Integration by substitution

Olivia Choi

November 7, 2024

Question 1

Use the substitutions given to perform the integration:

a)

$$\int 3x^2(x^3+1)^4 dx \text{ using } u(x) = x^3 + 1$$

b)

$$\int x^2 e^{x^3+1} dx \text{ using } u(x) = x^3 + 1$$

c)

$$\int \sin^4 x \cos x dx \text{ using } u(x) = \sin x$$

Question 2

Integrate by substitution:

a)

$$\int \frac{2x}{\sqrt{x^2+3}} dx$$

b)

$$\int \frac{x}{(1-x^2)^5} dx$$

c)

$$\int \sqrt{x^3 + x}(3x^2 + 1)dx$$

d)

$$\int -2e^{1-2x}dx$$

e)

$$\int 2xe^{x^2}dx$$

f)

$$\int \frac{2x}{x^2 + 1}dx$$

g)

$$\int \frac{x}{2 - x^2}dx$$

Question 3

Using the identities:

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x) \text{ and } \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

to help you, integrate with respect to x:

a)

$$\cos^2 x$$

b)

$$(1 + \cos x)^2$$

c)

$$1 + \cos^2(2x)$$

d)

$$3 - \sin^2(3x)$$

Question 4

Find f(x) given that:

a)

$$f'(x) = 2x - 1 \text{ and } f(0) = 3$$

b)

$$f'(x) = 3x^2 + 2x \text{ and } f(2) = 5$$

c)

$$f'(x) = e^x + \frac{1}{\sqrt{x}} \text{ and } f(1) = 1$$

d)

$$f'(x) = x - \frac{2}{\sqrt{x}} \text{ and } f(1) = 2$$