MA 109 Recap Week 1

Recap Slides

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Week 5

Logarithmic Function

$$\ln x = \int_1^x \frac{1}{t} dt \text{ for } x > 0$$

Exponential Function

$$\exp(x) = y \iff \ln y = x$$

$$\frac{d}{dx}(e^x) = e^x$$

Week 5

Area under a Curve

Let $f : [a, b] \to \mathbb{R}$ be a bounded function. Suppose $f \geq 0$ on $[a, b]$, let

$$R_f = \{(x, y) \in \mathbb{R}^2 : a \leq x \leq b \text{ and } 0 < y < f(x)\}$$

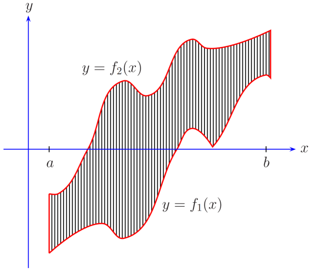
We say that R_f has an area if f is Reimann integrable, and we define

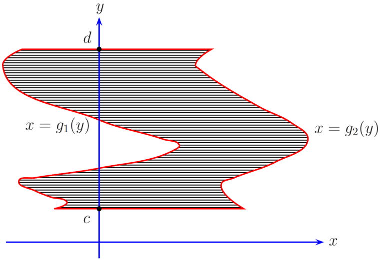
$$\text{Area}(R_f) = \int_a^b f(x)dx$$

Area between Curves

Let $f_1, f_2 : [a, b] \to \mathbb{R}$ be integrable functions such that $f_1 < f_2$. Let $R = \{(x, y) \in \mathbb{R}^2 : a \leq x \leq b \text{ and } f_1(x) \leq y \leq f_2(x)\}$ be the region between the curves $y = f_1(x)$ and $y = f_2(x)$. Define

$$\text{Area}(R) = \text{Area}(R_{f_2 - f_1}) = \int_a^b (f_2(x) - f_1(x))dx$$





Curve given by Polar Coordinates

Let R denote the region bounded by the curve $r = p(\theta)$ and the rays $\theta = \alpha$ and $\theta = \beta$ where $-\pi \leq \alpha < \beta \leq \pi$.

$$R = \{(r\,cos\theta, r\,sin\theta) : \alpha \leq \theta \leq \beta \text{ and } 0 \leq r \leq p(\theta)\}$$

$$\text{Area}(R) = \frac{1}{2}\int_{\alpha}^{\beta} p(\theta)^2 d\theta = \int_{\alpha}^{\beta} r^2 d\theta$$

Week 5

Volume of a Solid

Let D be a bounded subset of $\mathbb{R}^3$. A cross-section of D obtained by cutting D by a plane in $\mathbb{R}^3$ is called a slice of D.

Let $a < b$, and suppose D lies between the planes $x = a$ and $x = b$, which are perpendicular to the x-axis. For $s \in [a, b]$, consider the slice of D by the plane $x = s$, namely $(x, y, z) \in D : x = s$, and suppose it has an *"area"* $A(s)$. Then,

$$\text{Vol}(D) = \int_a^b A(x)dx$$

Week 5

Parametrized Curve

A parametrized curve or a path C in $\mathbb{R}^2$ is given by $(x(t), y(t))$, where $x, y : [\alpha, \beta] \to \mathbb{R}$ are continuous functions.

Arc length of a smooth curve

Arc Length of C is given by,

$$\text{length}(C) = \int_{\alpha}^{\beta} \sqrt{x'(t)^2 + y'(t)^2}dt$$

Arc Length in Polar Coordinates

Let C be given by a polar equation $r = p(\theta)$, $\theta \in [\alpha, \beta]$. As a parametrized curve, C is given by $(x(\theta), y(\theta))$, where

$$x(\theta) = p(\theta)cos\theta \text{ and } y(\theta) = p(\theta)sin\theta,\ \theta \in [\alpha, \beta]$$

Suppose the function p is continuously differentiable on $[\alpha, \beta]$

$$\text{length}(C) = \int_{\alpha}^{\beta} \sqrt{p(\theta)^2 + p'(\theta)^2}d\theta$$