Algebra 1 Workbook Solutions

Systems of two equations

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2-STEP PROBLEMS

1. Why can't you solve the following 2-step problem?

If $2(x-1)-3=9+x$, what is $y+2$?

Solution:

You can't solve the problem because there is no variable y in the first equation, so you can't get a value from the first equation to plug into $y+2$.

2. If $5-2x=17$, what is $x-1$?

Solution:

Solve the first equation for x.

$$5-2x=17$$

$$-2x=12$$

$$x=-6$$

Now plug $x=-6$ into the second equation.

$$x-1$$

$$-6-1$$

$$-7$$

3. Describe in words how you would solve the following 2-step problem.

If $x-3=5$, what is $x+5$?

Solution:

First, we would solve the first equation for x by adding 3 to each side of $x-3=5$ to get $x=8$. Then, to find the value of $x+5$, we plug in $x=8$ to get a final value of 13.

4. If $3(2-x)+5=-(4x-2)$, what is $(x/2)+1$?

Solution:

Solve the first equation for x.

$$3(2-x)+5=-(4x-2)$$

$$6-3x+5=-4x+2$$

$$11-3x=-4x+2$$

$$9 = -x$$

$$x = -9$$

Now plug $x = -9$ into the second expression.

$$\frac{x}{2} + 1$$

$$\frac{-9}{2} + 1$$

$$-\frac{9}{2} + \frac{2}{2}$$

$$-\frac{7}{2}$$

5. What are the two steps of a 2-step problem?

Solution:

The first step is to solve for the variable in the equation. The second step is to use this solution to solve for the value of the expression.

6. If $2(x + y) - 6 = 3$, what is $x + y - 1$?

Solution:

Solve the equation for x.

$$2(x+y)-6=3$$

$$2(x+y)=9$$

$$x+y=\frac{9}{2}$$

Now substitute this value into the expression.

$$x+y-1$$

$$\frac{9}{2}-1$$

$$\frac{9}{2}-\frac{2}{2}$$

$$\frac{7}{2}$$

7. What went wrong in solving the following 2-step problem?

If $2x+3=7$, what is $x/3$?

$$2x+3=7$$

$$2x=4$$

$$\frac{x}{3}=\frac{4}{3}$$

Solution:

The variable x was not completely solved for. In the second step, it should be $2x = 4$ gives $x = 2$. Then $x = 2$ should get plugged into the expression $x/3$.

■ 8. If $a + 2b = 6 - a$ and $b = 1$, what is $a/2$?

Solution:

First, plug $b = 1$ into $a + 2b = 6 - a$ and solve for a.

$$a + 2(1) = 6 - a$$

$$2a = 4$$

$$a = 2$$

Then plug $a = 2$ into $a/2$.

$$\frac{a}{2}$$

$$\frac{2}{2}$$

$$1$$

SOLVING WITH SUBSTITUTION

1. Find the unique solution to the system of equations.

$$-x + 2y = 6$$

$$3x = y - 10$$

Solution:

Solve for x in the second equation.

$$3x = y - 10$$

$$x = \frac{y - 10}{3}$$

Plug this value for x into the first equation, then solve for y.

$$-x + 2y = 6$$

$$-\frac{y - 10}{3} + 2y = 6$$

$$-y + 10 + 6y = 18$$

$$5y = 8$$

$$y = \frac{8}{5}$$

Plug $y = 8/5$ back into the equation we found for x.

$$x = \frac{y - 10}{3}$$

$$x = \frac{\frac{8}{5} - 10}{3}$$

$$x = \frac{\frac{8}{5} - \frac{50}{5}}{3}$$

$$x = -\frac{42}{5} \cdot \frac{1}{3}$$

$$x = -\frac{14}{5}$$

The unique solution to the system is

$$\left(-\frac{14}{5}, \frac{8}{5}\right)$$

2. What is the easiest variable to get by itself? Set up but do not solve the substitution.

$$2y - x = 7$$

$$3x = 9 - 18y$$

Solution:

It is easiest to solve for the variable x in the second equation by dividing both sides by 3 and then simplifying.

$$x = \frac{9 - 18y}{3}$$

$$x = \frac{9}{3} - \frac{18y}{3}$$

$$x = 3 - 6y$$

■ 3. Find the unique solution to the system of equations.

$$-5x + y = 8$$

$$y = 3x - 8$$

Solution:

Taking the value for y given in the second equation as $y = 3x - 8$, we'll substitute for y in the first equation.

$$-5x + y = 8$$

$$-5x + (3x - 8) = 8$$

$$-5x + 3x - 8 = 8$$

$$-2x = 16$$

$$x = -8$$

Now substitute $x = -8$ into the second equation to find a value for y.

$$y = 3x - 8$$

$$y = 3(-8) - 8$$

$$y = -32$$

The unique solution to the system is

$$(-8, -32)$$

4. Find the unique solution to the system of equations.

$$3 - y = 2x$$

$$-4x + 10 = 2y$$

Solution:

Solving the second equation for y.

$$-4x + 10 = 2y$$

$$-2x + 5 = y$$

Plug $y = -2x + 5$ into the first equation.

$$3 - y = 2x$$

$3 - (-2x + 5) = 2x$

$3 + 2x - 5 = 2x$

$-2 + 2x = 2x$

$-2 = 0$

Since this is not true, there is no solution to the system.

■ 5. Fill in the blanks with the correct variables x and y if the solution to the system of equations is $(-1, 3)$.

$-2\_ + \_ = 5$

$2\_ = 7 - 3\_$

Solution:

The first equation will either be $-2x + y = 5$ or $-2y + x = 5$. The second equation will either be $2x = 7 - 3y$ or $2y = 7 - 3x$. If we try each combination to try finding a solution of $(-1, 3)$, we find that the correct system is

$-2x + y = 5$

$2x = 7 - 3y$

■ 6. What went wrong in the following substitution?

$y = x - 2$

$2y - x = 7$

Substitution: $2x - 2 - x = 7$

Solution:

When substituting $y = x - 2$ into the second equation, we get

$2y - x = 7$

$2(x - 2) - x = 7$

$2x - 4 - x = 7$

Therefore, in the substitution given, the 2 was not distributed to the -2.

7. Find the unique solution to the system of equations.

$5y = 6 - 2x$

$6x + 15y = 18$

Solution:

Solve for y in the first equation.

$5y = 6 - 2x$

$$y = \frac{6 - 2x}{5}$$

Plug this value for y into the second equation.

$$6x + 15y = 18$$

$$6x + 15\left(\frac{6 - 2x}{5}\right) = 18$$

$$6x + 3(6 - 2x) = 18$$

$$6x + 18 - 6x = 18$$

$$18 = 18$$

Since this equation is true, there are infinitely many solutions.

SOLVING WITH ELIMINATION

1. What is the easiest way to set up the elimination method for the system of equations? Set up but do not solve the elimination.

$$6y - 3x = 8$$

$$x - 4y = 5$$

Solution:

The easiest way to solve the elimination is the multiply the second equation by 3 to get

$$x - 4y = 5$$

$$3x - 12y = 15$$

Then add the two equations together to eliminate x from the system and get

$$6y - 3x + (3x - 12y) = 8 + (15)$$

$$6y - 12y = 8 + 15$$

$$-6y = 23$$

2. Find the unique solution to the system of equations.

$$2x - y = 5$$

$$-3x + y = 7$$

Solution:

If we add the two equations together to eliminate y, we get

$$2x - y + (-3x + y) = 5 + (7)$$

$$2x - 3x = 12$$

$$-x = 12$$

$$x = -12$$

Plug $x = -12$ back into the second equation.

$$-3x + y = 7$$

$$-3(-12) + y = 7$$

$$y = -29$$

The solution to the system is

$$(-12, -29)$$

3. Would it be easier to solve the system of equations using the substitution method or the elimination method?

$$7x - 3y = 2$$

$$3y - x = 11$$

Solution:

The elimination method would be easier, because adding the two equations together eliminates the $-3y$ in the first equation and the $3y$ in the second equation.

4. What went wrong in the following elimination?

$$-4x + 3y = 7$$

$$-4x - y = 4$$

Elimination: $2y = 3$

Solution:

When subtracting the two equations, $-y$ in the second equation was added, instead of subtracted. The elimination method should have produced $4y = 3$.

5. Find the unique solution to the system of equations.

$$x = 2y - 5$$

$$-3x + 6y = 15$$

Solution:

Multiplying the first equation by 3 gives

$$x = 2y - 5$$

$$3x = 6y - 15$$

Then adding $3x = 6y - 15$ to $-3x + 6y = 15$ gives

$$3x - 6y + (-3x + 6y) = -15 + (15)$$

$$3x - 6y - 3x + 6y = -15 + 15$$

$$-6y + 6y = -15 + 15$$

$$0 = 0$$

This is always true, so there are infinitely many solutions to the system of equations.

6. Fill in the blanks with the correct variables x and y if the solution to the system of equations is $(2/7, -18/7)$.

$$3\_\_\_ - \_\_\_ = -8$$

$-\_\_\_ = 10 + 4\_\_\_$

Solution:

The first equation will either be $3x - y = -8$ or $3y - x = -8$. The second equation will either be $-x = 10 + 4y$ or $-y = 10 + 4x$. If we try each combination to try finding a solution of $(2/7, -18/7)$, we find that the correct system is

$$3y - x = -8$$

$$-x = 10 + 4y$$

■ 7. Find the unique solution to the system of equations.

$$4 - 2x = 6y$$

$$7 = x + 3y$$

Solution:

Multiplying the second equation by -2 gives

$$7 = x + 3y$$

$$-14 = -2x - 6y$$

Then adding the two equations together gives

$$2x + 6y + (-2x - 6y) = 4 + (-14)$$

$$2x + 6y - 2x - 6y = 4 - 14$$

$$0 = -10$$

Since this is not true, there is no solution to the system of equations.

8. Would it be easier to solve the system of equations using the substitution method or the elimination method?

$$5y - x = 3$$

$$x = 7y - 10$$

Solution:

Substitution would be easier, since the second equation is already solved for x.

9. Find the unique solution to the system of equations.

$$x = 2y - 8$$

$$3y = x + 5$$

Solution:

Rewriting the second equation gives

$$3y = x + 5$$

$$-x = -3y + 5$$

Adding this to the first equation gives

$$x + (-x) = 2y - 8 + (-3y + 5)$$

$$x - x = 2y - 8 - 3y + 5$$

$$0 = -y - 3$$

$$y = -3$$

Substitute $y = -3$ into the first equation to solve for x.

$$x = 2y - 8$$

$$x = 2(-3) - 8$$

$$x = -6 - 8$$

$$x = -14$$

Therefore, the solution to the system of equations is

$$(-14, -3)$$

SOLVING THREE WAYS

1. Explain why using the graphing method would make the following system of equations easy to solve.

$$y = 3x - 4$$

$$y - 3 = 2(x + 1)$$

Solution:

The first equation is easy to graph because it's in the slope-intercept form $y = mx + b$. And the second equation is easy to graph because it's in the point-slope form $y - y_1 = m(x - x_1)$.

2. Find the unique solution to the system of equations using the elimination method.

$$2y = x + 5$$

$$3x - 2y = 11$$

Solution:

Adding the two equations and solving for x gives

$$-x + 2y + (3x - 2y) = 5 + (11)$$

$$-x + 2y + 3x - 2y = 5 + 11$$

$$2x = 16$$

$$x = 8$$

Substitute $x = 8$ into the first equation.

$$2y = x + 5$$

$$2y = 8 + 5$$

$$y = \frac{13}{2}$$

Therefore the unique solution to the system of equations is

$$\left(8, \frac{13}{2}\right)$$

3. In words, describe the graphical solution to a system of equations.

Solution:

The solution to a system of equations on a graph is the intersection point of the two graphs.

4. Find the unique solution to the system of equations using the substitution method.

$$5y + x = 4$$

$$3y - 3x = 6$$

Solution:

Solve the first equation for x.

$$5y + x = 4$$

$$x = 4 - 5y$$

Substitute this into the second equation.

$$3y - 3x = 6$$

$$3y - 3(4 - 5y) = 6$$

$$3y - 12 + 15y = 6$$

$$18y = 18$$

$$y = 1$$

Plug $y = 1$ into the equation for x.

$$x = 4 - 5y$$

$$x = 4 - 5(1)$$

$$x = -1$$

Therefore the solution to the system of equation is

$$(-1,1)$$

5. Explain why using the substitution method would make the system of equations easy to solve.

$$2y = 6 - 4x$$

$$7 - y = 3x$$

Solution:

If we divide the first equation by 2, we get

$$2y = 6 - 4x$$

$$y = 3 - 2x$$

Then we can use the substitution to plug this value into the second equation, $7 - y = 3x$.

6. In words, describe the solution to a system of equations.

Solution:

The solution to a system of equations is a value(s) (x, y) that satisfies every equation in the system, such that, when you plug (x, y) into each equation, all of the equations are true.

7. Explain why using the elimination method would make the system of equations easy to solve.

$$3y - 2x = 7$$

$$2x = 4 - 6y$$

Solution:

If we add the two equations, the x terms cancel out, making it a very easy elimination method problem.

8. Find the unique solution to the system of equations using the graphing method.

$$y - 2 = -(x + 1)$$

$$y = x + 1$$

Solution:

As you can see from the graphs of the two functions, the intersection point is at $(0,1)$, which means $(0,1)$ is the solution to the system of equations.