Lecture 13

Viscous flow in PIPE :-

Internal incompressible viscous flow :-

- Flows completely bounded by solid surfaces are called Internal flows. This includes flows through pipes, ducts, nozzles, diffusers, sudden contractions and expansions, valves and fittings.

Internal flow

Laminar   Turbulent.

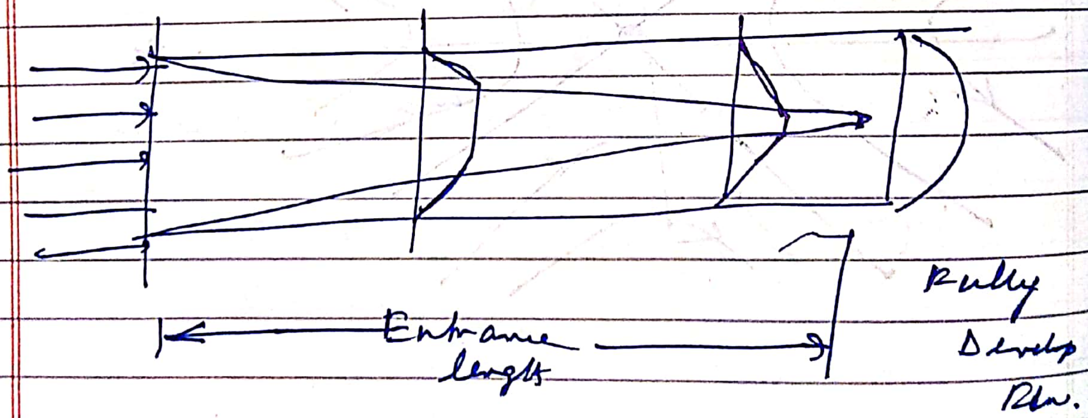
Gas $\underline{M < 0.3}$ $\approx 100\ m/s \rightarrow$ Incompressible flow.

Newtonian flow

$$Re = \frac{\rho V D}{\mu}$$

Reynold's experiments.

Transition takes place under normal condition at $Re = 2300$.



no slip ~~condition~~ condition

$$V_{av} = \bar{V} = \frac{1}{A}\int_{Area} V\,dA$$

For laminar flow entrance length

$$L = f(Re)$$

$$\frac{L}{D} \cong 0.06 \frac{\rho V_{av} D}{\mu}$$

$$Re \leq 2300$$

$$L \cong 0.06\, Re\, D$$

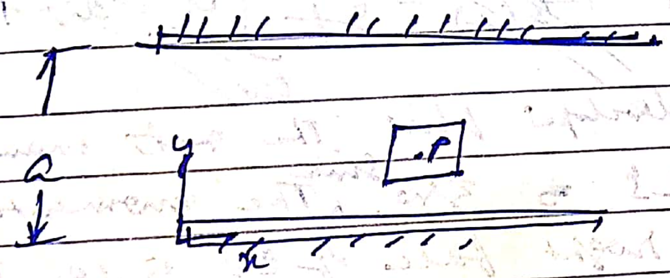
$$\leq 0.06(2300)D = 138D.$$

FULLY DEVELOPED LAMINAR FLOW.

(a) FDLF between infinitely parallel plates.

(1) Both plates static (stationary).

Example → leakage flow.



z → infinity

Assume (1) No variation of any fluid property in this direction

(2) steady (3) Incompressible

No slip condition

⇒ at $y = 0$, $u = 0$ | B.C.

$y = a$, $u = 0$

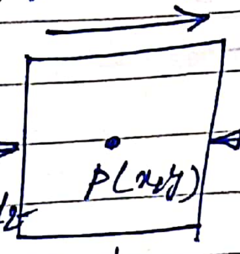
for fully Developed flow $u \neq f(x)$
only $u = f(y)$.

There is no component of velocity in y & z direction

differential control volume

$$d\forall = dx\,dy\,dz$$

$$\left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\left(\frac{dy}{2}\right)\right] dx\,dz$$



$$\left[p + \frac{\partial p}{\partial x}\left(-\frac{dx}{2}\right)\right] dy\,dz \qquad p(x,y) \qquad \left[p + \frac{\partial p}{\partial x}\left(\frac{dx}{2}\right)\right] dy\,dz$$

dx

$$\left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\left(-\frac{dy}{2}\right)\right] dx\,dz$$

RTT
B = momentum

$$F_{Sx} + \cancel{F_{B(x)}} = \cancel{\frac{\partial}{\partial t}\int_{CV} u \rho \, d\forall} + \int_{CS} u \rho \vec{V}\cdot d\vec{A}$$

For fully developed flow the net momentum flux through CS is zero. Since the momentum flux through right face of CS is equal to in magnitude but opposite in sign to the momentum flux through left face of

$$F_{S(x)} = 0.$$

So doing $\Sigma F_x = 0$ in the diagram

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = 0$$

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$$\frac{d}{dy}\tau_{yx} = \frac{\partial p}{\partial x} = \text{constant}$$

~~i.e. $\tau_{yx} = f(y)$.~~

$$\tau_{yx} = \left(\frac{\partial p}{\partial x}\right) y + C_1$$

$$\tau_{yx} = \mu \frac{du}{dy}$$

$$\mu \frac{du}{dy} = \left(\frac{\partial p}{\partial x}\right) y + C_1$$

$$\frac{du}{dy} = \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right) y + \frac{C_1}{\mu}$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) y^2 + \frac{C_1}{\mu} y + C_2 .$$

Applying B.C. $u = 0$, $y = 0$

$C_2 = 0$.

$u = 0$, $y = a$

$$0 = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) a^2 + \frac{C_1}{\mu} a$$

$$C_1 = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right) a .$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - ay) = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left(\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right)$$

velocity profile

$$= \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - ay).$$

Shear stress distribution

$$\tau_{yx} = \left(\frac{\partial p}{\partial x}\right) y + c_1$$

$$= \left(\frac{\partial p}{\partial x}\right) y - \frac{1}{2}\left(\frac{\partial p}{\partial x}\right) a$$

$$= a\left(\frac{\partial p}{\partial x}\right)\left(\frac{y}{a} - \frac{1}{2}\right).$$

volume flow rate

$$Q = \int_A \vec{V} \cdot d\vec{A}$$

For depth l in z direction

$$= \int_0^a u \, l \, dy$$

$$\frac{Q}{l} = \int_0^a \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - ay)\, dy$$

$$\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right) a^3.$$

Flow rate as a fn of pressure drop

$$\frac{\partial p}{\partial x} = \text{const.} = \frac{p_2 - p_1}{L} = \frac{-\Delta p}{L}$$

$$\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{-\Delta p}{L}\right) a^3$$

$$= \frac{a^3 \Delta p}{12\mu L}$$

Average velocity

$$\bar{V} = \frac{Q}{A} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)\frac{a^3 l}{la}$$

$$= -\frac{1}{12}\left(\frac{\partial p}{\partial x}\right) a^2$$

Point of Max velocity

$$\frac{du}{dy} = 0$$

$$\Rightarrow y = a/2$$

$$u|_{a/2} = u_{max} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)a^2 = \frac{3}{2}\bar{V}_{av} = \frac{3}{2}\bar{V}$$

~~Transformation of coordinate~~

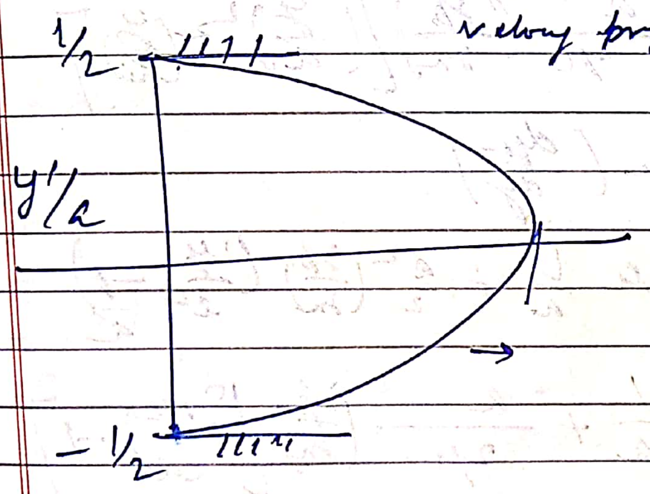
In eqn of u,

putting $y = y' + \frac{a}{2}$

$$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y'}{a}\right)^2 - \frac{1}{4}\right]$$

transfer of origin

velocity profile



$$\frac{u}{u_{max}} \longrightarrow$$

$$= \frac{u}{-\frac{a^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)}$$

Ex 8.1 Fox — Leakage

(b) upper plate moving with constant velocity U.

$$u = 0 \quad \text{at } y = 0$$
$$u = U \quad \text{at } y = a$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) y^2 + \frac{C_1}{\mu} y + C_2$$

$\underline{C_2 = 0}$

$y = a \cdot \quad u = U$

$$U = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) a^2 + \frac{C_1}{\mu} a$$

$$C_1 = \frac{U\mu}{a} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) a$$

$$u = \frac{Uy}{a} + \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$

$$\tau_{yx} = \mu\left(\frac{du}{dy}\right)$$

$$= \mu \frac{U}{a} + \frac{a^2}{2}\left(\frac{\partial p}{\partial x}\right)\left(\frac{2y}{a^2} - \frac{1}{a}\right)$$

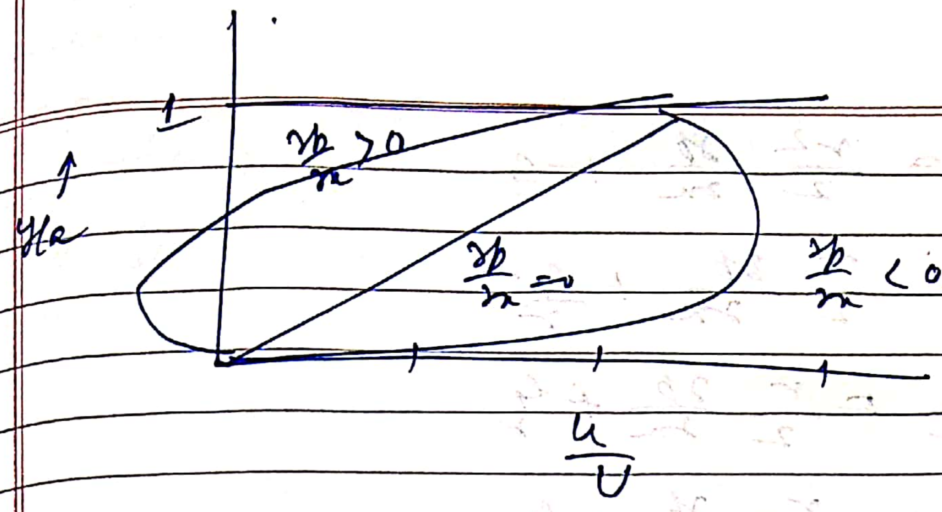
$$= \frac{U}{a} + \frac{a}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[2\left(\frac{y}{a}\right) - 1\right]$$

$$\frac{du}{dy} = 0 \Rightarrow y = \frac{a}{2} - \frac{U/a}{(1/\mu)\left(\frac{\partial p}{\partial x}\right)}$$

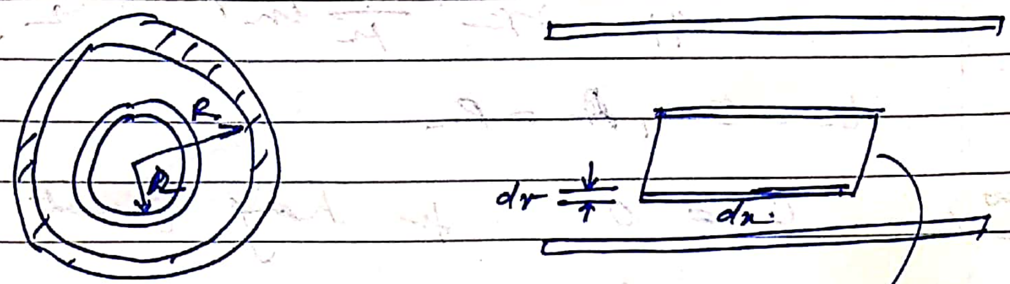
for $u = u_{max}$

Example

Journal Bearing problem

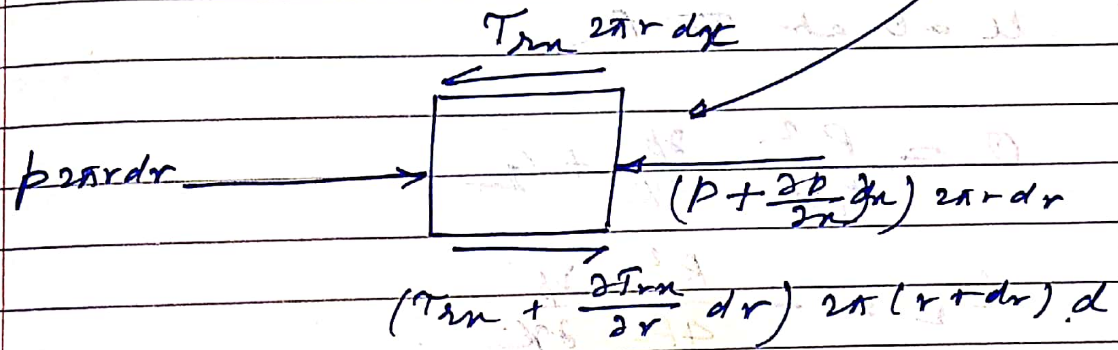


FDLF in a pipe :-



for Annular differential CV.

$F_{sx} = 0$.



$\Sigma F_x = 0$

$$\Rightarrow -\frac{\partial p}{\partial x} 2\pi r\, dr\, dx + \tau_{rx} 2\pi\, dr\, dx + \frac{\partial \tau_{rx}}{dr} 2\pi r\, dr\, dx = 0$$

$$\frac{\partial p}{\partial x} = \frac{\tau_{rx}}{r} + \frac{\partial \tau_{rx}}{\partial r}$$

$$= \frac{1}{r} \frac{d}{dr} (r \tau_{rx})$$

$$\frac{d(r \tau_{rx})}{dr} = r \frac{\partial p}{\partial x}$$

$$r\tau_{rx} = \frac{r^2}{2}\frac{\partial p}{\partial x} + C_1$$

$$\tau_{rx} = \frac{r}{2}\frac{\partial p}{\partial x} + \frac{C_1}{r}$$

$$\mu\frac{du}{dr} = \frac{r}{2}\frac{\partial p}{\partial x} + \frac{C_1}{r}$$

$$\frac{du}{dr} = \frac{r}{2\mu}\frac{\partial p}{\partial x} + \frac{C_1}{\mu r}$$

$$u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} + \frac{C_1}{\mu}\ln r + C_2.$$

$$u = 0 , \cancel{R} = R$$

for $r=0$, $C_1 = 0$ for having finite value

$$u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} + C_2$$

$u = 0$ at $r = R$

$$0 = \frac{R^2}{4\mu}\frac{\partial p}{\partial x} + C_2$$

$$C_2 = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}$$

$$u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(r^2 - R^2)$$

$$= -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$\tau_{rx} = \mu\frac{\partial u}{\partial r} = \frac{r}{2}\left(\frac{\partial p}{\partial x}\right).$$

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volume flow rate

$$Q = \int_{A} \vec{V} \cdot d\vec{A}$$

$$= \int_0^R u \; 2\pi r \, dr$$

$$= \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\int_0^R (r^2 - R^2)\, dr \; 2\pi r$$

$$= -\frac{\pi R^4}{8\mu}\left(\frac{\partial p}{\partial x}\right).$$

Flow rate as a fn of pressure drop.

$$\frac{\partial p}{\partial x} = \frac{p_2 - p_1}{L} = -\frac{\Delta p}{L}$$

$$Q = \frac{\pi(\Delta p)R^4}{8\mu L} = \frac{\pi(\Delta p)\, D^4}{128\mu L}$$

$$V_{av} = \bar{V} = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

$$= -\frac{R^2}{8\mu}\left(\frac{\partial p}{\partial x}\right).$$

for Maxm velocity

$$\frac{du}{dr} = 0 \Rightarrow r = 0$$

$$u_{max} = U_{max} = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right) = 2\bar{V}$$