Algebra 1 Workbook Solutions

Inequalities

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TRICHOTOMY

1. Solve the inequality.

$$2(x+1) \nleq -(8-x)$$

Solution:

By the trichotomy law, the inequality can be rewritten, and then simplified.

$$2(x+1) > -(8-x)$$

$$2x+2 > -8+x$$

$$2x > -10+x$$

$$x > -10$$

2. If $x \not< y$ and $x \not> y$, by the law of trichotomy, what do we know about the relationship between x and y?

Solution:

By the law of trichotomy, we know that $x = y$.

3. Give two ways to write the following sentence in mathematical notation.

"x^2 is not greater than $4y$ and is also not equal to $4y$."

Solution:

The two ways to express the statement are

$x^2 \ngeq 4y$ and $x^2 < 4y$

4. Solve the inequality.

$$x(3x-2) \ngeq 3(x+x^2)+10$$

Solution:

By the trichotomy law, the inequality can be rewritten, and then simplified.

$$x(3x-2) < 3(x+x^2)+10$$

$$3x^2-2x < 3x+3x^2+10$$

$$-2x < 3x+10$$

$$-5x < 10$$

$$x > -2$$

5. Give the three possible relationships in the law or trichotomy.

Solution:

The three statements of the trichotomy law are

If $a \ngeq b$ then $a < b$.

If $a \nleq b$ then $a > b$.

If $a \ngtr b$ and $a \nless b$ then $a = b$.

6. Find a way to express the following relationships as one equality or inequality.

$x^2 + x \nless 2$ and $x^2 + x \ngtr 2$

Solution:

By the law of trichotomy, we can rewrite the two statements as

$$x^2 + x = 2$$

7. Give two ways to write the following statement in mathematical notation.

"$3(x+1)$ is not less than $-x-5$ and is also not equal to $-x-5$."

Solution:

The two ways to write the statement are

$$3(x+1) \nleq -x-5 \text{ and } 3(x+1) > -x-5$$

8. Solve the following statement.

$$-3(1-x) \ngtr 3(7-x)-2x \text{ and } -3(1-x) \nless 3(7-x)-2x$$

Solution:

By the law of trichotomy, we can rewrite the statement, and then solve for the value of the variable.

$$-3(1-x) = 3(7-x)-2x$$

$$-3+3x = 21-3x-2x$$

$$-3+8x = 21$$

$$8x = 24$$

$x = 3$

INEQUALITIES AND NEGATIVE NUMBERS

1. Solve the inequality.

$$-3x + 4 < 22$$

Solution:

The inequality is solved as

$$-3x < 18$$

$$x > -6$$

2. What is the only difference between solving inequalities and solving equations? Give an example.

Solution:

The only difference is that when we multiply or divide both sides of the inequality by a negative number, we have to flip the inequality sign.

3. What went wrong in the following set of steps?

$$-5x + 6 < 9 - 2x$$

$$-3x < 3$$

$$x < -1$$

Solution:

When the inequality was divided by -3, the inequality sign was not flipped. The solution should be $x > -1$.

4. Solve the inequality.

$$-(5 - 2x) \geq 3(x - 3) + 2x$$

Solution:

The inequality is solved as

$$-5 + 2x \geq 3x - 9 + 2x$$

$$4 + 2x \geq 5x$$

$$4 \geq 3x$$

$$\frac{4}{3} \geq x$$

5. Of $<$, $>$, or $=$, which sign is unaffected when solving inequalities? Give an example.

Solution:

The $=$ sign is unaffected. For example, when solving $-2x \geq 4$, we divide both sides by -2 to get $x \leq -2$.

6. Solve the inequality.

$$-6x + 7 > -3x + 2$$

Solution:

The inequality is solved as

$$-3x > -5$$

$$x < \frac{5}{3}$$

7. What went wrong in the following set of steps?

$$-2(x + 1) \geq 3(2 + x)$$

$$-2x - 2 \geq 6 + 3x$$

$$-2x - 3x - 2 \leq 6$$

Solution:

The inequality sign was flipped when $3x$ was subtracted from each side, but it should have remained the same and not been flipped.

8. Solve the inequality.

$$7(1 - x) \leq 2x$$

Solution:

The inequality is solved as

$$7 - 7x \leq 2x$$

$$7 \leq 9x$$

$$\frac{7}{9} \leq x$$

GRAPHING INEQUALITIES ON A NUMBER LINE

1. Give two expressions that, when graphed, have open circles at 3.

Solution:

There are many correct solutions. For example, $x < 3$ and $x > 3$.

2. Graph the inequality on a number line.

$$-2x < 4$$

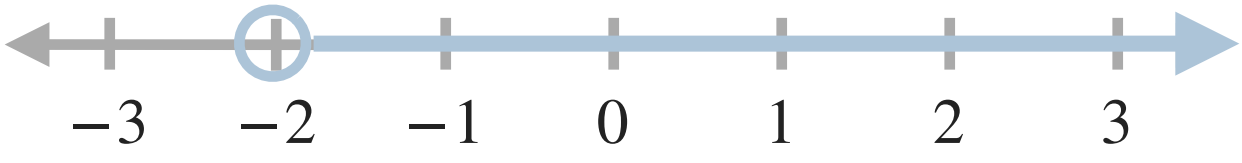
Solution:

Rewrite the inequality.

$$-2x < 4$$

$$x > -2$$

This inequality is graphed as

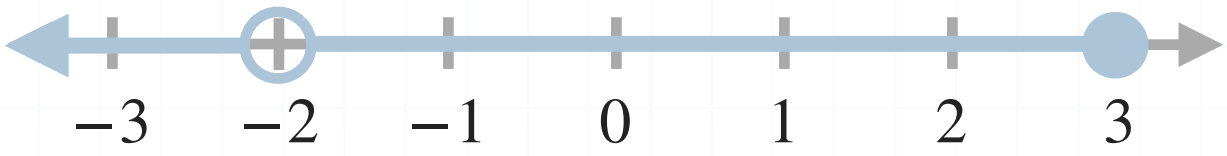


3. Graph the values of x that satisfy the following expressions.

$x \leq 3$ and $x \neq -2$

Solution:

This inequality is graphed as



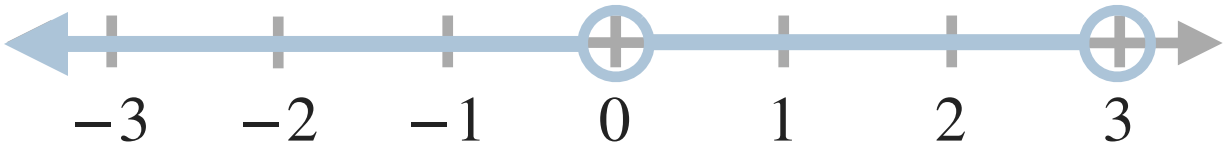
4. Give two expressions that, when graphed, have closed circles at -1.

Solution:

There are many correct solutions. For example, $x \leq -1$ and $x \geq -1$.

5. What is wrong with the graph of the following inequality?

$x \leq 3$ and $x \neq 0$



Solution:

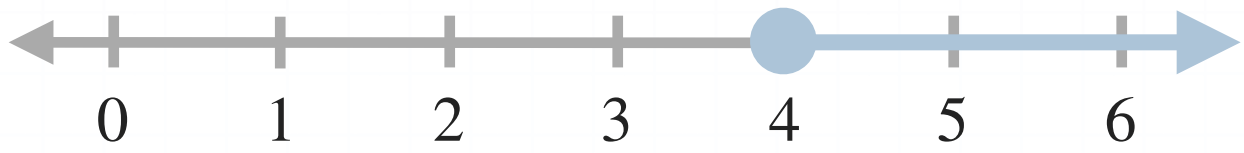
There should be a closed circle at 3 since the inequality $x \leq 3$ includes the value $x = 3$.

6. Graph the inequality on a number line.

$$x - 1 \geq 3$$

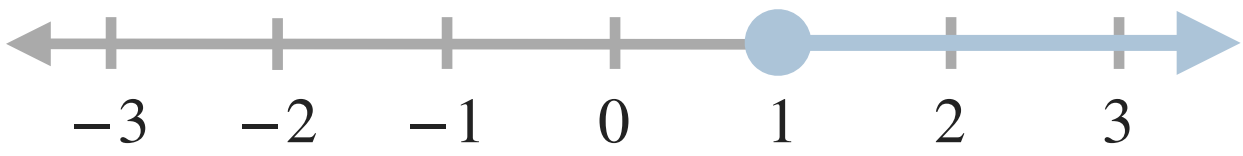
Solution:

This inequality is $x \geq 4$ which is graphed as



7. What is wrong with the graph of the following inequality?

$$x > 1$$



Solution:

There should be an open circle at 1, since the inequality $x > 1$ does not include the value $x = 1$.

GRAPHING CONJUNCTIONS ON A NUMBER LINE

1. Write the inequality that takes away the absolute value sign.

$$|3x - 7| \geq 2$$

Solution:

Taking away the absolute value sign gives

$$3x - 7 \geq 2 \text{ and } 3x - 7 \leq -2$$

2. Graph the inequality.

$$-8 \leq -2x < 10$$

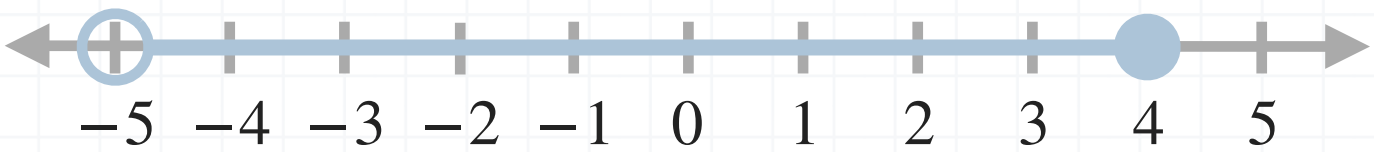
Solution:

Simplify the conjunction.

$$-8 \leq -2x < 10$$

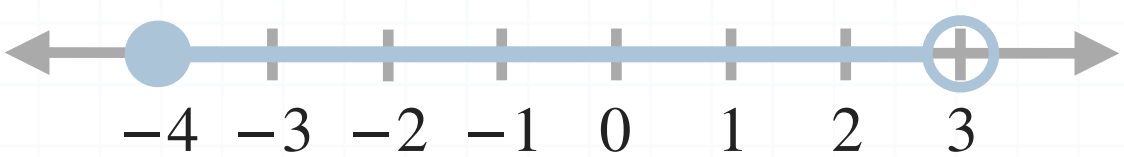
$$4 \geq x > -5$$

Then we can graph the conjunction on a number line.



3. What is wrong with the graph of the following inequality?

$x \leq 3$ and $x > -4$



Solution:

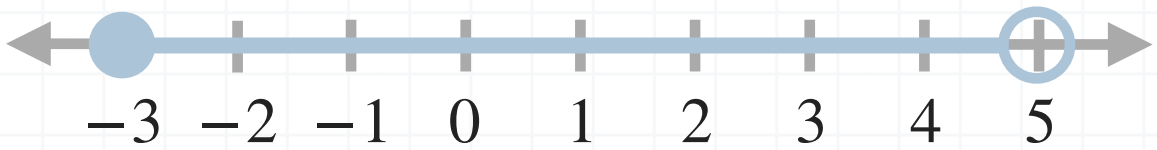
There should be an open circle at -4 since the inequality $x > -4$ does not include $x = -4$, and there should be a closed circle at 3 since the inequality $x \leq 3$ includes $x = 3$.

4. Graph the inequality.

$x < 5$ and $x \geq -3$

Solution:

The conjunction inequality is graphed as



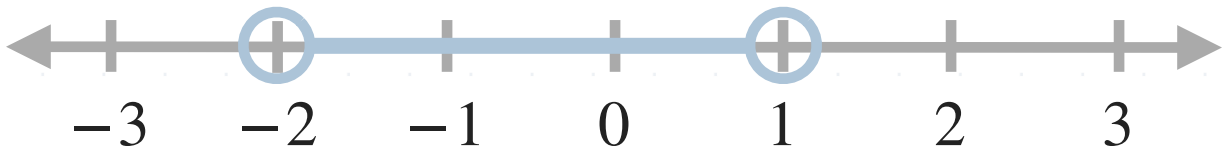
5. Give an example of a conjunction for which the graph is a line segment connecting two points.

Solution:

There are many possible solutions. For example $-1 \leq x \leq 4$.

6. What is wrong with the graph of the following inequality?

$x < -2$ and $x > 1$



Solution:

The graph is showing the conjunction inequality $-2 < x < 1$, instead of $x < -2$ and $x > 1$.

7. Graph the inequality.

$$|6 - 2x| \leq 4$$

Solution:

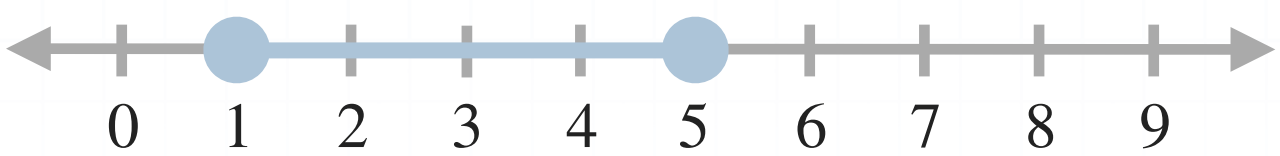
Taking away the absolute value sign, we get

$$-4 \leq 6 - 2x \leq 4$$

$$-10 \leq -2x \leq -2$$

$$5 \geq x \geq 1$$

This conjunction inequality is graphed as



8. Graph the inequality.

$2x - 1 \geq 3$ and $-x \geq -9$

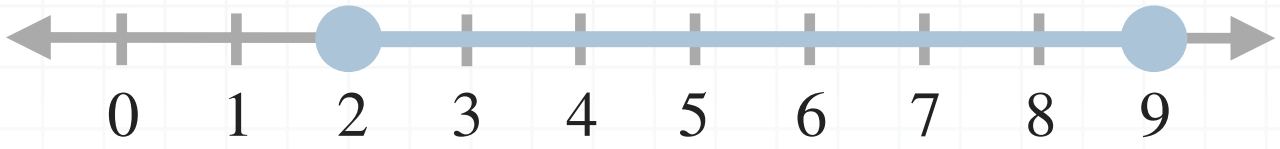
Solution:

This conjunction inequality is

$2x \geq 4$, which is $x \geq 2$

$-x \geq -9$, which is $x \leq 9$

So $x \geq 2$ and $x \leq 9$ is graphed as

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