Algebra 2 Workbook Solutions

Imaginary numbers

krista king MATH

IMAGINARY NUMBERS

■ 1. Simplify the imaginary expression.

$$2 - 6i - 4 + 9i$$

Solution:

Group, then combine like terms.

$$2 - 4 - 6i + 9i$$

$$-2 + 3i$$

■ 2. Simplify the imaginary expression.

$$-3 - 7i + 8 + 3i$$

Solution:

Group, then combine like terms.

$$-3 + 8 - 7i + 3i$$

$$5 - 4i$$

■ 3. Simplify the imaginary expression.

$$\sqrt{-4} + ii + 5i - 2i^3$$

Solution:

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\sqrt{4 \cdot -1} + i^2 + 5i - 2i^2 i$$

$$\sqrt{4}\sqrt{-1} + (-1) + 5i - 2(-1)i$$

$$2i - 1 + 5i + 2i$$

Group, then combine like terms.

$$-1 + 2i + 2i + 5i$$

$$-1 + 9i$$

■ 4. Simplify the imaginary expression.

$$\sqrt{27} - 3ii + 2i - 7i^3 + \sqrt{-36}$$

Solution:

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\sqrt{9 \cdot 3} - 3i^2 + 2i - 7i^2 i + \sqrt{36 \cdot -1}$$

$$\sqrt{9}\sqrt{3} - 3(-1) + 2i - 7(-1)i + \sqrt{36}\sqrt{-1}$$

$$3\sqrt{3} + 3 + 2i + 7i + 6i$$

Group, then combine like terms.

$$3 + 3\sqrt{3} + 15i$$

■ 5. Simplify the imaginary expression.

$$\sqrt{-9} + 2i^3 + 6i - \sqrt{25}\sqrt{-25} - 2\sqrt{-16}$$

Solution:

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\sqrt{9 \cdot -1} + 2i^2 i + 6i - 5\sqrt{25 \cdot -1} - 2\sqrt{16 \cdot -1}$$

$$\sqrt{9}\sqrt{-1} + 2(-1)i + 6i - 5\sqrt{25}\sqrt{-1} - 2\sqrt{16}\sqrt{-1}$$

$$3i - 2i + 6i - 5(5)i - 2(4)i$$

Simplify, then combine like terms.

$$i + 6i - 25i - 8i$$

$$7i - 33i$$

$$-26i$$

■ 6. Simplify the imaginary expression.

$$\sqrt{-4} + 2i^4 + 6i^5 - \sqrt{-49} - 2i^6$$

Solution:

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\sqrt{4 \cdot -1} + 2i^2i^2 + 6i^2i^2i - \sqrt{49 \cdot -1} - 2i^2i^2i^2$$

$$\sqrt{4}\sqrt{-1} + 2(-1)(-1) + 6(-1)(-1)i - \sqrt{49}\sqrt{-1} - 2(-1)(-1)(-1)$$

$$2i + 2 + 6i - 7i + 2$$

Group, then combine like terms.

$$2 + 2 + 2i + 6i - 7i$$

$$4 + 8i - 7i$$

$$4 + i$$

RATIONALIZING COMPLEX DENOMINATORS

■ 1. Use the conjugate method to simplify the imaginary expression.

$$\frac{2+6i}{3-i}$$

Solution:

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $3-i$ is $3+i$, so multiply the expression by $(3+i)/(3+i)$.

$$\frac{2+6i}{3-i} \cdot \frac{3+i}{3+i}$$

$$\frac{(2+6i)(3+i)}{(3-i)(3+i)}$$

$$\frac{6+2i+18i+6i^2}{9+3i-3i-i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{6+20i+6(-1)}{9-(-1)}$$

$$\frac{6-6+20i}{9+1}$$

$$\frac{20i}{10}$$

$$2i$$

■ 2. Use the conjugate method to simplify the imaginary expression.

$$\frac{5-2i}{7+3i}$$

Solution:

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $7+3i$ is $7-3i$, so multiply the expression by $(7-3i)/(7-3i)$.

$$\frac{5-2i}{7+3i} \cdot \frac{7-3i}{7-3i}$$

$$\frac{(5-2i)(7-3i)}{(7+3i)(7-3i)}$$

$$\frac{35-15i-14i+6i^2}{49-21i+21i-9i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{35-29i+6(-1)}{49-9(-1)}$$

$$\frac{35-6-29i}{49+9}$$

$$\frac{29-29i}{58}$$

$$\frac{1}{2}-\frac{1}{2}i$$

■ 3. Use the conjugate method to simplify the imaginary expression.

$$\frac{2-2i}{4i-1}$$

Solution:

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $4i-1$ is $4i+1$, so multiply the expression by $(4i+1)/(4i+1)$.

$$\frac{2-2i}{4i-1}\cdot\frac{4i+1}{4i+1}$$

$$\frac{(2-2i)(4i+1)}{(4i-1)(4i+1)}$$

$$\frac{8i+2-8i^2-2i}{16i^2+4i-4i-1}$$

$$\frac{6i+2-8i^2}{16i^2-1}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{6i+2-8(-1)}{16(-1)-1}$$

$$\frac{6i+2+8}{-16-1}$$

$$\frac{6i+10}{-17}$$

$$-\frac{10}{17}-\frac{6i}{17}$$

■ 4. Use the conjugate method to simplify the imaginary expression.

$$\frac{3i+2i^2}{5i^3+4i^4}$$

Solution:

Simplify the expression first.

$$\frac{3i+2i^2}{5i^3+4i^4}$$

$$\frac{3i+2i^2}{5i^2i+4i^2i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{3i+2(-1)}{5(-1)i+4(-1)(-1)}$$

$$\frac{-2+3i}{4-5i}$$

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $4-5i$ is $4+5i$, so multiply the expression by $(4+5i)/(4+5i)$.

$$\frac{-2+3i}{4-5i}\cdot\frac{4+5i}{4+5i}$$

$$\frac{(-2+3i)(4+5i)}{(4-5i)(4+5i)}$$

$$\frac{-8-10i+12i+15i^2}{16+20i-20i-25i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{-8+2i+15(-1)}{16-25(-1)}$$

$$\frac{-8-15+2i}{16+25}$$

$$\frac{-23+2i}{41}$$

$$-\frac{23}{41}+\frac{2}{41}i$$

■ 5. Use the conjugate method to simplify the imaginary expression.

$$\frac{2i+4i^2}{6-6i}$$

Solution:

Simplify the expression first by plugging in -1 for i^2.

$$\frac{2i+4i^2}{6-6i}$$

$$\frac{2i+4(-1)}{6-6i}$$

$$\frac{-4+2i}{6-6i}$$

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $6-6i$ is $6+6i$, so multiply the expression by $(6+6i)/(6+6i)$.

$$\frac{-4+2i}{6-6i}\cdot\frac{6+6i}{6+6i}$$

$$\frac{(-4+2i)(6+6i)}{(6-6i)(6+6i)}$$

$$\frac{-24-24i+12i+12i^2}{36+36i-36i-36i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{-24-12i+12(-1)}{36-36(-1)}$$

$$\frac{-24-12-12i}{36+36}$$

$$\frac{-36-12i}{72}$$

$$-\frac{1}{2}-\frac{1}{6}i$$

■ 6. Use the conjugate method to simplify the imaginary expression.

$$\frac{8i-3i^2}{5i-6i^2}$$

Solution:

Simplify the expression first by plugging in -1 for i^2.

$$\frac{8i-3i^2}{5i-6i^2}$$

$$\frac{8i-3(-1)}{5i-6(-1)}$$

$$\frac{3+8i}{6+5i}$$

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $6+5i$ is $6-5i$, so multiply the expression by $(6-5i)/(6-5i)$.

$$\frac{3+8i}{6+5i}\cdot\frac{6-5i}{6-5i}$$

$$\frac{(3+8i)(6-5i)}{(6+5i)(6-5i)}$$

$$\frac{18-15i+48i-40i^2}{36-30i+30i-25i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{18+33i-40(-1)}{36-25(-1)}$$

$$\frac{18+40+33i}{36+25}$$

$$\frac{58+33i}{61}$$

$$\frac{58}{61}+\frac{33}{61}i$$

■ 7. Use the conjugate method to simplify the imaginary expression.

$$\frac{\sqrt{-5}\sqrt{-5}-7i^3}{3+i}$$

Solution:

Simplify the expression.

$$\frac{\sqrt{5}^2 i^2 - 7i^2 i}{3 + i}$$

$$\frac{5i^2 - 7i^2 i}{3 + i}$$

Remember that $\sqrt{-1} = i$ and $i^2 = -1$.

$$\frac{5(-1) - 7(-1)i}{3 + i}$$

$$\frac{-5 + 7i}{3 + i}$$

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $3 + i$ is $3 - i$, so multiply the expression by $(3 - i)/(3 - i)$.

$$\frac{-5 + 7i}{3 + i} \cdot \frac{3 - i}{3 - i}$$

$$\frac{(-5 + 7i)(3 - i)}{(3 + i)(3 - i)}$$

$$\frac{-15 + 5i + 21i - 7i^2}{9 + 3i - 3i - i^2}$$

$$\frac{-15 + 26i - 7i^2}{9 - i^2}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{-15+26i-7(-1)}{9-(-1)}$$

$$\frac{-15+26i+7}{9+1}$$

$$\frac{-8+26i}{10}$$

$$-\frac{4}{5}+\frac{13}{5}i$$

■ 8. Use the conjugate method to simplify the imaginary expression.

$$\frac{\sqrt{-2}\sqrt{-2}+3i^3}{i-4}$$

Solution:

Simplify the expression.

$$\frac{\sqrt{2}^2 i^2+3i^2 i}{i-4}$$

$$\frac{2i^2+3i^2 i}{i-4}$$

Remember that $\sqrt{-1}=i$ and $i^2=-1$.

$$\frac{2(-1)+3(-1)i}{i-4}$$

$$\frac{-2-3i}{i-4}$$

Use the conjugate method the get the imaginary number, i, out of the denominator. The conjugate of $i-4$ is $i+4$, so multiply the expression by $(i+4)/(i+4)$.

$$\frac{-2-3i}{i-4}\cdot\frac{i+4}{i+4}$$

$$\frac{(-2-3i)(i+4)}{(i-4)(i+4)}$$

$$\frac{-2i-8-3i^2-12i}{i^2+4i-4i-16}$$

$$\frac{-14i-8-3i^2}{i^2-16}$$

Plug in -1 for i^2 and combine like terms.

$$\frac{-14i-8-3(-1)}{(-1)-16}$$

$$\frac{-14i-8+3}{-17}$$

$$\frac{-14i-5}{-17}$$

$$\frac{5}{17}+\frac{14}{17}i$$