Algebra 2 Workbook Solutions

Advanced equations

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MATH

DIRECT VARIATION

■ 1. If $2k = 12$ and $kx = 48$, find x.

Solution:

Solve the first equation for k.

$$2k = 12$$

$$\frac{2k}{2} = \frac{12}{2}$$

$$k = 6$$

Plug $k = 6$ into the second equation and solve for x.

$$kx = 48$$

$$6x = 48$$

$$\frac{6x}{6} = \frac{48}{6}$$

$$x = 8$$

■ 2. If $10k = 5$ and $kx = 3$, find x.

Solution:

Solve the first equation for k.

$$10k = 5$$

$$\frac{10k}{10} = \frac{5}{10}$$

$$k = \frac{1}{2}$$

Plug $k = 1/2$ into the second equation and solve for x.

$$kx = 3$$

$$\frac{1}{2}x = 3$$

$$\frac{2}{1} \cdot \frac{1}{2}x = 3 \cdot \frac{2}{1}$$

$$x = 6$$

■ 3. If x and y vary directly and the constant of variation, k, equals 4, what is the value of y when $x = 13$?

Solution:

The general form of direct variation is $y = kx$. Plug in the values for k and x.

$$y = 4(13)$$

$$y = 52$$

■ 4. If x and y vary directly and the constant of variation, k, equals $1/3$, what is the value of y when $x = 54$?

Solution:

The general form of direct variation is $y = kx$. Plug in the values for k and x.

$$y = \frac{1}{3}(54)$$

$$y = 18$$

■ 5. If x varies directly with y and $y = 32$ when $x = 2$, what is the value of the constant of variation, k?

Solution:

The general form of direct variation is $y = kx$. Plug in the values for x and y.

$$32 = k(2)$$

$$\frac{32}{2} = \frac{2k}{2}$$

$$k = 16$$

■ 6. If x varies directly with y and $y = 4$ when $x = 20$, what is the value of the constant of variation, k?

Solution:

The general form of direct variation is $y = kx$. Plug in the values for x and y.

$$4 = k(20)$$

$$\frac{4}{20} = \frac{20k}{20}$$

$$k = \frac{1}{5}$$

■ 7. If x varies directly with y and $y = 15$ when $x = 5$, what is the value of x when $y = 36$?

Solution:

First, we need to find the constant of variation. The general form of direct variation is $y = kx$. Plug in the values for x and y to solve for k.

$$15 = k(5)$$

$$\frac{15}{5} = \frac{5k}{5}$$

$$k = 3$$

Now that we have the constant of variation, plug in $k = 3$ and $y = 36$ to solve for x.

$$36 = 3x$$

$$\frac{36}{3} = \frac{3x}{3}$$

$$x = 12$$

■ 8. If x varies directly with y and $y = 7$ when $x = 42$, what is the value of y when $x = 54$?

Solution:

First, we need to find the constant of variation. The general form of direct variation is $y = kx$. Plug in the values for x and y to solve for k.

$$7 = k(42)$$

$$\frac{7}{42} = \frac{42k}{7}$$

$$k = \frac{1}{6}$$

Now that we have the constant of variation, plug in $k=1/6$ and $x=54$ to solve for y.

$$y=\frac{1}{6}(54)$$

$$y=9$$

INVERSE VARIATION

■ 1. If $k/3 = 6$ and $k/x = 2$, find x.

Solution:

Solve the first equation for k.

$$\frac{k}{3} = 6$$

$$k = 18$$

Plug $k = 18$ into the second equation and solve for x.

$$\frac{k}{x} = 2$$

$$\frac{18}{x} = 2$$

$$2x = 18$$

$$x = 9$$

■ 2. If $k/5 = 4$ and $k/x = 10$, find x.

Solution:

Solve the first equation for k.

$$\frac{k}{5} = 4$$

$$k = 20$$

Plug $k = 20$ into the second equation and solve for x.

$$\frac{k}{x} = 10$$

$$\frac{20}{x} = 10$$

$$10x = 20$$

$$x = 2$$

■ 3. If x and y vary inversely and the constant of variation, k, equals 12, what is the value of y when $x = 4$?

Solution:

The general form of inverse variation is $y = k/x$. Plug in the values for k and x.

$$y = \frac{12}{4}$$

$$y = 3$$

■ 4. If x and y vary inversely and the constant of variation, k, equals $1/3$, what is the value of y when $x = 8$?

Solution:

The general form of inverse variation is $y = k/x$. Plug in the values for k and x.

$$y = \frac{\frac{1}{3}}{8}$$

$$y = \frac{1}{3} \div \frac{8}{1}$$

$$y = \frac{1}{3} \cdot \frac{1}{8}$$

$$y = \frac{1}{24}$$

■ 5. If x varies inversely with y and $y = 5$ when $x = 6$, what is the value of the constant of variation, k?

Solution:

The general form of inverse variation is $y = k/x$. Plug in the values for x and y.

$$5 = \frac{k}{6}$$

$$k = 30$$

■ 6. If x varies inversely with y and $y = 7$ when $x = 3$, what is the value of the constant of variation, k?

Solution:

The general form of inverse variation is $y = kx$. Plug in the values for x and y.

$$7 = \frac{k}{3}$$

$$k = 21$$

■ 7. If x varies inversely with y and $y = 4$ when $x = 2$, what is the value of x when $y = 1/2$?

Solution:

First, we need to find the constant of variation. The general form of inverse variation is $y = k/x$. Plug in the values for x and y to solve for k.

$$4 = \frac{k}{2}$$

$$k = 8$$

Now that we have the constant of variation, plug in $k = 8$ and $y = 1/2$ to solve for x.

$$\frac{1}{2} = \frac{8}{x}$$

$$1 \cdot x = 2 \cdot 8$$

$$x = 16$$

■ 8. If x varies inversely with y and $y = 3$ when $x = 9$, what is the value of y when $x = 1/4$?

Solution:

First, we need to find the constant of variation. The general form of inverse variation is $y = k/x$. Plug in the values for x and y to solve for k.

$$3 = \frac{k}{9}$$

$$k = 27$$

Now that we have the constant of variation, plug in $k = 27$ and $x = 1/4$ to solve for y.

$$y = \frac{27}{\frac{1}{4}}$$

$$y = 27 \div \frac{1}{4}$$

$$y = 27 \cdot 4$$

$$y = 108$$

DECIMAL EQUATIONS

■ 1. Solve the decimal equation.

$$0.2x + 4 = 10$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 10.

$$(0.2x + 4 = 10)10$$

$$0.2x(10) + 4(10) = 10(10)$$

$$2x + 40 = 100$$

$$2x = 60$$

$$x = 30$$

■ 2. Solve the decimal equation.

$$0.34x - 0.62 = 1.25$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 100.

$$(0.34x - 0.62 = 1.25)100$$

$$0.34x(100) - 0.62(100) = 1.25(100)$$

$$34x - 62 = 125$$

$$34x = 187$$

$$x = 5.5$$

■ 3. Solve the decimal equation.

$$2.1a - 1.4a = 2.8$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 10.

$$(2.1a - 1.4a = 2.8)10$$

$$2.1a(10) - 1.4a(10) = 2.8(10)$$

$$21a - 14a = 28$$

$$7a = 28$$

$$a = 4$$

■ 4. Solve the decimal equation.

$$4a + 6a = 1.7$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 10.

$$(4a + 6a = 1.7)10$$

$$4a(10) + 6a(10) = 1.7(10)$$

$$40a + 60a = 17$$

$$100a = 17$$

$$a = \frac{17}{100}$$

$$a = 0.17$$

■ 5. Solve the decimal equation.

$$0.12n + 3.6 = 4.8$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 100.

$$(0.12n + 3.6 = 4.8)100$$

$$0.12n(100) + 3.6(100) = 4.8(100)$$

$$12n + 360 = 480$$

$$12n = 120$$

$$n = 10$$

■ 6. Solve the decimal equation.

$$5n - 6.1 = -2.9$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 10.

$$(5n - 6.1 = -2.9)10$$

$$5n(10) - 6.1(10) = -2.9(10)$$

$$50n - 61 = -29$$

$$50n = 32$$

$$n = \frac{32}{50}$$

$$n = 0.64$$

■ 7. Solve the decimal equation.

$$3.2x + 2.6 = 1.8x - 4.4$$

Solution:

To solve, get rid of the decimals by multiplying the equation by 10.

$$(3.2x + 2.6 = 1.8x - 4.4)10$$

$$3.2x(10) + 2.6(10) = 1.8x(10) - 4.4(10)$$

$$32x + 26 = 18x - 44$$

$$14x + 26 = -44$$

$$14x = -70$$

$$x = -5$$

FRACTIONAL EQUATIONS

■ 1. Solve for the variable.

$$\frac{2}{5}x = 6$$

Solution:

Multiply both sides by the reciprocal of the fractional coefficient.

$$\frac{5}{2} \cdot \frac{2}{5}x = 6 \cdot \frac{5}{2}$$

$$x = \frac{30}{2}$$

$$x = 15$$

■ 2. Solve for the variable.

$$\frac{4}{3}x = 18$$

Solution:

Multiply both sides by the reciprocal of the fractional coefficient.

$$\frac{3}{4} \cdot \frac{4}{3}x = 18 \cdot \frac{3}{4}$$

$$x = \frac{54}{4}$$

$$x = \frac{27}{2}$$

3. Solve for the variable.

$$\frac{1}{3}x + 3 = 12$$

Solution:

Multiply every term in the equation by the denominator of the fraction.

$$\left(\frac{1}{3}x + 3 = 12\right)3$$

$$\frac{1}{3}x(3) + 3(3) = 12(3)$$

$$x + 9 = 36$$

$$x = 27$$

4. Solve for the variable.

$$\frac{4}{7}x + \frac{1}{7} = \frac{10}{7}$$

Solution:

Multiply every term in the equation by the denominator of the fraction.

$$\left(\frac{4}{7}x + \frac{1}{7} = \frac{10}{7}\right)7$$

$$\frac{4}{7}x(7) + \frac{1}{7}(7) = \frac{10}{7}(7)$$

$$4x + 1 = 10$$

$$4x = 9$$

$$x = \frac{9}{4}$$

RATIONAL EQUATIONS

■ 1. Solve the abstract equation for n, if $n \neq 0$.

$$\frac{2m}{n} + xy - 3ab = z$$

Solution:

Multiply every term of the equation by the denominator of the fraction.

$$\left(\frac{2m}{n} + xy - 3ab = z\right) n$$

$$\frac{2m}{n}(n) + xy(n) - 3ab(n) = z(n)$$

$$2m + nxy - 3abn = nz$$

Move all the terms containing n to the left side of the equation, and move all the terms that don't contain n to the right side of the equation.

$$2m - 2m + nxy - 3abn - nz = nz - nz - 2m$$

$$nxy - 3abn - nz = -2m$$

Factor out n from the left side of the equation.

$$n(xy - 3ab - z) = -2m$$

Divide both sides by $(xy - 3ab - z)$.

$$n = -\frac{2m}{xy - 3ab - z}$$

■ 2. Solve the abstract equation for x, if $x \neq 0$.

$$\frac{1}{x} - z = y$$

Solution:

Multiply every term of the equation by the denominator of the fraction.

$$\left(\frac{1}{x} - z = y\right)x$$

$$\frac{1}{x}(x) - z(x) = y(x)$$

$$1 - xz = xy$$

Move all the terms containing x to the left side of the equation, and move all the terms that don't contain x to the right side of the equation.

$$1 - 1 - xz - xy = xy - xy - 1$$

$$-xz - xy = -1$$

Factor out -1 from the left side of the equation.

$$-1(xz + xy) = -1$$

$$xz + xy = 1$$

Factor out x from the left side of the equation.

$$x(z + y) = 1$$

Divide both sides by $(z + y)$.

$$\frac{x(z + y)}{(z + y)} = \frac{1}{(z + y)}$$

$$x = \frac{1}{z + y}$$

■ 3. Solve the abstract equation for y, if $x \neq 0$.

$$\frac{y}{x} + 3x = 2z$$

Solution:

Multiply every term of the equation by the denominator of the fraction.

$$\left(\frac{y}{x} + 3x = 2z\right)x$$

$$\frac{y}{x}(x) + 3x(x) = 2z(x)$$

$$y + 3x^2 = 2xz$$

Move all the terms containing y to the left side of the equation, and move all the terms that don't contain y to the right side of the equation.

$$y + 3x^2 - 3x^2 = 2xz - 3x^2$$

$$y = 2xz - 3x^2$$

■ 4. Solve the abstract equation for a, if $a \neq 0$ and $b \neq 0$.

$$\frac{bc}{a} - cxy = \frac{z}{b}$$

Solution:

Multiply every term of the equation by the denominator of both fractions.

$$\left(\frac{bc}{a} - cxy = \frac{z}{b}\right) ab$$

$$\frac{bc}{a}(ab) - cxy(ab) = \frac{z}{b}(ab)$$

$$b^2c - abcxy = az$$

Move all the terms containing a to the left side of the equation, and move all the terms that don't contain a to the right side of the equation.

$$b^2c - b^2c - abcxy - az = az - az - b^2c$$

$$-abcxy - az = -b^2c$$

Factor out -1 from the left side of the equation.

$$-1(abcxy + az) = -b^2c$$

$$abcxy + az = b^2c$$

Factor out a from the left side of the equation.

$$a(bcxy + z) = b^2c$$

Divide both sides by $(bcxy + z)$.

$$\frac{a(bcxy + z)}{(bcxy + z)} = \frac{b^2c}{(bcxy + z)}$$

$$a = \frac{b^2c}{bcxy + z}$$

■ 5. Solve the abstract equation for x, if $x \neq 0$ and $y \neq 0$.

$$\frac{a}{x} - \frac{b}{y} = c$$

Solution:

Multiply every term of the equation by the denominator of both fractions.

$$\left(\frac{a}{x} - \frac{b}{y} = c\right)xy$$

$$\frac{a}{x}(xy) - \frac{b}{y}(xy) = c(xy)$$

$$ay - bx = cxy$$

Move all the terms containing x to the left side of the equation, and move all the terms that don't contain x to the right side of the equation.

$$ay - ay - bx - cxy = cxy - cxy - ay$$

$$-bx - cxy = -ay$$

Factor out -1 from the left side of the equation.

$$-1(bx + cxy) = -ay$$

$$bx + cxy = ay$$

Factor out x from the left side of the equation.

$$x(b + cy) = ay$$

Divide both sides by $(b + cy)$.

$$\frac{x(b + cy)}{(b + cy)} = \frac{ay}{(b + cy)}$$

$$x = \frac{ay}{b + cy}$$

■ 6. Solve the abstract equation for y, if $y \neq 0$, $b \neq 0$, and $n \neq 0$.

$$\frac{1}{y} + \frac{a}{b} = \frac{m}{n}$$

Solution:

Multiply every term of the equation by the denominator of each fraction.

$$\left(\frac{1}{y} + \frac{a}{b} = \frac{m}{n}\right) bny$$

$$\frac{1}{y}(bny) + \frac{a}{b}(bny) = \frac{m}{n}(bny)$$

$$bn + any = bmy$$

Move all the terms containing y to the left side of the equation, and move all the terms that don't contain y to the right side of the equation.

$$bn - bn + any - bmy = bmy - bmy - bn$$

$$any - bmy = -bn$$

Factor out y from the left side of the equation.

$$y(an - bm) = -bn$$

Divide both sides by $(an - bm)$.

$$\frac{y(an - bm)}{(an - bm} = \frac{-bn}{(an - bm)}$$

$$y = -\frac{bn}{an - bm}$$

■ 7. Solve the abstract equation for x, if $z \neq 0$, $n \neq 0$, and $b \neq 0$.

$$\frac{2x + y}{z} - \frac{m}{n} = \frac{a}{b}$$

Solution:

Multiply every term of the equation by the denominator of each fraction.

$$\left(\frac{2x + y}{z} - \frac{m}{n} = \frac{a}{b}\right) bnz$$

$$\frac{2x + y}{z}(bnz) - \frac{m}{n}(bnz) = \frac{a}{b}(bnz)$$

$$bn(2x + y) - bmz = anz$$

Distribute bn through $(2x + y)$.

$$2bnx + bny - bmz = anz$$

Move all the terms containing x to the left side of the equation, and move all the terms that don't contain x to the right side of the equation.

$$2bnx + bny - bny - bmz + bmz = anz - bny + bmz$$

$$2bnx = anz - bny + bmz$$

Divide both sides by $2bn$.

$$\frac{2bnx}{2bn}=\frac{anz-bny+bmz}{2bn}$$

$$x=\frac{anz-bny+bmz}{2bn}$$

■ 8. Solve the abstract equation for x, if $x\neq 0$ and $y+z\neq 0$.

$$\frac{1}{x}+\frac{2}{y+z}=3$$

Solution:

Multiply every term of the equation by the denominator of each fraction.

$$\left(\frac{1}{x}+\frac{2}{y+z}=3\right)\cdot x(y+z)$$

$$\frac{1}{x}\cdot x(y+z)+\frac{2}{y+z}\cdot x(y+z)=3\cdot x(y+z)$$

$$y+z+2x=3x(y+z)$$

Distribute $3x$ through $(y+z)$.

$$y+z+2x=3xy+3xz$$

Move all the terms containing x to the left side of the equation, and move all the terms that don't contain x to the right side of the equation.

$$y - y + z - z + 2x - 3xy - 3xz = 3xy - 3xy + 3xz - 3xz - y - z$$

$$2x - 3xy - 3xz = -y - z$$

Factor out x on the left side of the equation.

$$x(2 - 3y - 3z) = -y - z$$

Divide both sides by $(2 - 3y - 3z)$.

$$x = -\frac{y + z}{2 - 3y - 3z}$$

RADICAL EQUATIONS

■ 1. Solve the radical equation for the variable.

$$\sqrt{x} - 4 = 5$$

Solution:

To solve the radical equation, first isolate the radical by adding 4 to both sides of the equation.

$$\sqrt{x} - 4 = 5$$

$$\sqrt{x} - 4 + 4 = 5 + 4$$

$$\sqrt{x} = 9$$

Square both sides.

$$(\sqrt{x})^2 = 9^2$$

$$x = 81$$

Plug the solution back into the original equation to make sure it satisfies it.

$$\sqrt{81} - 4 = 5$$

$$9 - 4 = 5$$

$$5 = 5$$

So $x = 81$ is a solution to the radical equation.

■ 2. Solve the radical equation for the variable.

$$2\sqrt{x} = 14$$

Solution:

To solve the radical equation, first isolate the radical by dividing both sides of the equation by 2.

$$2\sqrt{x} = 14$$

$$\frac{2\sqrt{x}}{2} = \frac{14}{2}$$

$$\sqrt{x} = 7$$

Square both sides.

$$(\sqrt{x})^2 = 7^2$$

$$x = 49$$

Plug the solution back into the original equation to make sure it satisfies it.

$$2\sqrt{49} = 14$$

$$2(7) = 14$$

$$14 = 14$$

So $x = 49$ is a solution to the radical equation.

■ 3. Solve the radical equation for the variable.

$$\sqrt{x+1} - 3 = 2$$

Solution:

To solve the radical equation, first isolate the radical by adding 3 to both sides of the equation.

$$\sqrt{x+1} - 3 = 2$$

$$\sqrt{x+1} - 3 + 3 = 2 + 3$$

$$\sqrt{x+1} = 5$$

Square both sides.

$$(\sqrt{x+1})^2 = 5^2$$

$$x + 1 = 25$$

Subtract 1 from both sides.

$$x + 1 - 1 = 25 - 1$$

$$x = 24$$

Plug the solution back into the original equation to make sure it satisfies it.

$$\sqrt{24+1}-3=2$$

$$\sqrt{25}-3=2$$

$$5-3=2$$

$$2=2$$

So $x=24$ is a solution to the radical equation.

■ 4. Solve the radical equation for the variable.

$$\sqrt{x-5}+4=6$$

Solution:

To solve the radical equation, first isolate the radical by subtracting 4 from both sides of the equation.

$$\sqrt{x-5}+4=6$$

$$\sqrt{x-5}+4-4=6-4$$

$$\sqrt{x-5}=2$$

Square both sides.

$$(\sqrt{x-5})^2=2^2$$

$$x - 5 = 4$$

Add 5 to both sides.

$$x - 5 + 5 = 4 + 5$$

$$x = 9$$

Plug the solution back into the original equation to make sure it satisfies it.

$$\sqrt{9-5} + 4 = 6$$

$$\sqrt{4} + 4 = 6$$

$$2 + 4 = 6$$

$$6 = 6$$

So $x = 9$ is a solution to the radical equation.

■ 5. Solve the radical equation for the variable.

$$3x + \sqrt{x+3} = 1$$

Solution:

To solve the radical equation, first isolate the radical by subtracting $3x$ from both sides of the equation.

$$3x + \sqrt{x+3} = 1$$

$$3x - 3x + \sqrt{x+3} = 1 - 3x$$

$$\sqrt{x+3} = 1 - 3x$$

Square both sides of the equation. Remember to FOIL the right side of the equation.

$$(\sqrt{x+3})^2 = (1 - 3x)^2$$

$$x + 3 = 1 - 3x - 3x + 9x^2$$

$$x + 3 = 9x^2 - 6x + 1$$

Move all terms to one side of the equation.

$$x - x + 3 - 3 = 9x^2 - 6x + 1 - x - 3$$

$$9x^2 - 7x - 2 = 0$$

$$(x - 1)(9x + 2) = 0$$

Then

$$x - 1 = 0$$

$$x = 1$$

or

$$9x + 2 = 0$$

$$x = -\frac{2}{9}$$

Plug both solutions back into the original equation to make sure they satisfy it.

$$3(1)+\sqrt{1+3}=1$$

$$3+\sqrt{4}=1$$

$$3+2=1$$

$$5=1$$

and

$$3\left(-\frac{2}{9}\right)+\sqrt{-\frac{2}{9}+3}=1$$

$$-\frac{6}{9}+\sqrt{-\frac{2}{9}+\frac{27}{9}}=1$$

$$-\frac{6}{9}+\sqrt{\frac{25}{9}}=1$$

$$-\frac{6}{9}+\frac{5}{3}=1$$

$$-\frac{6}{9}+\frac{15}{9}=1$$

$$\frac{9}{9}=1$$

$$1=1$$

So $x=-2/9$ is the only solution.

■ 6. Solve the radical equation for the variable.

$$\sqrt{1-x}-x=5$$

Solution:

To solve the radical equation, first isolate the radical by adding x from both sides of the equation.

$$\sqrt{1-x}-x=5$$

$$\sqrt{1-x}-x+x=5+x$$

$$\sqrt{1-x}=x+5$$

Square both sides of the equation. Remember to FOIL the right side of the equation.

$$(\sqrt{1-x})^2=(x+5)^2$$

$$1-x=x^2+5x+5x+25$$

$$1-x=x^2+10x+25$$

Move all terms to one side of the equation.

$$1-1-x+x=x^2+10x+25+x-1$$

$$x^2+11x+24=0$$

$$(x+8)(x+3)=0$$

Then

$$x+8=0$$

$$x=-8$$

or

$$x+3=0$$

$$x=-3$$

Plug both solutions back into the original equation to make sure they satisfy it.

$$\sqrt{1-(-8)}-(-8)=5$$

$$\sqrt{1+8}+8=5$$

$$\sqrt{9}+8=5$$

$$3+8=5$$

$$11=5$$

and

$$\sqrt{1-(-3)}-(-3)=5$$

$$\sqrt{1+3}+3=5$$

$$\sqrt{4}+3=5$$

$$2 + 3 = 5$$

$$5 = 5$$

So $x = -3$ is the only solution.

■ 7. Solve the radical equation for the variable.

$$\sqrt{x^2 - 2x + 4} + 4 = x$$

Solution:

To solve the radical equation, first isolate the radical by subtracting 4 from both sides of the equation.

$$\sqrt{x^2 - 2x + 4} + 4 = x$$

$$\sqrt{x^2 - 2x + 4} + 4 - 4 = x - 4$$

$$\sqrt{x^2 - 2x + 4} = x - 4$$

Square both sides of the equation. Remember to FOIL the right side of the equation.

$$(\sqrt{x^2 - 2x + 4})^2 = (x - 4)^2$$

$$x^2 - 2x + 4 = x^2 - 4x - 4x + 16$$

$$x^2 - 2x + 4 = x^2 - 8x + 16$$

Move all terms to one side of the equation.

$$x^2 - x^2 - 2x + 8x + 4 - 16 = x^2 - x^2 - 8x + 8x + 16 - 16$$

$$6x - 12 = 0$$

Solve for x.

$$6x - 12 + 12 = 0 + 12$$

$$6x = 12$$

$$x = 2$$

Plug the solution back into the original equation to make sure it satisfies it.

$$\sqrt{2^2 - 2(2) + 4} + 4 = 2$$

$$\sqrt{4 - 4 + 4} + 4 = 2$$

$$\sqrt{4} + 4 = 2$$

$$2 + 4 = 2$$

$$6 = 2$$

So there’s no solution to the radical equation.

MULTIVARIABLE EQUATIONS

■ 1. Solve for b if $xy = -2abc$.

Solution:

To solve the equation for b, divide both sides by $-2ac$.

$$xy = -2abc$$

$$\frac{xy}{-2ac} = \frac{-2abc}{-2ac}$$

$$-\frac{xy}{2ac} = b$$

$$b = -\frac{xy}{2ac}$$

■ 2. Solve for x if $y = z/x$.

Solution:

Multiply both sides by x.

$$y = \frac{z}{x}$$

$$x \cdot y = \frac{z}{x} \cdot x$$

$$xy = z$$

Divide both sides by y.

$$\frac{xy}{y} = \frac{z}{y}$$

$$x = \frac{z}{y}$$

■ 3. Solve for t if $4s - 3t + u = 5$.

Solution:

Get $-3t$ by itself by subtracting $4s$ and u from both sides of the equation.

$$4s - 3t + u = 5$$

$$4s - 4s - 3t + u = 5 - 4s$$

$$-3t + u = 5 - 4s$$

$$-3t + u - u = 5 - 4s - u$$

$$-3t = 5 - 4s - u$$

Divide both sides by -3.

$$\frac{-3t}{-3} = \frac{5 - 4s - u}{-3}$$

$$t = -\frac{5 - 4s - u}{3}$$

4. Solve for z if $2x - 3y + 4z = 10$.

Solution:

Get $4z$ by itself by subtracting $2x$ and adding $3y$ to both sides of the equation.

$$2x - 3y + 4z = 10$$

$$2x - 2x - 3y + 4z = 10 - 2x$$

$$-3y + 4z = 10 - 2x$$

$$-3y + 3y + 4z = 10 - 2x + 3y$$

$$4z = 10 - 2x + 3y$$

Divide both sides by 4.

$$\frac{4z}{4} = \frac{10 - 2x + 3y}{4}$$

$$z = \frac{10 - 2x + 3y}{4}$$

■ 5. Solve for y if $z - x + 4y = 3x + z$.

Solution:

Get $4y$ by itself by subtracting z and adding x to both sides of the equation.

$$z - x + 4y = 3x + z$$

$$z - z - x + 4y = 3x + z - z$$

$$-x + 4y = 3x$$

$$-x + x + 4y = 3x + x$$

$$4y = 4x$$

Divide both sides by 4.

$$\frac{4y}{4} = \frac{4x}{4}$$

$$y = x$$

■ 6. Solve for c if $2a - b + 3c = 2b - 4a + c$.

Solution:

Get $3c$ by itself by subtracting $2a$ and adding b to both sides of the equation.

$$2a - b + 3c = 2b - 4a + c$$

$$2a - 2a - b + 3c = 2b - 4a - 2a + c$$

$$-b + 3c = 2b - 6a + c$$

$$-b + b + 3c = 2b + b - 6a + c$$

$$3c = 3b - 6a + c$$

Get all c-terms to the same side of the equation by subtracting c from both sides.

$$3c - c = 3b - 6a + c - c$$

$$2c = 3b - 6a$$

Divide both sides by 2.

$$\frac{2c}{2} = \frac{3b - 6a}{2}$$

$$c = \frac{3b - 6a}{2}$$

■ 7. Solve for u if $u + 5v - 3w = 4$.

Solution:

Get u by itself by subtracting $5v$ and adding $3w$ to both sides of the equation.

$$u + 5v - 3w = 4$$

$$u + 5v - 5v - 3w = 4 - 5v$$

$$u - 3w = 4 - 5v$$

$$u - 3w + 3w = 4 - 5v + 3w$$

$$u = 4 - 5v + 3w$$

■ 8. Solve for y if $2x - y + z = 3x$.

Solution:

Get $-y$ by itself by subtracting $2x$ and z from both sides of the equation.

$$2x - y + z = 3x$$

$$2x - 2x - y + z = 3x - 2x$$

$$-y + z = x$$

$$-y + z - z = x - z$$

$$-y = x - z$$

Multiply both sides by -1.

$$-1 \cdot -y = -1(x - z)$$

$$y = -x + z$$

$$y = z - x$$

9. Solve for a if $x + y = 3ab + c$.

Solution:

Get $3ab$ by itself by subtracting c from both sides of the equation.

$$x + y = 3ab + c$$

$$x + y - c = 3ab + c - c$$

$$x + y - c = 3ab$$

Divide both sides by $3b$.

$$\frac{x + y - c}{3b} = \frac{3ab}{3b}$$

$$\frac{x + y - c}{3b} = a$$

$$a = \frac{x + y - c}{3b}$$

DISTANCE, RATE, AND TIME

■ 1. The car traveled 124 miles in 2 hours. What was the car's average rate in m/hr?

Solution:

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Identify what we know and what we need to find.

$$D = 124 \text{ mi}$$

$$R = ?$$

$$T = 2 \text{ hr}$$

Plug these values into the distance formula.

$$124 \text{ mi} = R \cdot 2 \text{ hr}$$

Divide both sides by 2 hr to solve for the rate.

$$\frac{124 \text{ mi}}{2 \text{ hr}} = \frac{R \cdot 2 \text{ hr}}{2 \text{ hr}}$$

$$R = 62 \text{ mi/hr}$$

■ 2. A train travels at an average rate of 35 mph for 45 minutes. How many miles did the train travel?

Solution:

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Identify what we know and what we need to find.

$$D = ?$$

$$R = 35 \text{ mi/hr}$$

$$T = 45 \text{ min}$$

Notice that the rate is in hours and the time is in minutes. These units need to be the same in order to use the distance formula. Convert the time into hours by multiplying by $(1 \text{ hr})/(60 \text{ min})$.

$$T = 45 \text{ min} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = 0.75 \text{ hr}$$

$$T = 0.75 \text{ hr}$$

Plug these values into the distance formula.

$$D = 35 \text{ mi/hr} \cdot 0.75 \text{ hr}$$

$$D = 35 \cdot 0.75 \frac{\text{mi}}{\text{hr}} \cdot \text{hr}$$

$$D = 26.25 \text{ mi}$$

■ 3. Alan runs an average rate of 5 mph for 3 miles. How many minutes did Alan run?

Solution:

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Identify what we know and what we need to find.

$$D = 3 \text{ mi}$$

$$R = 5 \text{ mi/hr}$$

$$T = ?$$

Plug these values into the distance formula.

$$3 \text{ mi} = 5 \text{ mi/hr} \cdot T$$

Divide both sides by 5 mi/hr to solve for time.

$$\frac{3 \text{ mi}}{5 \text{ mi/hr}} = \frac{5 \text{ mi/hr} \cdot T}{5 \text{ mi/hr}}$$

$$T = \frac{3}{5} \text{ hr}$$

Notice that our answer is in hours and we were asked to find the number of minutes Alan ran. Convert hours to minutes by multiplying by $(60 \text{ min})/(1 \text{ hr})$.

$$T = \frac{3}{5} \text{ hr} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$$

$$T = 36 \text{ min}$$

■ 4. The train traveled 420 miles at 48 mph and arrived 1 hour and 45 minutes late. How fast should the train have traveled to have arrived on time?

Solution:

First we need to find how much time it took the train to travel 420 miles at 48 mph. Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Identify what we know and what we need to find.

$D = 420$ mi

$R = 48$ mi/hr

$T = ?$

Plug these values into the distance formula.

$420 \text{ mi} = 48 \text{ mi/hr} \cdot T$

Divide both sides by 48 mi/hr to solve for time.

$$\frac{420 \text{ mi}}{48 \text{ mi/hr}} = \frac{48 \text{ mi/hr} \cdot T}{48 \text{ mi/hr}}$$

$T = 8.75$ hr

If the train was 1 hour 45 minutes late (or 1.75 hr), then the train needs to travel $8.75 - 1.75 = 7$ hrs to arrive on time. Use the distance formula again with the new time to solve for the new rate.

Identify what we know and what we need to find.

$D = 420$ mi

$R = ?$

$T = 7$ hr

Plug these values into the distance formula.

$420 \text{ mi} = R \cdot 7 \text{ hr}$

Divide both sides by 7 hr to solve for rate.

$$\frac{420 \text{ mi}}{7 \text{ hr}} = \frac{R \cdot 7 \text{ hr}}{7 \text{ hr}}$$

$$R = 60 \text{ mi/hr}$$

■ 5. Brittany ran for 2 hours at 4 mph, but ended up 5 miles short of her goal. If she tried the next day and increased her speed by 2 mph, how long would it take her to reach her goal?

Solution:

First we need to find how many miles Brittany ran the first day. Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Identify what we know and what we need to find.

$$D = ?$$

$$R = 4 \text{ mi/hr}$$

$$T = 2 \text{ hr}$$

Plug these values into the distance formula.

$$D = 4 \text{ mi/hr} \cdot 2 \text{ hr}$$

$$D = 8 \text{ mi}$$

If Brittany was 5 miles short of her goal, then her goal is $8 + 5 = 13$ mi. The next day she increases her speed 2 mph, so her new rate is $4 + 2 = 6$ mi/hr. Use the distance formula again with the new rate and distance to solve for the new time.

Identify what we know and what we need to find.

$$D = 13 \text{ mi}$$

$$R = 6 \text{ mi/hr}$$

$$T = ?$$

Plug these values into the distance formula.

$$13 \text{ mi} = 6 \text{ mi/hr} \cdot T$$

Divide both sides by 6 mi/hr to solve for time.

$$\frac{13 \text{ mi}}{6 \text{ mi/hr}} = \frac{6 \text{ mi/hr} \cdot T}{6 \text{ mi/hr}}$$

$$T = 2\frac{1}{6} \text{ hr}$$

Convert $1/6$ hr to minutes by multiplying by $(60$ min$)/(1$ hr$)$.

$$\frac{1}{6} \text{ hr} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$$

$$10 \text{ min}$$

It will take Brittany $T = 2$ hr 10 min to reach her goal.

■ 6. Adeline and Ellie live 10 miles away from each other. Adeline started walking towards Ellie at 1:00 p.m. Ellie left 1 hour later and walked 4 mph. If they met at 3:00 p.m., how fast did Adeline walk?

Solution:

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts for two distance formulas, one for Adeline and one for Ellie.

$$D_A = R_A T_A$$

$$D_E = R_E T_E$$

We know that in order to meet, they covered the 10 miles between them.

$$D_A + D_E = 10 \text{ mi}$$

Substitute $D_A = R_A T_A$ and $D_E = R_E T_E$

$$D_A + D_E = 10 \text{ mi}$$

$$R_A T_A + R_E T_E = 10 \text{ mi}$$

We know that Adeline walks from 1:00 p.m. to 3:00 p.m., or 2 hours. We also know that Ellie walks from 2:00 p.m. to 3:00 p.m., or 1 hour, at a rate of 4 mph.

$$T_A = 2 \text{ hr}$$

$$T_E = 1 \text{ hr}$$

$$R_E = 4 \text{ mi/hr}$$

Then we can say

$$R_A T_A + R_E T_E = 10 \text{ mi}$$

$$R_A(2 \text{ hr}) + (4 \text{ mi/hr})(1 \text{ hr}) = 10 \text{ mi}$$

$$2 \text{ hr}\, R_A + 4 \text{ mi} = 10 \text{ mi}$$

Subtract 4 mi from both sides and divide by 2 hr.

$$2 \text{ hr}\, R_A + 4 \text{ mi} - 4 \text{ mi} = 10 \text{ mi} - 4 \text{ mi}$$

$$2 \text{ hr}\, R_A = 6 \text{ mi}$$

$$R_A = 3 \text{ mi/hr}$$

Adeline walks at a rate of 3 mph.

■ 7. Sophia and Cooper live 20 miles away from each other. Sophia started walking towards Cooper at a rate of 3 mph at 8:00 a.m. Cooper left 2 hours later and they met at 12:00 p.m. How fast did Cooper walk?

Solution:

Use the formula for distance.

Distance = Rate · Time

$$D = RT$$

Use subscripts for two distance formulas, one for Sophia and one for Cooper.

$$D_S = R_S T_S$$

$$D_C = R_C T_C$$

We know that in order to meet, they covered the 20 miles between them.

$$D_S + D_C = 20 \text{ mi}$$

Substitute $D_S = R_S T_S$ and $D_C = R_C T_C$.

$$D_S + D_C = 20 \text{ mi}$$

$$R_S T_S + R_C T_C = 20 \text{ mi}$$

We know that Sophia's rate is 3 mph and that she walks from 8:00 a.m. to 12:00 p.m., or 4 hours. We also know that Cooper walks from 10:00 a.m. to 12:00 p.m., or 2 hours.

$$R_S = 3 \text{ mi/hr}$$

$$T_S = 4 \text{ hr}$$

$$T_C = 2 \text{ hr}$$

Then we can say

$$R_S T_S + R_C T_C = 20 \text{ mi}$$

$$3 \text{ mi/hr}(4 \text{ hr}) + R_C(2 \text{ hr}) = 20 \text{ mi}$$

$$12 \text{ mi} + 2 \text{ hr } R_C = 20 \text{ mi}$$

Subtract 12 mi from both sides and divide by 2 hr.

$$12 \text{ mi} - 12 \text{ mi} + 2 \text{ hr } R_C = 20 \text{ mi} - 12 \text{ mi}$$

$$2 \text{ hr } R_C = 8 \text{ mi}$$

$$R_C = 4 \text{ mi/hr}$$

Cooper walks at a rate of 4 mph.

■ 8. Eric and Evan live 35 miles away from each other. Eric started walking towards Evan at a rate of 5 mph at 9:00 a.m. If Evan walks at a rate of 3 mph, what time does he need to leave in order for them to meet at 1:00 p.m.?

Solution:

Use the formula for distance.

Distance = Rate · Time

$$D = RT$$

Use subscripts with two distance formulas, one for Eric (E) and one for Evan (V).

$$D_E = R_E T_E$$

$$D_V = R_V T_V$$

We know that in order to meet, they covered the 35 miles between them.

$$D_E + D_V = 35 \text{ mi}$$

Substitute $D_E = R_E T_E$ and $D_V = R_V T_V$.

$$D_E + D_V = 35 \text{ mi}$$

$$R_E T_E + R_V T_V = 35 \text{ mi}$$

We know that Eric's rate is 5 mph and that he walks from 9:00 a.m. to 1:00 p.m., or 4 hours. We also know that Evan's rate is 3 mph.

$$R_E = 5 \text{ mi/hr}$$

$$T_E = 4 \text{ hr}$$

$$R_V = 3 \text{ mi/hr}$$

Then when we can say

$$R_E T_E + R_V T_V = 35 \text{ mi}$$

$$5 \text{ mi/hr}(4 \text{ hr}) + (3 \text{ mi/hr})T_V = 35 \text{ mi}$$

$$20 \text{ mi} + 3 \text{ mi/hr } T_V = 35 \text{ mi}$$

Subtract 20 mi from both sides and divide by 3 mi/hr.

$$20 \text{ mi} - 20 \text{ mi} + 3 \text{ mi/hr } T_V = 35 \text{ mi} - 20 \text{ mi}$$

$$3 \text{ mi/hr } T_V = 15 \text{ mi}$$

$$T_V = 5 \text{ hr}$$

Evan walks for 5 hours, so he needs to leave at 8:00 a.m. to arrive at 1:00 p.m.

■ 9. Clay and Beth live 32 miles away from each other. Clay started walking towards Beth at a rate of 4 mph. Beth starts walking at 2:00 p.m., and she also walks at a rate of 4 mph. What time does Clay need to leave in order for them to meet at 5:00 p.m.?

Solution:

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts with two distance formulas, one for Clay and one for Beth.

$$D_C = R_C T_C$$

$$D_B = R_B T_B$$

We know that in order to meet, they covered the 32 miles between them.

$$D_C + D_B = 32 \text{ mi}$$

Substitute $D_C = R_C T_C$ and $D_B = R_B T_B$.

$$D_C + D_B = 32 \text{ mi}$$

$$R_C T_C + R_B T_B = 32 \text{ mi}$$

We know that Clay's rate is 4 mph. We know that Beth's rate is also 4 mph, and that she walks from 2:00 p.m. to 5:00 p.m., or 3 hours.

$$R_C = 4 \text{ mi/hr}$$

$$R_B = 4 \text{ mi/hr}$$

$$T_B = 3 \text{ hr}$$

Then we can say

$$R_C T_C + R_B T_B = 32 \text{ mi}$$

$$4 \text{ mi/hr } T_C + (4 \text{ mi/hr})(3 \text{ hr}) = 32 \text{ mi}$$

$$4 \text{ mi/hr } T_C + 12 \text{ mi} = 32 \text{ mi}$$

Subtract 12 mi from both sides and divide by 4 mi/hr.

$$4 \text{ mi/hr } T_C + 12 \text{ mi} - 12 \text{ mi} = 32 \text{ mi} - 12 \text{ mi}$$

$$4 \text{ mi/hr } T_C = 20 \text{ mi}$$

$$T_C = 5 \text{ hr}$$

Cooper walks for 5 hours, so he needs to leave at 12:00 p.m. in order to arrive at 5:00 p.m.

■ 10. Brian starts walking towards Diane at 8:30 a.m. at 5 mph. Diane starts walking towards Brian at 10:30 a.m. at a rate of 3 mph. If they meet at 1:00 p.m., how far apart do they live?

Solution:

Use the formula for distance.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$D = RT$$

Use subscripts with two distance formulas, one for Brian and one for Diane.

$$D_B = R_B T_B$$

$$D_D = R_D T_D$$

We're looking for the total distance between them.

$$D_B + D_D = D$$

Substitute $D_B = R_B T_B$ and $D_D = R_D T_D$.

$$D_B + D_D = D$$

$$R_B T_B + R_D T_D = D$$

We know that Brian's rate is 5 mph and that he walks from 8:30 a.m. to 1:00 p.m., or 4.5 hours. We know that Diane's rate is 3 mph, and that she walks from 10:30 a.m. to 1:00 p.m., or 2.5 hours.

$$R_B = 5 \text{ mi/hr}$$

$$T_B = 4.5 \text{ hr}$$

$$R_D = 3 \text{ mi/hr}$$

$$T_D = 2.5 \text{ hr}$$

Then we can say

$$R_B T_B + R_D T_D = D$$

$$5 \text{ mi/hr}(4.5 \text{ hr}) + (3 \text{ mi/hr})(2.5 \text{ hr}) = D$$

$$22.5 \text{ mi} + 7.5 \text{ mi} = D$$

$$D = 30 \text{ mi}$$

They live 30 miles away from each other.