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# Quantum Mechanics

$$\text{Debye nature of matter, de-Broglie wavelength, } \lambda = \frac{h}{mv} \quad \text{---(1)}$$

The momentum of the photon:

$$\text{For electrons/protons moving with } v \\ \Rightarrow \gamma = \frac{h}{p} = 0 \quad p = mv.$$

The expression for the wavelength at the matter wave as de-Broglie did can also be derived using a general equation of relativity for the purpose. Let's consider a material particle such as an electron or proton as a standing wave system in the regional space.

Let  $(\psi)$  be the quantity that undergoes periodic changes during rise to matter waves than the value of  $(\psi)$  at any instant  $t$  at a point  $(x, y, z)$  is given by

$$\Psi = \Psi_0 \sin \omega t$$

$\Psi = \Psi_0 \sin 2\pi \nu t$ ,  $\Psi_0$  is amplitude and  $\nu$  is frequency.

Let the particle at  $x = 0$  is moving along the  $x$ -axis. Then the value of  $y$  under this condition is given by

$$\varphi = \varphi_0 \sin(2\pi f t) \left( \frac{1}{2} + \frac{m}{C_m} \right)$$

Date / /

The standard equation of wave motion is given by

$$y = A \sin \left[ \frac{2\pi}{T} (t + \frac{x}{v}) \right] \quad (1)$$

where  $A$  is the amplitude.

$T'$  is the periodictime.

$v'$  is the phase velocity.

Comparing eq (1) and (1)

$$v' = \frac{c}{\nu}, \frac{1}{T'} = \nu' = \frac{\omega'}{2\pi} \quad (2)$$

According to the wave energy relation -

$$E = m c v = h D \Rightarrow v = \frac{mc}{E}$$

$$\Rightarrow v' = \frac{mc}{E} = \frac{mc}{hD} = \frac{m}{h} \nu \quad (3)$$

$$\frac{x - \text{vel}}{\text{frequency}} = \frac{\omega'}{\nu'} = \frac{c/v}{m/v} = \frac{p}{m} \nu \quad (4)$$

$$\boxed{\lambda = \frac{h}{m\nu}} \quad (5)$$

### Group velocity and phase velocity.

Let's assume that the two waves travel with the same amplitude but slightly different frequencies and phase velocities so that they can be represented by equation

$$E_1 = A \cos \omega_1 (t - \nu_1 x) \quad (1)$$

$$E_2 = A \cos \omega_2 (t - \nu_2 x) \quad (2)$$

The resultant amplitude of the wave may be obtained by adding (1) and (2) that is

$$E = E_1 + E_2$$

Date / /

$$E = A \cos \omega_1 (t - \nu_1 x) + A \cos \omega_2 (t - \nu_2 x)$$

$$E = 2A \cos \left[ \left( \frac{\omega_1 + \omega_2}{2} \right) t - \frac{\nu_1 + \nu_2}{2} x \right] \cos \left[ \frac{(\omega_1 - \omega_2)}{2} t + \frac{(\nu_2 - \nu_1)}{2} x \right]$$

Let us write.

$$\omega_1 = \omega = 2\pi\nu$$

$$\omega_2 = \omega + d\omega = 2\pi(\nu + d\nu)$$

$$\text{then } \omega_1 - \omega_2 = -d\omega$$

$$\omega_1 + \omega_2 = 2\omega \text{ as } d\omega \ll \omega$$

$$\text{Similarly } \frac{\omega_1}{\nu_1} + \frac{\omega_2}{\nu_2} = \frac{2\omega}{\nu}.$$

Then eq (1) becomes

$$E = 2A \cos \omega \left[ t - \frac{d\omega}{2} x \right] \cos \left[ \frac{1}{2} \left( \frac{\omega_2 - \omega_1}{\nu} \right) x \right]$$

$$\text{Now: } \frac{\omega_2 - \omega_1}{\nu} = \frac{d\omega}{\nu} \quad (6)$$

So that:

$$E = 2A \cos \frac{d\omega}{2} \left[ t - \frac{d\omega}{2\nu} (\nu x) \right] \cos \omega \left( t - \frac{\nu x}{2} \right) \quad (7)$$

$$= 2A \cos \frac{d\omega}{2} \left[ t - \frac{\nu}{2} x \right] \cos \omega \left( t - \frac{\nu x}{2} \right) \quad (8)$$

$$\text{where } C_1 = \frac{d\omega}{2} = \frac{d(2\pi\nu)}{2} = \frac{1}{2} (2\pi\nu) = \frac{d(2\pi\nu)}{2\nu} \quad (6)$$

In equation (8) a wave is travelling with phase velocity  $v'$  and its amplitude is.

$$2A \cos \frac{d\omega}{2} \left[ t - \frac{\nu x}{2} \right]$$

Date / /  
 This equation clearly represents that the amplitude itself is being propagated along  $n$ -axis with velocity is signal velocity. Thus group velocity.

Now eqn ② can be rewritten as

$$\frac{1}{c_1} = \frac{d(\frac{1}{x})}{dv} \rightarrow ⑦$$

Let  $E, v$  be the total energy and potential energy at the particle then,

$$\text{Kinetic energy } K = \frac{1}{2}mv^2 = (E - v)$$

$$\Rightarrow v = \sqrt{\frac{2(E-v)}{m}} \quad ⑧$$

Using the de-Broglie wavelength  $\lambda = \frac{h}{mv}$

$$\frac{1}{x} = \frac{mv}{h} = \frac{m}{h} \left\{ 2 \left( \frac{E-v}{m} \right) \right\}^{1/2}$$

Putting the value of  $\frac{1}{x}$  in eqn ⑦

$$\begin{aligned} \frac{1}{c_1} &= \frac{d}{dv} \left[ \frac{m}{h} \left\{ 2 \left( \frac{E-v}{m} \right) \right\}^{1/2} \right] \\ &= \frac{d}{dv} \left[ \frac{m}{h} \left\{ 2 \left( h^2 m v - v^2 \right) \right\}^{1/2} \right] \\ &= \frac{1}{h} \frac{d}{dv} \left[ \frac{2m(h^2 m v - v^2)^{1/2}}{h^2 m} \right] \\ &= \frac{1}{h} \times \frac{1}{2} \left[ 2m(h^2 m v - v^2)^{-1/2} \right] \times 2hv \\ &= \frac{m}{h^2} \left[ 2m(h^2 m v - v^2)^{-1/2} \right] \times \frac{1}{2} \end{aligned}$$

$$= \frac{1}{v}$$

Date / /

$$\Rightarrow \frac{1}{c_1} = \frac{1}{v} \Rightarrow c_1 = v$$

\* Group velocity is equal to particle velocity.

\* Relation between phase velocity and group velocity in non-relativistic condition.

Let  $v'$  be the phase and group velocity at the particle respectively.

Let ' $\lambda'$ ' is the de-Broglie wavelength at the particle and  $v$  be the frequency at the wave

$$\text{then phase velocity } = v = \lambda v' \quad ①$$

According to de-Broglie wavelength

$$\lambda = \frac{h}{mv} \quad ②$$

Let  $E$  be the total energy and the kinetic energy at the particle is

$$\begin{aligned} E &= KE = \frac{1}{2}mv^2 \\ E &\approx h\nu \Rightarrow \nu = E/h = \frac{1}{2}mv \end{aligned} \quad ③$$

$$v = \frac{h\nu}{m} = \frac{1}{2}mv$$

$$\Rightarrow v = \frac{\nu}{2}$$

$$\Rightarrow U = \frac{\nu}{2}$$

① ~~Time independent Schrodinger Equation~~

② Time dependent Schrodinger Equation.

③ Time independent Schrodinger Equation.

Date / /

Consider a system of stationary waves to be associated with a particle. Let  $(x, y, z)$  be the coordinates of the particle and  $\psi$  the wave function, displacement for the de-Broglie's waves at any time  $t$ .

Then the differential equation of wave motion in three dimension can be written in classical way as,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad (1)$$

where  $v$  is wave velocity.

The solution of eqn (1) can be written as:

$$\psi(x, y, z, t) = \psi_0(x, y, z) e^{-i\omega t} \quad (2)$$

Where  $\psi_0$  is the amplitude at the point considered. It is a function of  $(x, y, z)$

$$\psi(x, t) = \psi_0(x) e^{-i\omega t} \quad (3)$$

Differentiating twice on (3) w.r.t  $t$ , we get

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0(x) e^{-i\omega t} = -\omega^2 \psi$$

Putting value of  $\frac{\partial^2 \psi}{\partial t^2}$  in eqn (1)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{\omega^2}{v^2} \psi \quad (4)$$

$$\text{But } \omega = 2\pi\nu \Rightarrow \frac{\omega}{v} = \frac{2\pi}{\lambda} \quad (5)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{4\pi^2}{\lambda^2} \psi \quad \text{Laplacian operator}$$

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad (6)$$

Date / /



So far we have not introduced wave mechanics concept and so the treatment is general for us. Introducing the concept of wave mechanics as quantum mechanics de-Broglie's equation to be

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad (7)$$

$$\nabla^2 \psi + \frac{8\pi^2 m}{\hbar^2} (E - v) \psi = 0$$

Let  $E$  and  $v$  are the total energy and the potential energy of the particle respectively then-

$$K.E. = \frac{1}{2} m v^2 = E - v$$

$$\Rightarrow m v^2 = 2m(E - v)$$

$$\Rightarrow \nabla^2 \psi + \frac{8\pi^2 m}{\hbar^2} (E - v) \psi = 0$$

$$\Rightarrow \nabla^2 \psi + \frac{8\pi^2 m}{\hbar^2} (E - v) \psi = 0 \quad \left( \text{where } \hbar = \frac{h}{2\pi} \right)$$

$$\boxed{\nabla^2 \psi + \frac{8\pi^2 m}{\hbar^2} (E - v) \psi = 0} \quad (8)$$

### Time-dependent Schrödinger Equation:

Deriving eqn (8) w.r.t  $t$ )

$$\frac{d\psi}{dt} = -i\omega \psi \left( \frac{2\pi}{\lambda} \right) e^{-i\omega t} = -2\pi i \nu \psi$$

$$\frac{d\psi}{dt} = -2\pi i \frac{E}{\hbar} \psi = -iE \frac{\psi}{\hbar} \quad (9)$$

$$E\psi = i\hbar \frac{d\psi}{dt} \quad (10)$$

Date / /

Putting in eqn (10)

$$\nabla^2 \psi + \frac{2m}{\hbar^2} \left( i\hbar \frac{\partial \psi}{\partial t} - V \psi \right) = 0 \quad (11)$$

$$\nabla^2 \psi = -\frac{2m}{\hbar^2} \left( i\hbar \frac{\partial \psi}{\partial t} - V \psi \right)$$

$$\Rightarrow \frac{1}{2m} \nabla^2 \psi + V \psi = i\hbar \frac{\partial \psi}{\partial t} \quad (12)$$

$$\Rightarrow \left( \frac{-\hbar^2}{2m} \nabla^2 + V \right) \psi = i\hbar \frac{\partial \psi}{\partial t} \quad (13)$$

$$\Rightarrow [ \hat{H}(\psi) = E(\psi) ] \quad (13)$$

Physical Interpretation of wave function  $\psi$ 

Schrodinger himself attempted the physical interpretation  $\psi$  in terms of  $V$ . It is analogous with the electromagnetic radiation. In a EM radiation if  $A$  is amplitude then the energy density that is the number of energy per unit volume  $= A^2$ . To that the number of photons per unit volume, i.e. photon density is equal to  $\frac{A^2}{h\nu}$ .

This means that photon density is proportional to  $A^2$  since  $h\nu$  is a constant quantity. In a similar way if  $\psi$  is the amplitude of matter waves that the number of material particles per unit volume, i.e. particle density must be proportional to  $\psi^2$ .

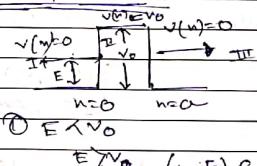


Date / /

Now if  $\psi$  is the charge associated with the particle then it will be charge density. Thus the quantity  $\psi^*$  is then measure of charge density as Schrodinger considered usually.  $\psi^* \psi$  is written instead of  $\psi \psi^*$  where  $\psi^*$  is the complex conjugate of  $\psi$ .

Uncertainty Principle

$$\Delta q \Delta p \geq \hbar/2$$

Rectangular Potential Barrier:

Region - I.

$$\begin{cases} \psi(n) \neq 0 \text{ for } n < 0 \\ \psi(n) = 0 \text{ for } 0 \leq n \leq a \\ \psi(n) \neq 0 \text{ for } n > a \end{cases} \quad (1)$$

Schrodinger eqn in region I

$$\frac{\partial^2 \psi}{\partial n^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad (\text{since } V=0) \quad (2)$$

Schrodinger eqn in region II

$$\frac{\partial^2 \psi}{\partial n^2} + \frac{2m}{\hbar^2} (E - V_0) \psi = 0 \quad (3)$$

Schrodinger eqn in region III

$$\frac{\partial^2 \psi}{\partial n^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad (4)$$

Date / /



Where,  $\psi_1, \psi_2, \psi_3$  are the wave function in the region I, II and III respectively.

The general sol<sup>n</sup> at any "②, ③ and ④ can be written as

$$\psi_1 = A_1 e^{i P_1 n/h} + B_1 e^{-i P_1 n/h} \quad (5)$$

$$\psi_2 = A_2 e^{i P_2 n/h} + B_2 e^{-i P_2 n/h} \quad (6)$$

$$\psi_3 = A_3 e^{i P_3 n/h} + B_3 e^{-i P_3 n/h} \quad (7)$$

Where,  $P_1$  and  $P_2$  are the momenta of the particle in corresponding region.

$$\begin{aligned} P_1 &= \sqrt{q/mE} \\ P_2 &= \sqrt{2m(E - V_0)} \end{aligned} \quad (8)$$

$A_1, B_1, A_2, B_2$  and  $A_3, B_3$  are the constants determined by the boundary conditions.

In eqn ⑤ The first term represent the wave incident wave and second term represent the reflected wave in region I at  $n=0$ .

In eqn ⑥ The first term represents the transmitted wave and the second term represent the wave reflected at  $n=a$ .

In eqn ⑦ the 1st term represents the transmitted wave at  $n=a$ , and the 2nd term represents the wave traveling in  $x$ -axis in  $n$ -direction but no wave travels back from infinity to 3rd region. Consequently

Date / /



$B_3 = 0$ , so that the solution of eqn 4 i.e. eqn ⑦ can be written as

$$\psi_3 = A_3 e^{i P_3 n/h} \quad (9)$$

$\psi$  must be continuous at the boundary boundaries i.e.

$$\psi_1 = \psi_2 \text{ at } n=0 \quad (10)$$

$$\psi_2 = \psi_3 \text{ at } n=a \quad (11)$$

The other condition  $\frac{d\psi}{dn}$  also be continuous at the boundary

$$\frac{d\psi_1}{dn} = \frac{d\psi_2}{dn} \text{ at } n=0 \quad (12) \quad (11)$$

$$\frac{d\psi_2}{dn} = \frac{d\psi_3}{dn} \text{ at } n=a \quad (13)$$

Applying boundary condition (10) to the eqn ⑤

$$A_1 + B_1 = A_2 + B_2 \quad (14)$$

Applying boundary condition (11) to the eqn ⑥

(15) Applying the boundary condition (10) to eqn ⑦

$$A_2 e^{i P_2 a/h} + B_2 e^{-i P_2 a/h} = A_3 e^{i P_3 a/h} \quad (16)$$

Diff eqn ⑤, ⑥ and ⑦, we will get

$$\frac{d\psi_1}{dn} = \frac{i P_1}{h} [A_1 e^{i P_1 n/h} - B_1 e^{-i P_1 n/h}] \quad (17)$$

$$\frac{d\psi_2}{dn} = \frac{i P_2}{h} [B_2 e^{i P_2 n/h} - A_2 e^{-i P_2 n/h}] \quad (18)$$

$$\frac{d\psi_3}{dn} = \frac{i P_3}{h} A_3 e^{i P_3 n/h} \quad (19)$$

Page No.

Date / /

Applying the boundary condition (11) & (12) to these equations will get,

$$P_1(A_1 - B_1) = P_2(A_2 - B_2)$$

$$P_2 \left[ A_2 e^{i P_2 a/h} - B_2 e^{-i P_2 a/h} \right] = P_1 \left( A_3 e^{i P_1 a/h} \right)$$

$$\Rightarrow (A_1 - B_1) = P_2 (A_2 - B_2) \quad (13)$$

$$\Rightarrow A_2 e^{i P_2 a/h} - B_2 e^{-i P_2 a/h} = \frac{P_1}{P_2} A_3 e^{i P_1 a/h} \quad (14)$$

Solving eqns (12) and (13) for  $A_1$  and  $B_2$ , we get

$$A_1 = \frac{A_2}{2} \left( 1 + \frac{P_2}{P_1} \right) + \frac{B_2}{2} \left( 1 - \frac{P_2}{P_1} \right) \quad (15)$$

$$B_2 = \frac{B_2}{2} \left( 1 - \frac{P_2}{P_1} \right) + \frac{A_2}{2} \left( 1 + \frac{P_2}{P_1} \right) \quad (16)$$

Solving eqns (13) and (15) for  $A_1$  and  $B_2$ , we find the volume of  $A_2$  and  $B_2$ .

$$A_2 = \frac{A_2}{2} \left( 1 + \frac{P_1}{P_2} \right) e^{i(P_1 + P_2)a/h} \quad (17)$$

$$B_2 = \frac{B_2}{2} \left( 1 - \frac{P_1}{P_2} \right) e^{i(P_1 + P_2)a/h} \quad (18)$$

Putting the value of  $A_2$  and  $B_2$  from these eqns in (12) and (13)

$$A_1 = \frac{A_2}{2} e^{i P_1 a/h} \left[ \left( 1 + \frac{P_2}{P_1} \right) \left( 1 + \frac{P_1}{P_2} \right) e^{i P_2 a/h} + \left( 1 - \frac{P_2}{P_1} \right) \left( 1 - \frac{P_1}{P_2} \right) e^{i P_2 a/h} \right]$$

Page No. [ ]

Date / /

$$B_1 = \frac{B_2}{2} e^{i P_2 a/h} \left[ \left( 1 - \frac{P_2}{P_1} \right) \left( 1 + \frac{P_1}{P_2} \right) e^{i P_2 a/h} + \left( 1 + \frac{P_2}{P_1} \right) \left( 1 - \frac{P_1}{P_2} \right) e^{i P_2 a/h} \right] \quad (19)$$

Eqn (23) maybe can be written as

$$\frac{A_3}{A_1} = \frac{e^{i P_1 a/h}}{\dots}$$

$$= \frac{e^{i P_1 a/h}}{\left\{ \left( 1 + \frac{P_2}{P_1} \right) \left( 1 + \frac{P_1}{P_2} \right) e^{i P_2 a/h} + \left( 1 + \frac{P_2}{P_1} \right) \left( 1 - \frac{P_1}{P_2} \right) e^{i P_2 a/h} \right\}}$$

$$= \frac{e^{i P_1 a/h}}{(P_1 + P_2) e^{-i P_2 a/h} - (P_1 - P_2)^2 e^{i P_2 a/h}}$$

$$= \frac{e^{i P_1 a/h}}{(P_1 + P_2) \left( \frac{e^{-i P_2 a/h} - e^{i P_2 a/h}}{2} \right) + 2P_1 P_2 \left( \frac{e^{-i P_2 a/h} + e^{i P_2 a/h}}{2} \right)}$$

$$P_L = \sqrt{2m(E - \nu_0)}$$

$$\frac{A_3}{A_1} = \frac{2P_1 e^{-i P_1 a/h}}{(P_1 + P_2) \sinh(i P_1 a/h) + 2P_1 P_2 \cosh(i P_2 a/h)}$$

$$\Rightarrow \frac{A_3}{A_1} = \frac{2P_1 e^{-i P_1 a/h}}{\text{Cosec}(P_1 a/h) [ (P_1 + P_2)^2 \tanh(i P_2 a/h) + 2P_1 P_2 ]} \quad (20)$$

$$= \frac{2P_1 e^{-i P_1 a/h}}{(P_1 + P_2) \tanh(i P_2 a/h) + 2P_1 P_2} \quad (21)$$

Taking the complex conjugate of eqn (23)

$$\frac{A_3^*}{A_1^*} = \frac{2P_1 P_2 \text{sech}(-i P_2 a/h) e^{i P_1 a/h}}{(P_1 + P_2)^2 \text{tanh}(-i P_2 a/h) + 2P_1 P_2} \quad (22)$$

$P_2^* = i \sqrt{2m(\nu_0 - E)}$
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$\Rightarrow P_2^* = -P_2$
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$\Rightarrow P_2^{*\prime} = P_2$
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$\Rightarrow P_2^{*\prime\prime} = P_2$
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Page No. [ ]

Date / /

$$\frac{A_3''}{A_1''} = \frac{-2P_1 P_2 \operatorname{sech}(iP_2 a/h) e^{iP_1 a/h}}{(P_1^2 + P_2^2) \tanh(iP_2 a/h) - 2iP_2} \quad (27)$$

$\tau = \text{mag. of the transmitted current}$   
 $\tau = \text{mag. of incident current}$

$$= \frac{C A_3 A_1''}{(P_1^2 + P_2^2) h/m} = \frac{A_3 A_1''}{P_1^2 + P_2^2}$$

$$\tau = \frac{A_3}{A_1} \times \frac{A_1''}{A_1}$$

$$\Rightarrow \tau = \frac{\{2P_1 P_2 \operatorname{sech}(iP_2 a/h) e^{iP_1 a/h}\}^2 \{2P_1 P_2 \operatorname{sech}(iP_2 a/h)\}^2}{\{2P_1 P_2 \operatorname{sech}(iP_2 a/h) e^{iP_1 a/h}\}^2}$$

$$= \frac{\{P_1^2 + P_2^2\} \tanh(iP_2 a/h) + 2iP_1 P_2}{\{P_1^2 + P_2^2\} \tanh(iP_2 a/h) - 2iP_1 P_2}$$

$$\Rightarrow \tau = \frac{-4P_1 P_2 \operatorname{sech}(iP_2 a/h)}{(P_1^2 + P_2^2) \tanh(iP_2 a/h) - 2iP_1 P_2} \quad (28)$$

$$R = \frac{B_1}{A_1} \times \frac{B_1''}{A_1''}$$

$$\Rightarrow R = \frac{(P_1^2 + P_2^2) \tanh(iP_2 a/h)}{(P_1^2 + P_2^2)^2 \tanh^2(iP_2 a/h) - 4P_1^2 P_2^2}$$



Page No. [ ]

Date / /

### Particle in a Box.

Let a single particle that is a nucleon or meson be confined within a rectangular potential box with edges parallel to  $x$ ,  $y$  and  $z$ -axis, with faces  $0x$ ,  $0y$  and  $0z$  respectively. The particle can move within the region.

$$\begin{aligned} &0x \\ &0y \\ &0z \end{aligned}$$

i.e. Inside the box a potential function  $V(x, y, z)$  is equal to zero but the rises suddenly to a very large value and the boundaries of the region remaining infinitely large everywhere outside the boundaries. Therefore, the particle will rebounded when it will strike either of the boundaries.

If  $m$  is the mass of the particle and  $E$  is the total energy the Schrödinger eqn for the case under consideration may be written as:

$$\nabla^2 \psi + \frac{2m}{\hbar^2} E \psi = 0 \quad (\text{since } V=0)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad (1)$$

Page No. [ ]

Date / /

Q) Eqn ① is a 2nd-order partial differential equation and can be solved by the method of variable separable method, so we may write the solution at the above eqn in form

$$\psi(m, y, z) = u(n) \cdot y(y) \cdot z(z) \quad \text{--- ②}$$

putting the value of  $\psi$  from eqn ② in eqn ① and dividing by  $u(n) y(y) z(z)$  so we will get

$$\frac{1}{n} \frac{\partial u}{\partial n} + \frac{1}{y} \frac{\partial y}{\partial y} + \frac{1}{z} \frac{\partial z}{\partial z} + \frac{2m}{\pi^2} E = 0 \quad \text{--- ③}$$

$$\Rightarrow \frac{1}{n} \frac{\partial u}{\partial n} = -\frac{1}{y} \frac{\partial y}{\partial y} - \frac{1}{z} \frac{\partial z}{\partial z} - \frac{2m}{\pi^2} E \quad \text{--- ④}$$

$$\text{Let } \frac{1}{n} \frac{\partial u}{\partial n} = kn \quad \text{--- ⑤}$$

$$kn = -\frac{1}{y} \frac{\partial y}{\partial y} - \frac{1}{z} \frac{\partial z}{\partial z} - \frac{2m}{\pi^2} E \quad \text{--- ⑥}$$

Similarly,

$$ky = \frac{1}{y} \frac{\partial y}{\partial y} \quad \text{--- ⑦}$$

$$kz = \frac{1}{z} \frac{\partial z}{\partial z} \quad \text{--- ⑧}$$

$$kn + ky + kz = -\frac{2m}{\pi^2} E \quad \text{--- ⑨}$$

$$kn = -\frac{2m}{\pi^2} E_n \quad \text{--- ⑩}$$

$$ky = \frac{2m}{\pi^2} Ey \quad \text{--- ⑪}$$

$$kz = \frac{2m}{\pi^2} E_z \quad \text{--- ⑫}$$

Date / /

from eqn ②

$$\frac{\partial u}{\partial n} + \frac{2m}{\pi^2} E_n = 0 \quad \text{--- ⑬}$$

$$\frac{\partial y}{\partial y} + \frac{2m}{\pi^2} Ey = 0 \quad \text{--- ⑭}$$

$$\frac{\partial z}{\partial z} + \frac{2m}{\pi^2} E_z = 0 \quad \text{--- ⑮}$$

The general solution at eqn ⑬ will be sine function of arbitrary amplitude-frequency and phase

$$u(n) = A \sin(\alpha n + c) \quad \text{--- ⑯}$$

$$|u(n)| = 0 \text{ for } n=0, n \in \mathbb{N}$$

$$u(n) = 0 \text{ when } n=0, n \in \mathbb{N}$$

$$\sin c = 0$$

$$c = 0$$

$$\text{and } \sin(\alpha n + c) = 0$$

$$\sin c = 0$$

$$\Rightarrow \sin(\alpha n) = 0$$

$$\Rightarrow \alpha n = \pi k \quad [k = \frac{n\pi}{\alpha}]$$

$$u(n) = A \sin \frac{n\pi}{\alpha}$$

$$I(n)$$

$$\int_0^{ln} |u(n)|^2 dn = 1$$

$$\Rightarrow \int_0^{ln} |A \sin \frac{n\pi}{\alpha}|^2 dn = 1$$

Date / /

$$\Rightarrow \frac{\Delta m}{2} = 1.$$

$$\Rightarrow A = \sqrt{\frac{2}{m}} \quad (13)$$

$$m\psi = \sqrt{\frac{2}{m}} \sin \frac{m\pi x}{a} \quad (14)$$

$$\frac{\partial m\psi}{\partial x} = -\left(\frac{m\pi}{a}\right) m\psi(n) \quad (15)$$

Putting values of eqn (14) and (15) in eqn (11(a)) we have.

$$\left(\frac{m\pi x}{a}\right)^2 m\psi(n) + \frac{2m}{\hbar^2} E\psi(n) = 0$$

$$\Rightarrow E_n = \frac{1}{2m} \left( \frac{m\pi n}{a} \right)^2 = \frac{n^2 \pi^2 h^2}{8ma^2}$$

Similarly solving eqn 11(b) and 11(c) there we get

$$E_y = \frac{m\pi^2 y}{8ma^2} \quad (16)$$

$$E_z = \frac{n^2 \pi^2}{8ma^2} \quad (17)$$

Now,

$$E = E_x + E_y + E_z$$

$$= \frac{A^2}{m} \left( \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{a^2} \right) \quad (18)$$

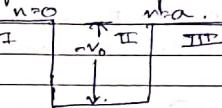
where  $x, y$ , and  $z$  are set at positive numbers.

$$\text{then } \psi_{mnxyz}(x, y, z) = X(x)Y(y)Z(z)$$

Date / /

$$= \frac{2}{\sqrt{L_x L_y L_z}} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \sin \frac{z\pi z}{L_z} \quad (19)$$

The square well potential.



$$\begin{cases} V_0 & \text{for } x < 0 \\ 0 & \text{for } 0 < x < a \\ V_0 & \text{for } x > a \end{cases} \quad (20)$$

The Schrödinger eqn in region (I) is given by,

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{2m}{\hbar^2} (E + V_0) \psi_1 = 0 \quad (21)$$

Region - II

$$\frac{\partial^2 \psi_2}{\partial x^2} + \frac{2m}{\hbar^2} (E + V_0) \psi_2 = 0 \quad (22)$$

Region - III

$$\frac{\partial^2 \psi_3}{\partial x^2} + \frac{2m}{\hbar^2} E \psi_3 = 0 \quad (23)$$

Where  $\psi_1, \psi_2, \psi_3$  are the wave function in region I, II, III respectively.

The general solution of eqn (21), (22), (23) may be written as,

$$\psi_1 = A_1 e^{ip_1 x/\hbar} + B_1 e^{-ip_1 x/\hbar} \quad (24)$$

$$\psi_2 = A_2 e^{ip_2 x/\hbar} + B_2 e^{-ip_2 x/\hbar} \quad (25)$$

$$\psi_3 = A_3 e^{ip_3 x/\hbar} + B_3 e^{-ip_3 x/\hbar} \quad (26)$$

Where  $p_1$  and  $p_2$  are the momenta in the region (I, II) and III respectively.

Date / /

$$\begin{aligned} P_1 &= \sqrt{2mE} \\ P_2 &= f_{mn}(E) \end{aligned} \quad \text{--- (4)}$$

Now  $A_1, B_1, A_2, B_2$  and  $A_3, B_3$  are the constants to be determined by the boundary conditions.

$$\text{So eqn (2) can be written as} \\ \psi_3 = A_3 e^{ip_3 n/k} \quad \text{--- (5)}$$

For calculating  $A_1, B_1, A_2, B_2$  and  $A_3, B_3$  we will apply boundary condition at  $n=0$  and  $n=a$ .

$$\begin{aligned} \psi_1 &= \psi_2 \text{ at } n=0 \quad \text{--- (6)} \\ \psi_2 &= \psi_3 \text{ at } n=a \quad \text{--- (7)} \end{aligned} \quad \text{--- (11)}$$

Differentiating eqn (3), (6) and (7) we will get

$$\frac{d\psi_1}{dn} = \frac{iP_1}{h} [A_1 e^{ip_1 n/k} - B_1 e^{-ip_1 n/k}] \quad \text{--- (8)}$$

$$\frac{d\psi_2}{dn} = \frac{iP_2}{h} [A_2 e^{ip_2 n/k} - B_2 e^{-ip_2 n/k}] \quad \text{--- (9)}$$

$$\frac{d\psi_3}{dn} = \frac{iP_3}{h} [A_3 e^{ip_3 n/k}]. \quad \text{--- (10)}$$

Applying boundary conditions 10(8) and 10(9) to the eqn (3), (6) and (7) will be written

$$A_1 + B_1 = A_2 + B_2 \quad \text{--- (11)}$$

$$A_2 e^{ip_2 a/k} + B_2 e^{-ip_2 a/k} = A_3 e^{ip_3 a/k} \quad \text{--- (12)}$$

Date / /

Applying boundary condition 11(7) and 12(11) we will get

$$A_1 + B_1 = \frac{P_2}{P_1} (A_2 - B_2) \quad \text{--- (13)}$$

$$A_2 e^{ip_2 a/k} - B_2 e^{-ip_2 a/k} = \frac{P_1}{P_2} A_3 e^{ip_3 a/k} \quad \text{--- (14)}$$

Solving eqn (13) and (14) for  $A_1$  and  $B_1$ , we get

$$A_1 = \frac{A_2}{2} \left( 1 + \frac{P_2}{P_1} \right) + \frac{B_2}{2} \left( 1 + \frac{P_2}{P_1} \right) \quad \text{--- (15)}$$

$$B_1 = \frac{A_2}{2} \left( 1 - \frac{P_2}{P_1} \right) + \frac{B_2}{2} \left( 1 - \frac{P_2}{P_1} \right) \quad \text{--- (16)}$$

Solving eqn (15) and (16) for  $A_2$  and  $B_2$ , we get

$$A_2 = \frac{A_3}{2} \left( 1 + \frac{P_1}{P_2} \right) e^{i(P_1 + P_2)a/k} \quad \text{--- (17)}$$

$$B_2 = \frac{A_3}{2} \left( 1 - \frac{P_1}{P_2} \right) e^{i(P_1 + P_2)a/k} \quad \text{--- (18)}$$

Putting the value of  $A_2$  and  $B_2$  from eqn (17) and (18) to eqn (9) and (10)

$$\begin{aligned} \psi_2 &= \frac{A_3}{h} e^{ip_2 a/k} \cdot \left[ \left( 1 + \frac{P_2}{P_1} \right) \left( 1 + \frac{P_1}{P_2} \right) e^{-iP_2 a/k} \right. \\ &\quad \left. + \left( 1 - \frac{P_2}{P_1} \right) \left( 1 - \frac{P_1}{P_2} \right) e^{iP_2 a/k} \right] \quad \text{--- (22)} \end{aligned}$$

$$\begin{aligned} \psi_2 &= \frac{A_3}{h} e^{ip_2 a/k} \left[ \left( 1 - \frac{P_2}{P_1} \right) \left( 1 + \frac{P_1}{P_2} \right) e^{-iP_2 a/k} \right. \\ &\quad \left. + \left( 1 + \frac{P_2}{P_1} \right) \left( 1 - \frac{P_1}{P_2} \right) e^{iP_2 a/k} \right] \quad \text{--- (22)} \end{aligned}$$

Date / /

Problem 2.3

$$\rightarrow \frac{A_3}{A_1} = \frac{e^{-ip_1\alpha/\hbar}}{[C + \frac{p_2}{p_1}] [C + \frac{p_1}{p_2}]}$$

$$= \frac{e^{-ip_1\alpha/\hbar}}{\left(\frac{p_1}{p_2} + 1\right) \left(C^{-ip_2\alpha/\hbar} + e^{ip_2\alpha/\hbar}\right) + 2 \left(C^{-ip_2\alpha/\hbar} + C^{ip_2\alpha/\hbar}\right)}$$

$$e^{-ip_1\alpha/\hbar}$$

$$= \frac{-i \left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right)}{2i} \left( \frac{-e^{ip_2\alpha/\hbar} + e^{ip_2\alpha/\hbar}}{2i} \right) + \frac{\left(C^{-ip_2\alpha/\hbar} + C^{ip_2\alpha/\hbar}\right)}{2}$$

$$\rightarrow \frac{A_3}{A_1} = \frac{e^{-ip_1\alpha/\hbar}}{-i \left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right) \sinh\left(\frac{p_2\alpha}{\hbar}\right) + i \cosh\left(\frac{p_2\alpha}{\hbar}\right)}$$

$$T = \frac{\text{magnitude of transmitted current}}{\text{magnitude of incident current}} \quad (2)$$

$$= \frac{|A_3|^2}{|A_1|^2}$$

$$= \frac{1}{4} \left( \frac{p_1}{p_2} - \frac{p_2}{p_1} \right)^2 \sinh^2\left(\frac{p_2\alpha}{\hbar}\right)$$

$$[P^2 + T^2 = 1]$$



Date / /

Potential prob.

A potential A particle in three dimensional.  
Initially deep potential well.

$$\Psi(0, y, z) = \Psi(n, 0, z) = \Psi(n, y, 0) = 0 \quad (1)$$

$$\Psi(x, 0, z) = \Psi(n, x, z) = \Psi(n, y, z) = 0 \quad (2)$$



The schrodinger eqn of the particle  
inside the box is

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{2m}{\hbar^2} E \Psi = 0 \quad (3)$$

$$\Psi(n, y, z) = X(n) Y(y) Z(z) \quad (4)$$

Putting the value of  $\Psi(n, y, z)$  from eqn 3 in eqn 3 and dividing by xyz.

$$\rightarrow \frac{1}{n} \frac{\partial^2 X}{\partial n^2} + \frac{1}{y} \frac{\partial^2 Y}{\partial y^2} + \frac{1}{z} \frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar^2} E = 0 \quad (5)$$

The eqn may be written as

$$\frac{1}{n} \frac{\partial^2 X}{\partial n^2} = -\frac{1}{y} \frac{\partial^2 Y}{\partial y^2} = -\frac{1}{z} \frac{\partial^2 Z}{\partial z^2} = -\frac{2m}{\hbar^2} E \quad (6)$$

$$\text{Let } -\frac{1}{n} \frac{\partial^2 X}{\partial n^2} = -k^2 \quad (7)$$

$$\text{Similarly } -\frac{1}{y} \frac{\partial^2 Y}{\partial y^2} = -k_y^2 \quad (8)$$

$$\text{and } -\frac{1}{z} \frac{\partial^2 Z}{\partial z^2} = -k_z^2 \quad (9)$$

Date No. \_\_\_\_\_

Page No. \_\_\_\_\_

Date / /

$$E = \frac{t^m}{2m} (K_x n^2 K_y^2 + K_z^2) - (2)$$

The general solution of  $\nabla^2 E = 0$  and  $E$  can be written as

$$X = A_1 \sin K_x n + B_1 \cos K_x n$$

$$Y = A_2 \sin K_y y + B_2 \cos K_y y$$

$$Z = A_3 \sin K_z z + B_3 \cos K_z z$$

By applying (1A)

$$B_1 = B_2 = B_3 = 0$$

$$X = A_1 \sin K_x n - (A)$$

$$Y = A_2 \sin K_y y - (B) \quad \left. \begin{array}{l} \\ \end{array} \right\} (1B)$$

$$Z = A_3 \sin K_z z - (C)$$

Applying boundary condition (1B)

$$\sin K_x a = 0$$

$$\sin K_y b = 0$$

$$\sin K_z c = 0 \quad \text{since } A_1 \neq A_2 \neq A_3$$

$$K_x a = n_x \pi, \quad K_y b = n_y \pi, \quad K_z c = n_z \pi$$

$$k_x = \frac{n_x \pi}{a}$$

$$k_y = \frac{n_y \pi}{b}, \quad k_z = \frac{n_z \pi}{c} \quad \left. \begin{array}{l} \\ \end{array} \right\} (1)$$

Date / /

$$x = n_x \sin \frac{\pi x}{a}$$

$$y = n_y \sin \frac{\pi y}{b}$$

$$z = n_z \sin \frac{\pi z}{c}$$

$$\psi(n_x, n_y, n_z) = A_1 A_2 A_3 \sin \frac{\pi n_x x}{a} \sin \frac{\pi n_y y}{b} \sin \frac{\pi n_z z}{c} \quad (13)$$

$$E_{mnz} = \frac{t^m}{2m} \left[ \frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right] \quad (14)$$

Then if  $n_x = n_y = n_z = 1$ .

$$\text{then } E_{111} = \frac{t^{111}}{2 \cdot 1} [n_x^2 + n_y^2 + n_z^2] \quad (15)$$

$$E_{mnz} = \frac{t^m}{2m} \left[ n_x^2 + n_y^2 + n_z^2 \right] \quad (16)$$

$$E_{111} = \frac{t^{111}}{2 \cdot 1} \quad (17)$$

$$\psi(n_x, n_y, n_z) = A_1 \sin \frac{\pi n_x x}{a} \sin \frac{\pi n_y y}{b} \sin \frac{\pi n_z z}{c} \quad (18)$$

Date / /

## LASER

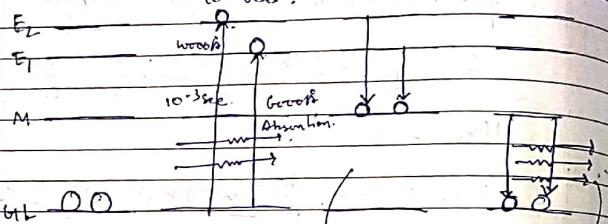
Ruby Laser

(Solid state laser)

$\text{Al}_2\text{O}_3$  (alumina/pink ruby + 0.5% Cr)

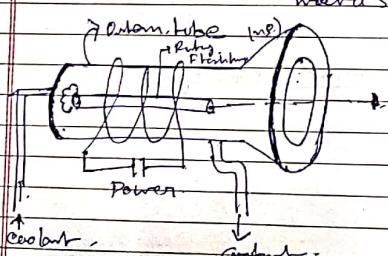
0 → 0.5 sec.  
10<sup>-4</sup> sec → 1 sec.

10<sup>-8</sup> sec.



Cg: Energy diagram. Then give energy to crystal (to to to end at meta state.)

Simulated emission



Date / /

## Excitation or pumping

→ Optical pumping

→ Electrical discharge

→ Induction & atomic atomic collision

→ Direct power

→ in optical-pumping sources.

→ in optical pumping a source of light is used to supply energy to the active medium. Most of the energy comes in the short pulses of light, a method used by "Maser's" ruby laser, widely used to during solid state lasers. The laser generally placed inside the helical xenon flash lamp.

→ Electric discharge:- Another method of pumping is by direct electric-pumping excitation as it occurs in an electric discharge. the method is preferred for gas laser of gaseous organic lasers. For example, if the electric field the by pair (vertical lines) causes electron emitted by the cathode to be accelerated to the anode. Some of its electron will impinge on the atom of the active medium and because them to the excited state. As a result the population inversion is active in the active medium.

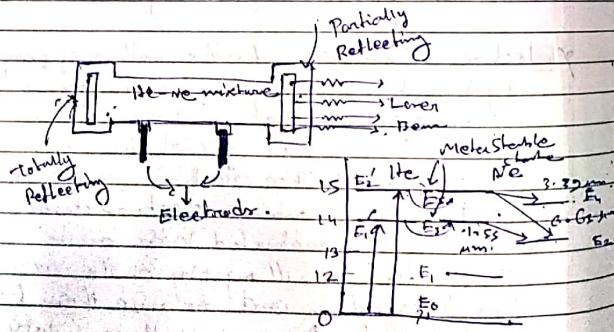
Date / /

⇒ In-elastic atomic collision :- If this process method of pumping the electric discharge provides the initial excitation which causes one type of atoms to their excited states. This atom subsequently collides in in-elastic way with another types of atom. The energy transferred in-in-elasticity may raise the later types of atoms to the excited state and there are the atoms which provides the population inversion. He-Ne laser such pumping process is employed.

⇒ Direct conversion:- A direct conversion in electrical energy into radiation occurs in LED. Such LED is used for pumping by direct conversion in semiconductor lasers.

#### Helium-Deuterium Laser

(Gronian Laser)  
(10:1) ratio of helium - deuterium



Date / /

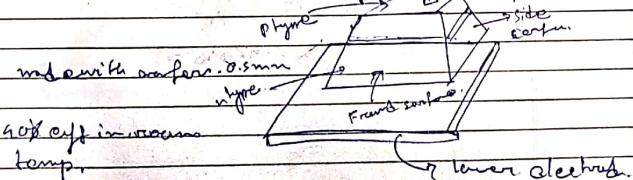


#### Semiconductor Laser

Gra-Si material

In the case of P-N junction pumping is passively diode lasers are injected from P region and e<sup>-</sup> from N-P region. The electrons and holes recombined and transfer of energy takes place in very near junction region the amount of energy emitted the energy gap depends upon a particular type of semi-conductor in one of form Ga or Si semiconductor format if the energy is released in the shape of heat because the recombination of carrier of opposite sign laser pulses through interaction with the atom of the crystals

In case of Gra-Si and after the energy is released as light because the atom of the crystals are not involved in transfer of energy. Protons emitted at the moment of recombination of an electron when a hole will stimulate recombination of other carriers of the electric charges the result will be stimulated emission of radiation.



If it is capable of multi mode output with file of light wave so we use it.

Page No. [ ]

Date / /

→ If laser action occurs by transitions from excited to ground state, properties 693 nm wavelength. What is the energy of excited state to take the energy to be zero.

$$C_E = 0$$

$$C = 3 \times 10^8 \text{ m}^{-1}$$

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{693 \times 10^{-9} \times 1.6 \times 10^{-19}} = 1.75 \text{ eV}$$

Assume that an atom has 2 energy levels separated by an energy corresponding to a frequency  $4.7 \times 10^{14} \text{ Hz}$ , as in the He-Ne laser. Let us assume that all the atoms are located in one of the other these two states. Calculate the fraction of atoms in upper state at room temp  $T = 300 \text{ K}$ .

$$V = 4.7 \times 10^{14} \text{ Hz}$$

$$T = 300 \text{ K} \Rightarrow 27^\circ \text{C}$$

$$\rightarrow E = hc$$

$$\rightarrow E = 6.626 \times 10^{-34} \times 4.7 \times 10^{14} =$$

$$= 31.14 \times 10^{-20} \text{ J}$$

$$\frac{N_2}{N_1} = C \frac{(E_2 - E_1)}{k_B T}$$

$$= 2.42 \times 10^{-33}$$

Date / /

→ Absolute temp  $T$  for an arbitrary source of light is of order of  $10^3 \text{ K}$ , so that in such sources the process of spontaneous emission will dominate over stimulated emission over length of light  $\lambda$ .

$T = \text{order of } 10^3 \text{ K}$

$$\lambda = 6000 \text{ nm}$$

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-9}}$$

$$= 3.15 \times 10^{-19}$$

$$\frac{N_2}{N_1} = C \left( \frac{e^{-E/k_B T}}{1 + e^{-E/k_B T}} \right)$$

$$= C \left( \frac{e^{-3.15 \times 10^{-19}}}{1 + e^{-3.15 \times 10^{-19}}} \right)$$

$$= 2.40917 \times 10^3$$

$$\approx 2.4 \times 10^3$$

$$\frac{N_2}{N_1} = \frac{e^{-E/k_B T}}{1 + e^{-E/k_B T}}$$

$$= e^{-E/k_B T}$$

$$= e^{-3.15 \times 10^{-19}}$$

$$= 2.54 \times 10^{10}$$

It is a very large value. The spontaneous emission dominates over stimulated emission.

→ A pulse laser we fire welding per second 1000 J of power during 10 ms. Calculate the total energy delivered in the world.

Date / /

$$P = 100 \text{ W}$$

$$t = 10 \text{ ms}$$

$$= 10 \times 10^{-3} \text{ s}$$

$$P = \frac{E}{T} \Rightarrow E = (10^{-3} \times 100) = 1 \text{ J}$$

Imagine we chop a continuous laser beam (100W) into 0.1 ms pulses. Using same stroboscopic cutter. Compute the resultant band width and coherence length.

$$\Delta = 623 \text{ nm} \quad 0.1 \text{ ms} = 1 \times 10^{-2} \text{ sec}$$

$$= 623 \times 10^{-9} \text{ m}$$

$$E = \frac{hc}{\Delta} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{623 \times 10^{-9}}$$

$$= 3.19 \times 10^{-19} \text{ J}$$

$$T = 0.1$$

$$P = \frac{E}{T} = \frac{3.19 \times 10^{-19}}{0.1 \times 10^{-9}} = 3.19 \times 10^9 \text{ watt}$$

$$\Delta \omega = c \times \Delta t$$

$$= 3 \times 10^8 \times 0.1 \times 10^{-9}$$

$$= 0.03 \text{ rad/s}$$

$$\text{Bandwidth } \Delta \theta = \frac{1}{\Delta t} = \frac{1}{0.1 \times 10^{-9}} = 10^{10} \text{ Hz}$$

Page No. [ ]

Date / /

$$\text{Line width width: } \Delta \lambda = \frac{c \times \Delta t}{c}$$

$$= 1.295 \times 10^{-11}$$

$$= 1.295 \times 10^{-11}$$

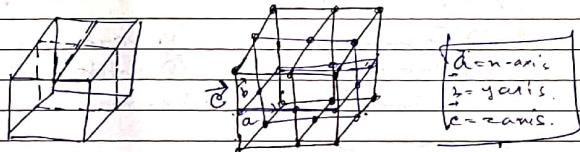
### Solid state physics

Crystalline structures  
(Material)

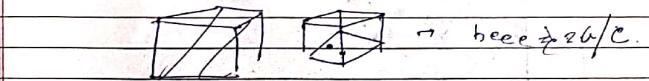
↓  
Crystalline      Amorphous  
↓  
Polycrystalline

Primitive cell and non-primitive cell

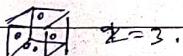
Primitive cell.



non primitive cell.



Fcc:



$$Z = 4$$

$$a = \sqrt{2} r$$

$$c/a = \sqrt{3}$$

$$V = a^3$$

$$M = Z \cdot N_A \cdot M_r$$

$$= 4 \cdot N_A \cdot M_r$$

$$= 4 \cdot 6.02 \times 10^{23} \cdot 40$$

$$= 9.63 \times 10^{25} \text{ g/mol}$$

Date \_\_\_\_\_ / \_\_\_\_\_ / \_\_\_\_\_ ~~Brutinol lattice transformation reaction~~  
(barbie)

$$\left. \begin{array}{l} \vec{c}_1 = \frac{\alpha}{2} (\vec{i} + \vec{j} - \vec{k}) \\ \vec{b}_2 = \frac{\alpha}{2} (\vec{i} + \vec{j} + \vec{k}) \\ \vec{c}_1 = \frac{\alpha}{2} (\vec{i} - \vec{j} + \vec{k}) \end{array} \right\} \text{ } \alpha \text{ is the side of the cube}$$

~~fun fact~~

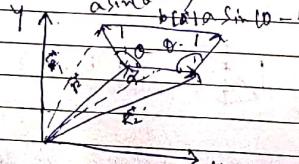
$$\left. \begin{aligned} \vec{a}^1 &= \frac{\alpha}{2} (\vec{i} + \vec{j}) \\ \vec{b}_r &= \frac{\alpha}{2} (\vec{j} + \vec{k}) \\ \vec{c}^1 &= \frac{\alpha}{2} (\vec{k} + \vec{i}) \end{aligned} \right\} \text{inside of cube}$$

o) Symbolic operation in 2-dim crystal.

- (i) Translation : (ii) Screw
  - (iii) Rotation . (iv) Glides.
  - (v) Transformation center.

$$\vec{r} = m_1 \vec{a} + m_2 \vec{b} + m_3 \vec{c}$$

$\theta = \frac{2\pi}{n}$  where  $n=1, 2, 3, 4, 6$ , will satisfy condition.



Date \_\_\_ / \_\_\_ / \_\_\_

$$\vec{a} = \vec{P}_0 - \vec{R}_1 \quad \theta = 2\pi/n.$$

$$p[\vec{a}] = \vec{r}_1 - \vec{r}_2$$

$$|P[\vec{a}]| = |P\vec{a}^*| = |\vec{a}^*| = 2|\vec{a}| \sin(\theta - \varphi_2) \\ \Rightarrow |\vec{a}^*| (1 - 2 \cos \varphi)$$

$$P = 1 - 2 \cos \theta$$

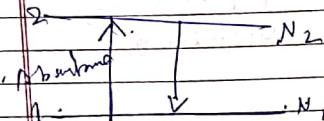
$$Q+P = -\left| \frac{L-P}{2} \right| < L$$

$$P = 3, 2, 1, 0, -1,$$

$S_0 = \$50$ ,  $n \in \{1, 2, 3, 4, 6\}$  (~~Allow~~, Allow values of  $n$ )

Date / /

Laser. light-amplifiable  
of stimulated  
emission.



Consider an atom having two states and let  $N_1$  and  $N_2$  be no of atoms per unit  $\nu(v)$  which represent the energy density of atomic radiation i.e.  $U(v)$  also represents the energy density in the frequency interval b/w  $v$  and  $v + dv$ . Now, the rate of absorption of radiation of frequency  $v$  is assumed to be proportional to the radiation energy density  $U(v)$  thus if the proportionality constant is denoted by  $B_{12}$  then  $N_1 B_{12} U(v)$  will represent the no of absorption per unit time.

The no of spontaneous emission will be proportional to  $N_2$  and if the proportionality constant is denoted by  $B_{21}$  then  $N_2 A_{21}$  will represent the no of spontaneous emission per unit volume per unit time. Finally, the no of stimulated loss

Date / /

emission will be proportional to  $N_2$  and also the radiation energy density  $U(v)$  if the proportionality constant is denoted by  $B_{21}$  then  $N_2 B_{21} U(v)$  will be represent the number of stimulated emission per unit time. The quantities  $B_{21}$ ,  $N_1$  and  $N_2$  are known as Einstein's coefficient and are determined by atomic system.

Now at thermal equilibrium

$$N_1 B_{12} U(v) = N_2 A_{21} + N_2 B_{21} U(v) \quad (1)$$

$$U(v) = \frac{A_{21}}{N_2 B_{12} - B_{21}}$$

at thermal equilibrium  $N_1$  and  $N_2$  will be related by the Boltzmann Boltzmann's const law.

$$\frac{N_2}{N_1} = e^{-\frac{(E_2 - E_1)}{kT}} \quad (2)$$

$$U(v) = \frac{B_{21}}{e^{\frac{(E_2 - E_1)}{kT}} B_{12} - B_{21}} \quad (3)$$

Now from Planck's law of heat radiation:

$$U(v) = \frac{8\pi h v^3}{c^3 \cdot \epsilon^4 \cdot \text{const}} \quad (4)$$

From eqn (3) and (4)

$$B_{12} = B_{21} \quad (5)$$

Date \_\_\_/\_\_\_/\_\_\_

$$8 \text{th } 23 \\ c_3 - B_{21} = A_{21} - 7$$

Car (1) and (2) are known as Einstein's predictions. Here  $A_{21}$ ,  $B_{12}$  and  $B_{21}$  are the Einstein's coefficients.

Date \_\_\_/\_\_\_/\_\_\_



Page No.

Page No.

Date / /

Date / /



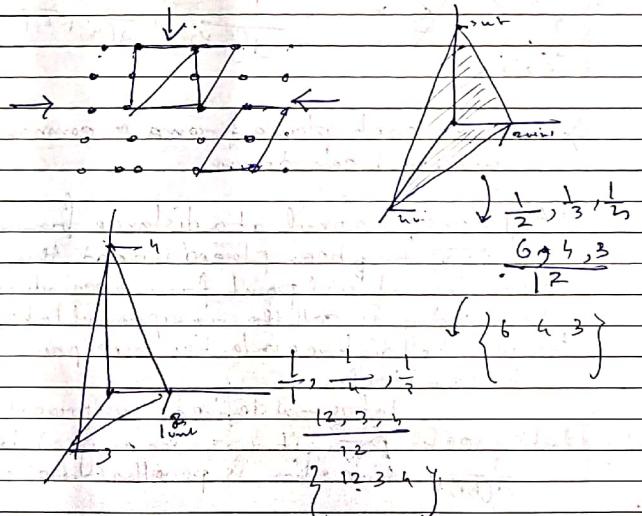
### Inversion center

Every bravais lattice permits the existence of inversion symmetry.

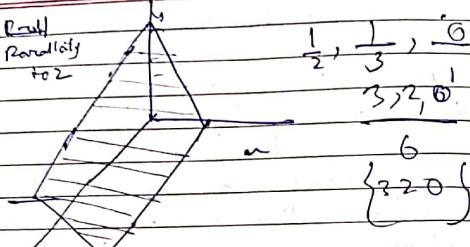
COCW :- A combination of rotation and translation is known as screw operation.

Elide :- Elide is a combination of reflection and translations.

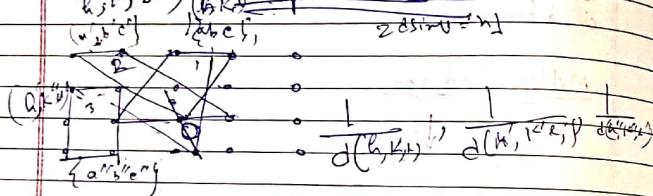
### Miller Indices



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Reciprocal lattices



→ normal to each plane forms a common straight line, been drawn.

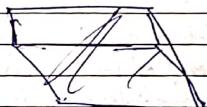
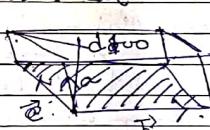
→ A point of normal at a distance from origin =  $\frac{1}{d(h, k, l)}$  has been placed indeed the collection of such point from a periodic array. This array is call the reciprocal lattice because the distance in the reciprocal lattices is the characteristic.

Let us now define reciprocal unit lattices vector. It is vector reciprocal direction is parallel to the normal of hkl plane.

Date / /

The parallelogram p.t.d. spanned by not non coplanar reciprocal lattice vector.

$\vec{v}_{100}, \vec{v}_{101}, \vec{v}_{010}$  is the unit cell in reciprocal lattices to its volume is inversely directly reciprocal to the unit cell (volume) in the cristal.



(1)  $\vec{v}_{100}$

$\vec{v}_{100}$   
 $\vec{v}_{001}$

volume = area  $\times d_{100}$  right.

$$\text{Group } \frac{1}{d_{100}} = \frac{\text{area}}{\text{volume}} = \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}} \quad [ \vec{a} \text{ is the unit vector direction of } \vec{b}, \vec{c} ]$$

$$\vec{v}_{100} = \frac{1}{d_{100}} \vec{a} = \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}} \vec{a}$$

$$\vec{v}_{001} = \frac{1}{d_{001}} \vec{a} = \frac{\vec{a} \times \vec{b}}{\vec{a} \cdot \vec{b} \times \vec{c}} \vec{c}$$

$$\vec{a} \cdot \vec{b} = 0$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{a} = G$$

$$\vec{c} \cdot \vec{b} = 0$$

$$\vec{c} \cdot \vec{a} = 1$$

Date / /

### Free electron theory of metal

In case of Na atom there is one electron which is free and it will move the nature is discrete. When the atoms are brought together to form solid the orbital of different atoms overlap and interact. It is considered that the interaction is so strong that quantization is completely vanished. Electrons are free and move freely in crystal and it will behave as a perfect gas. The Na atoms which are densely packed electrons develop a positive ion.

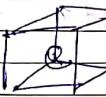
There exists a force of attraction between  $e^-$  and  $Na^+$ . This will help to form minicells and form within the crystal the repulsive force b/w  $e^-$  and ion are neglected. The electrons are free to move and confined with in the walls the potential energy is confined to be negative as the electron reaches at the boundary it becomes infinite potential.

Drude was the first who also had given the idea of the theory. He said after that it was well supported by Lorentz. The theory was called Drude-Lorentz theory. Drude and Lorentz assumed that the free electrons are treated as a perfect gas and obey Maxwell-Boltzmann statistics. The free electron theory. This theory is good to derive Ohm's law relation b/w current density, electric field and conductivity but failed to explain paramagnetic susceptibility.

Date / /



### Sommerfeld theory of free electron of metal (quantum treatment)



Let an electron of mass  $m$  is confined to a one dim box of length  $L$  by barrier at the two ends.

$$\psi(n) = 0 \text{ for } x < 0 \text{ and } x > L \\ = \phi \sin \frac{n\pi x}{L}, 0 \leq x \leq L$$

$$\frac{\partial^2 \psi_n}{\partial x^2} + \frac{2m}{\hbar^2} E \psi_n = 0$$

where  $E_n$  is the total energy of the electron in the  $n$ th state.

The general sol of the eqn.

$$\psi_{nn}(x) = A e^{ikx} + B e^{-ikx}$$

$$\text{when } E_n = \left(\frac{2m}{\hbar^2}\right) E_n \text{ and } A, B \text{ are too constant}$$

$$\psi_n(0) = 0 \text{ or } \psi_n(L) = 0.$$

$$\psi_n(x) = B \sin \left( \frac{n\pi x}{L} \right)$$

$$E_n = \frac{\hbar^2}{2m} \left( \frac{n\pi}{L} \right)^2 m \frac{d^2}{dx^2} \left( \frac{n\pi}{L} \right)^2$$

$$\int_0^L \psi_n(x) \psi_n(x) dx = 1$$

$$\int_0^L |\psi_n(x)|^2 dx = 1.$$

$$\Delta = \sqrt{2/L}$$

Date

$$\psi_n(\vec{r}) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$\psi_{n,k}(\vec{r}) = A \sin \left( \frac{n\pi x}{L} \right)$$

 $E_n$ 

$$\nabla^2 \psi_k(\vec{r}) + \frac{2m}{\hbar^2} E_k \psi_k(\vec{r}) = 0$$

$$\psi_k(\vec{r}) = \left( \frac{8}{L^3} \right)^{1/2} \sin \left( \frac{n\pi x}{L} \right) \sin \left( \frac{m\pi y}{L} \right) \sin \left( \frac{p\pi z}{L} \right)$$

$$\text{Area } K = \left( \frac{2m}{\hbar^2} \right)^{1/2} k$$

Normalisation condition.

$$\int_0^L \psi_k^*(\vec{r}) \psi_k(\vec{r}) d\vec{r} = 1$$

$$A = \left( \frac{L}{\lambda} \right)^3$$

$$\psi_k(\vec{r}) = \left( \frac{1}{V} \right)^{1/2} e^{i \vec{k} \cdot \vec{r}}$$

$$f(E) = \frac{1}{4\pi \epsilon_0 \left( \frac{E - E_F}{k_B T} \right)}$$

Date \_\_\_\_\_  
 Density of available electron states  
 $D(E)$  and filled electron state  $N(E)$

Date \_\_\_\_\_  
 Density of available electron states  
 $D(E)$  and filled electron state  $N(E)$   
 $\left( \frac{2\pi}{L}, \frac{2\pi}{L}, \frac{2\pi}{L} \right)$  cut along a unit cell and  $\left( \frac{2\pi}{a} \right)^3 \left( \frac{2\pi}{a} \right)^3 \left( \frac{2\pi}{a} \right)^3$   
 $= \frac{8\pi^3}{a^3} V = L^3$

Dividing known volume by the volume associated with a single lattice point. The procedure leads to the value.

$$D(E) dE = \frac{4\pi^2 V dK}{8\pi^3 / a^3}$$

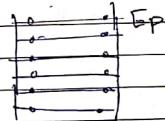
$$= \frac{K V}{2\pi^2} dE$$

$$\text{as known } E = \frac{h^2 k^2}{8m} \frac{1}{V}$$

$$D(E) dE = 2 \times \frac{V}{4\pi^2} \left( \frac{2m}{h^2} \right)^{3/2} E^{1/2} dE$$

$$D(E) = \frac{V}{2\pi^2} \left( \frac{2m}{h^2} \right)^{3/2} E^{1/2}$$

$$N(E) = D(E) \times f(E)$$



a) Band theory of semiconductor and insulator

Date 1/1

$$f(E) = \frac{1}{T} \exp\left(\frac{E - E_F}{kT}\right)$$

$$f(E) = \frac{1}{2}$$

$$f(E) = \frac{2}{3} \text{ states below } E_F$$

$$f(E) = 0 \text{ for } E > E_F$$

Crys.

Crys.

VB

In insulation the magnitude of forbidden gap  $E_F$  is large. But there is practically little thermal excitation of the electrons in conduction band with the result that the conduction band remains considerably empty. The valence band even full in at finite temp. There are very few interesting property of pure insulation.

In the case of insulation the energy of usually located somewhere between valence and conduction band i.e. in general  $E_F$  is not an allowed state which can be occupied by electrons. We will now prove that  $E_F$  at  $T=0$  in insulator exactly between half wave between valence and conduction band.

Date 1/1

At temp different from zero  
Hodensity of electron in the conduction band given by,

$$n_C = \frac{D(E)}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Similarly the density of conation in the VB

$$n_V = \frac{D(E)}{1 + \exp\left(\frac{E_F - E}{kT}\right)}$$

$$D(E) = n_C n_V$$

$$D(E) - n_V = \frac{D(E)}{1 + \exp\left(\frac{E_F - E}{kT}\right)}$$

$$\Rightarrow \frac{D(E)}{1 + \exp\left(\frac{E_F - E}{kT}\right)} = D(E) - \frac{D(E)}{1 + \exp\left(\frac{E_F - E}{kT}\right)}$$

$$E_F = \frac{E_C + E_V}{2}$$

Date \_\_\_\_\_

### Band theory of semiconductors

$$I = J \cdot eV / \text{vol}$$

A semiconductor is a solid having the energy band model. This is similar to an insulator, that is absolute  $E_F$ . It has a filled valence band which is separated from an empty conduction band by a forbidden gap. This gap is usually quite narrow in semiconductors, so that at finite temp some of  $e^-$  of Fermi level are excited to fill the gap in the conduction band where they become free. This is called free electrons. As the temperature rises more and more electrons are freed in this manner and there are more free electrons. Conclusion: In this material it is possible to control carrier concentration. On the other hand in metal and insulators such a control is not possible by any means.

At finite temp., it is observed that some of  $e^-$  are present in the conduction band. These electrons are free, so thus it is effective mass  $m_e$ .

Now density of states in  $E_F$ , C.B.

$$D(E) = \frac{1}{2\pi^2} \left( \frac{2m_e}{\pi^2} \right)^{1/2} (E - E_F)^{1/2}$$

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

$$S(E) = e^2 N_p (E_F - E)^{1/2}$$

Date \_\_\_\_\_

Now the number of  $e^-$  present per unit volume in C.B.

$$n_c(E)dE = D(E)f(E)dE$$

$$n_c = \int_{E_F}^{\infty} n_c(E)dE = \int_{E_F}^{\infty} D(E)f_c(E)dE$$

$$= \frac{1}{2\pi^2} \left( \frac{2m_e}{\pi^2} \right)^{3/2} e^{-E_F/kT} \int_{E_F}^{\infty} (E - E_F)^{1/2} e^{-E/kT} dE$$

which is to integrate to

$$n_c = 2 \left( \frac{2m_e}{\pi^2} \right)^{3/2} e^{-E_F/kT} (E_F - E_F)/kT$$

The basic density of holes static in volume v. n. is given by:-  $D_h(E)dE = \frac{1}{2\pi^2} \left( \frac{2m_h}{\pi^2} \right)^{3/2} (E_F - E)^{1/2} dE$

$$f_h(E) = 1 - f_c(E)$$

$$f_h(E) = e^{-E_F/kT}$$

$$[eq] n_h(E)dE = D_h(E)f_h(E)dE$$

$$n_h = \int_{E_F}^{\infty} n_h(E)dE = \int_{E_F}^{\infty} D_h(E)f_h(E)dE$$

$$= 2 \left( \frac{2m_h}{\pi^2} \right)^{3/2} e^{-E_F/kT}$$

In this case of pure semiconductor

$$n_e = n_h$$

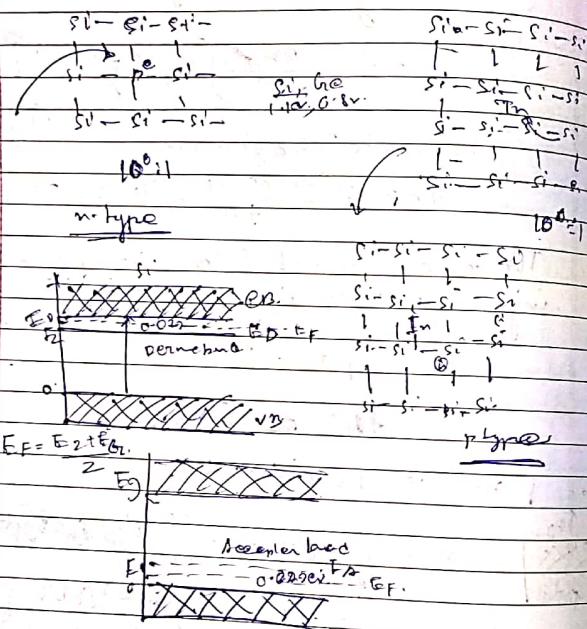
$$m_e^{3/2} e^{-E_F/kT} = m_h^{3/2} e^{-E_F/kT}$$

$$E_F = \frac{E_F}{2} + \frac{3}{4} kT \ln \left( \frac{m_e}{m_h} \right)$$

Date / /

$$\text{At } T=0, E_F = \frac{E_g}{2}$$

### Extensive Semiconductors.

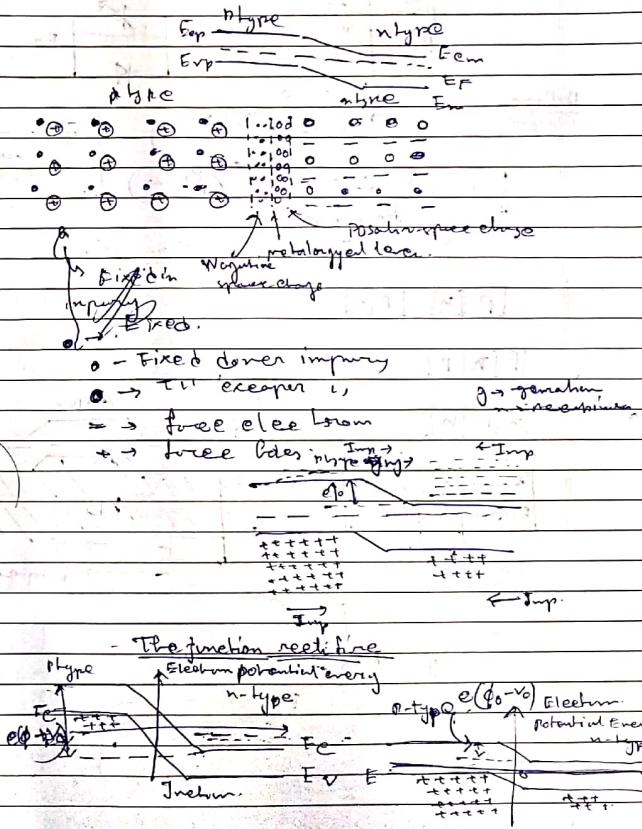


$$E_F = E_g/2$$

Date / /

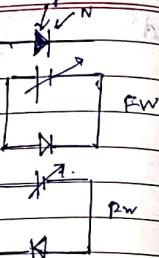
$$d^3 = \frac{\text{Total No. of Valence}}{\text{Volume}}$$

### P-n Junction



Date / /

$$\begin{aligned} P\text{N} & (C_0 - C_{V0}) F_W \\ & (C_0 + C_{V0}) R_W \end{aligned}$$

Half wave  
rectifier

Forward

$$I = I_0 \left( e^{\frac{C_V}{kT}} - 1 \right)$$

Input

I = I\_0 \left( e^{\frac{C\_V}{kT}} - 1 \right)

Reverse

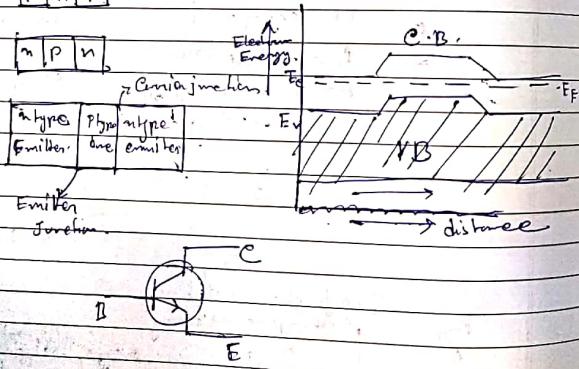
I = I\_0 \left( e^{\frac{C\_V}{kT}} - 1 \right)

I\_0 = reverse saturation current

The Junction Transistor

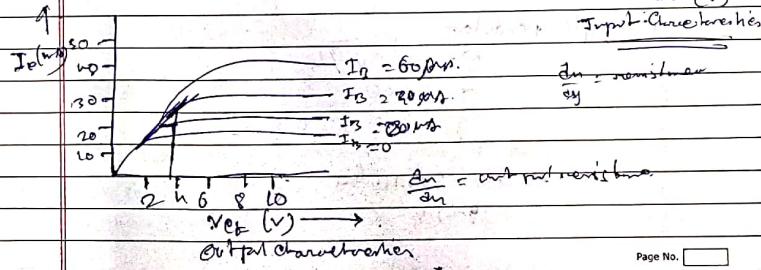
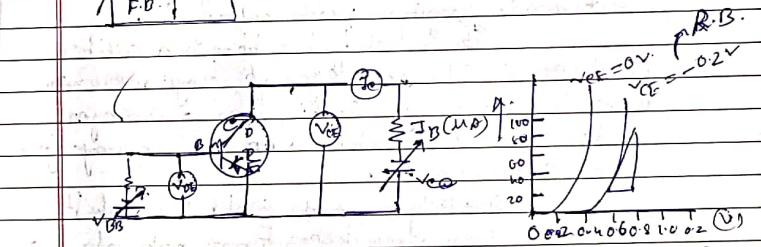
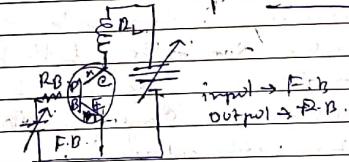
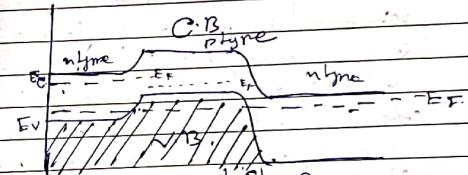
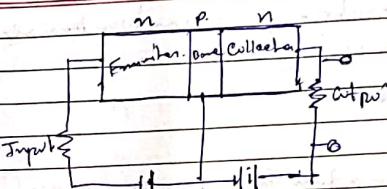
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Page No. [ ]

Date / /



Date / /

## Introductory to Quantum Mechanics for nano science

→ Application to real world

→ There are often advantages of making things smaller especially in certain semiconductor industry. What miniaturization of device often leads to divergence of efficient performance. Theoretically devices can be miniaturized to atomic level where sizes can be reduced from fraction of nanometer to few hundred of nanometer. The ideal miniaturization of size scale is fundamentally interesting due to the fact that as sizes approach to atomic scale the relevant physics associated with the system changes from classical domain to quantum mechanical regime.

In fact both natural and artificial could can now be categorized from micro world view to nano view. Starting from micro view to nano structure the nature is revolutionizing nano scale structure.

Similarly man made could is now shifting its attention from micro devices towards materials.

### (1) Simple pendulum.

$$\tau = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$T = 2\pi\sqrt{L}$$

$$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{L}} = 0.5^{\circ} \text{ Hz}$$

$$L = 1.8 \times 10^{-6} \text{ m.}$$

$$f = 29.8$$

Date / /

$$v = \frac{1}{2\pi} \sqrt{\frac{g}{L \times 10^6}}$$

$$v = 500$$

### (2) Thermal Time Constant decreases in smaller system

If a hot body of heat capacity by per unit volume is connected to a cold body ( $T=0\text{K}$ ) through a heat channel channel of length  $L$ . Cross section  $A$  and thermal heat capacity  $KTH$ . The rate of flow of heat  $\frac{dQ}{dT}$  from the hot to the cold body can be written as  $\frac{dQ}{dT} = \frac{KTH A T}{L}$  (1)

and it is steady state, this equals to the rate of loss of heat from the hot body.

$$\frac{dQ}{dT} = -Cv \frac{dT}{dt} \quad (2)$$

Therefore in the vol of the hot body changing indicate the loss of heat from hot body

$$\Rightarrow -Cv \frac{dT}{dt} = \frac{KTH A T}{L}$$

$$\Rightarrow \frac{dT}{dt} = - \left( \frac{KTH A}{L Cv} \right) dt \quad (3)$$

$$T = T_0 e^{-t/T_{\text{Th}}}$$

where after  $T_{\text{Th}} = \frac{L Cv}{KTH A}$  is the thermal time constant which under isothermal environment of the dia of  $L$  there is the thermal time const.

strongly decreases with the size of the system. This

Date / /

Terminis is used for data storage system.

Potential box Trapped parido (2D) (Nanodot)

$$-\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi(r) + V(r)\psi(r) = E\psi(r)$$

$$\psi_n(r,y,z) = \left(\frac{2}{L}\right)^{3/2} \sin\left(\frac{n_x r}{L_x}\right) \sin\left(\frac{n_y y}{L_y}\right) \sin\left(\frac{n_z z}{L_z}\right)$$

$$E_n = \left(\frac{\hbar^2}{2mL^2}\right)(n_x^2 + n_y^2 + n_z^2)$$

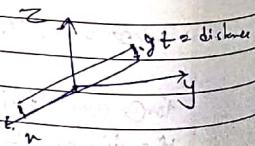
$$E_{nxyz} = \left(\frac{\hbar^2}{2mL^2}\right)(n_x^2 + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2})$$

Electron trapped in 2D plane.

$$\psi(r,y,z) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n_x r}{L_x}\right) \psi(y,z)$$

$$E_F = \left[\frac{\hbar^2}{2mL^2}\right] n_x^2 + \frac{\hbar^2 k_y^2}{2mL_y^2} + \frac{\hbar^2 k_z^2}{2mL_z^2}$$

Electron moving in 3D dim. (Nanowire or nanorib)



Date / /

$$\psi(r,y,z) = \left(\frac{2}{L}\right) \sin\left(\frac{n_x r}{L_x}\right) \sin\left(\frac{n_y y}{L_y}\right) e^{ik_z z}$$

$$E_n = \left(\frac{\hbar^2}{2mL^2}\right) (n_x^2 + n_y^2 + \frac{k_z^2}{L_z^2})$$

① Topdown process  $\rightarrow$  Ball milling process

② Bottomup process

③ Sol gel process

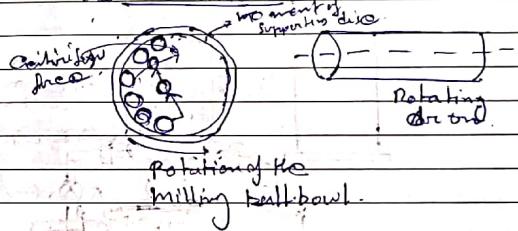
④ Electrodeposition

⑤ Physical and chemical process (PPC, CVD)

⑥ Epitaxial growth

⑦ Laser ablation etc.

Ball milling method for synthesis of nano materials



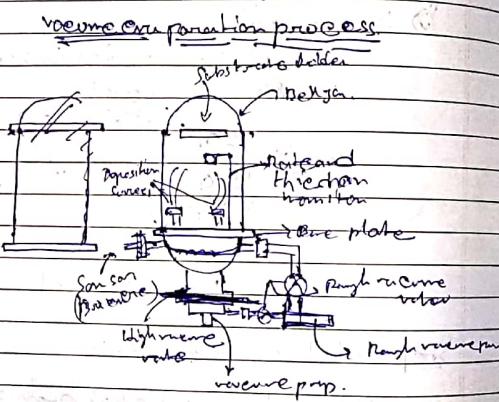
Steps in ball milling method:-

① The ball milling method consists of an enclosed chamber. It is filled over all a ball wheel contains a stainless steel container may small iron boron steel, silicon carbide or tungsten carbide balls are made to rotate inside a drum. The powder of a material is taken inside a steel container. This powder will be made into nano size using ball milling technique. A magnet is placed outside the container.

Page No. [ ]

Date / /

provide pulling force to material and this magnetic force increases the milling energy when the milling chamber contains the metal ball & the ball to material mass ratio is normally maintained 2:1. This silicon carbide balls provide very large amount of energy to the material powder, and the powder thus get crushed this process of ball milling done approximately 120-150 hr to get uniform fine powder. Ball milling is a mechanical process and small structural and chemical changes are produced by mechanical energy.



Page No. [ ]

Date / /

To maintain vacuum a chamber is needed to keep out normal atmospheric pressure normally 11 mm Hg steel or glass chambers are used because they don't corrode. (1) fig shows the basic vacuum system used for thin film deposition the system consists of vacuum chamber pumping system deposition sysytem, rotary pump system. the glass or stainless steel bell jar are fitted to stainless steel base plates with rubber gaskets. The base plate not necessarily provides a leak proof part of pumping and array of small ports for deposition source of and components.

The above system can be heated periodically to a few wcel this is usually referred to as baking the system. This process evaporates and removes any impurity and moisture remaining inside the bell jar it is most simple film deposition technique by evaporation. The physical vapor depoition has average the low deposition rate, expensively and relative cost to use. vacine evaporation is use for deposition of conductor material in electronic circuits and devices.

The rate of deposition can be expressed by Bental's Works kindred as

$$\frac{dN}{dt} \propto \frac{1}{A} \frac{\alpha (P'' - P)}{\sqrt{2\pi M k T}}$$

where  $\frac{dN}{dt}$  = rate of deposition with surface area  $A$ .

$\alpha$  = gas evaporation coeff.

$M$  = molecular weight of compound.

$K$  = Boltzmann const.

Page No. [ ]

Date / /

T<sub>g</sub> temperature.

P<sup>II</sup> = vapour pressure at the evaporant surface,  
r = hydrostatic presur. acting on the source surface.

Electro deposition:

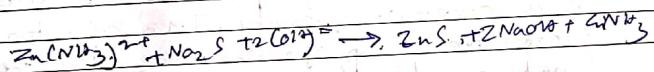
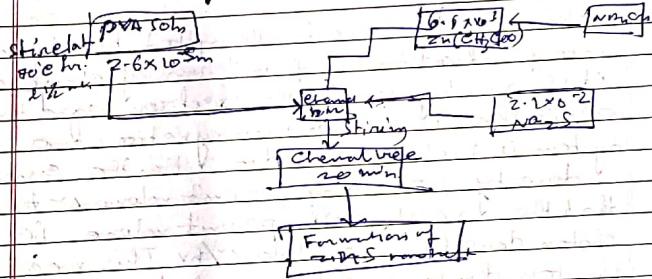
It is a liquid based deposition process where an electric current is pass through a polar liquid by applying a direct potential by battery between 2 electrodes. Due to this injection of liquid between them and other charge compound and depending on the charge and the ion formed and potential of the electrode, this charge material deposition on the electrodes. This technique has been used for long time to make electroplated materials. The primary application of electroplating is deposition of a layer of metal having some desired property (absorption, wire tension, corrosion protection, etc.). On today we have following that property.

Electrodeposition of zinc  
Control of film thickness uniformly and deposition rate and it is specially attractive owing to its low evaporation cost and low energy requirement. Deposition follows Faraday's law of electrolysis.

Date / /

Chemical Bath Deposition (C.B.D)

Synthesis of ZnS nano material in a polymer (PVA)  
in form of drop chemical bath den.



Date / /

### Physical Interpretation Of Wave Function

Schrödinger himself attempted the physical interpretation of  $\psi$  in terms of charge density. It is analogous to the electron against the radiation. In an electromagnetic wave system if  $A$  is the magnitude of the energy density i.e. energy per unit volume  $A^2$  to that the no. of photons per unit volume i.e. photon density is equal to  $A^2/h\nu$ . This means that photon density is proportional to  $A^2$  since  $h\nu$  is a constant quantity. In a similar way it will be the case of matter wave that the matter wave density per unit volume must be proportional to  $\psi^2$ . Now it is the charge associated with the particle then it will be the charge density. Thus the quantity  $\psi$  is the measure of charge density. The quantity  $\psi^*$  is the measure of charge density or from Dirac's theory  $\psi^*$  is written instead of  $\psi$  where  $\psi^*$  is the complex conjugate of  $\psi$ .

$$20 - 5I_1 - 3I_1 + 3I_2 - 5V = 0$$

Date / /

### New material

Thermal time constant decreases in smaller system. If a hot body of heat capacity  $C$  and volume  $V$  is connected to a cold body at  $(T=0)$  through a heat channel of length  $L$ , area  $A$  and thermal heat capacity  $K_{TH}$ , the rate of flow of heat  $Q/dt$  from the hot to the cold body can be written as,

$$\frac{dQ}{dt} = \frac{K_{TH} \Delta T}{L} \rightarrow \text{---(1)}$$

and at the steady state, this equals to the rate of loss of heat from the hot body

$$\frac{dQ}{dt} = -Cv \frac{dT}{dt}$$

where  $Cv$  is the vol. of the hot body as  $-ve$  sign indicates the loss of heat from hot body

$$-Cv \frac{dT}{dt} = \frac{K_{TH} \Delta T}{L}$$

$$\Rightarrow \frac{dT}{T} = -\frac{2K_{TH}}{Lcv} dt$$

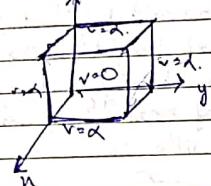
$$T = T_0 e^{-t/T_{th}}$$

where  $T_{th} = \frac{Lcv}{K_{TH} a}$  is the thermal time const.

Therefore the thermal time const. strongly decreases with the size of the system. This phenomenon is used for data storage systems.

Date / /

Potential box



Trapped particle in  
3D (nanodots)

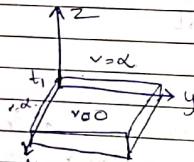
$$\left(\frac{\hbar^2}{2m}\right) \nabla^2 \psi(\vec{r}) + V(\vec{r}) \psi(\vec{r}) = E \psi(\vec{r})$$

$$\psi_n(x, y, z) = \left(\frac{2}{L}\right)^{3/2} \sin\left(\frac{n_1 \pi x}{L}\right) \sin\left(\frac{n_2 \pi y}{L}\right) \sin\left(\frac{n_3 \pi z}{L}\right)$$

$$E_n = \left(\frac{\hbar^2}{8mL^2}\right) (n_1^2 + n_2^2 + n_3^2)$$

$$E_{n_1, n_2, n_3} = \left(\frac{\hbar^2}{8m}\right) \left[ \frac{n_1^2 \pi^2}{L_x^2} + \frac{n_2^2 \pi^2}{L_y^2} + \frac{n_3^2 \pi^2}{L_z^2} \right]$$

(2) Electron trapped in 2D Nanosheet

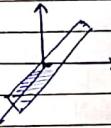


$$\psi_{n_1, n_2}(x, y) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n_1 \pi x}{L}\right) \sin\left(\frac{n_2 \pi y}{L}\right)$$

$$E_n = \left[\frac{\hbar^2}{8mL^2}\right] n_1^2 + \frac{\hbar^2 k n_2^2}{2m} + \frac{\hbar^2 k^2 y^2}{2mL^2}$$

Date / /

(3) Electron moving in one dim.



$$\psi_{n_1}(x) = \left(\frac{2}{L}\right) \sin\left(\frac{n_1 \pi x}{L}\right) e^{ikx}$$

$$E_n = \left(\frac{\hbar^2}{8mL^2}\right) (n_1^2 \pi^2) \frac{k^2}{\sin^2(kL)}$$