20	28.5	0.415
400	0.08	32.5	0.217



12.9 IN THE UNSTEADY WAKE REGION

$$1 < Re < 10^3$$

$$@ 20^\circ \text{C} \quad \nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Re = Dv/\nu$$

$$@ Re = 1 = \frac{0.0127 v}{1.505 \times 10^{-5}}$$

$$v = 0.00119 \text{ m/s}$$

$$@ Re = 10^3 \quad v = 1.185 \text{ m/s}$$

THESE ARE THE LOWER  
UPPER BOUNDS FOR  $v$

12.10 for Air @ 80°F

$$\rho = 0.169 \times 10^{-3} \text{ lbm/ft}^3$$

$$g = 0.0735 \text{ lbm/lb ft}^3$$

$$Re = \frac{(0.2/12)(88)}{0.169 \times 10^{-3}} = 8680$$

From Fig 12.2  $C_D \approx 1.2$

$$F_D = C_D A \frac{v^2}{2}$$

$$= 1.2 \left( \frac{0.2}{12} \right) \left( \frac{3}{2} \right) \left( \frac{0.0735}{32.2} \right) \left( \frac{88^2}{2} \right)$$

$$= 0.530 \text{ lb}_F$$

$$12.11 \quad F_D = C_D A \frac{v^2}{2}$$

$$Air @ 20^\circ \text{C} \quad g = 1.2048 \text{ kg/m}^3$$

$$\nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

$$F_D = 0.26 (2.33) (1.2048) (30)^2$$

$$= 657 \text{ N}$$

$$\text{POWER} = F_D v = (657)(30) = 19.7 \text{ kW}$$

WITH A HEADWIND OF 6 m/s

$$F_D = 0.26 (2.33) (1.2048) (36)^2 = 28.4 \text{ kW}$$

WITH A TAILWIND OF 6 m/s

$$F_D = (24)^2 = 12.6 \text{ kW}$$

FOR STILL AIR  $P = 15.9 \text{ hp}$

WITH HEADWIND  $P = 37.4 \text{ "}$

WITH TAILWIND  $P = 16.9 \text{ "}$

$$12.12 \quad 100 \text{ mi/hr} = 44.7 \text{ m/s}$$

$$F_L = C_D A S V^2 / 2$$

$$= 0.18 (2.33) (1,2048) (44.7)^2 / 2$$

$$= \underline{\underline{589 \text{ N}}}$$

$$12.13 \quad \text{IF } C_D = 1$$

$$F_L = 589 \left( \frac{1}{0.18} \right) = \underline{\underline{2805 \text{ kN}}}$$

$$12.14 \quad F_D = C_D A S V^2 / 2$$

IN SAME ENVIRONMENT AT SAME SPEED

$$C_D A |_{\text{CAR}} = C_D A |_{\text{PLATE}}$$

$$C_D A |_{\text{CAR}} = 0.18 (2.33)$$

$$C_D A |_{\text{PLATE}} = 1.1 \pi D^2$$

$$D = \left[ \frac{0.18 (2.33) (4)}{(1.1) (\pi)} \right]^{1/2}$$

$$= \underline{\underline{0.837 \text{ m}}}$$

$$12.15 - \text{CIRCULAR SIGN} - D = 8 \text{ FT}$$

$$V = 120 \text{ MPH}$$

$$(176 \text{ FT/S})$$

$$F_D = C_D A S V^2 / 2$$

$$\text{AT } 80^\circ \text{ F} - \rho = 0.0735 \text{ lbm/ft}^3$$

$$F_D = \frac{1.1 \left( \frac{\pi}{4} \right) (8)^2 (0.0735) (176)^2}{32.2 (2)}$$

$$= \underline{\underline{1955 \text{ lbf}}}$$

$$12.16 \quad \text{FOR AIR @ } 100^\circ \text{ F} - \rho = 0.0710 \text{ lbm/ft}^3$$

$$0^\circ \text{ F} - \rho = 0.0862 \text{ "}$$

$$C_D = 0.18 \quad A = 2.14 \text{ m}^2 = 25.83 \text{ ft}^2$$

$$P = F_D S = C_D A S V^3 / 2$$

$$= \frac{0.18 (25.83) (0.0710) (102.7)^3}{2 (32.2) (550)}$$

$$= \underline{\underline{15.7 \text{ kip}}} - \text{AT } 100^\circ \text{ F}$$

$$= 15.7 \left( \frac{0.0862}{0.0710} \right) = \frac{19.1 \text{ kip}}{\text{AT } 0^\circ \text{ F}}$$

$$12.17 \quad \text{SPHERE} - D = 9.25 \text{ IN} = 2.94 \text{ m}$$

$$\text{WT} = 5.25 \text{ OUNCES}$$

$$\text{AT } V = 95 \text{ MPH} \quad (139.3 \text{ FT/S})$$

$$\text{AT } 80^\circ \text{ F} \quad \rho_{\text{AIR}} = 0.0735 \text{ lbm/ft}^3$$

$$D_{\text{AIR}} = 0.169 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$\text{a) } Re = \frac{DV}{D} = \frac{(2.94/12)(139.3)}{0.169 \times 10^{-3}} = \underline{\underline{2.02 \times 10^5}}$$

$$F_D = C_D A S V^2 / 2$$

$$\text{b) AT } Re = 2.02 \times 10^5 \quad C_D \approx 0.4$$

$$F_D = \frac{0.4 \left( \frac{\pi}{4} \right) \left( \frac{2.94}{12} \right)^2 (0.0735) (139.3)^2}{2 (32.2)}$$

$$= \underline{\underline{0.418 \text{ lbf}}}$$

$$\text{c) Flow IS NEAR TRANSITION - BUT STILL IN LAMINAR RANGE}$$

12.18

$Re \cdot 10^{-4}$	7.5	10	15	20	25
$C_D$	0.48	0.38	0.22	0.12	0.10

$$\text{At } 80^\circ\text{F} - D = 0.169 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$g = 0.0135 \text{ ft m/ft}^3$$

$$\text{For } Re = 7.5 \times 10^4 = Du/D$$

$$U = \frac{(7.5 \times 10^4)(0.169 \times 10^{-3})}{1.165/2}$$

$$= 92.18 \text{ ft/s}$$

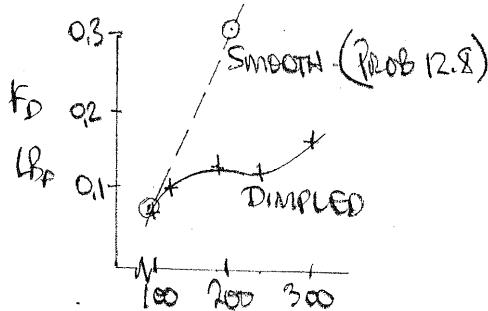
$$F_D = C_D A S U^2 / 2$$

$$= \frac{0.48 \left(\frac{\pi}{4}\right) \left(\frac{1.165}{12}\right)^2 (0.0135) (92.18)^2}{2(32.2)}$$

$$= 0.069 \text{ lbf}$$

DOING THIS CALCULATION FOR ALL GIVEN CONDITIONS WE GENERATE THE FOLLOWING!

$Re \times 10^4$	$U$	$C_D$	$F_D, \text{lbf}$
7.5	92.2	0.48	0.069
10	122.9	0.48	0.100
15	184.4	0.47	0.129
20	245.8	0.44	0.125
25	307.3	0.40	0.164

12.19  $W_T = 5.25 \text{ OUNCES} = 0.328 \text{ lbf}$ 

$$F_L = W_T = C_D A S U^2 / 2$$

$$A = \frac{\pi}{4} \left(\frac{2.94}{12}\right)^2 = 0.04714 \text{ ft}^2$$

$$C_D = 0.224$$

From Problem Statement

$$C_L \approx 0.24 \frac{R_D}{U} - 0.05$$

So for this case

$$\frac{R_D}{U} = \frac{0.224 + 0.05}{0.24} = 1.142$$

$$U = 110 \text{ mph} = 161.3 \text{ ft/s}$$

$$\Omega = \frac{1.142 (161.3)}{0.385} = 478 \text{ Rad/s}$$

$$= 76.1 \text{ Rev/s}$$

$$\text{To travel } 60.5 \text{ ft} \quad t = \frac{60.5}{161.3} = 0.3755$$

$$\text{No of Revolutions} = 76.1(0.375) = \underline{\underline{28.5}}$$

12.20 BLASIUS EQUATION FOR LAMINAR BOUNDARY LAYER FLOW IS

$$8 \frac{DU_x}{Dt} = \frac{dP}{dx} + \rho \nu U_x^2$$

OR, WRITTEN AS

$$U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y} = -\frac{1}{8} \frac{dP}{dx} + \nu \left[ \frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_x}{\partial y^2} \right]$$

AT  $y=0$  -  $U_x=0$  BUT, IN THIS CASE,  $U_y \neq 0$   
THE RESULTANT FORM IS

$$U_y \frac{\partial U_x}{\partial y} = -\frac{1}{8} \frac{dP}{dx} + \nu \frac{\partial^2 U_x}{\partial y^2}$$

↑  
THIS TERM IS NOT PRESENTFOR  $U_y(0)=0$  {EQU (12-33) CASE}

12.21 TURBULENCE INTENSITY

$$I = \frac{[(\bar{U}_x^2 + \bar{U}_y^2 + \bar{U}_z^2)/3]^{1/2}}{U_p}$$

$$\text{KINETIC ENERGY} = \frac{U_p^2 + \bar{U}_x^2 + \bar{U}_y^2 + \bar{U}_z^2}{2}$$

$$= \frac{U_p^2 (1 + 3I^2)}{2}$$

FOR  $I=0.1$   $\frac{\text{K.E.}}{\text{TOTAL}} = \frac{U_p^2}{2} (1.03)$

WHILE  $\text{K.E.TURB} = \frac{U_p^2}{2} (0.03)$

FRACTION DUE TO TURBULENCE

$$= \frac{0.03}{1.03} = 2.91\%$$

12.22  $\dot{V} = 2 \text{ gpm} = 0.4416 \times 10^{-2} \text{ ft}^3/\text{s}$

$$U = \frac{\dot{V}}{A} = \frac{0.4416 \times 10^{-2}}{\pi/4 (0.75/12)^2} = 1.45 \text{ ft/s}$$

FOR  $H_2O @ 120^\circ F$   $D = 0.162 \times 10^{-5} \text{ ft}^2/\text{s}$

@  $45^\circ F$   $D = 1.57 \times 10^{-5}$  "

@  $120^\circ F$   $Re = \frac{(0.75)(1.45)}{0.162 \times 10^{-5}}$

$$= 14,600 \quad (a)$$

@  $45^\circ F$   $Re = \frac{(0.75/12)(1.45)}{1.57 \times 10^{-5}}$

$$= 5770 \quad (b)$$

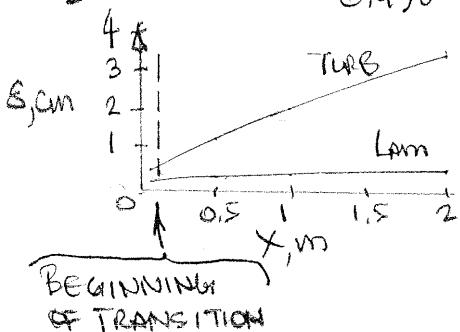
12.23 Laminar Flow:  $\frac{\delta}{x} = 5 Re_x^{-1/4}$

Turbulent "  $\frac{\delta}{x} = 0.376 Re_x^{-0.2}$

FOR AIR @  $20^\circ C$   $D = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$

$$Re_x = \frac{30 \times}{1.505 \times 10^{-5}} = 2 \times 10^6 \times$$

$x, \text{m}$	$Re_x$	$\delta_L, \text{cm}$	$\delta_t, \text{cm}$
0	0	0	0
0.1	$2 \times 10^5$	0.111	0.327
0.5		0.249	1.126
1		0.352	2.063
2		0.498	3.591



12.24  $\dot{V} = 0.006 \text{ m}^3/\text{s}$

$$U = \frac{0.006}{(\pi/4)(0.15)^2} = 0.34 \text{ m/s}$$

TO CALCULATE  $y_{max}^+ \& U_{max}^+$ :

$$\frac{J}{g} = 0.0225 \frac{U_{max}^2}{U_{x,MAX}^2} \left[ \frac{D}{U_{x,MAX} y_{max}} \right]$$

FROM RESULTS OF  $1/7$  POWER LAW:

$$\bar{U} = 0.817 U_{x,MAX} \sim U_{x,MAX} = 0.416 \text{ m/s}$$

$$y_{max} = 0.075 \text{ m}$$

AT  $20^\circ C$   $D = 0.995 \times 10^{-4} \text{ m}^2/\text{s}$

SUBSTITUTING INTO  $J/g$  EXPRESSION

$$\sqrt{J/g} = 0.071 \text{ m/s}$$

## 12.24 CONTINUED

- LAMINAR SUBLAYER:

$$y^+ = \frac{\sqrt{3/8} \cdot 4}{\lambda} \approx 5$$

$$y = \frac{5(0.995 \times 10^{-6})}{0.0171} = \underline{0.291 \text{ mm}}$$

BUFFER LAYER -

EXTENDS FOR  $5 \leq y^+ \leq 30$ 

$$@ y^+ = 30 \quad y = 1.746 \text{ mm}$$

$$\text{THICKNESS}_{B.L.} = \underline{1.455 \text{ mm}}$$

TURBULENT CORE EXTENDS

FROM  $y = 1.455 \text{ mm}$ TO  $y = 75 \text{ mm}$ 

$$\text{THICKNESS}_{T.C.} = \underline{73.55 \text{ mm}}$$

## 12.25 EQU. (12-68)

$$\frac{f_0}{8} = 0.0225 \left( \frac{U_{AV}}{U_{MAX}} \right)^2 \left[ \frac{R}{U_{MAX} y_{MAX}} \right]^{1/4}$$

$$C_{fx} = \frac{f_0/8}{U_{AV}^2/2} \\ = 0.045 \left( \frac{U_m}{U_{AV}} \right)^2 \left[ \frac{R}{U_{AV} y_{MAX}} \right]^{1/4}$$

$$\left[ \frac{R}{U_{AV} y_{MAX}} \right]^{1/4} = \left( \frac{U_m}{U_{AV}} \right)^{-1/4} (2^{1/4})^{-1/4} R_{D0}$$

$$\text{GIVEN } C_{fx} = 0.0535 \left( \frac{U_m}{U_{AV}} \right)^{7/4} R_{D0}^{-1/4}$$

Now - To find  $\frac{U_m}{U_{AV}}$  FOR PIPE FLOW

## 12.25 CONTINUED

$$U_m (R^2) = \int_A U \delta A$$

$$= 2\pi \int_U r dr$$

$$U_{AV} = \frac{2}{R^2} \int_0^R U_{MAX} \left( 1 - \frac{r}{R} \right)^4 r dr$$

$$\text{POINT TAC MATH: } \frac{U_m}{U_{AV}} = 1.225$$

$$C_{fx} = 0.0763 R_{D0}^{-1/4}$$

$$12.26 \quad Re_L = \frac{L U}{\nu} = \frac{0.5(40)}{0.159 \times 10^{-3}} = 125,800$$

$$C_{fL} = \frac{1}{L} \int_0^L C_{fx} dx = \frac{1}{L} \int_0^L \frac{0.05764}{Re_L^{1/4}} dx \\ = 0.072 Re_L^{-1/5}$$

$$C_{fL} = 0.072 (125,800)^{-1/5} = 6.877 \times 10^{-3}$$

for 2 SIDES  $\frac{1}{2}$   $60^\circ$  AIR

$$C_D = 2C_{fL} A \frac{U^2}{2} \\ = \frac{2(6.877 \times 10^{-3})(1.5)(0.0164)(40)^2}{2(32.2)} \\ = \underline{0.0392 \text{ Lbf}} \quad (a)$$

FOR LAMINAR FLOW

$$C_{fL} = 1.328 Re_L^{-1/2} = 0.00375$$

$$F_D = 2C_{fL} A \frac{U^2}{2}$$

$$= \underline{0.0213 \text{ Lbf}} \quad (b)$$

12.27  $Re = 10^5$ 

$$\text{LAMINAR flow} - \delta_L = 5 Re_x^{-1/2}$$

$$\text{TURBULENT } " \quad \delta_t = 0.375 Re_x^{-0.2}$$

$$\text{for } Re = 10^5 \quad \frac{\delta_t}{\delta_L} = 1.38$$

From CHAPTER 5: MOMENTUM  $\sim \delta u_\infty^2$ " " 6: ENERGY  $\sim \delta u_\infty^3$ 

$$\text{for } U = U_p f\left(\frac{y}{\delta}\right)$$

$$\text{MOMENTUM} = \delta u_\infty^2 f^2\left(\frac{y}{\delta}\right)$$

$$\text{ENERGY} = \frac{\delta u_\infty^3}{2} f^3\left(\frac{y}{\delta}\right)$$

$$\frac{M}{\delta u_\infty^2} = f^2\left(\frac{y}{\delta}\right)$$

$$\frac{E}{\delta u_\infty^3/2} = f^3\left(\frac{y}{\delta}\right)$$

FOR LAMINAR CASE:

$$\frac{M}{\delta u_\infty^2} = \sin^2\left(\frac{y}{\delta_L} \frac{\pi}{2}\right)$$

$$\frac{E}{\delta u_\infty^3/2} = \sin^3\left(\frac{y}{\delta_L} \frac{\pi}{2}\right)$$

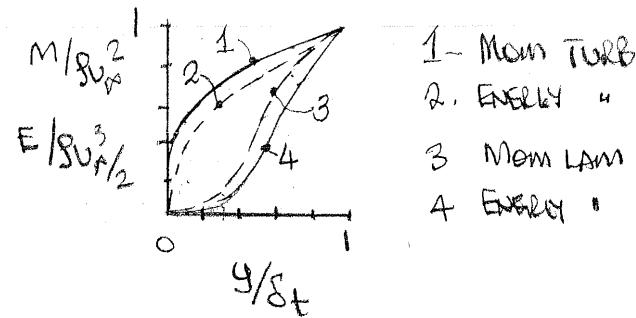
$y/\delta_L$	$\sin \frac{y \pi}{\delta_L 2}$	$\frac{M}{\delta u_\infty^2}$	$\frac{E}{\delta u_\infty^3/2}$
0	0	0	0
0.1	0.156	0.0244	0.0038
0.3	0.455	0.207	0.094
0.5	0.701	0.50	0.355
0.7	0.89	0.795	0.708
0.9	0.99	0.98	0.97
1	1.0	1.0	1.0

12.27 CONTINUED

FOR TURBULENT CASE:

$$\frac{M}{\delta u_\infty^2} = \left(\frac{y}{\delta_t}\right)^2 \quad \frac{E}{\delta u_\infty^3/2} = \left(\frac{y}{\delta_t}\right)^{3/2}$$

$y/\delta_t$	$\frac{M}{\delta u_\infty^2}$	$\frac{E}{\delta u_\infty^3/2}$
0	0	0
0.1	0.518	0.573
0.3	0.709	0.600
0.5	0.820	0.743
0.7	0.903	0.858
0.9	0.970	0.956
1	1	1



$$12.28 \quad f_D = C_{fL} A \delta U^2/2$$

$$A = (7)(40)(2) = 560 \text{ ft}^2 \quad \{2 \text{ sides}\}$$

$$U = 140 \text{ mph} = 205.3 \text{ ft/s}$$

$$@ 560 \text{ ft} - \delta = 0.0660 \text{ lbm/ft}^3$$

$$\mu = 1.165 \times 10^{-5} \text{ lbm/ft.s}$$

$$\rho_e = \frac{L_U}{N} = \frac{7(205.3)}{1.165 \times 10^{-5} / 0.0660} = 8.14 \times 10^6$$

12.28 (CONTINUED -

a) LAMINAR

$$C_{fL} = 1.328 \text{ } Re_L^{-1/2} = 0.000465$$

$$F_D = \frac{(0.000465)(560)(0.0000)(205.3)^2}{2(32.2)} = \underline{11.26 \text{ lbf}}$$

b) TURBULENT

$$C_{fL} = 0.012 \text{ } Re_L^{-0.2} = 0.00226$$

$$F_D = \underline{72.3 \text{ lbf}}$$

$$12.29 \quad Re_x = 10^6$$

B.L. THICKNESS -

$$\text{Lam: } \delta = 5 \times Re_x^{-1/2}$$

$$\text{Turb: } \delta = 0.376 \times Re_x^{-1/5}$$

$$\delta_t/\delta_L = \frac{0.376}{5} Re_x^{0.3} = \underline{4.74}$$

COEFF. OF SKIN FRICTION:

$$\text{Lam: } C_{fx} = 0.664 \text{ } Re_x^{-1/2}$$

$$\text{Turb: } C_{fx} = 0.0576 \text{ } Re_x^{-0.2}$$

$$\frac{C_{fxt}}{C_{fxL}} = \frac{0.0576}{0.664} Re_x^{0.3} = \underline{5.47}$$

12.30 FOR TURBULENT B.L. ~ WATER

$$@ 60^\circ \text{ F } \delta = 62.3 \text{ ft m/ft}^3$$

$$\rho = 1.22 \times 10^{-5} \text{ ft}^2/\text{s}$$

$$U = 20 \text{ ft/s}$$

$$Re_L = \frac{20(20)}{1.22 \times 10^{-5}} = 3,28 \times 10^7$$

12.30 (CONTINUED)

$$\delta = 0.376 \times Re_L^{-0.2}$$

$$= 0.376(20)(3.28 \times 10^7)^{-0.2} = \underline{0.2310 \text{ ft}}$$

$$C_{fL} = 0.012 \text{ } Re_L^{-0.2} = 0.00226$$

$$F_D = C_{fL} A g U^2/2$$

$$= \frac{(0.00226)(100)(2)(62.3)(20)^2}{2(32.2)} = \underline{350 \text{ lbf}}$$

$$= \underline{350 \text{ lbf}}$$

IF FLOW IS LAMINAR -

$$C_{fL} = 1.328 \text{ } Re_L^{-1/2} = 2.319 \times 10^{-4}$$

$$F_D = \underline{35.91 \text{ lbf}}$$

12.31 EXPANDING  $U_x(x,y)$  IN TAYLOR SERIES:

$$\begin{aligned} U_x'(x,y) &= U_x'(0,0) + x \frac{\partial U}{\partial x}(0,y) + y \frac{\partial U}{\partial y}(x,0) \\ &\quad + \frac{x^2}{2} \frac{\partial^2 U}{\partial x^2}(0,y) + \frac{y^2}{2} \frac{\partial^2 U}{\partial y^2}(x,0) \\ &\quad + \frac{\partial^2 U}{\partial x \partial y}(0,0) + \dots \end{aligned}$$

AS  $y \rightarrow 0 \quad x' \rightarrow 0$ THE EXPRESSION FOR  $U_x'$  REDUCES TO

$$U_x'(x,y) = a_1 y + a_2 y^2 + a_3 x y + \dots$$

WHERE  $a_1 = \frac{\partial U_x}{\partial y}|_{(0,0)}$  - ETC.

SIMILARLY:

$$U_y'(x,y) = b_1 y + b_2 y^2 + b_3 x y + \dots$$

WHERE  $b_1 = \frac{\partial U_y}{\partial y}|_{(0,0)}$  - ETC.

12.31 CONTINUED

CONTINUITY EQUATION REQUIRES THAT

$$\frac{\partial U_x'}{\partial x} + \frac{\partial U_y'}{\partial y} = 0$$

GIVEN:

$$a_3y + b_1 + 2b_2y + b_3x = 0$$

COEFFICIENTS OF LINEAR FRS OF  $x^2y$

REQUIRE  $a_3 + 2b_2 = 0$

$$b_1 = b_3 = 0$$

$$\text{So } U_x'(x, y) = a_1y + a_2y^2 + a_3xy$$

$$U_y'(x, y) = -a_3y^2$$

$$U_x' U_y' = -a_3a_1y^3 - a_3a_2y^4$$

TAKING TIME AVERAGE

$$\overline{U_x' U_y'} = -a_3a_1y^3 + \dots$$

$$\text{i.e. } \overline{U_x' U_y'} \approx y^3$$

WHILE MIXING LENGTH THEORY

$$\text{SAYS } \overline{U_x' U_y'} \approx y^2$$

12.32 Power Law Profiles

$$\frac{U_x}{U_{\max}} = \left(\frac{y}{R}\right)^{1/n}$$

$$\frac{\partial U_x}{\partial y} = \frac{U_{\max}}{n} \frac{y^{1/n-1}}{R^{1/n}}$$

$$\text{As } y \rightarrow 0 \quad \frac{\partial U_x}{\partial y} \rightarrow \infty$$

$$\text{As } y \rightarrow R \quad \frac{\partial U_x}{\partial y} \rightarrow \frac{U_{\max}}{nR}$$

$$12.33 \quad \overline{U} = 0.0225 \frac{R}{U_{\max}} \left( \frac{y}{R} \right)^{1/4} \quad (1)$$

$$\text{For } \frac{U_x}{U_{\max}} = \left(\frac{y}{R}\right)^{1/n}$$

$$\frac{U_x}{U_{\max}^2} = \frac{1}{n} \int_0^y \left(1 - \frac{U_x}{U_{\max}}\right) dy$$

$$= \frac{1}{n} \int_0^y \left[ \left(\frac{y}{R}\right)^{1/n} - \left(\frac{y^2}{R^2}\right)^{1/n} \right] d\left(\frac{y}{R}\right)$$

$$= \left[ \frac{1}{1+1/n} - \frac{1}{1+2/n} \right] \frac{dy}{dx}$$

EQUATING WITH (1) & DOING ALGEBRA

$$0.0225 \left( \frac{R}{U_{\max} S} \right)^{1/4} = \frac{n}{(n+1)(n+2)} \frac{ds}{dx}$$

$$0.0225 \left( \frac{R}{U_{\max}} \right)^{1/4} \int_0^y ds = \frac{n}{(n+1)(n+2)} \int_0^y ds^{1/4}$$

BECOMES

$$\left(\frac{S}{X}\right)^{5/4} = \frac{(n+1)(n+2)}{n} (0.0281) Re_x^{-1/4}$$

& FINALLY

$$\frac{S}{X} = \left[ 0.0281 \frac{(n+1)(n+2)}{n} \right]^{0.8} Re_x^{-0.2}$$

## CHAPTER 13

13.1 OIL  $\rho = 0.08 \times 10^{-3} \text{ lb}^2/\text{s}$   
 $\rho = 57 \text{ lbm/ft}^3$   
 $\dot{V} = 10 \text{ gal/sec}$

TUBE - Diam = 0.24 in., L = 50 ft

$$V = \frac{\dot{V}}{\pi D^2/4} = 1.18 \text{ ft/s}$$

$$Re = \frac{DV}{\nu} = \frac{(0.24/12)(1.18)}{0.08 \times 10^{-3}} = 295$$

*Laminar*

$$\begin{aligned} \Delta P &= h_L = 2 f_F \frac{L}{D} V^2 \quad \{ f_F = 16/Re \} \\ &= 2 \frac{(16)}{295} \frac{50}{0.24/12} (1.18)^2 \\ &= 377.6 \text{ ft}^2/\text{s}^2 \end{aligned}$$

$$\Delta P = \frac{57(377.6)}{32.2} = 668 \text{ lbf/ft}^2$$

13.2 OIL - SAME PROPERTIES AS IN PROB 13.1

TUBE - D = 0.1 m, L = 30 m.

$$\Delta P = 15 \text{ kPa/m}^2$$

FOR LAMINAR FLOW - USE H-P. EQU.

$$\Delta P = 32 \mu V \Delta x / D^2$$

or  $\frac{\Delta P}{\rho g} = \frac{32 \mu V \Delta x}{g D^2}$

$$\begin{aligned} V &= \frac{(15)(144)(0.1/12)^2 (30/12)}{32(57)(0.08 \times 10^{-3})(30/12)} \\ &= 13.24 \text{ ft/s} \end{aligned}$$

$$\dot{V} = V A = 7.22 \times 10^{-4} \text{ ft}^3/\text{s}$$

$$Re = \frac{(0.1/12)(13.24)}{0.08 \times 10^{-3}} = 1379$$

*Laminar flow*  
*O.K.*

13.3  $\Delta P = 2 f_F \frac{L}{D} V^2$

FOR A SPECIFIED PIPE:  $\Delta P = f_F V^2$   
 IF FULLY TURBULENT -  $f_F \sim e/D$  ONLY  
 $\therefore \Delta P \sim V^2$

FOR  $H_2O$   $\Delta P = 13 \text{ psi}$  FOR  $\dot{m} = 28.3 \text{ lbm/s}$

FOR LOX  $\rho = 70 \text{ lbm/ft}^3$   $\dot{m} = 35 \text{ lbm/s}$

$$\begin{aligned} \frac{\Delta P_{Lox}}{\Delta P_{H2O}} &= \frac{(\dot{m}/SA)_{Lox}}{(\dot{m}/SA)_{H2O}}^2 \\ &= \left(\frac{35}{70}\right)^2 \left(\frac{62.4}{28.3}\right)^2 = 1.21 \end{aligned}$$

FOR LOX -  $\Delta P = 13(1.21) = 15.8 \text{ psi}$

13.4 ENERGY EQU:

$$\frac{dE_{ws}}{dt} = \dot{m} \left[ \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g \Delta y + h_L \right]$$

$$\text{OIL: } \rho = 810 \text{ kg/m}^3 \quad D = 4.5 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\dot{V} = 0.56 \text{ m}^3/\text{s}$$

$$\text{LINE: } D = 0.162 \text{ m} \quad y_2 - y_1 = -250 \text{ m}$$

$$\Delta P = 101.3 - 300 = -198.1 \text{ kPa}$$

COMMERCIAL STEEL

$$C = \dot{V}/A = \frac{0.56}{\pi/4(0.162)^2} = 1.855 \text{ m/s}$$

$$\frac{P_2 - P_1}{\rho} = \frac{+198.1(1000)}{810} = +248.3 \text{ m}^2/\text{s}^2$$

$$\frac{V_2^2 - V_1^2}{2} = 0$$

$$g \Delta y = 9.81(-250) = -2452 \text{ m}^2/\text{s}^2$$

### 13.4 CONTINUED -

$$Re = \frac{Dv}{\nu} = \frac{0.42(1855)}{4.5 \times 10^{-6}} = 256,000$$

for THIS Re VALUE & COMMERCIAL STEEL -  $\frac{\epsilon}{D} \approx 0.00075$

$$\text{Fig 13.1} - f_f \approx 0.0045$$

$$h_L = 2 \left( 0.0045 \right) \frac{(280000)}{0.42} (1855)^2$$

$$= 13590 \text{ m}^2/\text{s}^2$$

$$-\frac{dW_s}{dt} = (810)(0.56) \left[ +245.3 - 2452 + 13590 \right]$$

$$= \underline{5.34 \text{ MW}}$$

13.5 SAME CONDITIONS AS IN PROB 13.4 EXCEPT

2 PIPES IN SERIES -

270 KM OF ORIGINAL PIPE

10 KM OF NEW PIPE  
WITH  $D = 0.42 \text{ m}$

FOR THE NEW SYSTEM:

$$-\frac{dW_s}{dt} = m \left[ \frac{P_2 - P_1}{g} + \frac{V_2^2 - V_1^2}{2} + g\Delta y + h_L \right]$$

$$\frac{P_2 - P_1}{g} = \{ \text{SAME} \} = 245.3 \text{ m}^2/\text{s}$$

$$\frac{\Delta V^2}{2} = \{ \text{SAME} \} = 0$$

$$g\Delta y = \{ \text{SAME} \} = -2452 \text{ m}^2/\text{s}$$

$$h_L = h_{L1} + h_{L2}$$

1 → ORIGINAL

2 → NEW

### 13.5 CONTINUED -

$$h_L = \frac{270000}{280000} (13590) = 13490 \text{ m}^2/\text{s}^2$$

FOR NEW SECTION:

$$v = \frac{0.54}{\pi/4 (0.42)^2} = 4.04 \text{ m/s}$$

$$Re = \frac{0.42(4.04)}{4.5 \times 10^{-6}} = 3,773 \times 10^5$$

$$\frac{\epsilon}{D} = 0.00012 \quad f_f \approx 0.0038$$

$$h_L = 2 \left( 0.0038 \right) \frac{10000}{0.42} (4.04)^2$$

$$= 2753 \text{ m}^2/\text{s}^2$$

$$\text{TOTAL } h_L = 13490 + 2753 = 16440 \text{ m}^2/\text{s}^2$$

NEW GASE -

$$-\frac{dW_s}{dt} = (810)(0.56) [245.3 - 2452 + 16440]$$

$$= \underline{6.46 \text{ MW}}$$

13.6 STEADY FLOW BETWEEN PUMPING STATIONS

$$0 = \frac{P_2 - P_1}{g} + \frac{V_2^2 - V_1^2}{2} + g\Delta y + h_L$$

$$\Delta V^2/2 = 0$$

$$\Delta y = 0$$

$$\text{so } \frac{\Delta P}{g} = h_L = 2 f_f \frac{L}{D} V^2$$

$$Re = \frac{DV}{\nu} = \frac{(0.71)(1.1)}{6.7 \times 10^{-6}} = 1,166 \times 10^5$$

$$\frac{\epsilon}{D} = 0.000068 \quad f_f \approx 0.0046$$

$$h_L = 2 \left( 0.0046 \right) \frac{(320 \times 10^3)}{0.71} (1.1)^2$$

$$= 5017 \text{ m}^2/\text{s}^2$$

$$\Delta P = h_L/g = \underline{511 \text{ m OF OIL}}$$

13.6 CONTINUED -

$$\begin{aligned} -\frac{\delta ws}{dt} &= \dot{m}gh_L \\ &= 801 \left(\frac{\pi}{4}\right)(0.71)^2(1.1)(9.81)(511) \\ &= \underline{\underline{1,749 \text{ MW}}} \end{aligned}$$

13.7 ENERGY EQUATION IN STEADY FLOW.

$$\frac{P_2 - P_1}{g} + \frac{V_2^2 - V_1^2}{2} + g\Delta y + h_L = 0$$

$$-\frac{\Delta P}{g} = -\frac{60(144)(32.2)}{62.4} = -4460 \text{ ft}^2/\text{s}$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{V_2^2}{2}$$

$$g\Delta y = 0$$

$$h_L = 2f_f \frac{L}{D} V^2 + \sum K \frac{V^2}{2}$$

$$= V^2 \left[ 2f_f \frac{L}{D} + \frac{1}{2} \sum K \right]$$

$$\sum K = (6)(0.7) + 3.8 + 7.5 = 15.5$$

$$2f_f \frac{L}{D} = 2(0.001) \frac{160}{0.75/12} = 35.84$$

$$h_L = V^2 \left[ 35.84 + 7.75 \right] = 43.6 V^2$$

$$V_2 = \frac{VA}{A_2} = V \left( \frac{D}{D_2} \right)^2 = V \left( \frac{0.75}{0.11} \right)^2 = 56.25 V$$

ENERGY EQUATION BECOMES

$$-4460 + \frac{1}{2} (56.25 V)^2 + 43.6 V^2 = 0$$

$$V^2 = \frac{4460}{1465} = 2.74 \text{ ft}^2/\text{s}^2$$

$$V = 1.656 \text{ ft/s}$$

13.7 CONTINUED

NOW TO CHECK  $f_f$ :

$$\begin{aligned} Re &= \left(\frac{0.75}{12}\right)(1.656)/(1.12 \times 10^{-5}) \\ &= 8480 \end{aligned}$$

$$\frac{f_f}{D} = \frac{5 \times 10^{-6} (R)}{0.75} = 0.00008$$

$$\text{FIGURE 13.1} - f_f \approx 0.75$$

THIS MAKES A NEGLIGIBLE CHANGE  
IN THE  $h_L$  CALCULATION - ..

$$V = 1.656 \text{ ft/s}$$

$$\dot{V} = \frac{\pi}{4} \left(\frac{0.75}{12}\right)^2 (1.656) = 0.0051 \text{ ft}^3/\text{s}$$

13.8

FOR THIS CASE -

$$\frac{\Delta P}{g} + \frac{\Delta V^2}{2} + g\Delta y + h_L = 0$$

$$-\frac{\Delta P}{g} = -\frac{4.55(144)(32.2)}{62.4} = -338.1 \text{ ft}^2/\text{s}^2$$

$$\Delta V^2 = 0$$

$$g\Delta y = 0$$

$$h_L = 2f_f \frac{L}{D} V^2$$

$$V = \frac{118 \text{ ft}^3/\text{m}}{(60) \pi/4 (D^2)} = \frac{2.50}{D^2}$$

$$h_L = 2f_f \frac{2.50}{D} \left( \frac{2.50}{D^2} \right)^2 = \frac{3125 f_f}{D^5}$$

GOVERNING EQUATION IS

$$-338.1 + \frac{3125}{D^5} f_f = 0$$

$$f_f = 0.1082 D^5$$

OTHER CONSTRAINT IS  $f_f(Re) \sim \text{FIGURE 13.1}$

13.8 CONTINUED -

$$Re = \frac{DV}{\nu} = \frac{DV}{\pi D^2/4} = \frac{118}{60(\pi)D/4(1.22 \times 10^{-5})} = \frac{2.052 \times 10^5}{D}$$

TRIAL & ERROR -

ASSUME  $f_f = 0.004$

$$D = \left[ \frac{0.004}{0.0062} \right]^{1/5} = 0.517 \text{ ft}$$

$$Re = \frac{2.052 \times 10^5}{0.517} = 3.97 \times 10^5$$

Fig 13.1 -  $f_f = 0.00325$

USING THIS VALUE -

$$D = 0.496 \text{ ft} \quad Re = 4.137 \times 10^5$$

$$f_f = 0.0031$$

$$\Rightarrow D = 0.491 \text{ ft} \quad (5.9 \text{ in})$$

13.9 ENERGY EQU - STEADY FLOW

$$\frac{\Delta P}{g} + \frac{\Delta V^2}{2} + gAy + h_L = 0$$

LOCATION 1 - SUMP ( $V_1=0$ )

2 - PUMP INLET

$$\frac{\Delta P}{g} = \frac{P_2 - P_{atm}}{g} = \frac{P_{2g}}{g}$$

$$V_2 = \frac{V}{\pi D^2/4} = \frac{500}{(1.48)(60)(\frac{\pi}{4})(6/12)^2} = 5.67 \text{ ft/s}$$

$$\Delta V^2/2 = 16.1 \text{ ft}^2/\text{s}^2$$

13.9 CONTINUED -

$$gAy = 9.81(3) = 29.4 \text{ ft}^2/\text{s}^2$$

$$h_L = 2f_f \frac{L}{D} V^2$$

$$Re = \frac{(6/12)(5.67)}{1.22 \times 10^{-5}} = 2.32 \times 10^5$$

$$e/D = 0.003$$

$$f_f (Fig 13.1) \approx 0.0066$$

$$h_L = 2(0.0066)\left(\frac{6}{0.5}\right)(5.67)^2 = 5.05 \text{ ft}^2/\text{s}^2$$

SUBSTITUTING INTO ENERGY EQU:

$$-\frac{P_{2g}}{g} = 16.1 + 29.4 + 5.05 = 50.6 \text{ ft}^2/\text{s}^2$$

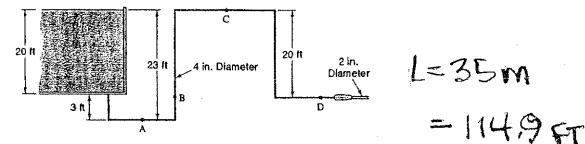
$$-\frac{P_{2g}}{g} = \frac{62.4(50.6)}{32.2} = \underline{\underline{98.0 \text{ ft}^2/\text{s}^2}}$$

$$P_2 = -0.681 \text{ PSIG}$$

$$P_2 \text{ absolute} = 14.7 - 0.681$$

$$= \underline{\underline{14.02 \text{ PSIA}}}$$

13.10



BETWEEN RESERVOIR SURFACE (1)  
& NOZZLE EXIT (2)

$$\frac{P_2 - P_1}{g} + \frac{V_2^2 - V_1^2}{2} + gAy + h_L = 0$$

$$\Delta P = 0$$

$$V_1^2 = 0$$

$$gAy = -(32.2)(20) = +644 \text{ ft}^2/\text{s}^2$$

$$\text{IN Pipe: } V_p = V_2 \frac{A_2}{A_1} = V_2 \left( \frac{D_2}{D_1} \right)^2 = \frac{V_2}{4}$$

$$98 \quad V_p^2 = V_2^2/16$$

## 13.10 CONTINUED

$$h_L = 2 f_F \frac{L}{D} U_p^2 + \sum K \frac{U_p^2}{2}$$

$$= U_p^2 \left[ 2 f_F \frac{114.9}{4/12} + \frac{\sum K}{2} \right]$$

$$\sum K = (5)(0.7) + 1$$

ELBOWS ENTRANCE

$$h_L = U_p^2 [689.4 f_F + 2.25]$$

ENERGY EQUATION IS

$$\frac{U_p^2}{2} - 644 + U_p^2 [ ] = 0$$

$$\text{OR } U_p^2 [689.4 f_F + 2.25] = 644$$

TRIAL  $\frac{1}{2}$  ERROR -

$$\text{ASSUME } f_F = 0.005$$

$$U_p = 10.29 \text{ FT/s}$$

$$Re = \frac{(4/12)(10.29)}{1.22 \times 10^{-5}} = 2,811 \times 10^5$$

$$\epsilon/D = 0.0005$$

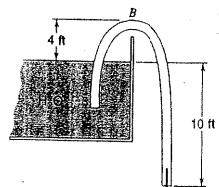
$$\text{From 13.1} - f_F \approx 0.0045$$

$$\text{WITH } f_F = 0.0045 \quad U_p = 10.6 \text{ FT/s}$$

Re CHECKS:

$$\frac{1}{2} \dot{V} = \frac{\pi}{4} \left( \frac{4}{12} \right)^2 (10.6) = 0.925 \text{ FT}^3/\text{s}$$

13.11



$$L = 23 \text{ FT}$$

$$D = 1 \text{ INCH}$$

BETWEEN RESERVOIR SURFACE (1)  
& EXIT (2)

## 13.11 CONTINUED

$$\frac{P_B - P_1}{\rho g} + \frac{U_B^2 - U_1^2}{2} + g \Delta y + h_L = 0$$

$$\Delta P = 0$$

$$U_1 = 0$$

$$g \Delta y = 32.2(-10) = -322 \text{ FT}^2/\text{s}^2$$

$$h_L = 2 f_F \frac{L}{D} U^2 + \sum K \frac{U^2}{2}$$

$$= 2 f_F \frac{23}{1/2} U^2 + U^2 \sum K / 2$$

 $\sum K = 1$  - ENTRANCE LOSS

$$h_L = U^2 [552 f_F + 0.5]$$

ENERGY EQUATION IS

$$\frac{U^2}{2} - 322 + U^2 [552 f_F + 0.5] = 0$$

$$U^2 [552 f_F + 1] = 322$$

TRIAL  $\frac{1}{2}$  ERROR:

$$\text{ASSUME } f_F = 0.005$$

$$U = 9.25 \text{ FT/s}$$

$$Re = \frac{(1/2)(9.25)}{1.22 \times 10^{-5}} = 6,32 \times 10^4$$

From 13.1 - SMOOTH TUBE -  $f_F = 0.0047$ For  $f_F = 0.0047 \quad U = 9.46 \text{ FT/s}$ 

$$Re = 6,446 \times 10^4 \quad f_F = 0.0047$$

$$\dot{V} = \frac{\pi}{4} \left( \frac{1}{2} \right)^2 (9.46) = 0.0516 \text{ FT}^3/\text{s}$$

BETWEEN (2) &amp; B

$$\frac{P_B - P_2}{\rho g} + \frac{U_B^2 - U_2^2}{2} + g \Delta y + h_L = 0$$

13.11 CONTINUED -

$$\frac{P_B - P_2}{8} = \frac{P_{fg}}{8}$$

$$\frac{V_B^2 - V_2^2}{2} = 0$$

$$g\Delta y = 32.2(14) = 450.8 \text{ FT}^2/\text{s}^2$$

$$h_L = 2f_F \frac{L}{D} V^2 + \sum K \frac{V^2}{2}$$

$$= \frac{2(0.004f)(14)(9.46)^2}{1/12}$$

$$= 141.3 \text{ FT}^2/\text{s}^2$$

INTO ENERGY EQUATION:

$$\frac{P_{fg}}{8} = -450.8 - 141.3 = -592.1 \text{ FT}^2/\text{s}^2$$

$$P_{Bfg} = -\frac{(592.1)(62.4)}{32.2} = -1147 \text{ PSF}$$

$$= -7.91 \text{ PSI}$$

$$P_{B\text{ABSOLUTE}} = 14.7 - 7.91 = 6.73 \text{ PSI}$$

13.12 RECTANGULAR DUCT - 8" x 8" x 25 FT

$$\dot{V} = 600 \text{ FT}^3/\text{m} \text{ STP AIR}$$

$$D_{eq} = \frac{4(8)(8)}{4/8} = 8 \text{ IN}$$

$$V = \frac{600/60}{8(8)/144} = 22.5 \text{ FT/s}$$

ENERGY EQUATION REDUCES TO

$$\frac{\Delta P}{8} = 2f_F \frac{L}{D} V^2$$

$$Re = \frac{(8/12)(22.5)}{1.56 \times 10^{-5}} = 9.59 \times 10^4$$

13.12 CONTINUED -

$$e/D = \frac{0.0005}{8/12} = 0.00075$$

$$\text{Fig 13.1} - f_F \approx 0.0054$$

$$\frac{\Delta P}{8} = 2(0.0054) \frac{25}{8/12} (22.5)^2 = 105 \text{ FT}^2/\text{s}^2$$

$$\Delta P = \frac{105(0.0766)}{32.2} = 0.4876 \text{ PSF}$$

$$= 6.366 \text{ FT AIR} = 76.4 \text{ IN AIR}$$

$$= (76.4) \frac{0.0766}{62.4} = 0.0938 \text{ IN H}_2\text{O}$$

13.13 ENERGY EQUATION:

$$\frac{\Delta P}{8} + \frac{\Delta V^2}{2} + g\Delta y + h_L = 0$$

$$g\Delta y = (32.2)(175) = 5635 \text{ FT}^2/\text{s}^2$$

$$\dot{V} = 3 \times 10^6 \frac{\text{GAL}}{\text{DAY}} = 4.642 \text{ FT}^3/\text{s}$$

$$V = \frac{4.642}{\frac{\pi}{4} D^2} = \frac{5.91}{D^2} \text{ FT/s}$$

$$h_L = 2f_F \frac{L}{D} V^2 = 2f_F \frac{10560}{D} \frac{V^2}{2}$$

$$= 2.112 \times 10^4 \frac{f_F V^2}{D^2}$$

$$\text{FOR 10-10 PIPE: } V = \frac{5.91}{(10/12)^2} = 8.51 \text{ FT/s}$$

$$f_F = \frac{(10/12)(8.51)}{1.22 \times 10^{-5}} = 5.81 \times 10^5$$

$$e/D = 0.00011 - f_F = 0.0051$$

$$h_L = \frac{2.112 \times 10^4 (0.0051)(8.51)^2}{(10/12)^2}$$

$$= 19410 \text{ FT}^2/\text{s}^2$$

### B.13 (CONTINUED)

For 12" pipe:  $V = 5.91 \text{ ft/s}$   
 $Re = 4,84 \times 10^5$   $c_f = 0.00085$   
 $f_f \approx 0.0048$   $h_L = 3540 \text{ ft}^2/\text{s}^2$

For 14" pipe:  $V = 4.34 \text{ ft/s}$   
 $Re = 4,15 \times 10^5$   $c_D = 0.00073$   
 $f_f \approx 0.0047$   $h_L = 865 \text{ ft}^2/\text{s}^2$

$$\begin{aligned} \text{Cost/yr} &= (\text{Power}) + \left\{ \frac{1}{20} \left( \frac{\text{Initial}}{\text{lost}} \right) \right. \\ &\quad \left. + 0.06 \left( \frac{\text{Initial}}{\text{lost}} \right) \right\} \\ &= (\text{Power lost}) + 0.11 \left\{ \frac{\text{Initial}}{\text{lost}} \right\} \end{aligned}$$

$\text{Power lost} = \$ \frac{0.07}{\text{kWh}} (P)$

$$\begin{aligned} P &= m(h_L + g\Delta y)(1.356)(3.65)(24) \\ &= 106,860(h_L + g\Delta y) \text{ kWh} \end{aligned}$$

For 10-in pipe:

$$\begin{aligned} \text{CPY} &= 0.11 (\$11.40)(2)(5280) \\ &\quad + \$0.07 (106,860)(19410 + 5635) \\ &= \$260,580 \end{aligned}$$

For 12" pipe -

$$\begin{aligned} \text{CPY} &= 0.11 (\$14.70)(2)(5280) \\ &\quad + \$0.07 (106,860)(3540 + 5635) \\ &= \$85,670 \end{aligned}$$

For 14" pipe -

$$\begin{aligned} \text{CPY} &= 0.11 (\$16.80)(2)(5280) \\ &\quad + \$0.07 (106,860)(865 + 5635) \\ &= \$68,108 \quad \leftarrow \text{GREATEST} \end{aligned}$$

### B.14 ENERGY EQUATION Reduces to

$\frac{\Delta P}{g} + 2f_f \frac{L}{D} V^2 = 0$

$$\begin{aligned} -\frac{\Delta P}{g} &= \frac{P_1 - P_{\text{ATM}}}{g_w} = \frac{P_{14}}{g_w} = \frac{40 \text{ psi}}{g_w} \\ &= \frac{40(144)(32.2)}{62.4} = 2970 \text{ ft}^2/\text{s}^2 \end{aligned}$$

$2f_f \frac{L}{D} V^2 = 2f_f \frac{50}{0.5/12} V^2 = 2400 f_f V^2$

~ For 1/2-in. diam hose -

into Energy Eqn:  $f_f V^2 = 1,2375$

TRIAL & ERROR:

ASSUME  $f_f = 0.005$   $V = 15.73 \text{ ft/s}$

$Re = \frac{(0.5)(15.73)}{1.22 \times 10^{-5}} = 5,373 \times 10^4$

fit B.1 - ASSUME Smooth -  $f_f = 0.0049$

WITH  $f_f = 0.0049$   $V = 15.89 \text{ ft/s}$

$Re = 5,427 \times 10^4 \rightarrow f_f = 0.0049$

∴ for 1/2-in. hose -  $V = 15.89 \text{ ft/s}$

$V = 15.89 \left( \frac{\pi}{4} \right) \left( \frac{0.5}{12} \right)^2 = 0.0217 \text{ ft}^3/\text{s}$

For 3/4-in diam hose:

$h_L = 1600 f_f V^2 \sim f_f V^2 = 1,856$

- ASSUME  $f_f = 0.004$  -  $V = 21.54 \text{ ft/s}$

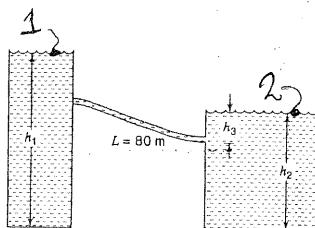
$Re = \frac{(0.75/2)(21.54)}{1.22 \times 10^{-5}} = 1,1035 \times 10^5 \quad f_f = 0.0042$

WITH  $f_f = 0.0042$   $V = 21.02 \text{ ft/s}$

$Re = 1,077 \times 10^5 \quad f_f = 0.00425$

$V = 20.9 \text{ ft/s} \quad V = 0.0641 \text{ ft}^3/\text{s}$

13.15



$$h_1 = 60 \text{ m}, h_2 = 30 \text{ m}, h_3 = 8 \text{ m}$$

$$L = 80 \text{ m} \quad D = 0.35 \text{ m}$$

ENERGY EQUATION - BETWEEN 1 &amp; 2

$$\frac{\Delta P}{\rho} + \frac{\Delta U^2}{2} + g \Delta y + h_L = 0$$

$$\frac{\Delta P}{\rho} = 0 \quad \frac{\Delta U^2}{2} = 0$$

$$g \Delta y = -(9.81)(30) = -294.3 \text{ m}^2/\text{s}^2$$

$$h_L = 2f_F \frac{L}{D} U^2 + \sum K \frac{U^2}{2}$$

$$= 2(0.004) \left( \frac{80}{0.35} \right) U^2$$

$$+ 0.5 U^2$$

$$= 2,329 U^2$$

INTO ENERGY EQUATION:

$$2,329 U^2 = 294.3$$

$$U = 11.24 \text{ m/s}$$

$$\textcircled{a}) \quad \dot{V} = 11.24 \left( \frac{\pi}{4} \right) (0.35)^2 = 1,082 \text{ m}^3/\text{s}$$

$$\text{FOR } C_D = 0.004$$

$$h_L = 2f_F \frac{80}{0.35} U^2 = 457 f_F U^2$$

INTO ENERGY EQUATION:

$$457 f_F U^2 = 294.3 \quad f_F U^2 = 0.6438$$

TRIAL &amp; ERROR:

$$\text{ASSUME } f_F = 0.0072$$

$$U = 9.46 \text{ m/s}$$

13.15 CONTINUED-

$$Re = \frac{(0.35)(9.46)}{0.995 \times 10^{-6}} = 3,328 \times 10^6$$

$$\text{fully TURBULENT} - f_F = 0.0072$$

$$\therefore U = 9.46 \text{ m/s} \quad \dot{V} = 0.910 \text{ m}^3/\text{s}$$

13.16 ENERGY EQUATION:

$$\frac{\Delta P}{\rho} + \frac{\Delta U^2}{2} + g \Delta y + h_L = 0$$

$$\frac{\Delta U^2}{2} = 0$$

$$g \Delta y = (9.81)(-6.8) = -6553 \text{ m}^2/\text{s}^2$$

$$h_L = 2f_F \frac{L}{D} U^2$$

$$\dot{V} = 90 \text{ m}^3/\text{s} \quad U = \frac{90}{\sqrt[4]{(5)^2}} = 4.584 \text{ m/s}$$

$$Re = \frac{5(4.584)}{0.995 \times 10^{-6}} = 1.3 \times 10^7$$

$$C_D \approx \frac{(0.003 \text{ ft})(0.3048)}{5} = 0.00018$$

$$f_F \approx 0.0034$$

$$h_L = 2(0.0034) \left( \frac{8000}{5} \right) (4.584)^2 = 228.6 \text{ m}^2/\text{s}^2$$

INTO ENERGY EQUATION:

$$\frac{\Delta P}{\rho} = 6553 - 228.6 = 6324 \text{ m}^2/\text{s}^2$$

$$\Delta P = 6324 (1000) = 6324 \times 10^3 \text{ N/m}^2 \\ = 6,324 \text{ MPa}$$

13.17 GATE VALVE -

$$\frac{\Delta P}{g} = K \frac{V^2}{2}$$

$$P_1 = 234 \text{ kPa} \quad P_2 = P_{\text{atm}} = 101.4 \text{ kPa}$$

$$\Delta P = 134.6 \text{ kPa} \quad \frac{\Delta P}{g} = 134.6 \frac{\text{m}^2}{\text{s}^2}$$

a) VALVE FULLY OPEN:  $K = 0.15$

$$V = \left[ \frac{(134.6)^2}{0.15} \right]^{1/2} = 42.36 \text{ m/s}$$

$$\dot{V} = (42.36) \left( \frac{\pi}{4} \right) (0.2)^2 = 1331 \text{ m}^3/\text{s}$$

b) VALVE 1/4 CLOSED -  $K = 0.85$

$$V = 17.8 \text{ m/s} \quad \dot{V} = 0.559 \text{ m}^3/\text{s}$$

c) VALVE 1/2 CLOSED -  $K = 4.4$

$$V = 7.82 \text{ m/s} \quad \dot{V} = 0.246 \text{ m}^3/\text{s}$$

d) VALVE 3/4 CLOSED -  $K = 20$

$$V = 3.67 \text{ m/s} \quad \dot{V} = 0.115 \text{ m}^3/\text{s}$$

13.18  $h_L = 2 f_f \frac{L}{D} V^2$

$$Re = \frac{DV}{\nu} = \frac{(0.18)(34)}{0.995 \times 10^{-6}} = 6.15 \times 10^6$$

$$\epsilon_D = 0.0014 \quad f_f = 0.0053$$

$$h_L = 2(0.0053) \frac{400}{0.18} (34)^2$$

$$= 27230 \text{ m}^2/\text{s}^2$$

$$= 2716 \text{ m of H}_2\text{O}$$

13.19  $H_2\text{O} @ 15^\circ\text{C}$   $\frac{\Delta P}{g} = 0.50 \text{ m}$

$$L = 300 \text{ m} \quad D = 2.20 \text{ m}$$

$$\nu = 1.195 \times 10^{-6} \text{ m}^2/\text{s}$$

$$h_L = 2 f_f \frac{L}{D} V^2$$

13.19 (CONTINUED)

$$Re = \frac{DV}{\nu} = \frac{(2.2)(V)}{1.195 \times 10^{-6}} = 1.841 \times 10^6 V$$

$$h_L = 9.81(0.5) = 2 f_f \frac{300}{2.2} V^2$$

$$f_f V^2 = 0.0799$$

TRIAL & ERROR -

Assume TURBULENT FLOW - SMOOTH PIPE

Assume  $f_f = 0.003$

$$V = 2.448 \text{ m/s} \quad Re = 4.508 \times 10^6$$

$$f_{16.1} - f_f = 0.0022$$

$$V = 2.86 \text{ m/s} \quad Re = 5.26 \times 10^6$$

$$f_{16.1} - f_f \approx 0.0021$$

$$V = 2.93 \text{ m/s} \rightarrow \text{CLOSE ENOUGH}$$

$$\dot{V} = 2.93 \left( \frac{\pi}{4} \right) (2.2)^2 = 11.13 \text{ m}^3/\text{s}$$

13.20 ENERGY EQUATION:

$$\frac{\Delta P}{g} + \frac{\Delta V^2}{2} + g \Delta y + h_L = 0$$

$$\frac{\Delta P}{g} = 0$$

$$\frac{\Delta V^2}{2} = \frac{V_2^2}{2}$$

$$g \Delta y = 9.81(-16.9) = -165.8 \text{ m}^2/\text{s}^2$$

$$h_L = 2 f_f \frac{L}{D} V^2 = 2 f_f \frac{30}{0.6} V^2 = 600 f_f V^2$$

INTO ENERGY EQN:

$$V^2 [600 f_f + 0.5] = 165.8$$

TRIAL & ERROR -

0.6-m CAST IRON PIPE -  $\epsilon_D = 0.00045$

13.20 CONTINUED -

$$Re = \frac{0.16V}{0.995 \times 10^{-6}} = 6.03 \times 10^5 V$$

ASSUME  $f_f = 0.003$   $V = 6.36 \text{ m/s}$

$$Re = 3.83 \times 10^6 \quad f_f = 0.0041$$

THIS IS IN FLOW TURBULENT REGION

$\therefore f_f$  IS  $0.0041 \nparallel V = 7.48 \text{ m/s}$

$$\dot{V} = (7.48) \left(\frac{\pi}{4}\right) (0.16)^2 = 2.116 \text{ m}^3/\text{s}$$

13.21  $D = 0.15 \text{ m}$   $L = 100 \text{ m}$

$$20^\circ \text{C } H_2O - \lambda = 0.995 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\Delta P = 30 \text{ kPa} \sim \frac{\Delta P}{8} = 30 \text{ m}^2/\text{s}^2$$

WEIGHT IRON PIPE  $\frac{\rho}{D} = 0.00035$

$$Re = \frac{(0.15)V}{0.995 \times 10^{-6}} = 1.507 \times 10^5 V$$

ENERGY EQUATION:  $\frac{\Delta P}{8} + h_L = 0$

$$2f_f \frac{100}{0.15} V^2 = 30 \quad f_f V^2 = 0.0225$$

TRIAL  $\nparallel$  ERROR

ASSUME  $f_f = 0.004$   $V = 2.37 \text{ m/s}$

$$Re = 3.574 \times 10^5 \quad f_f \approx 0.0042$$

$\therefore f_f = 0.0042$   $V = 2.31 \text{ m/s}$

$$f_f = 3.488 \times 10^5 \quad f_f = 0.0042$$

$$\dot{V} = (2.31) \left(\frac{\pi}{4}\right) (0.15)^2 = 0.0408 \text{ m}^3/\text{s}$$

13.22  $\Delta P = 1.3 \text{ m H}_2O$   $L = 10 \text{ m}$

$$D = 0.2 \text{ m} \quad \epsilon = 0.0004 \text{ m}$$

ASSUME  $20^\circ$   $\lambda = 0.995 \times 10^{-6} \text{ m}^2/\text{s}$

$$\frac{\Delta P}{8} = 1.3(9.81) = 12.75 \text{ m}^2/\text{s}^2$$

13.22 CONTINUED -

$$\text{BROWNIEN FORM} - \frac{\Delta P}{8} = 2 f_f \frac{L}{D} V^2$$

$$12.75 = 2 f_f \frac{10}{0.2} V^2 = 100 f_f V^2$$

$$f_f V^2 = 0.1275$$

$$Re = \frac{0.2V}{0.995 \times 10^{-6}} = 2.01 \times 10^5 V$$

ASSUME SMOOTH PIPE -

①  $f_f = 0.004$   $V = 5.646 \text{ m/s}$

$$f_f = 1.135 \times 10^6 \quad f_f \approx 0.002565$$

②  $f_f = 0.003$   $V = 6.52 \text{ m/s}$

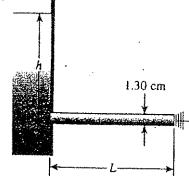
$$f_f = 1.31 \times 10^6 \quad f_f \approx 0.0027$$

③  $f_f = 0.0027$   $V = 6.87 \text{ m/s}$

$$f_f = 1.381 \times 10^6 \quad f_f \approx 0.0027$$

$$\dot{m} = (6.87) \left(\frac{\pi}{4}\right) (0.2)^2 (1000) = 0.216 \text{ kg/s}$$

13.23



$$\dot{V} = 5.675 \times 10^{-4} \text{ m}^3/\text{s}$$

$$L = 20 \text{ m}$$

PIPE IS 6cm STEEL

$$\frac{\rho}{D} = 2.446 \times 10^{-5}$$

$$\frac{\Delta P}{8} = h_L + \frac{\Delta K V^2}{2}$$

$$= 2 f_f \frac{L}{D} V^2 + 0.5 V^2 \quad \left\{ \text{FOR ENTRANCE} \atop K=1 \right.$$

$$V = \frac{5.675 \times 10^{-4}}{\pi/4 (0.013)^2} = 4.275 \text{ m/s}$$

$$Re = \frac{(0.013)(4.275)}{0.995 \times 10^{-6}} = 55900$$

Fig 13.1 -  $f_f \approx 0.0049$

13.23 CONTINUED

$$\frac{\Delta P}{g} = V^2 \left[ 2(0.0049) \left( \frac{20}{0.03} \right) + 0.5 \right]$$

$$= 284.7 \text{ m}^2/\text{s}^2$$

$$h = \frac{284.7}{g} = \underline{29.02 \text{ m}}$$

13.24  $\dot{V} = 0.25 \text{ m}^3/\text{s}$ 

$$\text{PIPE 1: } V = \frac{0.25}{\frac{\pi}{4}(0.16)^2} = 12.43 \text{ m/s}$$

$$\text{PIPE 2 } V = \frac{0.25}{\frac{\pi}{4}(0.18)^2} = 9.82 \text{ m/s}$$

$$\text{PIPE 3 } V = \frac{0.25}{\frac{\pi}{4}(0.2)^2} = 7.96 \text{ m/s}$$

$$\text{PIPE 1 - } \frac{\Delta P}{g} = 2f_f \frac{L}{D} V^2$$

$$Re = \frac{0.16(12.43)}{0.995 \times 10^{-6}} = 1.998 \times 10^6$$

$$\epsilon/D = 0.0055 - f_f = 0.0019$$

$$\frac{\Delta P}{g} = \frac{2(0.0019)(900)(12.43)^2}{0.16(9.81)} = \underline{1400 \text{ m}}$$

PIPE 2 -

$$Re = \frac{0.18(9.82)}{0.995 \times 10^{-6}} = 1.776 \times 10^6$$

$$\epsilon/D = 0.005 \quad f_f = 0.0075$$

$$\frac{\Delta P}{g} = \frac{2(0.0075)(1500)(9.82)^2}{0.18(9.81)} = \underline{1241 \text{ m}}$$

PIPE 3 -

$$Re = \frac{0.2(7.96)}{0.995 \times 10^{-6}} = 1.6 \times 10^6$$

$$\epsilon/D \approx 0.0045 \quad f_f \approx 0.0073$$

$$\frac{\Delta P}{g} = \frac{2(0.0073)(800)(7.96)^2}{0.2(9.81)} = \underline{377 \text{ m}}$$

13.25

Pipe	Length, m	Diameter, cm	Roughness, mm
1	125	8	0.240
2	150	6	0.120
3	100	4	0.200

PIPES IN SERIES -  $H_2O @ 20^\circ C - V = 0.995 \times 10^6 \text{ m}^2/\text{s}$ 

$$V_1 = \dot{V} / \pi D_1^2 = 199 \text{ m/s}$$

$$V_2 = \dots = 354 \text{ m/s}$$

$$V_3 = \dots = 796 \text{ m/s}$$

$$\frac{\Delta P}{g} = h_{L1} + h_{L2} + h_{L3} + 5(9.81)$$

$$h_{L1} = 2f_f \frac{125}{0.08} (199)^2 = 1.238 \times 10^8 f_f V^2$$

$$h_{L2} = 2f_f \frac{150}{0.06} (354)^2 = 6.266 \times 10^8 f_f V^2$$

$$h_{L3} = 2f_f \frac{100}{0.04} (796)^2 = 3.176 \times 10^9 f_f V^2$$

$$\text{PIPE 1 - } \epsilon/D = \frac{0.24}{80} = 0.003$$

Assume Fully Turbulent -  $f_f = 0.0065$ 

$$\text{PIPE 2 - } \epsilon/D = \frac{0.12}{60} = 0.002$$

~ SAME ASSUMPTION -  $f_f = 0.00585$ 

$$\text{PIPE 3 } \epsilon/D = \frac{0.20}{40} = 0.005$$

~ SAME ASSUMPTION  $f_f = 0.0077$ 

$$\sum h_L = \dot{V}^2 \left[ 8.047 \times 10^5 + 36.66 \times 10^5 + 244.55 \times 10^5 \right] = 289.3 \times 10^5 \text{ m}^2/\text{s}^2$$

$$\sum h_L = \frac{\Delta P}{g} + gA \frac{V}{2} = 180 + 9.81 \cdot 276.3 = 276.3 \text{ m}^2/\text{s}^2$$

Solving -  $\dot{V} = 0.00309 \text{ m}^3/\text{s}$ 

$$V_1 = 0.615 \text{ m/s } Re_1 = 4.94 \times 10^4 \quad f_f = 0.0071$$

$$V_2 = 1.094 \text{ m/s } Re_2 = 6.160 \times 10^4 \quad f_f = 0.0065$$

$$V_3 = 2.440 \text{ m/s } Re_3 = 9.89 \times 10^4 \quad f_f = 0.0077$$

13.25 CONTINUED -

USING NEW VALUES FOR  $f_F$  -

$$\sum h_L = \left[ (8.79 + 40.73 + 245) \times 10^5 \right] \frac{V^2}{D}$$

$$= 294.5 \times 10^5 \frac{V^2}{D} = 276.3$$

$$\dot{V} = 0.00306 \text{ m}^3/\text{s}$$

13.26 CONCRETE PIPES IN SERIES

$$H_2O @ 20^\circ\text{C} - \dot{V} = 0.18 \text{ m}^3/\text{s}$$

$$h_{L1} + h_{L2} = 18 \text{ m} = 176.6 \text{ m}^2/\text{s}^2$$

$$\text{for Pipe 1} - h_{L1} = 2f_F \frac{L}{D} V_1^2$$

$$V_1 = \frac{0.18}{(\frac{\pi}{4})(0.3)^2} = 2.55 \text{ m/s}$$

$$Re = \frac{(0.3)(2.55)}{0.995 \times 10^{-6}} = 7,678 \times 10^5$$

$$e/D = \frac{0.0035}{0.3} = 0.00117$$

$$f_F \approx 0.0051$$

$$\frac{\Delta P}{g} = h_L = 2(0.0051) \frac{312.5}{0.3} (2.55)^2$$

$$= 69.09 \text{ m}^2/\text{s}^2$$

THIS REQUIRES  $h_L$  FOR PIPE 2

$$\text{TO BE } 176.6 - 69.09 = 107.5 \text{ m}^2/\text{s}^2$$

$$(107.5 = 2f_F \frac{312.5}{D} V^2)$$

$$\frac{f_F V^2}{D} = 0.172$$

$$\therefore V = \frac{\dot{V}}{\pi D^2/4} = \frac{0.18}{\pi D^2/4} = \frac{0.2292}{D^2}$$

$$\therefore \frac{f_F}{D^5} = 3,275$$

13.26 CONTINUED -

$$Re = \frac{DV}{\nu} = \frac{D(0.2292)}{0.02(0.995 \times 10^{-6})}$$

$$= 2,304 \times 10^5$$

D

TRUE  $\neq$  ERROR -

ASSUME  $f_F = 0.006 - D = 0.2835 \text{ m}$

$$e/D = 0.0123 \quad Re = 8,127 \times 10^5$$

$$f_F \approx 0.01$$

$$D = 0.314 \text{ m} \quad e/D = 0.0111$$

$$Re = 7,338 \times 10^5 \quad f_F = 0.01$$

$$\sim D = 0.314 \text{ m}$$

13.27 2 PIPES IN PARALLEL:

$$\text{PIPE 1} - D = 0.2 \text{ m} \quad L = 150 \text{ m}$$

$$\text{CAST IRON} - e/D = 0.0013$$

$$\text{PIPE 2} - D = 0.067 \text{ m} \quad L = 150 \text{ m}$$

$$\text{STEEL} - e/D = 0.0007$$

$$\Delta P = 210 \text{ kPa} \quad \frac{\Delta P}{g} = 210 \text{ m}^2/\text{s}^2$$

$$\text{PIPE 1: } \frac{\Delta P}{g} = 2f_F \frac{L}{D} \frac{V^2}{2}$$

ASSUME FULLY TURBULENT -  $f_F \approx 0.0055$

$$210 = 2(0.0055) \frac{150}{0.2} \frac{V^2}{2} - V = 5.045 \text{ m/s}$$

$$Re = \frac{0.2(5.045)}{0.995 \times 10^{-6}} = 1,014 \times 10^6 -$$

THIS CONFIRMS FULLY TURBULENT &  
 $V_1 = 5.045 \text{ m/s}$

PIPE 2: AGAIN ASSUME FULLY TURBULENT

$$f_F = 0.0045 \sim V_2 = 3.228 \text{ m/s}$$

## 13.27 CONTINUED -

$$Re_2 = \frac{0.06(3.22)}{0.995 \times 10^{-6}} = 2.173 \times 10^5$$

fin 13.1: REvised  $f_F \approx 0.0049$

WITH THIS VALUE  $V_2 = 3.094 \text{ m/s}$

$$Re = 2.083 \times 10^5 \quad f_F \approx 0.0049$$

$$\therefore V_2 = 3.049 \text{ m/s}$$

$$\dot{V} = 5.045 \left(\frac{\pi}{4}\right)(0.2)^2$$

$$+ 3.049 \left(\frac{\pi}{4}\right)(0.06)^2$$

$$= 0.1585 \text{ ft}^3/\text{s} + 0.0107 \text{ ft}^3/\text{s}$$

$$\overset{\circ}{V}_1 = 0.1585 \text{ ft}^3/\text{s} \quad \overset{\circ}{V}_2 = 0.0107 \text{ ft}^3/\text{s}$$

## 13.28 3 PIPES IN PARALLEL

Pipe	Length, m	Diameter, cm	Roughness, mm
1	100	8	0.240
2	150	6	0.120
3	80	4	0.200

$$\text{TOTAL } h_L = 24 \text{ m} = 235.4 \text{ m}^2/\text{s}^2$$

$$\text{PIPE 1: } 2f_F L \frac{V_1^2}{D} = 2f_F \frac{100}{0.08} V_1^2$$

$$\sim f_F V_1^2 = 0.0824$$

$$Re_1 = \frac{0.08 V_1}{0.995 \times 10^{-6}} = 8.04 \times 10^5 V_1$$

TRIPLE  $\frac{1}{4}$  ERROR -

$$e/D = 0.24/100 = 0.0024$$

ASSUME FULLY TURBULENT -

$$f_F = 0.0063 - V_1 = 3.617 \text{ m/s}$$

$$Re = \frac{(0.08)(3.617)}{0.995 \times 10^{-6}} = 2.91 \times 10^5$$

$$f_F = 0.0062 -$$

REvised VALUE -  $V_1 = 3.65 \text{ m/s}$

## 13.28 CONTINUED

$$\text{PIPE 2: } 235.4 = 2f_F \frac{150}{0.06} V_2^2$$

$$f_F V_2^2 = 0.0412$$

$$Re_2 = \frac{(0.06) V_2}{0.995 \times 10^{-6}} = 6.03 \times 10^4 V_2$$

$$e/D = 0.002 - \text{Assume } f_F = 0.006$$

$$\therefore V_2 = 2.162 \text{ m/s} \quad Re = 1.58 \times 10^5$$

$$\sim f_F = 0.0061$$

REvised VALUE FOR  $V_2$ :  $V_2 = 2.160 \text{ m/s}$

$$\text{PIPE 3: } 235.4 = 2f_F \frac{80}{0.04} V_3^2$$

$$f_F V_3^2 = 0.059$$

$$Re = \frac{(0.04) V_3}{0.995 \times 10^{-6}} = 4.02 \times 10^4 V_3$$

$$e/D = 0.005 - \text{Assume } f_F = 0.008$$

$$V_3 = 2.716 \text{ m/s} \quad Re = 1.092 \times 10^5$$

$$f_F = 0.0077$$

$$\sim \text{REvised VALUE: } V_3 = 2.77 \text{ m/s}$$

TOTAL SYSTEM FLOW RATE:

$$\dot{V} = 3.65 \left(\frac{\pi}{4}\right)(0.08)^2 + (2.160) \left(\frac{\pi}{4}\right)(0.06)^2$$

$$+ 2.77 \left(\frac{\pi}{4}\right)(0.104)^2$$

$$= 0.0292 \text{ ft}^3/\text{s}$$

## CHAPTER 14

### 14.1 CENTRIFUGAL PUMP:

$$\begin{aligned}\dot{V} &= 0,2 \text{ m}^3/\text{s} & \omega &= 850 \text{ rpm} \\ r_2 &= 0,225 \text{ m} & \rho &= 1000 \text{ kg/m}^3 \\ L &= 0,05 \text{ m} \\ \beta_2 &= 24^\circ\end{aligned}$$

TORQUE - Eqn. 14.9

$$M_2 = 8\dot{V}r_2 \left[ r_2\omega - \frac{\dot{V}}{2\pi r_2 L} \cot\beta_2 \right]$$

$$\sim$$

$$\omega = 850 \left( \frac{2\pi}{60} \right) = 89,0 \text{ rad/s}$$

$$\sim$$

$$M_2 = (1000)(0,2)(0,225) \times$$

$$\left[ (0,225)(89) - \frac{0,2 \cancel{\omega} 24}{2\pi(0,225)(0,05)} \right]$$

$$= 615 \text{ N.m} \quad \text{a)}$$

$$\dot{W} = M_2 \omega = 615(89)$$

$$= 54,75 \text{ kW} \quad \text{a)}$$

$$\left. \frac{\Delta P}{g} \right|_{\max} = - \frac{\dot{W}}{\dot{V}} = - \frac{\dot{W}}{8\dot{V}}$$

$$\Delta P_{\max} = - \frac{54,75 \times 10^3 \text{ N.m/s}}{0,2 \text{ m}^3/\text{s}}$$

$$= - 274 \text{ kPa} \quad \text{b)}$$

### 14.2 CENTRIFUGAL PUMP:

$$\begin{aligned}\rho &= 680 \text{ kg/m}^3 & r_1 &= 0,075 \text{ m} \\ L &= 0,09 \text{ m} & r_2 &= 0,114 \text{ m} \\ \beta_1 &= 25^\circ & \beta_2 &= 40^\circ \\ \omega &= (1200) \left( \frac{2\pi}{60} \right) = 125,7 \text{ rad/s}\end{aligned}$$

### 14.2 CONTINUED

$$\begin{aligned}\dot{V} &= 2\pi r_1^2 L \omega \tan\beta_1 \\ &= 2\pi(0,075)^2(0,09)(125,7) \tan 25^\circ \\ &= 0,186 \text{ m}^3/\text{s} \quad \text{a)} \\ \dot{W} &= M_2 \omega = 8\dot{V}r_2\omega \left[ r_2\omega - \frac{\dot{V} \cot\beta_2}{2\pi r_2 L} \right] \\ &= (680)(0,186)(0,114)(125,7) \times \\ &\quad \left[ (0,114)(125,7) - \frac{0,186 \cancel{\omega} 40^\circ}{2\pi(0,114)(0,09)} \right] \\ &= 32,94 \text{ kW} \quad \text{b)}\end{aligned}$$

$$\left. \frac{\Delta P}{g} \right|_{\max} = \frac{\dot{W}}{8g\dot{V}}$$

$$= \frac{32,94 \times 10^3}{680(9,81)(0,186)}$$

$$= 26,5 \text{ m} \quad \text{c)}$$

### 14.3 CENTRIFUGAL PUMP -

$$r_2 = 0,21 \text{ m} \quad L = 0,05 \text{ m} \quad \beta_2 = 33^\circ$$

$$\omega = 1200 \left( \frac{2\pi}{60} \right) = 125,7 \text{ rad/s}$$

$$\begin{aligned}\frac{\Delta P}{g} &= 52 \text{ m H}_2\text{O} \\ \dot{W} &= 8\dot{V}r_2\omega \left[ r_2\omega - \frac{\dot{V} \cot\beta_2}{2\pi r_2 L} \right] \\ &= \frac{\dot{m} \Delta P}{g} = \dot{V} \Delta P\end{aligned}$$

EQUATIONS:

$$\Delta P = 8r_2\omega \left[ r_2\omega - \frac{\dot{V} \cot\beta_2}{2\pi r_2 L} \right]$$

### 14.3 (CONTINUED)

$$\begin{aligned}\Delta P &= 52(1000)(9.81) = 490 \text{ kPa} \\ &= (1000)(0.21)(125.7) \times \\ &\quad \left[ (0.21)(125.7) - \frac{\dot{V} \cot 33^\circ}{2\pi(0.21)(0.05)} \right] \\ &= 26400 [26.4 - 23.34 \dot{V}]\end{aligned}$$

EQUATING:

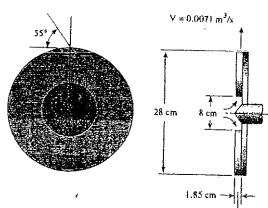
$$\begin{aligned}18.56 &= 26.4 - 23.34 \dot{V} \\ \dot{V} &= 0.336 \text{ m}^3/\text{s} \quad a)\end{aligned}$$

$$\begin{aligned}\dot{W} &= \dot{V} \Delta P \\ &= 0.336 (490 \times 10^3) \\ &= 164.6 \text{ kW} \quad b)\end{aligned}$$

### 14.4

Pump DEPICTED

$$\begin{aligned}\omega &= 1020 \text{ rpm} \\ &= 106.8 \text{ rad/s}\end{aligned}$$



$$\begin{aligned}r_1 &= 0.04 \text{ m} & \dot{V} &= 0.0071 \text{ m}^3/\text{s} \\ r_2 &= 0.14 \text{ m} & \beta_2 &= 55^\circ\end{aligned}$$

$$L = 0.0185 \text{ m} \quad g = 1000 \text{ kg/m}^3$$

$$\begin{aligned}\dot{W} &= 8\dot{V}r_2\omega \left[ r_2\omega - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right] \\ &= (1000)(0.0071)(0.14)(106.8) \times \\ &\quad \left[ (0.14)(106.8) - \frac{0.0071 \cot 55^\circ}{2\pi(0.14)(0.0185)} \right] \\ &= 1555 \text{ W} = 1.555 \text{ kW}\end{aligned}$$

### 14.5 CENTRIFUGAL PUMP -

$$\begin{aligned}\rho &= 1000 \text{ kg/m}^3 & \dot{V} &= 0.018 \text{ m}^3/\text{s} \\ \dot{W} &= 4.5 \text{ kW} & \eta &= 63\%\end{aligned}$$

$$\eta = \frac{\dot{m} \Delta P}{\dot{W}}$$

$$\Delta P = \frac{\eta \dot{W}}{\dot{m}} = \eta \frac{\dot{W}}{\dot{V}}$$

$$= \frac{0.63 (4500)}{0.018} = 157.5 \text{ kPa}$$

$$\frac{\Delta P}{g f} = \frac{157500}{(1000)(9.81)} = 16.05 \text{ m H}_2\text{O}$$

### 14.6

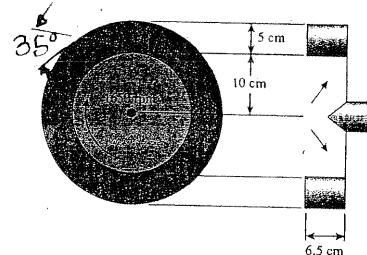
Pump DEPICTED

$$\dot{V} = 0.032 \text{ m}^3/\text{s}$$

$$\rho = 680 \text{ kg/m}^3$$

$$\omega = 1650 \text{ rpm}$$

$$= 172.8 \text{ rad/s}$$



$$\dot{W} = 8\dot{V}r_2\omega \left[ r_2\omega - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right]$$

$$\begin{aligned}&= (680)(0.032)(0.15)(172.8) \times \\ &\quad \left[ (0.15)(172.8) - \frac{(0.032) \cot 35^\circ}{2\pi(0.15)(0.05)} \right]\end{aligned}$$

$$= 14.2 \text{ kW} = 19.0 \text{ hp} \quad a)$$

$$\Delta P = \frac{\dot{W}}{\dot{V}} = \frac{14200}{0.032} = 444 \text{ kPa}$$

$$= \frac{444000}{(680)(9.81)} = 66.5 \text{ m} \quad b)$$

#### 14.6 CENTRIFUGAL

$$\dot{V} = 2\pi r_1^2 L \omega \tan \beta_1$$

$$\tan \beta_1 = \frac{0.032}{2\pi(0.10)^2(0.005)(172.8)} = 0.0453$$

$$\underline{\beta_1 = 2.6^\circ} \quad c)$$

#### 14.7 CENTRIFUGAL PUMP

$$\rho = 1000 \text{ kg/m}^3 \quad r_1 = 0.12 \text{ m}$$

$$\beta_1 = 32^\circ \quad r_2 = 0.20 \text{ m}$$

$$\beta_2 = 20^\circ \quad L = 0.042 \text{ m}$$

$$\omega = 1500 \text{ rpm} = 157.1 \text{ rad/s}$$

$$\begin{aligned} \dot{V} &= 2\pi r_1^2 L \omega \tan \beta_1 \\ &= 2\pi (0.12)^2 (0.042) (157.1) \tan 32^\circ \\ &= \underline{0.373 \text{ m}^3/\text{s}} \quad (a) \end{aligned}$$

$$\begin{aligned} \dot{W} &= \dot{V} \bar{r}_2 \omega \left[ r_2 w - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right] \\ &= (1000)(0.373)(0.2)(157.1) \times \\ &\quad \left[ (0.2)(157.1) - \frac{0.373 \cot 20^\circ}{2\pi(0.2)(0.042)} \right] \\ &= 140.7 \text{ kW} = \underline{189 \text{ kW}} \quad (b) \end{aligned}$$

$$\Delta p = \frac{\dot{W}}{\dot{V}} = \frac{140.7}{0.373} = 377 \text{ kPa}$$

$$\frac{\Delta p}{\rho g} = \frac{377 \times 10^3}{(1000)(9.81)} = \underline{38.5 \text{ m H}_2\text{O}}$$

#### 14.8 H<sub>2</sub>O @ 15°C

$$\rho = 999 \text{ kg/m}^3$$

$$D = 0.45 \text{ m}$$

$$\omega = 1600 \left( \frac{2\pi}{60} \right) = 167.6 \text{ rad/s}$$

@  $\eta_{\text{MAX}}$

$$C_Q \approx 0.012$$

$$C_H \approx 0.0515$$

$$C_P \approx 0.0068$$

$$\eta \approx 0.89$$

$$C_H = \frac{g h}{h^2 D^2} \sim$$

$$h = \frac{(0.0515)(167.6)^2 (0.45)^2}{9.81} = \underline{29.9 \text{ m}} \quad (a)$$

$$C_Q = \frac{\dot{V}}{\rho D^3}$$

$$\dot{V} = 0.012 (167.6) (0.45)^3 = \underline{0.183 \text{ m}^3/\text{s}} \quad (b)$$

$$\Delta p = \rho g h = (999)(9.81)(29.9) = \underline{293 \text{ kPa}} \quad (c)$$

$$C_P = \frac{\dot{W}}{8 \pi^3 D^5}$$

$$\begin{aligned} \dot{W} &= (0.0068)(999)(167.6)^3 (0.45)^5 \\ &= 587 \text{ kW} \end{aligned}$$

$$BHP = \frac{587 \times 10^3}{0.89} = 659.6 \text{ kW}$$

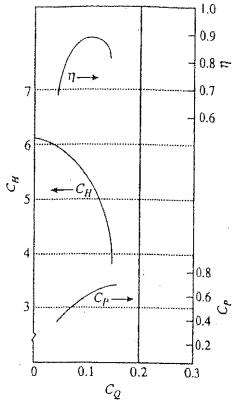
$$= \underline{884 \text{ kW}} \quad (d)$$

#### 14.9 CENTRIFUGAL PUMP WITH SAME CHARACTERISTICS AS IN PROB 14.8

$$\dot{V} = 0.2 \text{ m}^3/\text{s}$$

$$\omega = 1400 \left( \frac{2\pi}{60} \right) = 146.6 \text{ rad/s}$$

$$\rho = 1000 \text{ kg/m}^3$$



## 14.9 CONTINUED -

$$\text{AT } \eta_{\text{MAX}}: C_Q \approx 0.012$$

$$C_H \approx 0.0515$$

$$C_P \approx 0.0068$$

$$C_Q = \frac{\dot{V}}{nD^3} \quad D^3 = \frac{0.2}{(146.6)(0.012)}$$

$$D = 0.485 \text{ m} \quad (\text{a})$$

$$C_H = \frac{g h}{n^2 D^2} \quad h = \frac{(0.0515)(146.6)^2 (0.0068)^2}{9.81}$$

$$= 26.5 \text{ m}$$

$$P_{\text{MAX}} + \rho g h = (1000)(9.81)(26.5)$$

$$= 260 \text{ kPa} \quad (\text{b})$$

14.10 SAME PUMP FAMILY AS IN PROB 14.8 BUT:

$$D = 0.4 \text{ m}$$

$$\omega = 2200 \left( \frac{2\pi}{60} \right) = 230.4 \text{ Rad/s}$$

$$\rho = 999 \text{ kg/m}^3$$

$$\textcircled{a} \quad \eta_{\text{MAX}} \approx 0.89$$

$$C_Q \approx 0.012$$

$$C_H \approx 0.0515$$

$$C_P \approx 0.0068$$

$$C_H = \frac{g h}{n^2 D^2}$$

$$h = \frac{(0.0515)(230.4)^2 (0.4)^2}{9.81}$$

$$= 44.6 \text{ m H}_2\text{O} \quad (\text{a})$$

## 14.10 CONTINUED

$$C_Q = \frac{\dot{V}}{nD^3} \quad \dot{V} = (0.012)(230.4)(0.4)^3$$

$$= 0.177 \text{ m}^3/\text{s} \quad (\text{b})$$

$$\Delta P = \rho g h = (999)(9.81)(44.6)$$

$$= 437 \text{ kPa} \quad (\text{c})$$

$$C_H = \frac{\dot{W}}{\rho n^3 D^5}$$

$$\dot{W} = (0.0068)(999)(230.4)^3 (0.4)^5$$

$$= 850.8 \text{ kW}$$

$$BHP = \frac{850.8}{(0.89)(0.746)} = 1280 \text{ Bhp} \quad (\text{d})$$

## 14.11 SAME PUMP FAMILY AS IN PROB 14.8

$$D = 0.35 \text{ m} \quad \omega = 2400 \left( \frac{2\pi}{60} \right) = 251.3 \text{ rad/s}$$

$$\rho = 999 \text{ kg/m}^3$$

$$\eta_{\text{MAX}} = 0.89$$

$$C_Q \approx 0.012$$

$$C_H \approx 0.0515$$

$$C_P \approx 0.0068$$

$$C_H = \frac{g h}{n^2 D^2} \quad h = \frac{(0.0515)(251.3)^2 (0.35)^2}{9.81}$$

$$= 40.61 \text{ m H}_2\text{O} \quad (\text{a})$$

$$C_Q = \frac{\dot{V}}{nD^3} \quad \dot{V} = (0.012)(251.3)(0.35)^3$$

$$= 0.129 \text{ m}^3/\text{s} \quad (\text{b})$$

$$\Delta P = (999)(9.81)(44.6)$$

$$= 437 \text{ kPa} \quad (\text{c})$$

14.11 CONTINUED -

$$C_Q = \frac{\dot{W}}{g n^3 D^5}$$

$$\dot{W} = (0.0068)(0.99)(251.3)^3(0.39)^5$$

$$= 566 \text{ kW}$$

$$BHP = \frac{566}{(0.89)(0.746)} = 853 \text{ HP} \quad (\text{a})$$

14.12 - SAME PUMP FAMILY AS  
IN PROB 14.8

$$\dot{V} = 0.30 \text{ m}^3/\text{s} \quad n = 1800 \left(\frac{2\pi}{60}\right) = 188.5 \text{ r/s}$$

$$@ \eta_{MAX} = 0.89 \quad C_Q \approx 0.12$$

$$C_H \approx 0.0515$$

$$C_P \approx 0.0068$$

$$C_Q = \frac{\dot{V}}{n D^3} \quad D = \left[ \frac{0.30}{(188.5)(0.12)} \right]^{1/3}$$

$$D = 0.430 \text{ m} \quad (\text{a})$$

$$C_H = \frac{gh}{n^2 D^2}$$

$$h = \frac{(0.0515)(188.5)^2}{9.81} (0.43)^2$$

$$= 34.49 \text{ m H}_2\text{O}$$

$$\Delta P = \rho g h = (1000)(9.81)(34.49)$$

$$= 338 \text{ kPa} \quad (\text{b})$$

14.13 - SAME PUMP FAMILY AS IN  
PROB 14.8

$$\dot{V} = 0.201 \text{ m}^3/\text{s} \quad \omega = (1800)(2\pi/60)$$

$$= 188.5 \text{ r/s}$$

14.13 - CONTINUED

$$@ \eta_{MAX} = 0.89$$

$$C_Q \approx 0.12$$

$$C_H \approx 0.0515$$

$$C_P \approx 0.0068$$

$$C_Q = \frac{\dot{V}}{n D^3} \quad D = \left[ \frac{0.201}{(188.5)(0.12)} \right]^{1/3}$$

$$= 0.207 \text{ m} \quad (\text{a})$$

$$C_H = \frac{gh}{n^2 D^2} \quad h = \frac{(0.0515)(88.5)^2}{9.81} (0.207)^2$$

$$= 7.99 \text{ m H}_2\text{O}$$

$$\Delta P = \rho g h = (1000)(9.81)(7.99)$$

$$= 78.4 \text{ kPa} \quad (\text{b})$$

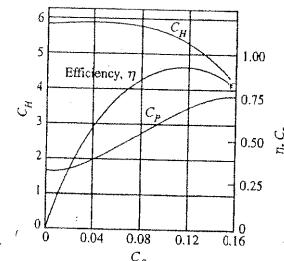
14.14

$$@ \eta_{MAX} \approx 0.88$$

$$C_Q \approx 0.12$$

$$C_H \approx 5.3$$

$$h = 90 \text{ m H}_2\text{O}$$



$$C_H = \frac{gh}{n^2 D^2} = 5.3$$

$$\omega^2 = \frac{gh}{C_H D}$$

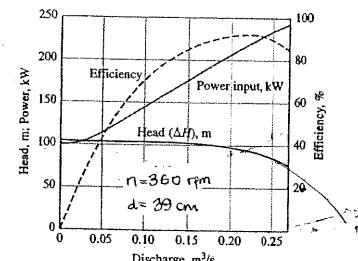
$$= \frac{9.81(90)}{5.3(0.39)^2}$$

$$\omega = 33.1 \text{ rad/s} = 316 \text{ rpm} \quad (\text{a})$$

$$C_Q = 0.12 = \frac{\dot{V}}{n D^3}$$

$$\dot{V} = (0.12)(33.1)(0.39)^3 = 0.236 \text{ m}^3/\text{s}$$

$$= 0.236 \text{ m}^3/\text{s} \quad (\text{b})$$



14.15 SAME PUMP FAMILY AS IN 14.14

$$\text{New Pump: } n = 400 \text{ rpm} \\ = 41.89 \text{ rad/s}$$

$$D_{\text{NEW}} = 6 \text{ m D_1}$$

$$\text{At } \eta_{\text{MAX}} - C_Q \approx 0.12 = \dot{V}/nD^3 \\ C_H \approx 5.3 = gh/n^2 D^2 \\ C_P \approx 0.7 = P/gn^3 D^5 \\ Q_1 = 0.70 (1000)(37.70)^3 (0.371)^5 \\ = 263.6 \text{ kW}$$

$$P_{\text{NEW}} = P_1 \left( \frac{\omega_2}{\omega_1} \right)^3 \left( \frac{D_2}{D_1} \right)^5 \\ = 263.6 \left( \frac{400}{360} \right)^3 (6)^5 \\ = \underline{281 \text{ MW}} \quad (a)$$

$$h_1 = \frac{5.3 (37.70)^2 (0.371)^2}{9.81} = 105.7 \text{ m}$$

$$h_2 = h_1 \left( \frac{\omega_2}{\omega_1} \right)^2 \left( \frac{D_2}{D_1} \right)^2 \\ = 105.7 \left( \frac{400}{360} \right)^2 (6)^2 \\ = \underline{4.7 \text{ km}} \quad (b)$$

$$\dot{V}_1 = 0.12 n_1 D_1^3 \\ = 0.12 (37.70)(0.371)^3 \\ = 0.231 \text{ m}^3/\text{s}$$

$$\dot{V}_2 = \dot{V}_1 \left( \frac{n_2}{n_1} \right) \left( \frac{D_2}{D_1} \right)^3 \\ = 0.231 \left( \frac{400}{360} \right) (6)^3 \\ = \underline{55.4 \text{ m}^3/\text{s}} \quad (c)$$

14.16 SAME PUMP FAMILY AS IN PROB 14.14

$$\text{New } n = 1000 \text{ rpm}$$

$$C_Q \approx 0.12 = \dot{V}/nD^3 \quad C_P \approx 0.7 = P/gn^3 D^5$$

$$\dot{V} = 0.12 \left( 1000 \times \frac{2\pi}{60} \right) (0.371)^3 \\ = \underline{0.642 \text{ m}^3/\text{s}} \quad (a)$$

$$P = 0.7 (1000) \left( 1000 \times \frac{2\pi}{60} \right)^3 (0.371)^5 \\ = \underline{5.65 \text{ MW}} \quad (b)$$

14.17 SAME PUMP FAMILY AS IN PROB 14.14

$$\text{New } \omega = 800 \text{ rpm} = 83.8 \text{ rad/s}$$

$$h = 410 \text{ m}$$

$$C_H = \frac{gh}{n^2 D^2} = \frac{9.81 (410)}{(83.8)^2 (0.371)^2} = 4.161$$

$$\text{AT THIS VALUE OF } C_H, C_Q \approx 0.16$$

$$C_Q = 0.16 = \dot{V}/nD^3 \\ \dot{V} = 0.16 (83.8)(0.371)^3 \\ = \underline{0.685 \text{ m}^3/\text{s}}$$

14.18 SAME PUMP FAMILY AS PROB 14.14

$$D_2 = 3D_1, \quad n_2 = 0.5n_1$$

$$\textcircled{a} \quad \eta_{\text{MAX}} \quad C_Q \approx 0.12 = \dot{V}/nD^3 \\ C_H \approx 5.3 = gh/n^2 D^2$$

$$\frac{\dot{V}_2}{\dot{V}_1} = \left( \frac{n_2}{n_1} \right) \left( \frac{D_2}{D_1} \right)^3 = \frac{1}{2} (3)^2 = 13.5$$

$$\frac{h_2}{h_1} = \left( \frac{n_2}{n_1} \right)^2 \left( \frac{D_2}{D_1} \right)^2 = \left( \frac{1}{2} \right)^2 (3)^2 = 2.25$$

### 14.18 (CONTINUED)

$$\dot{V}_1 = 0.12(37.7)(0.371)^3 = 0.231 \text{ m}^3/\text{s}$$

$$\dot{V}_2 = 0.231(13.5) = \underline{3.12 \text{ m}^3/\text{s}}$$

$$h_1 = \frac{5.3(37.7)^2(0.371)^2}{9.81} = 105.7 \text{ m}$$

$$h_2 = (105.7)(2.25) = \underline{238 \text{ m}}$$

14.19 Pump Performance AS IN  
PROB 14.14 -  $\Delta y = 95 \text{ m}$

$H_2O$  Pump -  $D = 0.28 \text{ m}$   
 $L = 550 \text{ m}$   
 $e = 0.457 \times 10^{-4} \text{ m}$

$$\epsilon/D = 0.000163$$

BETWEEN FDN:

$$-\dot{W}_s = \dot{m} \left[ \frac{\Delta P}{g} + \frac{\Delta V^2}{2} + g\Delta y + h_f \right]$$

$$h_f = 2f_f \frac{L}{D} \frac{V^2}{2}$$

- ASSUME FLOW TURBULENT

$$f_f \approx 0.0033$$

$$h_f = 2(0.0033) \frac{(550)}{(0.28)} \frac{V^2}{2}$$

$$= 12.96 V^2$$

1st LAW EXPRESSION BECOMES -

$$-\dot{W}_s = \dot{m} \left[ 90g + 12.96 V^2 \right]$$

SYSTEM HEAD -

$$-\frac{\dot{W}_s}{mg} = h = 90 + 1.32 V^2 \quad (1)$$

THIS MUST MATCH PUMP PERFORMANCE -

### 14.19 (CONTINUED)

SYSTEM PERFORMANCE - EQUATION (1)

$\dot{V}$	$h$
0.10	93.48
0.15	95.93
0.20	100.54
0.25	106.5

$$\dot{V} = \frac{\pi}{4} D^2 U = 0.0616 U$$

SYSTEM & PUMP PERFORMANCE IN TELSTEC  
AT  $\dot{V} \approx 0.21 \text{ m}^3/\text{s}$  -  $U = 3.41$

$$Re = \frac{(0.28)(3.41)}{0.995 \times 10^{-4}} = 9.59 \times 10^5$$

$f_f = 0.0035 \sim$  CLOSE ENOUGH

SO: WITHIN ACCURACY OF READING PLOTS

$$\dot{V} = 0.21 \text{ m}^3/\text{s}$$

### 14.20

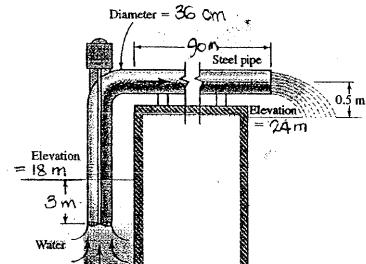
FOR STEEL

$$e = 0.457 \times 10^{-4} \text{ m}$$

$$\epsilon/D = 0.000127$$

FOR Fully-Turbulent

$$f_f \approx 0.0031$$



$$\text{ENERGY EQUATION: } -\dot{W}_s = \dot{m} \left[ \frac{\Delta P}{g} + \frac{\Delta V^2}{2} + g\Delta y + h_f \right]$$

BETWEEN RESERVOIR SURFACE (1)  
& DISCHARGE (2) -

$$\frac{\Delta P}{g} = V_1^2 = 0$$

$$g\Delta y = 6.5 g$$

$$\frac{\Delta V^2}{2} = V^2/2$$

14.20 CONTINUED-

$$h_r = 2f_f \frac{L}{D} V^2 + \sum K \frac{V^2}{2}$$

$$= 2(0.0031) \frac{99.5}{0.36} V^2 + \frac{V^2}{2}$$

$$= 2.21 V^2$$

ENTRANCE

ENERGY EQN NOW BECOMES:

$$\dot{W} = \dot{m} [0.5 V^2 + 6.5 g + 2.21 V^2]$$

$$-\frac{\dot{W}}{\dot{m} g} = h_{\text{SYST}} = 6.5 + 0.276 V^2 \quad (1)$$

- SYSTEM PERFORMANCE - EQN (1)

$$V \quad h_{\text{SYST}}$$

0.20	7.27
0.25	7.71
0.30	8.24
0.35	8.87

PUMP & SYSTEM AREN'T WELL MATCHED - PUMP PERFORMANCE HEAD CURVE MUST BE EXTRAPOLATED

$$V \approx 0.33 \text{ m}^3/\text{s}$$

$$\dot{W} \approx 8.8 (1000)(0.33)(9.81)$$

$$= 28.5 \text{ kW}$$

14.21 SAME PUMP FAMILY AS IN PROB 14.14

$$\frac{h_2}{h_1} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$$

$$h_2 = h_1 \left(\frac{900}{360}\right)^2 = 6.25 h_1$$

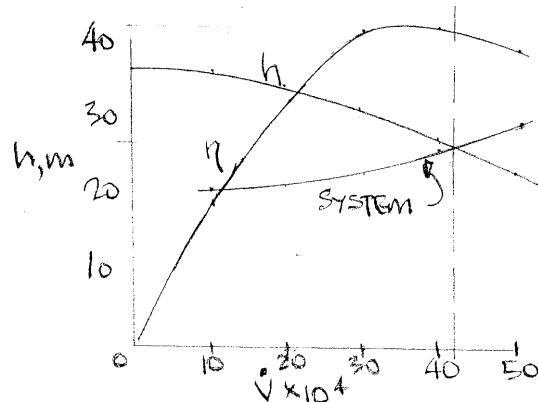
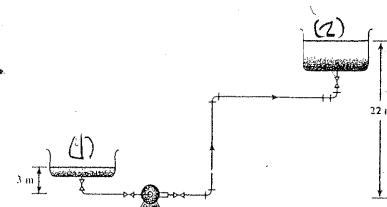
TOTAL MISMATCH -

14.22.

PUMP  
PERFORMANCE  $\rightarrow$

	Capacity, $\text{m}^3/\text{s} \times 10^4$	Developed head, m	Efficiency, %
0	36.6	0	
10	35.9	19.1	
20	34.1	32.9	
30	31.2	41.6	
40	27.5	42.2	
50	23.3	39.7	

SYSTEM  
CONFIGURATION



SYSTEM - INLET -  $D = 0.06 \text{ m}$   
 $L = 8.5 \text{ m}$

DISCHARGE -  $D = 0.06 \text{ m}$   
 $L = 60 \text{ m}$

STEEL -  $C = 0.457 \times 10^{-4} \text{ m}$

$$\frac{C}{D} = 0.000762$$

MINOR LOSSES - 4 VALVES  $K = 10$   
4 ELBOWS  $K = 0.3$   
1 CONTRACTION  $K = 1.1$

BETWEEN RESERVOIRS - (1)  $\leq$  (2)

$$\dot{W} = \dot{m} \left[ \frac{\Delta P}{g} + \frac{\Delta V^2}{2} + g \Delta y + h_L \right]$$

$$\frac{\Delta P}{g} = \frac{\Delta V^2}{2} = 0$$

$$g \Delta y = 19 g \text{ m}^2/\text{s}^2$$

$$h_L = 2 f_f \frac{L}{D} V^2 + \sum K \frac{V^2}{2}$$

14.22 CONTINUED -

ASSUME FLOW IS FULLY TURBULENT

$$f_f \approx 0,0046$$

$$\sum K = 4(10) + 4(0.3) + 1 = 42.2$$

$$h_L = \left[ 2(0,0046) \frac{68.5}{0,06} + \frac{42.2}{2} \right] V^2$$

$$= 31.6 V^2$$

ENERGY EQUATION BECOMES:

$$-\frac{\dot{W}}{mg} = \Delta y + \frac{h_L}{g} = h$$

$$= 19 + 3,22 V^2$$

$$V \times 10^4 \quad h, m$$

20	20,61
30	22,63
40	25,45
50	29,07

INTERSECTION OCCURS AT

$$\dot{V} \approx 42 \times 10^{-4} \text{ m}^3/\text{s}$$

$$\text{AT } \dot{V} = 42 \times 10^{-4} \text{ m}^3/\text{s}$$

$$V = 1,485 \text{ m/s}$$

$$Re = \frac{(0,06)(1,485)}{0,995 \times 10^{-4}} = 8,957 \times 10^5$$

USING FIG 13.1 -

CONDITIONS ARE VERY CLOSE TO  
FULLY TURBULENT FLOW -

INITIAL ASSUMPTION FOR  $f_f$   
WAS OK.

$$\dot{V} = 0,0042 \text{ m}^3/\text{s}$$

14.23 PUMP -  $D = 0,25 \text{ m}$

$$N = 1000 \text{ rpm}$$

$$\dot{V} = 0,065 \text{ m}^3/\text{s}$$

$$V_{INLET} = 6,1 \text{ m/s}$$

$$H_2O @ 20^\circ C \sim P_v = 2,34 \text{ kPa}$$

GRAVITATION OCCURS AT  $P_i = 82,7 \text{ kPa}$

$$NPSH + \frac{P_v}{\rho g} = \frac{V_i^2}{2g} + \frac{P_i}{\rho g}$$

$$NPSA = \frac{V_i^2}{2g} + \frac{P_i - P_v}{\rho g}$$

$$= \frac{(6,1)^2}{2(9,81)} + \frac{(82,7 - 2,34) \times 10^3}{1000(9,81)}$$

$$= 10,09 \text{ m H}_2\text{O}$$

14.24 - SAME PUMP AS DESCRIBED IN  
PROB 14.23 -

NEW TEMP IS  $80^\circ C$  ( $P_v = 47,35 \text{ kPa}$ )

$$NPSA = \frac{V_i^2}{2g} + \frac{P_i - P_v}{\rho g}$$

$$= \frac{(6,1)^2}{2(9,81)} + \frac{(82,7 - 47,35) \times 10^3}{1000(9,81)}$$

$$= 5,50 \text{ m H}_2\text{O}$$

CHANGE FROM  $20^\circ C$  CASE IS

$$\Delta = 10,09 - 5,50 = 4,59 \text{ m}$$

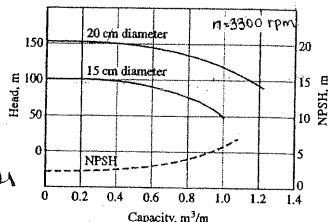
14.25

Pump -

$$D = 0.18 \text{ m}$$

INLET AT  $y = 3.8 \text{ m}$ ABOVE SUPPLY  
RESERVOIR.

$$\dot{V} = 0.760 \text{ m}^3/\text{s}$$

BETWEEN RESERVOIR SURFACE &  
PUMP INLET -  $h_L = 1.8 \text{ m H}_2\text{O}$ 

ENERGY EQUATION:

$$NPSH = \frac{P_{atm} - P_v}{\rho g} - y_2 - h_L$$

$$\text{AT } 30^\circ\text{C: } P_v = 2.34 \text{ kPa}$$

$$\frac{P_{atm} - P_v}{\rho g} = \frac{(101.3 - 2.34) \times 10^3}{(1000)(9.81)} = 10.09 \text{ m}$$

$$NPSH = 10.09 - 3.8 - 1.8 = 4.49 \text{ m H}_2\text{O}$$

from PERFORMANCE CURVE -

$$@ \dot{V} = 0.760 \text{ m}^3/\text{s}$$

$$NPSH \approx 3.9 \text{ m}$$

CAVITATION SHOULD NOT OCCUR

$$14.26 \quad \dot{V} = 220 \text{ m}^3/\text{s} = 3.487 \times 10^6 \text{ gpm}$$

$$h = 420 \text{ m} = 1318 \text{ ft}$$

$$N_S = \frac{(400)(3.487 \times 10^6)^{1/2}}{(1318)^{3/4}} = 3302$$

ACCORDING TO FIG 14.11

THIS IS PROBABLY A HIGH  
CAPACITY CENTRIFUGAL PUMP14.27 Pump To Deliver 60,000 gpm  
With  $h = 300 \text{ m}$  @ 2000 rpm.

$$N_S = \frac{(2000)(6 \times 10^6)^{1/2}}{(300/0.3048)^{3/4}} \approx 2790$$

USING FIG 14.11 - PUMP IS PROBABLY  
A HIGH-CAPACITY CENTRIFUGAL  
PUMP.14.28 AXIAL FLOW PUMP -  $N_S = 6.0$ 

$$N_S = \frac{C_Q^{1/2}}{C_H^{3/4}} = \frac{\dot{V}^{1/2} w}{h^{3/4} g^{3/4}} \quad (1)$$

THIS RATIO IS (OBVIOUSLY) DIMENSIONLESS -

BY CONVERTING TO UNITS ON ABSCISSA  
OF FIG 14.11 -THE RATIO OF  $N_S$  GIVEN BY (1)  
TO THE VALUE ON FIG 14.11 IS  
2733- SO A VALUE OF 6 FOR EQUATION (1)  
IS EQUIVALENT TO 6 (2733) =  $1.64 \times 10^4$   
ON ABSCISSA OF FIG 14.11.

$$1.64 \times 10^4 = \frac{n(2400)^{1/2}}{(18)^{3/4}}$$

$$n = 2925 \text{ rpm}$$

14.29 Pump @ 520 rpm

$$\dot{V} = 3.3 \text{ m}^3/\text{s}$$

$$h = 16 \text{ m}$$

$$\dot{V} = (3.3) \left( \frac{1}{0.3048} \right)^3 (7.48)(60)$$

$$= 52302 \text{ gpm}$$

$$h = (16) / 0.3048 = 42.65 \text{ FT}$$

$$N_S = \frac{(520)(5.23 \times 10^5)^{1/2}}{(42.65)^{3/4}}$$

$$= 22532$$

FIGURE 14.11: AXIAL FLOW

14.30  $n = 2400 \text{ rpm}$

$$\dot{V} = 3.2 \text{ m}^3/\text{s}$$

$$h = 21 \text{ m}$$

$$\dot{V} = 3.2 \left( \frac{1}{0.3048} \right)^3 (7.48)(60)$$

$$= 5.072 \times 10^4 \text{ gpm}$$

$$h = \frac{21}{0.3048} = 68.9 \text{ FT}$$

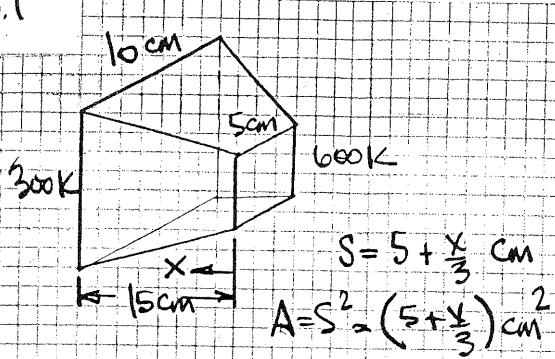
$$N_S = \frac{(5.072 \times 10^4)^{1/2} (2400)}{(68.9)^{3/4}}$$

$$= 22601$$

FIGURE 14.11 AXIAL FLOW

# CHAPTER 15

15.1



$$\dot{q} = -\frac{kA \Delta T}{dx}$$

$$\int \dot{q} dx = -kA \frac{\Delta T}{dx}$$

$$\int_0^{15} \frac{dx}{(5+x/3)^2} = k \int_{300}^{600} dT$$

$$\int \left[ -\frac{3}{5+x/3} \right]_0^{15} = -300 \text{ K}$$

$$\int [0.6 - 0.3] = 300 (0.173 \text{ W/m·K})$$

$$\dot{q} = \underline{1.73 \text{ W}}$$

15.2 SAME VALUE AS IN PREVIOUS PROBLEM EXCEPT HEAT FLOWS IN OPPOSITE DIRECTION

$$\dot{q} = \underline{1.73 \text{ W}}$$

15.3

$$\int_0^{15} \frac{dx}{(5+x/3)^2} = -k_o \int_{300}^{600} (1+fT) dT$$

$$\int \left[ -\frac{3}{5+x/3} \right]_0^{15} = k_o \Delta T \left[ 1 + \frac{f}{2} (T_1 + T_2) \right]_{300}^{600}$$

$$\int [0.3 \text{ cm}^{-1}] = \left[ 0.135 \text{ W/m·K} \right] (300 \text{ K})$$

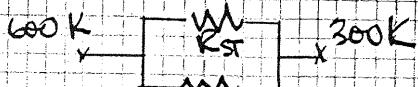
$$\times \left[ 1 + 1.95 \times 10^{-4} \right] (450)$$

$$\dot{q} = \underline{1.50 \text{ W}}$$

15.4

$$\dot{P}_{BOLT} = \frac{L}{kA} = \frac{0.15}{(40)(\pi/4)(0.00905)^2} = 13.16 \text{ K/W}$$

NEGLECTING CHANGE IN CROSS-SECTIONAL AREA OF ASBESTOS:



$R_{AB}$

$$R_{ASB} = \frac{\Delta T}{\dot{q}} = \frac{300}{1.73} = 173.4 \text{ K/W}$$

$$\frac{1}{R_{TOT}} = \frac{1}{13.16} + \frac{1}{173.4} = \frac{1}{12.23}$$

$$\dot{q} = \frac{\Delta T}{R_{TOT}} = \underline{14.5 \text{ W}}$$

15.5

$$\dot{q} = \frac{kA}{L} \Delta T$$

$$\Delta T = \frac{4000 \text{ W} (0.02 \text{ m})}{(0.12 \text{ W/m·K})(2.97 \text{ m}^2)} = 122.4 \text{ K}$$

$$T_c = 55 + 122.4 = \underline{177.4 \text{ C}}$$

15.6  $\dot{q} = \frac{\Delta T}{\sum R}$

$$\sum R = \frac{L}{kA} + \frac{1}{hA}$$

$$= \frac{0.02}{(0.12)(2.97)} + \frac{1}{(284)(2.97)}$$

$$= 4.246 \times 10^{-2} \text{ K/W}$$

$$\Delta T_{TOT,AVG} = (4000)(4.246 \times 10^{-2}) = 169.9 \text{ K}$$

$$T_{HOT} = 30 + 169.9 = \underline{199.9 \text{ C}}$$

$$T_{SURF} = 30 + \frac{4000}{(284)(2.97)} = \underline{77.4 \text{ C}}$$

15.7



$$\begin{aligned} q_{\max} &= -k \frac{\Delta T}{L} \Big|_{\max} \\ &= (1.35 \text{ W/mK})(15 \text{ K/cm})(100 \text{ cm/m}) \\ &= 2025 \text{ W/m}^2 = \Delta T/R \end{aligned}$$

$$\Delta T = 2025 \left(\frac{1}{5}\right) = 405 \text{ K}$$

$$T_{\min} = 850 - 405 = \underline{445 \text{ K}}$$

$$15.8 \quad q_{\max} (\text{from previous prob}) = 2025 \text{ W/m}^2$$

$$= \frac{\Delta T}{R} + \sigma (T_{\text{surf}}^4 - T_A^4)$$

$$= \frac{850 - T}{1/5} + 5.676 \left[ 8.5^4 - \left( \frac{T}{100} \right)^4 \right]$$

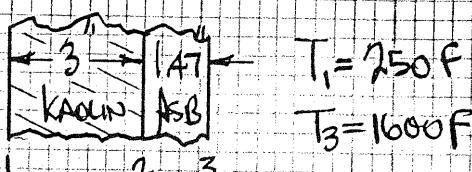
$$\text{By trial & error: } T = \underline{836 \text{ K}}$$

15.9

$$\frac{q}{A} = \frac{k \Delta T}{L} \quad \text{or} \quad L = \frac{k \Delta T}{\frac{q}{A}}$$

$$L = \frac{(0.10 \text{ Btu}/\text{hr ft F})((100 \text{ F})}{900 \text{ Btu}/\text{hr ft}^2} = \underline{0.122 \text{ ft}} = \underline{1.47 \text{ in.}}$$

15.10 ADDING 3 IN. OF KAOLIN:



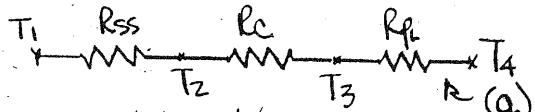
$$\begin{aligned} \frac{q}{R} &= \frac{\Delta T}{\sum R} = \frac{1350}{1.47/12 + \frac{3/12}{0.10}} \\ &= \underline{281 \text{ Btu/hr ft}^2} \quad (\text{a}) \end{aligned}$$

15.10 CONTINUED -

$$\frac{q}{R} = \frac{1600 - T_2}{1.47/12} = \frac{T_2 - 250}{3/12} \quad \frac{0.10}{0.07}$$

$$T_2 = 1254 \text{ F} \quad \leftarrow (\text{b})$$

15.11



$$R_{ss} = \frac{L}{k_{ss}} = \frac{1/48}{10} = 0.0028 \quad \leftarrow (\text{a})$$

$$R_c = \frac{L}{k_c} = \frac{3/12}{0.025} = 10 \quad \leftarrow (\text{b})$$

$$R_p = \frac{L}{k_p} = \frac{1/24}{1.5} = 0.0278 \quad \leftarrow (\text{c})$$

$$\frac{q}{R} = \frac{\Delta T}{\sum R} = \frac{170}{10.03} = \underline{16.95 \text{ Btu/hr ft}^2}$$

$$\frac{1600 - T_2}{0.0028} = \frac{T_2 - T_3}{10} = \frac{T_3 - 80}{0.0278} = 16.95$$

$$T_2 = \underline{1249.95 \text{ F}} \quad T_3 = \underline{8047 \text{ F}} \quad (\text{d})$$

15.12

$$R_{ins} = \frac{1}{40} = 0.025 \text{ hr ft}^2 \text{ Btu}$$

$$R_{out} = \frac{1}{S} = 0.20 \quad " \quad (\text{a})$$

$$\frac{q}{R} = \frac{\Delta T}{\sum R} = \frac{180}{10.225} = 17.6 \text{ Btu/hr ft}^2 \quad (\text{b})$$

$$= \frac{T_3 - T_0}{0.0278 + 0.2} \quad T_3 = 740 \text{ F} \quad (\text{b})$$

CONTROLLING RESISTANCE IS  
THE CORK BOARD.



15.18 (CONTINUED) -

WITH RADIATION FROM TOP

WITHOUT " "

$$\text{EQU ①: } T_B = 1.019T - 11.53$$

$$\text{② } T_B = 0.980T + 7.65$$

$$T = 582 \text{ K} = 122^\circ\text{F}$$

$$15.19 \quad A = 2[(0.3)(0.25) + (0.3)(0.5) + (0.25)(0.5)] \\ = 0.7 \text{ m}^2$$

$$q = \frac{kA}{L} \Delta T \quad L = \frac{kA \Delta T}{q}$$

$$L = \frac{(0.30 \text{ W/mK})(0.7 \text{ m})(43 \text{ K})}{400 \text{ W}}$$

$$= 0.0226 \text{ m} = 2.26 \text{ cm}$$

$$15.20 \quad A = 0.7 \text{ m}^2$$

$$q = \Delta T / \sum R$$

$$R_i = 1/h_i A = \frac{1}{16(0.7)} = 8.93 \times 10^{-2} \text{ K/W}$$

$$R_{cond} = L/kA = \frac{L}{(0.30)(0.7)} = \frac{L}{0.21}$$

$$R_o = 1/h_o A = \frac{1}{32(0.7)} = 4.46 \times 10^{-2}$$

$$\sum R = 0.1340 + \frac{L}{0.21} = \frac{43 \text{ K}}{400 \text{ W}}$$

FOR THESE CONDITIONS - NO INSULATION IS NECESSARY & THE SYSTEM CANNOT TRANSFER 400 W.

15.20 (CONTINUED)

$$q = \frac{\Delta T}{\sum R} = \frac{43 \text{ K}}{0.1340} = 321 \text{ K}$$

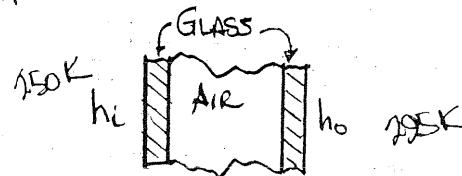
EFFECTIVE WALL TEMP IS

$$q = h_i(0.7)\Delta T_i = h_o(0.7)\Delta T_o$$

$$\Delta T_i = \frac{321}{16(0.7)} = 28.6 \text{ K}$$

$$T_{wall} = 18.6^\circ\text{C}$$

15.21



$$T_L \xrightarrow{R_i} \xrightarrow{R_{air}} \xrightarrow{R_a} \xrightarrow{R_o} T_o$$

$$q = \frac{\Delta T}{\sum R} \quad R_i = \frac{1}{(20)(1.83)(3.66)} = 7.45 \times 10^{-3}$$

$$R_{air} = \frac{0.0032}{(0.18)(1.83)(3.66)} = 6.125 \times 10^{-4}$$

$$R_a = \frac{0.008}{(0.0245)(1.83)(3.66)} = 0.0488$$

$$R_o = \frac{1}{(15)(1.83)(3.66)} = 9.95 \times 10^{-3}$$

$$\sum R = 0.06744$$

$$q = \frac{\Delta T}{\sum R} = \frac{45}{0.06744} = 667 \text{ W}$$

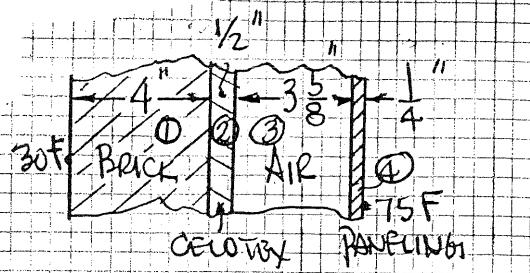
15.22 FOR 1 PANE OF GLASS ONLY

$$\sum R = R_i + R_{air} + R_o$$

$$= 0.0180$$

$$q = \frac{\Delta T}{\sum R} = \frac{45}{0.0180} = 2500 \text{ W}$$

15.23



$$\frac{q}{A} = -k \frac{dT}{dx} = k \frac{\Delta T}{\Delta x} = \frac{\Delta T}{R}, R = \frac{\Delta x}{k}$$

$$\frac{q}{A} = \frac{75 - 30}{0.38 + \frac{1/24}{0.028} + \frac{29/96}{0.05} + \frac{1/48}{0.012}} F$$

$$= \frac{45}{0.876 + 1.49 + 202 + 0.174} F$$

$$= \underline{1.98 \text{ BTU/HR-FT}^2} \rightarrow (a)$$

$$R_{AIR} = \frac{\Delta x}{k} = \frac{1}{1.8} = 0.555$$

$$\sum R = 0.876 + 1.49 + 0.555 + 0.174$$

$$= 3.095$$

$$\frac{q}{A} = \frac{45}{3.095} = \underline{14.54 \text{ BTU/HR-FT}^2} \rightarrow (b)$$

$$R_{GLASS WOOL} = \frac{29/96}{0.025} = 12.1$$

$$\sum R = 14.64$$

$$\frac{q}{A} = \frac{45}{14.64} = \underline{3.07 \text{ BTU/HR-FT}^2} \rightarrow (c)$$

15.24

$$\frac{q}{A} = \Delta T / \sum R$$

$$R_{INSIDE} = \frac{1}{h_i} = \frac{1}{1} \frac{\text{HR.F-FT}^2}{\text{BTU}}$$

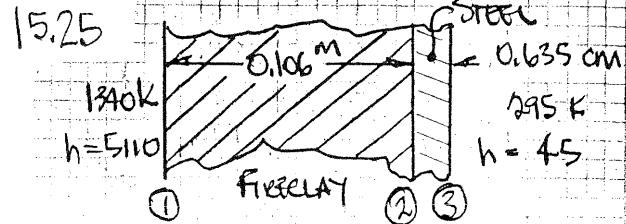
$$R_{OUTSIDE} = \frac{1}{h_o} = \frac{1}{2} "$$

15.24 CONTINUED

$$\text{PART a)} \sum R = 23.4 \quad \frac{q}{A} = 1.92 \frac{\text{BTU}}{\text{HR-FT}^2}$$

$$\text{b)} \sum R = 3.74 \quad \frac{q}{A} = 12.04 "$$

$$\text{c)} \sum R = 15.28 \quad \frac{q}{A} = 2.94 "$$



$$R_i = \frac{1}{A_i h_i} = \frac{1}{(5110)(1)} = 1.97 \times 10^{-4}$$

$$R_0 = \frac{1}{A_0 h_0} = \frac{1}{(45)(1)} = 0.22$$

$$R_1 = \frac{\Delta x}{k_A} = \frac{0.106}{(1.13)(1)} = 0.0938$$

$$R_2 = \frac{0.00635}{42.9} = 1.48 \times 10^{-4}$$

$$\sum R = 0.1161$$

$$q = \frac{1340 - 295}{0.1161} = 9000 \text{ W}$$

$$q_{000} = \frac{1340 - T_1}{R_i} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2}$$

$$= \frac{T_3 - 295}{R_0}$$

$$T_1 = 1338 \text{ K} \quad T_2 = 494 \text{ K} \quad T_3 = 493 \text{ K}$$

15.26

From previous problem

$$R_i + R_0 + R_1 + R_2 + R_{CELOTEX}$$

$$= 0.1161 + \frac{1}{0.069}$$

$$= 0.1161 + 14.49 \text{ L}$$

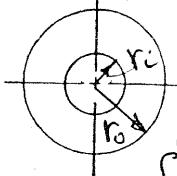
15.26 CONTINUED

$$q_f = \frac{340 - 295}{0.1022} = 2027 \text{ W}$$

$$2027 = \frac{340 - 295}{0.1161 + 14.49 L}$$

$$L = 0.0276 \text{ m}$$

15.27



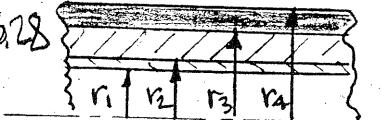
$$\int \frac{q}{A} dr = \int k dT$$

$$q \int_{r_i}^{r_o} \frac{dr}{2\pi r L} = -0.08 \int_{400}^{300} (1 - 0.0003T) dT$$

$$q \frac{\ln r_o/r_i}{2\pi L} = 65.52$$

$$q = \frac{2\pi (65.52)}{\ln 4} = 297 \frac{\text{Btu}}{\text{hr ft}}$$

15.28



For UNIT LENGTH:

$$R_{ins} = \frac{1}{2\pi r_i h_i} = 0.0238$$

$$R_1 = \frac{\ln r_2/r_1}{2\pi k_1} = 0.00105$$

$$R_2 = \frac{\ln r_3/r_2}{2\pi k_2} = 0.115/k_2$$

$$R_3 = \frac{\ln r_4/r_3}{2\pi k_3} = 0.059/k_3$$

$$R_4 = \frac{1}{2\pi r_4 h_o} = 0.1296$$

15.28 CONTINUED

$$\sum R = 0.1545 + 0.115/k_2 + 0.059/k_3$$

CASE 1:  $k_2$  for MAGNESIR

$$\sum R = 6.134$$

CASE 2:  $k_2$  for GLASS WOOL

$$\sum R = 7.096$$

GLASS WOOL CASE IS BEST

$$q = \frac{\Delta T / \sum R}{7.096} = \frac{60}{7.096} = 8.47 \frac{\text{Btu}}{\text{hr ft}}$$

$$q = \frac{8.47}{2\pi (2.95/2)} = 5.55 \frac{\text{Btu}}{\text{hr ft}^2}$$

15.29 for BARE PIPE:

$$q = \pi D_i h \Delta T = \pi \left( \frac{1.315}{12} \right) (1.5)(310)$$

$$= 160 \frac{\text{Btu}}{\text{hr ft}}$$

for INSULATED PIPE:

$$q_0 = \frac{T_s - T_o}{\frac{\ln D_2/D_1}{2\pi k} + \frac{1}{\pi D_2 h}}$$

$$\frac{\pi h}{2\pi k} \ln \frac{D_2}{D_1} + \frac{1}{D_2} = \frac{\pi h}{80} \Delta T$$

$$12.5 \ln \frac{D_2}{0.1905} + \frac{1}{D_2} = 18.15$$

By TRIAL & ERROR:  $D_2 = 0.382 \text{ ft}$ 

$$2t = D_2 - D_1 = 3.1265 \text{ in}$$

$$t = 1.63 \text{ in.}$$

15.30

FOR BARE PIPE, PER FOOT:

$$q = h A \Delta T = \frac{0.575}{(1.315/12)^{1/4}} \pi (1.315/12) (310)$$

$$= 106.7 \text{ BTU/HR}$$

WITH INSULATION -  $q = 53.3 \text{ W}$ 

$$q = \frac{\Delta T}{\frac{\ln D_o/D_i}{2\pi k} + \frac{1}{\pi D_o h_o}}$$

$$53.3 = \frac{310}{\frac{\ln D_o/1.315}{2\pi (0.06)} + \frac{(D_o/12)^{1/4}}{0.575 \pi D_o/12}}$$

BY TRIAL & ERROR:  $D_o = 9.22 \text{ IN}$ 

$$\text{INSULATION THICKNESS} = \frac{9.22 - 1.315}{2} = \underline{\underline{3.95 \text{ IN}}}$$

15.31

$$q_o = \frac{\Delta T}{\sum R}$$

WITHOUT INSULATION:

$$\sum R_{wo} = \frac{1}{2\pi L} \left[ \frac{\ln 137/12.5}{17.3} + \frac{1}{(12)(0.137)} \right]$$

$$= \frac{0.6136}{2\pi L} \text{ K/W}$$

WITH INSULATION:

$$\sum R_w = \frac{1}{2\pi L} \left[ \frac{\ln 137/12.5}{17.3} + \frac{1}{12r_o} + \frac{\ln r_o/0.137}{0.13} \right]$$

$$= \frac{1}{2\pi L} \left[ 5.299 \times 10^{-3} + \frac{1}{12r_o} + \frac{\ln r_o/0.137}{0.13} \right]$$

$$\therefore q_w = q_{wo} / 4$$

$$\sum R_w = 4 \sum R_{wo} = \frac{2.454}{2\pi L}$$

$$\therefore 5.299 \times 10^{-3} + \frac{1}{12r_o} + \frac{\ln r_o/0.137}{0.13} = 2.454$$

15.31 (CONTINUED -

BY TRIAL &amp; ERROR:

$$r_o = 0.177 \text{ m}$$

INSULATION THICKNESS =  $r_o - r_i$ 

$$= 0.177 - 0.137 = \underline{\underline{0.04 \text{ m}}}$$

$$= \underline{\underline{4 \text{ CM}}}$$

## CHAPTER 16

16.1 IN CYLINDRICAL COORDINATES: (a)

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = 0 \quad \text{OR} \quad \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = 0$$

$$r \frac{\partial T}{\partial r} = C_1$$

$$T = C_1 \ln r + C_2$$

$$\text{B.C. } T_i = C_1 \ln r_i + C_2$$

$$T_o = C_1 \ln r_o + C_2$$

$$C_1 = -\frac{T_i - T_o}{\ln r_o / r_i} \quad C_2 = T_i - C_1 \ln r_i$$

$$T = T_i - (T_i - T_o) \frac{\ln r / r_i}{\ln r_o / r_i} \quad (b)$$

$$q = -kA \frac{\partial T}{\partial r} = -k(2\pi r L) \frac{\partial T}{\partial r}$$

$$= -2\pi k L C_1 = \frac{2\pi k L}{\ln r_o / r_i} (T_i - T_o) \quad (c)$$

16.2  $\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) = 0 \quad (a)$

$$r^2 \frac{\partial T}{\partial r} = C_1 \quad T = -\frac{C_1}{r} + C_2$$

$$\text{B.C. } T_i = -\frac{C_1}{r_i} + C_2$$

$$T_o = -\frac{C_1}{r_o} + C_2$$

$$C_1 = \frac{T_i - T_o}{\frac{1}{r_o} - \frac{1}{r_i}} \quad C_2 = T_i + C_1 r_i$$

$$T = T_i - \frac{\frac{1}{r} - \frac{1}{r_i}}{\frac{1}{r_o} - \frac{1}{r_i}} (T_i - T_o) \leftarrow (b)$$

$$q = -k \left( 4\pi r^2 \right) \frac{\partial T}{\partial r} = -4\pi k C_1$$

$$= \frac{4\pi k}{\frac{1}{r_o} - \frac{1}{r_i}} (T_i - T_o) \leftarrow (c)$$

16.3  $\frac{\partial^2 T}{\partial \theta^2} = 0 \quad (a)$

$$T = C_1 \theta + C_2$$

$$\text{B.C. } T_o = C_2$$

$$T_\pi = C_1 \pi + C_2$$

$$C_1 = \frac{T_\pi - T_o}{\pi} \quad C_2 = T_o$$

$$T = T_o - \frac{\theta}{\pi} (T_o - T_\pi) \quad (b)$$

$$dq_\theta = -k \Delta A \frac{\partial T}{\partial n}$$

$$= -k(L dr) \frac{\partial T}{\partial r} = -kL \frac{\partial T}{\partial r} dr$$

$$\int_0^{r_o} dq_\theta = -kL C_1 \int_{r_i}^{r_o} \frac{\partial r}{r}$$

$$q_\theta = -kLC_1 \ln \frac{r_o}{r_i}$$

$$= \frac{kL}{\pi} \ln \frac{r_o}{r_i} (T_o - T_\pi) \quad (c)$$

16.4 PROBLEM STATEMENT REQUIRES THAT WE DEMONSTRATE

$$\frac{\delta Du}{Dt} + \frac{\delta D}{Dt} (\vec{g} \cdot \vec{y}) + \vec{v} \cdot \vec{\delta g} = \delta c_v \frac{DT}{DE}$$

$$\text{FOR } c_v \text{ CONSTANT: } \frac{\delta Du}{Dt} = \delta c_v \frac{DT}{DE} \quad (1)$$

$$\frac{\delta D}{Dt} (\vec{g} \cdot \vec{y}) = \delta \left[ \frac{\partial}{\partial t} (\vec{g} \cdot \vec{y}) + v_x \frac{\partial}{\partial x} (\vec{g} \cdot \vec{y}) + v_y \frac{\partial}{\partial y} (\vec{g} \cdot \vec{y}) + v_z \frac{\partial}{\partial z} (\vec{g} \cdot \vec{y}) \right]$$

$$= \delta \vec{g} \cdot \vec{v} \quad (2)$$

$$\vec{v} \cdot \vec{\delta g} = \vec{v} \cdot \vec{\delta g} \vec{e}_y = -\delta v_y g \quad (3)$$

SUBSTITUTING (1), (2), & (3)

THE DESIRED RESULT IS OBTAINED

16.5 EQUATION (16.7) WITH CONSTANT  $k$ : 16.5 (cont.) - By continuity  $\frac{1}{S} \frac{DS}{DT} = \nabla \cdot \vec{V}$

$$k\nabla^2 T + \dot{q}_D + L = \nabla \cdot S\vec{V} + S \frac{D}{DT} \left( \frac{V^2}{2} \right) + S \frac{DU}{DT} + S \frac{D}{DT} (gy)$$

For  $\dot{q}_D = 0$   $\Rightarrow$  NO VISCOUS DISSIPATION

$$k\nabla^2 T + \vec{V} \cdot \mu \nabla^2 \vec{V} = \nabla \cdot S\vec{V} + S \frac{D}{DT} \left( \frac{V^2}{2} \right) + S \frac{DU}{DT} + S \frac{D}{DT} (gy) \quad (1)$$

from NAVIER-STOKES:

$$S \frac{D\vec{V}}{DT} = S\vec{g} - \nabla P + \mu \nabla^2 \vec{V} \quad (9-19)$$

DOT PRODUCT WITH  $\vec{V}$  YIELDS

$$S \frac{D}{DT} \left( \frac{V^2}{2} \right) = \vec{V} \cdot S\vec{g} - \vec{V} \cdot \nabla P + \vec{V} \cdot \mu \nabla^2 \vec{V}$$

SUBSTITUTING INTO (1)  $\Rightarrow$  CANCELLING:

$$k\nabla^2 T = P \nabla \cdot \vec{V} + \vec{V} \cdot S\vec{g} + S \frac{DU}{DT} + S \frac{D}{DT} (gy)$$

FOR A POTENTIAL FUNCTION  $\phi = gy$

$$\nabla \phi = -\vec{g}$$

$$\text{THEN } S \frac{D}{DT} (gy) = S \frac{D}{DT} \phi = S \left( \frac{\partial \phi}{\partial t} + \vec{V} \cdot \nabla \phi \right)$$

COMBINING WITH THE ENERGY EQUATION:

$$k\nabla^2 T = P \nabla \cdot \vec{V} + S \frac{DU}{DT}$$

Now - From THERMODYNAMICS:

$$u = u(V, T) \quad du = \left( \frac{\partial u}{\partial V} \right)_T dV + \left( \frac{\partial u}{\partial T} \right)_V dT$$

$$\Rightarrow S \frac{DU}{DT} = S \left( \frac{\partial u}{\partial V} \right)_T \frac{DV}{DT} + S \left( \frac{\partial u}{\partial T} \right)_V \frac{dT}{DT}$$

GIVING:

$$k\nabla^2 T = P \nabla \cdot \vec{V} + S c_V \frac{DT}{DT} + S \frac{DV}{DT} \left[ -P + T \left( \frac{\partial P}{\partial T} \right)_V \right]$$

$$S \frac{DV}{DT} = S \frac{D}{DT} \left( \frac{1}{S} \right) = -\frac{1}{S} \frac{DS}{DT}$$

$$\text{so } k\nabla^2 T = P \nabla \cdot \vec{V} + S c_V \frac{DT}{DT} - P \nabla \cdot \vec{V} + T \left( \frac{\partial P}{\partial T} \right)_V \nabla \cdot \vec{V}$$

FOR INCOMPRESSIBLE FLOW  $\nabla \cdot \vec{V} = 0$

$$k\nabla^2 T = S c_V \frac{DT}{DT}$$

Q.E.D.

16.6 @ STEADY STATE  $\nabla^2 T + \frac{\dot{q}}{k} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} \frac{1}{\epsilon} = 0$$

$$\frac{\partial T}{\partial x} = \frac{\dot{q}}{k} \frac{L}{\epsilon} + C_1$$

$$T = -\frac{\dot{q}}{k} \frac{L^2}{\epsilon} + C_1 x + C_2$$

$$\text{B.C. } T(0) = T_0 \quad T_b = -\frac{\dot{q}}{k} \frac{L^2}{\epsilon} + C_2$$

$$T(L) = T_L \quad T_L = -\frac{\dot{q}}{k} \frac{L^2}{\epsilon} + C_1 L + C_2$$

$$T = T_0 + (T_L - T_0) \frac{x}{L} + \frac{\dot{q}}{k} \frac{L^2}{\epsilon} \left[ \frac{1 - e^{-\frac{x}{L}}}{1 - e^{-\frac{L}{\epsilon}}} \right] - \frac{x}{L} \left( 1 - e^{-\frac{L}{\epsilon}} \right)$$

16.7 SAME PROBLEM EXCEPT 1<sup>ST</sup> B.C. IS

$$\frac{\partial T}{\partial x}(L) = 0 \Rightarrow 0 = \frac{\dot{q}}{k} \frac{L}{\epsilon} - \frac{P}{\epsilon} + C_1$$

$$T = T_0 + \frac{\dot{q}}{k} \frac{L^2}{\epsilon} \left( \frac{1 - e^{-\frac{L}{\epsilon}}}{1 - e^{-\frac{P}{\epsilon}}} \right)$$

16.8. Same problem as 16.6 but 2nd

B.C.  $\frac{dT}{dx}(L) = \xi$  ( $\xi$  constant)

$$\xi = \frac{q_0}{K} \frac{L}{\beta^2} e^{-\beta L} + C_2$$

$$T = T_0 + \xi x - \frac{q_0}{K} \frac{L^2}{\beta^2} \left( \frac{\beta x}{L} e^{-\beta x/L} - e^{-\beta L} \right)$$

16.9  $TdS = dh - \frac{dp}{\rho} = du + pdV$

FORMING SUBSTANTIAL DERIVATIVES

$$\frac{TdS}{dt} = \frac{Du}{dt} + P \frac{dV}{dt} = \frac{Du}{dt} + \frac{P}{\xi^2} \frac{dS}{dt}$$

BY CONTINUITY /  $\frac{dS}{dt} = -S \nabla \cdot \vec{v}$

So  $\frac{Du}{dt} = T \frac{dS}{dt} + \frac{P}{S} \nabla \cdot \vec{v}$

From THE ENERGY EQUATION

$$\frac{Du}{dt} = \nabla \cdot k \nabla T + \dot{q} + \phi$$

$$\therefore T \frac{dS}{dt} = \nabla \cdot k \nabla T + \dot{q} + \phi + \frac{P}{S} \nabla \cdot \vec{v}$$

SINCE  $\phi$  IS ALWAYS  $> 0$  ITS EFFECT IS ALWAYS TO INCREASE S

SINCE  $\nabla \cdot k \nabla T$  CAN BE EITHER + OR - AT TX CAN EITHER INCREASE OR DECREASE S

16.10 For  $\frac{V_x}{V_{y0}} = \frac{3}{2} \frac{y}{8} - \frac{1}{2} \left( \frac{y}{8} \right)^3$

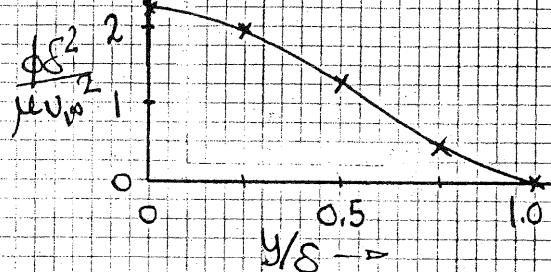
WITH  $V_y = V_z = 0$  THE DISSIPATION FUNCTION REDUCES TO

$$\phi = \mu \left[ \frac{\partial V_x}{\partial y} \right]^2$$

FOR THIS GASE

$$\frac{\phi \delta^2}{\mu v_{y0}^2} = \frac{9}{4} \left[ 1 - \left( \frac{y}{8} \right)^2 \right]$$

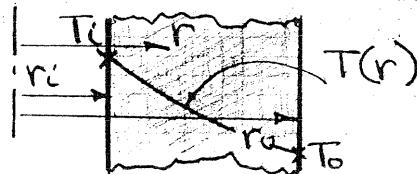
16.10 (CONT.)



16.11 SPHERICAL COORDINATES

$$\frac{1}{r} \frac{d}{dr} \left( r^2 \frac{\partial T}{\partial r} \right) = 0$$

$$r^2 \frac{\partial T}{\partial r} = \text{CONST} \quad \frac{\partial T}{\partial r} = \frac{\text{CONST}}{r}$$



AS  $r$  INCREASES  $\frac{\partial T}{\partial r}$  DECREASES

16.12 FOR THE TRUNCATED CONE:

$$A_1 = \pi r_1^2 \quad A_2 = \pi r_2^2 \quad r = r_1 + \frac{r_2 - r_1}{L} x$$

$$\Rightarrow A = A_0 \left[ 1 + \left( \frac{r_2 - r_1}{r_1} \right) x \right]^2 = A_0 \left( 1 + \beta x \right)^2$$

$$\beta = \left( \frac{r_2 - r_1}{r_1} \right) \frac{1}{L}$$

SINCE  $\dot{q} = -kA \frac{\partial T}{\partial x}$  WE HAVE

$$\dot{q}_b = -kA_0 \left( 1 + \beta x^2 \right) \frac{\partial T}{\partial x}$$

$$\int_0^L \frac{\partial x}{1 + \beta x^2} = -kA_0 \int_{T_1}^{T_2} \frac{dT}{x}$$

$$\dot{q}_b = kA_0 \left[ \tan^{-1}(\sqrt{\beta} L) \right] (T_1 - T_2)$$

### 16.12 CONTINUED -

IF, IN ADDITION,  $k = k_0 - \alpha T$

WE HAVE

$$q = -(k_0 - \alpha T)(1 + \beta x^2) \frac{dT}{dx}$$

$$\oint_0^L \frac{dx}{(1 + \beta x^2)} = - \int_{T_1}^{T_2} (k_0 - \alpha T) dT$$

$$q = A_0 \left[ \tan^{-1} \left( \beta L \right) \right] \left[ k_0 - \alpha (T_1 + T_2) \right]$$

$$\times (T_1 - T_2)$$

### 16.13. HT GENERATION IN PLANE WALL

$$\dot{q} = q_{\text{max}} \left( 1 - \frac{x}{L} \right)$$

FOURIER FIELD EQUATION FOR STEADY STATE 1-D CONDUCTION, REDUCES TO

$$\frac{d^2 T}{dx^2} + \frac{q_{\text{max}}}{k} \left( 1 - \frac{x}{L} \right) = 0$$

1<sup>ST</sup> INTEGRATION:

$$\frac{dT}{dx} + \frac{q_{\text{max}}}{k} \left( x - \frac{x^2}{2L} \right) = C_1$$

SYMMETRY;  $\frac{dT}{dx} = 0 @ x=0 \therefore C_1 = 0$

SECOND INTEGRATION:

$$\int_{T_c}^{T_o} dT + \frac{q_{\text{max}}}{k} \int_0^x \left( x - \frac{x^2}{2L} \right) dx = 0$$

$$T_o - T_c + \frac{q_{\text{max}}}{k} \left( \frac{x^2}{2} - \frac{x^3}{6L} \right) \Big|_0^L = 0$$

$$T_c - T_o = \frac{q_{\text{max}} L^2}{3k}$$

### 16.14 HT GENERATION IN A CYLINDER

$$\dot{q} = \dot{q}_{\text{max}} \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right]$$

FOURIER FIELD EQUATION REDUCES TO

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{q_{\text{max}}}{k} \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right] = 0$$

SEPARATING VARIABLES - 1<sup>ST</sup> INTEGRATION

$$\int d \left( r \frac{dT}{dr} \right) + \frac{\dot{q}_{\text{max}}}{k} \left[ r - \frac{r^3}{3r_0} \right] dr = 0$$

$$r \frac{dT}{dr} + \frac{\dot{q}_{\text{max}}}{k} \left( \frac{r^2}{2} - \frac{r^4}{4r_0^2} \right) = C_1$$

SYMMETRY:  $\frac{dT}{dr} = 0 @ r=0 \therefore C_1 = 0$

SECOND SEPARATION & INTEGRATION

$$\int_{T_c}^{T_o} dT + \frac{\dot{q}_{\text{max}}}{k} \int_0^r \left( \frac{r}{2} - \frac{r^3}{4r_0^2} \right) dr = 0$$

$$T_c - T_o = \frac{\dot{q}_{\text{max}}}{k} \frac{3}{16} \frac{r_0^2}{L^2}$$

### 16.15 HT GENERATION IN A SPHERE

FOURIER FIELD EQUATION REDUCES TO:

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) + \frac{\dot{q}_{\text{max}}}{k} \left[ 1 - \left( \frac{r}{r_0} \right)^3 \right] = 0$$

1<sup>ST</sup> INTEGRATION YIELDS      0 - SYMMETRY

$$r^2 \frac{dT}{dr} + \frac{\dot{q}_{\text{max}}}{k} \left( \frac{r^3}{3} - \frac{r^6}{6r_0^3} \right) = C_1$$

SECOND INTEGRATION

$$\int_{T_c}^{T_o} dT + \frac{\dot{q}_{\text{max}}}{k} \int_0^r \left( \frac{r}{3} - \frac{r^4}{6r_0^3} \right) dr = 0$$

$$T_c - T_o = \frac{\dot{q}_{\text{max}}}{k} \frac{2}{15} \frac{r_0^2}{L^2}$$

## CHAPTER 17

### 17.1 STEADY-STATE X-DIRECTIONAL CONDUCTION THROUGH A PLANE WALL

$$q_x = -kA \frac{\Delta T}{\Delta x} = \frac{kA}{L} (T_1 - T_2)$$

$$\text{For } T_1 - T_2 = 75 \text{ K}$$

$$q = (30 \text{ W/m}\cdot\text{K})(1 \text{ m}^2)(75 \text{ K})$$

$$= 0.30 \text{ m}$$

$$= 7500 \text{ W/m}^2$$

$$\Delta T/\Delta x = \frac{\Delta T}{L} = \frac{75 \text{ K}}{0.30 \text{ m}} = 250 \text{ K/m}$$

$$\text{For } T_1 = 300 \text{ K} \quad q = -2000 \text{ W/m}^2$$

$$\Delta T/\Delta x = -\frac{q}{kA} = \frac{2000 \text{ W/m}^2}{(30 \text{ W/m}\cdot\text{K})} = 66.7 \text{ K/m}$$

$$\Delta T = \frac{q_0 L}{kA} = \frac{(2000)(0.3)}{(30)} = -200 \text{ K}$$

$$T_2 = 320 \text{ K}$$

$$\text{For } T_2 = 350 \text{ K} \quad \Delta T/\Delta x = -300 \text{ K/m}$$

$$q = -(30)(-300) = 9000 \text{ W/m}^2$$

$$\Delta T = -300 \text{ K/m}(0.3 \text{ m}) = 90 \text{ K}$$

$$T_1 = 440 \text{ K}$$

$$\text{For } T_1 = 250 \text{ K} \quad \Delta T/\Delta x = 200 \text{ K/m}$$

$$q = -(30)(200 \text{ K/m}) = -6000 \text{ W/m}^2$$

$$\Delta T = -(200)(0.3) = -60 \text{ K} \quad T_2 = 310 \text{ K}$$

$$17.2 \quad q = \frac{k\bar{A}}{r_0 r_i} \Delta T = \frac{k}{r_0 r_i} 2\pi \frac{(r_0 - r_i)}{\ln r_0/r_i} \Delta T$$

$$= \frac{2\pi k}{\ln r_0/r_i} \Delta T \quad (a)$$

$$\% \text{ ERROR} = \frac{A_m - A_{pm}}{A_m} \times 100$$

### 17.2 (CONTINUED)

$$\begin{aligned} & \frac{2\pi(r_0 - r_i)}{\ln r_0/r_i} - \pi(r_0 + r_i) \\ & = \frac{2\pi(r_0 - r_i)/r_0}{\ln r_0/r_i} \times 100 \\ & = \left| 1 - \frac{(r_0 + r_i)\ln r_0/r_i}{2(r_0 - r_i)} \right| \times 100 \\ & = \left| 1 - \frac{(r_0/r_i + 1)\ln r_0/r_i}{2(r_0/r_i - 1)} \right| \times 100 \end{aligned}$$

$$\text{For } \frac{r_0}{r_i} = 1.5 \quad \% \text{ ERROR} = 1.3 \%$$

$$3 \quad " = 10.0 \%$$

$$5 \quad " = 20.7 \%$$

17.3

$$q = \frac{4\pi k r_0 r_i}{r_0 - r_i} \Delta T \quad \bar{A} = 4\pi r_0 r_i \quad (a)$$

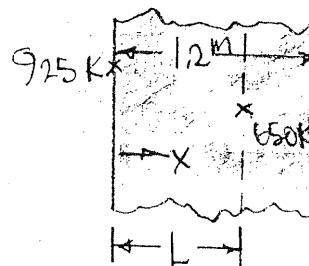
$$A_m = \frac{4\pi(r_0^2 + r_i^2)}{2} = 2\pi(r_0^2 + r_i^2)$$

$$\% \text{ ERROR} = \frac{4\pi r_0 r_i - 2\pi(r_0^2 + r_i^2)}{4\pi r_0 r_i}$$

$$= 1 - \frac{1}{2} \left( \frac{r_0}{r_i} - \frac{r_i}{r_0} \right)$$

$$\begin{aligned} r_0/r_i &= 1.5 \quad \% \text{ ERROR} = 8.3 \% \\ 3 \quad " &= 66.6 \% \\ 5 \quad " &= 160 \% \end{aligned} \quad (b)$$

17.4



$$T_\infty = 300 \text{ K}$$

17.4 (CONTINUED)

$$q'' = -k \frac{dT}{dx} = -k_0(1+bT) \frac{dT}{dx}$$

From 0 TO 1.2:

$$\int_0^{1.2} q'' dx = -k_0(1+bT) dT$$

$$q'' = \frac{k_0}{1.2} \left[ T + \frac{b}{2} T^2 \right]_{q_{25}}^{q_{125}} = 23(T-300)$$

SOLVING:  $T_{\text{RH WALL}} = 307.1 \text{ K}$

$$q'' = 163.3 \text{ W/m}^2$$

From 0 TO L:

$$q'' \int_0^L = -k_0 \left[ (1+bT) dT \right]_{q_{25}}^{q_{650}}$$

$$L = \frac{k_0}{q''} \left[ T + \frac{b}{2} T^2 \right]_{650}^{q_{125}}$$

SOLVING:

$$L = 0.646 \text{ m.}$$

17.5 GOVERNING EQUATION:  $\nabla \cdot k \nabla T = 0$

IN ONE DIMENSION:  $\frac{d}{dx}(k \frac{dT}{dx}) = 0$

FOR CONSTANT k:

$$\frac{d^2T}{dx^2} = 0$$

$$\frac{dT}{dx} = C_1$$

$$T = C_1 x + C_2$$

$$T(0) = T_0 = C_1(0) + C_2 \quad C_2 = T_0$$

$$T(L) = T_L = C_1 L + C_2 \quad C_1 = \frac{T_L - T_0}{L}$$

FOR VARIABLE k:  $\frac{d}{dx} k_0(1+\beta T) \frac{dT}{dx} = 0$

$$(1+\beta T) \frac{dT}{dx} = C_3$$

$$T + \frac{\beta T^2}{2} = C_3 x + C_4$$

$$T(0) = T_0 \quad T_0 + \frac{\beta T_0^2}{2} = C_4$$

$$T(L) = T_L \quad T_L + \frac{\beta T_L^2}{2} = C_3 L + C_4$$

17.5 (CONTINUED)

$$C_3 = \frac{T_L}{L} \left( 1 + \frac{\beta T_L}{2} \right) - \frac{C_4}{L}$$

$$T^2 + \frac{1}{\beta} T - \frac{1}{\beta} C_3 x - \frac{1}{\beta} C_4$$

$$T^2 + BT - C = 0 \quad B = \frac{1}{\beta}$$

$$T = -\frac{B}{2} \pm \sqrt{\frac{B^2}{4} - C} \quad C = \frac{1}{\beta} (C_3 x + C_4)$$

NOW - THE TEMPERATURE DIFFERENCE WE'RE SEEKING IS:

$$\Delta = C_1 x + C_2 - \left[ -\frac{B}{2} \pm \sqrt{\frac{B^2}{4} - C} \right]$$

MAXIMUM IS WHERE  $\frac{d}{dx} \Delta = 0$

$$\frac{d\Delta}{dx} = C_1 \mp \left( \frac{B^2}{4} - C \right)^{-1/2} \left( -\frac{2C_3}{\beta} \right) = 0$$

$$\frac{C_1 \beta}{2C_3} \mp \left( \frac{B^2}{4} - C \right)^{-1/2} = 0$$

$$\frac{B^2}{4} - C = \frac{4C_3^2}{\beta^2 C_1^2}$$

$$C = \frac{B^2}{4} - \frac{4C_3^2}{\beta^2 C_1^2}$$

$$\frac{2}{\beta} C_3 x + \frac{2}{\beta} C_4 = \frac{1}{\beta^2} - \frac{4C_3^2}{\beta^2 C_1^2}$$

$$x = \frac{1}{2\beta C_3} - \frac{2}{\beta C_1^2} - \frac{C_4}{C_3}$$

$C_1, C_3, \frac{1}{\beta} C_4$  ARE AS DETERMINED ABOVE

17.6 SAME GENERAL PROCEDURE AS PREVIOUS PROBLEM:

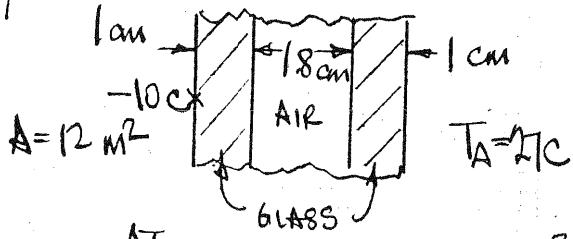
D.E. IS  $\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0$

FOR CONSTANT k:  $\frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0$

FOR VARIABLE k:  $\frac{d}{dr} \left[ r \left( 1 + \beta T \right) \frac{dT}{dr} \right] = 0$

- MESSY BUT STRAIGHTFORWARD -

17.7



$$\dot{q} = \frac{\Delta T}{\sum R} \quad R_{GL} = \frac{0.01}{(0.78)(12)} = 1.068 \times 10^{-3}$$

$$R_{AIR} = \frac{0.018}{0.0262(12)} = 5.725 \times 10^{-2}$$

$$R_{CONV} = \frac{1}{12(12)} = 6.944 \times 10^{-3}$$

$$\sum R = 2(1.068 \times 10^{-3}) + R_{AIR} + R_{CONV} = 0.06633 \text{ K/W}$$

$$\dot{q} = 37 / 0.06633 = 585 \text{ W}$$

$$T_i = 27 - \frac{585}{(12)(12)} = 22.94 \text{ C}$$

17.8 Brick Stack = 9" x 4.5' x 3"

$$\text{Brick #1 } k = 0.44 \text{ BTU/HPTF } T_{max} = 1500 \text{ F}$$

$$\#2 \quad k = 0.94 \text{ " } T_{max} = 2100 \text{ "}$$

MOST ECONOMICAL ARRANGEMENT IS TO USE AS MUCH OF #1 AS POSSIBLE (LOW  $k$ ). USE #2 NEXT TO HIGHEST TEMP SUCH THAT ITS COOLER SURFACE HAS  $T \leq 1500 \text{ F}$ .

$$\dot{q}'' = \frac{2000 - T}{L/k} \quad L_2 = \frac{k(2000 - T_m)}{200} = 2.35 \text{ FT} = 28.2 \text{ IN.}$$

$$L_{ACTUAL} = 28.5 \text{ in. } (9 \times 2 + 4.5 + 2 \times 3)$$

$$T_{INTERFACE} = T_H - \frac{\dot{q}''}{k_2/L_2} = 1495 \text{ F}$$

$$L_{MIN} = \frac{0.44(1495 - 300)}{200} = 2.63 \text{ FT} = 31.6 \text{ IN.}$$

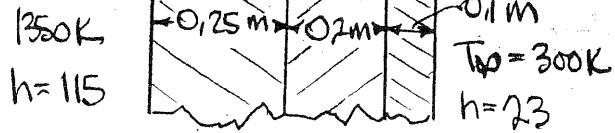
$$L_{ACT} = 33 \text{ in.}$$

17.8 CONTINUED -

MOST ECONOMICAL:

$$\begin{aligned} L_1 &= 33 \text{ in.} \\ L_2 &= 28.5 \end{aligned}$$

17.9



$$T_i = T_1 - \frac{\dot{q}}{h} = T_1 - \frac{1070}{0.115} = 8.696 \times 10^5 \text{ K/W}$$

$$R_1 = 0.25 / 1.13 = 0.221 \text{ "}$$

$$R_2 = 0.20 / 1.45 = 0.138 \text{ "}$$

$$R_3 = 0.10 / 0.166 = 0.152 \text{ "}$$

$$\dot{q} = \frac{1}{23} = 0.0435 \text{ K/W}$$

$$\sum R = 0.563$$

$$\dot{q} = \frac{\Delta T}{\sum R} = \frac{1070}{0.563} = 1900 \text{ W/m}^2$$

$$= 116.5 \text{ W/in}^2$$

$$\dot{q} = 23(T_o - 300) \quad T_o = 363 \text{ K}$$

17.10



$$T_b = 325 \text{ K} \quad \dot{q} = 23(325 - 300) = 575 \text{ W/m}^2$$

$$R_i = \frac{1}{115} = 8.696 \times 10^5 \text{ K/W}$$

$$R_1 = 0.25 / 1.13 = 0.221 \text{ "}$$

$$R_2 = 0.20 / 1.45$$

$$R_3 = 0.10 / 0.166 = 0.152 \text{ "}$$

132

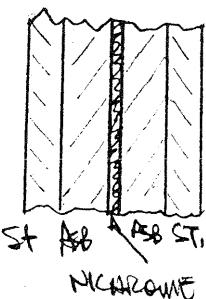
## 17.10 CONTINUED

$$\Sigma R = 0.416 + \frac{1}{1.45} = \frac{\Delta T}{q}$$

$$0.416 + \frac{1}{1.45} = \frac{1045}{575}$$

$$L = 2.03 \text{ m}$$

17.11

 $T_{\infty} \approx 70 \text{ F}$  (ASSUME)

$$L_{st} = \frac{1}{8} \text{ in} \quad k = 10 \text{ BTU/HR-FT}^2$$

$$L_{ASB} = \frac{1}{8} \text{ in} \quad k = 0.15 \text{ in}$$

$$q = \frac{\Delta T}{\Sigma R} = \frac{100 - T_{\infty}}{\frac{0.125/12}{0.15} + \frac{0.125/12}{10} + \frac{1}{3}}$$

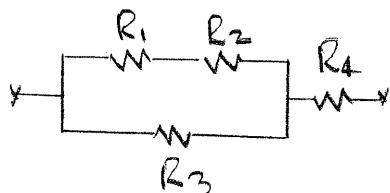
$$= \frac{930}{0.403} = 2305 \text{ BTU/HR-FT}^2 \text{ PER SIDE}$$

$$q = \frac{2305(2)}{3.413} = 1351 \text{ W/ft}^2 \text{ (a)}$$

$$2305 = h \Delta T = 3 \Delta T$$

$$\Delta T = 768 \text{ F} \quad T_{surf} = 838 \text{ F (b)}$$

17.12



$$R_1 = \frac{0.125/12}{0.15} = 0.0694$$

$$R_2 = \frac{0.125/12}{10} = 0.00104$$

$$R_3 = \frac{0.25/12}{(2)(2)(\pi/4)(0.75/2)^2}$$

$$= 0.154$$

## 17.12 CONTINUED

$$R_{CONDUCTION, EQUIV} = \frac{1}{\frac{1}{k_1 + R_2} + \frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{0.01044} + \frac{1}{0.154}} = 0.0483$$

$$\Sigma R_{PDL \text{ SIDE}} = 0.0483 + \frac{1}{3} = 0.3817$$

$$\text{NEW HT FLUX} = \frac{930}{0.3817} = 2437 \text{ BTU/HR-FT}^2$$

$$\text{INCREASE} = \frac{2437 - 2305}{2305}$$

$$= 0.057 = 5.7 \%$$

17.13



$$T_{\infty} = 295 \text{ K}$$

$$h_1 = 12 \text{ W/m}^2\text{K}$$

$$h_2 = "$$

$$k_1 = 2.42 \text{ W/mK} \quad k_2 = 209$$

a) APPLIED TO PLASTIC:

$$q = 12(T_1 - 295) + \frac{2.42}{0.025}(T_1 - 325)$$

$$\frac{229}{0.05}(325 - T_2) = 12(J_2 - 295)$$

$$T_2 = 324.9 \quad T_1 = 328.7$$

$$q = 359 + 404 = 763 \text{ W}$$

b) APPLIED TO AL:

$$q = 12(T_2 - 295) + \frac{229}{0.05}(J_2 - 325)$$

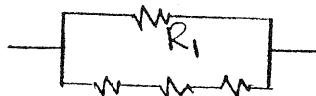
$$\frac{229}{0.05}(T_2 - 325) = \frac{2.42}{0.025}(325 - T_1)$$

$$T_1 = 322 \text{ K}$$

$$T_2 = 325$$

$$q = 320 + 361 = 681 \text{ W}$$

17.14



BOLTS IN A SQUARE ARRAY  
WITH 4 EQUV. BOLTS/FT<sup>2</sup>

$$R_4(\text{BOLTS}) = \frac{L}{kA} = \frac{3.75/2}{(10)(4)(\pi/4)(1/4)} = 5.7 \text{ KRF/BTU} - \text{STEEL}$$

$$= 0.475 \text{ " } - \text{ALM.}$$

$$R_{2ss} = \frac{1/48}{(10)(1)} = 0.0021$$

$$R_{3cb} = \frac{3/2}{(0.25)(1)} = 10$$

$$R_{4pl} = \frac{1/24}{(1.5)(1)} = 0.0278$$

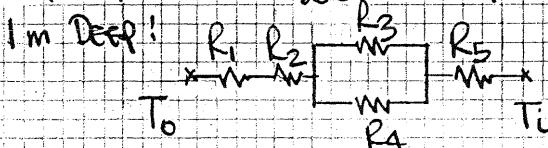
$$\sum R = 10.03$$

PER FT<sup>2</sup> OF X-SECTION

$$R_{\text{equiv}} = \frac{1}{1/5.7 + 1/10.03} = 3.63 \frac{\text{HRF}}{\text{BTU}} \quad (\text{a})$$

$$= \frac{1}{1/0.475 + 1/10.03} = 0.454 \text{ " } \quad (\text{b})$$

17.15 FOR A SECTION 36 CM WIDE  $\frac{1}{2}$



$$R_1 = \frac{1}{A \cdot h_0} = \frac{1}{(0.36)(20)} = 0.139 \text{ K/W}$$

$$R_2 = \frac{L}{kA} = \frac{0.02}{(0.814)(0.36)} = 0.0683 \text{ "}$$

$$R_3 = \frac{L}{kA} = \frac{0.15}{0.15(0.06)} = 16.67 \text{ "}$$

17.15 (CONTINUED)

$$R_4 = \frac{L}{kA} = \frac{0.15}{0.035(0.36)} = 14.28 \text{ "}$$

$$R_5 = \frac{1}{A \cdot h_0} = \frac{1}{(0.36)(10)} = 0.278 \text{ "}$$

$$R_{\text{stud wall equiv}} = \frac{1}{1/6.67 + 1/14.28} = 7.691 \text{ "}$$

$$q_f = \frac{\Delta T}{\sum R_{\text{equiv}}} = \frac{35 \text{ K}}{8.176 \text{ K/W}} = 4.28 \text{ W}$$

$$q_f^3 = \frac{\Delta T_{sw}}{R_3} \quad q_f^4 = \frac{\Delta T_{sw}}{R_4}$$

$$\Delta T_{sw} = \frac{q_f}{1/R_3 + 1/R_4} = 32.92 \text{ K}$$

$$q_f^3 = \frac{32.92}{16.67} = 1.975 \text{ W}$$

$$q_f^4 = \underline{2.305 \text{ W}}$$

17.16

$$q_{\text{loss}} = q_f^1 + q_f^2$$

$$q_f^1 = HT \text{ TO AIR} = hAAT$$

$$\begin{aligned} &= (23 \text{ W/m}^2 \cdot \text{K})(2.5 \times 0.1 \times 2 \text{ m}^2 \\ &+ 2.5 \times 0.05 \times 2 \text{ m}^2 + 0.1 \times 0.05 \times 2 \text{ m}^2 \\ &- (0.08)^2 \times 2 \text{ m}^2)(T-300) \end{aligned}$$

$$= 17.19 (T-300)$$

$$q_f^2 = HT \text{ THROUGH PESTALS}$$

$$= 2kA m \theta_0 \left[ 1 - 2 \frac{e^{-\theta_0} - e^{-mL}}{e^{mL} - e^{-mL}} \right]$$

$$m = \left[ \frac{hP}{kA} \right]^{\frac{1}{2}} = \left[ \frac{(23)(0.08)(4)}{(2.6)(0.08)(0.08)} \right]^{\frac{1}{2}} = 21$$

$$e^{mL} = 23.45 \quad e^{-mL} = 0.0427$$

17.16 (CONTINUED -

$$q_2 = 2(24)(0.08)^2 (21)(T-300) \times \\ \times \left[ 1 - 2 \frac{-0.0421}{23.45} \right] \\ = 0.696(T-300)$$

$$1000 = (17.19 + 0.696)(T-300)$$

$$\underline{\underline{T = 355.9 \text{ K}}}$$

17.17

$$q_{\text{loss}} = q_1 + q_2 + q_3$$

$$q_1 = 17.19(T-300) \quad \text{From Prev. Prob.}$$

$q_2$  = Same Expression

$$A = (0.08)(0.08) - \frac{\pi}{4}(0.019)^2 \\ = 0.00612 \quad \left\{ \begin{array}{l} \text{Previously} \\ 0.0064 \end{array} \right\}$$

For PEDESTAL MATTL:

$$q_2 \approx 0.7(T-300)$$

$q_3$  = CONDUCTION THROUGH BOLTS

$$= \frac{kA}{L} \Delta T = 42.9 \frac{\pi}{4} \frac{(0.019)^2}{0.15} \Delta T \\ = 0.081(T-300)$$

$$1000 = [17.19 + 0.7 + 0.081](T-300)$$

$$\underline{\underline{T = 355.6 \text{ K}}}$$

17.18

$$q = \Delta T / \sum R$$

$$\Delta T = 292.7 - 70 = 222.7 \text{ F}$$

Room Temp (Assumed)

$$\text{For } 2\text{-in SCH 40} \quad ID = 2.067 \text{ in} \\ OD = 2.375 \text{ in}$$

$$R_{ST} = \frac{\ln \frac{D_o}{D_i}}{2\pi k L} = 1.475 \times 10^{-5}$$

$$R_{INS} = \frac{\ln \frac{D_o}{D_i}}{2\pi k L} = 0.0529$$

$$R_{BOLTS} = \frac{1}{h A_0} = 0.00537 \text{ w/o INSUL} \\ = 0.00237 \text{ w/ INSUL}$$

$$\sum R_{\text{WITHL}} = 0.0553 \quad q = 4030 \text{ BTU/hr}$$

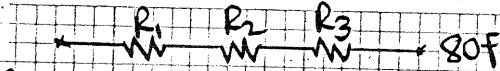
$$\sum R_{\text{WITHOUT}} = 0.00537 \quad q = 41,500 \text{ "}$$

$$\Delta q = 37470 \text{ BTU/hr}$$

$$(lost) = 37470 \left( \frac{0.68}{10^5} \right) = \$ 0.255/\text{hr}$$

$$\text{TIME} = \frac{(60 \text{ ft})(0.75/\text{ft})}{\$ 0.255/\text{hr}} = \underline{\underline{177 \text{ Hours}}}$$

17.19



267.25 F

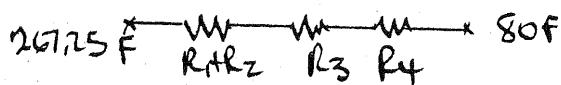
$$R_1 = \frac{1}{\pi D h_i L} = \frac{1}{\pi (1.755/2)(1500)(10)} = 0.0015$$

$$R_2 = \frac{\ln \frac{D_o}{D_i}}{2\pi k L} = \frac{\ln 1.078}{2\pi (0.04)(10)} = 6.19 \times 10^{-5}$$

$$R_3 = \frac{1}{\pi (1.755/2)(12)} = 0.0725$$

$$\sum R = 0.072 \text{ HRF/BTU}$$

$$q = \frac{\Delta T}{\sum R} = \frac{187.25}{0.072} = 2530 \text{ BTU/hr} \quad (a)$$



$$R_1 + R_2 = 0.00156 \text{ HRF/BTU}$$

$$R_3 = \frac{\ln \frac{5.755}{1.755}}{2\pi (0.04)(10)} = 0.461 \frac{\text{HrF}}{\text{BTU}}$$

17.19 CONTINUED -

$$R_4 = \frac{1}{\pi(5.155/12)(3)(10)} = 0.022 \quad (b)$$

$$\sum R = 0.485 \quad q = \frac{\Delta T}{\sum R} = 386 \frac{\text{BTU}}{\text{HR}}$$

for Bare Pipe:  $m_{\text{steam}} = \dot{q}/h_{\text{fg}}$

$$= \frac{3530 \text{ BTU/HR}}{9337 \text{ BTU/LB}_m} = 2.71 \frac{\text{LB}_m}{\text{HR}} \quad (c)$$

17.20

$$\dot{q} \frac{\pi D^2}{4} L = h \pi D L \Delta T$$

$$\frac{I^2 R}{\pi D^2/4 L} \frac{D}{4} = h \Delta T$$

$$I^2 R = 10 \text{ kW}$$

$$h = 850 \text{ W/m}^2 \cdot \text{K}$$

$$L = 0.6 \text{ m} \quad \Delta T = 1280 \text{ K}$$

$$D = \frac{4(10000 \text{ W})}{(1280 \text{ K})(\pi)(0.6 \text{ m})(850 \text{ W/m}^2 \cdot \text{K})}$$

$$= 0.0195 \text{ m} = 1.95 \text{ cm} \quad (a)$$

$$(A \text{ GAGE} = 0.004 \text{ m}, D_{\text{AM}} = 1.626 \times 10^{-3} \text{ m})$$

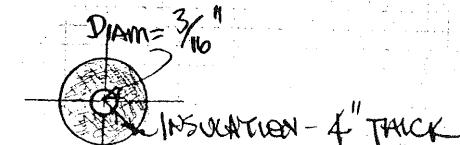
$$L = 7.2 \text{ m} \quad (b)$$

For  $h = 1150 \text{ W/m}^2 \cdot \text{K}$ 

$$D = 1.44 \text{ cm} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad (c)$$

$$L = 5.32 \text{ m} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad (c)$$

17.21



$$\frac{q}{L} = \frac{2\pi k}{\ln r_o/r_i} (T_i - T_o)$$

$$= \frac{2\pi (0.14)}{\ln \frac{8.094}{0.1875}} (120 - 70)$$

$$= 11.72 \frac{\text{BTU}}{\text{HR-FT}} = 3.43 \text{ W}$$

17.21 CONTINUED -

$$I^2 R = 3.43 \text{ W}$$

$$R = \frac{PL}{A} = 2.95 \times 10^4 \Omega$$

$$I^2 = \frac{3.43}{2.95 \times 10^4} \quad I = 107.9 \text{ Amp}$$

17.22

$$\dot{q} = \frac{120-70}{\ln \frac{8.094}{0.1875}} + \frac{1}{\pi (131/16 \times 12) A}$$

$$I = 106.1 \text{ Amp}$$

$$11.34 = 4 \left[ \frac{131}{16 \times 12} \right] \pi (T - 70)$$

$$T = 71.3 \text{ F}$$

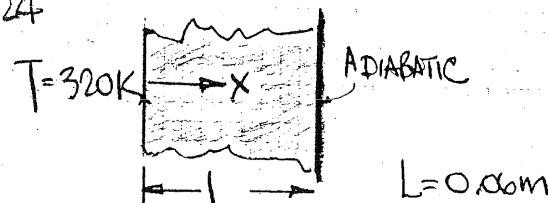
17.23

SAME AS PROB 17.21 EXCEPT  
WIRE IS ALUMINUM =

$$R = 4.85 \times 10^4 \Omega$$

$$I = 83.9 \text{ Amp}$$

17.24



$$\dot{q} = q_f [1 - x/L]$$

POISSON EQUN APPLIES: (ENERGY BALANCE)

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$\frac{d^2 T}{dx^2} = - \frac{q_f}{k} \left[ 1 - \frac{x}{L} \right]$$

$$\frac{dT}{dx} = - \frac{q_f}{k} \left[ x - \frac{x^2}{2L} \right] + C_1$$

17.24 CONTINUED

$$\frac{dT}{dx}(L) = 0 \quad C_1 = \frac{q_0 L}{k^2}$$

$$\frac{dT}{dx} = \frac{q_0}{k} \frac{L}{2} \left[ 1 - \frac{x}{L} + \frac{x^2}{L^2} \right]$$

$$T = \frac{q_0 L}{k^2} \left[ x - \frac{x^2}{L} + \frac{x^3}{3L^2} \right] + C_2$$

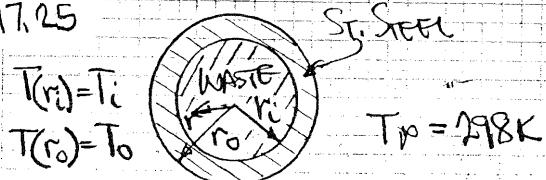
$$T(0) = T_0 = C_2$$

$$T = T_0 + \frac{q_0}{k} \frac{L}{2} \left[ x - \frac{x^2}{L} + \frac{x^3}{3L^2} \right] \quad (a)$$

(b)  $T = T_{\max}$  where  $\frac{dT}{dx} = 0$ , i.e. @  $x = L$

$$T(L) = T_0 + \frac{180}{0.6} \left( \frac{0.06}{2} \right) \left( \frac{0.06}{3} \right) (1000) \\ = 320 + 180 = \underline{500 \text{ K}} \quad (c)$$

17.25



$$\text{In WASTE MATERL} \quad q = q \left( \pi r_i^2 L \right) \quad (1)$$

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{q}{k} = 0$$

$$x \frac{dT}{dr} + \frac{q r^2}{2k} = C_1/r$$

$$\frac{dT}{dx}(0) = 0 \Rightarrow C_1 = 0$$

$$T + \frac{q r^2}{4k} = C_2$$

$$T(r_i) = T_i \quad C_2 = T_i + \frac{q r_i^2}{4k}$$

$$T = T_i + \frac{q}{4k} (r_i^2 - r^2)$$

for ST. STEEL

$$q = \frac{2\pi k L}{4\pi r_i^2 h} (T_i - T_0) = 2\pi r_i L h (T_i - T_0)$$

EQUATING WITH EQUATION (1)

$$\frac{2\pi k L}{4\pi r_i^2 h} (T_i - T_0) = 2\pi r_i L h (T_i - T_0) = \frac{q}{4k} r_i^2 K$$

17.25 CONTINUED

$$\text{PUTTING IN VALUES} - T_0 = 339.7 \text{ K}$$

$$T_i = 339.7 + 303 = \underline{\underline{642.7 \text{ K}}} \quad (a)$$

@ CENTER OF WASTE MATERL:

$$T = 642.7 + \frac{q}{4k} r_i^2$$

$$= 642.7 + 625 = \underline{\underline{1268 \text{ K}}} \quad (b)$$

17.26

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{q}{k} = 0$$

$$\frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{q r}{k} = 0$$

$$x \frac{dT}{dr} + \frac{q r^2}{2k} = \frac{C_1}{r}$$

$$\frac{dT}{dr}(0) = 0 \Rightarrow C_1 = 0$$

$$T + \frac{q r^2}{4k} = C_2$$

$$T(r) = T_R \quad C_2 = T_R + \frac{q r^2}{4k}$$

$$T - T_R = \frac{q}{4k} (R^2 - r^2) \quad (a)$$

$$T_{\max} = T @ r = 0$$

$$T_{\max} = T_R + \frac{q R^2}{4k}$$

$$= T_R + \frac{(51.7 \times 10^6)(0.107)}{4(339)}$$

$$= T_R + 442$$

$$q = \frac{q}{4} V = \frac{q}{4} \frac{\pi D L}{4}$$

$$= \frac{(51.7 \times 10^6)(\pi)(0.107)^2}{4} (0.107)$$

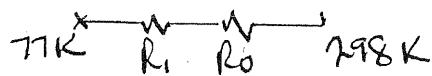
$$= h (\pi) (0.107) (0.107) \Delta T$$

$$\Delta T = 307 \text{ K}$$

$$T_{\text{surf}} = 332 \text{ C} \quad T_{\max} = 740 \text{ C}$$

$$(a) \uparrow \quad (b) \uparrow$$

17.27 ASSUME THIN-WALLED INNER VESSEL IS 77 K THROUGHOUT



$$R_i = \frac{1}{4\pi k} \frac{1/r_i + 1/r_o}{\eta_f} = \frac{1/0.5 - 1/0.55}{4\pi(0.002)} = 7.234$$

$$R_o = \frac{1}{4\pi r_o^2 h} = \frac{1}{4\pi(0.55)^2(18)} = 0.046$$

$$q = \frac{\Delta T}{\sum R} = \eta_f h q_f$$

$$\dot{m} = \frac{221/7.234}{2 \times 105} = 1.524 \times 10^{-4} \text{ kg/s}$$

17.28 FOR  $q = \frac{1}{2}$  OR VALUE IN 17.27

$$\sum R = 14.58 = R_{\text{conv}} + R_{\text{ins}}$$

$$R_{\text{ins}} = \frac{1/0.5 - 1/r_o}{4\pi(0.002)} \quad R_{\text{conv}} = \frac{1}{4\pi r_o^2(18)}$$

$$14.58 = \frac{1}{4\pi} \left[ 500 \left( \frac{1}{0.5} - \frac{1}{r_o} \right) + \frac{1}{18 r_o^2} \right]$$

$$r_o = 0.611 \text{ m} \quad \text{INS. THICKNESS} = 0.0055 \text{ m}$$

$$\text{ADDED THICKNESS} = 0.0055 \text{ m} \quad \text{OR } 5.5 \text{ mm}$$

17.29 PER FOOT OF BASE PIPE:

$$q = h A \Delta T = \left( 6 \frac{\text{Btu}}{\text{ft} \cdot \text{hr} \cdot \text{F}} \right) \left( \frac{1}{12} \text{ ft} \right)^2 (170 \text{ F}) \\ = 267 \text{ Btu/ft hr}$$

FOR LONGITUDINAL FINS:

$$A_f = 12 \left( \frac{3}{4} \right) \left( \frac{1}{12} \right) (2)(1) = 1.5 \text{ ft}^2$$

$$A_0 = \left( \frac{\pi}{12} \right) (1) - 12 \left( \frac{3}{32} \right) \left( \frac{1}{12} \right) = 0.168 \text{ ft}^2 \quad (\text{ENDS NEGLECTED})$$

$$L \left( \frac{h}{k t} \right)^{1/2} = \left( \frac{3/4}{12} \right) \left[ \frac{6}{24.8 \left( \frac{3}{64} \right) \left( \frac{1}{12} \right)} \right]^{1/2} \\ = 0.539 \quad \eta_f \approx 0.92$$

17.29 CONTINUED

$$q = h (A_0 + \eta_f A_f) \Delta T \\ = 6 \left( 0.168 + 0.92 \times 1.5 \right) (170) = 1580 \frac{\text{Btu}}{\text{hr}}$$

$$\text{INCREASE} = 1580 - 267 = 1310 \frac{\text{Btu}}{\text{hr}}$$

$$\% \text{ INCR} = 491 \%$$

FOR CIRCULAR FINS:

$$\text{PER FIN: } A_f = 2 \frac{\pi}{4} \left[ \frac{2.5^2 - 1^2}{144} \right] = 0.053 \text{ ft}^2$$

$$\text{PER FOOT} \quad n = 1.5 / 0.053 = 116 \text{ FINS}$$

$$A_0 = \frac{\pi}{12} - \left( \frac{\pi}{12} \times \frac{3}{32} \times \frac{1}{12} \times 16 \right) = 0.209 \text{ ft}^2$$

$$(r_o - r_i) \left[ \frac{h}{k t} \right]^{1/2} = \frac{3/4}{12} \left[ \frac{6}{24.8 \left( \frac{3}{64} \right) \left( \frac{1}{12} \right)} \right]^{1/2} \\ = 0.539 \quad \eta_f \approx 0.88$$

$$q = 6 \left[ 0.209 + (0.88) (2.6) (0.053) \right] (170) \\ = 1560 \frac{\text{Btu}}{\text{hr}}$$

$$\text{INCREASE} = 1560 - 267 = 1293 \frac{\text{Btu}}{\text{hr}}$$

$$\% \text{ INCR} = \frac{1293}{267} = 484 \%$$

17.30  $h = 60 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot \text{F}}$

$$\text{LONGITUDINAL CASE} \quad L \left( \frac{h}{k t} \right)^{1/2} = 1.70$$

$$\eta_f \approx 0.56$$

$$q = 10820 \frac{\text{Btu}}{\text{hr}}$$

$$\text{CIRCULAR CASE: } (r_o - r_i) \left( \frac{h}{k t} \right)^{1/2} = 1.70$$

$$\eta_f \approx 0.52$$

$$q = 10180 \frac{\text{Btu}}{\text{hr}}$$

$$116 \text{ FINS} \quad q = 2670 \frac{\text{Btu}}{\text{hr}}$$

INCREASE:

$$\text{Long:} \quad \underline{8150 \frac{\text{Btu}}{\text{hr}}} \quad \underline{305 \%}$$

$$\text{Arc:} \quad \underline{7510 \text{ "}} \quad \underline{281 \%}$$

17.31

SOLUTION FOR  $\theta = \frac{T - T_{\infty}}{T_0 - T_{\infty}}$

IS IN FORM  $\theta = C_1 e^{-mx} + C_2 e^{-nx}$

$$m = \left[ \frac{(17 \text{ W/m}^2 \cdot \text{K})(\pi)(0.03 \text{ m})}{k \pi (0.03 \text{ m})^2} \right]^{\frac{1}{2}} = \frac{47.6}{k^{\frac{1}{2}}}$$

LONG FIN APPROXIMATION:  $\theta = C_2 e^{-mx}$

$$\theta_1 = 99 = C_2 e^{-mx}$$

$$\theta_2 = 65 = C_2 e^{-m(x_1 + 0.076)}$$

$$\frac{\theta_2}{\theta_1} = \frac{65}{99} = e^{-m(x_1 + 0.076 - x_1)}$$

$$m = 5.536 = \frac{47.6}{k^{\frac{1}{2}}}$$

$$k \approx 74 \text{ W/m}^2 \cdot \text{K}$$

17.32

$$\eta_f = \text{fn of } L \left[ \frac{h}{kt} \right]^{\frac{1}{2}}$$

AIRSIDE:  $= 0.75 \left[ \frac{2(3)}{0.05/12 \times 229} \right]^{\frac{1}{2}}$   
 $= 0.156 \quad \eta_f \approx 0.98$

WATERSIDE:  $= 0.75 \left[ \frac{2(25)}{0.5/12 \times 229} \right]^{\frac{1}{2}}$   
 $= 0.143 \quad \eta_f \approx 0.98$

for 1 ft<sup>2</sup>  $A_0 = 1 - 150 \left( 1 \times \frac{0.05}{12} \right) = 0.375 \text{ ft}^2$

$$A_f = 150(2) \left( \frac{0.75}{12} \right) (1) + 0.625 = 19.35 \text{ in}^2$$

W/FINS!  $q = 25 \Delta T_w = 3 \Delta T_A = \frac{\Delta T_{\infty A}}{\frac{25}{12} + \frac{1}{3}}$

W/OUT FINS!

WATERSIDE  $q = 25 \Delta T_w (0.375 + 18.96) = 483 \Delta T_w$

AIRSIDE  $q = 3 \Delta T_A (0.375 + 18.96) = 58.0 \Delta T_A$

FINS ADDS TO AIRSIDE ONLY!

17.32 CONTINUED

$$q = \frac{\Delta T_{\infty A}}{\frac{1}{12} + \frac{1}{3}} = 17.47 \Delta T_{\infty A}$$

$$\% \text{ GAIN} = 549\%$$

TO WATERSIDE:

$$q = \frac{\Delta T_{\infty A}}{\frac{1}{483} + \frac{1}{3}} = 198 \Delta T_{\infty A}$$

$$\text{GAIN} = 11.2\%$$

BOTH SIDES:

$$q = \frac{\Delta T_{\infty A}}{\frac{1}{483} + \frac{1}{58}} = 51.78 \Delta T_{\infty A}$$

$$\text{GAIN} = 1832\%$$

17.33

ONE-DIM. CONDUCTION WITH INTERNAL HT GENERATION

$$\frac{d^2 T}{dx^2} = -\frac{\dot{q}}{k}$$

$$\frac{dT}{dx} = -\frac{\dot{q}}{k} x + C_1$$

$$T = -\frac{\dot{q}}{k} \frac{x^2}{2} + C_1 x + C_2$$

$$T(0) = T_0 = C_2$$

$$T(L) = T_L = -\frac{\dot{q}}{k} \frac{L^2}{2} + C_1 L + T_0$$

$$C_1 = \frac{T_L - T_0}{L} + \frac{\dot{q} L}{k^2}$$

$$T = \frac{\dot{q}}{k} \left[ \frac{Lx}{2} - \frac{x^2}{2} \right] + \frac{T_L - T_0}{L} x + T_0$$

OR  $T - T_0 = \frac{\dot{q} L^2}{2k} \left( \frac{x}{L} - \left( \frac{x}{L} \right)^2 \right) + (T_L - T_0) \left( \frac{x}{L} \right)$

AT  $\frac{L}{2}$ :  $T - T_0 = \frac{\dot{q} L^2}{2k} \left( \frac{1}{4} \right) + \frac{T_L - T_0}{2}$

$$\dot{q} = \frac{I^2 R}{V} = \frac{I^2 \sigma k}{A^2 L}$$

$$= \frac{(10)^2 (2 \times 10^{-5})}{\pi / 4 (0.01)^2} = 2547 \text{ W/m}^3$$

## 17.33 CONTINUED

MID-PT. TEMP:

$$T_{\text{m.p.}} = \frac{25.47 \text{ W/m}^3 (0.04 \text{ m})^2}{8 (\text{2 W/m}\cdot\text{K})} + 50 \\ \approx 50.00 \text{ -C}$$

$$q = -kA \frac{\partial T}{\partial x} = -kA \left[ -\frac{C_1}{R} x + C_1 \right] \\ = -kA \left[ -\frac{C_1}{R} x + T_L - T_0 + \frac{qL}{k} \frac{L}{2} \right]$$

$$@x=0 \quad q = -kA \left[ \frac{T_L - T_0}{L} + \frac{qL}{2k} \right] \\ = -0.393 \text{ W}$$

$$\text{AT } x=L \quad q = -kA \left[ -\frac{C_1 L}{2R} + \frac{T_L - T_0}{L} \right] \\ = +0.393 \text{ W}$$

$$17.34 \quad m^2 = \frac{hP}{kA} = \frac{(40)(\pi)(0.019)(4)}{(54)(\pi)(0.019)^2} \\ = 2885 \text{ m}^{-2}$$

$$\theta = \theta_0 \cosh mx$$

$$\cosh mx / \lambda$$

$$\frac{d\theta}{dx} = m\theta_0 \frac{\sinh mx}{\cosh mx / \lambda} \Rightarrow \sinh mx = 0 @ x=0$$

$$\theta_{\text{max}} = \frac{\theta_0}{\cosh mx / \lambda} = \frac{145 \text{ K}}{\cosh 12.08}$$

$$T \approx 625 \text{ K}$$

## 17.35 ENERGY BALANCE:

$$\frac{d^2\theta}{dx^2} - m^2\theta = -\frac{W}{kA} \quad \left\{ \begin{array}{l} W \text{ IN BTU} \\ \text{FT} \end{array} \right.$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx} + \frac{W}{hP}$$

$$\theta(0) = 0 \quad C_1 = \frac{-W/hP (1 - e^{-mL})}{e^{mL} + e^{-mL}}$$

$$\theta(L) = 0 \quad C_2 = \frac{-W/hP (e^{mL} - 1)}{e^{mL} - e^{-mL}}$$

## 17.35 CONTINUED

PUTTING IN VALUES:

$$m = 2.28 \quad C_1 = -0.0017 \text{ W/hP}$$

$$C_2 = -0.999$$

$$\theta = \theta_{\text{max}} @ 1.5 \text{ ft}$$

$$W_{\text{max}} = I_{\text{max}}^2 R$$

$$R = \frac{8L}{A} = \frac{(172 \times 10^{-6})(3)}{\frac{\pi}{4}(1/48)^2(30.48)} \\ = 4.97 \times 10^{-4} \Omega/\text{ft}$$

$$\theta_{\text{max}} = 90 \left[ -0.00107 \right] \frac{2.28(1.5)}{0.999} \\ = -0.228(1.5)$$

$$+ 1 \left[ \frac{W}{hP} \right]$$

$$\frac{W}{hP} = 96.3 \quad W = 96.3(\omega)(\pi)(0.25/12)$$

$$= 37.8 \text{ BTU/HP FT}$$

$$= 11.08 \text{ W/ft}$$

$$I_{\text{max}}^2 = \frac{11.08}{4.97 \times 10^{-4}} = 2.23 \times 10^4 \text{ A}^2$$

$$I_{\text{max}} = 150 \text{ A}$$

## 17.36

$$m^2 = \frac{hP}{kA} = \frac{45(2)}{42(0.0159)} = 134.8 \text{ m}^{-2}$$

$$m = 11.61 \text{ m}^{-1}$$

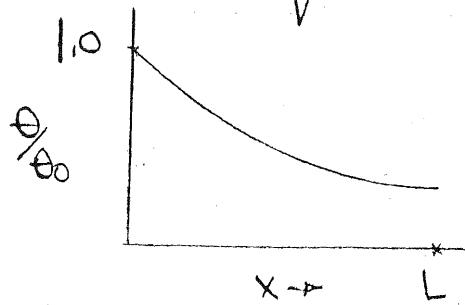
$$q = kA m \theta_0 \frac{\sinh mL + h/mk \cosh mL}{\cosh mL + h/mk \sinh mL}$$

$$kA m \theta_0 = (42)(0.0159)(11.61)(300) \\ = 2330$$

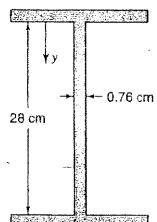
$$\sinh mL = 2.01 \quad \cosh mL = 1.75$$

$$h/mk = \frac{45}{11.61(42)} = 0.0923$$

17.36 (CONTINUED)

SUBSTITUTING:  $q = 2.25 \text{ kW}$ 

17.37



$$\frac{\theta}{\theta_0} = \left[ \frac{\theta}{\theta_0} - e^{-mx} \right] \left[ \frac{e^{mx} - e^{mL}}{e^{mL} - e^{m0}} \right] + e^{mx}$$

$$\theta_0 = 400 \quad m = 19.6 \text{ m}^{-1}$$

$$mL = 569 \quad e^{mL} = 242$$

$$e^{m0} = 0.0041$$

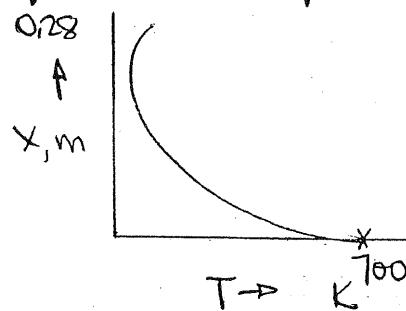
SUBSTITUTING

$$\theta = 0.262 e^{-19.6x} + 399.7 e^{-19.6x}$$

$$q = -kA \frac{dT}{dx} \quad \frac{dT(0)}{dx} = -1829$$

$$\frac{dT(L)}{dx} = 1305$$

$$q_0 = -2.32 \text{ kW} \quad q_L = 387 \text{ W}$$



17.38

For aluminum, I-beam:

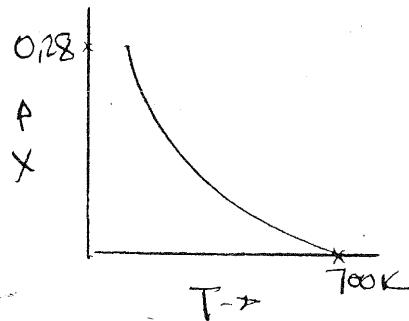
Same procedure as previous prob.

$$m = 8.08 \quad e^{mL} = 9.61$$

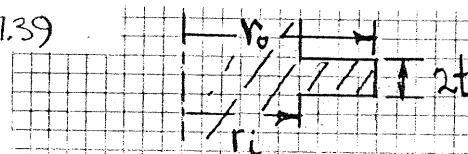
$$e^{m0} = 0.104$$

$$\theta = 3e^{8.08x} + 688e^{-8.08x}$$

$$q_0 = 551 \text{ kW} \quad q_L = 170 \text{ W}$$



17.39



$$r_e = \frac{r_0 + r_i}{2} = 0.15 \text{ m} \quad r_0 = 0.15 + 0.02 = 0.17 \text{ m}$$

$$t = 0.003 \text{ m} = 0.0015 \text{ m}$$

$$A_{PEA} = 2\pi(r_0^2 - r_i^2) + A_{END}$$

$$= 2\pi(0.17^2 - 0.15^2) + 2\pi(0.17)(0.003)$$

$$= 0.0434 \text{ m}^2$$

$$(r_0 - r_i)\sqrt{h/k}t = 0.02 \left[ \frac{12}{(46.4)(0.0015)} \right]^{\frac{1}{2}}$$

$$= 0.163$$

$$r_0/r_i = 1.13$$

From fig 17.11  $\eta_F \approx 0.96$ 

$$q = A_F \eta_F h (T_0 - T_\infty)$$

$$= (0.0434)(0.96)(12)(270)$$

$$= 135 \text{ W per fin}$$

17.39 CONTINUED -

for A 30% 3 kW ENGINE

$$Q_{in} = \frac{3 \text{ kW}}{0.3} = 10 \text{ kW}$$

$$Q_{out} = Q_{in} - W = 7 \text{ kW}$$

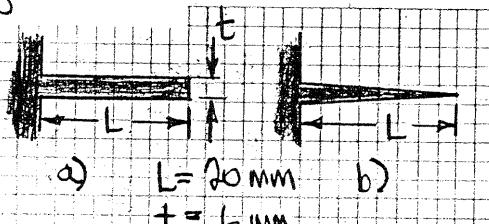
$$\text{AMOUNT Tx from FINS} = 0.5(7) \\ = 3.5 \text{ kW}$$

No. of Fins req'd:

$$n = \frac{3500 \text{ W}}{135 \text{ W/fm}} = 25.92$$

26 FINS REQUIRED

17.40



$$a) L = 20 \text{ mm} \quad b) t = 6 \text{ mm}$$

for Beta Gases:  $T_b = 120^\circ\text{C}$ 

$$T_\infty = 20^\circ\text{C}$$

$$h = 60 \text{ W/m}^2\text{K}, k_{ss} = 15.3 \text{ W/mK}$$

for CASE a) STRAIGHT FIN

$$q = \eta_f h A_f \theta_b$$

USING TEXT - fin 17.11

$$(L + \frac{t}{2})^{3/2} \left[ \frac{h}{k t (L + \frac{t}{2})} \right]^{1/2}$$

$$= (0.02 + 0.003)^{3/2} \left[ \frac{60}{(15.3)(0.006)(0.023)} \right]^{1/2}$$

$$= 0.588 \quad \eta_f \approx 0.80$$

P.S. m of WIND TURBINE (NEAREST ENDS)

$$q_f = 0.80(60)(100)(2)(0.020)$$

$$= 192 \text{ W/m}$$

17.40 CONTINUED -

for CASE b) - TRIANGULAR

$$L^{3/2} \left[ \frac{h}{k t L} \right]^{1/2}$$

$$= 0.02^{3/2} \left[ \frac{60}{(15.3)(0.003)(0.02)} \right]^{1/2}$$

$$= 0.723 \quad \eta_f \approx 0.81$$

$$q = \eta_f A_f h \theta_b$$

$$= (0.81)(2)(0.02)(60)(100)$$

$$= 194.4 \text{ W/m}$$

17.41

w/out fins:

$$q_0 = h \Delta T \pi \left( \frac{2}{12} \right) = 712 \text{ BTU/hr}$$

for LONGITUDINAL FINS:

$$L \left( \frac{h}{k t} \right)^{1/2} = \frac{1}{12} \left[ \frac{8}{10 \left( \frac{1}{12} \right)} \right] = 1.46$$

$$\eta_f \approx 0.6$$

$$A_0 = \pi D_0 - 16(2t) = 0.440 \text{ ft}^2$$

$$A_f = 16(2)(\frac{1}{12}) + 16(\frac{1}{16})(\frac{1}{12}) \approx 2.67 \text{ ft}^2$$

$$q = h \Delta T [A_0 + \eta_f A_f]$$

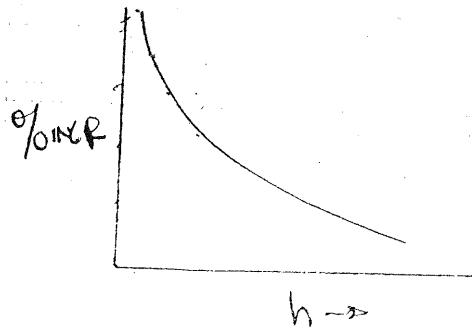
$$= 8(712)[0.440 + 0.6(2.67)] \approx 2780 \frac{\text{BTU}}{\text{hr}}$$

$$\text{INCREASE} = \frac{2068 \frac{\text{BTU}}{\text{hr}}}{712 \frac{\text{BTU}}{\text{hr}}} \cdot \% \text{ INCR} = 290\% \text{ a)}$$

for VARYING VALUES OF  $h$ :

$h$	$L \sqrt{h/k t}$	$\eta_f$	% INCR
2	0.731	0.84	412
5	1.156	0.71	346
8	1.462	0.60	290
15	2.00	0.48	229
50	3.65	0.27	122
100	5.17	0.19	81

## 17.41 CONTINUED



i.e. FINS ARE MOST EFFECTIVE WHEN  $h$  IS SMALL

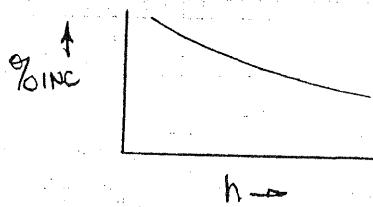
17.42 Same as 17.41 except MAT'L IS ALUMINUM

$$\text{w/o fins } \dot{Q} = 712 \text{ Btu/hr}$$

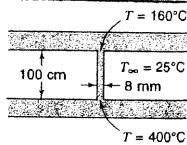
$$L \left[ \frac{h}{kt} \right]^2 = 0.121 \quad \eta_f \approx 0.99 \quad (a)$$

$$\dot{Q} \approx 4300 \text{ Btu/hr} \quad \% \text{ INCR} = 503\%$$

$h$	2	5	15	50	100
% INCR	488	471	4448	370	303



## 17.43



FOR KNOWN END TEMPS:

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \left[ \frac{\theta_L}{\theta_0} - e^{-mL} \left[ \frac{e^{mX} - e^{-mX}}{e^{mL} - e^{-mL}} \right] + e^{-mX} \right]$$

$$\theta_0 = 160 - 25 = 135$$

$$\theta_L = 400 - 25 = 375 \quad \frac{\theta_L}{\theta_0} = 2.78$$

$$m = \left[ \frac{hP}{kA} \right]^{\frac{1}{2}} = \left[ \frac{300(2)}{(0.008)(229)} \right]^{\frac{1}{2}} = 18.1 \text{ m}^{-1}$$

$$e^{mL} = 6.11$$

$$e^{-mL} = 0.164$$

## 17.43 CONTINUED

SUBSTITUTING:

$$\frac{\theta}{\theta_0} = 0.440 e^{\frac{18.1x}{18.1}} + 0.560 e^{-\frac{18.1x}{18.1}}$$

$$\frac{d\theta}{dx} = \theta_0 \left[ 7.964 e^{\frac{18.1x}{18.1}} - 10.14 e^{-\frac{18.1x}{18.1}} \right]$$

$$@ x=0 \quad \frac{d\theta}{dx} = -2.176 \theta_0$$

$$@ x=L \quad \frac{d\theta}{dx} = 47.0 \theta_0$$

$$\frac{F}{L} = -k + \frac{d\theta}{dx} \quad \underline{\underline{F}(0) = -294 \text{ N/m}}$$

$$\underline{\underline{F}(L) = 6,345 \text{ kW/m}}$$

## 17.44 USING TABLE 17.1

$$S = \frac{2\pi}{\cosh^{-1} \left( \frac{1+\beta^2-\epsilon^2}{2\beta} \right)}$$

$$\beta = 0.5 \quad \epsilon = 1/6$$

$$S = \frac{2\pi}{\cosh^{-1} \left( \frac{1+\frac{1}{4}-\frac{1}{36}}{1} \right)} = \frac{2\pi}{\cosh^{-1} \frac{11}{9}}$$

$$= 96$$

$$\underline{\underline{q}} = k S \Delta T = (0.023)(9.6)(300) \\ = 66.3 \text{ Btu/hr-ft}$$

## 17.45 TABLE 17.1

$$S = \frac{2\pi}{\cosh^{-1}(\beta/r)} = \frac{2\pi}{\cosh(2.5)} = 1.311$$

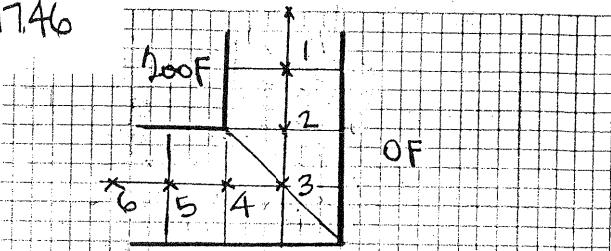
$$\underline{\underline{q}} = k S A T$$

L

$$= (0.341)(1.311)(600)$$

$$= \underline{\underline{26.83 \text{ W/m}}}$$

17.46



$$200 + 0 + 2T_2 - 4T_1 = 0$$

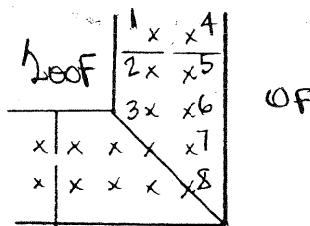
$$200 + 0 + T_1 - T_3 - 4T_2 = 0$$

$$0 + 0 + 2T_2 - 4T_3 = 0$$

$$T_1 = 91.7 \text{ F} \quad T_2 = 83.3 \text{ F} \quad T_3 = 41.7 \text{ F}$$

$$\dot{Q} = 8k \left[ \frac{200-91.7}{2} + \frac{200-83.3}{2} \right] = 205 \text{ BTU/HR-FT}$$

17.47



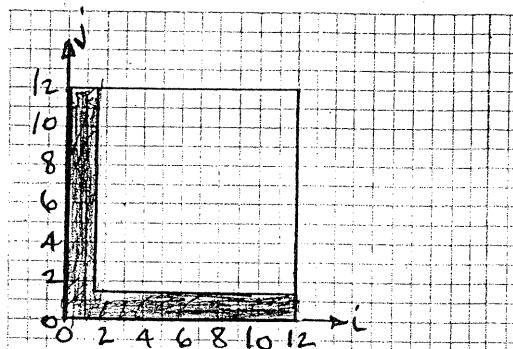
By ITERATION:

NODE	T, F
1	123
2	112
3	74
4	58
5	51.5
6	36
7	18

$$\dot{Q} = 8k \left[ (200-123) + (200-112) \right]$$

$$= 198 \text{ BTU/HR-FT}$$

17.48



17.48 CONTINUED-

NUMERICAL SOLUTION USING A

12x12 MESH YIELDS THE

FOLLOWING:

$$\dot{Q} \approx 1400 \text{ BTU/HR-FT}^2 \text{ PER FT.}$$

$$T_{\min} \approx 91.8 \text{ F} \text{ AT } i,j = 12,12$$

17.49

$$\begin{aligned} S &= \frac{2\pi}{\mu r c - 0.271} \\ &= \frac{2\pi}{\ln(5/0.5) - 0.271} \\ &= 8.17 \\ \dot{Q} &= kSLAT \\ &= (0.037)(8.17)(300)(50) \\ &= 2180 \text{ BTU/HR} \end{aligned}$$

$$\begin{aligned} \text{STEAM CONDENSED} &= \frac{\dot{Q}}{h_{fg}} \\ &= \frac{2180}{824} = 2.65 \text{ LBm/HR} \end{aligned}$$

17.50

$$\begin{aligned} S &= \frac{2\pi}{\ln^{-1}(8/r)} = \frac{2\pi}{\ln^{-1} 1.2} \\ &= 3.17 \end{aligned}$$

$$\begin{aligned} \dot{Q} &= kSLAT \\ &= (0.066 \text{ W/m.K})(3.17)(90 \text{ K})(1.45 \text{ m}) \\ &= 273 \text{ W} \end{aligned}$$

## CHAPTER 18

$$18.1 \quad Bi = \frac{hV/s}{k} = \frac{3}{10} \frac{3/488}{0.5} = 0.0369$$

∴ CAN USE LUMPED PARAMETERS

$$\text{By 1ST LAW: } \dot{q}_V - hA\theta = \dot{q}_{cp}V \frac{d\theta}{dt}$$

$$\text{WHERE } \theta = T - T_{\infty}$$

$$\frac{d\theta}{dt} = \frac{\dot{q}_V}{\dot{q}_{cp}V} - \frac{hA\theta}{\dot{q}_{cp}V} = a - b\theta$$

$$\text{SOLN: } t = \frac{1}{b} \ln \frac{a}{a - b\theta} = \frac{1}{b} \ln \frac{1}{1 - \frac{b}{a}\theta}$$

$$a = \frac{\dot{q}_V}{\dot{q}_{cp}} = \frac{(500)(3.413)}{3(0.11)} = 5170 \text{ W/m}^2$$

$$b = \frac{hA}{\dot{q}_{cp}V} = \frac{3(0.05)}{(0.11)(3)} = 4.54 \text{ K}^{-1}$$

$$\Rightarrow t = \frac{1}{4.54} \ln \frac{1}{1 - 4.54 \cdot \frac{1}{5170}(160)}$$

$$= 0.0333 \text{ hours} = 2.0 \text{ MIN.}$$

18.2

$$V = \frac{\pi D^2}{4} L = \frac{\pi}{4} (0.0001)^2 (0.005)$$

$$= 3.93 \times 10^{-11} \text{ m}^3$$

$$A = \frac{2\pi D^2}{4} + \pi DL$$

$$= \frac{\pi}{2} (0.0001)^2 + \pi (0.0001)(0.005)$$

$$= 1.587 \times 10^{-4} \text{ m}^2$$

$$\frac{hV}{RA} = \frac{10}{20} \frac{(3.93 \times 10^{-11})}{(1.587 \times 10^{-4})} \approx 1.24 \times 10^{-5}$$

CLEARLY A LUMPED PARAMETER CASE

ENERGY BALANCE:

$$\dot{q} - hA(T - T_{\infty}) = \dot{q}_{cp}V \frac{dT}{dt}$$

$$\text{LET } \theta = T - T_{\infty} :$$

$$\frac{d\theta}{dt} = \frac{\dot{q}}{\dot{q}_{cp}V} - \frac{hA}{\dot{q}_{cp}V} \theta$$

$$= A - B\theta$$

## 18.2 CONTINUED -

$$\int_0^\theta \frac{d\theta}{A - B\theta} = \int_0^t dt$$

$$-\frac{1}{B} \ln |A - B\theta| = t$$

$$\Rightarrow t = \frac{1}{B} \ln \frac{A}{A - B\theta}$$

$$A = \frac{\dot{q}}{\dot{q}_{cp}V} = \frac{9(0.1)}{k/\alpha(V)} = \frac{1.8}{(20/5 \times 10^{-5})(3.93 \times 10^{-11})} = 1.145 \times 10^5$$

$$B = \frac{hA}{\dot{q}_{cp}V} = \frac{10(1.587 \times 10^{-4})}{(20/5 \times 10^{-5})(3.93 \times 10^{-11})} = 1.010$$

$$t = \frac{1}{1.01} \ln \frac{1.145 \times 10^5}{1.145 \times 10^5 - (1.01)(810)} = 7.63 \times 10^{-3} \text{ s} = 7.63 \text{ ms}$$

## 18.3 ALUMINUM WIRE:

$$D = 0.794 \text{ mm} \quad R = 0.0572 \text{ m}^2/\text{m}$$

$$k = 229 \text{ W/m.K} \quad \rho = 2701 \text{ kg/m}^3$$

$$c_p = 938 \text{ J/kg.K} \quad \alpha = 9.16 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Bi = \frac{hV}{ks} = \frac{h}{k} \frac{\pi D K}{4 \pi D K}$$

$$= \frac{(500)(0.794 \times 10^{-3})}{(229)(4)} = 4.33 \times 10^{-4}$$

- LUMPED PARAMETER SOLN IS OK.

STEADY STATE CASE - PER M

$$I_2 R = hA \Delta T$$

$$\Delta T = \frac{(100)^2 (0.0572) \text{ W}}{(550 \text{ W/m}^2 \cdot \text{K}) \pi (0.794 \times 10^{-3}) \text{ m}^2} = 416.9 \text{ K}$$

$$T_{max} = 25 + 416.9 = 441.9 \text{ C}$$

18.3 CONTINUED -

$$\begin{aligned} & \text{TRANSIENT CASE -} \\ & \frac{ScpV}{dt} = I^2 R - hS(T-T_{\infty}) \\ & \frac{d\theta}{dt} = \frac{I^2 R}{ScpV} - \frac{hS}{ScpV} \theta = A - B\theta \\ & A = \frac{I^2 R}{ScpV} = \frac{(100)^2 (0.05 \Omega)}{(2701)(938) \frac{\pi}{4} (0.714 \times 10^{-3})^2} \\ & = 456 \text{ K/s} \\ & B = \frac{hS}{ScpV} = \frac{(550)(\pi)(0.714 \times 10^{-3})}{(2701)(938) \frac{\pi}{4} (0.714 \times 10^{-3})^2} \\ & = 1.094 \text{ s}^{-1} \\ & \frac{d\theta}{dt} = A - B\theta \\ & \int_0^\theta \frac{d\theta}{\theta - B\theta} = \int_0^t dt \\ & -\frac{1}{B} \ln \frac{A - B\theta}{A} = t \\ & t = \frac{1}{B} \ln \frac{A}{A - B\theta} \\ & = \frac{1}{1.094} \ln \frac{456}{456 - 1.094(4119)} \\ & = 4.06 \text{ s} \end{aligned}$$

$$18.4 Bi = \frac{hV}{kS} = \frac{6}{0.151} \left[ \frac{(0.1)(0.3)(0.45)}{0.6(0.45)(2)} + \frac{0.6(0.3)(2)}{0.3(0.45)(1)} \right] = 2.75$$

A DISTRIBUTED PARAMETER Prob,

$$\begin{aligned} \frac{T-T_{\infty}}{T_0-T_{\infty}} &= \frac{320-297}{420-297} = 0.187 = Y_A Y_B Y_C \\ M_X &= \frac{0.151}{6(0.15)} = 0.168 \quad X_x = \frac{dt}{0.15^2} = 2.75 \times 10^{-4} t \\ M_Y &= 0.119 \quad X_y = 1.22 \times 10^{-6} t \\ M_Z &= 0.042 \quad X_z = 1.72 \times 10^{-6} t \end{aligned}$$

18.4 CONTINUED -

By TRIAL & ERROR  
 $t \approx 80 \text{ hours}$

$$\begin{aligned} 18.5 \quad Bi &= \frac{hV}{kS} = \frac{16}{23} \left( \frac{6}{6} \right) = 0.058 \\ & \text{LUMPED PARAMETER!} \\ F_0 &= \frac{\alpha t}{(V/S)^2} = \frac{23}{460(0.10) \text{ hr}} \frac{t}{\left( \frac{6}{6} \right)^2} \\ & = 72t \\ \frac{T-T_{\infty}}{T_0-T_{\infty}} &= \frac{600}{2000} - (0.058)(72t) \\ t &= \underline{0.288 \text{ hr}} = \underline{17.3 \text{ min}} \end{aligned}$$

18.6 LUMPED PARAMETER SOLN APPLIES

$$\begin{aligned} \text{IF } Bi &= \frac{hV}{kS} < 0.1 \text{ or } h < 0.1 \text{ kS/V} \\ \frac{kS}{V} &= \frac{k \pi D^2}{\pi D^3 / 6} = 0.47(6) = 2.82 \\ \text{So } h &\text{ MUST BE } < 0.1(2.82) = 0.282 \text{ W/m}^2\text{K} \\ \text{But } h &= 15 \Rightarrow \text{USE DISTRIBUTED PARAM. SOLN.} \\ \frac{dt}{r_0^2} &= \frac{(0.47) t}{(940)(3800)(0.05)^2} = 5.26 \times 10^{-5} t \\ \frac{T_s - T_{\infty}}{T_0 - T_{\infty}} &= 0.5 \quad \frac{k}{h r_0} = \frac{0.47}{15(0.05)} = 0.127 \\ X &\approx 0.17 = 5.26 \times 10^{-5} t \\ t &= \underline{3230 \text{ s}} = \underline{53.9 \text{ MIN}} \end{aligned}$$

$$18.7 \quad Bi \approx 0.005$$

USE LUMPED PARAMETER SOLN.

$$\begin{aligned} \frac{T-T_{\infty}}{T_0-T_{\infty}} &= \frac{h}{k S} \sqrt{t} \\ \frac{T-T_{\infty}}{T_0-T_{\infty}} &= 2 \\ t &= \frac{Scp V}{h S} \ln \frac{T_0 - T_{\infty}}{T - T_{\infty}} \\ &= \frac{51 \sqrt{V}}{4 A \times r_0} (0.12) = 0.658 \frac{V}{A} \end{aligned}$$

## 18.7 CONTINUED -

CASE	L (IN)	$V/A$ IN INCHES	
		$V/A$	t (min)
a	3	0.5	19.7
b	6	0.6	23.6
c	12	0.67	26.3
d	24	0.706	27.9
e	60	0.732	28.9

18.8

$$\frac{T_c - T_{\infty}}{T_0 - T_{\infty}} = \frac{500 - 1000}{70 - 1000} = 0.538$$

$$\frac{V}{A} = \frac{D}{4 + 2D/L} = \frac{1}{17}$$

$$Bi = \frac{h V/S}{k} = \frac{4/\pi}{k}$$

a) Cu -  $Bi \leq 0.1$  - LUMPED

$$t = \frac{8C_p V}{h A} \ln \frac{1}{0.538} = \underline{27.9 \text{ MIN}}$$

b) Al  $Bi \leq 0.1$ 

$$t = 0.345 \text{ HR} = \underline{20.7 \text{ MIN}}$$

c) Zn  $Bi < 0.1$ 

$$t = 0.381 \text{ HR} = \underline{22.9 \text{ MIN}}$$

d) STEEL  $Bi < 0.1$ 

$$t = 0.502 \text{ HR} = \underline{30.2 \text{ MIN}}$$

18.9 WATER IS WELL-STIRRED  
 $\therefore$  LUMPED  $\sim T = T(t)$  only

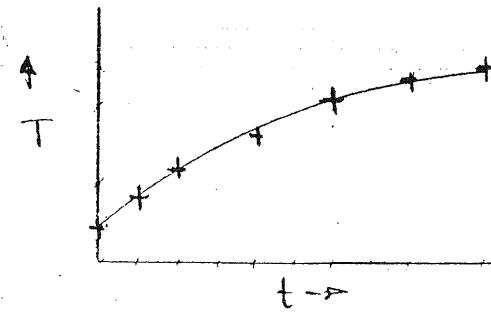
$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = \exp \left( -\frac{hAt}{8C_p V} \right)$$

$$= \exp \left[ - \frac{40(\pi)(15)(2)t}{62.4(1)(\pi)(1.5^2/4)(2)} \right]$$

$$T = 300 - 260 e^{-171t}$$

$t, \text{HR}$	0	0.1	0.2	0.4	0.6	0.8	1.0
$T, F$	40	81	115	169	210	237	253

## 18.9 CONTINUED



18.10

$$Bi = \frac{h V/S}{k} = \frac{h}{k} \frac{\pi D^2 L / 4}{\pi D L + 2\pi D^2 / 4}$$

$$= \frac{h D}{4k(L+D/2)} = \frac{85(0.6)(0.06)}{(229)(4)(0.9)} = 0.0371$$

~ LUMPED PARAMETER (CASE)

TEMP MAY BE CONSIDERED

UNIFORM AT ANY TIME

$$f_0 = \frac{\pi t}{(V/S)^2} = \frac{(9.16 \times 10^{-5})(3600)}{(0.10)^2}$$

$$= 32.98$$

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = e^{-Bi f_0} = e^{-(0.0371)(32.98)} = 0.294$$

$$T = 345 + 0.294(130)$$

$$= \underline{383.2 \text{ K}}$$

18.11

$$Bi = \frac{h V/S}{k} = \frac{15 \left[ \frac{\pi D^2}{4} L / \pi D L \right]}{12.4} = 0.151$$

MUST USE DISTRIBUTED PARAMETER  
 SOLN. FIG F.8

$$\frac{I_s - I}{I_s - I_0} = \frac{2300 - 1500}{2300 - 200} = 0.381$$

$$I_s - I_0 = 2300 - 200$$

$$\frac{xt}{x^2} = \frac{0.15}{(0.25)^2} t = 24t$$

18.11 CONTINUED -

$$\eta = \frac{x}{x_1} = 0 \quad m = \frac{k}{hx_1} = 3.31$$

$$X \approx 1.7 \Rightarrow t = \frac{1.7}{2.4} = 0.708 \text{ HR}$$

$$\text{VELOCITY} = \frac{20}{0.708} = \underline{\underline{28.2 \frac{\text{FT}}{\text{HR}}}} = \underline{\underline{0.47 \frac{\text{FT}}{\text{m}}}}$$

$$18.12 \frac{T_s - T_p}{T_0 - T_p} = \frac{410 - 435}{295 - 435} = 0.179$$

$$\frac{xt}{x_1^2} = \frac{(6.19 \times 10^{-8})t}{(0.015)^2} = 2.75 \times 10^{-4} t$$

$$m \approx 0 \text{ from CHART } X \approx 0.8$$

$$t = \frac{0.8}{2.75 \times 10^{-4}} = \underline{\underline{29125}} = \underline{\underline{48.5 \text{ min}}}$$

18.13

$$B_c = \frac{hv}{ks} = \frac{40 \pi \delta^3 / 6}{19.3 \pi D^2} = 0.00575$$

LUMPED PARAMETERS!

$$f_0 = \frac{xt}{(v/s)^2} = \frac{0.8(15/3600)}{[0.2/2(6)]^2} = 432$$

$$\frac{T - T_p}{T_0 - T_p} = \frac{-B_c f_0}{2} = 0.0834$$

$$T = \underline{\underline{115.9 \text{ F}}}$$

18.14

$$x_1 = 0.15 \text{ m} \quad x = 0.05 \text{ m}$$

$$\frac{x}{x_1} = 1/3$$

$$\frac{T - T_p}{T_0 - T_p} = \frac{100 - 380}{125 - 380} = 0.789$$

$$m = k/hx_1 = \frac{0.20}{140(0.15)} = 0.00952$$

$$xt/x_1^2 = \frac{1.1 \times 10^{-7} t}{(0.15)^2} = 4.89 \times 10^{-6} t$$

18.14 CONTINUED -

{USING CHART FOR CYL}

$$@ \frac{x}{x_1} = 0.2 \quad \frac{xt}{x_1^2} = 0.10$$

$$@ \quad 0.4 \quad " = 0.07$$

$$\text{INTERPOLATING} @ 0.33 \quad \frac{xt}{x_1^2} \approx 0.08$$

$$t \approx 0.08 \approx 16360 \text{ S}$$

$$4.89 \times 10^{-6} = 273 \text{ MIN} = 4.54 \text{ HR}$$

$$18.15 \quad B_c = \frac{hv}{k} = \frac{228 \pi D^2}{0.19 \pi D L} = 3.9$$

~ DISTRIBUTED PARAMETER SOLN:

$$\frac{xt}{x_1^2} = \frac{0.19 t}{(580)(0.050)(0.065)^2} = 7.38 \times 10^{-5} t$$

$$\frac{k}{hx_1} = \frac{0.19}{22.8(0.065)} = 0.128$$

$$\frac{T - T_p}{T_0 - T_p} = \frac{530 - 810}{295 - 810} = 0.544 \quad n = 0$$

$$\text{from CHART } X \approx 0.23 = 7.38 \times 10^{-5} t$$

$$t = \underline{\underline{3114 \text{ S}}} = \underline{\underline{51.9 \text{ MIN}}}$$

18.16

USING CHART SOLN:

$$\frac{T_s - T}{T_s - T_0} = \frac{100 - 250}{100 - 400} = 0.5$$

$$Y_A Y_B Y_C = 0.5 \quad n_A = n_B = n_C = 0$$

$$m_A = m_B = m_C = 0 \quad X_A = 442 t$$

$$X_B = 707 t$$

$$X_C = 0.884 t$$

$$Y_C \approx 1$$

$$\text{BY TRIAL \& ERROR, } t \approx 8.4 \times 10^{-4} \text{ HR}$$

$$\approx 3.05 \text{ S}$$

18.17

$$\frac{B_i}{k_s} = \frac{hV}{k_s} = \frac{130}{0.151} (0.6)(0.3)(0.45) [ \text{see Prob 18.7} ] \\ = 105 \quad \left\{ \begin{array}{l} \text{DISTRIBUTED} \\ \text{PARAMETER} \end{array} \right.$$

$$\frac{T_s - T_p}{T_0 - T_p} = 0.187 = Y_A Y_B Y_C$$

$$M_x = \frac{0.151}{130(0.151)} = 4.377 \times 10^{-3}$$

$$M_y = 2.92 \times 10^{-3}$$

$$M_z = 1.09 \times 10^{-3}$$

$$X_x = \frac{(6.19 \times 10^{-8})t}{(0.15)^2} = 2.75 \times 10^{-6}t$$

$$X_y = 1.22 \times 10^{-6}t$$

$$X_z = 1.09 \times 10^{-6}t$$

TRIAL  $\frac{1}{2}$  ERROR:  $t \approx \underline{62 \text{ hours}}$

$$18.18 \quad B_i = \frac{hV}{k_s} = \frac{(90 \text{ W/m}^2 \cdot \text{K})(\pi D^2 K/4)}{(0.5 \text{ W/m} \cdot \text{K})(\pi D K)} \\ = 0.9 \quad \left\{ \begin{array}{l} \text{DISTRIBUTED} \\ \text{PARAMETER} \end{array} \right.$$

$$\frac{T_s - T_p}{T_0 - T_p} = \frac{80 - 100}{5 - 100} = 0.211$$

$$m = \frac{k}{h x_i} = \frac{0.5}{90(0.01)} = 0.556$$

$$n=0 \quad X = \frac{xt}{x_i^2} = \frac{0.5t}{880(3250)(0.10)^2} \\ = 1.70 \times 10^{-3}t$$

from CHART  $X \approx 0.76 = 1.70 \times 10^{-3}t$

$$t = \underline{447 \text{ s}} = \underline{7.45 \text{ min.}}$$

18.19



$$T_{r_0} \quad 2L = 2r_0 \\ V = \pi r_0^2 (2L) \\ = \frac{2.25}{991} = 2.27 \times 10^{-3} \text{ m}^3$$

$$L = r_0 = \frac{2.27 \times 10^{-3}}{\pi} = 0.0712 \text{ m}$$

for A FINITE CYLINDER:

$$\frac{T(0,t) - T_p}{T_i - T_p} = R(0,t) X(0,t)$$

$$\frac{T(0,0,t) - T_p}{T_i - T_p} = \frac{95 - 190}{5 - 190} = 0.514$$

FOR BOTH  $r \neq x$  DIRECTIONS:

$$\frac{h r_0}{k} = \frac{h L}{k} = \frac{15(0.0712)}{0.675} = 1.582$$

$$\frac{xt}{r_0^2} = \frac{xt}{L^2} = \frac{0.167 \times 10^{-6}t}{(0.0712)^2} = 3.29 \times 10^{-5}t$$

TRIAL  $\frac{1}{2}$  ERROR  $\frac{xt}{L^2} = \frac{xt}{r_0^2} \approx 0.34$   
 {USING CHARTS}

$$t = 0.34 / 3.29 \times 10^{-5} = 10330 \text{ s} \\ = \underline{172 \text{ MIN}} = \underline{2.87 \text{ HR}}$$

18.20 SAME CYL AS IN PROB 18.15  
 BUT H VARIES -

AS  $H/D \rightarrow \infty \quad t = 3114 \text{ s} = 51.9 \text{ MIN.}$

WITH ENDS CONSIDERED:  $D = \text{CONST}$

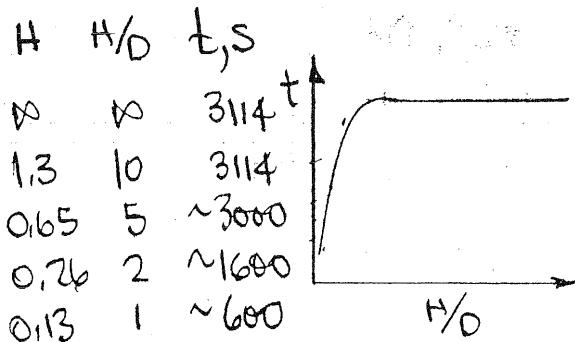
$$\frac{xt}{x_i^2} \Big|_{\text{CYL}} = 7.38 \times 10^{-5}t \quad \frac{k}{h x_i k_{\text{ext}}} \Big|_{\text{CYL}} = 0.128 \\ = 13 \text{ cm}$$

$$Y = 0.544 = Y_{\text{CYL}} Y_{\text{PL}}$$

$$\text{for PLANE: } \frac{xt}{x_i^2} = \frac{1.25 \times 10^{-5}t}{H^2}$$

$$k/h x_i = 0.0167/H$$

18.20 (CONTINUED -



18.21 USE SEMI-INFINITE WALL SOLN.

$$\frac{T_p - T}{T_p - T_0} = \operatorname{erf} \eta + \exp\left(\frac{\beta^2}{4\eta^2}\right) \left[ 1 - \operatorname{erf}\left(\frac{\beta}{2\eta} + \eta\right) \right]$$

$$\eta = \frac{y}{\sqrt{2kt}} \quad \beta = \frac{h\gamma}{k} \quad \frac{\beta}{2\eta} = \frac{h\sqrt{kt}}{k}$$

@ SURFACE  $\sim x=0$ 

$$\frac{T_p - T}{T_p - T_0} = \exp\left(\frac{\beta^2}{4\eta^2}\right) \left(1 - \operatorname{erf}\left(\frac{\beta}{2\eta}\right)\right)$$

$$\frac{h^2 \alpha t}{k^2} = \frac{(200)^2 (0.35)}{(1.73)^2} t = 4678 t$$

$t$  IN HOURS

$$\frac{h\sqrt{kt}}{k} = 684 t^{1/2}$$

$$0.1 = e^{684 t^{1/2}} (1 - \operatorname{erf} 4678 t)$$

APPROXIMATION: USE 1ST TERM IN SERIES EXPANSION:

$$\frac{T_p - T_0}{T_p - T_s} = 0.1 = \frac{2\eta}{\sqrt{\pi}} \frac{1}{\beta} \frac{k}{h\sqrt{kt}}$$

$$t = 6.80 \times 10^{-3} \text{ hr} = \underline{24.5 \text{ s}}$$

AT THIS TIME  $\eta = 0.427 \quad \beta = 4.81$ 

SOLVING FOR T:

$$\underline{T = 1413 \text{ F}}$$

18.22

USE SEMI-INFINITE SOLN.

$$\frac{T - T_0}{T_0 - T_s} = \operatorname{erf} \frac{y}{\sqrt{2kt}} = \frac{2}{\sqrt{\pi}} \int_0^{\frac{y}{\sqrt{2kt}}} e^{-\beta^2} d\beta$$

$$\text{GIVEN: } \alpha = 0.0456 \text{ ft}^2/\text{hr} \quad T_s = 0 \text{ F}$$

$$\frac{\partial T}{\partial y}(0) = 0.02 \text{ F/ft} \quad T_0 = 7000 \text{ F}$$

$$\frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[ \frac{2}{\sqrt{\pi}} \int_0^{\frac{y}{\sqrt{2kt}}} e^{-\beta^2} d\beta \right] (T_0 - T_s)$$

$$= \frac{2}{\sqrt{\pi}} \left[ -\frac{y^2}{4kt} e^{-\frac{y^2}{8kt}} \frac{1}{2\sqrt{kt}} \right] (T_0 - T_s)$$

$$\text{for } y=0 \quad \frac{\partial T}{\partial y}(0) = \frac{T_0 - T_s}{\sqrt{\pi k t}}$$

$$t = \frac{(T_0 - T_s)^2}{\pi \alpha (0.02)^2} = \frac{(7000)^2}{\pi (0.0456)(4 \times 10^{-4})}$$

$$\underline{= 9.75 \times 10^7 \text{ YEARS}}$$

18.23 USE SEMI-INFINITE WALL SOLN.

$$\frac{T - T_0}{T_p - T_0} = \operatorname{erfc} \frac{x}{2\sqrt{kt}} - \left[ \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \right] \operatorname{erfc}\left(\frac{x}{2\sqrt{kt}} + \frac{h\sqrt{kt}}{k}\right)$$

@  $x=0$  THIS REDUCES TO

$$\frac{T - T_0}{T_p - T_0} = 1 - e^{-z^2} (1 - \operatorname{erfc} z)$$

$$\text{WHERE } z = \frac{h\sqrt{kt}}{k}$$

$$\alpha = \frac{k}{scp} = \frac{0.17}{(545)(2385)} = 1.3 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\frac{T - T_0}{T_p - T_0} = \frac{400 - 21}{900 - 21} = 0.431$$

$$z = \frac{h\sqrt{kt}}{k} = \frac{30}{0.17} \sqrt{1.3 \times 10^{-6} t} = t^{1/2}$$

$$= 0.0636 t^{1/2}$$

$$z^2 = 0.00405 t$$

18.23 CONTINUED

SUBSTITUTING,  $z^2$

$$0.431 = 1 - e^{-z^2} (1 - \operatorname{erf} z)$$

$$e^{-z^2} (1 - \operatorname{erf} z) = 0.569$$

BY TRIAL & ERROR:  $z \approx 0.6$

$$t = \frac{z^2}{0.00405} = \frac{88.93}{0.00405} = 1.48 \text{ MIN}$$

$$18.24. \frac{T - T_s}{T_0 - T_s} = \frac{2}{\sqrt{\pi}} \int_0^{x/\sqrt{2xt}} e^{-\beta^2} d\beta$$

$$\frac{\partial T}{\partial t} = (T_0 - T_s) \frac{2}{\sqrt{\pi}} e^{-x^2/4xt} \left[ \frac{x}{4\sqrt{x + 3t/2}} \right]$$

$$= - \frac{(T_0 - T_s)}{t \sqrt{\pi}} \frac{x}{2\sqrt{xt}} e^{-x^2/4xt}$$

$$\frac{\partial T}{\partial t} = - \frac{(T_0 - T_s)}{t \sqrt{\pi}} \frac{-z^2}{z^2 e^{-z^2/2\sqrt{xt}}} \quad \{ z = \frac{x}{2\sqrt{xt}} \}$$

So  $\left| \frac{\partial T}{\partial t} \right|$  is MAX when  $z^2 e^{-z^2/2\sqrt{xt}}$  is MAX

$$\frac{\partial}{\partial z} \left( z^2 e^{-z^2/2\sqrt{xt}} \right) = z^2 - 2z^2 e^{-z^2/2\sqrt{xt}} = 0$$

$$z = 1/\sqrt{2} \text{ or } \infty$$

$$\Rightarrow \frac{x}{2\sqrt{xt}} = \frac{1}{\sqrt{2}} \text{ or } x = \sqrt{2xt}$$

$$x = [2(0.045b)(9.8 \times 10^7)(24)(365)]^{1/2}$$

$$= 2.8 \times 10^5 \text{ FT} = 53 \text{ MILES}$$

18.25 This is A Semi-Infinite (ass)

$$\frac{T_s - T}{T_0 - T_s} = \operatorname{erf} \frac{x}{2\sqrt{xt}}$$

$$\frac{x}{2\sqrt{xt}} = \frac{0.25 \text{ m}}{2\sqrt{(5.16 \times 10^{-7} \text{ m}^2/\text{s})(1800 \text{ s})}}^{1/2}$$

$$= 1.297 \quad \operatorname{erf} 1.297 \approx 0.934$$

$$\frac{100 - T}{100 - 280} = 0.934 \quad T = 334 \text{ K}$$

18.26 Solid Is Amenable To Either Numerical or Analytical Approach

$$\frac{T_s - T_p}{T_0 - T_p} = \exp\left(\frac{h^2 at}{k^2}\right) \left[ \operatorname{erfc} \frac{h\sqrt{at}}{k} \right]$$

$$\Delta T_s = T_s - T_p \quad h = 0.44 (T - T_p)^{1/3}$$

$$\Delta T_s = 900 \exp\left(0.00732 \Delta T_p^{2/3} t\right) \times \left[ \operatorname{erfc} \left(0.0856 \Delta T_p^{1/3} t^{1/2}\right) \right]$$

TRIAL & ERROR - AT EACH  $t$

$t$	1 HR	6 HR	24 HR
$T_{s,F}$	580	396	275

18.27

$$\frac{T - T_0}{T_p - T_0} = \operatorname{erfc} \frac{x}{2\sqrt{xt}} = \left[ \exp\left(\frac{hv}{k}\right) + \frac{h^2 at}{k^2} \right] \times \left[ \operatorname{erfc} \left( \frac{x}{2\sqrt{xt}} + \frac{h\sqrt{at}}{k} \right) \right]$$

$$\frac{T - T_0}{T_p - T_0} = \frac{400 - 25}{800 - 25} = 0.484$$

18.27 CONTINUED

@ SURFACE ( $x=0$ )

$$\frac{x}{2\sqrt{kt}} = 0 \quad \text{erf}(0) = 0 \quad \text{erfc}(0) = 1$$

$$\frac{h\sqrt{xt}}{k} = \frac{20}{0.21} (1.07 \times 10^{-7})^{1/2} t^{1/2}$$

$$= 0.0311 t^{1/2} = z$$

GOVERNING EXPRESSION BECOMES:

$$0.484 = 1 - e^{-z^2} (1 - \text{erf}z)$$

$$\text{TRIAL } \nexists \text{ ERROR: } z \approx 0.73 = 0.0311t^{1/2}$$

$$t = \underline{551 \text{ s}} = \underline{9.18 \text{ MIN}}$$

18.28

FOR GLASS:

$$x = \frac{k}{\rho c_p} = \frac{0.45}{(170)(0.2)} = 0.0132 \text{ ft}^2/\text{hr}$$

USING SEMI-INFINITE WALL EXPRESSION:

$$\frac{T-T_0}{T_s-T_0} = \frac{32-30}{65-30} = 0.0572 = \text{erfc} \frac{x}{2\sqrt{kt}}$$

$$\frac{x}{2\sqrt{kt}} = 1.38$$

$$\underline{t = 3.9 \text{ s}}$$

18.29 CHARTS APPLY BUT ARE

DIFFICULT TO READ -

CHECK VALIDITY OF INFINITE WALL SOLN

$$\frac{L}{2\sqrt{kt}} = \frac{1 \text{ ft}}{2(0.0231t)^{1/2}} > 2$$

WORKS FOR  $t > 27$  HOURS

$$\frac{x}{2\sqrt{kt}} = \frac{1}{2(0.0231t)^{1/2}} = \frac{3.29}{t^{1/2}}$$

$$\frac{T_s-T}{T_s-T_0} = \text{erf} \frac{3.29}{t^{1/2}} \quad \underline{t \approx 5.2 \text{ HR}}$$

18.30 APPROPRIATE EXPRESSION IS

$$\frac{T_s-T}{T_s-T_0} = \text{erf} A + \exp\left(\frac{hx}{k} + B^2\right) \left[ 1 - \text{erf}(A+B) \right]$$

$$A = \frac{x}{2\sqrt{kt}} = \frac{0.05}{2(0.444 \times 10^5 t)^{1/2}}$$

$$= 11.92 t^{-1/2}$$

$$\frac{hx}{k} = \frac{22(0.05)}{17.3} = 0.0636$$

$$B = \frac{h\sqrt{xt}}{k} = \frac{22}{17.3} (0.444 \times 10^5 t)^{1/2}$$

$$= 0.00267 t^{1/2}$$

$$B^2 = 7.11 \times 10^{-6} t$$

$$\text{TRIAL } \nexists \text{ ERROR: } t \approx 49000 \text{ S}$$

$$\approx \underline{13.6 \text{ hours}}$$

18.31

$$\frac{T_s-T}{T_s-T_0} = \text{erf} A + \exp\left(\frac{hx}{k} + B^2\right) \left[ 1 - \text{erf}(A+B) \right]$$

$$@ x=0, t=180 \text{ s}$$

$$A = \frac{x}{2\sqrt{kt}} = 0 \quad \frac{hx}{k} = 0$$

$$B = \frac{h\sqrt{xt}}{k} = \frac{110}{17.3} \sqrt{180} \times 0.180$$

$$\frac{T_s-T}{T_s-300} \approx 0.96$$

$$T = 20 + 268 = \underline{288 \text{ C}}$$

$$\text{AT } x = 50 \text{ mm}$$

$$A = 0.89$$

$$\frac{hx}{k} = 0.1318$$

$$B = 0.180 \quad B^2 = 0.0324$$

$$\frac{T_s-T}{T_s-300} = 0.974 \quad T = \underline{293 \text{ C}}$$

18.32.

Eqn (18-33)

$$\frac{q_x}{A} = \frac{d}{dt} \int_0^{\delta} S c_p T dx - S c_p T_0 \frac{dS}{dt}$$

$$\text{For } \frac{T-T_0}{T_s-T_0} = \phi \left( \frac{x}{\delta} \right)$$

$$\frac{\partial T}{\partial x} \Big|_{x=0} = (T_s - T_0) \frac{1}{\delta} \frac{\partial \phi}{\partial x} \Big|_{x=0}$$

$$\frac{\partial \phi}{\partial x}(0) = K \quad (\text{A CONSTANT})$$

$$\frac{q_x}{A} = -k \frac{\partial T}{\partial x}(0) = F(t)$$

$$\therefore T_s - T_0 = \frac{F(t)}{kK} \delta$$

$$F(t) = \frac{d}{dt} \int_0^{\delta} T dx - T_0 \frac{d\delta}{dt}$$

$$\frac{d}{dt} \int_0^{\delta} T dx = T_0 \frac{d\delta}{dt} + \frac{d}{dt} (T_s - T_0) \int_0^{\delta} \phi dx$$

$$= T_0 \frac{d\delta}{dt} + \frac{d}{dt} [(T_s - T_0) B \delta]$$

$\{B \text{ A CONSTANT}\}$

$$\Rightarrow \frac{F(t)}{S c_p} = \frac{d}{dt} [(T_s - T_0) B \delta]$$

$$= \frac{d}{dt} \left[ \frac{B \delta^2 F(t)}{kK} \right]$$

$$\frac{k}{S c_p} \frac{K}{B} F(t) = \frac{d}{dt} [\delta^2 F(t)]$$

$$\delta^2 F(t) = \frac{K}{B} \times \int_0^t F(t) dt$$

$$\delta = (\text{CONSTANT}) \sqrt{\alpha} \left[ \int_0^t F(t) dt \right]^{1/2}$$

18.33

NUMERICAL SOIN PROB

INITIAL TEMP PROFILE-

$$T = 35 + 0.5x \quad T \text{ IN } ^\circ\text{F}, x \text{ IN FT}$$

ALGORITHMS -

FOR ALL NODES EXCEPT SURFACE:

$$T_i^{t+1} = \frac{T_{i+1}^t + T_{i-1}^t}{2}$$

FOR SURFACE NODE:

$$T_0^{t+1} = T_1^t - \frac{hA_K}{K} T_0^t$$

$$\sim \frac{\Delta t}{\Delta x^2} = \frac{1}{2} \frac{2h\Delta t}{S c_p \Delta x} = \frac{hA_K}{K} \quad T_0 = 0$$

RESULT - USING SPREADSHEET

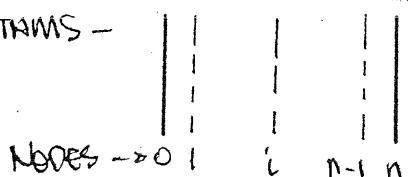
OR PROGRAM

TIME  $\cong$  1800 hours

18.34

NUMERICAL SOIN PROB

ALGORITHMS -



$$\text{NODE 1: } T_1^{t+1} = \frac{T_0^t + T_2^t}{2}$$

$$T_{n-1}^{t+1} = \frac{T_{n-2}^t + T_n^t}{2}$$

$$T_i^{t+1} = \frac{T_{i+1}^t + T_{i-1}^t}{2}$$

$$\text{for } \frac{\Delta t}{\Delta x^2} = \frac{1}{2} : \text{ If } \Delta x = 0.25 \text{ FT}$$

$$\Delta t = 1.95 \text{ HR}$$

NO OF INCREMENTS  $\cong 7.4$ TIME = 7.4 (1.95)  $\cong$  14.4 HR

$$FB-35. T = 520 + 330 \sin \frac{\pi x}{L}$$

$$\alpha = \frac{k}{\rho c_p} = \frac{0.66}{(1670)(8138)} = 4.72 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2}$$



NODES -> 0 1 i n-1 n

SAME ALGORITHMS AS PROB (FB-34)

$$\text{FOR } \frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2} \quad \text{IF } \Delta x = 0.225 \text{ m}$$

$$\Delta t = 536 \text{ s}$$

NO OF INCREMENTS  $\approx 2.4$

$$\text{TIME} \approx 2.4(536) = 1286 \text{ s}$$

$$= 21.4 \text{ MIN}$$

AT THIS TIME:  $T_{\text{surf}} \approx \underline{360 \text{ K}}$

## CHAPTER 19

19.1 For A Plane Wall:

VARIABLES Dimensions

$$\begin{array}{ll} T & T \\ T_0 & T \\ T_\infty & T \\ x & L \\ L & L \\ \alpha & L^2/T \\ k & Q/LT \\ t & t \\ h & Q/L^2t \end{array}$$

$$i = n - r = 5$$

If Temps Are Grouped As

$$T - T_\infty, T_0 - T_\infty \quad i = n - r = 4$$

$$\pi_1 = \Delta T^a L^b k^c \alpha^d (T - T_\infty)$$

$$\pi_2 = (\quad) (x)$$

$$\pi_3 = (\quad) (t)$$

$$\pi_4 = (\quad) (h)$$

$$\pi_1 = \frac{T - T_\infty}{T_0 - T_\infty} \quad \pi_2 = \frac{x}{L} \quad \pi_3 = \frac{\alpha t}{L^2} \quad \pi_4 = \frac{hL}{k}$$

19.2 Air H<sub>2</sub>O Benz Hg Gyc

$$\rho = 1.9 \times 10^{-3} \quad 0.474 \times 10^{-5} \quad 0.473 \times 10^{-5} \quad 1.06 \times 10^{-6} \quad 0.18 \times 10^{-2}$$

$$cp = 1.008 \times 10^3 \quad 1.0 \quad 0.45 \quad 0.083 \quad 0.598$$

$$k = 0.0293 \quad 0.383 \quad 0.0762 \quad 5.03 \quad 0.165$$

$$Re = 23 \times 10^5 \quad 1.02 \times 10^7 \quad 1.02 \times 10^7 \quad 4.57 \times 10^7 \quad 37,800$$

$$\Pr = 0.699 \quad 2.72 \quad 5.21 \quad 0.021 \quad 13.1$$

$$Nu = 348 \quad 154 \quad 77.3 \quad 1.17 \quad 35.7$$

$$St = 2.16 \times 10^{-3} \quad 5.55 \times 10^{-7} \quad 1.45 \times 10^{-6} \quad 1.22 \times 10^{-5} \quad 7.21 \times 10^{-5}$$

19.3 ~ PLOTS ~

19.4 Air@ 310K: Pr=0.705

$$k = 27 \times 10^{-2} \text{ W/m.K}$$

$$\frac{fg}{\lambda^2} = 1.161 \times 10^8 / m^3 \cdot K$$

$$Gr = \frac{fg}{\lambda^2} X^3 AT = (1.161 \times 10^8)(110) X^3$$

$$\delta = 3.94 \frac{Pr^{-1/2}}{X} (Pr + 0.954)^{1/4} Gr_x^{-1/4}$$

$$X = 15 \text{ cm} \quad 30 \text{ cm} \quad 1.5 \text{ m}$$

$$Gr_x = 4.31 \times 10^7 \quad 3.45 \times 10^8 \quad 4.31 \times 10^{10}$$

$$\delta = 0.985 \text{ cm} \quad 1.17 \text{ cm} \quad 1.75 \text{ cm}$$

$$Nu = 30.5 \quad 51.2 \quad 171.3$$

$$h_x = 5.48 \text{ W/m.K}^{1/2} \quad 4.61 \quad 3.08$$

19.5

$$h_x = \frac{k}{x} 0.332 Re_x^{1/2} Pr^{1/3}$$

$$= 0.055 \frac{k}{x} Re_x^{1/2} Pr^{1/3}$$

$$T = 30 \text{ F} \quad T_f = 55 \text{ F} \quad h_x = 0.4 \frac{Re_x}{x}^{1/2}$$

x	$Re_x^{1/2}/x$	$h_x$
0	00	00
0.5	10.92	8.74
1	15.46	6.19
1.5	18.92	5.05
2	21.85	4.37

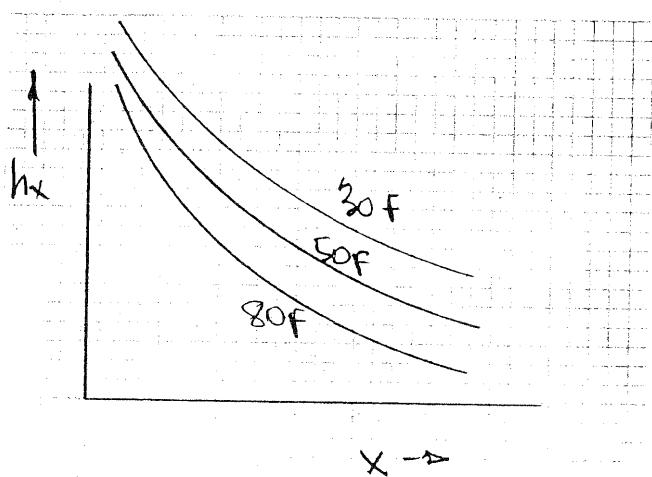
$$T = 50 \text{ F} \quad T_f = 75 \text{ F} \quad h_x = 0.1256 \frac{Re_x}{x}^{1/2}$$

x	$Re_x^{1/2}/x$	$h_x$
0	00	00
0.5	22.2	11.37
1	31.45	8.06
1.5	38.5	6.57
2	44.5	5.70

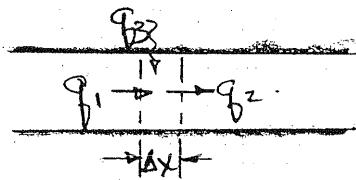
19.5 CONTINUED -

$$T = 80F \quad T_f = 105 \quad h_x = 0.119 \frac{F}{x}^{\frac{1}{2}}$$

x	$\frac{F}{x}^{\frac{1}{2}}$	$h_x$
0	∞	∞
0.5	67.5	16.06
1	95.2	11.32
1.5	117	9.29
2	135	8.03



19.6



AS PER DEVELOPMENT IN TEXT

$$q_2 - q_1 - q_3 = 0$$

$$\frac{\partial V Q_p z T}{\partial x} \Big|_{x+\Delta x} - \frac{\partial V Q_p z T}{\partial x} \Big|_x - \frac{q}{A} (z \Delta x) = 0$$

$$\frac{\partial V Q_p z}{\partial x} \frac{T|_{x+\Delta x} - T|_x}{\Delta x} - \frac{q}{A} = 0$$

19.6 CONTINUED -

IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{\partial V Q_p z}{\partial x} \frac{\partial T}{\partial x} - \frac{q}{A} = 0$$

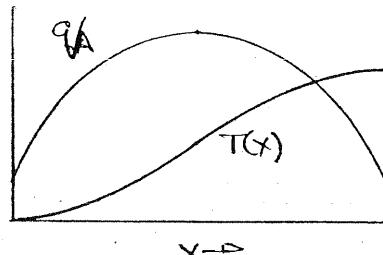
$$\int_0^T \frac{\partial T}{\partial x} dx = \frac{z}{8VQ_p z} \int_0^x (x + \beta \sin \frac{\pi x}{L}) dx$$

$$T - T_0 = \frac{2}{8VQ_p z L} \left[ \beta x + \frac{\beta L}{\pi} \left( 1 - \cos \frac{\pi x}{L} \right) \right]$$

$$\frac{2}{8VQ_p z} = \frac{1}{30} \frac{\text{HR PT F}}{\text{BTU}}$$

$$\beta L / \pi = 1910 \text{ BTU/HR-PT}$$

x	$q/A$	T-120
0	250	0
1	1310	27
2	1750	80.3
3	1310	134
4	250	161



19.7

FOR A SINUSOIDAL 4-FT-LONG PLATE:

$$q = w \int_0^L (x + \beta \sin \frac{\pi x}{L}) dx$$

$$= w \left( \alpha x + \frac{\beta L}{\pi} \sin \frac{\pi x}{L} \right)$$

$$= WL \left( \alpha + 2 \beta / \pi \right)$$

$$= 16 \text{ ft}^2 \left( 250 + \frac{2(1500)}{\pi} \right)$$

$$= 19300 \text{ BTU/HR}$$

FOR STACK OF PLATES:

$$q = 19300 \frac{(640)}{16} = \underline{112,000 \text{ BTU/HR}}$$

$$19.8 \quad \frac{q}{A} = a + b \sin \frac{\pi x}{L} = 900 + 2500 \sin \frac{\pi x}{1.22}$$

$\frac{q}{A}$  IN  $W/m^2$ ,  $x$  IN m.

$\frac{q}{A}$

$\frac{q_1}{A}$   $\frac{q_2}{A}$

$\Delta x$

ENERGY BALANCE:  $q_2 - q_1 - q_3 = 0$

STANDARD PROCEDURE

RESOLUTION EXPRESSIONS

$$\frac{dT}{dx} = \frac{2 \frac{q}{A}}{8VcpD(1)} - C \frac{q}{A} \quad \left\{ \begin{array}{l} C \propto A \\ \text{CONSTANT} \end{array} \right.$$

$$\int_{T_E}^T dT = C \int_0^x \left( a + b \sin \frac{\pi x}{L} \right) dx$$

$$T - T_E = C \left[ ax + \frac{b}{\pi} \left( 1 - \cos \frac{\pi x}{L} \right) \right]$$

$$C = \frac{2}{8VcpD(1)} = \frac{(7.5)(1034)(0.003)(1)}{2} = 0.086 \text{ m} \cdot \text{kg}/\text{W}$$

$$T - T_E = 77.4x + 83.5 \left( 1 - \cos \frac{\pi x}{1.22} \right)$$

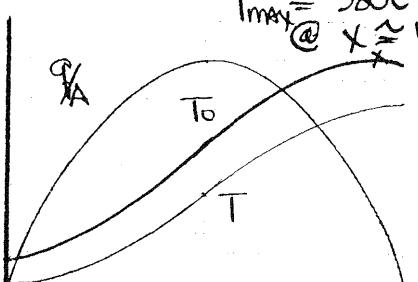
for  $h = 56 \text{ W/m}^2 \cdot \text{K}$   $\frac{q}{A} = h(T_0 - T)$

$$T_0 = T + \frac{\frac{q}{A}}{56}$$

$x$	$\frac{q}{A}$	$T$	$T_0$
0	900	100	116
0.4	3040	171	226
0.8	3110	285	340
1.2	1030	360	378
1.22	900	361	377

$$T_{max} \approx 380^\circ C$$

@  $x \approx 1.15m$



$$19.9 \quad q = \int_0^L h_x \Delta T dx = \int_0^L N_u x \frac{k}{x} \Delta T dx$$

$$= k \Delta T^{5/4} (0.508) Pr^{1/2} (Pr + 0.954)^{-1/4}$$

$$\times \left( \frac{Pr}{V^2} \right)^{1/4} \left( \frac{4}{3} L^{3/4} \right)$$

$$= 995 \text{ W PER M OF WIDTH}$$

19.10 for  $V = a + by + cy^2$

B.C.  $V(0) = 0$

$$V(\delta) = V_p \quad \frac{V}{V_p} = 2 \frac{y}{\delta} - \left( \frac{y}{\delta} \right)^2$$

$$\frac{dy}{dy} (\delta) = 0$$

for  $T - Ts = x + \beta y + \gamma y^2$

B.C.  $(T - Ts)|_0 = 0$

$$(T - Ts)_{\delta t} = T_p - Ts \quad \frac{T - Ts}{T_p - Ts} = 2 \frac{y}{\delta t} - \left( \frac{y}{\delta t} \right)^2$$

$$\frac{d}{dy} (T - Ts) \Big|_{\delta t} = 0$$

INTO MOMENTUM EQU - TO GET

$$\delta t^2 = 30 \frac{V_p}{V_p} \quad (1)$$

INTO ENERGY EQU.:

$$\delta t \frac{d}{d\xi} \left( \delta t^2 - \frac{1}{5} \delta t^3 \right) = 12 \frac{d}{dx} \quad (2)$$

WHERE  $\xi = \delta t / \delta$

SOLN GIVES  $\delta \approx Pr^{-1/3}$

$$\therefore \delta_t = Pr^{-1/3} \delta$$

SINCE  $\frac{q}{A} = -k \frac{dT}{dy}(0) = h(J_s - T_p)$

$$\frac{h}{k} = \frac{2}{\delta t} = \frac{2 Pr^{1/3}}{\delta} = 0.365 Pr^{1/3} \left( \frac{V_p}{\delta x} \right)^{1/2}$$

OR;  $N_u x = \frac{hx}{k} = 0.365 Pr^{1/3} Re_x^{1/2}$

$$\begin{aligned}
 19.11 \quad & \frac{q}{A} = \alpha + \beta \sin \frac{\pi x}{L} \\
 & = \pi D \int_0^L (\alpha + \beta \sin \frac{\pi x}{L}) dx \\
 & = \pi D \left[ \alpha L + 2 \frac{\beta L}{\pi} \right] \\
 & = \pi \left( \frac{15}{12} \right) \left[ 250 + 2 \frac{3600}{\pi} \right] = 4730 \frac{\text{Btu}}{\text{hr}}
 \end{aligned}$$

$$\begin{aligned}
 T_{\text{ext}} &= T_b + \frac{q}{SAVcp} = 60 + \frac{4730}{60(1)(0.062)(3600)} \\
 &= \underline{60.3 \text{ F}}
 \end{aligned}$$

$$T_w = 60.3 + \frac{250}{976} \approx \underline{60.6 \text{ F}}$$

$$19.12 \quad T_o = 300 - 240 e^{-720 St}$$

$$\begin{aligned}
 q &= \pi D L S_c V (T_s - T_b) St \\
 &= 471 (S_c V) St
 \end{aligned}$$

$$\begin{aligned}
 \text{FOR AIR @ } 180\text{F} \quad & S_c = 0.0622 \\
 & cp = 0.241
 \end{aligned}$$

$$\begin{aligned}
 q &= 471 (0.0622)(0.241)(15 \times 3600) St \\
 &= 3,82 \times 10^5 \text{ St}
 \end{aligned}$$

$$V = 0.228 \times 10^{-3}$$

$$Re = \frac{DV}{\nu} = 5480$$

$$f = 88 \times 10^{-3}$$

$$\text{FREUDENST. } S_f = 0.00444$$

$$T_o = 190\text{F} \quad q = 937 \frac{\text{Btu}}{\text{hr}}$$

$$\text{COURBET} \quad -23$$

$$S_f = 0.00444 \text{ fr}^{-2} = 0.00467$$

$$T_o = 298\text{F} \quad q = 959 \frac{\text{Btu}}{\text{hr}}$$

$$\begin{aligned}
 19.13 \quad & N_2 \text{ AT } 100\text{F} \quad 200\text{F} \quad 150\text{F} \\
 & S_p \quad 0.069 \quad 0.0583 \\
 & \nu \text{ (m/s)} \quad 1.71 \times 10^{-3} \quad 0.736 \times 10^{-3} \quad 0.209 \times 10^{-3} \\
 & k \quad 0.0154 \quad 0.0174 \quad 0.0164 \\
 & Pr \quad 0.71 \quad 0.71 \quad 0.71 \\
 & Re = \frac{LV}{\nu} = \frac{4 \pi (10 \text{ ft/s})}{0.71} = 1.91 \times 10^5 \\
 & a) S = \frac{5x}{Re^{1/2}} = \frac{5(4)}{(1.91 \times 10^5)^{1/2}} = 0.0457 \text{ PT} \\
 & = 0.549 \text{ N} \\
 & b) S_f = \frac{8}{Pr^{1/3}} = \frac{8}{(0.71)^{1/3}} = 0.165 \text{ N} \\
 & c) C_f = 0.00444 Re^{-1/2} = 0.0052 \\
 & d) C_{FL} = 1.328 Re_L^{-1/2} = 0.00304 \\
 & e) h_x = 0.332 \frac{k}{x} Re^{1/2} Pr^{1/3} = 0.531 \frac{\text{Btu}}{\text{hr ft}^2 \text{ F}} \\
 & f) h = 0.4604 \frac{k}{L} Re_L^{-1/2} Pr^{1/3} = 1.06 \text{ "} \\
 & g) f_d = A C_f \frac{SV^2}{2} = \frac{2(0.00304)(0.003)(10)}{2(32.2)} \\
 & = 5.95 \times 10^{-4} \text{ lbf} \\
 & h) q = h A \Delta T = 1.06(2)(100) \\
 & = 212 \frac{\text{Btu}}{\text{hr}}
 \end{aligned}$$

$$19.14 \quad \text{FOR AIR AT } T_f = 325\text{K} :$$

$$p = 1.087 \text{ kg/m}^3 \quad \nu = 1.807 \times 10^{-5} \text{ m}^2/\text{s}$$

$$cp = 1.008 \text{ kJ/kg.K} \quad Pr = 0.702$$

$$k = 2.816 \text{ W/m.K}$$

$$(a) C_{FL} = 1.328 Re_L^{-1/2}$$

$$Re = \frac{LV}{\nu} = \frac{(1\text{m})(1.8\text{ m/s})}{1.807 \times 10^{-5} \text{ m}^2/\text{s}} = 1.55 \times 10^5$$

$$C_{FL} = (1.328)(1.55 \times 10^5)^{-1/2} = 0.00337$$

## 19.14 (CONTINUED)

$$(b) F_D = C_{FL} A \frac{\rho v^2}{2}$$

$$= (0.00337)(0.25)(1)(1.081)(2.8)^2$$

$$= 3.59 \times 10^{-3} \text{ N}$$

$$(c) \dot{q} = h A \Delta T$$

USING Colburn Analogy:

$$St_L = \frac{C_{FL}}{2} Pr^{2/3}$$

$$= \frac{(0.00337)}{2} (0.702)^{2/3}$$

$$= 2.133 \times 10^{-3}$$

$$= h / \rho c_p V$$

$$h = (2.133 \times 10^{-3})(1.081)(1008)(2.8)$$

$$= 6.54 \text{ W/m}^2 \cdot \text{K}$$

$$\dot{q} = 6.54 (1)(0.25)(55)$$

$$= 90.0 \text{ W}$$

## 19.15

Momentum Theorem ~ X DIR.

$$\sum F_x = \iint_{CS} v_x s (\vec{j} \cdot \vec{n}) dA + \frac{\partial}{\partial x} \iiint_{CV} v_x s dV$$

AT Low Velocity:  $\rho \approx \text{CONST}$ ,  $v_{\infty} = 0$

& STEADY STATE

$$\text{LHS: } \sum F_x = (\text{Buoyant Force}) - (\text{Viscous Force})$$

Buoyant force (B.F.)  $\propto \Delta \rho \{ \text{per unit volume} \}$

$$f = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P = -\frac{1}{V} \left( \frac{\partial \rho}{\partial T} \right)_P \approx \frac{1}{V} \frac{\Delta \rho}{\Delta T}$$

$$\therefore \Delta \rho \approx -\beta \Delta T$$

$$\text{B.F.} = \Delta x f g \int_0^{\delta t} (T - T_{\infty}) dy$$

## 19.15 (CONTINUED)

$$\text{Viscous force (V.F.)} = \Delta x \mu \frac{\partial v_x}{\partial y}(0)$$

$$\text{R.A.S.} \iint_{CS} v_x s (\vec{j} \cdot \vec{n}) dA = \int_0^{\delta} \delta v_x^2 dy |_{x+\Delta x}$$

$$- \int_0^{\delta} \delta v_x^2 dy |_x - v_{\infty}^2 \text{ outside}$$

$$\text{Equating: } (\text{LHS}) = (\text{RHS}) \xi \text{ Div. By } \Delta x;$$

$$\int_0^{\delta} \delta \int_0^t (T - T_{\infty}) dy - \mu \frac{\partial v_x}{\partial y}(0)$$

$$= \int_0^{\delta} \delta v_x^2 dy |_{x+\Delta x} - \int_0^{\delta} \delta v_x^2 dy |_x$$

$$\text{In LIMIT As } \Delta x \rightarrow 0 \quad \xi \quad \delta = \text{constant}$$

$$\int_0^{\delta} \int_0^t (T - T_{\infty}) dy - \nu \frac{\partial v_x}{\partial y}(0) = \frac{d}{dy} \int_0^{\delta} v_x^2 dy$$

ENERGY EQUATION: SAME FOR BOTH NATURAL & FORCED CONVECTION

$$\times \frac{\partial T}{\partial y}(0) = \frac{d}{dy} \int_0^{\delta t} (T_{\infty} - T) v_x dy$$

$$19.16 \quad f_D = \frac{(2 \rho t)(10 \rho T_b)}{V} = 20/\omega$$

LAMINAR FLOW FOR ALL VALUES OF  $V$

$$C_{FL} = \frac{1328}{Re^{1/2}} \quad f_D = A C_F S \frac{V^2}{2}$$

$$f_D = \frac{4(1328)(S)(100)}{Re^{1/2}} = 8.25 \frac{S}{Re^{1/2}}$$

$$Re = 0.604 Re^{1/2} Pr^{1/3}$$

$$h = \frac{k}{L} (0.604) Re^{1/2} Pr^{1/3}$$

19.16 CONTINUED -

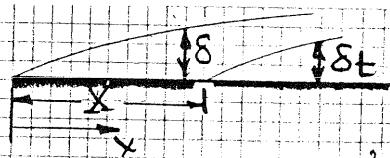
T, F	S	N	Re	$f_r$	$f_p$
30	79.5	$6.61 \times 10^{-2}$	303	37.6	1.00
50	79.0	$1.52 \times 10^{-2}$	1320	17.9	"
80	78.2	$0.13 \times 10^{-2}$	15400	5.2	"

$$T_a T_r T_f \rightarrow Re^{1/2} Pr^{1/3}$$

$$\begin{aligned} 30 & 80 55 0.0119 21.9 7.25 \\ 50 & 100 75 0.0101 44.6 4.64 \\ 80 & 130 105 0.0011 134.8 2.16 \end{aligned}$$

T	$h_u$	$g/A$
30	$874 \frac{\text{Btu}}{\text{ft}^2 \text{F}}$	$437 \frac{\text{Btu/lb ft}^2}{}$
50	11.4 "	570 "
80	16.0 "	800 "

19.17



$$\text{Assuming } T - T_s = \alpha + \beta y + \gamma y^2 + \delta y^3$$

$$\text{B.C. } T(0) = T_s \quad \frac{\partial T}{\partial y}(0) = 0$$

$$T(\delta_t) = T_p \quad \frac{\partial^2 T}{\partial y^2}(0) = 0$$

Temp Profile becomes

$$\frac{T - T_s}{T_p - T_s} = \frac{3}{2} \frac{y}{\delta_t} - \frac{1}{2} \left( \frac{y}{\delta_t} \right)^3 \quad (1)$$

Similarly for Velocity:

$$\frac{V}{V_\infty} = \frac{3}{2} \frac{y}{\delta_t} - \frac{1}{2} \left( \frac{y}{\delta_t} \right)^3 \quad (2)$$

INTO INTEGRAL EXPRESSION:

$$\alpha \frac{\partial T}{\partial y}(0) = \frac{d}{dx} \int_0^{\delta_t} (T_p - T) V dy$$

(LHS) ~ LEFT-HAND SIDE:

$$\alpha \frac{\partial T}{\partial y}(0) = \alpha (T_p - T_s) \left( \frac{3}{2} / \delta_t \right)$$

19.17 (CONTINUED)

$$\begin{aligned} (\text{RHS}) \int_0^{\delta_t} (T_p - T_s) V dy &= \int_0^{\delta_t} (T_p - T_s) V_p \int_0^y \left( 1 - \frac{T - T_s}{T_p - T_s} \right) dy \\ &= (T_p - T_s) V_p \left[ \frac{3}{20} \frac{\delta_t^2}{\delta} - \frac{3}{80} \frac{\delta_t^4}{\delta^3} \right] \end{aligned}$$

SUBST EQUATIONS (1) & (2):

$$= (T_p - T_s) V_p \left[ \frac{3}{20} \frac{\delta_t^2}{\delta} - \frac{3}{80} \frac{\delta_t^4}{\delta^3} \right]$$

NEGLECT

$$\text{GIVING: } (\text{RHS}) - (T_p - T_s) V_p \frac{d}{dx} \left( \frac{3}{20} \frac{\delta_t^2}{\delta} \right)$$

EQUATION:  $(\text{LHS}) = (\text{RHS})$

$$\delta_t \frac{d}{dx} \frac{\delta_t^2}{\delta} = 10 \times$$

$$\frac{1}{\delta} \text{ LETTING } \xi = \frac{\delta_t}{\delta} : \quad V_p$$

$$\delta \xi d(\xi^2) = 10 \times \frac{V_p}{\delta} dx$$

$$\delta = \frac{4.64}{\sqrt{V_p}} x^{1/2}$$

SUBSTITUTION  $\downarrow$  SOME ALGEBRA (GIVE)

$$4.31 \times \xi^2 d\xi = \left( \frac{x}{\delta} - 1.07 \xi^3 \right) dx$$

SEPARATING VARIABLES,

$$\frac{4.31 \xi^2 d\xi}{\frac{1}{\delta} - 1.07 \xi^3} = \frac{dx}{x}$$

$$\xi^3 = \frac{1}{1.07} \frac{1}{4.31} \left[ 1 - \left( \frac{x}{\delta} \right)^{3/4} \right] \quad \downarrow \text{SOLVING}$$

$$\xi = \frac{\delta_t}{\delta} \approx \frac{1}{4.31} \left[ 1 - \left( \frac{x}{\delta} \right)^{3/4} \right]^{1/3}$$

19.17 (CONTINUED -

Nusselt No.

$$\begin{aligned} \text{G} &= -k \frac{\partial T}{\partial y}(0) = -\frac{3}{2} k \frac{T_p - T_s}{\delta_t} \\ &= \frac{3}{2} k \frac{T_s - T_p}{\delta} Re_x^{1/2} \left[ \frac{Pr}{1 - (\frac{V_x}{x})^{3/4}} \right]^{1/3} \\ &= h(T_s - T_p) \\ N_{ux} &= \frac{h_x}{k} = 0,323 Re_x^{1/2} \left[ \frac{Pr}{1 - (\frac{V_x}{x})^{3/4}} \right]^{1/3} \end{aligned}$$

$$19.18 N_{ux} = \frac{h_x x}{k} = 0,508 Pr^{1/2} \left( Pr + 0,954 \right)^{-1/4} \left( \frac{\Delta T}{x} \right)^{1/4}$$

for Air:  $Pr = 0,72$   $k = 0,025 \frac{\text{W}}{\text{mK}}$

$$h = k (0,38) \left( 2 \times 10^8 \frac{\Delta T}{x} \right)^{1/4} = 0,445 \left( \frac{\Delta T}{x} \right)^{1/4}$$

$$q = h L A \Delta T = \int_0^L h_x A \Delta T dx$$

$$h_L = \frac{1}{L} \int_0^L h_x dx = 0,286 \left( \frac{\Delta T}{L} \right)^{1/4} = K \left( \frac{\Delta T}{L} \right)^{1/4}$$

$$\Rightarrow \alpha = 0,286 \quad b = 1/4$$

$$19.19 \quad \frac{V_x}{U_x} = \frac{y}{\delta} \left( 1 - \frac{y}{\delta} \right)^2 \quad \frac{T - T_p}{T_s - T_p} = \left( 1 - \frac{y}{\delta} \right)^2$$

INTO ENERGY EQUATION

$$\alpha \frac{\partial T}{\partial y}(0) = \frac{d}{dx} \int_0^y U_x (T_p - T) dy$$

$$\text{cont. } LHS = -\frac{2\alpha (T_s - T_p)}{\delta}$$

$$RHS = (T_p - T_s) \frac{d}{dx} \left( \frac{8U_x}{30} \right)$$

$$\text{Equating: } \frac{2\alpha}{\delta} = \frac{d}{dx} \frac{8U_x}{30}$$

19.19 (CONTINUED -

INTO MOMENTUM EQUATION:

$$Bg \int_0^y (T_s - T_p) dy - \frac{dU_x}{dy}(0) = \frac{d}{dx} \int_0^y U_x^2 dy$$

$$LHS = Bg (T_s - T_p) \delta - \frac{dU_x}{\delta}$$

$$RHS = \frac{d}{dx} \left( \frac{8U_x^2}{105} \right)$$

EQUATING:

$$Bg (T_s - T_p) \delta - \frac{dU_x}{\delta} = \frac{d}{dx} \int_0^y U_x^2 dy$$

$$\text{LETTING } \delta = Ax^a \quad U_x = Bx^b$$

PREVIOUS TWO EQUATIONS BECOME:

$$\frac{2\alpha}{A} x^{-a} = \frac{AB(a+b)}{30} x^{a+b-1}$$

$$-\frac{dB}{A} x^{-a+b} + Bg \Delta T \frac{A}{3} x^a = AB(a+b) \frac{a+2b-1}{105} x^{a+2b-1}$$

EXPONENTS ON X MUST AGREE

$$\Rightarrow -a = a+b-1$$

$$-a+b = a = a+2b-1$$

$$\text{GIVING } a = 1/4 \quad b = 1/2$$

SO EQUATIONS FOR A & B BECOME

$$\frac{2\alpha}{A} = \frac{AB}{30} \left( \frac{3}{4} \right)$$

$$\frac{dB}{A} + Bg \Delta T A \frac{1}{3} = \frac{AB^2}{105} \left( \frac{5}{4} \right)$$

SO USE THESE

$$A = \left[ 240 \left( \frac{D}{Bg \Delta T} \right) \left( \frac{a^2}{b^2} \right) \left( \frac{20}{21} + \frac{D}{x} \right) \right]^{1/4}$$

$$B = \frac{80 \alpha}{A}$$

19.19 (CONTINUED -

" FINALLY - UPON SUBSTITUTION!

$$\delta = \frac{1}{4} x^{\frac{1}{4}} = 3.94 \Pr^{-\frac{1}{2}} (\Pr + 0.954) Gr_x^{\frac{1}{4}} x^{\frac{1}{4}}$$

$$\frac{d}{dx} = -k \frac{\partial T}{\partial y}(0) = \frac{2k}{\delta} (T_s - T_w) = h (T_s - T_w)$$

$$\Rightarrow Nu_x = 0.508 \Pr^{\frac{1}{2}} (\Pr + 0.954) Gr_x^{-\frac{1}{4}} x^{\frac{1}{4}}$$

19.20

$$\frac{\delta_t}{\delta} = \frac{1}{\Pr^{\frac{1}{3}}} \left[ 1 - \left( \frac{x}{x} \right)^{\frac{3}{4}} \right]^{\frac{1}{3}}$$

$$Nu_x = 0.33 \left[ \frac{\Pr}{1 - \left( \frac{x}{x} \right)^{\frac{3}{4}}} \right]^{\frac{1}{3}} Re_x^{\frac{1}{2}}$$

$$Re_x = \frac{0.4(5)}{1.569 \times 10^{-5}} = 127,500$$

$$\delta = \frac{5x}{Re_x^{\frac{1}{2}}} = \frac{5(40)}{(1275 \times 10^5)^{\frac{1}{2}}} = 0.56 \text{ cm}$$

$$\delta_t = \frac{0.56}{0.708^{\frac{1}{3}}} \left[ 1 - \left( \frac{1}{2} \right)^{\frac{3}{4}} \right]^{\frac{1}{3}} = 0.465 \text{ cm}$$

$$C_{fx} = \frac{0.1604}{Re_x^{\frac{1}{2}}} = 1.86 \times 10^{-6}$$

$$Nu_x = 0.33 \left[ \frac{0.708}{1 - \left( \frac{1}{2} \right)^{\frac{3}{4}}} \right]^{\frac{1}{3}} (1275 \times 10^5)^{\frac{1}{2}}$$

$$= 143$$

$$h_x = k_x (143) = 9.38 \text{ W/m}^2 \cdot \text{K}$$

$$19.21 \quad v = a + by \quad BC, v(0) = 0$$

$$v(\delta) = v_\infty$$

$$\therefore \frac{v}{v_\infty} = \frac{y}{\delta}$$

$$T - T_s = K + \beta y$$

$$BC, (J - T_s)_0 = 0$$

$$(J - T_s)_{\delta_t} = T_p - T_s$$

$$\therefore \frac{T - T_s}{T_p - T_s} = \frac{y}{\delta_t}$$

19.21 (CONTINUED)

INTO MOMENTUM EQN:

$$\frac{d}{dy} \frac{\delta}{\delta_t} (0) = \frac{d}{dx} \int_0^\delta (v_\infty - v) dy$$

$$LHS = \frac{d}{dy} \frac{\delta}{\delta_t}$$

$$RHS = -v_\infty \frac{d}{dx} \int_0^\delta \left( 1 - \frac{v}{v_\infty} \right) \left( \frac{v}{v_\infty} \right) dy$$

EQUATING &amp; SOLVING!

$$\delta^2 = \frac{12 \Pr x}{v_\infty} \quad (1)$$

ENERGY EQN:

$$\frac{d}{dy} \frac{\delta}{\delta_t} (0) = \frac{d}{dx} \int_0^\delta (J_p - T) v dy$$

SUBSTITUTING &amp; SOLVING!

$$\frac{6 \alpha}{v_\infty \delta \xi} = \frac{d}{dx} (\delta \xi^2) \quad \left\{ \xi = \frac{\delta_t}{\delta} \right\} \quad (2)$$

$$\delta_t = 0 \quad \text{for } x = \infty$$

$$\xi^3 = \frac{x}{\delta} \left[ 1 - \left( \frac{x}{\delta} \right)^{\frac{3}{4}} \right]$$

$$\delta_t = \Pr^{-\frac{1}{3}} \left[ 1 - \left( \frac{x}{\delta} \right)^{\frac{3}{4}} \right]^{\frac{1}{3}}$$

$$\frac{d}{dx} \frac{\delta}{\delta_t} (0) = -k \frac{(T_s - T_w)}{\delta_t} = h (T_s - T_w)$$

GIVEN:

$$Nu_x = \frac{h_x}{k} = 0.288 \left[ \frac{\Pr}{1 - \left( \frac{x}{\delta} \right)^{\frac{3}{4}}} \right]^{\frac{1}{3}} Re_x^{\frac{1}{2}}$$

IF  $x = 0$ 

$$Nu_x = 0.288 \Pr^{\frac{1}{3}} Re_x^{\frac{1}{2}}$$

$$19.22 \quad V = \alpha \sin \beta y \quad T - T_s = \alpha \sin \beta y$$

$$\text{BC}, \quad V(0) = 0 \quad (T - T_s)|_0 = 0$$

$$V(\delta) = V_p \quad (T - T_s)|_{\delta} = T_p - T_s$$

$$\Rightarrow \frac{V}{V_p} = \sin \frac{\pi y}{2\delta} \quad \frac{T - T_s}{T_p - T_s} = \sin \frac{\pi y}{2\delta}$$

INTO ENERGY EQU.

$$\delta_t \frac{dS_t}{dx} = \frac{\alpha \pi}{V_p} \left( \frac{\pi}{4 - \pi} \right)$$

{PRESUMES  $\delta = \delta_t$  FOR INTEGRATION}

$$\frac{q}{A} = -k \frac{dT}{dy}(0) = h(T_s - T_p)$$

$$\frac{k\pi}{2\delta_t} = h \quad \text{or} \quad h = \frac{k\pi}{2\delta_t}$$

$$\Rightarrow N_{ux} = \frac{hx}{k} = 0.327 \Pr^{1/3} \operatorname{Re}_{\text{ex}}^{1/2}$$

19.23

$$\frac{V}{V_p} = \left( \frac{y}{\delta} \right)^{1/4} \quad \frac{T - T_s}{T_p - T_s} = \left( \frac{y}{\delta} \right)^{1/4}$$

ENERGY EQUON:

$$\alpha \frac{dT}{dy}(0) = V_p(T_p - T_s) \frac{d}{dx} \int_0^{\delta_t} V_p \left( 1 - \frac{T - T_s}{T_p - T_s} \right) dy$$

$$LHS = \frac{0.0225(T_p - T_s)V_p}{\delta^{1/\alpha}} \left( \frac{y}{\delta} \right)^{1/4}$$

$$RHS = V_p(T_p - T_s) \frac{7}{72} \frac{dS}{dx}$$

{ASSUMES  $\delta = \delta_t$  FOR INTEGRATION}

EQUATING  $\frac{7}{72}$  SOME ALGEBRA!

$$\frac{\delta}{x} = 0.371 \Pr^{4/5} \operatorname{Re}_{\text{ex}}^{-1/5}$$

19.23 CONTINUED -

$$\frac{q}{A} = -k \frac{dT}{dy}(0) = -\frac{k(0.0225)(\Delta T)V_p}{\delta} \left( \frac{y}{\operatorname{Re}_{\text{ex}}} \right)^4$$

$$= h \Delta T$$

$$N_{ux} = \frac{hx}{k} = 0.10288 \Pr^{19/20} \operatorname{Re}_{\text{ex}}^{1/5}$$

$$19.24 \quad q = h A \Delta T$$

$$\frac{q}{A} = 184 - 95 = 889 \text{ W/m}^2$$

$$\Delta T = 8 \text{ K} \quad A = (1)(18.3) = 18.3 \text{ m}^2$$

$$h = \frac{889}{8} = 110.625 \text{ W/m}^2 \cdot \text{K}$$

FOR CONDITIONS SPECIFIED:

$$\operatorname{Re}_L = \frac{(18.3 \text{ m}) V}{1.5689 \times 10^{-5} \text{ m}^2/\text{s}} = 1.166 \times 10^6 V$$

PROBABLY TURBULENT B.L.

$$\text{USE COUBERN ANALOGY: } S_t = C_f \frac{\Pr}{2}$$

FROM COUBERN - FOR TURB. B.L.

$$C_f = 0.0576 \operatorname{Re}_{\text{ex}}^{-1/5}$$

$$C_f L = \frac{1}{L} \int_0^L C_f x dx$$

$$= 0.072 \operatorname{Re}_{\text{ex}}^{-1/5} \quad \begin{array}{l} \text{(ASSUMING ALL} \\ \text{SURFACE EXPOSED} \\ \text{TO TURB. B.L.)} \end{array}$$

$$S_t = \frac{h}{8 C_f V_p} = \frac{0.072 \operatorname{Re}_{\text{ex}}^{-1/5} \Pr^{-1/3}}{2} = 1/5 \quad \frac{1}{15} \quad \frac{1}{3}$$

$$= 0.036 \left[ 1.166 \times 10^6 V \right] (0.708)$$

$$= 0.00277 V^{1/5}$$

$$h = 8 C_f V (0.00277 V^{1/5})$$

$$= (1.177)(1000)(0.00277) V^{4/5} \text{ W/m}^2 \cdot \text{K}$$

$$= 3.280 V^{4/5} = 13.63$$

$$V = 11.8 \text{ m/s}$$

19.25

$$\frac{T - T_s}{T_0 - T_s} = \exp\left(-St \frac{4\pi}{D}\right)$$

$$\frac{T - 300}{60 - 300} = \exp\left[-St \frac{4(15)}{12}\right] = \exp(-720 St)$$

$$v = 12.25 \text{ ft/s}$$

REYNOLDS ANALOGY: ASSUME  $T_L = 240^\circ F$ 

$$T_{avg} = 150^\circ F \quad T_f = 225^\circ F$$

$$Re = \frac{1/12 (12.25)}{3.07 \times 10^{-5}} = 3.32 \times 10^5$$

$$f = 0.0036 \quad St = 0.0018$$

$$T = 300 - (240)e^{-(0.0018)(720)}$$

$$= 234.5^\circ F,$$

CLOSE ENOUGH - DOING OVER  
WITH  $T_L = 234.5$  WILL YIELD  
 $T_L \approx 234.5$  AS A RESULT.COLBURN ANALOGY: ASSUME  $T = 100^\circ F$ 

$$T_{avg} = 130 \quad T_f = 215$$

$$Re = \frac{1/12 (12.25)}{0.324 \times 10^{-5}} = 3.18 \times 10^5$$

$$f = 0.0036 \quad St = 0.0018 (1.79)$$

$$= 0.00122$$

$$T = 300 - (240)e^{-(0.00122)(720)}$$

$$= 201^\circ F$$

$$q = \dot{m}cp\Delta T = \frac{30}{7.48} (62.3)(0.999)\Delta T$$

$$= 250 \Delta T \text{ BTU/min}$$

SUMMARY  $\Delta T, F$   $q, \text{ BTU/min}$ 

REYNOLDS 174 43,500

COLBURN 141 35,300

19.26

$$q = \frac{500 \text{ BTU}}{\text{HR} \cdot \text{FT}^2} (\pi)(15/12) \text{ FT}^2$$

$$= 1960 \text{ BTU/HR}$$

$$q = \dot{m}cp\Delta T = 1960$$

$$\Delta T = \frac{1960}{(30/1.48)(62.3)(60)(0.999)}$$

$$= 0.131^\circ F$$

$$T_{exit} = 60.13^\circ F$$

FROM COLBURN ANALOGY:  $T \approx 60^\circ F$ 

$$Re = \frac{(1/12)(30/1.48)(144 \times 4)}{0.76 \times 10^{-3}} (\pi) (160)$$

$$= 1344 \quad \{ \text{LAMINAR} \}$$

$$f = \frac{16}{1344} = 0.0119$$

$$St = \frac{0.0119}{2} (8.07)^{2/3} = 0.00148$$

$$h = 623 \left( \frac{30 \times 144 \times 4}{7.48 \times \pi \times 60} \right) (0.24)(3000)(St)$$

$$= 976 \text{ BTU/HR FT}^2 F$$

$$T_{exit} = 60.13 + \frac{500}{976} = 60.6^\circ F$$

19.27

 $q$  = SAME AS IN PROB 19.26

$$= 1960 \text{ BTU/HR}$$

$$T = T_0 + \frac{q}{\dot{m}cp}$$

$$= 600 + \frac{1960}{(0.0764)(\frac{\pi}{4} \times \frac{1}{144})(15 \times 300)(0.24)}$$

$$= 423^\circ F$$

19.28

$$T_L = T_0 - \Delta T e^{-St \frac{44}{D}}$$

$$= 300 - 100 e^{-St \frac{44}{D}}$$

$$V = \frac{30(144)}{7.48(60)(\pi/4)} = 12.25 \text{ ft/s}$$

$$f_e = \frac{DV}{D} = \frac{(1/2)(12.25)}{7.43 \times 10^{-6}} = 137,100$$

$$f_r = 0.0118 \quad f_f = 0.0044$$

REYNOLDS  $S_f = 0.0022$

ANALOGY:

$$T_L = 300 - 100 e^{-1585} = \underline{279.5 \text{ F}}$$

$$q = \dot{S}AVcp \Delta T = \underline{100.5 \text{ BTU/s}}$$

CARBON !  $S_f = 0.0022(0.0118)^{4/3}$

ANALOGY !  $= 0.0424$

$$T_L = 300 - 100 e^{-30.1} \cong 300 \text{ F}$$

$$\underline{q = 126.4 \text{ BTU/s}}$$

19.29

for constant  $\dot{Q}/A$ 

$$T = T_0 + \frac{\dot{Q}/A}{\dot{S}Vcp}$$

$$= 200 + \frac{500}{(58.1)(12.25 \times 3600)(0.0022)}$$

$$\cong \underline{300 \text{ F}}$$

## CHAPTER 20

$$20.1 \quad \frac{q}{A} = \frac{750(3413)}{\pi(3/48)(1/2)} = 26100 \frac{\text{Btu}}{\text{HR ft}^2}$$

ENDS ARE NEGLECTED

$$h = \frac{k}{L} Nu = \frac{k}{L} \left[ 0.825 + \frac{0.387 Ra^{1/6}}{1 + (0.492 \frac{g}{Pr})^{9/16}} \right]^2$$

FOR VERTICAL ORIENTATION

BY TRIAL & ERROR:  $\Delta T \approx 103 \text{ F}$

HTR SURFACE TEMP = 198 F

HORIZONTAL ORIENTATION

$$h = \frac{k}{D} \left[ 0.60 + \frac{0.387 Ra^{1/6}}{1 + (0.559 \frac{g}{Pr})^{9/16}} \right]^2$$

TRIAL & ERROR:  $\Delta T = 99 \text{ F}$

HTR SURFACE TEMP = 194 F

20.2 BISMUTH  $T_p = 700 \text{ F}$

$$\text{AS IN PROBLEM 20.1} \quad \frac{q}{A} = 26100 \frac{\text{Btu}}{\text{HR ft}^2}$$

VERTICAL - SAME FORMULA AS ABOVE

TRIAL & ERROR:  $\Delta T \approx 57 \text{ F}$

$T_{\text{SURF}} \approx 757 \text{ F}$

HORIZONTAL - SAME FORMULA AS ABOVE

TRIAL & ERROR:  $\Delta T \approx 44 \text{ F}$

$T_{\text{SURF}} \approx 744 \text{ F}$

## 20.2 CONT. HYDRAULIC FLUID

SAME FORMULAS & PROCEDURES

$$\text{VERTICAL: } \Delta T \approx 630 \quad T_{\text{SURF}} \approx 630 \text{ F}$$

$$\left. \begin{array}{l} \text{PROPERTIES USED AT} \\ 700 \text{ F - HIGHEST} \end{array} \right\} \text{HORIZONTAL TEMP IN TABLES}$$

$$\Delta T \approx 580 \quad T_{\text{SURF}} \approx 580 \text{ F}$$

FOR VERTICAL ORIENTATION

$$20.3 \quad \frac{q}{A} = \frac{3413}{0.344} = 9900 \frac{\text{Btu}}{\text{HR ft}^2}$$

16 cm (0.525 ft) - VERTICAL AT.

$$T_p = 71 \text{ F}$$

$$h = \frac{k}{L} \left[ 0.825 + \frac{0.387 Ra^{1/6}}{1 + (0.492 \frac{g}{Pr})^{9/16}} \right]^2$$

TRIAL & ERROR:  $\Delta T = 62 \text{ F}$   $T_s = 133 \text{ F}$

FOR 10 cm (0.328 ft) - HORIZONTAL

TRIAL & ERROR:  $\Delta T = 60 \text{ F}$   $T_s = 131 \text{ F}$

ENGLISH UNITS USED - TABLES  
EASIER TO USE

20.4 FOR  $T_f = 100 \text{ F}$

$$Gr_L = (107 \times 10^6) \left( \frac{1}{2} \right)^3 (100) = 1,337 \times 10^9$$

$$Pr = 4.51 \quad Ra = 6.03 \times 10^9$$

$$h_{\text{SIDES}} = \frac{k}{L} \left[ 1 + \frac{0.387 Ra^{1/6}}{1 + (0.492 \frac{g}{Pr})^{9/16}} \right]^2$$

$$= 190 \frac{\text{Btu}}{\text{HR ft}^2 \text{ F}}$$

$$R_i = \frac{hV/A}{k} = \frac{190}{200} (0.0357) = 0.0308$$

USE LUMPED PARAMETER

Part (c) cont. for  $T_{\text{surf}} = 150^\circ\text{F}$

$$\frac{T - T_p}{T_0 - T_p} = \frac{50}{150} = \frac{1}{3} = \frac{-Bi_f o}{2}$$

$$f_o = \frac{\kappa t}{(V/A)^2} = \frac{398 t}{(0.0357)^2} = 3120 t$$

t IN hours

$$-Bi_f o = \ln \frac{1}{3}$$

$$t = 0.0114 \text{ hr} = 0.686 \text{ min} = 41.15$$

SINCE LUMPED PARAMETER SOLN IS  
VALID - ANSWERS TO PARTS (a) & (b)  
ARE THE SAME

WHEN  $T_c = 100^\circ\text{F}$   $T_{\text{surf}} \approx 100^\circ\text{F}$

No. 5  $T_s = 140^\circ\text{C}$   $T_p = 25^\circ\text{C}$   $T_f = 82.5^\circ\text{C}$

$$\text{AIR @ } 355 \text{ K: } \frac{f_o}{D^2} = 0.625 \times 10^8 \text{ (m}^3\text{.K)}^{-1}$$

$$f_o = (0.625 \times 10^8)(0.035)^3 (115) = 3.08 \times 10^5$$

HORZ. CYLINDER:

$$Nu = \left\{ 0.60 + \frac{0.387 \frac{f_o}{D}^{1/6}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{2/3} \right]^{8/27}} \right\}^2$$

$$\text{for } Pr = 0.696 \quad Nu_D = 10.47$$

$$q = hA\Delta T = \frac{k}{D} (\pi D L) (\Delta T)$$

$$= (0.0304) (\pi) (0.8) (115) (10.47)$$

$$= 92.0 \text{ W}$$

REMAINDER OF 100W INPUT GOES  
TO ELECTRICAL & CONDUCTION LOSSES  
& TO ILLUMINATION

No. 6 for a HORIZ. CYLINDER

$$Nu = \left\{ 0.60 + \frac{0.387 \frac{f_o}{D}^{1/6}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{2/3} \right]^{8/27}} \right\}^2$$

$$D = 27 \text{ W/m} = hA\Delta T = \frac{k}{D} Nu A \Delta T$$

$$= \frac{k}{D} Nu (\pi D) \Delta T$$

$$27 = \pi k Nu \Delta T$$

TRIAL & ERROR  $\Delta T \approx 9.8 \text{ K}$

$$T_{\text{surf}} \approx 39.8^\circ\text{C}$$

No. 7

for Cu CYLINDER WITH

$$HT = 20.3 \text{ cm, DIAM} = 2.54 \text{ cm}$$

$$\frac{V}{A} = \frac{(\pi D^2 / 4)L}{\pi D L + \pi D^2 / 2} = \frac{DL / 4}{L + D / 2} = 0.598 \text{ cm}$$

$$\text{for } Bi = \frac{h V / A}{k} = 0.1 \quad T_f \approx 16^\circ\text{C}$$

$$h = \frac{0.1 (379)}{0.00598} = 6340 \text{ W/m}^2\text{.K}$$

for AN h Nusselt  $\approx 6340$

$Bi < 0.1 \therefore$  LUMPED PARAM.

$$\frac{T - T_p}{T_0 - T_p} = \frac{-Bi_f o}{2}$$

$$f_o = \frac{\kappa t}{(V/A)^2} = \frac{(0.27 \times 10^{-5})(180)}{(0.00598)^2} = 5169$$

$$\frac{4.8 - (-1)}{32.5 - (-1)} = 0.173 = \frac{-Bi_f o}{2}$$

$$Bi_f o = 1.754 \quad Bi = 3.393 \times 10^{-3}$$

$$h = Bi (379) = 2.15 \text{ W/m}^2\text{.K}$$

$$20.8 \text{ for A SPHERE: } Nu_0 = 2 + 0.43 Ra_0^{1/4}$$

$$T_{\infty} = 240K \quad T_f = 320K \quad k = 2.78 \times 10^{-2}$$

$$T_{\infty} = 295K \quad Pr = 0.703$$

$$Ra_0 = (0.994 \times 10^8) D^3 (45)(0.703)$$

$$= 3.144 \times 10^9 D^3$$

D, cm	h	k/hx,
7.5	615	0.0104
5	710	0.0135
1.5	1180	0.0271

L.E., SURFACE RESISTANCE IS VERY SMALL ::  $T_s = T_{\infty}$  & FAULTS TO THIS VALUE ALMOST INSTANTANEOUSLY  
 $\sim T_{\text{TIME}} \approx 0$

20.9 FOR  $T_c$  TO REACH 320K - USE VALUES CALCULATED IN PROB 20.8

$$\alpha = k/f_c p = 2.1 \times 10^{-7} \text{ m}^2/\text{s}$$

D, cm	h	$\Delta t / x^2$	t
7.5	615	0.16	1.19 AR
5	710	0.16	31.7 MIN
1.5	1180	0.16	2.86 "

$T_{\text{surf}} \approx 295K$  AT ALL TIMES

$$20.10 \quad T_s = 240F \quad T_{\infty} = 60F \quad T_f = 150F$$

$$\Delta T = 180F \quad \frac{f_g}{v^2} = 1.22 \times 10^6 \quad Pr = 0.6918$$

$$k = 0.0167$$

a) HORIZONTAL:

$$h = \frac{k}{D} \left[ 0.6 + \frac{0.387 Ra_0^{1/6}}{\left\{ 1 + \left( \frac{0.559}{Pr} \right)^{9/16} \right\}^{8/27}} \right]^2$$

20.10 (CONT.)

$$h = 1.51 \frac{\text{Btu}}{\text{HR ft}^2 \text{ F}}$$

$$q = h \Delta T = 271 \frac{\text{Btu}}{\text{HR ft}^2 \text{ F}^2}$$

VERTICAL:

$$h = \frac{k}{D} \left[ 0.825 + \frac{0.387 Ra_0^{1/6}}{\left\{ 1 + \left( \frac{0.492}{Pr} \right)^{9/16} \right\}^{8/27}} \right]^2$$

$$h = 1.0 \quad q = 180 \frac{\text{Btu}}{\text{HR ft}^2}$$

20.11 SAME CONDITIONS AS PROB 20.10 EXCEPT FLUID IS  $H_2O @ 60F$

$$\frac{f_g}{v^2} = 403 \times 10^6 \quad Pr = 2.72 \quad k = 0.383$$

$$\text{HORIZ: } h = 316 \frac{\text{Btu}}{\text{HR ft}^2 \text{ F}} \quad q = 57000 \frac{\text{Btu}}{\text{HR ft}^2}$$

$$\text{VERT: } h = 281 \quad q = 50,600 \frac{\text{Btu}}{\text{HR ft}^2}$$

20.12 SPHERICAL TANK  $D = 0.6m$

$$T_c = 78K \quad T_{\infty} = 278K \quad q = \frac{\Delta T}{D} \frac{A}{4R}$$

FOR SPHERE:

$$h = \frac{k}{D} Nu = \frac{k}{D} \left[ 2 + 0.43 Ra_0^{1/4} \right]$$

$$h = \frac{0.0247}{70} \left[ 2 + 0.43 \left( 2.57 \times 10^8 \right) (0.7)^3 (8)(0.72) \right]^{1/4}$$

$$= 2.86 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \quad \text{- PROPERTIES @ } 260K$$

$$R_{\text{conv}} = \frac{1}{2.86 (4\pi)(0.35)^2} = 0.127$$



20.16

$$\begin{aligned} T_{\text{surf}} &= 1300 \text{ K} \\ T_p &= 270 \text{ K} \end{aligned}$$

$T_f = 785 \text{ K}$

$$\frac{q}{A} = h \Delta T \quad h = \frac{k}{D} \left( 2 + 0.43 Ra^{\frac{1}{4}} \right)$$

$$k = 5.68 \times 10^{-2} \text{ W/m}\cdot\text{K} \quad Pr = 0.688$$

$$Ra = (2.015 \times 10^6) (0.15)^3 (1030) (0.688) \\ = 4.82 \times 10^6$$

$$h = 8.4 \text{ W/m}^2 \cdot \text{K}$$

$$\frac{q}{A} = 8.4 (1030) = \underline{\underline{8550 \text{ W/m}^2}}$$

20.17

From Prob 20.16

$$h = \frac{k}{D} Nu = \frac{k}{D} \left[ 2 + 0.43 Ra^{\frac{1}{4}} \right]$$

$T_{\text{surf}}$	$T - T_p$	$T_f$	$Re^{\frac{1}{4}}$	$h$
1300	1030	785	4.68	8.40
1000	730	635	5.03	7.65
700	430	486	6.31	8.10
420	150	345	5.72	6.69

$$h_{\text{avg}} \approx 7.71 \text{ W/m}^2 \cdot \text{K}$$

$$Nu = \frac{hV/A}{k} = \frac{7.71 (0.15)}{39.8} = 0.00484$$

{LUMPED PARAMETER IS OK!}

$$\frac{T - T_p}{T_0 - T_p} = \frac{600 - 270}{1300 - 270} = 0.32 = \frac{\theta_0 - \theta_p}{\theta_0 - \theta_f}$$

$$R_c f_0 = 1.1394 \quad f_0 = 235.4 = \frac{\Delta t}{(\sqrt{A})^2}$$

$$t = \frac{(0.15)^2 (235.4)}{0.001125} = \underline{\underline{130.8 \text{ s}}}$$

20.18

$$q = I^2 R = (400)^2 8 \Omega / A$$

$$R = \frac{(1.72 \times 10^{-6})(100)}{\pi (0.5)^2} = 8.76 \times 10^4 \Omega / \text{m}$$

$$\frac{q}{A} = \frac{400^2 (8.76 \times 10^4)}{\pi (0.018)(1)} = 2480 \text{ W/m}$$

$$= h \Delta T \quad q = 140 \text{ W/m}$$

$$h = \frac{k}{D} C Ra^n$$

$$\frac{q}{A} = \frac{k}{D} C \left[ \left( \frac{\beta g}{\nu^2} D^3 Pr \right) \Delta T \right]^{1+n}$$

By TRIAL & ERROR:  $\Delta T \approx 220 \text{ K}$ 

$$h = \underline{\underline{11.0 \text{ W/m}^2 \cdot \text{K}}}$$

$$T_{\text{surf}} = 290 + 220 = \underline{\underline{510 \text{ K}}}$$

RESISTANCE OF INSULATION

$$= \frac{\ln \frac{D/2i}{2R}}{2\pi k} = \frac{\ln \frac{0.018}{0.005}}{2\pi (0.242)} = 0.842$$

$$\frac{q}{A} = \frac{\Delta T}{R} \quad \Delta T = \frac{140}{0.842} = 166 \text{ K}$$

$$T_{\text{interior}} = 510 + 166 = \underline{\underline{676 \text{ K}}}$$

20.19

$$R_M = \frac{2.83 \times 10^6}{1.72 \times 10^6} (8.76 \times 10^4)$$

$$= 1.44 \times 10^{-3} \Omega / \text{m}$$

$$\frac{q}{A} = (400)^2 (1.44 \times 10^{-3}) = 231 \text{ W/m}$$

$$\frac{q}{A} = \frac{231}{\pi (0.018)} = 4080 \text{ W/m}^2$$

$$= \frac{k}{D} C \left[ \left( \frac{\beta g}{\nu^2} D^3 Pr \right) \Delta T \right]^{1+n}$$

## 20.19 CONTINUED -

TRIAL & ERROR:  $\Delta T \approx 336 \text{ K}$ 

$$h = 12.1 \text{ W/m}^2 \cdot \text{K}$$

$$T_{\text{surf}} = 290 + 336 = 626 \text{ K}$$

$$R_{\text{INSUL}} = 0.842 \quad \left\{ \text{Prob 20A3} \right\}$$

$$\Delta T = \frac{q}{R} = \frac{231}{0.842} = 274 \text{ K}$$

$$T_{\text{INTERFACE}} = 626 + 274 = 900 \text{ K}$$

$$20.20 \quad q_{\text{TOTAL}} = q_{\text{CONV}} + q_{\text{RAD}}$$

Assume  $T_{\text{INSIDE}} = T_{\text{SURFACE}}$ 

$$q = h_i A_i (T_{\text{STB}} - T) = h_o A_o (T - T_p) \\ + \sigma A_o \left[ \left( \frac{T}{100} \right)^4 - \left( \frac{T_p}{100} \right)^4 \right]$$

 $T \propto R$ TRIAL & ERROR:  $T = 1147 \text{ K}$ 

$$= 687 \text{ F}$$

$$h_o = \frac{k}{D} \left[ 0.6 + \frac{0.387 P_e^{1/6}}{\left\{ 1 + \left( \frac{0.559}{P_r} \right)^{9/16} \right\}^{8/27}} \right]^2$$

$$q_A = 33 (1260 - 1147) = 3730 \text{ BTU/hr ft}^2$$

$$q_b = 3730 (\pi) \left( \frac{81625}{12} \right) (20) \\ = 168,000 \text{ BTU/hr}$$

## 20.21 FORCED CONVECTION OUTSIDE

$$\frac{q}{A} = 33 (1260 - T) = h_o (T - 530) \\ + \sigma \left[ \left( \frac{T}{100} \right)^4 - 53^4 \right]$$

$$h_o = \frac{k}{D} B \frac{le}{Re} l^{1/3} \quad \left\{ \begin{array}{l} B - \text{functions} \\ n - \text{of Re} \end{array} \right\}$$

Assume  $T \approx 650 \text{ F} = 1110 \text{ K} \quad T_p = 360 \text{ F}$ 

$$Re = \frac{\left( \frac{81625}{12} \right) (65) (3.281)}{0.348 \times 10^{-3}} = 4.40 \times 10^4$$

$$\text{TABLE 20.3} \quad B = 0.021 \quad n = 0.805$$

$$@ \text{THIS TEMP} \quad h_o = 4.31$$

$$LHS = 4950 \quad RHS = 4923 \quad \left\{ \begin{array}{l} \text{Pretty} \\ \text{good} \end{array} \right\}$$

$$q \approx 4950 \frac{Btu}{hr ft^2} \left( \frac{81625}{12} \right) (\pi) (20)$$

$$= \underline{224000 \text{ BTU/hr}}$$

$$= \underline{65.5 \text{ kW}}$$

20.22 INSULATION ON OUTSIDE  $\text{by}$ 

NATURAL CONV. ON SURFACE

$$R_{\text{INSULATION}} = \frac{ln r_o / r_i}{2 \pi k} = 1.401 \text{ m}$$

per ft

$$\sum R = \frac{1}{A_i h_i} + 1.401 + \frac{1}{A_o h_o} \\ = \frac{1}{33 \pi \left( \frac{81625}{12} \right)} + 1.401 + \frac{1}{h_o \pi \frac{141625}{12}} \\ = 1.414 + 0.261/h_o$$

$$\frac{q}{L} = \frac{\Delta T}{\sum R} = \frac{730}{1.414 + 0.261/h_o} = \frac{800 - T}{1.414}$$

$$\text{WITH} \quad h_o = \frac{k}{D} \left[ 0.6 + \frac{0.387 P_e^{1/6}}{\left\{ 1 + \left( \frac{0.559}{P_r} \right)^{9/16} \right\}^{8/27}} \right]$$

## 20.22 CONTINUED

TRIAL  $\frac{1}{2}$  ERROR:  $T \approx 190\text{ F}$ 

$$\dot{q} = \frac{800 - 190}{1.414} \frac{\text{Btu}}{\text{HRFT}} (20 \text{ ft})$$

$$= 8630 \frac{\text{Btu}}{\text{HR}} = 253 \text{ kW}$$

20.23  $\dot{q} = \Delta T / R$

$$= \frac{800 - T_i}{1/\pi D_i (33)} = \frac{2\pi k}{\ln D_o/D_i} (T_i - 250) = \frac{800 - 250}{\Sigma R}$$

$$R_{\text{sum}} = \frac{1}{33(\pi)(8,625)} = 0.0134$$

$$R_{\text{insul}} = \frac{\ln D_o / 8,625}{2\pi(0.0134)} = 2.65 \ln D_o / 8,625$$

EQUATIONS TO BE SOLVED ARE:

$$\dot{q} = \frac{800 - T}{0.0134} = \frac{T - 250}{2.65 \ln D_o / 8,625} = \frac{550}{\Sigma R}$$

TRIAL  $\frac{1}{2}$  ERROR:  $T \approx 750\text{ F}$ 

$$D_o \approx 9.08 \text{ in.}$$

INSULATION THICKNESS

$$= \frac{9.08 - 8.625}{2} = 0.228 \text{ in.}$$

## 20.24

$$\dot{q} = \frac{800 - T_1}{R_{\text{ins}}} = \frac{T_1 - T_2}{\ln D_o / D_i} = \pi D_o h_o (T_2 - T_1) \left( \frac{1}{0.0134} \right)$$

$$\frac{800 - T_1}{0.0134} = (T_1 - T_2) \frac{0.377}{\ln D_o / D_i} = 3.70 D_o h_o (T_2 - T_1)$$

$$= \frac{730}{0.0134 + 2.65 \ln D_o / 8,625 + \frac{0.27}{D_o h_o}}$$

TRIAL  $\frac{1}{2}$  ERROR PROBLEM (LENGTHY)

## 20.24 CONTINUED -

ANSWER - APPROXIMATELY

$$T_1 = 793.8 \text{ F} \quad T_2 = 178.4 \text{ F}$$

$$\dot{q} = 465 \frac{\text{Btu}}{\text{HRFT}} (20 \text{ ft}) = 9290 \frac{\text{Btu}}{\text{HR}} = 2.72 \text{ kW}$$

$$\frac{800 - 793.8}{0.0134} = \frac{(793.8 - 178.4)}{0.377} \ln D_o / D_i$$

$$\ln D_o / D_i = 0.50 \quad \frac{D_o}{D_i} = 1.651$$

$$D_o = 1.651 (8.625) = 14.24 \text{ in.}$$

$$\text{THICKNESS} = \frac{14.24 - 8.625}{2}$$

$$= 2.81 \text{ INCHES}$$

## 20.25 FOR NATURAL CONVECTION CASE

PLANE UPWARD-FACING HOT SURFACE

$$Nu_L = 0.14 Ra_L^{1/3} \text{ IF } 1 \times 10^7 < Ra_L < 10^{10}$$

ASSUME TOP SURFACE IS SQUARE

$$\sim A = L^2$$

$$\dot{q} = h A \Delta T = h L^2 \Delta T$$

$$= k \frac{L}{L} Nu_L L^2 \Delta T$$

$$= k \left[ 0.14 Ra_L^{1/3} \right] L \Delta T$$

$$Ra_L = \frac{bg}{v^2} L^3 \Delta T \cdot Pr$$

$$\dot{q} = k \left[ 0.14 \left( \frac{bg}{v^2} \Delta T \right)^{1/3} Pr^{1/3} L^2 \right]$$

$$@ T_s = 45^\circ C \quad T_p = 20^\circ C \quad T_f = 32.5^\circ C$$

$$k = 0.02663 \frac{W}{m \cdot K} \quad \frac{bg}{v^2} = 1.244 \times 10^{18} \frac{N^2}{m^2 K^4}$$

## 20.25 CONTINUED

$$Ra = (1.244 \times 10^8)(25)(0.707) L^3$$

$$= 2.199 \times 10^9 L^3$$

$$40 W = 0.0243 \left[ 0.14 \left( 1.244 \times 10^8 \right)^{1/3} \left( 0.707 \right)^{1/3} L^2 (25) \right]$$

$$L^2 = 0.965 \quad L \approx 0.982 \text{ m}$$

$$Ra = 2.08 \times 10^9 \sim \infty$$

Now - for SAME HT LOSS  $\frac{1}{L} = 0.982 \text{ m}$

$$q = hA\Delta T = \frac{k}{L} Nu_c A \Delta T \quad \begin{cases} \text{forced} \\ \text{conv.} \end{cases}$$

ASSUME  $T_{\text{surf}} \approx 20^\circ \text{C}$

$$k = 2.569 \times 10^{-2}$$

$$\rho = 1.506 \times 10^5$$

$$Re = \frac{DV}{\nu} = \frac{0.965(20)}{1.506 \times 10^{-5}} = 1.28 \times 10^6$$

TRANSITION REGIME -

Assume LAMINAR  $b_i L$ ,

$$Nu = 0.664 Re_L^{1/2} Pr^{1/3}$$

$$40W = \frac{0.02569}{0.965} \left( 0.965 \right)^2 \left( 0.664 \right) \left( 1.28 \times 10^6 \right)^{1/2} \times (0.71)^{1/3} \Delta T$$

$$\Delta T = 2.4 \text{ F} \quad T_{\text{surf}} \approx 22.4 \text{ C}$$

$$20.26 \quad q = \frac{\Delta T}{\Sigma R} = \frac{319 - 301}{\Sigma R} \text{ K}$$

$$f_0 = \frac{1}{h_0 \pi D_L} = \frac{1}{(6800) \pi (0.0165)} = 2.464 \times 10^{-3}$$

$$R_i = \frac{1}{h_i \pi D_L} = \frac{1}{(200) \pi (0.0165)} = 3.71 \times 10^{-3}$$

$$R_{cu} = \frac{f_{cu} D_L}{2 \pi k_L} = \frac{f_{cu} 1.9 \cdot 1.65}{2 \pi (385)} = 5.83 \times 10^{-5}$$

## 20.26 CONTINUED

$$\Sigma R = 6.232 \times 10^{-3}$$

$$\dot{m} = \frac{18}{(\Sigma R) h_f g} = \frac{18}{6.232 \times 10^{-3} (2390)} = 1.21 \text{ kg/s}$$

$$20.27 \quad \dot{m} \text{ for TUBE} = 0.49 \text{ kg/s}$$

$$Re = \frac{0.49(4)}{\pi(0.0209)(79 \times 10^{-3})} = 3780$$

USE ANALOGY OR ASSUME TURBULENT

$$\ln \frac{T_L - T_S}{T_0 - T_S} = -4 \frac{L}{D} St \quad St = \frac{f}{2} \Pr^{-2/3}$$

$$\text{Assume } T_{\text{exit}} = 314 \text{ K} \quad T_{\text{avg}} = 307 \text{ K}$$

$$\Pr \approx 1.21 \quad St = \frac{0.01}{2} (121)^{-2/3} = 2.04 \times 10^{-4}$$

$$f \approx 0.01 \quad -2.04 \times 10^{-4} (4)(5) / 0.0209$$

$$T_L = 372 - 72 \approx$$

$$= 313 \text{ K} \sim \text{CHECK}$$

$$q = 1.47 (2000)(13) = 38.2 \text{ kW}$$

$$20.28 \quad \text{Assume } T_L = 235 \text{ F} \quad T_{\text{avg}} = 148 \text{ F}$$

$$Re = \frac{DV}{\nu} = \frac{(0.87/12)(40)}{0.209 \times 10^{-3}} = 1.39 \times 10^4$$

{TURBULENT}

$$St = 0.023 f_{eq}^{-0.8} \Pr^{-0.2} = 4.33 \times 10^{-3}$$

$$\frac{T_L - T_S}{T_0 - T_S} = e^{-4 \frac{L}{D} St}$$

$$T_L = 240 - 180(0.0083)$$

$$= 129 \text{ F} \quad \begin{cases} \text{CLOSE} \\ \text{ENOUGH} \end{cases}$$

$$20.29 \quad f = h\Delta T = 180 \text{ h}$$

a) flow parallel to tube

$$Re = \frac{Lr}{D} = \frac{L(40)}{0.201 \times 10^{-3}} = 1.91 \times 10^5 L$$

If  $x \leq 1.5$  B.L. is laminar

If  $x = 10$  B.L. is in transition

If laminar over total length

$$h = \frac{k}{L} (0.027) Re^{1/2} Pr^{1/3}$$

$$= \frac{0.167}{10} (0.027) (1.91 \times 10^5)^{1/2} (0.72)^{1/3}$$

$$= 13.74 \text{ Btu/HR FT}^2 F$$

$$q_f = 13.74 (180) = 2470 \text{ Btu/HR FT}^2$$

b) crossflow GASE

$$Re = \frac{Dv}{\nu} = 1.59 \times 10^4$$

$$h = \frac{k}{D} \left[ 0.193 (1.59 \times 10^4)^{0.618} \right]^{1/3}$$

$$= 137 \text{ Btu/HR FT}^2 F$$

$$q_f = 137 (180) = 24,600 \text{ Btu/HR FT}^2$$

20.30 WATER:

a) parallel to tube  $T_f = 150 F$

$$Re = \frac{10(40)}{0.40 \times 10^{-5}} = 8,44 \times 10^7 \quad \{ \text{TURBULENT} \}$$

$$h = \frac{k}{L} (0.027) Re^{4/5} Pr^{1/3}$$

$$= \frac{0.383}{10} (0.027) (8,44 \times 10^7)^{4/5} (2.72)^{1/3}$$

$$= 4220 \text{ Btu/HR FT}^2 F$$

$$q_f = 4220 (180) = 7.6 \times 10^5 \text{ Btu/HR FT}^2$$

20.30 CONTINUED -

b) Crossflow

$$Re = 7,03 \times 10^5$$

$$h = \frac{k}{D} (0.027) (7,03 \times 10^5)^{0.805} (2.72)^{1/3}$$

{ TABLE 20.3 VALUES @ HIGHEST RE }

$$h = 8820 \text{ Btu/HR FT}^2 F$$

$$q_f = 8820 (180) = 1.59 \times 10^6 \text{ Btu/HR FT}^2$$

20.31 a) parallel

$$Re = \frac{10(40)}{5.45 \times 10^{-5}} = 7.34 \times 10^6 \quad \{ \text{TURB} \}$$

$$h = \frac{k}{D} (0.027) (7.34 \times 10^6)^{4/5} (80.5)^{1/3}$$

$$= 312 \text{ Btu/HR FT}^2 F$$

$$q_f = 312 (180) = 56,100 \text{ Btu/HR FT}^2$$

b) crossflow  $Re = 61,000$

$$h = \frac{k}{D} (0.027) (61,000)^{0.805} (80.5)^{1/3}$$

$$= 642 \text{ Btu/HR FT}^2 F$$

$$q_f = 642 (180) = 115,600 \text{ Btu/HR FT}^2$$

20.32  $Re = \frac{GP}{\mu} \quad T_f = 186 F$

$$Re = \frac{(0.385/12)(20)}{0.379 \times 10^{-5}} = 169,000$$

From FIGURE 20.13  $J \approx 10^{-3}$

{ EXTRAPOLATION }

20.32 CONTINUED

$$h = 10^{-3} \Pr^{-\frac{1}{3}} \left( \frac{\mu_w}{\mu_{ref}} \right)^{0.14}$$

$$= 10^{-3} (2.14) \left( \frac{0.273}{0.492} \right)^{0.14} = 0.631 \times 10^{-3}$$

$$h = (0.631 \times 10^{-3}) (1.01) (60.4) (3600)$$

$$= \underline{135 \text{ BTU/HR.FT}^2 \text{ F}}$$

20.33  $q = hA\Delta T$

$$= 135 (48) (\pi) \left( \frac{0.387}{12} \right) (5)$$

$$= \underline{3280 \text{ BTU/HR}}$$

20.34  $T_{surr} = 380 \text{ K}$

$$T_\infty = 295 \text{ K} \quad T_f = 337.5 \text{ K}$$

$$\rho = 980.6 \text{ kg/m}^3 \quad \nu = 0.453 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 0.661 \text{ W/m.K} \quad \Pr = 1.90$$

a) HORIZONTAL NATURAL CONV.

$$h = \frac{k}{D} \left\{ 0.600 + \frac{0.387 Ra_w^{1/6}}{\left[ 1 + (0.559 \Pr)^{1/10} \right]^{8/27}} \right\}^2$$

$$Ra_w = (27.54 \times 10^9) (0.012) (85) (2.9)^3$$

$$= 1.358 \times 10^7$$

$$h = 1898 \text{ W/m}^2 \cdot \text{K}$$

$$q = hA\Delta T = (1898) \pi (0.012) (0.075) (85)$$

$$= \underline{479 \text{ W}}$$

20.34 CONTINUED -

b) VERTICAL NATURAL CONV.

$$h = \frac{k}{L} \left\{ 0.825 + \frac{0.387 Ra_w^{1/6}}{\left[ 1 + (0.492 \Pr)^{1/10} \right]^{8/27}} \right\}^2$$

$$Ra_w = (27.54 \times 10^9) (0.012)^3 (85) (2.9)$$

$$= 1.864 \times 10^9$$

$$h = 1351 \text{ W/m}^2 \cdot \text{K}$$

$$q = (1351) \pi (0.012) (0.075) (85)$$

$$= \underline{341 \text{ W}}$$

c) CROSSFLOW

$$Re = \frac{DV}{D} = \frac{(0.012)(1.5)}{0.453 \times 10^{-6}} = 41700$$

$$h = \frac{k}{D} \left\{ 0.3 + \frac{0.62 Re^{1/2} \Pr^{1/3}}{\left[ 1 + \left( \frac{0.4}{\Pr} \right)^{2/3} \right]^{1/4} \left[ 1 + \left( \frac{Re}{282000} \right)^{4/5} \right]} \right\}$$

$$= 11030 \text{ W/m}^2 \cdot \text{K}$$

$$q = (11030) \pi (0.012) (0.075) (85) = \underline{2.78 \text{ kW}}$$

$$20.35 \quad Re = \frac{0.15(150)}{7.98 \times 10^{-5}} = 282,000$$

$$h = \frac{k}{D} Nu = \frac{0.0566}{0.15} (400)$$

from Fig 20.11

$$= 151 \text{ W/m}^2 \cdot \text{K}$$

$$q = 151 (1030) = \underline{155 \text{ kW/m}^2}$$

$$20.36 \quad q = 140 \text{ W/m} \quad \left\{ \begin{array}{l} \text{from Prob} \\ \text{Pr}_A = 2480 \text{ W/m}^2 \end{array} \right\} \quad 20.18$$

$$k = \frac{0.018(9)}{1.569 \times 10^{-3}} = 0.300 \quad \left\{ T_f \approx 300 \text{ K} \right\}$$

$$Pr = 0.708 \quad k = 0.0262$$

$$h = \frac{k}{D} \left[ 0.193 \left( 10300 \right)^{0.18} \left( 0.708 \right)^{1/3} \right]$$

$$= 75.6 \text{ W/m}^2 \cdot \text{K}$$

$$\Delta T = 2480 / 75.6 = 32.8$$

$\left\{ T_c = 323, T_f = 326, \text{ close enough} \right\}$

$$h = 75.6 \text{ W/m}^2 \cdot \text{K}$$

$$\Delta T = 32.8 \quad T_{\text{surf}} = 323 \text{ K}$$

INSUL. RESISTANCE =  $0.842 \left\{ \text{Prob 20.13} \right\}$

$$\Delta T = 140 / 0.842 = 166 \text{ K}$$

$$T_{\text{intef.}} = 323 + 166 = 489 \text{ K}$$

20.37 SPHERES:  $D = 0.075 \text{ m}$

$$T_{b0} = 25^\circ \text{C} \quad T_s = 145^\circ \text{C}$$

$$\lambda = 1.59 \times 10^{-5} \quad \mu_p = 1837 \times 10^{-5}$$

$$k = 0.0261 \quad \mu_s = 2429 \times 10^{-5}$$

$$Pr = 0.708$$

$$q = h \Delta T = h (\pi) (0.075)^2 (120)$$

$$Re = \frac{Dv}{\nu} = \frac{(0.075)(0.5)}{1.551 \times 10^{-5}} = 2418$$

$$h = \frac{k}{D} \left[ 2 + \left( 0.4 \text{Re}^{1/2} + 0.001 \text{Re}^{2/3} \right) \text{Pr} \left( \frac{\mu_p}{\mu_s} \right)^{0.4} \right]$$

$$= \frac{0.0261}{0.075} \left\{ 2 + \left[ 0.4(2418)^{1/2} + 0.001(2418)^{2/3} \right] \times (0.708)^{0.4} \left( \frac{1.837}{2429} \right)^{1/4} \right\}$$

20.37 CONTINUED -

$$= 8.99 \text{ W/m}^2 \cdot \text{K}$$

$$q = 8.99 (\pi) (0.075)^2 (120)$$

$$= \underline{\underline{19.07 \text{ W}}}$$

$$20.38 \quad G = 8 \text{ vs} = 3.64 \text{ lbm/s} \cdot \text{ft}^2$$

$$Re = \frac{GD}{\mu} = \frac{3.64 (0.1622)}{0.29 \times 10^{-3}} = 650 \quad (\text{LAMINAR})$$

use Sieder-Tate fcn. ASSUME  $T_{b,\text{AVG}} = 150^\circ \text{F}$

$$Nu = 1.86 \left( Re Pr \frac{D}{L} \right)^{1/3} \left( \frac{\mu_p}{\mu_s} \right)^{0.14}$$

$$h = \frac{k}{D} Nu = \frac{0.383 (1.86)}{0.1622 / 12} \left[ (650) (272) \frac{0.1622}{12 (5)} \right]^{1/3} \times (0.708)^{0.14}$$

$$= 32.9 \text{ Btu/hr ft}^2 \text{ F}$$

$$St = \frac{Nu}{Re Pr} = 0.00252$$

$$\frac{T_c - T_s}{T_c - T_b} = \frac{-4 \frac{L}{D} St}{C} = \frac{-0.912}{C} = 0.378$$

$$T = 80 + 0.378(100) = 117.8 \text{ F}$$

$$T_{b,\text{AVG}} = \frac{117.8 + 180}{2} = 149 \text{ F} - \text{OK}$$

$$q = \dot{m} c_p \Delta T = 3.64 (0.0021) (149.7 - 86.7)$$

↑  
from Steam Tables

$$= \underline{\underline{1710 \text{ Btu/lb}}}$$

$$20.39 \quad G = 60.6 (35) = 2120 \text{ lbm/s} \cdot \text{ft}^2$$

$$Re = 307,000 \quad \left\{ \text{TURBOVENT} \right\}$$

$T_f \approx 180^\circ \text{F}$  - use Colburn fcn:

$$St = 0.023 (307000)^{0.2} (344)^{-2/3}$$

$$= 0.000807$$

## 20.39 (CONTINUED)

$$T = 80 + 180 e^{-St(4)(5)/0.622}$$

$$= 153.3 \text{ F} \quad \text{- FIRST GUESS}$$

$$T_f = \left[ \frac{80 + 153.3}{2} + 180 \right] / 2 \approx 149 \text{ F}$$

AT THIS TEMP:  $Re = 443,000$   $Fr = 4.51$

$$St = 0.000625 \quad T \approx 155 \text{ F}$$

20.40  $GA = 10,000 \text{ lbm/hr}$

$$G = \frac{10,000}{0.276} = 37400 \text{ lbm/HR PT^2}$$

$$Re = \frac{37400 \left( \frac{1.001}{12} \right)}{1.63 \times 10^{-5} (3600)} = 3.72 \times 10^5 \quad \{ \text{TURBULENT} \}$$

USE DITRUS-BOEHLER EQUATION:

$$h = \frac{k}{D} (0.023) Re^{0.8} Fr^{0.3}$$

$$= \frac{0.0321}{(1.001/12)} (0.023) (3.72 \times 10^5)^{0.8} (0.912)^{0.3}$$

$$= 35.2 \text{ BTU/HR PT^2 F}$$

20.41  $m_{\text{TOTAL}}^0 = 1.47 \text{ kg/s} \approx 0.245 \text{ kg/s}$

PER TUBE

$$Re = \frac{DvS}{\mu} = \frac{\dot{m}4}{\pi D \mu} = \frac{0.245(4)}{\pi(0.0209)(7.9 \times 10^{-3})}$$

$$= 1890 \quad \{ \text{LAMINAR} \}$$

$$\ln \frac{T_L - T_S}{T_0 - T_S} = - \frac{4L}{D} St$$

$$St = 1.86 \left( \frac{D}{L} \right)^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{-2/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

## 20.41 (CONTINUED)

ASSUME  $T_{\text{EXT}} = 305 \text{ K}$   $T_{\text{avg}} \approx 302 \text{ K}$

$$St = 1.86 \left( \frac{0.029}{2.5} \right)^{1/3} \left( \frac{1890 \times 12}{L} \right)^{1/3} \left( \frac{0.0414}{3.72 \times 10^{-5}} \right)^{0.14}$$

$$= 1.414 \times 10^{-3}$$

$$T_L = 372 - 72 e^{- (1.414 \times 10^{-3})(4)(25) / 0.0209}$$

$$= 304.7 \quad \sim \text{GOOD AGREEMENT}$$

$$q = \dot{m} c_p \Delta T = 1.47 (1.84 \times 10^3)(4.7)$$

$$= 12740 \text{ J/s} = \underline{12.74 \text{ kW}}$$

20.42  $\frac{T - T_S}{T_0 - T_S} = \frac{-4 \frac{L}{D} St}{1}$

$$T_0 = 160 \text{ C} \quad \text{ASSUME } T \approx 140 \text{ C}$$

$$T_S = 100 \text{ C} \quad T_{\text{bavg}} = 150 \text{ C}$$

$$Re = \frac{4m}{\pi D \mu} = \frac{4(136)}{(3600)(0.015)\pi \mu_w}$$

$$@ 423 \text{ K} \quad \mu = (0.0008 \times 10^{-3})(812)$$

$$Re = 1160 \quad \sim \text{LAMINAR}$$

$$St = \frac{Nu}{Re Pr} = \frac{1.86}{(Re Pr)^{2/3}} \left( \frac{D}{L} \right)^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

$$= \frac{1.86}{[(116)(600)]^{2/3}} \left( \frac{0.015}{15} \right)^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

$$= 0.00053$$

$$e^{-4L/5} St = 0.654$$

$$T = 100 + 0.654(60) = 139.3 \text{ C}$$

GOOOD!

$$T_{\text{ext}} \approx 139 \text{ C}$$

20.43

FOR THIS CASE

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = e^{-4 \frac{L}{D} \frac{U}{h_i c_p}}$$

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = \frac{30}{40} = 0.75$$

$$0.2877 = 4 \frac{L}{D} \frac{U}{h_i c_p}$$

$$U = \frac{1}{\frac{1}{h_i A_i} + \frac{1}{h_o A_o}} = \frac{\pi D L}{\frac{1}{h_i} + \frac{1}{h_o}}$$

$$h_o = 500 \text{ W/m}^2 \cdot \text{K} \quad h_i = \frac{k}{D} Nuss$$

$$Re_D = \frac{4V}{\pi D \nu} = \frac{4(0.006)}{\pi(0.0025)(7 \times 10^{-7})(3600)} = 123 \quad \left. \right\} \text{LAMINAR}$$

$$Pr = \frac{\mu c_p}{k} = \frac{(1000)(7 \times 10^{-7})(4000)}{0.5} = 5.6$$

$$h_i = \frac{k}{D} (1.86) \left( \frac{Re}{L} \frac{Pr}{L} \right)^{1/3} \left( \frac{k_e}{k_w} \right)^{0.14}$$

$$= \frac{0.5}{0.0025} (1.86) \left[ (123)(5.6) \left( \frac{0.0025}{L} \right) \right]^{1/3}$$

$$= 956 \text{ L}^{-1/3}$$

$$U = \frac{\pi D L}{\frac{1}{h_i} + \frac{L}{h_o}} = \frac{L}{0.255 + 0.133 L^{1/3}}$$

PUTTING EVERYTHING TOGETHER:

$$0.2877 = \frac{0.001178 L^2}{0.255 + 0.133 L^{1/3}}$$

By TRIAL &amp; ERROR:

$$\underline{L \approx 11.7 \text{ m}}$$

20.44

 $T_0 = 320 \text{ K}$ 

$$\frac{q}{\dot{m}} \frac{\Delta T}{\Sigma R} = \frac{\Delta T}{\frac{1}{h_i A_i} + \frac{\ln r_o/r_i}{2\pi k}}$$

{ASSUME  $T_0$  IS OUTSIDE TUBE TEMP}

$$\frac{1}{h_i A_i} = \frac{1}{(700)(\pi)(0.01656)} = 1.13 \times 10^{-2}$$

$$\frac{\ln r_o/r_i}{2\pi k} = \frac{\ln 1.905 / 1.156}{2\pi(110)} = 2.03 \times 10^{-4}$$

$$\Sigma R = 1.15 \times 10^{-2}$$

$$\frac{q}{\dot{m}} = \frac{320 - 290}{1.15 \times 10^{-2}} = 2,606 \text{ kW/m}$$

$$\dot{m}_{\text{CONO}} = \frac{2606 \text{ kW/m}}{1393 \text{ kJ/kg}} = 1.09 \text{ g/s}$$

$$= \underline{3920 \text{ g/hr}} = \underline{3.92 \text{ kg/hr}}$$

20.45 OUTSIDE OF TUBE INSULATED

∴ ALL HEAT GENERATED GOES INTO  $H_2O$ 

$$q = \dot{m} c_p \Delta T = (0.12 \text{ kg/s}) (4171 \text{ J/kg.K}) (70 - 25) \text{ K}$$

$$= 22.56 \text{ kW}$$

$$\{ q = \dot{q} V = 1.5 \times 10^6 \text{ W/m}^2 \left[ \frac{\pi}{4} (0.045^2 - 0.025^2) \right] L$$

$$= 1,649 \text{ L.kW}$$

$$L = \underline{13.68 \text{ m}}$$

$$\text{CONSTANT AT } f_{\text{fun}} = \frac{22560 \text{ W}}{\pi (0.025)(3.14) \text{ m}^2}$$

$$= 21000 \text{ W/m}^3$$

$$h = \frac{21000}{110 - 70} = \underline{525 \text{ W/m}^2 \cdot \text{K}}$$

$$20.46 \quad T - T_s = \frac{-4L}{D} St$$

$$\frac{T - T_s}{T_0 - T_s} = e^{-\frac{4L}{D} St}$$

$$St = \frac{D}{4L} \ln \frac{60 - 120}{100 - 120} = \frac{0.0717}{L}$$

a)  $V = 15 \text{ ft/s}$

$$Re = \left(0.15/\nu\right)(25) = 8640$$

$$\nu = 0.181 \times 10^{-3}$$

USE D'ARSONVAN ANALOGY:  $St = \frac{C_f}{2} Fr^{-\frac{1}{3}}$

$$St = \frac{0.0018}{2} (1.257) = 0.0049$$

$$L = \frac{0.0717}{0.0049} = \underline{\underline{3.5 \text{ ft}}}$$

b)  $V = 15 \text{ ft/s}$   $Re = 5190$

$$St = 0.005660 \quad L = \underline{\underline{3.03 \text{ ft}}}$$

$$20.47 \quad D_{\text{WIV}} = \frac{4(2)(4)}{(2)(6)} = \frac{16}{6} \text{ ft}$$

$$Re = \frac{(16/6)(6)}{1.28 \times 10^{-5}} = 1.25 \times 10^6$$

$$St = 0.023 Re^{-0.2} Fr^{-\frac{1}{3}}$$

$$= 0.023 (1.25 \times 10^6)^{-0.2} (0.703)^{-\frac{1}{3}}$$

$$= 0.00176$$

FOR THE SHORT DISTANCE INVOLVED:

$$h = St (8\nu C_D) = (0.00176)(6)(0.24)$$

$$= 2.53 \times 10^{-3} \frac{\text{Btu}}{\text{s ft}^2 \text{ F}}$$

$$q = h A \Delta T = (2.53 \times 10^{-3})(12)(40)$$

$$= 1.215 \frac{\text{Btu}}{\text{s}} \text{ per ft} = 4370 \frac{\text{Btu}}{\text{hr}} \text{ per ft}$$

$$\Delta T = \frac{q}{m c_p} = \frac{1.215}{(0.24)(6)(8)} = \underline{\underline{0.105 \text{ F per ft}}}$$

20.48

$$T_{in} = 290 \text{ K} \quad \text{ASSUME } T_{out} = 350$$

$$T_{surf} = 370 \text{ K} \quad T_{base} = 320 \text{ K}$$

$$N = 0.596 \times 10^{-6} \text{ m}^2 \text{ s} \quad Pr = 3.87$$

$$\frac{T - T_s}{T_0 - T_s} = e^{-\frac{4L}{D} St}$$

$$Re = \frac{(0.0254)(1.5)}{0.596 \times 10^{-6}} = 63900$$

USE DITTUS-BOECKER EQUATION:

$$St = 0.023 Re^{-0.2} Fr^{-0.6}$$

$$= 0.023 (63900)^{-0.2} (3.87)^{0.6}$$

$$= 0.00112$$

$$e^{-\frac{4L}{D} (St)} = 0.414$$

$$T_{out} = 370 - (0.414)(80)$$

$$\cong 337$$

SECOND TRY -  $T_{out} = 337$

$$T_{base} = 313.5$$

$$N = 0.663 \times 10^{-6} \quad Pr = 4.33$$

$$Re = 57400 \quad St = 0.00107$$

$$e^{-\frac{4L}{D} (St)} = 0.432$$

$$T_{out} = 370 - (0.432)(80) = \underline{\underline{335 \text{ K}}}$$

$$q = \dot{m} q \Delta T$$

$$= (972) \frac{\pi}{4} (0.0254)^2 (1.5) (4175) (45)$$

$$= \underline{\underline{141.6 \text{ kW}}}$$

20.49 Rect. Duct  $0.61\text{m} \times 1.22\text{m}$

$$De_{\text{Duct}} = \frac{4(0.61)(1.22)}{2(0.61+1.22)} = 0.813 \text{ m}$$

$$q = hA\Delta T$$

use Dittus Boelter Eqn.

$$Re = \frac{DG}{\mu} = \frac{(0.813)(29.4)}{1948 \times 10^{-5}} = 1,227 \times 10^6$$

$$Pr = 0.703$$

$$h = \frac{k}{D} (0.023) Re^{0.8} Pr^{0.3}$$

$$= \frac{0.0279}{0.813} (0.023) (1,227 \times 10^6)^{0.8} (0.703)^{0.3}$$

$$= 52.8 \text{ W/m}^2 \cdot \text{K}$$

$$q = hA\Delta T = 52.8 (2)(0.61 \times 1.22)(22)$$

$$= \underline{\underline{4250 \text{ W/m}}}$$

$$q = \dot{m}c_p \Delta T = 6A c_p \Delta T$$

$$\Delta T = \frac{4250}{29.4 (0.61)(1.22)(1007)}$$

$$= \underline{\underline{0.193 \text{ K per m}}}$$

20.50 Duct:  $7.5\text{cm} \times 15\text{cm}$

$$\frac{T-T_s}{T_o-T_s} = \frac{4L}{D} St \quad L = 6 \text{ m} \quad De_{\text{Duct}} = 10 \text{ cm}$$

$$\frac{T-T_s}{T_o-T_s} = \frac{30-70}{10-70} = 0.667$$

$$4\frac{L}{D} St = 0.405$$

$$St = \frac{0.405(0.10)}{4(6)} = 0.00169$$

20.50 CONTINUED -

use Corrsen Eq.

$$St = 0.023 Re^{-0.2} Pr^{-0.3}$$

$$Re = \frac{DU}{\nu} = \frac{(0.10)(U)}{1.739 \times 10^{-5}} = 5750 \text{ V}$$

$$Pr = 0.704$$

$$0.00169 = 0.023 (5750)^{-0.2} (0.704)^{-0.3}$$

$$\underline{\underline{U = 261 \text{ m/s}}}$$

UNREALISTIC BUT MATHEMATICALLY CORRECT

20.51 FLS 20.12 & 20.13 APPLY STRICTLY FOR LIQUIDS FLOWING THROUGH TUBE BANKS BUT THEY WILL BE USED FOR LACK OF OTHER RESOURCES.

Using FLS 20.13 ~

$$Re = \frac{D_U U}{\nu} = \frac{(0.018)(60)}{1.505 \times 10^{-5}} = 7.17 \times 10^3$$

AT THIS Re:  $j \approx 0.01$

$$h = 0.01 c_p G Pr^{-2/3} \left( \frac{h_{\text{N}}}{h_{\text{lb}}} \right)^{-0.14}$$

$$= 0.01 (10055) (1.205) (6) (0.707)^{-2/3} \left( \frac{1.117}{1.813} \right)^{-0.14}$$

$$= 89.6 \text{ W/m}^2 \cdot \text{K}$$

FOR A BANK OF 10 TUBES, 10 ROWS DEEP

$$A = 100(\pi)(0.018)(1.8) = 10.18 \text{ m}^2$$

$$= (89.6)(10.18)(65) = \underline{\underline{593 \text{ kW}}}$$

20.52 FOR SAME CONDITIONS AS PROB. 20.51 EXCEPT FOR STAGGERED TUBE ARRANGEMENT -

$$Re = 7.17 \times 10^3$$

$\frac{1}{2}$  BOTH ARRANGEMENTS GIVE SAME VALUE for  $j$

$$\therefore \underline{j = 59.3 \text{ kW}}$$

20.53 USING FIG 20.12

SAME CAVEATS AS FOR PROB 20.51 -

$$D_{\text{down}} = \frac{4}{\pi(0.013)} \left[ (0.032)(0.032) - \frac{\pi}{4} (0.013)^2 \right]$$

$$= 0.0813 \text{ m}$$

$$Re = \frac{(0.0813)(1.25)}{1.569 \times 10^{-5}} = 6.95 \times 10^3$$

- OUT OF LAMINAR RANGE -

MUST USE FIG 20.13

$$Re = \frac{0.013(1.25)}{1.569 \times 10^{-5}} = 1.04 \times 10^3$$

FOR IN-LINE CONFIGURATION -

$$j \approx 0.017$$

$$h = 0.017(1.0063)(1.17)(1.25)$$

$$\times (0.708)^{-2/3} \left( \frac{2.143}{1.813} \right)^{-0.14}$$

$$= 30.95 \text{ W/m}^2 \cdot \text{K}$$

20.53 (CONTINUED -

$$A = 64(\pi)(0.013)(18) = 4.70 \text{ m}^2$$

$$q_f = h A \Delta T$$

$$= 30.95(4.70)(63)$$

$$= \underline{9.164 \text{ kW}}$$

20.54 SAME CONDITIONS AS PROB 20.53 EXCEPT TUBES ARE IN STAGGERED CONFIGURATION.

ALL CALCULATIONS THE SAME AS IN PROB 20.54 EXCEPT  $j = 0.035$

$$\text{GIVEN } h = 63.7 \text{ W/m}^2 \cdot \text{K}$$

$$\frac{1}{2} q_f = 63.7(4.70)(63)$$

$$= \underline{18.87 \text{ kW}}$$

## CHAPTER 21

21.1 PLATE IS ASSUMED TO BE COPPER

FOR  $H_2O @ 323 K$   $L = 0.565 \text{ FT}$

$$\frac{q}{A} = 1.26 \times 10^7 (\text{F.PT})^{-1} \quad Pr = 1.81$$

$$\mu_L = 0.702 \frac{\text{lbm}}{\text{HR.FT}} \quad C_L = 1.01 \frac{\text{BTU}}{\text{lbm.F}}$$

$$k = 0.393 \frac{\text{BTU}}{\text{HR.FT}^2} \quad h_{fg} = 970 \frac{\text{BTU}}{\text{lbm}}$$

$$S_L - S_V \approx S_L = 60 \frac{\text{BTU}}{\text{lbm.FT}^3}$$

NATURAL CONVECTION:  $\frac{q}{A} = h \Delta T$

$$\frac{q}{A} = \frac{k}{L} \left[ 0.68 + \frac{0.67 \frac{Pr}{4}}{1 + (0.492)^{9/16}} \right] \Delta T$$

$$= 60 \left[ 0.68 + 89.6 \Delta T^{1/4} \right] \Delta T \quad (1)$$

NUCLEATE BOILING:

$$\frac{C_L \Delta T}{h_{fg} Pr^{1/7}} = C_{sf} \left[ \frac{q/A}{\mu_L h_{fg}} \left( \frac{g_c \sigma}{g [S_L - S_V]} \right)^{1/2} \right]^{1/3}$$

$$\sigma = 3.79 \times 10^3 \frac{\text{lb}_f}{\text{FT}} \quad C_{sf} = 0.013$$

$$\text{LHS: } \frac{C_L \Delta T}{h_{fg} Pr^{1/7}} = 3.80 \times 10^4 \Delta T$$

$$\text{RHS: } C_{sf} \left[ \frac{q}{A} \right]^{1/3} = 0.013 \left[ \frac{q/A}{0.702(970)} \right]^{3/16} \frac{60}{60}^{1/3}$$

$$= 294 \times 10^{-4} \left( \frac{q}{A} \right)^{1/3}$$

$$\frac{q}{A} = 214 \Delta T^3 \quad (2)$$

EQUATING: (1) = (2)

$$2.14 \Delta T^3 = 0.6 \left[ 0.68 + 89.6 \Delta T^{1/4} \right]$$

$$\Delta T \approx 6.3 \text{ F}$$

PART (b): Plot  $q/A$  from (1),  
 $q/A$  from (2),  $\frac{q}{A}$  from sum

$$21.2 \frac{C_L \Delta T}{h_{fg} Pr^{1/7}} = C_{sf} \left[ \frac{q/A}{\mu_L h_{fg}} \left( \frac{g_c \sigma}{g [S_L - S_V]} \right)^{1/2} \right]^{1/3}$$

IN FAMOUS UNITS:

$$C_L = 1.03 \quad Pr = 1.74$$

$$h_{fg} = 970 \quad \mu_L = 0.195 \times 10^{-3}$$

$$T_{SAT} = 212 \quad \Delta \sigma = 60.2$$

$$A = \frac{C_L \Delta T}{h_{fg}} \quad B = \frac{1}{\mu_L h_{fg} \sqrt{g \Delta \sigma}}$$

$$\frac{q}{A} = \left[ \frac{A}{C_{sf} B (2.74)} \right]^{1/3}$$

$$\text{For Ni } \frac{q}{A} \text{, BRASS } C_{sf} = 0.006$$

$$\text{" Cu } \frac{q}{A} \text{, PT } C_{sf} = 0.013$$

$$T_S(K) \quad \Delta T \quad \Delta T(F) \quad A \quad 0 \times 10^3 \quad B$$

390	17	31	0.033	5.04	0.364
420	47	85	0.090	4.67	0.360
450	77	139	0.148	3.79	0.355

T <sub>S</sub>	$\frac{q}{A}$	$h \text{ W/m.K}$	$\frac{q}{A}$	h
390	168	$2 \times 10^5$	165	$0.533 \times 10^5$
420	3516	85 "	346	4.07 "
450	16340	24 "	1610	1610 "

Ni, BRASS

Cu, PT

$$21.3 \frac{C_L \Delta T}{h_{fg}} = 0.0709$$

$$\frac{q}{A} = \left( \frac{0.0709}{0.01235} \right)^3 = 190 \frac{\text{BTU}}{\text{s.ft}^2}$$

$$= 680,000 \frac{\text{BTU}}{\text{HR.FT}^2}$$

$$q = 680,000 (\pi) (1/4)^2 = 178,000 \frac{\text{BTU}}{\text{HR}}$$

$$h = \frac{680,000}{68} = 10,000 \frac{\text{BTU}}{\text{HR.FT}^2}$$

21.4 Boiling A,  $\rho$  @ 1 ATM; BURNOUT  
POINT IS  $\Delta T \approx 100 F$   $T_s = 312 F$   
AS IT LOOKS THE CYLINDER IS IN

Film Boiling  $500 < T_s < 312$

Nucleate  $312 < T_s < 240$

Film Boiling Part:

$$h = 0.62 \left[ k_f^3 g_f (\Delta s) g (h_{fg} + 0.4 C_p \Delta T_s) \right]^{1/4}$$

$$k_f = 0.0145 \frac{Btu}{HRFT}$$

$$g_f = 0.0372 \frac{lbm/ft^3}{}$$

$$S_L = 60.0$$

$$\mu_f = 3.12 \times 10^{-3} \frac{lbm/HRFT}{}$$

$$h_{fg} = 970 \frac{Btu}{lbm}$$

$$C_p = 0.451 \frac{Btu}{lbm}$$

SUBSTITUTING INTO FORMULA!

$$h = 35.9 \frac{Btu}{HRFT^2} \quad \{ \text{AND } \Delta T \approx 194 F \}$$

$$\frac{q}{A} = h(194) = 35.9(194) = 6960 \frac{Btu}{HRFT^2}$$

Nucleate Boiling Part:

$$\frac{C_s \Delta T}{h_{fg}} = C_{sf} \left[ \frac{g/A}{\mu_L h_{fg}} \left( \frac{\sigma}{g \Delta s} \right)^{1/2} \right]^3 Pr^{1/7}$$

$$\Rightarrow \frac{q}{A} = \left( \frac{0.0168 \Delta T}{C_{sf}} \right)^3 = 4.8 \times 10^{-6} \left( \frac{\Delta T}{C_{sf}} \right)^3$$

$$\text{WITH } \Delta T = 64 F \quad C_{sf} = 0.013 \sim \text{Cu} \\ = 0.006 \sim \text{Br, Ni}$$

$$\frac{q}{A} = 5.72 \times 10^5 \sim \text{Cu}$$

$$= 5.82 \times 10^6 \sim \text{Brass, Ni}$$

$$\frac{q}{A} = \frac{g V C_p}{A} \frac{\Delta T}{\Delta t}$$

$$t = \frac{g(V)}{A} C_p \left[ \frac{\int_{312}^{500} \Delta T}{V/A_f} + \frac{\int_{240}^{312} \Delta t}{V/A_{nuc}} \right]$$

21.4 CONT.

$$t = \frac{g D}{4} C_p \left( \frac{188}{V/A_f} + \frac{72}{V/A_{nuc}} \right)$$

$$\frac{g D C_p}{4} = 555 \left( \frac{1}{96} \right) (0.092) = 0.532 \quad \{ \text{Cu}$$

$$= 532 \left( \frac{1}{96} \right) (0.091) = 0.503 \quad \{ \text{Br}$$

$$= 556 \left( \frac{1}{96} \right) (0.111) = 0.643 \quad \{ \text{Ni}$$

$$\text{Copper: } t = 0.532 \left[ \frac{188}{6960} + \frac{72}{5.72 \times 10^5} \right] = 52.5$$

$$\text{Br: } t = 0.503 \left[ \frac{188}{6960} + \frac{72}{5.8 \times 10^5} \right] = 48.95$$

$$\text{Ni: } t = 0.643 \left[ \frac{188}{6960} + \frac{72}{5.8 \times 10^5} \right] = 62.65$$

21.5 USING ENGLISH UNITS:

$$A = \pi D L + 2 \frac{\pi D^2}{4} = \pi (0.02)(0.15)$$

$$+ \frac{\pi}{4} (2)(0.02)^2$$

$$= 0.1082 \frac{ft^2}{}$$

$$\frac{q}{A} = \frac{500 (3.413)}{0.1082} = 15800 \frac{Btu}{HRFT^2}$$

ASSUME NUCLEATE BOILING:

$$\frac{1 \Delta T}{(1.8)^{1.7} (970)} = 0.006 \left[ \frac{15800}{(0.195 \times 10^3)(970)(3600)} \right. \\ \left. \times \left( \frac{379 \times 10^{-3}}{60} \right)^{1/2} \right]^{1/3}$$

$$\Delta T = 9.0 F$$

$$\text{SURFACE TEMP.} = \frac{221 F}{}$$

$$h = \frac{15800}{9} = \frac{1760 \frac{Btu}{HRFT^2}}{}$$

$$21.6 \quad h = 0.62 \left[ k_v^3 g \Delta \bar{g} g (h_{fg} + 0.4 C_p \Delta T) \right]^{1/4}$$

$$= 0.62 \left[ \frac{(0.0153)^3 (0.0341 \times 10^4 / 14.7) (58.9)}{\frac{1}{12} (0.914 \times 10^{-5}) (933)} \right]^{1/4}$$

$$\times 32.2 (3600) (934 + 0.4 \times 0.483 \times 933)$$

$$= 26.9 \text{ BTU/HR FT}^2 \text{ F}$$

$$q = h \Delta T = 26.9 (1200 - 247) = 25,000 \text{ BTU/HR FT}^2$$

$$21.7 \quad \Delta T = 2200 - 240 = 1960 \text{ F } \left\{ \begin{array}{l} \text{FILM} \\ \text{BOILING} \end{array} \right\}$$

$$h = 0.62 \left[ \frac{(0.0153)^3 (0.035) (58.9) (32.2) (3600)}{(0.02/12) (1.53 \times 10^{-5}) (1960)} \right]$$

$$\times (952 + 0.4 \times 0.483 \times 1960) \right]^{1/4}$$

$$= 43.3 \text{ BTU/HR FT}^2 \text{ F}$$

$$q = h A \Delta T = 43.3 (\pi) \left( \frac{0.2}{12} \right) (1) (1960)$$

$$= 444 \text{ BTU/HR FT}^2 \text{ F}$$

$$21.8 \quad 2000 \text{ W} = 6826 \text{ BTU/HR}$$

Per Plate:  $A = 2(0.05)(0.1) = 0.01 \text{ m}^2$

$$= 0.1076 \text{ FT}^2$$

$$\frac{q}{A} = 20,000 \text{ W/m}^2 = 63400 \text{ BTU/HR FT}^2$$

IT APPEARS THAT NUCLEATE BOILING ON ONE PLATE CAN ACHIEVE THIS.

$$T_{SAT} = 242 \text{ F}$$

$$\frac{C_p \Delta T}{h_{fg} Pr^{1/7}} = C_{sf} \left[ \frac{q/A}{\mu_L h_{fg}} \left( \frac{\sigma}{g \Delta S} \right)^{1/2} \right]^{1/3}$$

21.8 CONT.

$$\frac{\Delta T}{945(2.12)} = 0.013 \left[ \frac{63400}{0.167 \times 10^{-3} (970) (3600)} \right]$$

$$\times \left( \frac{379 \times 10^{-3}}{59.1} \right)^{1/3}$$

$$\underline{\Delta T = 24.9 \text{ F}} - \left\{ \begin{array}{l} \text{OK} \\ \text{1 PLATE} \\ \text{WILL DO IT} \end{array} \right\}$$

### 21.9 PLOTS REQUIRED

PROCEDURE:

$T_{sat}(k)$   $\Delta T(k)$

600	227	Film Boiling*
500	127	" "
400	27	Nucleate **

\* USE EAN (21-7)  
\*\* USE EAN (21-5)

$$q = h \Delta T$$

$h_{film}$ ,  $b$ , VARIES AS  $\Delta T^{-1/4}$

$h_{nuc}$ ,  $b$ , " "  $\Delta T^2$

$b$ ,  $A_{film}$  VARIES AS  $\Delta T^{3/4}$

$b$ ,  $A_{nuc}$  " "  $\Delta T^3$

PLATE IS LUMPED

$$\frac{q}{A} = \dot{m} C_p \frac{\Delta T}{\Delta t} \quad \Delta T = \frac{q}{\dot{m} C_p A} \Delta t$$

21.10 IDEA HERE IS TO RETARD BOILING SUCH THAT INTERNAL PRESSURE WILL NOT BE TOO LARGE

$$q_f = \left(3 \times 10^6 \frac{\text{Btu}}{\text{HR} \cdot \text{ft}^2}\right) \pi \left(\frac{3}{48}\right)(4) = 555 \frac{\text{Btu}}{\text{s}}$$

3600

$$\text{FOR } P=1 \text{ ATM} \quad h_{fg} = 970 \frac{\text{Btu}}{\text{lbm}}$$

$$\dot{m}_{\text{EVAPORATION}} = \frac{655}{970} = 0.676 \frac{\text{lbm}}{\text{s}}$$

$$= \frac{0.676}{0.0372} = 18.2 \frac{\text{ft}^3}{\text{s}}$$

VOLUME OF PIPE { ALSO VOLUME OF FLUID }

$$= \frac{\pi}{4} \left(\frac{3}{48}\right)^2 (4) = 0.0123 \text{ ft}^3$$

$$\text{FLOW AREA} = \frac{\pi}{4} \left(\frac{3}{48}\right)^2 = 0.0031 \text{ ft}^2$$

FOR  $\dot{V}_w \left\{ \frac{\text{ft}^3}{\text{s}} \right\}$  OF FLUID ENTERING

$\dot{V}_w + 18.2$  TOTAL FLUID EXISTS

$$\dot{V}_w = 18.2 \left( \frac{0.0372}{59.5} \right) \approx 0.0144 \frac{\text{ft}^3}{\text{s}}$$

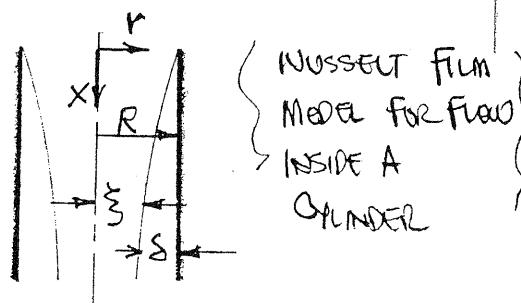
$\dot{V}_w + 18.2 \approx 18.2 \frac{\text{ft}^3}{\text{s}}$  OUT

$$V_{\text{exit}} = \frac{18.2}{0.0031} = 5870 \text{ ft/s}$$

SUPersonic!

INCREASE PIPE DIAMETER, OR ADD ADDITIONAL PIPES IN PARALLEL, OR DECREASE  $q_f$

21.11



21.11 (CONT.)

$$\text{AT EQUILIBRIUM} \quad \sum f_x = 0$$

$$\text{UPWARD force} = T \Delta x 2\pi r = -\mu \frac{dV}{dr} 2\pi r \Delta x$$

(viscous)

$$\text{DOWNWARD force} = \rho g \pi (r^2 - \xi^2) \Delta x$$

$$\Rightarrow \rho g (r^2 - \xi^2) = -\mu \frac{dV}{dr} 2r$$

$$\text{SEPARATING VARIABLES: } dV = -\frac{\rho g}{2\mu} (r - \xi^2) dr$$

$$V = -\frac{\rho g}{2\mu} \left( \frac{r^2}{2} - \xi^2 \ln r \right) + C$$

$$\text{B.C., } V(R) = 0 \Rightarrow C = \frac{\rho g}{\mu} \left( \frac{R^2}{2} - \xi^2 \ln R \right)$$

$$V = \frac{\rho g}{2\mu} \left[ \frac{R^2 - r^2}{2} - \xi^2 \ln \frac{R}{r} \right]$$

AT ANY X, MASS FLOW RATE IS

$$\Gamma = \int_0^R SV(2\pi r) dr$$

$$= \frac{\pi \rho g}{\mu} \int_0^R \left[ \frac{R^2}{2} - \frac{r^2}{2} - \xi^2 r \ln r \right] dr$$

$$= \frac{\pi \rho g}{\mu} \left[ \frac{R^4}{8} - \frac{\xi^2 R^2}{2} + \frac{3\xi^4}{8} + \frac{\xi^2}{2} \ln R - \frac{\xi^4}{2} \ln \xi \right]$$

RATE OF HEAT FLOW TO WALL - THROUGH CONDENSATE -

$$= \frac{2\pi k}{\rho u^2 g} dx (T_v - T_w)$$

AMOUNT OF CONDENSATE IN DISTANCE dx

$$= \frac{\partial \Gamma}{\partial x} dx = \frac{\partial \Gamma}{\partial \xi} \frac{\partial \xi}{\partial x} dx = \frac{\partial \Gamma}{\partial \xi} dx$$

21.11 CONT

$$\frac{\partial \Gamma}{\partial x} = \frac{2\pi g^2}{\mu} h_{fg} \left[ -\xi R + \xi^3 + 2\xi^2 \ln R - 2\xi^2 \ln \xi \right]$$

RATE OF HEAT FLOW TO COOL WAll

$$= \frac{2\pi g^2}{\mu} h_{fg} \left[ -\xi R + \xi^3 + 2\xi^2 \ln R - 2\xi^2 \ln \xi \right] d\xi$$

EQUATING HEAT FLOW RATES -

$$\frac{k}{\mu r/g} \frac{dx}{x} (T_b - T_w) = g^2 \frac{h_{fg}}{\mu} \left[ -\xi R + \xi^3 + 2\xi^2 \ln R - 2\xi^2 \ln \xi \right] d\xi$$

SEPARATING VARIABLES  $\xi$  INTEGRATING

$$A \int_0^x \frac{dx}{x} = \int_R^\xi \left[ \dots \right] d\xi$$

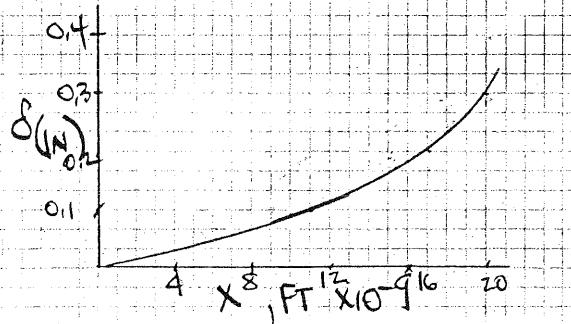
$$Ax = f(\xi/R) = f(\eta)$$

SOLVING {messy} WE GET, FOR X(η)

$$X(FT) = (2.42 \times 10^9) \left[ 1.23 - 3.14 \ln \eta \right] \eta^4 + (1.84 - 0.5 \ln \eta) \eta^2 - 9.07$$

$\eta$	$\ln \eta$	$\eta^2$	$\eta^4$	$X \times 10^9$ (FT)	$\delta_{(IN)}$
0.9	-0.1	0.81	0.654	6.3	0.05
0.8	-0.22	0.64	0.409	11.2	0.10
0.6	-0.508	0.36	0.1294	17.3	0.20
0.4	-0.913	0.16	0.0256	20.2	0.30
0.2	-1.607	0.04	0.0016	21.6	0.40
0				22.0	0.50

21.11 CONT



21.12 a) VERTICAL TUBE

$$h = 0.943 \left\{ \frac{g_L g k^3 \Delta \theta \left[ h_{fg} + \frac{3}{8} C_p L (\Delta T) \right]}{L \mu \Delta T} \right\}^{1/4}$$

$$= 0.943 \left\{ \frac{961.2 (9.81) (0.679)^3 (0.961)}{1 (297 \times 10^{-6}) (9)} \times \right. \\ \left. \times \left[ 2.25 \times 10^6 + \frac{3}{8} (4206)(9) \right] \right\}^{1/4}$$

$$= 6600 \text{ W/m}^2 \cdot \text{K}$$

$$\dot{m}_{cond.} = \frac{6600 (9) (\pi) (15) (1)}{225 \times 10^6} = 0.0125 \text{ kg/s}$$

b) HORIZONTAL

$$h = 0.725 \left\{ \frac{g_L g \Delta \theta k^3 \left[ h_{fg} + \frac{3}{8} C_p L \Delta T \right]}{\mu \Delta T} \right\}^{1/4}$$

$$= 7100 \text{ W/m}^2 \cdot \text{K} \quad \dot{m} = 0.0146 \text{ kg/s}$$

21.13 NEGLECT  $\Delta T$  ACROSS TUBE

$$h_i = \frac{k}{D_i} 0.023 P_e^{0.8} \rho^{4/3} \Pr^{4/3}$$

$$h_o = 0.725 \left\{ \frac{g_L g \Delta \theta k^3 \left[ h_{fg} + \frac{3}{8} C_p L \Delta T \right]}{\mu \Delta T} \right\}^{1/4}$$

$$\text{for } h_i \quad T_{in} = 20^\circ \text{C} \quad T_w = ? \quad T_b = \frac{20 + T_{out}}{2}$$

$$\text{for } h_o \quad \text{PROPERTIES EVALUATED AT } T_f = \frac{T_0 + T_w}{2}$$

2.13 (CONT.)

$$q = \frac{\Delta T_{\text{overall}}}{\sum R} = \frac{\Delta T_i}{R_i} = \frac{\Delta T_o}{R_o}$$

A MESSY TRIAL & ERROR PROBLEM  
AFTER QUITE A BIT OF WORK

$$\text{ASSUMING } T_{i,\text{out}} = 36^\circ\text{C} = 309\text{ K}$$

$$\text{and } T_{w,\text{avg}} = 58^\circ\text{C} = 331\text{ K}$$

$$\text{GIVING } T_{b,\text{avg}} = 28^\circ\text{C} = 301\text{ K}$$

$$R_i = \frac{DVS}{\mu} = \frac{4 \text{ m}}{\pi D \mu} = \frac{4(4000)}{\pi(0.0165)(863 \times 10^{-6})} \times (3600) \\ = 99000$$

$$Pr = 5.95 \quad k = 0.611$$

$$h_i = \frac{0.611}{0.0165} (0.023)(99000)^{0.8} (5.95)^{0.4}$$

$$= 17200 \text{ W/m}^2\text{K}$$

$$h_o = 0.725 \left\{ \frac{97.8(98)(97.8)(0.673)}{(352 \times 10^{-6})(0.019)(42)} \right\}^{1/4} \times \left[ 2.25 \times 10^6 + \frac{3}{8}(4194)(42) \right]^{1/4} \\ = 8960 \text{ W/m}^2\text{K}$$

$$R_i = \frac{1}{17200(\pi)(0.0165)(2)} = 5.61 \times 10^{-4}$$

$$R_o = \frac{1}{8960(\pi)(0.019)(2)} = 9.35 \times 10^{-4}$$

$$\sum R = 14.96 \times 10^{-4}$$

$$\Delta T_{\text{TOTAL}} = 72 \text{ K} \quad \Delta T_i \approx 27 \text{ K} \quad \Delta T_o \approx 45 \text{ K}$$

$$q = \frac{27}{R_i} = \frac{45}{R_o} = 48000 \text{ W}$$

2.13 (CONT.)

$$q = mc_p \Delta T = \frac{4000}{3600} (4180) \Delta T$$

$$\Delta T \approx 10.4 \text{ K}$$

$$\Rightarrow T_{w,\text{out}} \approx 303.5 \text{ K} \quad T_{b,\text{avg}} \approx 298 \text{ K}$$

~ CLOSE TO ORIGINAL ASSUMPTION

$$\text{FINALY! } h_{H_2O} = 17200 \text{ W/m}^2\text{K} \quad (\text{a})$$

$$h_{\text{condens}} = 8690 \text{ "} \quad (\text{b})$$

$$T_{w,\text{out}} = 303.5 \text{ K} \quad (\text{c})$$

$$m_{\text{cond}} = \frac{48000}{2.25 \times 10^6} = 0.0213 \text{ kg/s} \quad (\text{d})$$

$$2.14 \text{ Flow Rate} = \frac{0.042}{0.586} = 0.0717 \text{ m}^3/\text{s}$$

per m

$$\text{ALLOWABLE WIDTH} = \frac{0.0717}{(550)} = 0.00478 \text{ m} \\ = 0.478 \text{ m}$$

$$0.478 + 2\delta = 1 \text{ cm}$$

$$\delta = 0.261 \text{ cm} = 0.00261 \text{ m}$$

Film Model:

$$S^4 = \left[ \frac{4k \mu x \Delta T}{S_L g \Delta S (h_f + \frac{3}{8} c_p \Delta T)} \right] \\ = \frac{4(0.392)(0.195 \times 10^{-3})(0.5)}{(59.5)(321)(59.5)(93.5)(3600)} \times \\ \sim \{ \text{ALL ENGLISH UNITS} \}$$

$$S^4 = 4.425 \times 10^{14} \times$$

$$x = \frac{121,500 \text{ FT}}{}$$

$$= 37000 \text{ m}$$

$$21.15 \quad \frac{q}{A} = k_L \frac{\Delta T}{y} = 8 h_{fg} \frac{dy}{dt}$$

{ AT A GIVEN DEPTH,  $y$  }

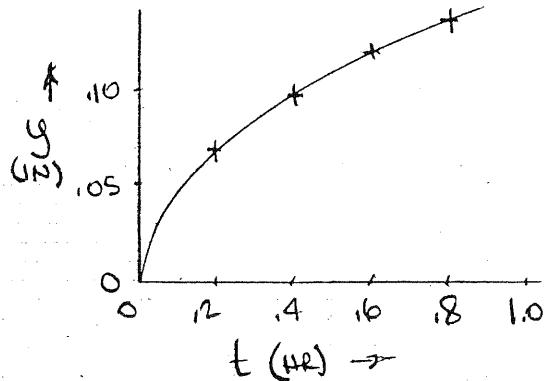
$$\int_0^y dy = \frac{k_L \Delta T}{8 h_{fg}} \int_0^t dt$$

$$\frac{y^2}{2} = \frac{k_L \Delta T}{8 h_{fg}} t$$

$$y^2 = \frac{2(0.392)(12)}{60.1(970)} t = 1.614 \times 10^{-4} t$$

$t$ , hours       $y$       inches

	0	0	0
0.2	0.586	0.0681	
0.4	0.804	0.0964	
0.6	1.0	0.12	
0.8	1.14	0.137	
1.0	1.27	0.152	



### 21.16 HORIZONTAL CYLINDER

$$h_H = 0.725 \left\{ \frac{8.9 \Delta S k^3 (h_{fg} + \frac{3}{8} C_p \Delta T)}{\mu D \Delta T} \right\}^{1/4}$$

$$= 0.725 \left\{ \frac{61.3 (32.2) (41.3) (0.0562) (2100) (997)}{0.29 \times 10^{-3} (0.0656) (45)} \right\}^{1/4}$$

$$= 397 \frac{\text{BTU}}{\text{HRFT}^2} = 2250 \text{ W/m}^2 \cdot \text{K}$$

$$h_V = h_H \left[ 1.3 \left( \frac{D}{L} \right)^{1/4} \right] = 1000 \text{ W/m}^2 \cdot \text{K}$$

$$21.16 \text{ CONT. } \dot{m}_{\text{COND}} = \frac{h A \Delta T}{h_{fg}}$$

$$= 2250(\pi)(0.02)(1.5)(25)$$

$$= 2.32 \times 10^6$$

$$= 2.29 \times 10^{-3} \text{ kg/s}$$

$$= 1.02 \times 10^{-3} \text{ m}$$

HORIZONTAL  
VERTICAL

$$21.17 \quad h_{\text{AVG}} = \bar{h} \left( \frac{1}{8} \right)^{1/4} = \frac{h}{1.681}$$

HORIZONTAL TUBE GATE { SEE PROB }  
21.16

$$h_{\text{HORIZ}} = 2250 \text{ W/m}^2 \cdot \text{K}$$

$$h_{\text{AVG}} = 2250 / 1.681 = 1341 \text{ W/m}^2 \cdot \text{K}$$

$$q = h_{\text{AVG}} A \Delta T$$

$$= 1341 (8)(\pi)(0.02)(1.5)(25)$$

$$= 25.3 \text{ kW}$$

### 21.18 SINKING HORIZONTAL TUBE:

$$h = 0.725 \left\{ \frac{8.9 \Delta S k^3 (h_{fg} + \frac{3}{8} C_p \Delta T)}{D \mu \Delta T} \right\}^{1/4}$$

$$= 0.725 \left\{ \frac{(60.1)(32.2)(60.1)(0.392)(1015)(100)}{5/96 (0.29 \times 10^{-3})(100)} \right\}^{1/4}$$

$$= 1600 \frac{\text{BTU}}{\text{HRFT}^2 \text{F}}$$

$$21.19 \quad h_{\text{AVG}} = h \left( \frac{1}{8} \right)^{1/4} = \frac{h}{1.681}$$

$$h_{\text{HORIZ}} = 2250 \text{ W/m}^2 \cdot \text{K} \quad \{ \text{from Prob} \} \quad 21.16$$

$$\text{For BANK: } h_{\text{AVG}} = \frac{2250}{1.681} = 1341 \text{ W/m}^2 \cdot \text{K}$$

21.19 (cont.) for n Tubes -

$$\dot{q} = h_{\text{ave},n} n A_{\text{TUBE}} \Delta T$$

$$= h_{\text{ave},n-1} (n-1) A_{\text{TUBE}} \Delta T + h_n A_{\text{TUBE}} \Delta T$$

$$h_n n A \Delta f = h_{n-1} (n-1) A \Delta f + h_n A \Delta T$$

$$n^{\text{th}} \text{TUBE: } h_n = n \bar{h}_n - (n-1) \bar{h}_{n-1}$$

$$\text{TOP TUBE: } h_1 = 2250 \text{ W/m}^2 \cdot \text{K}$$

$$3^{\text{RD}} \text{TUBE: } \bar{h}_2 = 1890 \text{ W/m}^2 \cdot \text{K}$$

$$\bar{h}_3 = 1710 \text{ "}$$

$$h_3 = 3(1710) - 2(1890) = 1350 \text{ W/m}^2 \cdot \text{K}$$

$$8^{\text{th}} \text{TUBE: } \bar{h}_8 = 1341 \text{ W/m}^2 \cdot \text{K}$$

$$\bar{h}_7 = 1383 \text{ "}$$

$$h_8 = 8(1341) - 7(1383) = 1047 \text{ W/m}^2 \cdot \text{K}$$

$$21.20 \frac{4A\Gamma_c}{P\mu_f} = Re_c = 2000$$

$$\frac{4A\Gamma_c}{P\mu_f} = \frac{4A}{P} \frac{m}{A} \frac{1}{\mu} = \frac{4}{P\mu} h_f L \Delta T$$

$$L = \frac{\mu h_f g}{4h \Delta T} (2000)$$

$$= \frac{(0.0206 \times 10^{-3})(970)(2000)}{4(100)} \left[ 2250(1.3 \times 0.02) \right]^{1/4} L^{(3600)}$$

$$L^{3/4} = 3.27 \quad L = 4.85 \text{ FT}$$

$$21.21 \quad \dot{q} = 0.943 \left[ S_g \Delta S \left( h_{fg} + \frac{3}{8} C_p \Delta T \right) \right]^{1/4}$$

$$= 0.943 \left[ \frac{37.2(32.2)(37.2)(0.294)(505)}{2(14 \times 10^5)(25)} \right]^{1/4}$$

$$= 694 \text{ BTU/HR FT}^2 \text{ F}$$

$$\dot{q} = h A \Delta T = 694(2)(25)$$

$$= 34,700 \text{ BTU/HR PER FOOT OF WIDTH}$$

$$21.22 \quad q = \frac{k \Delta y}{y} = S h_{fg} \frac{dy}{dt} \quad \text{FOR THICKNESS}$$

$$\int_0^t dt = \frac{S h_{fg}}{k A T} \int_0^y y dy = \frac{S h_{fg}}{k A T} \frac{y^2}{2}$$

$$t = 60.2 (972 \times 0.02 / 0.3048)^2$$

$$0.390 (39.6)(2)$$

$$= 16.3 \text{ Hours} \quad \text{IF PAN IS HORIZONTAL}$$

FOR PAN INCLINED:

L



$$\text{PER UNIT DEPTH: } Vol = \frac{1}{2} \frac{0.02}{\tan \theta} (0.02)$$

$$Vol = \frac{2 \times 10^{-4} L}{\tan \theta} \text{ m}^3$$

$$\text{IF } \theta = 10^\circ \quad Vol = 1.34 \times 10^{-3} \text{ m}^3$$

$$" = 3.46 \times 10^{-4} \text{ "}$$

LENGTH OF SURFACE EXPOSED:

IF  $\theta = 10^\circ$  LENGTH = 40 TO 28.7 cm

$\theta = 30^\circ$  " 40 TO 36.5 "

21.22 CONT.

ASSUME: ACCUMULATION OF CONDENSATE DUE PRINCIPALLY TO CONDENSATION ON EXPOSED SURFACE

$$h = 0.943 \left[ \frac{0.9 \sin \theta k \Delta S (h_{fg} + \frac{3}{8} q_{\text{sat}})}{L \mu \Delta T} \right]^{1/4}$$

$$= \frac{0.943}{(L \sin \theta)^{1/4}} \left[ \frac{(0)(32.2)(0.792)^3 (60)(995)(3600)}{0.201 \times 10^{-3} (40)} \right]$$

$$= 1252 \left( \frac{L}{\sin \theta} \right)^{-1/4}$$

$$@ \theta = 10^\circ \quad L_{\text{avg}} = \frac{40 + 28.7}{2} = 33.8 \text{ cm} = 1.109 \text{ ft}$$

$$30^\circ \quad " = \frac{40 + 36.5}{2} = 38.3 \text{ " } = 1.257 \text{ "}$$

$$h_{10} = 788 \text{ BTU/HR FT}^2 F \quad h_{30} = 994 \text{ BTU/HR FT}^2 F$$

$$\dot{m} = \frac{h_{\text{sat}} - h}{h_{fg}} = \frac{h_{\text{sat}}(40)}{980}$$

$$\dot{m}_{10} = 35.7 \text{ lbm/HR} = 0.572 \text{ FT}^3/\text{sec}$$

$$\dot{m}_{30} = 51.0 \text{ " } = 0.817 \text{ "}$$

$$t = V / \dot{m}$$

$$t_{10} = \frac{1.34 \times 10^{-3}}{0.572} / (0.3048)^2 = 0.0252 \text{ HR}$$

$$= 1.51 \text{ MIN}$$

$$\approx 91 \text{ S}$$

$$t_{30} = 0.00456 \text{ HR} = 0.274 \text{ MIN}$$

$$\approx 16.4 \text{ S}$$

21.23

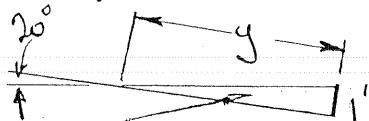
$$h = 0.943 \left[ \frac{0.9 \sin \theta k \Delta S (h_{fg} + \frac{3}{8} q_{\text{sat}})}{L \mu \Delta T} \right]^{1/4}$$

$$= 0.943 \left[ \frac{(0.601)(32.2)(\sin 20^\circ)(0.792)^3 (60)}{10.125/12 (0.206 \times 10^{-3}) (30)} \times (981/3600) \right]^{1/4}$$

$$= 1050 \text{ BTU/HR FT}^2 F$$

$$\dot{m} = \frac{h_{\text{sat}} - h}{h_{fg}}$$

$$= \frac{1050(1/30)}{970} = 32.5 \text{ lbm/HR}$$



$$V_{\text{vol}} = \frac{1}{2} \left( \frac{1}{2} \right) \left( \frac{1}{\tan 20^\circ} \right) \left( \frac{1}{2} \right)$$

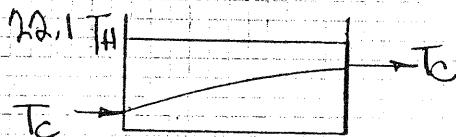
$$= 0.00955 \text{ FT}^3$$

$$t = \frac{V}{\dot{m}} = \frac{0.00955}{32.5/62.4}$$

$$= 0.0183 \text{ hour}$$

$$= 1.10 \text{ MIN.}$$

## CHAPTER 22



$$q = \dot{m} c_p \Delta T_{\text{H2O}} = U A \Delta T_{\text{LM}}$$

$$U A = \text{CONST.} = \frac{\dot{m}}{c_p} \frac{T}{\Delta T_{\text{LM}}}$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{A_L}{2\pi k} \ln \frac{r_o}{r_i} + \frac{A'}{h_o}} \approx h_i$$

NEGL.                    NEGL.

$$\Rightarrow h_i A_i = \text{CONST.}$$

USING DITTUS-BOEHLER CORRELATION

$$h_i = \frac{k}{D} (\text{CONST}) Re^{0.8} Pr^{0.4} = K \frac{(Re)^{0.8}}{D}$$

$$= \frac{K (D Q / \pi D^2)^{0.8}}{D} = (\text{CONST.}) D^{-1.8}$$

$$\Rightarrow h A = \text{CONST.} = A (\text{CONST.}) D^{-1.8}$$

AS DIAMETER INCREASES THE REQUIRED AREA INCREASES AS  $D^{1.8}$

$$22.1 \quad q = \dot{m} c_p \Delta T_w = \dot{m} c_p \Delta T_{\text{PROF}}$$

$$10^5 (1)(60) = 10^5 (0.24) \Delta T_{\text{PROF}}$$

$$\Delta T_{\text{PROF}} = 250 \text{ F} = 800 - T_{\text{PROF OUT}}$$

$$T_{\text{PROF OUT}} = 550 \text{ F}$$

$$\Delta T_{\text{LM}} = \frac{600 - 400}{\ln \frac{600}{400}} = 500 \text{ F}$$

$$A = \frac{q}{U \Delta T_{\text{LM}}} = \frac{10^5 (1)(60)}{12(500)} = 1000 \text{ ft}^2$$

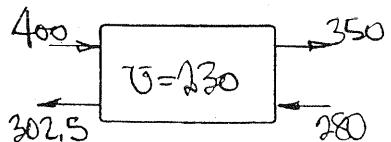
$$22.3 \quad \text{Oil: } T_{\text{IN}} = 400 \text{ K} \quad T_{\text{OUT}} = 350 \text{ K}$$

$$\dot{m} = 2 \text{ kg/s} \quad q = 1880 \text{ J/kg.K}$$

$$q = \dot{m} c_p \Delta T = 2(1880)(50) = 188000 \text{ W}$$

$$\Delta T_w = \frac{q}{\dot{m} c_p} = \frac{188000}{2(4187)} = 22.5 \text{ K}$$

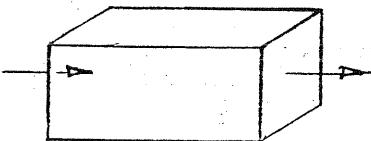
$$T_{\text{wIN}} = 280 \text{ K} \quad T_{\text{wOUT}} = 302.5 \text{ K}$$



$$\Delta T_{\text{LM}} = \frac{97.5 - 70}{\ln \frac{97.5}{70}} = 83 \text{ K}$$

$$A = \frac{q}{U \Delta T_{\text{LM}}} = \frac{188000}{230(83)} = 9.85 \text{ m}^2$$

22.4



$$A_{\text{EQUIV}} = \frac{4(0.1)(0.2)}{2(0.1+0.2)} = 0.0667 \text{ m}^2$$

$$T_{\text{bANG}} = 295 \text{ K} \quad T_f = 345 \text{ K}$$

$$k_e = 0.0667 \frac{S \cdot U}{\mu} = 0.0667 \frac{\dot{m}}{A \mu}$$

$$q = h A \Delta T_{\text{LM}}$$

$$\Delta T_{\text{LM}} = \frac{105 - 95}{\ln \frac{105}{95}} = 99.9 \text{ K}$$

## 22.4 CONTINUED

ASSUMING TURBULENT FLOW:

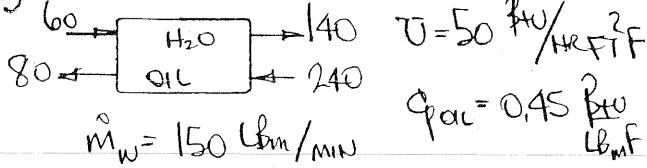
$$\dot{m}c_p \Delta T = 815 c_f [0.023 f_r^{-0.2} \Pr^{-2/3}] A_s \Delta T_{LM}$$

$$\dot{m} = \frac{\dot{m}}{A} \left[ 0.023 \left( \frac{0.0667}{0.205 \times 10^5} \right)^{-0.2} \times (0.698)^{-2/3} \right] Q (0.3) (25) 999$$

SOLVING FOR  $\dot{m}$ :  $\dot{m} = 105 \text{ kg/s}$

$$q = \dot{m} c_p \Delta T = 105 (1009)(60) = 1060 \text{ kW}$$

## 22.5



a)  $q = 150(1)(80) = 12000 \text{ BTU/MIN}$   
 $= UA \Delta T_{LM}$

$$\Delta T_{LM} = \frac{100 - 20}{\ln \frac{100}{20}} = 49.7 \text{ F}$$

$$A = \frac{q}{U \Delta T_{LM}} = \frac{12000(60)}{50(49.7)} = 290 \text{ ft}^2$$

b) WATER IN SHELL; OIL IN 2 PASSES

$$Y = \frac{80 - 240}{60 - 240} = 0.889$$

$$Z = \frac{60 - 140}{80 - 240} = 0.5$$

$A = \infty \sim \text{CANT BE DONE}$

c)  $q = \dot{m} c_p \Delta T_w = \dot{m} c_p \Delta T_o$

$$C_o = \frac{12000(60)}{160} = 75(60)$$

$$= 4500 \text{ BTU/HR·F}$$

## 22.5 CONTINUED

$$C_o = C_{MIN}$$

$$\frac{UA}{C_{MIN}} = \frac{50(290)}{4500} = 3.22$$

$$\frac{C_{MIN}}{C_{MAX}} = 0.625$$

$$\text{Fig 22.12} \sim \varepsilon \approx 0.86$$

$$q = \varepsilon C_{MIN}(180) = 7200 \Delta T$$

$$\Delta T_w = \frac{0.86(4500)(180)}{7200} \approx 97 \text{ F}$$

$$T_w \text{ EXIT} = 157 \text{ F}$$

22.6  $\Delta T_w = 340 - 255 = 85 \text{ K}$

$$\Delta T_o = 350 - 305 = 45 \text{ K}$$

$$q = \varepsilon C_{MIN}(350 - 255) = C_w(85)$$

$$\varepsilon = \frac{85}{95} = 0.895$$

22.7 WATER  $T_{IN} = 50 \text{ F}$

$$\dot{m} = 400 \text{ lbm/HR}$$

$$C_p = 1 \text{ BTU/lbm F}$$

QW:  $T_{IN} = 150 \text{ F}$   $C_p = 0.45 \text{ BTU/lbm F}$   
 $\dot{m} = ?$

$$U = 60 \text{ BTU/HR°F}^2\text{F}$$

$$A = 18 \text{ ft}^2$$

$$T_{w \text{ OUT, MAX}} = 212 \text{ F}$$

$$T_{o \text{ OUT, MAX}} = 160 \text{ F}$$

$$q = \dot{m}_w C_p w \Delta T_w = \dot{m}_o C_p o \Delta T_o$$

$$= 400(1)(Tw - 50) = \dot{m}_o(0.45)(250 - To)$$

$$= \varepsilon C_{MIN}(200)$$

## 22.7 CONTINUED -

For  $T_{w\text{out}} = 212 \Delta T_w = 162$

$$\dot{Q} = 400(162) = 64800$$

$$= C_o \Delta T_o = \epsilon C_{min}(200)$$

$$\left. \begin{array}{l} \left\{ \dot{Q} = \dot{m}_o C_p \Delta T_o, \therefore \text{MAX } \dot{m}_o \text{ will} \right. \\ \text{BE ASSOCIATED WITH MINIMUM } \Delta T_o \end{array} \right\}$$

If  $T_{o\text{out max}} = 160 \sim \Delta T_{o\text{ min}} = 90$

For  $H_2O$  as MINIMUM fluid:

$$C_{min} = 400$$

$$400(162) = \epsilon(400)(200)$$

$$\epsilon = 0.81$$

$$NTU = \frac{\epsilon A}{C_{min}} = \frac{60(18)}{400} = 2.7$$

$$\left\{ \text{FIG 22.12 a} \quad C_{min}/C_{max} \approx 0.65 \right\}$$

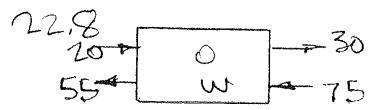
$$C_{max} = \dot{m}_o C_p = \frac{400}{0.65} = 615$$

$$\Delta T_o = \frac{\dot{Q}}{C_o} = \frac{64800}{615} = 105.3 \text{ F}$$

$$T_{o\text{ out}} = 144.7 - \text{OK}$$

$$\text{Finally: } \dot{m}_{max} C_p = 615$$

$$\dot{m}_{max} = \frac{615}{0.45} = \underline{1370 \text{ lbm/hr}}$$



$$\begin{aligned} \dot{m}_o &= 12 \text{ kg/s} \\ C_p &= 2.2 \text{ kJ/kg.K} \\ C_{pw} &= 4180 \text{ J/kg.K} \\ U &= 1080 \text{ W/m}^2 \text{ K} \end{aligned}$$

(PROBLEM STATEMENT)  
USES  $C_H$  AS  $C_{pw}$  &  
 $C_C$  AS  $C_p$ )

$$\dot{Q} = UAF \Delta T_{lm}$$

$$= \dot{m}_o C_p \Delta T_o = \dot{m}_w C_{pw} \Delta T_w$$

$$= (12)(2200)(10) = \dot{m}_w(4180)(30)$$

$$\rightarrow \dot{m}_w = 3.16 \text{ kg/s}$$

$$\Delta T_{lm} = \frac{45-35}{\ln \frac{45}{35}} = 39.8$$

$$\text{To find F: } Y = \frac{10}{55} = 0.182$$

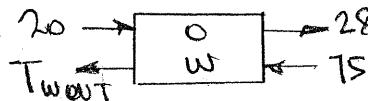
$$Z = 20/10 = 2$$

$$\text{From 22.9 a: } F \approx 1$$

$$FAT_{lm} = \underline{39.8 \text{ C}}$$

$$A = \frac{\dot{Q}}{UAT_{lm}} = \frac{(12)(2200)(10)}{1080(39.8)} = \underline{6.14 \text{ m}^2}$$

22.9 SAME EXCHANGER & ENTRANCE CONDITIONS AS PROB 22.8 -



(PROBLEM STATEMENT)  
SHOULD SAY  
 $T_{o\text{ out}} = 28^\circ\text{C}$

$$\dot{Q} = \dot{m}_o C_p \Delta T_o = 12(2200)(8)$$

$$= \dot{m}_w C_{pw} \Delta T_w = 3.16(4180)\Delta T_w$$

$$\Delta T_w \approx 16^\circ\text{C} \quad T_{w\text{out}} \approx 59^\circ\text{C}$$

22.9 CONTINUED -

$$\Delta T_{lm} = \frac{47-39}{\ln \frac{47}{39}} = 42.9^\circ C$$

$$\text{TO FIND } F: Y = \frac{8}{55} = 0.145 \\ Z = 160/8 = 2$$

FIG 22.9a -  $F \approx 1$

$$U = \frac{q}{A F \Delta T_{lm}} = \frac{(12)(2200)(8)}{6.14(42.9)} \\ = 802 \text{ W/m}^2 \cdot K$$

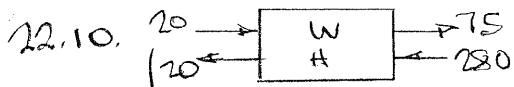
$$|UA|_0 = \frac{1}{\sum R_0} \quad |UA|_1 = \frac{1}{\sum R_1}$$

$$\sum R_0 = \frac{1}{(1080)(6.14)} = 1.508 \times 10^{-4}$$

$$\sum R_1 = \frac{1}{(802)(6.14)} = 2.031 \times 10^{-4}$$

FOURING RESISTANCE

$$= \sum R_1 - \sum R_0 = 0.523 \times 10^{-4} \text{ K/W}$$



$$\dot{m}_w = 2.7 \text{ kg/s} \quad U = 160 \text{ W/m}^2 \cdot K$$

$$q = \dot{m}_w c_p w \Delta T_w = (2.7)(4200)(55) \\ = \dot{m}_w (1200)(160)$$

$$\dot{m}_w = 3.25 \text{ kg/s} \quad a)$$

$$q = UA F \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{110-65}{\ln \frac{110}{65}} = 146.3^\circ C$$

22.10 CONTINUED -

$$\text{TO FIND } F: Y = \frac{55}{260} = 0.211 \\ Z = 160/55 = 2.91$$

FIG 22.10 a -  $F \approx 0.96$

$$A = \frac{q}{U F \Delta T_{lm}} = \frac{(2.7)(4200)(55)}{(160)(0.96)(146.3)} \\ = 27.8 \text{ m}^2 \quad b)$$

$$\dot{m}_0 = 11 \text{ kg/s} \\ c_p 0 = 2.2 \text{ kJ/kg.K} \\ U = 1200 \text{ W/m}^2 \cdot K$$

$$\Delta T_{lm} = \frac{68-35}{\ln \frac{68}{35}} = 49.7^\circ C$$

$$q = \dot{m}_w c_p w \Delta T_w = \dot{m}_w (4200)(22) \\ = \dot{m}_w c_p \Delta T_0 = (11)(1200)(45)$$

$$\dot{m}_w = 11.79 \text{ kg/s} \quad a)$$

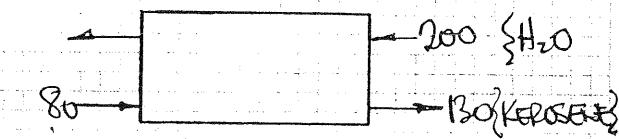
$$\text{TO FIND } F: Y = \frac{110-65}{110-20} = 0.5$$

$$Z = \frac{42-20}{110-65} = 0.489$$

FIG 22.9a ~  $F \approx 1$

$$A = \frac{q}{U F \Delta T_{lm}} = \frac{11(1200)(45)}{(1200)(1)(49.7)} \\ = 18.26 \text{ m}^2 \quad b)$$

22.12



$$\dot{m}_k = 2500 \text{ lbm/hr} \quad \dot{m}_w = 900 \text{ lbm/hr}$$

$$\bar{U} = 260 \text{ BTU/hr ft}^2 \text{ F}$$

$$\Delta T_w = \frac{2500(130 - 80)(0.5)}{900(1)} = 70.8$$

$$T_{wout} = 129 \text{ F} \quad \Delta T_{lm} = \frac{70 - 49}{\ln \frac{70}{49}} = 58.9 \text{ F}$$

$$AF = \frac{\dot{q}}{\bar{U} \Delta T_{lm}} = \frac{2500(50)(0.5)}{58.9(260)} = 4.16$$

FIGURE 22.9 a

$$Y = \frac{130 - 80}{200 - 80} = \frac{50}{120} = 0.416 \\ Z = \frac{200 - 129}{130 - 80} = \frac{71}{50} = 1.4 \quad \left. \right\} F \approx 0.83$$

$$A = 4.16 / 0.83 = 5.01 \text{ ft}^2$$

22.13 INPUT DATA - See Prob 22.3

$$\bar{U} = 230 \text{ W/m}^2 \text{ K} \quad T_{wout} = 400 \text{ K}$$

$$A = 9.85 \text{ m}^2 \quad T_{win} = 280 \text{ K}$$

$$(a) \text{CROSSFLOW: } C_o = \dot{m} c_p = 3760 \text{ J/kg} \cdot \text{K}$$

$$C_w = 8374$$

$$\frac{\dot{q}}{A} = \frac{230(985)}{3760} = 0.603 \quad \left. \right\} \epsilon = 0.43$$

$$C_{min} / C_{max} = 0.45$$

$$q = \epsilon C_{min} (400 - 280) = 194000 \text{ W}$$

$$T_{wout} = 280 + \frac{194000}{8374} = 303 \text{ K}$$

$$T_{oin} = 400 - \frac{194000}{3760} = 248 \text{ K}$$

22.13 CONTINUED -

(b) SHELL AND TUBE

$$\text{SAME DATA} - \epsilon = 0.40$$

$$\dot{q} = 0.4(3760)(120) = 180500 \text{ W}$$

$$T_{wout} = 301.5 \text{ K} \quad T_{oin} = 352 \text{ K}$$

22.14



$$\dot{q} = \dot{m} c_p \Delta T_w = C_w \Delta T_w$$

$$C_w = S A \nu c_p = \frac{2 \times 10^8}{18} = 1.11 \times 10^7$$

$$\frac{\dot{q}}{A} / C_{min} = \frac{4600 \text{ A}}{1.11 \times 10^7} = 4.14 \times 10^{-4} \text{ A}$$

$$A = n \pi D L = n \pi (1.37/12) L = 0.359 \text{ m}^2$$

$$\frac{\dot{q}}{C_{min}} = 1.485 \times 10^{-4} \text{ m}^2$$

NEGLECTING TUBE FERST

$$\bar{U} = \frac{1}{h_i + h_o} = \frac{1}{4600 + 10600} = 1$$

$$h_i = 8130 \quad Nu_i = \frac{8130 (1.37/12)}{0.615} = 1509$$

USING DITTUS-BOEHLER EQN.

$$Nu = 1509 = 0.023 \left[ \frac{8130 (1.37/12)}{825 \times 10^{-6}} \right]^{0.8} \left( \frac{1}{5.65} \right)^{0.4}$$

$$\delta V = \frac{\dot{q}}{A C_p \Delta T} = \frac{2 \times 10^8}{n \pi D^2 (4179) (18)} = 3192$$

$$n = 81 \text{ TUBES}$$

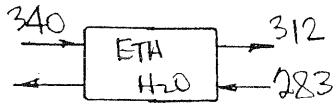
22.14 (CONTINUED -

$$q = \dot{m} C_{min}(65) \quad \dot{m} = 0,277$$

$$T_{in} = 22,2^{\circ}C \quad \frac{UA}{C_{min}} \cong 0,38$$

$$L = \frac{0,38}{(1,48 \times 10^{-4})(81)} = \underline{\underline{31,7 \text{ m}}}$$

22.15



$$\dot{m}_{ETH} = 6,93 \text{ kg/s}$$

$$q = \dot{m}_{ETH} c_p \Delta T = 6,93 (3810)(28)$$

$$= \dot{m} c_p \Delta T_W = 6,93 (4182) \Delta T$$

$$\Delta T_W = 28,1$$

(a) COUNTERFLOW:  $\Delta T_{LM} \cong 29 F$

$$A = \frac{q}{U \Delta T_{LM}} = \frac{6,93(3810)(28)}{568(29)} = \underline{\underline{44,9 \text{ m}^2}}$$

(b) PARALLEL flow:

$$\Delta T_{LM} = \frac{57 - 0,9}{\ln \frac{57}{0,9}} = 13,52$$

$$A = \frac{q}{568(13,52)} = \underline{\underline{96,3 \text{ m}^2}}$$

(c) CROSSFLOW:

$$C_{mixeo} = \dot{m} c_{pw} = 26350$$

$$C_{unmixed} = \dot{m} c_{pe} = 26403$$

$$Y = \frac{312 - 340}{283 - 340} = 0,491 \quad Z = \frac{28,1}{28} \cong 1$$

$$F \cong 0,85 \quad A = \frac{44,9}{0,85} = \underline{\underline{52,8 \text{ m}^2}}$$

22.16

$$q = C_A \Delta T_A = C_W \Delta T_W = 95000 \frac{\text{Btu}}{\text{hr}}$$

$$C_W = \frac{95000}{35} = 2720 \frac{\text{Btu}}{\text{hr}}$$

$$C_A = \frac{95000}{80} = 1188 " \sim C_{min}$$

$$\frac{C_{min}}{C_{max}} = \frac{1188}{2720} = 0,437 \quad \xi = \frac{q}{C_{min}(110)} \cong 0,73$$

a) COUNTERFLOW:  $\frac{UA}{C_{min}} \cong 1,65$

$$V = \frac{A}{130} = \frac{1,65(1188)}{30(130)} = \underline{\underline{0,502 \text{ ft}^3}}$$

b) GROSSFLOW - AIR MIXED

$$\frac{UA}{C_{min}} \cong 2$$

$$V = \frac{A}{100} = \frac{2(1188)}{40(100)} = \underline{\underline{0,593 \text{ ft}^3}}$$

c) CROSSFLOW - BOTH MIXED

$$\frac{UA}{C_{min}} \cong 1,75$$

$$V = \frac{A}{90} = \frac{1,75(1188)}{50(90)} = \underline{\underline{0,462 \text{ ft}^3}}$$

CONFIGURATION (c) IS MOST COMPACT

22.17  $T_5 \rightarrow \dot{m} = 5000 \rightarrow 220$

$$T_{Hout} \leftarrow \dot{m} = 2400 \leftarrow 400$$

$$\bar{U} = 300 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} = 52,8 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot \text{F}}$$

$$q = 5000(1)(445) = 2400(1) \Delta T_H$$

$$\Delta T_H = 302 \quad T_{Hout} = 98 F$$

$$\Delta T_{LM} = \frac{180 - 23}{\ln \frac{180}{23}} = 76,3 F$$

22.17 CONTINUED -

FOR COOL FLUID IN TUBES:

$$\left. \begin{aligned} Y &= \frac{220-75}{400-75} = 0.446 \\ Z &= \frac{302}{145} = 2.08 \end{aligned} \right\} F \approx ?$$

HOT FLUID IN TUBES:

$$\left. \begin{aligned} Y &= \frac{-302}{-325} = 0.929 \\ Z &= \frac{145}{302} = 0.480 \end{aligned} \right\} F \approx ?$$

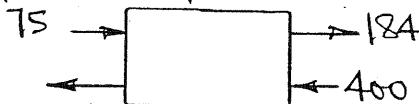
BOTH ARE OFF THE CHARTS

NEITHER IS POSSIBLE ~  
CAN'T USE THIS CONFIGURATION

22.18 . IF COUNTERFLOW :

$$AU_{OLD} = \frac{1}{\left[ \frac{1}{A_{ch,i}} + R_f + \frac{1}{A_{ch,o}} \right]} = 36.7(300)$$

FOR NEW OPERATING CONDITIONS:



$$q = 5000(1)(184-75) = 2400(1)\Delta T_H$$

$$= 545000 \quad \Delta T_H = 227 \quad T_{hot,out} = 173$$

$$\Delta T_{LM} = \frac{216-98}{\ln \frac{216}{98}} = 149.3$$

$$AU_{NEW} = \frac{q}{\Delta T_{LM}} = \frac{545000}{149.3} = 3650$$

$$= \frac{1}{\left[ \frac{1}{A_{ch,i}} + R_f + \frac{1}{A_{ch,o}} \right] + R_F}$$

$$AU_{NEW} = \frac{1}{1/36.7(300) + R_f}$$

22.18 CONTINUED -

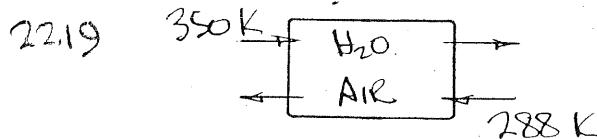
$$\frac{1}{36.7(300)} + R_f = \frac{1}{3650}$$

$$R_f = 1.83 \times 10^{-4} \text{ K/W}$$

$$\text{OR } AR_f = 36.7 R$$

$$= 6.72 \times 10^{-3} \text{ m}^2 \cdot \text{K/W}$$

22.19



$$h_w = 470 \text{ W/m}^2 \cdot \text{K} \quad \dot{m}_w = 10 \text{ kg/s}$$

$$h_A = 210 \quad " \quad \dot{m}_A = 16 \quad "$$

$$q = (10 \text{ kg/s})(4181 \text{ J/kg.K})(350 - T_{w,out})$$

$$= (16 \text{ kg/s})(1007 \text{ J/kg.K})(T_{A,out} - 288)$$

$$C_w = 41810 \quad C_A = 16112 = C_{MIN}$$

$$C_{MIN} / C_{MAX} = 0.385$$

$$A = \pi D L (50) = \pi (0.026)(6.7)(50) \\ = 27.36 \text{ m}^2$$

{ ASSUMES TOTAL LENGTH OF EACH }  
TUBE IS 6.7 M

$$U = \frac{1}{\frac{1}{h_i} + R_{f,NO} + \frac{1}{h_o}} = \frac{1}{\frac{1}{470} + \frac{1}{210}} = 145$$

NEG<sub>i</sub>

$$UA / C_{MIN} = \frac{145(27.36)}{16112} = 0.246$$

$$\underline{\underline{\epsilon \approx 0.20}}$$

22.19 (CONTINUED -

$$q = \dot{E} C_{\min} (T_{w,i} - T_{w,o}) \\ = 0,2 (16112)(62) = \underline{\underline{199800 \text{ W}}}$$

$$T_{w,out} = \underline{\underline{345,2 \text{ K}}} \quad T_{A,out} = \underline{\underline{300,4 \text{ K}}}$$

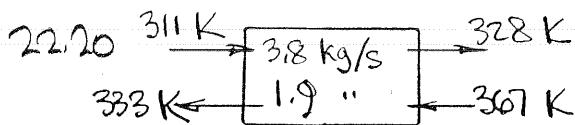
For FOULING RESISTANCE = 0.0021

$$U = \frac{1}{\frac{1}{410} + \frac{1}{210} + 0,0021} = 111,2$$

$$\frac{T_{A,i}}{C_{\min}} = 0,188 \quad \underline{\underline{\dot{E} \approx 0,10}}$$

$$q = 0,1 (16112)(62) = \underline{\underline{99900 \text{ W}}}$$

$$T_{w,out} = \underline{\underline{347,6 \text{ K}}} \quad T_{A,out} = \underline{\underline{294,2 \text{ K}}}$$



$$U = 1420 \text{ W/m}^2 \cdot \text{K}$$

$$\text{TUBES: } D = 0,01905 \text{ m}$$

$$V = 0,366 \text{ m/s}$$

$$L_{\max} = 244 \text{ m}$$

$$\dot{q} = \dot{m} c_p \Delta T_{\text{shell}} = \dot{m} c_p \Delta T_{\text{TUBES}}$$

$$\Delta T_s = \frac{3,8(17)}{1,9} = 34$$

$$C_{\min} = 1,9 (4180) = 7942 \text{ W/K}$$

$$\rho = 983 \text{ kg/m}^3$$

$$\dot{m} = \rho A V = 3,8 = n (983) \left(\frac{\pi}{4}\right) (0,01905) (0,366)$$

$$n = \underline{\underline{37 \text{ TUBES}}}$$

22.20 (CONTINUED -

$$q = \dot{E} C_{\min} (367 - 311) \quad \frac{C_{\min}}{C_{\max}} = 0,5$$

$$\dot{E} = \frac{C_{\min} (34)}{C_{\min} (56)} = 0,607$$

$$\frac{T_{A,i}}{C_{\min}} \approx 1,3 \quad \left\{ \text{fig. 22.12 c} \right\}$$

$$A = \frac{7942(1,3)}{1420} = 7,27 \text{ m}^2$$

$$= \sqrt{\pi (0,01905)} L$$

$$L = 1,64 \text{ m}$$

2 TUBE PASSES WILL WORK

37 TUBES PER PASS

L = 1,64 m per pass

22.21 NTU = 1,25

$$\frac{C_{\min}}{C_{\max}} = 0 \quad \dot{E} \approx 0,72$$

$$\dot{q} = \dot{E} C_{\min} (T_{w,in} - T_{c,in})$$

$$= 0,72 (0,07) (4,18) (93)$$

$$= 19,59 \text{ kW}$$

$$= C_w \Delta T_w = 4,18 (0,07) \Delta T_w$$

$$\Delta T_w = \frac{0,72 (0,07) (4,18) (93)}{4,18 (0,07)} = 67 \text{ K}$$

$$T_{w,out} = 280 + 67 = \underline{\underline{347 \text{ K}}}$$

22.2.1 CONTINUED -

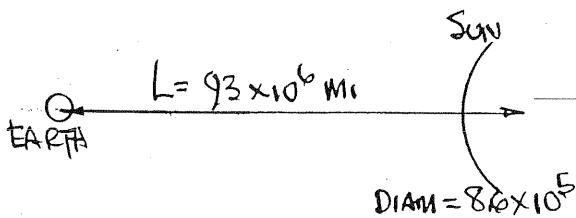
STEAM CONDENSATION RATE

$$\dot{m}_{\text{COND}} = \frac{\dot{q}}{h_{fg}} = \frac{1959}{2256}$$

$$= 8.68 \times 10^{-3} \text{ kg/s}$$

## CHAPTER 23

23.1



$$\text{RADIANT EMISSION FROM SUN} = \pi S E_{\text{BS}}$$

ALL PASSES THROUGH A SPHERICAL SURFACE OF RADIUS, L.

AT THE EARTH

$$I = \frac{\pi D_s^2}{4\pi L^2} E_{\text{BS}} = \left(\frac{D}{2L}\right)^2 E_{\text{BS}}$$

$$\text{FLUX AT EARTH} = 360 + 90 = 450 \text{ BTU HR PT}^2$$

$$450 = \left[ \frac{8.6 \times 10^5}{2(93 \times 10^6)} \right]^2 \sigma T_s^4$$

$$T_s = 10530 \text{ K}$$

23.2

$$0 < \lambda < 0.35 \mu \quad \tau = 0$$

$$0.35 < \lambda < 1.7 \mu \quad \tau = 0.92$$

$$1.7 < \lambda \quad \tau = 0$$

FOR T = 5800 K

$$\lambda_1 T = 2030 \quad F = 0.072$$

$$\lambda_2 T = 15660 \quad F = 0.972$$

$$\Delta F = 0.90$$

$$\text{PER CENT TX} = 0.90(0.92) = 0.828 \\ \underline{\underline{\approx 83\%}}$$

FOR T = 300 K:

$$\begin{aligned} \lambda_1 T = 105 & \quad F \approx 0 \\ \lambda_2 T = 810 & \quad F \approx 0 \end{aligned} \quad \left. \right\} \Delta F \approx 0$$

$$\text{PERCENT TX} = \underline{\underline{0}}$$

23.3

FROM WIEN'S DISPLACEMENT LAW:

$$\lambda_{\text{MAX}} T = 5215.6 \text{ } \mu\text{K}$$

$$\lambda_{\text{MAX}} = \frac{5215.6}{4000} = \underline{\underline{1.304 \mu}}$$

FRACTION IN VISIBLE BAND:

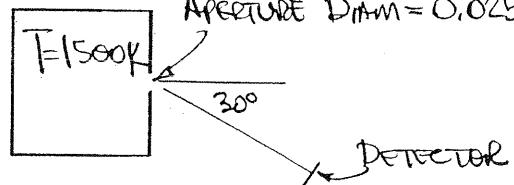
$$= \frac{\int_{\lambda_1}^{\lambda_2} E_b d\lambda}{\int_0^{\infty} E_b d\lambda} = \frac{1}{\sigma T^4} \left( E_b \Big|_0^{\lambda_2 T} - E_b \Big|_0^{\lambda_1 T} \right)$$

$$\left\{ \text{TABLE 23.1} \right\} \lambda_1 T = 667 \mu\text{K} \quad F \approx 0$$

$$\lambda_2 T = 1667 \text{ " } \quad F = 0.0256$$

$$\text{OR } \underline{\underline{2.56\%}}$$

23.4



$$(a) I = \frac{q}{A_A \omega \theta_A w}$$

$$I = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi} = 9.137 \times 10^4 \text{ W/m}^2 \cdot \text{sr}$$

$$\omega = \frac{A_D}{r^2} = \frac{0.001 \text{ m}^2}{1 \text{ m}^2} = 0.001 \text{ Sr}$$

$$q = I A_A \omega \theta_A w$$

$$= (9.137 \times 10^4) \left( \frac{\pi}{4} \right) (0.025)^2 \cos 30^\circ \\ \times (0.001)$$

$$= \underline{\underline{3.88 \times 10^{-2} \text{ W}}}$$

23.4 CONTINUED -

(b) WITH WINDOW

$$q = \int I A_{\lambda} \omega d\lambda \omega$$

$$I = \frac{\int_0^{\infty} T_x G_x d\lambda}{\int_0^{\infty} G_x d\lambda}$$

$$= \frac{\int_0^{\infty} T_x E_{\lambda, b} d\lambda}{\int_0^{\infty} E_{\lambda, b} d\lambda}$$

$$= 0.8 \int_0^2 \frac{E_{\lambda, b}}{E_b} d\lambda + 0 \int_2^{\infty} \sim$$

$$= 0.8 F(0-2\mu\text{m})$$

$$\lambda T = 2(1500) = 3000$$

$$F_{0-2\mu\text{m}} = 0.273$$

$$T = 0.8(0.273) = 0.218$$

$$q = 0.218 (3.88 \times 10^{-2})$$

$$= \underline{8.47 \times 10^{-3} \text{ W}}$$

$$\begin{aligned} 23.5 \quad T_{\text{SOLAR}} &= \int \lambda F_{\lambda, \lambda_1, \lambda_2} \\ &= \int \lambda (F_{0-\lambda_2} - F_{0-\lambda_1}) \end{aligned}$$

FOR SOLAR IRRADIATION:

PLAIN GLASS :

$$\lambda_1 T = 0.3(5800) = 1740 \quad F = 0.033$$

$$\lambda_2 T = 2.5(5800) = 14500 \quad F = 0.966$$

23.5 CONTINUED -

$$T_s = 0.9 (0.966 - 0.033) = \underline{0.84}$$

TINTED GLASS :

$$\lambda_1 T = 0.5(5800) = 2900 \quad F = 0.25$$

$$\lambda_2 T = 1.5(5800) = 8700 \quad F = 0.881$$

$$T = 0.9 (0.881 - 0.25) = \underline{0.568}$$

IN THE VISIBLE RANGE :

$$\lambda_1 = 0.38 \mu\text{m} \quad \lambda_2 = 0.76 \mu\text{m}$$

{ FOR TINTED GLASS }  $\lambda_1 = 0.5 \mu\text{m}$

$$\lambda_1 = 0.38 \quad F_{0-\lambda_1 T} = 0.1017$$

$$\lambda_1 = 0.5 \quad F_{0-\lambda_1 T} = 0.250$$

$$\lambda_2 = 0.76 \quad F_{0-\lambda_2 T} = 0.550$$

$$\begin{aligned} \text{PLAIN GLASS: } T &= 0.9 (0.550 - 0.1017) \\ &= \underline{0.404} \end{aligned}$$

$$\begin{aligned} \text{TINTED GLASS: } T &= 0.9 (0.550 - 0.250) \\ &= \underline{0.27} \end{aligned}$$

$$\begin{aligned} 23.6 \quad \lambda_1 &= 0.8 \mu\text{m} \\ \lambda_2 &= 5 \mu\text{m}, \quad f_{\lambda_1 T - \lambda_2 T} = F_{0-\lambda_2 T} - F_{0-\lambda_1 T} \end{aligned}$$

T, K	$F_{0-\lambda_1 T}$	$F_{0-\lambda_2 T}$	$F_{\lambda_1 T - \lambda_2 T}$
500	-0	0.1613	0.1613
2000	0.0197	0.9142	0.8945
3000	0.1402	0.9689	0.8287
4500	0.4036	0.9894	0.586



23.7  $T = 5800 \text{ K}$

$$\text{for } \lambda_1 = 0.4 \mu\text{m} \quad \lambda_1 T = 2320$$

$$\lambda_2 = 0.7 \mu\text{m} \quad \lambda_2 T = 4060$$

$$F_{0-\lambda_1 T} = 0.1220 \quad F_{0-\lambda_2 T} = 0.4916$$

$$\left. \begin{array}{l} \text{FRACTION IN} \\ \text{VISIBLE RANGE} \end{array} \right\} = \underline{0.3696}$$

IN UV RANGE:  $0.01 < \lambda < 0.4$

$$F_{0-\lambda_1 T} = 0 \quad F_{0-\lambda_2 T} \approx 0.12$$

$$\left. \begin{array}{l} \text{FRACTION IN} \\ \text{UV RANGE} \end{array} \right\} \approx \underline{0.12}$$

IN IR RANGE  $0.4 < \lambda < 10^2$

$$F_{0-\lambda_1 T} = 0.12 \quad F_{0-\lambda_2 T} = 100$$

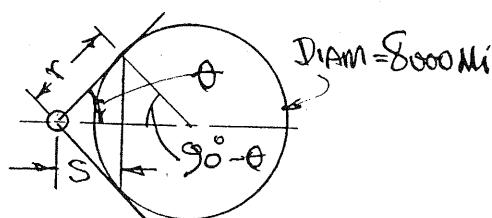
$$\left. \begin{array}{l} \text{FRACTION IN} \\ \text{IR RANGE} \end{array} \right\} \approx \underline{0.88}$$

WIEN'S LAW -

$$\lambda_{\max} T = 2897.6 \mu\text{m} \cdot \text{K}$$

$$\lambda_{\max} \approx \underline{0.500 \mu\text{m}}$$

23.8



$$\theta = \sin^{-1} \frac{4000}{4500} = 62.7^\circ = 1.096 \text{ RAD}$$

$$S = 4500 - 4000 \cos(90 - \theta) = 945 \text{ mi}$$

$$r = \frac{S}{\cos \theta} = 2060 \text{ mi}$$

23.8 CONTINUED -

AREA SUBTENDED BY EARTH

$$\Delta A = \int_0^{\pi/2} 2\pi r^2 \sin \theta d\theta = 2\pi r^2 (1 - \cos \theta)$$

$$= 2\pi r^2 (0.541) \text{ mi}^2$$

$$F_{S-E} = \frac{\Delta A}{4\pi r^2} = 0.271$$

$$f_{S-\text{SPACE}} = 0.729$$

$$\text{INCIDENT SOLAR ENERGY} = 450 \frac{\text{BTU}}{\text{HR} \cdot \text{FT}^2}$$

{from Prob 23.1}

$$q_{\text{SUN-SAT}} = 450 \left(\frac{\pi}{4}\right) \left(\frac{50}{12}\right)^2 = 6150 \frac{\text{BTU}}{\text{HR}}$$

$$q_{\text{ABSORBED BY SAT}} = 0.3 (6150) = 1845 \text{ "}$$

$$q_{\text{REFLECTED}} = 4305 \frac{\text{BTU}}{\text{HR}}$$

$$q_{E-\text{SAT}} = \epsilon A_s f_{S-E} (T_E)^4$$

$$= 0.195 \left[\left(\frac{\pi}{12}\right)^2\right] (0.271) (0.1714) (51)^4$$

$$= 164 \frac{\text{BTU}}{\text{HR}}$$

$$q_{\text{ABSORBED}} = 905 (164) = 8.2 \frac{\text{BTU}}{\text{HR}}$$

$$q_{\text{REFLECTED}} = 155.8 \frac{\text{BTU}}{\text{HR}}$$

$$q_{\text{EMITTED BY SAT.}} = 0.05 (0.1714) \left(\frac{T_S}{100}\right)^4 \pi \left(\frac{50}{12}\right)^2$$

$$= 0.467 \left(\frac{T_S}{100}\right)^4$$

ENERGY BALANCE:

$$6150 + 164 = 4305 + 155.8 + 0.467 \left(\frac{T_S}{100}\right)^4$$

$$T_S = \underline{794} \text{ R} = \underline{334} \text{ F}$$

23.9

$$\begin{aligned} \frac{q}{A}_{\text{NET}} &= \frac{q}{A}_{\text{IN}} - \frac{q}{A}_{\text{OUT}} \\ &= 1000 - h(T_s - T_\infty) - \epsilon\sigma \left[ \left(\frac{T_s}{100}\right)^4 - \left(\frac{T_\infty}{100}\right)^4 \right] \\ &= 1000 - 12(30-20) - 5.676(0.3)(100) \\ &= \underline{\underline{862 \text{ W/m}^2}} \end{aligned}$$

23.10

ENERGY BALANCE FOR COLLECTOR:

$$q_{\text{IN}} = 800 \text{ A W}$$

$$\begin{aligned} q_{\text{OUT}} &= q_{\text{RAD}} + q_{\text{COND}} \\ &= \sigma A (T^4 - T_\infty^4) + h A (T - T_\infty) \end{aligned}$$

EQUATING:

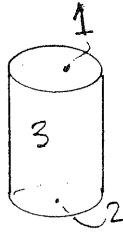
$$\begin{aligned} 800 &= \sigma (T^4 - T_\infty^4) + h (T - T_\infty) \\ &= 5.676 \left(\frac{T}{100}\right)^4 - 5.676(3,03)^4 \\ &\quad + 35T - 10605 \\ \left(\frac{T}{100}\right)^4 + 6.17T &= 2094 \end{aligned}$$

By TRIAL & ERROR:  $T = \underline{\underline{322 \text{ K}}}$ 

$$\begin{aligned} q_{\text{RAD}} &= \sigma A (T^4 - T_\infty^4) \\ &= 5.676(60) \left[ 3,22^4 - 3,03^4 \right] \\ &= \underline{\underline{7910 \text{ W}}} \end{aligned}$$

23.11

$$f_{12} = 0,12 \left( \begin{matrix} f_{16} \\ f_{23,14} \end{matrix} \right)$$



$$f_{13} = 0,88$$

$$\begin{aligned} f_{31} &= f_{32} = \frac{A_1 f_{13}}{A_3} \\ &= \frac{\pi D^2}{4} \frac{0,88}{\pi D L} = 0,17 \end{aligned}$$

$$F_{3-\text{SURF}} = 0,17$$

$$\begin{aligned} q_{3-\text{SURF}} &= \pi (0,075)(0,1)(0,17)(5,676)(T_{-3,1}^4) \\ &= \underline{\underline{105 \text{ W}}} \end{aligned}$$

23.12 ENTIRE HOLE (INTERIOR IS SURF 2  
OPENING (SURROUNDINGS) IS  $\times 1$ 

$$f_{12} = 1 \quad A f_{12} = \frac{\pi D^2}{4} (1)$$

$$\begin{aligned} q_{21} &= A_2 f_{21} \epsilon \sigma (T_2^4 - T_1^4) = A f_{12} \epsilon \sigma (T_2^4 - T_1^4) \\ &= \frac{\pi}{4} (0,075)^2 (1)(5,676)(T_{-3,1}^4) \\ &= \underline{\underline{57,9 \text{ W}}} \end{aligned}$$

$$23.13 \quad \frac{q}{A} = \frac{1200 \text{ W}}{5(0,49 \text{ m}^2)} = 490 \text{ W/m}^2$$

$$= \epsilon \sigma \left[ \left(\frac{T}{100}\right)^4 - 2,8^4 \right]$$

$$490 = 0,7(5,676) \left[ \left(\frac{T}{100}\right)^4 - 2,8^4 \right]$$

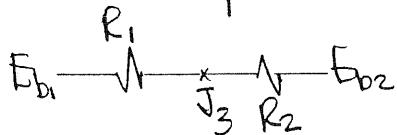
$$T = \underline{\underline{369 \text{ K}}}$$

23.14 WITH NO INTERVENING PLATE:

$$q_{f12} = A_f J_{12} \sigma (T_1^4 - T_2^4)$$

$$\frac{q}{A} = 5,676 \left[ 9^4 - 5,8^4 \right] = \underline{\underline{30,8 \text{ kW/m}^2}}$$

WITH INTERVENING PLATE PRESENT:



PER UNIT AREA:

$$\frac{q}{A} = \frac{E_{B1} - E_{B2}}{\frac{1}{F_{13}} + \frac{1}{F_{23}}} = \frac{E_{B1} - E_{B2}}{2}$$

$$= \underline{\underline{15,4 \text{ kW/m}^2}}$$

$$\frac{q}{A} = (E_{B1} - J_3) \frac{1}{F_{13}} = (J_3 - E_{B2}) \frac{1}{F_{23}}$$

$$J_3 = \sigma T_1^4 - \frac{q}{A} = \underline{\underline{15,4 \text{ kW/m}^2}}$$

$$= \sigma T_3^4$$

$$T_3^4 = \frac{15,4 \times 10^3}{\sigma} \quad T_3 = \underline{\underline{722 \text{ K}}}$$

{EMISSIVITY OF INTERVENING PLATE}  
HAS NO EFFECT

23.15 FILAMENT AT 2910 K

$$q = 100 \text{ W}$$

$$\lambda_{MAX} = \frac{2897,6}{2910} = \underline{\underline{0,999 \mu\text{m}}} \text{ a)}$$

VISIBLE RANGE:  $0,38 < \lambda < 0,76$

$$\lambda T_1 = 0,38(2910) = 1102 \quad F_{0-\lambda} = 0,0009$$

$$\lambda T_2 = 0,76(2910) = 2204 \quad F_{0-\lambda} = 0,1017$$

$$\text{FRACTION IN V.R.} = \underline{\underline{0,1008}} \quad \text{b)}$$

23.16 FOR SURROUNDINGS AT 0 K:

$$E_B = \sigma T^4 = (5,676)(20)^4 = 9,08 \times 10^5 \text{ W/m}^2$$

$$100 \text{ W} = 9,08 \times 10^5 \text{ A}$$

$$A = 1,109 \times 10^{-2} \text{ m}^2 = \pi D^2 / 4$$

$$D = 0,01188 \text{ m} = 1,188 \text{ cm} \quad \text{a)}$$

IN VISIBLE RANGE:  $0,4 < \lambda < 0,7 \mu\text{m}$

$$\lambda T_1 = 2000(0,4) = 800 \quad F \approx 0$$

$$\lambda T_2 = 2000(0,7) = 1400 \quad F = 0,0078$$

$$\text{FRACTION} = \underline{\underline{0,0078}} \quad \text{b)}$$

IN UV RANGE:  $0 < \lambda < 0,4$

$$\lambda T_1 = 0 \quad \text{FRACTION} = \underline{\underline{0}} \quad \text{c)}$$

IN IR RANGE:  $0,4 < \lambda < 100$

$$\lambda T_1 = 0,0078$$

$$\lambda T_2 \approx 1,0 \quad \text{FRACTION} = \underline{\underline{0,992}} \quad \text{d)}$$

23.17

$q = 8 \text{ W}$  THROUGH HOLE WITH  $D = 0,0025 \text{ m}^2$

$$E_B = \frac{8}{0,0025} = 3200 \text{ W/m}^2 = \sigma T^4$$

$$T = 487 \text{ K}$$

23.18

$$\lambda_{MAX} T = 2897,6 \mu\text{m} \cdot \text{K}$$

SUN	570 K
L. BOULB	2910 K
SURFACE	1550 K
SKIN	308 K

$\lambda_{MAX}$
1998 $\mu\text{m}$
1,004 "
0,535 "
0,1063 "

23.19  $T = 1500K$  Freehole  $D = 10\text{ cm}$

$$\sigma = 0.78 \text{ for } 0 < \lambda < 3.2 \mu\text{m}$$

$$0.08 \quad 3.2 < \lambda < \infty$$

$$q_{\max} = 5.676 \left(\frac{\sigma}{4}\right) \left(\frac{\pi}{4}\right) (90)^2$$

$$= 22.57 \text{ W}$$

$$\text{for } \Delta T_1 = 0 \quad f_{0-\Delta T_1} = 0$$

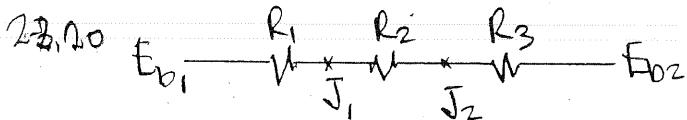
$$\Delta T_2 = 4800 \quad f_{0-\Delta T_2} = 0.6075$$

$$\Delta T_3 = \infty \quad f_{0-\Delta T_3} = 1$$

TOTAL HT LOSS

$$= 22.57 \left[ 0.78(0.6075) + 0.08(0.3925) \right]$$

$$= \underline{\underline{11.40 \text{ W}}}$$



1 IS INNER CYLINDER

2 " OUTER "

$$E_{b1} = \sigma (\pi)^4 = 2.0 \text{ W/m}^2$$

$$E_{b2} = \sigma (300)^4 = 4600 \text{ "}$$

$$R_1 = \frac{S_1}{A_1 E_1} = \frac{0.8}{\pi (0.02)(1)(0.2)} = 63.7 \text{ m}^{-2}$$

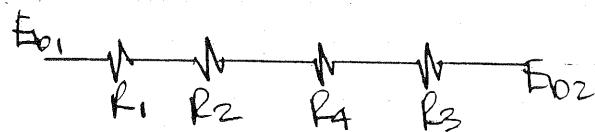
$$R_2 = \frac{1}{A_2 f_{\beta 2}} = \frac{1}{\pi (0.02)(1)} = 15.9 \text{ "}$$

$$R_3 = \frac{S_2}{A_2 E_2} = \frac{0.95}{\pi (0.05)(1)(0.05)} = 121 \text{ "}$$

$$\sum R = 201 \text{ m}^{-2}$$

$$q = \frac{E_{b2} - E_{b1}}{\sum R} = \frac{4600 - 2}{201} = \underline{\underline{228 \text{ W/m}}}$$

23.20 CONT. - WITH RADIATION SHIELD



$$R_1 = 63.7$$

$$R_3 = 121$$

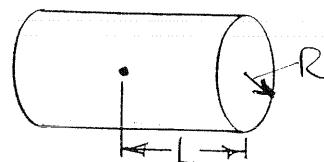
$$R_2 = 15.9$$

$$R_4 = \frac{1}{A_2 f_{\beta 2}} = \frac{1}{\pi (0.05)(1)} = 9.09$$

$$\sum R = 209.7$$

$$q = \frac{460 - 2}{209.7} = \underline{\underline{2.18 \text{ W/m}}}$$

23.21 ASSUMING THERMOCOUPLE AT GEOMETRIC CENTER OF DUCT



SOLID ANGLE OF DUCT OPENING

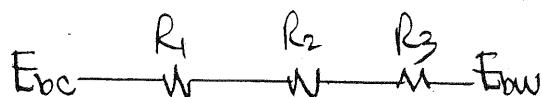
$$\Omega \cong \frac{\pi R^2}{\pi R_0^2} = \frac{\text{DUCT AREA}}{\text{HEMISPHERE SURFACE}}$$

$$= \frac{(15/12)^2}{1^2} = 0.0156$$

{ THERMOCOUPLE SEES DUCT, PRIMARILY }

FOR THERMOCOUPLE:  $\dot{q}_{\text{RAD}} = q_{\text{FOW}}$

$$A_f \dot{q}_{\text{FOW}} (E_{bc} - E_{ow}) = hA (T_b - T_c)$$



23.21 CONT.

$$A_c \frac{f_{cw}}{f_{cw}} = \frac{1}{\frac{s_c}{A_c E_c} + \frac{1}{A_c f_{cw}} + \frac{s_w}{A_w E_w}}$$

$$\frac{f_{cw}}{f_{cw}} = \frac{1}{\frac{s_c/E_c}{s_c/E_c + 1/f_{cw} + s_w/A_w E_w}}$$

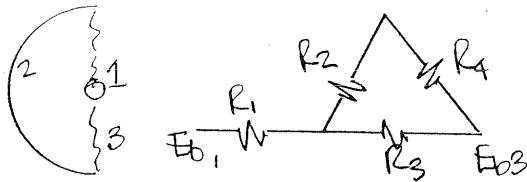
$$f_{cw} \approx 1 \quad A_c/A_w \approx 0$$

$$\therefore \frac{f_{cw}}{1 - E_c} = \frac{1}{\frac{E_c}{1 - E_c} + 1} \approx E_c = 0.6$$

$$30(T_b - T_c) = E_c(0.1714) \left[ \left(\frac{T_c}{100}\right)^4 - \left(\frac{T_w}{100}\right)^4 \right]$$

$$T_c = 316 F$$

23.22



$$R_1 = \frac{s_1}{A_1 E_1} = \frac{0.2}{\pi/6(0.8)} = \frac{1.5}{\pi}$$

$$R_2 = \frac{1}{A_1 F_{12}} = \frac{1}{\pi/6(0.5)} = \frac{12}{\pi}$$

$$R_3 = \frac{1}{A_2 F_{13}} = \frac{1}{\pi/6(0.5)} = \frac{12}{\pi}$$

$$R_4 = \frac{1}{A_2 F_{23}} = \frac{1}{A_3 F_{32}} = \frac{1}{1.5 - 0.167} = 0.75$$

$$R_{\text{equiv}} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_2 + R_4}} = 2.08$$

23.22 CONT.

$$\sum R = 1.5/\pi + 2.08 = 2.557$$

$$q = \frac{0.1714 (24.6^4 - 5.3^4)}{2.557} = 24,500 \frac{\text{Btu}}{\text{HR ft}} \quad (a)$$

WITH NO REFLECTOR:

$$q = \epsilon A (E_{b1} - E_{b3})$$

$$= 0.8 \left(\frac{\pi}{6}\right) (0.1714) (24.6^4 - 5.3^4) \\ = 25,500 \frac{\text{Btu}}{\text{HR ft}} \quad (b)$$

23.23



$$f_1 = \frac{s_p}{A_p E_p} = \frac{0.3}{\pi(1/4)(1)} = 0.546$$

$$R_2 = \frac{1}{A_p f_{pw}} = \frac{1}{\pi(1/4)(1)} = 1.273$$

$$R_3 = \frac{s_w}{A_w E_w} = \frac{0.2}{(A_w) E_w} \approx \text{Very Small}$$

$$q = 0.1714 (6.65^4 - 5.3^4) \\ = 1.819$$

$$= 110 \frac{\text{Btu}}{\text{HR ft}} \text{ per foot} \left\{ \begin{array}{l} \text{RADIANT} \\ \text{LOSS} \end{array} \right\}$$

CONVECTION:

$$q = h A \Delta T$$

$$h = \frac{k}{D} \left\{ 0.60 + \frac{0.387 R_a^{1/4}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{1/6} \right]^{1/27}} \right\}^2$$

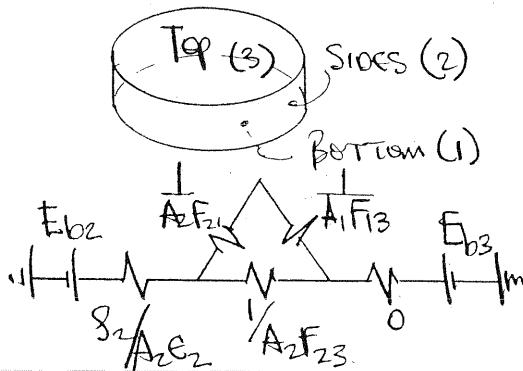
23.23 (CONTINUED -

$$\text{for } T_f = Bf_f$$

$$h = 1,21 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot \text{F}}$$

$$q_{\text{flow}} = 1,21 (\pi \times \frac{1}{4})(15) = 128 \frac{\text{Btu}}{\text{hr}}$$

23.24



$$\{ \text{Fig 23.14}\} \quad F_{13} = 0,38 \quad F_{12} = 0,62$$

$$A_2 f_{12} = A_2 f_{21} \quad F_{21} = f_{23} = A_1 f_{12} / A_2$$

$$\frac{1}{A_2 f_{12}} = \frac{1}{A_1 f_{12}} = \frac{1}{\pi(6)^2(0,62)}$$

$$\frac{1}{A_2 f_{13}} = \frac{1}{\pi(6)^2(0,38)}$$

$$\frac{1}{A_2 f_{23}} = \frac{1}{A_3 f_{32}} = \frac{1}{A_1 f_{12}} = \frac{1}{A_2 f_{21}} = \frac{1}{\pi(6)^2(0,62)}$$

$$\begin{aligned} \frac{1}{P_{\text{flow}}} &= A_2 f_{23} + \frac{1}{A_2 f_{21}} + \frac{1}{A_1 f_{13}} \\ &= \pi(6)^2(0,62) + \frac{1}{\pi(6)^2(0,62)} + \frac{1}{\pi(6)^2(0,38)} \\ &= \pi(6)^2(0,856) \end{aligned}$$

23.24 (CONTINUED -

$$\frac{g_e}{A_2 e_2} = \frac{0,2}{\pi(6)^2(0,856)} = \frac{1}{\pi(6)^2(8)}$$

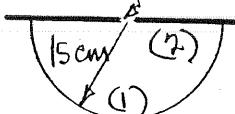
$$\begin{aligned} \sum R &= \frac{1}{\pi(6)^2(0,856)} + \frac{1}{\pi(6)^2(8)} \\ &= \frac{1,29}{\pi(6)^2} \end{aligned}$$

$$q_f = \frac{\sigma (T_2^4 - T_1^4)}{\sum R}$$

$$= \frac{0,1714 (\pi \times 6)^2}{1,29} (10^4 - 5^4)$$

$$= 140,900 \frac{\text{Btu}}{\text{hr}}$$

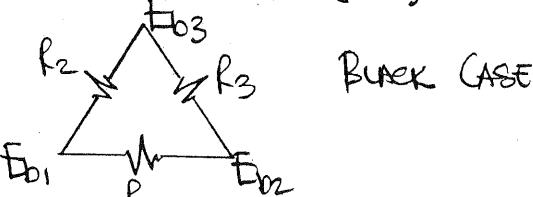
23.25 (3) Diam of hole = 5 cm



$$f_{11} + f_{12} + f_{13} = 1 \quad f_{24} = f_{31} = 1$$

$$f_{12} = f_{21} \frac{A_2}{A_1} = \frac{\pi (0,15^2 - 0,025^2)}{2\pi (0,15^2)} = 0,486$$

$$f_{13} = f_{31} \frac{A_3}{A_1} = \frac{\pi (0,25^2)}{2\pi (0,15^2)} = 0,0139$$



$$R_1 = \frac{1}{A_1 f_{12}} = \frac{1}{2\pi (0,15^2)(0,486)} = 14,55$$

$$R_2 = \frac{1}{A_1 f_{13}} = \frac{1}{2\pi (0,15^2)(0,0139)} = 509$$

$$R_3 = \infty$$

23.25 (CONTINUED -

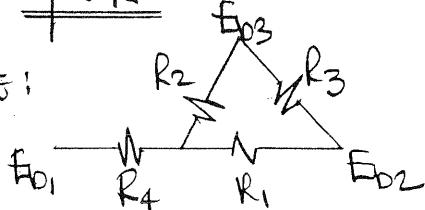
$$q_{f3} = A_f F_{f3} E_{b1} = A_3 F_{31} E_{b3}$$

$$= \frac{\pi (0.05)^2}{4} (1)(5.676)(7^4)$$

$$= \underline{26.7 \text{ W}}$$

$$T_2 = T_1 = \underline{700 \text{ K}}$$

GRAY CASE:

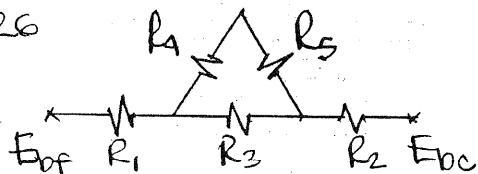


$$R_4 = \frac{S_1}{A_1 G_1} = \frac{0.3}{2\pi(0.15^2)(0.7)} = 3.03$$

$$q_f = \frac{E_{b1} - 0}{\sum R} = \frac{0.714}{512.03} = \underline{26.7 \text{ W}}$$

$$T_2 = T_1 = \underline{700 \text{ F}}$$

23.26



{WALLS ASSUMED TO BE AT  
A UNIFORM TEMPERATURE}

$$R_1 = \frac{0.2}{12(20)(0.8)} = 0.00104$$

$$R_2 = S_{AW} = 0.00104$$

$$R_3 = \frac{1}{A_f F_f - C} = \frac{1}{(12)(20)(0.45)} = 0.00903$$

$$R_4 = \frac{1}{A_f F_f - W} = \frac{1}{(12)(20)(0.55)} = 0.0076$$

$$R_5 = R_4 = 0.0076$$

23.26 (CONTINUED)

$$R_{EQVN} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4 + R_5}} = 0.0058$$

$$\sum R = R_1 + R_2 + R_{EQVN} = 0.00785$$

$$q_f = \frac{0.714 (7^4 - 5.25^4)}{0.00785} = \underline{2680 \text{ BTU/HR}}$$

23.27



{EQUIVALENT CIRCUIT}

$$R_1 = \frac{S_1}{A_1 G_1}, \quad R_2 = \frac{1}{A_2 F_{12}} = \frac{1}{A_2 F_{21}}, \quad R_3 = \frac{S_2}{A_2 G_2}$$

$$T_1 = 300 \text{ K} \quad T_2 = 78 \text{ K}$$

$$A_1 = \pi D_1^2 = \pi (1.3)^2 = 1.69\pi \text{ m}^2$$

$$A_2 = \pi D_2^2 = \pi (1)^2 = \pi \text{ m}^2$$

$$R_1 = \frac{0.8}{(1.69\pi)(0.2)} = \frac{1.37}{\pi} \text{ m}^{-1}$$

$$R_2 = \frac{1}{\pi (1)^2 (1)} = \frac{1}{\pi} \text{ m}^{-1}$$

$$R_3 = \frac{0.8}{\pi (0.2)} = \frac{4}{\pi} \text{ m}^{-1}$$

$$\sum R = \frac{1.37}{\pi} = 2.35 \text{ m}^{-1}$$

23.27 (CONTINUED -

$$q_f = \frac{E_{b1} - E_{b2}}{\sum R} = \sigma (T_1^4 - T_2^4)$$

$$= \frac{5.676 (3^4 - 0.78^4)}{2.35} = 194.8 \text{ W}$$

Boil-off rate:  $\dot{m} = q_f / h_{fg}$

$$\dot{m} = \frac{194.8}{2 \times 10^5} = 9.74 \times 10^{-4} \text{ kg/s}$$

$$= 3.51 \text{ kg/hr}$$

23.28 OPENING DIAM = 5 mm

(a) EQUIV. SURFACE ① SEES

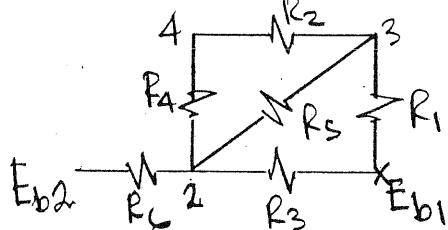
INTERIOR AS A SINGLE SURFACE

$$q_f = \frac{E_{b2} - E_{b1}}{R} = \sigma T_2^4 - 0$$

$$= \frac{\pi}{4} (5)^2 (5.676) (6)^4 \left(10^{-6} \frac{\text{m}^2}{\text{mm}^2}\right)$$

$$= 0.144 \text{ W}$$

b) ANALOG CIRCUIT:



$$f_1 = \frac{1}{A_1 F_{13}} = 0.060 \text{ mm}^{-2} \quad F_{12} \approx 0.15$$

$$F_2 = \frac{1}{A_2 F_{24}} = 1.61 \times 10^{-3} \quad F_{13} = 0.85$$

$$F_3 = \frac{1}{A_3 F_{34}} = 0.339 \quad F_{23} \approx 0.1$$

$$F_4 = \frac{1}{A_4 F_{24}} = 0.0148 \quad F_{24} = 4.167 \times 10^{-3}$$

23.28 (CONTINUED -

$$F_5 = \frac{1}{A_2 F_{23}} = 1.57 \times 10^{-3} \quad F_{24} = 0.0958$$

$$F_6 = \frac{S_2}{A_2 e_2} = 9.43 \times 10^4 \quad F_{42} = 0.0985$$

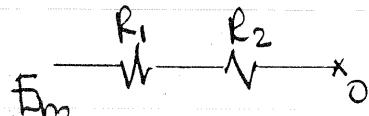
$$F_{43} = 0.9015$$

$$\frac{1}{R_{23\text{EQUIV}}} = \frac{1}{R_5} + \frac{1}{R_4 + R_2} \quad R_{23\text{EQ}} = 1433 \times 10^{-3}$$

$$\frac{1}{R_{21\text{EQUIV}}} = \frac{1}{R_3} + \frac{1}{R_5 + R_1} \quad R_{21\text{EQ}} = 0.052$$

$$q_f = \frac{\sigma T^4}{R_6 + R_{21\text{EQUIV}}} = 0.139 \text{ W}$$

(c) ALL INTERIOR MAY BE CONSIDERED  
A SINGLE SURFACE -



$$R_2 = \frac{1}{A_1 F_{12}} = \frac{1}{A_1} = 0.0509$$

$$R_1 = \frac{S}{Ae} = \frac{0.4}{A_1 (0.6)} = 1.291 \times 10^{-4}$$

$$q_f = \frac{\sigma T^4}{\sum R} = 0.144 \text{ W}$$

(SLIGHTLY LESS THAN  
IN PART (a))

23.29 PROBLEM STATEMENT ASKS FOR  
RADIANT ENERGY REACHING TANK  
BOTTOM - I.E., THE IRRADIATION

a)  $q_f^{\text{TOTAL}} = q_f^{\text{from HTR}} + q_f^{\text{from SPACE}}$

23.29 CONTINUED

$$q_{\text{HTR}} = A_1 F_{12} E_{b1}$$

$$\left\{ f_{12} \approx 0,39 - \text{fig 23.15} \right\}$$

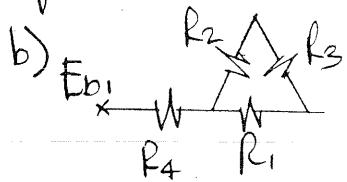
$$= \frac{\pi}{4} (0,2)^2 (0,39) \sigma T_1^4$$

$$= 1826 \text{ W}$$

$$q_{\text{space}} = \frac{\pi}{4} (0,2)^2 (0,61) \sigma T_2^4$$

$$= 8,8 \text{ W}$$

$$q_{\text{TOTAL}} \approx \underline{1835 \text{ W}}$$



$$R_1 = \frac{1}{A_1 F_{12}} = \frac{1}{\frac{\pi}{4} (0,2)^2 (0,39)} = 81,62$$

$$R_2 = \frac{1}{A_1 F_{13}} = \frac{1}{\frac{\pi}{4} (0,2)^2 (0,61)} = 52,18$$

$$R_3 = \frac{1}{A_2 F_{23}} = R_2 = 52,18$$

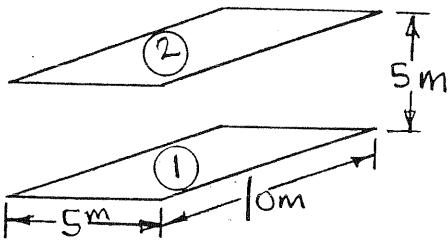
$$R_4 = \frac{S_1}{A_1 E_1} = \frac{0,4}{\frac{\pi}{4} (0,2)^2 (0,6)} = 21,22$$

$$\frac{1}{R_{12 \text{ equiv}}} = \frac{1}{R_1} + \frac{1}{R_2 + R_3} \quad R_{12 \text{ equiv}} = 45,8$$

$$\sum R = R_4 + R_{12 \text{ equiv}} \approx 67$$

$$q_{\text{REACHING SURF}} = \frac{\sigma T^4}{67} = \underline{2224 \text{ W}}$$

23.30

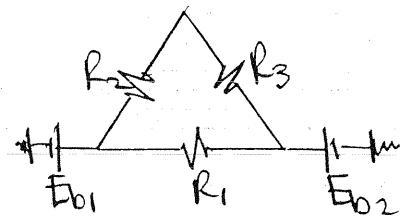


SURROUNDINGS ARE CONSIDERED AN EQUIVALENT SURFACE ③ AT 0K

$$T_1 = 100 \text{ K} \quad f_{12} \approx 0,28 \quad \left\{ \text{fig 23.14} \right\}$$

$$T_2 = 200 \text{ K} \quad f_{13} = 0,72$$

$$T_3 = 0 \text{ K} \quad f_{23} = 0,72$$



$$R_1 = \frac{1}{50(0,28)} = 0,0714$$

$$R_2 = R_3 = \frac{1}{50(0,72)} = 0,0278$$

$$(a) q_{12} = \frac{E_{b1} - E_{b2}}{R_1} = \frac{\sigma (T_1^4 - T_2^4)}{R_1} = \underline{-1192 \text{ W}}$$

$$(b) q_1 = q_{12} + q_{13}$$

$$q_{12} = -1192$$

$$q_{13} = \frac{E_{b1} - 0}{R_2} = \frac{E_{b1}}{R_2} = 204 \text{ W}$$

$$q_1 = \underline{-988 \text{ W}}$$

23.30 CONTINUED-

$$q_2 = q_{f21} + q_{f23}$$

$$q_{f21} = 1192$$

$$q_{f23} = \frac{E_{b2}-0}{R_3} = 3270 \text{ W}$$

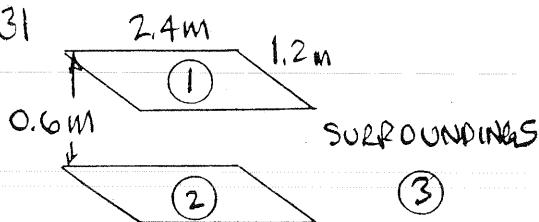
$$q_2 = \underline{4462 \text{ W}}$$

c) To SURROUNDINGS

$$q_{f13} = \underline{204 \text{ W}}$$

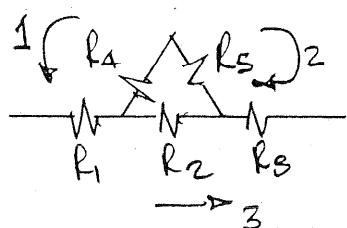
$$q_{f23} = \underline{3270 \text{ W}}$$

23.31



$$\epsilon_1 = 0.6 \quad T_1 = 1000 \text{ K} \quad A_1 = 2.88 \text{ m}^2$$

$$\epsilon_2 = 0.9 \quad T_2 = 400 \text{ K} \quad A_2 = "$$



$$\left\{ f_{12} \approx 0.5 \right\}$$

{NOTE 3 "Loops"}

$$R_1 = \frac{\epsilon_1}{A_1 \epsilon_1} = 0.231 \quad R_2 = \frac{1}{A_1 f_{12}} = 0.694$$

$$R_3 = \frac{\epsilon_2}{A_2 \epsilon_2} = 0.039 \quad R_4 = R_5 = \frac{1}{A_2 f_{13}} = 0.694$$

23.31 CONTINUED-

WRITING EQUATIONS FOR LOOPS AS SHOWN:

$$E_{b1}-0 = (I_1+I_3)R_1 + I_1R_4$$

$$E_{b2}-0 = (I_2-I_3)R_3 + I_2R_5$$

$$E_{b1}-E_{b2} = (I_1+I_3)R_1 + I_3R_2 + (I_3-I_2)R_3$$

SUBSTITUTING VALUES & SOLVING SIMULTANEOUS EQUATIONS.

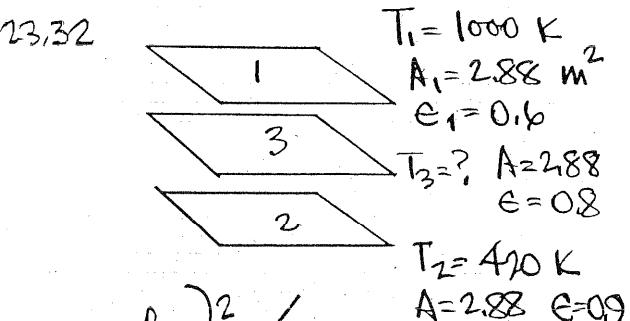
$$I_1 = 59550 \quad I_2 = 4695 \quad I_3 = 42970$$

$$Q_{NET} = I_1 + I_3 = \underline{102.5 \text{ kW}}$$

$$Q_{f12} = I_3 = \underline{4297 \text{ kW}}$$

{ THESE RESULTS PRESUME NO HT TX FROM OTHER SIDES OF PLATES }

23.32



$$T_1 = 1000 \text{ K}$$

$$A_1 = 2.88 \text{ m}^2$$

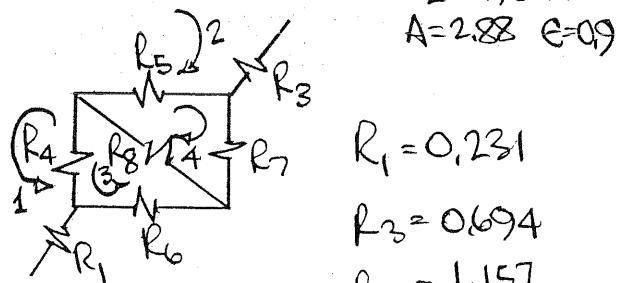
$$\epsilon_1 = 0.6$$

$$T_3 = ? \quad A = 2.88$$

$$\epsilon = 0.8$$

$$T_2 = 420 \text{ K}$$

$$A = 2.88 \quad \epsilon = 0.9$$



$$R_1 = 0.231$$

$$R_3 = 0.694$$

$$R_4 = 1.157$$

$$R_7 = 0.496$$

$$R_5 = 1.157$$

$$R_8 = 0.579$$

$$R_6 = 0.496$$

23.32 (CONTINUED -

EQUATIONS for Loops Shown:

$$E_{b1} - 0 = I_1 R_1 + (I_1 - I_3) R_4$$

$$E_{b2} - 0 = I_2 R_3 + (I_2 - I_4) R_5$$

$$0 = (I_3 - I_1) R_4 + I_3 R_6 + (I_3 + I_4) R_8$$

$$0 = (I_3 + I_4) R_8 + I_4 R_7 + (I_4 - I_2) R_5$$

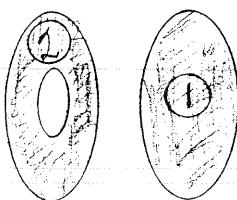
$$\text{SOLVING: } I_1 = 62100 \quad I_2 = 25500$$

$$I_3 = 16940 \quad I_4 = 25600$$

$$Q_f = \underline{62.1 \text{ kW}}$$

23.33

Pole (3)



From figs 23.14 & 23.15

$$F_{(2+3)} - 1 = f_{1-(2+3)} \approx 0.04$$

$$f_{31} \approx 0.04$$

$$f_{13} = 0.04 \frac{(2.5)^2}{(4)^2} = 0.0156$$

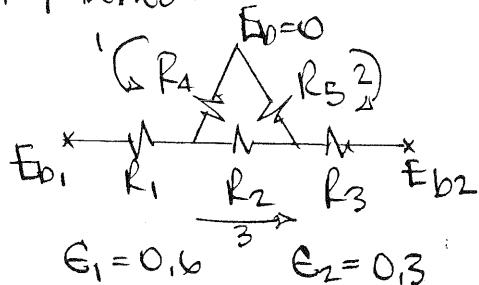
$$f_{12} = 0.04 - 0.0156 = 0.0244$$

for BACK DISKS:

$$\begin{aligned} Q_{12} &= A_1 f_{12} \sigma (T_4 - T_2^4) \\ &= \frac{\pi}{4} \left(\frac{4}{12}\right)^2 (0.0244) (0.1714) \\ &\quad \times (9.6^4 - 6.7^4) \\ &= \underline{2.36 \text{ BTU/HR}} \quad (\text{a}) \end{aligned}$$

23.33 (CONTINUED -

for GRAY bodies:



$$R_1 = \frac{S_1}{A_1 \epsilon_1} = \frac{0.4}{\frac{\pi}{4} \left(\frac{4}{12}\right)^2 (0.6)} = 76.4$$

$$R_2 = \frac{1}{A_1 f_{12}} = 470$$

$$R_3 = \frac{S_2}{A_2 \epsilon_2} = 65$$

$$R_4 = \frac{1}{A_2 f_{1-S}} = 11.74$$

$$R_5 = \frac{1}{A_2 f_{2-S}} = 19.16$$

for Loops Shown:

$$E_{b1} - 0 = I_1 R_1 + I_1 R_4 + I_3 R_1$$

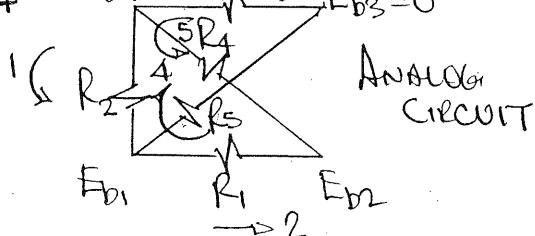
$$E_{b2} - 0 = I_2 R_3 + I_2 R_5 - I_3 R_3$$

$$E_{b1} - E_{b2} = (I_1 + I_3) R_1 + I_3 R_2 + (I_3 - I_2) R_3$$

SOLVING SIMULTANEOUSLY!

$$Q_{1-2} = I_3 = \underline{1.67 \text{ BTU/HR}} \quad (\text{b})$$

23.34  $E_{b4} = R_3 \rightarrow 3 \quad E_{b3} = 0$



23.34 CONTINUED-

$$R_1 = 470$$

$$E_{b1} = 14600$$

$$R_2 = 11.94$$

$$E_{b2} = 345$$

$$R_3 = 30.6$$

$$E_{b3} = 0$$

$$R_4 = 735$$

A IS ADIABATIC

$$R_5 = 19.6$$

FOR BLACK SURFACES:

$$q_{f12} = \frac{E_{b1} - E_{b2}}{R_{\text{ADUV},12}}$$

$$R_{\text{ADUV},12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_4}} = 288$$

$$q_{f12} = \frac{1460 - 345}{288} = 3.87 \text{ BTU}$$

$$q_{\text{FROST}} \text{ THROUGH FROST} = q_{f13} = \frac{E_{b1} - 0}{R_{\text{ADUV}}}$$

$$R_{\text{ADUV}} = \frac{1}{\frac{1}{R_5} + \frac{1}{R_1 + R_2}} = 18.83$$

$$q_{\text{FROST}} = \frac{1460}{18.83} = 77.5 \text{ BTU}$$

FOR GREY SURFACES:  $\epsilon_1 = 0.6$   $\epsilon_2 = 0.3$

ADDITIONAL RESISTANCES  $R_A$ ,  $R_B$

$$R_A = \frac{\epsilon_1}{A \epsilon_1} = \frac{0.4}{\frac{\pi}{4} \left(\frac{d_2}{12}\right)^2 0.6} = 7.64$$

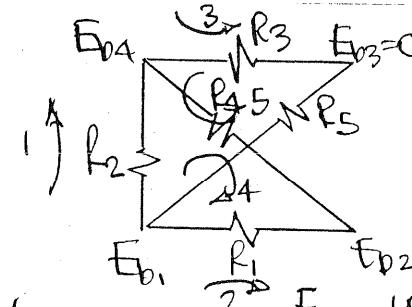
$$R_B = \frac{0.3}{\frac{\pi}{4} \left[\left(\frac{d_2}{12}\right)^2 - \left(\frac{d_5}{12}\right)^2\right] 0.1} = 8.06$$

23.34 CONTINUED-

$$q_{f12} = \frac{1460 - 345}{288 + 7.64 + 8.06} = 3.67 \text{ BTU}$$

$$q_{\text{FROST}} = \frac{1460}{18.83 + 7.64} = 55.2 \text{ BTU}$$

23.35



$$\begin{aligned} R_1 &= 470 & E_{b1} &= 1460 \\ R_2 &= 11.94 & R_4 &= 735 & E_{b2} &= 345 \\ R_3 &= 30.6 & R_5 &= 19.6 & E_{b3} &= 0 \\ &&&&& E_{b4} = 738 \end{aligned}$$

VERTICAL LOOP EQUATIONS:

$$1: E_{b1} - E_{b4} = R_2(I_1 + I_4 + I_5)$$

$$2: E_{b4} - 0 = R_3(I_3 - I_5)$$

$$3: E_{b1} - E_{b2} = R_1(I_2 - I_4)$$

$$4: 0 = R_1(I_4 - I_2) + R_2(I_4 + I_1 - I_5) + I_4 R_4$$

$$5: 0 = R_3(I_5 - I_3) + R_2(I_5 - I_1 - I_4) + I_5 R_5$$

SOLVING:

$$I_1 = 134.8 \quad I_3 = 98.7 \quad I_5 = 74.6$$

$$I_2 = 2.84 \quad I_4 = 0.47$$

$$q_{f12} = I_2 = 2.84 \text{ BTU}$$

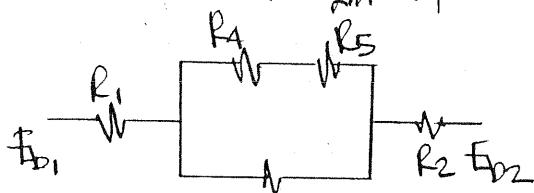
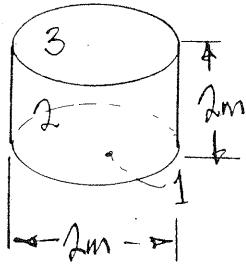
$$q_{\text{FROST}} = I_3 + I_5 = 173.3 \text{ BTU}$$

23.36

{fig 23.14}

$$F_{13} = 0.18$$

$$F_{12} = 0.82$$



$$R_1 = \frac{S_1}{A_1 E_1} = \frac{0.69}{\frac{\pi}{4}(2)^2(0.31)} = 0.708 \text{ m}^{-2}$$

$$R_2 = \frac{S_2}{A_2 E_2} = 0.996 \text{ m}^{-2}$$

$$R_3 = \frac{1}{A_1 F_{12}} = 0.388 \text{ m}^{-2}$$

$$R_4 = \frac{1}{A_2 F_{13}} = 1.77 \text{ m}^{-2}$$

$$R_5 = 0.388 \text{ m}^{-2}$$

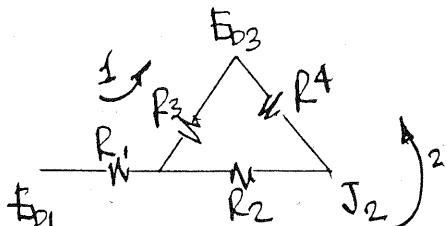
$$R_{\text{parallel}} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4 + R_5}} = \frac{1}{\frac{1}{0.388} + \frac{1}{1.77 + 0.388}} = 0.329$$

$$\sum R = 0.708 + 0.329 + 0.996 = 2.03 \text{ m}^{-2}$$

$$q = \frac{E_{D1} - E_{D2}}{\sum R} = \frac{5.676}{2.03} (7.55^4 - 3.95^4)$$

$$= 8400 \text{ W} = 8.4 \text{ kW}$$

23.37



{SURFACE 3 IS SURROUNDED}

23.37 (CONTINUED)

$$A_1 = A_2 = \frac{\pi}{4} (0.15)^2 = 0.0177 \text{ m}^2$$

$$R_1 = \frac{S_1}{A_1 E_1} = \frac{0.2}{A(0.8)} = 14.12$$

{fig 23.14 }  $F_{12} \approx 0.37 \Rightarrow F_{13} = 0.18$ 

$$R_2 = \frac{1}{A_2 F_{12}} = 153$$

$$f_3 = \frac{1}{A F_{13}} = 89.7$$

$$f_4 = \frac{1}{A_2 f_{23}} = 89.7$$

$$E_{D3} = 5 (3.5)^4 = 852 \text{ W/m}^2$$

Loop EQUATIONS:

$$E_{D1} - E_{D3} = (I_1 + I_2) R_1 + I_1 R_3$$

$$E_{D1} - E_{D3} = (I_1 + I_2) R_1 + I_2 (R_2 + R_4)$$

$$\text{SOLVING: } I_1 = 2.706 I_2$$

$$I_1 + I_2 = 300$$

$$\therefore I_2 \approx 81 \quad I_1 = 219$$

$$J_2 = E_{D2} + I_2 R_4$$

$$= 57.9 + 81(89.7) = 7319$$

$$J_1 = J_2 + I_2 R_2$$

$$J_1 = 7319 + 81(153) = 19710$$

$$E_{D1} = J_1 + 300(R_1)$$

$$= 19710 + 300(14.12)$$

$$= 123950$$

23.37 CONTINUED -

$$\text{FINALLY: } T_1 = \left(\frac{E_{D1}}{S}\right)^{\frac{1}{4}} = \underline{806 \text{ K}} \quad (a)$$

$$T_2 = \left(\frac{J_2}{S}\right)^{\frac{1}{4}} = \underline{599 \text{ K}} \quad (b)$$

$$q_{f, \text{SOAR}}^{\text{TO}} = I_1 + I_2 = \underline{300 \text{ W}} \quad (c)$$

$$q_{f,1-2} = I_2 = \underline{81 \text{ W}} \quad (d)$$

23.37 ALTERNATE SOLUTION USING EQUATIONS 23.37 & 23.38

APPLYING THEM TO EACH NODE:

$$300 = J_1 - f_{12}J_2 - f_{13}J_3$$

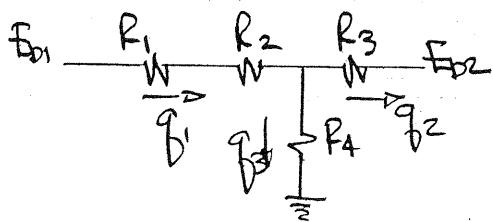
$A_1$

$$0 = J_2 - f_{21}J_1 - f_{23}J_3$$

$$E_{D3} = J_3$$

SOLVING THESE EQUATIONS SIMULTANEOUSLY GIVES SAME RESULTS AS ABOVE

23.38 Test Specimen is 1  
TUBE is 2  
VIEWING PORT W



$$R_1 = \frac{S_1}{A_1 E_1} = \frac{0.2}{0.833(0.8)} = 0.30$$

$$R_2 = \frac{1}{A_1 f_{12}} = \frac{1}{A_1} = 1.133 \quad \{f_{12} \approx 1\}$$

23.38 CONTINUED -

$$R_3 = \frac{S_2}{A_2 E_2} = \frac{0.77}{340(0.23)} = 9.85 \times 10^{-4}$$

$$R_4 = \frac{1}{A_W f_W}$$

$$A_1 = 0.883 \text{ in}^2 \quad A_W = 0.049 \text{ in}^2$$

$$A_2 = \frac{\pi}{4}(16) + 8\pi + 4\pi(24) = 340 \text{ in}^2$$

$$q_f^1 = q_f^2 + q_f^3$$

$$q_f^2 = A_2 E_2 \left( J_2 - E_{D2} \right)$$

$$q_f^3 = A_W J_2$$

$$q_f^1 = \frac{E_{D1} - J_1}{R_1 + R_2} = A_1 E_1 (E_{D1} - J_1)$$

$J_2$  BECOMES:

$$J_2 = \frac{E_{D1} + \frac{A_1}{A_2} \frac{E_2}{S_2 E_1} E_{D2}}{1 + \frac{A_2}{A_1} \frac{E_2}{S_2 E_1} + \frac{A_W}{A_1 E_1}}$$

$$E_{D1} = 131,500 \frac{\text{Btu}}{\text{HR ft}^2} \quad E_{D2} = 16$$

$$\Rightarrow J_2 = 344 \frac{\text{Btu}}{\text{HR ft}^2}$$

$$q_f^1 = \frac{0.883}{144} (0.2) (131,500 - 344)$$

$$= \underline{161 \frac{\text{Btu}}{\text{ft}^2}} \quad (b)$$

~ FROM SPECIMEN

$$q_f^3 = \frac{0.049}{144} (344) = 0.177 \frac{\text{Btu}}{\text{ft}^2} \quad (c)$$

~ LOSS THROUGH WINDOW

$$f_W \approx \frac{A_W f_W}{A_1} = \frac{\frac{1}{4}(16)}{2\pi(144)} \approx \underline{5 \times 10^{-5}} \quad (a)$$

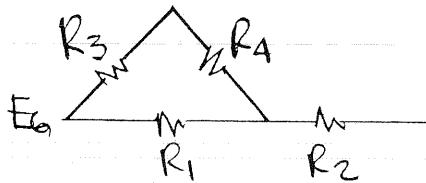
23.39  $q_{\text{gas-wall direct}}$   
 $= A_1 F_{1G} \alpha_G \sigma (T_G^4 - T_1^4)$

$q_{\text{gas to reradiating walls}}$   
 $= A_2 F_{2G} \alpha_G \sigma (T_G^4 - T_2^4)$

$q_{\text{reradiating walls to 1}}$   
 $= A_1 F_{12} \bar{J}_G \sigma (T_2^4 - T_1^4)$

$q_{f_{12}} = q_{f_{21}} = q_{f_R} = \frac{\sigma (T_G^4 - T_1^4)}{\frac{1}{A_1 F_{12} \bar{J}_G} + \frac{1}{A_2 F_{2G} \alpha_G}}$

$q_{\text{TOTAL TO 1}} = q_{f_{G1}} + q_{f_R}$



$$L = \frac{3A(0.2)(0.2)\chi(1)}{4(0.2)(1)} = 0.17 \text{ m}$$

$$p = 1 \text{ ATM}$$

$$\alpha_G = 0.22$$

$$P_L = 0.558 \text{ ATM-FT} \quad \bar{J}_G = 0.78$$

$$R_1 = \frac{1}{0.2(1)(0.22)} = 22.7$$

$$R_2 = \frac{0.2}{0.2(1)(0.8)} = 1.25$$

$$R_3 = \frac{1}{3(0.2)(1)(0.22)} = 7.58$$

$$R_4 = \frac{1}{0.2(1)(1)(0.78)} = 6.41$$

$$R_{\text{EQUIN}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_3 + R_4}} = 8.66$$

23.39 (CONTINUED -)

$$\sum R = 8.66 + 1.25 = 9.91$$

$$q = \frac{5.676 (6^4 - 4.2^4)}{9.91} = \frac{564 \text{ W}}{\text{per m}}$$

23.40  $q_{\text{NET}} = \sigma A (\epsilon_G T_G^4 - \alpha_G T_w^4)$

$A = 4\pi r^2 = \pi (3\text{m})^2 = 28.27$

$T_G = 1000 \text{ K} \quad T_w = 600 \text{ K}$

$L = 2/3 D = 2 \text{ m}$

$pL = 0.15 (5) (6.56) = 4.92 \text{ ATM-FT}$

$$\alpha_G = 0.18 \quad \epsilon_G = 0.22$$

$$q_{\text{NET}} = 5.676 (9\pi) [0.22(10^4) - 0.18(6)]$$

$$= \underline{\underline{316 \text{ kW}}}$$

23.41

$$\frac{q_{f_1} - \frac{q_{f_2} + q_{f_3}}{\Delta x}}{q_{f_3}}$$

$$q_{f_2} - q_{f_1} = A \nu S C_p (T_{x+\Delta x} - T_x)$$

$$q_3 = h P \Delta x (T - T_w)$$

$$+ A w F_{wG} \sigma \epsilon_w [E_G T_G^4 - \alpha_G T_w^4]$$

$$q_1 = q_2 + q_3$$

$$A \nu S C_p \frac{T_{x+\Delta x} - T_x}{\Delta x} + h P (T - T_w)$$

$$+ P F_{wG} \sigma \epsilon_w (E_G T_G^4 - \alpha_G T_w^4) = 0$$

23.41 (CONTINUED) -

IN LIMIT AS  $\Delta x \rightarrow 0$ :

$$ASV_{cp} \frac{\Delta T}{\Delta x} + hP(T-T_w) + P_{EW0}(\epsilon_g T_g^4 - \epsilon_g T_w^4) = 0$$

$$P_c = 0,20 \quad L = \frac{3,4 \text{ WD}}{4 \text{ WD}} = 0,425$$

$$P_c L = 0,085 \sim @ 2000 \text{ F} \quad \epsilon_g \approx 0,035$$

$$@ 1000 \quad \epsilon_g \approx 0,065$$

$$T_w = 1260 \text{ R} \quad \alpha_g = 0,071$$

$$\begin{aligned} \frac{\Delta T}{\Delta x} &= \left[ -\frac{hP}{ASV_{cp}}(T-T_w) - \frac{P_{EW0}\sigma(\epsilon_g T_g^4 - \epsilon_g T_w^4)}{ASV_{cp}} \right] \frac{1}{A} \\ &= \left\{ \frac{-1,5(2)(T-1260)}{0,4(0,28)} - \frac{2(0,9)(0,071)}{(0,4)(0,28)} \right. \\ &\quad \left. \times \left[ \epsilon_g \left( \frac{T_g}{100} \right)^4 - 0,071 \left( \frac{1260}{100} \right)^4 \right] \right\} \frac{4}{3600} \end{aligned}$$

$$\frac{\Delta T}{\Delta x} = \frac{1}{900} \left\{ -26,8(T-1260) - 2,75 \left[ \epsilon_g \left( \frac{T_g}{100} \right)^4 - 1772 \right] \right\}$$

$$\Delta x = -\left[ \frac{1}{-26,8(T-1260) - 2,75 \epsilon_g \left( \frac{T_g}{100} \right)^4 + 4870} \right] \times 900 \Delta T$$

BY GRAPHICAL INTEGRATION

$$x = \int_{1000}^{2000} \left[ \frac{1}{-26,8(T-1260) - 2,75 \epsilon_g \left( \frac{T_g}{100} \right)^4 + 4870} \right] 900 \Delta T = 35,2 \text{ ft} \quad (a)$$

$$Q_{TOTAL} = ASV_{cp} \Delta T$$

$$= 0,4 \left( \frac{1}{4} \right) (0,28) (1000)$$

$$= 28 \text{ BTU/s}$$

23.41 (CONTINUED) -

$$q_{conv} = hP(x)(T-800)$$

$$= \frac{1,5(2)x(T-800)}{3600} = x \frac{T-800}{1200}$$

$T_w$	$x$	$q_{conv,i}$
1900	3,2	2,93
1700	4,0	3,0
1500	5,4	3,15
1300	8,0	3,33
1100	14,6	3,65

$$q_{conv} = \sum q_i = 16,06 \text{ BTU/s}$$

$$q_{RAD} = 28 - 16,06 = 11,94 \text{ BTU/s}$$

$$\text{RADIANT FRACTION} = \frac{11,94}{28} = 0,43 \quad (b)$$

FOR U DOUBLED:

$$\Delta x = \left[ \frac{900}{-13,4(T-1260) - 1,375 \left[ \epsilon_g \left( \frac{T_g}{100} \right)^4 - 1772 \right]} \right] \Delta T$$

INTEGRATE GRAPHICALLY UNTIL  $x = 35,2$

AT THIS LOCATION  $T = 1265 \text{ F}$

## CHAPTER 24

### 24.1 BASIS 1g mole LNG

	g MOLE	MW	g	WT FRACTION
CH <sub>4</sub>	0.935	16	14.96	0.871
C <sub>2</sub> H <sub>6</sub>	0.046	30	1.38	0.080
C <sub>3</sub> H <sub>8</sub>	0.012	44	0.528	0.031
CO <sub>2</sub>	0.007	44	0.308	0.018

$$17.176 \quad 1.00$$

$$\text{WT FRACTION ETHANE} = \underline{0.080} \quad a)$$

$$\text{Ave. M.WT} = \underline{17.176 \text{ g/mole}} \quad b)$$

DENSITY:

$$\rho = \frac{PM}{RT} = \frac{1.4 \times 10^5 (17.176)}{8.314(900)}$$

$$= 1397 \text{ g/m}^3 = \underline{1.397 \text{ kg/m}^3} \quad c)$$

$$P_{\text{CH}_4} = y_{\text{CH}_4} P = (0.935)(1.4 \times 10^5) \\ = \underline{131 \text{ kPa}} \quad d)$$

MASS FRACTION CO<sub>2</sub>

$$= \frac{0.308}{17.176} = \underline{0.0179} \quad e)$$

	BASIS - 1 kg MOLE			
	kg/MOLE	M.W.	kg	WT FRACTION
SiCl <sub>4</sub>	0.40	321.2	12.85	0.914
H <sub>2</sub>	0.60	2.02	1.21	0.086

↓ a

### 24.2 CONTINUED -

$$\text{M.WT.} = \underline{14.06 \text{ kg/kg mole}} \quad b)$$

$$C_A, \text{SiCl}_2 = y_A C$$

$$P = \frac{60}{760} (1.013 \times 10^5) = 7.99 \times 10^3 \text{ Pa}$$

$$C = \frac{P}{RT} = \frac{7.99 \times 10^3}{8.314(900)} = 1.068 \text{ mole/m}^3$$

$$C_A = (0.40)(1.068) = \underline{0.1427 \text{ mole/m}^3} \quad c)$$

### 24.3 Basis - 1g MOLE

	y	MOLE	M.WT.	g
O <sub>2</sub>	0.21	0.21	32	6.72
N <sub>2</sub>	0.79	0.79	28	22.12

1.0      28.84

$$\text{MOLE FRACTION OF O}_2 = \underline{0.21} \quad a)$$

$$\text{VOLUME " n " = } \underline{0.21} \quad b)$$

$$\text{WT OF MIXTURE} = \underline{28.84 \text{ g}} \quad c)$$

$$\text{Vol/mole} = \frac{RT}{P} = \frac{8.314(400)}{1.013 \times 10^5} \\ = 0.0328 \text{ m}^3/\text{mole}$$

$$\rho_{O_2} = \frac{6.72}{0.0328} = \underline{204.9 \text{ g/m}^3} \quad d)$$

$$\rho_{N_2} = \frac{22.12}{0.0328} = \underline{674.4 \text{ "}} \quad e)$$

$$\rho_{\text{mix}} = \underline{879 \text{ "}} \quad f)$$

$$\text{M.WT OF MIXTURE} = \underline{28.84} \quad g)$$

24.4

$$N_{A2} = -CD_{AB} \frac{dy_A}{dz} + y_A (N_{A2} + N_{B2})$$

$$N_{B2} = -CD_{BA} \frac{dy_B}{dz} + y_B (N_{A2} + N_{B2})$$

ADDING:

$$N_{A2} + N_{B2} = -CD_{AB} \frac{dy_A}{dz} - CD_{BA} \frac{dy_B}{dz} + (y_A + y_B)(N_{A2} + N_{B2})$$

$$CD_{AB} \frac{dy_A}{dz} + CD_{BA} \frac{dy_B}{dz} = 0$$

$$y_A + y_B = 1$$

$$\therefore \frac{dy_A}{dz} + \frac{dy_B}{dz} = 0 \quad \therefore \frac{dy}{dz} = -\frac{dy_B}{dz}$$

GIVINH  $D_{AB} = D_{BA}$

IN AIRSAFELEDOF EEN:

$$\sigma_{AB}^2, \sigma^2 \left( \frac{1}{M_A} + \frac{1}{M_B} \right)^{1/2}$$

WILL BE THE SAME FOR  $D_{AB}, D_{BA}$ 

∴ AGREEMENT — Q.E.D.

$$24.5 \quad \vec{N}_A = -CD_{AB} \nabla y_B + y_A (\vec{N}_A + \vec{N}_B)$$

 $C = \text{CONST.}$ ; MULTIPLY BY  $M_A$ 

$$\vec{N}_A M_A = -D_{AB} M_A \nabla C_A + y_A M_A (\vec{N}_A + \vec{N}_B)$$

$$w_A = \frac{y_A M_A}{x_A M_A + x_B M_B} = \frac{y_A M_A}{M_A w_B}$$

$$\vec{N}_A = -D_{AB} \nabla p_A + w_A (\vec{N}_A + \vec{N}_B)$$

$$\therefore \vec{N}_A = -D_{AB} \nabla w_A + w_A (\vec{N}_A + \vec{N}_B)$$

24.5 CONTINUED

$$\vec{N}_A = -D_{AB} \nabla C_A + C_A \left[ \frac{C_A \vec{V}_A + C_B \vec{V}_B}{C} \right]$$

$$C_A \vec{V}_A = -D_{AB} \nabla C_A + C_A \vec{V}$$

$$C_A (\vec{V} - \vec{J}) = -D_{AB} \nabla C_A$$

$$\therefore \vec{J}_A = -D_{AB} \nabla C_A \quad b)$$

$$24.6: \quad \vec{N}_A + \vec{N}_B = \left[ C_A \vec{V}_A + C_B \vec{V}_B \right] \frac{C}{C}$$

$$= \underline{\underline{CV}} \quad a)$$

$$n_A + n_B = (S_A \vec{V}_A + S_B \vec{V}_B) \underline{\underline{S}}$$

$$= \underline{\underline{SV}} \quad b)$$

$$\vec{J}_A + \vec{J}_B = -D_{AB} S \nabla w_A - D_{AB} S \nabla w_B$$

AS  $w_A + w_B = 1$ ;  $\nabla w_A + \nabla w_B = 0$ 

$$\therefore \underline{\underline{J}_A + \underline{\underline{J}_B}} = 0 \quad c)$$

24.7

$$-\frac{dC_A}{dz} = \beta_{AB} \frac{p_A}{M_A} \frac{p_B}{M_B} (v_{A2} - v_{B2})$$

$$+ \beta_{AC} \frac{p_A}{M_A} \frac{p_C}{M_C} (v_{A2} - v_{C2}) + \dots$$

$$\text{AS } C = \frac{p_i}{M_i} = \frac{p_i}{RT}$$

$$-\frac{1}{RT} \frac{dp_A}{dz} = \beta_{AB} \left[ \frac{p_B}{RT} C_A v_{A2} - \frac{p_A}{RT} C_B v_{B2} \right] \\ + \beta_{AC} \left[ \frac{p_C}{RT} C_A v_{A2} - \frac{p_A}{RT} C_C v_{C2} \right] + \dots$$

24.7 CONTINUED -

$$\begin{aligned}
 -\frac{1}{RT} \frac{dP_A}{dz} &= \frac{\beta_{AB}}{RT} \left( P_B N_{BZ} - P_A N_{BZ} \right) \\
 &\quad + \frac{\beta_{AC}}{RT} \left( P_C N_{CZ} - P_A N_{CZ} \right) + \dots \\
 -\frac{dP_A}{dz} &= \left[ \beta_{AB} P_B + \beta_{AC} P_C + \dots \right] N_{BZ} \\
 &\quad - P_A \left[ \beta_{AB} N_{BZ} + \beta_{AC} N_{CZ} + \dots \right]
 \end{aligned}$$

LET  $D_{Ai} = \frac{RT}{\beta_{Ai} P}$  or  $\beta_{Ai} = \frac{RT}{D_{Ai} P}$

$$\begin{aligned}
 -\frac{dP_A}{dz} &= \left[ \frac{RT}{P} \left[ \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \dots \right] N_{BZ} \right. \\
 &\quad \left. - \frac{P_A RT}{P} \left[ \frac{N_{BZ}}{D_{AB}} + \frac{N_{CZ}}{D_{AC}} + \dots \right] \right]
 \end{aligned}$$

for A DIFFUSING THROUGH NON-DIFFUSING B,C,D,...

$$N_{BZ} = N_{CZ} = N_{DZ} = \dots = 0$$

GIVEN

$$-\frac{dP_A}{dz} = \frac{RT}{P} \left( \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \dots \right) N_{BZ}$$

$$\therefore \frac{P}{RT N_{BZ}} \left( -\frac{dP_A}{dz} \right) = \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \dots \quad (1)$$

Now - consider a BINARY CASE

$$\text{WITH } N_{BZ} = 0$$

$$N_{BZ} = -CD_{AB} \frac{dy_A}{dz} + y_A N_{AZ}$$

24.7 CONTINUED -

$$\begin{aligned}
 N_{AZ} &= -\frac{P}{RT} \frac{D_{AB}}{1-y_A} \frac{dy_A}{dz} \\
 &= -\frac{P}{RT} \frac{D_{AB}}{P-P_A} \frac{dP_A}{dz} \\
 \text{or } \frac{P-P_A}{D_{AB}} &= \frac{P}{RT} \left[ \frac{-dP_A/dz}{N_{AZ}} \right] \quad (2)
 \end{aligned}$$

COMBINING (1) & (2)

$$\begin{aligned}
 \frac{P-P_A}{P_{A-mix}} &= \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \frac{P_D}{D_{AD}} + \dots \\
 \therefore P_{A-mix} &= \frac{P-P_A}{\frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \frac{P_D}{D_{AD}} + \dots}
 \end{aligned}$$

DIVIDING NUMERATOR & DENOMINATOR  
BY P WE GET

$$P_{A-mix} = \frac{1-y_A}{y_B/D_{AB} + y_C/D_{AC} + y_D/D_{AD} + \dots}$$

DIVIDING NUMERATOR & DENOMINATOR  
BY  $1-y_A$  & DESIGNATING  $y'_i = y_i / (1-y_A)$

WE HAVE, FINALLY,

$$P_{A-mix} = \frac{1}{y'_B/D_{AB} + y'_C/D_{AC} + y'_D/D_{AD} + \dots}$$

24.8 CO<sub>2</sub> IN AIR @ 310 K, 1.5 × 10<sup>5</sup> Pa

APPENDIX J:  $D_{AB}P = 1378 \text{ m}^2/\text{s Pa}$

$$D_{AB} @ T_2 P_2 = D_{AB} \left| \frac{P_1}{T_1} \right| \left( \frac{T_2}{T_1} \right)^{3/2} \Omega_{DT_2}$$

CO<sub>2</sub>:  $\epsilon_{AB}/k = 190$

AIR:  $\epsilon/k = 97$

$$\epsilon_{AB}/k = \sqrt{190}(97) = 135.76$$

T<sub>1</sub>:  $\frac{TK}{\epsilon_{AB}} = \frac{273}{135.76} = 2.011 \quad \Omega_{D0} = 1.673$

T<sub>2</sub>:  $\frac{TK}{\epsilon_{AB}} = \frac{310}{135.76} = 2.283 \quad \Omega_{D0} = 1.028$

$$D_{AB} = \frac{1.378}{1.5 \times 10^5} \left( \frac{310}{273} \right)^{3/2} \frac{1.673}{1.028}$$

$$= 1.16 \times 10^{-5} \text{ m}^2/\text{s} \quad (\text{a})$$

ETHANOL IN AIR @ 325 K 2 NO<sup>S</sup> Pa

SAME PROCEDURE AS ABOVE -

$$D_{AB}P \Big|_{298} = 1337 \text{ m}^2/\text{s Pa}$$

$\epsilon_{AB}/k = 194.7 \quad \Omega_{DT1} = 1.188 \quad \Omega_{DT2} = 1.148$

$$D_{AB} = \frac{1.377}{2 \times 10^5} \left( \frac{325}{298} \right)^{3/2} \left( \frac{1.188}{1.148} \right)$$

$$= 1.88 \times 10^{-6} \text{ m}^2/\text{s} \quad (\text{b})$$

24.8 (CONTINUED) -

CO IN AIR @ 310 K, 1.5 × 10<sup>5</sup> Pa

MUST USE HIRSCHFELDER EQUATION

$$D_{AB} = \frac{0.001858 T^{3/2} \left[ \frac{1}{m_A} + \frac{1}{m_B} \right]^{1/2}}{P \sigma_{AB}^2 \Omega_D}$$

VALUES:  $\left[ \frac{1}{m_A} + \frac{1}{m_B} \right]^{1/2} = 0.265$

P = 1.4807 ATM

$$\sigma_{AB}^2 = 12.985 \quad \epsilon_{AB}/k = 103.29$$

$$TK/\epsilon_{AB} = 3.0 \quad \Omega_D = 0.949$$

SUBSTITUTING & SOLVING:

$$D_{AB} = 1.47 \times 10^{-5} \text{ m}^2/\text{s} \quad (\text{c})$$

CCl<sub>4</sub> IN AIR @ 298 K, 1.913 × 10<sup>5</sup> Pa

AGAIN - HIRSCHFELDER EQUATION - SEE PART (C)

VALUES:  $\left[ \frac{1}{m_A} + \frac{1}{m_B} \right]^{1/2} = 0.202$

P = 1.888 ATM

$$\sigma_{AB}^2 = 22.553 \quad \epsilon_{AB}/k = 178.1$$

$$TK/\epsilon_{AB} = 1.67 \quad \Omega_D = 1.148$$

SUBSTITUTING & SOLVING:

$$D_{AB} = 3.95 \times 10^{-6} \text{ m}^2/\text{s} \quad (\text{d})$$

24.9 n BUTANE - i BUTANE @ 673 K  
2.0 ATM

USE HIRSHFELDER EQUATION - SEE PROB 24.8

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{\frac{1}{2}} = 0.1857$$

$$\sigma_{AB}^2 = 16,718 \quad \epsilon_{AB}/k = 358.2$$

$$TK/\epsilon_{AB} = 1.88 \quad \Omega_D = 1.098$$

SUBSTITUTING  $\nmid$  SOLVING:

$$D_{AB} = 1.03 \times 10^{-5} \text{ m}^2/\text{s}$$

FULLER-SCHAFER-GIDDINGS

$$D_{AB} = 10^{-3} T^{1.73} \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{\frac{1}{2}} \\ P \left[ \left( \sum \nu_A \right)^{\frac{1}{3}} + \left( \sum \nu_B \right)^{\frac{1}{3}} \right]^2$$

$$\sum \nu_A = \sum \nu_B = \left[ A(14.8) + 10(3.7) \right] \\ = 96.2$$

SUBSTITUTING VALUES  $\nmid$  SOLVING

$$D_{AB} = 9.9 \times 10^{-6} \text{ m}^2/\text{s}$$

24.10 CH<sub>4</sub> IN AIR, 373 K, 1.5 × 10<sup>5</sup> Pa

HIRSHFELDER EQUATION - SEE PROB 24.8

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{\frac{1}{2}} = 0.311$$

$$\sigma_{AB}^2 = 13.834 \quad \epsilon_{AB}/k = 115.07$$

$$TK/\epsilon_{AB} = 3.24 \quad \Omega_D = 0.930$$

SUBSTITUTING  $\nmid$  SOLVING:

$$D_{AB} = 2.19 \times 10^{-5} \text{ m}^2/\text{s} \quad (\text{a})$$

24.10 (CONTINUED) -

$$\text{WILKE EQUATION: } D_{A-\text{mix}} = \frac{1}{\frac{0.21}{D_{A-\text{O}_2}} + \frac{0.79}{D_{A-\text{N}_2}}} \\ A = \text{CH}_4$$

MUST USE HIRSHFELDER EQUATION FOR D<sub>A-i</sub>  
- SEE PROB 24.8 FOR EQUATION.

FOR D<sub>A-O<sub>2</sub></sub>:

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{\frac{1}{2}} = 0.306$$

$$\sigma_{AB}^2 = 13.159 \quad \epsilon_{AB}/k = 124.19$$

$$TK/\epsilon_{AB} = 3.0 \quad \Omega_D = 0.949$$

$$\text{SUBSTITUTING: } D_{A-\text{O}_2} = 2.22 \times 10^{-5} \text{ m}^2/\text{s}$$

FOR D<sub>A-N<sub>2</sub></sub>:

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{\frac{1}{2}} = 0.313$$

$$\sigma_{AB}^2 = 14.074 \quad \epsilon_{AB}/k = 111.76$$

$$TK/\epsilon_{AB} = 3.337 \quad \Omega_D = 0.923$$

$$\text{SUBSTITUTING: } D_{A-\text{N}_2} = 2.19 \times 10^{-5} \text{ m}^2/\text{s}$$

FOR MIXTURE: (WILKE EQUATION)

$$D_{A-\text{air}} = \frac{1}{\frac{0.21}{2.22} + \frac{0.79}{2.19}}$$

$$= 2.19 \times 10^{-5} \text{ m}^2/\text{s} \quad (\text{b})$$



24.13 H<sub>2</sub>S in MIXTURE 350 K, 1 atm



for A into B: USE H.E. (Prob. 24.8)

$$\text{VALUES: } \left( \frac{1}{M_A} + \frac{1}{M_B} \right)^{1/2} = 0.255$$

$$\sigma_{AB}^2 = 14.27 \quad \epsilon_{AB}/k = 162.2$$

$$KT/\epsilon_{AB} = 2.158 \quad -D_B = 1.048$$

SUBSTITUTION  $\nless$  SWINSON

$$D_{AB} = 2.07 \times 10^{-5} \text{ m}^2/\text{s}$$

for A into C: ~ SAME PROCEDURE

$$\text{VALUES: } \left( \frac{1}{M_A} + \frac{1}{M_B} \right)^{1/2} = 0.212$$

$$\sigma_{AB}^2 = 16.66 \quad \epsilon_{AB}/k = 269.1$$

$$KT/\epsilon_{AB} = 1.30 \quad -D_B = 1.273$$

$$D_{AC} = 1.20 \times 10^{-5} \text{ m}^2/\text{s}$$

MIXTURE:  $y_A = 0.03 \quad y_B = 0.92 \quad y_C = 0.05$

$$y'_B = 0.948 \quad y'_C = 0.0515$$

$$1 \times 10^{-5}$$

$$D_{H_2S-\text{MIX}} = \frac{\frac{0.948}{2.07} + \frac{0.0515}{1.20}}{1.00} = 2.00 \times 10^{-5} \text{ m}^2/\text{s}$$

$$24.14 \quad D_{AB} = \frac{kT}{6\pi r \mu_B} \sim r = \frac{kT}{6\pi D_{AB} \mu_B}$$

$$\text{Given } D_{AB} = 5.94 \times 10^{-11} \text{ m}^2/\text{s}$$

$$T = 293 \text{ K} \quad \mu_B = 998 \times 10^{-6} \text{ Pa}\cdot\text{s}$$

$$\text{SUBSTITUTION: } r = 3.637 \text{ nm}$$

24.15 O<sub>2</sub> IN C<sub>2</sub>H<sub>5</sub>OH 293 K

$$\text{for C}_2\text{H}_5\text{OH: } \rho = 1.25 \text{ g} \quad M_B = 46 \quad \phi_B = 1.5$$

$$V_{O_2} = 25.6$$

$$D_{AB} = \frac{T}{\mu_B} \frac{(7.4 \times 10^{-8})(\phi_B M_B)^{1/2}}{V_A^{0.16}} \quad (a)$$

$$\text{SUBSTITUTION VALUES: } D_{AB} = 2.06 \times 10^{-9} \text{ m}^2/\text{s}$$

C<sub>2</sub>H<sub>5</sub>OH IN H<sub>2</sub>O, 288 K

$$\mu_B = 1.14 \text{ g} \quad M_B = 46 \quad \phi_B = 2.26$$

$$V_{CH_3OH} = 14.8 + 4(3.7) + 7.4 = 37$$

SUBSTITUTION INTO Eq (24-52)  $\nless$  SEE PART A

$$D_{AB} = 1.336 \times 10^{-9} \text{ m}^2/\text{s} \quad (b)$$

H<sub>2</sub>O IN C<sub>2</sub>H<sub>5</sub>OH 288 K

$$\mu_B = 0.62 \text{ g} \quad M_B = 32 \quad \phi_B = 1.9$$

$$V_A = 18.9$$

SUBSTITUTION INTO EoN (24-52)

$$D_{AB} = 4.59 \times 10^{-9} \text{ m}^2/\text{s} \quad (c)$$

C<sub>2</sub>H<sub>5</sub>OH IN H<sub>2</sub>O 288 K

$$\mu_B = 1.14 \text{ g} \quad M_B = 46 \quad \phi_B = 2.26$$

SUBSTITUTION INTO EoN (24-52)

$$D_{AB} = 7.37 \times 10^{-10} \text{ m}^2/\text{s} \quad (d)$$

From TEXT - Appendix J

$$D_{AB} = 7.7 \times 10^{-10} \text{ m}^2/\text{s}$$

24.16 Cl<sub>2</sub> in H<sub>2</sub>O 289 K

$$\mu_B = 1.13 \text{ cp} \quad M_B = 18 \quad \phi_B = 2.26 \\ V_A = 484$$

SUBSTITUTION INTO FON (24-52)

$$D_{AB} = 1.17 \times 10^{-9} \text{ m}^2/\text{s}$$

USING FON (24-53)

$$D_{AB} = (3.26 \times 10^{-5}) \mu_B^{-1.14} V_A^{-0.589} \\ = 1.14 \times 10^{-9} \text{ m}^2/\text{s}$$

APPENDIX J:  $D_{AB} = 1.26 \times 10^{-9} \text{ m}^2/\text{s}$

24.17 C<sub>6</sub>H<sub>6</sub> IN C<sub>2</sub>H<sub>5</sub>OH 288 K

$$\mu_B = 1.3 \text{ cp} \quad M_B = 46 \quad \phi_B = 1.5 \\ V_A = 96$$

SUBSTITUTION INTO FON (24-52)

$$D_{AB} = 8.81 \times 10^{-10} \text{ m}^2/\text{s}$$

C<sub>2</sub>H<sub>5</sub>OH INTO C<sub>6</sub>H<sub>6</sub>

$$\mu_B = 0.75 \text{ cp} \quad M_B = 78 \quad \phi_B = 1.0 \\ V_A = 59.2$$

SUBSTITUTION INTO FON (24-52)

$$D_{AB} = 2.17 \times 10^{-9} \text{ m}^2/\text{s}$$

24.18 O<sub>2</sub> in H<sub>2</sub>O (g) 288 K

$$FON (24-52) - \mu_B = 1.14 \text{ cp}$$

$$M_B = 18 \quad \phi_B = 2.26 \quad V_A = 256$$

SUBSTITUTION:  $D_{AB} = 1.70 \times 10^{-9} \text{ m}^2/\text{s}$

FON (24-53)

$$D_{AB} = 1.69 \times 10^{-9} \text{ m}^2/\text{s}$$

24.19 P IN Si (s)

$$@ 1316 \text{ K} \quad D_{AB} = 1 \times 10^{-17} \text{ m}^2/\text{s}$$

$$1408 \text{ K} \quad D_{AB} = 1 \times 10^{-16} \text{ m}^2/\text{s}$$

$$D_i = D_0 e^{-Q/RT}$$

$$\ln D_i = \ln D_0 - Q/RT$$

SUBSTITUTION:  $Q_1/R = 4.645 \times 10^4$

$$D_0 = 213.31$$

$$@ 1373 \text{ K} \quad \ln D_i = -18.47$$

$$D_{AB} = 4.32 \times 10^{-17} \text{ m}^2/\text{s}$$

24.20 C IN FCC Fe 1000 K

$$D_0 = 2.5 \times 10^{-6} \text{ m}^2/\text{s} \quad Q = 144.2 \text{ kJ/mol}$$

$$D_i = D_0 e^{-Q/RT} = 7.34 \times 10^{-10} \text{ m}^2/\text{s}$$

C IN BCC Fe

$$D_0 = 2.0 \times 10^{-6} \text{ m}^2/\text{s} \quad Q = 84.1 \text{ kJ/mol}$$

$$D_i = D_0 e^{-Q/RT} = 8.09 \times 10^{-9} \text{ m}^2/\text{s}$$

24.21 EFFECTIVE DIFFUSION OF H<sub>2</sub> IN N<sub>2</sub> 373 K, 1 ATM

STREAMLINED FLOW (D=100 Å) IN PARALLEL

$$d_p = 1 \times 10^{-8} \text{ m}$$

$$\text{EQN (24-58)} \quad D_{KA} = 4850 d_p \sqrt{T/M_A}$$

$$= 4850 (10^{-8}) \left[ \frac{373}{2.015} \right]^{1/2}$$

$$= 6.6 \times 10^{-8} \text{ m}^2/\text{s}$$

$$\text{AT } 288 \text{ K} \quad D_{AB} = 0.743 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{AT } 373 \text{ K} \quad D_{AB} = 1.095 \times 10^{-6} \text{ m}^2/\text{s}$$

ASSUMING DILUTE N<sub>2</sub>

$$\text{DEFECTIVE} = \frac{1 \times 10^{-6}}{\frac{1}{0.743} + \frac{1}{0.095}}$$

$$= 0.062 \times 10^{-6} \text{ m}^2/\text{s} \quad \textcircled{a}$$

RANDOM POROSITY - VOID FRACTION = 0.4

$$D_{eff} = \epsilon^2 D_E = (0.4)^2 (0.062 \times 10^{-6})$$

$$= 9.62 \times 10^{-7} \text{ m}^2/\text{s} \quad \textcircled{b}$$

RANDOM POROSITY 1000 Å E=0.4

$$\text{EQN (24-58)} \quad D_{KA} = 0.062 \times 10^{-6} \text{ m}^2/\text{s}$$

$$D_{AE} = 0.383 \times 10^{-6} \text{ m}^2/\text{s}$$

$$D_{AE}' = (0.4)^2 (0.383 \times 10^{-6})$$

$$= 0.0614 \times 10^{-6} \text{ m}^2/\text{s} \quad \textcircled{c}$$

24.21 (CONTINUED) -  $d_p = 20,000 \text{ \AA}$  PARALLEL

$$\text{EQN (24-58)} \quad D_{KA} = 1.3197 \times 10^{-5} \text{ m}^2/\text{s}$$

$$D_{AE} = \frac{1 \times 10^{-6}}{\frac{1}{0.743} + \frac{1}{1.3197}}$$

$$= 1.011 \times 10^{-6} \text{ m}^2/\text{s} \quad \text{(d)}$$

24.22 A = CH<sub>4</sub> ~ 20 mol %

B = H<sub>2</sub>O ~ 80 "

USE H.E. - EQN. (24-33)

$$\text{VALUES } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.3436$$

$$\Omega_{AB}^2 = 10.468 \quad \epsilon_{AB}/\kappa = 220.4$$

$$kT/E_{AB} = 2.60 \quad \Omega_0 = 0.9878$$

$$\text{SUBSTITUTING: } D_{AB} = 1.694 \times 10^{-6} \text{ m}^2/\text{s}$$

$$D_{AE} = \frac{1}{\frac{1}{D_{AB}} + \frac{1}{D_K}}$$

$$D_K = 4850 (2 \times 10^{-7} \text{ m}) \sqrt{\frac{573}{16}}$$

$$= 0.580 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{SUBSTITUTING: } D_{AE} = 0.432 \times 10^{-6} \text{ m}^2/\text{s}$$

KNUSSEN DIFFUSION IS ~ 75% OF TOTAL

24.23 H<sub>2</sub>O INTO CO 353 K 2 ATM

$$A = H_2O \quad B = CO$$

$$D_{AB} @ 273 K, 1 \text{ atm} = 0.1651 \times 10^{-4} \text{ m}^2/\text{s}$$

$$\frac{1}{\sqrt{\frac{M_A}{M_B}} \sqrt{\frac{T_2}{T_1}}} D_{AB} = 0.1651 \left( \frac{353}{273} \right)^{\frac{1}{2}} \left( \frac{1}{2} \right) \\ = 0.479 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D'_{AE} = 0.036 \times 10^{-4} \text{ m}^2/\text{s} \\ = (0.3)^2 D_{AB}$$

$$D_{AE} = 0.4 \text{ m}^2/\text{s} \\ 0.4 = \frac{1 \times 10^{-4}}{\frac{1}{0.479} + \frac{1}{D_{AK}}}$$

$$D_{AK} = 2.425 \times 10^{-4} \text{ m}^2/\text{s}$$

From Eqn (24-58)

$$2.425 \times 10^{-4} = 4850 d_p \left[ \frac{353}{2.0158} \right]^{\frac{1}{2}}$$

$$\underline{d_p = 3.78 \times 10^{-7} \text{ m}}$$

24.24 O<sub>2</sub> INTO HE ~ A INTO B

$$d_p = 5 \times 10^{-6} \text{ m} \quad P = 300 \text{ Pa}$$

$$T = 373 \text{ K} \quad M_A = 32 \quad M_B = 4$$

$$C = \frac{P}{RT} = \frac{300}{8.314(373)} = 0.0967 \text{ mol/m}^3$$

$$C_{CO_2} = 0.01(0.0967) = \underline{9.67 \times 10^{-5} \text{ mol/m}^3}$$

24.24 CONTINUED -

$$\ln \text{Pores} - D_{eff} = \frac{1}{\frac{1}{D_{AB}} + \frac{1}{D_{AK}}}$$

USE Eqn (24-33) TO FIND D<sub>AB</sub>:

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{\frac{1}{2}} = 0.530$$

$$\sigma_{AB}^2 = 9.027 \quad \epsilon_{AB}/k = 33.98$$

$$kT/\epsilon_{AB} = 10.98 \quad \Omega_0 = 0.8161$$

$$\text{SUBSTITUTION: } D_{AB} = 0.0325 \text{ m}^2/\text{s}$$

USE Eqn (24-58) TO FIND D<sub>AK</sub>

$$D_{AK} = 8.28 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D_{AE} = \frac{1 \times 10^{-4}}{\frac{1}{325} + \frac{1}{8.28}} = \underline{8.08 \times 10^{-4} \text{ m}^2/\text{s}}$$

24.25 Cu<sub>6</sub> IN H<sub>2</sub>O (l)

$$d_p = 1.50 \times 10^{-7} \text{ m} \quad \mu_B = 0.95 \text{ cP}$$

$$\epsilon = 0.4 \quad \phi_B = 2.26 \quad M_B = 18$$

$$V_A = 96.38$$

SUBSTITUTION INTO Eqn (24-52)

$$D_{AB} = 0.955 \times 10^{-9} \text{ m}^2/\text{s}$$

USE Eqn (24-58) TO GET D<sub>AK</sub>

$$D_{AK} = 0.142 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D_{AE} = \frac{1 \times 10^{-9}}{\frac{1}{14200} + \frac{1}{0.955}} \approx 0.955 \times 10^{-9} \text{ m}^2/\text{s}$$

$$D_{AE}' = \epsilon^2 D_{AE} = \underline{1.528 \times 10^{-10} \text{ m}^2/\text{s}}$$

24.26 CO IN H<sub>2</sub>



$$d_p = 1.5 \times 10^{-8} \text{ m} \quad \epsilon = 0.10$$

$$T = 673 \text{ K} \quad P = 5.0 \text{ atm}$$

$$\text{APPENDIX J: } D_{AB} = 0.651 \times 10^{-4} \text{ m}^2/\text{s} @ 1 \text{ atm}$$

$$\sim D_{AB} = 0.130 \times 10^{-4} \text{ m}^2/\text{s} @ 5 \text{ atm}$$

$$\epsilon_{AB}/k = 60.52$$

$$@ 273 \text{ K} - kT/\epsilon_{AB} = 4.51 \quad D_p = 0.8606$$

$$@ 673 \text{ K} - kT/\epsilon_{AB} = 11.12 \quad D_p = 0.7345$$

$$D_{AB} \Big|_{673} = 0.130 \times 10^{-4} \left( \frac{673}{273} \right)^{3/2} \left( \frac{0.8606}{0.7345} \right)$$

$$= 0.5891 \times 10^{-4} \text{ m}^2/\text{s}$$

OBTAIN D<sub>AK</sub> FROM EQUATION (24-58)

$$D_{AK} = 4850 (1.5 \times 10^{-8}) \sqrt{\frac{673}{273}}$$

$$= 0.0357 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D_{PK} = \frac{1 \times 10^{-4}}{1/0.5891 + 1/0.0357}$$

$$= 0.337 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D'_{AE} = (0.1)^2 (0.337 \times 10^{-4})$$

$$= \underline{\underline{0.337 \times 10^{-4} \text{ m}^2/\text{s}}}$$

$$K.D. = \frac{0.0357}{0.5891 + 0.0357} \approx \underline{\underline{5.7\%}}$$

24.27 Glucose (A) IN A<sub>2</sub>O

$$T = 303 \text{ K} \quad d_p = 3 \times 10^{-9} \text{ m}$$

$$d_A = 0.86 \times 10^{-9} \text{ m}$$

$$\mu_B = 825 \text{ g/cm.s}$$

USE STO克斯-EINSTEIN EQUATION (24-50)

$$D_{AB} = \frac{kT}{6\pi\mu_B r_A} = \frac{(1.38 \times 10^{-16})(303)}{6\pi(825)(0.86 \times 10^{-9})}$$

$$= 6.25 \times 10^{-15} \text{ m}^2/\text{s}$$

USE EQUATION (24-62) TO OBTAIN D<sub>AE</sub>

$$\phi = \frac{8.6 \times 10^{-10} \text{ m}}{30 \times 10^{-10} \text{ m}} = 0.2867$$

$$f_1 = (1-\phi)^2 = 0.508$$

$$f_2 = 1 - 2.104(0.2867) + 2.09(0.2867)^3$$

$$- 0.95(0.2867)^5 = 0.444$$

$$D_{AE} = D_{AB} f_1 f_2 = (6.25 \times 10^{-15})(0.508)(0.444)$$

$$= \underline{\underline{1.41 \times 10^{-15} \text{ m}^2/\text{s}}}$$

24.28 UREA (A) INTO SUPPORT (B)

$$D_{AB} = 3.46 \times 10^{-11} \text{ m}^2/\text{s}$$

$$d_{molecule} = 12.38 \text{ nm} \quad d_p = 100 \text{ nm}$$

$$\phi = \frac{12.38}{100} = 0.1238$$

$$f_1(\phi) = (1 - 0.1238)^2 = 0.7677$$

$$f_2(\phi) = 1 - 2.104(0.1238) + 2.09(0.1238)^3$$

$$- 0.95(0.1238)^5$$

$$= 0.743$$

24.28 (CONTINUED) ~

$$D_{AE} = (3.46 \times 10^{-7})(0.7157)(0.743)$$
$$= \underline{1.97 \times 10^{-11} \text{ m}^2/\text{s}}$$

24.29 RIBONUCLEASE (A)  
INTO SUPPORT (B)

$$D_{AB} = 5.0 \times 10^{-11} \text{ m}^2/\text{s}$$

$$D_{AB} = 1.19 \times 10^{-10} \text{ m}^2/\text{s}$$

$$d_m = 3.6 \text{ nm}$$

$$D_{AE} = D_{AB} f_1(\phi) f_2(\phi)$$

$$f_1(\phi) f_2(\phi) = \frac{5.0 \times 10^{-11}}{1.19 \times 10^{-10}}$$

$$= 0.4202$$

TRIAL  $\frac{1}{2}$  ERROR ~

$$\phi \approx 0.183$$

$$f_1(\phi) = (-0.183)^2 = 0.06675$$

$$f_2(\phi) = 1 - 1.104(0.183)$$
$$+ 2.09(0.183)^3 - 0.95(0.183)^5$$
$$= 0.6276$$

$$f_1 f_2 \approx 0.4190$$

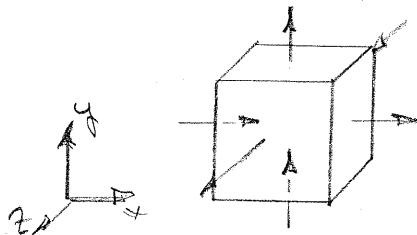
CLOSE ENOUGH ~

$$d_p = \frac{3.6 \text{ nm}}{0.183} = \underline{19.67 \text{ nm}}$$

## CHAPTER 25

### 25.1 CONSERVATION OF MASS:

$$\iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{CV} \rho dV = 0$$



MASS FLOW:

$$N_A X \Delta Y \Delta Z |_{x+AX} - N_A X \Delta Y \Delta Z |_x \\ + N_A Y \Delta X \Delta Z |_{y+AY} - N_A Y \Delta X \Delta Z |_y \\ + N_A Z \Delta X \Delta Y |_{z+AZ} - N_A Z \Delta X \Delta Y |_z$$

ACCUMULATION:  $\frac{\partial C_A}{\partial t} A X \Delta Y \Delta Z$

PRODUCTION:  $R_A A X \Delta Y \Delta Z$

PROCEDURE:

1. RELATE ACCORDING TO BASIC EQU.
2. DIVIDE THROUGH BY  $\Delta X \Delta Y \Delta Z$
3. CANCEL  $\Delta$  TERMS WHERE APPLICABLE
4. TAKE LIMIT AS  $\Delta X \Delta Y \Delta Z \rightarrow 0$

RESULT!

$$\vec{\nabla} \cdot \vec{n}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

$$25.2 \quad \vec{v} \cdot \vec{n}_A + \frac{\partial S_A}{\partial t} - v_A = 0$$

FOR  $S_A \notin D_{AB}$  CONSTANT

$$\vec{n}_A = -D_{AB} \nabla S_A + S_A \vec{v}$$

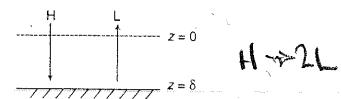
$$\nabla \cdot \vec{n}_A = -D_{AB} \nabla^2 S_A + \nabla \cdot S_A \vec{v}$$

SUBSTITUTION YIELDS:

$$\underline{\frac{\partial S_A}{\partial t} - D_{AB} \nabla^2 S_A + \nabla \cdot S_A \vec{v}} = R_A$$

### 25.3

$$\vec{\nabla} \cdot \vec{n}_H + \frac{\partial C_H}{\partial t} = R_H$$



ONE-DIRECTIONAL, STEADY STATE, NO HOMOGENEOUS REACTION

$$\underline{\frac{d}{dz} N_{H2} = 0} \quad (a)$$

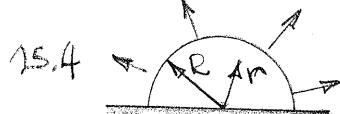
$$N_{H2} = -CD_{HL} \frac{dy_H}{dz} + y_H (N_{H2} + N_{L2})$$

$$\text{AS } N_{L2} = -2N_{H2}$$

$$N_{H2} = -CD_{HL} \frac{dy_H}{dz} + y_H (N_{H2} - 2N_{H2})$$

$$N_{H2} (1 + y_H) = -CD_{HL} \frac{dy_H}{dz}$$

$$\underline{N_{H2} = -\frac{CD_{HL}}{1 + y_H} \frac{dy_H}{dz}}$$



- 25.4
1. T, P CONSTANT; C = CONSTANT
  2. STEADY STATE
  3. NO HOMOGENEOUS REACTION,  $R_t = 0$
  4. ONE DIRECTIONAL DIFFUSION
  5. CONCENTRATION CONSTANT @  $r=R$
  6.  $N_{AIR} = 0$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - \dot{F}_A^0 = 0$$

$$\frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = 0 \quad r^2 N_{Ar} = \text{CONST.}$$

$$N_{Ar} = -CD_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

$$\underline{N_{Ar} = -CD_{AB} \frac{dy_A}{1-y_A} dr}$$

25.5  $O_2 \sim A \quad H_2O \sim B$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - \dot{F}_A^0 = 0$$

FOR Z-DIRECTION:

$$\frac{d}{dz} N_{Az} = 0 \quad N_{Az} = \text{CONST}$$

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} + y_A (N_{Az} + N_{Bz})$$

SINCE DILUTE:  $y_A \approx 0, C \approx \text{CONST}$

$$\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}$$

FOR  $R_A = -k C_A$

$$\underline{\frac{\partial C_A}{\partial t} - D_{AB} \frac{\partial^2 C_A}{\partial z^2} + k C_A = 0}$$

25.6



(a)

1. DIFFUSION IN r-DIRECTION ONLY
2. NO HOMOGENEOUS REACTION,  $R_t = 0$
3.  $C_A @ r = R + 10$  IS KNOWN & CONSTANT
4.  $C_A @ r = R$  IS CONSTANT,  $y_A = P_A/P$
5. MOLECULAR DIFFUSION ONLY
6. STEADY STATE

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - \dot{F}_A^0 = 0$$

$$\underline{\sim \frac{1}{r} \frac{d}{dr} (r N_{Ar}) = 0} \quad (b)$$

$$\frac{d}{dr} (r N_{Ar}) = 0$$

$$\sim \underline{r N_{Ar} = \text{CONSTANT}} \quad (c)$$

$$N_{Ar} = -CD_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

$$= -CD_{AB} \frac{dy_A}{dr} + y_A N_{Ar}$$

$$\underline{N_{Ar} = -CD_{AB} \frac{dy_A}{1-y_A} dr}$$

FOR DILUTE CONCENTRATION:  $y_A \approx 0$

$$\underline{N_{Ar} = -CD_{AB} \frac{dy_A}{dr}}$$

25.7  $\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - \dot{F}_A^0 = 0$

ASSUMPTIONS / CONDITIONS:

1. STEADY STATE
2. NO HOMOGENEOUS REACTION
3. DIFFUSION IN X & Y DIRECTIONS
4.  $U_y = 0$
5. CONSTANT C, DAB
6.  $U_x = Qy$

## 25.7 CONTINUED -

$$\frac{\partial N_{AY}}{\partial y} + \frac{\partial N_{AY}}{\partial z} = 0$$

$$N_{AY} = -D_{AB} \frac{\partial C_A}{\partial y} + \alpha y C_A$$

$$N_{AY} = -D_{AB} \frac{\partial C_A}{\partial y}$$

SUBSTITUTION:

$$-D_{AB} \frac{\partial^2 C_A}{\partial y^2} + \alpha y \frac{\partial C_A}{\partial y} - D_{AB} \frac{\partial C_A}{\partial z^2} = 0$$

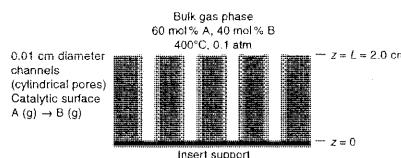
$$D_{AB} \left[ \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} \right] = \alpha y \frac{\partial C_A}{\partial y}$$

B.C.  $C_A(0, y) = 0$

$C_A(x, 0) = C_{AS}$

$C_A(x, R) = 0$

25.8



1. DIFFUSION IN  $r \frac{1}{2} z$  DIRECTIONS
2. STEADY STATE
3. NO HOMOGENEOUS REACTION

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - \dot{P}_A = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r N_A) + \frac{\partial}{\partial z} N_A = 0$$

IN BOTH DIRECTIONS:

$$N_{Ar} = -N_{Br} \quad N_{Az} = -N_{Bz}$$

~ FICKIAN COUNTERDIFFUSION

## 25.8 CONTINUED -

$$\therefore N_{Ar} = -CD_{AB} \frac{\partial y_A}{\partial r} = -D_{AB} \frac{\partial C_A}{\partial r}$$

$$N_{Az} = -CD_{AB} \frac{\partial y_A}{\partial z} = -D_{AB} \frac{\partial C_A}{\partial z}$$

INTO MASS CONSERVATION EQU:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( -D_{AB} r \frac{\partial C_A}{\partial r} \right) + \frac{\partial}{\partial z} \left( -D_{AB} \frac{\partial C_A}{\partial z} \right) = 0$$

$$\underline{\underline{\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_A}{\partial r} \right) + \frac{\partial^2 C_A}{\partial z^2} = 0}}$$

B.C.  $\frac{\partial C_A}{\partial r}(0, z) = 0$

$C_A(0, 0.005 \text{ cm}, z) = 0$

$C_A(R, 2.0 \text{ cm}) = 0.6 \text{ C}$

25.9  $\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - \dot{P}_A = 0 \quad \{ \text{A is O}_2 \}$

STEADY STATE,  $R_A = -m$

$$\nabla^2 N_A + m = 0$$

DIFFUSION IN  $r$ -DIRECTION ONLY

$$\underline{\underline{\frac{1}{r} \frac{\partial}{\partial r} (r N_A) + m = 0}}$$

EQUIMOLAR (UNSTEADY)

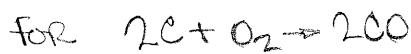
$$N_{Ar} = -CD_{AB} \frac{\partial y_A}{\partial r} + y_A (N_{Ar} + N_{Bz})$$

$$\therefore N_{Ar} = -CD_{AB} \frac{\partial y_A}{\partial r} = -D_{AB} \frac{\partial C_A}{\partial r}$$

OR -  $\underline{\underline{N_{Ar} = -\frac{D_{AB}}{RT} \frac{d P_A}{dr}}}$

25.10  $\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$   
 STEADY STATE  $\{ A \text{ IS } O_2 \}$   
 NO HOMOGENEOUS REACTION  
 ONE-D (SPHERICAL) DIFFUSION

$$\underline{\underline{\frac{d}{dr}(r^2 N_{Ar}) = 0}}$$



$$2N_{Ar} = N_{Br}$$

$$y_A (N_{Ar} + N_{Br}) = -y_A N_{Ar}$$

$$N_{Ar} = -CD_{AB} \frac{dy_A}{dr} - y_A N_{Ar}$$

$$\underline{\underline{= - \frac{CD_{AB}}{1+y_A} \frac{dy_A}{dr}}}$$



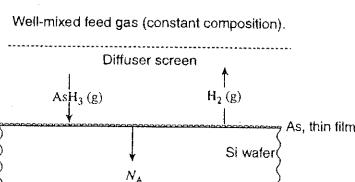
$$y_A (N_{Ar} + N_{Br}) = 0$$

$$\underline{\underline{N_{Ar} = -CD_{AB} \frac{dy_A}{dr}}}$$

B.C.  $y_A(R) = 0$

$$y_A(\infty) = 0.21$$

25.11



ASSUMPTIONS:

1. Temp = CONST,  $D_{AB} \approx P_S$  CONSTANT
2. No Homogeneous Reaction

25.11 (CONTINUED)

3. Silicon Treats As Semi-Infinite

$$4. C_A(z, 0) = 0$$

5. MOLECULAR DIFFUSION IN SOLID

6. ONE DIRECTIONAL (z) DIFFUSION

GENERAL MASS CONSERVATION EQUATION IS

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} = 0$$

$$\therefore N_A = -D_{AB} \frac{\partial C_A}{\partial z}$$

lomb BiN Nbi:  $\underline{\underline{\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}}}$

B.C.  $C_A(z, 0) = 0$

$$C_A(0, t) = C_{AS}$$

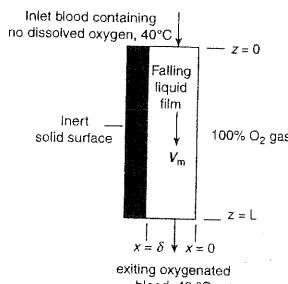
$$C_A(\infty, t) = 0$$

25.12  $A \text{ IS } O_2$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

ST, ST.  $\downarrow$  NO  $R_A$

$$\underline{\underline{\frac{\partial N_{Ar}}{\partial x} + \frac{\partial N_{A2}}{\partial z} = 0}}$$



$$N_{Ar} = -D_{AB} \frac{\partial C_A}{\partial x} + y_A (N_{Ar} + N_{A2})$$

LO - DILUTE CONCENTR.

$$N_{Ar} = -D_{AB} \frac{\partial C_A}{\partial x}$$

$$N_{A2} = -D_{AB} \frac{\partial C_A}{\partial z} + C_A G_M$$

BULK FLOW & MOLECULAR FLOW  
IN 2-DIRECTION

## 25.12 CONTINUED -

SUBSTITUTING INTO MASS BALANCE EQU.

$$\frac{\partial}{\partial z} \left( -D_{AB} \frac{\partial C_A}{\partial y} \right) - \frac{\partial}{\partial z} (C_A V_m) = 0$$

$$-D_{AB} \frac{\partial^2 C_A}{\partial y^2} + V_m \frac{\partial C_A}{\partial z} = 0$$

$$\text{B.C. } C_A(x, 0) = 0,$$

$$C_A(0, z) = C_A^*$$

$$\frac{\partial C_A}{\partial x}(8, z) = 0$$

$$25.13 \quad \nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

ONE-DIMENSIONAL ( $r$ ) DIFFUSION  
IN SPHERICAL GEOMETRY

NO HOMOGENEOUS REACTION

$$-\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 N_{Ar} \right) + \frac{\partial C_A}{\partial t} = 0$$

FICK'S LAW:

$$N_{Ar} = -D_{AB} \frac{\partial C_A}{\partial r} + C_A \frac{\partial}{\partial r}$$

COMBINING:

$$-\frac{D_{AB}}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial C_A}{\partial r} \right) + \frac{\partial C_A}{\partial t} = 0$$

$$\text{B.C. } C_A(R, t) = 0$$

$$C_A(r, 0) = C_{AO}$$

$$\frac{\partial C_A}{\partial r}(0, t) = 0$$

## 25.14 SPHERICAL GEOMETRY -

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

No Homog. Rx

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 N_{Ar} \right) + \frac{\partial C_A}{\partial t} = 0$$

FICK'S LAW:

$$N_{Ar} = -D_{A, \text{eff}} \frac{\partial C_A}{\partial r} + C_A \frac{\partial}{\partial r}$$

O - NO BULK CONTRIB.

COMBINING:

$$-\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 D_{A, \text{eff}} \frac{\partial C_A}{\partial r} \right) + \frac{\partial C_A}{\partial t} = 0$$

$$\frac{\partial C_A}{\partial t} = \frac{D_{A, \text{eff}}}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial C_A}{\partial r} \right)$$

$$C_A(r \leq R, 0) = C_{AO}$$

$$C_A(R, t) = C_A^*$$

$$\frac{\partial C_A}{\partial r}(0, t) = 0$$

## 25.15 INTO AIR: {A - Herbicide}

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

O-S.T.S. No Homog. Rx

ONE-DIMENSIONAL ( $z$ ) DIFFUSION

$$\frac{d N_A}{d z} = 0$$

FICK'S LAW

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} + y_A (\text{NET } N_{Az})$$

$$N_{Az} = -\frac{CD_{AB}}{1-y_A} \frac{dy_A}{dz} \quad (a)$$

INTO Soil -

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

No Rx

$$N_{A2} = -D_{AB} \frac{\partial C_A}{\partial z} + \gamma$$

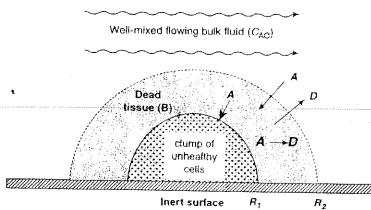
No bulk limit.

$$\frac{\partial N_{A2}}{\partial z} + \frac{\partial C_A}{\partial t} = 0$$

(COMBINE):

$$\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}$$

25.16



$$R_D = -k C_A$$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

SPHERICAL GEOMETRY - STATIONARY STATE

$$\frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) + k C_A = 0$$

$$N_{Ar} = -D_{p-mix} \frac{d C_A}{d r} + \gamma \neq 0$$

(COMBINE):

$$-D_{p-mix} \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d C_A}{d r} \right) + k C_A = 0$$

## CHAPTER 26

26.1 ARNOLD CELL

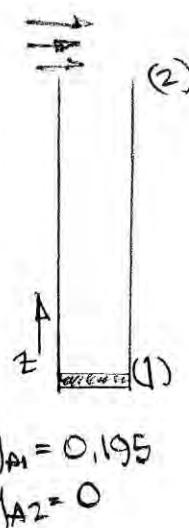
$$d = 1 \text{ cm}$$

$$T = 308 \text{ K}$$

$$P_A^0 = 0.195 \text{ atm}$$

$$\rho_L = 0.85 \text{ g/cm}^3$$

$$M_A = 78$$



$$y_{A1} = 0.195$$

$$y_{A2} = 0$$

EQUATION (26-19) APPLIES

$$D_{AB} = \frac{P_{AL} y_{BL,m.}/M_A}{C(y_{A1} - y_{A2}) t} \left( \frac{r_0^2 - r_i^2}{2} \right)$$

$$y_{BL,m.} = \frac{1.0 - 0.805}{9 \ln \frac{1}{0.805}} = 0.899$$

$$C = \frac{P}{RT} = \frac{1}{(82.06)(308)} = 3.956 \times 10^{-5} \text{ mol/cm}^2$$

$$t = 72 \text{ h} = 2.592 \times 10^5 \text{ s}$$

SUBSTITUTING -

$$\underline{D_{AB} = 9.6 \times 10^{-6} \text{ m}^2/\text{s}}$$

From APPENDIX J.1.

$$\text{AT } 298 \text{ K: } D_{AB} = 9.62 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{AT } 308 \text{ K: } D_{AB} = (9.62 \times 10^{-5}) \left( \frac{308}{298} \right)^{3/2} \\ = 1.01 \times 10^{-5} \text{ m}^2/\text{s}$$

- IN EXPERIMENT - EDDIES AT TOP OF CELL WOULD ALTER DIFFUSION MECHANISM

26.2 CYLINDRICAL GEOMETRY:

STOY STATE, NO HOMOGENEOUS REACTION -

$$\frac{1}{r} \frac{d}{dr} (r N_{Ar}) = 0$$

$$N_{Ar} = - C D_{AB} \frac{dy_A}{dr} + y_A \frac{P_{AL}}{P_{Ai}} \quad \text{O- DILUTE}$$

$$= - \frac{D_{AB} dy_A}{r_0 RT dr}$$

$$r N_{Ar} \int_{r_i}^{r_0} \frac{dr}{r} = - \frac{D_{AB}}{RT} \int_{P_{Ai}}^{P_{AL}} \frac{dP_A}{P_A}$$

$$r N_{Ar} \ln \frac{r_0}{r_i} = \frac{D_{AB}}{RT} (P_{AL} - P_{Ai})$$

$$\text{TABLE J.3. AT } 293 \text{ K } D_{AB} = 4.49 \times 10^{-15} \text{ m}^2/\text{s}$$

SUBSTITUTING VALUES IN SOLVING:

$$\underline{N_{Ar} = 3.92 \times 10^{-11} \text{ mol/m}^2 \cdot \text{s}}$$

TO GET CONCENTRATION PROFILE:

$$\frac{d}{dr} (r N_{Ar}) = \frac{d}{dr} (-r D_{AB} \frac{dc_A}{dr}) = 0$$

$$\frac{d}{dr} (r \frac{dc_A}{dr}) = 0$$

$$\text{SOLVING: } r \frac{dc_A}{dr} = C_1$$

$$c_A = C_1 \ln r + C_2$$

$$\text{AT } r_i = 5 \text{ mm}$$

$$c_{AL} = \frac{P_{AL}}{RT} = \frac{1.5 \times 10^5}{8.314(293)} = 61.58 \text{ mol/m}^3$$

$$\text{AT } r_0 = 8 \text{ mm}$$

$$c_{AO} = \frac{1.0 \times 10^5}{(8.314)(293)} = 41.05 \text{ mol/m}^3$$

### 26.2 CONTINUED -

UNITS:  $C_A, \text{ mol/m}^3$      $r, \text{ mm}$

$$@r_1 \quad 61.58 = C_1 \ln 5 + C_2$$

$$@r_0 \quad 41.05 = C_1 \ln 8 + C_2$$

3

$$C_1 = -43.64 \quad C_2 = 131.8$$

$$\underline{C_A = -43.64 \ln r + 131.8}$$

### 26.3

ONE DIRECTIONAL

STEADY STATE

B INSOLUBLE IN A  $\frac{1}{\delta}$  STATIONARY

$$N_{AB} = -CD_{AB} \frac{dy_B}{dz} + y_A(N_{A2} + N_{B2})$$

$$= \frac{CD_{AB}}{z_2 - z_1} \ln \left( \frac{1 - y_A^2}{1 - y_A} \right) \quad \text{(Eq. 24-5)}$$

$$y_A(3.0) = 1.0 \quad y_A(0.5) = \frac{1.0}{7.60} = 0.131$$

$$C = \frac{P}{RT} = \frac{1.0}{(82.06)(303)} = 4.02 \times 10^{-5} \frac{\text{mol}}{\text{cm}^3}$$

$$\text{App. J: } @ 298 \text{ K} - D_{AB} = 1.62 \times 10^{-4} \text{ m}^2/\text{s}$$

$$\text{AT } 303 \text{ K} \quad D_{AB} = (1.62 \times 10^{-4}) \left( \frac{303}{298} \right)^{3/2}$$

$$= 1.66 \times 10^{-4} \text{ m}^2/\text{s}$$

SUBSTITUTING VALUES  $\frac{1}{\delta}$  SOLVING!

$$N_{A2} = 6.42 \times 10^{-5} \frac{\text{mol}}{\text{m}^2 \cdot \text{s}}$$

$$W_A = (6.42 \times 10^{-5})(32)(3600)(24) \text{ A}$$

$$A = \pi / 4 (1)^2 = 0.785 \text{ m}^2$$

$$\underline{W = 139 \frac{\text{g}}{\text{day}}} \quad (\text{a})$$

### 26.3 CONTINUED

IF TEMPERATURE IS 313 K:

$$D_{AB} = (1.62 \times 10^{-4}) \left( \frac{313}{298} \right)^{3/2} = 1.74 \times 10^{-4} \text{ m}^2/\text{s}$$

$$y_A = \frac{265}{760} = 0.349$$

ALL OTHER VALUES REMAIN THE SAME -

SOLVING:

$$\underline{W_A = 260.6 \frac{\text{g}}{\text{day}}}$$

### 26.4 C<sub>2</sub>H<sub>5</sub>OH (A) THROUGH SPONGE Aqueous H<sub>2</sub>O (B)

ONE DIMENSIONAL, STEADY DIFFUSION

DILUTE CONCENTRATION:  $y_A \sim \text{small}$

$$N_{AB} = -D_{AB} \frac{dy_A}{dz} = \frac{D_{AB}}{\delta} (C_{A1} - C_{A2})$$

TO EVALUATE  $D_{AB}$  - USE EQU. (24-53)

$$\left\{ \begin{array}{l} V_A = 2(14.8) + 6(37) + 7.4 = 59.2 \text{ cm}^3/\text{mol} \\ \mu_B = 1.45 \text{ cP} \end{array} \right.$$

$$D_{AB} = (13.26 \times 10^{-9}) (1.45) \left( \frac{-1.14}{59.2} \right)^{-0.589}$$

$$= 7.82 \times 10^{-10} \text{ m}^2/\text{s}$$

$$C_{A1} = 0.1 \text{ mol/m}^3 \quad C_{A2} = 0.02 \text{ mol/m}^3$$

SUBSTITUTING  $\frac{1}{\delta}$  SOLVING!

$$\underline{N_{A2} = 1.56 \times 10^{-12} \frac{\text{mol}}{\text{m}^2 \cdot \text{s}}}$$

TO DETERMINE  $C_A(z)$ :

$$\nabla \cdot \vec{N}_A = 0 \sim \frac{dN_A}{dz} = 0$$

$$\text{GIVEN: } \frac{d^2 N_A}{dz^2} = 0$$

## 26.4 CONTINUED

$$C_A = C_2 + C_1$$

BC.  $C_A(0) = 0.1 \text{ mol/m}^3$

$$C_A(0.004) = 0.02 \text{ "}$$

$$C_2 = 0.1 \quad C_1 = \frac{(0.02 - 0.1)}{0.004} = -20$$

$$\underline{C_A = 0.1 - 20z} \quad \left\{ \begin{array}{l} C_A, \text{ mol/m}^3 \\ z, \text{ m} \end{array} \right.$$

FOR  $\text{C}_2 + \text{SO}_2(\text{A}) \text{ IN AIR(B) } 283\text{K}$

$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{8.314(283)} = 43.05 \text{ mol/m}^3$$

$$Y_{A1} = C_{A1}/C = \frac{0.1}{43.05} = 2.32 \times 10^{-3}$$

$$Y_{A2} = \frac{0.02}{43.05} = 4.64 \times 10^{-4}$$

$$D_{AB} = (1.32 \times 10^{-5}) \left( \frac{283}{298} \right)^{3/2} \\ = 1.72 \times 10^{-5} \text{ m}^2/\text{s}$$

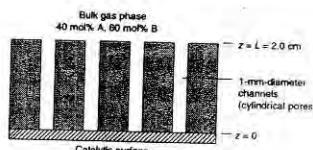
SAME EQUATION FOR  $N_{A2}$  AS IN PART (a)

$$N_{A2} = \frac{(1.72 \times 10^{-5})(0.1 - 0.02)}{4 \times 10^{-3}}$$

$$= 2.44 \times 10^{-4} \text{ mol/m}^2\text{s}$$

## 26.5

$$\begin{aligned} P &= 2 \text{ atm} \\ T &= 373 \text{ K} \\ M_A &= 58 \end{aligned}$$



STEADY STATE, 1D DIFFUSION

## 26.5 CONTINUED

$$\frac{\partial N_{A2}}{\partial r} = 0 \quad N_{A2} = -N_{B2} = \frac{C_D A_B}{S} (Y_{A1} - Y_{A2})$$

$$Y_{A1}(0) = 0.4$$

$$Y_{A2}(0.02 \text{ m}) = 0$$

$$C = \frac{P}{RT} = \frac{2}{(8.314)(373)} = 6.53 \times 10^{-5} \text{ mol/cm}^3$$

$$D_{AB} = 0.1 \left( \frac{373}{298} \right)^{3/2} \left( \frac{1}{2} \right) = 0.07 \text{ cm}^2/\text{s}$$

SUBSTITUTING INTO  $N_{A2}$  EXPRESSION

$$N_{B2} = -N_{A2} = 1.829 \times 10^{-6} \text{ g-mol/cm}^2\text{s}$$

$$N_A = N_{A2} \cdot S$$

$$0.01 \text{ mol/min} = (1.829 \times 10^{-6})(60) S$$

$$S = \text{SURFACE AREA} = 91.12 \text{ cm}^2$$

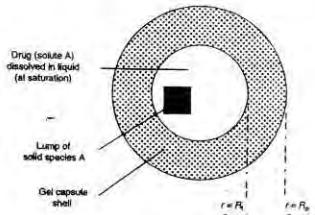
$$\text{PER CHANNEL} - S' = \frac{1}{4}(0.1)^2 = 0.00785$$

$$\text{NO. CHANNELS} = \frac{91.12}{0.00785} = 11608$$

$$26.6 \quad D_{AB} = 1.5 \times 10^{-9} \text{ m}^2/\text{s}$$

$$C_{AS} = C_A(R_i) = 0.01 \text{ mol/cm}^3$$

$$C_{AO} = C_A(R_o)$$



STEADY STATE, NO HOMOGENEITY -

$$\nabla \cdot \vec{N}_A = 0 \quad N_A = -D_{AB} \frac{dC_A}{dr}$$

$$\frac{d}{dr}(r^2 N_A) = 0 \quad \text{or } r^2 N_A \text{ IS CONST.}$$

$$\frac{d}{dr}(r^2 \frac{dC_A}{dr}) = 0$$

26.6 CONTINUED -

$$r^2 \frac{dc_A}{dr} = g \quad \frac{dc_A}{dr} = r^2 c_1$$

$$c_A = -\frac{c_1}{r} + c_2$$

USING B.C.

$$c_{A1} = 0.01 = -\frac{c_1}{0.2} + c_2$$

$$c_{A0} = -\frac{c_1}{0.35} + c_2$$

$$\text{SUBTRACTING} - c_1 = \frac{c_{A0} - 0.01}{0.466}$$

$$W = 4\pi r^2 N_{A2} = 4\pi (-D_{AB}) c_1$$

$$= -4\pi D_{AB} (c_{A0} - 0.01)$$

$$= -\frac{4\pi (1.5 \times 10^{-5})}{0.466} (c_{A0} - 0.01)$$

$$= -4.045 \times 10^{-4} (c_{A0} - 0.01) \text{ mol/s}$$

$W_A$  IS MAX FOR  $c_{A0} = 0$

$$= 4.045 \times 10^{-4} \text{ mol/s}$$

$$= \underline{\underline{1.456 \text{ mol/H}}}$$

26.7 SPHERICAL GEOMETRY

STEADY STATE, NO HOMO FLUX

A INTO STAGNANT B

$$\nabla \cdot \vec{N}_A = 0$$

$$\frac{d}{dr}(r^2 N_{Ar}) = 0$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

26.7 - CONTINUED -

$$\nabla \cdot \vec{N}_A = 0 \Rightarrow \frac{d}{dr}(r^2 N_{Ar}) = 0$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

$$N_{Ar} = -\frac{C D_{AB}}{1-y_A} \frac{dy_A}{dr}$$

$$N_{Ar} (4\pi r^2) \int_r^R \frac{dy_A}{r^2} = -4\pi C D_{AB} \int_{y_{A0}}^{y_A} \frac{dy_A}{1-y_A}$$

$$W_A \left( \frac{1}{r} \Big|_R \right) = -4\pi C D_{AB} \left( \ln \frac{1}{1-y_A} \Big|_{y_{A0}} \right)$$

$$W_A = 4\pi C D_{AB} R \ln \left( \frac{1}{1-y_{A0}} \right)$$

MASS BALANCE FOR A:

$$4\pi C D_{AB} R \ln \frac{1}{1-y_{A0}} = \frac{S}{M_A} \frac{dN}{dt}$$

$$4\pi C D_{AB} R \ln \frac{1}{1-y_{A0}} = \frac{4\pi S}{M_A} R^2 \frac{dr}{dt}$$

SEPARATING VARIABLES  $\frac{1}{1-y_{A0}}$  INTEGRATING:

$$C D_{AB} \ln \left( \frac{1}{1-y_{A0}} \right) = \frac{S}{M_A} \left( R_1^2 - R_2^2 \right)$$

VALUES:  $D_{AB} = 8.19 \times 10^{-6} \text{ m}^2/\text{s}$

$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{8.314(347)} = 35.11 \text{ g/mol/m}^3$$

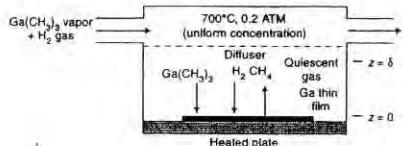
$$P_S = \frac{1.145}{128} \times (100)^3 = 8945 \text{ g/mol/m}^3$$

SOLVING FOR t:

$$t = \frac{(8945) \left( \frac{10^{-4}}{2} - \frac{0.0625 \times 10^{-4}}{2} \right)}{(35.11)(8.19 \times 10^{-6}) \ln \left( \frac{1}{0.993} \right)}$$

$$= \underline{\underline{2.076 \times 10^5 \text{ s}}} = \underline{\underline{57 \text{ H}}}$$

26.8



PSEUDO STEADY STATE -

NO HEATING,  $\dot{F} = 0$ 

ONE (2) DIRECTIONAL DIFFUSION

$$\nabla \cdot \vec{N}_A = 0 \quad \frac{dN_{A2}}{dz} = 0$$

$$N_{B2} = -CD_{AB} \frac{dy_A}{dz} + y_A(N_{A2} + N_{B2} + N_{C2})$$

$$\left\{ \begin{array}{l} H_2: \quad N_{B2} = 3/2 N_{A2} \\ Ga: \quad N_{C2} = -3N_{A2} \end{array} \right.$$

$$N_{A2} = -CD_{AB} \frac{dy_A}{dz} + y_A N_{A2} (1 + 3/2 - 3)$$

$$= -CD_{AB} \frac{dy_A}{(1+y_A/2) dz}$$

$$N_{A2} \int_0^\delta dz = -CD_{AB} \int_{y_0}^0 \frac{dy_A}{1+y_A/2} \quad (a)$$

FOR DILOUTED A  $\sim y_A$  SMALL

$$N_{A2} = -CD_{AB} \frac{dy_A}{dz}$$

$$N_{A2} dz = -CD_{AB} \int_0^{0.0002} dy_A$$

$$C = \frac{P}{RT} = \frac{0.20}{82.06(973)} = 2.5 \times 10^{-6} \text{ g mol/cm}^3$$

$$D_{AB} \left|_{T_2, P_2} \right. = 2.0 \text{ cm}^2/\text{s} \left( \frac{1}{0.2} \right)^{3/2} \left( \frac{973}{1023} \right)^{3/2}$$

$$= 9.276 \text{ cm}^2/\text{s} \quad (c)$$

IN TERMS OF  $\delta$ :

$$\underline{\underline{N_{A2} = \frac{CD_{AB}}{\delta} (0.0002)}} \quad (b)$$

$$26.9 \quad f = 303.9 \text{ Pa} \quad T = 873 \text{ K}$$

$$y_A = y_{AS} = 0 \text{ @ } z = 0$$

$$y_A = 0.2 \text{ @ } z = \delta = 6 \text{ cm}$$

$$M_A = 78$$

FOR  $D_{AB}$  - FICK'S FELDER FORM.

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.716$$

$$\sigma_{AB}^2 = 12A18 \quad E_{AB}/k = 6913$$

$$E_{AB}/k_T = 7.92 \quad \Omega_0 = 0.8556$$

$$\sim \text{SUBSTITUTING } \underline{\underline{D_{AB} = 0.0221 \text{ m}^2/\text{s}}} \quad (a)$$

PHYSICAL SITUATION IS EQUIVALENT TO CASE EXAMINED IN EXAMPLE 2, CH 25

$$N_{A2} = \frac{CD_{AB}}{\delta} \ln \left( \frac{1+y_{A0}}{1+y_{AS}} \right)$$

$$C = \frac{P}{RT} = \frac{3 \times 10^{-3}}{82.06(873)} = 4188 \times 10^{-3} \text{ mol/cm}^3$$

$$N_{A2} = \frac{(4.188 \times 10^{-3})(221 \text{ cm}^2/\text{s})}{6 \text{ cm}} \ln \left( \frac{1.2}{1} \right)$$

$$= 2.814 \times 10^{-7} \text{ mol/cm}^2 \cdot \text{s}$$

$$W_A = N_{A2} A = N_{A2} \left( \frac{\pi}{4} \right) b^2$$

$$= (2.814 \times 10^{-7}) \left( \frac{\pi}{4} \right) (15)^2 (60)(78)$$

$$= 0.2327 \text{ g/m}$$

26.10 HEMISPHERICAL DROPLET ON A PLANE SURFACE

STEADY STATE, NO HOMOGENEOUS FIX - DNE D

$$\nabla^2 \vec{N}_A = \frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = 0$$

$r^2 N_{Ar} \sim \text{CONSTANT}$

$$N_{Ar} = -CD_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

$$N_{Ar} = -\frac{CD_{AB}}{1-y_A} \frac{dy_A}{dr}$$

$$\underbrace{2\pi r^2 N_{Ar}}_{W_A} \int_0^r \frac{dy_A}{r^2} = 2\pi C D_{AB} \int_0^r \frac{dy_A}{1-y_A}$$

$$@t=0 \quad r=0.005 \text{ m}$$

$$y_{A1} = \frac{31.824}{760} = 0.0419$$

$$W_A \left[ -\frac{1}{r} \right]_R^0 = 2\pi C D_{AB} R \ln \left( \frac{1}{0.958} \right)$$

$$W_A = 2\pi C D_{AB} R \ln (1.043) \quad (1)$$

FOR DROPLET ~ PSEUDO S.S.

$$W_A = -\frac{\dot{N}_A}{M_A} \frac{dN}{dt}$$

$$= -\frac{1}{18} \left( 2\pi R^2 \frac{dR}{dt} \right) \quad (2)$$

EQUATING (1) & (2) & INTEGRATING:

$$CD_{AB} R \ln (1.043) t = 0.0556 \left( \frac{R_i^2 - R_f^2}{2} \right)$$

$$\tilde{C} = \frac{P}{RT} = \frac{1.03 \times 10^5}{8.314(303)} = 4.021 \times 10^{-7} \text{ g mol/cm}^3$$

26.10 CONTINUED -

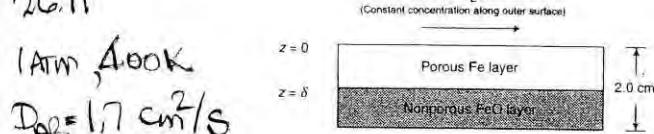
$$D_{AB} = 0.260 \left( \frac{303}{298} \right)^{3/2} = 0.2166 \text{ cm}^2/\text{s}$$

$$R_i = 0.5 \text{ cm} \quad P_0 = 0$$

SUBSTITUTE & SOLVE -

$$t = 1.517 \times 10^8 \text{ s} = 421.4 \text{ h}$$

26.11



$$\delta_{FeO} = 2.5 \text{ g/cm}^3$$

$$M_{FeO} = 71.85$$

STEADY STATE  
NO HOMOGENEOUS FIX

$$\nabla^2 \vec{N}_A = \frac{dN_{A2}}{dz} = 0 \sim N_{A2} \sim \text{CONST.}$$

$$N_{A2} = -CD_{AB} \frac{dy_A}{dz} + y_A (N_{A2} + N_{B2})$$

$$\text{AS } N_{B2} = -N_{A2}, \quad y_A (N_{A2} + N_{B2}) = 0$$

$$N_{A2} \int_0^z dz = -CD_{AB} \int_{1.0}^{y_A} dy_A$$

$$N_{A2} = \frac{CD_{AB}}{\delta} \quad (a)$$

$$C = \frac{P}{RT} = \frac{1}{82.06(400)} = 3.047 \times 10^{-5} \text{ g mol/cm}^3$$

$$N_{A2} = 5.18 \times 10^{-5} \text{ g mol/cm}^2 \cdot \text{s} \quad (b)$$

FOR  $0.1 < \delta < 0.2$

$$W = N_{A2} (1) = S \frac{dS}{dt}$$

$$\frac{MB}{S_B} D_{AB} \int_0^t dt = \int_{S_1}^{S_2} S dS$$

26.11 (CONTINUED) -

$$\frac{M_B D_{ABC}}{S_B} t = \frac{s_2^2 - s_1^2}{2}$$

SUBSTITUTING NUMERICAL VALUES:

$$t = 1007 \text{ s} = 16.78 \text{ min.}$$

26.12

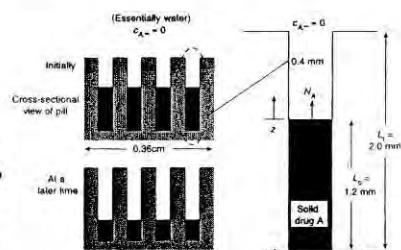
$$T = 310 \text{ K}$$

$$D_{AB} = 2 \times 10^{-5} \text{ cm}^2/\text{s}$$

$$\rho_A = 1.10 \text{ g/cm}^3$$

$$M_A = 120$$

$$C_A^* = 2.0 \times 10^{-4} \text{ g mol/cm}^3$$



PSEUDO STOICHIOMETRIC, NO HOMOGEN. RX

ONE-DIMENSIONAL - DILUTE SOLN

$$\nabla \cdot \vec{N}_A = \frac{dN_{A2}}{dz} = 0 \sim N_{A2} \text{ CONST.}$$

$$N_{A2} = -D_{AB} \frac{dC_A}{dz}$$

$$N_{A2} \int_{z_1}^{z_2} dz = -D_{AB} \int_{C_A}^{0} \frac{dC_A}{C_A^*}$$

$$N_{A2} = \frac{D_{ABC}^*}{z_2 - z_1} \quad (\alpha)$$

for A Pore:

$$W_{A2} = N_{A2} A = \frac{D_{ABC}^* A}{z_2 - z_1}$$

$$W_A = \frac{(2 \times 10^{-5})(2.0 \times 10^{-4})(\pi/4)(0.04)^2}{0.2 - 0.12}$$

$$= 6.3 \times 10^{-11} \text{ g mol/s per pore}$$

26.12 (CONTINUED) -

for 1 Pill ~ 16 pores ~

$$W_A = (6.3 \times 10^{-11})(16) = 1.008 \times 10^{-9} \text{ g mol/s}$$

TIME TO DISSOLVE -

$$\frac{S_B}{M_B} \frac{dS}{dt} = \frac{D_{ABC}^*}{S}$$

$$\int_{0.02}^{0.08} \frac{dS}{S} = \frac{D_{ABC}^* M_B}{S_B} \int_0^t dt$$

$$\frac{S^2}{2} \Big|_{0.02}^{0.08} = \frac{D_{ABC}^* M_B}{S_B} t$$

$$t = 3.16 \times 10^4 \text{ s} = 10.14 \text{ h}$$

26.13 for CONDITIONS DESCRIBED -

$$\nabla \cdot \vec{N}_A = \frac{dN_A}{dz} = 0 \quad N_{A2} \sim \text{CONSTANT}$$

$$N_{A2} = -CD_{AB} \frac{dC_A}{dz} + y_A(N_{A2} + N_{B2})$$

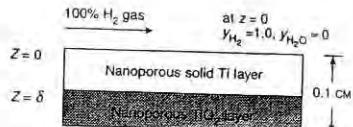
IN EACH REACTION -  $N_{B2} = -N_{A2}$

$$\therefore N_{A2} = -CD_{AB} \frac{dC_A}{dz}$$

$$N_{A2} = \frac{CD_{AB}(y_A - 0)}{S}$$

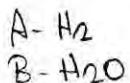
ALL REACTIONS INVOLVE EQUAL MOL DIFFUSION -

26.14



$$T = 900\text{ K}$$

$$P = 1 \text{ atm}$$



FOR CONDITIONS STATED:

$$\nabla \cdot \vec{N}_A = \frac{dN_{A2}}{dz} = 0 \quad N_{A2} \text{ CONST.}$$

$$N_{A2} = -CD_{AB} \frac{dy_A}{dz} + y_A(N_{A2} - N_{B2})$$

$$\text{Since } N_{A2} - N_{B2} = 0$$

$$N_{A2} = -CD_{AB} \frac{dy_A}{dz}$$

INTEGRATING:

$$N_{A2} = \frac{CD_{AB} y_{A0}}{\delta}$$

$$C = \frac{P}{RT} = \frac{1}{(82.06)(900)} = 1.354 \times 10^{-5} \text{ mol/cm}^3$$

For  $\delta = 0.05 \text{ cm}$ 

$$N_{A2} = \frac{(0.031)(1.354 \times 10^{-5})}{0.05} = 8.39 \times 10^{-6} \text{ mol/cm}^2 \cdot \text{s} \quad (2)$$

By STOICHIOMETRY:

$$\left\{ \begin{array}{l} \text{RATE OF} \\ \text{Ti DEPOSITED} \end{array} \right\} = \frac{1}{2} \left\{ \begin{array}{l} \text{RATE OF} \\ \text{A}_2 \text{ DIFFUSED} \end{array} \right\}$$

$$\frac{S_T}{M_{Ti}} \frac{d\delta}{dt} = \frac{1}{2} \frac{D_{AB}}{\delta} C_{A0}$$

$$\frac{S_T}{M_{Ti}} \int_0^{0.05} d\delta = \frac{D_{AB} C_{A0}}{2} \int_0^t dt$$

$$\delta = \left[ \frac{M_{Ti} D_{AB} C_{A0}}{S_T} \right]^{1/2} t^{1/2}$$

26.14 CONTINUED -

INSERTING VALUES - FOR  $\delta = 0.1 \text{ cm}$ 

$$t = 12935 = 0.359 \text{ h} \quad (b)$$

$$@ \delta = 0.05 \text{ cm} ; N_A = 8.39 \times 10^{-6} \text{ mol/cm}^2 \cdot \text{s}$$

 $= A \text{ (A CONSTANT)}$ 

$$A \int_0^z dz = -D_{AB} \int_{C_{A0}}^{C_A} \frac{dC_A}{C_{A0}}$$

$$C_A - C_{A0} = -\left[ A / D_{AB} \right] z$$

$$C_A = C_{A0} - \frac{A}{D_{AB}} z \quad (c)$$

26.15 ACETONE (A) DIFFUSING IN AIR (B)

$$D_{AB}|_{298K} = 0.093 \text{ cm}^2/\text{s}$$

$$D_{AB}|_{323K} = 0.093 \left( \frac{323}{298} \right)^{3/2} = 0.105 \text{ cm}^2/\text{s}$$

STEADY STATE - NO HOMOGENEOUS RX

$$\nabla \cdot \vec{N}_A = \frac{dN_{A2}}{dz} = 0 \quad N_{A2} \text{ CONST.}$$

FOR  $T \neq P$  CONSTANT  $N_{A2} = -NB_2$ 

$$N_{A2} = -CD_{AB} \frac{dy_A}{dz} = \frac{CD_{AB}}{z_2 - z_1} (y_{A1} - y_{A2})$$

$$C = \frac{P}{RT} = \frac{1 + \text{TM}}{82.06(323)} = 3.77 \times 10^{-5} \text{ mol/cm}^3$$

$$z_2 - z_1 = 500 \text{ cm} \quad y_{A1} - y_{A2} = 0.5$$

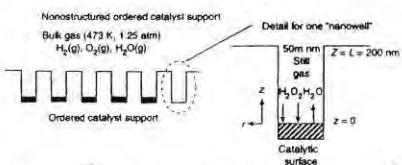
$$\text{SUBSTITUTING: } N_{A2} = 3.96 \times 10^{-9} \text{ mol/cm}^2 \cdot \text{s}$$

$$W_A = N_{A2}(A)$$

$$= (3.96 \times 10^{-9}) \left( \frac{\pi}{4} \right) (10 \text{ cm})^2$$

$$= 3.11 \times 10^{-7} \text{ mol/s}$$

26.16



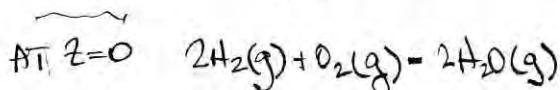
$$d = 5 \times 10^{-6} \text{ cm} \quad \Delta z = 2 \times 10^{-5} \text{ cm}$$

$$T = 473 \text{ K} \quad P = 1.25 \text{ atm}$$

ASSUMPTIONS - STEADY  
NO HOMOGENEOUS RX  
ONE DIMENSIONAL

$$\nabla \cdot \vec{N}_A = \frac{d}{dz} N_{AZ} = 0 \quad N_{AZ} \text{ CONST.}$$

$$N_{AZ} = -CD_{A-MIX} \frac{dy_A}{dz} + y_A(N_{AZ} + N_{BZ} + N_{EZ})$$



H<sub>2</sub>(A), O<sub>2</sub>(B), H<sub>2</sub>O(C)

$$N_{BZ} = \frac{1}{2} N_{AZ} \quad N_{EZ} = -N_{AZ}$$

$$N_{AZ} = -CD_{AB} \frac{dy_A}{dz} + \frac{1}{2} y_A N_{AZ}$$

$$N_{AZ} \int_0^L dz = CD_{AB} \int_0^{0.01} \frac{dy_A}{1-y_A/2}$$

$$N_{AZ} L = 2CD_{AMIX} \ln \frac{1-0.01/2}{1-0}$$

$$N_{AZ} = \frac{2CD_{AMIX}}{L} (-0.0050)$$

$$C = \frac{P}{RT} = \frac{1.25}{82.06(473)} = 3.22 \times 10^{-5} \text{ mol/m}^3$$

$$D_{AB} = \frac{0.697}{1.25} \left( \frac{473}{273} \right)^{3/2} = 1.212 \text{ cm}^2/\text{s}$$

$$D_{AC} = \frac{0.850}{1.25} \left( \frac{473}{373} \right)^{3/2} = 1.551 \text{ "}$$

26.16 CONTINUED -

$$D_{A-L-MIX} = \frac{y_A/D_{AB} + y_B/D_{AC}}{y_A + y_B} = 1.274 \text{ cm}^2/\text{s}$$

SUBSTITUTING NUMERICAL VALUES!

$$\underline{N_{AZ} = -0.0205 \text{ mol/cm}^2 \cdot \text{s}}$$

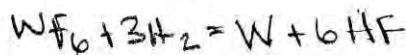
26.17 WF<sub>b</sub>(A)

KNUDSEN DIFFUSION

VERY DILUTE

$$\nabla \cdot \vec{N}_A = \frac{dN_{AZ}}{dz} = 0 \quad N_{AZ} \text{ - CONSTANT}$$

$$N_{AZ} = -D_{AB} \frac{dy_A}{dz} = \frac{D_{AB} C_{AO}}{\delta}$$



RATE OF FORMATION OF W

$$= N_{AZ}(A) = \frac{D_{AB} C_{AO}(A)}{\delta} = \frac{s_w}{M_w} \frac{d(A\delta)}{dt}$$

$$\frac{P_w}{M_w} \frac{d\delta}{dt} = \frac{D_{AB} C_{AO}}{\delta}$$

$$\delta^2 = 2D_{AB} M_w C_{AO} \frac{s_w}{M_w}$$

$$\delta = \left[ 2D_{AB} \frac{M_w C_{AO}}{s_w} \right]^{1/2} t^{1/2}$$

FOR KNUDSEN DIFFUSION - EQUATION (24-58)

$$D_{KA} = 4850 \delta p \sqrt{\frac{T}{M_w F_b}}$$

$$\delta p = 2.5 \times 10^{-5} \text{ cm}, T = 700 \text{ K}, M = 298$$

$$D_{KA} = 0.1858 \text{ cm}^2/\text{s}$$

26.17 (CONTINUED)

$$C = \frac{P}{RT} = \frac{75 \text{ Pa}}{8,314(100)} = 0,0129 \text{ mol/m}^3$$

$$c_{AO} = y_{AO} C = 1,29 \times 10^{-3} \text{ mol/cm}^3$$

SUBSTITUTING INTO EQUATION FOR S(t)

$$\underline{t = 8,80 \times 10^4 \text{ s} = 24,44 \text{ h}}$$

26.18 C<sub>6</sub>H<sub>6</sub>(A) IN C<sub>7</sub>H<sub>8</sub>

$$h_{fg}(A) = 30 \text{ kJ/mol}$$

$$h_{fg}(B) = 33 \text{ "}$$

$$\nabla \cdot \vec{N}_A = \frac{d N_{A2}}{dz} = 0 \quad N_{A2} \text{ CONST.}$$

$$\underline{N_{A2} = -CD_{AB} \frac{dy_A}{dz} + y_A(N_{A2} + N_{B2})}$$

$$N_{A2}(30) = N_{B2}(33)$$

$$N_{B2} = -0,909 N_{A2}$$

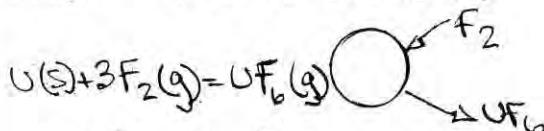
$$N_{A2} = -CD_{AB} \frac{dy_A}{dz} + y_A N_{A2}(1-0,909)$$

$$N_{A2} \int_0^8 \frac{dy_A}{1-0,909 y_A} = -CD_{AB} \int_{y_{A0}}^{y_A} \frac{dy_A}{1-0,909 y_A}$$

$$N_{A2} \delta = \frac{CD_{AB}}{0,091} \ln \left[ \frac{1-0,091 y_A}{1-0,091 y_{A0}} \right]$$

$$N_{A2} = \frac{CD_{AB}}{0,091 \delta} \ln \left[ \frac{1-0,091 y_A}{1-0,091 y_{A0}} \right]$$

26.19 SPHERICAL GEOMETRY -



$$T = 1000 \text{ K} \quad P = 1 \text{ atm}$$

$$D_{AB} = 0,273 \text{ cm}^2/\text{s} \quad \delta = 0,4 \text{ cm}$$

STEADY STATE, NO HOMOGENEITY

$$\nabla \cdot \vec{N}_A = \frac{1}{r^2} \frac{d}{dr} r^2 N_{Ar} = 0 \quad r^2 N_{Ar} \sim \text{CONST.}$$

$$\underline{N_{Ar} = -CD_{AB} \frac{dy_A}{dr} + y_A(N_{Ar} + N_{Br})}$$

$$N_{Br} = -3N_{Ar} \Rightarrow N_{Ar} + N_{Br} = -2N_{Ar}$$

$$\underline{N_{Ar} = -\frac{CD_{AB}}{1+2y_A} \frac{dy_A}{dr}}$$

$$\underline{4\pi r^2 N_{Ar} \int_R^P \frac{dr}{r^2} = 4\pi C D_{AB} \int_{1,0}^P \frac{dy_A}{1+2y_A}}$$

$$W_A(1) = \frac{4\pi C D_{AB}}{2} \ln \frac{3,0}{1,0}$$

$$W_A = 2\pi R C D_{AB} \ln 3$$

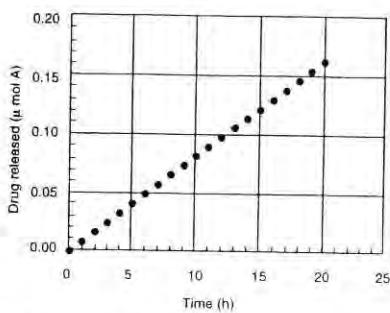
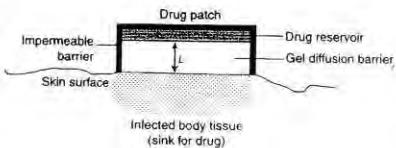
$$C = \frac{P}{RT} = \frac{1,013 \times 10^5}{8,314(1000)} = 12,18 \text{ mol/m}^3$$

$$= 1,218 \times 10^{-5} \text{ mol/cm}^3$$

SUBSTITUTING VALUES:

$$\underline{W_A = 4,59 \times 10^{-6} \text{ mol/s}}$$

24,20

Slope of Plot is  $W_A$ :

$$W_A \approx \frac{0.15 \text{ μmol}}{18.5 \text{ h}} \left( \frac{1}{3600} \right) \\ = 2.25 \times 10^{-12} \text{ mol/s}$$

$$A_s = 9 \text{ cm}^2$$

$$N_{A2} = 2.503 \times 10^{-13} \text{ mol/s} \cdot \text{cm}^2$$

Since surface is linear -

All transport is diffusion

$$\therefore N_{A2} = D_{AB} \frac{C_{A1} - C_{A2}}{z_2 - z_1}$$

$$D_{AB} = \frac{2.503 \times 10^{-13}}{0.5 \times 10^{-6}} (0.2) \\ = 1.00 \times 10^{-7} \text{ cm}^2/\text{s}$$

MODIFIED WILKE-CRANK-EQN (24-54)

$$\frac{D_{AB} \mu}{T} = \text{CONST}$$

$$D_{AB|35} = D_{AB|20} \left( \frac{293}{308} \right) \left( \frac{\mu_{H_2O|35}}{\mu_{H_2O|20}} \right) \\ = 1.0 \times 10^{-7} \left( \frac{293}{308} \right) \left( \frac{0.00593}{0.00742} \right) \\ = 1.273 \times 10^{-7} \text{ cm}^2/\text{s}$$

24,20 CONTINUED -

ALL OTHER TERMS REMAIN THE SAME -

$$W_A|_{35} = W_A|_{20} \frac{D_{AB|35}}{D_{AB|20}} \\ = (2.25 \times 10^{-12}) \frac{1.273 \times 10^{-7}}{1 \times 10^{-7}} \\ = 2.864 \times 10^{-12} \text{ mol/s} \\ = 2.475 \times 10^{-7} \text{ mol/day}$$

$$24,21 \quad J_{A2} = -CD_{AB} \frac{dc_A}{dz} = \frac{D_{AB}}{z} (c_{A1} - c_{A2})$$

$$c_{A1} - c_{A2} = k \left( \frac{1}{2} P_{A1} - \frac{1}{2} P_{A2} \right)$$

$$\Rightarrow J_{A2} = D_{AB} k \left( \frac{1}{2} P_{A1} - \frac{1}{2} P_{A2} \right)$$

$$\text{AT } 1 \text{ atm} \quad c_{A1} = k P_A^{1/2}$$

$$= \frac{7 \text{ cm}^3}{100 \text{ g}} \left( \frac{9.8}{\text{cm}^3} \right) = 0.63$$

$$k = 0.63 \text{ atm}^{-1/2}$$

$$D_{AB} = 6 \times 10^{-5} \text{ cm}^2/\text{s} \quad P_{A1} = 8 \text{ atm} \quad P_{A2} = 0$$

$$\Delta z = 0.2 \text{ cm}$$

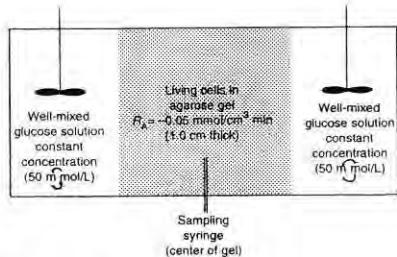
$$\text{SUBSTITUTION!} \quad J_{A2} = 5.346 \times 10^{-4} \text{ cm/s}$$

$$W_A = J_{A2} A = (5.346 \times 10^{-4})(8)$$

$$= 4.277 \text{ cm}^3/\text{s}$$

$$= 15.4 \text{ cm}^3/\text{day}$$

26.22



$$\nabla \cdot \vec{N}_A - R_A = 0$$

$$\frac{dN_{A2}}{dz} - R_A = 0$$

FOR NO BULK CONTRIBUTION

$$N_{A2} = -D_{AB} \frac{dc_A}{dz}$$

$$\frac{d}{dz} \left( -D_{AB} \frac{dc_A}{dz} \right) = R_A$$

$$d \left( -D_{AB} \frac{dc_A}{dz} \right) = R_A dz$$

$$-D_{AB} \frac{dc_A}{dz} = R_A z + C_1$$

$$-D_{AB} c_A = R_A \frac{z^2}{2} + C_1 z + C_2$$

$$B.C., c_A(0.5 \text{ cm}) = C_0$$

$$\frac{dc_A}{dz}(0) = 0 \Rightarrow C_1 = 0$$

$$C_2 = -D_{AB}(c_0) - \frac{R_A}{2}(0.5)^2$$

$$c_A = -\frac{1}{D_{AB}} \left[ R_A \frac{z^2}{2} + C_1 z + C_2 \right]$$

WITH VALUES SUBSTITUTED

$$c_A = C_0 - \frac{R_A}{D_{AB}} \left( \frac{z^2}{2} - 0.125 \right)$$

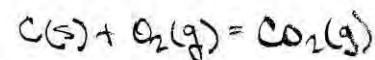
## 26.23 CYLINDRICAL GEOMETRY -

$$T = 1100 \text{ K} \quad P = 2 \text{ atm}$$

$$S(B,L, \text{film}) = 5 \text{ mm} \quad L = 25 \text{ cm}$$

$$d = 2 \text{ cm}$$

$$\nabla \cdot \vec{N}_A = \frac{1}{r} \frac{d}{dr} (r N_A) = 0 \quad r N_A \sim \text{CONST.}$$



$$O_2 \text{ is } A \quad N_{Br} = -N_{Ar}$$

$$CO_2 \text{ is } B \quad N_{Ar} = -C D_{AB} \frac{dy_A}{dr}$$

$$\underbrace{2\pi L r N_{Ar}}_{W_A} \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi L C D_{AB} \int_0^{0.21} dy_A$$

$$W_A \ln \frac{r_2}{r_1} = -2\pi L C D_{AB} (0.21)$$

$$W_A = -\frac{2\pi L C D_{AB} (0.21)}{\ln(r_2/r_1)}$$

$$C = \frac{P}{RT} = \frac{2.026 \times 10^5}{(8.314)(1100)} = 22.1 \text{ mol/m}^3$$

$$= 2.21 \times 10^5 \text{ mol/cm}^3$$

$$D_{AB} = 0.175 \left( \frac{1}{2} \sqrt{\frac{1100}{273}} \right)^{3/2} = 0.708 \text{ cm}^2/\text{s}$$

$$At t=0 \quad r_2/r_1 = 1.5$$

SOLVING FOR  $W_A$  AT  $t=0$ 

$$\underline{W_A = -1.28 \times 10^{-3} \text{ mol/s}}$$

FOR  $t > 0 \sim r$ , DECREASES

$$W_A = -\frac{2\pi L C D_{AB} (0.21)}{\ln(\frac{r+0.5}{r})}$$

26.23 (CONTINUED)

$$\text{FOR SOLID C: RATE OF DEPOSITION} = \frac{\rho}{M} \frac{dN}{dt}$$

$$\text{PER MOLE: } W_A = -\frac{\rho}{M} \frac{dN}{dt}$$

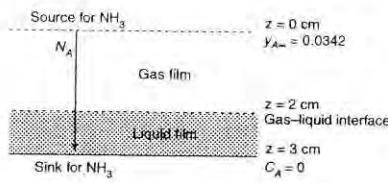
$$\frac{2\pi L c D_{AB} (0,2)}{\ln \left[ \frac{(r+0,5)/r}{r} \right]} = \frac{\rho}{M} (2\pi r L) \frac{dr}{dt}$$

INTEGRATION BETWEEN  $r=r_i$  @  $t=0$   
 $r=0$  @  $t$

THE SOLUTION ~ A BIT MESSY ~

$$t = 18000 \text{ s (E h)}$$

26.24



EQUILIBRIUM DATA  $\rightarrow$

$P_A (\text{mmHg})$	5.0	10.0	15.0	20.0	25.0	30.0
$C_A (\text{mol/m}^3)$	6.1	11.9	20.0	32.1	53.6	84.8

$\text{NH}_3 (\text{A})$  DIFFUSES IN SERIES, THROUGH GAS & LIQUID LAYERS

$$\text{THROUGH GAS: } N_{AZ} = \frac{C D_{AB} \ln(1-y_{Ai})}{\delta_L} \frac{1}{1-y_{A1}}$$

$$\text{LIQUID: } N_{AZ} = \frac{D_L}{\delta_L} (C_{Ai} - C_{As})$$

$$D_{AB} = 0.198 \left( \frac{288}{273} \right)^{3/2} = 0.215 \text{ cm}^2/\text{s}$$

$$D_L = 1.77 \times 10^{-5} \text{ cm}^2/\text{s}$$

$$N_{AG} = N_{AL}$$

$$\frac{C D_{AB}}{\delta_L} \ln \frac{1-y_{Ai}}{1-y_{A1}} = \frac{D_L}{\delta_L} (C_{Ai} - C_{As})$$

26.24 (CONTINUED)

$$C = \frac{P}{R_T} = \frac{1.013 \times 10^5}{(8314)(288)} = 42.3 \text{ mol/m}^3$$

$$= 4.23 \times 10^{-5} \text{ mol/cm}^3$$

INSERTING VALUES:

$$0.257 \ln \frac{1-y_{Ai}}{0.9658} = C_{Ai}$$

VALUES OF  $y_{Ai}$ ,  $C_{Ai}$  MUST AGREE WITH PLOT OF DATA --

TRIAL & ERROR IS NECESSARY --

~ RESULT IS  $P_{Ai} = 25.88 \text{ mm}$

$$\sim y_{Ai} = \frac{25.88}{760} = 0.0339$$

$$C_{Ai} = 5.58 \times 10^{-5} \text{ mol/cm}^3$$

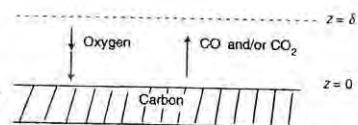
WITH THESE VALUES --

$$N_{AZ} = \frac{(1.77 \times 10^{-5})(5.58 \times 10^{-5})}{1}$$

$$= 9.88 \times 10^{-10} \text{ mol/cm}^2 \cdot \text{s}$$

26.25

CONSTITUENT A IS  $\text{O}_2$



$$\nabla \cdot \vec{N}_A = \frac{d N_{AZ}}{dz} = 0 \quad N_{AZ} = \text{CONSTANT}$$

$$N_{AZ} = -C D_{AB} \frac{dy_A}{dz} + y_A (N_{AZ} + N_{BZ})$$

REACTION AT SURFACE IS



$$\sim 2 N_{AZ} = -N_{BZ}$$

$$y_A (N_{AZ} + N_{BZ}) = y_A N_{AZ} (-1)$$

26.25 (CONTINUED →

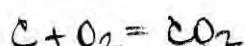
$$N_{A2}(1+y_A) = -CD_{AB} \frac{dy_A}{dz}$$

SEPARATION VARIABLES & INTEGRATION!

From  $z=0$  TO  $\delta$        $y_A$  from 0 TO 0.21

$$\underline{N_{A2} = -\frac{CD_{AB}}{\delta} \ln 1.21} \quad (a)$$

IF REACTION AT SURFACE IS



$$\text{THEN } y_A(N_{A2} + N_{B2}) = 0$$

& SOLUTION IS

$$\underline{N_{A2} = -\frac{CD_{AB}}{\delta} (0.21)} \quad (b)$$

IF REACTION AT SURFACE IS



$$\text{THEN } N_{B2} = N_C - \frac{2}{3} N_{A2}$$

$$y_A(N_{A2} + N_{B2} + N_C) = y_A N_{A2} \left(-\frac{1}{3}\right)$$

FICK'S LAW EXPRESSION BECOMES

$$N_{A2} = -CD_{AB} \frac{dy_A}{dz} - y_A \frac{N_{A2}}{3}$$

$$N_{A2} \left(1 + \frac{y_A}{3}\right) = -CD_{AB} \frac{dy_A}{dz}$$

& SOLUTION IS

$$\begin{aligned} N_{A2} &= -\frac{CD_{AB}}{\delta} \left[ 3 \ln 1.07 \right] \\ &= \frac{CD_{AB}}{\delta} (0.203) \quad (c) \end{aligned}$$

26.26

Time (h)	Measured SiO <sub>2</sub>		film thickness (μm)
	(100) Si	(111) Si	
1	0.049	0.070	
2	0.078	0.105	
4	0.124	0.154	
7	0.180	0.212	
16	0.298	0.339	

SYSTEM CONSIDERED WAS EVALUATED IN TEXT - EXAMPLE 2.

$$\delta^2 = \frac{2MBD_{AB}C_{AS}}{S_B} t$$

FROM DATA IN TABLE -

$$\frac{d}{dt} \delta^2 = \frac{2MBD_{AB}C_{AS}}{S_B} = A$$

$$D_{AB} = \frac{AS_B}{2MC_{AS}}$$

A EVALUATED FROM DATA VARIES FROM 0.0049 TO 0.00718 ~ TAKE MIDDLE VALUE (CONDITION @ t=4h)

$$A = 0.00593 \mu\text{m}^2/\text{h}$$

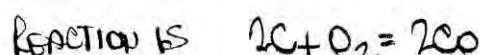
$$= 1.646 \times 10^{-14} \text{ cm}^2/\text{s}$$

$$\begin{aligned} D_{AB} &= \frac{(1.646 \times 10^{-14})(2.27)}{2(100)(9.68 \times 10^{-8})} \\ &= 3.24 \times 10^{-9} \text{ cm}^2/\text{s} \end{aligned}$$

26.27 CYLINDRICAL GEOMETRY -

$$\nabla \cdot \vec{N}_A = \frac{1}{r} \frac{d}{dr} (rN_{Ar}) = 0 \quad rN_{Ar} \text{ (const.)}$$

$$\begin{aligned} A \sim O_2 \\ B \sim CO \end{aligned} \quad N_A = -CD_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$



$$N_{Br} = -2N_{Ar}$$

26.27 (CONTINUED) -

$$N_{Ar} = - \frac{c D_{AB}}{1+y_A} \frac{dy_A}{dr}$$

$$\frac{2\pi L r N_{Ar}}{W_A} \int_{r_1}^{r_2} \frac{dy_A}{r} = - 2\pi L c D_{AB} \int_0^{0.4} \frac{dy_A}{1+y_A}$$

$$W_A = - \frac{2\pi L c D_{AB} \ln(1.4)}{\ln(r_2/r_1)}$$

$$c = \frac{P}{RT} = \frac{1 \text{ ATM}}{(82.04)(1145)} = 1.065 \times 10^{-5} \text{ mol/cm}^3$$

$$D_{AB} = 1.0 \times 10^{-5} \text{ m}^2/\text{s} = 0.10 \text{ cm}^2/\text{s}$$

SUBSTITUTING NUMERICAL VALUES -

$$W_A = -4.099 \times 10^{-5} \text{ mol/s} \quad (1)$$

$$W_B = W_C = -2W_A = 8.20 \times 10^{-5} \text{ mol/s}$$

FOR CONCENTRATION PROFILE:

$$\frac{d}{dr} r N_{Ar} = \frac{d}{dr} \left[ -r \frac{c D_{AB}}{1+y_A} \frac{dy_A}{dr} \right] = 0$$

INTEGRATE ONCE:

$$\frac{r}{1+y_A} \frac{dy_A}{dr} = C_1$$

$$\therefore \text{ AGAIN: } \ln(1+y_A) = C_1 \ln r + C_2$$

$$\text{B.C. } y_A(r_1=0.5) = 0$$

$$y_A(r_2=1.5) = 0.4$$

$$\text{SOLVING: } C_1 = 0.304 \quad C_2 = 0.212$$

Now - For  $r = 1$

26.27 (CONTINUED) -

$$\ln(1+y_A) = C_1 \ln(1) + C_2$$

$$y_A = e^{0.212} - 1 = 0.236$$

26.28 PROBLEM STATEMENT REFERS TO EXAMPLE 4 IN CHAPTER FOR SPHERICAL GEOMETRY -

$$\frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = -k_1 c_A$$

WITH PURE DIFFUSION:

$$N_{Ar} = -D_{AB} \frac{dc_A}{dr}$$

$$\therefore - \frac{D_{AB}}{r^2} \frac{d}{dr} \left( r^2 \frac{dc_A}{dr} \right) = -k_1 c_A$$

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dc_A}{dr} \right) = \frac{k_1}{D_{AB}} c_A \quad (1)$$

$$\text{LETTING } y = c_A r \sim \frac{dy}{dr} = c_A + r \frac{dc_A}{dr}$$

$$\text{WE HAVE: } r^2 \frac{dc_A}{dr} = r \frac{dy}{dr} - y \quad (2)$$

SUBSTITUTING (2) INTO (1) WE HAVE

$$\frac{d^2 y}{dr^2} - \frac{k_1}{D_{AB}} y = 0$$

SOLN. IS

$$y = C_1 \sinh(\sqrt{\frac{k_1}{D_{AB}}} r) + C_2 \cosh(\sqrt{\frac{k_1}{D_{AB}}} r)$$

$$\text{B.C. } y(0) = 0 \quad y(R) = C_0$$

$$\therefore C_1 = 0 \quad C_2 = \frac{C_0 R}{\sinh(\sqrt{\frac{k_1}{D_{AB}}} R)}$$

26.28 (CONTINUED -

$$\frac{C_A}{C_{A0}} = \frac{R}{r} \frac{\operatorname{Sinh}(\sqrt{k_1 D_{AB}} r)}{\operatorname{Sinh}(\sqrt{k_1 D_{AB}} R)} \quad (a)$$

To EVALUATE  $N_{Ar}$  - MUST KNOW  $\frac{dC_A}{dr}$

From (a)

$$\begin{aligned} \frac{dC_A}{dr} &= \frac{C_{A0} R}{\operatorname{Sinh}(\sqrt{k_1 D_{AB}} R)} \times \\ &\left[ -\frac{1}{r^2} \operatorname{Sinh}(\sqrt{k_1 D_{AB}} r) \right. \\ &\left. + \frac{\sqrt{k_1 D_{AB}} \operatorname{cosech}(\sqrt{k_1 D_{AB}} r)}{r} \right] \end{aligned}$$

EVALUATING AT  $r = R$ :

$$\frac{dC_A}{dr} \Big|_{r=R} = \frac{C_{A0}}{R} + C_{A0} \sqrt{\frac{k_1}{D_{AB}}} \operatorname{coth}(\sqrt{\frac{k_1}{D_{AB}}} R)$$

∴ FINALLY -

$$N_{Ar} = \frac{D_{AB} C_{A0}}{R} \left[ 1 - R \sqrt{\frac{k_1}{D_{AB}}} \operatorname{coth}(\sqrt{\frac{k_1}{D_{AB}}} R) \right]$$

from EXAMPLE 4:  $D_{AB} = 2 \times 10^{-10} \text{ m}^2/\text{s}$

$$R = 0.002$$

$$C_{A0} = 0.02 \text{ mol/m}^2$$

$$k_1 = 0.019 \text{ s}$$

SUBSTITUTING:

$$N_{Ar} = 1.02 \times 10^{-12} \text{ mol/m}^2 \cdot \text{s}$$

26.29 FLAT CATALYTIC SURFACE:

$$\frac{\partial N_{A2}}{\partial z} - k_1 y_B = 0 \quad (1)$$

$$\frac{\partial N_{B2}}{\partial z} + k_1 y_B = 0 \quad (2)$$

$$\text{AT POINT } z: \frac{d}{dz} (N_{A2} + N_{B2}) = 0 \quad \{ N_{B2} = -N_{A2} \}$$

$$N_{A2} = -C D_{AB} \frac{dy_A}{dz}$$

EQN (1) BECOMES:

$$C D_{AB} \frac{d^2 y_B}{dz^2} - k_1 y_B = 0$$

$$\text{OR } \frac{d^2 y_B}{dz^2} - \frac{k_1}{C D_{AB}} y_B = 0$$

$$\text{LETTING } b^2 = \frac{k_1}{C D_{AB}}$$

$$\frac{d^2 y_B}{dz^2} - b^2 y_B = 0$$

∴ SOLUTION IS

$$y_B = C_1 \operatorname{cosh} bz + C_2 \operatorname{sinh} bz$$

$$\text{B.C. } y(0) = y_{B0} \quad y(\infty) = 1$$

$$\text{GIVING } C_1 = y_{B0}$$

$$C_2 = \frac{1 - y_{B0} \operatorname{cosh} bS}{\operatorname{sinh} bS}$$

DOING THE MATH:

$$N_{A2} = b C D_{AB} \left[ \frac{1 - y_{B0} \operatorname{cosh} bS}{\operatorname{sinh} bS} \right]$$

26.30 Same configuration as in  
Prob 26.29 except in film  
 $A \xrightarrow{k_1} B$        $R_A = k_1 y_B - k'_1 y_A$

$$\text{Fick's Law: } N_{AB} = -CD_{AB} \frac{dy_A}{dz}$$

CONSERVATION OF MASS:

$$\frac{dN_{AB}}{dz} - k_1 y_B + k'_1 y_A = 0$$

$$-CD_{AB} \frac{d^2 y_A}{dz^2} - k_1 (1-y_A) + k'_1 y_A = 0$$

WITH A LITTLE ALGEBRA WE GET

$$\frac{d^2 y_A}{dz^2} - \frac{k_1 + k'_1}{CD_{AB}} y_A = -\frac{k_1}{CD_{AB}}$$

$$\text{DEFINING } M^2 = \frac{k_1 + k'_1}{CD_{AB}}$$

$$N^2 = k_1 / CD_{AB}$$

our EQN for  $y_A(z)$  is

$$\frac{d^2 y_A}{dz^2} - M^2 y_A = -N^2$$

SOLN IS:

$$y_A = C_1 \cosh Mz + C_2 \sinh Mz + \frac{N^2}{M^2}$$

$$\text{B.C. } y_A(0) = y_{A0}$$

$$y_A(\delta) = 0$$

~ DOING THE MATH ~

$$C_1 = y_{A0} - \frac{N^2}{M^2}$$

26.30 (CONTINUED)

$$C_2 = \frac{\left( \frac{N^2}{M^2} - y_{A0} \right) \cosh M\delta - \frac{N^2}{M^2}}{\sinh M\delta}$$

## CHAPTER 27

### 27.1 - SEMI INFINITE BODY OF LIQUID

- THIS CASE IS DISCUSSED IN TEXT  
EQUATION (27-9) APPLIES

$$\frac{P_A - P_{AO}}{P_{AS} - P_{AO}} = 1 - \operatorname{erf}\left(\frac{z}{2\sqrt{D_{AB}t}}\right)$$

FOR O<sub>2</sub> IN H<sub>2</sub>O - WILKE-CRANZ,  
EQUATION (4-52) APPLIES

$$D_{AB} = \frac{7.4 \times 10^{-8} (\phi_B M_B)^{1/2} T}{(V_B)^{0.6} \mu_B}$$

VALUES:  $\phi_B = 2.26$   $M_B = 18$   
 $T = 283$   $V_B = 25.6$

$$\mu_B = 0.01394 \text{ cP}$$

$$D_{AB} = 1.469 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$@ t = 3600 \text{ s} - \frac{z}{2\sqrt{D_{AB}t}} = 4.60$$

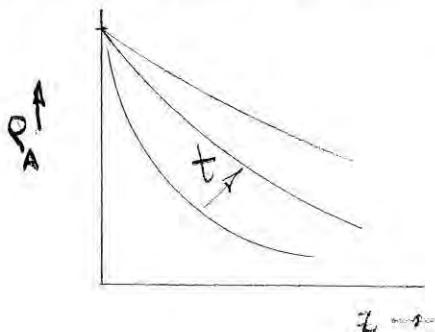
$$@ 36000 - \frac{z}{2\sqrt{D_{AB}t}} = 14.54$$

$$@ 360000 - \frac{z}{2\sqrt{D_{AB}t}} = 46.10$$

EQUATION - @ 3600 S -

$$\frac{P_A - z}{g - z} = 1 - \operatorname{erf}\left(\frac{z}{46.0}\right)$$

CONC. PROFILES APPEAR AS



### 27.2 O<sub>2</sub> DISSOLVING INTO POLYMER FILM

$$C_{AS} = 3.16 (1.5) = 4.74 \text{ g mol/m}^3$$

FOR t = 10 s - VERY SHORT PENETRATION -  
EQUATION (27-11) APPLIES

$$\begin{aligned} N_A Z &= \sqrt{\frac{D_{AB}}{\pi t}} (C_{AS} - C_{AO}) \\ &= \left[ \frac{1 \times 10^{-11}}{\pi (10)} \right]^{1/2} (4.74 - 0.39) \\ &= \underline{\underline{2.45 \text{ g mol/m}^2 \cdot \text{s}}} \quad a) \end{aligned}$$

FOR C<sub>A</sub> = 3 g mol/m<sup>3</sup> @ z = 4 mm

$$\frac{C_A - C_{AO}}{C_{AS} - C_{AO}} = 1 - \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{3 - 0.39}{4.74 - 0.39} = 0.6 = 1 - \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}} = 0.4$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 0.372$$

$$\begin{aligned} t &= 2.89 \times 10^6 \text{ s} \\ &= 802.8 \text{ h} \\ &= 33.4 \text{ DAYS} \end{aligned}$$

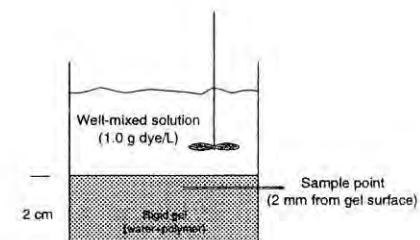
### 27.3

$$C_{AO} = 0$$

$$C_{AS} = 1.0$$

$$\text{FOR } t = 24 \text{ h}$$

$$C_A = 0.203 @ z = 2 \text{ mm}$$



### 27.3 CONTINUED -

$$\frac{C_{AS} - C_A}{C_{AS} - C_{AO}} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{1.0 - 0.203}{1.0} = 0.797 = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

For  $t = 24 \text{ h} = 86400 \text{ s}$

$$z = 0.2 \text{ cm}$$

$$D_{AB} = 1.43 \times 10^{-7} \text{ cm}^2/\text{s} \quad \text{a)}$$

#### ASSUMPTIONS:

1. SURFACE CONCENTRATION CONSTANT  $\sim \neq C_{AS}(t)$
2. ONE DIRECTIONAL DIFFUSION
3. CONSTANT  $D_{AB}$  b)

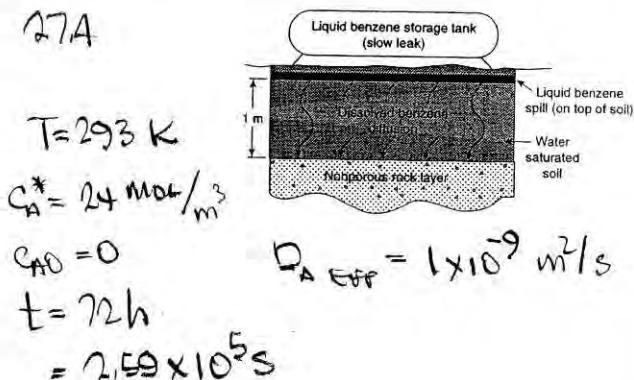
USING WILKE-CAPANIS EQUATION (24-54)

$$D_{AB}/T_2 = D_{AB}/T_1 \frac{\mu_{B1} T_2}{\mu_{B2} T_1}$$

$$= 1.43 \times 10^{-7} \left( \frac{9.93 \times 10^{-4}}{6.58 \times 10^{-4}} \right) \left( \frac{313}{293} \right)$$

$$= 2.30 \times 10^{-7} \text{ cm}^2/\text{s} \quad \text{(c)}$$

### 27.4



### 27.4 CONTINUED -

$$\frac{z}{2\sqrt{D_{AB}t}} = \frac{0.05}{2[4 \times 10^9 (2.59 \times 10^5)]}$$

$$= 1.553$$

$$\operatorname{erf}(1.553) = 0.972 = \frac{C_{AS} - C_A}{C_{AS} - C_{AO}}$$

$$\underline{C_A = 0.672 \text{ mol/m}^3}$$

### 27.5 H<sub>2</sub> INTO Fe

$$D_{AB} = 1.24 \times 10^{-11} \text{ cm}^2/\text{s}$$

$$C_{AO} = 0$$

$$C_A @ 0.1 \text{ cm} = 1.76 \times 10^{-7} \text{ mol/g Fe}$$

$$C_{AS} = 2.2 \times 10^{-7} \text{ mol/g Fe}$$

$$T = 373 \quad P = 1 \text{ atm}$$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{AO}} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{2.2 - 1.76}{2.2 - 0} = 0.2 = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 0.179$$

$$t = 6.27 \times 10^5 \text{ s} = 174 \text{ h}$$

### 27.6 HERBICIDE INTO SOIL

$$D_{AB} = 1 \times 10^{-8} \text{ m}^2/\text{s} \quad t = 1800 \text{ s}$$

$$C_{AS} = 1 \quad C_{AO} = 0 \quad C_A = 0.001$$

$$\frac{C_A - C_{AO}}{C_{AS} - C_{AO}} = \frac{0.001}{1} = 1 - \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

## 27.6 CONTINUED -

$$\operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}} = 0.999$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 2.25$$

$$z = 2(2.25) \left[ (1 \times 10^{-3})(1800) \right]^{1/2}$$

$$= \underline{6.037 \text{ m}}$$

## 27.7 BORON DIFFUSING INTO Si

$$C_B = 5 \times 10^{20} \text{ atoms/cm}^3$$

$$C_{B2} = 0.17 \times 10^{20} \text{ " } @ z = 3 \times 10^{-7} \text{ m}$$

$$C_{B0} = 0 \quad t = 1800 \text{ s}$$

$$\frac{C_{B2} - C_{B0}}{C_{B0} - C_{B2}} = \frac{(5 - 0.17) \times 10^{20}}{5 \times 10^{20}} = 0.966$$

$$= \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

From APPENDIX L -  $\frac{z}{2\sqrt{D_{AB}t}} = 1.5$

$$D_{AB} = \left( \frac{2 \times 10^{-7}}{3} \right)^2 (1800)^{-1}$$

$$= 2.469 \times 10^{-18} \text{ m}^2/\text{s}$$

AS STATED -  $D_{AB} = D_0 e^{-Q_0/RT}$

$$\ln \frac{D_{AB}}{D_0} = -\frac{Q_0}{RT}$$

$$T = \frac{R \ln \frac{D_0}{D_{AB}}}{Q_0}$$

$$= \frac{2.74 \times 10^5}{(8.314) \ln \frac{19 \times 10^{-18}}{2.469 \times 10^{-18}}}$$

$$= \underline{1204 \text{ K}}$$

## 27.8 CARBON DIFFUSING INTO STEEL

$$w_A = 0.007 \left\{ \frac{0.007 - w_A}{0.007 - 0.002} \right\} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{0.007 - w_A}{0.005} = \operatorname{erf} \left[ \frac{z}{2(1 \times 10^{-11})(3600)} \right]$$

$$w_A = 0.007 - 0.005 \operatorname{erf} \left[ \frac{z}{3.7 \times 10^{-4} \text{ m}} \right]$$

for  $z = 0.01 \text{ cm}$

$$\operatorname{erf} [ ] = \operatorname{erf}(0.244) = 0.291$$

$$w_A = 0.007 - 0.005(0.291)$$

$$= \underline{0.55 \text{ wt \% C}}$$

for  $z = 0.02 \text{ cm}$

$$\operatorname{erf} [ ] = \operatorname{erf}(0.528) = 0.545$$

$$w_A = 0.007 - 0.005(0.528) = \underline{0.417 \text{ wt \% C}}$$

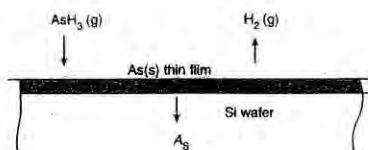
for  $z = 0.04 \text{ cm}$

$$\operatorname{erf} [ ] = \operatorname{erf}(1.056) = 0.866$$

$$w_A = 0.007 - 0.005(1.056) = \underline{0.267 \text{ wt \% C}}$$

27.9

AS INTO Si:



$$\frac{C_{As} - C_A}{C_{As} - C_{A0}} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = \frac{2 \times 10^{-4}}{2[(5 \times 10^{13})(3600)]^{1/2}} = 2.357$$

27.9 (CONTINUED)

$$\operatorname{erf}(2.357) = 0.9990 = \frac{2 \times 10^{21} - C_A}{2 \times 10^{21} - 1 \times 10^{12}}$$

$$C_A = 2.0 \times 10^{18} \text{ Atoms/cm}^3$$

$$N_{A2} = \left[ \frac{D_{AB}}{\pi t} \right]^{1/2} (C_{AS} - C_{AO})$$

$$= \left[ \frac{5 \times 10^{13}}{\pi (3600)} \right]^{1/2} (2 \times 10^{21} - 1 \times 10^{12})$$

$$= 1.33 \times 10^{13} \text{ Atoms/cm}^2 \cdot \text{s}$$

27.10 Same Situation as Prob 27.9

$$C_{AS} = 2 \times 10^{21} \text{ Atoms/cm}^3$$

$$C_{AO} = 1 \times 10^{12} \text{ "}$$

$$C_A = 2 \times 10^{17} \text{ "}$$

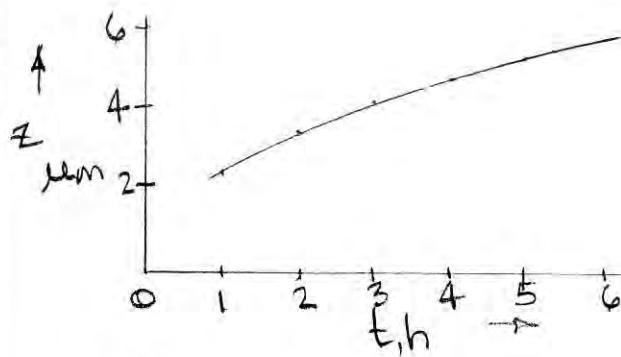
$$\operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}} = \frac{2 \times 10^{21} - 2 \times 10^{17}}{2 \times 10^{21} - 1 \times 10^{12}} = 0.9999$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 2.8$$

$$z = 5.6 \left( 5 \times 10^{13} \right)^{1/2} t^{1/2} \\ = 3.960 \times 10^{-6} t^{1/2}$$

<u>t(s)</u>	<u><math>t^{1/2}</math></u>	<u><math>z(\text{cm}) \times 10^4</math></u>
3600	60	2.3440
7200	84.8	3.36
10800	103.9	4.115
14400	120	4.75
18000	134.2	5.31
21600	147.0	5.82

27.10 - (CONTINUED)



$z(t^{1/2})$  IS OBVIOUSLY LINEAR  
WITH SLOPE = 0.0396  $\mu\text{m}/\text{s}^{1/2}$

27.11  $\text{O}_2(\text{A}) \rightarrow \text{H}_2\text{O}(\text{B})$

$$T = 298 \text{ K} \quad P = 2 \text{ atm}$$

$$C_{AO} = 10 \text{ g/m}^3$$

$$C_{AS} = \frac{10}{0.8} = 12.5 \text{ mol/m}^3 \\ = 12.5(32) = 80 \text{ g/m}^3$$

100%  $\text{O}_2$  gas  
298 K 2.0 atm  
Sealed tank  
deep liquid water  
initial dissolved  $\text{O}_2$   
100 g  $\text{O}_2/\text{m}^3$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{AO}} = \frac{80 - 20}{80 - 10} = 0.857 = \operatorname{erf} \left[ \frac{z}{2\sqrt{D_{AB}t}} \right]$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 1.038$$

$$t = \left[ \frac{z}{2(1.038)} \right]^2 \left[ \frac{1}{D_{AB}} \right]$$

$$= \left[ \frac{0.3}{2(1.038)} \right]^2 \left[ \frac{1}{2.1 \times 10^{-5}} \right]$$

$$= 994 \text{ s} = 16.6 \text{ m}$$

27.12 A DIFFUSING INTO SEMI-INFINITE MEDIUM

$$C_{AS} = 2 \text{ mol/m}^3$$

$$C_{AO} = 0 \text{ "}$$

$$C_A = 0.2 \text{ " } @ z = 5 \text{ mm}$$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{AO}} = \frac{2 - 0.2}{2} = g_D = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 1.165$$

$$t = \left[ \frac{z}{2(1.165)} \right]^2 (D_{AB})^{-1}$$

$$= \left[ \frac{0.5}{2(1.165)} \right]^2 (1 \times 10^{-6})^{-1}$$

$$= 46053 \text{ s} = 12.79 \text{ h}$$

27.13 REFER TO PROB 27.4

C<sub>6</sub>H<sub>6</sub> DIFFUSING INTO H<sub>2</sub>O SATURATED SOIL,

ANALYTICAL SOLN GIVEN BY EQ (27-16):

$$\frac{C_A - C_{AS}}{C_{AO} - C_{AS}} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \left( \frac{n\pi z}{L} \right) e^{-(n\pi/2)x_D}$$

$$\text{FOR } n \text{ ODD; } X_D = D_{AB}t/x_D^2$$

$$\text{IN THE PRESENT CASE: } X_D = \frac{L}{2} = 1 \text{ m}$$

CALCULATIONS MADE USING SPREAD SHEET - R.H. COLUMN

PROCEDURE IS TO GUESS A VALUE OF t & SOLVE CONTINUOUSLY UNTIL C\_A = 1 g/m<sup>3</sup> @ z = 4,

27.13 CONTINUED -

EXCEL SPREADSHEET:

$$T = 293 \text{ K}$$

$$M_A = 78 \quad D_{AB} = 1 \times 10^{-9} \text{ m}^2/\text{s}$$

$$C_{AS} = 24.0 \text{ mol/m}^3$$

$$C_{AO} = 0$$

n	Term	Summation	% Change
1	1.16E+00	1.160E+00	
3	-1.84E-01	9.763E-01	18.8
5	2.50E-02	1.001E+00	2.5
7	-1.92E-03	9.994E-01	0.2
9	7.67E-05	9.995E-01	0.0
11	-1.53E-06	9.995E-01	0.0
13	1.50E-08	9.995E-01	0.0
15	-7.19E-11	9.995E-01	0.0
17	1.67E-13	9.995E-01	0.0
19	-1.86E-16	9.995E-01	0.0
21	1.00E-19	9.995E-01	0.0
23	-2.59E-23	9.995E-01	0.0
25	3.21E-27	9.995E-01	0.0

$$\text{RESULT: } t = 3.763 \times 10^7 \text{ s}$$

$$= 10452 \text{ h}$$

$$= 435.5 \text{ DAYS}$$

27.14 REFER TO PROBLEM 27.4

FLUX EXPRESSION GIVEN AS EQN (27-17).

$$N_{A2} = \frac{4D_{AB}}{L} (C_{AS} - C_{AO}) \sum_{n=1}^{\infty} \cos \left( \frac{n\pi z}{L} \right) e^{-\frac{n^2 \pi^2 D_{AB} t}{4x_D^2}}$$

$$\text{WITH } X_D = \frac{L}{2} = 1 \text{ m} \quad N_{A2} = 0 @ z = 1$$

$$M_A(t) = W^2 \int_0^t N_A(z) \Big|_{z=0} dt$$

27.14 CONTINUED -

## SPREADSHEET FOR SUMMATION:

n	Term	Summation	% Change
1	1.17E-01	1.168E-01	
3	6.79E-02	1.847E-01	36.7
5	3.18E-02	2.164E-01	14.7
7	1.65E-02	2.330E-01	7.1
9	1.00E-02	2.430E-01	4.1
11	6.70E-03	2.497E-01	2.7
13	4.80E-03	2.545E-01	1.9
15	3.60E-03	2.581E-01	1.4
17	2.80E-03	2.609E-01	1.1
19	2.25E-03	2.631E-01	0.9
21	1.84E-03	2.650E-01	0.7
23	1.53E-03	2.665E-01	0.6
25	1.30E-03	2.678E-01	0.5
27	1.11E-03	2.689E-01	0.4
29	9.64E-04	2.699E-01	0.4
31	8.43E-04	2.707E-01	0.3
33	7.44E-04	2.715E-01	0.3
35	6.62E-04	2.721E-01	0.2
37	5.92E-04	2.727E-01	0.2
39	5.33E-04	2.733E-01	0.2
41	4.82E-04	2.737E-01	0.2
43	4.38E-04	2.742E-01	0.2
45	4.00E-04	2.746E-01	0.1
47	3.67E-04	2.749E-01	0.1
49	3.38E-04	2.753E-01	0.1
51	3.12E-04	2.756E-01	0.1

RESULT: For  $t = 2 \times 10^7 \text{ s}$ 

$$M_A = 516 \text{ grams}$$

27.15. CONCENTRATION PROFILE IN A SLAB w/ NO SURFACE RESISTANCE IS EXPRESSED BY EQU (27-16),

$$\frac{C_A - C_{AS}}{C_{AO} - C_{AS}} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi t}{L} e^{-\left(\frac{n\pi}{2}\right)^2 X_D}$$

n opp -

27.15 CONTINUED -

$$\text{MEAN CONCENTRATION } \bar{C} = \frac{\int_0^L C_A dz}{\int_0^L dz}$$

SUBSTITUTING:

$$\bar{C}_A = \frac{4}{\pi} (C_{AO} - C_{AS}) \left[ \sum_{n=1}^{\infty} \frac{1}{n} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} \times \sin \left( \frac{n\pi t}{L} \right) dz + C_{AS} dz \right]$$

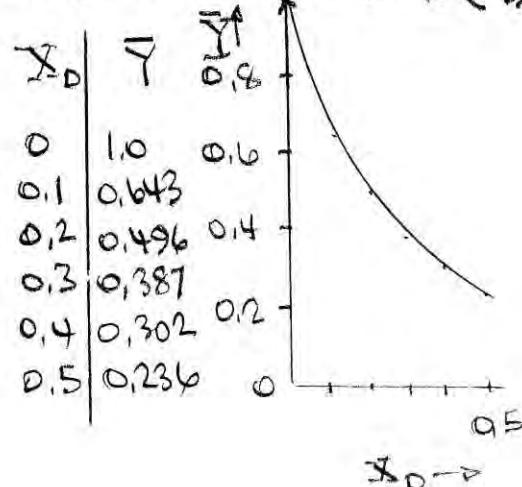
$$= -\frac{4}{\pi} (C_{AO} - C_{AS}) \left[ \sum_{n=1}^{\infty} \frac{1}{n} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} \times \cos \left( \frac{n\pi t}{L} + C_{AS} L \right) \right]$$

$$\frac{C_A - C_{AS}}{C_{AO} - C_{AS}} = -\frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} [-1 - (-1)]$$

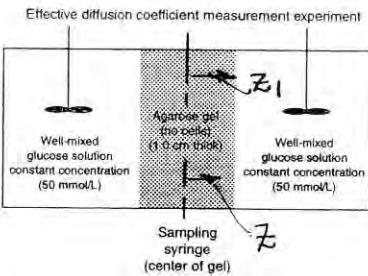
$$= \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} - n \text{ opp}$$

$$\text{FOR } \bar{Y} = \frac{\bar{C}_A - C_{AS}}{C_{AO} - C_{AS}}$$

$$\bar{Y} = 0.8106 \left[ e^{(T/2)X_D} + \frac{1}{9} e^{-\left(\frac{3\pi}{2}\right)^2 X_D} + \frac{1}{25} e^{-\left(\frac{5\pi}{2}\right)^2 X_D} + \dots \right]$$

DOSER THE CALCULATION FOR  $\bar{Y}(X_D)$ :

27.16



$$C_{A0} = 0$$

$$C_A|_{t=42\text{ h}} = 48.5 \text{ mmol/L}$$

GOVERNING DIFFERENTIAL EQUATION:

$$\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2} \quad (\text{a})$$

CHARTS WILL BE USED -

$$Y = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \frac{50 - 48.5}{50} = 0.03$$

FIGURE F.4

$$\text{AT } \frac{z}{z_1} = 0 \quad X = \frac{D_{AB} t}{X_1^2} \approx 1.6$$

$$D_{AB} = \frac{1.6 (0.5)^2}{42 (3600)} = 1.64 \times 10^{-6} \text{ cm}^2/\text{s}$$

## 27.17 CYLINDRICAL GEOMETRY

$$D_{AB} = 4 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$L = 5 \text{ cm}$$

$$C_A' = 2.0 \text{ g mol/m}^3$$

$$r = 0.5 \text{ "}$$

$$K = 1.5 \frac{\text{cm}^3 \text{ FLUID}}{\text{cm}^3 \text{ ABSORBENT}}$$

$$C_A = K C_A'$$

$$C_{A0} = 0$$

$$C_{AS} = 1.5 C_A' = 3.0 \text{ g mol/m}^3$$

$$C_A = 2.94 \text{ g mol/m}^3 @ X = 0.1 \text{ cm}$$

27.17 (CONTINUED)

USING CHARTS - SINCE  $L \gg r$  WE ASSUME DIFFUSION IS ONLY SIGNIFICANT IN THE  $r$ -DIRECTION

$$Y = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \frac{3.0 - 2.94}{3.0} = 0.02$$

$$\text{FIGURE F.2 } @ \frac{X}{X_1} = 0.2 \quad m = 0$$

$$X \approx 0.75 = \frac{D_{AB} t}{r^2}$$

$$t = \frac{0.75 (0.5)^2}{4 \times 10^{-7}} = \frac{4.69 \times 10^5 \text{ s}}{= 130.3 \text{ h}}$$

$$= 5.428 \text{ DAYS}$$

## 27.18 SPHERICAL GEOMETRY

$$D_{AB} = 1.5 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$\text{AT } r = 0 \quad C_A = 0.02$$

$$C_{A0} = 0.2$$

$$r_1 = 0.05 \text{ cm}$$

$$\frac{C_A - C_{AS}}{C_{A0} - C_{AS}} = 0.10$$

USING CHARTS - FIGURE F.9

$$n = \frac{r}{r_1} = 0 \quad m = 0$$

$$X = 0.3 = \frac{D_{AB} t}{r_1^2}$$

$$t = \frac{0.3 (0.05)^2}{1.5 \times 10^{-7}} = \frac{5 \times 10^3 \text{ s}}{= 1.389 \text{ h}}$$

## 27.19 - TRANSIENT DRYING OF A SLAB

$$D_{AB} = 1.3 \times 10^{-4} \text{ cm}^2/\text{s}$$

@  $t=0$   $w = 15\%$  by wt

@  $x=x_1$ ,  $w = 4\%$  by wt

DESIRABLE  $w @ x = \frac{x_1}{2} = 10\%$  by wt.

MOISTURE CONCENTRATIONS MUST BE EXPRESSED IN CONSISTENT TERMS ~

- WT  $H_2O$  PER WT DRY SOLID ~

$$\therefore w'_0 = \frac{0.15}{1-0.15} = 0.1765 \frac{\text{gH}_2\text{O}}{\text{gDS}}$$

$$w'_S = \frac{0.04}{1-0.04} = 0.0417 \quad "$$

$$w'_A = \frac{0.10}{1-0.10} = 0.111 \quad "$$

FOR CHART SOLUTION:

$$Y = \frac{0.111 - 0.0417}{0.1765 - 0.0417} = 0.515$$

$$n = \frac{x}{x_1} = 0.5$$

$$m = 0$$

$$\text{fig. f.7} - @ n=0.4 \frac{D_{AB}t}{x_1^2} \approx 0.24$$

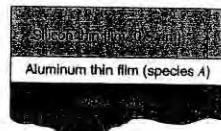
$$@ n=0.6 \quad " \quad \approx 0.16$$

$$\therefore @ n=0.5 \quad \frac{D_{AB}t}{x_1^2} \approx 0.20$$

$$t = \frac{0.20(5)^2}{1.3 \times 10^{-4}} = \frac{38400 \text{ s}}{= 10.62 \text{ h}}$$

## 27.20

Al DIFFUSES INTO Si



$$T = 1250 \text{ K} \quad t = 10 \text{ h} = 3.6 \times 10^4 \text{ s}$$

$$\text{fig. 24.6} \sim D_{AB} \approx 1.1 \times 10^{-13} \text{ cm}^2/\text{s}$$

$$x_1 = 0.5 \mu\text{m}$$

$$\text{CONDITION SOUGHT IS } w_A @ \frac{x}{x_1} = 0.5$$

CHART SOLUTION ISN'T POSSIBLE SINCE B.C. ON TOP Si SURFACE IS UNKNOWN

PRESUMING Si THICKNESS IS LARGE COMPARED TO PENETRATION DEPTH - CONSIDER THIS A SEMI-INFINITE SITUATION:

$$\frac{w_{AS} - w_A}{w_{AS} - w_{AO}} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}} \quad \{ \text{eq. (27-10)}$$

$$\frac{z}{2[D_{AB}t]^{1/2}} = \frac{0.25 \times 10^{-4}}{2[(1.1 \times 10^{-13})(3.6 \times 10^4)]^{1/2}} \approx 0.1986$$

$$\operatorname{erf}(0.1986) \approx 0.2212$$

$$= \frac{0.01 - w_A}{0.01} \quad w_A = 0.00779$$

$$\approx 0.779\%$$

## 27.21 SPHERICAL GEOMETRY

$$D_{AB} = 2 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$r_1 = 0.25 \text{ cm}$$

$$C_{AS} = 0.1 (150) = 15 \text{ mol/m}^3$$

$$C_A(r=0) = 12 \text{ mol/m}^3$$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{AO}} = \frac{15 - 12}{15} = 0.2$$

$$n = \frac{x}{x_1} = 0 \quad m = 0$$

$$\text{Fig. F.9} \quad X_0 = \frac{D_{AB} t}{r_1^2} \approx 0.25$$

$$t = \frac{0.25 (0.25)^2}{2 \times 10^{-6}} = \frac{7812.5}{2 \times 10^{-6}} = 3.90625 \times 10^9 \text{ s}$$

## 27.22 RECTANGULAR BLOCK - EDGES SEALED -

$$C_{AO} = 64 \text{ mol/cm}^3$$

$$C_{AS} = 0$$

$$D_{AB} = 3 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$V = (1 \text{ cm})(0.652 \text{ cm}) L$$

L = OTHER DIMENSION

$$\frac{41.7}{64} = (1)(0.652)L$$

$$L = 1 \text{ cm}$$

$$\frac{D_{AB} t}{r_1^2} = \frac{(3 \times 10^{-7})(9.6)(3600)}{(0.652/2)^2}$$

$$= 0.976$$

## 27.22 CONTINUED -

$$n = 0 \quad m = 0$$

$$\text{fig F.7} \quad Y \approx 0.11$$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{AO}} = \frac{0 - C_A}{0 - 64} = 0.11 \quad C_A = 7.04 \text{ mol/cm}^3$$

## 27.23 CYLINDER: $r = 1.25 \text{ cm}$ $L = 80 \text{ cm}$

$$C_{AO} = 30 \text{ wt\%} = \frac{0.3}{1-0.3} = 0.429 \text{ g}_A/\text{g}_{\text{ps.}}$$

$$C_{AS} = 1 \text{ wt\%} = \frac{0.01}{1-0.01} = 0.0101 \text{ "}$$

SINCE  $L \gg r$  - VIRTUALLY ALL DIFFUSION IS IN  $r$ -DIRECTION

$$C_A(r=0) = 18 \text{ wt\%} = \frac{0.18}{1-0.18} = 0.2195$$

$$\text{AT } t = 36000 \text{ s (10h)}$$

$$Y = \frac{w_A' - w_{AS}}{w_{AO}' - w_{AS}} = \frac{0.219 - 0.0101}{0.429 - 0.0101} \approx 0.50$$

$$\text{FOR } n = m = 0 \quad X = \frac{D_{AB} t}{r^2} = 0.2$$

$$\frac{D_{AB}}{r^2} = \frac{0.2}{10h}$$

$$\text{AFTER 15 h: } X = \frac{D_{AB}(15)}{r^2} = 0.3$$

$$\text{fig F.8} \quad Y \approx 0.29 = \frac{w_A' - 0.0101}{0.429 - 0.0101}$$

$$w_A' = 0.1316$$

$$\text{WT\%} = \frac{w_A}{1-w_A} = 0.1316 \quad \underline{w_p = 11.6\%}$$

## 27.24 SPHERICAL GEOMETRY -

$$r = 0.1 \text{ cm}$$

For H<sub>2</sub>O in Air -  $D_{AB} = 0.260 \text{ cm}^2/\text{s}$   
 @ 298 K, 1 atm

$$w_{AO} = 0$$

$$w_A(r=0) = 0.9 w_{AS}$$

$$\bar{Y} = \frac{w_{AS} - w_A}{w_{AS} - w_{AO}} = \frac{1 - 0.9}{1} = 0.1$$

Fig F.9 -  $n=m=0$

$$X_D = \frac{D_{AB}t}{r^2} \approx 0.3$$

$$t = \frac{0.3(0.1)^2}{0.260} = \underline{\underline{0.0115 \text{ s}}}$$

## 27.25 RECTANGULAR SODIUM

$$10 \text{ cm} \times 10 \text{ cm} \times 45 \text{ cm}$$

H<sub>2</sub>O diffuses:  $D_{AB} = 1.04 \times 10^{-5} \text{ cm}^2/\text{s}$

$$C_{AO} = 45 \text{ wt\%}, w_{AO} = \frac{0.45}{1-0.45} = 0.818$$

$$C_{AS} = 15 \text{ "}, w_{AS} = \frac{0.15}{1-0.15} = 0.176$$

$$C_A = 25 \text{ "}, w_A = \frac{0.25}{1-0.25} = 0.333$$

$$\bar{Y} = \frac{w_{AS} - w_A}{w_{AS} - w_{AO}} = \frac{0.176 - 0.333}{0.176 - 0.818} = 0.244$$

$$= Y_1 Y_2 = Y_s^2 \quad \begin{matrix} \text{SINCE SIDES} \\ \text{HAVE SAME} \\ \text{DIMENSIONS} \end{matrix}$$

$$\therefore Y_s = (0.244)^{\frac{1}{2}} = 0.494$$

## 27.25 (CONTINUED)

USING FIGURE F.7

$$n=m=0 \quad X_D \approx 0.39 = \frac{D_{AB}t}{r^2}$$

$$t = \frac{0.39(5)^2}{1.04 \times 10^{-5}} = \frac{9.375 \times 10^5 \text{ s}}{= 260.4 \text{ h}} \\ = \underline{\underline{10.85 \text{ DAYS}}}$$

IF ALL DIFFUSION IS FROM ENDS:

$$\bar{Y} = 0.244$$

$$X_D \approx 0.72 = \frac{D_{AB}t}{r^2}$$

$$t = \frac{0.72(22.5)^2}{1.04 \times 10^{-5}} = \frac{3.50 \times 10^7 \text{ s}}{= 9734 \text{ h}} \\ = \underline{\underline{405.6 \text{ DAYS}}}$$

## CHAPTER 28

28.1 FOR O<sub>2</sub> DIFFUSING IN AIR  
@ 300 K, 1 ATM

$$D_{AB}(273K) = 0.175 \text{ cm}^2/\text{s}$$

$$D_{AB}(300K) = 0.175 \left(\frac{300}{273}\right)^{3/2} = 0.202 \text{ cm}^2/\text{s}$$

$$@ 300 K - \nu_{H_2} = 0.1569 \text{ cm}^2/\text{s}$$

$$S_C = \frac{0.1569}{0.202} = \underline{\underline{0.777}} \quad (\text{a})$$

FOR O<sub>2</sub> IN H<sub>2</sub>O @ 300 K

$$D_{AB} = 1.5 \times 10^{-9} \text{ m}^2/\text{s}$$

$$\nu_{H_2O} = 0.880 \times 10^{-6} \text{ m}^2/\text{s}$$

$$S_C = \frac{0.880 \times 10^{-6}}{1.5 \times 10^{-9}} = \underline{\underline{587}} \quad (\text{b})$$

28.2 SiH<sub>4</sub> IN He  
(A) 900 K  
(B) 100 Pa  
 $\gamma_{SiH_4} = 0.01$

$$D_{AB} @ 298 K, 101.3 kPa = 0.518 \text{ cm}^2/\text{s}$$

$$D_{AB} T, P = \left(0.518\right) \left(\frac{P_1}{P_2}\right) \left(\frac{T_2}{T_1}\right)^{3/2} \left(\frac{\Omega_D}{\Omega_D}\right)$$

$$\text{VALUES: } E_{AB}/k = 46.06$$

$$@ 298 K \quad E_{AB}/kT = 6470 \quad \Omega_D = 0.802$$

$$@ 900 K \quad E_{AB}/kT = 19.54 \quad \Omega_D = 0.1668$$

$$D_{AB} T, P = \left(0.518\right) \left(\frac{1.013 \times 10^5}{1100}\right) \left(\frac{900}{298}\right)^{3/2} \times \frac{0.802}{0.1668}$$

28.2 CONTINUED -

$$D_{AB} T, P = 3.31 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$\nu_{He} @ 900 K = 6 \times 10^{-3} \text{ ft}^2/\text{s} \cdot \left(\frac{0.3048 \text{ m}}{\text{ft}}\right)^2 \\ = 5.574 \times 10^{-4} \text{ m}^2/\text{s} \\ = 5.574 \text{ cm}^2/\text{s}$$

$$S_C = \frac{5.574}{3210} = \underline{\underline{0.001684}}$$

28.3 Cl<sub>2</sub> IN SiCl<sub>4</sub> (l)  
(A) (B)

FOR D<sub>AB</sub> - USE WILKE-CRANG EQU.  
- Eq (24-52)

$$D_{AB} = \frac{7.4 \times 10^{-8} (M_B \phi_B)^{1/2} T}{V_A^{0.6} \frac{\mu_B}{\mu_B}}$$

$$\text{VALUES: } \phi_B = 1.0 \quad M_B = 170$$

$$\mu_B = 5.2 \times 10^{-4} \text{ kg/m.s} \\ = 0.52 \text{ cP}$$

$$V_A = 48.4$$

SUBSTITUTION:  $D_{AB} = 5.395 \times 10^{-5} \text{ cm}^2/\text{s}$

$$\lambda = \frac{\mu}{S} = \frac{5.2 \times 10^{-4} \text{ kg/m.s}}{1.47 \text{ g/cm}^3} = 3.54 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$S_C = \frac{3.54 \times 10^{-3}}{5.395 \times 10^{-5}} = \underline{\underline{65.6}}$$

$$28.4 \quad S_t = \frac{k_c}{\sigma_{DAB}}$$

$$= \frac{k_c L}{D_{DAB}} \frac{\nu}{L \nu_p} \frac{D_{DAB}}{\nu}$$

$$= \underline{\underline{Sh / \rho_e Sc}}$$

$$\rho_e = \frac{V_{DAB} L}{D_{DAB}}$$

$$= \underline{\underline{\frac{V_{DAB} L}{\nu} \frac{\nu}{D_{DAB}} = Re Sc}}$$

28.5

VARIABLE	SYMBOL	DIM.
MASS TX COEF	$k_c$	$L^{-1}$
LENGTH	$L$	$L$
VELOCITY	$\nu$	$L t^{-1}$
VISCOSEITY	$\mu$	$M L^{-1} t^{-1}$
DIFFUSIVITY	$D_{DAB}$	$L^2 t^{-1}$
DENSITY	$\rho$	$M L^{-3}$

$$L = n - r = 6 - 3 = 3 \text{ \pi groups}$$

CORE =  $D_{DAB}, \rho, L$

$$\Pi_1 = D_{DAB} \rho L k_c$$

$$= \left(\frac{L^2}{t}\right) \left(\frac{M}{L^3}\right) L \frac{C}{t}$$

$$M: \alpha = b$$

$$L: \alpha = 2a - 3b + c + 1$$

$$t: \alpha = -a - 1$$

$$a = -1 \quad b = 0 \quad c = 1$$

$$\underline{\underline{\Pi_1 = k_c L / D_{DAB} = Sh}}$$

28.5 (CONTINUED)

$$\Pi_2 = \frac{d \epsilon f}{D_{DAB} \rho L} V = \left(\frac{L^2}{t}\right) \left(\frac{M}{L^3}\right) L \frac{F}{t}$$

$$M: \alpha = \epsilon$$

$$L: \alpha = 2d - 3\epsilon + f + 1$$

$$t: \alpha = -d - 1$$

$$d = -1 \quad \epsilon = 0 \quad f = 1$$

$$\underline{\underline{\Pi_2 = \frac{\nu L}{D_{DAB}}}}$$

$$\Pi_3 = D_{DAB} \rho L \nu e = \left(\frac{L^2}{t}\right)^g \left(\frac{M}{L^3}\right)^h L^i \frac{M}{L^j}$$

$$M: \alpha = h + 1$$

$$L: \alpha = 2g - 3h + i - 1$$

$$t: \alpha = -g - 1$$

$$g = -1 \quad h = -1 \quad i = 0$$

$$\underline{\underline{\Pi_3 = \frac{\mu}{\rho D_{DAB}} = Sc}}$$

$$-\text{NOTE THAT } \Pi_2 / \Pi_3 = \frac{\nu \rho L}{\mu} = Re$$

28.6

VARIABLE	SYMBOL	DIMENSIONS
MASS	$M$	$M$
DIAMETER	$D$	$L$
SURFACE TENSION	$\gamma$	$L/t^2$
DENSITY ( $\rho$ )	$\rho$	$M/L^3$
VISCOSITY ( $\eta$ )	$\eta$	$M/Lt$
VELOCITY ( $\nu$ )	$\nu$	$L/t$
DENSITY ( $\rho_g$ )	$\rho_g$	$M/L^3$
VISCOSITY ( $\eta_g$ )	$\eta_g$	$M/Lt$

$$i = n - r = 9 - 3 = 6 \text{ \pi groups}$$

CORE =  $\rho, \mu, D$

28.6 (CONTINUED) -

$$\Pi_1 = S_L^a \mu_L^b D^c U = \left(\frac{m}{L^3}\right)^a \left(\frac{m}{L^2 t}\right)^b \left(\frac{L}{t}\right)^c$$

$$\begin{array}{ll} M: 0 = a+b & a=1 \\ L: 0 = -3a - b + c + 1 & c=1 \\ t: 0 = -b - 1 & b=-1 \end{array}$$

$$\underline{\Pi_1 = S_L D U / \mu_L = Re}$$

$$\Pi_2 = S_L^a \mu_L^b D^c g = \left(\frac{m}{L^3}\right)^a \left(\frac{m}{L^2 t}\right)^b \left(\frac{L}{t^2}\right)^c$$

$$\begin{array}{ll} M: 0 = a+b & a=2 \\ L: 0 = -3a - b + c + 1 & c=3 \\ t: 0 = -b - 2 & b=-2 \end{array}$$

$$\underline{\Pi_2 = S_L^2 D^3 g / \mu_L^2 = D^3 g / V^2}$$

$$\Pi_3 = S_L^a \mu_L^b D^c \sigma = \left(\frac{m}{L^3}\right)^a \left(\frac{m}{L^2 t}\right)^b \left(\frac{M}{t^2}\right)^c$$

$$\begin{array}{ll} M: 0 = a+b+1 & a=1 \\ L: 0 = -3a - b + c & c=1 \\ t: 0 = -b - 2 & b=-2 \end{array}$$

$$\underline{\Pi_3 = S_L D \sigma / \mu_L^2}$$

$$\Pi_4 = S_L^a \mu_L^b D^c \mu_g$$

$$\sim \text{By INSPECTION} - \underline{\Pi_4 = \mu_L / \mu_g}$$

$$\Pi_5 = S_L^a \mu_L^b D^c g$$

$$\sim \text{By INSPECTION} \quad \underline{\Pi_5 = g / S_L}$$

$$\Pi_6 = S_L^a \mu_L^b D^c M = \left(\frac{m}{L^3}\right)^a \left(\frac{m}{L^2 t}\right)^b \left(\frac{L}{t}\right)^c M$$

$$\begin{array}{ll} M: 0 = a+b+1 & a=-1 \\ L: 0 = -3a - b + c & c=-3 \\ t: 0 = -b & b=0 \end{array}$$

$$\underline{\Pi_6 = M / S_L D^3}$$

28.7 VARIABLE

SYMBOL

DIMENSIONS

MASS TR COEF.	$k_c$	$L/t$
VELOCITY	$U$	$L/t$
RE DIA	$D_L$	$L$
PIPE "	$D_o$	$L$
DENSITY	$\rho$	$M/L^3$
VISCOSEITY	$\mu$	$M/Lt$
DIFFUSIVITY	$D_{AB}$	$L^2/t$

$$L = n - r = 7 - 3 = 4 \text{ GROUPS}$$

$$\text{CONST} = D_{AB}, S, D_o$$

$$\Pi_1 = D_{AB} S D_o k_c = \left(\frac{L^2}{t}\right)^a \left(\frac{m}{L^3}\right)^b \left(\frac{L}{t}\right)^c$$

$$\begin{array}{ll} M: 0 = b & b=0 \\ L: 0 = 2a - 3b + c + 1 & c=1 \\ t: 0 = -a - 1 & a=-1 \end{array}$$

$$\underline{\Pi_1 = k_c D_o / D_{AB} = Sh}$$

$$\Pi_2 = D_{AB}^a S^b D_o^c U = \left(\frac{L^2}{t}\right)^a \left(\frac{m}{L^3}\right)^b \left(\frac{L}{t}\right)^c$$

SAME FORM AS  $\Pi_1$  -

$$\underline{\Pi_2 = D_o U / D_{AB}}$$

28.7 CONTINUED -

$$\Pi_3 = \frac{a}{D_{AB}} \frac{b}{S} \frac{c}{D_B} \mu = \left(\frac{L^2}{t}\right)^a \left(\frac{m}{L^3}\right)^b \left(\frac{m}{L^2 t}\right)^c$$

$$M: 0 = b+1$$

$$b = -1$$

$$L: 0 = 2a - 3b + c - 1$$

$$c = 0$$

$$t: 0 = -a - 1$$

$$a = -1$$

$$\underline{\Pi_3 = \frac{\mu}{S D_{AB}} = \frac{D}{D_{AB}} = Sc}$$

$$\Pi_4 = \frac{a}{D_{AB}} \frac{b}{S} \frac{c}{D_B} \frac{c}{D_C}$$

$$\sim \text{By Inspection} - \underline{\Pi_4 = \frac{D}{D_C}}$$

$$\text{NOTE THAT } \frac{\Pi_2}{\Pi_3} = \frac{D_{AB} S}{\mu} = Pe$$

28.8 VARIABLE SYMBOL DIMENSION

CONCENTRATION DIFFERENCE	$C_{A0} - C_{Ar}$	$M/L^3$
OVERALL "	$C_{A0} - C_{Ar}$	$M/L^3$
RADIUS	$r$	$L$
REFERENCE RADIUS	$R$	$L$
DIFFUSIVITY	$D_{AB}$	$L^2/t$
MASS TX. COEFF	$k_c$	$L/t$
TIME	$t$	$t$

$$l = n - r = 7 - 3 = 4$$

CORE  $\rightarrow C_{A0} - C_{Ar}, R, D_{AB}$

$$\Pi_1 = \frac{a}{(C_{A0} - C_{Ar})^a} \frac{b}{R^b} \frac{c}{D_{AB}^c} (C_A - C_{Ar})$$

$$\text{By INSPECTION} - \underline{\Pi_1 = \frac{C_A - C_{Ar}}{C_{A0} - C_{Ar}}}$$

28.8 CONTINUED

$$\Pi_2 = (C_{A0} - C_{Ar})^a R^b D_{AB}^c r$$

$$\sim \text{By INSPECTION} - \underline{\Pi_2 = \frac{r}{R}}$$

$$\Pi_3 = (C_{A0} - C_{Ar})^a R^b D_{AB}^c k_c = \left(\frac{m}{L^3}\right)^a \left(\frac{L^2}{t}\right)^b \left(\frac{L}{t}\right)^c$$

$$M: 0 = a$$

$$a = 0$$

$$L: 0 = -3a + b + 2c + 1$$

$$b = 1$$

$$t: 0 = -c - 1$$

$$c = -1$$

$$\underline{\Pi_3 = k_c R / D_{AB}}$$

$$\Pi_4 = (C_{A0} - C_{Ar})^a R^b D_{AB}^c t = \left(\frac{m}{L^3}\right)^a \left(\frac{L^2}{t}\right)^b \left(\frac{L}{t}\right)^c t$$

$$M: 0 = a$$

$$a = 0$$

$$L: 0 = -3a + b + 2c$$

$$b = -2$$

$$t: 0 = -c + 1$$

$$c = 1$$

$$\underline{\Pi_4 = D_{AB} t / R^2}$$

28.9 B.L. EQUATIONS:

$$\text{LAMINAR: } \frac{k_c x}{D_{AB}} = 0.332 Re_x^{1/2} Sc^{1/3}$$

$$\text{TURBULENT: } \frac{k_c x}{D_{AB}} = 0.0292 Re_x^{4/5} Sc^{1/3}$$

$$Re_x l_{tr} = 2 \times 10^5$$

$$\left. \begin{array}{l} \text{FRACTION OF} \\ \text{MASS TX} \\ \text{WHICH IS} \\ \text{LAMINAR} \end{array} \right\} = \frac{N_{AL}}{N_{AL} + N_{AT}} = \frac{\bar{k}_{CL}}{\bar{k}_{CL} + \bar{k}_{CT}}$$

28.9 CONTINUED

$$\begin{aligned}
 F_{CL} &= \frac{1}{L} \int_0^{L_{tr}} k_{ct} x dx \\
 &= \frac{1}{L} (0.332) \left( \frac{U_x}{\omega} \right)^{1/2} S_c^{1/3} \int_0^{L_{tr}} x^{1/2} dx \\
 &= \underline{0.664} \quad Re_{tr}^{1/2} S_c^{1/3} \\
 \bar{k}_{ct} &= \frac{1}{L} \int_{L_{tr}}^L k_{ct} x dx \\
 &= \frac{1}{L} (0.0292) \left( \frac{U_x}{\omega} \right)^{4/5} S_c^{1/3} \int_{L_{tr}}^L x^{4/5} dx \\
 &= \underline{0.0365} \left( Re_L^{4/5} - Re_{tr}^{4/5} \right) S_c^{1/3}
 \end{aligned}$$

$$\begin{aligned}
 \text{LAMINAR} &= \frac{0.664 Re_{tr}^{1/2}}{0.664 Re_{tr}^{1/2} + 0.0365 (Re_L^{4/5} - Re_{tr}^{4/5})} \\
 Re_{tr}^{1/2} &= (2 \times 10^5)^{1/2} = 447.2 \\
 Re_{tr}^{4/5} &= (2 \times 10^5)^{4/5} = 17411 \\
 Re_L^{4/5} &= (3 \times 10^5)^{4/5} = 151950
 \end{aligned}$$

SUBSTITUTING  $\frac{1}{2}$  SOLVING

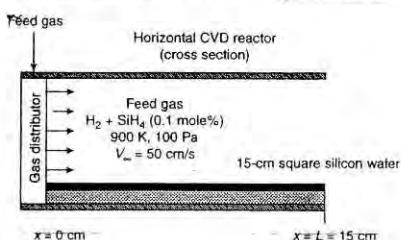
$$\text{LAMINAR} = 0.051 = 5.1\%$$

28.10

AT SURFACE



$$\begin{aligned}
 T &= 900 K \quad P = 100 Pa \quad U_{in} = 50 \text{ cm/s} \\
 L &= 15 \text{ cm} \quad D_{AB} = 0.4036 \times 10^{-4} \text{ cm}^2/\text{s} \\
 Y_{AB} &= 0.061
 \end{aligned}$$



28.10 CONTINUED

$$\begin{aligned}
 Sc &= \frac{\nu}{D_{AB}} = \frac{1.8 \times 10^{-4}}{2.167 \times 10^{-8}} \left( \frac{1}{0.4036 \times 10^{-4}} \right) = \underline{1.67} \\
 Re &= \frac{(50)(15)(2.167 \times 10^{-8})}{1.8 \times 10^{-4}} = \underline{0.1125} \quad (\text{LAMINAR})
 \end{aligned}$$

$$Sh = \frac{\bar{k}_{ct} L}{D_{AB}} = 0.664 Re^{1/2} Sc^{1/3} = \underline{0.264}$$

$$k_C = \frac{0.4036 \times 10^{-4}}{15} (0.264) = 71.1 \text{ cm}^2/\text{s}$$

$$c = \frac{P}{RT} = \frac{100 / 1.0135 \times 10^5}{(82.06)(500)} = 1.336 \times 10^{-8} \text{ mol/cm}^3$$

$$C_{AB} = 0.001 (1.336 \times 10^{-8}) = 1.336 \times 10^{-11}$$

$$\begin{aligned}
 W_A = N_A A &= k_C (C_{AB} - C_{AS})(15)(15) \\
 &= (71.1)(1.336 \times 10^{-11})(225) \\
 &= 2.136 \times 10^{-7} \text{ mol/s} \\
 &= \underline{1.28 \times 10^{-5} \text{ mol/m}}
 \end{aligned}$$

THICKEST Si LAYER WILL OCCUR  
WHERE  $k_{ct}$  IS LARGEST

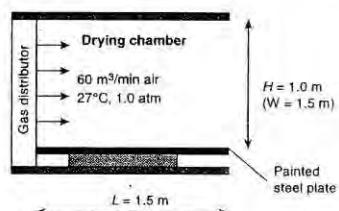
$\sim$  AT  $X=0$

28.11

$$T = 300 K$$

$$\dot{f} = 1 \text{ atm}$$

$$P_{AS} = 0.137 \text{ atm}$$



$$\begin{aligned}
 U &= \frac{V}{(W \cdot H)} = \frac{60}{(1 \times 1.5 \times 1.0)} = 0.67 \text{ m/s} \\
 &= 67 \text{ cm/s}
 \end{aligned}$$

### 28.11 (CONTINUED)

$$D_{AB} = 0.0962 \left( \frac{300}{293} \right)^{3/2} = 0.0972 \text{ cm}^2/\text{s}$$

$$\lambda = 1.569 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{0.0972}{1.569 \times 10^{-5} \times 10^4} = 1.614$$

$$Re = \frac{\rho L}{\lambda} = \frac{(67)(50)}{0.1569} = 6.41 \times 10^4 \quad (\text{LAMINAR})$$

$$k_c = 0.1204 \frac{(0.0972)}{150} (6.41 \times 10^4)^{1/2} (1.614)^{1/3} = 0.128 \text{ cm/s}$$

$$C_{AS} = \frac{P_{AS}}{RT} = \frac{0.137}{(82.06)(300)} = 5.965 \times 10^{-4} \text{ mol/cm}^3$$

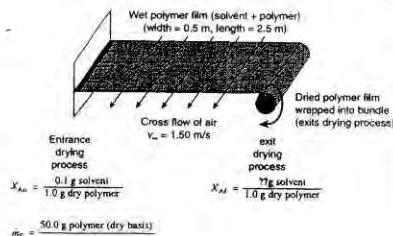
$$W_A = N_A \times A = k_c (C_{AS} - C_{A, \infty})(A) = (0.128)(5.965 \times 10^{-4} - 0)(50)(150) = 0.0160 \text{ mol/s} = 4500 \text{ g/h}$$

### 28.12

$$T = 293K$$

$$P = 1 \text{ ATM}$$

$$V = 150 \text{ cm/s}$$



$$D_{AB} = 0.080 \text{ cm}^2/\text{s}$$

$$P_w = 0.16 \text{ ATM}$$

$$V_{AIR} = 0.15 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.15}{0.080} = 1.815$$

### 28.12 (CONTINUED)

$$Re_l = \frac{V_w L}{\lambda} = \frac{150(50)}{0.15} = 50,000$$

(LAMINAR)

$$k_c = 0.664 \frac{D_{AB}}{W} Re_l^{1/2} Sc^{1/3}$$

$$= 0.664 \frac{0.080}{50} (5 \times 10^4)^{1/2} (1.815)^{1/3} = 0.293 \text{ cm/s}$$

$$W_A = N_A(A) = k_c \Delta C_A (2WL)$$

$$C_{AS} = \frac{P_w}{RT} = \frac{0.16}{(82.06)(293)} = 6.65 \times 10^{-4} \text{ mol/cm}^3$$

$$W_A = (0.293)(6.65 \times 10^{-4} - 0)(2 \times 50 \times 250)$$

$$= 0.0487 \text{ mol/s}$$

$$= (0.0487)(86) = \underline{4.19 \text{ g/s}} \quad (\text{b})$$

$$Sh = \frac{k_c W}{D_{AB}} = \frac{(0.293)(50)}{0.080} = \underline{183.1} \quad (\text{a})$$

$$\text{INPUT} = \frac{0.1 \text{ g SOLVENT}}{\text{g DRY SOL.}} \left( \frac{50 \text{ g DRY SOL.}}{\text{s}} \right)$$

$$= 5 \text{ g/s SOLVENT}$$

$$\text{OUTPUT} = 4.19 \text{ g/s} + \underline{m \text{ g/s (IN Polymer)}}$$

$$m = 5 - 4.19 = 0.81 \text{ g/s}$$

$$X = \frac{0.81}{50} = 0.0162 \frac{\text{g SOLVENT}}{\text{g DRY SOLID}} \quad (\text{c})$$

### 28.13 ACETONE (A) IN AIR (B)

$$T = 298 \text{ K}$$

$$P = 1.013 \times 10^5 \text{ Pa} \quad P_A^\circ = 3.066 \times 10^4 \text{ Pa}$$

$$U = 600 \text{ cm/s} \quad L = 100 \text{ cm}$$

$$D_{AB} = 0.93 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{AT } 298 \text{ K} \dots \overline{D}_{AB} = 1.55 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{1.55 \times 10^{-5}}{0.93 \times 10^{-5}} = 1.67$$

$$f_{ex} = \frac{(6)(0.4)}{1.55 \times 10^{-4}} = 1.548 \times 10^5 \quad (\text{LAMINAR})$$

$$k_{ex}^x = 0.332 \overline{Re}_x^{1/2} Sc^{1/3}$$

$D_{AB}$

$$k_c = \frac{0.93 \times 10^{-5}}{0.4} (0.332) (1.548 \times 10^5)^{1/2} \times (1.67)^{1/3}$$

$$= \underline{\underline{3.6 \times 10^{-3} \text{ m/s}}} \quad (\text{a})$$

for  $L = 1 \text{ m}$

$$\overline{Re}_L = \frac{(6)(U)}{1.55 \times 10^{-5}} = 3.87 \times 10^5$$

TURBULENT for  $\overline{Re}_x > 2 \times 10^5$

ASSUMING B,L, IS

LAMINAR for  $0 < k_{ex}^x < 2 \times 10^5$

TURBULENT for  $2 \times 10^5 < k_{ex}^x$

$$\begin{aligned} \overline{k}_c &= \frac{0.604}{L} \frac{D_{AB} \overline{Re}_{tr}^{1/2} Sc^{1/3}}{+ 0.0365 \frac{D_{AB}}{L} Sc^{1/3} \left[ \overline{Re}_L^{4/5} - \overline{Re}_{tr}^{4/5} \right]} \end{aligned}$$

### 28.13 (CONTINUED)

$$\overline{Re}_{tr} = 2 \times 10^5$$

$$\overline{Re}_L = 3.87 \times 10^5$$

SUBSTITUTING  $\frac{1}{3}$  SOLVING

$$\overline{k}_c = 8.15 \times 10^{-3} \text{ m}^2/\text{s}$$

$$W_A = \overline{k}_c A (C_{AS} - C_{AO})$$

$$C_{AS} = \frac{P_A^\circ}{RT} = \frac{3.066 \times 10^4}{(8.314)(298)} = 12.37 \text{ mol/m}^3$$

$$W_A = (8.15 \times 10^{-3})(12.37 - 0)(1)$$

$$= 0.101 \text{ mol/s}$$

$$= (0.101)(58) = \underline{\underline{5.86 \text{ g/s}}}$$

### 28.14 GAS STREAM CONTAINING CO (A) O<sub>2</sub> (B) CO<sub>2</sub> (C)

$$y_A = 0.009 \quad T = 300 \text{ K}$$

$$y_B = 0.001 \quad P = 1 \text{ atm}$$

$$y_C = 0.99$$

$$y'_A = \frac{0.009}{0.999} = 0.00901$$

$$y'_C = \frac{0.99}{0.999} = 0.991$$

$$D_{AB} = 0.213 \text{ cm}^2/\text{s} \quad D_A = 0.158 \text{ cm}^2/\text{s}$$

$$D_{AC} = 0.155 \quad D_B = 0.159 \text{ "}$$

$$D_{BC} = 0.166 \quad D_C = 0.0832 \text{ "}$$

28.14 CONTINUED -

$$D_{B-MN} = \frac{1}{\frac{y_A}{D_{BA}} + \frac{y_C}{D_{BC}}}$$

SUBSTITUTING VALUES

$$D_{B-MN} = 0.166 \text{ cm}^2/\text{s}$$

USE VISCOSITY  $\sim \eta_C$  - THE DOMINANT COMPONENT

$$Sc = \frac{D}{D_{AB}} = \frac{0.0832}{0.166} = 0.501 \quad (a)$$

$$Re = \frac{\rho L}{D} = \frac{(1200)(300)}{0.0832} = 4.327 \times 10^6$$

{ VERY MUCH INTO TURBULENT REGIME }

PRESUMING B.L. FLOW TO BE

LAMINAR FOR  $0 < Re < 2 \times 10^5$

TURBULENT  $2 \times 10^5 < Re$

$$F_c = 0.1664 \frac{D_{AB}}{L} Sc^{1/3} Re_L^{1/2} + 0.0365 \frac{D_{AB} Sc^{1/3}}{L} [Re_L^{4/5} - Re_{tr}^{4/5}]$$

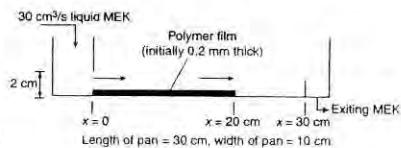
SUBSTITUTING VALUES

$$\underline{F_c = 3.277 \text{ cm/s}} \quad (c)$$

TURBULENT EFFECTS DOMINATE (b)

28.15

SOLUTE (A)  
INTO MEK (B)



28.15 CONTINUED -

$$C_{AB} = 0 \quad D_{AB} = 3 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$D = 6 \times 10^{-3} \text{ "}$$

$$\rho_{solid} = 1.05 \text{ g/cm}^3 \quad \rho_L = 0.80 \text{ g/cm}^3$$

$$\rho_A^* = 0.04 \text{ g/cm}^3 \quad V = 30 \text{ cm}^3/\text{s}$$

$$V = \frac{30}{(1.98)(10)} = 1.515 \text{ cm/s} \quad \{ \text{UNIT DEPTH} \}$$

$$Sc = \frac{6 \times 10^{-3}}{3 \times 10^{-6}} = 2000$$

$$Re = \frac{(1.515)(20)}{6 \times 10^{-3}} = 5050 \quad \{ \text{LAMINAR} \}$$

$$F_c = 0.1664 \frac{D_{AB}}{L} Re_L^{1/2} Sc^{1/3} = 88.94 \times 10^{-6} \text{ cm/s}$$

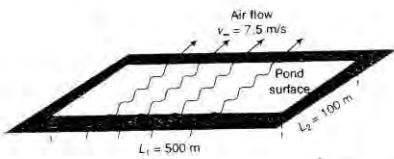
$$n_A = k_c (\rho_A^* - \rho_{AB}) = (88.94 \times 10^{-6})(0.04) = \frac{3.558 \times 10^{-6} \text{ g/cm}^2 \cdot \text{s}}{7.116 \times 10^{-4}} \quad (a)$$

$$N_A = n_A A = (3.558 \times 10^{-6})(10 \times 20) = \underline{7.116 \times 10^{-4} \text{ g/s}}$$

$$m_{solid} = (0.02)(10)(20)(1.05) = 4.20 \text{ g}$$

$$t = \frac{4.20}{7.116 \times 10^{-4}} = \frac{5902 \text{ s}}{1.64 \text{ h}} \quad (b)$$

28.16



## METHYLENE CHLORIDE (A) IN AIR (B)

$$T = 293 \text{ K} \quad P = 1 \text{ atm}$$

$$D_{AB} = 0.085 \text{ cm}^2/\text{s} \quad V = 7.5 \text{ m/s}$$

$$\bar{V} = 0.15 \text{ cm}^2/\text{s}$$

$$Sc = \frac{\bar{V}}{D} = \frac{0.15}{0.085} = 1.765$$

$$Re = \frac{VL}{\bar{V}} = \frac{(7.5)(100)}{0.15 \times 10^{-4}} = 5 \times 10^7$$

Assume B.L. Flow To Be

LAMINAR FOR  $0 < Re < 2 \times 10^5$ TURBULENT FOR  $2 \times 10^5 < Re$ 

$$\bar{k}_c = 0.664 \frac{D_{AB}}{L} Sc^{1/3} Re_{tr}^{1/2} + 0.0365 \frac{D_{AB}}{L} Sc^{1/3} [Re_L^{4/5} - Re_{tr}^{4/5}]$$

$$Re_{tr} = 2 \times 10^5 \quad Re_L = 5 \times 10^7$$

SUBSTITUTING  $\bar{k}_c$  SOLVING

$$\bar{k}_c = 0.5372 \text{ cm/s} \quad (b)$$

$$\text{For } Re = 2 \times 10^5 = \frac{VL_{tr}}{\bar{V}}$$

$$L_{tr} = \frac{(2 \times 10^5)(0.15 \times 10^{-4})}{7.5} = 0.4 \text{ m} \quad (a)$$

THIS IS THE EXTENT OF  
THE LAMINAR B.L.

$$Sc(L) = \frac{0.010}{1.07 \times 10^{-5}} = 934 \quad (c)$$

## 28.17 LUBRICATING OIL (A) IN AIR (B)

$$T = 386 \text{ K} \quad P = 1 \text{ atm} \quad V = 50 \text{ m/s}$$

$$X_{tr} = 0.097 \text{ m} \quad D_{AB} = 0.040 \text{ cm}^2/\text{s}$$

$$\mu = 2.23 \times 10^{-5} \text{ kg/m.s}$$

$$\rho = 0.917 \text{ kg/m}^3$$

$$P_A^o = 0.20 \text{ Pa}$$

$$\bar{V} = \frac{2.23 \times 10^{-5}}{0.917} = 2.43 \times 10^{-5} \text{ m}^2/\text{s}$$

$$= 0.243 \text{ cm}^2/\text{s}$$

$$Sc = \frac{\bar{V}}{D_{AB}} = \frac{0.243}{0.040} = 6.075 \quad (a)$$

$$\bar{k}_c = 0.664 \frac{D_{AB}}{L} Sc^{1/3} Re_{tr}^{1/2} + 0.0365 \frac{D_{AB}}{L} Sc^{1/3} [Re_L^{4/5} - Re_{tr}^{4/5}]$$

$$Re_{tr} = \frac{(5000)(9.7)}{0.243} \quad Re_L = \frac{(5000)(200)}{0.243}$$

$$\approx 2 \times 10^5 \quad = 4.115 \times 10^6$$

SUBSTITUTING VALUES INTO  $\bar{k}_c$  EXPRESSION:

$$\bar{k}_c = 2.48 \text{ cm/s} \quad (b)$$

AT  $X = 120 \text{ cm}$ 

$$Re_x = \frac{(5000)(120)}{0.243} = 2.469 \times 10^6$$

$$\bar{k}_{cy} = \frac{0.0292 D_{AB}}{X} Sc^{1/3} Re_x^{1/2}$$

$$= \frac{0.0292(0.040)(6.075)^{1/3} (2.469 \times 10^6)^{4/5}}{120}$$

$$= 2.307 \text{ cm/s} \quad (c)$$

$$28.18 \quad \text{for } k_{ex} = 70,000 = \frac{XU}{V}$$

$$X = 70,000 \frac{V}{U}$$

$$\frac{k_e X}{D_{AB}} = 0.332 Re_x^{1/2} Sc^{1/3}$$

$$k_c = 0.332 \frac{(70,000)^{1/2} Sc^{1/3} D_{AB} U}{70,000^{2/3}} \quad (a)$$

$$\text{for } k_{ex} = 70,000 = L U / V$$

$$L = 70,000 \frac{V}{U}$$

$$\frac{k_e L}{D_{AB}} = 0.664 Re_L^{1/2} Sc^{1/3}$$

$$= 0.664 \frac{(70,000)^{1/2} Sc^{1/3} D_{AB} U}{70,000} \quad (b)$$

$$= 0.0025 U Sc^{-2/3}$$

$$\text{for } Re_x = 7 \times 10^5$$

$$X = 7 \times 10^5 \frac{V}{U}$$

$$\frac{k_e X}{D_{AB}} = 0.0292 Re_x^{4/5} Sc^{1/3}$$

$$k_c = 0.0292 \frac{(7 \times 10^5)^{4/5} Sc^{1/3} D_{AB} U}{(7 \times 10^5)} \quad (c)$$

$$= 0.00198 U Sc^{-2/3}$$

28.19 - VON KARMAN BL. ANALYSIS -

$$\text{GIVEN } U = \alpha + \beta y^{1/7}$$

$$\text{B.C. } U(y=0) = 0 \quad \alpha = 0$$

$$U(y=\delta) = U_\infty \quad \beta = \frac{U_\infty}{\delta^{1/7}}$$

$$U_x = U_\infty \left( \frac{y}{\delta} \right)^{1/7}$$

$$\text{Given } C_A - C_{AP} = \eta + \xi y^{1/7}$$

$$\text{at } y=0 \quad C_A - C_{AP} = C_{AS} - C_{AP}$$

$$\text{at } y=\delta_C \quad C_A - C_{AP} = 0$$

$$\eta = C_{AS} - C_{AP}$$

$$\xi = -\frac{\eta}{\delta_C^{1/7}} = -\frac{C_{AS} - C_{AP}}{\delta_C^{1/7}}$$

$$\text{Now. } \frac{C_A - C_{AP}}{C_{AS} - C_{AP}} = 1 - \left( \frac{y}{\delta_C} \right)^{1/7}$$

EQU (28.29)

$$\frac{d}{dx} \int_0^{Sc} (C_A - C_{AP}) U dy = k_c (C_{AS} - C_{AP})$$

DIVIDE BY  $(C_{AS} - C_{AP}) U_\infty$

$$\frac{d}{dx} \int_0^{Sc} \left( \frac{C_A - C_A}{C_{AS} - C_{AP}} \right) \frac{U}{U_\infty} dy = \frac{k_c}{U_\infty}$$

EVALUATING THE INTEGRAL -

$$\int_0^{Sc} \left[ 1 - \left( \frac{y}{\delta_C} \right)^{1/7} \right] \left( \frac{y}{\delta} \right)^{1/7} dy$$

$$\int_0^{Sc} \left[ \left( \frac{y}{\delta} \right)^{1/7} - \frac{y^{2/7}}{\delta_C^{1/7} \delta^{1/7}} \right] dy$$

$$\left[ \frac{7}{8} \frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{9} \frac{y^{9/7}}{\delta_C^{1/7} \delta^{1/7}} \right]_0^{Sc}$$

28.19 CONTINUED

INTEGRAL EVALUATION - BETWEEN 0 &  $\delta_c$

$$\frac{7}{8} \frac{\delta_c^{8/7}}{8^{1/7}} - \frac{7}{9} \frac{\delta_c^{8/7}}{8^{1/7}} = \frac{7}{92} \frac{\delta_c^{8/7}}{8^{1/7}}$$

BACK INTO GOVERNING EQU.

$$\frac{d}{dx} \left[ \frac{7}{92} \frac{\delta_c^{8/7}}{8^{1/7}} \right] = \frac{k_c}{V_p}$$

LETTING  $\delta_c = 1$  -  $\delta_c = \delta$

$$\frac{7}{92} \frac{d\delta_c}{dx} = \frac{k_c}{V_p}$$

$\therefore$  SINCE WE KNOW  $\delta = \frac{0.311x}{Rex}$

$$\frac{d\delta_c}{dx} = \frac{d\delta}{dx} = \frac{0.311}{(V_p/x)^{1/5}} \left( \frac{4}{5} x^{-1/5} \right)$$

$$= 0.291 Rex^{-1/5}$$

WE NOW HAVE

$$\frac{7}{92} (0.291 Rex^{-1/5}) = k_c / V_p$$

$\therefore$ , FINALLY  $k_c = 0.0189 V_p Rex^{-1/5}$

28.20  $V_x = a + bx$

B.C.  $V_x(0) = 0$

$$V_x(\delta) = V_p$$

$$V_x = V_p \left( \frac{y}{\delta} \right)$$

$$C_A = s + fy$$

B.C.  $C_A = C_{AS} @ y=0$

$$C_A = C_{AP} @ y=\delta_c$$

28.20 CONTINUED

$$\frac{C_A - C_{AS}}{C_{AP} - C_{AS}} = \frac{y}{\delta_c}$$

ANOTHER PHYSICAL SITUATION THAT A PROFILE SHOULD PROVIDE IS

$$\frac{dC_A}{dy} = 0 @ y=0$$

THE LINEAR MODEL DOES YIELD THIS RESULT & WILL NOT SATISFY ALL OF THE PHYSICAL REQUIREMENTS

28.21 FOR A SPHERICAL PELLET ( $d=1\text{ cm}$ )

$$Nu = 0.37 Re^{0.6} Pr^{1/3}$$

From DATA REQUIRED IN PROBLEM STATEMENT:

$$Re_{\text{app}} = \frac{V_p d}{\nu} = \frac{(1)(0.01)}{1.569 \times 10^{-5}} = 637$$

$$Pr = \frac{\lambda}{\alpha} = \frac{1.569 \times 10^{-5}}{2.216 \times 10^{-5}} = 0.708$$

$$h = Nu \frac{k}{d} = \frac{0.317(637)}{0.01} \left( \frac{0.708}{0.02642} \right)^{1/3}$$

$$= 41.64 \text{ W/m}^2 \quad (\text{a})$$

AT  $\frac{1}{2}$  MASS TX ANALOGUE:

$$\frac{h}{8 C_p \nu} Pr^{2/3} = \frac{k_c}{V_p} \frac{s^{2/3}}{\delta_c^{2/3}}$$

$$k_c = \frac{h}{8 C_p} \left( \frac{Pr}{\delta_c} \right)^{2/3}$$

$$\delta_c = \frac{V}{T_{AB}} = \frac{1.569 \times 10^{-5}}{9.462 \times 10^{-6}} = 1.63$$

28.21 CONTINUED -

$$k_c = \frac{41.64}{(1.17)(1.006)} \left( \frac{0.708}{1.163} \right)^{2/3}$$

$$= \underline{\underline{0.020 \text{ m/s}}} \quad (b)$$

$$N_A = k_c (C_{AS} - C_{AP})$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{1.27 \times 10^4}{(8.314)(300)} = 5.09 \text{ mol/m}^3$$

$$C_{AP} = 0 \quad (c)$$

$$N_A = (0.020)(5.09) = \underline{\underline{0.102 \text{ mol/m}^2 \cdot \text{s}}}$$

28.22 - GIVEN IN PROBLEM STATEMENT

$$\frac{h d_1}{k} = 0.031 Re_{d_1}^{0.8} Pr^{1/3} \left( \frac{\mu}{\mu_{vis}} \right)^{0.14} \left( \frac{d_2}{d_1} \right)^{0.15}$$

THAT  $\frac{1}{3}$  MASS IN PIPE RELATED BY

$$\frac{h}{Sc \nu} Pr^{2/3} = \frac{k_c}{\nu} Sc^{2/3}$$

SUBSTITUTION  $\frac{1}{3}$  SOLVING:

$$Sh = \frac{k_c d_1}{D_{AB}}$$

$$= 0.031 Re_{d_1}^{0.8} Sc^{1/3} \left( \frac{\mu}{\mu_{vis}} \right)^{0.14} \left( \frac{d_2}{d_1} \right)^{0.15}$$

28.23 AS GIVEN:

$$Nu = \frac{h d_p}{k} = 0.37 Re_{dp}^{0.6} Pr^{1/3}$$

$\frac{1}{3}$  FROM GUILTON - CORBURN ANDREY

$$\frac{h}{Sc \nu} Pr^{2/3} = \frac{k_c}{\nu} Sc^{2/3}$$

28.23 CONTINUED

COMBINING EQUATIONS -

$$Sh = \frac{k_c d_p}{D_{AB}} = 0.37 Re_{dp}^{0.6} Sc^{1/3}$$

FOR SLOW FLOW - NO BULK CONTRIBUTION:

$$\text{STEADY STATE: } \frac{d}{dr} (r^2 N_A) = 0$$

$$N_A = - D_{AB} \frac{dC_A}{dr}$$

$$r^2 N_A \int_R^\infty \frac{dr}{r^2} = - D_{AB} \int_{CAS}^{C_{AP}} dC_A$$

$$r^2 N_A \left( -\frac{1}{r} \right) \Big|_R^\infty = D_{AB} (C_{AS} - C_{AP})$$

$$\text{AT } r=R: N_A = \frac{D_{AB}}{R} (C_{AS} - C_{AP}) = k_c \Delta C_A$$

$$k_c = \frac{D_{AB}}{R} = \frac{2 D_{AB}}{d_p}$$

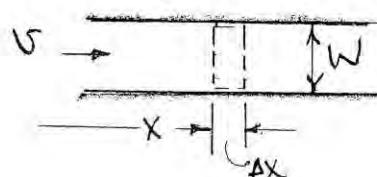
$$\text{GIVEN: } Sh = \frac{k_c d_p}{D_{AB}} = 2$$

MODIFIED EQUATION IS, THUS

$$Sh = 2 + 0.37 Re_{dp}^{0.6} Sc^{1/3}$$

28.24

FOR FLOW IN A CHANNEL BETWEEN  
2 PLANES (PER UNIT DEPTH)



28.24 CONTINUED -

MASS BALANCE FOR CONTROL VOLUME

$$c_A \nu W |_x + 2 k_c (c_{AS} - c_A) \Delta x = c_A \nu W |_{x+\Delta x}$$

$$\frac{c_A|_{x+\Delta x} - c_A|_x}{\Delta x} = \frac{2 k_c}{W \nu} (c_{AS} - c_A)$$

IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{dc_A}{dx} = \frac{2}{W \nu} k_c (c_{AS} - c_A)$$

$$\text{LET } \theta = c_A - c_{AS} \sim \frac{dc_A}{dx} = \frac{d\theta}{dx}$$

$$\frac{d\theta}{dx} = - \frac{2}{W \nu} k_c \theta$$

$$\int_{\theta_0}^{\theta_L} \frac{d\theta}{\theta} = - \frac{2}{W \nu} k_c \int_0^L dx$$

$$\ln \frac{\theta_L}{\theta_0} = - \frac{2 k_c L}{W \nu}$$

$$\frac{\theta_L}{\theta_0} = \exp \left( - \frac{2 k_c L}{W \nu} \right)$$

$$\frac{c_{AL} - c_{AS}}{c_{A0} - c_{AS}} = \exp \left( - \frac{2 k_c L}{W \nu} \right)$$

FOR  $c_{A0} = 0$

$$c_{AL} = c_{AS} \left[ 1 - e^{- \frac{2 k_c L}{W \nu}} \right]$$

NAPHTHALENE IN AIR:

$$T = 273 K \quad P = 1.013 \times 10^5 \text{ Pa}$$

$$P_A^o = 1 \text{ Pa} \quad D_{AB} = 5.14 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Sc = 2.5 \quad \lambda = Sc D_{AB} = 1.32 \times 10^{-5} \text{ "}$$

$$\nu = 15 \text{ m/s}$$

28.24 CONTINUED

$$C_{AS} = \frac{P_A}{RT} = \frac{1}{(8.314)(273)} = 4.406 \times 10^{-4} \text{ mol/m}^3$$

USING REYNOLDS ANALOGY

$$Re = \frac{Dequiv \nu}{D}$$

$$Dequiv = \frac{4 (1)(w)}{2} = 2w$$

$$Re = \frac{2 (0.0075) (15)}{1.32 \times 10^{-5}} = 1.7 \times 10^4$$

$$\text{From (13.1)} \quad f_r = C_f \approx 0.0064$$

$$\frac{k_c}{\nu} = \frac{0.0064}{2} = 0.0032$$

$$c_{AL} = (4.406 \times 10^{-4}) \left[ 1 - e^{-2(0.0032)(0.1 / 0.0075)} \right]$$

$$= 3.60 \times 10^{-5} \text{ mol/m}^3 \quad (a)$$

USING THE VON KARMAN ANALOGY:

$$\frac{k_c}{\nu} = \frac{C_{f/2}}{1 + 5(C_{f/2})^{1/2} [Sc - 1 + \ln(1 + \frac{5}{6} Sc)]}$$

$$C_{f/2} = 0.0032 \quad Sc = 2.5$$

$$\frac{k_c}{\nu} = 0.00184$$

$$c_{AL} = (4.406 \times 10^{-4}) \left[ 1 - e^{-2(0.00184)(0.1 / 0.0075)} \right]$$

$$= 2.11 \times 10^{-5} \text{ mol/m}^3 \quad (b)$$

CHILTON COLBURN:  $\frac{k_c}{\nu} = \frac{C_f Sc^{-2/3}}{2}$

$$k_c / \nu = 0.00174 - \frac{10^{-5}}{Sc} \quad (c)$$

$$\text{GIVING } c_{AL} = 2.0 \times 10^{-5} \text{ mol/m}^3$$

28.24 CONTINUED -

PARTS (a), (b) & (c) COMPARE CONCENTRATIONS AT/NEAR STARTING CONDITIONS - BEFORE NAPHTHALENE SHEETS HAVE CHANGED DIMENSIONS.

AFTER EXTENDED TIME -

ORIGINAL VOL. OF NAP

$$= (10)(10)(0.25) = 25 \text{ cm}^3$$

WHEN  $\frac{1}{2}$  OF VOLUME HAS BEEN SUBLIMED -  $12.5 \text{ cm}^3$  REMAIN &  $12.5 \text{ cm}$  ARE GONE -

NEW CHANNEL WIDTH =  $0.00875 \text{ m}$

AT AVERAGE CONDITIONS -

$$D_{COVW} = 2W = 2(0.00875) \\ = 0.01625 \text{ m}$$

$$Re = \frac{(0.01625)(15)}{1.32 \times 10^{-5}} = 1.85 \times 10^4$$

$$C_f = f_f \approx 0.0064 \quad \frac{C_f}{2} = 0.0032$$

1. AT AVERAGE CONDITIONS THE ANSWERS TO PARTS (a), (b) & (c) BECOME -

$$\text{REYNOLDS} \quad C_{AL} = 3.34 \times 10^{-5} \text{ mol/m}^3$$

$$\text{VON KARMAN} \quad = 1.95 \times 10^{-5} "$$

$$\text{C-COLBURN} \quad = 1.84 \times 10^{-5} "$$

THESE ARE PROBABLY MORE REPRESENTATIVE VALUES -

28.24 CONTINUED

TOTAL NAPHTHALENE LOST -

$$= (12.5 \text{ cm}^3)(1.145 \text{ g/cm}^3) \left( \frac{\text{mol}}{128.1 \text{ g}} \right)$$

$$= 0.1117 \text{ mol}$$

$$W_A = C_{AL} S A$$

$$= C_{AL}(15 \text{ m/s})(0.1 \text{ m})(0.00875 \text{ m})$$

$$= 0.0122(C_{AL}) \text{ mol/s}$$

$$t = \frac{0.1117}{0.0122 C_{AL}}$$

USING CORRECTED RESULTS FOR  $C_{AL}$

$$\text{REYNOLDS: } t = \frac{0.1117}{(0.0122)(3.34 \times 10^{-5})} \\ = 2.744 \times 10^5 \text{ s} \\ = \underline{\underline{76.2 \text{ h}}}$$

$$\text{VON-KARMAN: } t = \underline{\underline{130.6 \text{ h}}}$$

$$\text{C-COLBURN: } t = \underline{\underline{138.4 \text{ h}}}$$

28.25 SPHERICAL DROP IN AIR -

$$D_{AIR} = 15689 \times 10^{-5} \text{ m}^2/\text{s} \quad \rho_{AIR} = 1.177 \text{ kg/m}^3$$

$$\times_{AIR} = 2.2156 \times 10^{-5} " \quad k = 2.64 \times 10^{-2} \text{ W/m.K}$$

$$D_{DB} = 2.63 \times 10^{-5} " \quad c_p = 1006 \text{ kJ/kg.K}$$

$$T_s = 290 \text{ K} \quad \lambda = 2461 \text{ J/g}$$

$$\rho_w = 1940 \text{ Pa} \quad T_p = 310 \text{ K}$$

28.25 CONTINUED -

### ENERGY BALANCE --

$$\left. \begin{array}{l} \text{HT TO DROP} \\ \text{BY CONVECTION} \end{array} \right\} = \left. \begin{array}{l} \text{HT LOST BY} \\ \text{EVAPORATION} \end{array} \right\}$$

$$h(T_p - T_s) = \lambda k_c (C_{AS} - C_{AV}) M$$

USING GALTAN-CALBURN ANALOGY

$$\frac{k_c}{v_p} Sc^{2/3} = \frac{h}{8c_p v_p} Pr^{2/3}$$

$$\frac{h}{k_c} = \left( \frac{Sc}{Pr} \right)^{2/3} 8 c_p$$

$$C_{AS} - C_{AV} = \frac{h}{k_c} \frac{(T_p - T_s)}{\lambda M}$$

$$= \left( \frac{Sc}{Pr} \right)^{2/3} \frac{R_c (T_p - T_s)}{\lambda M}$$

$$= \left( \frac{0.60}{0.708} \right)^{2/3} \frac{(117)(1,000)(20)}{2461(18)}$$

$$= 0.478 \text{ mol/m}^3$$

$$C_{AS} = \frac{P}{RT} = \frac{1940}{(8,314)(2910)} = 0.805 \text{ mol/m}^3$$

$$C_{AV} = 0.805 - 0.478 = 0.326 \text{ mol/m}^3$$

28.26 - THIS IS THE SAME PHYSICAL PROCESS AS IN TEXT EXAMPLE b

$$T_m = \frac{\lambda_{TS}}{8c_p} \left( \frac{Pr}{Sc} \right)^{1/3} (C_{AS} - C_{AV}) + T_s$$

28.26 CONTINUED -

$$T_s = 298 \text{ K}$$

$$C_{AIR} = 1 \text{ J/g.K}$$

$$\mu = 1.84 \times 10^4 \text{ g/cm.s.}$$

$$\rho = 1.17 \times 10^{-3} \text{ g/cm}^3$$

$$P_w = 1300 \text{ Pa}$$

$$k = 2.62 \times 10^4 \text{ W/cm.K}$$

$$v_p = 0.22 \text{ m/s}$$

$$D_{AB} = 3 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Fr = \frac{\mu c s}{k} = \frac{(1.84 \times 10^4)(1)}{2.62 \times 10^4} = 0.70$$

$$Sc = \frac{\mu}{k D_{AB}} = \frac{(1.84 \times 10^4)}{(1.17 \times 10^{-3})(3 \times 10^{-5})} = 0.524$$

$$C_{AS} = \frac{P}{RT} = \frac{1300}{(8,314)(298)} = 0.525 \text{ mol/m}^3$$

$$C_{AV} = 0$$

$$T_m = \frac{(2450)(18)}{(1.17 \times 10^{-3})(1)} \left( \frac{0.70}{0.524} \right)^{1/3} (0.525 \times 10^{-6}) + 298$$

$$= 218 + 298 = \underline{\underline{319.8 \text{ K}}}$$

28.27 H<sub>2</sub>O (A) INTO AIR (B)

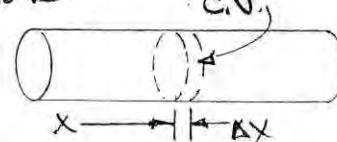
$$T = 310 \text{ K}$$

$$D = 0.15 \text{ m}$$

$$P = 1.013 \times 10^5 \text{ Pa}$$

$$D/\epsilon = 10^4$$

$$T_s = 290 \text{ K}$$



MASS BALANCE FOR CN, SHOWN:

28.27 (CONTINUED -

$$C_A \frac{\pi D^2}{4} |_x + k_c (C_{AS} - C_A) \pi D \Delta x = \\ C_A \frac{\pi D^2}{4} |_{x+\Delta x}$$

$$\frac{C_A|_{x+\Delta x} - C_A|_x}{\Delta x} = 4 \frac{k_c}{D \nu} (C_{AS} - C_A)$$

IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{dC_A}{dx} = \frac{4}{D \nu} k_c (C_{AS} - C_A)$$

$$\text{LET } \Theta = C_A - C_{AS} - \frac{d\Theta}{dx} = \frac{dC_A}{dx}$$

$$\frac{d\Theta}{dx} = - \frac{4}{D \nu} k_c \Theta$$

$$\int_{\Theta_0}^{\Theta} \frac{d\Theta}{\Theta} = - \frac{4}{D \nu} k_c \int_0^L dx$$

$$\ln \frac{\Theta}{\Theta_0} = - \frac{4}{D \nu} k_c L$$

$$\frac{C_A - C_{AS}}{C_{AO} - C_{AS}} = e^{- \frac{4}{D \nu} k_c L}$$

CHILTON-COLBURN ANALOGY

$$\frac{k_c S_c^{2/3}}{\nu} = \frac{C_f}{2} = \frac{f}{2}$$

$$S_c = \frac{\nu}{D_{AB}}$$

$$\nu = 1.569 \times 10^5 \text{ m}^2/\text{s} @ T_f = 300 \text{ K}$$

28.27 (CONTINUED -

$$D_{AB} = \frac{2.634}{1.013 \times 10^5} \left( \frac{300}{298} \right)^{3/2} = 2.626 \times 10^{-5} \text{ m}^2/\text{s}$$

$$S_c = \frac{\nu}{D_{AB}} = \frac{1.569 \times 10^5}{2.626 \times 10^{-5}} = 0.597$$

$$Re = \frac{DV}{\nu} = \frac{(0.15)(1.5)}{1.569 \times 10^{-5}} = 1.43 \times 10^4$$

$$\text{Fig. 13.1 } f_f \approx 0.0066$$

$$\frac{k_c}{\nu} = \frac{0.0066}{2} (0.597)^{-2/3} = 4.65 \times 10^{-3}$$

$$\frac{C_{AO} - C_{AS}}{C_{AO} - C_{AS}} = \frac{-\frac{4}{D \nu} (4.65 \times 10^{-3})}{0.15} = 0.474$$

$$C_{AS} = \frac{P}{RT} = \frac{1895}{(8.314)(290)} = 0.786 \text{ mol/m}^3$$

$$C_{AO} = C_{AS} (1 - 0.474)$$

$$= 0.786 (0.526) = \underline{\underline{0.413 \text{ mol/m}^3}}$$

28.28 SAME PHYSICAL SITUATION AS  
IN PROB 28.27

$$\text{IN } \frac{C_{AO} - C_{AS}}{C_{AO} - C_{AS}} = - \frac{4}{D \nu} k_c L$$

$$Re = \frac{DV}{\nu} = \frac{(0.025)(1.5)}{1.415 \times 10^{-5}} = 2.65 \times 10^4$$

$$\text{Fig 13.1 } - f_f = C_f = 0.0058$$

$$S_c = \frac{\nu}{D_{AB}} = \frac{1.415 \times 10^{-5}}{540 \times 10^{-6}} = 2.62$$

28, 28 (CONTINUED)

USE GILTON-COLBURN ANALOGY

$$\frac{k_C}{V} = \frac{C_e/2}{S_C^{2/3}} = \frac{0.0058/2}{2.62^{2/3}}$$

$$= 0.00153$$

$$\ln \left[ \frac{C_{AL} - C_{AS}}{C_{AO} - C_{AS}} \right] = - \frac{4}{0.00153} L$$

$$= -0.245 \text{ L}$$

$$C_{AS} = \frac{P^o}{RT} = \frac{3}{(8.314)(283)} = 1.275 \times 10^{-3} \text{ mol/m}^3$$

$$C_{AL} = 4.75 \times 10^{-4} \text{ mol/m}^3 \quad C_{AO} = 0$$

$$\ln \frac{4.75 - 12.75}{-12.75} = -0.466$$

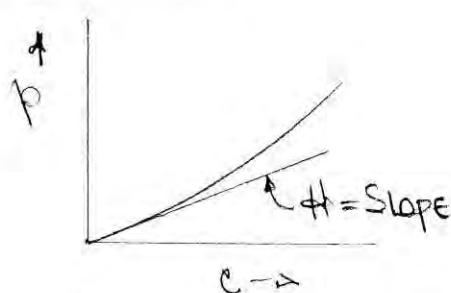
$$L = \frac{0.466}{0.245} = \underline{\underline{1.90 \text{ m}}}$$

## CHAPTER 29 -

### 29.1 EQUILIBRIUM DATA - $\text{Cl}_2$ IN $\text{H}_2\text{O}$

$p, \text{Cl}_2 \text{ kg/m}^3 \text{ mol/m}^3$

6666	0.438	6.17
1330	0.575	8.10
4000	0.937	13.20
6600	1.210	
13200	1.773	



A CAREFUL PLOT WILL YIELD

$$H \approx 62 \text{ Pa/(mol/m}^3)$$

### 29.2 EQUILIBRIUM DATA FOR TCE IN $\text{H}_2\text{O}$

$p, \text{TCE}$	$C$
ATM	$\text{mol/m}^3$
0.000	0
0.050	5.0
0.150	15.0
0.200	20.0

PLOT  $\frac{1}{C}$  OBSERVATION WILL SHOW LINEAR BEHAVIOR -

$$H = \frac{\Delta p}{\Delta C} = 0.010 \text{ atm/(mol/m}^3)$$

29.3 BENZENE (B) — 49 MOLES  
TOLUENE (T) — 21 MOLES  
 $\sum = \frac{70}{70} \text{ mol}$

$$\text{AT } 363 \text{ K, } P = 1.013 \times 10^5 \text{ Pa}$$

$$P_B = 1.344 \times 10^5 \text{ Pa}$$

$$P_T = 5.38 \times 10^5 \text{ Pa}$$

B	X <sub>B</sub>	$P_B \times 10^{-5}$	$P_T \times 10^{-5}$
		1.344	$1.344 X_B$
T	$1-X_B$	5.38	$5.38 (1-X_B)$

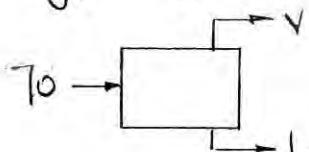
$$P_B + P_T = 1.013 \times 10^5$$

$$1.344 \times 10^5 X_B + 5.38 \times 10^5 (1-X_B) \\ = 1.013 \times 10^5$$

$$X_B = 0.589 \quad X_T = 0.411 \quad (a)$$

$$Y_B = \frac{1.344 \times 10^5}{1.013 \times 10^5} (0.589) = \underline{\underline{0.783}}$$

$$Y_T = 1-Y_B = \underline{\underline{0.217}}$$



MASS BALANCE —

$$\text{TOTAL: } 70 = V + L$$

$$B: 49 = 0.783V + 0.589L$$

$$= 0.783(70-L) + 0.589L$$

$$L = 29.95 \text{ mol} \quad (b)$$

29.4 BASIS - 100 kg H<sub>2</sub>O

	kg	M	mol	X <sub>i</sub>
O <sub>2</sub>	2 × 10 <sup>-3</sup>	32	6.25 × 10 <sup>-5</sup>	1.126 × 10 <sup>-5</sup>
H <sub>2</sub> O	100	18	5.55	

$$P_{O_2} = \gamma_{O_2} P = 0.21 (1.013 \times 10^5) = 0.2127 \times 10^5$$

$$P_{O_2}^* = H X_{O_2} = (4.06 \times 10^9) (1.126 \times 10^{-5}) \\ = 4.57 \times 10^4$$

As  $P_{O_2}^* > P_{O_2}$  (a)  
SOLUTION WILL LOSE O<sub>2</sub>

$$P_{O_2} = H X_{O_2}$$

$$2.127 \times 10^4 = 4.06 \times 10^9 X_{O_2}$$

$$X_{O_2} = 5.24 \times 10^{-6}$$

FROM TABLE I TOTAL MOLES = 5.55

IN EQUILIBRIUM SOLUTION :

$$Z_{O_2} = (5.24 \times 10^{-6})(5.55) \\ = 2.91 \times 10^{-5} \text{ kg mol}$$

BY MASS :

$$(2.91 \times 10^{-5} \text{ kg mol})(32)$$

$$100 \text{ kg H}_2\text{O}$$

$$= 9.3 \times 10^{-4} \text{ kg O}_2 / 100 \text{ kg H}_2\text{O} \quad (b)$$

29.5

$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{H k_A}$$

$$P_A = H C_A = H C_X A$$

$$C = (1 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)(\frac{\text{kg}}{1000 \text{ g}})(\frac{\text{kg mol}}{18 \text{ kg}})$$

$$= 55.56 \text{ kg mol/m}^3$$

$$H = \frac{4.06 \times 10^9}{55.56} = 7.3 \times 10^7 \frac{\text{Pa}}{\text{kg mol/m}^3}$$

$$\frac{1}{K_L} = \frac{1}{2.15 \times 10^{-5}} + \frac{1}{(9.28 \times 10^{-8})(7.3 \times 10^7)} \\ = 4.65 \times 10^5 + 0.148 \approx 4.65 \times 10^5$$

$$K_L = 2.15 \times 10^{-5} \text{ m/s}$$

ALL (100%) OF RESISTANCE IS IN GAS

29.6 INTERPHASE TRANSPORT

CO<sub>2</sub> - AIR - H<sub>2</sub>O

$$P = 1.5 \text{ ATM} \quad H = 7.7 \times 10^4 \text{ ATM/(g mol/m}^3)$$

$$\gamma_P = 0.0401 \quad \gamma_L = 992.3 \text{ kg/m}^3$$

$$\gamma_A = 0.00040$$

AT EQUILIBRIUM :

$$P_A = H C_A$$

$$= \gamma_A P = 0.04 (1.5) = 0.06 \text{ ATM}$$

$$C_A = \gamma_A C = \frac{0.0004 (992.3)}{18}$$

$$= 0.022 \text{ kg mol/m}^3 = 22.0 \text{ g mol/m}^3$$

$$C_A^* = \frac{P_A}{H} = \frac{0.06}{7.7 \times 10^4} = 77.9 \text{ g mol/m}^3$$

29.6 CONTINUED -

SINCE  $C_A^* > C_A$  - ABSORPTION (a)

maximum  $C_A = C_A^* = 77.9 \text{ g mol/m}^3$  (b)

$$k_y = 1.0 \text{ g mol/m}^2 \cdot \text{s}$$

$$kg = 0.010 \text{ g mol/m}^2 \cdot \text{s} \cdot \text{dm}$$

$$ky = kgP = 0.010(1.5) = 0.015 \text{ g mol/m}^2 \cdot \text{s}$$

$$\frac{1}{k_y} = \frac{1}{kg} + \frac{H}{k_x}$$

$$p_A = H C_A$$

$$\frac{p_A}{P} = y_A = \frac{CH}{P} x_A = H x_A$$

$$H' = \frac{CH}{P} = \frac{(55.13)(7.7 \times 10^{-4})}{1.5}$$

$$= 0.0283$$

$$\frac{1}{k_y} = \frac{1}{0.015} + \frac{0.0283}{1.0}$$

$$k_y = 0.015 \text{ g mol/m}^2 \cdot \text{s} \quad (c)$$

$$N_A = k_y (y_{A0} - y_A^*)$$

$$y_{A0} = 0.06$$

$$y_A^* = \frac{p_A^*}{P} = \frac{7.7 \times 10^{-4}}{1.5} \quad (22)$$

$$= 0.0113$$

$$N_A = (0.015)(0.06 - 0.0113)$$

$$= 7.30 \times 10^{-4} \text{ g mol/m}^2 \cdot \text{s} \quad (d)$$

$$29.7 N_A = K_L (C_A^* - C_{AL})$$

$$C_A^* = \frac{P_{A0}}{H} = \frac{1.013 \times 10^4}{1.674 \times 10^3} = 6.05 \text{ kg mol/m}^3$$

$$N_A = (1.26 \times 10^{-6})(6.05 - 4) = 2.58 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s}$$

$$\frac{1}{K_L} = \frac{K_L}{K_L} = 0.53 \quad (d)$$

$$k_L = \frac{K_L}{0.53} = \frac{1.26 \times 10^{-6}}{0.53} = 2.38 \times 10^{-6} \quad (a)$$

{UNITS ARE  $\text{kg mol/m}^2 \cdot \text{s} \cdot (\text{mol/m}^3)$ }

$$N_A = (2.58 \times 10^{-6}) = 2.38 \times 10^{-6} (C_{AL} - 4)$$

$$C_{AL} = 5.08 \text{ kg mol/m}^3 \quad (c)$$

$$N_A = kg (P_{A0} - P_{AL})$$

$$P_{AL} = H C_{AL} = (1.674 \times 10^3)(5.08) \\ = 8.50 \times 10^3 \text{ Pa}$$

$$k_y = \frac{N_A}{P_{A0} - P_{AL}}$$

$$= \frac{2.58 \times 10^{-6}}{1.013 \times 10^4 - 0.850 \times 10^4}$$

$$= 1.58 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa} \quad (b)$$

## 29.8 STRIPPING TA FROM WASTEWATER

$$T = 293 \text{ K} \quad P = 1,25 \text{ atm}$$

$$H' = 400 \text{ atm} \quad p_A = H' x_A$$

$$p_A = H' x_A$$

$$y_A = \frac{H'}{P} x_A \quad \frac{H'}{P} = \frac{400}{1,25} = 320 \frac{\text{kg mol}}{\text{m}^3}$$

$$p_A = \frac{H'}{C} c_A \quad H = \frac{H'}{C} \quad (\text{a})$$

$$c = \frac{P}{RT} = \frac{1,25}{(0,08206)(293)} = 0,0520 \frac{\text{kg mol}}{\text{m}^3}$$

$$H = \frac{400}{0,0520} = 7690 \frac{\text{atm}}{\text{kg mol/m}^3} \quad (\text{b})$$

GIVEN -  $k_C = 0,01 \text{ mol/s}$

$$k_g = \frac{k_C}{RT} = \frac{0,01}{(0,08206)(293)} = 4,16 \times 10^{-4} \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s} \cdot \text{atm}} \quad (\text{c})$$

$$N_A = k_C \Delta C_A = C k_C \frac{\Delta C_A}{C} = k_g \Delta y_A$$

$$k_g = C k_C = (0,0520)(0,01) = 5,2 \times 10^{-4} \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s} \cdot \Delta y}$$

$$k_g = C k_L \quad (\text{d})$$

$$\left\{ \begin{array}{l} \text{IN} \\ \text{LIQUID} \end{array} \right\} C = S_w = \frac{9982}{18} = 55,46 \frac{\text{kg mol}}{\text{m}^3}$$

$$k_g = (55,46)(0,01) = 0,5546 \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s}} \quad (\text{d})$$

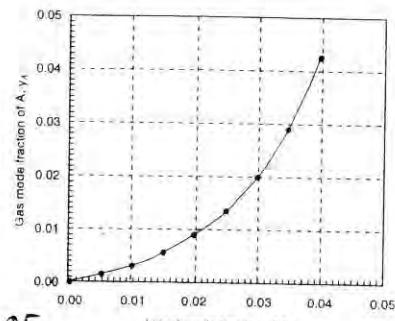
## 29.9

$$T = 300 \text{ K}$$

$$P = 2 \text{ atm}$$

$$y_A = 0,101$$

$$x_A = 0,035$$



80% OF RESISTANCE  
IS IN LIQUID

$$N_A = k_x (x_{A\text{in}} - x_{A\text{L}}) - K_x (y_{A\text{in}} - y_A^*)$$

$$\frac{1/k_x}{1/K_x} = \frac{K_x}{k_x} = 0,8 = \frac{x_{A\text{in}} - x_{A\text{L}}}{y_{A\text{in}} - y_A^*} = \frac{0,035 - 0,021}{0,035 - 0,021}$$

$$x_{A\text{L}} = 0,035 - 0,8(0,014) = 0,0238 \quad (\text{a})$$

FROM EQUILIBRIUM DIAGRAM ABOVE

$$y_{A\text{in}} = 0,0122 \quad (\text{a})$$

$$\frac{1/k_y}{1/K_y} = \frac{K_y}{k_y} = 0,2$$

$$K_y = 0,2(1,25) = 0,25 \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s} \cdot \text{atm}} \quad (\text{b})$$

$$K_y = \frac{K_C}{P} = \frac{0,25}{2} = 0,125 \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s} \cdot \text{atm}} \quad (\text{c})$$

$$\frac{K_y}{P} = \frac{K_C}{RT} \sim K_C = K_y \frac{RT}{P}$$

$$K_C = 0,25 \frac{(82,06)(300)}{2}$$

$$= 3077 \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s}}$$

29.10 SOLUTE A REMOVED FROM GAS STREAM

$$\begin{aligned} T &= 300 \text{ K} & x_{\text{A}} &= 0.01 \\ P &= 2 \text{ ATM} & y_{\text{A}} &= 0.035 \\ \text{EQUILIBRIUM:} & & y_{\text{A}}^* &= 0.3x_{\text{A}} \end{aligned}$$

$$\frac{1/k_y}{1/k_y} = \frac{k_y}{k} = 0.6$$

$$y_{\text{A}} - y_{\text{A}}^* = 0.6$$

$$y_{\text{A}}^* - y_{\text{A}}^* =$$

$$y_{\text{A}}^* = 0.3 x_{\text{A}} = 0.3(0.01) = 0.003$$

$$\frac{0.035 - y_{\text{A}}^*}{0.035 - 0.003} = 0.6$$

$$\underline{y_{\text{A}}^* = 0.0158}$$

$$x_{\text{A}} = \frac{y_{\text{A}}^*}{0.2} = \underline{0.0791} \quad (\text{a})$$

$$\begin{aligned} k_y &= 0.6 k_y = 0.6(1.25) \\ &= 0.75 \text{ g mol/m}^2 \cdot \text{s} \cdot \Delta x \end{aligned} \quad (\text{b})$$

$$\left\{ \begin{array}{l} \frac{1}{k_x} = \frac{1}{H' k_y} + \frac{1}{k_x} \\ \frac{1}{k_y} = \frac{1}{k_y} + \frac{H'}{k_x} \end{array} \right\} \rightarrow \frac{1}{k_x} = \frac{H'}{k_y}$$

$$\begin{aligned} k_x &= \frac{k_y}{H'} = \frac{0.75}{0.3} \\ &= \underline{2.5 \text{ g mol/m}^2 \cdot \text{s} \cdot \Delta x} \quad (\text{c}) \end{aligned}$$

29.11 PERCOLATION OF  $\text{H}_2\text{O}$

$$\begin{aligned} T &= 293 \text{ K} & y_{\text{A}} &= 0.21 \\ P &= 2 \text{ ATM} & \\ P_{\text{H}_2\text{O}} &= 1000 \text{ kg/m}^3 \end{aligned}$$

$$H' = 40,100 \text{ ATM } \Delta y / \Delta x$$

$$P_{\text{O}_2} = 0.21(2) = 0.42 \text{ ATM}$$

$$= (40,100) x_{\text{O}_2}$$

$$x_{\text{O}_2}^* = \frac{0.42}{40,100} = \underline{1.047 \times 10^{-5} \frac{\text{mol O}_2}{\text{mol H}_2\text{O}}} \quad (\text{a})$$

$$c_A^* = (1.047 \times 10^{-5}) \left( \frac{1000}{18} \right)$$

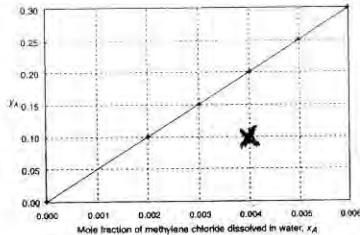
$$= \underline{5.82 \times 10^{-4} \text{ kg mol/m}^3} \quad (\text{b})$$

AS SYSTEM PRESSURE INCREASES

$x^* \& c^*$  WILL INCREASE (c)

29.12

EQUILIBRIUM FOR  
SPECIES A IN AIR  
↓ SPECIES A →  
DISSOLVED IN  $\text{H}_2\text{O}$



$$T = 293 \text{ K}$$

$$P = 2.20 \text{ ATM}$$

$$P_{\text{H}_2\text{O}} = 992.3 \text{ kg/m}^3$$

$$k_y = 0.0109 \text{ g mol/m}^2 \cdot \text{s}$$

$$k_x = 0.125 \text{ "}$$

THIS IS A STRIPPING PROCESS a)

29.12 (CONTINUED) -

$$y_A = H' x_A'$$

$$P_A = \frac{H' P}{C} x_{AC}^* = H C_A^* \quad H = \frac{H' P}{C}$$

$$C = \frac{992.3}{18} = 55.13 \text{ kg mol/m}^3$$

$$H = 50 \quad \text{--- from Diagram}$$

$$H = \frac{50(2.2)}{55.13} = 1.996 \text{ atm/}(\text{kg mol/m}^3) \quad (b)$$

$$y_A = H' x_A' = 0.10 = 50 x_A^* \\ x_A^* = 0.002$$

$$C_A^* = x_A^* C = 0.002(55.13) \\ = 0.110 \text{ kg mol/m}^3$$

$$C_{AP} = x_{AP} C = (0.004)(55.13) \\ = 0.220 \text{ kg mol/m}^3$$

$$\frac{1}{K_X} = \frac{1}{k_X} + \frac{1}{H' k_Y} = \frac{1}{0.125} + \frac{1}{(50)(0.01)} \\ = 8 + 2 = 10$$

$$k_X = 0.10 \text{ g mol/m}^2 \cdot \text{s} \quad (c)$$

$$k_L = \frac{k_X}{C} = \frac{0.10}{55.13} = 1.81 \times 10^{-6} \text{ m/s}$$

$$N_A = K_L (C_{AP} - C_A^*)$$

$$= (1.81 \times 10^{-6})(0.220 - 0.110)$$

$$= 8.91 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s}$$

29.12 (CONTINUED) -

$$N_A = (8.91 \times 10^{-6}) = k_X (x_{AP} - x_{AI})$$

$$= (0.125)(0.004 - x_{AI})$$

$$x_{AI} = \frac{3.93 \times 10^{-3}}{50} \quad (d)$$

$$y_{AI} = H' x_{AI} = 50 (3.93 \times 10^{-3}) = 0.196$$

29.13 HEPTANE (A) ABSORBED FROM AIR

$$T = 273 \text{ K}$$

$$P = 1.5 \text{ atm}$$

$$S_L = 0.80 \text{ g/cm}^3$$

$$M_L = 180$$

$$y_{AP} = 0.015 \text{ atm}$$

$$x_{AP} = 0.05$$

$$k_Y = 0.02 \text{ kg mol/m}^2 \cdot \text{s}$$

$$k_X = 0.01 \quad "$$

$$C = \frac{S_L}{M} = \frac{0.80}{180} = 4.44 \times 10^{-3} \text{ g mol/cm}^3$$

$$p_A = 0.15 x_A^*$$

$$\frac{1}{K_X} = \frac{1}{k_X} + \frac{1}{H' k_Y} = \frac{1}{0.01} + \frac{1}{(0.15)(0.02)}$$

$$K_X = 2.31 \times 10^3 \text{ kg mol/m}^2 \cdot \text{s}$$

$$K_L = \frac{K_X}{C} = \frac{2.31 \times 10^3 (10)^{-3}}{4.44 \times 10^{-3} (10)^4}$$

$$= 0.52 \times 10^7 \text{ cm/s} \quad (a)$$

$$k_Y (y_{AP} - y_{AI}) = k_X (x_{AP} - x_{AI})$$

$$0.02(0.015 - y_{AI}) = 0.01(x_{AI} - 0.05)$$

$$\text{Also: } y_{AI} = \frac{p_{AI}}{P} = \frac{0.15}{P} x_{AI}$$

$$y_{AI} = \frac{0.15}{1.5} = 0.10 x_{AI}$$

29.13 (CONTINUED)

COMBINING THESE EXPRESSIONS -

$$Y_{AI} = \underline{0.0667} \quad (b)$$

$$Y_{AI} = 0.1 X_{AI} = \underline{0.00667}$$

29.14 SO<sub>2</sub> ABSORBED INTO H<sub>2</sub>O

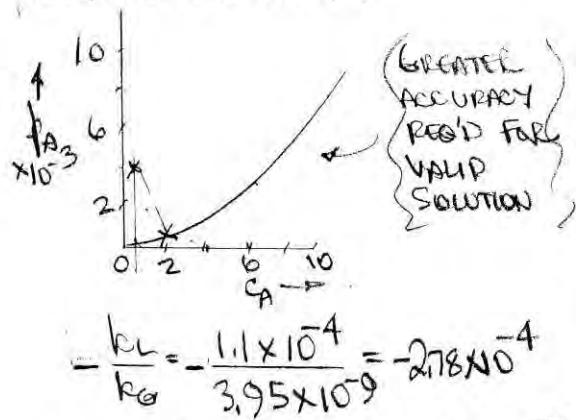
$$P_{AO} = 4 \times 10^3 \text{ Pa}$$

$$C_{AO} = 0.55 \text{ kg mol/m}^3$$

$$k_g = 3.95 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$k_L = 1.1 \times 10^{-4} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

EQUILIBRIUM DATA - GIVEN IN  
PROBLEM STATEMENT -



From Plot -  $\underline{P_{AI} \approx 213 \text{ Pa}} \quad (a)$   
 $\underline{C_{AI} \approx 0.69 \text{ kg mol/m}^3}$

$$N_A = \lg(P_{AI} - P_{AO})$$

$$= 3.95 \times 10^{-9} (4000 - 213)$$

$$= 1.496 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{s}$$

29.14 (CONTINUED)

FROM EQUILIBRIUM PLOT -

$$\text{for } C_{AL} = 0.55 \quad P_A^* \approx 164$$

$$K_g = \frac{N_A}{P_{AO} - P_A^*} = \frac{1.496 \times 10^{-5}}{4000 - 164}$$

$$= 3.9 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

ALSO FROM EQUILIBRIUM PLOT -

$$\text{for } P_{AO} = 4000 \quad C_A^* \approx 6.9$$

$$K_L = \frac{N_A}{C_A^* - C_{AL}} = \frac{1.496 \times 10^{-5}}{6.9 - 0.55}$$

$$= 2.36 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

Summary :

$$k_g = 3.95 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$k_L = 1.1 \times 10^{-4} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

$$K_g = 3.9 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$K_L = 2.36 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

$$P_{AO} - P_{AI} = 3781 \text{ Pa} \quad (b)$$

$$C_{AI} - C_{AL} = 0.14 \text{ kg mol/m}^3$$

$$P_{AO} - P_A^* = 3836 \text{ Pa}$$

$$C_A^* - C_{AL} = 6.35 \text{ kg mol/m}^3$$

$$\frac{1/\text{kg}}{1/\text{kg}} = \frac{K_g}{K_L} = \frac{3.9 \times 10^{-9}}{3.95 \times 10^{-9}} = 0.987$$

$\sim 98.7\% \text{ OF RESISTANCE IS IN GAS } \phi \quad (c)$

29.15  $\text{Cl}_2$  from GAS STREAM into LIQUID

$$P = 1.013 \times 10^5 \text{ Pa} \quad y_{\text{A}i} = 0.002$$

$$k_y = 1.0 \text{ kg mol/m}^2 \cdot \text{h} \cdot \Delta y$$

$$k_y = 10 \text{ kg mol/m}^2 \cdot \text{h} \cdot \Delta x$$

$$H = 6.13 \times 10^4 \text{ Pa/(kg mol/m}^3)$$

$$C_{\text{AL}} = 2.6 \times 10^{-3} \text{ kg mol/m}^3$$

$$\rho_{\text{AI}} = H C_{\text{AI}} \quad y_{\text{AI}} = \frac{H C}{P} x_A = H' x_A$$

$$C_L = \frac{1000}{18} = 55.55 \text{ kg mol/m}^3$$

$$H' = \frac{H C}{P} = \frac{(6.13 \times 10^4)(55.55)}{1.013 \times 10^5} = 33.6$$

$$\frac{1}{K_x} = \frac{1}{k_y} + \frac{1}{H' k_y}$$

$$= \frac{1}{10} + \frac{1}{33.6(1)}$$

$$K_x = 7.71 \text{ kg mol/m}^2 \cdot \text{h} \cdot \Delta x \quad (\text{a})$$

$$x_{\text{AI}} = \frac{C_{\text{AL}}}{C} = \frac{2.6 \times 10^{-3}}{55.55} = 4.68 \times 10^{-5}$$

$$x_A^* = \frac{y_{\text{A}i}}{H'} = \frac{0.002}{33.6} = 5.95 \times 10^{-5}$$

$$N_A = K_x (x_A^* - x_{\text{AI}}) = 7.71 (5.95 - 4.68) \times 10^{-5}$$

$$= 9.79 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{h} \quad (\text{b})$$

$$N_A = k_y (y_{\text{AI}} - y_{\text{AL}})$$

$$9.79 \times 10^{-5} = 10 (y_{\text{AI}} - 4.68 \times 10^{-5})$$

$$y_{\text{AI}} = 5.66 \times 10^{-5} \quad (\text{c})$$

29.15 CONTINUED -

$$y_{\text{AI}} = A' x_{\text{AI}} = 33.6 (5.66 \times 10^{-5})$$

$$= \underline{\underline{1.90 \times 10^{-3}}} \quad (\text{c})$$

FRACTION OF RESISTANCE IN LIQUID &

$$\frac{1/k_x}{1/K_x} = \frac{K_x}{k_x} = \frac{7.71}{10} = \underline{\underline{0.771}}$$

$$= \underline{\underline{77.1\%}} \quad (\text{d})$$

29.16 COMPONENT A - FROM LIQ TO GAS

$$T = 290 \text{ K} \quad P_{\text{AG}} = 4000 \text{ Pa}$$

$$P = 1.013 \times 10^5 \text{ Pa} \quad C_{\text{AL}} = 4 \text{ kg mol/m}^3$$

60% of RESISTANCE IS IN GAS PHASE

$$K_G = 246 \times 10^{-8} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$H = 1400 \text{ Pa/(kg mol/m}^3)$$

$$\frac{1/k_g}{1/K_g} = \frac{K_g}{k_g} = \frac{0.10}{0.6} \quad k_g = \frac{2.46 \times 10^{-8}}{0.6}$$

$$= \underline{\underline{4.1 \times 10^{-8} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}}} \quad (\text{e})$$

$$P_A^* = H C_{\text{AL}} = 1400 (4) = 5600 \text{ Pa}$$

$$N_A = K_g (P_A^* - P_{\text{AG}}) = (2.46 \times 10^{-8})(5600 - 4000)$$

$$= 3.94 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{s}$$

$$= k_g (P_{\text{AI}} - P_{\text{AG}}) = (4.1 \times 10^{-8})(P_{\text{AI}} - 4000)$$

$$P_{\text{AI}} = 4961 \text{ Pa} \quad (\text{f})$$

29.16 (CONTINUED) -

$$C_{Ai} = \frac{P_{Ai}}{H} = \frac{4901}{1400} = 3.54 \text{ kg/mol/m}^3$$

$$N_A = k_L (C_{AL} - C_{Ai})$$

$$3.94 \times 10^{-5} = k_L (4 - 3.54)$$

$$k_L = 8.50 \times 10^{-5} \text{ kg/mol/m}^2 \cdot \text{s} \quad (\text{b})$$

$$N_A = K_L (C_{AL} - C_A^*)$$

$$C_A^* = \frac{P_{A*}}{H} = \frac{4000}{1400} = 2.86$$

$$K_L = \frac{3.94 \times 10^{-5}}{4 - 2.86} \quad (\text{d})$$

$$= 3.46 \times 10^{-5} \text{ kg/mol/m}^2 \cdot \text{s. (kg/mol/m}^3\text{)}$$

29.17 Cl<sub>2</sub> from GAS PHASE INTO WATER

(EQUILIBRIUM DATA FOR THIS SYSTEM GIVEN IN PROB 29.1)

$$T = 293 \text{ K} \quad P_{AG} = 4 \times 10^4 \text{ Pa}$$

$$P = 1.013 \times 10^5 \text{ Pa} \quad C_{AL} = 1 \text{ kg/m}^3$$

75% OF RESISTANCE IS IN LIQUID -

$$\frac{P_{AG} - P_{Ai}}{P_{AG} - P_A^*} = 0.25$$

FROM PLOT OF PROB 29.1 DATA

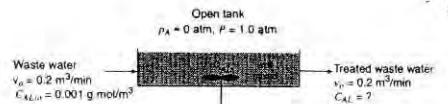
$$P_A^* = 4480 \text{ Pa}$$

$$\frac{40000 - P_{Ai}}{40000 - 4480} = 0.25 \quad P_{Ai} = 3.12 \times 10^4 \text{ Pa}$$

$$\xi \text{ FROM PLOT} \quad - \quad C_{Ai} = 3.0 \text{ kg/m}^3$$

29.18

SYSTEM →



$$\frac{1}{k_L} = \frac{1}{k_L'} + \frac{1}{H \cdot \xi}$$

$$k_L = 0.01 \text{ kg/mol/m}^2 \cdot \text{s. atm}$$

$$H = 10 \text{ atm/(kg/mol/m}^3\text{)}$$

$$k_L = 5 \times 10^4 \text{ (kg/mol/m}^2\text{)}$$

$$\frac{1}{K_L} = \frac{1}{5 \times 10^4} + \frac{1}{(10)(0.01)} = 2.010$$

$$K_L = 4.975 \times 10^{-4} \text{ kg/mol/m}^2 \cdot \text{s. (kg/mol/m}^3\text{)}$$

FRACTION OF RESISTANCE IN LIQUID

$$= \frac{1/k_L}{1/k_L'} = \frac{k_L'}{k_L} = \frac{4.975 \times 10^{-4}}{5 \times 10^4} = \underline{\underline{0.995}}$$

$$= \underline{\underline{99.5\%}}$$

29.19 Tx from Benzene phase to aqueous

$$C_A' = 170 \text{ C}_B'' \quad (\text{a})$$

$$k_L' = 3.5 \times 10^{-6} \text{ kg/mol/m}^2 \cdot \text{s. (kg/mol/m}^3\text{)}$$

$$k_L' = 2.5 \times 10^{-5} \text{ "}$$

$$\frac{1}{K_L'} = \frac{1}{k_L'} + \frac{A}{k_L''} = \frac{1}{3.5 \times 10^{-6}} + \frac{170}{2.5 \times 10^{-5}} \quad (\text{a})$$

$$K_L' = 1.41 \times 10^{-7} \text{ kg/mol/m}^2 \cdot \text{s. (kg/mol/m}^3\text{)}$$

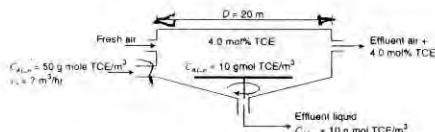
29.19 (CONTINUED -

$$\begin{aligned} \frac{1}{K_L''} &= \frac{1}{k_L'} + \frac{1}{H' k_L'} \\ &= \frac{1}{25 \times 10^{-5}} + \frac{1}{(170)(3.5 \times 10^{-6})} \quad (b) \\ K_L'' &= 2.40 \times 10^{-5} \text{ kg mole/m}^2 \cdot \text{s} \cdot (\text{kg mole/m}^3) \end{aligned}$$

FRACTION OF RESISTANCE IN AQ FILM

$$\begin{aligned} \frac{1/k_L''}{1/K_L''} &= \frac{k_L''}{K_L''} = \frac{2.40 \times 10^{-5}}{250 \times 10^{-5}} = 0.96 \\ &= \underline{\underline{96\%}} \quad (c) \end{aligned}$$

29.20



TCE TRANSFERRED FROM LIQUID TO GAS PHASE -

$$\begin{aligned} T &= 293 \text{ K} & C_{Ain} &= 10 \text{ g mole/m}^3 \\ P &= 1 \text{ ATM} & y_{Ain} &= 0.04 \\ D &= 20 \text{ m} & k_F &= 200 \text{ g mole/m}^2 \cdot \text{s} \\ A &= \frac{\pi}{4}(20)^2 & k_{fg} &= 0.1 \text{ "} \\ &= 314 \text{ m}^2 & H &= 550 \text{ atm/}\Delta x \\ & & C_L &= 66 \text{ g mole/m}^3 \end{aligned}$$

$$P_A = H y_A$$

$$y_A = \frac{P_A}{P} = \frac{H}{P} y_A = H' y_A$$

$$H' = \frac{550}{1} = 550$$

29.20 (CONTINUED -

$$\frac{1}{K_X} = \frac{1}{k_X} + \frac{1}{H' k_Y} = \frac{1}{200} + \frac{1}{(550)(0.1)}$$

$$K_X = 43.10 \text{ g mole/m}^2 \cdot \text{s}$$

$$K_L = \frac{K_X}{C} = \frac{43.10}{66} = \underline{\underline{0.653 \text{ m/s}}} \quad (a)$$

$$N_A = K_X (x_{Ain} - x_A^*)$$

$$x_{Ain} = \frac{C_{Ain}}{C} = \frac{10}{66} = 0.1515$$

$$y_{Ain} = 550 x_A^*$$

$$x_A^* = \frac{0.04}{550} = 7.27 \times 10^{-5}$$

$$N_A = (43.10)(0.1515 - 7.27 \times 10^{-5})$$

$$= \underline{\underline{6.53 \text{ g mole/m}^2 \cdot \text{s}}} \quad (b)$$

$$\begin{aligned} W_A &= N_A \cdot A = 6.53 (314) \\ &= 2050 \text{ g mole/s} \end{aligned}$$

MASS BALANCE FOR LIQUID -

$$\dot{V}_0 C_{Ain} = 2050 (3600) + \dot{V}_0 C_{Aout}$$

$$\dot{V}_0 = \frac{2050 (3600)}{50 - 10}$$

$$= 1.845 \times 10^5 \text{ g mole/h}$$

$$= \frac{1.845 \times 10^5}{66} \text{ m}^3/\text{h}$$

$$= \underline{\underline{2795 \text{ m}^3/\text{h}}} \quad (c)$$

29.21 NH<sub>3</sub> & H<sub>2</sub>S STRIPPED FROM H<sub>2</sub>O

FOR BEMAN -

$$k_G = 3.20 \times 10^9 \text{ kg mol/m}^2 \cdot \text{s. Pa}$$

$$k_L = 1.73 \times 10^9 \text{ kg mol/m}^2 \cdot \text{s. (kg mol/m}^3\text{)}$$

$$H_{NH_3} = 1.36 \times 10^3 \text{ Pa/(kg mol/m}^3\text{)}$$

$$H_{H_2S} = 8.81 \times 10^5 \text{ "}$$

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{1}{k_L}$$

$$NH_3: \frac{1}{K_G} = \frac{1}{3.20 \times 10^9} + \frac{1.36 \times 10^3}{1.73 \times 10^9}$$

$$K_G = 2.556 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s. Pa}$$

$$H_2S: \frac{1}{K_G} = \frac{1}{3.20 \times 10^9} - \frac{8.81 \times 10^5}{1.73 \times 10^9}$$

$$K_G = 1.95 \times 10^{-11} \text{ kg mol/m}^2 \cdot \text{s. Pa}$$

$$\frac{K_G \text{ NH}_3}{K_G \text{ H}_2S} = \frac{2.556}{1.95} = \underline{\underline{131 \text{ TO } 1}}$$

29.22 NH<sub>3</sub> ABSORBED

$$K_G = 3.12 \times 10^9 \text{ kg mol/m}^2 \cdot \text{s. Pa}$$

$$C_{PAI} = 4 \text{ kg mol/m}^3$$

$$\bar{P}_{AG} = 3040 \text{ Pa}$$

$$\bar{P}_{AI} = (1360 \text{ Pa/(kg mol/m}^3\text{)}) C_{AI}$$

75% OF RESISTANCE IS IN THE Q -

29.22 (CONTINUED) -

$$\frac{1/k_A}{1/k_G} = \frac{k_G}{k_A} = 0.75 = \frac{3.12 \times 10^9}{k_A}$$

$$k_A = 4.16 \times 10^9 \text{ kg mol/m}^2 \cdot \text{s. Pa} \quad (a)$$

$$K_L = H K_G = (1360)(3.12 \times 10^9) \quad (c)$$

$$= 4.24 \times 10^{16} \text{ kg mol/m}^2 \cdot \text{s. (kg mol/m}^3\text{)}$$

25% OF RESISTANCE IN LIQUID PHASE

$$0.25 = \frac{K_L}{k_L} \quad k_L = \frac{4.24 \times 10^{16}}{0.25}$$

$$k_L = 16.96 \times 10^{16} \text{ kg mol/m}^2 \cdot \text{s. (kg mol/m}^3\text{)} \quad (b)$$

$$N_A = K_G (\bar{P}_A^* - \bar{P}_{AG})$$

$$\bar{P}_A^* = H C_{AI} = (1360)(4) = 5440 \text{ Pa}$$

$$N_A = (3.12 \times 10^9)(5440 - 3040)$$

$$= 7.488 \times 10^{16} \text{ kg mol/m}^2 \cdot \text{s}$$

$$= k_A (\bar{P}_{AI} - \bar{P}_{AG})$$

$$= (4.16 \times 10^9)(4840 - 3040)$$

$$\bar{P}_{AI} = 4840 \text{ Pa}$$

$$C_{AI} = \frac{\bar{P}_{AI}}{H} = \frac{4840}{1360} = \underline{\underline{3.56 \text{ kg mol/m}^3}} \quad (d)$$

29.23

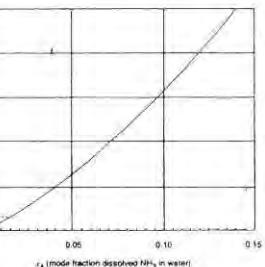
NH<sub>3</sub> REMOVAL

$T = 303 \text{ K}$

$P = 1 \text{ ATM}$

$C_L = 55.6 \text{ kg mol/m}^3$

$X_{AL} = 0.04$



$P_{AG} = 0.2 \text{ ATM}$

$k_G = 1.0 \text{ kg mol/m}^2 \cdot \text{s.ATM}$

$v_L = 0.045 \text{ m/s}$

$$k_x = C k_L = (55.6)(0.045) \\ = 2.50 \text{ kg mol/m}^2 \cdot \text{s.ATM} \quad (a)$$

## VALUES FROM EQUILIBRIUM CURVE

$P_A = 0.02 \text{ ATM} \quad X_A = 0.018$

$$C_A = C X_A = 55.6(0.018) \\ = 1.0 \text{ kg mol/m}^3$$

$$H = P_A/C_A = 0.02/1 \\ = 0.02 \text{ ATM/(kg mol/m}^3)$$

$\frac{1}{Kg} = \frac{1}{k_G} + \frac{H}{k_L} = \frac{1}{1.0} + \frac{0.02}{0.045}$

$K_G = 0.692 \text{ kg mol/m}^2 \cdot \text{s.ATM} \quad (c)$

FRACTION OF RESISTANCE IN GAS  $\phi$ :

$\frac{1/Kg}{1/Kg} = \frac{Kg}{Kg} = \frac{0.692}{1} = 0.692$

29.23 CONTINUED -

FOR OPERATING POINT AT  $X_A = 0.04$ ,

$P_A = 0.2 \rightarrow P_A^* = 0.050 \text{ ATM}$

$\frac{P_{AG} - P_{Ai}}{P_{AG} - P_A^*} = 0.692 = \frac{0.20 - P_{Ai}}{0.20 - 0.05}$

$P_{Ai} = 0.096 \text{ ATM} \quad X_{Ai} = 0.067$

(FPTM CURVE)

$y_{Ai} = \frac{P_{Ai}}{P} = \frac{0.096}{2} = 0.048 \quad (b)$

$N_A = K_G (P_{AG} - P_A^*)$

$= (0.692)(0.20 - 0.05)$

$= 0.104 \text{ kg mol/m}^2 \cdot \text{s} \quad (d)$

29.24 ABSORPTION TOWER - SOURCE (A)  
SOLVENT (B)

$P_{AG} = 1.519 \times 10^4 \text{ Pa}$

$C_{AL} = 1.0 \times 10^3 \text{ kg mol/m}^3$

$N_A = 4 \times 10^5 \text{ kg mol/m}^2 \cdot \text{s}$

$K_G = 3.95 \times 10^9 \text{ kg mol/m}^2 \cdot \text{s. Pa}$

$H = \frac{3.04 \times 10^3}{1 \times 10^{-3}} = 3.04 \times 10^6 \text{ Pa/(kg mol/m}^3)$

$N_A = k_G (P_{AG} - P_{Ai})$

$4 \times 10^5 = (3.95 \times 10^9)(1.519 \times 10^4 - P_{Ai})$

$P_{Ai} = 5070 \text{ Pa}$

$P_{AG} - P_{Ai} = 15190 - 5070 = 10120 \text{ Pa}$

29.24 (CONTINUED)

$$N_A = k_L (C_{Ai} - C_{AL})$$

$$C_{Ai} = \frac{P_{Ai}}{H} = \frac{5.07 \times 10^3}{3.04 \times 10^4}$$

$$= 1.67 \times 10^{-3} \text{ kg mol/m}^3$$

$$4 \times 10^{-5} = k_L (1.67 \times 10^{-3} - 1.0 \times 10^{-3})$$

$$\underline{k_L = 0.0597 \text{ m/s}}$$

$$C_{AL} - C_{AL} = 1.67 \times 10^{-3} - 1.0 \times 10^{-3}$$

$$= 0.67 \times 10^{-3} \text{ kg mol/m}^3$$

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{H}{k_L} = \frac{1}{3.95 \times 10^{-9}} + \frac{3.04 \times 10^4}{0.0597}$$

$$\underline{K_G = 3.29 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}}$$

$$N_A = K_G (P_{AG} - P_A^*)$$

$$P_{AG} - P_A^* = \frac{4 \times 10^{-5}}{3.29 \times 10^{-9}}$$

$$= 1.216 \times 10^4 \text{ Pa}$$

$$\frac{1}{k_L} = \frac{1}{k_G} + \frac{1}{H k_G}$$

$$= \frac{1}{0.0597} + \frac{1}{(3.95 \times 10^{-9})(3.04 \times 10^4)}$$

$$\underline{K_L = 0.010 \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)}$$

29.24 (CONTINUED)

$$N_A = k_L (C_A^* - C_{AL})$$

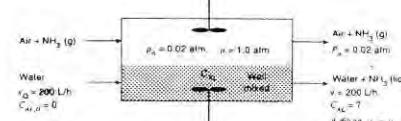
$$C_A^* - C_{AL} = \frac{4 \times 10^{-5}}{0.01} = \underline{4 \times 10^{-3} \text{ kg mol/m}^3}$$

FRACTION OF RESISTANCE IN LIQUID &

$$= \frac{1/k_L}{1/k_L} = \frac{k_L}{k_L} = \frac{0.010}{0.0597} = \underline{0.167} \\ (16.7\%)$$

29.25

NH<sub>3</sub> INTO H<sub>2</sub>O



$$T = 293 \text{ K}$$

$$P = 1 \text{ atm}$$

$$D = 4 \text{ m}$$

$$P_{AG} = 0.02 \text{ atm}$$

$$\dot{V} = 200 \text{ L/h} = 0.2 \text{ m}^3/\text{h}$$

$$H = 0.02 \text{ atm}/(\text{kg mol/m}^3)$$

$$K_G = 1.25 \text{ kg mol/m}^2 \cdot \text{h} \cdot \text{atm}$$

$$k_L = 0.05 \text{ m/h}$$

$$\frac{1}{k_G} = \frac{1}{k_L} + \frac{H}{k_L} = \frac{1}{1.25} + \frac{0.02}{0.05}$$

$$\underline{K_G = 0.833 \text{ kg mol/m}^2 \cdot \text{h} \cdot \text{atm}} \quad (a)$$

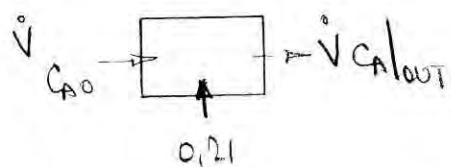
$$\frac{1}{k_G} = \frac{K_G}{k_G} = \frac{0.0833}{1.25} = \frac{P_{AG} - P_{Ai}}{P_{AG} - P_A^*}$$

$$\frac{0.02 - P_{Ai}}{0.02 - 0} = 0.666 \quad \underline{P_{Ai} = 0.0067} \quad (b)$$

29.25 (CONTINUED -

$$\begin{aligned}W_A &= N_A \cdot A_x = K_0 (S_{A0} - p_A^*) A \\&= (0,833)(0,02-0)\left(\frac{\pi}{4}\right)(4)^2 \\&= 0,21 \text{ kg mol/h} \quad (\text{c})\end{aligned}$$

MASS BALANCE:



$$\text{for } \text{NH}_3: \dot{V}(0) + 0,21 = \dot{V} c_{A\text{out}}$$

$$c_{A\text{out}} = \frac{0,21}{0,2} = \underline{1,05 \text{ kg mol/m}^3} \quad (\text{d})$$

## CHAPTER 30

### 30.1 SOLVENT EVAPORATION INTO AIR

$$T_s = 312 \text{ K} \quad P = 1 \text{ atm} \\ T_{\infty} = 293 \text{ K} \quad T_f = 303 \text{ K} \quad P_s = 0.05 \text{ atm} \\ C_{\infty} = 0.001 \text{ mol/cm}^3$$

$$\text{Air @ } 303 \text{ K: } D = 0.158 \text{ cm}^2/\text{s} \\ \rho = 1.17 \times 10^{-3} \text{ g/cm}^3$$

$$Re = \frac{Lu_p}{D} = \frac{(20)(5.0)}{0.158} = 633 \quad \text{(LAMINAR)}$$

$$D_{AB} = 0.1 \left( \frac{303}{298} \right)^{3/2} = 1.025 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.158}{1.025} = 0.154 \quad \text{(a)}$$

$$Sh = \bar{k}_c L = 0.664 Re^{1/2} Sc^{1/3}$$

$$D_{AB} = 9.16 \quad \text{(a)}$$

$$\bar{k}_c = \frac{9.16 (1.025)}{70} = 0.469 \text{ cm/s}$$

$$C = \frac{P}{RT} = \frac{111}{(82.06)(303)} = 4.02 \times 10^{-5} \text{ g/mol/cm}^3$$

$$\bar{k}_c = C \bar{k}_c = (4.02 \times 10^{-5})(0.469) \quad \text{(b)} \\ = 1.89 \times 10^{-5} \text{ g/mol/cm}^2 \cdot \text{s} \cdot \Delta P$$

$$W_A = \bar{k}_c (y_A^* - y_{A\infty}) A_c$$

$$y_A^* = \frac{P_{\infty}}{P} = 0.05$$

$$W_A = (1.89 \times 10^{-5})(0.05 - 0)(20 \times 10) \\ = 1.89 \times 10^{-4} \text{ g/mol/s}$$

### 30.1 (CONTINUED)

$$\text{Moles of Solvent} = PV$$

$$= (0.001)(20)(10)(0.01) = 0.002$$

$$t = \frac{0.002}{1.89 \times 10^{-4}} = 10,58 \text{ s} \quad \text{(c)}$$

### 30.2 NAPHTHALENE SUBLIMING INTO AIR

$$T_s = 290 \text{ K} \quad T_f = 300 \text{ K} \quad P_A^0 = 26 \text{ Pa} \\ T_{\infty} = 310 \text{ K} \quad @ 290 \text{ K}$$

$$D = 1.569 \times 10^{-5} \text{ m}^2/\text{s} \quad V = 20 \text{ m/s}$$

$$D_{AB} = 5.61 \times 10^{-6} \left( \frac{300}{290} \right)^{3/2} = 5.90 \times 10^{-6} \text{ m}^2/\text{s}$$

$$At \ x = 3 \text{ m} \quad Re = \frac{0.3(20)}{1.569 \times 10^{-5}} = 382 \times 10^5$$

$$Sc = \frac{1.569 \times 10^{-5}}{5.90 \times 10^{-6}} = 266$$

$$\bar{k}_{c4} = \frac{D_{AB}}{x} Re^{4/5} Sc^{1/3}$$

$$= \frac{5.90 \times 10^{-6}}{0.3} (382 \times 10^5)^{4/5} (2.66)^{1/3}$$

$$= 0.0232 \text{ m/s} \quad \text{(d)}$$

From 0.5m < x < 0.75m

$$\bar{k}_c = \frac{0.0365 D_{AB} Sc^{1/3}}{0.75 - 0.5} \left[ Re_{0.75}^{4/5} - Re_{0.5}^{4/5} \right]$$

$$Re_{0.75} = \frac{0.75(20)}{1.569 \times 10^{-5}} = 9.56 \times 10^5$$

$$Re_{0.5} = \frac{0.5(20)}{1.569 \times 10^{-5}} = 6.31 \times 10^5$$

30.2 CONTINUED-

SUBSTITUTING VALUES:

$$\bar{k}_c = 0.020 \text{ m/s}$$

$$W_A = N_A A = \bar{k}_c (C_{AS} - C_{AR}) A$$

$$C_{AS} = \frac{P}{RT} = \frac{26}{(8.314)(289)} = 0.0108 \text{ mol/m}^3$$

$$C_{AR} = 0$$

$$W_A = (0.020)(0.0108)(0.25)(1) \\ = 5.4 \times 10^{-5} \text{ mol/s} \\ = 0.1944 \text{ mol/h}$$

30.3 C<sub>2</sub>H<sub>5</sub>OH INTO AIR

$$T_f = \frac{289 + 303}{2} = 296 \text{ K} \quad P_{AO} = 6.45 \times 10^{-2} \text{ atm}$$

$$D_{AB} = 1.32 \times 10^{-5} \text{ m}^2/\text{s}$$

$$N = 1.53 \times 10^{-5}$$

$$Re_L = \frac{U_{AO} L}{\nu} = \frac{(3)(2)}{1.53 \times 10^{-5}} = 3.92 \times 10^5$$

TAKING FLOW AS LAMINAR FOR  $Re \leq 2 \times 10^5$   
OR TURBULENT FOR  $Re > 2 \times 10^5$

$$\bar{k}_c = \frac{D_{AB}}{L} \left[ 0.004 Re_L^{1/2} Sc^{1/3} + 0.0365 Sc^{1/3} (Re_L^{4/5} - Re_L^{1/5}) \right]$$

$$Re_L = 2 \times 10^5 \quad Re_L = 3.92 \times 10^5$$

SUBSTITUTING VALUES:

$$\bar{k}_c = 5.16 \times 10^{-3} \text{ m/s}$$

30.3 CONTINUED-

$$W_A = N_A A = \bar{k}_c (C_{AS} - C_{AR}) A$$

$$C_{AS} = \frac{P}{RT} = \frac{(6.45 \times 10^{-2})(1.03 \times 10^5)}{(8.314)(289)} = 2.72 \text{ mol/m}^3$$

$$W_A = (5.16 \times 10^{-3})(2.72 - 0)(2 \times 4) \\ = 0.112 \text{ mol/s} \\ = 0.112(46) = 5.15 \text{ g/s}$$

30.4 MOLECULAR DIFFUSION THROUGH GRAVEL - THEN CONVECTIVE TRANSFER TO AIR -

$$T = 288 \text{ K} \quad U = 2 \text{ cm/s}$$

$$P_A = 1039 \text{ Pa} \quad L = 10 \text{ m} \quad \text{GRAVEL DEPTH} = 1 \text{ m}$$

$$\text{THROUGH GRAVEL} \quad N_A = \frac{D_{AB}}{S} (C_{A1} - C_{A2})$$

$$\text{AT SURFACE} \quad N_A = \bar{k}_c (C_{A2} - C_{AR})$$

$$Re_L = \frac{LU_{AO}}{\nu} = \frac{(10)(0.02)}{1.46 \times 10^{-5}} = 1.37 \times 10^6$$

$$\bar{k}_c = \frac{D_{AB}}{L} (0.004) Re_L^{1/2} Sc^{1/3}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.46 \times 10^{-5}}{5.72 \times 10^{-6}} = 2.55$$

$$\bar{k}_c = \frac{5.72 \times 10^{-6}}{10} (0.004) (1.37 \times 10^6)^{1/2} 2.55^{1/3}$$

$$= 6.07 \times 10^{-5} \text{ m/s}$$

$$C_{A1} = \frac{P}{RT} = \frac{1039}{(8.314)(288)} = 0.434 \text{ mol/m}^3$$

30.4 CONTINUED -

AT STEADY STATE -

$$N_A = \frac{D_{AB}}{\delta} (C_{A1} - C_{A2}) = \bar{k}_c (C_{A2} - 0)$$

$$\frac{5.72 \times 10^{-6}}{1} (0.434 - C_{A2}) = 6.07 \times 10^{-5} C_{A2}$$

$$C_{A2} = 0.0374 \text{ mol/m}^3$$

$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{(8.314)(288)} = 42.31 \text{ mol/m}^3$$

$$y_{A2} = \frac{0.0374}{42.31} = 8.84 \times 10^{-4} \quad (\text{a})$$

$$N_A = (6.07 \times 10^{-5})(0.0374) = 2.27 \times 10^{-6} \text{ mol/m}^2\text{s}$$

FOR SAME CONFIGURATION & PROCESS

BUT  $V_B = 50 \text{ cm/s}$

$$Re_L = \frac{(40)(50)}{1.416 \times 10^{-5}} = 3.42 \times 10^5$$

{ INTO TURBULENT  
FLOW REGIME }

FOR  $Re \leq 2 \times 10^5$  LAMINAR B.L.

$Re > "$  TURBULENT "

$$\bar{k}_c = \frac{D_{AB} S_c^{1/3}}{L} \left[ 0.064 Re_{tr}^{1/2} + 0.0365 (Re_L^{4/5} - Re_{tr}^{4/5}) \right]$$

$$Re_{tr} = 2 \times 10^5$$

$$Re_L = 3.42 \times 10^5$$

SUBSTITUTING & SOLVING:

$$\bar{k}_c = 4.98 \times 10^{-4} \text{ m/s}$$

30.4 CONTINUED -

$$\frac{5.72 \times 10^{-6}}{1} (C_{A1} - C_{A2}) = 4.98 \times 10^{-4} (C_{A2} - 0)$$

$$C_{A2} = 4.93 \times 10^{-3} \text{ mol/m}^3$$

$$y_{A2} = \frac{4.93 \times 10^{-3}}{42.31} = \underline{\underline{1.16 \times 10^{-4}}} \quad (\text{b})$$

$$N_A = (4.98 \times 10^{-4})(4.93 \times 10^{-3}) = 2.45 \times 10^{-6} \text{ mol/m}^2\text{s}$$

MEAN TX BIOT NO. =  $\frac{\bar{k}_c \delta}{D_{AB}}$

$$= \frac{(6.07 \times 10^{-5})(1)}{5.72 \times 10^{-6}} = 10.61 \quad \{ \text{Case (a)} \}$$

$$\frac{1/\bar{k}_c}{1/D_{AB}/\delta} = \frac{1}{10.61} = 0.094$$

- 9.4% RESISTANCE IN FLOWING STREAM

$$\text{for Case (b)} - Bi = \frac{\bar{k}_c \delta}{D_{AB}} = 87.06$$

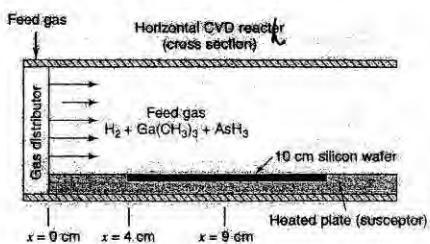
$$\frac{1}{87.06} = 0.0115$$

- 1.15% RESISTANCE IN AIR STREAM

30.5 REFER TO CHAPTER - EXAMPLE 1 - FOR PROBLEM SPECIFICATIONS -

ACSIINE (A)

TMB (B)



$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{(8.314)(800)} = 15.23 \text{ mol/m}^3$$

30.5 CONTINUED -

$$C_{A\text{pp}} = C_{B\text{pp}} = 0.001(15.23) \\ = 0.0152 \text{ mol/m}^3$$

FOR TM6 - 1800 K

$$\begin{aligned} D_{A2} &= 5.686 \text{ cm}^2/\text{s} \\ D_{AB} &= 1.65 \text{ cm}^2/\text{s} \end{aligned} \quad \left. \begin{aligned} Sc &= 3.67 \\ \end{aligned} \right\}$$

AT  $x = 4 \text{ cm}$   $Sh_x = k_c = N_A = 0$

$x = 9 \text{ cm}$  - SEE EXAMPLE 1 -

$$\begin{aligned} Sh_x &= 8.375 \quad k_c = 0.0144 \text{ m/s} \\ N_B &= 0.0144(0.0152) \\ &= 2.19 \times 10^{-4} \text{ mol B/m}^2 \cdot \text{s} \end{aligned}$$

$x = 14 \text{ cm}$

$$Re_x = \frac{V_{Dx}x}{D} = \frac{(100)(14)}{5.686} = 2416$$

$$k_{CB} = \frac{D_{AB}}{x} \left[ 0.332 Re_x^{1/2} \left( \frac{Sc}{1 + [x]^{1/4}} \right)^{1/3} \right]$$

$x/x = 1/4$  - OTHER QUANTITIES KNOWN -

SUBSTITUTING  $\frac{1}{4}$  FOR UNKNOWN:

$$k_c = 0.0144 \text{ m/s}$$

$$N_B = (0.0144)(0.0152) = 1.59 \times 10^{-4} \text{ mol/m}^2 \cdot \text{s}$$

For A:

AT  $x = 4 \text{ cm}$   $Sh_x = k_c = N_A = 0$

30.5 CONTINUED -

$$x = 9 \text{ cm} \quad Re_x = \frac{(100)(9)}{5.686} = 158.3$$

$$k_{CA} = \frac{3.17}{9} \left[ 0.332(158.3) \left( \frac{1.784}{1 - (4/9)^{1/4}} \right)^{1/3} \right] \\ = 0.0232 \text{ m/s}$$

$$N_A = (0.0232)(0.0152) = \underline{\underline{3.53 \times 10^{-4} \text{ mol/m}^2 \cdot \text{s}}}$$

AT  $14 \text{ cm}$   $Re_x = \frac{(100)(14)}{5.686} = 2416$

SAME FORMULA BUT  $x = 14 \quad Re = 2416$

$$k_{CA} = 0.0169 \text{ m/s}$$

$$N_A = (0.0169)(0.0152) = \underline{\underline{2.57 \times 10^{-4} \text{ mol/m}^2 \cdot \text{s}}}$$

TO PRODUCE  $\frac{N_A}{N_B} = 1 \Rightarrow \frac{k_{CA}(C_{AS} - C_{AP})}{k_{CB}(C_{BS} - C_{BP})} = 1$

@ 9 cm:  $\frac{k_{CA}}{k_{CB}} = \frac{0.0232}{0.0144} \frac{\Delta C_A}{\Delta C_B}$

$$= 1 \quad \text{IF} \quad \frac{\Delta C_A}{\Delta C_B} = 0.162$$

@ 14 cm:  $\frac{\Delta C_A}{\Delta C_B} = \frac{k_{CA}}{k_{CB}} = \frac{0.0169}{0.0144} = 0.162$

FOR BOTH CASES -

$$\Delta C_B = C_{BS} - C_{BP} = 0.0152 - C_{BP}$$

$$\Delta C_A = C_{AS} - C_{AP} = 0.0152$$

$$\text{SO } C_{BP} \text{ SHOULD BE } \frac{0.0152 - C_{BP}}{0.0152} = 0.62$$

$$\text{OR } C_B = 0.00578 \text{ mol/m}^3$$

### 30.5 (CONTINUED) -

THICKNESS OF GAFS FILM AFTER 120S

$$- @ x = 4 \text{ cm} \quad \delta = 0$$

$$@ x = 9 \text{ cm}$$

GAFS DEPOSITED -

$$= (2.19 \times 10^{-4}) (144) (120) = 378 \text{ g/m}^2$$

$$P_s = 5.8 (180)^3 = 5.8 \times 10^6 \text{ g/m}^2$$

$$\delta = \frac{3.78}{5.68 \times 10^6} = 0.652 \times 10^{-6} \text{ m}$$

$\sim 0.652 \mu\text{m}$

AT  $x = 14 \text{ cm}$

$$\delta = \frac{(1.59 \times 10^{-4})(144)(120)}{5.68 \times 10^6}$$

$$= 0.474 \times 10^{-6} \text{ m} = \underline{\underline{0.474 \mu\text{m}}}$$

### 30.6 MASS TRANSFER FROM SPHERICAL SURFACE -

$$D = 1 \text{ cm}$$

$$P_A^0 = 1.17 \times 10^4 \text{ Pa}$$

$$T = 298 \text{ K}$$

$$M_A = 78$$

$$\bar{P} = 1 \text{ atm}$$

$$D_{AB} = 0.0962 \text{ cm}^2/\text{s}$$

$$\text{Mass of Solvent} = (\rho, 12 \text{ g/cm}^3) A$$

$$= 0.12(\pi)(1)^2 = 0.377 \text{ g}$$

$$= 3.77 \times 10^{-4} \text{ kg}$$

$$W_A = N_A A = k_C (C_{AS} - C_{AR}) \pi D^2$$

$$D = \frac{\mu}{P} = \frac{1.85 \times 10^{-4}}{1.18 \times 10^{-3}} = 0.1568 \text{ cm}^2/\text{s}$$

$$D_{AB} = 0.0962 \text{ cm}^2/\text{s}$$

### 30.6 (CONTINUED) -

$$Sc = \frac{D}{D_{AB}} = \frac{0.1568}{0.0962} = 1.63$$

$$\text{IN STAGNANT AIR} - \frac{k_c D}{D_{AB}} = 2$$

$$k_C = \frac{2(0.0962)}{1} = 0.1924 \text{ cm/s}$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{1.17 \times 10^4}{(8.314)(298)} = 4.72 \text{ mol/m}^3$$

$$W = (0.1924)(4.72 - 0) \pi (1)(18)(10^{-6})$$

$$= 2.225 \times 10^{-4} \text{ g/s} = 2.225 \times 10^{-7} \text{ kg/s}$$

$$t = \frac{3.77 \times 10^{-4}}{2.225 \times 10^{-7}} = \frac{1694 \text{ s}}{0.471 \text{ h}} \quad (a)$$

for  $U_p = 1 \text{ m/s}$

$$Re = \frac{1(100)}{0.1568} = 638$$

$$Sh = \frac{k_c D}{D_{AB}} = 2 + 0.552 Re^{1/2} Sc^{1/3}$$

$$k_C = \frac{D_{AB}}{D} \left( \frac{Re}{Sc} \right)^{1/2} = 1.77 \text{ cm/s}$$

$$W_A = 2.225 \times 10^{-4} \left( \frac{1.77}{0.1924} \right)$$

$$= 20.43 \times 10^{-7} \text{ kg/s}$$

$$t = \frac{3.77 \times 10^{-4}}{20.43 \times 10^{-7}} = \underline{\underline{184 \text{ s}}} \quad (b)$$

30.7 A DIFFUSING THROUGH STAGNANT B  $\sim N_B = 0$

$$N_{Ar} = - \frac{CD_{AB}}{1-y_A} \frac{dy_A}{dr}$$

$$\nabla^2 N_A = \frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = 0$$

$$\sim r^2 N_{Ar} = \text{CONSTANT}$$

$$N_{Ar} R \int_0^R \frac{dy_A}{r^2} = C D_{AB} \left( \frac{y_{Ar}}{1-y_A} - \frac{y_{Ar}}{y_{AS}} \right)$$

$$N_{Ar} R^2 \left[ \frac{1}{R} - 0 \right] = C D_{AB} \ln \left( \frac{1-y_{Ar}}{1-y_{AS}} \right)$$

$$N_{Ar} R = \frac{C D_{AB}}{y_{BLM.}} (y_{AS} - y_{Ar0})$$

$$N_{Ar} \frac{D}{2} = \frac{C D_{AB}}{y_{BLM.}} (y_{AS} - y_{Ar0})$$

$$N_{Ar} = \frac{2 D_{AB}}{y_{BLM.}} (C_{AS} - C_{Ar0})$$

$$\Rightarrow k_c = \frac{2 D_{AB}}{(y_{BLM.}) D}$$

$\frac{1}{2}$  for  $y_{BLM.} \approx 1$  (DILUTE SOLN OF A)

$$Sh = \frac{k_c D}{D_{AB}} = 2$$

30.8 SPHERICAL PENET IN CROSSFLOW

$$T = 293 K \quad D = 9.95 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$D = 1 \text{ cm} \quad D_{AB} = 1.2 \times 10^{-5} "$$

$$U_p = 5 \text{ cm/s} \quad C_{Ar0} = 0$$

30.8 CONTINUED -

$$\frac{k_c D}{D_{AB}} = 2.0 + 0.552 Re^{1/2} Sc^{1/3}$$

$$Re = \frac{DU_p}{\nu} = \frac{(1)(5)}{9.95 \times 10^{-3}} = 502$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{9.95 \times 10^{-3}}{1.2 \times 10^{-5}} = 9.37$$

Solving for  $k_c$ :  $k_c = 0.00141 \text{ cm/s}$  (2)

$$V = \frac{\pi D^3}{6} \quad \frac{dN}{dt} = 2\pi D^2 \frac{dI}{dt}$$

$$W_A = \frac{\rho_A}{M_A} \frac{dN}{dt} = k_c (C_{AS} - C_{Ar0}) \pi D^2$$

$$\frac{\rho_A}{M_A} 2\pi D^2 \frac{dI}{dt} = \pi D^2 k_c C_{AS}$$

$$\frac{dI}{dt} = \frac{(0.00141)(7 \times 10^{-4})(110)}{2(2)}$$

$$= 2.71 \times 10^{-5} \text{ cm/s} = 0.0977 \text{ cm/h}$$

$$\frac{df}{dt} = 0.105 \text{ cm/h}$$

for  $D = 0.5 \text{ cm}$

$$Re_p = 251 \quad k_c = 2.0 + 0.552 Re^{1/2} Sc^{1/3} \\ = 0.00201 \text{ cm/s}$$

$$\frac{W_A|_{1.0}}{W_A|_{0.5}} = \frac{0.00141}{0.00201} \frac{C_{AS} \pi (1)^2}{C_{AS} \pi (0.5)^2}$$

$$= 2.804 \quad (c) \quad \left\{ \begin{array}{l} \text{INCREASE BY} \\ \text{THIS FACTOR} \end{array} \right\}$$

30.9 Glucose (A) INTO AQUEOUS STREAM 30.10  $\text{C}_2\text{H}_5\text{OH}$  (A) INTO LIQUID (B)

$$T = 298 \text{ K}$$

$$V = 0.15 \text{ m/s}$$

$$D_{AB} = 6.9 \times 10^{-10} \text{ m}^2/\text{s}$$

$$D = 0.3 \text{ cm}$$

(SPACER)

FOR  $T_x$  INTO A LIQUID STREAM:

$$Re_D = \frac{DV}{\nu}$$

$$\nu = \frac{0.00091 \text{ kg/m.s}}{997 \text{ kg/m}^3} = 9.127 \times 10^{-7} \text{ m}^2/\text{s}$$

$$Re = \frac{(0.003)(0.15)}{9.127 \times 10^{-7}} = 493$$

$$Sc = \frac{9.127 \times 10^{-7}}{6.9 \times 10^{-10}} = 1322$$

$$Pe = Re Sc = 6521 \times 10^5$$

EQN. (30-8) APPLIES

$$k_L D = 1.01 \frac{1}{Re} \frac{1}{Sc}$$

$$D_{AB}$$

$$k_L = \frac{1.01 (6521 \times 10^5)^{1/3}}{0.003} \frac{1}{6.9 \times 10^{-10}} \\ = 1.20 \times 10^5 \text{ m/s}$$

$$k_L \sim \frac{1}{D} \left( D^{1/3} V^{1/3} \right) \sim \frac{V^{1/3}}{D^{2/3}}$$

FOR  $D$  INCREASING  $k_L$  DECREASES

FOR  $V$  " -  $k_L$  INCREASES

LARGER EFFECT IS  $\Delta D$  (b)

BUBBLE  $D = 0.002 \text{ m}$

$$\rho_B = 1.47 \text{ g/cm}^3$$

$$\mu_B = 5.2 \times 10^{-4} \text{ kg/m.s}$$

$$D_{AB} = 5.6 \times 10^{-5} \text{ cm}^2/\text{s}$$

$$H = 6.76 \text{ atm}/\Delta x_A$$

FOR  $D < 2.5 \text{ mm}$  - Eqn (30-14a) APPLIES -

$$k_C = \frac{D_{AB}}{D} \left[ 0.131 \left( \frac{\rho_L}{\rho_B} \right)^{1/3} Sc^{1/3} \right]$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{5.2 \times 10^{-4}}{(1.47 \times 10^3)(5.6 \times 10^{-5})}$$

$$= 63.2$$

$$G_r = \frac{D^2 \rho_L (\rho_L - \rho_g) g}{\mu_L^2}$$

$$\rho_g = \frac{PM}{RT} = \frac{(1.013 \times 10^5)(71)}{(8.314)(298)} = 2.9 \text{ kg/m}^3$$

$$\rho_L = 1470 \text{ kg/m}^3$$

SUBSTITUTING VALUES

$$k = 1.295 \times 10^4 \text{ m/s}$$

$$N_A = k_C (C_{AS} - C_{AR})^0$$

$$C_{AS} = X_A C_L = \frac{P_A}{H} C_L = \frac{1}{676} C_L$$

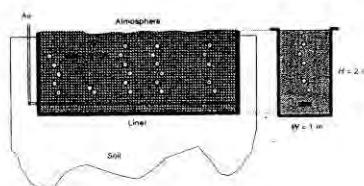
$$C_L = \frac{(1470)(1000)}{169.8} = 8660 \text{ mol/m}^3$$

$$N_A = (1.295 \times 10^4) \left( \frac{8660}{676} \right)$$

$$= 0.378 \text{ mol/m}^2 \text{ s}$$

30.11

TCE (A)

BEVELY  
STRIPPER -

MASS BALANCE FOR A:

$$\left\{ \begin{array}{l} \text{(RATE OF Tx)} \\ \text{from H}_2\text{O} \end{array} \right\} = \left\{ \begin{array}{l} \text{(RATE OF DEPLETION)} \\ \text{IN A}_2\text{O PHASE} \end{array} \right\}$$

$$N_A A_i = - \frac{dC_A}{dt} \quad \left\{ \text{PER m}^3 \right\}$$

$$K_L A_i C_A = - \frac{dC_A}{dt}$$

FOR LIQUID PHASE Tx CONTROLLING

$$k_L \approx K_L$$

$$-\frac{dC_A}{dt} = k_L A_i C_A$$

$$-\int_{C_A^0}^{C_A} \frac{dC_A}{C_A} = k_L A_i \int_0^t dt$$

$$\ln\left(\frac{C_A^0}{C_A}\right) = k_L A_i t \quad (a)$$

$$T = 293 K$$

$$M_A = 131.4$$

$$\mu_L = 9.93 \times 10^{-4} \text{ kg/m.s}$$

$$\rho_L = 998.2 \text{ kg/m}^3 \quad D = 9.95 \times 10^{-7} \text{ m}^2/\text{s}$$

$$\rho_G = 1.19 \text{ kg/m}^3$$

$$H = 997 \text{ Atm/(kg.m)}^2/\text{m}^3$$

$$D_{AB} = 8.9 \times 10^{-10} \text{ m}^2/\text{s}$$

$$Sc = \frac{9.95 \times 10^{-7}}{8.9 \times 10^{-10}} = 1117$$

$$\text{BUBBLE DIAM} \approx 0.005 \text{ m}$$

30.11 CONTINUED -

$$A_i (\text{in}^2) = 0.015 \frac{\text{m}^3 \text{ AIR}}{\text{m}^3 \text{ H}_2\text{O}} \frac{6}{0.005} \frac{\text{m}^2}{\text{m}^3} = 18 \text{ m}^2/\text{m}^3$$

EQUATION (30-14b) APPLIES:

$$k_L = \frac{D_{AB}}{dp} (0.42) Gr^{\frac{1}{3}} Sc^{\frac{1}{2}}$$

$$Gr = \frac{dp^3 \rho_L g (\rho_L - \rho_G)}{dp^2}$$

SUBSTITUTING VALUES -

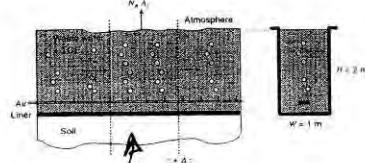
$$k_L = 2.682 \times 10^{-4} \text{ m/s}$$

$$k_L A_i = (2.682 \times 10^{-4})(18) = 0.00482 \text{ s}^{-1}$$

$$\ln\left(\frac{50}{0.005}\right) = 0.00482 t$$

$$\underline{t = 1911 \text{ s}}$$

30.12

SAME SYSTEM AS  
IN PROB 13.11

MASS BALANCE FOR C.N. {CONSTITUENT A}

$$C_A V_{1/2} - C_A V_{1/2,eq} = N_A A_i A_{eq}$$

DIVIDE BY A<sub>eq</sub> & EVALUATE IN LIMIT  $\Delta z \rightarrow 0$ 

$$-\frac{V}{\Delta z} \frac{dC_A}{dx} = K_L A_i (C_A - C_A^*) \quad (a)$$

$$C_A^* = \frac{p_A}{H} = 0 \quad K_L = K_L - \left\{ \begin{array}{l} \text{LIQUID} \\ \text{PHASE} \\ \text{CONTROLS} \end{array} \right\}$$

30.12 (CONTINUED) -

$$-\int_{C_0}^{C_A} \frac{dC_A}{C_A} = \frac{k_L A_i}{V} \int_0^L dz$$

$A_i = 18 \text{ m}^2/\text{m}^3$  {SEE SOLN TO PROB 30.11}

$$\ln \frac{C_A}{C_0} = k_L V A_i L$$

$$k_L = 2.682 \times 10^{-4} \text{ m/s}$$

{SEE PROB 30.11}  
FOR DETAILS

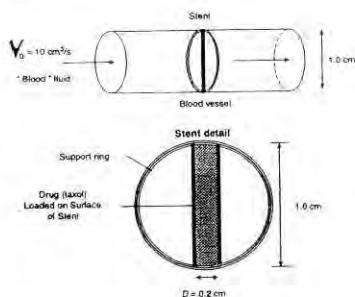
SUBSTITUTE VALUES & SOLVE -

$$L = 191.5 \text{ m}$$

30.13

THICKNESS OF COATING = 0.01 CM

MASS OF COATING  
= 5 mg



$$\mu_f = 0.040 \text{ g/cm.s} \quad \dot{V} = 10 \text{ cm}^3/\text{s}$$

$$\rho_s = 1.05 \text{ g/cm}^3$$

$$D_{AB} = 1 \times 10^{-6} \text{ cm}^2/\text{s} \quad M = 18$$

$$V = \frac{\dot{V}}{A} = \frac{10}{\pi/4(1)^2} = 12.73 \text{ cm/s}$$

for A SINKING CYLINDER - EQUATION (30-16) -

$$\frac{k_L S_C}{V} = 0.281 R_e^{-0.4}$$

30.13 (CONTINUED)

$$S_C = \frac{0.040}{(1.05)(1 \times 10^{-6})} = 3.8 \times 10^4$$

$$R_e = \frac{(0.12)(12.73)(1.05)}{0.04} = 66.8 \quad (a)$$

SUBSTITUTING VALUES:  $k_L = 0.00181 \text{ cm/s}$

$$W = N_A A = k_L (C_A^* - C_{AB})(\pi D L)$$

$$C_A^* = 2.5 \times 10^{-4} \text{ mg/cm}^3$$

$$W = (0.00181)(2.5 \times 10^{-4})(\pi)(0.2)(1)$$

$$= 2.84 \times 10^{-7} \text{ mg/s}$$

$$t = \frac{5}{2.84 \times 10^{-7}} = 1.76 \times 10^7 \text{ s}$$

$$= 4890 \text{ h}$$

$$\approx 204 \text{ DAYS}$$

30.14

$$P_A = 428 \text{ Pa}$$

$$M_A = 106$$

$$T_f = \frac{298 + 313}{2} = 305.5 \text{ K}$$

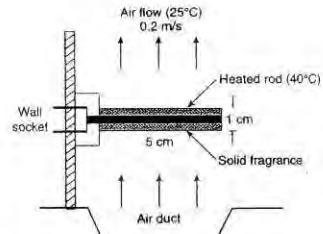
$$\rho_{air} = 1.156 \text{ kg/m}^3$$

$$D_{AB} = \left(0.08 \left(\frac{305.5}{313}\right)^{3/2}\right) = 0.077 \text{ cm}^2/\text{s}$$

$$\text{INITIALLY: } Re = \frac{(1)(20)}{0.01621} = 1234$$

$$S_C = \frac{2}{D_{AB}} = \frac{0.01621}{0.077} = 2.104$$

EQUATION (30-16) APPLIES



30.14 CONTINUED -

$$k_0 \frac{P_{Sc}^{0.56}}{6m} = 0.281 Re^{-0.4}$$

$$P = 1.013 \times 10^5 \text{ Pa}$$

$$G_m = \frac{g}{M} = \frac{(1.156)(0.2)}{29}$$

$$= 0.00797 \text{ kg mol/m}^2\text{s}$$

SUBSTITUTING VALUES

$$k_0 = 2.12 \times 10^{-6} \text{ kg mol/m}^2\text{s Pa}$$

$$N_A = k_0 A (P_{AS} - P_{Ar})^0 \\ = (2.12 \times 10^{-6})(4.28) = 9.07 \times 10^{-7} \text{ kg mol/m}^2\text{s}$$

$$W_A = N_A A$$

$$= (9.07 \times 10^{-7})(\pi)(0.0)(0.05)$$

$$= 1.425 \times 10^{-9} \text{ kg mol/s}$$

$$= (1.425 \times 10^{-9})(100) = 1.51 \times 10^{-7} \text{ kg/s}$$

$$= 0.544 \text{ g/h} \quad (\text{a})$$

WHEN A IS DEPLETED

$$D = 0.5 \text{ cm} \sim Re_D = 61.7$$

same procedure as in part (a)

NEW VALUES:

$$k_0 = 2.79 \times 10^{-6} \text{ kg mol/m}^2\text{s Pa}$$

$$N_A = 1.19 \times 10^{-6} \text{ kg mol/m}^2\text{s}$$

30.14 CONTINUED -

$$W_A = (1.19 \times 10^{-6})(\pi)(0.005)(0.05) \\ = 9.35 \times 10^{-10} \text{ kg mol/s}$$

$$W_{A,avg} = \frac{(1.425 + 0.935) \times 10^{-9}}{2}$$

$$= 1.18 \times 10^{-9} \text{ kg mol/s}$$

TOTAL MASS OF A DEPLETED -

$$m = \frac{\pi}{4} (D_i^2 - D_f^2) (1.1)(0.4) \frac{10^6}{10^6}$$

{ 0.4 IS FRACTION OF A IN SOLID }

$$= 0.0122 \text{ g mol}$$

$$t = \frac{0.0122}{(1.18 \times 10^{-9})(1000)} = \frac{10340 \text{ s}}{1.27 \text{ h}}$$

30.15 CONSTITUENT A INTO WATER (B)

CYLINDRICAL FILM -  $D_i = 1.8 \text{ cm}$

$$D_o = 2.0 \text{ "}$$

$$T = 293 \text{ K}$$

$$\dot{V}_{H_2O} = 314 \text{ cm}^3/\text{s} \quad U = \frac{314}{\pi/4 (1.8)^2} = 123 \text{ cm/s}$$

$$\dot{N}_{H_2O} = 0.00995 \text{ cm}^2/\text{s}$$

$$D_{AB} = 1.2 \times 10^{-9} \text{ m}^2/\text{s}$$

$$Sc = \frac{0.00995}{(1.2 \times 10^{-9})(100)} = 829$$

$$Re = \frac{(1.8)(123)}{0.00995} = 22,250$$

30.15 (CONTINUED -

EQN (30-18) APPLIES

$$k_L = \frac{D_{AB}}{D} (0.023) Re^{0.83} Sc^{1/3}$$

SUBSTITUTING VALUES:  $k_L = 0.00583 \text{ cm/s}$

WHEN SCALE HAS BEEN REMOVED

$$D = 2 \text{ cm}$$

$$V = \frac{\pi/4(2)^2}{\pi/4(2)^2} = 100 \text{ cm/s}$$

SAME PROCEDURE AS ABOVE -

NEW VALUES:

$$Re = 20,100$$

$$k_L = 0.00482 \text{ cm/s}$$

$$k_{L, \text{AVG}} = \frac{0.00583 + 0.00482}{2} \\ = 0.005325 \text{ cm/s}$$

$$W_A = k_L (C_{AS} - C_0) (\pi D L)$$

$$= (0.005325)(0.14 \times 10^{-6})(\pi)(2)(100) \\ = 4.68 \times 10^{-7} \text{ g mol/s}$$

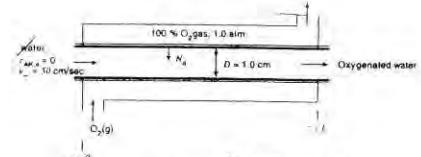
MASS OF  $\text{CaCO}_3$  REMOVED -

$$m = \rho V = \frac{(2.7)(\pi)}{100} (2^2 - 1.8^2)(100) \\ = 1.611 \text{ g mol}$$

$$t = \frac{1.611}{4.68 \times 10^{-7}} = 3.44 \times 10^6 \text{ s} \\ = 956 \text{ h}$$

30.16

$\text{O}_2$  INTO  $\text{H}_2\text{O}$ :



$$T = 298 \text{ K}$$

$$P = 1 \text{ ATM}$$

$$D = 9.12 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$D_{AB} = 2.10 \times 10^{-5} \text{ " } \quad V = 50 \text{ cm/s}$$

$$L = 500 \text{ cm}$$

$$H = 0.78 \text{ atm/(mol/m}^3)$$

FOR MASS TR FROM CYLINDRICAL

INTERFACE IN A PIPE:



MASS BALANCE FOR C.N. YIELDS

$$\ln \frac{C_{AS} - C_{AO}}{C_{AS} - C_{AL}} = \frac{4L}{D} \frac{k_L}{V}$$

$$Sc = \frac{D}{D_{AB}} = \frac{9.12 \times 10^{-3}}{2.10 \times 10^{-5}} = 434 \quad (a)$$

$$Re = \frac{1(50)}{9.12 \times 10^{-3}} = 5482$$

$$\text{EQN (30-18)} \quad Sh = 0.023 Re^{0.83} Sc^{1/3}$$

SUBSTITUTING VALUES -  $Sh = 220$  (a)

$$k_L = \frac{D_{AB} Sh}{D} = \frac{2.10 \times 10^{-5}}{1}(220) \\ = 0.00463 \text{ cm/s}$$

$$4L \frac{k_L}{D} \frac{1}{V} = 4 \frac{(500)(0.00463)}{1 \quad 50} = 0.185$$

$$\ln \left[ \frac{1.28 - 0}{1.28 - C_{AL}} \right] = 0.185$$

$$C_{AL} = 0.213 \text{ mol/m}^3$$

30.16 (CONTINUED)

FOR  $C_{AL} = 0.6 \text{ CAS}$

$$\ln \frac{C_{AS}}{C_{AS} - 0.6 C_{AS}} = \ln 2.5 = 0.916$$

$$0.916 = \frac{4L}{1} \frac{(0.00463)}{50}$$

$$L = 2470 \text{ cm} = \underline{\underline{24.7 \text{ m}}}$$

30.17

NAPHTHALENE - AIR

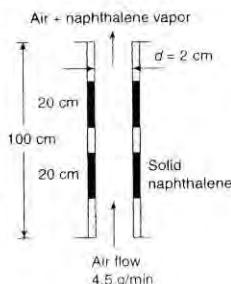
$$T = 373 \text{ K} \quad P = 1 \text{ atm}$$

$$\rho_0 = 1 \text{ mm Hg}$$

$$D_{AB} = 0.086 \text{ cm}^2/\text{s}$$

$$A_B = 0.25 \text{ cm}^2/\text{s}$$

$$P_B = 9.5 \times 10^{-4} \text{ g/cm}^3$$



MASS BALANCE FOR A IN X (UP) DIRECTION

$$C_A \nu \frac{\pi D^2}{4} \Big|_x + k_C (C_{AS} - C_A) \pi D \Delta x \\ = C_A \nu \frac{\pi D^2}{4} \Big|_{x+\Delta x}$$

DO ALGEBRA & EVALUATE IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{dC_A}{C_{AS} - C_A} = \frac{4}{D \nu} \frac{k_C}{\pi} dx$$

LEFT-HAND-SIDE:

$$\int \frac{dC_A}{C_{AS} - C_A} = \int_0^{C_{A1}} \frac{dC_A}{C_{AS} - C_A} + \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{C_{AS} - C_A} \\ = \ln \frac{C_{AS} - 0}{C_{AS} - C_{A1}} + \ln \frac{C_{AS} - C_{A1}}{C_{AS} - C_{A2}} \\ = \ln \frac{C_{AS}}{C_{AS} - C_{A2}}$$

30.17 (CONTINUED)

RIGHT-HAND-SIDE -

$$4 \frac{k_C}{D} \int_0^x dx = \frac{4}{D} k_C \left[ \int_0^{20} dx + \int_x^{20+4} dx \right] \\ = \frac{4}{D} k_C (40)$$

FINAL EXPRESSION IS:

$$\ln \frac{C_{AS}}{C_{AS} - C_{AL}} = \frac{4}{D \nu} k_C (40) \quad (a)$$

$$C = \frac{1}{RT} = \frac{1}{(82.06)(373)} = 3.267 \times 10^{-5} \text{ g mol/cm}^3$$

$$C_{AL} = y_{AL} C = 0.806 y_0 (C) = 2.5 \times 10^{-7} \text{ "}$$

$$C_{AS} = \frac{P_{AS}}{RT} = \frac{0.0314}{(82.06)(373)} = 4.29 \times 10^{-7} \text{ "}$$

$$\nu = 4.5 \text{ g/m} \left( \frac{\text{m}}{60 \text{ s}} \right) \left( \frac{\text{cm}^3}{9.5 \times 10^{-4} \text{ g}} \right) \left( \frac{4}{\pi} \right) \left( \frac{20 \text{ m}}{2} \right)^2 \\ = 15.13 \text{ cm/s}$$

SUBSTITUTING VALUES & SOLVING:

$$\underline{k_C = 0.120 \text{ cm/s}} \quad (b)$$

$$R_C = \frac{D \nu}{D} = \frac{(2)(15.13)}{0.25} = 201$$

{ Laminar }

$$Sc = \frac{D}{DAB} = \frac{0.25}{0.086} = 2.91$$

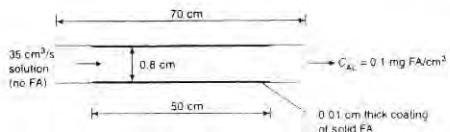
FOR (30-19) APPLIES:

$$k_C = \frac{D}{D_{AB}} (1.86) \left[ \frac{D}{L} Re Sc \right]^{1/3}$$

WITH VALUES SUBSTITUTED:

$$\underline{k_C = 0.146 \text{ cm/s}} \quad (c)$$

30.18



A INTO SOLVENT -

$$C_A^* = 20 \text{ mg/cm}^3 \quad \rho_A = 1.10 \text{ g/cm}^3$$

$$D = 0.02 \text{ cm}^2/\text{s} \quad C_{A1} = 0.1 \text{ mg/cm}^3$$

$$\rho_{\text{soln}} = 1.04 \text{ g/cm}^3$$

USUAL MASS BALANCE FOR A  
TRANSFERRING FROM TUBE WALL -  
SEE PROB 30.17

$$\int_0^{C_A} \frac{dC_A}{C_{A1}-C_A} = \frac{4}{D} \frac{k_c}{V} \int_0^L dx$$

$$\ln \frac{C_A}{C_{A1}-C_A} = \frac{4L k_c}{D V} \quad (\alpha)$$

$$\ln \frac{20}{20-0.1} = 0.00501$$

$$= \frac{4(50)}{0.8} \frac{k_c}{V}$$

$$k_c = 2.005 \times 10^5 \text{ V}$$

$$V = 35 \left( \frac{4}{\pi} \right) (0.8)^2 = 69.63 \text{ cm/s}$$

$$k_c = 1.389 \times 10^{-3} \text{ cm/s}$$

$$Re = \frac{DV}{D} = \frac{(0.8)(69.63)}{0.02} = 2793$$

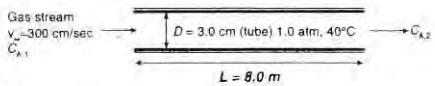
USE EQU. 30-18

$$k_c = \frac{D_{AB}}{D} (0.023) Re \left( \frac{2}{D_{AB}} \right)^{1/3}$$

SUBSTITUTING VALUES!

$$D_{AB} = 5.36 \times 10^{-5} \text{ cm}^2/\text{s} \quad (\beta)$$

30.19

H<sub>2</sub>O INTO AIRWater adsorbent liner  
(1.0 mm thick, 0.6 g H<sub>2</sub>O/cm<sup>3</sup> liner initially)

T = 313 K

$$P = 1 \text{ atm}$$

$$P_A^0 = 55.4 \text{ mm Hg} = 0.0729 \text{ atm}$$

$$\mu_{A1,2} = 1.91 \times 10^{-4} \text{ g/cm.s} \quad D = 0.169 \text{ cm}^2/\text{s}$$

$$\rho = 1.13 \times 10^{-3} \text{ g/cm}^3$$

$$D_{AB} = 0.240 \left( \frac{313}{298} \right)^{3/2} = 0.280 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.169}{0.280} = 0.60 \quad (\alpha)$$

USUAL MASS BALANCE - SEE PROB 30.17

$$\int_{C_{A1}}^{C_{A2}} \frac{dC_A}{C_A^* - C_A} = \frac{4}{D} \frac{k_c}{V} \int_0^L dx$$

$$\ln \frac{C_A^* - C_{A1}}{C_A^* - C_{A2}} = \frac{4L k_c}{D V} \quad (\beta)$$

$$Re = \frac{(3)(300)}{0.169} = 5325$$

$$\text{EQN (30-18): } k_c = \frac{D_{AB}}{D} (0.023) Re^{0.83} Sc^{1/3}$$

SUBSTITUTING VALUES -  $k_c = 2.24 \text{ cm/s}$ 

$$\ln \frac{C_A^* - C_{A1}}{C_A^* - C_{A2}} = \frac{4(800)}{3} \frac{2.24}{300}$$

$$\frac{C_A^* - C_{A1}}{C_A^* - C_{A2}} = 2877$$

$$C_A^* = \frac{P_A^0}{RT} = \frac{0.0729}{(82.06)(313)} = 2.8 \times 10^{-4} \text{ mol/cm}^3$$

$$C_{A1} = \bar{y}_c = 0.01 \left[ \frac{1}{(82.06)(313)} \right]$$

$$= 3.89 \times 10^{-7} \text{ mol/cm}^3$$

30.19 (CONTINUED) -

$$\text{SUBSTITUTING VALUES: } C_{A2} = 2.8 \times 10^{-5} \text{ mol/cm}^3$$

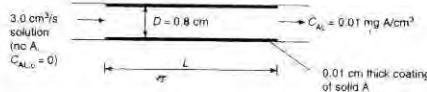
MASS OF  $H_2O$  ABSORBED -

$$\begin{aligned} m &= 0.6(0.10)(\pi)(800)3/8 \\ &= 25.13 \text{ g mol} \\ &= (C_{A2} - C_{A1})V\left(\frac{\pi D^2}{4}\right)t \\ &= (28 - 3.89)(10^{-7})(300)\left(\frac{\pi}{4}\right)^2 t \\ &= 5.112 \times 10^{-3} \text{ g mol/s} \end{aligned}$$

$$t = \frac{25.13}{5.112 \times 10^{-3}} = \underline{4916 \text{ s}}$$

$$\underline{1.366 \text{ h}}$$

30.20



$$C_{AS} = 20 \text{ mg/cm}^3 \quad D = 0.02 \text{ cm}^2/\text{s}$$

$$C_{AL0} = 0.01 \quad D_{AB} = 4 \times 10^{-5} \text{ "}$$

$$\rho_{solid} = 1.10 \text{ g/cm}^3$$

$$V = \frac{V}{A} = \frac{(3)(A)}{\pi(0.8)^2} = 5.968 \text{ cm/s}$$

$$Re = \frac{(0.8)(5.968)}{0.02} = 238.7 \quad \{ \text{LAMINAR} \}$$

$$Sc = \frac{D}{D_{AB}} = \frac{0.02}{4 \times 10^{-5}} = 500$$

$$\text{From (30-18)} \quad k_c = \frac{D_{AB}}{D} (1.816) \left[ \frac{D}{Re} Sc \right]^{1/3}$$

$$\Rightarrow k_c = 4.234 \times 10^{-4} L^{-1/3}$$

30.20 (CONTINUED) -

~ USUAL MASS BALANCE -

$$\text{In } \frac{C_A^* - C_A^0}{C_A^* - C_{AL0}} = \frac{4Lk_c}{DV}$$

$$\begin{aligned} \ln \frac{20-0}{20-0.01} &= 5.00 \times 10^{-4} \\ &= \frac{4(4.234 \times 10^{-4})L^{2/3}}{0.8(5.968)} \\ L &= \underline{1.67 \text{ cm}} \end{aligned}$$

30.21 WETTED WALL COLUMN

$$P = 1 \text{ ATM} \quad D_i = 5 \text{ cm}$$

$$T = 300 \text{ K} \quad L = 600 \text{ cm}$$

$$D = 0.157 \text{ cm}^2/\text{s} \quad P_A^0 = 0.035 \text{ ATM}$$

For  $H_2O$  IN AIR @ 298 K

$$D_{AB} = \frac{240}{2} \left( \frac{300}{298} \right)^{3/2} = \underline{0.131 \text{ cm}^2/\text{s}} \quad (a)$$

$$V = \frac{V}{A} = \frac{4000}{(\pi/4)(5)^2} = 203 \text{ cm/s}$$

$$Re = \frac{DV}{D} = \frac{(5)(203)}{0.157} = 6487$$

$$\text{From (30-18)} \quad k_c = \frac{D_{AB}}{D} (0.023)^{0.83} Sc^{1/3}$$

$$Sc = \frac{0.157}{0.131} = 1.198 \quad k_c = 0.934 \text{ cm/s}$$

$$Sh = \frac{(0.934)(5)}{0.131} = \underline{35.65} \quad (b)$$

$$k_G = \frac{k_c}{RT} = \frac{0.934}{(82.04)(300)} = \underline{3.79 \times 10^{-5} \text{ g mol/cm} \cdot \text{s} \cdot \text{atm}} \quad (c)$$

30.21 (CONTINUED -

$$C_A^* = \frac{P_A}{RT} = \frac{0.035}{(42.00)(300)} = 1.42 \times 10^{-6} \text{ g mol/cm}^3 \quad (\text{d})$$

USUAL MASS BALANCE FOR OIL,

$$\int_{C_{AO}}^{C_{AL}} \frac{dc_A}{C_A^* - C_A} = \frac{4}{D} \frac{kc}{V} \int_0^L dz$$

$$\ln \frac{C_A^* - C_{AL}}{C_A^* - C_{AO}} = \frac{4L}{D} \frac{kc}{V}$$

$$\ln \frac{1.42 \times 10^{-6}}{1.42 \times 10^{-6} - C_{AL}} = \frac{4(600)(0.934)}{(5)(203)}$$

$$C_{AL} = 1.263 \times 10^{-6} \text{ g mol/cm}^3$$

30.22 WETTED WALL COLUMN  $\text{CO}_2 - \text{H}_2\text{O}$

$$T = 293 \text{ K} \quad P_L = 998.2 \text{ kg/m}^3$$

$$P = 2.54 \text{ atm} \quad \mu_L = 993 \times 10^6 \text{ kg/m.s}$$

$$H = 25.5 \text{ atm/(kg mol/m}^3)$$

$$L = 2 \text{ m} \quad \dot{m}_{H_2O} = 2 \text{ g mol/s}$$

$$D = 6 \text{ cm} \quad \dot{m}_{CO_2} = 0.5 \text{ "}$$

$$\dot{m}_{CO_2} = H C_{CO_2}^* \quad C_{CO_2}^* = \frac{2.54}{25.5} = 0.1 \text{ kg mol/m}^3 \quad (\text{a})$$

$$Re_w = \frac{4 \dot{m}_w}{\pi D \mu_w}$$

$$= \frac{4(2)(12)}{\pi(6)(993 \times 10^6)} = 769$$

EQUATION (30-20) APPLIES

$$k_L = \frac{D_{AB}}{z} (0.433) S_C \left( \frac{g_L z^3}{D_L^2} \right)^{1/6} k_e^{0.4}$$

30.22 (CONTINUED -

FOR  $\text{CO}_2$  IN  $\text{H}_2\text{O}$  @ 293K

$$D_{AB} = 1.77 \times 10^{-9} \text{ m/s}$$

$$S_C = \frac{993 \times 10^{-6}}{(998.2)(1.77 \times 10^{-9})} = 562$$

$$\frac{g_L^3}{D_L^2} = \frac{(9.81)(2)^3}{[993 \times 10^{-6}/998.2]} = 79.2 \times 10^{12} \quad (b)$$

$$\text{SUBSTITUTING VALUES: } k_e = 2.686 \times 10^{-3} \text{ cm/s}$$

USUAL MASS BALANCE:

$$\int_{C_{AO}}^{C_{AL}} \frac{dc_A}{C_A^* - C_A} = \frac{4}{D} \frac{kc}{V} \int_0^L dz$$

$$Re = 769 = \frac{DV}{D} \sim V = 0.0127 \text{ m/s}$$

$$\ln \frac{C_A^*}{C_A^* - C_A} = \frac{4(2)}{0.06} \left( \frac{2.686 \times 10^{-3}}{0.0127} \right)$$

$$\frac{C_A^*}{C_A^* - C_A} = 1.3258 = \frac{0.1}{0.1 - C_A}$$

$$C_A = 0.0246 \text{ kg mol/m}^3 \quad (c)$$

30.23 FALLING FILM TO TEOS (A) INTO  $\text{H}_2$

$$V = 2000 \text{ cm}^3/\text{s} \quad T = 333 \text{ K}$$

$$D_G = 5 \text{ cm} \quad D_G = 1.47 \text{ cm}^2/\text{s}$$

$$L = 2 \text{ m} \quad D_{AB} = 1315 \text{ cm}^2/\text{s}$$

$$P_A = 2133 \text{ Pa}$$

$$V = \frac{V}{A} = \frac{2000}{\frac{\pi}{4}(5)^2} = 101.9 \text{ cm/s}$$

30.23 (CONTINUED)

$$Re = \frac{DU}{\nu} = \frac{(5)(101.9)}{1.47} = 346.5$$

$\left\{ \text{LAMINAR} \right\}$

$$\text{from (30-19)} \quad k_c = \frac{D_{AB}}{D} (1.86) \left( \frac{D}{L} Re Sc \right)^{1/3}$$

$$Sc = \frac{1.47}{1.315} = 1.118$$

SUBSTITUTING VALUES:  $k_c = 1.042 \text{ cm/s}$

$$k_b = \frac{k_c}{RT} = \frac{1.042}{(8206)(333)} = 3.813 \times 10^{-5} \text{ mol/cm}^2 \text{ atm}$$

(a)

USUAL MASS BALANCE:

$$\int_{C_{AO}}^{C_{AS}} \frac{dc_A}{C_{AS}-C_A} = \frac{4}{D} \frac{k_c}{U} \int_0^L dx$$

$$\ln \frac{C_{AS}-C_A}{C_{AS}-C_{AO}} = \frac{4L}{D} \frac{k_c}{U}$$

$$= 4 \left( \frac{200}{5} \right) \frac{1.042}{101.9} = 1.637$$

$$\frac{C_{AS}-0}{C_{AS}-C_{AO}} = 5.1387$$

$$C_{AO} = 0.805 C_{AS}$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{2133}{(8206)(333)} = 0.770 \text{ mol/m}^3$$

$$C_{AL} = 0.805 (0.770) = \underline{\underline{0.620 \text{ mol/m}^3}}$$

(b)

AT BOTTOM OF COLUMN:  $C_{AS}-C_{AL} = 0.770 - 0 \text{ mol/m}^3$

AT TOP:  $C_{AS}-C_{AL} = 0.770 - 0.620 \text{ "}$

30.23 (CONTINUED)

$$(C_{AS}-C_{AL})_{LM} = \frac{0.770 - 0.150}{4 \times 0.770 / 0.150} = 0.380 \text{ mol/m}^3$$

$$N_A = k_c (C_{AS}-C_{AL})_{LM}$$

$$= \frac{1.042 (0.380)}{(100)^3} = 3.96 \times 10^{-7} \text{ mol/cm}^3 \text{ s}$$

$$W_A = N_A A = (3.96 \times 10^{-7})(\pi)(5)(200)$$

$$= 1.24 \times 10^{-3} \text{ mol/s}$$

$$\dot{m}_{H_2} = \frac{P_V}{RT} = \frac{(1.013 \times 10^5)(2 \times 10^{-3})}{(8206)(333)}$$

$$= 0.0732 \text{ mol/s}$$

$$y_A = \frac{1.24 \times 10^{-3}}{1.24 \times 10^{-3} + 0.0732} = \underline{\underline{0.0167}} \quad (b)$$

$$\dot{m}_A = (1.24 \times 10^{-3})(208.33)$$

$$= \underline{\underline{0.258 \text{ g/s}}} \quad (c)$$

30.24 WETTED-WALL COLUMN

- ETHYL ACETATE (A) INTO AIR

$$U = 0.2 \text{ m/s} \quad T = 300 \text{ K} \quad P = 1 \text{ atm}$$

$$D = 0.05 \text{ m} \quad P_A^0 = 0.080 \text{ atm}$$

$$L = 10 \text{ m}$$

$$D_{AIR} = 1.569 \times 10^{-5} \text{ m}^2/\text{s}$$

$$D_{AB} = 0.0709 \left( \frac{300}{273} \right)^{3/2} = 0.0817 \text{ cm}^2/\text{s}$$

$$Sc = \frac{(1.569 \times 10^{-5})(100)}{0.0817} = 1.92$$

$$Re = \frac{DU}{\nu} = \frac{(0.05)(0.2)}{1.569 \times 10^{-5}} = 637$$

$\left\{ \text{LAMINAR} \right\}$

30,2A (CONTINUED) -

$$\text{EQUATION (30-19): } k_c = \frac{D_{AB}}{D} (186) \left[ \frac{2}{L} k_{eSc} \right]^{1/3}$$

$$\text{SUBSTITUTION VALUES: } k_c = 555 \times 10^{-4} \text{ m/s} \quad (a)$$

THE USUAL MASS BALANCE YIELDS:

$$\int_{C_{A0}}^{C_A} \frac{dC_A}{C_{AS}-C_A} = \frac{4}{D} \frac{k_c}{V} \int_0^t dy$$

$$\ln \frac{C_{AS}-0}{C_{AS}-C_A} = \frac{4L}{DV} k_c t$$

$$C_{AS} = \frac{P_A}{RT} = \frac{0.08}{(82.06)(300)} = 3.25 \times 10^{-4} \text{ g mol/cm}^3$$

$$= 3.25 \text{ g mol/m}^3$$

$$\ln \frac{3.25}{3.25-C_A} = \frac{4(10)}{0.05} \frac{5.55 \times 10^{-4}}{0.2}$$

$$\text{GIVING } C_A = 1.90 \text{ g mol/m}^3$$

$$\dot{m} = C_A V A$$

$$= 1.90(0.2)\left(\frac{\pi}{4}\right)(0.05)^2(3000)$$

$$= 4.1 \text{ g mol/h}$$

30,25 OZONE BUBBLED INTO H<sub>2</sub>O

$$T = 293 \text{ K} \quad V_{TANK} = 2 \text{ m}^3$$

$$P = 1 \text{ atm}$$

$$C_A = 4 \text{ g mol/m}^3 \text{ AFTER 10 M}$$

$$H = 6.67 \times 10^{-2} \text{ atm/(g mol/m}^3\text{)}$$

30,25 (CONTINUED) -

FOR A WELL-MIXED PROCESS:  
AN OZONE IS DISSOLVED -

MASS BALANCE ON OZONE (A)

$$k_c a (C_A^* - C_A) = \frac{dC_A}{dt}$$

$$\int_0^t \frac{dC_A}{C_A^* - C_A} = k_c a \int_0^t dt$$

$$\ln \frac{C_A^*}{C_A^* - C_A} = k_c a t$$

$$P_A^* = \frac{1}{4} C_A^* = 6.67 \times 10^{-2} C_A^* = 1$$

$$C_A^* = 14.99 \text{ g mol/m}^3$$

$$\ln \frac{14.99}{14.99 - 1} = k_c a (10)(60)$$

$$k_c a = 5.14 \times 10^{-5} \text{ s}^{-1}$$

30,26 USING EQUATION (30-21)  $J_D = 1.17 Re^{-0.415}$

$$J_D = \frac{k_c}{V_{TP}} S_C^{2/3} = 1.17 Re^{-0.415}$$

$$k_c = 1.17 V_{TP} Re^{-0.415} S_C^{-2/3}$$

$$\text{AT } T = 311 \text{ K} \quad D = 1.673 \times 10^{-5} \text{ m}^2/\text{s}$$

$$D_{AB} = \frac{2.634}{1.013 \times 10^{-5}} \left( \frac{311}{298} \right)^{3/2} = 2.772 \times 10^{-5} \text{ m}^2/\text{s}$$

$$S_C = \frac{1.673 \times 10^{-5}}{2.772 \times 10^{-5}} = 0.60$$

30,26 (CONTINUED) =

$$Re = \frac{Dg}{\nu} = \frac{Dg}{\eta_D} = \frac{Dg}{\mu}$$

$$\text{AT } 311 \text{ K} - \mu_{\text{AIR}} = 1.897 \times 10^{-5} \text{ Pa.s}$$

$$Re = \frac{(0.00571)(0.816)}{1.897 \times 10^{-5}} = 246$$

SUBSTITUTING VALUES:  $k_c = 0.120 \text{ m/s}$

$$k_a = \frac{k_c}{RT} = \frac{0.120}{(8.314)(311)} = 4.64 \times 10^{-5} \text{ kg mol/m}^2 \text{ s Pa}$$

$$= 4.64 \times 10^{-8} \text{ kg mol/m}^2 \text{ s Pa}$$

$$= 4.70 \times 10^{-3} \text{ kg mol/m}^2 \text{ s atm}$$

COMPARED WITH EXPERIMENTAL VALUE

$$\Delta = 0.28 \times 10^{-3} \text{ kg mol/m}^2 \text{ s atm} \sim 633 \%$$

METHOD 2:

$$Ej_D = \frac{2.06}{Re^{0.575}} = E \cdot \frac{k_c}{Sc} Sc^{2/3}$$

$$Re = 246 \quad Sc = 0.6 \quad E = 0.75$$

$$k_c = \frac{2.06 (0.75) Re^{-0.575}}{0.75} Sc^{2/3}$$

$$V = G/f = \frac{0.816}{1.136} = 0.718 \text{ m/s}$$

SUBSTITUTING VALUES:  $k_c = 0.117 \text{ m/s}$

$$k_a = \frac{k_c}{RT} = \frac{0.117}{(8.314)(311)} = 4.52 \times 10^{-8} \text{ kg mol/m}^2 \text{ s Pa}$$

$$= 4.58 \times 10^{-3} \text{ kg mol/m}^2 \text{ s atm}$$

$\sim 6.32\%$  DIFFERENT FROM  
EXPERIMENT

30,27 FOR O<sub>2</sub> TRANSFER

$$k_a a = 300 \text{ h}^{-1}$$

FOR D<sub>O<sub>2</sub>-H<sub>2</sub>O</sub> - EAN: (24-52)

$$\frac{D_{AB} \mu_B}{T} = \frac{7.4 \times 10^{-8} (\phi_B M_B)^{1/2}}{V_A^{0.16}}$$

VALUES:  $\phi_B = 2.26 \quad M_B = 18 \quad T = 283 \text{ K}$

$$Y_A = 7.4 \quad \mu_B = 1.45 \text{ cp}$$

$$D_{O_2-H_2O} = 2.77 \times 10^{-5} \text{ cm}^2/\text{s}$$

TABLE J.2  $D_{CO_2-H_2O} = 1.46 \times 10^{-5} \text{ cm}^2/\text{s}$

FILM THEORY -  $k_a a \sim D_{AB}^{1/2}$

$$k_a a_{CO_2} = k_a a_{O_2} \left[ \frac{D_{CO_2-H_2O}}{D_{O_2-H_2O}} \right]^{1/2}$$

$$= 300 \left[ \frac{1.46 \times 10^{-5}}{2.77 \times 10^{-5}} \right]^{1/2} = 158 \text{ h}^{-1}$$

BOUNDARY-LAYER THEORY:  $k_a a \sim D_{AB}^{2/3}$

$$k_a a_{CO_2} = 300 \left[ \frac{1}{2} \right]^{2/3} = 195.8 \text{ h}^{-1}$$

PENETRATION THEORY:  $k_a a \sim D_{AB}^{1/2}$

$$k_a a_{CO_2} = 300 \left[ \frac{1}{2} \right]^{1/2} = 217.8 \text{ h}^{-1}$$

30,28 CO<sub>2</sub> INTO H<sub>2</sub>O IN PACKED BED

$$\dot{m}_w = 5 \text{ kg mol/m} \quad P = 2 \text{ atm}$$

$$\dot{m}_{CO_2} = 1 \quad T = 293 \text{ K}$$

$$D = 0.25 \text{ m}$$

$$P_{H_2O} = 55.5 \text{ kg mol/m}^3$$

$$= 998.2 \text{ kg/m}^3$$

$$\mu_w = 993 \times 10^{-6} \text{ kg/m.s}$$

$$A = 2\pi D \text{ m} / (\text{kg mol/m}^3)$$

$$\text{EQUATION (30-33)} \quad \frac{k_L a}{D_A} = \alpha \left( \frac{L}{\mu} \right)^{1-n} S_C^{1/2}$$

for 1-in. FISCHER RINGS:  $\alpha = 100$

$$n = 0.22$$

$D_{AB} \sim CO_2 \text{ in } H_2O @ 293 \text{ K}$

$$= 1.77 \times 10^{-9} \text{ m}^2/\text{s}$$

$$S_C = \frac{913 \times 10^{-6}}{(998.2)(1.77 \times 10^{-9})} = 562$$

$$A_x = \frac{\pi}{4}(0.25)^2 = 0.0491 \text{ m}^2$$

$$= 0.529 \text{ ft}^2$$

$$L = 5(18)(60) \frac{2.2}{0.529}$$

$$= 22500 \text{ lb.m/h.ft}^2$$

$$D_{AB} = 1.77 \times 10^{-9} (0.3048)^2$$

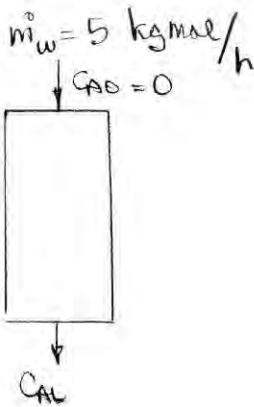
$$= 6.86 \times 10^{-5} \text{ ft}^2/\text{h}$$

SUBSTITUTION VALUES:

$$\underline{k_L a = 0.0566 \text{ s}^{-1} \quad (a)}$$

MASS BALANCE FOR A

$$C_A^* = \frac{P_{CO_2}}{RT} = \frac{2}{25.4} = 0.075 \text{ kg mol/m}^3$$



$$C_A = 0.95 C_A^*$$

$$= 0.95(0.075)$$

$$= 0.07125 \text{ kg mol/m}^3$$

MASS BALANCE -

$$k_L (C_A^* - C_A) = V \frac{dC_A}{dz}$$

$$\int_0^V \frac{dC_A}{C_A^* - C_A} = \frac{k_L a f_L}{V} dz$$

$$\ln \frac{C_A^*}{C_A^* - 0.95 C_A^*} = \frac{k_L a L}{V}$$

$$L = \frac{V}{k_L a} \ln 20$$

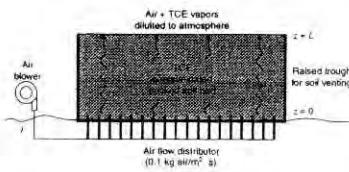
V (ASSUMING EMPTY X-SECTION)

$$= \frac{V}{8A} = \frac{5}{(60)(55.5)(1/4)(0.25)^2}$$

$$= 0.0306 \text{ m/s}$$

$$L = \frac{(0.0306)(\ln 20)}{0.0566} = \underline{1.62 \text{ m}} \quad (b)$$

30,29



TCE (A) IN AIR

$$D_p = 3 \text{ mm}$$

$$\epsilon = 0.5$$

$$T = 293 \text{ K}$$

$$G_B = 0.1 \text{ kg/m}^2 \text{ s}$$

$$D_A = 58 \text{ mm}$$

$$\rho = 1,206 \text{ kg/m}^3$$

$$D_{AB} = 8.08 \times 10^{-6} \text{ m}^2/\text{s}$$

$$D_B = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{1505 \times 10^{-5}}{8.08 \times 10^{-6}} = 1.863$$

$$-Re = \frac{D_p G}{\mu} = \frac{(0.003)(0.1)}{1.863 \times 10^{-5}} = 16,53$$

EQUATION (30,23) APPLIES:

$$\epsilon \frac{k_c}{V} Sc^{2/3} = 0.25 Re^{-0.31}$$

$$k_c = \frac{0.25 V Re^{-0.31}}{\epsilon Sc^{2/3}}$$

$$V = \frac{G}{\rho} = \frac{0.1}{1,206} = 0.083 \text{ m/s}$$

SUBSTITUTING VALUES:  $k_c = 0.011 \text{ m/s}$   
(a)

MASS BALANCE FOR A:

$$k_c \frac{A}{V} (C_A^* - C_A) = V \frac{dC_A}{dz}$$

$$\int_0^{C_A^*} \frac{dC_A}{C_A^* - C_A} = \frac{k_c A}{V} \int_0^L dz$$

30,29 (CONTINUED) -

$$\text{for } \frac{C_A^*}{C_A^* - 0.9C_A^*} = \frac{k_c A}{V} \frac{L}{V}$$

$$L = \frac{V}{k_c A} \ln 10$$

$\frac{V}{A}$  IS VOLUME TO SURFACE AREA RATIO OF PARTICLES IN TOWER

FOR A SPHERICAL PARTICLE -

$$A = \pi D^2$$

OCCUPYING A SPACE WITH  $V = D^3$ 

IN TOWER -

$$\sim \frac{V}{A} = \frac{D^3}{\pi D^2} = \frac{D}{\pi}$$

$$L = \frac{V}{k_c} \frac{D}{\pi} \ln 10$$

$$= \frac{0.083 (0.003)}{0.011 (\pi)} \ln 10$$

$$= 0.0166 \text{ m} = 1.66 \text{ cm} \quad (b)$$

## CHAPTER 31

### 31.1 AERATION TANK w/ SPARGERS -

THIS PROBLEM IS SIMILAR TO EXAMPLE 2 IN SECTION 31.2.

FOR A WELL MIXED TANK - EQUATION (31-1) APPLIES & THE FINAL O<sub>2</sub> LEVEL IS DESCRIBED BY

$$C_p = C_A^* - (C_A^* - C_{A0}) e^{k_L a t} (-K_L a t)$$

$$\dot{V}_{air} = 0.0078 \text{ m}^3/\text{s}$$

$$= \frac{(0.0078)(60)}{(0.3048)} = 15 \text{ cfm}$$

FOR 6 SPARGERS - & FIGURE 31.7  
@ 15 cfm & 15 FT DEPTH

$$k_L a = k_L \frac{A}{V} = 1200 \text{ (6)}$$

$$= \frac{(1200)(6)}{10,000} = 0.72 \text{ h}^{-1}$$

$$P_{atm} = \frac{P_{top} + P_{bottom}}{2}$$

$$\frac{1 + [1 + 14.93(0.0295)]}{2} = 1.22 \text{ ATM}$$

$$X_{O_2}^* = \frac{P}{N} = \frac{0.21(1.22)}{3.27 \times 10^4} = 7.83 \times 10^{-6}$$

$$C = \frac{1000}{18} = 55.56 \text{ g mol/l}$$

$$C_A^* = (7.83 \times 10^{-6})(55.56)$$

$$= 4.35 \times 10^{-4} \text{ g mol/l}$$

SUBSTITUTING VALUES:

$$\text{For } t = 9000 \text{ s} = 2.5 \text{ h}$$

$$C_{O_2 t} = 3.64 \times 10^{-4} \text{ g mol/l}$$

### 31.2 OZONE / H<sub>2</sub>O TREATMENT USING SPARGERS

SYSTEM IS ANALOGOUS TO EXAMPLE 2 -

$$t = \ln \left( \frac{C_A^* - C_{AO}}{C_A^* - C_{At}} \right) \frac{1}{k_L a}$$

$$\text{for } V_b = 17.8 \text{ m}^3/h = 4.9 \times 10^{-3} \text{ m}^3/s = 10.4 \text{ cfm}$$

$$\& \text{DEPTH} = 3.2 \text{ m} = 10.5 \text{ FT}$$

$$\text{FIGURE 31.7 GIVES } K_L A/V \approx 400 \text{ CFH}$$

$$K_L a = \frac{(400)(8)}{(80)/(0.3048)^3} = 1.132 \text{ h}^{-1}$$

BY PENETRATION THEORY:

$$\frac{K_L a \log}{K_L a \log} = \frac{\left[ \frac{P_{O_2 - H_2O}}{P_{O_2 + H_2O}} \right]^{\frac{1}{2}}}{\left[ \frac{1.7 \times 10^{-5}}{2.14 \times 10^{-5}} \right]^{\frac{1}{2}}} = 0.891$$

$$|K_L a|_{O_3} = (1.132)(0.891) = 1.01 \text{ h}^{-1} \quad (a)$$

$$P_{atm} = \frac{1 + (3.2)(0.0295)}{0.3048 + 1}$$

$$= 1.155 \text{ ATM}$$

$$P_{O_3} = 0.04(1.155) = 0.0462 \text{ ATM}$$

$$C_{O_3}^* = \frac{0.0462}{0.0667} = 0.682 \text{ g mol/m}^3$$

$$= 0.682 \left( \frac{48}{1000} \right) = 32.7 \text{ mg/l}$$

$$C_{At} = 0.15 \text{ g mol/m}^3 = 7.2 \text{ mg/l}$$

SUBSTITUTING VALUES:

$$t = 0.246 \text{ h} = 886 \text{ s} \quad (b)$$

31.3 WASTEWATER TREATMENT USING 10 SPARGERS -

$$V = 425 \text{ m}^3 = 15000 \text{ ft}^3$$

$$\dot{V} = 7.08 \times 10^3 \text{ m}^3/\text{s} = 15 \text{ cfm}$$

$$\text{DEPTH} = 3.2 \text{ m} = 10.5 \text{ ft.}$$

ANALYSIS PRECEDES EXAMPLE 2.

$$t = \ln \left( \frac{C_{O_2}^* - C_{O_2,t}}{C_{O_2}^* - C_{O_2,i}} \right) \left[ \frac{1}{K_a} \right]$$

$$\text{Fig. (31.7)} \quad K_a V = 800 \text{ ft}^3/\text{h}$$

$$K_a = \frac{(800)(10)}{15000} = 0.533 \text{ h}^{-1}$$

$$P_{\text{Top}} = 1 \text{ atm}$$

$$P_{\text{Bottom}} = 1 + (10.5)(0.0295) = 1.31 \text{ atm}$$

$$P_{\text{AVG}} = 1.155 \text{ atm}$$

$$P_{O_2} = 0.21 (1.155) = 0.2425 \text{ atm}$$

$$X_{O_2}^* = \frac{P_{O_2}}{P} = \frac{0.2425}{3.27 \times 10^4} = 7.42 \times 10^{-6}$$

$$C_L = \frac{1000}{18} = 55.56 \text{ mol/l}$$

$$C_{O_2}^* = (7.42 \times 10^{-6})(55.56)$$

$$= 4.12 \times 10^{-4} \text{ mol/l}$$

$$t = \ln \left[ \frac{4.12 \times 10^{-4} - 8 \times 10^{-5}}{4.12 \times 10^{-4} - 2 \times 10^{-4}} \right] \left( \frac{1}{0.533} \right)$$

$$= 0.841 \text{ h} = 3028 \text{ s}$$

31.4 O<sub>2</sub> ABSORPTION USING 1 SPARGER

$$V = 28.3 \text{ m}^3 = 1000 \text{ ft}^3$$

$$\dot{V} = 7.08 \times 10^3 \text{ m}^3/\text{s} = 15 \text{ cfm}$$

$$\text{DEPTH} = 3.2 \text{ m} = 10.5 \text{ ft}$$

FROM ANALYSIS ACCOMPANYING EXAMPLE 2

$$t = \frac{1}{K_a} \ln \frac{C_{O_2}^* - C_{O_2,t}}{C_{O_2}^* - C_{O_2,i}}$$

$$C_{O_2,i} = 0.04 \text{ mmol/l}$$

$$C_{O_2,t} = 0.25 \text{ "}$$

FOR 10.5 FT DEPTH:

$$P_{\text{Top}} = 1 \text{ atm} \quad P_{\text{Bottom}} = 1 + (10.5)(0.0295) = 1.31 \text{ atm}$$

$$P_{\text{AVG}} = 1.155 \text{ atm}$$

$$P_{O_2} = 0.21 (1.155) = 0.2426 \text{ atm}$$

$$X_{O_2}^* = \frac{P_{O_2}}{P} = \frac{0.2426}{3.27 \times 10^4} = 7.42 \times 10^{-6}$$

$$C = \frac{1000}{18} = 55.56 \text{ g/mol/l}$$

$$C_{O_2}^* = (7.42 \times 10^{-6})(55.56) = 4.12 \times 10^{-4} \text{ g/mol/l}$$

$$= 0.412 \text{ mmol/l}$$

FOR  $t = 4 \text{ h}$  - SUBSTITUTION YIELDS

$$K_a \Big|_{O_2} = 0.208 \text{ h}^{-1} \quad \text{FOR 1 SPARGER} \quad (a)$$

$$\frac{K_a \Big|_{H_2S}}{K_a \Big|_{O_2}} \sqrt{\frac{D_{H_2S-H_2O}}{D_{O_2-H_2O}}} = \sqrt{\frac{1.4 \times 10^{-5}}{2.14 \times 10^{-5}}} = 0.809$$

$$K_a \Big|_{H_2S} = 0.809 (0.208) = 0.168 \text{ h}^{-1} \quad (b)$$

### 31.4 CONTINUED -

FOR  $H_2S$  - 10 SPALERS,  $K_{La} = 1.68 \text{ h}^{-1}$

$$V = 425 \text{ m}^3 = 15000 \text{ ft}^3$$

$$\text{DEPTH} = 10.5 \text{ FT} \quad t = 4 \text{ h}$$

$$Y_{H_2S} = P_{H_2S} * C_{H_2S}^* = 0$$

$$t = 4 = \frac{1}{1.68} \ln \left[ \frac{0 - C_{H_2Si}}{0 - C_{H_2St}} \right]$$

$$C_{H_2Si} = 0.03 \text{ mmol/l} \quad (c)$$

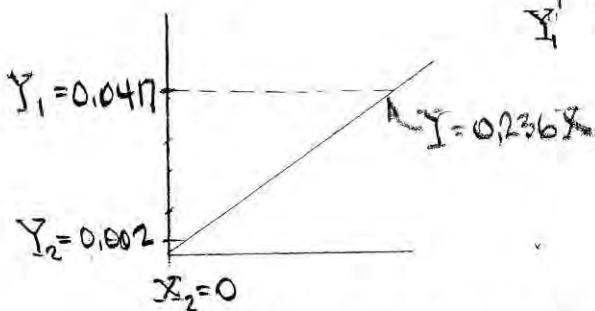
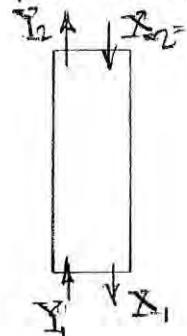
$$\text{SOLN} : C_{H_2St} = 3.61 \times 10^{-4} \text{ g/mol/l}$$

### 31.5 COUNTERCURRENT ABSORPTION TOWER

$$X_1 = ? \quad X_2 = 0$$

$$Y_1 = \frac{0.04}{0.96} = 0.0417$$

$$Y_2 = \frac{0.002}{0.998} = 0.002$$



$$\left| \frac{L_s}{G_s} \right|_{\text{MIN}} = \frac{Y_1 - Y_2}{X_1^* - X_2} = \frac{0.0417 - 0.002}{0.177 - 0} = 0.224$$

$$\left| \frac{L_s}{G_s} \right|_{\text{ACTUAL}} = (0.224)(1.5) = 0.336 \frac{\text{mol SOW}}{\text{mol C.G.}}$$

### 31.5 CONTINUED -

$$0.336 = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0417 - 0.002}{X_1 - 0}$$

$$X_1 = \frac{0.0397}{0.336} = 0.118$$

$$X_1 = \frac{X_1}{1 + X_1} = \frac{0.118}{1.118} = 0.106$$

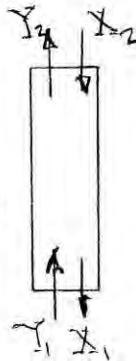
### 31.6 TCE STRIPPED FROM $H_2O$ NA COUNTERCURRENT TOWER

For Dilute Streams

$$X \approx Y \approx y$$

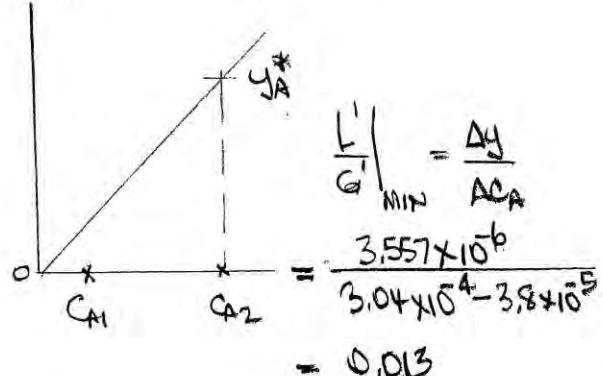
$$C_{A2} = \frac{40 (1000)}{131.5} = 3.04 \times 10^4 \text{ mol/m}^3$$

$$C_{A1} = \frac{5 (1000) (10^{-6})}{131.5} = 3.80 \times 10^{-5} \text{ "}$$



$$Y_A = 0$$

$$Y_A^* = \frac{L}{P} C_{A2} = \frac{11.7 \times 10^{-3}}{1} (3.04 \times 10^4) = 3.557 \times 10^{-6}$$

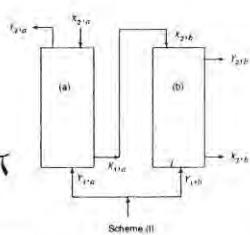


$$\left| \frac{L}{G} \right|_{\text{ACTUAL}} = \frac{0.013}{3} = 4.33 \times 10^{-3} = \frac{Y_{A2} - 0}{2.166 \times 10^{-4}}$$

$$Y_{A2} = 1.15 \times 10^{-6}$$

$$P_{A2} = Y_{A2} P = 1.15 \times 10^{-6} \text{ atm}$$

31.7



TOWER (a)

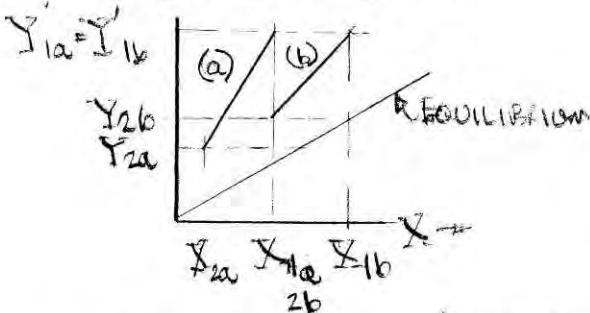
- COUNTERCURRENT

TOWER (b)

- COUNTERCURRENT

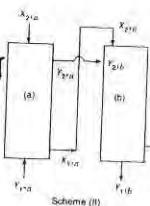
$$Y_{1a} = Y_{1b} > Y_{2b} > Y_{2a}$$

$$X_{1b} > X_{1a} = X_{2b} > X_{2a}$$



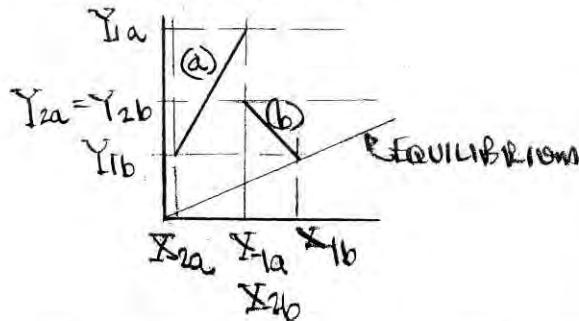
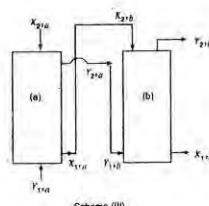
TOWER (a) - COUNTERCURRENT

" (b) - COCURRENT



$$X_{1b} > X_{1a} = X_{2b} > X_{2a}$$

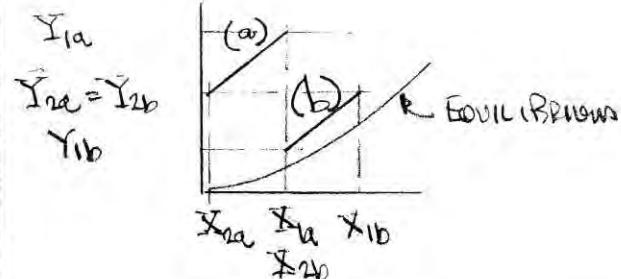
$$Y_{1a} > Y_{2a} = Y_{2b} > Y_{1b}$$

BOTH TOWERS ARE  
COUNTERCURRENT

$$X_{1b} > X_{1a} = X_{2b} > X_{2a}$$

$$Y_{1a} > Y_{2a} = Y_{1b} > Y_{2b}$$

31.7 CONTINUED

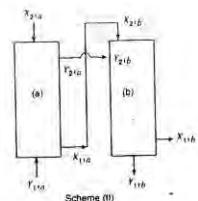
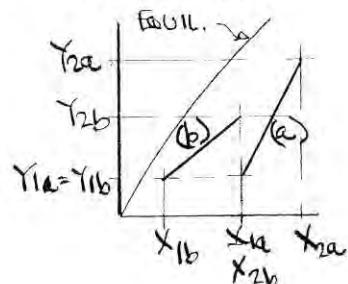
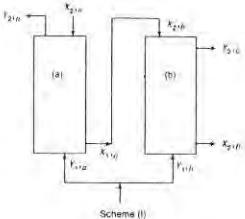


31.8 SAME FLOW SCHEMES AS IN PROB 31.7  
EXCEPT PROCESSES ARE NOW  
DESORPTION / STRIPPING

BOTH ARE COUNTERCURRENT

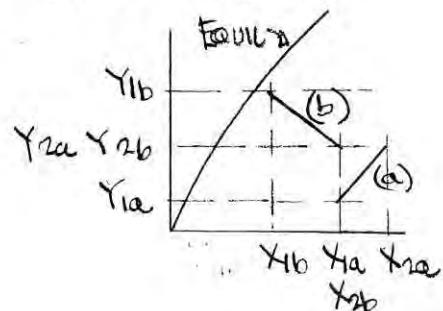
$$X_{2a} > X_{1a} = X_{2b} = X_{1b}$$

$$Y_{2a} > Y_{2b} > Y_{1a} = Y_{1b}$$



$$X_{2a} > X_{1a} = X_{2b} > X_{1b}$$

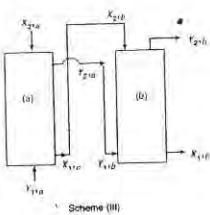
$$Y_{1b} > Y_{2a} = Y_{2b} > Y_{1a}$$



31.7

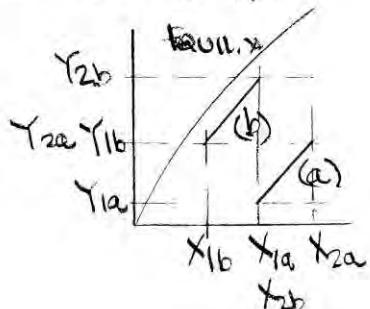
### 31.8 CONTINUED

BETA TOWERS ARE COUNTERCURRENT



$$X_{2a} > X_{1a} = X_{2b} > X_{1b}$$

$$Y_{2b} > Y_{2a} = Y_{1b} > Y_{1a}$$



### 31.9

$$G_1 = 136 \text{ mol/m}^2 \cdot \text{s}$$

$$G_S = G_1(1-y_1)$$

$$= 136(0.95)$$

$$= 129.2 \text{ mol air/m}^2 \cdot \text{s}$$

$$L_S = L_1 = \frac{3400}{18}$$

$$= 188.9 \text{ mol H}_2\text{O/m}^2 \cdot \text{s}$$

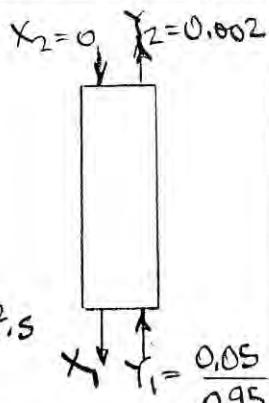
$$L_S(X_1 - X_2) = G_S(Y_1 - Y_2)$$

$$188.9(X_1 - 0) = 129.2(0.0526 - 0.002)$$

$$X_1 = 0.0346$$

$$Y_1 = \frac{X_1}{1+X_1} = \frac{0.0346}{1.0346} = \underline{\underline{0.033}} \quad (\text{a})$$

$$\left| \frac{L_S}{G_S} \right|_{\text{ACTUAL}} = \frac{188.9}{129.2} = 1.462$$

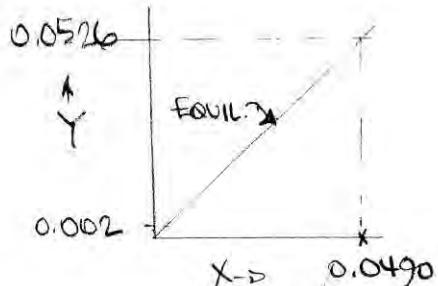


### 31.9 CONTINUED

EQUILIBRIUM DATA:  $y = 1.075x$

$$Y = \frac{y}{1-y} \quad X = \frac{x}{1-x}$$

$Y$	$y$	$X$	$x$
0	0	0	0
0.0054	0.0054	0.005	0.005
0.0109	0.0108	0.01	0.0101
0.0220	0.0215	0.02	0.0204
0.0333	0.0323	0.03	0.0309
0.0449	0.0430	0.04	0.0417
0.0568	0.0538	0.05	0.0526



$$\left| \frac{L_S}{G_S} \right|_{\text{MIN}} = \frac{0.0526 - 0.002}{0.0490 - 0} = 1.033$$

$$\frac{\left| \frac{L_S}{G_S} \right|_{\text{ACT}}}{\left| \frac{L_S}{G_S} \right|_{\text{MIN}}} = \frac{1.462}{1.033} = 1.415 \quad (\text{b})$$

MASS BALANCE - REFERENCE IS TOP ~ (2)

$$G_S(Y_1 - Y_2) = L_S(X_1 - X_2)$$

$$Y = \frac{y_2}{1-y_2} = \frac{0.02}{0.98} = 0.0204$$

$$129.2(0.0204 - 0.002) = 188.9 X_2$$

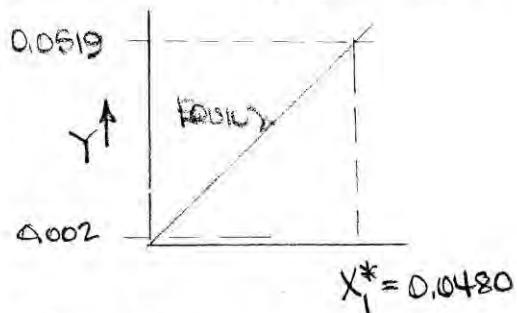
$$X_2 = 0.01258$$

$$X_2 = \frac{0.01258}{1.01258} = \underline{\underline{0.0124}} \quad (\text{c})$$

31,10

$$\begin{aligned}
 & y_2 = 0.802 \\
 & x_2 = 0 \\
 & x_2 = 0 \quad \textcircled{2} \quad \begin{array}{|c|c|}\hline & Y_2 = 0.002 \\ \hline \end{array} \\
 & G_{11} = 136 \text{ mol/m}^2 \cdot s \\
 & \begin{array}{|c|c|}\hline x_1 & y_1 \\ \hline x_1 & \textcircled{1} \quad \begin{array}{|c|c|}\hline & y_1 \\ \hline \end{array} \\ \hline \end{array} \\
 & y_1 = 0.0493 \\
 & Y_1 = \frac{0.0493}{0.9501} = 0.0519 \\
 & G_3 = 6 \cdot (1 - y_1) = 136 \cdot (0.9501) \\
 & = 129.3 \text{ mol/m}^2 \cdot s
 \end{aligned}$$

EQUILIBRIUM DATA - SEE TABLE  
for Prob 31.9



$$\frac{L_s}{G_s} \Big|_{\min} = \frac{\gamma_1 - \gamma_2}{X_1^* - X_2} = \frac{0.0519 - 0.002}{0.0480 - 0} = 1.0396$$

$$\frac{Ls}{Gg} = 1.4(1.0396) = 1.455$$

ACTUAL

$$= \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0,0519 - 0,002}{X_1 - 0}$$

$$x = 0.0345$$

$$\text{Molar flux of } \text{NH}_3 = 6s(Y_{A1} - Y_{A2})$$

### 31.10 CONTINUED

$$= G_S (T_{A1} - T_{B2})$$

$$= 129,3 (0,0519 - 0,002)$$

$$= 6.45 \text{ g/mol/m}^2\text{-s} \times \frac{17}{1000}$$

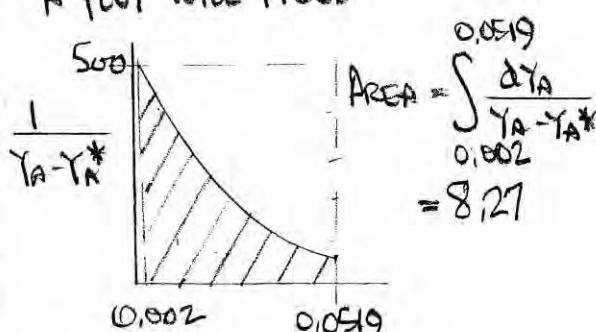
$$= 0,1096 \text{ kg/m}^2\text{.s} \quad (\text{a})$$

$$\text{Height of Reservoir: } z = \frac{65}{K_{\text{Ra}}} \int \frac{dY_A}{Y_A - Y_p} + 0,0519$$

SINCE INTEGRAND INFORMATION IS NOT  
IN ANALYTIC FORM - EVALUATION OF  $\int$   
MUST BE DONE GRAPHICALLY OR  
NUMERICALLY -

$\gamma_A$	$\gamma_A^*$	$\gamma_A - \gamma_A^*$	$(\gamma_A - \gamma_A^*)$
0.002	0	0.002	500
0.010	0.0057	0.0043	232.6
0.015	0.0095	0.0055	181.8
0.020	0.0132	0.0068	147.6
0.025	0.0170	0.0080	125.0
0.030	0.0208	0.0092	108.7
0.035	0.0247	0.0103	97.1
0.040	0.0284	0.0116	86.2
0.045	0.0321	0.0119	77.5
0.050	0.0358	0.0142	70.4
0.0519	0.0372	0.0147	68.0

## A PLOT WILL YIELD



31.10 (CONTINUED -

$$z = \frac{6s}{K_A} \int_{0.002}^{0.05} dY_A$$

$$= \frac{129.3 (8,2)}{107} \approx 10 \text{ m}$$

31.11

$$\begin{array}{l} X_2^* = Y_1 = \frac{0.0384}{48.3} = 0.000795 \\ \quad = 7.95 \times 10^{-4} \end{array}$$

$X_2 = 0$   
 $X_2 = 0$

$Y_2 = 0.002$   
 $\downarrow$   
 $Y_2 = 0.002$

$y_1 = 0.037$   
 $\downarrow$   
 $y_1 = 0.037$

$Y_1 = \frac{0.037}{0.963} = 0.0384$

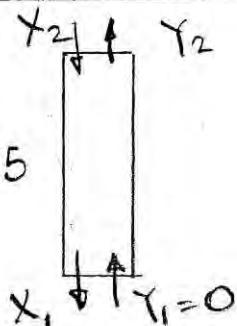
$$\begin{aligned} \frac{L_s}{G_s \text{ min}} &= \frac{Y_1 - Y_2}{X_1^* - X_2} \\ &= \frac{0.0384 - 0.002}{7.95 \times 10^{-4}} = 45.8 \\ \frac{L_s}{G_s \text{ ACT}} &= 1.5(45.8) = \underline{68.7 \text{ mol H}_2\text{O / molec C}_6} \quad (\text{a}) \\ &= \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0384 - 0.002}{X_1 - 0} \end{aligned}$$

$$X_1 = 5.30 \times 10^{-4}$$

$$X_1 = \frac{X_1}{1+X_1} = 5.30 \times 10^{-4} \quad (\text{b})$$

31.12

$$X_2 = \frac{0.07}{0.93} = 0.075$$



31.12 (CONTINUED -

EQUILIBRIUM

DATA →

x mole benzene	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14
y mole wash oil	0.00	0.07	0.14	0.22	0.31	0.405	0.515	0.65

$$L_s = 6.94 \text{ mol wash oil / s}$$

$$\begin{aligned} \text{BENZENE IN } L_2' &= 0.075(6.94) \\ &= 0.52 \text{ mol Benz / s} \end{aligned}$$

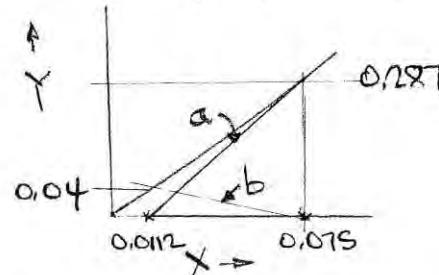
$$\begin{aligned} \text{BZ TO BE REMOVED} &= (0.52)(0.86) \\ &= 0.442 \text{ mol / s} \end{aligned}$$

$$\text{BZ REMAINING IN LIQUID} = 0.078 \text{ "}$$

$$X_1 = \frac{0.078}{6.94} = 0.0112 \frac{\text{mol BZ}}{\text{mol W.O.}}$$

FOR COUNTERCURRENT FLOW STREAMS:

$$\frac{G_s}{L_s \text{ min}} = \frac{X_2 - X_1}{Y_2^* - Y_1} = \frac{0.075 - 0.0112}{0.287 - 0}$$



$$G_s \text{ min} = 0.222(6.94) = 1.54 \text{ mol / s}$$

$$G_s \text{ ACTUAL} = 1.4(1.54) = \underline{2.16 \text{ mol / s}} \quad (\text{a})$$

COCURRENT FLOW:

$$\frac{G_s}{L_s \text{ min}} = \frac{X_2 - X_1}{Y_1^* - Y_2} = \frac{0.075 - 0.0112}{0.040 - 0} = 1.595$$

$$G_s \text{ min} = 1.595(6.94) = 11.07 \text{ mol / s}$$

$$G_s \text{ ACT} = 1.4(11.07) = \underline{15.5 \text{ mol / s}} \quad (\text{b})$$

31.13

$$X_{A1} = \frac{0,0365}{0,9635} = 0,0379$$

$$Y_{A1} = \frac{0,05}{0,95} = 0,0526$$

$$X_{A1} = 0,0365 \quad Y_{A1} = 0,0526$$

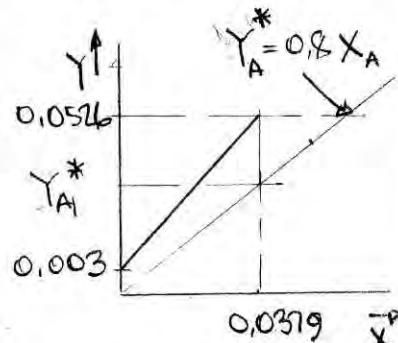
$$Y_{A2} = \frac{0,003}{0,997} \approx 0,003$$

$$G_1' = \dot{V} \frac{P}{RT} = \frac{(15/60)(1,013 \times 10^5)}{(8,314)(289)} \\ = 10,54 \text{ g mol/s}$$

$$G_S' = G_1' (1 - y_{A1}) = (10,54)(0,95) \\ = 10,01 \text{ g mol/s}$$

$$\frac{G_S'}{L_S} = \frac{X_{A1} - X_{A2}}{Y_{A1} - Y_{A2}} = \frac{0,0379 - 0}{0,0526 - 0,003}$$

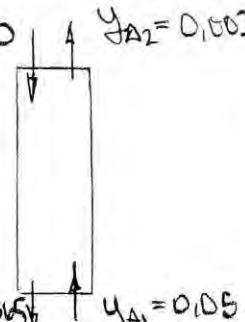
$$L_S' = 13,1 \text{ g mol/s}$$



$$Y_{A1}^* = 0,8 X_{A1} = 0,8(0,0379) \\ = 0,0303$$

$$Y_{A1} - Y_{A1}^* = 0,0526 - 0,0303 \\ = 0,0223$$

$$Y_{A2} - Y_{A2}^* = 0,003 - 0 = 0,003$$



31.13 CONTINUED-

$$Z = \frac{G_S}{K_A} \frac{Y_{A1} - Y_{A2}}{(Y_{A1} - Y_{A1}^*)_{L.M.}}$$

$$(Y_{A1} - Y_{A1}^*)_{L.M.} = \frac{0,0223 - 0,003}{\ln \frac{0,0223}{0,003}} \\ = 0,0096$$

$$G_S = \frac{G_S}{A} = \frac{10,01}{0,2} = 50,05 \text{ g mol/m}^2 \cdot \text{s}$$

$$Z = \frac{(50,05)(0,0223 - 0,003)}{(52)(0,0096)}$$

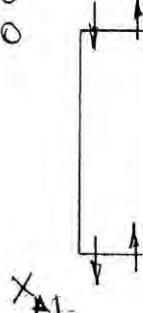
$$= 4,91 \text{ m}$$

31.14

$$X_{A2} = 0 \quad Y_{A2} = 0$$

$$Y_{A2} = 0,003$$

$$Y_{A2} = \frac{0,003}{0,997} \approx 0,003$$



$$Y_{A1} = 0,0379$$

$$Y_{A1} = \frac{0,0379}{0,964} = 0,0373$$

$$A = \frac{\pi}{4}(0,15)^2 = 0,0177 \text{ m}^2$$

$$L_1 = L_S = \frac{14,5}{0,0177} = 819,2 \text{ mol/m}^2 \cdot \text{s}$$

$$G_1 = \frac{8}{0,0177} = 452 \text{ mol/m}^2 \cdot \text{s}$$

$$G_S = G_1 (1 - y_J) = 452 (0,964)$$

$$= 435,7 \text{ mol/m}^2 \cdot \text{s}$$

$$\left| \frac{L_S}{G_S} \right|_{ACT} = \frac{819,2}{435,7} = 1,88$$

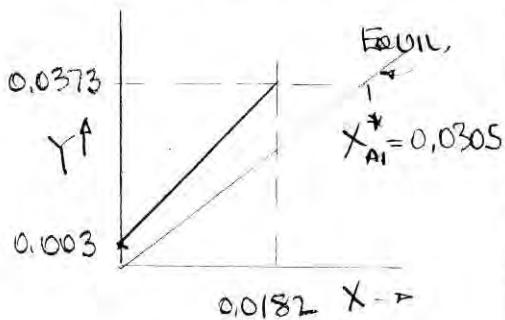
31.14 (CONTINUED)

$$\frac{L_s}{G_s} = 1.88 = \frac{Y_{A1} - Y_{A2}}{X_{A1} - X_{A2}} = \frac{0.0373 - 0.003}{X_{A1} - 0}$$

$$X_{A1} = 0.0182$$

DATA  $\rightarrow$

x	mole NH <sub>3</sub> mole NH <sub>3</sub> -free water	0.00	0.0164	0.0252	0.0349	0.0445	0.0722
y	mole NH <sub>3</sub> mole NH <sub>3</sub> -free air	0.00	0.021	0.032	0.042	0.053	0.080



$$\left| \frac{L_s}{G_s} \right|_{\text{MIN}} = \frac{Y_{A1} - Y_{A2}}{X_{A1}^* - X_{A2}} = \frac{0.0373 - 0.003}{0.0305 - 0} = 1.125$$

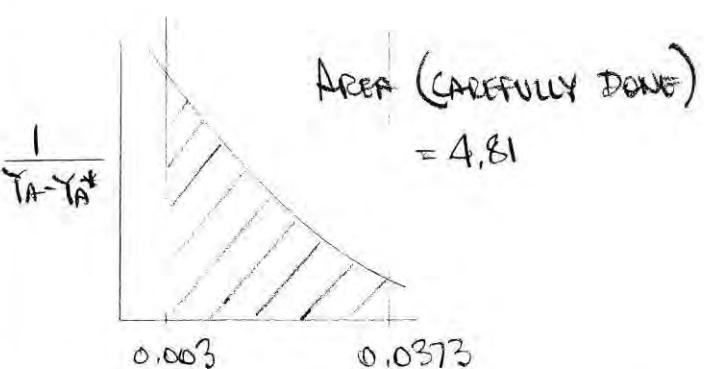
$$\frac{L_s/G_s \text{ ACT}}{L_s/G_s \text{ MIN}} = \frac{1.88}{1.125} = \underline{\underline{1.67}} \quad (\text{a})$$

USING GRAPHICAL INTEGRATION:

$$z = \frac{L_s}{K_y a} \int_{0.003}^{0.0373} \frac{dY_A}{Y_A - Y_A^*}$$

Y <sub>A1</sub>	Y <sub>A</sub> *	Y <sub>A</sub> - Y <sub>A</sub> *	(Y <sub>A1</sub> - Y <sub>A</sub> *) <sup>-1</sup>
0.003	0	0.003	333.3
0.010	0.0048	0.0052	192.3
0.015	0.0083	0.0067	149.2
0.020	0.0116	0.0084	119.1
0.025	0.0150	0.010	100.0
0.030	0.0183	0.0117	85.5
0.0373	0.023	0.0143	69.9

31.14 (CONTINUED)



$$z = \frac{435.7}{71} (4.81) = \underline{\underline{29.52 \text{ m}}} \quad (\text{b})$$

31.15 SAME Specs of SYSTEM AS  
IN PROB 31.14

from Prob 31.14 SOLUTION:

$$G_s = 435.7 \text{ Mol/m}^2 \cdot \text{s}$$

$$K_y a = 71 \text{ Mol/m}^2 \cdot \text{s} \cdot \Delta Y_A$$

$$Y_{A1} - Y_{A1}^* = 0.0373 - 0.023 = 0.0143$$

$$Y_{A2} - Y_{A2}^* = 0.003$$

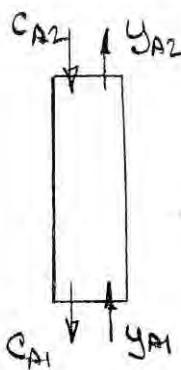
$$(Y_A - Y_A^*)_{L.M.} = \frac{0.0143 - 0.003}{\ln \frac{0.0143}{0.003}} = 0.0072$$

$$z = \frac{G_s}{K_y a} \frac{Y_{A1} - Y_{A2}}{(Y_A - Y_A^*)_{L.M.}}$$

$$= \frac{435.7}{71} \frac{(0.0373 - 0.003)}{0.0072}$$

$$= \underline{\underline{29.2 \text{ m}}}$$

31.16



$$c_{A2} = 0.0394 \times 10^3 \text{ g/mol/g}$$

$$= 0.0394 \text{ mol/m}^3$$

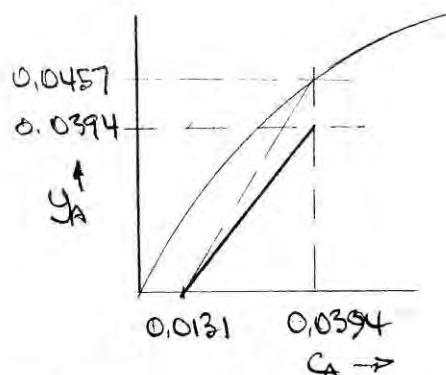
$$c_{A1} = 0.0131 \times 10^3 \text{ g/mol/g} = 0.0131 \text{ mol/m}^3$$

$$y_{A1} = 0$$

$$Q'_L = \frac{5000(3785)}{1000} = 18,92 \text{ m}^3/\text{h}$$

$$G' = \frac{\dot{V}_L'}{1.5} = \frac{18,92}{1.5} = 12.62 \text{ mol/h}$$

EQUILIBRIUM DATA →	$c_A$ , moles VOC/m <sup>3</sup>	0.014	0.0240	0.0349	0.0498
	y <sub>A</sub> , VOC	0.018	0.030	0.042	0.053



SINCE STREAMS ARE RELATIVELY DILUTE

$Q'_L$  &  $G'$  ARE CONSTANT

$$Q'_L(c_{A1} - c_{A2}) = G'(y_{A1} - y_{A2})$$

31.16 CONTINUED -

$$\frac{Q'_L}{A} = \frac{y_{A2}^* - y_{A1}}{c_{A2} - c_{A1}} = \frac{0.0457 - 0}{0.0394 - 0.0131}$$

$$= 1,738$$

$$G'_{\min} = \frac{18,92}{1,738} = \underline{\underline{10.89 \text{ mol/h}}} \quad (a)$$

$$Q'_L(c_{A1} - c_{A2}) = G'_{\min}(y_{A1} - y_{A2})$$

$$18,92(0.0131 - 0.0394) = 12.62(0 - y_{A2})$$

$$y_{A2} = 0.0394$$

$$\text{TOWER HEIGHT: } Z = \frac{\dot{V}_L}{k_a} \frac{c_{A2} - c_{A1}}{(c_A - c_A^*)_{\text{L.M.}}}$$

$$(c_A - c_A^*)_{\text{L.M.}} = 0.0394 - 0.0305 = 9.0 \times 10^{-3}$$

$$(c_A - c_A^*)_{\text{I}} = 1.31 \times 10^{-2} - 0 = 0.0131$$

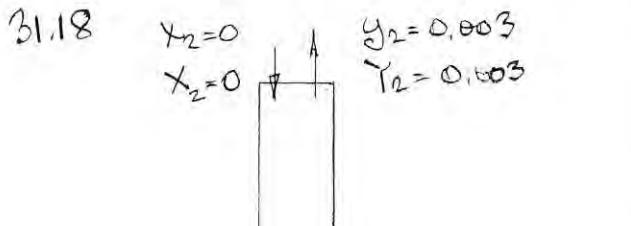
$$(c_A - c_A^*)_{\text{L.M.}} = \frac{0.0131 - 0.009}{\ln \frac{0.0131}{0.009}} = 0.0109$$

$$\frac{Q'_L}{A} = \frac{18,92}{\frac{1}{4}(0.6)^2(3600)} = 0.0186 \text{ m/s}$$

$$Z = \frac{0.0186(0.0394 - 0.0131)}{0.01(0.0109)}$$

$$= \underline{\underline{4.49 \text{ m}}} \quad (b)$$

31.17 THE EXOTHERMIC REACTION WILL CAUSE THE TEMPERATURE IN THE TOWER TO INCREASE, WHICH, IN TURN, WILL CAUSE THE EQUILIBRIUM LINE TO SHIFT UPWARD. THE RESULT WILL BE A SMALLER DRIVING FORCE,  $Y_A - Y_A^*$ . A TALLER TOWER WILL BE REQUIRED RELATIVE TO ONE OPERATING ISOTHERMALLY.



$$X_1 = 0.0305$$

$$X_1 = \frac{0.0305}{0.9695} = 0.0315$$

$$Y_1 = 0.05$$

$$Y_1 = \frac{0.05}{0.95} = 0.0526$$

$$G_s' = \frac{VR}{RT} = \frac{(0.2316)(1.013 \times 10^5)}{(8.314)(293)}$$

$$= 9.814 \text{ g mol/s}$$

$$G_s' = G'(1-g_s) = (9.814)(0.95)$$

$$= 9.323 \text{ g mol/s}$$

$$L_s' = \frac{G_s'(Y_1 - Y_2)}{X_1 - X_2}$$

$$= \frac{9.323(0.0526 - 0.003)}{0.0315 - 0}$$

$$= 14.68 \text{ g mol/s}$$

### 31.18 CONTINUED

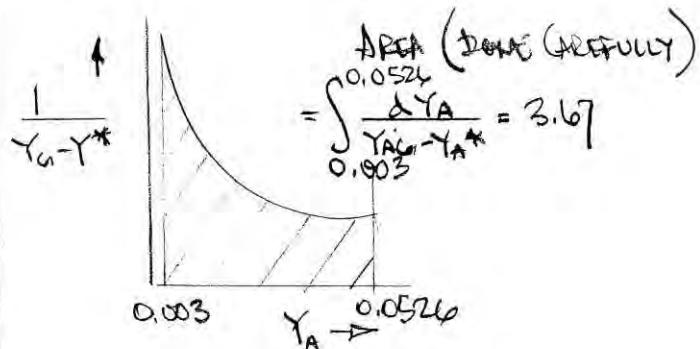
EQUILIBRIUM DATA

X	mole mercaptan mole mercaptan-free solvent	0.00	0.01	0.02	0.03	0.04
Y	mole mercaptan	0.00	0.0045	0.0145	0.0310	0.0545
	mole mercaptan-free air					

TOWER HEIGHT:  $Z = \frac{G_s}{k_{ya}} \int_{Y_2}^{Y_1} \frac{dY_A}{Y_A - Y_A^*}$

USING EQUILIBRIUM DATA PROVIDED:

$Y_a$	$Y^*$	$Y_a - Y^*$	$(Y_a - Y^*)^{-1}$
0.003	0	0.003	333.3
0.010	0.0016	0.0084	119.05
0.016	0.0038	0.0122	81.91
0.020	0.0057	0.0143	69.93
0.028	0.0100	0.0180	55.56
0.034	0.0140	0.0200	50.0
0.040	0.0193	0.0207	48.31
0.048	0.0281	0.0199	50.25
0.0526	0.0340	0.0186	53.76



$$Z = \frac{9.323(3.67)}{(0.2)(40)} = \underline{\underline{4.28 \text{ m}}}$$

31.19 Same System & Feed Streams  
As in Prob 31.18

From Prob 31.18 Solution -

$$G_1' = 9,814 \text{ gmol/s}$$

$$L_s' = 14,68 \text{ "}$$

IN THIS CASE -

$$Z = 4.5 \text{ m} \quad C_f = 155$$

1-IN RASCHIG RUNGS

$$\text{FOR GAS: } \Delta P_{12} = 300 \text{ N/m}^2$$

PARAMETERS for Fig 31.25:

$$\frac{L'}{G'} \left[ \frac{\rho_g}{\rho_L - \rho_g} \right]^{1/2}$$

$$G_1' = \frac{(9,814)(30.1)}{1000} = 0,295 \text{ kg/s}$$

$$L_1' = \frac{L_s}{1-X_1} = \frac{14,68}{1-0,0315}$$

$$= 15,16 \text{ gmol/s}$$

$$L_1' = \frac{(15,16)(180)}{1000} = 2,729 \text{ kg/s}$$

$$\rho_g = \frac{P}{RT} M = \frac{(1,013 \times 10^5)(30.1)}{(8,314)(293)(1000)} \\ = 1,252 \text{ kg/m}^3$$

$$\rho_L = 0,81(1000) = 810 \text{ kg/m}^3$$

SUBSTITUTING VALUES -

$$\frac{L'}{G'} \left[ \frac{\rho_g}{\rho_L - \rho_g} \right]^{1/2} = 0,364$$

31.19 (CONTINUED)

FROM FIG 31.25 -

$$\frac{G'^2 C_g \mu_L^{0.1}}{\rho_g (\rho_L - \rho_g) g_c} = 0.03$$

SUBSTITUTING VALUES

$$\mu_L = 0,0039 \text{ Pa.s}$$

$$g_c = 1$$

OTHERS ALREADY CALCULATED

$$G' = 0,592 \text{ kg/m}^2 \cdot \text{s}$$

$$\text{TOWER AREA} = \frac{G_1'}{G'} = \frac{0,295}{0,592} \\ = 0,498 \text{ m}^2$$

$$D = \left( \frac{0,498}{\pi/4} \right)^{1/2} = \underline{\underline{0,796 \text{ m}}} \\ \text{OR } \underline{\underline{0,8 \text{ m}}}$$

31.20  $X_{A2}=0$

$$X_{A2}=0$$

$$Y_{A2}=0,004$$

$$Y_{A2} = \frac{0,004}{0,996} \approx 0,004$$



$$X_{A1}$$

$$Y_{A1}=0,125$$

$$Y_{A1} = \frac{0,125}{0,875} = 0,143$$

FRACTION OF HCl REMOVED -

$$= \frac{G_S(Y_{A1} - Y_{A2})}{G_S(Y_{A1})} = \frac{0,143 - 0,004}{0,143}$$

$$= 0,972 \sim 97,2\% \text{ (a)}$$

31.20 CONTINUED -

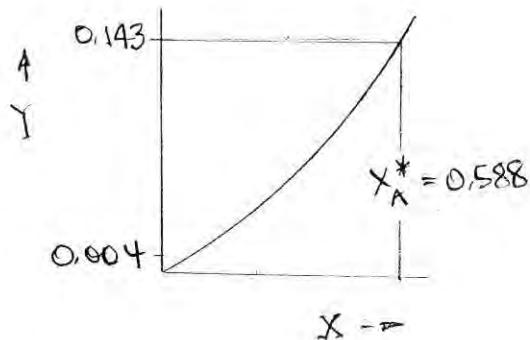
$x_{\text{HCl}}$	0.210	0.243	0.287	0.330	0.353	0.375	0.400	0.425
$y_{\text{HCl}}$	0.0023	0.00956	0.0215	0.0523	0.0852	0.1135	0.203	0.322

EQUILIB.

DATA →

DETERMINE EQUILB. VALUES IN TERMS  
OF  $X_A, Y_A$

$Y_A$	$X_A$	$Y_A$	$Y_A$
0.210	0.746	0.0023	0.0023
0.243	0.321	0.0095	0.0096
0.287	0.403	0.0215	0.0220
0.330	0.493	0.0523	0.0552
0.353	0.546	0.0852	0.0931
0.375	0.600	0.135	0.156
0.400	0.666	0.203	0.255
0.425	0.739	0.322	0.475



$$\left| \frac{L_s}{G_s} \right|_{\text{MIN}} = \frac{Y_{A1} - Y_{A2}}{X_{A1}^* - X_{A2}} = \frac{0.143 - 0.004}{0.588 - 0} = 0.236$$

$$\left| \frac{L_s}{G_s} \right|_{\text{ACT}} = 0.236 (1.04) = 0.387$$

$$= \frac{Y_{A1} - Y_{A2}}{X_{A1} - X_{A2}} = \frac{0.143 - 0.004}{X_{A1} - 0}$$

$$X_{A1} = 0.359$$

$$X_{A1} = \frac{X_{A1}}{1 + X_{A1}} = \frac{0.359}{1.359} = 0.264 \quad (b)$$

31.20 CONTINUED -

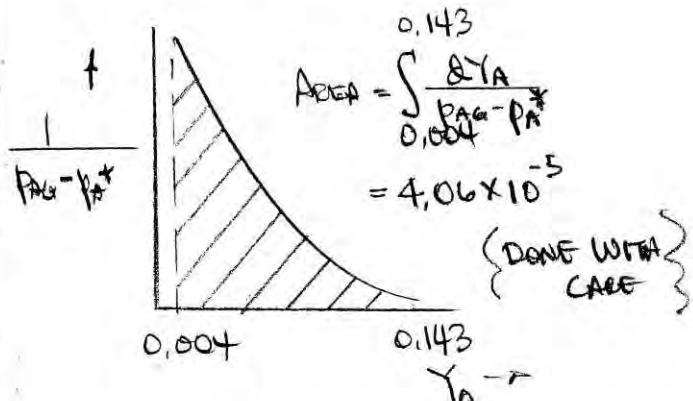
$$K_{GA} = 8.8 \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

WITH  $K_{GA}$  EXPRESSED IN THIS MANNER  
DRIVING FORCE MUST BE IN  $\text{Pa} - \text{Pa}^*$

$$Z = \frac{L_s}{K_{GA}} \int_{0.004}^{0.143} \frac{\Delta Y_A}{P_{AG} - P_A^*}$$

$Y_{AG}$	$Y_A^*$	$Y_{AG}$	$Y_A^*$	$P_{AG}$
0.004	0	0.004	0	405
0.02	0.0005	0.0196	0.0005	1985
0.04	0.0010	0.0385	0.0010	3900
0.06	0.0015	0.0566	0.0015	5734
0.08	0.0019	0.0741	0.0019	7506
0.10	0.0021	0.0909	0.0021	9208
0.12	0.0060	0.1071	0.0060	10850
0.143	0.015	0.1250	0.0148	12662

$\Sigma P_A^*$	$P_{AG} - P_A^*$	$(P_{AG} - P_A^*)^{-1} \times 10^4$
0	405	24.7
50.6	1935	5.17
101	3799	2.63
152	5582	1.79
192	7314	1.37
213	8996	1.11
608	10240	0.98
1499	11160	0.90



31.20 (CONTINUED -

$$\dot{V} = 5 \text{ m}^3/\text{m}$$

$$G_1' = \frac{\dot{V} P}{RT} = \frac{(5)(1,013 \times 10^5)}{(8,314)(293)(60)}$$

$$= 3,465 \text{ mol/s}$$

$$G_S' = G_1' (1 - y_{A1}) = (3,465)(1 - 0,125)$$

$$= 3,03 \text{ mol/s}$$

$$G_S = \frac{3,03}{(\pi/4)(0,6)^2} = 10,72 \text{ mol/m}^2\text{s}$$

$$k g a = 8,8 \times 10^{-8} \text{ kg mol/m}^3\text{s Pa}$$

$$= 8,8 \times 10^{-5} \text{ mol/m}^2\text{s Pa}$$

SUBSTITUTING:

$$z = \frac{10,72 (4,06 \times 10^{-5})}{8,8 \times 10^{-5}}$$

$$= \underline{\underline{4,94 \text{ m}}}$$