Algebra 1 Workbook Solutions

Factoring

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GREATEST COMMON FACTOR OF TRINOMIALS

1. Factor out the greatest common factor.

$$3x^2y^3 + 12x^3y^2 - 9x^4y^4$$

Solution:

The greatest common factor is $3x^2y^2$, so the expression is factored as

$$3x^2y^2(y + 4x - 3x^2y^2)$$

2. Fill in the blank with the correct term.

$$4a^3b - 10ab^2 + \_\_\_\_ = 2ab(2a^2 - 5b + 3a^2b^2)$$

Solution:

The blank should be filled in with $6a^3b^3$. We can see this by distributing the $2ab$ across the parentheses.

$$2ab(2a^2 - 5b + 3a^2b^2)$$

$$4a^3b - 10ab^2 + 6a^3b^3$$

3. What went wrong in the following factoring?

$$10x^3y^4 - 5x^4y^2 - 20x^6y^3$$

$$x^3y^2(10y^2 - 5x - 20x^3y)$$

Solution:

There is a factor of 5 in each term that was not factored out. The factoring should have been

$$5x^3y^2(2y^2 - x - 4x^3y)$$

4. Factor out the greatest common factor.

$$2x + 8xy^2 - 16bx^2$$

Solution:

The greatest common factor is $2x$, so the expression is factored as

$$2x(1 + 4y^2 - 8bx)$$

5. Fill in the blank with the correct term.

$$6axy + \_\_\_\_ - 2abx^2y = ax(6y + 3ab - 2bxy)$$

Solution:

The blank should be filled in with $3a^2xb$. We can see this by distributing the ax across the parentheses.

$$ax(6y + 3ab - 2bxy)$$

$$6axy + 3a^2bx - 2abx^2y$$

6. Give an example of a trinomial in the variable x.

Solution:

There are many correct answers. Here are a few examples:

$$3x^2 + 4xa - a^2$$

$$x^4 - 5x^2 + 2x$$

$$x^2 - 2x - 3$$

7. Factor out the greatest common factor.

$$16ab^2c^3 + 8a^3b^3c - 12a^2b^3c^2$$

Solution:

The greatest common factor is $4ab^2c$, so the expression is factored as

$$4ab^2c(4c^2 + 2a^2b - 3abc)$$

GREATEST COMMON FACTOR OF POLYNOMIALS

■ 1. Factor the expression.

$$\frac{4x^2+6x-4}{2}$$

Solution:

First, simplify.

$$\frac{4x^2+6x-4}{2}$$

$$\frac{2(2x^2+3x-2)}{2}$$

$$2x^2+3x-2$$

The simplified expression can be factored as

$$(2x-1)(x+2)$$

■ 2. Fill in the blank with the correct term.

$$\frac{3x^3-12x}{3x}=x^2-\underline{\hspace{2em}}$$

Solution:

The blank should be filled in with 4.

$$\frac{3x^3 - 12x}{3x}$$

$$\frac{3x(x^2 - 4)}{3x}$$

$$(x^2 - 4)$$

3. What is the greatest common factor of the polynomial?

$$9s^3t^2 + 15s^2t^5 - 24s^5t + 6s^4t^2$$

Solution:

The greatest common factor is $3s^2t$.

4. What is the difference between a trinomial and a polynomial?

Solution:

A trinomial is a polynomial with exactly three terms, whereas a polynomial can have any number of terms.

5. Factor the expression.

$$\frac{4x^4 - 8x^3 - 32x^2}{4x^2}$$

Solution:

First, simplify.

$$\frac{4x^4 - 8x^3 - 32x^2}{4x^2}$$

$$\frac{4x^2(x^2 - 2x - 8)}{4x^2}$$

$$x^2 - 2x - 8$$

The simplified expression can be factored as

$$(x - 4)(x + 2)$$

6. What went wrong in the following factoring.

$$3x^3 - 3x^2 - 6x$$

$$x(3x^2 - 3x - 6)$$

$$x(3x + 3)(x - 2)$$

Solution:

The 3 was not factored out in the greatest common factor in the first step of the factoring. If we factor it out as we should, we'd end up with

$$3x(x+1)(x-2)$$

FACTORING QUADRATIC POLYNOMIALS

1. Factor the quadratic expression.

$$2x^2 + x - 3$$

Solution:

The quadratic expression is factored as

$$2x^2 + x - 3$$

$$(2x + 3)(x - 1)$$

2. What went wrong in the following factoring?

$$x^2 - 4x + 3$$

$$x^2 - 4x - 3 - 1$$

Solution:

The term $-2x$ should be split up instead of the constant term. That is, to factor $x^2 - 4x + 3$, we would first write it as $x^2 - x - 3x + 3$.

3. Factor the quadratic expression.

$$3x^2 + 5x - 2$$

Solution:

The quadratic expression is factored as

$$3x^2 + 5x - 2$$

$$(3x - 1)(x + 2)$$

4. Factor the quadratic expression.

$$x^2 - 9x + 18$$

Solution:

The quadratic expression is factored as

$$x^2 - 9x + 18$$

$$(x - 3)(x - 6)$$

5. Fill in the blank with the correct term.

$$2x^2 - \underline{\quad\quad} - 4 = (2x + 1)(x - 4)$$

Solution:

The blank should be the term $7x$.

6. Factor the quadratic expression.

$$x^2 - x - 2$$

Solution:

The quadratic expression is factored as

$$x^2 - x + 2$$

$$(x - 2)(x + 1)$$

FACTORING THE DIFFERENCE OF TWO SQUARES

1. Factor the expression.

$$4y^2 - 36$$

Solution:

The expression can be rewritten as

$$4y^2 - 36$$

$$(2y)^2 - (6)^2$$

and factored as

$$(2y - 6)(2y + 6)$$

2. What went wrong in the following set of steps?

$$9a^4 - 25b^2$$

$$(9a^2 - 25b)(9a^2 + 25b)$$

Solution:

The coefficients were not taken into consideration when factoring the expression. It should be first written as

$$9a^4 - 25b^2$$

$$(3a^2)^2 - (5b)^2$$

and then factored as the difference of squares.

$$(3a^2 - 5b)(3a^2 + 5b)$$

■ 3. Factor the expression.

$$49x^6y^2 - 36z^4$$

Solution:

The expression can be rewritten as

$$49x^6y^2 - 36z^4$$

$$(7x^3y)^2 - (6z^2)^2$$

and factored as

$$(7x^3y - 6z^2)(7x^3y + 6z^2)$$

■ 4. Fill in the blank with the correct term.

$$\_\_\_\_ - 25y^2 = (2xz^2 - 5y)(2xz^2 + 5y)$$

Solution:

The blank should be filled in with $4x^2z^4$.

COMPLETING THE SQUARE

1. Solve for x by completing the square.

$$x^2 - 6x + 5 = 0$$

Solution:

Completing the square gives

$$x^2 - 6x = -5$$

$$x^2 - 6x + 9 = -5 + 9$$

$$(x-3)^2 = 4$$

$$x - 3 = \pm 2$$

$$x = 3 \pm 2$$

$$x = 1, 5$$

2. Fill in the blank with the correct term.

$$x^2 - \underline{\qquad} + \frac{9}{4} = -2 + \frac{9}{4}$$

Solution:

The blank should be the term $3x$.

3. Complete the square in the following expression, but do not solve.

$$3y^2 - 12y + 3 = 0$$

Solution:

To complete the square, we first write the expression as

$$3y^2 - 12y = -3$$

$$y^2 - 4y = -1$$

Now complete the square as

$$y^2 - 4y + 4 = -1 + 4$$

$$(y-2)^2 = 3$$

4. Solve for a by completing the square.

$$2a^2 + 8a = -4$$

Solution:

Completing the square gives

$$a^2 + 4a = -2$$

$$a^2 + 4a + 4 = -2 + 4$$

$$(a + 2)^2 = 2$$

$$a + 2 = \pm\sqrt{2}$$

$$a = -2 \pm \sqrt{2}$$

5. What is your first and second step in solving the problem by completing the square?

$$4x^2 - 16x + 28 = 0$$

Solution:

The first step is to move the 28 over to the other side. The second step is to divide everything by 4. These steps could be done in the opposite order, but they are the first two steps you must take before completing the square.

6. Explain when and why completing the square is used for factoring.

Solution:

Completing the square is used when it's not possible to solve for the roots by factoring.

7. Solve for y by completing the square.

$$3y^2 + 9y = 3$$

Solution:

Completing the square gives

$$3y^2 + 9y = 3$$

$$y^2 + 3y = 1$$

$$y^2 + 3y + \frac{9}{4} = 1 + \frac{9}{4}$$

$$\left(y + \frac{3}{2}\right)^2 = \frac{13}{4}$$

$$y = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

$$y = -\frac{3 \pm \sqrt{13}}{2}$$

8. Fill in the blank with the correct term.

$$\_\_\_\_ - 4x = 6$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{22}{9}$$

Solution:

The blank should be filled in with $3x^2$. We can work backwards from the second equation.

$$\left(x - \frac{2}{3}\right)^2 = \frac{22}{9}$$

$$\left(x - \frac{2}{3}\right)\left(x - \frac{2}{3}\right) = \frac{22}{9}$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{22}{9}$$

$$3x^2 - 4x + \frac{4}{3} = \frac{22}{3}$$

$$3x^2 - 4x = \frac{22}{3} - \frac{4}{3}$$

$$3x^2 - 4x = \frac{18}{3}$$

$$3x^2 - 4x = 6$$

COMPLETING THE SQUARE WITH COMPLEX ROOTS

1. Solve for x by completing the square.

$$x^2+6x+11=0$$

Solution:

Solving by completing the square.

$$x^2+6x=-11$$

$$x^2+6x+9=-11+9$$

$$(x+3)^2=-2$$

$$x=-3\pm i\sqrt{2}$$

2. What went wrong in the following set of steps toward completing the square?

$$3x^2-12x-7=0$$

$$3x^2-12x+36=7+36$$

Solution:

The coefficient in front of the x^2 term was not divided through before completing the square. The equation should have been rewritten as

$$x^2 - 4x + 4 = \frac{7}{3} + 4$$

3. Fill in the blank with the correct term.

$$-2y^2 - 12y = 9$$

$$y^2 + 6y + \_\_\_\_ = -\frac{9}{2} + \_\_\_\_$$

Solution:

Both blanks should be filled in with a 9.

4. Solve for x by completing the square.

$$2x^2 + 8x + 35 = 0$$

Solution:

Complete the square.

$$2x^2 + 8x = -35$$

$$x^2 + 4x = -\frac{35}{2}$$

$$x^2 + 4x + 4 = -\frac{35}{2} + 4$$

$$(x+2)^2 = -\frac{27}{2}$$

$$x = -2 \pm i\sqrt{\frac{27}{2}}$$

5. Complete the square but do not solve.

$$x^2 + 12x + 20 = 0$$

Solution:

Rewrite the expression.

$$x^2 + 12x + 20 = 0$$

$$x^2 + 12x = -20$$

Complete the square.

$$x^2 + 12x + 36 = -20 + 36$$

$$(x+6)^2 = 16$$

6. What is the difference in the problem when completing the square with complex roots compared to real roots?

Solution:

There's no difference in the way we solve these problems. The only difference is that with complex roots, we're taking the square root of a negative number. Whereas with real roots, we're taking the square root of a positive number.

7. Solve for z by completing the square.

$$z^2 - 8z + 25 = 0$$

Solution:

Solve by completing the square.

$$z^2 - 8z + 25 = 0$$

$$z^2 - 8z = -25$$

$$z^2 - 8z + 16 = -25 + 16$$

$$(z - 4)^2 = -9$$

$$z = 4 \pm 3i$$

8. Fill in the blank with the correct term.

$$z^2 + \_\_\_\_ z + \frac{25}{4} = -7 + \frac{25}{4}$$

Solution:

Name the coefficient on z with the variable b. Then follow the process of completing the square, dividing b by 2 to get $b/2$, and then squaring that to get $(b/2)^2$. That should be equal to the value we've added to both sides, $25/4$.

$$\left(\frac{b}{2}\right)^2 = \frac{25}{4}$$

$$\sqrt{\left(\frac{b}{2}\right)^2} = \sqrt{\frac{25}{4}}$$

$$\frac{b}{2} = \frac{5}{2}$$

$$b = 5$$

9. Give an example of a quadratic equation that would have complex roots.

Solution:

There are many correct solutions. Two examples are

$$(x+1)^2 = -14$$

$$x^2 - 6x + 9 = -24$$

■ 10. Solve for z.

$$z^2 - 4z = -5$$

Solution:

Solve by completing the square.

$$z^2 - 4z = -5$$

$$z^2 - 4z + 4 = -5 + 4$$

$$(z-2)^2 = -1$$

$$z - 2 = \pm\sqrt{-1}$$

$$z = 2 \pm i$$

QUADRATIC FORMULA

1. Solve for x using the quadratic formula.

$$4x^2 - 8x - 15 = 0$$

Solution:

The quadratic formula for the expression is

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-15)}}{2(4)}$$

$$x = \frac{8 \pm \sqrt{64 + 240}}{8}$$

$$x = \frac{8 \pm \sqrt{304}}{8}$$

$$x = \frac{8 \pm 4\sqrt{19}}{8}$$

$$x = \frac{2 \pm \sqrt{19}}{2}$$

2. Write the quadratic formula for the following quadratic equation.

$$x^2 - 5x - 24 = 0$$

Solution:

The quadratic formula for the expression is

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-24)}}{2(1)}$$

We could continue to simplify to solve for the roots.

$$x = \frac{5 \pm \sqrt{25 + 96}}{2}$$

$$x = \frac{5 \pm \sqrt{121}}{2}$$

$$x = \frac{5 \pm 11}{2}$$

$$x = -3, 8$$

3. What went wrong in the way the quadratic formula was applied?

$$3x^2 - 5x + 10 = 0$$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(3)(10)}}{2(3)}$$

Solution:

The $-b$ at the beginning of the quadratic formula is written as -5, but $b = -5$. Which means it should be written as $-(-5)$.

4. Solve for z using the quadratic formula.

$$z^2 = z + 3$$

Solution:

Rewrite the expression as

$$z^2 = z + 3$$

$$z^2 - z - 3 = 0$$

Then the quadratic formula gives

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

$$z = \frac{1 \pm \sqrt{13}}{2}$$

5. Fill in the blank with the correct term if the quadratic formula below was built from the quadratic equation.

$$\_\_\_\_ x^2 + 3x - 5 = 0$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(-5)}}{2(-2)}$$

Solution:

The blank should be the term -2.

6. Simplify the expression.

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

Solution:

The expression is simplified as

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

$$\frac{8 \pm \sqrt{64-56}}{2}$$

$$\frac{8 \pm \sqrt{8}}{2}$$

$$\frac{8 \pm 2\sqrt{2}}{2}$$

$$4 \pm \sqrt{2}$$

7. What are two ways to solve a quadratic equation when you cannot easily factor?

Solution:

You can either use the method of completing the square or the quadratic formula.

8. What went wrong if the quadratic formula below was built from the quadratic equation?

$$x^2 + 2x = 7$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(7)}}{2(1)}$$

Solution:

The expression was not written in the correct form before using the quadratic formula. It should be written as $x^2 + 2x - 7 = 0$, for which the quadratic formula would then be

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-7)}}{2(1)}$$

9. Solve for t using the quadratic formula.

$$4t^2 - 1 = -8t$$

Solution:

Rewrite the expression as

$$4t^2 - 1 = -8t$$

$$4t^2 + 8t - 1 = 0$$

Then the quadratic formula is

$$t = \frac{-(8) \pm \sqrt{(8)^2 - 4(4)(-1)}}{2(4)}$$

$$t = \frac{-8 \pm \sqrt{64 + 16}}{8}$$

$$t = \frac{-8 \pm 4\sqrt{5}}{8}$$

$$t = \frac{-2 \pm \sqrt{5}}{2}$$

FACTORING TO FIND A COMMON DENOMINATOR

1. Simplify the expression by combining the two fractions.

$$\frac{x+1}{2x^2+5x-3}+\frac{2}{x+3}$$

Solution:

The expression can be rewritten as

$$\frac{x+1}{(2x-1)(x+3)}+\frac{2}{x+3}$$

Then we can multiply the second fraction's numerator and denominator by $2x-1$ to get a common denominator, and combine the fractions as

$$\frac{x+1+2(2x-1)}{(2x-1)(x+3)}$$

$$\frac{5x-1}{(2x-1)(x+3)}$$

2. What is the one thing you need in order to add or subtract rational expressions?

Solution:

You must have a common denominator in order to add or subtract rational expressions.

3. What is the common denominator of the rational expressions?

$$\frac{x^2-1}{x^2-4} \text{ and } \frac{x+1}{3x^2-3x-6}$$

Solution:

The first denominator can be factored as

$$x^2-4$$

$$(x-2)(x+2)$$

The second denominator can be factored as

$$3x^2-3x-6$$

$$3(x^2-x-2)$$

$$3(x-2)(x+1)$$

Therefore the common denominator is

$$3(x-2)(x+1)(x+2)$$

4. Simplify the expression by combining the two fractions.

$$\frac{3}{x-2} - \frac{x-4}{x^2-5x+6}$$

Solution:

The expression can be rewritten as

$$\frac{3}{x-2} - \frac{x-4}{(x-2)(x-3)}$$

Then we can multiply the first fraction's numerator and denominator by $x-3$ to get a common denominator, and combine the fractions as

$$\frac{3(x-3)-(x-4)}{(x-2)(x-3)}$$

$$\frac{3x-9-x+4}{(x-2)(x-3)}$$

$$\frac{2x-5}{(x-2)(x-3)}$$

5. What went wrong in the following simplification?

$$\frac{1}{x+1} + \frac{3x-1}{(2x-2)(x+1)}$$

$$\frac{1}{(x+1)(2x-2)}+\frac{3x-1}{(2x-2)(x+1)}$$

Solution:

When finding the common denominator, the first term of the expression was only multiplied by $(2x-2)$ in the denominator instead of both the numerator and denominator. It should instead read

$$\frac{1(2x-2)}{(x+1)(2x-2)}+\frac{3x-1}{(2x-2)(x+1)}$$

■ 6. Fill in the blank with the correct term.

$$\frac{2}{\underline{\qquad\qquad}}-\frac{x-2}{x^2-9}=\frac{2(x-3)-4(x-2)}{4(x-3)(x+3)}$$

Solution:

The blank should be filled in with $4(x+3)$.

■ 7. Simplify the expression by combining the two fractions.

$$\frac{4}{(x+1)(x-3)}-\frac{1}{(x+4)(x+1)}$$

Solution:

The expression can be rewritten as

$$\frac{4(x+4)}{(x+1)(x-3)(x+4)} - \frac{1(x-3)}{(x+4)(x+1)(x-3)}$$

$$\frac{4x+16-x+3}{(x+1)(x-3)(x+4)}$$

$$\frac{3x+19}{(x+1)(x-3)(x+4)}$$

8. What went wrong in the following simplification?

$$\frac{3}{x^2-25} - \frac{1}{x+5}$$

$$\frac{3-x-5}{(x-5)(x+5)}$$

Solution:

The negative sign in the second term was not distributed. It should be

$$\frac{3-x+5}{(x-5)(x+5)}$$