Algebra 2 Workbook Solutions

Rational functions

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LONG DIVISION OF POLYNOMIALS

■ 1. Find the quotient.

$$\frac{x^2+2x-1}{x+3}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r|l}
 & x-1+\frac{2}{x+3} \\
\hline
x+3 & x^2+2x-1 \\
 & -(x^2+3x) \\
\cline{2-2}
 & \quad -x-1 \\
 & \quad -(-x-3) \\
\cline{2-2}
 & \qquad\qquad 2
\end{array}
$$

■ 2. Find the quotient.

$$\frac{2x^3-x^2-4x+5}{x-2}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r l}
 & 2x^2+3x+2+\frac{9}{x-2} \\
x-2 & \overline{\big)\,2x^3-x^2-4x+5} \\
 & -(2x^3-4x^2) \\
\hline
 & \quad 3x^2-4x \\
 & \quad -(3x^2-6x) \\
\hline
 & \qquad 2x+5 \\
 & \qquad -(2x-4) \\
\hline
 & \qquad\quad 9
\end{array}
$$

■ 3. Find the quotient.

$$\frac{2x^4+4x^3-x^2+5x-150}{x+4}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r|l}
 & 2x^3-4x^2+15x-55+\frac{70}{x+4} \\
\hline
x+4 & 2x^4+4x^3-x^2+5x-150 \\
 & -(2x^4+8x^3) \\
\hline
 & -4x^3-x^2 \\
 & -(-4x^3-16x^2) \\
\hline
 & 15x^2+5x \\
 & -(15x^2+60x) \\
\hline
 & -55x-150 \\
 & -(-55x-220) \\
\hline
 & 70
\end{array}
$$

■ 4. Find the quotient.

$$\frac{3x^3 - x^2 - 7x + 5}{x - 1}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r}
3x^2+2x-5 \\
x-1 \enclose{longdiv}{3x^3-x^2-7x+5} \\
-(3x^3-3x^2) \\
\hline
2x^2-7x \\
-(2x^2-2x) \\
\hline
-5x+5 \\
-(-5x+5) \\
\hline
0
\end{array}
$$

■ 5. Find the quotient.

$$\frac{-x^2+3x+15}{x+5}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r|l}
 & -x+8-\frac{25}{x+5} \\
\hline
x+5 & -x^2+3x+15 \\
 & -(-x^2-5x) \\
\hline
 & \quad 8x+15 \\
 & \quad -(8x+40) \\
\hline
 & \qquad -25
\end{array}
$$

■ 6. Find the quotient.

$$\frac{x^4+x-3}{x-2}$$

Solution:

Use long division to find the quotient. Remember to represent the missing x^3 and x^2 terms.

$$
\begin{array}{r}
x^3+2x^2+4x+9+\frac{15}{x-2} \\
x-2\ \overline{\big)\ x^4+0x^3+0x^2+x-3} \\
-(x^4-2x^3) \qquad\qquad\qquad \\
\hline
2x^3+0x^2 \qquad\qquad \\
-(2x^3-4x^2) \qquad\quad \\
\hline
4x^2+x \qquad \\
-(4x^2-8x) \quad \\
\hline
9x-3 \\
-(9x-18) \\
\hline
15
\end{array}
$$

7. Find the quotient.

$$\frac{x^3+6}{x+6}$$

Solution:

Use long division to find the quotient. Remember to represent the missing x^2 and x terms.

$$
\begin{array}{r}
x^2-6x+36-\dfrac{210}{x+6} \\
x+6 \enclose{longdiv}{x^3+0x^2+0x+6} \\
-(x^3+6x^2) \qquad\qquad\quad \\
\hline
-6x^2+0x \qquad \\
-(-6x^2-36x) \qquad \\
\hline
36x+6 \\
-(36x+216) \\
\hline
-210
\end{array}
$$

■ 8. Find the quotient.

$$\frac{x^2+x}{x-3}$$

Solution:

Use long division to find the quotient. Remember to represent the missing constant term.

$$
\begin{array}{r|l}
 & x+4+\frac{12}{x-3} \\
\hline
x-3 & x^2+x+0 \\
 & -(x^2-3x) \\
\hline
 & \quad\quad 4x+0 \\
 & \quad -(4x-12) \\
\hline
 & \quad\quad\quad 12
\end{array}
$$

■ 9. Find the quotient.

$$\frac{x^4-2x^2}{x-4}$$

Solution:

Use long division to find the quotient. Remember to represent the missing terms.

$$
\begin{array}{r|l}
 & x^3+4x^2+14x+56+\frac{224}{x-4} \\
\hline
x-4 & x^4+0x^3-2x^2+0x+0 \\
 & -(x^4-4x^3) \\
\hline
 & 4x^3-2x^2 \\
 & -(4x^3-16x^2) \\
\hline
 & 14x^2+0x \\
 & -(14x^2-56x) \\
\hline
 & 56x+0 \\
 & -(-56x-224) \\
\hline
 & 224
\end{array}
$$

■ 10. Find the quotient.

$$\frac{-2x^3+8x}{x+2}$$

Solution:

Use long division to find the quotient. Remember to represent the missing terms.

$$
\begin{array}{r|l}
 & -2x^2+4x \\
\hline
x+2 & -2x^3+0x^2+8x+0 \\
 & -(-2x^3-4x^2) \\
\hline
 & \qquad\qquad 4x^2+8x \\
 & \qquad\quad -(4x^2+8x) \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

SIMPLIFYING RATIONAL FUNCTIONS

■ 1. Reduce the fraction to its lowest terms.

$$\frac{4x^3 + 12x^2 + 8x}{12x^2}$$

Solution:

Look for a common factor in each term. In this case the common factor is $4x$.

$$\frac{4x^3 + 12x^2 + 8x}{12x^2}$$

$$\frac{4x(x^2 + 3x + 2)}{4x(3x)}$$

$$\frac{x^2 + 3x + 2}{3x}$$

■ 2. Reduce the fraction to its lowest terms.

$$\frac{10x^2 - 5x + 20}{15x^2}$$

Solution:

Look for a common factor in each term. In this case the common factor is 5.

$$\frac{10x^2 - 5x + 20}{15x^2}$$

$$\frac{5(2x^2 - x + 4)}{5(3x^2)}$$

$$\frac{2x^2 - x + 4}{3x^2}$$

■ 3. Reduce the fraction to its lowest terms.

$$\frac{3x^4 + 6x^3 - 18x^2}{12x^3}$$

Solution:

Look for a common factor in each term. In this case the common factor is $3x^2$.

$$\frac{3x^4 + 6x^3 - 18x^2}{12x^3}$$

$$\frac{3x^2(x^2 + 2x - 6)}{3x^2(4x)}$$

$$\frac{x^2 + 2x - 6}{4x}$$

■ 4. Reduce the fraction to its lowest terms.

$$\frac{18y^2 + 6y}{8y}$$

Solution:

Look for a common factor in each term. In this case the common factor is $2y$.

$$\frac{18y^2 + 6y}{8y}$$

$$\frac{2y(9y + 3)}{2y(4)}$$

$$\frac{9y + 3}{4}$$

■ 5. Simplify each expression in the difference.

$$\frac{3ab + 2a^2b^2}{5ab} - \frac{12a^3b^3 + 3a^2b^2}{6a^2b^2}$$

Solution:

Look for a common factor in each term. In this case the common factor of the first fraction is ab and the common factor of the second fraction is $3a^2b^2$.

$$\frac{3ab + 2a^2b^2}{5ab} - \frac{12a^3b^3 + 3a^2b^2}{6a^2b^2}$$

$$\frac{ab(3 + 2ab)}{ab(5)} - \frac{3a^2b^2(4ab + 1)}{3a^2b^2(2)}$$

$$\frac{3 + 2ab}{5} - \frac{4ab + 1}{2}$$

■ 6. Simplify each expression in the sum.

$$\frac{2ab^2 + 3a^2b^3}{a^3b^3} + \frac{2ab^3 + b^4}{ab^3}$$

Solution:

Look for a common factor in each term. In this case the common factor of the first fraction is $2ab^2$ and the common factor of the second fraction is b^3.

$$\frac{2ab^2 + 3a^2b^3}{a^3b^3} + \frac{2ab^3 + b^4}{ab^3}$$

$$\frac{ab^2(2 + 3ab)}{ab^2(a^2b)} + \frac{b^3(2a + b)}{b^3(a)}$$

$$\frac{2+3ab}{a^2b}+\frac{2a+b}{a}$$

■ 7. Simplify each expression in the sum.

$$\frac{8mn+20m}{4mn}+\frac{m^3n-3m^2n}{6m^2n}$$

Solution:

Look for a common factor in each term. In this case the common factor of the first fraction is $4m$ and the common factor of the second fraction is m^2n.

$$\frac{8mn+20m}{4mn}+\frac{m^3n-3m^2n}{6m^2n}$$

$$\frac{4m(2n+5)}{4m(n)}+\frac{m^2n(m-3)}{m^2n(6)}$$

$$\frac{2n+5}{n}+\frac{m-3}{6}$$

■ 8. Simplify each expression in the difference.

$$\frac{21x^2y^2}{14x^3y}-\frac{24xy+12y}{96y}$$

Solution:

Look for a common factor in each term. In this case the common factor of the first fraction is $7x^2y$ and the common factor of the second fraction is $12y$.

$$\frac{21x^2y^2}{14x^3y} - \frac{24xy + 12y}{96y}$$

$$\frac{7x^2y(3y)}{7x^2y(2x)} - \frac{12y(2x+1)}{12y(8)}$$

$$\frac{3y}{2x} - \frac{2x+1}{8}$$

ADDING AND SUBTRACTING RATIONAL FUNCTIONS

■ 1. Simplify the expression.

$$\frac{2}{3ab}+\frac{b}{4a}+\frac{ab}{6}$$

Solution:

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficients	a-terms	b-terms
3ab	3	a	b
4a	2*2	a	
6	2*3		

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get $2\cdot 2\cdot 3=12$. The largest common multiple in the a column is a and the largest common multiple in the b column is b. The least common multiple is therefore $12ab$.

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator $12ab$.

$$\frac{2}{3ab} \cdot \frac{4}{4} + \frac{b}{4a} \cdot \frac{3b}{3b} + \frac{ab}{6} \cdot \frac{2ab}{2ab}$$

$$\frac{8}{12ab} + \frac{3b^2}{12ab} + \frac{2a^2b^2}{12ab}$$

$$\frac{8 + 3b^2 + 2a^2b^2}{12ab}$$

■ 2. Simplify the expression.

$$\frac{1}{2xy} + \frac{2}{3x^2} + \frac{3}{4xy^2}$$

Solution:

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficients	x-terms	y-terms
2xy	2	x	y
3x^2	3	x^2	
4xy^2	2*2	x	y^2

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get $2 \cdot 2 \cdot 3 = 12$. The largest common multiple in the x column is x^2 and the largest common multiple in the y column is y^2. The least common multiple is therefore $12x^2y^2$.

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator $12x^2y^2$.

$$\frac{1}{2xy} \cdot \frac{6xy}{6xy} + \frac{2}{3x^2} \cdot \frac{4y^2}{4y^2} + \frac{3}{4xy^2} \cdot \frac{3x}{3x}$$

$$\frac{6xy}{12x^2y^2} + \frac{8y^2}{12x^2y^2} + \frac{9x}{12x^2y^2}$$

$$\frac{6xy + 8y^2 + 9x}{12x^2y^2}$$

■ 3. Simplify the expression.

$$\frac{a}{3xy} + \frac{b}{15y^2} + \frac{c}{5x^3y^2}$$

Solution:

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficients	x-terms	y-terms
3xy	3	x	y
$15x^2$	3*5		y^2
$5x^3y^2$	5	x^3	y^2

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get $3 \cdot 5 = 15$. The largest common multiple in the x column is x^3 and the largest common multiple in the y column is y^2. The least common multiple is therefore $15x^3y^2$.

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator $15x^3y^2$.

$$\frac{a}{3xy} \cdot \frac{5x^2y}{5x^2y} + \frac{b}{15y^2} \cdot \frac{x^3}{x^3} + \frac{c}{5x^3y^2} \cdot \frac{3}{3}$$

$$\frac{5ax^2y}{15x^3y^2} + \frac{bx^3}{15x^3y^2} + \frac{3c}{15x^3y^2}$$

$$\frac{5ax^2y + bx^3 + 3c}{15x^3y^2}$$

■ 4. Simplify the expression.

$$\frac{x}{2x^2y} + \frac{y}{3z} + \frac{z}{5yz^2}$$

Solution:

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficients	x-terms	y-terms	z-terms
$2x^2y$	2	x^2	y	
$3z$	3			z
$5yz^2$	5		y	z^2

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get $2 \cdot 3 \cdot 5 = 30$. The largest common multiple in the x column is x^2 and the largest common multiple in the y column is y. The largest common multiple in the z column is z^2. The least common multiple is therefore $30x^2yz^2$.

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator $30x^2yz^2$.

$$\frac{x}{2x^2y} \cdot \frac{15z^2}{15z^2} + \frac{y}{3z} \cdot \frac{10x^2yz}{10x^2yz} + \frac{z}{5yz^2} \cdot \frac{6x^2}{6x^2}$$

$$\frac{15xz^2}{30x^2yz^2} + \frac{10x^2y^2z}{30x^2yz^2} + \frac{6x^2z}{30x^2yz^2}$$

$$\frac{15xz^2 + 10x^2y^2z + 6x^2z}{30x^2yz^2}$$

5. Simplify the expression.

$$\frac{3ab}{4c}+\frac{2bc}{6a^3}+\frac{5}{8ab^2c^3}$$

Solution:

In order to combine the three fractions in the expression we need to find a common denominator. Make a chart of with each denominator to find the least common multiple.

	Coefficient s	a-terms	b-terms	c-terms
4c	2*2			c
6a³	2*3	a³		
8ab²c³	2*2*2	a	b²	c³

To find the least common multiple, find the largest multiple from each column. For the coefficients multiply each unique factor together to get $2\cdot2\cdot2\cdot3=24$. The largest common multiple in the a column is a^3 and the largest common multiple in the b column is b^2. The largest common multiple in the c column is c^3. The least common multiple is therefore $24a^3b^2c^3$.

Now we need to multiply each fraction by a well-chosen 1 that will make the denominator $24a^3b^2c^3$.

$$\frac{3ab}{4c}\cdot\frac{6a^3b^2c^2}{6a^3b^2c^2}+\frac{2bc}{6a^3}\cdot\frac{4b^2c^3}{4b^2c^3}+\frac{5}{8ab^2c^3}\cdot\frac{3a^2}{3a^2}$$

$$\frac{18a^4b^3c^2}{24a^3b^2c^2}+\frac{8b^3c^4}{24a^3b^2c^3}+\frac{15a^2}{24a^3b^2c^3}$$

$$\frac{18a^4b^3c^2+8b^3c^4+15a^2}{24a^3b^2c^3}$$

MULTIPLYING RATIONAL FUNCTIONS

■ 1. Simplify the expression.

$$\frac{25x^2-4}{x^2-36}\cdot\frac{x+6}{5x-2}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(5x-2)(5x+2)}{(x-6)(x+6)}\cdot\frac{x+6}{5x-2}$$

$$\frac{5x+2}{x-6}$$

We cancelled factors of $5x-2$ and $x+6$ which means

$5x-2\neq 0$, or $x\neq 2/5$

$x+6\neq 0$, or $x\neq -6$

So the simplified expression is

$\dfrac{5x+2}{x-6}$ with $x\neq -6, \dfrac{2}{5}$

■ 2. Simplify the expression.

$$\frac{4x^2-49}{9x^2-16}\cdot\frac{3x+4}{2x+7}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(2x-7)(2x+7)}{(3x-4)(3x+4)}\cdot\frac{3x+4}{2x+7}$$

$$\frac{2x-7}{3x-4}$$

We cancelled factors of $2x+7$ and $3x+4$ which means

$2x+7\neq 0$, or $x\neq -7/2$

$3x+4\neq 0$, or $x\neq -4/3$

So the simplified expression is

$$\frac{2x-7}{3x-4} \text{ with } x\neq -\frac{7}{2},\ \ -\frac{4}{3}$$

■ 3. Simplify the expression.

$$\frac{x^2-25}{81x^2-64}\cdot\frac{9x-8}{x-5}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(x-5)(x+5)}{(9x-8)(9x+8)} \cdot \frac{9x-8}{x-5}$$

$$\frac{x+5}{9x+8}$$

We cancelled factors of $x-5$ and $9x-8$ which means

$x-5 \neq 0$, or $x \neq 5$

$9x-8 \neq 0$, or $x \neq 8/9$

So the simplified expression is

$\dfrac{x+5}{9x+8}$ with $x \neq \dfrac{8}{9}, 5$

■ 4. Simplify the expression.

$$\frac{x^2+8x+16}{9x^2+36x+36} \cdot \frac{3x+6}{x+4}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(x+4)(x+4)}{(3x+6)(3x+6)} \cdot \frac{3x+6}{x+4}$$

$$\frac{x+4}{3x+6}$$

We cancelled factors of $x+4$ and $3x+6$ which means

$x+4 \neq 0$, or $x \neq -4$

$3x+6 \neq 0$, or $x \neq -2$

So the simplified expression is

$\frac{x+4}{3x+6}$ with $x \neq -4, \ -2$

However, the remaining factor of $3x+6$ in the denominator still shows us that $x \neq -2$, so we don't need to exclude that value in our answer.

$\frac{x+4}{3x+6}$ with $x \neq -4$

■ 5. Simplify the expression.

$$\frac{16x^2+16x+4}{x^2+18x+81} \cdot \frac{x^2-81}{16x^2-4}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(4x+2)(4x+2)}{(x+9)(x+9)} \cdot \frac{(x-9)(x+9)}{(4x-2)(4x+2)}$$

$$\frac{(4x+2)(x-9)}{(x+9)(4x-2)}$$

We cancelled factors of $4x+2$ and $x+9$ which means

$4x+2 \neq 0$, or $x \neq -1/2$

$x+9 \neq 0$, or $x \neq -9$

So the simplified expression is

$$\frac{(4x+2)(x-9)}{(x+9)(4x-2)} \text{ with } x \neq -9, \ -\frac{1}{2}$$

However, the remaining factor of $x+9$ in the denominator still shows us that $x \neq -9$, so we don't need to exclude that value in our answer.

$$\frac{(4x+2)(x-9)}{(x+9)(4x-2)} \text{ with } x \neq -\frac{1}{2}$$

■ 6. Simplify the expression.

$$\frac{x^2+5x-14}{x^2+2x-3} \cdot \frac{x^2+4x-5}{x^2+9x+14}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(x-2)(x+7)}{(x+3)(x-1)} \cdot \frac{(x-1)(x+5)}{(x+7)(x+2)}$$

$$\frac{(x-2)(x+5)}{(x+3)(x+2)}$$

We cancelled factors of $x+7$ and $x-1$ which means

$x+7 \neq 0$, or $x \neq -7$

$x-1 \neq 0$, or $x \neq 1$

So the simplified expression is

$$\frac{(x-2)(x+5)}{(x+3)(x+2)} \text{ with } x \neq -7,\ 1$$

■ 7. Simplify the expression.

$$\frac{x^2+3x+2}{x^2-7x+12} \cdot \frac{x^2+2x-15}{x^2-4x-12}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(x+1)(x+2)}{(x-3)(x-4)} \cdot \frac{(x-3)(x+5)}{(x+2)(x-6)}$$

$$\frac{(x+1)(x+5)}{(x-4)(x-6)}$$

We cancelled factors of $x+2$ and $x-3$ which means

$$x+2 \neq 0, \text{ or } x \neq -2$$

$$x-3 \neq 0, \text{ or } x \neq 3$$

So the simplified expression is

$$\frac{(x+1)(x+5)}{(x-4)(x-6)} \text{ with } x \neq -2, 3$$

■ 8. Simplify the expression.

$$\frac{x^2+x-56}{x^2-x-90} \cdot \frac{x^2+8x-9}{x^2-5x-14}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(x-7)(x+8)}{(x+9)(x-10)} \cdot \frac{(x+9)(x-1)}{(x-7)(x+2)}$$

$$\frac{(x+8)(x-1)}{(x-10)(x+2)}$$

We cancelled factors of $x-7$ and $x+9$ which means

$x - 7 \neq 0$, or $x \neq 7$

$x + 9 \neq 0$, or $x \neq -9$

So the simplified expression is

$$\frac{(x+8)(x-1)}{(x-10)(x+2)} \text{ with } x \neq -9,\ 7$$

■ 9. Simplify the expression.

$$\frac{2x^2 - 13x - 24}{3x^2 - x - 4} \cdot \frac{3x^2 - 7x + 4}{x^2 - 6x - 16}$$

Solution:

Factor and cancel whatever you can, then simplify.

$$\frac{(2x+3)(x-8)}{(3x-4)(x+1)} \cdot \frac{(3x-4)(x-1)}{(x+2)(x-8)}$$

$$\frac{(2x+3)(x-1)}{(x+1)(x+2)}$$

We cancelled $x - 8$ and $3x - 4$ which means

$x - 8 \neq 0$, or $x \neq 8$

$3x - 4 \neq 0$, or $x \neq 4/3$

So the simplified expression is

$$\frac{(2x+3)(x-1)}{(x+1)(x+2)} \text{ with } x \neq \frac{4}{3}, 8$$

DIVIDING RATIONAL FUNCTIONS

1. Simplify the expression.

$$\frac{2x+16}{9x^2+27x} \div \frac{3x+24}{x+3}$$

Solution:

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{2(x+8)}{9x(x+3)} \div \frac{3(x+8)}{x+3}$$

Consider restrictions. The denominator of the dividend gives $x \neq -3,0$, the denominator of the divisor gives $x \neq -3$, and the numerator of the divisor gives $x \neq -8$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -8, -3,0$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{2(x+8)}{9x(x+3)} \cdot \frac{x+3}{3(x+8)}$$

$$\frac{2}{9x} \cdot \frac{1}{3}$$

$$\frac{2}{27x}$$

This resulting quotient shows that $x \neq 0$, so we can eliminate that from our list of restrictions. Then the final answer is

$$\frac{2}{27x} \text{ with } x \neq -8, -3$$

2. Simplify the expression.

$$\frac{6x+15}{12x^2+24x} \div \frac{2x+5}{5x+10}$$

Solution:

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{3(2x+5)}{12x(x+2)} \div \frac{2x+5}{5(x+2)}$$

Consider restrictions. The denominator of the dividend gives $x \neq -2,0$, the denominator of the divisor gives $x \neq -2$, and the numerator of the divisor gives $x \neq -5/2$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -5/2, -2,0$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{3(2x+5)}{12x(x+2)} \cdot \frac{5(x+2)}{2x+5}$$

$$\frac{3}{12x} \cdot \frac{5}{1}$$

$$\frac{15}{12x}$$

$$\frac{5}{4x}$$

This resulting quotient shows that $x \neq 0$, so we can eliminate that from our list of restrictions. Then the final answer is

$$\frac{5}{4x} \text{ with } x \neq -5/2, -2$$

■ 3. Simplify the expression.

$$\frac{3x^3 - 3x^2 - 6x}{2x^2 - 14x + 24} \div \frac{3x^2 + 21x}{x^2 - 8x + 15}$$

Solution:

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{3x(x^2 - x - 2)}{2(x^2 - 7x + 12)} \div \frac{3x(x+7)}{(x-3)(x-5)}$$

$$\frac{3x(x+1)(x-2)}{2(x-3)(x-4)} \div \frac{3x(x+7)}{(x-3)(x-5)}$$

Consider restrictions. The denominator of the dividend gives $x \neq 3,4$, the denominator of the divisor gives $x \neq 3,5$, and the numerator of the divisor gives $x \neq -7,0$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -7,0,3,4,5$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{3x(x+1)(x-2)}{2(x-3)(x-4)} \cdot \frac{(x-3)(x-5)}{3x(x+7)}$$

$$\frac{(x+1)(x-2)(x-5)}{2(x-4)(x+7)}$$

This resulting quotient shows that $x \neq -7,4$, so we can eliminate those from our list of restrictions. Then the final answer is

$$\frac{(x+1)(x-2)(x-5)}{2(x-4)(x+7)} \text{ with } x \neq 0,3,5$$

■ 4. Simplify the expression.

$$\frac{2x^2-13x-7}{12x+6} \div \frac{3x-2}{3x^2-17x+10}$$

Solution:

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(2x+1)(x-7)}{6(2x+1)} \div \frac{3x-2}{(3x-2)(x-5)}$$

Consider restrictions. The denominator of the dividend gives $x \neq -1/2$, the denominator of the divisor gives $x \neq 2/3,5$, and the numerator of the divisor gives $x \neq 2/3$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -1/2,2/3,5$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(2x+1)(x-7)}{6(2x+1)} \cdot \frac{(3x-2)(x-5)}{3x-2}$$

$$\frac{(x-7)(x-5)}{6}$$

This resulting quotient doesn't show any restrictions, so we need to keep our entire list of them. Then the final answer is

$$\frac{(x-7)(x-5)}{6} \text{ with } x \neq -1/2,2/3,5$$

■ 5. Simplify the expression.

$$\frac{4x^2+13x+10}{3x+6} \div \frac{3x-1}{3x^2-13x+4}$$

Solution:

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(4x+5)(x+2)}{3(x+2)} \div \frac{3x-1}{(3x-1)(x-4)}$$

Consider restrictions. The denominator of the dividend gives $x \neq -2$, the denominator of the divisor gives $x \neq 1/3,4$, and the numerator of the divisor gives $x \neq 1/3$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -2,1/3,4$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(4x+5)(x+2)}{3(x+2)} \cdot \frac{(3x-1)(x-4)}{3x-1}$$

$$\frac{(4x+5)(x-4)}{3}$$

This resulting quotient doesn't show any restrictions, so we need to keep our entire list of them. Then the final answer is

$$\frac{(4x+5)(x-4)}{3} \text{ with } x \neq -2,1/3,4$$

■ 6. Simplify the expression.

$$\frac{x^2-9}{x^2+x-2} \div \frac{x^2+4x+3}{x^2-16}$$

Solution:

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(x-3)(x+3)}{(x+2)(x-1)} \div \frac{(x+3)(x+1)}{(x-4)(x+4)}$$

Consider restrictions. The denominator of the dividend gives $x \neq -2,1$, the denominator of the divisor gives $x \neq -4,4$, and the numerator of the divisor gives $x \neq -3,-1$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -4,-3,-2,-1,1,4$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(x-3)(x+3)}{(x+2)(x-1)} \cdot \frac{(x-4)(x+4)}{(x+3)(x+1)}$$

$$\frac{(x-3)(x-4)(x+4)}{(x+2)(x-1)(x+1)}$$

This resulting quotient shows that $x \neq -2,-1,1$, so we can eliminate those from our list of restrictions. Then the final answer is

$$\frac{(x-3)(x-4)(x+4)}{(x+2)(x-1)(x+1)} \text{ with } x \neq -4,-3,4$$

■ 7. Simplify the expression.

$$\frac{4x^2-9}{x^2+12x+36} \div \frac{4x^2-12x+9}{x^2+7x+6}$$

Solution:

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{(2x-3)(2x+3)}{(x+6)(x+6)} \div \frac{(2x-3)(2x-3)}{(x+6)(x+1)}$$

Consider restrictions. The denominator of the dividend gives $x \neq -6$, the denominator of the divisor gives $x \neq -6, -1$, and the numerator of the divisor gives $x \neq 3/2$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -6, -1, 3/2$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{(2x-3)(2x+3)}{(x+6)(x+6)} \cdot \frac{(x+6)(x+1)}{(2x-3)(2x-3)}$$

$$\frac{(2x+3)(x+1)}{(x+6)(2x-3)}$$

This resulting quotient shows that $x \neq -6, 3/2$, so we can eliminate those from our list of restrictions. Then the final answer is

$$\frac{(2x+3)(x+1)}{(x+6)(2x-3)} \text{ with } x \neq -1$$

■ 8. Simplify the expression.

$$\frac{15x^2+75x+90}{5x^2+50x+125} \div \frac{x^2-3x+2}{x^2-25}$$

Solution:

Factor the numerator and denominator of both fractions as completely as possible.

$$\frac{15(x^2+5x+6)}{5(x^2+10x+25)} \div \frac{(x-1)(x-2)}{(x-5)(x+5)}$$

$$\frac{15(x+3)(x+2)}{5(x+5)(x+5)} \div \frac{(x-1)(x-2)}{(x-5)(x+5)}$$

Consider restrictions. The denominator of the dividend gives $x \neq -5$, the denominator of the divisor gives $x \neq -5,5$, and the numerator of the divisor gives $x \neq 1,2$. So the set of restrictions we should keep in mind until the end of the problem is $x \neq -5,1,2,5$.

Now turn the division problem into a multiplication problem and cancel common factors.

$$\frac{15(x+3)(x+2)}{5(x+5)(x+5)} \cdot \frac{(x-5)(x+5)}{(x-1)(x-2)}$$

$$\frac{3(x+3)(x+2)(x-5)}{(x+5)(x-1)(x-2)}$$

This resulting quotient shows that $x \neq -5, 1, 2$, so we can eliminate those from our list of restrictions. Then the final answer is

$$\frac{3(x+3)(x+2)(x-5)}{(x+5)(x-1)(x-2)} \text{ with } x \neq 5$$

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