Online Appendix

Appendix A. Data Construction

I use Compustat North America annual data from 1963 until 2016. Following Imrohoroglu and Tuzel (2014) I remove all financial and regulated firms (i.e. all firms with SIC codes between 6000 and 6999, and between 4900 and 4999) and I exclude all observations that have missing or non-positive values for sales; total assets; number of employees; gross property, plant, and equipment; depreciation; operating income; and capital expenditures.[1] There are also two observations who have an apparent "Compustat age" (i.e. number of years that they have been tracked by Compustat) that is negative (likely due to re-use of a firm identifier), and three observations that have negative (calculated) stock of R&D (due to having recorded negative R&D expenditures in certain years) that are also dropped, . This leaves a total of 252,864 observations across 23,110 firms for the years 1963-2016, and 201,569 observations over 21,225 firms for the period from 1980 onward.

My modification of the De Loecker (2013) adaptation of Ackerberg, Caves and Frazer (2015) estimation procedure requires the calculation of a firm's value added as well as its materials costs. The Compustat dataset does not include either of these variables, but I follow Imrohoroglu and Tuzel (2014)'s approach in obtaining estimates of them from variables that are included in Compustat. First, I calculate a firm's labour expenses by multiplying the number of employees recorded in Compustat by the US mean wage for each year obtained from the Social Security Administration. I then obtain a firm's total expenses by subtracting its operating income before depreciation and amortisation from its total sales. A firm's materials costs are then its total expenses minus its labour expenses, while a firm's value added is its sales minus its materials costs. Both the value added and materials costs variables are then deflated using the GDP deflator.

The variables containing a firm's stock of R&D are calculated using the perpetual inventory method, following Hall (1993). That is, a firm's stock of R&D $R_{i,t}$ is obtained via $R_{i,t} = (1-\delta_r)R_{i,t-1} + I_{R,i,t-1}$ where δ_r is the depreciation rate of R&D (set at 15% as per Hall) and $I_{R,i,t-1}$ is a firm's R&D expenditure, deflated using the BEA overall price series for gross private fixed non-residential investment. The use of the perpetual inventory method requires specifying $R_{i,0}$ the starting level of a firm's R&D stock for the first period in which they appear in the dataset. I use a range of different values - first, letting $R_{i,0}$ equal 10^{-6} for all firms; second, setting $R_0 = \frac{I_{R,i,0}}{\delta_r}$; and finally setting $R_0 = \frac{I_{R,i,0}}{\delta_r+g}$

[1] The removal of observations with missing or negative capital expenditures (i.e. investment) drops only 3% of observations (8,355 out of 276,324) and so would not necessarily preclude using the Olley-Pakes investment inversion approach.

where g is the average growth rate of R&D and taken to be 8% from Hall (1993). This treatment of R&D as a stock has previously been part of models that take a "knowledge capital" approach to R&D, beginning with Griliches (1979). Moreover, individual firm investment in R&D is highly persistent from year to year (the one-year auto-correlation in a firm's R&D investment is 0.98), indicating that a firm's stock of R&D captures a firm's continued R&D efforts well and my results are robust to these different approaches to determining the initial R&D stock.

I also use two different approaches to calculate the capital stock. The first is based on Bloom et al. (2018) and deflates the net property, plant, and equipment variable in Compustat by industry-specific conversions from historic to market value (in constant 2012 USD). This "BEA capital stock" approach necessitates matching the NAICS code contained in Compustat to the industry-identification codes used by BEA. The resulting market value of capital stock is then lagged one year so that it represents the capital stock at the beginning of a period. Due to the fact that some firms in Compustat do not have a NAICS code associated with them, this method results in some firms being excluded from analyses that rely on this approach. Specifically, 12% (29,779 of 252,864) observations covering 17% (3,966 of 23,110) of firms over the period 1963-2016 cannot be matched to a BEA industry code (for the period from 1980 onward, these proportions are 4.6% of observations and 10% of firms).[2]

As a robustness check, I also use Imrohoroglu and Tuzel (2014)'s method for calculating the capital stock. This "average age" approach calculates the average age of capital by dividing accumulated depreciation as recorded in Compustat by current depreciation, which is then smoothed by taking a three-year rolling average. The value for gross property, plant, and equipment is then deflated using the aggregate BEA price index for gross private fixed non-residential investment from the year relevant to the average age of the firm's capital stock. The resulting market value of capital stock is then lagged one year so that it represents the capital stock at the beginning of a period. The advantage of this approach is that it does not exclude any observations for which it is not possible to match to a BEA industry, but its disadvantage is that it does not account for potential disinvestments as well as does the "BEA capital stock" approach.

Table A.1 presents summary statistics for the sample used in the estimation. The first two rows of the table report the summary statistics of the capital stocks estimated using the two different methods, with the average age method tending to lead to a higher estimated capital stock than the BEA method, albeit with a higher dispersion. As one would expect

[2] The method used in Bloom et al. (2018) also makes use of the perpetual inventory method because their data had capital expenditures rather than the book value of capital stock for most of their observations. As my data has the book value of capital for the vast majority of observations, I am able to use just the conversion of book to market value via the BEA conversion calculation without needing also to convert capital expenditures.

given that Compustat predominantly contains publicly-listed firms, a typical firm-year observation will have a physical capital stock of almost \$1.5 billion (or \$2.4 billion depending on the method used). The typically large size of firms covered by Compustat is also shown by the sales and employee numbers in the subsequent two rows.

Table A.1: Summary statistics

	Mean	Median	Standard deviation
Capital stock (BEA method)	1.46	0.07	7.38
Capital stock (average age method)	2.38	0.10	13.44
Sales	3.31	0.32	14.63
Employees (Thousands)	10.50	1.35	39.59
R&D stock ($R_0 = 10^{-6}$)	0.30	0.00	2.19
R&D stock ($R_0 = \frac{i_{r,0}}{\delta}$)	0.32	0.00	2.24
R&D stock ($R_0 = \frac{i_{r,0}}{\delta+g}$)	0.31	0.00	2.22
Investment in physical capital	0.25	0.01	1.37
Expenditure on R&D	0.07	0.00	0.46
Proportion of observations with positive R&D expenditure		45.4%	

Using Compustat data between 1980 and 2016. Numbers are in 2012 \$Billion unless otherwise stated. The "BEA method" of calculating the capital stock uses the Compustat book-value of a firm's capital stock and deflates it using BEA industry-specific deflators. The "average age" method of calculating the capital stock uses the firm's recorded accumulated and current depreciation to estimate its average age of capital before using the GDP deflator for that year to deflate the book value of capital. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate δ of 15% and setting R_0 according to the specification shown (with $g = 8\%$) and where $i_{r,0}$ is the firm's R&D expenditure in the first year it is present in Compustat.

Also as expected, the mean R&D stock (shown in rows 5-7) is much smaller than than the mean stock of physical capital. Interestingly, the median firm-year in Compustat has no R&D. This means that to ensure that observations with zero stock of R&D are not dropped when using the log of the variable, I add one to each firm's stock of R&D in each year when estimating the production function. This preserves the relationship between changes in a firm's stock of R&D and its idiosyncratic productivity while keeping in the more than 40% of firms that do not have any R&D stock.

The eighth and ninth rows show, respectively, that investment in physical capital is on average higher and more dispersed than is expenditure on R&D. Note, however, that the median investment in physical capital is very close to zero, indicating that the distribution of investment in physical capital is highly skewed.

Table A.2 shows the means of a firm's physical capital stock, investment in physical capital, sales, and employees split according to whether or not the firm has a stock of R&D (firms with more than a stock of R&D of \$50 million or more is included in the group of firms "with R&D", and any firm with an R&D less than that is classified as not having a stock of R&D). This table indicates that firms with stocks of R&D tend to be larger in terms of all four factors than are firms that do not have R&D stocks. In particular, firms with R&D stocks tend to have roughly 6 times higher stocks of physical capital (using the BEA method), investment in physical capital, and sales, as well as 4 times higher numbers of employees.

Table A.2: Characteristics of firms with and without R&D stocks

	Without R&D	With R&D
Capital stock (BEA method)	0.77	4.41
Capital stock (average age method)	1.09	7.68
Investment in physical capital	0.13	0.79
Sales	1.62	10.43
Employees (Thousands)	6.66	26.75

Using Compustat data between 1980 and 2016. Any firm with more than a stock of R&D of $50 million or more is included in the group of firms "with R&D", and any firm with an R&D less than that is classified as not having a stock of R&D. Numbers are in 2012 $Billion unless otherwise stated. The "average age" method of calculating the capital stock uses the firm's recorded accumulated and current depreciation to estimate its average age of capital before using the GDP deflator for that year to deflate the book value of capital. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate δ of 15% and setting R_0 according to the specification shown (with $g = 8\%$).

Appendix B. Production Function Estimation

Appendix B.1. Estimation Procedure

Let there be a continuum of firms, that produce according to the Cobb-Douglas production function:

$$y_{i,t} = z_t \varepsilon_{i,t} k_{i,t}^{\alpha} \ell_{i,t}^{\gamma}$$

where $k_{i,t}$ is the firm's pre-determined stock of capital, $\ell_{i,t}$ is its labour hired for that period, z_t is aggregate productivity, and $\varepsilon_{i,t}$ is the firm's idiosyncratic productivity. Taking logs and letting $\beta_k = \alpha$ and $\beta_\ell = \gamma$ gives

$$y_{i,t} = \beta_l \ell_{i,t} + \beta_k k_{i,t} + \underbrace{z_t + \varepsilon_{i,t}}_{\nu_{i,t}} + v_{i,t}$$

Note that the aggregate and idiosyncratic productivity are written separately in this expression. Previous approaches have grouped idiosyncratic and aggregate productivity together as $\nu_{i,t}$ when estimating firm-level production functions, but in the approach set out below (which is an extension of De Loecker (2013)) I separate out aggregate productivity from firm idiosyncratic productivity which enables me to isolate solely the latter component.

As demonstrated by Olley and Pakes (1996) and Ackerberg, Caves and Frazer (2015) (henceforth ACF) the fact that the productivity terms z_t and $\varepsilon_{i,t}$ are observed by the firm before making (some of) its decisions, but cannot be observed by the researcher, means that it is not possible to estimate the above equation via OLS without the estimates being subject to a simultaneity bias. The Olley-Pakes and ACF procedures try to resolve this bias by approximating the productivity process noting that a firm's investment choice (in the case of Olley-Pakes) or a firm's labour and materials choices (in ACF) are determined after a firm observes its productivity, thereby making it possible to "invert" this relationship.

In particular, Olley-Pakes use the fact that a firm chooses its investment in physical capital after observing its productivity to approximate a firm's productivity via $\nu_{i,t} = f_t^{-1}(i_{i,t}, k_{i,t})$ where $i_{i,t}$ is a firm's investment in physical capital. ACF note that a firm also chooses its labour after observing its productivity and (following the point made in Levinsohn and Petrin (2003) that the investment inversion is highly model-specific) use the fact that a firm chooses its materials costs after observing its productivity to implement an inversion that depends on a firm's labour $\ell_{i,t}$ and its materials costs $m_{i,t}$ as well as its pre-determined physical capital stock, such that the inversion used in ACF can be written as $\nu_{i,t} = f_t^{-1}(m_{i,t}, k_{i,t}, \ell_{i,t})$, and is to be approximated in the first step of the estimation procedure.[3]

In the next step of their estimation procedures, both Olley-Pakes and ACF assume a specific form for the unobserved productivity process that lets them recover the production function coefficients via GMM estimation. Specifically, they assume that it follows an exogenous AR(1) Markov process and then use that functional form to obtain an error term that is then used to evaluate sample moment conditions. However, as shown in De Loecker (2013), the estimates of the coefficients on capital and labour are likely to be biased if there are relevant terms omitted from the productivity process.

Therefore, I adapt De Loecker's approach of including other explanatory terms in the determinants of a firm's productivity. In particular, I use the following specification for productivity:

$$\nu_{i,t} = g(R_{i,t}, \bar{R}_t, \nu_{i,t-1}, \beta_g) + \sum_{t=1}^{T} \gamma_t year_t + \xi_{i,t} \tag{B.1}$$

where $R_{i,t}$ is a firm's own pre-determined stock of R&D, $\bar{R}_t$ is the pre-determined aggregate stock of R&D, $\nu_{i,t-1}$ represents a polynomial in the one-period lag of a firm's productivity, β_g are the coefficients of the g function that are to be estimated, $\sum_{t=1}^{T} \gamma_t year_t$ are annual dummy variables to capture aggregate productivity z_t, and $\xi_{i,t}$ is an i.i.d shock that is (by assumption) uncorrelated with any of the firm's choice variables. The inclusion of annual dummy variables allows me to separate aggregate productivity z_t from a firm's idiosyncratic productivity $\varepsilon_{i,t}$ (which consists of the other components in the expression above).

Hence, my estimation procedure is as follows. First, I use ACF's inversion method to recover $\hat{\phi}_{i,t}$ from a regression of value added $y_{i,t}$ on a cubic polynomial in a firm's capital stock $k_{i,t}$, its labour hiring $\ell_{i,t}$, its material costs $m_{i,t}$, and annual dummy variables $\sum_t \alpha_t year_t$ - in other words, I obtain $\hat{\phi}_{i,t}$ as the fitted values from a regression of $y_{i,t} = \beta_0 + \beta_k k_{i,t} + \beta_\ell \ell_{i,t} + f_t^{-1}(m_{i,t}, k_{i,t}, \ell_{i,t}) + \sum_t \alpha_t year_t + v_{i,t}$ where $f_t^{-1}(m_{i,t}, k_{i,t}, \ell_{i,t})$ is the inversion and is approximated via a cubic polynomial in each of its terms and interactions between them. I test the validity of the scalar unobservable and monotonicity assumptions that make this inversion possible in Appendix B.5.

Next, I guess values for β_k and β_ℓ and calculate $\hat{\nu}_{i,t} = \hat{\phi}_{i,t} - \beta_1 k_{i,t} - \beta_2 \ell_{i,t}$. I then obtain the $\hat{\xi}_{i,t}$ by choosing the specification of $g(R_{i,t}, \bar{R}_t, \nu_{i,t-1}, \beta_g)$ and estimating equation (B.1).

[3] The ACF procedure relies on the assumption that productivity is the only unobserved variable and is strictly monotonic in materials (without these assumptions, the inversion procedure used to approximate the productivity process in the first stage is not valid. Shenoy (2021) devises a test of both of these assumptions. The estimation procedure passes this test, such that the assumptions of a scalar unobservable and strict monotonicity hold, and the inversion and estimation procedure are valid. The test and the results as applied to this model are discussed more in Appendix B.5

The main specification is $g(R_{i,t}, \bar{R}_{i,t}, \nu_{i,t-1}, \beta_g) = \beta_{g1}R_{i,t} + \beta_{g2}R_{i,t}^2 + \beta_{g3}R_{i,t}\bar{R}_t + \beta_{g4}\nu_{i,t-1}^2 + \beta_{g5}\nu_{i,t-1}^3 + \beta_{g6}\nu_{i,t-1}$, but alternative specifications with different combinations of $R_{i,t}$ and $\bar{R}_t$ are also used for robustness checks. Note that it is possible to include a firm's R&D $R_{i,t}$ and aggregate R&D $\bar{R}_t$ in period t because both are determined before a firm observes its own productivity (i.e. a firm chose last period its R&D for this period).

Using the $\hat{\xi}_{i,t}$ I then evaluate the sample moment conditions $\frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\xi}_{i,t}k_{i,t}$ and $\frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\xi}_{i,t}\ell_{i,t-1}$, iterating on guesses of β_k and β_ℓ until both sample moments are sufficiently close to zero. The use of $k_{i,t}$ and $\ell_{i,t-1}$ as instruments relies on the assumption that neither are affected by a firm's productivity in period t: the firm's capital stock for the current period is pre-determined (i.e. chosen in the previous period before current-period productivity is known) and the firm's labour expenses in the previous period does not depend on the firm's productivity in the current period.

I then retrieve $\hat{\beta}_g$ by calculating $\hat{\nu}_{i,t} = y_{i,t} - \hat{\beta}_k k_{i,t} - \hat{\beta}_\ell \ell_{i,t}$ and using $\hat{\nu}_{i,t}$ as the dependent variable in estimating equation (B.1) via OLS. The coefficients on $R_{i,t}$ obtained from this regression show the effect of a firm's own R&D on its productivity and allow us to identify whether or not there are decreasing returns to an individual firm's stock of R&D while the coefficient on $R_{i,t}\bar{R}_t$ show the impact aggregate R&D has on the effectiveness of a firm's own stock of R&D on its productivity. This latter coefficient allows me to identify whether increases in aggregate R&D has a "crowding out" effect on an individual firm's R&D effort (a negative coefficient on this interaction term indicating that higher aggregate R&D reduces the effect of an individual firm's own R&D stock on its productivity) or a positive spillover effect (a positive coefficient on the interaction term indicating that higher aggregate R&D boosts the effect of a firm's own R&D stock on its productivity).

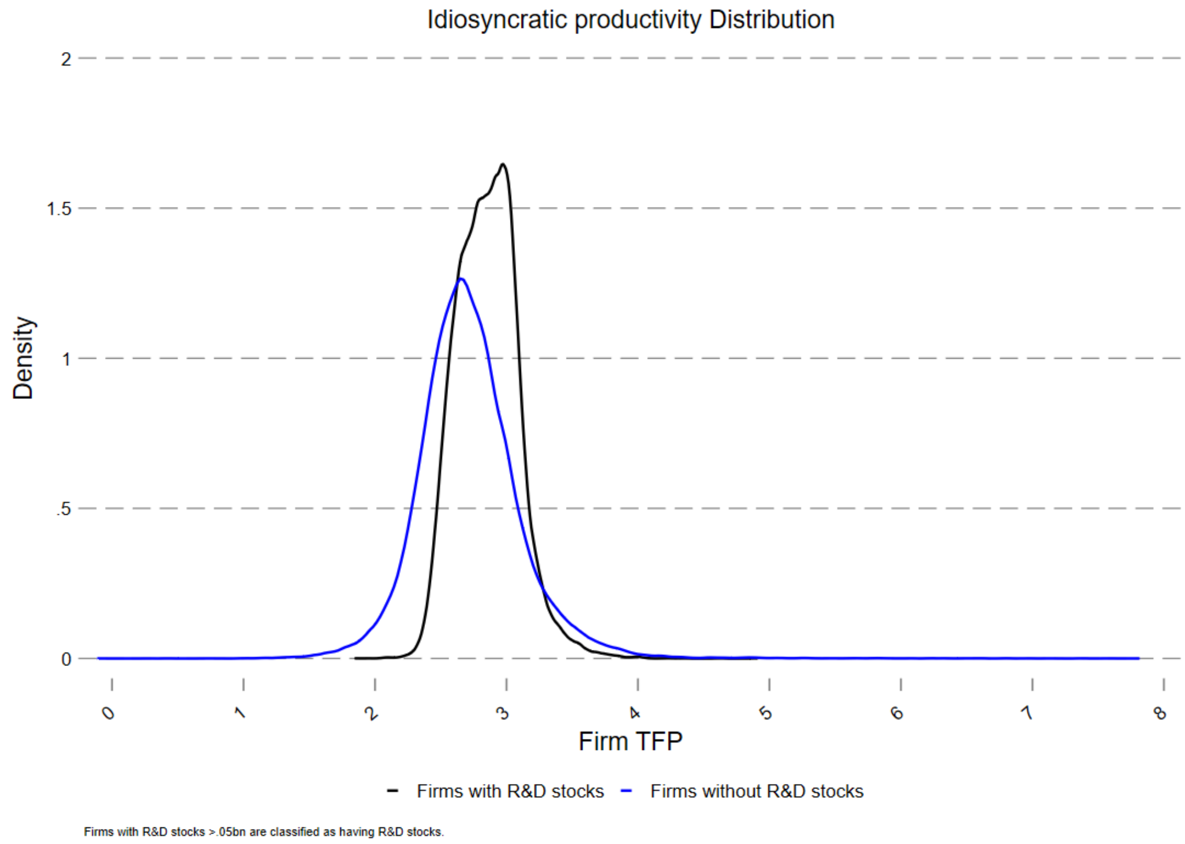
Finally, I obtain standard errors for the production function and productivity relationship coefficients by block bootstrapping.

Appendix B.2. Estimation Results

In this Section, I present the results of the effect of R&D on a firm's idiosyncratic productivity. All of the results presented here (except where specified) use the Compustat data over the period 1980-2016.

The importance of R&D on a firm's productivity is hinted at by comparing the distribution of firm productivity between firms that have stocks of R&D and those that do not. Specifically, I estimate a firm's idiosyncratic productivity assuming that current productivity depends only on lagged productivity (i.e. is an AR(1) Markov process) and annual dummy variables, such that it is exogenous and assumed unaffected by R&D. The result of this approach is shown in Figure B.1, which plots the distribution of firm idiosyncratic productivity obtained via the aforementioned approach (and accounting for the effect of aggregate productivity as represented by the annual dummy variables).

Figure B.1: Distribution of exogenously estimated idiosyncratic productivity split by firms with and without R&D stocks



Using Compustat data between 1980 and 2016. Estimated using ACF procedure where the productivity process is determined by an AR(1) Markov process and annual dummy variables. The idiosyncratic firm productivity removes the effect of aggregate productivity as captured by the annual dummy variables. The capital stock was obtained by deflating the Compustat book-value of a firm's capital stock by BEA industry-specific deflators. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate of 15% and setting $R_0 = 10^{-6}$.

In this graph, the black line shows the distribution for firms with R&D stocks (defined as a firm with a stock of R&D of at least \$50 million constant 2012 USD and representing roughly 19% of observations) while the black line shows the distribution for all other firms (i.e. firms without any R&D). It is immediately clear that firms with R&D have a higher mean and median idiosyncratic productivity than do firms without R&D. Moreover, the range of productivities for firms with R&D is much smaller than the range for those firms without R&D, suggesting that R&D not only increases a firm's productivity but also provides it with more certainty as to the value its productivity could take.

Therefore, Table B.3 presents the results of the determinants of productivity using different specifications of $g(R_{i,t}, \bar{R}_{i,t}, \nu_{i,t-1}, \beta_g)$ that depend on R&D, using the BEA approach to obtain the capital stock and with the R&D stock obtained by setting $R_0 = 10^{-6}$(each specification includes year dummies and a cubic polynomial in lagged productivity as controls variables). In particular column (1) includes just the log of a firm's own stock of R&D and suggests that a 1% increase in a firm's stock of R&D in any one year is associated with an idiosyncratic productivity that is 0.004% higher in that year. Column (2) also includes the square of a firm's own stock of R&D and shows that there are decreasing returns to a firm's own stock of R&D.

Table B.3: Estimates of endogenous productivity process under different specifications

	(1)	(2)	(3)	(4)
Firm's Research Stock	0.00444*** (0.000575)	0.0100*** (0.00157)	0.0130*** (0.00123)	0.0157*** (0.00161)
Square of Firm's R&D		-2.29e-03*** (4.86e-04)		-1.56e-03** (4.78e-04)
Interaction with Aggregate R&D			-0.0623*** ((0.00760)	-0.0540*** (0.00779)
Adjusted R^2	0.907	0.907	0.907	0.907
Joint Significance F Test	17.8	21.0	15.5	14.0
AIC	-181,720	-181,693	-181,779	-181,759
Observations	134,112	134,112	134,112	134,112

Standard errors (obtained via block-bootstrap with 60 repetitions) in parentheses. * p<0.05 ** p<0.01 *** p<0.001. Using Compustat data between 1980 and 2016. All columns estimated via specifying determinants of the endogenous productivity function, always including cubic polynomial in lagged productivity and annual dummy variables as controls. The capital stock was obtained by deflating the Compustat book-value of a firm's capital stock by BEA industry-specific deflators. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate of 15% and setting $R_0 = 10^{-6}$. The dependent and independent variables shown are in logs (with 1 added to each individual firm's stock of R&D before taking logs to ensure that those firms without any stocks of R&D are still included).

Column (3) includes an interaction between the aggregate stock of R&D in each year with a firm's own stock of R&D and indicates that higher aggregate R&D reduces the extent to which a firm increasing its own stock of R&D would increase its idiosyncratic productivity.[4] [5] This indicates that rather than have positive spillovers and allowing firms to make increasingly productive innovations, higher aggregate R&D "crowds out" the effect of an individual firm's own stock of R&D. This echoes the "ideas are getting harder to find" result found in Bloom et al. (2020) (since higher aggregate R&D leads to an individual firm getting a lower return on its own stock of R&D) and is consistent with the product-market rivalry explanation highlighted by Lucking, Bloom and Van Reenen (2019) (i.e. that higher aggregate R&D reflects firms trying to develop more rival products, thereby increasing competition and reducing the profits available for an innovation

[4] Here, the aggregate stock of R&D $\bar{R}$ is measured as the mean of (the log of) individual firms' stocks of R&D to account for the fact that the number of firms in Compustat changes substantially over time.

[5] The coefficient measuring the spillover from aggregate R&D is identified even in the presence of year fixed-effects due to the variation in a firm's own stock of R&D, which means that there is variation in the interaction term within an individual year. Moreover, the multiplicative nature of the interaction term means that there is variation across years as well, due to the fact that both the aggregate and individual stocks of R&D are changing over time. However, including the annual dummy variables means that I cannot separately discern the effect of the level of aggregate R&D on a firm's productivity via this procedure. Nonetheless, in Appendix B.3 I use a modified version of this estimation procedure to enable the inclusion of the level of aggregate R&D and find that the spillovers from the level of aggregate R&D are not statistically significantly different from zero.

by an individual firm).[6]

Moreover, there are other negative spillovers, beyond just product-market rivalry, that might result from higher aggregate R&D. For example, higher aggregate levels of R&D could mean that there is increased demand for inputs (such as scientists and researchers) into R&D, increasing the cost of those inputs, thereby creating an incentive for some firms to reduce their R&D in order to prevent their R&D costs from increasing a lot.[7] Hence, in contrast to Lucking, Bloom and Van Reenen (2019) and Bloom, Schankerman and Reenen (2013)'s focus on a single potential source of positive spillovers and a single potential source of negative spillovers, my approach accounts for all of possible sources of positive and negative spillovers.

My finding of negative R&D spillovers is also consistent with Dieppe and Mutl (2013) (which finds that higher aggregate R&D has a negative effect on aggregate productivity across a range of countries) and Kim and Choi (2019) (who find that negative market rivalry spillovers dominate positive technology spillovers from multi-national firms operating in South Korea).

Column (4) includes all three variables and shows that this specification does not change the overall picture. In particular, a firm's idiosyncratic productivity is increasing in its own stock of R&D, albeit with decreasing returns. Moreover, the rate at which a firm's productivity increases in its own stock of R&D is decreasing in aggregate R&D, with a similar interpretation as described above. In this specification, the coefficients imply that the mean firm from 1980 until today would experience an increase in productivity of 0.003% if it increased its own stock of R&D by 1%, which translates into an increase in the mean firm's value added of 0.005% (in other words, for the average firm, a $3,000 increase in a firm's stock of R&D is associated with an increase in their value added of roughly $4,900).[8] The estimated coefficients in this specification are of a similar order of magnitude to those found by Kehrig, Miao and Xu (2022). In all four specifications,

[6] Lucking, Bloom and Van Reenen (2019) finds that positive diffusion spillovers (from one firm's innovation being adopted by other firms in the same or similar industries) outweigh the negative product-market rivalry spillovers. This difference compared to my results is driven by the fact that I use the aggregate stock of R&D in the economy to capture spillovers whereas Lucking, Bloom and Van Reenen (2019) uses a measure of "technological proximity" that only captures spillovers that specifically relate to "technological proximity" and "product-market rivalry" (under the assumption that firms in similar industries are likely to have higher spillovers). My approach enables me to capture more general spillovers that result from R&D rather than the highly-specific types of spillovers that are examined in Lucking, Bloom and Van Reenen (2019).

[7] Berger (1993) finds that this is what happened after the introduction of the R&D Tax Credit in the USA in the 1980s. In particular, the R&D subsidy increased the demand of inputs into R&D (such as researcher wages), thereby increasing the costs of conducting R&D, reducing the profitability of R&D for some firms.

[8] Although not explicitly modelled as part of my estimation of the effect of a firm's R&D on its productivity, the impact of "jump-ahead" innovations on productivity are captured by the estimated coefficients on the R&D variables as part of my approach. This is because any such innovation still contributes to a firm's productivity as a result of the R&D stock used to obtain them. Moreover, the uncertainty regard-

the R&D terms are individually significant at the 1% level (and most are individually significant at the 0.1% level) and are also jointly significant at the 1% level as shown by the results of the F-test presented in the ante-penultimate row of the table (and the adjusted R^2 does not change substantially across the different specifications).

Moreover, the penultimate row of the table presents the Akaike Information Criterion (AIC) obtained under each specification.[9] As a smaller (i.e. more negative) value of the AIC indicates a model that fits the data better than a model with a higher AIC, the AIC suggests that the specifications in either model (3) or (4) fit the data best. Given that the difference in the AICs between the two specifications is small and the squared term of a firm's own R&D in specification (4) is statistically significant at the 1% level, I rely on the specification in column (4), and refer to this as the "benchmark specification".

These results are robust to different methods of obtaining the capital and R&D stocks, as well as to different time periods used. Moreover, a firm's R&D stock is an important determinant of a firm's productivity (the R&D stock terms are jointly significant, even if individual coefficients are not) when estimating this relationship for individual industries separately. These results are also robust to potential measurement error in both the R&D stock and the stock of physical capital. Each of these robustness checks is discussed in more detail below.

For completeness, Table B.4 presents the coefficients on capital and labour estimated for the production function. The first column of this table shows the results obtained assuming that productivity is given by the exogenous AR(1) Markov process with annual dummies (and no R&D variables) that was used to estimate the productivities shown in Figure B.1, while each subsequent column corresponds to the relevant model as shown in Table B.3. The results in this table show that the endogenisation (comparing the "Exog" column against the other columns in the table) of a firm's idiosyncratic productivity to depend on R&D has little effect on the estimated capital stock coefficient and a slight effect on the estimated labour coefficient. Nonetheless, the individual and joint significance of the R&D terms in a firm's idiosyncratic productivity demonstrates the importance of R&D in that aspect even if it has only a slight effect on the production function coefficients.

Moreover, the effect of R&D on a firm's idiosyncratic productivity is robust to the different ways of calculating the capital and R&D stocks, as shown in Table B.5. The first

ing whether or not an investment in R&D will actually lead to a successful innovation is also captured in these coefficients (as an investment in R&D that is unsuccessful would increase a firm's stock of R&D without boosting its productivity substantially, resulting in the estimated effect of an investment in R&D on a firm's productivity being closer to zero).

[9] The AIC is calculated as $-2 \log L + 2k$ where L is the value of the log-likelihood, and k is the number of parameters included in the model.

Table B.4: Estimated production function coefficients under different specifications of the productivity process

	Exog	(1)	(2)	(3)	(4)
Capital Stock	0.330***	0.330***	0.329***	0.329***	0.329***
	(0.00585)	(0.00587)	(0.00588)	(0.00586)	(0.00587)
Labour	0.643***	0.640***	0.638***	0.639***	0.638***
	(0.00996)	(0.0103)	(0.0103)	(0.0103)	(0.0103)
Constant	1.549***	1.538***	1.536***	1.536***	1.535***
	(0.0426)	(0.0371)	(0.0292)	(0.0338)	(0.0258)
Observations	134,112	134,112	134,112	134,112	134,112

Standard errors (obtained via block-bootstrap with 60 repetitions) in parentheses. * p<0.05 ** p<0.01 *** p<0.001. Using Compustat data between 1980 and 2016. All columns estimated via specifying determinants of $g(.)$, always including cubic polynomial in lagged productivity and annual dummy variables as controls. The capital stock was obtained by deflating the Compustat book-value of a firm's capital stock by BEA industry-specific deflators. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate of 15% and setting $R_0 = 10^{-6}$.

column of this table reproduces column (4) from Table B.3, whose estimates were obtained using the BEA method and an R&D stock setting the initial R&D stock of a firm equal to 10^{-6}. The second and third columns in this table use the same BEA method for calculating the capital stock, but use a different initial value of a firm's stock of R&D. Specifically, column (2) sets the initial value of a firm's stock of R&D equal to their R&D expenditure in that initial period $i_{r,0}$ divided by the rate of R&D depreciation $\delta_r = 0.15$, while column (3) uses a denominator of the same R&D depreciation of 15% per year but also incorporates the average growth rate of R&D of 8% as used by Hall (1993). Column (4) uses the benchmark calculation of an individual firm's stock of R&D, but takes the aggregate stock of R&D to be that for a firm's own (GIC) industry rather than for the economy as a whole. Finally, column (5) uses the average age method of calculating a firm's capital stock and sets the initial R&D stock of a firm equal to 10^{-6}.

The coefficients on all three R&D variables are very similar across the different ways of calculating a firm's or aggregate R&D stock, and only differ very slightly between the two different methods of calculating a firm's capital stock. In other words, the estimated effect of R&D on a firm's productivity are robust to different methods of calculating a firm's capital stock and to different methods of calculating a firm's stock of R&D.

This importance of R&D in determining a firm's productivity also holds across different industries, as can be seen in Table B.6 for a selection of industries. In particular, although the individual significance of the R&D variables varies across industries, in almost all industries (including ones not shown here), the R&D variables are jointly significant at the 5% level as shown by the penultimate row of the table (the critical value for an F-test of three variables with 1,000 observations is roughly 2.6). Of the industries shown in the table, only for Energy Equipment Services are the R&D terms not jointly significant. However, this does not mean that R&D is not important in determining a typical firm's

Table B.5: Estimates of endogenous productivity process using different capital and R&D stock measures

	(1) Benchmark BEA deflation & $R_0 = 10^{-6}$	(2) BEA deflation & $R_0 = \frac{i_{r,0}}{\delta}$	(3) BEA deflation & $R_0 = \frac{i_{r,0}}{\delta+g}$	(4) Industry aggregate R&D	(5) Average age of capital $R_0 = 10^{-6}$
Firm's Research Stock	0.0157*** (0.00161)	0.0158*** (0.00160)	0.0157*** (0.00161)	0.00992*** (0.00147)	0.00938*** (0.00158)
Square of Firm's R&D	-1.56e-03** (4.78e-04)	-1.51e-03** (4.71e-04)	-1.53e-03** (4.73e-04)	-2.01e-03*** (4.69e-04)	-9.05e-05 (6.33e-04)
Interaction with Aggregate R&D	-0.0540*** (0.00776)	-0.0543*** (0.00773)	-0.0543*** (0.00776)	-0.00104 (0.000772)	-0.0568*** (0.00886)
Adjusted R^2	0.907	0.907	0.907	0.907	0.882
Joint Significance F Test	14.0	15.6	15.6	12.04	4.6
Observations	134,112	134,112	134,112	134,112	138,760

Standard errors (obtained via block-bootstrap with 60 repetitions) in parentheses. * p<0.05 ** p<0.01 *** p<0.001. Using Compustat data between 1980 and 2016. All columns estimated via specifying determinants of $g(.)$, always including cubic polynomial in lagged productivity and annual dummy variables as controls. The "average age" method of calculating the capital stock uses the firm's recorded accumulated and current depreciation to estimate its average age of capital before using the GDP deflator for that year to deflate the book value of capital. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate δ of 15% and setting R_0 according to the specification shown (with $g = 8\%$) and where $i_{r,0}$ is the firm's R&D expenditure in the first year it is present in Compustat.

productivity for most industries.

Table B.6: Estimates of endogenous productivity process for specific industries

	(1) Oil, Gas, & Consumable Fuels	(2) Chemicals	(3) Semi-conductors	(4) Energy Equipment Services	(5) Pharmaceuticals
Firm's Research Stock	0.0446*** (0.0154)	-0.0502 (0.0333)	0.0304 (0.0141)	-0.00314 (0.0377)	0.0129 (0.0223)
Square of Firm's R&D	1.98e-03 (8.01e-03)	2.27e-02 (1.47e-02)	4.13e-03 (9.14e-03)	2.93e-02 (5.93e-02)	-1.27e-03 (5.90e-03)
Interaction with Aggregate R&D	-0.437*** (0.103)	0.137 (0.101)	-0.126 (0.00698)	-0.255 (0.454)	0.130 (0.158)
Adjusted R^2	0.930	0.979	0.907	0.908	0.966
Joint Significance F Test	5.42	11.70	9.29	0.68	6.93
Observations	7,041	4,128	3,493	3,048	1,962

Standard errors (obtained via block-bootstrap with 200 repetitions) in parentheses. * p<0.1 ** p<0.05 *** p<0.01. Using Compustat data between 1980 and 2016. All columns estimated via specifying determinants of $g(.)$, always including cubic polynomial in lagged productivity and annual dummy variables as controls. The capital stock was obtained by deflating the Compustat book-value of a firm's capital stock by BEA industry-specific deflators. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate of 15% and setting $R_0 = 10^{-6}$.

Finally, I also estimate my chosen specification over a longer time period of Compustat data. Specifically, I also estimate this specification using data over the period 1963-2016, and the results of this are shown in column (2) Table B.7 (with column (1) containing the results from the benchmark specification and time period for comparison). The estimated coefficients are very similar across the two different sample timeframes, indicating that these results are also robust to changing the time period over which the relationship between R&D and a firm's idiosyncratic productivity is estimated.

Table B.7: Estimates of endogenous productivity process using longer time period

	(1) Data for 1980-2016	(2) Data for 1963-2016
Firm's Research Stock	0.0157***	0.0146***
	(0.00161)	(0.00163)
Square of Firm's R&D	-1.56e-03**	-1.16e-03*
	(4.78e-04)	(5.10e-04)
Interaction with Aggregate R&D	-0.0540***	-0.0585***
	(0.00776)	(0.00715)
Adjusted R^2	0.907	0.912
Joint Significance F Test	14.0	16.7
Observations	134,112	161,917

Standard errors (obtained via block-bootstrap with 60 repetitions) in parentheses. * p<0.05 ** p<0.01 *** p<0.001. The time period used as the estimation sample differs across columns. All columns estimated via specifying determinants of $g(.)$, always including cubic polynomial in lagged productivity and annual dummy variables as controls. The capital stock was obtained by deflating the Compustat book-value of a firm's capital stock by BEA industry-specific deflators. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate of 15% and setting $R_0 = 10^{-6}$.

Appendix B.3. Including the level of aggregate R&D

The use of annual dummy variables in all other specifications precludes including the level of aggregate R&D due to its collinearity with those annual dummies. In particular, the level of aggregate R&D only varies over time and does not vary across firms, such that the annual dummy variables and the level of aggregate R&D are collinear and cannot both be included in the regressions at the same time.

However, it is possible to examine the effect of aggregate R&D at the same time as accounting for any other annual aggregate effects on a firm's productivity via a slight modification in the estimation procedure described in Section **??** of the main text. In particular, rather than immediately regress the recovered estimate of firm productivity on the chosen specification that contains the R&D variables (as per step 3 in the main estimation procedure), I first regress the recovered estimate of firm productivity on the set of annual dummy variables and then regress the residuals from that on the chosen specification that contains the R&D variables. These R&D variables can now include the aggregate level of R&D itself as any other annual aggregate effects have been accounted for and do not need to be included in this sub-step.

The second column in Table B.8 contains the results for the regression using all four explanatory variables: the level and square of a firm's own stock of R&D; the interaction between the aggregate stock of R&D and a firm's own stock of R&D; and the aggregate level of R&D. The third column presents the results for the regression that includes the level of aggregate R&D instead of the interaction between aggregate R&D and firm R&D.

Table B.8: Estimates of endogenous productivity including aggregate R&D

	(1) Level and interaction of aggregate R&D	(2) Just the level of aggregate R&D
Firm's Research Stock	0.00468**	0.00123
	(0.00151)	(0.00157)
Square of Firm's R&D	1.64e-04	-4.10e-04*
	(5.06e-04)	(4.96e-04)
Interaction with Aggregate R&D	-0.0465***	
	(0.00787)	
Level of Aggregate R&D	0.0181*	0.0122
	(0.00786)	(0.00727)
Adjusted R^2	0.868	0.868
Joint Significance F Test	2.19	1.13
Observations	134,112	134,112

Standard errors (obtained via block-bootstrap with 60 repetitions) in parentheses. * p<0.05 ** p<0.01 *** p<0.001. Using Compustat data between 1980 and 2016. All columns estimated via specifying determinants of $g(.)$, always including cubic polynomial in lagged productivity and annual effects having been accounted for by regressing recovered productivity estimates on annual dummy variables in a previous stage.. The capital stock was obtained by deflating the Compustat book-value of a firm's capital stock by BEA industry-specific deflators. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate of 15% and setting $R_0 = 10^{-6}$.

In both cases, the coefficient on the level of aggregate R&D is not statistically significant at the 1% level. Indeed, the third column of the table shows that using the aggregate level of R&D, despite itself being statistically insignificant, renders the other R&D terms (the level and square of a firm's own stock of R&D) to be statistically insignificant, while all three R&D terms are jointly statistically insignificant as well. Moreover, even though the coefficient on the level of aggregate R&D is (slightly) positive in the regression containing all four R&D variables (the second column of the table), the coefficient on the interaction term remains negative and statistically significant.

This means that including level of aggregate R&D does not affect the sign of the spillover that arises via the interaction term, and that any spillover arising from the level of aggregate R&D itself is not statistically significantly different from zero.

Appendix B.4. Accounting for Measurement error

The potential for measurement error to result in downwardly-biased estimates of coefficients that have been measured with error is well-document. For example, Collar-Wexler and De Loecker (2021) show that mis-measurement of a firm's stock of physical capital (perhaps due to the method used for depreciating the stock of physical capital not being accurate for some firms) can bias the estimated coefficient of capital in a production function estimation procedure downwards, while biasing upwards the estimated labour coefficient and firm productivity.

This potential measurement error of the stock of physical capital could affect my estimates of the impact of R&D stock on a firm's productivity. In particular, if the stock of physical capital is measured with error, then the estimated firm productivity (as a residual) could be too large, which will then lead to more variation in firm productivity that needs to be explained by a firm's stock of R&D. This could result in the R&D coefficients in the estimated productivity regression being biased upwards.

However, there is also the potential for measurement error in the stock of R&D, particularly if the depreciation rate for a firm's stock of R&D obtained using the perpetual inventory method is substantially different from the actual rate of depreciation the a firm experienced. This would bias downwards any estimated effect of a firm's R&D on its productivity.

As such, it is ambiguous as to the direction of bias of the estimated effect of R&D on productivity caused by measurement error in this case. Collar-Wexler and De Loecker (2021) suggests an easy method of accounting for measurement error: rather than using the value of the stock in a particular period, use as an instrument the level of investment in that stock in the previous period (i.e. instrument for k_t using i_{t-1}, and instrument for R_t using $I_{R,t-1}$). This requires modifying the GMM moment condition for capital in the ACF estimation procedure to be $\frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\xi}_{i,t}i_{i,t-1}$ rather than $\frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\xi}_{i,t}k_{i,t}$, and conducting a two-stage least squares regression (rather than OLS) using $I_{R,t-1}$as an instrument for R_t in order to recover the effect of R&D on firm productivity.

Table B.9 presents the results obtained when correcting for measurement error. Model (1) shows the estimated coefficients for R&D's effect on productivity for the baseline case where no correction for potential measurement error has been made. Models (2) and (3) show the estimated coefficients when potential measurement error in, respectively, the stock of physical capital and the stock of R&D, have been made. Finally, Model (4) shows the results obtained accounting for potential measurement error in both the stock of physical capital and the stock of R&D at the same time (I instrument for the aggregate stock of R&D using lagged aggregate R&D investment).

The estimated coefficients are of roughly the same magnitude across all specifications, albeit with a slightly smaller coefficient on the level of a firm's own R&D and a more negative coefficient on the interaction with aggregate R&D. In addition, the square of a firm's own R&D becomes less statistically significant when potential measurement error in a firm's R&D stock is accounted for (although its coefficient is of roughly the same size as before). Overall, therefore, it is unlikely that any potential measurement error in the stock of physical capital and / or R&D substantially affect my results.

Table B.9: Estimates of endogenous productivity process accounting for measurement error

	(1) Benchmark	(2) Just instrument for K	(3) Just instrument for R&D	(4) Instrument both K and R&D
Firm's Research Stock	0.0157*** (0.00161)	0.0138*** (0.00161)	0.0132*** (0.00161)	0.0098*** (0.00158)
Square of Firm's R&D	-1.56e-03** (4.78e-04)	-1.24e-03** (4.11e-04)	-1.49e-03* (5.86e-04)	-6.85e-04 (4.93e-04)
Interaction with Aggregate R&D	-0.0540*** (0.00776)	-0.0703*** (0.0115)	-0.0565*** (0.00997)	-0.0563*** (0.0116)
Adjusted R^2	0.907	0.906	0.907	0.906
Joint Significance F Test	14.0	12.4	9.24	5.27
Observations	134,112	134,112	134,112	134,112

Standard errors (obtained via block-bootstrap with 60 repetitions) in parentheses. * p<0.05 ** p<0.01 *** p<0.001. The time period used as the estimation sample differs across columns. All columns estimated via specifying determinants of $g(.)$, always including cubic polynomial in lagged productivity and annual dummy variables as controls. The capital stock was obtained by deflating the Compustat book-value of a firm's capital stock by BEA industry-specific deflators. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate of 15% and setting $R_0 = 10^{-6}$.

Appendix B.5. Test for monotonicity and scalar unobservable

As mentioned in Appendix B.1, the ACF estimation procedure relies crucially on the assumptions that productivity is the only unobservable (i.e. there is a scalar unobservable) and that it is strictly monotonic in materials. Shenoy (2021) argues that in some samples (particularly ones that contain firms that are financially constrained), these assumptions might not hold due to the presence of some "market frictions" and therefore develops a test to see if the assumptions are indeed valid.

This test involves using a firm's materials costs as a share of its output as the dependent variable and regressing that on the inversion function $f_t^{-1}(m_{i,t}, k_{i,t}, \ell_{i,t})$ in used to approximate productivity as $\nu_{i,t} = f_t^{-1}(m_{i,t}, k_{i,t}, \ell_{i,t})$ when recovering the fitted values $\hat{\phi}_{i,t}$ along with some "instruments" for whether or not a firm is subject to any frictions that might render the scalar unobservable and monotonicity assumptions invalid. If the assumptions hold, then the additional instruments should not be jointly statistically significant (Shenoy suggests using a 1% significance level for this specification test).

Shenoy suggests using the set of lags for $k_t$and ℓ_t that are used to construct the moments for the GMM stage of the ACF estimation procedure. In other words, Shenoy suggests using k_t and ℓ_{t-1} in a simple OLS regression if one is not worried about the presences of measurement error, or doing two-stage least squares using i_{t-1}as the instrument for k_t (alongside using ℓ_{t-1} in the top-level regression) if there might be measurement error.

Therefore, I run both versions of the test, without and with accounting for measurement error. When not accounting for measurement error, the p-value for the test of joint significance of the two instruments is 0.016 while when accounting for potential measurement error it is 0.55. Therefore, using Shenoy's suggested 1% threshold for the test, we cannot reject the null hypothesis of no joint significance of the two "instruments" and, as such, cannot reject the assumptions of strict monotonicity and a scalar unobservable.

Appendix C. Solving the model

In this Appendix I set out how I solve my model. I first cover how I obtain firm decision rules by extendingKhan and Thomas (2013) and Jo (2021) to allow for a firm choice over R&D as well as physical capital and debt. I then describe the pseudo-algorithm I use to solve for the steady-state.

Appendix C.1. Obtaining firm decision rules

In each period all incumbent firms optimally choose their labour demand depending on the wage rate, their capital stock, and their output productivity (determined by the endogenous firm-specific stock of R&D and the distribution over firms, and the exogenous idiosyncratic productivity shock). In other words, their optimal choice of labour is given by $\ell = \left(\frac{w(\mu)}{\gamma e^{f(R,\varepsilon_o,\mu)}}\right)^{\frac{1}{\gamma-1}} K^{\frac{-\alpha}{\gamma-1}}$.

In order to derive firms' remaining decision rules, following Khan and Thomas (2013) and Jo (2021), I distinguish between constrained and unconstrained firms. Unconstrained firms are those firms that have saved enough such that the collateral constraint will not bind now or in any future time period. Therefore these firms are indifferent between paying dividends or saving even more and, hence, these firms will pay positive dividends.[10] In contrast, constrained firms are those that might at some point in the future have a binding collateral constraint (even if it does not bind today) and, as such, will pay zero dividends to ensure that they continue to save enough so they can eventually become unconstrained. The solutions for each of these types of firms are discussed in turn below.

Appendix C.1.1. Decisions of unconstrained firms

The collateral constraint does not (and never will) bind for unconstrained firms, and this is an absorbing state (once a firm becomes unconstrained it will remain unconstrained until it dies). Therefore, an unconstrained firm's choice of K' conditional on its choice of R' is obtained by maximising of Equation (**??**) in the main text with respect to K' and applying the Benveniste-Scheinkman condition to replace $W_1(m', R', \varepsilon_o', \varepsilon_r', \mu')$. This gives the analytical expression for the optimum unconstrained choice of capital K^U:

$$K^U = \left(\frac{\frac{p(\mu)}{\beta} - p(\mu')(1-\delta_k)}{E_{\pi_{\varepsilon_o}\pi_{\varepsilon_r}}\left[\alpha e^{f(R',\varepsilon_o',\mu')} \left(\frac{w(\mu')}{\gamma e^{f(R',\varepsilon_o',\mu')}} \right)^{\frac{\gamma}{\gamma-1}} \right]} \right)^{\frac{\gamma-1}{1-\alpha-\gamma}} \tag{C.1}$$

[10] This indifference is because these firms' shadow value of further saving is exactly equal to the household's marginal utility of consumption.

The presence of $e^{f(R',\varepsilon_o',\mu')}$ in this expression means that the optimum choice of K^U depends on the firm's choice of R'. In other words, a firm's choice of physical capital investment depends on its stock of R&D next period as this stock of R&D affects the firm's expected return to physical capital through its effect on a firm's output productivity.

Given the firm's choice of ℓ, K^U, and R', an unconstrained firm chooses its debt to ensure that the borrowing constraint does not bind next period nor in any possible future period. This results in the firm following a minimum savings policy B^U obtained via the recursive solution to the problem below, in which $\tilde{B}$ represents an unconstrained firm's level of debt held at the start of a period in which they face exogenous idiosyncratic shocks $(\varepsilon_{o_i}, \varepsilon_{r_j})$.[11] Then, given the optimum choice of capital K^U (itself conditional on the choice of R'), B^U is the highest level of debt that a firm can choose for the next period while still being able to choose the unconstrained capital and R&D stocks in all possible future realisations of $\varepsilon_{o_i}, \varepsilon_{r_j}$. The dependence of the unconstrained choice of capital K^U on the firm's choice of R' means that the minimum savings policy B^U is also conditional on the firm's choice of R'.

$$B^U(R', \varepsilon_o, \varepsilon_r, \mu) = \min_{\varepsilon_o', \varepsilon_r'} \tilde{B}(K^U, R', R', \varepsilon_o', \varepsilon_r', \mu') \tag{C.2}$$

$$\begin{aligned}\tilde{B}(K, R, R', \varepsilon_o, \varepsilon_r, \mu') =& (1-\tau_c)[e^{f(R,\varepsilon_o,\mu)} K^\alpha \ell^\gamma - w(\mu')\ell - \xi \\ &+ (1-\delta_k)K - K'^U(R', \varepsilon_o, \varepsilon_r) - \frac{(1-\tau_r)I_r}{\varepsilon_r} \\ &+ q(\mu') \min\{B^U(R', \varepsilon_o, \varepsilon_r, \mu), \theta K'^U(R', \varepsilon_o, \varepsilon_r)\}]\end{aligned} \tag{C.3}$$

This means that the problem faced by unconstrained firms can now be written as follows, with decision rules for capital K^U and debt B^U determined as above.

$$\begin{aligned}W^U(m, R, \varepsilon_o, \varepsilon_r, \mu) = & \max_{R', D^U} p(\mu)(1-\tau_c)D^U \\ &+ \beta E_{\pi_{\varepsilon_o}\pi_{\varepsilon_r}} W(m', R', \varepsilon_o', \varepsilon_r', \mu')\end{aligned} \tag{C.4}$$

subject to

[11] As firms are subject to corporation taxes levied at rate τ_c, the level of debt (or savings) a firm can hold must also have those taxes applied.

$$
\begin{aligned}
D^U &= m - K' - \frac{(1-\tau_r)I_r}{\varepsilon_r} + q(\mu)B' - \xi \\
B' &= B^U(R', \varepsilon_o, \varepsilon_r, \mu) \\
K' &= K^U(R', \varepsilon_o, \varepsilon_r, \mu) \\
I_r &= R' - (1-\delta_r)R \geq 0 \\
\mu' &= \Gamma(\mu) \\
m' &= m(K', R', B', \varepsilon_o', \varepsilon_r', \mu') \\
&= \max_{\ell'} e^{f(R', \varepsilon_o', \mu')} K'^{\alpha} \ell'^{\gamma} - w(\mu')\ell' + (1-\delta_k)K' - B'
\end{aligned}
$$

and where $W(m', R', \varepsilon_o', \varepsilon_r', \mu')$ is given by equations (**??**) and (**??**) in the main text.

In this way, I have been able to reduce a problem in three variables K', B', and R' down to one that can be solved via a non-linear search over just one variable R'. For potential entrants that choose to enter, their choice of K' given their choice of R' is given by equation (C.1) scaled by a fraction κ that is calibrated to target moments regarding entrant size.

Appendix C.1.2. Decisions of constrained firms

Unlike unconstrained firms, constrained firms are either at the binding collateral constraint or know that there is a non-zero probability that the collateral constraint might bind at some point in the future. Therefore, constrained firms choose to pay zero dividends and instead increase their savings / reduce their debt. Setting $D = 0$ for the first constraint in Equation (C.4) gives $m = K' + \frac{(1-\tau_r)I_r}{\varepsilon_r} - q(\mu)B' + \xi$ (where $I_r = R' - (1-\delta_r)R \geq 0$), implying that a constrained firm's choice of physical capital, R&D stock, and debt next period must sum to its current cash-on-hand.

Constrained firms pay zero dividends. Supposing that the collateral constraint binds (i.e. $B' = \theta K'$), we can obtain a "capital threshold" of $K^T = \frac{m - \frac{(1-\tau_r)I_r}{\varepsilon_r} - \xi}{1 - q(\mu)\theta}$ via substitution and re-arranging the expression for dividend. This gives the maximum possible choice of capital for a firm with cash-on-hand m and R&D investment I_r. Using this capital threshold, and following Jo (2021), I can distinguish between two types of constrained firms.

The first type of constrained firms are those for which the collateral constraint does not currently bind but could at some point in the future. For these firms, the unconstrained choice of capital is less than K^T so they set their choice of capital next period next period equal to the unconstrained level $K' = K^U$ and their choice of debt is given by $B' = \frac{K^U + \frac{(1-\tau_r)I_r}{\varepsilon_r} - m + \xi}{q(\mu)}$.

The other set of constrained firms have a currently-binding collateral constraint. This means that the unconstrained choice of capital is greater than that which they can afford as indicated by the capital threshold (i.e. $K^U > K^T$). These firms set $K' = K^T$ and choose debt such that $B' = \theta K'$.

To obtain the value function W^O (the value function of continuing firms) across all firms, I define W^O as the unconstrained firm's value function W^U at all points where the unconstrained firm's choice of R', K', and B' imply non-negative dividends and the constrained firm's value function W^C elsewhere. In other words, W^O is given by

$$W^O(m, R, \varepsilon_o, \varepsilon_r, \mu) = \begin{cases} W^U(m, R, \varepsilon_o, \varepsilon_r, \mu) & \text{if} D^U \geq 0 \\ W^C(m, R, \varepsilon_o, \varepsilon_r, \mu) & \text{otherwise} \end{cases}$$

where W^C is given by the following (noting that a constrained firm pays zero dividends)

$$W^C(m, R, \varepsilon_o, \varepsilon_r, \mu) = \max_{R'} \beta E_{\pi_{\varepsilon_o} \pi_{\varepsilon_r}} V(m', R', \varepsilon_o', \varepsilon_r', \mu') \tag{C.5}$$

subject to

$$\begin{aligned}
K' &= \begin{cases} K^U(R', \varepsilon_o, \varepsilon_r, \mu) & \text{if } K^U \leq K^T \\ K^T(R', \varepsilon_o, \varepsilon_r, \mu) & \text{if } K^U > K^T \end{cases} \\
B' &= \frac{K' + \frac{(1-\tau_r)I_r}{\varepsilon_r} - m + \xi}{q} \\
I_r &= R' - (1 - \delta_r)R \geq 0 \\
\mu' &= \Gamma(\mu) \\
m' &= m(K', R', B', \varepsilon_o', \varepsilon_r', \mu') \\
&= \max_{\ell'} e^{f(R', \varepsilon_o', \mu')} K'^{\alpha} \ell'^{\gamma} - w\ell' + (1 - \delta_k)K' - B'
\end{aligned}$$

As with unconstrained firms, what was a three-variable problem in terms of K', B', and R' can now be solved via the choice of just R', with a firm's choice of K' and B' given conditional on their choice of R'. To determine a firm's optimum choice of R' I take these decision rules for K' and B' as given and use golden section search to determine the firm's optimum choice of R'.

Appendix C.2. Solution Algorithm

In order to solve the model I discretise the output productivity shock ε_o using the Rouwenhorst algorithm with 5 gridpoints, while the research productivity shock ε_r is discretised on 7 gridpoints. I define the value functions V, V^1, V V^U, and V^C on a grid of $m \times R \times \varepsilon_o \times \varepsilon_r$. For the base case, I set the R&D subsidy $\tau_r = 0.05$ and the corporation tax $\tau_c = 0.35$, while for the other values of the R&D subsidy I guess a value of τ_c (recall that when changing the level of the R&D subsidy I alter the corporation tax so that the government surplus Λ obtained in the base case is maintained across all scenarios).

I guess an aggregate level of R&D $\bar{R}$ and a consumption C, the latter implying a wage w through the household's labour-leisure condition. I guess an initial value for V. First, I solve V^U by carrying out a Golden Section search to find R' for unconstrained firms (recall that the choice of R' implies a choice of K^U and B^U) and iterating on V^U and V until they converge (at each iteration checking whether V^X or V^O is higher, and updating V accordingly).

Next, I take V^U as the starting guess of V^C and identify where the dividends implied by the unconstrained choices are negative. Any point where the unconstrained choice provides non-negative dividends indicates that firms are unconstrained at that point. At the points where the unconstrained choice results in negative dividends, I carry out a Golden Section search to find R' for the constrained firms.

Note that a firm's choice of R' affects the capital threshold K^T that determines whether or not the firm can afford the unconstrained choice of physical capital and also whether or not it is at the collateral constraint. However, this does not cause an issue for the Golden Section search due to that fact that a firm's problem remains continuous in its choice of R': a slight increase in R', with a resulting higher I_R, that results in the implied physical capital threshold moving from just above the unconstrained choice (such that the firm could choose the unconstrained level of physical capital) to just below the constrained choice (such that the firm must choose the level of physical capital implied by the threshold) is a very small change in a firm's K' and also implies a very small change in a firm's B'.

Hence, a firm's (expected) cash-on-hand next period also only changes by a small amount, such that it remains continuous in the choice of R'. This means that the firm's value function is also continuous in its choice of R', and that a single Golden Section search over a firm's choice of R' also directly implies a K^C and B^C for constrained firms.

I use these solved decision rules to update the relevant values of V^C and V, iterating until convergence (at each iteration checking whether V^X or V^O is higher, and updating V accordingly).

I then solve the potential entrants' problem V^N using a Golden Section search to find their choice of R' and K' (with K' given by a proportion κ of the unconstrained choice of K' for a given choice of R').

I use the solved decision rules for R', K', and B', as well as firms' decisions on entry and exit alongside exogenous exits, to find the stationary distribution, which is then used to obtain values for implied $\bar{R}$ and C (and τ_c when relevant). The initial guesses of these variables are then updating using Broyden's algorithm with the updating-step obtained as per Santaeulalia-Llopis (2016).

Appendix D. Model validation against untargeted moments

In this Appendix, I discuss my model's performance against some non-targeted moments. Focusing solely on aggregates the private R&D stock-to-output ratio in the USA over the period 1980-2015 was roughly 0.101, while for my model it is 0.34.[12] However, a more appropriate comparison is one that accounts for the fact that the R&D components of my model are calibrated using Compustat data (Compustat firms account for roughly 92% of R&D spending but only 70% of private sector output in the USA). As such, my model obtaining a higher R&D stock-to-output ratio is to be expected. [13] Hence, a better comparison is the ratio of total R&D stock to capital stock just for Compustat firms, which averages 0.177 over the period 1980-2015, and is 0.16 in my model, which indicates that my model does well in matching Compustat-specific moments in the data. Moreover, my model also matches the roughly 5% ratio of R&D investments to investments in physical capital for the USA found by Bond and Cummins (2000).[14]

Moreover, as well as the model's performance in static measures, I also corroborate my model's performance at capturing the different potential ways in which firms grow. Recall that Figure **??** in the main text showed three different paths of firm accumulation of stocks of physical capital and R&D: the first, including firms such as as Pfizer / Microsoft, invest by increasing their R&D substantially with only minimal increases in their capital stocks; a second, including PetroChina and Nippon Telegraph, invest mostly through increasing capital stocks with minimal R&D stock increases; and a third, for firms such as VW and Toyota, invest by increasing their R&D substantially and adding to their capital stocks as well.

Figure D.2 highlights that my model captures each of these different investment paths (the x axis shows the model firms' capital stock and the y axis shows their R&D stock, in units of output from the model). These results are obtained by starting each firm with a cash holding of $m = 0.4$ (which is sufficiently high that these firms do not exit immediately, but low enough that they are not close to their full size) and using the solved decision rules from the steady-state to see how they grow over time if subject to different paths of the exogenous shocks ε_o and ε_r. Each cross represents a specific simulated firm

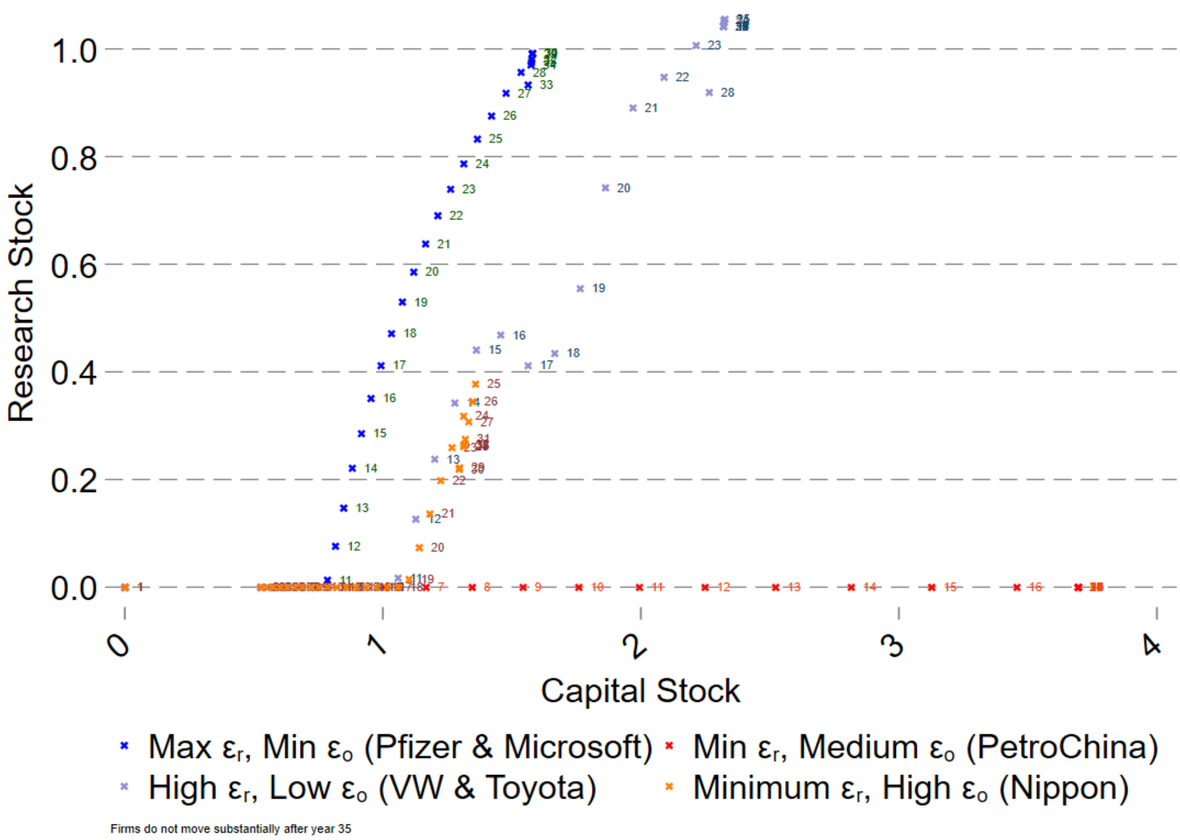
[12] The NSF provides data on annual total R&D investment in the US, split by various categories including into those investments made by private organisations and those made by government organisations. I calculate the aggregate stock of R&D using the same perpetual inventory method that I used to calculate individual firm's stocks of R&D presented in Section **??** in the main text, except I now apply it to aggregate R&D investments rather than individual firm R&D investments.

[13] Such data might be available from the US Census or from Orbis data relating to other countries.

[14] My model slightly overstates the amount of R&D spending accounted for by entrants / small firms compared to the data. In particular, new entrants account for roughly 14% of R&D spending in my model, compared to the at most 5% - 10% that such firms account for in the data (Compustat firms account for 90% - 95% of all private R&D spending in the US, such that new entrants, which would be part of the firms outside Compustat, account for at most 10% of such spending in the US).

in a specific year, with the number by the side of each cross indicating the year of the simulation. In each case, firms do not invest substantially in either R&D stock or capital stock after 35 simulation periods (years) have passed as they have reached their optimum, unconstrained, size.

Figure D.2: Model investment paths for certain types of firms



Each firm starts out with cash-holdings of 0.4 and are subject to the constant series of idiosyncratic research and output productivity shocks described in the legend of the graph. Their investment path is then simulated using the solved decision rules obtained in the stationary solution to my model.

In terms of my model capturing the different potential investment paths of the specific firms mentioned previously, the red crosses show a firm investing solely by increasing its physical capital without substantial investments in R&D (akin to PetroChina) while the orange crosses show a firm investing mostly in physical capital but then adding some R&D stock after a few years akin to Nippon Telegraph (albeit the path of these orange crosses has slightly too little physical capital compared to the path observed in the data for Nippon Telegraph). The investment paths of firms such as VW and Toyota are also captured as shown by the grey crosses. My model also captures the investment paths of firms that have little capital and mostly R&D as shown by the blue crosses (akin to firms like Pfizer in the data) as they have more R&D and less physical capital than do the grey crosses that represent firms like VW and Toyota.

Hence, the fact that my model captures the majority of these untargeted potential investment paths of different firms shows that it is suitable for assessing how R&D subsidies affect aggregate economic variables and firm growth over time.

Appendix E. Alternative R&D policies

The relatively small effect of increasing general R&D subsidies on output indicates that alternative policies are necessary to obtain large increases in output. Therefore, I examine the effect of a number of alternative policies that could increase output by affecting R&D: 1) taxing R&D investments instead of subsidising them; 2) "targeted" R&D subsidies where only firms that already have a stock of R&D above some threshold are eligible for higher R&D subsidies (to try to avoid the large increase in aggregate R&D caused by some firms that had no R&D under a 5% general R&D subsidy choosing to invest in R&D when the general subsidy is increased); and 3) allowing firms to use some proportion of their R&D stock as collateral alongside their physical capital stock (to reduce the financial constraints faced by new entrants and young firms).[15]

None of these alternative policies substantially differ in their impact on output compared to increasing the general R&D subsidy. This is because these policies all serve to affect the aggregate level of R&D, thereby exacerbating the effect of the negative spillover, and reducing any potential effect on output. I discuss the effect of each of these policies in more detail below.

Appendix E.1. Taxing R&D investments

The results in Section **??** of the main text demonstrate that output is increasing in the level of the general R&D subsidy, but that a household's utility is decreasing in the subsidy (driven by decreasing consumption and increasing hours worked). As such, if a government wished to increase household utility rather than just output, then it might want to tax firms' investments in R&D rather than subsidising them. Although rare, some countries apply a tax to R&D investments: an OECD report on R&D tax incentives (OECD (2022)) finds that countries such as Malta, Argentina, and even Japan in some circumstances, have taxes as high as 3% on R&D investments.

Therefore, in Table E.10 I present results obtained setting $\tau_r < 0$ (i.e. a general tax on R&D investments). The second column of this table contains the results from the baseline model with a general R&D subsidy of 5%, while columns 3 and 4 contain the results of scenarios when the R&D subsidy is at the level indicated in the first row of the table (with a negative value indicating a tax on R&D investments).

As one would expect, the results obtained by increasing the tax on R&D investments mirror those from increasing the general R&D subsidy. In particular, increasing the tax on

[15] In each alternative policy, as with general R&D subsidies in the previous Section, I allow the corporation tax to vary to ensure that the government surplus Λ from the base-case is constant in each scenario. Similar to the case with general R&D subsidies, the corporation tax only changes by a few percentage points at most. As a robustness check, I re-run each scenario keeping the corporation tax rate fixed and this does not substantially change my results due to the factors discussed previously.

Table E.10: Effect of taxes to R&D on aggregate variables

Aggregates	Baseline	$\tau_r = -0.05$	$\tau_r = -0.1$
R&D stock	0.22	96.7	95.1
Mass of firms	1.31	99.9	99.8
Output	0.50	99.98	99.97
Capital	1.04	99.99	99.98
Consumption	0.44	100.03	100.04
Labour	0.31	99.95	99.93
Average productivity	1.01	100.01	100.01
% of firms with zero R&D stock	42.3%	44.1%	44.7%
Compensating Variation (% of income)	N/A	-0.04%	-0.06%

Note: The results for the baseline model are with an R&D subsidy rate $\tau_r = 0.05$ applying to all firms and are the levels of each aggregate variable obtained in my calibrated model. The results for other values of τ_r are obtained by re-solving the stationary equilibrium for my model with that level of τ_r and allowing the level of the corporation tax τ_c to adjust so that the government surplus is kept the same as it was for the baseline scenario. The results for these other values of τ_r are presented relative to the baseline scenario except for the last two rows of the table, which are, respectively, the actual proportion of firms with zero R&D stock in each scenario and the percentage of income required to restore the representative household to its utility level in the baseline scenario (a positive compensating variation indicates that firms are worse off under that particular scenario, while a negative compensating variation indicates that firms are better off compared to the baseline).

a firm's R&D investments reduces the level of aggregate R&D and increases the number of firms with zero stock of R&D. This results in a small reduction in output and capital, predominantly due to the reduction in the mass of firms (as new entrants start off smaller and are less likely to survive). This decrease in the mass of firms reduces hours worked, while consumption is barely changed, such that the compensating variation is decreasing in the level of the tax (a negative compensating variation indicates that households are better off under the alternative scenario compared to the baseline).

Appendix E.2. Targeted R&D subsidies

An alternative policy that might reduce the dampening effect of negative spillovers to aggregate R&D would be to offer higher R&D subsidies only to certain firms. This could operate by ensuring that only those firms that already had high R&D stocks would be able to access the higher R&D subsidies, thereby reducing the impact of higher subsidies on firms that previously had zero R&D stocks but decided to invest in a small amount of R&D when the general subsidy increased. In theory, this could ameliorate the increase in aggregate R&D as fewer firms decide to increase their R&D investments, which in turn would reduce the effect of the negative spillovers.

Therefore, I present results in Table E.11 for targeted R&D subsidies where a 50% subsidy to R&D is applied only to firms that already have a stock of R&D above a certain threshold. In particular, I present results where firms with an R&D stock greater than the mean (the third column of the table) R&D stock in the baseline case are eligible for the higher 50% subsidy, and where that threshold is 1.5 times the mean (the fourth column) R&D stock in the baseline scenario. The second column of this table contains the results from the baseline model with a general R&D subsidy of 5%, while columns three and four contain the results of scenarios of these two alternative thresholds for targeted subsidies.

Table E.11: Effect of changing threshold of R that gets a 50% subsidy on aggregate variables

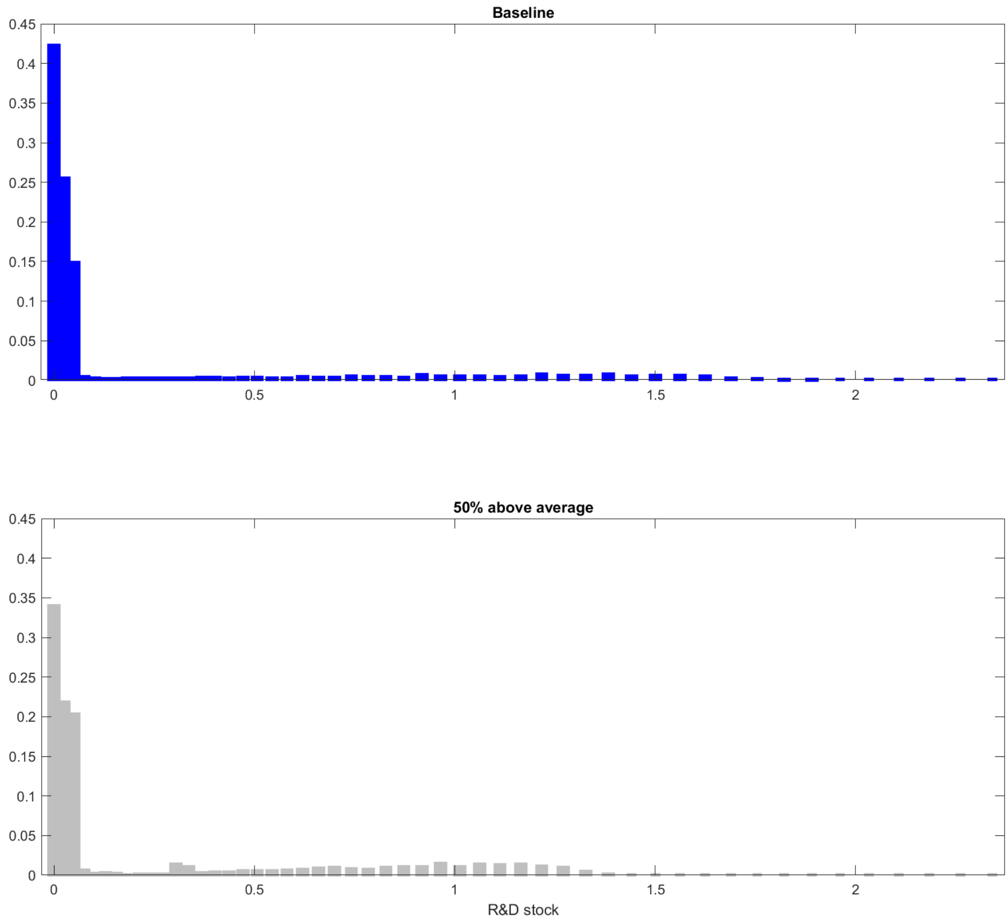
Aggregates	Baseline	> mean	>50% more
R&D stock	0.22	121.2	119.6
Mass of firms	1.31	101.0	100.8
Output	0.50	100.17	100.15
Capital	1.04	100.0	100.1
Consumption	0.44	99.78	99.81
Labour	0.31	100.40	100.35
Average productivity	1.01	99.92	99.92
% of firms with zero R&D stock	42.3%	29.7%	34.3%
Compensating Variation (% of income)	N/A	0.32%	0.28%

Note: The results for the baseline model are with an R&D subsidy rate $\tau_r = 0.05$ applying to all firms and are the levels of each aggregate variable obtained in my calibrated model. The results for changing the threshold of R at which a firm obtains a 50% subsidy obtained by re-solving the stationary equilibrium for my model with that level of τ_r and allowing the level of the corporation tax τ_c to adjust so that the government surplus is kept the same as it was for the baseline scenario. The results for these varying thresholds are presented relative to the baseline scenario except for the last two rows of the table, which are, respectively, the actual proportion of firms with zero R&D stock in each scenario and the percentage of income required to restore the representative household to its utility level in the baseline scenario (a positive compensating variation indicates that firms are worse off under that particular scenario, while a negative compensating variation indicates that firms are better off compared to the baseline).

The effect of targeted R&D subsidies is not the intended one. In particular, the increase in aggregate R&D is actually higher relative to the baseline economy (an increase of roughly 20%) than it is with an increase in the general R&D subsidy to 50% (an increase of about 18%). This is because some firms that would otherwise be just below the threshold under the baseline 5% general R&D subsidy now find it worthwhile to increase their stock of R&D above that threshold so that they can benefit from the higher subsidies that begin at that level of R&D stock. Meanwhile, the higher aggregate R&D means that large firms decide to reduce their stock of R&D due to the negative spillover caused by the higher aggregate R&D.

This can be seen in Figure E.3, which shows the distribution of firms over their stock of R&D in the baseline case (the top panel) and the case where firms with more than 1.5 times the mean R&D stock in that baseline are eligible for a 50% subsidy to their R&D investments (the bottom panel). The distribution of firms under targeted R&D subsidies has a mass of firms at around the 0.3 level of R&D stock that is not present in the baseline scenario, while there are far fewer firms with zero R&D stocks. This decrease in the proportion of firms with zero R&D stock is driven by the fact that more firms now want to invest in R&D sooner in their lifecycles so they can reach the threshold at which the higher subsidy kicks in, so they can obtain even cheaper R&D investments. Finally, firms that previously had large stocks of R&D no longer do so, as in the case with higher general R&D subsidies.

Figure E.3: Effect of targeted R&D subsidies on the distribution of R&D stock



Note: The results for the R&D subsidy rate $\tau_r = 0.05$ are the levels of each aggregate variable obtained in my calibrated model and is shown in the top panel. The results for other values of τ_r are obtained by re-solving the stationary equilibrium for my model with that level of τ_r and allowing the level of the corporation tax τ_c to adjust so that the government surplus is kept the same as it was for the $\tau_r = 0.05$ scenario. Each panel of the figure shows the steady-state proportion of firms at each level of the R&D stock. In the bottom panel, only firms with a stock of R&D 50% higher than the mean stock of R&D in the baseline case are given a 50% subsidy to their R&D investments while all other firms receive a 5% subsidy.

The higher targeted subsidies mean that new entrants with high research productivity can grow more quickly and are more likely to survive, resulting in a higher mass of firms under targeted subsidies. This serves to increase the level of output and capital stock slightly, while the slightly higher mass of firms means that the representative household's consumption is reduced and labour is increased. This means that the household would need to be compensated roughly 0.3% of their income to return them to the utility level they obtained under the baseline scenario. These effects are of roughly the same magnitude as the effect of increasing the general R&D subsidy to the same 50% level (which increased output by only 0.1%).

However, these effects do not appear to be monotonic. In particular, the output, the mass of firms, and the compensating variation required are higher when the threshold at which the 50% R&D subsidies start is at the mean R&D stock in the baseline scenario compared to when that threshold is 1.5 times that mean (although the differences are small).

This is due to the effect of the threshold on the number of firms with zero R&D stocks. When the threshold is low (e.g. at the mean of the baseline R&D stock) then more firms can afford to invest in R&D to get to that threshold in the first place, enabling them to benefit from the higher R&D subsidies, while fewer firms are dissuaded from investing in R&D at all (the proportion of firms with zero R&D stocks is only marginally higher under the low threshold as compared to the baseline case). Once the threshold is raised higher, more and more firms choose not to try obtaining sufficient R&D stocks to reach that threshold (such that the number of firms with zero R&D is much higher), while it is also harder for new entrants to obtain that threshold and access the higher subsidies, reducing the number of entrants that survive. This means that there are fewer firms and therefore output is lower.

Appendix E.3. Allowing firms to use R&D stocks as collateral

As mentioned in Section **??** of the main text, in the baseline model firms can only use the stock of physical capital as collateral against which they can borrow. However, there is some evidence that the use of intangibles (such as stocks of R&D, but also including brand loyalty, "goodwill", and trademarks) as collateral has been increasing over time. For example, Mann (2018) shows that 16% of patents owned by US firms have been pledged as collateral at some point, while Loumioti (2012) finds that as much as 21% of US secured loans use intangible assets for at least some of their collateral.

Therefore, I examine a scenario in which firms can use some proportion of their stocks of R&D as collateral alongside their stocks of physical capital. This means that the collateral constraint in Equation (**??**) in the main text becomes $B' \leq \theta K' + \theta_R R'$ (instead of $B' \leq \theta K'$), where θ_R is the proportion of a firm's stock of R&D that it can use as collateral. However, a firm's stock of R&D remains completely irreversible: in other words, they remain unable to sell any of their stock of R&D. This means that although there could be a risk of default (a firm exiting without paying back all of its debt) if a firm has to repay more than the value of its output and undepreciated capital stock, no firm in my model chooses to borrow more than the value of its undepreciated physical capital stock for any scenario of $\theta_R > 0$ presented here. In other words, firms still do not default in this scenario.

The results of this exercise are presented in Table E.12. The second column of this table contains the results from the baseline model with a general R&D subsidy of 5% and where firms cannot use any of their R&D stock as collateral, while columns 3 and 4 contain the results of scenarios varying the level of θ_R.

Table E.12: Effect of letting firms use R as collateral on aggregate variables

Aggregates	Baseline	$\theta_R = 0.05$	$\theta_R = 0.1$
R&D stock	0.22	102.7	111.6
Mass of firms	1.31	99.5	98.7
Output	0.50	99.9	99.7
Capital	1.04	100.0	99.8
Consumption	0.44	99.9	99.8
Labour	0.31	100.0	99.8
Average productivity	1.01	100.0	100.0
% of firms with zero R&D stock	42.3%	41.3%	39.3%
Compensating Variation (% of income)	N/A	0.04%	0.05%

Note: The results for the baseline model are with an R&D subsidy rate $\tau_r = 0.05$ applying to all firms, with firms not able to borrow against any proportion of their R&D stock, and are the levels of each aggregate variable obtained in my calibrated model. The results allowing firms to borrow against some portion of their stock of R&D i.e. $\theta_R > 0$) are obtained by re-solving the stationary equilibrium for my model with the same R&D subsidy rate of $\tau_r = 0.05$ and allowing the level of the corporation tax τ_c to adjust so that the government surplus is kept the same as it was for the baseline scenario (the only reason that τ_c changes in this scenario is because firms invest more in R&D, such that total government spending on the R&D subsidy increases relative to the baseline scenario even though the rate of the subsidy does not). The results for these different proportions of R allowed as collateral are presented relative to the baseline scenario except for the last two rows of the table, which are, respectively, the actual proportion of firms with zero R&D stock in each scenario and the percentage of income required to restore the representative household to its utility level in the baseline scenario (a positive compensating variation indicates that firms are worse off under that particular scenario, while a negative compensating variation indicates that firms are better off compared to the baseline).

As one would expect, letting firms use their stock of R&D as collateral incentivises additional investment in R&D. This means that there are fewer firms with zero R&D stock and the aggregate level of R&D also increases as the proportion of R&D stock that can be used as collateral increases.

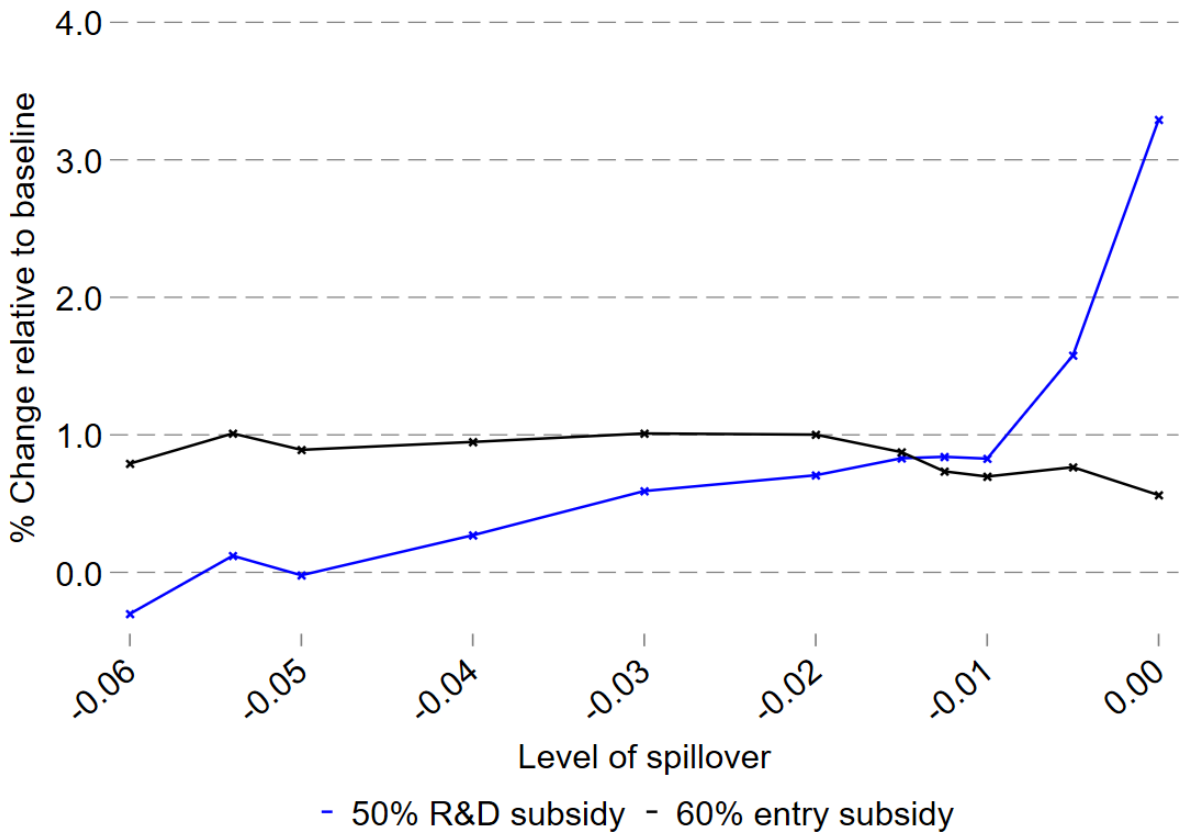
However, the mass of firms is slightly decreasing in θ_R as new entrants are less likely to survive. This is due to the fact that new entrants take on more debt than they do under the baseline scenario (as well as having substantially higher R&D stocks), which means that when they suffer a negative idiosyncratic output or research productivity shock they are more likely to find it profitable to just pay off their current debt and exit rather than gradually trying to recover from the impact of the negative shock.

This lower mass of firms combined with higher stocks of R&D and physical capital means that output is almost entirely unchanged when the amount of R&D stock that can be used as collateral changes. The decrease in consumption means that the compensating variation is decreasing in the amount of R&D stock that can be used as collateral θ_R, despite the slight reduction in hours worked as θ_R increases. In other words, the representative household is worse off when firms can borrow more against their stock of R&D.

Appendix F. The importance of the level of aggregate R&D spillovers

As covered in the main text, under the estimated coefficient of -0.054 for spillovers from aggregate R&D, increasing the R&D subsidy from 5% to 50% only increases output by 0.1%, compared to an increase in output of 3.2% when the spillover parameter is set to zero in the model. Figure F.4 shows how the effect on output of increases in the R&D subsidy from 5% to 50% (the blue line), and the implementation of a (cost equivalent) 60% entry subsidy (the black line) vary with the level of the spillover. The x axis shows the level of the spillover from aggregate R&D, while the y axis shows the percentage change relative to a baseline 5% R&D subsidy and no entry subsidy (where the level of the spillover is also that shown by the x axis). In this exercise, only the parameter for the level of the spillover varies; all other parameters are kept as in the model described in the main text.

Figure F.4: Effect of different spillover levels on policy outcomes



All results in this graph are obtained by keeping every parameter the same except for the parameter regarding the spillovers from aggregate R&D. The blue line shows the effect on output of increasing the R&D subsidy from 5% to 50%, while the black line shows the effect of a (cost equivalent) implementation of a 60% entry subsidy.

These results show that the effect of the entry subsidy on output is relatively stable as the level of the spillover becomes more positive: the entry subsidy increases output by roughly 1% in each case. However, as one would expect, the R&D subsidy is much more effective as the level of the spillover becomes more positive. In particular, the R&D subsidy's increase in aggregate R&D has less of a negative effect on firms as the level of the spillover becomes more positive, such that firms gain much more from having larger stocks of R&D. This means that firms' productivities increase much more under the higher R&D subsidy as the spillover becomes more positive, leading to higher aggregate output.

This also affects the comparison between the two subsidy policies. At a level of R&D spillovers that is more negative than roughly -0.015, entry subsidies remain more effective at boosting output than are R&D subsidies, as per the results in the main text. However, once the level of the R&D spillovers becomes more positive than -0.015, R&D subsidies are more effective at increasing output than are entry subsidies, and this difference increases substantially as the level of the R&D spillovers gets closer to zero.

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