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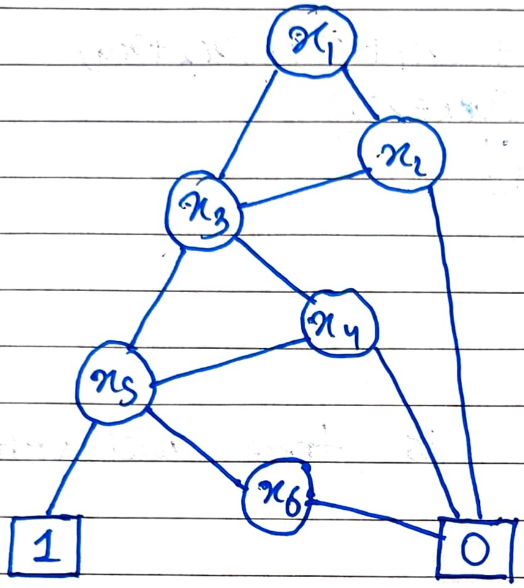
VLSI Testing Midsem



Q1) i)




3) $f = x_1 + x_2 + x_3 + x_4$ $\qquad c = x_1 + x_2$

$f \Downarrow c = x_1 (f_{x_1} \Downarrow c_{x_1}) + \bar{x}_1 (f_{\bar{x}_1} \Downarrow c_{\bar{x}_1})$ $\qquad \{c_{x_1} = 1\}$

$= x_1 \cdot 1 + \bar{x}_1 (f_{\bar{x}_1} \Downarrow c_{\bar{x}_1})$ $\qquad \{c_{\bar{x}_1 \bar{x}_2} = 0\}$

$= x_1 + \bar{x}_1 (f_{\bar{x}_1 x_2} \Downarrow c_{\bar{x}_1 x_2})$ $\qquad \{c_{\bar{x}_1 x_2} = 1\}$

$= x_1 + \bar{x}_1 \cdot 1$

$= x_1 + \bar{x}_1$

$\boxed{f \Downarrow c = 1}$

So $c \cdot (f \Downarrow c) = c \cdot 1 = c$

and $c \cdot f = (x_1 + x_2) \cdot (x_1 + x_2 + x_3 + x_4)$

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$$c\cdot f = (x_1+x_2)\cdot(x_1+x_2) + (x_1+x_2)\cdot(x_3+x_4)$$
$$= (x_1+x_2) + (x_1+x_2)(x_3+x_4)$$
$$= (x_1+x_2)[1+x_3+x_4]$$
$$= (x_1+x_2)\cdot 1$$
$$= x_1+x_2$$
$$= c$$

Hence $c\cdot(f \Downarrow c) = c\cdot f$ is true and verified

4) $j = a + \overline{b\cdot c}$
$= a + \bar{b} + \bar{c}$

$h = \overline{(b\cdot c)\cdot(d\cdot e)}$
$= \overline{(b\cdot c)} + \overline{(d\cdot e)}$
$= \bar{b} + \bar{c} + \bar{d} + \bar{e}$

$i = \overline{\overline{b\cdot c} + h}$
$= b\cdot c\cdot \bar{h}$
$= b\cdot c\cdot \overline{\overline{(b\cdot c)\cdot(d\cdot e)}}$
$= b\cdot c\cdot (b\cdot c)\cdot(d\cdot e)$
$= b\cdot c\cdot d\cdot e$

~~5) SAT for our required problem:~~

$(h\Leftrightarrow i)\cdot(i\Leftrightarrow j)\cdot(j\Leftrightarrow h)$
$= (h\Rightarrow i)\cdot(i\Rightarrow h)\cdot(i\Rightarrow j)\cdot(j\Rightarrow i)\cdot(j\Rightarrow h)\cdot(h\Rightarrow j)$
$= (\bar{h}+i)\cdot(\bar{i}+h)\cdot(\bar{i}+j)\cdot(\bar{j}+i)\cdot(\bar{j}+h)\cdot(\bar{h}+j)$

(basically $\Leftrightarrow$ is our XOR operation)

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5) SAT for our required problem.

$(h \Leftrightarrow \bar{i})\ (\bar{i} \Leftrightarrow \bar{j})\ (\bar{j} \Leftrightarrow h)\ (h \Leftrightarrow (\bar{b}+\bar{c}+\bar{d}+\bar{e}))\ (i \Leftrightarrow bcde)\ (j \Leftrightarrow (a+\bar{b}+\bar{c}))$

$= (\bar{h}+i)\ (\bar{i}+h)\ (\bar{i}+j)\ (\bar{j}+i)\ (\bar{j}+h)\ (\bar{h}+j)\ (\bar{h}+\bar{b}+\bar{c}+\bar{d}+\bar{e})\ (h+bcde)$
$(\bar{i}+bcde)\ (i+\overline{\bar{b}+\bar{c}+\bar{d}+\bar{e}})\ (\bar{j}+a+\bar{b}+\bar{c})\ (j+\overline{a+\bar{b}+\bar{c}})$ distributive

$= (\bar{h}+i)\ (h+\bar{i})\ (\bar{i}+j)\ (i+\bar{j})\ (\bar{j}+h)\ (j+\bar{h})\ (\bar{h}+\bar{b}+\bar{c}+\bar{d}+\bar{e})\ (h+b)\ (h+c)\ (h+d)\ (h+e)$
$(\bar{i}+b)\ (\bar{i}+c)\ (\bar{i}+d)\ (\bar{i}+e)\ (i+\bar{b}+\bar{c}+\bar{d}+\bar{e})\ (\bar{j}+a+\bar{b}+\bar{c})\ (j+\bar{a}bc)$

$(j+\bar{a})\ (j+b)\ (j+c)$

SAT: $(\bar{h}+i)(h+\bar{i})\ (\bar{i}+j)(i+\bar{j})\ (\bar{j}+h)\ (j+\bar{h})\cdot$
$(\bar{h}+\bar{b}+\bar{c}+\bar{d}+\bar{e})\ (h+b)(h+c)(h+d)(h+e)\cdot$
$(\bar{i}+b)(\bar{i}+c)\ (\bar{i}+d)\ (\bar{i}+e)\ (i+\bar{b}+\bar{c}+\bar{d}+\bar{e})\cdot$
$(\bar{j}+a+\bar{b}+\bar{c})\ (j+\bar{a})(j+b)\ (j+c)$

6) a) The states are $s_{qrd}$ where $q,r,d \in \{0,1\}$
Note that $y=q$
Thus, in total, there are 8 states $(2^3)$

b) Relationship (derived from graph on page 4)

$R = \{\{\bigcup_{r,d\in\{0,1\}} (s_{001}, s_{1rd})\} \cup \{\bigcup_{r,d\in\{0,1\}} (s_{101}, s_{1rd})\} \cup \{\bigcup_{\substack{r,d\in\{0,1\}\\(a,b,c)\in X}} (s_{abc}, s_{0rd})\}\}$

where $X = \{(0,0,0), (0,1,0), (0,1,1), (1,0,0), (1,1,0), (1,1,1)\}$

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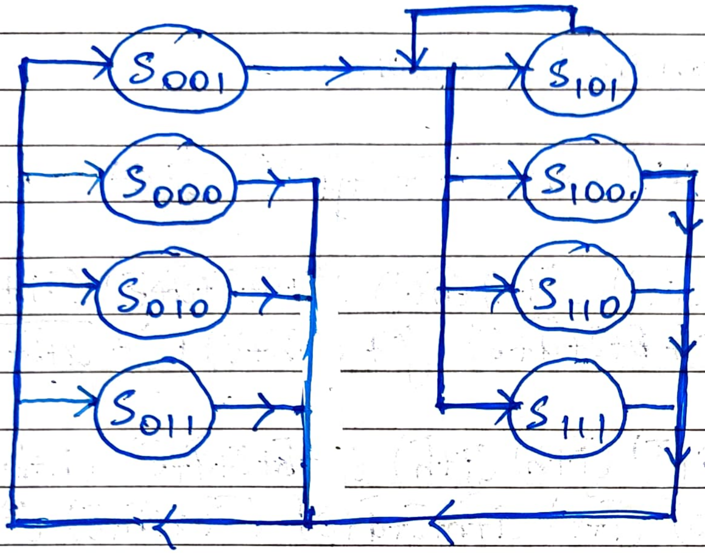



State transition graph.

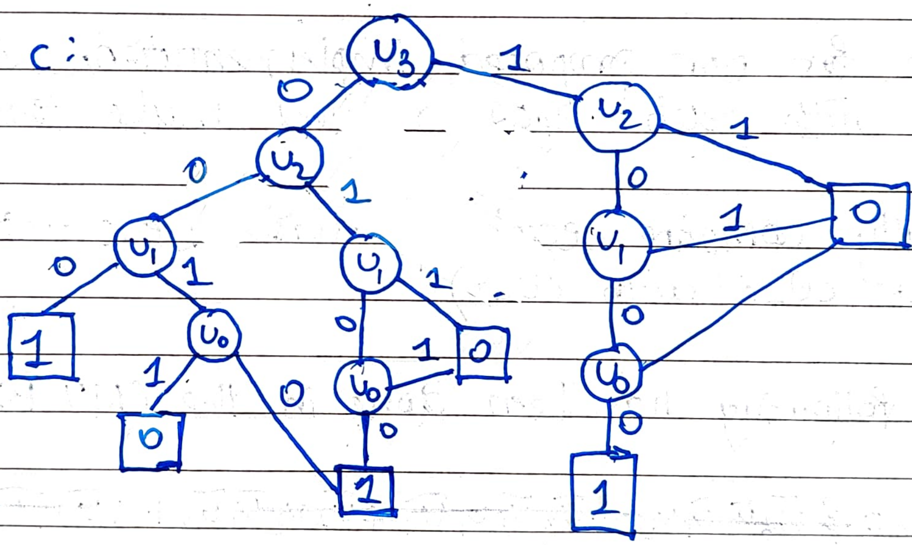
## Question 2

1) Care condition:

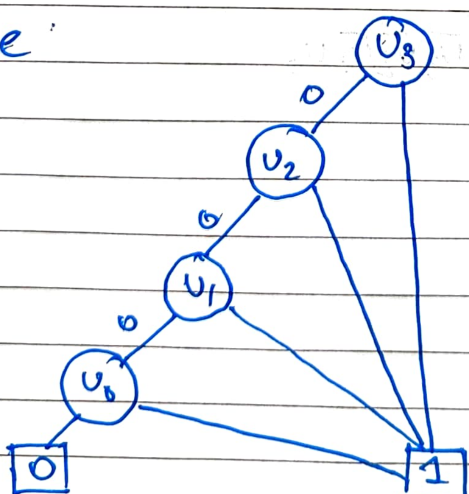
| $U_3U_2$ \ $U_1U_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 0 | 1 |
| 01 | 1 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 1 | 0 | 0 | 0 |

$$c = \overline{U_3}\,\overline{U_2}\,\overline{U_1} + \overline{U_3}\,\overline{U_2}\,\overline{U_0} + \overline{U_3}\,\overline{U_1}\,\overline{U_0} + \overline{U_2}\,\overline{U_1}\,\overline{U_0}$$

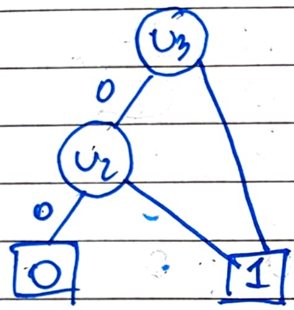
BDD of c:

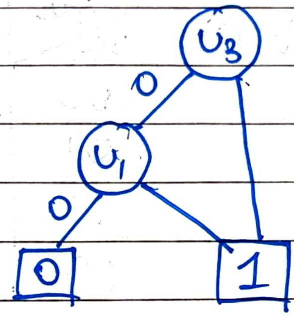


For f BDD of e:



For f | BDD of c1 | BDD of $c_0$





- Do a dot product of each with care (c).
- Then for our proposed implementation, dot the care (c) with each e, c1, c0 and make their BDDs.
- Finally compare each pairwise (e_c with e_t, c1_c with c1_t, c0_c with c0_t)

The following has been done in the CMU BDD package.

Note $\cancel{e_{spec} = f\ \bar{v}_3\bar{v}_2\bar{v}_1 v_0 + \bar{v}_3\bar{v}_2 v_1\bar{v}_0 + \bar{v}_3 v_2\bar{v}_1\bar{v}_0 + v_3\bar{v}_2\bar{v}_1\bar{v}_0}$

$e\_s = v_3 + \bar{v}_3 v_2 + \bar{v}_3\bar{v}_2 v_1 + \bar{v}_3\bar{v}_2\bar{v}_1 v_0$

$c1\_s = v_3 + \bar{v}_3 v_2$

$c0\_s = v_3 + \bar{v}_3 v_1$

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8+5+5+3+4+4+5+5+4+4+4+4+4.

## Question 2 Part 2

For e: $(e\_s \cdot c \Leftrightarrow e\_t \cdot c)$. to be done $\forall$ inputs

$e\_s \cdot c = f$, $e\_t \cdot c = g$ & check $(f \oplus g)$ SAT

$$(f \oplus g) \cdot (f \Leftrightarrow e\_s \cdot c) \cdot (g \Leftrightarrow e\_t \cdot c) \cdot (e\_s \Leftrightarrow (u_3 + \bar{u}_3 u_2 + \bar{u}_3 \bar{u}_2 u_1 + \bar{u}_3 \bar{u}_2 \bar{u}_1 u_0)) \cdot$$
$$(e\_t \Leftrightarrow u_3 + u_2 + u_1 + u_0) \cdot (c \Leftrightarrow \bar{u}_3 \bar{u}_2 \bar{u}_1 + \bar{u}_3 \bar{u}_2 \bar{u}_0 + \bar{u}_3 \bar{u}_1 \bar{u}_0 + \bar{u}_2 \bar{u}_1 \bar{u}_0)$$

$$= (\bar{f} + \bar{g})(f + g)(\bar{f} + e\_s \cdot c)(f + \overline{e\_s \cdot c})(\bar{g} + e\_t \cdot c)(g + \overline{e\_t \cdot c})$$
$$(e\_s \Leftrightarrow (u_3 + m_2 + m_1 + m_0)) \cdot (\overline{e\_t} + u_3 + u_2 + u_1 + u_0)(e\_t + \overline{u_3 + u_2 + u_1 + u_0})$$
$$(c \Leftrightarrow n_3 + n_2 + n_1 + n_0) \cdot (m_2 \Leftrightarrow \bar{u}_3 u_2)(m_1 \Leftrightarrow \bar{u}_3 \bar{u}_2 u_1)(m_0 \Leftrightarrow \bar{u}_3 \bar{u}_2 \bar{u}_1 u_0)$$
$$(n_3 \Leftrightarrow \bar{u}_3 \bar{u}_2 \bar{u}_1)(n_2 \Leftrightarrow \bar{u}_3 \bar{u}_2 \bar{u}_0)(n_1 \Leftrightarrow \bar{u}_3 \bar{u}_1 \bar{u}_0)(n_0 \Leftrightarrow \bar{u}_2 \bar{u}_1 \bar{u}_0)$$

$$= (\bar{f} + g)(f + \bar{g})(\bar{f} + e\_s)(\bar{f} + c)(f + \overline{e\_s} + \bar{c})(\bar{g} + e\_t)(\bar{g} + c)(g + \overline{e\_t} + \bar{c})$$
$$(\overline{e\_s} + u_3 + m_2 + m_1 + m_0)(e\_s + \bar{u}_3)(e\_s + \bar{m}_2)(e\_s + \bar{m}_1)(e\_s + \bar{m}_0)$$

SAT.

$$(\overline{e\_t} + u_3 + u_2 + u_1 + u_0)(e\_t + \bar{u}_3)(e\_t + \bar{u}_2)(e\_t + \bar{u}_1)(e\_t + \bar{u}_0)$$
$$(\bar{c} + n_3 + n_2 + n_1 + n_0)(c + \bar{n}_3)(c + \bar{n}_2)(c + \bar{n}_1)(c + \bar{n}_0)$$
$$(\bar{m}_2 + \bar{u}_3)(\bar{m}_2 + u_2)(m_2 + u_3 + \bar{u}_2)(\bar{m}_1 + \bar{u}_3)(\bar{m}_1 + \bar{u}_2)(\bar{m}_1 + \bar{u}_1)(m_1 + u_3 + u_2 + \bar{u}_1)$$
$$(\bar{m}_0 + \bar{u}_3)(\bar{m}_0 + \bar{u}_2)(\bar{m}_0 + \bar{u}_1)(\bar{m}_0 + u_0)(m_0 + u_3 + u_2 + u_1 + \bar{u}_0)$$
$$(\bar{n}_3 + \bar{u}_3)(\bar{n}_3 + \bar{u}_2)(\bar{n}_3 + \bar{u}_1)(n_3 + u_3 + u_2 + u_1)$$
$$(\bar{n}_2 + \bar{u}_3)(\bar{n}_2 + \bar{u}_2)(\bar{n}_2 + \bar{u}_0)(n_3 + u_3 + u_1 + u_0)$$
$$(\bar{n}_1 + \bar{u}_3)(\bar{n}_1 + \bar{u}_1)(\bar{n}_1 + \bar{u}_0)(n_1 + u_3 + u_1 + u_0)$$
$$(\bar{n}_0 + \bar{u}_2)(\bar{n}_0 + \bar{u}_1)(\bar{n}_0 + \bar{u}_0)(n_0 + u_2 + u_1 + u_0)$$

Simplifying this as 'care SAT'

$u_0 - 1$, $u_1 - 2$, $u_2 - 3$, $u_3 - 4$, $e\_s - 5$, $e\_t - 6$, $c - 7$, $n_0 - 8$, $n_1 - 9$, $n_2 - 10$, $n_3 - 11$
$f - 12$, $g - 13$, $m_0 - 14$, $m_1 - 15$, $m_2 - 16$

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$8+3+3+3+5+4\times4$ 

For c1 $f = c1\_s \cdot c \quad g = c1\_t \cdot c$

$(f \oplus g)\ (f \Leftrightarrow c1\_s \cdot c)\ (g \Leftrightarrow c1\_t \cdot c)\ (c1\_s \Leftrightarrow u_3 + \underbrace{\bar{u}_3 u_2}_{m_0})\ (c1\_t \Leftrightarrow u_3 + u_2)$
$(c \Leftrightarrow \bar{u}_3\bar{u}_2\bar{u}_1 + \bar{u}_3\bar{u}_2 u_0 + \bar{u}_3\bar{u}_1 u_0 + \bar{u}_2\bar{u}_1\bar{u}_0)$

$(\bar{f}+\bar{g})\ (f+g)\ (\bar{f}+c1\_s)\ (\bar{f}+c)\ (f+\overline{c1\_s}+\bar{c})\ (\bar{g}+c1\_t)\ (\bar{g}+c)\ (g+\overline{c1\_t}+\bar{c})$
$(\overline{c1\_s}+u_3+m_0)\ (c1\_s+\bar{u}_3)\ (c1\_s+\bar{m}_0)\ (\overline{c1\_t}+u_3+u_2)\ (c1\_t+\bar{u}_3)\ (c1\_t+\bar{u}_2)$
(care conditions)
$(\bar{m}_0+\bar{u}_3)\ (\bar{m}_0+u_2)\ (m_0+u_3+\bar{u}_2)$

$u_0-1,\ u_1-2,\ u_2-3,\ u_3-4,\ c1\_s-5,\ c1\_t-6,\ c-7,\ n_0-8,\ n_1-9,\ n_2-10,\ n_3-11$
$f-12,\ g-13,\ m_0-14$

For c0 Same assumptions as c1

$(\bar{f}+\bar{g})\ (f+g)\ (\bar{f}+c0\_s)\ (\bar{f}+c)\ (f+\overline{c0\_s}+\bar{c})\ (\bar{g}+c0\_t)\ (\bar{g}+c)\ (g+\overline{c0\_t}+\bar{c})$
$(\overline{c0\_s}+u_3+\overbrace{\bar{u}_3 u_1}^{m_0})\ (c0\_s+\bar{u}_3)\ (c0\_s+\bar{m}_0)$
$(\overline{c0\_t}+u_3+u_1)\ (c0\_t+\bar{u}_3)\ (c0\_t+\bar{u}_1)$
(care conditions)
$(\bar{m}_0+\bar{u}_3)\ (\bar{m}_0+u_1)\ (m_0+u_3+\bar{u}_1)$

Same notations $(m_0 = \bar{u}_3 u_1)$ & $c1 \to c0$ difference in code

All should output UNSATISFIABLE for SAT ~~solver~~.

Check MinSAT code further.

Question3

Part 1 - Seq. ckt given. We need till k=4

For circuit 1: Take 5 inputs each for $x$ & $r$
namely $x(0), x(1) \cdots x(4)$ for $k=0$ to $4$
$r(0), r(1) \cdots r(4)$

- Find BDD of $s0, s1, s2, s3$ for all states.
- Use this to find BDD of output. ~~for~~ all states

We thus get BDD_1 array.

For circuit 2: Same 5 inputs fed to this circuit

- Find BDD of $t0, t1$ for all states
- Use this to find BDD of output for all states

We thus get BDD-2 array.

Finally, compare each BDD from array
All should output true for equality, only then can we say the seq. circuits are equivalent for first 4 values of k

Note: The code shows that at $k=1$, the output fails.
This can be useful in calculation of SAT (show that SAT fails ~~for~~ $k=1$)

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The SAT looks like $\sum o1(i) \oplus o2(i)$
If anyone is 1, we get satisfiability.

$\because$ we know from previous ~~[illegible]~~ BDD analysis that at $k=1$, the BDDs mismatch.

~~o1(1)~~ $s3(1)=0 \quad s2(1)=0.$

$o1(1) = \overline{r(1)}\,[x(1)\,s3(1) + \overline{x(1)}\,s2(1)]$

$= \overline{r(1)}\,[x(1)\cdot 0 + \overline{x(1)}\cdot 0]$

$= 0.$

Similarly for $o2(1)$, $t1(1)=0 \quad t0(1)=0$

$o2(1) = \overline{r(1)}\,[x(1)\,(t1(1) \equiv t0(1)) + \overline{x(1)}\,(t1(1) \oplus t0(1))]$

$= \overline{r(1)}\,[x(1)\cdot 1 + \overline{x(1)}\cdot 0]$

$= \overline{r(1)}\,x(1)$

SAT equation: $[o1(1) \oplus o2(1)]\,(o1 \Leftrightarrow 0)$
$(o2 \Leftrightarrow \bar{r}x)$

$= (o1+o2)(\overline{o1}+\overline{o2})\,\overset{1}{(\cancel{o1+1})}\,\overset{\overline{o1}}{(\cancel{\overline{o1}+0})}$
$(\overline{o2}+\bar{r})(\overline{o2}+x)(o2+r+\bar{x})$

$o1 - 1 \quad o2 - 2 \quad r - 3 \quad x - 4$