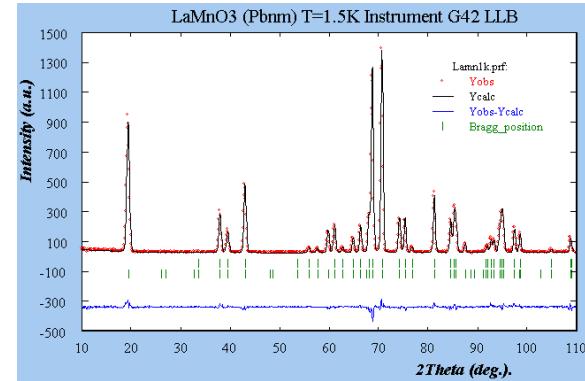


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# CRYSTALLOGRAPHIC and MAGNETIC STRUCTURES from NEUTRON DIFFRACTION: the POWER of SYMMETRIES (Lecture I)

**Béatrice GRENIER**

UGA & CEA, INAC/MEM/MDN

Grenoble, France

&

**Gwenaëlle ROUSSE**

UPMC & Collège de France,  
 Paris, France

## I- Crystallographic structures

**Point Group Symmetry:** *from the elementary point symmetries to the 32 point groups*

**Translation Symmetry:** *lattice, motif, unit cell, directions and net planes, twins*

**Space group symmetry:** *glide planes and screw axes, the 230 space groups, the ITC*

## II- Magnetic structures

**Description in terms of propagation vector:** *the various orderings, examples*

**Description in terms of symmetry:**

**Magnetic point groups:** *time reversal, the 122 magnetic point groups*

**Magnetic lattices:** *translations and anti-translations, the 36 magnetic lattices*

**Magnetic space groups** = Shubnikov groups

## III- Determination of nucl. and mag. structures from neutron diffraction

**Nuclear and magnetic neutron diffraction:** *structure factors, extinction rules*

**Examples in powder neutron diffraction**

**Examples in single-crystal neutron diffraction**

# Crystallography: *introduction*

The word **crystal** comes from Greek (*krustallas*) and means “solidified by the cold”.

**Crystallography = science of crystals**

→ external shape, internal structure, crystal growth, and physical properties.

Objective: determine the atomic positions in the unit cell.

Mean: diffraction techniques (X-rays, neutrons, electrons)

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**Curie's principle : The symmetry of a cause is always preserved in its effects**



Existence or not of some phenomena, symmetries of the possible ones

Examples: existence or not of ferroelectricity

relations between the various components of the stress tensor

...

# Crystallography: introduction

Crystallography → Link between structure and physical properties

## 1- Translation symmetry

Periodicity of the physical properties: Solid state physics

- *Phonons, magnons, ...*
- *Diffraction*

## 2- Point (group) symmetry

Anisotropy of the physical properties: macroscopic physics

→ reflects the point symmetry of crystals

- *External shape of crystals (natural faces)*
- *Electric conductivity, optical, mechanical, magnetic, .... properties*

To describe crystals: geometrical aspects,  
symmetries, atomic positions, ...

**Direct space**

To determine the crystal  
structure: diffraction  
**Reciprocal space**

# Crystallography: *Outline*

## 1. Point group symmetry

Elementary point symmetry operations

Crystallographic point groups: definition, international notation

Examples of point groups

The 32 crystallographic point groups and 11 Laue classes

## 2. Translation symmetry

Lattice and motif, Unit cell

The orientation symmetries of lattices:

the 6 conventional cells, 7 crystal systems and 14 Bravais lattices

Lattice directions and net planes

## 3. Space group symmetry

Glide planes and screw axes

The 230 space groups

The International Tables for Crystallography

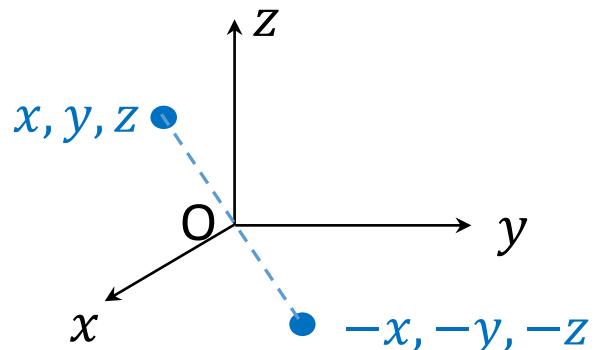
# 1. Point Group Symmetry: Elementary point symmetries

At the macroscopic & atomic (to within a translation) scales,  $\exists$  point symmetries, named **point symmetries**, that keep at least one point fixed, the origin.

Inversion (through a point)

→ *centrosymmetric* crystal

$\bar{1}$



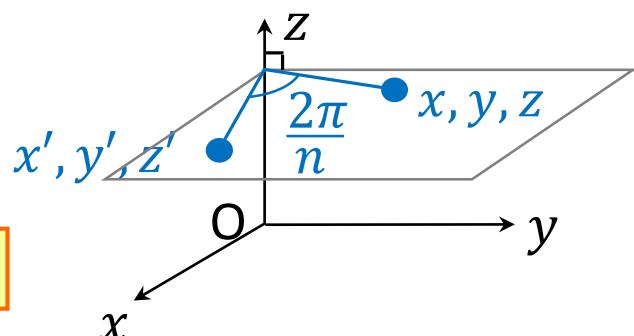
$$\alpha(\bar{1}) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Det = -1

Rotation (around an axis)

*Rotation of order n* = rotation by  $\frac{2\pi}{n}$

1, 2, 3, ...



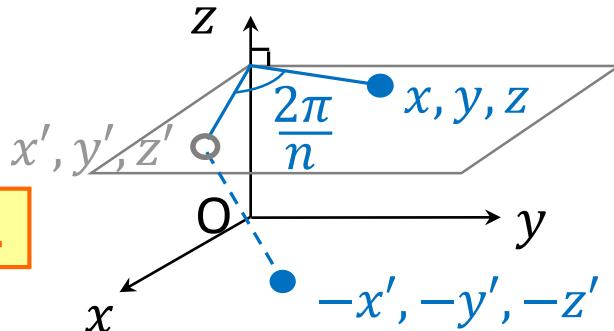
$$\alpha(n) = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Det = +1

# 1. Point Group Symmetry: Elementary point symmetries

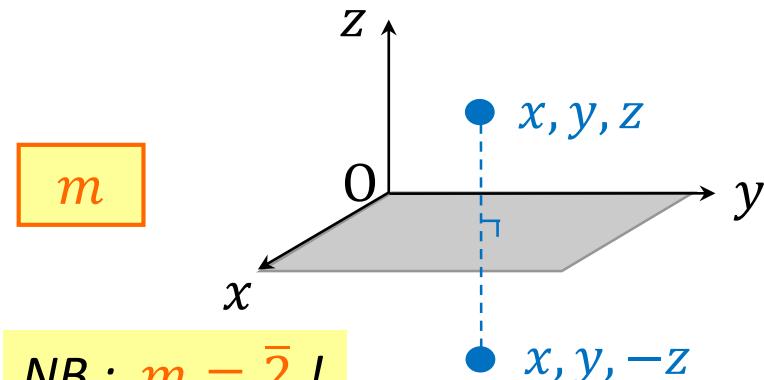
## Rotoinversion

(around an axis and through a point)



## Reflection

(through a mirror plane)



$$\alpha(n) = \begin{pmatrix} -\cos \phi & \sin \phi & 0 \\ -\sin \phi & -\cos \phi & 0 \\ 0 & 0 & -1 \end{pmatrix} \text{ Det} = -1$$

$$\alpha(m) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \text{ Det} = -1$$

Rotations compatible with the translation symmetry = those of orders 1, 2, 3, 4, 6

→ 10 elementary operations: point groups  $1, 2, 3, 4, 6, \bar{1}, \bar{2} = m, \bar{3}, \bar{4}, \bar{6}$

Proper (Det = 1)      Improper (Det = -1)

# 1. Point Group Symmetry: *Definition of a group*

The point symmetry operations form a group

A **group** ( $G, \times$ ) of order  $n$  is a set of distinct elements  $g_1, g_2, \dots, g_n$  equipped with an operation (**group multiplication**  $\times$ ) that combines any two elements to form a third element and that satisfies the four axioms:

**Closure:**  $g_i \times g_j \in G$

**Identity:**  $\exists ! e$  such that  $g \times e = e \times g = g$  → 1 (does nothing)

**Invertibility:** each element  $g$  has a unique inverse  $g^{-1}$  such that:  $g \times g^{-1} = g^{-1} \times g = e$  inverse of  $n$ :  $-n$   
(rotate in the other way)

**Associativity:**  $(g_i \times g_j) \times g_k = g_i \times (g_j \times g_k)$

**For point symmetry operations:**

$\times \leftrightarrow$  apply successively 2 symmetry operations

# 1. Point Group Symmetry: How to obtain and name all point groups?

## How to obtain all crystallographic point groups (= crystal classes) ?

Combine the 10 elementary symmetry operations, with the following constraints:

- all symmetry elements go through a common point,

- compatibility with the translation symmetry

⇒ constraints between the orientations of the various symmetry axes / planes

## Notation of the point groups – International (Hermann-Mauguin) symbol

Symmetry operations along 1, 2 or 3 directions (primary, secondary, tertiary), ordered with decreasing or equal degree of symmetry  
*(except for 2 cubic point groups)*

Examples :

$4/m$

$\frac{4}{m} \frac{2}{m} \frac{2}{m}$  (=  $4/mmm$ )

The direction of a rotation is given by its axis

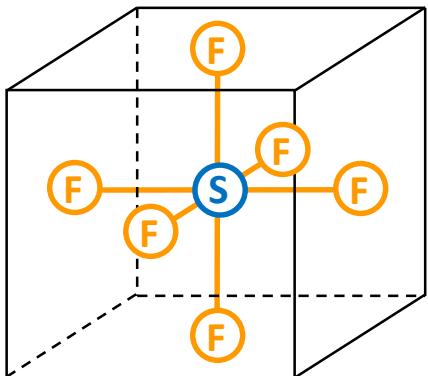
The direction of a mirror is given by its normal

' $n/m$ ' = axis  $n$  and normal to mirror  $m$  along same direction  
(i.e. plane of the mirror  $\perp$  to axis  $n$ )

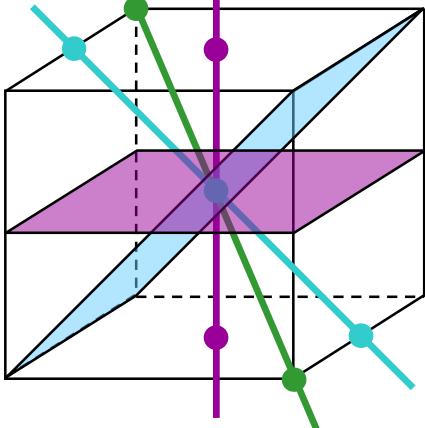
There exists another notation: **Schoenflies symbol** → widely used in spectroscopy

# 1. Point Group Symmetry: Elementary point symmetries

SF<sub>6</sub> molecule



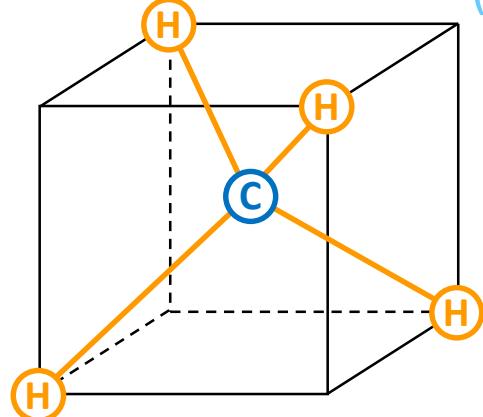
2 et  $m$  (tertiary)  
 $\bar{3}$  (secondary)  
4 et  $m$  (primary)



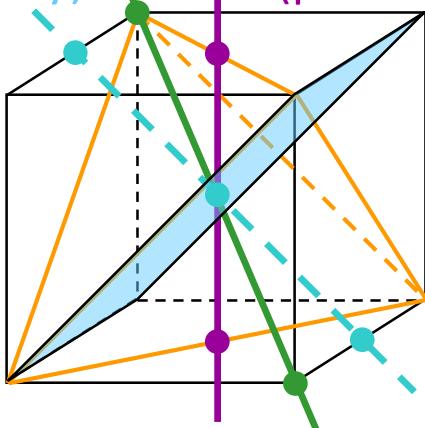
→ Point group:

$$\begin{matrix} 4 & \bar{3} & 2 \\ \hline m & 3 & m \end{matrix} (= m\bar{3}m)$$

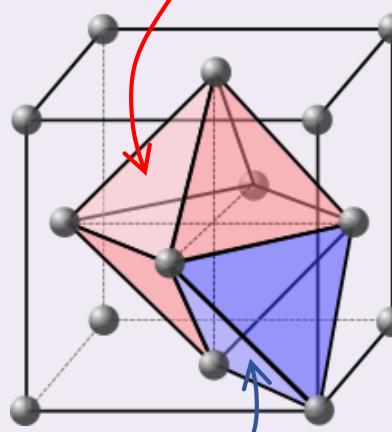
CH<sub>4</sub> molecule



$m$  (tertiary)  
3 (secondary)  
 $\bar{4}$  (primary)



Octahedral site :  $m\bar{3}m$  symmetry



Tetrahedral site :  $\bar{4}3m$  symmetry

→ Point group:  
 $\bar{4}3m$

# 1. Point Group Symmetry: Elementary point symmetries

Order of the point symmetry along the: primary direction	secondary direction	tertiary direction	Point groups (short symbols) and Laue classes
—	—	—	1, $\bar{1}$
2	—	—	2, $m$ , $2/m$
2	2	2	222, $2mm$ , $mmm$
3	—	—	3, $\bar{3}$
3	2	—	32, $3m$ , $\bar{3}m$
4	—	—	4, $\bar{4}$ , $4/m$
4	2	2	422, $4mm$ , $\bar{4}2m$ , $4/mmm$
6	—	—	6, $\bar{6}$ , $6/m$
6	2	2	622, $6mm$ , $\bar{6}2m$ , $6/mmm$
2	3	—	23, $m\bar{3}$
4	3	2	432, $\bar{4}3m$ , $m\bar{3}m$

# 1. Point Group Symmetry: Elementary point symmetries

- Example: dielectric properties

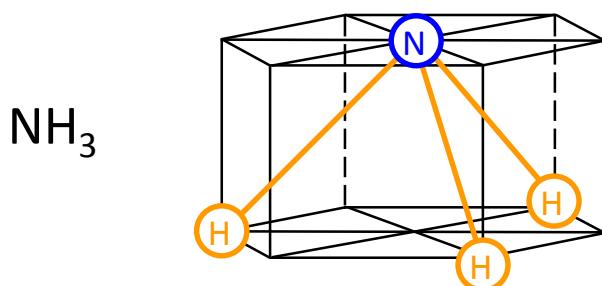
They can only be found for particular crystal symmetries

**Piezoelectricity** → point groups that do not possess inversion

**Ferroelectricity and pyroelectricity**

→ piezoelectric point groups (i.e. non centrosymmetric)  
with a unique polar axis ( $\vec{p} \parallel n$ -axis and contained in the plane of the mirror):

1, 2, m, 2mm, 3, 3m, 4, 4mm, 6, 6mm  
**polar groups**



Point group: 3m  
→  $\exists$  dipolar moment ( $p = 1.46$  Debye)

## 2. Translation Symmetry: *Lattice and motif*

At the atomic scale,  $\exists$  translation vectors  $\vec{T}$  that put the crystallographic structure in coincidence with itself.

$$\vec{T} = u\vec{a} + v\vec{b} + w\vec{c} \text{ with } u, v, w \text{ integers} \quad (\text{positive or negative})$$

$\vec{a}, \vec{b}$ , and  $\vec{c}$  are called the **basis vectors**

(non-coplanar elementary translation vectors defining a right-handed system).

The volume they define is called the **unit cell**.

Crystal = Lattice + Motif



The set of extremities of the  $\vec{T}$  vectors define an abstract network of points (= nodes): the **lattice**.

At each lattice node, one associates a group of atoms: the **motif**.

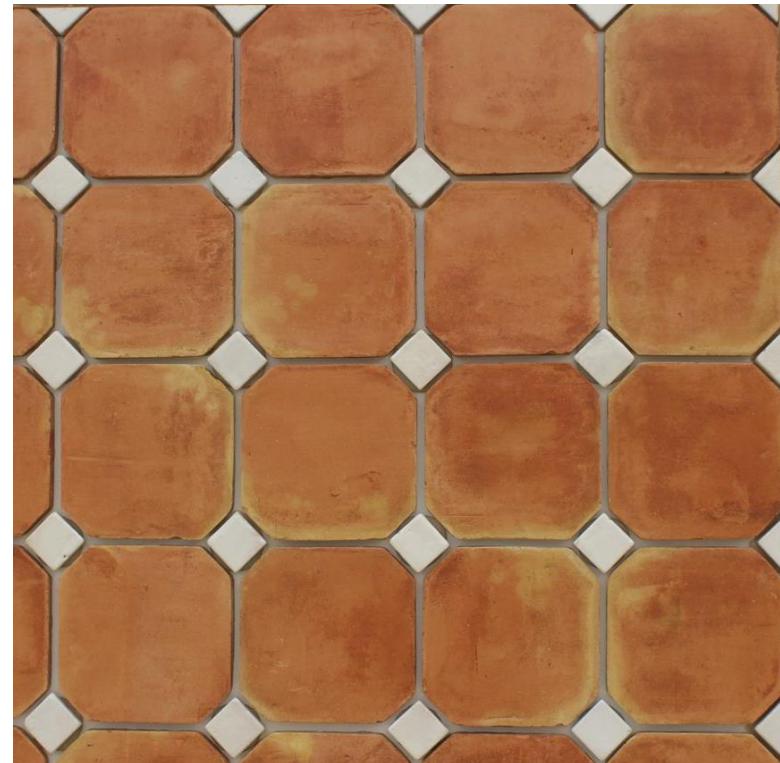
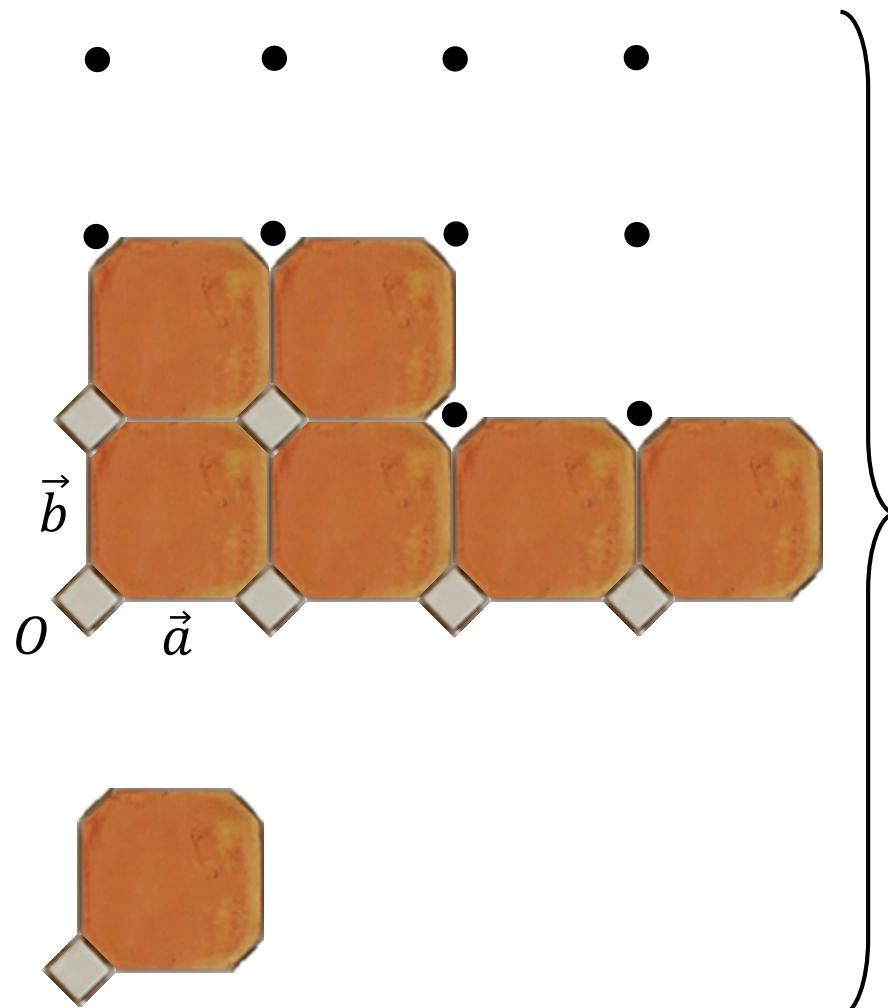
The knowledge of the lattice (basis vectors  $\vec{a}, \vec{b}, \vec{c}$ ) and of the motif (nature and positions  $x, y, z$  of the atoms in the cell) completely characterizes the crystalline structure.

N.B. :  $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$  ( $|x|, |y|, |z| < 1$ )

## 2. Translation Symmetry: *Lattice and motif*

Example 1 : terracotta floor tiles (2D)

Lattice



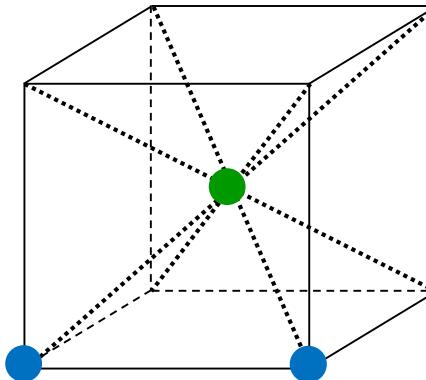
Motif

## 2. Translation Symmetry: Lattice and motif

Example 2 : CsCl single-crystal (3D)

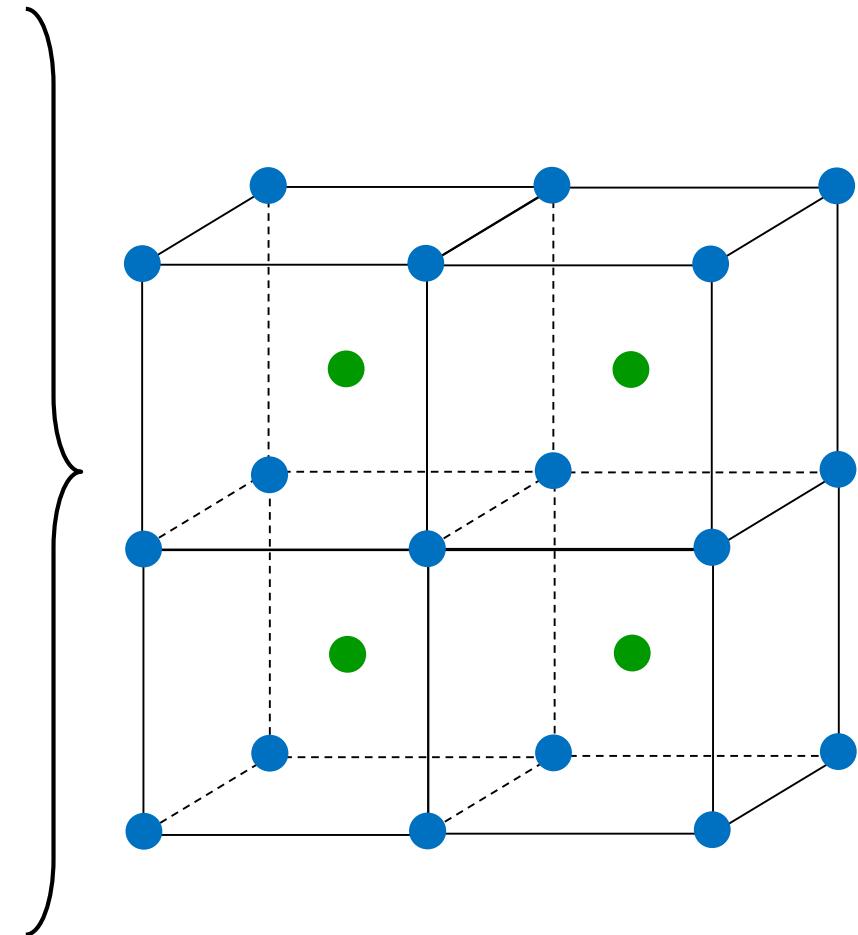
Unit cell:

cubic  
primitive



Cs<sup>+</sup> on the corner

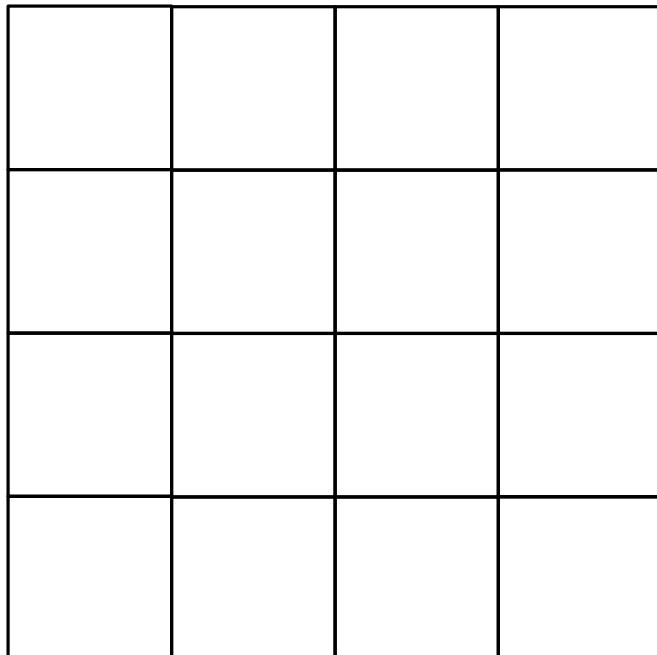
Cl<sup>-</sup> at the center



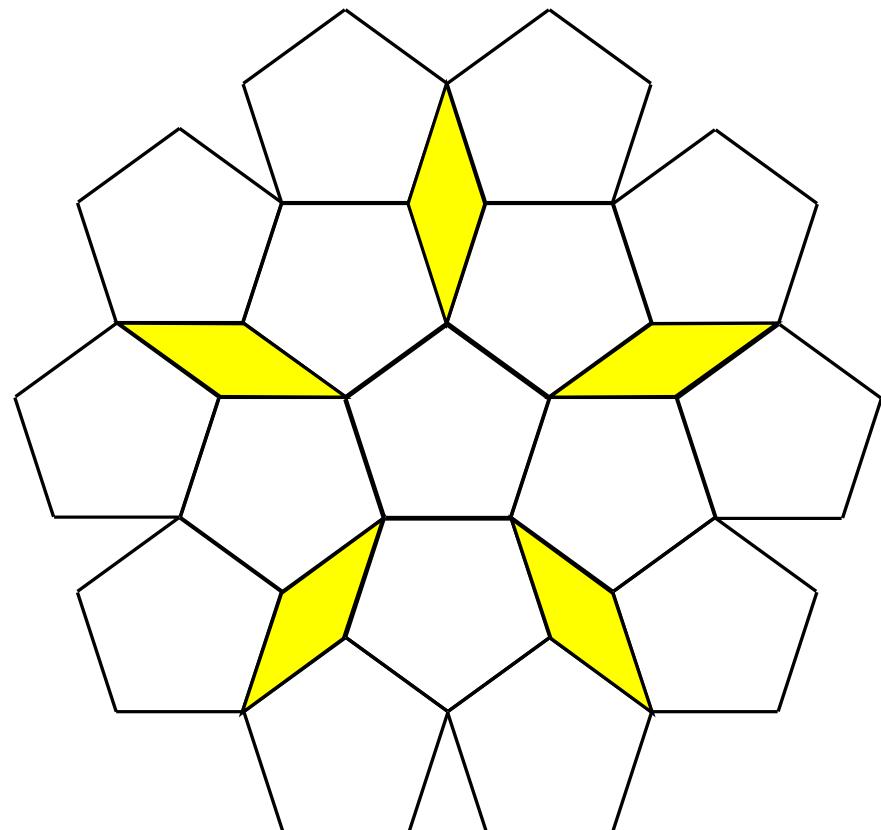
## 2. Translation Symmetry: *Unit cell*

The **unit cell** allows to pave the space with no empty space nor overlap, by applying the lattice translations.

Examples at 2D:

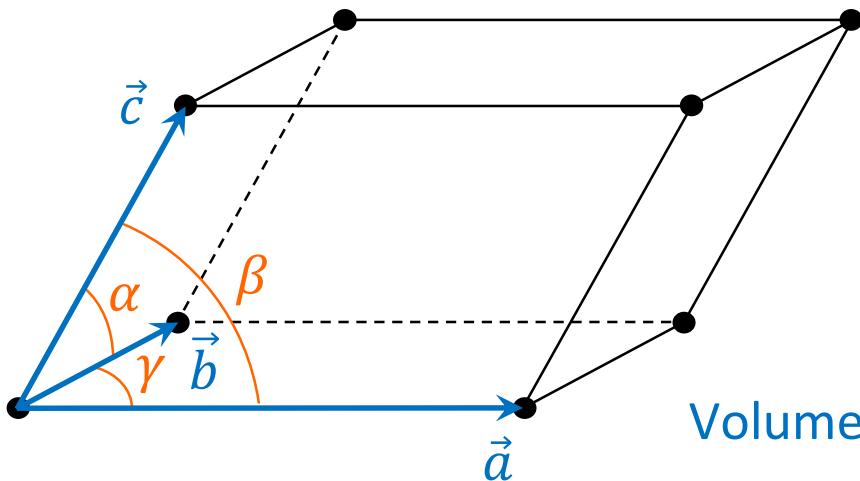


Rotation of order 4 : compatible with translation symmetry.



Rotation of order 5 : not compatible with translation symmetry → **quasicrystals**

## 2. Translation Symmetry: *Unit cell*



Lattice parameters:

Lengths	Angles
$a$	$\alpha = (\vec{b}, \vec{c})$
$b$	$\beta = (\vec{c}, \vec{a})$
$c$	$\gamma = (\vec{a}, \vec{b})$

Volume of the unit cell:

$$V = (\vec{a}, \vec{b}, \vec{c}) = (\vec{a} \wedge \vec{b}) \cdot \vec{c}$$

- Multiplicity  $m$  of a unit cell: Number of lattice nodes (and thus of motifs) per unit cell

How to count the number of lattice nodes per unit cell?

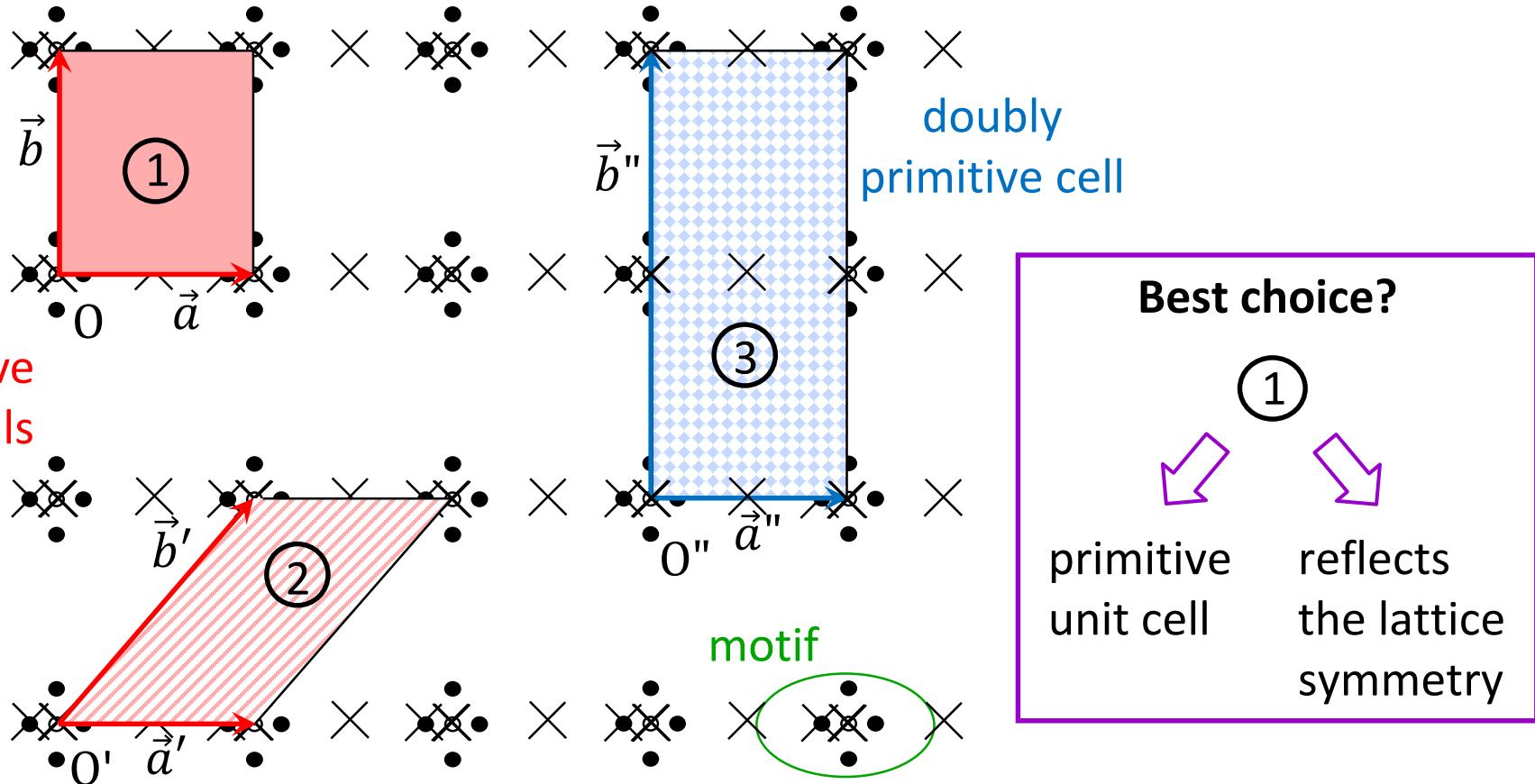
→ each lattice node counts for  $1/n$ , with  $n$  = number of unit cells to which it belongs

- Primitive unit cell:  $m = 1$

For a given lattice, all primitive unit cells have the same volume  $V$

- Centered unit cell:  $m = 2, 3$  or  $4$  (doubly, triply ... primitive) → Volume :  $V_m = m V$   
→ used only when more symmetrical than any primitive cell of the lattice

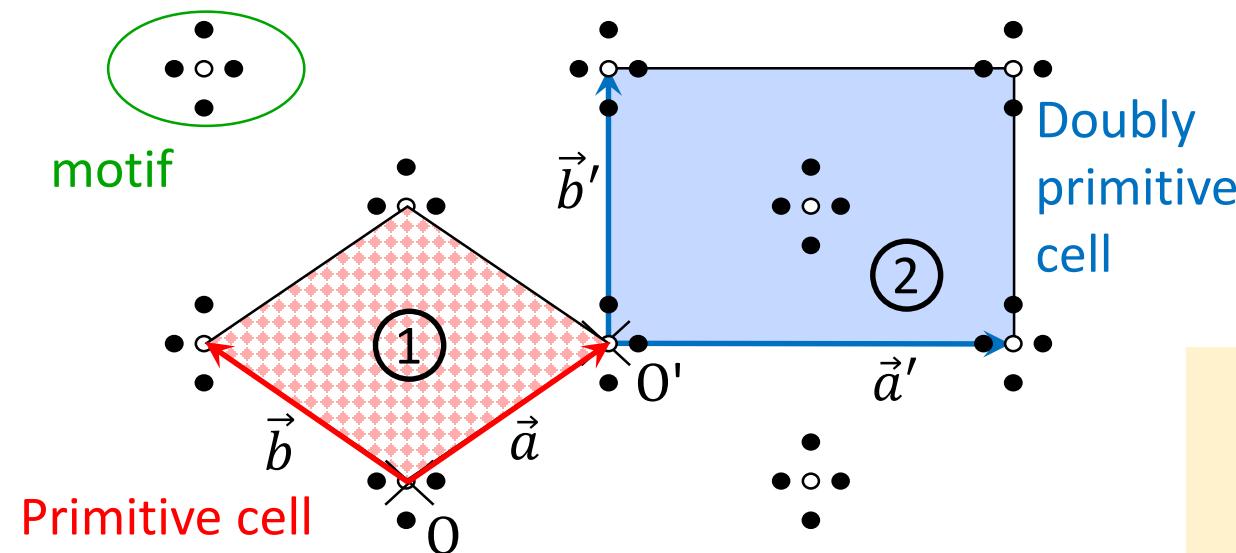
## 2. Translation Symmetry: *Unit cell*



Primitive cells: 4 lattice nodes (on corners)  $\in$  4 cells  $\rightarrow m = 4 \times 1/4 = 1$

Doubly primitive cell: 4 nodes (on corners)  $\in$  4 cells  $\rightarrow 4 \times 1/4 = 1$  }  
+ 2 nodes (on edges)  $\in$  2 cells  $\rightarrow 2 \times 1/2 = 1$  }  $m = 2$

## 2. Translation Symmetry: Unit cell



Cell 1 is primitive but does not reflect the  $\perp^{\text{ty}}$   
 Best choice: 2

Conventional unit cell  
 (basis vectors  $\parallel$  directions of symmetry of the lattice)

N.B.: For a primitive cell, the translation vectors  $\vec{T}$  are defined by:

$$\vec{T} = u\vec{a} + v\vec{b} + w\vec{c} \text{ with } u, v, w \text{ integers.}$$

For a non primitive cell of multiplicity  $m$ , one must add  $(m - 1)$  translation vectors such as:  $\vec{T} = u'\vec{a} + v'\vec{b} + w'\vec{c}$  with  $u', v', w'$  integers or fractionals

Ex.: For unit cell 2 ( $m = 2$ ):

$$\begin{cases} \vec{T}_1 = u\vec{a}' + v\vec{b}' \\ \vec{T}_2 = \vec{T}_1 + \frac{1}{2}(\vec{a}' + \vec{b}') = \left(u + \frac{1}{2}\right)\vec{a}' + \left(v + \frac{1}{2}\right)\vec{b}' \end{cases}$$

half integers

## 2. Translation Symmetry: The 6 conventional cells and 7 crystal systems

Translation and orientation (point) symmetries:

 The crystals can be classified into **6 conventional cells** and **7 crystal systems**  
each of them having a characteristic orientation symmetry

The 6 conventional cells are, by increasing degree of symmetry:

				Number of parameters
<i>a</i>	triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma$	6
<i>m</i>	monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90^\circ, \beta > 90^\circ$	4
<i>o</i>	orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	3
<i>t</i>	tetragonal or quadratic	$a = b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	2
<i>h</i>	hexagonal **	$a = b \neq c$	$\alpha = \beta = 90^\circ, \gamma = 120^\circ *$	2
<i>c</i>	cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$	1

\*  $\gamma = 120^\circ$  and not  $60^\circ$  (for the hexagonal reciprocal lattice:  $\gamma^* = 60^\circ$ )

\*\* The hexagonal conventional cell splits in two crystal systems:  
*trigonal* (axis 3) and *hexagonal* (axis 6); the 5 other ones are the same.

## 2. Translation Symmetry: *Crystal system vs point group*

Crystal system	Point groups and Laue classes	Primary direction	Secondary direction	Tertiary direction
triclinic	$1, \bar{1}$	—	—	—
monoclinic	$2, m, 2/m$	$\vec{b}$ (ou $\vec{c}$ )	—	—
orthorhombic	$222, 2mm, mmm$	$\vec{a}$	$\vec{b}$	$\vec{c}$

## 2. Translation Symmetry: Crystal system vs point group

Crystal system	Point groups and Laue classes	Primary direction	Secondary direction	Tertiary direction
triclinic	$1, \bar{1}$	—	—	—
monoclinic	$2, m, 2/m$	$\vec{b}$ (ou $\vec{c}$ )	—	—
orthorhombic	$222, 2mm, mmm$	$\vec{a}$	$\vec{b}$	$\vec{c}$
trigonal	$3, \bar{3}$ $32, 3m, \bar{3}m$	$\vec{c}$	$\vec{a}, \vec{b}, -\vec{a}-\vec{b}$	—
tetragonal or quadratic	$4, \bar{4}, 4/m$ $422, 4mm, \bar{4}2m, 4/mmm$	$\vec{c}$	$\vec{a}, \vec{b}$	$\vec{a}+\vec{b}, \vec{a}-\vec{b}$
hexagonal	$6, \bar{6}, 6/m$ $622, 6mm, \bar{6}2m, 6/mmm$	$\vec{c}$	$\vec{a}, \vec{b}, -\vec{a}-\vec{b}$	$2\vec{a}+\vec{b}, \dots$

## 2. Translation Symmetry: Crystal system vs point group

Crystal system	Point groups and Laue classes	Primary direction	Secondary direction	Tertiary direction
triclinic	$1, \bar{1}$	—	—	—
monoclinic	$2, m, 2/m$	$\vec{b}$ (ou $\vec{c}$ )	—	—
orthorhombic	$222, 2mm, mmm$	$\vec{a}$	$\vec{b}$	$\vec{c}$
trigonal	$3, \bar{3}$ $32, 3m, \bar{3}m$	$\vec{c}$	$\vec{a}, \vec{b}, -\vec{a}-\vec{b}$	—
tetragonal or quadratic	$4, \bar{4}, 4/m$ $422, 4mm, \bar{4}2m, 4/mmm$	$\vec{c}$	$\vec{a}, \vec{b}$	$\vec{a}+\vec{b}, \vec{a}-\vec{b}$
hexagonal	$6, \bar{6}, 6/m$ $622, 6mm, \bar{6}2m, 6/mmm$	$\vec{c}$	$\vec{a}, \vec{b}, -\vec{a}-\vec{b}$	$2\vec{a}+\vec{b}, \dots$
cubic	$23, m\bar{3}$ $432, \bar{4}3m, m\bar{3}m$	$\vec{a}, \vec{b}, \vec{c}$	$\vec{a}+\vec{b}+\vec{c}, \dots$	$\vec{a}+\vec{b}, \dots$

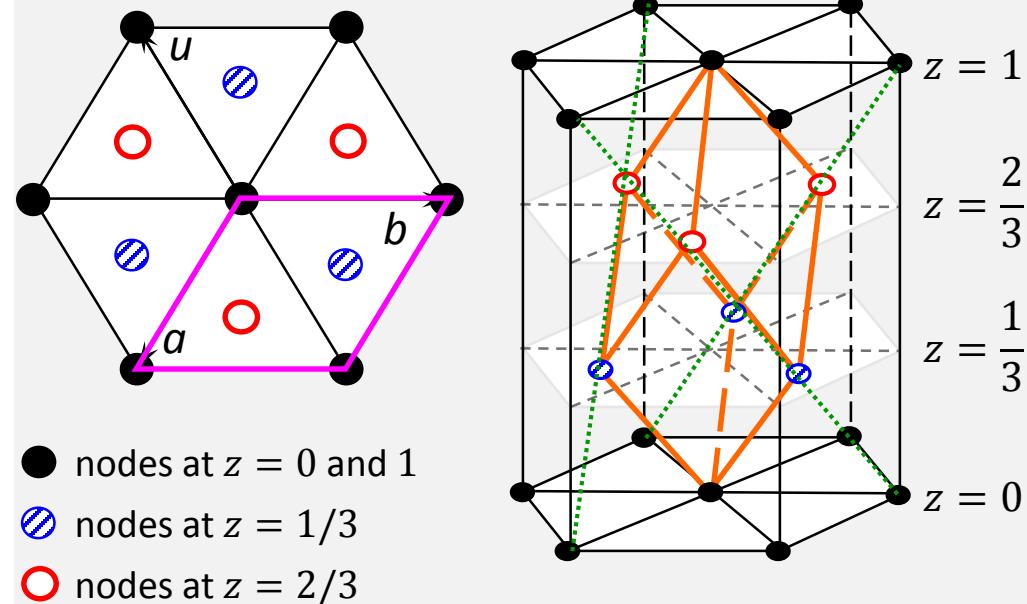
## 2. Translation Symmetry: The 14 Bravais lattices

- 6 primitive lattices (one for each of the 6 conventional cells),
- 8 non primitive ones, by adding nodes in the former cells, provided no symmetry element is lost & the centered cell is more symmetric than any primitive cell.

Symbol	Lattice mode	$m$
$P$	primitive	1
$I$	body centered	2
$F$	all face centered	4
$A, B, C$	$A$ -, $B$ -, $C$ -face centered: $(\vec{b}, \vec{c}), (\vec{a}, \vec{c}), (\vec{a}, \vec{b})$ respectively	2
$R$	rhombohedrally centered: additional lattice nodes at $1/3$ and $2/3$ of the long diagonal of the $h$ cell $(\rightarrow$ trigonal system)	3

N.B.: the primitive cell of the  $hR$  cell is a rhombohedral cell

$$(a = b = c, \alpha = \beta = \gamma \neq 90^\circ)$$



## 2. Translation Symmetry: The 14 Bravais lattices

Conventional cell	Lattice mode				
	P	I	F	C	R
triclinic					
monoclinic					
orthorhombic					
tetragonal					
hexagonal					
cubic					

**Reminder:**  
For centered cells,  
 $\exists$  additional lattice  
translations.

Example: I lattice

$$\begin{cases} \vec{T} = u\vec{a} + v\vec{b} + w\vec{c} \\ \vec{T}' = \vec{T} + \frac{1}{2}(\vec{a} + \vec{b} + \vec{c}) \end{cases}$$

with  $u, v, w$  integers

## 2. Translation Symmetry: Example – the diamond structure

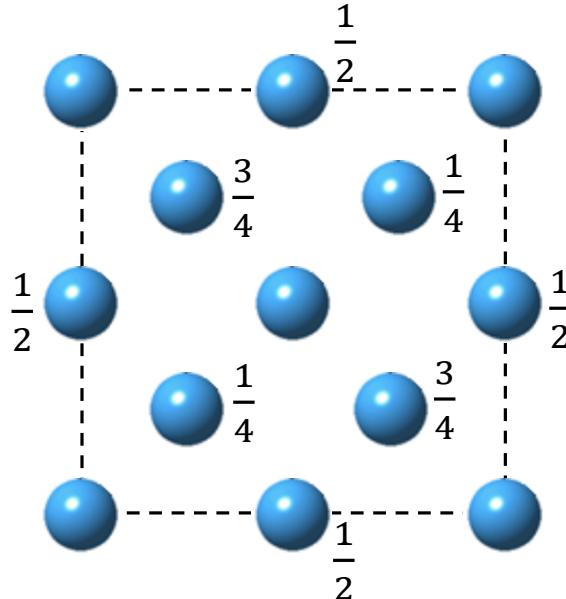
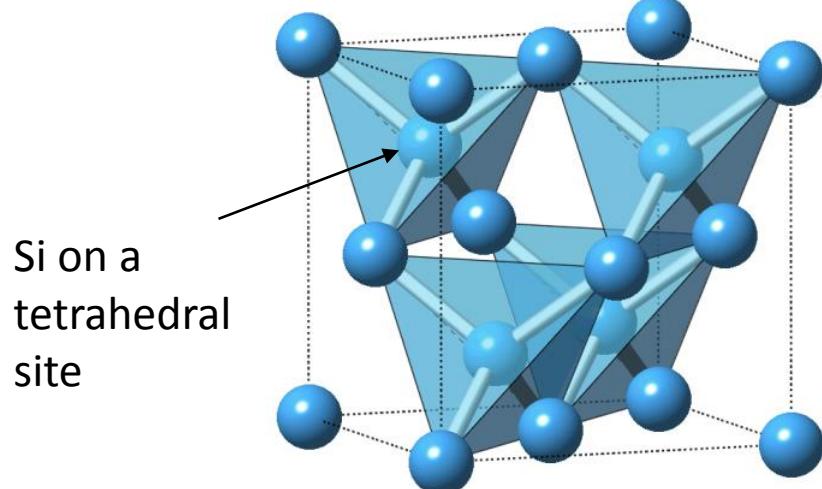
Si (diamond structure): cubic  $F$  lattice, motif = atoms at  $(0,0,0)$  and  $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

$F$  lattice ( $m = 4$ ) → lattice translations:

$$\vec{T}_1 = u\vec{a} + v\vec{b} + w\vec{c}, \vec{T}_2 = \vec{T}_1 + \frac{1}{2}(\vec{a} + \vec{b}), \vec{T}_3 = \vec{T}_1 + \frac{1}{2}(\vec{b} + \vec{c}), \vec{T}_4 = \vec{T}_1 + \frac{1}{2}(\vec{a} + \vec{c})$$

→  $4 \times 2 = 8$  Si atoms per unit cell with coordinates:

$$(0,0,0), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right), \text{ and } \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right), \left(\frac{3}{4}, \frac{3}{4}, \frac{1}{4}\right), \left(\frac{1}{4}, \frac{3}{4}, \frac{3}{4}\right), \left(\frac{3}{4}, \frac{1}{4}, \frac{3}{4}\right)$$



## 2. Translation Symmetry: *Lattice directions* [ $uvw$ ]

- Family of lattices directions

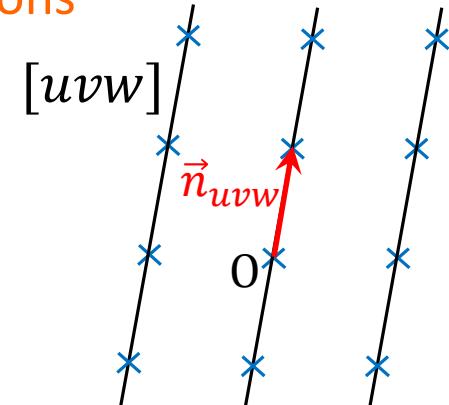
One can group all lattice nodes into parallel equidistant directions

labelled  $[uvw]$  along  $\vec{n}_{uvw} = u\vec{a} + v\vec{b} + w\vec{c}$

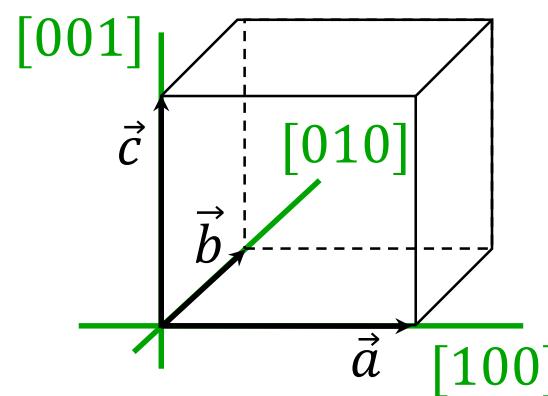
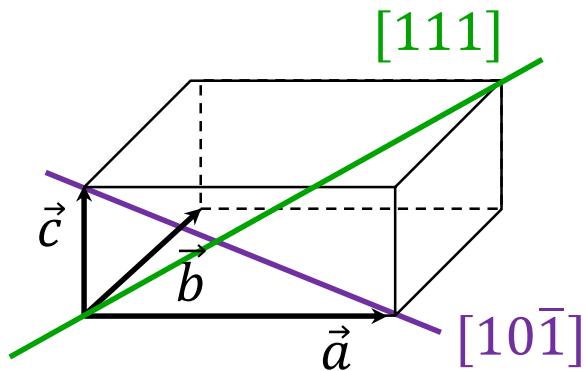
$n_{uvw}$  (length of the direction vector): direction parameter

$u, v, w$  (coprime integers): direction indices

A family of lattice directions contains all lattice points.



- Examples:



Cubic unit cell:  
→ directions  
symmetrically  
equivalent are  
labeled  $\langle 100 \rangle$

## 2. Translation Symmetry: Net planes ( $hkl$ )

- Family of net planes

One can group all lattice nodes into parallel equidistant net planes labelled  $(hkl)$  of equation:  $hx + ky + lz = m$  with  $m$  integer ( $> 0$  or  $< 0$ )

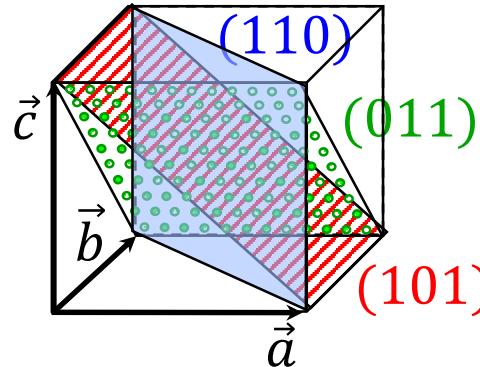
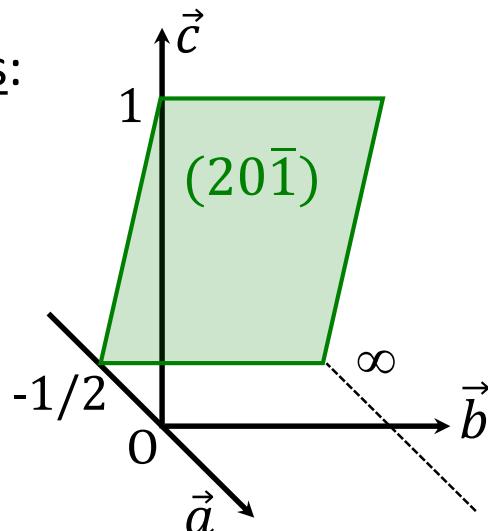
The plane the closest to the origin ( $m = 1$ ) intercepts the  $\vec{a}$  axis at  $1/h$ , the  $\vec{b}$  axis at  $1/k$ , and the  $\vec{c}$  axis at  $1/l$ .

$h, k, l$  (integers, which are coprime for a  $P$  lattice): Miller indices

$d_{hkl}$  (distance between 2 consecutive planes):  $d$ -spacing

A family of net planes contains all lattice points.

- Examples:



Cubic unit cell:  
→ planes  
symmetrically  
equivalent are  
labeled  $\{110\}$

# Symmetry relations between crystals: *twinned crystals*

**Twinned crystal:** association of identical single-crystals with different orientations, connected through a point group symmetry: reflection, rotation, or inversion.

## Formation of twinned crystals

- Growth twins: occurs during the crystal growth;
- Annealing or Transformation twins: upon cooling (phase transition)
- Deformation or gliding twins: result of stress after the crystal has formed



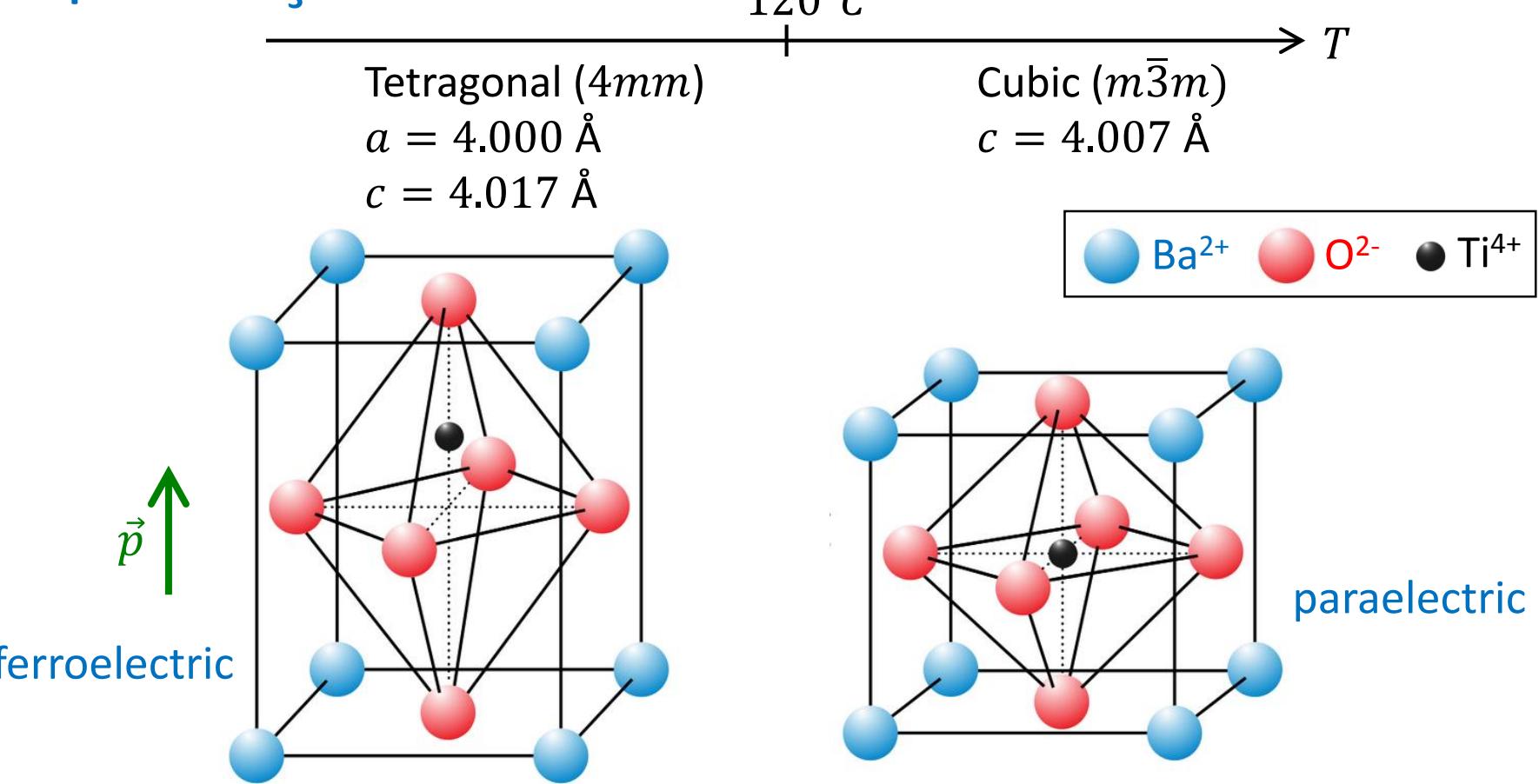
*Twinned pyrite crystal*



*Japanese twins of quartz*

# Symmetry relations between crystals: phase transitions

Example : BaTiO<sub>3</sub>



With no external stress (pressure, electric field, ...):

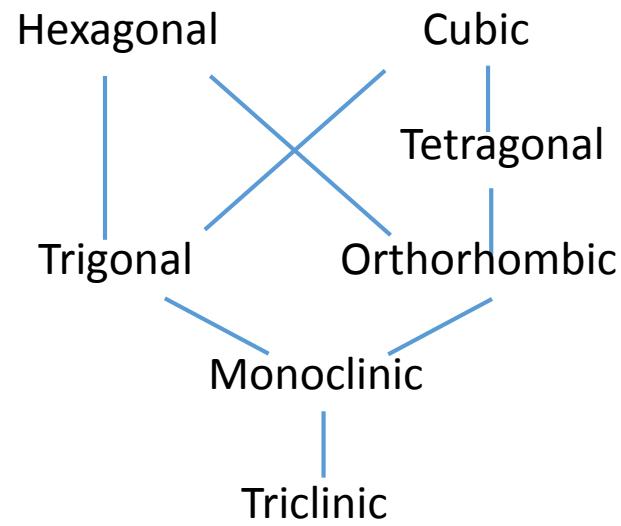
**3 different twins** with 2 domains at 180° each

# Symmetry relations between crystals: phase transitions

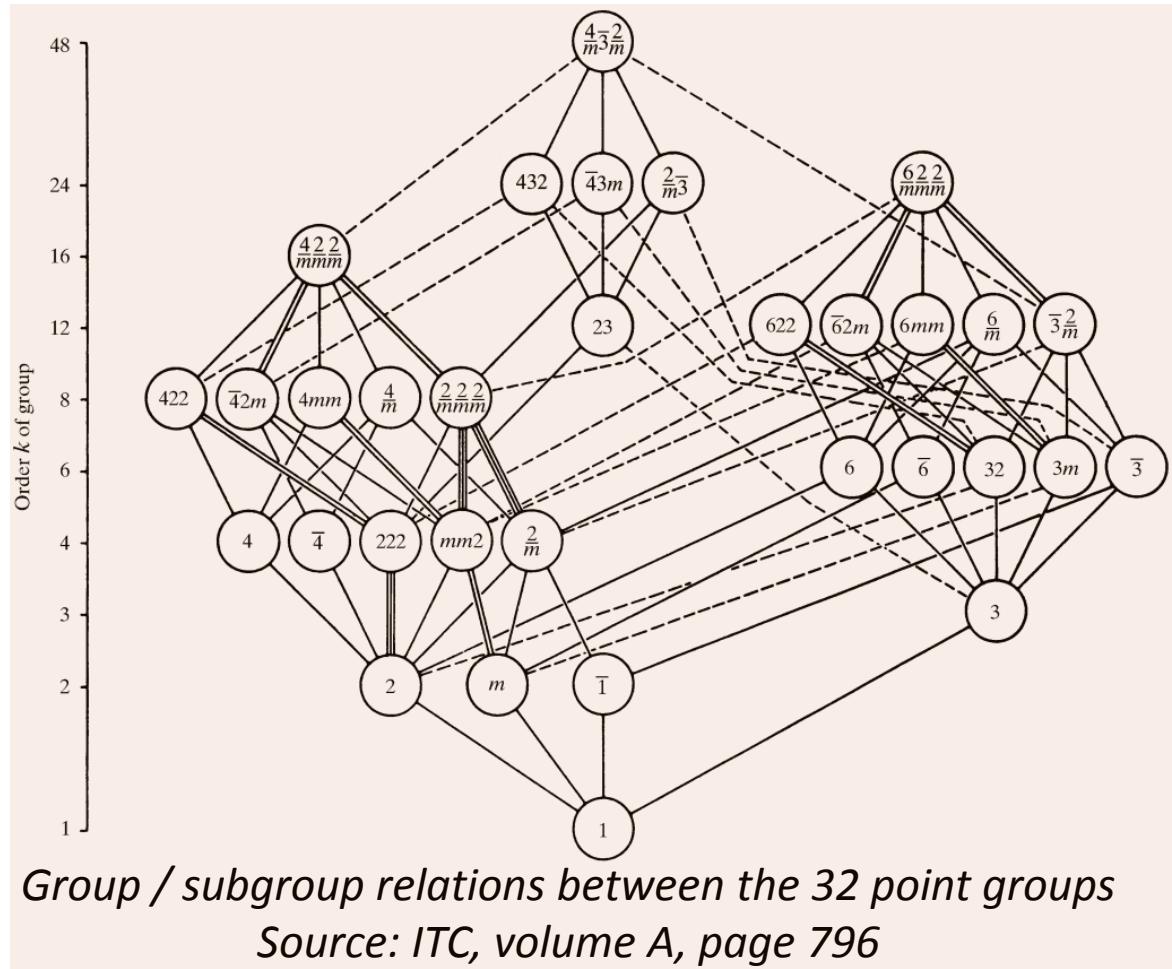
## 2<sup>nd</sup> order phase transition:

There exist a group / subgroup relation between the 2 phases

Example: cooling down → **symmetry lowers** (change of point group)



*Relation between  
the 7 crystal systems*



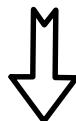
### 3. Space group symmetry

Crystal = **lattice** + **motif**

**translations  $\vec{T}$**

+

**$\exists$  symmetries acting inside the motif**  
(symmetry planes and axes)



14 Bravais lattices

Point symmetries  
(32 point groups)  
combined or not with  
a fractional translation

#### 230 SPACE GROUPS

- Describe the symmetry of the internal structure of crystals
  - Allow to classify all the crystals
- 
- International Tables for Crystallography (ITC) (<https://it.iucr.org>)
  - Bilbao Crystallographic Server (<http://www.cryst.ehu.es>)
  - A Hypertext Book of Crystallographic Space Group Diagrams and Tables (<http://img.chem.ucl.ac.uk/sgp/mainmenu.htm>)

### 3. Space group symmetry

INTERNATIONAL TABLES  
for CRYSTALLOGRAPHY



ⓘ 🔒 <https://it.iucr.org/resources/>

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INTERNATIONAL TABLES Resources

| [A](#) | [A1](#) | [B](#) | [C](#) | [D](#) | [E](#) | [F](#) | [G](#) |

[Home](#) > [Resources](#)

## International Tables for Crystallography Resources

The following resources are available as part of International Tables Online:

- Search for a crystallographic symmetry group

**Go to**  **No.**  **Go**

- Symmetry database
- Retrieve scattering factors for electron diffraction
- Plot scattering factors for electron diffraction
- Retrieve scattering lengths for neutron diffraction
- Resources for Volume D (*Tenxar* and *GI\*KoBo-1*)
- Superspace Group Finder
- CIF dictionaries
  - Core CIF Dictionary
  - Electron Density CIF Dictionary
  - Image CIF Dictionary
  - Macromolecular CIF Dictionary
  - Modulated Structures CIF Dictionary
  - Powder CIF Dictionary
  - Symmetry CIF Dictionary

### 3. Space group symmetry

(i) [www.cryst.ehu.es](http://www.cryst.ehu.es)

bilbao crystallographic server

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How to

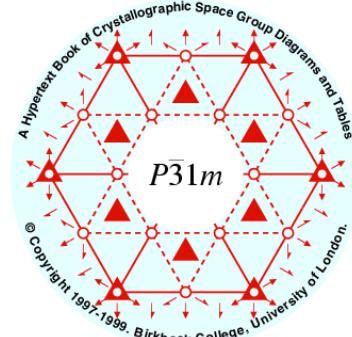
#### Space-group symmetry

GENPOS  
WYCKPOS  
HKLCOND  
MAXSUB  
SERIES  
WYCKSETS  
NORMALIZER  
KVEC  
SYMMETRY OPERATIONS  
IDENTIFY GROUP

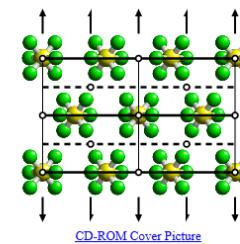
Generators and General Positions of Space Groups  
Wyckoff Positions of Space Groups  
Reflection conditions of Space Groups  
Maximal Subgroups of Space Groups  
Series of Maximal Isomorphic Subgroups of Space Groups  
Equivalent Sets of Wyckoff Positions  
Normalizers of Space Groups  
The  $k$ -vector types and Brillouin zones of Space Groups  
Geometric interpretation of matrix column representations of symmetry operations  
Identification of a Space Group from a set of generators in an arbitrary setting



(i) [img.chem.ucl.ac.uk/sgp/mainmenu.htm](http://img.chem.ucl.ac.uk/sgp/mainmenu.htm)



A Hypertext Book of  
**Crystallographic Space Group  
Diagrams and Tables**



CD-ROM Cover Picture



High-Resolution Space Group  
Diagrams and Tables  
(1280 × 1024 pixel screens)

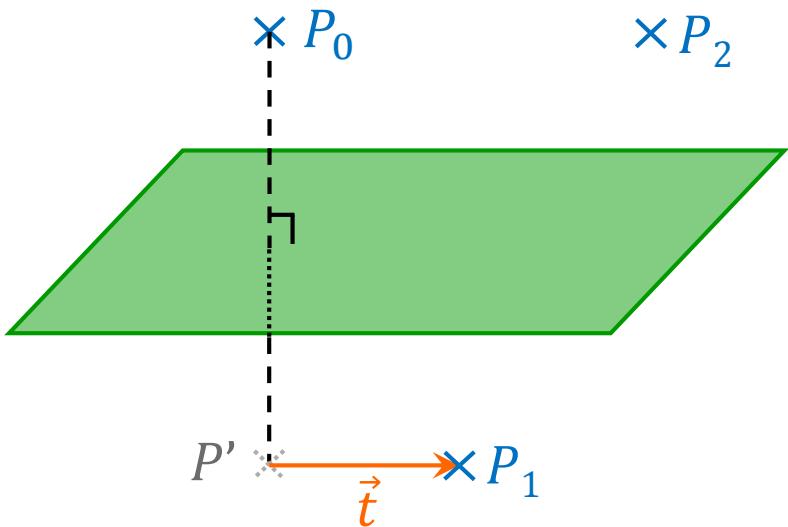


Medium-Resolution Space Group  
Diagrams and Tables  
(1024 × 768 pixel screens)

### 3. Space group symmetry: *symmetry planes*

- Glide plane

Combination of a **reflection** (through a plane) and a fractional translation  $\vec{t} \parallel$  plane  
*acting inside the unit cell*



$\alpha$ : point symmetry

$\vec{t}_\alpha$ : translation embedding the glide translation + the position of  $\alpha$

Example: glide plane  $a \perp \vec{c}$  at  $z = \frac{1}{4}$   
 $a \times a \rightarrow$  lattice translation

$$P_0P_2 = \vec{a} \rightarrow \boxed{\vec{t} = \frac{\vec{a}}{2}}$$

Seitz notation:  $\{\alpha | \vec{t}_\alpha\} = \{m_z | \frac{1}{2}, 0, \frac{1}{2}\}$

4 × 4 matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

### 3. Space group symmetry: symmetry planes

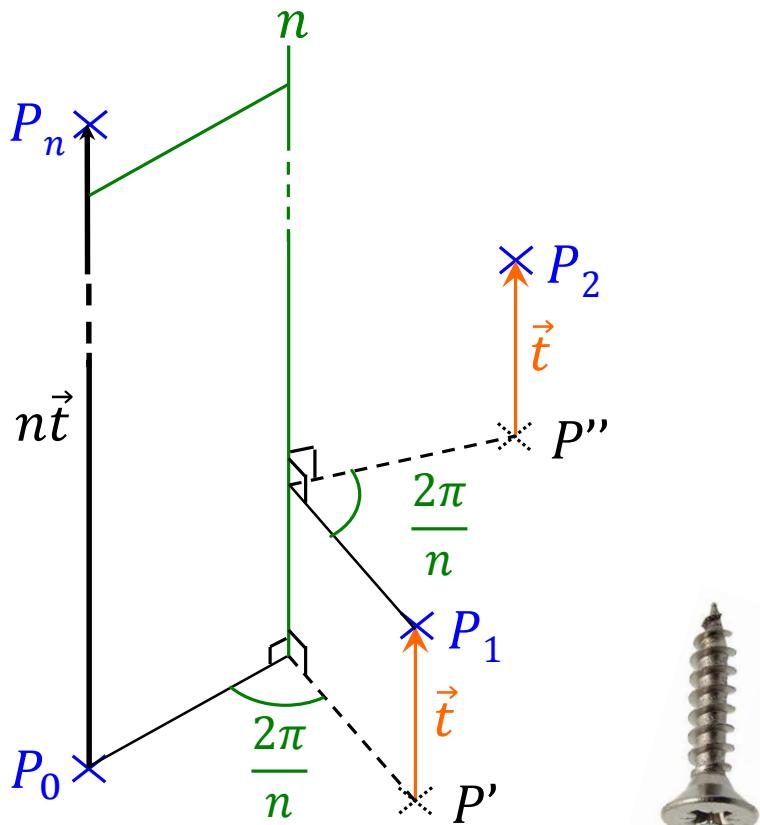
The various symmetry planes and their Hermann-Mauguin symbol

Printed symbol	Symmetry plane	Graphic symbol		Nature of the gliding (fractional translation $\vec{t}$ )
		⊥ projection plane	projection plane	
$m$	mirror	—	— / — \	none
$a, b, c$	Axial glide plane	$\vec{t} \parallel \text{proj. plane}$ $\vec{t} \perp \text{proj. plane}$	— ↓ ←	$a/2, b/2, \text{ or } c/2$ respectively
$e$	Double glide plane	— · · · · —	← ↓	$a/2$ and $b/2, b/2$ and $c/2$ , or $a/2$ et $c/2$ ; OR $(a \pm b)/2$ and $c/2$ , etc ... for $t$ and $c$ systems
$n$	Diagonal glide plane	— · · · · —	— ↗	$(a+b)/2, (b+c)/2$ or $(c+a)/2$ ; OR $(a+b+c)/2$ in some cases for $t$ and $c$ systems
$d$	Diamond glide plane	← · · · →	$\frac{1}{8}$ ↗ $\frac{1}{8}$	$(a+b)/4, (b+c)/4$ or $(c+a)/4$ ; OR $(a+b+c)/4$ in some cases for $t$ and $c$ systems

### 3. Space group symmetry: symmetry axes

- Screw axes

Combination of a **rotation** (around an axis  $n$ ) and a **fractional translation**  $\vec{t} \parallel$  axis



Example: screw axis  $n_p \parallel \vec{c}$

$\underbrace{n_p \times \cdots \times n_p}_{n \text{ times}} \rightarrow$  lattice translation

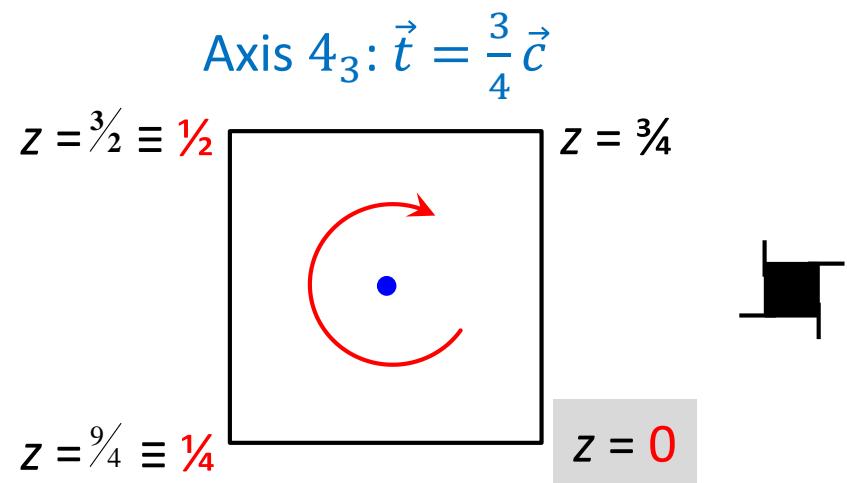
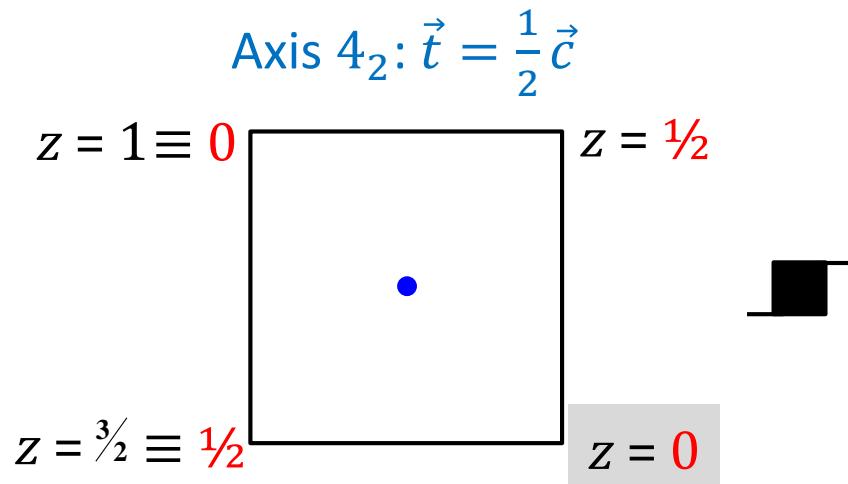
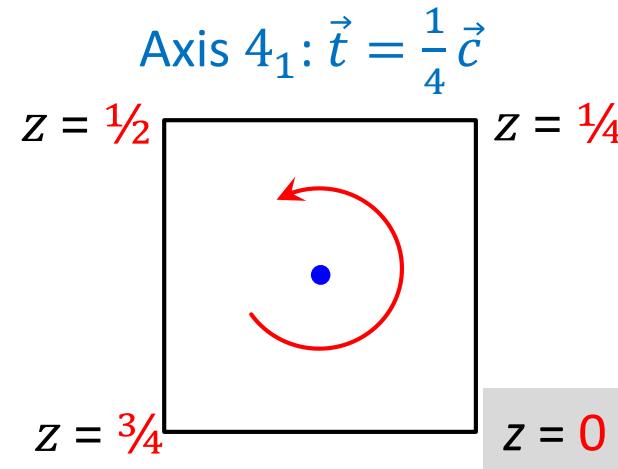
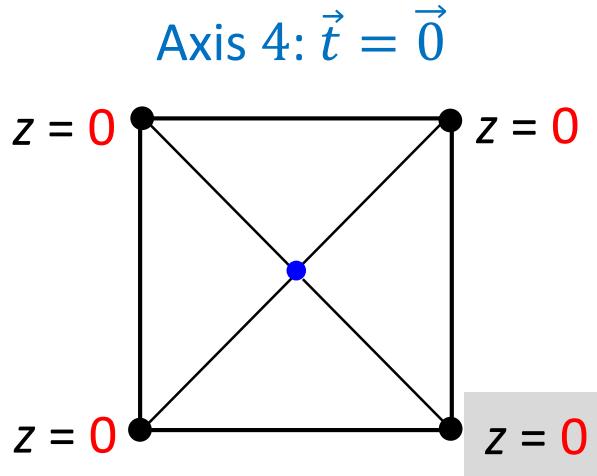
$$\overrightarrow{P_0 P_n} = n\vec{t} = p\vec{c}$$

with  $\begin{cases} n = 1, 2, 3, 4, \text{ or } 6 \\ p \text{ integer } < n \end{cases}$

$$\rightarrow \boxed{\vec{t} = \frac{p}{n} \vec{c}} \quad \text{with} \quad \boxed{p = 0, 1, \dots, n - 1}$$

### 3. Space group symmetry: symmetry axes

Example: screw axes  $4_p \parallel \vec{c}$



### 3. Space group symmetry: symmetry axes

The various symmetry axes and their Hermann-Mauguin symbol (*projection plane  $\perp \vec{c}$* )

Printed symbol	Symmetry axis	Graphic symbol	Gliding $\vec{t}$	Printed symbol	Symmetry axis	Graphic symbol	Gliding $\vec{t}$
1	Identity	none	none	4	4-fold rotat°		none
$\bar{1}$	Inversion		none	$4_1$	4-fold screw axes		$c/4$
2	2-fold rotation axis	( $\perp$ plan proj.)  ( $\parallel$ plan proj.) 	none	$4_2$			$2c/4$
				$4_3$			$3c/4$
$2_1$	2-fold screw axis	( $\perp$ plan proj.)  ( $\parallel$ plan proj.) 	$c/2$  $a/2$ ou $b/2$	$\bar{4}$	4-fold rotoinversion		none
3	3-fold rotation axis	$\perp$ plan proj. 	none	$6$	6-fold rotat°		none
$3_1$	2-fold screw axes		$c/3$	$6_1$			$c/6$
$3_2$			$2c/3$	$6_2$			$2c/6$
				$6_3$			$3c/6$
				$6_4$			$4c/6$
				$6_5$			$5c/6$
$\bar{3}$	3-fold rotoinversion		none	$\bar{6}$	6-fold rotoinversion		none

### 3. Space group symmetry: the 230 space groups

- International notation (Hermann-Mauguin symbol)

Ex.  $P4_2/mmc$

1<sup>st</sup> letter : capital letter designing the **lattice mode**  $P, I, F, A$  ( $B$  or  $C$ ),  $R$   
Following letters: **nature of the symmetry elements**

Symmetry axes (with  $n$  max and  $p$  min) and planes ( $m > e > a > b > c > n > d$ )

Along the primary, secondary, and tertiary directions: 3 non equivalent directions of symmetry (the same ones as point groups)

Conventional cell	Primary direction	Secondary direction	Tertiary direction
triclinic	A single symbol (1 or $\bar{1}$ ), thus no direction of symmetry		
monoclinic		A single direction of symmetry: $b$ or $c$ (order 2, unique axis)	
orthorhombic	$a$ (order 2)	$b$ (order 2)	$c$ (order 2)
tetragonal	$[001]$ (order 4)	$<100>$ , i.e. $a$ and $b$ (order 2)	$<110>$ , i.e. $a \pm b$ (order 2)
hexagonal	$c$ (order 6 or 3)	$<100>$ , i.e. $a, b, [1\bar{1}0]$ (order 2)	$<210>$ , i.e. $[210]$ , $[\bar{1}20], [1\bar{1}0]$ (order 2)
cubic	$<100>$ (order 4 or 2)	$<111>$ (order 3)	$<110>$ (order 2)

# 3. Space group symmetry: the 230 space groups

cryst. syst.	point group	space group No.	space group symbol	cryst. syst.	point group	space group No.	space group symbol	cryst. syst.	point group	space group No.	space group symbol
<i>a</i>	1	1	P1			51	Pmma			101	P4 <sub>2</sub> cm
	1	2	$\bar{P}1$			52	Pnna			102	P4 <sub>2</sub> nm
<i>m</i>	2	3	P2			53	Pnma			103	P4cc
	4	4	P2 <sub>1</sub>			54	Pcca			104	P4nc
	5	5	C2			55	Pham			105	P4 <sub>2</sub> mc
<i>m</i>	6	6	Pm			56	Pccn			106	P4 <sub>2</sub> bc
	7	7	Pc			57	Pbcm			107	I4mm
	8	8	Cm			58	Pnnm			108	I4cm
	9	9	Cc			59	Pnmm			190	I4 <sub>1</sub> md
<i>2/m</i>	10	10	P2/m			60	Pbcn			110	I4 <sub>1</sub> cd
	11	11	P2 <sub>1</sub> /m			61	Pbca				
	12	12	C2/m			62	Pnma				
	13	13	P2/c			63	Cmcm				
	14	14	P2 <sub>1</sub> /c			64	Cmce				
	15	15	C2/c			65	Cmmm				
<i>o</i>	222	16	P222			66	Cccm				
		17	P222 <sub>1</sub>			67	Cmme				
		18	P2 <sub>1</sub> 2 <sub>1</sub> 2			68	Ccce				
		19	P2 <sub>1</sub> 2 <sub>1</sub> 2 <sub>1</sub>			69	Fmmm				
		20	C222 <sub>1</sub>			70	Fddd				
		21	C222			71	Immm				
		22	F222			72	Ibam				
		23	I222			73	Ibca				
		24	I2 <sub>1</sub> 2 <sub>1</sub> 2 <sub>1</sub>			74	Imma				
<i>mm2</i>	25	Pnnm2		<i>t</i>	4	75	P4				
	26	Pmc <sub>2</sub> 1				76	P4 <sub>1</sub>				
	27	Pcc2				77	P4 <sub>2</sub>				
	28	Pma2				78	P4 <sub>3</sub>				
	29	Pca2 <sub>1</sub>				79	I4				
	30	Pnc2				80	I4 <sub>1</sub>				
	31	Pmn2 <sub>1</sub>			<i>4</i>	81	$\bar{P}4$				
	32	Pba <sub>2</sub> 1				82	I4				
	33	Pna2 <sub>1</sub>			<i>4/m</i>	83	P4/m				
	34	Pnn2				84	P4 <sub>2</sub> /m				
	35	Cnm2				85	P4/n				
	36	Cmc <sub>2</sub> 1				86	P4 <sub>2</sub> /n				
	37	Ccc2				87	I4/m				
	38	Anm2				88	I4 <sub>1</sub> /a				
	39	Aem2			<i>422</i>	89	P422				
	40	Ana2				90	P42 <sub>1</sub>				
	41	Aea2				91	P4 <sub>1</sub> 22				
	42	Fmm2				92	P4 <sub>1</sub> 2 <sub>1</sub> 2				
	43	Fdd2				93	P4 <sub>2</sub> 22				
	44	Imm2				94	P4 <sub>2</sub> 2 <sub>1</sub> 2				
	45	Iba2				95	P4 <sub>3</sub> 22				
	46	Ima2				96	P4 <sub>3</sub> 2 <sub>1</sub> 2				
<i>mmm</i>	47	Pnumm				97	I422				
	48	Pnnn				98	I4 <sub>1</sub> 22				
	49	Pccm			<i>4mm</i>	99	P4mm				
	50	Pban				100	P4bm				

6 conventional cells  
14 Bravais lattices (translation symmetry)

32 point groups

Symmetry at the macroscopic scale

230 space groups

Symmetry at the microscopic scale

cryst. syst.	point group	space group No.	space group symbol	cryst. syst.	point group	space group No.	space group symbol	cryst. syst.	point group	space group No.	space group symbol
		123	P4mmm			151	P3 <sub>1</sub> 12			178	P6 <sub>1</sub> 22
		124	P4/mcc			152	P3 <sub>1</sub> 21			179	P6 <sub>5</sub> 22
		125	P4/nbm			153	P3 <sub>2</sub> 12			180	P6 <sub>2</sub> 22
		126	P4/nnc			154	P3 <sub>2</sub> 21			181	P6 <sub>4</sub> 22
		127	P4/mbm			155	R32			182	P6 <sub>3</sub> 22
		128	P4/mnc			129	P4/nmm			183	P6mm
						130	P4/nnc			184	P6cc
					<i>3m</i>	131	P4 <sub>2</sub> /mmc			185	P6 <sub>3</sub> cm
						132	P4 <sub>2</sub> /mcm			186	P6 <sub>3</sub> mc
						133	P4 <sub>2</sub> /nbc			187	P6m2
						134	P4 <sub>2</sub> /nnm			188	P6 <sub>2</sub> c2
						135	P4 <sub>2</sub> /mbc			189	P6 <sub>2</sub> 2m
						136	P4 <sub>2</sub> /mmn			190	P6 <sub>2</sub> c2
						137	P4 <sub>2</sub> /nmc			191	P6/mmm
						138	P4 <sub>2</sub> /ncm			192	P6/mcc
						139	I4/mmm			193	P6/mcm
						140	I4/mcm			194	P6/mmc
					<i>h</i>	141	I4 <sub>1</sub> /amd			168	P6
						142	I4 <sub>1</sub> /acd			169	P6 <sub>1</sub>
						143	P3			170	P6 <sub>5</sub>
						144	P3 <sub>1</sub>			171	P6 <sub>2</sub>
						145	P3 <sub>2</sub>			172	P6 <sub>4</sub>
						146	R3			173	P6 <sub>3</sub>
					<i>3</i>	147	$\bar{P}3$			174	P6
						148	$\bar{R}3$			175	P6/m
						32	149	P312		176	P6 <sub>3</sub> /m
							150	P321		177	P622
					<i>c</i>	23			195	P23	
						196	F23		197	I23	
						198	P2 <sub>1</sub> 3		199	I2 <sub>3</sub>	
						200	Pm $\bar{3}$		201	Pn $\bar{3}$	
						202	Fm $\bar{3}$		203	Fd $\bar{3}$	
						204	Im $\bar{3}$		205	Pa $\bar{3}$	
					<i>m3m</i>	221	Pm $\bar{3}$ m		206	Ia $\bar{3}$	
						222	Pn $\bar{3}$ n		207	P432	
						223	Pm $\bar{3}$ n		208	P4 <sub>2</sub> 32	
						224	Pn $\bar{3}$ m		209	F432	
						225	Fm $\bar{3}$ m		210	F4 <sub>1</sub> 32	
						226	Fm $\bar{3}$ c		211	I432	
						227	Fd $\bar{3}$ m		212	P4 <sub>3</sub> 32	
						228	Fd $\bar{3}$ c		213	P4 <sub>1</sub> 32	
						229	Im $\bar{3}$ m		214	I4 <sub>1</sub> 32	
						230	Ia $\bar{3}$ d		215	P43m	

### 3. Space group symmetry: space group *Pnma* – ITC

**Non symmorphic SG**  
( $\exists$  glide translations)

Diagrams of symmetry operations

⚠  $\exists$  different settings  
(permutations of  $a, b, c$ )

Diagrams of equivalent positions

Location of the origin

Symmetry operations:  
(number), nature, location

page 1/2 of *Pnma*  
taken from the ITC,  
volume A



*Pnma*

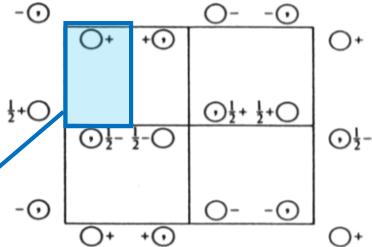
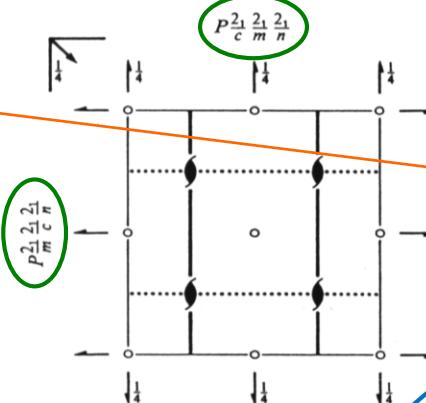
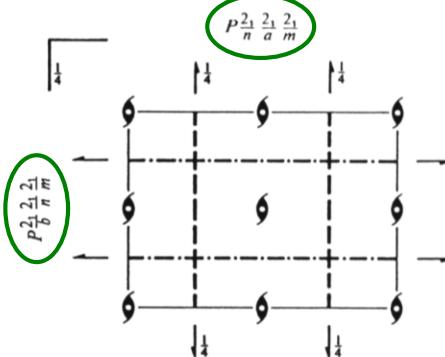
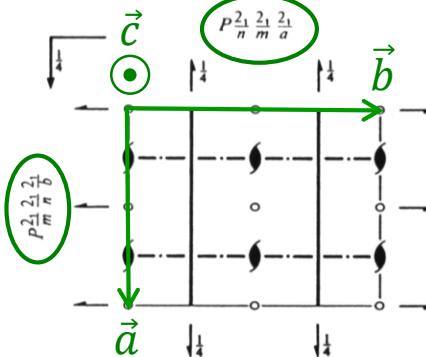
No. 62

*D*<sub>2h</sub><sup>16</sup>

*P* 2<sub>1</sub>/*n* 2<sub>1</sub>/*m* 2<sub>1</sub>/*a*

*m m m*

Orthorhombic



Origin at  $\bar{1}$  on 12<sub>1</sub>1

Asymmetric unit  $0 \leq x \leq \frac{1}{2}; 0 \leq y \leq \frac{1}{2}; 0 \leq z \leq 1$

Symmetry operations

(1) 1 1 0,0,0	(2) 2(0,0,½) ½,0,z	(3) 2(0,½,0) 0,y,0	(4) 2(½,0,0) x,½,½
(5) 1 0,0,0	(6) a x,y,½	(7) m x,½,z	(8) n(0,½,½) ½,y,z

Smallest volume of the unit cell  
containing all structural information

### 3. Space group symmetry: space group *Pnma* – ITC

Bravais lattice

Orthorhombic

*P*  $2_1/n$   $2_1/m$   $2_1/a$

Screw axis  $2_1 \parallel \vec{a}$   
Glide plane  $n \perp \vec{a}$

Screw axis  $2_1 \parallel \vec{c}$   
Glide plane  $a \perp \vec{c}$

Point group:  $\frac{2}{m} \frac{2}{m} \frac{2}{m}$   
 $(n, m, a \rightarrow m \text{ and } 2_1 \rightarrow 2)$

Screw axis  $2_1 \parallel \vec{b}$   
Mirror plane  $m \perp \vec{b}$

"Representation" of the symmetry operations

#### Symmetry operations

(1)  $1$   $0,0,0$   
(5)  $\bar{1}$   $0,0,0$

(2)  $2(0,0,\frac{1}{2})$   
(6)  $a$   $x,y,\frac{1}{4}$

(3)  $2(0,\frac{1}{2},0)$   $0,y,0$   
(7)  $m$   $x,\frac{1}{2},z$

(4)  $2(\frac{1}{2},0,0)$   $x,\frac{1}{2},\frac{1}{2}$   
(8)  $n(0,\frac{1}{2},\frac{1}{2})$   $\frac{1}{2},y,z$

2-fold rotation  
followed by  $\vec{t} = \frac{1}{2}\vec{c}$   
i.e. axis  $2_1 \parallel \vec{c}$

axis  $\parallel \vec{c}$  at  
 $x = \frac{1}{4}$  et  $y = 0$

Glide plane  $n$   
with  $\vec{t} = \frac{1}{2}(\vec{b} + \vec{c})$

plane  $(x, y)$ , i.e.  $\perp \vec{a}$   
with  $x = \frac{1}{4}$

### 3. Space group symmetry: space group *Pnma* – ITC

Arbitrary choice of generators for the SG

Identity  
Elementary translations  
+ some of the symmetry axes and planes

CONTINUED				No. 62
Generators selected (1); <i>t</i> (1,0,0); <i>t</i> (0,1,0); <i>t</i> (0,0,1); (2); (3); (5)				
<b>Positions</b>				<b>Coordinates</b>
Multiplicity, Wyckoff letter, Site symmetry				
8 <i>d</i> 1    (1) <i>x,y,z</i> (2) $\bar{x}+\frac{1}{2},\bar{y},z+\frac{1}{2}$ (3) $\bar{x},y+\frac{1}{2},\bar{z}$ (4) $x+\frac{1}{2},\bar{y}+\frac{1}{2},\bar{z}+\frac{1}{2}$ (5) $\bar{x},\bar{y},\bar{z}$ (6) $x+\frac{1}{2},y,\bar{z}+\frac{1}{2}$ (7) $x,\bar{y}+\frac{1}{2},z$ (8) $\bar{x}+\frac{1}{2},y+\frac{1}{2},z+\frac{1}{2}$				
4 <i>c</i> .m . <i>x,½,z</i> $\bar{x}+\frac{1}{2},\frac{1}{2},z+\frac{1}{2}$ $\bar{x},\frac{1}{2},\bar{z}$ $x+\frac{1}{2},\frac{1}{2},\bar{z}+\frac{1}{2}$				
4 <i>b</i> $\bar{1}$ $0,0,\frac{1}{2}$ $\frac{1}{2},0,0$ $0,\frac{1}{2},\frac{1}{2}$ $\frac{1}{2},\frac{1}{2},0$				
4 <i>a</i> $\bar{1}$ $0,0,0$ $\frac{1}{2},0,\frac{1}{2}$ $0,\frac{1}{2},0$ $\frac{1}{2},\frac{1}{2},\frac{1}{2}$				

#### Symmetry of special projections

Along [001]  $p\ 2g\ m$   
 $\mathbf{a}' = \frac{1}{2}\mathbf{a}$      $\mathbf{b}' = \mathbf{b}$   
Origin at  $0,0,z$

Along [100]  $c\ 2m\ m$   
 $\mathbf{a}' = \mathbf{b}$      $\mathbf{b}' = \mathbf{c}$   
Origin at  $x,\frac{1}{2},\frac{1}{2}$

#### Reflection conditions

General:

$$\begin{aligned} Okl : k+l &= 2n \\ hk0 : h &= 2n \\ h00 : h &= 2n \\ 0k0 : k &= 2n \\ 00l : l &= 2n \end{aligned}$$

Special: as above, plus  
no extra conditions

$$\begin{aligned} hkl : h+l, k &= 2n \\ hkl : h+l, k &= 2n \end{aligned}$$

Along [010]  $p\ 2gg$   
 $\mathbf{a}' = \mathbf{c}$      $\mathbf{b}' = \mathbf{a}$   
Origin at  $0,y,0$

See Lecture III

subgroups /supergroups,  
for crystallogr. transitions to a  
lower / higher symmetry phase  
See Lecture II

See also [www.cryst.ehu.es](http://www.cryst.ehu.es)

Maximal non-isomorphic subgroups	
I	[2] $P_{2_1}2_12_1$
	[2] $P_{112_1}/a$ ( $P_{2_1}/c$ )
	[2] $P_{12_1}/m$ ( $P_{2_1}/m$ )
	[2] $P_{2_1}/n$ 1 1 ( $P_{2_1}/c$ )
	[2] $Pnm_2_1$ ( $Pmn_2_1$ )
	[2] $Pn_2_1a$ ( $Pna_2_1$ )
	[2] $P_{2_1}ma$ ( $Pmc_2_1$ )
	1; 2; 3; 4
	1; 2; 5; 6
	1; 3; 5; 7
	1; 4; 5; 8
	1; 2; 7; 8
	1; 3; 6; 8
	1; 4; 6; 7

IIa none

IIb none

#### Maximal isomorphic subgroups of lowest index

IIc [3] $Pnma$  ( $\mathbf{a}' = 3\mathbf{a}$ ); [3] $Pnma$  ( $\mathbf{b}' = 3\mathbf{b}$ ); [3] $Pnma$  ( $\mathbf{c}' = 3\mathbf{c}$ )

#### Minimal non-isomorphic supergroups

I none

II [2] $Amm$  ( $Cmcm$ ); [2] $Bbmm$  ( $Cmcm$ ); [2] $Ccmb$  ( $Cmc$ ); [2] $Imma$ ; [2] $Pnmm$  ( $2\mathbf{a}' = \mathbf{a}$ ) ( $Pmmn$ );  
[2] $Pcm$  ( $2\mathbf{b}' = \mathbf{b}$ ) ( $Pbam$ ); [2] $Pbma$  ( $2\mathbf{c}' = \mathbf{c}$ ) ( $Pbcm$ )



page 2/2 of *Pnma*  
taken from the ITC,  
volume A

### 3. Space group symmetry: space group *Pnma* – ITC

Wyckoff sites: List of the different sites from the most general (*i.e.* less symmetrical) to the less general position (*i.e.* most symmetrical: special position)

Positions			Coordinates					
Multiplicity, Wyckoff letter, Site symmetry								
8 <i>d</i>	1		(1) $x, y, z$		(2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$		(3) $\bar{x}, y + \frac{1}{2}, \bar{z}$	(4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$
			(5) $\bar{x}, \bar{y}, \bar{z}$		(6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$		(7) $x, \bar{y} + \frac{1}{2}, z$	(8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$
4 <i>c</i>	<i>.m.</i>		$x, \frac{1}{4}, z$		$\bar{x} + \frac{1}{2}, \frac{3}{4}, z + \frac{1}{2}$		$\bar{x}, \frac{3}{4}, \bar{z}$	$x + \frac{1}{2}, \frac{1}{4}, \bar{z} + \frac{1}{2}$
4 <i>b</i>	$\bar{1}$		$0, 0, \frac{1}{2}$		$\frac{1}{2}, 0, 0$		$0, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, 0$
4 <i>a</i>	$\bar{1}$		$0, 0, 0$		$\frac{1}{2}, 0, \frac{1}{2}$		$0, \frac{1}{2}, 0$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

Site name

Multiplicity  
of the site

Wyckoff  
letter

Symmetry  
of the site

Coordinates of all  
equivalent positions

### 3. Space group symmetry: space group *Pnma* – ITC

Wyckoff sites: List of the different sites from the most general (*i.e.* less symmetrical) to the less general position (*i.e.* most symmetrical: special position)

Positions		Coordinates															
Multiplicity, Wyckoff letter, Site symmetry		(1)	$x, y, z$	(2)	$\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$	(3)	$\bar{x}, y + \frac{1}{2}, \bar{z}$	(4)	$x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$	(5)	$\bar{x}, \bar{y}, \bar{z}$	(6)	$x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	(7)	$x, \bar{y} + \frac{1}{2}, z$	(8)	$\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$
8	<i>d</i>	1															

#### Symmetry operations

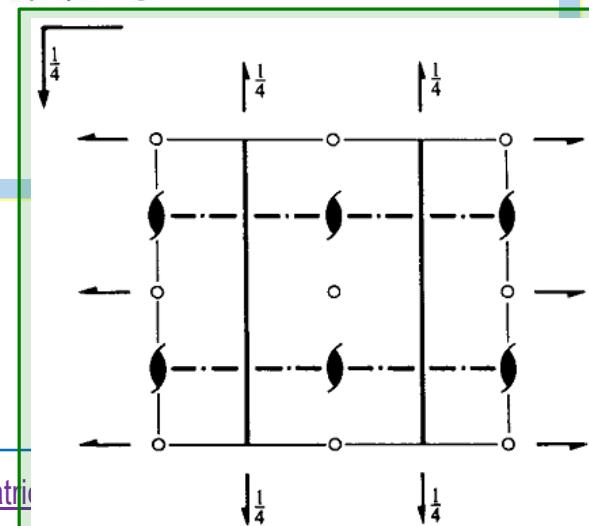
(1)	$1$	$0,0,0$	(2)	$2(0,0,\frac{1}{2})$	$\frac{1}{2},0,z$	(3)	$2(0,\frac{1}{2},0)$	$0,y,0$	(4)	$2(\frac{1}{2},0,0)$	$x,\frac{1}{2},\frac{1}{2}$	(5)	$\bar{1}$	$0,0,0$	(6)	$a$	$x,y,\frac{1}{2}$	(7)	$m$	$x,\frac{1}{2},z$	(8)	$n(0,\frac{1}{2},\frac{1}{2})$	$\frac{1}{2},y,z$
-----	-----	---------	-----	----------------------	-------------------	-----	----------------------	---------	-----	----------------------	-----------------------------	-----	-----------	---------	-----	-----	-------------------	-----	-----	-------------------	-----	--------------------------------	-------------------

### 3. Space group symmetry: space group *Pnma* – ITC

Wyckoff sites: List of the different sites from the most general (*i.e.* less symmetrical) to the less general position (*i.e.* most symmetrical: special position)

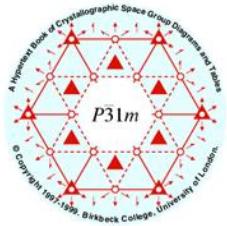
Positions		Coordinates			
Multiplicity, Wyckoff letter, Site symmetry					
8 <i>d</i>	1	(1) $x, y, z$	(2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$	(3) $\bar{x}, y + \frac{1}{2}, \bar{z}$	(4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$
		(5) $\bar{x}, \bar{y}, \bar{z}$	(6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	(7) $x, \bar{y} + \frac{1}{2}, z$	(8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$
4 <i>c</i>	. <i>m</i> .	$x, \frac{1}{4}, z$	$\bar{x} + \frac{1}{2}, \frac{3}{4}, z + \frac{1}{2}$	$\bar{x}, \frac{3}{4}, \bar{z}$	$x + \frac{1}{2}, \frac{1}{4}, \bar{z} + \frac{1}{2}$
4 <i>b</i>	$\bar{1}$	$0, 0, \frac{1}{2}$	$\frac{1}{2}, 0, 0$	$0, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, 0$
4 <i>a</i>	$\bar{1}$	0,0,0	$\frac{1}{2}, 0, \frac{1}{2}$	$0, \frac{1}{2}, 0$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

Origin at  $\bar{1}$  on  $12_1\bar{1}$

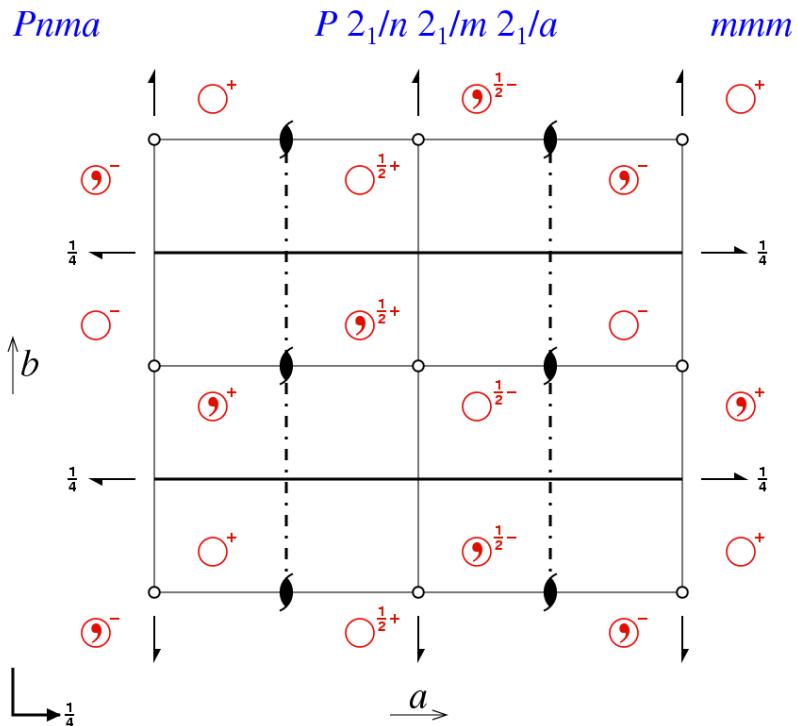


# 3. Space group symmetry: space group *Pnma*

<http://img.chem.ucl.ac.uk/sgp/>



*Pnma*



*P 2<sub>1</sub>/n 2<sub>1</sub>/m 2<sub>1</sub>/a*

*mmm*

No. 62



## Symmetry Operators

1	$x, y, z$	1
2	$\frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2} + z$	$n$ $(\frac{1}{4}, y, z)$ $[0, \frac{1}{2}, \frac{1}{2}]$
3	$x, \frac{1}{2} - y, z$	$m$ $(x, \frac{1}{4}, z)$
4	$\frac{1}{2} + x, y, \frac{1}{2} - z$	$a$ $(x, y, \frac{1}{4})$ $[\frac{1}{2}, 0, 0]$
5	$\bar{x}, \bar{y}, \bar{z}$	$\bar{1}$ $(0, 0, 0)$
6	$\frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2} - z$	$2_1$ $(x, \frac{1}{4}, \frac{1}{4})$ $[\frac{1}{2}, 0, 0]$
7	$\bar{x}, \frac{1}{2} + y, \bar{z}$	$2_1$ $(0, y, 0)$ $[0, \frac{1}{2}, 0]$
8	$\frac{1}{2} - x, \bar{y}, \frac{1}{2} + z$	$2_1$ $(\frac{1}{4}, 0, z)$ $[0, 0, \frac{1}{2}]$

*Careful: different order as compared to the ITC!*

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### 3. Space group symmetry: space group *Pnma*

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Space-group symmetry

WYCKPOS

#### Wyckoff Positions of Group 62 (*Pnma*)

Multiplicity	Wyckoff letter	Site symmetry	Coordinates
8	d	1	(x,y,z) (-x+1/2,-y,z+1/2) (-x,y+1/2,-z) (x+1/2,-y+1/2,-z+1/2) (-x,-y,-z) (x+1/2,y,-z+1/2) (x,-y+1/2,z) (-x+1/2,y+1/2,z+1/2)
4	c	.m.	(x,1/4,z) (-x+1/2,3/4,z+1/2) (-x,3/4,-z) (x+1/2,1/4,-z+1/2)
4	b	-1	(0,0,1/2) (1/2,0,0) (0,1/2,1/2) (1/2,1/2,0)
4	a	-1	(0,0,0) (1/2,0,1/2) (0,1/2,0) (1/2,1/2,1/2)

#### Wyckoff position and site symmetry group of a specific point

Specify the point by its relative coordinates (in fractions or decimals)

Variable parameters (x,y,z) are also accepted

x =

y =

z =

<http://www.cryst.ehu.es/>

If you want to see the Wyckoff position in other setting, click [here](#)

### 3. Space group symmetry: space group *Pnma*



- |              |                                 |   |   |   |
|--------------|---------------------------------|---|---|---|
| 8 <i>d</i> 1 | (1) $x, y, z$                   | (2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$ | (3) $\bar{x}, y + \frac{1}{2}, \bar{z}$ | (4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$ |
|              | (5) $\bar{x}, \bar{y}, \bar{z}$ | (6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$       | (7) $x, \bar{y} + \frac{1}{2}, z$       | (8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$       |

#### Symmetry operations

- |               |                          |                     |                          |                     |                                    |                               |
|---------------|--------------------------|---------------------|--------------------------|---------------------|------------------------------------|-------------------------------|
| (1) $1$       | (2) $2(0,0,\frac{1}{2})$ | $\frac{1}{2}, 0, z$ | (3) $2(0,\frac{1}{2},0)$ | $0, y, 0$           | (4) $2(\frac{1}{2},0,0)$           | $x, \frac{1}{2}, \frac{1}{2}$ |
| (5) $\bar{1}$ | (6) $a$                  | $x, y, \frac{1}{2}$ | (7) $m$                  | $x, \frac{1}{2}, z$ | (8) $n(0,\frac{1}{2},\frac{1}{2})$ | $\frac{1}{2}, y, z$           |

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#### Space-group symmetry

GENPOS

#### General Positions of the Group 62 (*Pnma*)

Click here to get the general positions in text format

No.	(x,y,z) form	Matrix form	Symmetry operation	
			ITA	Seitz ?
1	$x, y, z$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	1	{1 0}
2	$-x+1/2, -y, z+1/2$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	$2(0,0,1/2) 1/4, 0, z$	{2_001   1/2 0 1/2}

Same order



### 3. Space group symmetry: space group *Pnma* – $\text{LaMnO}_3$

Example:  $\text{LaMnO}_3$  (space group *Pnma*)

( $\equiv Pbnm$  if  $\vec{a} \rightarrow \vec{b} \rightarrow \vec{c} \rightarrow \vec{a}$ )

	$x$	$y$	$z$
La	0.518	0.25	0.007
Mn	0	0	0
O <sub>1</sub>	-0.005	0.25	0.075
O <sub>2</sub>	0.288	0.096	0.226

→ 4c  
→ 4a  
→ 4c  
→ 8d

→ Motif =  $\text{La}_4\text{Mn}_4\text{O}_{12}$

7 coordinates to determine  
out of  $(4+4+12) \times 3 = 60 !!!$

#### Positions

Multiplicity,  
Wyckoff letter,  
Site symmetry

#### Coordinates

$\text{O}_2$	8    d	1	(1) $x, y, z$	(2) $\bar{x} + \frac{1}{2}, \bar{y}, z + \frac{1}{2}$	(3) $\bar{x}, y + \frac{1}{2}, \bar{z}$	(4) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{2}$
			(5) $\bar{x}, \bar{y}, \bar{z}$	(6) $x + \frac{1}{2}, y, \bar{z} + \frac{1}{2}$	(7) $x, \bar{y} + \frac{1}{2}, z$	(8) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, z + \frac{1}{2}$

$\text{La, O}_1$  4    c .m.     $x, \frac{1}{2}, z$      $\bar{x} + \frac{1}{2}, \frac{1}{2}, z + \frac{1}{2}$      $\bar{x}, \frac{1}{2}, \bar{z}$      $x + \frac{1}{2}, \frac{1}{2}, \bar{z} + \frac{1}{2}$

4    b    1     $0, 0, \frac{1}{2}$      $\frac{1}{2}, 0, 0$      $0, \frac{1}{2}, \frac{1}{2}$      $\frac{1}{2}, \frac{1}{2}, 0$

Mn 4    a    1     $0, 0, 0$      $\frac{1}{2}, 0, \frac{1}{2}$      $0, \frac{1}{2}, 0$      $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

### 3. Space group symmetry: space group *Pnma* – *LaMnO<sub>3</sub>*



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<a href="#">Z' (min, max)</a>	<input type="text"/>			
chemical formula <a href="#">(in Hill notation)</a>	<input type="text"/>			
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NOT these elements	<input type="text"/>			
volume min and max	<input type="text"/>			
number of distinct elements min and max	3 <input type="text"/>			
filters	<input type="checkbox"/> has F <sub>obs</sub> <input type="checkbox"/> include <a href="#">duplicates</a> <input type="checkbox"/> include structures with errors <input type="checkbox"/> include theoretical structures			
<input type="button" value="Reset"/>	<input type="button" value="Send"/>			

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### 3. Space group symmetry: space group *Pnma* – $\text{LaMnO}_3$

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# This file is available in the Crystallography Open Database (COD),  
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#  
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_atom_site_fract_y  
_atom_site_fract_z  
_atom_site_occupancy  
atom site U iso or equiv  
Mn1 Mn+3 0 0 0.5 1 0.0  
O2 O-2 0.194 0.537 0.223 1 0.0  
La1 La+3 0.0495 0.25 -0.083 1 0.0  
O1 O-2 0.474 0.25 0.07 1 0.0
```

### 3. Space group symmetry: space group *Pnma* – *LaMnO<sub>3</sub>*

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Elements Element Count Chem/Mineral Name ANX/Pearson/S.Type Search Effacer  
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System Laue Class Centering Space Group Cell Size/Mass  
any any any Wyckoff Sequence  
Remarks Min. Distance Distance Select Co-ordin.  
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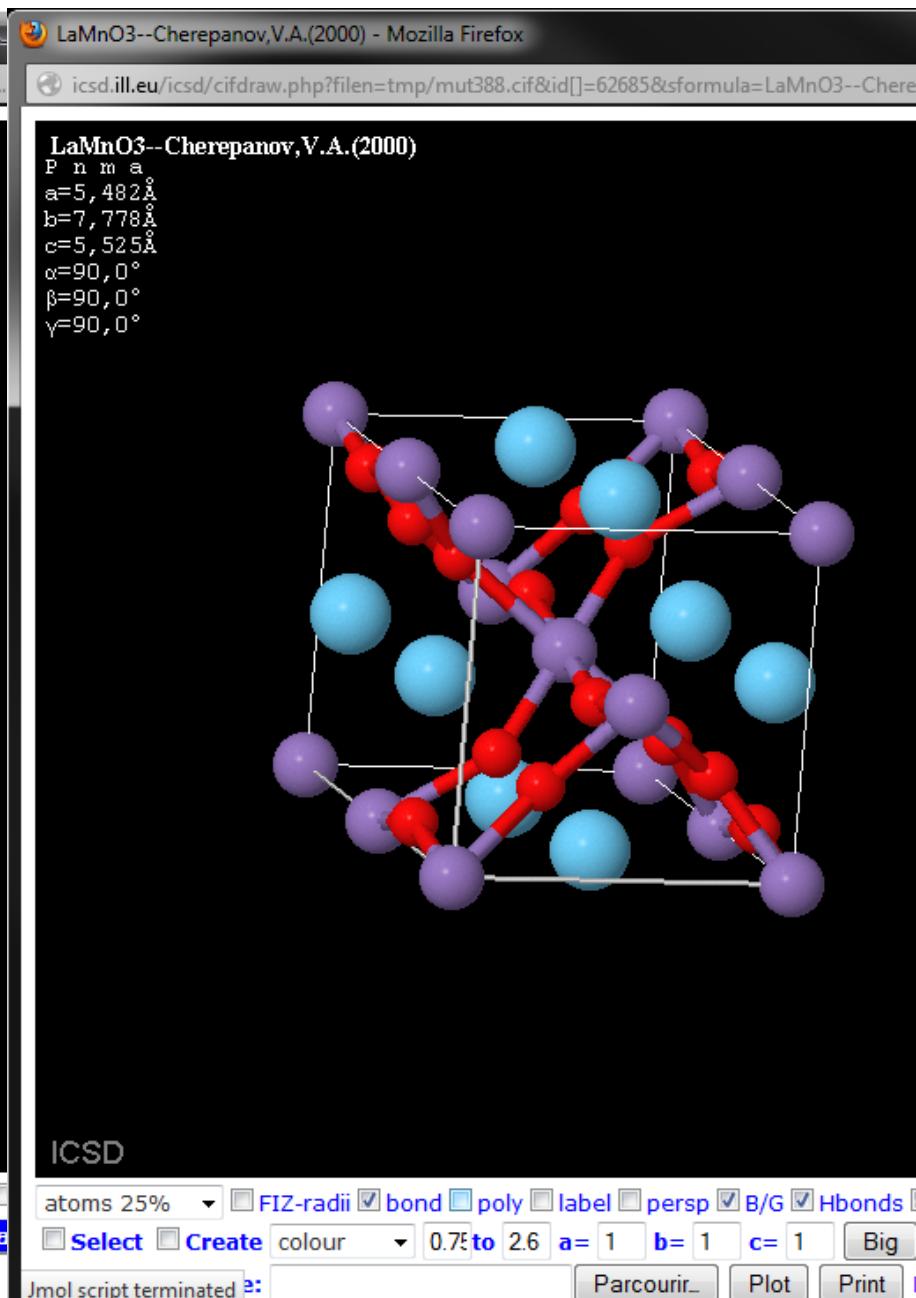
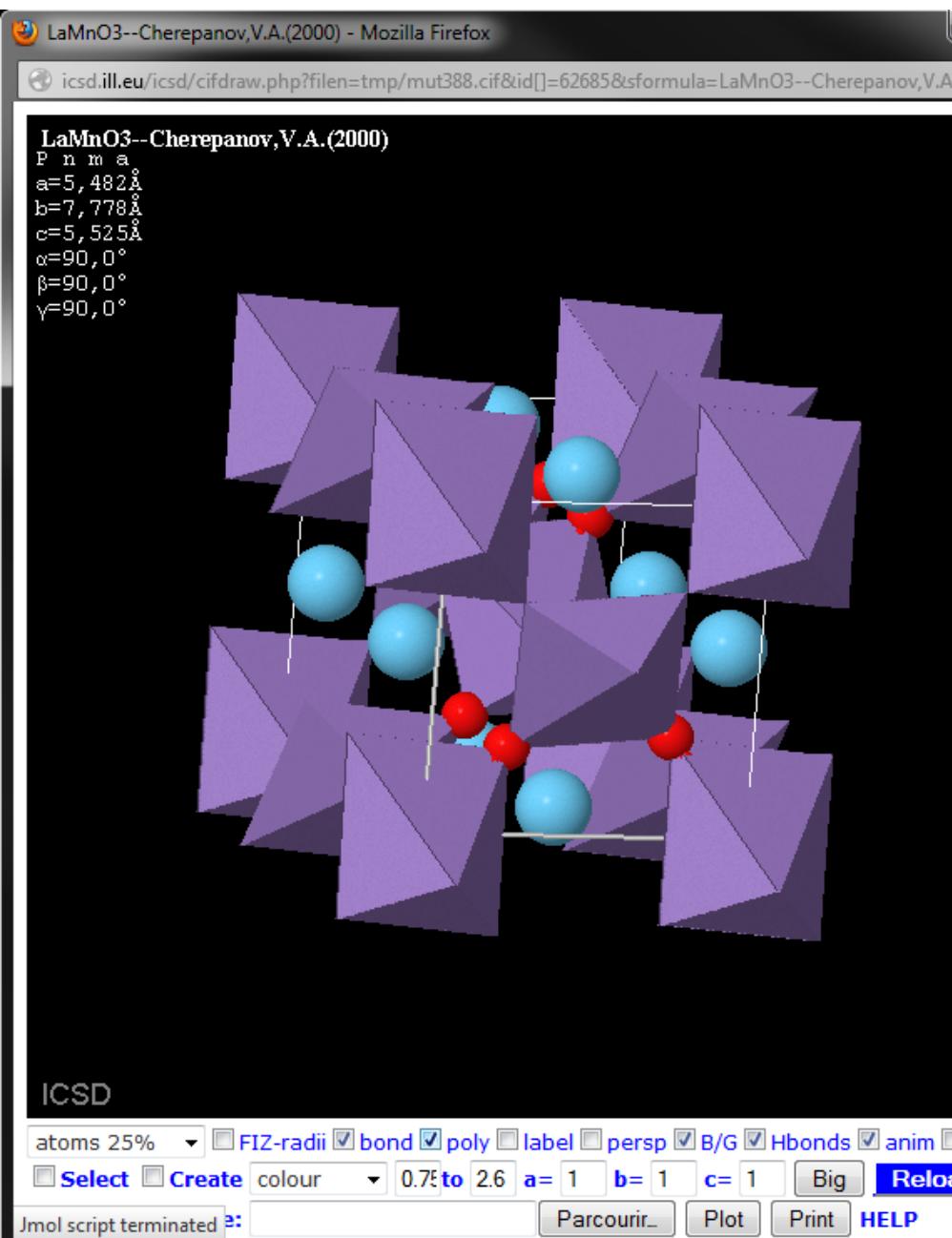
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Select All / None 8 Results References EndNote Details CIF Bonds Pattern Structure Jmol

Year	Authors	Title	Struct. Formula	sgr	Mineral
2004	Hansteen, O.H.; Breard, Y.; Fjellvag, H.; Hauback, B.C.	Divalent manganese in reduced La Mn O <sub>3-d</sub> - effect of oxygen nonstoichiometry on structural and magnetic properties	La (Mn O <sub>2.78</sub> )		PNMA
2000	Cherepanov, V.A.; Filonova, E.A.; Voronin, V.I.; Berger, I.F.	Phase equilibria in the (La Co O <sub>3</sub> ) - (La Mn O <sub>3</sub> ) -(Ba Co O <sub>2</sub> ) (Ba Mn O <sub>3</sub> ) system	La Mn O <sub>3</sub>		PNMA
1999	Taguchi, H.; Matsu-ura, S.-I.; Nagao, M.; Kido, H.	Electrical properties of perovskite-type La (Cr <sub>1-x</sub> Mn <sub>x</sub> ) O <sub>3+d</sub>	La <sub>0.951</sub> Mn <sub>0.951</sub> O <sub>3</sub>		R3-CR
1997	Ferris, V.; Goglio, G.; Brohan, L.; Joubert, O.; Molinie, P.; Ganne, M.; Dordor, P.	Transport properties and magnetic behavior in the polycrystalline lanthanum-deficient manganate perovskite (La <sub>(1-x)</sub> Mn O <sub>3</sub> )	La <sub>0.91</sub> (Mn <sub>0.99</sub> O <sub>3</sub> )		R3CR
1997	Alonso, J.A.; Martinez-Lopez, M.J.; Casais, M.T.; MacManus-Driscoll, J.L.; de Silva, P.S.I.P.N.; Cohen, L.F.; Fernandez-Diaz, M.T.	Non-stoichiometry, structural defects and properties of La Mn O <sub>3+d</sub> with high d values (0.11)	La <sub>0.969</sub> Mn <sub>0.93</sub> O <sub>3</sub>		R3-CH
1996	Shimura, T.; Hayashi, T.; Inaguma, Y.; Itoh, M.	Magnetic and electrical properties of Lay Ax Mn <sub>w</sub> O <sub>3</sub> (A = Na, K, Rb and Sr) with perovskite-type structure	La <sub>0.953</sub> Mn <sub>0.935</sub> O <sub>3</sub>		R3-CH
1996	Hauback, B.C.; Fjellvag, H.; Sakai, N.	Effect of nonstoichiometry on properties of La <sub>1-t</sub> Mn O <sub>3+d</sub> . III. Magnetic order studied by neutron powder diffraction	La <sub>0.92</sub> Mn O <sub>2.88</sub>		PNMA
1996	Hauback, B.C.; Fjellvag, H.; Sakai, N.	Effect of nonstoichiometry on properties of La <sub>1-x</sub> Mn O <sub>3+delta</sub> III. Magnetic order studied by powder neutron diffraction	La <sub>0.88</sub> Mn O <sub>2.82</sub>		PNMA

Page : [1](8 results) 10 results per page.

### 3. Space group symmetry: space group *Pnma* – $\text{LaMnO}_3$



### 3. Space group symmetry: space group *Pnma* – $\text{LaMnO}_3$

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Authors/Code Years Journal Title/Comment Help

Elements Element Count Chem/Mineral Name ANX/Pearson/S.Type Search Effacer

La Mn O 3

System Laue Class Centering Space Group Cell Size/Mass

any any any Wyckoff Sequence

Remarks Min. Distance Distance Select Distance Range Co-ordin.

Query : (EL\_COUNT = "3") AND icsd.sum\_form RLIKE 'LA[0-9]' AND icsd.sum\_form RLIKE 'MN[0-9]' AND icsd.sum\_form RLIKE BINARY 'O[0-9]'

Select All / None 8 Results References EndNote Details CIF Bonds Pattern Structure Jmol

Year	Authors	Title	Struct. Formula	sgr	Mineral
2004	Hansteen, O.H.; Breard, Y.; Fjellvag, H.; Hauback, B.C.;	Divalent manganese in reduced $\text{La Mn O}_3\text{-d}$ - effect of oxygen nonstoichiometry on structural and magnetic properties	$\text{La}(\text{Mn O}_{2.78})$		PNMA
2000	Cherepanov, V.A.; Filonova, E.A.; Voronin, V.I.; Berger, I.F.;	Phase equilibria in the $(\text{La Co O}_3)$ - $(\text{La Mn O}_3)$ -(Ba Co O <sub>z</sub> ) (Ba Mn O <sub>3</sub> ) system	$\text{La Mn O}_3$		PNMA
1999	Taguchi, H.; Matsu-ura, S.-I.; Nagao, M.; Kido, H.;	Electrical properties of perovskite-type $\text{La}(\text{Cr}_{1-x}\text{Mn}_x)\text{O}_{3+\delta}$	$\text{La}_{0.951}\text{Mn}_{0.951}\text{O}_3$		R3-CR
1997	Ferris, V.; Goglio, G.; Brohan, L.; Joubert, O.; Molinie, P.; Ganne, M.; Dordor, P.;	Transport properties and magnetic behavior in the polycrystalline lanthanum-deficient manganate perovskite $(\text{La}_{(1-x)}\text{Mn O}_3)$	$\text{La}_{0.91}(\text{Mn}_{0.99}\text{O}_3)$		R3CR
1997	Alonso, J.A.; Martinez-Lopez, M.J.; Casais, M.T.; MacManus-Driscoll, J.L.; de Silva, P.S.I.P.N.; Cohen, L.F.; Fernandez-Diaz, M.T.;	Non-stoichiometry, structural defects and properties of $\text{La Mn O}_{3+\delta}$ with high d values (0.11)	$\text{La}_{0.969}\text{Mn}_{0.93}\text{O}_3$		R3-CH
1996	Shimura, T.; Hayashi, T.; Inaguma, Y.; Itoh, M.;	Magnetic and electrical properties of $\text{La}_x\text{A}_{1-x}\text{Mn}_w\text{O}_3$ ( $\text{A} = \text{Na}, \text{K}, \text{Rb}$ and $\text{Sr}$ ) with perovskite-type structure	$\text{La}_{0.953}\text{Mn}_{0.935}\text{O}_3$		R3-CH
1996	Hauback, B.C.; Fjellvag, H.; Sakai, N.;	Effect of nonstoichiometry on properties of $\text{La}_{1-t}\text{Mn O}_{3+\delta}$ . III. Magnetic order studied by neutron powder diffraction	$\text{La}_{0.92}\text{Mn O}_{2.88}$		PNMA
1996	Hauback, B.C.; Fjellvag, H.; Sakai, N.;	Effect of nonstoichiometry on properties of $\text{La}_{1-x}\text{Mn O}_{3+\delta}$ . III. Magnetic order studied by powder neutron diffraction	$\text{La}_{0.88}\text{Mn O}_{2.82}$		PNMA

Page : [1](8 results) 10 results per page.

### 3. Space group symmetry: space group *Pnma* – *LaMnO<sub>3</sub>*

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## ICSD for WWW

### Details of the selected entries

1 entry selected.

CC=Collection Code: [AB2X4]=ANX Form: [cF56]=Pearson: [e d a]=Wyckoff Symbol: [Al2MgO4]=Structure Type:  
**\*\*\*Click the ANX, Pearson or Wyckoff Symbol to find structures with that symbol\*\*\*.**

CC=91175	<a href="#">Help</a> <a href="#">CIF</a> <a href="#">Export</a> <a href="#">Bonds</a> <a href="#">Pattern</a> <a href="#">Structure</a> <a href="#">Jmol</a>
Title	Phase equilibria in the (La Co O <sub>3</sub> ) - (La Mn O <sub>3</sub> ) -(Ba Co O <sub>2</sub> ) (Ba Mn O <sub>3</sub> ) system.
Authors	Cherepanov, V.A.; Filonova, E.A.; Voronin, V.I.; Berger, I.F.
Reference	<a href="#">Journal of Solid State Chemistry (2000) 153, 205-211</a> <a href="#">Link XRef SCOPUS SCIRUS Google</a>
Compound	La1 Mn1 O <sub>3</sub> - Lanthanum manganese trioxide <a href="#">[ABX3]</a> <a href="#">[oP20]</a> <a href="#">[d c2 a]</a> []
Cell	5.4820(9), 7.778(2), 5.5253(9), 90., 90., 90. <b>PNMA (62)</b> V=235.59
Remarks	RVP XDP At least one temperature factor missing in the paper. The coordinates are those given in the paper but the atomic distances do not agree with those calculated during testing. The coordinates are probably correct. No R value given in the paper.
Atom (site) Oxid.	x, y, z, B, Occupancy
La1 (4c)	3 0.5184(4) 0.25 0.007(2) 0 1
Mn1 (4a)	3 0 0 0 0 1
O1 (4c)	-2 -0.005(7) 0.25 0.075(1) 0 1
O2 (8d)	-2 0.288(9) 0.096(9) 0.23(2) 0 1

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**Demo database** (The Full database will be used if available after the first query is entered)  
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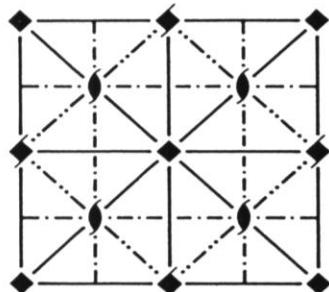


### 3. Space group symmetry: space group $I4mm$

Symmorphic SG

**$I4m\bar{m}$**

No. 107



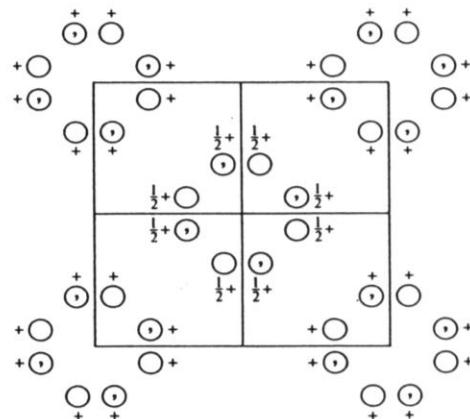
**$C_{4v}^9$**

$I4m\bar{m}$

$4m\bar{m}$

Tetragonal

Patterson symmetry  $I4/m\bar{m}m$



**Origin** on  $4m\bar{m}$

**Asymmetric unit**  $0 \leq x \leq \frac{1}{2}; 0 \leq y \leq \frac{1}{2}; 0 \leq z \leq \frac{1}{2}; x \leq y$

**Symmetry operations**

For  $(0,0,0)^+$  set

(1) 1  
(5)  $m$   $x,0,z$

(2) 2  $0,0,z$   
(6)  $m$   $0,y,z$

(3)  $4^+$   $0,0,z$   
(7)  $m$   $x,\bar{x},z$

(4)  $4^-$   $0,0,z$   
(8)  $m$   $x,x,z$

For  $(\frac{1}{2},\frac{1}{2},\frac{1}{2})^+$  set

(1)  $t(\frac{1}{2},\frac{1}{2},\frac{1}{2})$   
(5)  $n(\frac{1}{2},0,\frac{1}{2})$   $x,\frac{1}{2},z$

(2)  $2(0,0,\frac{1}{2})$   $\frac{1}{2},\frac{1}{2},z$   
(6)  $n(0,\frac{1}{2},\frac{1}{2})$   $\frac{1}{2},y,z$

(3)  $4^+(0,0,\frac{1}{2})$   $0,\frac{1}{2},z$   
(7)  $c$   $x+\frac{1}{2},\bar{x},z$

(4)  $4^-(0,0,\frac{1}{2})$   $\frac{1}{2},0,z$   
(8)  $n(\frac{1}{2},\frac{1}{2},\frac{1}{2})$   $x,x,z$

Bravais lattice: body centered ( $I$ ) tetragonal

Axis  $4 \parallel \vec{c}$ ; mirrors  $m \perp <100>$ ; mirrors  $m \perp <110>$

### 3. Space group symmetry: space group I4mm

#### Symmetry operations

For  $(0,0,0)+$  set

- (1) 1  
(5)  $m$   $x,0,z$

- (2) 2  $0,0,z$   
(6)  $m$   $0,y,z$

- (3)  $4^+$   $0,0,z$   
(7)  $m$   $x,\bar{x},z$

- (4)  $4^-$   $0,0,z$   
(8)  $m$   $x,x,z$

4-fold rotation applied 2, 1, and 3 times, resp.

For  $(\frac{1}{2},\frac{1}{2},\frac{1}{2})+$  set

- (1)  $t(\frac{1}{2},\frac{1}{2},\frac{1}{2})$   
(5)  $n(\frac{1}{2},0,\frac{1}{2})$   $x,\frac{1}{2},z$

- (2)  $2(0,0,\frac{1}{2})$   $\frac{1}{2},\frac{1}{2},z$   
(6)  $n(0,\frac{1}{2},\frac{1}{2})$   $\frac{1}{2},y,z$

- (3)  $4^+(0,0,\frac{1}{2})$   $0,\frac{1}{2},z$   
(7)  $c$   $x+\frac{1}{2},\bar{x},z$

- (4)  $4^-(0,0,\frac{1}{2})$   $\frac{1}{2},0,z$   
(8)  $n(\frac{1}{2},\frac{1}{2},\frac{1}{2})$   $x,x,z$

I lattice

Generators selected (1);  $t(1,0,0)$ ;  $t(0,1,0)$ ;  $t(0,0,1)$ ;  $t(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ ; (2); (3); (5)

#### Positions

Multiplicity,  
Wyckoff letter,  
Site symmetry

#### Coordinates

$(0,0,0)+$

$(\frac{1}{2},\frac{1}{2},\frac{1}{2})+$

I lattice

16

e

1

- |                   |                         |                         |                   |
|-------------------|-------------------------|-------------------------|-------------------|
| (1) $x,y,z$       | (2) $\bar{x},\bar{y},z$ | (3) $\bar{y},x,z$       | (4) $y,\bar{x},z$ |
| (5) $x,\bar{y},z$ | (6) $\bar{x},y,z$       | (7) $\bar{y},\bar{x},z$ | (8) $y,x,z$       |

8  $d$  . $m$ .  $x,0,z$   $\bar{x},0,z$   $0,x,z$   $0,\bar{x},z$

8  $c$  .. $m$   $x,x,z$   $\bar{x},\bar{x},z$   $\bar{x},x,z$   $x,\bar{x},z$

4  $b$  2 $mm$ .  $0,\frac{1}{2},z$   $\frac{1}{2},0,z$

2  $a$  4 $mm$   $0,0,z$

Add  $(0,0,0)$  and  $(\frac{1}{2} \frac{1}{2} \frac{1}{2})$

to these coordinates

→  $8 \times 2$  atomic coordinates

## Point group symmetry:

Allows to **predict** the existence or not of some **macroscopic physical properties**  
And in the case they do exist, the direction of the vectorial quantity or form of the tensor, ...

## Translation symmetry:

Responsible for diffraction → see lecture III

## Structure completely described by:

**Space group + lattice parameters + asymmetric unit**

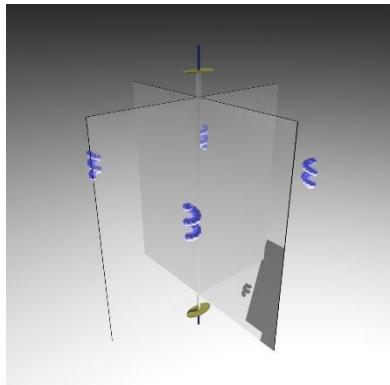
Then use Wyckoff positions to calculate the coordinates of the other atoms of the motif, and last the lattice translations

The same approach can be done for magnetic structures → see lecture II

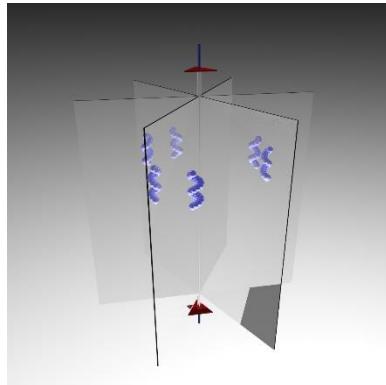
# Point group: to go further ...

From crystallographic point groups ...

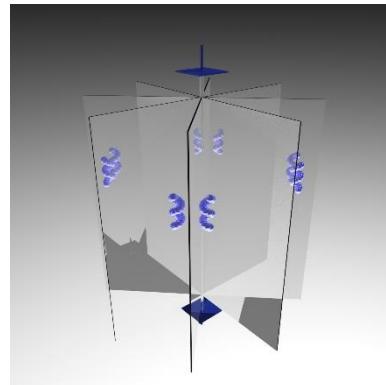
<http://materials.cmu.edu/degraef/pg/>



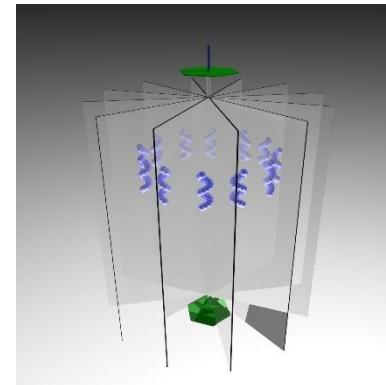
$2mm$



$3m$



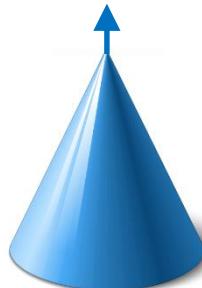
$4mm$



$6mm$

... to Curie point groups

$\infty m$



Thank you ...  
et bonne dégustation

