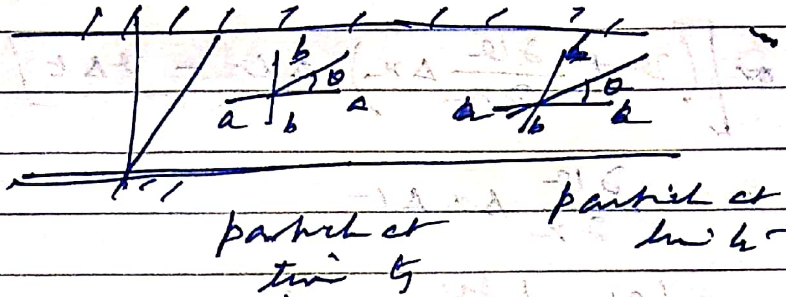
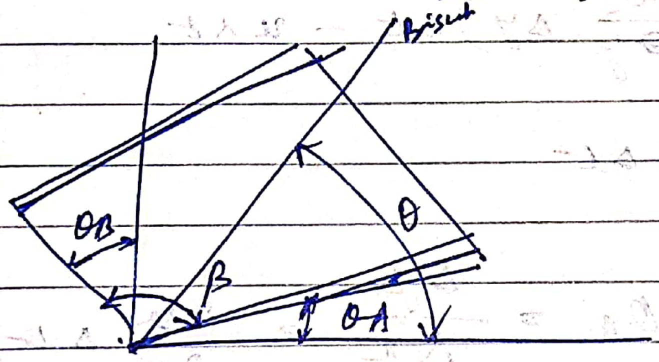
Class 16

Inviscid Flow → having zero viscosity ([illegible]).

IRROTATIONAL FLOW :-

Concept of rotation: Rotation of a fluid particle is defined as the average rotation of two initially mutually ⊥r faces of a fluid particle. The test is to look at the rotation of the line that bisects both faces. The

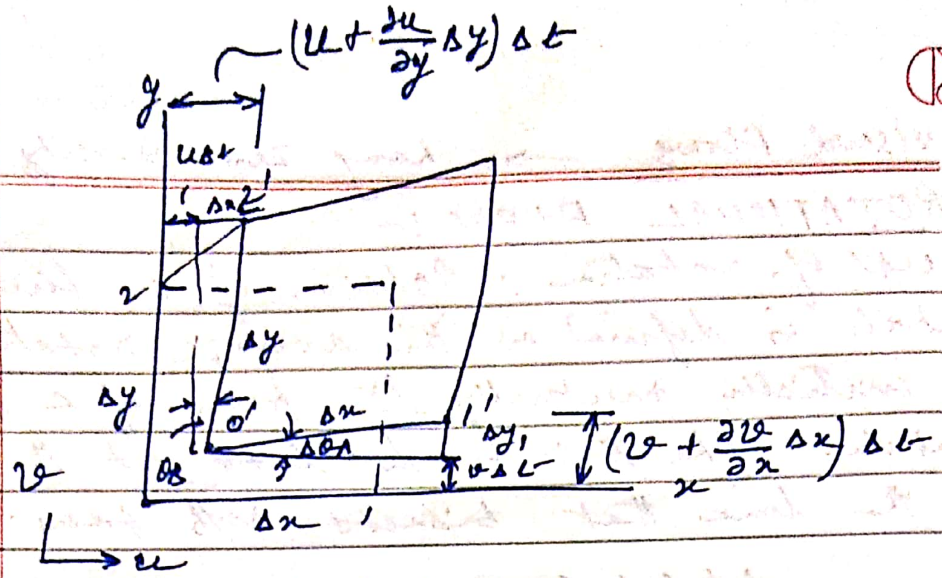




$$\theta = \frac{\beta}{2} + \theta_A = \frac{\pi}{4} + \frac{1}{2}(\theta_A + \theta_B)$$

$$\dot{\theta} = \frac{1}{2}(\dot{\theta}_A + \dot{\theta}_B)$$

If $\dot{\theta} = 0$ → flow is irrotational for all pt. in velocity field.



$$\Delta y_1 \approx \left[\left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta t - v\Delta t\right]$$

$$= \frac{\partial v}{\partial x}\Delta x \Delta t$$

$$\Delta x_2 \approx \left(u + \frac{\partial u}{\partial y}\Delta y\right)\Delta t - u\Delta t$$

$$= \frac{\partial u}{\partial y}\Delta y \Delta t$$

$$\sin \Delta\theta_A = \Delta\theta_A = \frac{\Delta y_1}{\Delta x} = \frac{\partial v}{\partial x}\Delta t$$

$$\sin \Delta\theta_B = \Delta\theta_B = \frac{\Delta x_2}{\Delta y} = \frac{\partial u}{\partial y}\Delta t$$

Limit $\Delta t \to 0$

$$\dot{\theta}_A = \frac{\partial v}{\partial x} \longrightarrow +ve$$

$$\dot{\theta}_B = \frac{\partial u}{\partial y} \longrightarrow -ve$$

The component of rotational velocity Ω_z

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

rate of rotation ~~velocit~~ vector.

(ω) $\Omega = \Omega_x \hat{i} + \Omega_y \hat{j} + \Omega_z \hat{k}$

An Irrotational flow ($\Omega = 0$) requiring that

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$$

for Ideal flow

Fin component Envisioned.

(ω) $\Omega = \frac{1}{2} \nabla \times \vec{V}$

$= \frac{1}{2} \text{curl } \vec{V}$

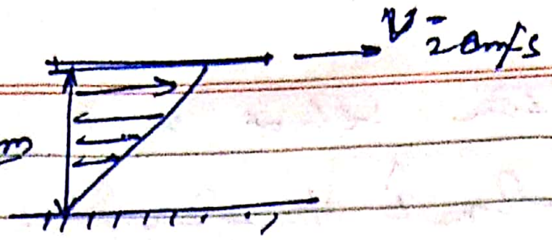
$$= \frac{1}{2} \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

vorticity. is ζ Create

$\vec{\zeta}$ ~~ω~~ $= 2 \vec{\Omega}$

$= \nabla \times \vec{V}.$

Problem



$\Omega = ?$ at 0.5 cm. after plate travel 4 cm

$$u = 0.02 \times \frac{y}{0.01} = 2y$$

$$(\omega_z)\ \Omega_z = \frac{1}{2}\left(\cancel{\frac{\partial v}{\partial x}}^{0} - \frac{\partial u}{\partial y}\right)$$

$$= -1 \text{ rad/s.}$$

$$u = 0.5 \times 2 = 0.01 \text{ m/s}$$

$$\Delta t = \frac{\Delta x}{\Delta u} = \frac{0.01}{0.01} = 1 \text{ s}$$

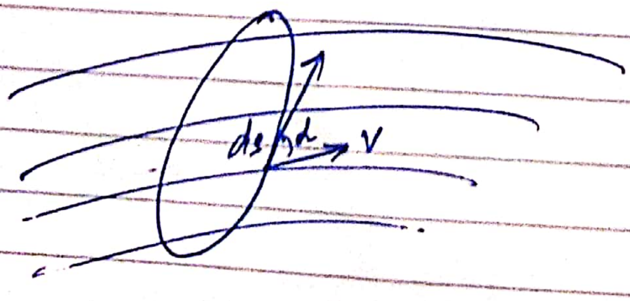
Amount of rotation

$$\Delta\theta = \Omega_z \Delta t$$

$$= -1 \times 1 = -1 \text{ rad. } \quad \text{cw}$$

CIRCULATION.

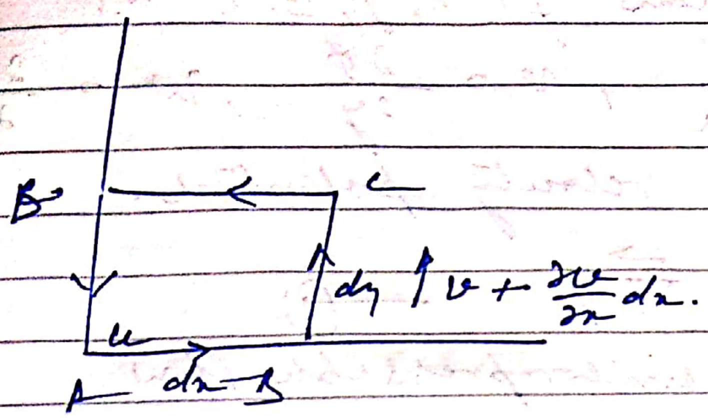
– Circulation Γ is the line integral of tangential velocity around a closed contour in the flow field.



$$\Gamma = \int V \cos\alpha \, ds$$

$$= \oint \vec{V} \cdot \vec{ds}$$

By convention, circulation is reckoned +ve in anticlockwise direction.



ABCDA

$$d\Gamma = u\,dx + \left(v + \frac{\partial v}{\partial x}dx\right)dy - \left(u + \frac{\partial u}{\partial y}dy\right)dx - v\,dy$$

$$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)dx\,dy$$

$$= \zeta_z\,dx\,dy$$

$$\Gamma = \oint \vec{V}\cdot d\vec{s} = \int_A \cancel{\vec{\Omega}\cdot d\vec{A}} \quad \int_A \cancel{\zeta\cdot dA} \quad \int \vec{\zeta}\cdot dA$$

Circulation per unit area equals vorticity in flow. For an irrotational flow vorticity is zero and so will be circulation around any closed path in irrotational flow.

Velocity Potential

For an irrotational flow, the velocity component can be expressed in form of scalar function $\phi(x, y, z, t)$ as

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z}$$

ϕ is called velocity potential.

$$\Rightarrow \vec{V} = \vec{\nabla} \phi \quad \text{--- (1)}$$

For an incompressible flow

$$\vec{\nabla} \cdot \vec{V} = 0 \quad \text{--- (1)}$$

$$\Rightarrow \nabla^2 \phi = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{--- (1)}$$

If ϕ exist $\rightarrow$ Irrotational flow.

ie If (1) is satisfied $\rightarrow$ Irr, Incomp, steady flow.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial x \partial y}\right)$$

$$= 0. \quad \text{Irr.}$$

Equipotential line

$$\phi = \text{const}$$

$$d\phi = 0$$

$$0 = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$= u\, dx + v\, dy$$

$$\left(\frac{dy}{dx}\right)_{eqp} = -\frac{u}{v} \quad \text{--- (2)}$$

Stream function

for 2D steady incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{(continuity eq}^n\text{)} \quad \text{--- (1)}$$

$\psi(x, y)$ is a f^n (stream f^n) such that

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

$\Rightarrow$ If ψ exist.

from eq (1)

$$\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0$$

continuity eq^n satisfy $\longrightarrow$ Incomp. steady flow.

$\psi = $ const $\longrightarrow$ Streamlines.

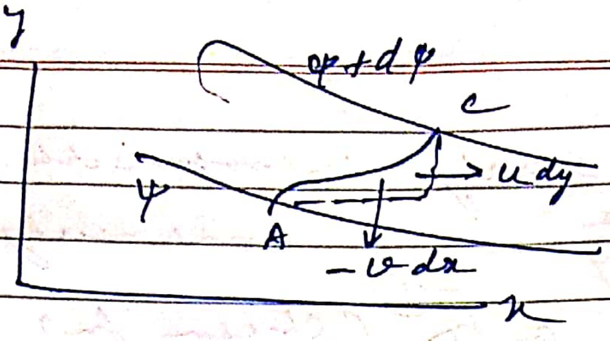
$$d\psi = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\text{const } s} = \frac{v}{u} \quad \text{--- (II)}$$

I & (II)

$$\underbrace{\left[\left(\frac{dy}{dx}\right)_{\text{equip}}\right]}_{\text{slope}} \times \underbrace{\left(\frac{dy}{dx}\right)_{\text{equi } s}}_{\text{slope}} = -1$$

$\Rightarrow$ Equipotential lines are equi Streamlines are orthogonal to ~~eq~~ each other.



$$dq = u\,dy - v\,dx$$

$$= \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial x} dx$$

$$dq = d\psi$$

The volume flow rate q between two streamlines such as ψ_1 and ψ_2

$$q = \int_{\psi_1}^{\psi_2} d\psi = \psi_2 - \psi_1$$

problem

$$\left.\begin{array}{l} u = 2y \\ v = 4x \end{array}\right\} \psi = ?$$

$$u = \frac{\partial \psi}{\partial y} = 2y$$

$$\psi = y^2 + f_1(x) \quad \text{—(I)}$$

$$v = -\frac{\partial \psi}{\partial x} = 4x$$

$$\psi = -x^2 + f_2(y) \quad \text{(II)}$$

$\psi = -2x^2 + y^2 + C$

for simplicity $C = 0$

$\psi = -2x^2 + y^2$

Streamlines

at $\psi = 0$ $\quad \psi = c$

$0 = -2x^2 + y^2$

$y = \pm\sqrt{2}\,x$

$C \neq 0$

$$\frac{y^2}{\psi} - \frac{x^2}{\psi/2} = 1 \qquad \psi > 0$$

$\psi < 0$