<u>Report on Preliminary Apollo 11 Predictions:</u>

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Introduction

In order to explain the importance of our calculations in the later sections, it is necessary to contextualize them with some background in the underlying physics. The first important concept is that of **gravitational potential**. The equation for the gravitational potential at a point in space due to an object is:

$$\Phi(r) = -\frac{GM}{r}$$

where M is the mass of the object, r is the distance from the object to the point in space, and G represents the gravitational constant. Intuitively, the gravitational potential tells you how much energy some object has by virtue of the gravitational field created by another object (for example, how much energy a ball has from the Earth's gravitational field).

This concept of potential leads into the idea of a **gravitational force** between two bodies, which is described by the equation:

$$\overrightarrow{F_{21}} = -G\frac{M_1 m_2}{|\overrightarrow{r_{21}}|^2}\,(\hat{r}_{21})$$

where M_1 and m_2 are the two masses, $\overrightarrow{r_{21}}$ is the vector pointing from the second mass to the first mass, and the hat corresponds with a unit vector. This can be explained simply. The gravitational force between two objects is attractive, proportional to the mass of each object, and inversely proportional to the square of the distance between the two objects.

Moving from this, we discuss some of the physics behind the calculation of the Saturn V rocket altitude. First, it is important to discuss how a rocket actually propels itself, and for that we need to understand **conservation of momentum**. For a simple example, when a moving billiard ball hits a stationary one, the combined momentum(mass times velocity) of the two balls needs to remain the same, which is why the first ball stops, and the second continues with the same speed as the original billiard. This informs us of how rocket propulsion works: when fuel is ejected out of the exhaust towards the Earth, the overall momentum of the system needs to be conserved, and hence the rocket must gain some momentum (and velocity) away from the Earth.

The Gravitational Potential of the Earth-Moon system

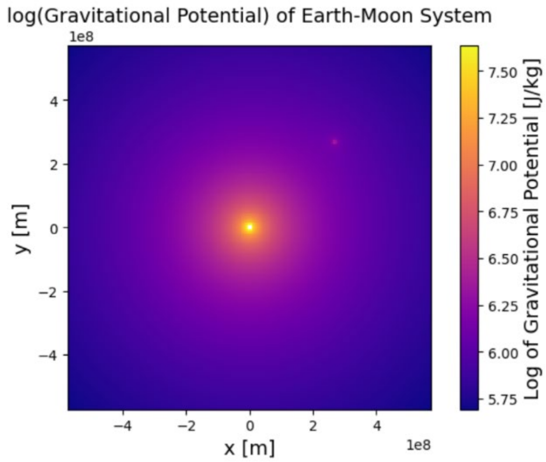

***Figure 1**: Gravitational Potential Colormesh*

To calculate the potential of the Earth-Moon system, we first created a function that, given an object's mass and location, could calculate the potential due to that object at any other point. We then created a 200 x 200 grid of points in space, where each grid axis ranged from $-1.5 * d_{em}$ to $1.5 * d_{em}$, where d_{em} is just the distance from the Earth to the Moon. We set the Earth to be at the center of our grid, and the Moon to be at the point $(\frac{d_{em}}{\sqrt{2}}, \frac{d_{em}}{\sqrt{2}})$. We then calculated the gravitational potential at each grid point, and plotted the resulting data as a colormesh in **Figure 1**. Each coordinate corresponds with the location of the point where we are finding the potential, and the color at that point corresponds with the base 10 logarithm of the gravitational potential (so the actual value of the potential is 10^ value given by the color). The potential spikes at two locations: $10^{7.5}$ J/kg close to the origin, where the Earth is, and about $10^{6.5}$ J/kg where the Moon is located. The potential falls off rapidly as one gets further from the Moon/Earth, and notably the potential of the Moon is very small compared to that of the Earth. From the plot, it is clear that the total gravitational potential is only affected by the Moon's potential when extremely close to the Moon, where its potential will dominate.

The Gravitational Force of the Earth-Moon system

The calculation for the force of the Earth-Moon system was implemented similarly to the previous calculation. First, a function was defined which, upon specifying the mass and location (as a coordinate) of two objects, would return the components (one in the x-direction, one in the y-direction) of gravitational force on one object from the other. Two forces were then calculated: the force of the Earth on the Apollo command module, and the force of the Moon on the Apollo command module, with the location of the Apollo command module set by the grid points defined in the previous section. The two forces were then added together componentwise to get the overall force on the command module based on its spatial location. This yielded the streamplot shown in **Figure 2**, where the direction of the arrows represents the direction of the force, and the

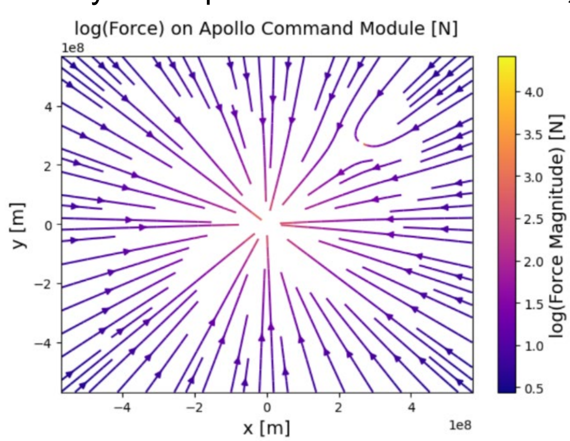

***Figure 2**: Gravitational Force Streamplot*

color of the arrows corresponds with the log of the force magnitude on the command module. The lines describe the way that the command module would “fall” if dropped at some coordinate. The plot shows that the module will fall into Earth unless it is very close to the Moon, meaning that the orbital radius of the Moon is orders of magnitude smaller than the distance between the Earth and Moon.

Altitude of the Saturn V Rocket

To calculate the altitude of the Saturn V rocket, we employed the Tsiolkovsky rocket equation:

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

which was derived from the earlier conservation of momentum arguments. The equation gives the change in the velocity of the rocket as a function of the rocket exhaust speed v_e, the wet mass m_0, acceleration due to gravity at Earth’s surface g, the time t, and $m(t) = m_0 - \dot{m}(t)$, where $\dot{m}$ is the fuel burn rate. After defining a function which would calculate $\Delta v(t)$, we defined the burn time $T = \frac{m_0 - m_f}{\dot{m}}$, where m_f is the dry mass of the rocket. This allowed us to generate a list of linearly spaced times from 0 to T (since after T seconds, all of the fuel is spent). In order to get the altitude, we integrated the function from 0 to T (multiplying the velocity by a small change in time to get a small change in distance, then adding up those small changes in distance to get the total altitude). This integration was done numerically using the scipy.integrate.quad function. It resulted in an altitude of 74135.63 m after a burn time of 157.69 s.

Discussion and Future Work

There are a number of approximations that have been made in doing these calculations. For one, the Moon’s orbit is elliptical, so further analysis would require using the existing developed functions to calculate the force/potential at all points along the orbit. The Earth is also not a perfect sphere, so further analysis would have to modify the potential calculation to account for slight differences in radius. Notably, the Saturn V test differs moderately in altitude compared to predictions (74 km predicted compared to 70 km in the test), despite the burn times being roughly equal (158 s vs 160 s). This difference is due to neglecting air resistance in the rocket equation, since air drag would slow the rocket down and reduce its max altitude. Future work would modify the rocket equation to include a time-dependent air-resistance term; this could then be integrated to achieve a more accurate predicted altitude. The discrepancy in the burn time can be attributed to “start lag” in the fuel burn rate (i.e., the rocket doesn’t begin burning fuel at $\dot{m}$ right after the engine is turned on, which leads to an overall longer burn time). This can be addressed in the future by making $\dot{m}$ slightly time-dependent at the beginning.