Boosting Innovation or Entry: What Works Best?*

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Abstract

Governments spend substantial sums subsidising firm investment in Research and Development (R&D) with the aim of increasing output. How effective are these subsidies? Could alternative policies boost output more? I develop a dynamic general equilibrium model that is the first to enable examination of fiscal policies' effects when heterogeneous firms choose R&D, physical capital, and debt subject to a collateral constraint distinguishing between physical capital and R&D. I use an of the estimate the relationship between R&D and a firm's productivity in my model alongside calibrating other parameters to match US economic aggregates and reproduce the joint distribution of firms over R&D and physical capital. I find that policies incentivising firm R&D have a small effect on output due to negative spillovers from higher aggregate R&D, whereas subsidising entry instead of R&D achieves ten times the increase in output because entry subsidies do not result in large negative R&D spillovers.

Key words: R&D, Spillovers, Industrial policy, Subsidies

JEL Codes: E22, E62, D22, D24, H32

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1. Introduction

How effective are government policies that aim to increase output by incentivising firm investment in Research & Development (R&D)? Could alternative policies, such as subsidising entry, boost output more? The US government spends billions on providing R&D subsidies to firms and, in recent years, has announced its intention to stimulate firm investment in R&D even further.[1] However, the presence of spillovers from firm R&D investments means it is not clear to what extent these policies could actually increase overall R&D or output.[2] Moreover, it is possible that alternative policies that do not focus on R&D could raise aggregate output more.

I demonstrate that there is substantial heterogeneity in the joint distribution of firms across their stocks of physical capital and R&D. I then construct a model that captures this rich heterogeneity across firms and use it to assess the effectiveness of alternative policies that could increase output using a dynamic general equilibrium model. My model includes four key components that enable me to do so. First, firms are heterogeneous in their choice of stocks of R&D, physical capital, and debt, to capture the rich heterogeneity I have found in the data, and to allow for potential heterogeneous firm responses to different policies. Second, I specify a productivity process that depends on a firm's own stock of R&D and the aggregate stock and estimate the relationship between productivity and R&D using US firm-level data. This allows me to capture the impact of any spillovers from aggregate R&D on an individual firm's productivity. Third, I include endogenous entry (and exit) so I can examine the effect of subsidising new, faster-growing firms, rather than older, slower-growing incumbents.

Finally, a firm's choice of debt is subject to a collateral constraint that depends only on its choice of physical capital in my model. This encapsulates a crucial difference between R&D and physical capital: the latter can be sold and used as collateral, whereas the former cannot.[3] This difference provides firms with an incentive to invest in physical

[1] See, for example, speeches by President Biden in July 2021 (https://www.whitehouse.gov/briefing-room/speeches-remarks/2021/07/27/remarks-by-president-biden-at-the-office-of-the-director-of-national-intelligence/) and May 2022 (https://www.whitehouse.gov/briefing-room/speeches-remarks/2022/05/06/remarks-by-president-biden-on-the-bipartisan-innovation-act/).

[2] These spillovers can include positive ones such as the diffusion of a new technology to rival firms reducing those firms' costs, or negative ones such as the innovation and introduction of a new product stealing sales from rivals.

[3] This difference between R&D and physical capital is due to the intangible nature of the former as described in Bloom, Van Reenen and Williams (2019) and Haskel and Westlake (2018).

capital rather than R&D, creating a possible role for fiscal policy to try to incentivise more R&D investments and potentially increase output. To my knowledge, I am the first to study the effect of R&D subsidies and other fiscal policies in a heterogeneous firm model with collateral constraints distinguishing between physical capital and R&D.

I specify a production function in which a firm's idiosyncratic productivity depends on both its own stock of R&D and the aggregate stock. My use of the stock of R&D provides a parsimonious way to capture the lags between R&D expenditures and their effect on a firm's productivity. I estimate this relationship between R&D and firm productivity using US firm-level data and find that a firm's productivity has decreasing marginal returns in the firm's own stock of R&D, and that there are negative spillovers from higher aggregate R&D.[4] A higher aggregate stock of R&D reduces the positive contribution a firm's own R&D stock has on that firm's productivity. Using these results as parameters in my model, I calibrate other model parameters to match aggregates for the US economy as well as the joint distribution of firms over their stocks of R&D and physical capital.

The negative spillovers from aggregate R&D drive my result that government policies to stimulate firm R&D only have a small effect on output. Increasing the level of the general R&D subsidy from 5% to 50% only increases output by 0.1%, despite raising the aggregate stock of R&D by 18%.[5] Without the negative spillovers, output would increase by 3.2% as a result of the same increase in the general R&D subsidy.

Higher R&D subsidies substantially increase the aggregate level of R&D, with the resulting negative spillovers eroding any benefits from an individual firm increasing its own stock of R&D. This leads to a heterogeneous firm response. Firms that have little or no stock of R&D when the R&D subsidy is low invest more in R&D when the subsidy is raised, but firms that had high stocks of R&D when the R&D subsidy is small reduce their stocks of R&D. I obtain similar results for R&D-focused policies that include increasing targeted R&D subsidies or letting firms use a portion of their stock of R&D as collateral. This means that the overall effect of higher R&D subsidies on economic

4 This is in contrast to the results obtained by Lucking, Bloom and Van Reenen (2019), but consistent with those obtained by Dieppe and Mutl (2013) and Kim and Choi (2019), suggesting that in this case the effect of product-market rivalry and other negative spillovers (such as increased demand for R&D increasing the cost of inputs into R&D, including scientists' wages) outweigh the positive spillover effect of technology diffusion.

5 OECD (2022) finds that the level of the R&D subsidy in the US has been roughly 5% for the past 10-20 years, while the highest level of the subsidy in OECD countries is roughly 50%.

aggregates such as output and consumption are relatively small, similar to the results obtained by Atkeson and Burstein (2019).

An alternative policy of subsidising entry leads to an increase in output of 1.0%; for the same cost, entry subsidies are ten times as effective at increasing output as are R&D subsidies. The entry subsidy avoids the negative spillovers that result from higher aggregate levels of R&D and results in a larger mass of output-producing firms. Potential entrants have an exogenous cash endowment such that some high productivity entrants are unable to afford the fixed cost of entry unless it is subsidised. Hence, entry subsidies allow more high productivity firms to enter and start producing, thereby increasing output. Moreover, the entry subsidies result in more, fast-growing, entrants (as per Luttmer (2011)'s findings) rather than mostly subsidising incumbents as happens with higher R&D subsidies.

My paper contributes to three main strands of literature. First, my paper contributes to the literature examining the effect of R&D subsidies. Relative to previous studies, I take into account the fact that firms can invest in either physical capital and R&D, and are subject to financial frictions that distinguish between those two stock variables. This enables me to make a meaningful comparison between R&D subsidies, other policies that try to incentivise investment in R&D, and entry subsidies. As in Acemoglu et al. (2018) my model incorporates heterogeneous research productivity although in my model firms can move from low to high productivity as well as from high to low. This reflects the possibility for a firm to have a useful idea / innovation many years after coming into existence. My result that R&D subsidies have a small effect on output is similar to their finding, albeit via a different mechanism.

My results also relate to the mis-allocation aspect of R&D subsidies noted by Akcigit, Hanley and Serrano-Velarde (2021). They are driven by the heterogeneous response of firms to the subsidy, reinforcing Galaasen and Irarrazabal (2021)'s result that it is crucial to account for firm heterogeneity when considering R&D. In particular, a higher subsidy increases the incentive and ability of firms that had zero stocks of R&D to invest in some R&D, while the higher level of aggregate R&D "crowds out" firms that previously had large stocks of R&D so that they reduce their holdings of R&D stocks.[6] Hence, my results

[6] The fact that some firms reduce their R&D after an increase in the R&D subsidy is consistent with Berger (1993)'s finding that some firms decreased their R&D expenditure when the US R&D Tax Credit was introduced.

add further to those that find R&D subsidies can have negligible effects on output, such as Atkeson and Burstein (2019) and Davidson and Segerstrom (1998).

My second contribution is to the literature on firm lifecycles, which I extend to account for firms' growth in terms of R&D as well as physical capital, subject to financial constraints. Closest to my model in this area are Ericson and Pakes (1995) and Pakes and Ericson (1998) in which firms have the opportunity to invest in R&D to boost their productivity and highlight the importance of taking R&D into account.[7] My focus on the effect of a policy on economic aggregates mirrors Hopenhayn and Rogerson (1993), albeit analysing the effects of different policies and incorporating the fact that firms can grow in more than one dimension. Sterk, Sedlacek and Pugsley (2020) highlights the importance of taking account of different types of heterogeneity (ex-ante vs persistent ex-post shocks) and my model finds another dimension of heterogeneity (research productivity versus output productivity, affecting whether firms grow by investing in R&D or in physical capital) matters for firms lifecycles.

Moreover, my inclusion of firms being subject to a borrowing constraint as they grow reflects Cabral and Mata (2003) and Dinlersoz et al. (2018)'s findings that binding financial constraints can have strong effects on how firms grow over time. The importance of these financial constraints is highlighted by Howell (2017)'s finding that the level of financial constraints a firm faces affects the effectiveness of any research grants they receive. My model also captures Gertler and Gilchrist (1994)'s finding that smaller firms are more likely to be subject to financial constraints than are larger firms. My modelling of the effect of R&D subsidies alongside a firm financial friction is a first for this literature and provides a potential role for government in incentivising individual firm R&D even in the presence of negative spillovers to aggregate R&D.

Finally, I also contribute to the literature regarding the effect of R&D on a firm's productivity. Syverson (2011)'s survey of the literature finds that there are many factors that might affect a firm's productivity. De Loecker (2013) estimates a production function with endogenous productivity to show that a firm's productivity is affected by how much it exports, while I apply this approach to show that a firm's stock of R&D matters for its productivity. This is similar to Doraszelski and Jaumandreu (2013)'s finding that

[7] Pakes and Ericson present their model as an alternative to the learning-by-doing mechanism covered in Jovanovic (1982).

a firm's R&D investment in the previous period affects firm productivity (although their use of R&D expenditure rather than the stock of R&D assumes full depreciation of R&D each period).

The estimated effect of R&D on a firm's productivity in my results is a similar magnitude to that found by Kehrig, Miao and Xu (2022).[8] In contrast to Lucking, Bloom and Van Reenen (2019), my production function estimation indicates that there are negative spillovers to higher aggregate R&D. This is due to my use of the aggregate stock of R&D in the economy as a more general way of capturing these spillovers rather than focusing specifically on product-market rivalry and technology diffusion. In this way, my results add support for the business-stealing effect of R&D in which one firm's R&D investments that lead to the introduction of new products steal sales away from rival firms, as described in Bloom, Schankerman and Reenen (2013).

The rest of this paper proceeds as follows. Section 2 presents the evidence for firms growing in terms of R&D as well as in terms of physical capital, as well as the evidence that a firm's stock of R&D matters for its productivity. Section 3 sets out my model, while Section 4 describes how I calibrate this model to match aspects of the US economy.

Section 5 contains the results of analysing the effect of changing R&D subsidies on economic aggregates as well as the growth of a simulated cohort of entrants, and a discussion of the mechanisms underlying those results, while Section 6 presents the same for subsidising entry. Section 7 concludes.

2. The importance of R&D

In this Section I demonstrate two empirical facts. First, I show that there is substantial heterogeneity in the growth of firms' stocks of R&D and physical capital, which leads to an asymmetric joint distribution of firms over these two stocks. Second, I find that firms with positive stocks of R&D tend to have higher idiosyncratic productivity than firms that have no R&D stock. I discuss each of these two facts in turn below.

[8] Kehrig, Miao and Xu (2022) use Census data and estimate the effect of a firm's R&D expenditures on their labour productivity (defined as the ratio of revenues to labour expenses) using a linear regression in which labour productivity depends on a firm's R&D investment and a set of control variables.

2.1. The asymmetric joint distribution of firms over R&D and physical capital

Firms tend to specialise either by investing in R&D or in physical capital. Figure 1 shows how some firms' stocks of physical capital and R&D grow over time, with six specific firms highlighted.[9] The change in each stock variable reflects firms' investments in that stock over time, thereby capturing any persistent differences in firms' investment decisions. Each cross in the figure is an individual firm-year observed in the data, and the numbers next to each cross are the number of years that firm has appeared in the Compustat data.[10] [11] Six firms have been highlighted to show that there are persistent differences in firm investment decisions. For example, PetroChina (the red crosses) and Nippon Telegraph (orange crosses) invest almost entirely by increasing their capital stocks over time, only accumulating a small amount of R&D stocks after having been present in the data for a number of years.

By contrast, Microsoft (cyan) and Pfizer (blue) invest by substantially increasing their stock of R&D while only increasing their capital stocks marginally over the entire period. As a sort of middle-ground VW (grey) and Toyota (purple) have added substantially to their R&D stocks while at the same time also increasing their capital stocks.[12] There are no firms in the top-right area of the graph, indicating that no firm in the data has very high stocks of both physical capital and R&D (i.e. no firm persistently invests heavily in both R&D and physical capital).

This persistent heterogeneity in firm investment decisions is also reflected in the joint distribution of firms over their stocks of capital and R&D, as shown in Figure 2 for 2007.[13] The y axis shows the distribution of firms over their stock of R&D while the x axis

[9] This figure and all results in this Section are created using Compustat data regarding publicly-listed firms in the USA from 1963 until 2016. Details regarding the data construction are provided in Online Appendix A.

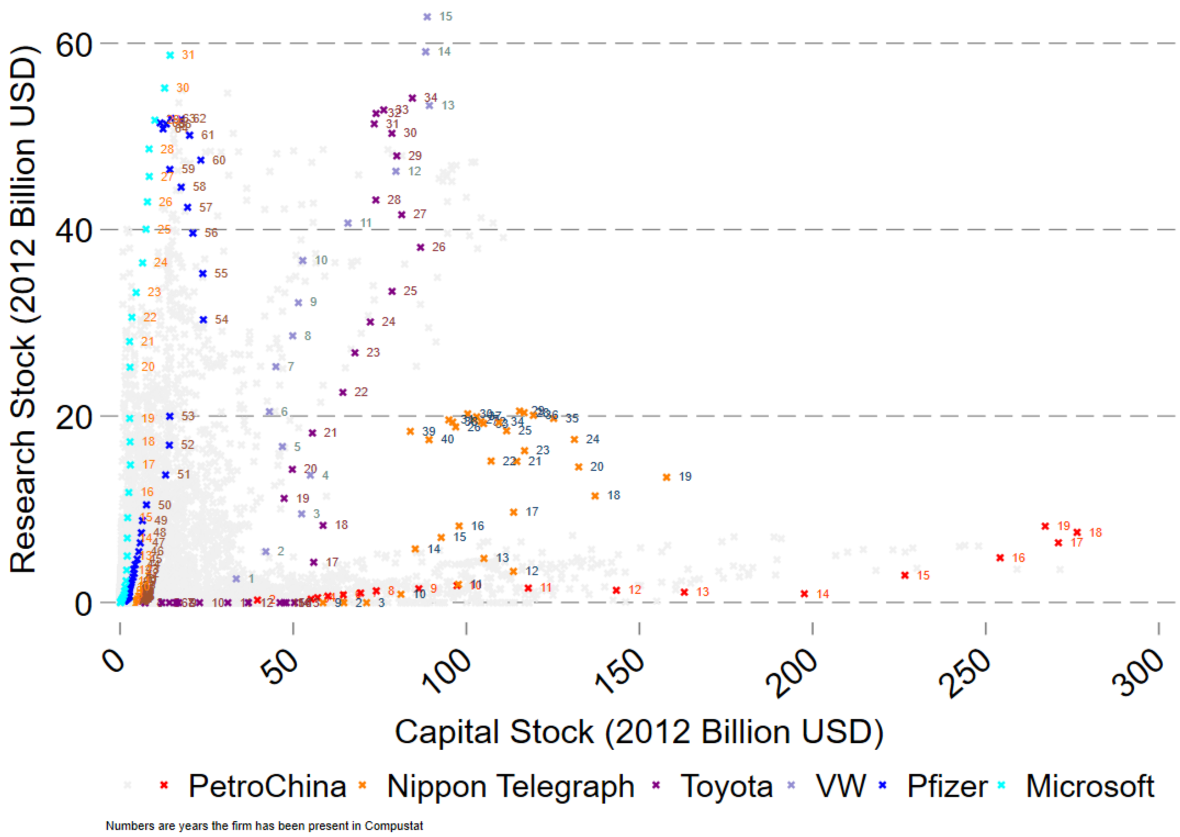
[10] Due to the nature of Compustat data, the actual firm age at each observation is likely to be higher than the number of years shown in the figure.

[11] Although my empirical results use Compustat data, such that they are based on a small subset (large, public, firms) of all firms, firms covered by Compustat account for more than 90% of private R&D spending in the USA and therefore are likely to capture accurately how firm's R&D investments and stocks affect their growth and productivity.

[12] The grey crosses in the graph below show all other firms present in Compustat (i.e. each grey cross is a firm-year combination for a non-highlighted firm), and the fact that the highlighted firms overlap with these non-highlighted firms shows that the firms I have highlighted here are not outliers, but reflect a general pattern of firms taking different paths in terms of how they grow over time.

[13] The joint distribution for other years follows a similar pattern as that for 2007.

Figure 1: Growth of firms' capital and R&D stocks over time

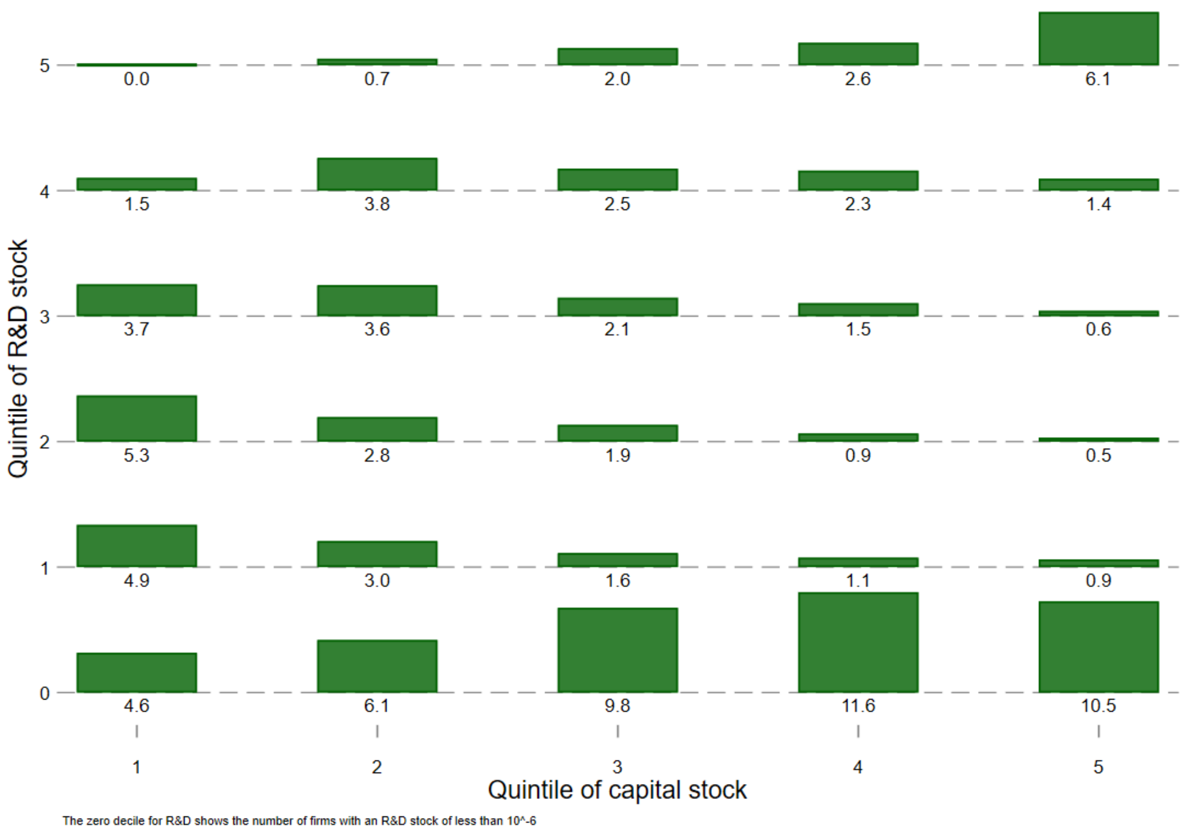


Note: Created using Compustat data from 1963-2016. Each cross is a single firm-year observation. The numbers by the side of each cross are the number of years a firm has been observed in the Compustat data. The capital stock is calculated by combining a firm's net property, plant, and equipment values with BEA industry-specific deflators to account for industry-specific rates of inflation. A firm's R&D stock is obtained using the perpetual inventory method on a firm's annual R&D expenditures along with an assumed annual depreciation rate of 15% and an initial R&D stock of 10^{-6} in the first year of a firm's appearance in the data.

shows the quintiles of the distribution of firms over their capital stock. The row labelled zero contains all firms with R&D stocks of less than $\$10^{-6}$ billion, and the rows labelled 1-5 referring to the quintiles of a firm's stock of R&D given that stock is greater than this. In this way, the bottom row of the graph shows the proportion of firms that have negligible (i.e zero) stocks of R&D, while the rows labelled 1-5 shows the distribution of firms given that they have some positive stock of R&D. The x axis shows the quintiles of the distribution of firms over their capital stock.

Firms in the bottom two quintiles of capital stock are more likely to have some non-zero amount of R&D stock than are firms at the top of the capital stock distribution. In particular, there are only 4.6% of firms in the bottom quintile for the stock of physical

Figure 2: Joint distribution of firms over Capital and R&D stocks for 2007



Note: Created using Compustat data from 1963-2016. The capital stock is calculated by combining a firm's net property, plant, and equipment values with BEA industry-specific deflators to account for industry-specific rates of inflation. A firm's R&D stock is obtained using the perpetual inventory method on a firm's annual R&D expenditures along with an assumed annual depreciation rate of 15% and an initial R&D stock of 10^{-6} in the first year of a firm's appearance in the data. The zero row for the R&D stock shows the proportion of firms with zero R&D, with the R&D quintiles then reflecting a firm's R&D stock given that that stock is greater than zero.

capital and no R&D stock, but 10.5% of firms are in the top physical capital quintile and no R&D stock. Of those firms that have some stock of R&D, those with a small amount of R&D are more likely to be firms towards the bottom of the capital distribution. For example, there are 5.3% of firms in the bottom quintile of the capital distribution and in the second quintile of the non-zero R&D distribution (column 1 and the row labelled 2), whereas there are 0.5% of firms that are at the top of both the capital distribution but in that same R&D "quintile" (column 5 and the row labelled 2).

On the other hand, the top of the R&D distribution is populated more by firms that also have a lot of capital. For example, 6.1% of firms are in the top quintiles of both R&D and capital, whereas a negligible proportion of firms are in the top quintile of R&D and the bottom quintile of the capital distribution.

The heterogeneous investment in, and distribution of firms over their, stocks of physical capital and R&D suggest that the effect of R&D subsidies is likely to be be heterogeneous across firms.[14] For example, the effect of a higher general R&D subsidy on a firm with lots of R&D and little physical capital is likely to be very different from its effect on a firm with no R&D and lots of physical capital.[15]

Therefore it is important to take into account this heterogeneity when assessing the effect of government policies that incentivise firm investments in R&D.

2.2. *The effect of R&D stocks on firm productivity*

The fact that some firms focus on increasing their stocks of R&D rather than their physical capital stock indicates that a firm's stock of R&D could be an important component in determining their productivity. In order to investigate the effect of a firm's stock of R&D on their output, I first estimate a firm's production function following Ackerberg, Caves and Frazer (2015) (ACF), which assumes that a firm's productivity is completely exogenous.[16]

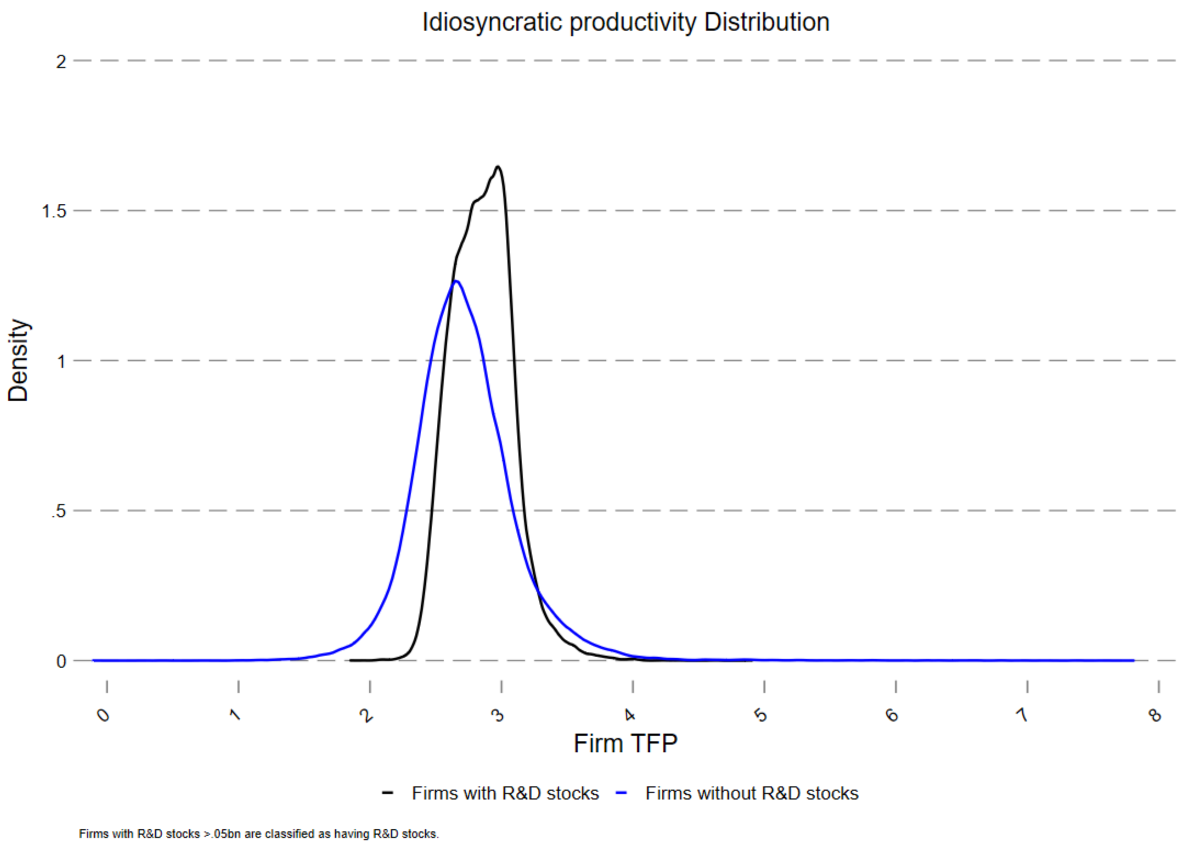
Figure 3 plots the kernel density of firm productivity estimated using this procedure separately for firms with non-zero stocks of R&D (the black line) and firms without any R&D stock (the blue line). The distribution of productivity for firms with non-zero stocks of R&D is further to the right (representing higher productivity) than is the distribution of productivity for firms without any stock of R&D. This indicates that a firm's stock of R&D is likely to affect its productivity and that, therefore, not including stocks of R&D in a firm's idiosyncratic productivity is likely to be crucial in capturing firms' decisions to invest in R&D in any model.

[14] The heterogeneity of firms over their R&D and physical capital stocks is in addition to Galaasen and Irarrazabal (2021)'s result that heterogeneity in a firm's research productivity matters for the effect of R&D subsidies on aggregate R&D investment.

[15] The majority of the heterogeneity of firms over these stocks is due to differences between individual firms rather than between industries. A decomposition of the variation in firms' stocks of physical capital and stocks of R&D indicate that generally 15% - 25% of the variation is due to the industry (depending on the industry classification system that is used), with the remainder being due to individual firm differences.

[16] The ACF procedure assumes that the idiosyncratic productivity process follows an AR(1) Markov process.

Figure 3: Distribution of exogenously estimated idiosyncratic productivity split by firms with and without R&D stocks



Using Compustat data between 1963 and 2016, with the estimated productivity obtained via a regression using data between 1980 and 2016. Estimated using ACF procedure where the productivity process is determined by an AR(1) Markov process and annual dummy variables. The idiosyncratic firm productivity removes the effect of aggregate productivity as captured by the annual dummy variables. The capital stock was obtained by deflating the Compustat book-value of a firm's capital stock by BEA industry-specific deflators. The R&D stock used was obtained via the perpetual inventory method with a depreciation rate of 15% and setting $R_0 = 10^{-6}$.

3. Model

In this Section I set out the features of my discrete-time general equilibrium model and characterise agents' optimisation problems. There are three categories of agents in my model: an infinitely-lived representative household, heterogeneous firms, and a government. The representative household has preferences over consumption C and leisure $1 - N$ and owns the firms (so receives all profits). It also provides one-period discount bonds for firms to borrow or save.

There are a large number of heterogeneous firms, each producing an homogeneous good.

In each period these firms could receive an exogenous death shock with probability π_d, and are subject to persistent idiosyncratic output productivity ε_o and research productivity ε_r shocks. Incumbent firms can also choose to exit after producing if they do not wish to pay a fixed continuation cost ξ. Continuing firms, and potential entrants that choose to enter, choose their physical capital stock K', R&D stock R' and debt B' (subject to a collateral constraint and in the form of a one-period discount bond, with $B' > 0$ indicating debt and $B' < 0$ representing savings) that they take into the next period. A firm's stock of R&D affects its output only through its effect on the firm's idiosyncratic output productivity. My model has three endogenous idiosyncratic state variables (capital stock K, stock of R&D R, debt holdings B) as well as exogenous idiosyncratic shocks to output productivity ε_o and research productivity ε_r over which there is a distribution μ of firms.

Physical capital and the stock of R&D differ in two crucial ways in my model, due to the intangible nature of R&D. This intangibility of R&D reflects the fact that the outputs of R&D are difficult to value and rarely come up for sale. Even the most easily measurable R&D output (patents) are very challenging to value due to their unique nature. This means that firms are less able to borrow against their R&D outputs than they are their tangible assets such as physical capital because 1) the lender would find it challenging to value any R&D output put up for collateral, thereby making it difficult for the bank to work out just how much a firm could borrow against its R&D output; and 2) if the firm were to default, a lender would run into difficulties when trying to recover funds via selling the R&D outputs. [17]

In support of this, Peters and Taylor (2017) find that the median firm only obtains 3% of its intangible capital via external purchases, with the remaining 97% being created internally (thereby indicating that intangible capital such as R&D is rarely traded and is unlikely to be viable as collateral). In addition, Mann (2018) finds that only 16% of patents owned by US firms have ever been pledged as collateral, indicating that it is rare for R&D to be used as collateral. The difficulty of using R&D as collateral against which a firm can borrow is also noted in Bloom, Van Reenen and Williams (2019).

Therefore, in my model, the first difference between physical capital and R&D is that

[17] See Haskel and Westlake (2018) for a detailed discussion of the other qualities held by intangible investments.

only physical capital can be used as collateral.[18] In other words, there is a collateral constraint that takes the form $B' \leq \theta K'$, where θ is the proportion of the firm's capital stock that it is able to borrow against (and a firm cannot borrow at all against its stock of R&D). Coupled with the fact that firms cannot use equity financing, this means that firms in my model face a constraint on how much investment they can make in physical capital and investment each period that would not be present if firms could finance those investments using equity. Hence, if a firm wishes to invest more in R&D, then it must reduce its investment in physical capital, resulting in firms facing a clear trade-off between investing in R&D.

My inclusion of this financial friction reflects Cabral and Mata (2003) and Dinlersoz et al. (2018)'s findings that financial constraints affect firm decisions and how they grow, as well as capturing Gertler and Gilchrist (1994)'s result that smaller firms are more likely to be subject to financial constraints than are larger firm. Moreover, my use of the collateral constraint allows my model to incorporate Howell (2017)'s finding that research grants have stronger effects for firms that are more financially constrained. The form of the financial friction in my model ensures that firms will be able to pay back all of its debt should it exit in the next period as the debt can be recouped when a firm sells its undepreciated capital (if θ is sufficiently less than 1). Nonetheless, as Loumioti (2012) finds that intangible assets such as R&D are gradually contributing more to the collateral that firms use when borrowing, I investigate the effect of allowing firms to use some proportion of their R&D as collateral in Online Appendix E.3.

The second difference between physical capital and R&D in my model is that although a firm's capital stock is fully reversible (i.e. a firm can sell its capital stock freely), its stock of R&D is completely irreversible. In other words, a firm's investment in R&D I_r each period must be non-negative. This reflects the fact that the 3% of intangible capital that is sourced externally as mentioned above is much smaller than the 28% of physical capital that is purchased second-hand (or "re-allocated") as found by Eisfeldt and Rampini (2006), indicating that physical capital is substantially more reversible than is a firm's stock of R&D. This means that a firm with "too much" stock of R&D cannot just sell its "excess" stock of R&D but instead must allow that stock to depreciate over time.

[18] This assumption that an intangible stock (such as R&D) cannot be used as collateral but physical capital can is common in models with collateral constraints (see, for example, Falato et al. (2020) and Döttling and Ratnovski (2020), both of which group all intangibles together rather than focusing on R&D as in my model)

A firm's physical capital stock depreciates at rate δ_k each period, while the depreciation rate of a firm's stock of R&D is given by δ_r. To my knowledge, I am the first to incorporate such differences between R&D and physical capital in a model with heterogeneous firms subject to financial frictions that is used to examine the effect of R&D subsidies.

These differences between physical capital and R&D mean that firms might favour investment in the former rather than the latter. This provides the government with a rationale to "step in" and provide a subsidy to firm investments in R&D such that investment in physical capital is not favoured quite as much in comparison to investment in R&D. Hence, in my model, the government provides a general subsidy at constant rate τ_r to all R&D investments and levies corporation taxes at constant rate τ_c on firm profits in order to fund it.[19] Any government surplus is rebated lump sum to households.

3.1. Household Problem

The unit measure of infinitely-lived identical households has preferences over consumption C and leisure $1 - N$ (with N representing the household's labour supply), with a subjective rate of time preference β. They earn wages $w(\mu)$ for each unit of labour they supply, own shares (represented by the vector λ) in the firms, and can borrow or save in one-period discount bonds ϕ. Households also receive a lump sum transfer of any surplus government tax revenues T over government expenditures G.

Letting the price of a one-period discount bond be $q(\mu)$; the ex-dividend price of shares in a firm with capital stock K, R&D stock R, debt B, and exogenous productivities ε_o and ε_r be $\rho_1(K', R', B', \varepsilon_o', \varepsilon_r', \mu)$; and the share price including dividend be $\rho_0(K, R, B, \varepsilon_o, \varepsilon_r, \mu)$, households maximise lifetime discounted utility via their choice of consumption, labour, bond, and share-holdings. In other words, households maximise Equation (1) subject to the household budget constraint Equation (2)

$$V^h(\lambda, \phi, \mu) = \max_{C,N,\phi',\lambda'} U(C, 1 - N) + \beta V^h(\lambda', \phi', \mu') \tag{1}$$

[19] The general nature of the R&D subsidy means that any firm making any level of investment in R&D receives the subsidy. Alternative R&D subsidy schemes might include firms only being eligible for the subsidy if they are of a certain type, or the R&D subsidy might only apply to R&D expenditure up to a certain amount.

$$\begin{aligned} C + q(\mu)\phi' + \int \rho_1(K', R', B', \varepsilon_o', \varepsilon_r', \mu)\lambda'(d[K' \times R' \times B' \times \varepsilon_o' \times \varepsilon_r')] \\ = w(\mu)N + \phi + \int \rho_0(K, R, B, \varepsilon_o, \varepsilon_r, \mu)\lambda(d[K \times R \times B \times \varepsilon_o \times \varepsilon_r) + (T - G) \end{aligned} \tag{2}$$

In equilibrium, the wage w is determined by the household's marginal rates of substitution between consumption and leisure, while the bond price (or loan discount factor) q is determined by the inverse of the (expected) gross real interest rate. This means that $w(\mu) = \frac{D_2 U(C,1-N)}{D_1 U(C,1-N)}$ and $q(\mu) = \beta \frac{D_1 U(C',1-N')}{D_1 U(C,1-N)}$ (as there is no aggregate uncertainty in this model, the expected real interest rate is $1/\beta$ in the steady state).

Finally, the household's share optimisation problem establishes the firm's stochastic discount factor due to the fact that household owns all firms. Hence, following Khan and Thomas (2013), I can re-write the firm problem to incorporate that stochastic factor in a more parsimonious way. In other words the firm problem is written with any current period profit scaled by $p(\mu) = D_1 U(C, 1 - N)$.

I use Rogerson-Hansen preferences such that $U(C, 1 - N) = \log C + S(1 - N)$ where S is the household's disutility of working.

3.2. *Firm problem*

Firms learn their persistent idiosyncratic output productivity and research productivity (with high values of research productivity meaning that the same amount of investment in R&D can be made more cheaply for those firms than it can for firms with low research productivity) at the start of the period. They also learn if they will be affected by the exogenous death shock or if they get a choice of continuing on to the next period.[20]

After these shocks are observed each period, firms hire labour ℓ, and produce using their stock of physical capital K via a Cobb-Douglas production function with weight α on capital and weight γ on labour. A firm's output productivity is determined by a function $e^{f(R,\varepsilon_o,\mu)}$ of their own R&D stock, their exogenous output productivity shock, and the

[20] The presence of an exogenous probability of death each period is necessary in my model to ensure that my model economy is not entirely populated by large firms. Without the exogenous death shock, all firms would survive until they reached their optimum size, such that the steady-state would consist only of large firms.

distribution μ of firms (which is summarised using the aggregate level of R&D $\bar{R}$). The output productivity function $f(R, \varepsilon_o, \mu)$ depends on a firm's own stock of R&D and the aggregate level of R&D and is estimated using Compustat data as described in more detail in Section 4.1. In this way, a firm's productivity is endogenous (as it depends on a firm's own and the aggregate stock of R&D) and R&D affects a firm's output only through its effect on that firm's productivity.

After production has occurred, those firms that did not receive the exogenous death shock and chose to pay the fixed continuation cost in order to continue onto the next period choose their capital stock K', R&D stock R', and debt B' (subject to the collateral constraint). Firms that received the exogenous death shock or chose to exit pay off their current debts, sell off their undepreciated capital stock, and pay out whatever is left as dividends to the household. Each period there is also a mass of potential entrants M that can choose to pay a fixed cost of entry ξ_e and then select their K', R', and B' with which they begin the next period.

As such, the timing within a period is as follows:

1. Incumbents find out their idiosyncratic output and research productivity for the current period, and whether or not they are subject to the exogenous death shock. Potential entrants discover their idiosyncratic cash endowment as well as their research and output productivities.
2. All incumbents hire labour, produce output, pay off their existing debt, sell their undepreciated physical capital, and pay any resulting dividends to households
3. Incumbents that did not receive the exogenous death shock decide whether or not to pay the fixed continuation cost
4. Any incumbent that received the exogenous death shock or chose not to pay the continuation cost exits
5. Incumbents continuing onto the next period, and potential entrants that decide to pay the fixed entry cost, choose their K', R', and B'.

3.2.1. An incumbent firm

In order to reduce the state-space over which a firm's value function is represented, following Jo (2021) it is possible to write the firm's value function in terms of their cash-on-hand m because a firm's choice of capital for the next period does not depend on its current level of capital (i.e. there are no adjustment costs or irreversibilities for the capital stock). This allows me to use a firm's cash-on-hand m to replace the stock of physical capital K and debt B in the set of state variables that characterise an individual firm. However, as a firm's stock of R&D is irreversible, R must remain as a distinct idiosyncratic state variable. Hence, under this approach, a firm's cash-on-hand is given by $m = \max_\ell e^{f(R,\varepsilon_o,\mu)}K^\alpha \ell^\gamma - w(\mu)\ell + (1-\delta_k)K - B$ each period, and the firm's value function can be represented by $W(m, R, \varepsilon_o, \varepsilon_r, \mu)$.

Let W represent the value of an incumbent firm at the start of the period, W^1 the value of a firm that does not receive the exogenous death shock, W^X the value of a firm that chooses to exit after production occurs, and W^O the value of a firm that chooses to pay the continuation cost and continue into the next period. The incumbent firm's optimisation problem can be defined recursively as

$$\begin{aligned} W(m,\varepsilon_o,\varepsilon_r,\mu) = \pi_d & \left[\max_\ell p(\mu)(1-\tau_c)[e^{f(R,\varepsilon_o,\mu)}K^\alpha \ell^\gamma - w(\mu)\ell + (1-\delta_k)K - B]\right] \\ & + (1-\pi_d)W^1(m,\varepsilon_o,\varepsilon_r,\mu) \end{aligned} \tag{3}$$

$$W^1(m,\varepsilon_o,\varepsilon_r,\mu) = \max\{W^X, W^O - \xi\} \tag{4}$$

where ξ is the fixed cost of continuing on to the next period. Equation (3) states that the firm takes the probability of an exogenous death into account. If the firm receives the exogenous death shock, it chooses its labour to maximise output and then pays off its debt, sells off its undepreciated capital and pays out whatever is left (after corporation tax is applied) as dividends to the household. If the firm does not receive the exogenous death shock, then it receives W^1, which is the higher of exiting endogenously W^X or of continuing onto the next period W^O, as per Equation (4). A firm that chooses to exit receives the same value as one that received the exogenous death shock (i.e. it receives its output and undepreciated capital having paid off its debt, net of corporation tax payments).

Letting D represent a firm's dividends, an incumbent firm that chooses to continue onto the next period and pays the fixed continuation cost faces the following problem: it hires labour to maximise output; chooses its capital stock next period, R&D stock next period, and debt next period to maximise

$$\begin{aligned} W^O(m, \varepsilon_o, \varepsilon_r, \mu) = \max_{K',R',B',D} & \; p(\mu)(1-\tau_c)D + \\ & \beta E_{\pi_{\varepsilon_o}\pi_{\varepsilon_r}} V(m', R', \varepsilon_o', \varepsilon_r', \mu') \end{aligned} \tag{5}$$

subject to

$$\begin{aligned} 0 \leq D &= m - K' - \frac{(1-\tau_r)I_r}{\varepsilon_r} + q(\mu)B' \\ B' &\leq \theta K' \\ I_r &= R' - (1-\delta_r)R \geq 0 \\ \mu' &= \Gamma(\mu) \\ m' &= m(K', R', B', \varepsilon_o', \varepsilon_r', \mu') \\ &= \max_{\ell'} e^{f(R', \varepsilon_o', \mu')} K'^{\alpha} \ell'^{\gamma} - w(\mu')\ell' + (1-\delta_k)K' - B' \end{aligned} \tag{6}$$

where $B' \leq \theta K'$ is the collateral constraint that firms can only borrow up to a fraction θ of their physical capital next period; $I_r = R' - (1-\delta_r)R \geq 0$ indicates that investment in R&D must be non-negative; and $\mu' = \Gamma(\mu)$ represents the law of motion of the distribution of firms.[21]

3.2.2. *Potential entrants*

There is also a mass M of potential entrants each period, each of which is endowed with an idiosyncratic level of cash m_e drawn from a distribution H. Each potential entrant also draws their persistent idiosyncratic research and output productivities (each is drawn from the ergodic distributions of those shocks).

They can then choose to pay the fixed cost of entry ξ_e. If they pay this cost, they must fund this fixed cost of entry, along with their choice of capital K' and research R' stocks

[21] In my model, changes in the principal component of a firm's debt (as well as the interest payments) affects a firm's profits and the corporation taxes that they pay. This preserves the reduction in the state-space achieved by converting the firm-problem to one based on a firm's cash-on-hand since there is no requirement to track a firm's debt from period-to-period. Notably, firms often do increase their debt levels in order to pay dividends (e.g. through dividend recapitalisation) such that this component of the model is plausible.

from their cash endowment and any debt B' (subject to the collateral constraint) that they choose to take on this period. Their choice of research stock is constrained to be no more than a maximum proportion κ_R of the aggregate research stock $\bar{R}$; and their capital stock no more than a proportion κ_K of that which an unconstrained firm would choose (denoted K'^*) given their choice of R' (unconstrained firms are discussed in more detail in Online Appendix C.1.1). Any of the cash endowment that is not used by entrants is returned lump-sum to households.

Hence, the value of entry W^N is given by

$$W^N(m_e, \varepsilon_o, \varepsilon_r) = \max_{K' \leq \kappa_K K'^*, R' \leq \kappa_R \bar{R}, B'} \beta E_{\pi_{\varepsilon_o} \pi_{\varepsilon_r}} W(m', \varepsilon'_o, \varepsilon'_r, \mu') \tag{7}$$

subject to

$$\begin{aligned} B' &\leq \theta K' \\ m_e &\geq K' + \frac{(1-\tau_r)R'}{\varepsilon_r} + \xi_e - q(\mu)B' \\ \mu' &= \Gamma(\mu) \\ m' &= m(K', R', B', \varepsilon'_o, \varepsilon'_r, \mu') \\ &= \max_{\ell'} e^{f(R', \varepsilon'_o, \mu')} K'^{\alpha} \ell'^{\gamma} - w(\mu')\ell' + (1-\delta_k)K' - B' \end{aligned} \tag{8}$$

with potential entrants only entering if $W^N > 0$.

Under this formulation, depending on the distribution H, there are some potential entrants with high research and / or output productivity that cannot afford to pay the fixed cost of entry from their cash endowment (plus debt that they take on) and so do not enter. This provides a role for entry subsidies to allow such high productivity potential entrants to enter, thereby ameliorating the financial constraint that these high productivity potential entrants face.

3.3. Competitive equilibrium

In this model, letting $\mathbb{I}^O$ represent those firms in the distribution that are continuing incumbent firms and $\mathbb{I}^{\mathbb{N}}$ representing those firms that are entrants that decide to enter, a

Stationary Equilibrium is a set of prices p, w, q, ρ_0, ρ_1, and functions for quantities K', ℓ, B', R', D, C, N, ϕ', λ', K^N, R^N and values V^h, W, W^1, W^O, W^X, W^N that solve the firm and household problems and clear markets as described below:

1. W, W^1, W^X, and W^O solve Equations (3), (4), and (5), and K', ℓ, B', R', and D are the associated policy functions for incumbent firms
2. V^h solves Equation (1) with C, N, ϕ, and λ being the household's associated policy functions
3. W^Nsolves Equation (7), with K^N and R^Nas the associated policy functions for potential entrants
4. The distribution of firms over $K \times R \times B \times \varepsilon_o \times \varepsilon_r$ is stationary and given by μ
5. The labour market clears $N = \int \ell(K, R, \varepsilon_o)\mu(d[K \times R \times B \times \varepsilon_o \times \varepsilon_r)$
6. The goods market clears $C = \int \Bigg(e^{f(R,\varepsilon_o,\mu)} K^\alpha \ell^\gamma + (1-\delta_k)K - \mathbb{I}^O[K' + \frac{I_r}{\varepsilon_r} + \xi] - \mathbb{I}^{\mathbb{N}}[K^{N\prime} + \frac{R^{N\prime}}{\varepsilon_r} + \xi_e] \Bigg) \mu(d[K \times R \times B \times \varepsilon_o \times \varepsilon_r])$
7. Price q clears the bond market
8. Prices ρ_0 and ρ_1 clear the market for shares
9. The government budget constraint is given by $\int [\tau_c D]\, \mu(d[K \times R \times B \times \varepsilon_o \times \varepsilon_r) - \int \left[\tau_r(\mathbb{I}^O \frac{I_r}{\varepsilon_r} + \mathbb{I}^{\mathbb{N}} \frac{R^{N\prime}}{\varepsilon_r}) + \mathbb{I}^{\mathbb{N}} \tau_c D^N \right] \mu(d[K \times R \times B \times \varepsilon_o \times \varepsilon_r) = \Lambda$, where Λ is a constant.

4. Quantitative strategy

In this Section, I set out how I calibrate the values of parameters in my model to match target moments, and present evidence that the model with these parameter values performs well in matching these moments.[22] The number of target moments is roughly double the number of calibrated parameters, such that my model is overidentified.

4.1. Parameters set outside the model

I set the value of 8 parameters outside the model, and jointly calibrate 20 others to match an overidentified set of 38 targeted moments related to aggregate variables and the joint distribution of firms over their stocks of capital and R&D.

[22] Details regarding the approach I take to solve my model are contained in Online Appendix C.

The parameters governing the relationship between a firm's stock of R&D and its productivity are obtained as follows. I estimate a firm's production function with endogenous productivity that depends on a firm's stock of R&D by modifying De Loecker (2013)'s extension of the ACF estimation algorithm to allow for this relationship between a firm's stock of R&D and its productivity.[23] This allows the effect of a firm's stock of R&D on its productivity to also affect the marginal productivities of physical capital and labour, thereby allowing the more accurate estimation of all the effects contained in this procedure.

Specifically, in a first stage, I recover the residuals from a regression of a firm's value added on a cubic polynomial of its labour, capital stock, and materials expenses, as well as a set of annual dummy variables. I then use De Loecker's extension that allows the productivity process to be endogenous and determined by a set of explanatory variables (rather than assumed to be an AR(1) Markov process as in OP and ACF) in a GMM estimation procedure that involves: 1) guessing the production function coefficients for labour and capital; 2) recovering the firm-specific productivity implied by these guesses; 3) obtaining the error term implied by regressing the recovered estimate of firm productivity on the chosen specification that contains the R&D variables; 4) using the error term that results from this to construct moment conditions that depend on a firm's labour last period and their stock of capital this period (since neither of these instruments are affected by a firm's current level of productivity); and 5) updating the guess of the labour and capital coefficients until the moment conditions are minimised.[24]

I examine a range of potential specifications for $f(R, \varepsilon_o, \mu)$, with a final estimated specification taking the form $f(R, \varepsilon_o, \mu) = \nu_1 R + \nu_2 R^2 + \nu_3 R\bar{R}$.[25] I find that a firm's productivity is increasing in its own stock of R&D (a positive value of ν_1, the coefficient on the level of a firm's own stock of R&D), but with decreasing marginal returns (a negative value of ν_2, the coefficient on the square of a firm's own stock of R&D).

[23] Due to the fact that my dependent variable is value-added rather than gross output (i.e. materials is not an input in the Cobb-Douglas production function in my empirical or modelling framework), I do not use the estimation procedure developed by Gandhi, Navarro and Rivers (2020).

[24] To ensure that observations with zero stock of R&D are not dropped when using the log of the variable, I add one to each firm's stock of R&D in each year. This preserves the relationship between changes in a firm's stock of R&D and its idiosyncratic productivity while keeping in the more than 40% of firms that do not have any R&D stock.

[25] In the calibrated model, the actual specification used is $f(R, \varepsilon_o, \mu) = \chi(\nu_1 R + \nu_2 R^2 + \nu_3 R\bar{R}) + \varepsilon_o$, with χ and the parameters governing ε_o calibrated to match target moments of the data.

I also find that there are negative spillovers from higher aggregate R&D (a negative value of ν_3, the coefficient on an interaction between a firm's own stock of R&D and the aggregate stock of R&D). In other words, higher aggregate R&D reduces the extent to which a firm increasing its own stock of R&D would increase its idiosyncratic productivity.[26] [27] This indicates that rather than positive spillovers allowing firms to make increasingly productive innovations, higher aggregate R&D "crowds out" the effect of an individual firm's own stock of R&D. This echoes the "ideas are getting harder to find" result found in Bloom et al. (2020) (since higher aggregate R&D leads to an individual firm getting a lower return on its own stock of R&D) and is consistent with the product-market rivalry explanation highlighted by Lucking, Bloom and Van Reenen (2019) (i.e. that higher aggregate R&D reflects firms trying to develop more rival products, thereby increasing competition and reducing the profits available for an innovation by an individual firm).[28]

Moreover, there are other negative spillovers, beyond just product-market rivalry, that might result from higher aggregate R&D. For example, higher aggregate levels of R&D could mean that there is increased demand for inputs (such as scientists and researchers) into R&D, increasing the cost of those inputs, thereby creating an incentive for some firms to reduce their R&D in order to prevent their R&D costs from increasing a lot.[29] Hence, in

[26] Here, the aggregate stock of R&D $\bar{R}$ is measured as the mean of (the log of) individual firms' stocks of R&D to account for the fact that the number of firms in Compustat changes substantially over time.

[27] The coefficient measuring the spillover from aggregate R&D is identified even in the presence of year fixed-effects due to the variation in a firm's own stock of R&D, which means that there is variation in the interaction term within an individual year. Moreover, the multiplicative nature of the interaction term means that there is variation across years as well, due to the fact that both the aggregate and individual stocks of R&D are changing over time. However, including the annual dummy variables means that I cannot separately discern the effect of the level of aggregate R&D on a firm's productivity via this procedure. Nonetheless, in Online Appendix B.3 I use a modified version of this estimation procedure to enable the inclusion of the level of aggregate R&D and find that the spillovers from the level of aggregate R&D are not statistically significantly different from zero.

[28] Lucking, Bloom and Van Reenen (2019) finds that positive diffusion spillovers (from one firm's innovation being adopted by other firms in the same or similar industries) outweigh the negative product-market rivalry spillovers. This difference compared to my results is driven by the fact that I use the aggregate stock of R&D in the economy to capture spillovers whereas Lucking, Bloom and Van Reenen (2019) uses a measure of "technological proximity" that only captures spillovers that specifically relate to "technological proximity" and "product-market rivalry" (under the assumption that firms in similar industries are likely to have higher spillovers). My approach enables me to capture more general spillovers that result from R&D rather than the highly-specific types of spillovers that are examined in Lucking, Bloom and Van Reenen (2019).

[29] Berger (1993) finds that this is what happened after the introduction of the R&D Tax Credit in the USA in the 1980s. In particular, the R&D subsidy increased the demand of inputs into R&D (such as researcher

contrast to Lucking, Bloom and Van Reenen (2019) and Bloom, Schankerman and Reenen (2013)'s focus on a single potential source of positive spillovers and a single potential source of negative spillovers, my approach accounts for all of possible sources of positive and negative spillovers. My finding of negative R&D spillovers is also consistent with Dieppe and Mutl (2013) (which finds that higher aggregate R&D has a negative effect on aggregate productivity across a range of countries) and Kim and Choi (2019) (who find that negative market rivalry spillovers dominate positive technology spillovers from multi-national firms operating in South Korea).

Additional details regarding this estimation procedure and robustness checks are described in Online Appendix B.1.

Other parameters set outside the model are determined as follows. The depreciation rate of capital δ_k is set to match the average investment-to-capital ratio during the postwar US period, while the depreciation rate of a firm's stock of R&D δ_r is taken from Hall (1990) and Hall (1993). I set β, the household's rate of time preference, so that the (expected) annual interest rate is 4%. The corporation tax rate τ_c is taken from the USA tax schedule for 1993-2007, while the effective R&D subsidy rate τ_r is the value for the USA in OECD (2019). The values of the parameters determined outside the model are summarised in Table 1.

Table 1: Parameter values set outside the model

Parameter	Description	Source / reason	Value
ν_1	Coefficient on R in $f(R, \varepsilon_o, \mu)$	Production function estimation	0.01566
ν_2	Coefficient on R^2 in $f(R, \varepsilon_o, \mu)$	Production function estimation	$-1.56\text{x}10^{-3}$
ν_3	Coefficient on $R * \bar{R}$ in $f(R, \varepsilon_o, \mu)$	Production function estimation	-0.0540
δ_k	Capital depreciation rate	Mean investment:capital ratio 1954-2002	0.069
δ_r	R&D stock depreciation rate	Hall (1990, 1993)	0.15
β	Household time preference	4% annual interest rate	0.96
τ_c	Corporation tax	US Tax schedule for 1993-2007	0.35
τ_r	R&D subsidy	OECD effective R&D subsidy for the USA	0.05

wages), thereby increasing the costs of conducting R&D, reducing the profitability of R&D for some firms.

4.2. Calibrated parameters

The aggregate moments I target, and my model's performance in matching them, are shown in Table 2. Although the parameters are all determined jointly, the rows of the table show what parameters are most-closely associated with each target. The exponent on capital in the production function α is set to obtain a capital-to-output ratio of roughly 2.3 (the average private capital-to-output ratio between 1954 and 2002 as per Khan and Thomas (2013)), while the exponent on labour γ is set to target the 60% labour share of output from Cooley and Prescott (1995).[30] A household's disutility of working S is set so that the total hours worked is one-third. The value of the collateral constraint θ is determined by targeting the average aggregate debt-to-asset ratio for the USA between 1954 and 2006.[31]

The entry rate is determined predominantly by the exogenous probability of death π_d and the fixed cost of entry ξ_e, which are chosen jointly to target the 10.6% average entry rate over the period 1980-2007 implied by the BDS data on firm age. The moments for the relative size of entrants and the proportion of employment accounted for by entrants are also taken from these BDS data, and are targeted mainly using, respectively, the proportions κ_R of aggregate research and κ_K unconstrained capital stocks that potential entrants can choose, the parameters governing the distribution H of the idiosyncratic cash endowment given to potential entrants, and the value for the measure of potential entrants M. Targeting these moments is necessary to ensure that my analysis of the effect of entry subsidies captures the size and effect that any newly-encouraged entrants would have on the economy. The distribution of cash endowment H is a pareto distribution with minimum value $m_{e,low}$, maximum value $m_{e,high}$, shape parameter Ξ.

The value of χ in my specification of $f(R, \varepsilon_o, \mu) = \chi(\nu_1 R + \nu_2 R^2 + \nu_3 R\bar{R}) + \varepsilon_o$ is set along with (the support, persistence, and variance of) the terms governing a firm's exogenous

30 Although the results from estimating a production function as presented in Section 2.2 also provides estimated coefficients for the exponents on physical capital and labour, that estimation is based on Compustat data (covering only about 70% of private sector output in the US) whereas my calibration targets are based on economic aggregates. Hence, I calibrate those two exponents to match those aggregate moments. Nonetheless, the calibrated values are relatively similar to those obtained from the estimation: the calibrated exponent on labour is 0.61 compared to an estimated coefficient of 0.64, and the calibrated exponent on physical capital is 0.29 compared to an estimated coefficient of 0.33.

31 I use the total private non-residential fixed assets from the NIPA and the nonfinancial corporate businesses financial assets from the FRB as the aggregate assets, while the total debt in the economy is taken as the total liabilities for nonfinancial corporate business excluding foreign direct investment from the FRB.

Table 2: Comparison of model and targeted moments

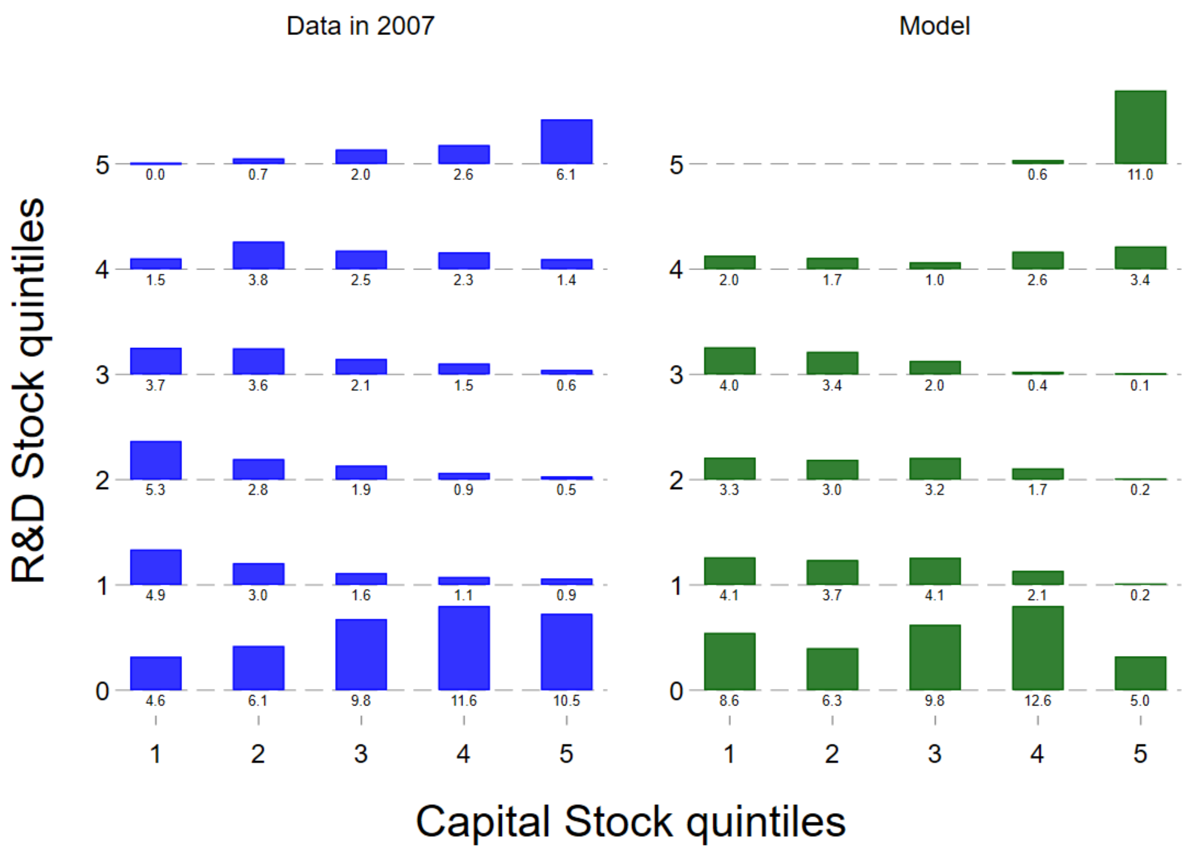
Moment	Main parameter	Source	Model	Data
Capital: Output ratio	Cobb-Douglas Exponent α	Literature	2.11	2.1
Labour Share	Cobb-Douglas Exponent γ	Cooley & Prescott (1995)	61%	60%
% of time worked	Household disutility of working S	Literature	31%	33%
Aggregate debt-to-Capital ratio (1954-2006)	Collateral constraint θ	FRB and NIPA	0.34	0.37
Entry rate	Fixed entry cost ξ_e, exogenous death probability π_d	BDS	10.4%	10.6%
Relative average size of entrants vs incumbents	Maximum choice of capital and research stock κ_R and κ_K	BDS	24.7%	25.6%
Proportion of total employment by entrants	Distribution of potential entrants' cash endowment H, and measure of potential entrants M	BDS	2.8%	3.0%
Proportion of firms with 0 R&D	χ, Idiosyncratic shock terms for ε_o and ε_r, fixed continuation cost ξ	Compustat	42.3%	42.6%
Joint distribution of Capital and R&D	χ, Idiosyncratic shock terms for ε_o and ε_r, fixed continuation cost ξ	Figure 4		

persistent idiosyncratic output productivity shocks ε_o and research productivity shocks ε_r to match the proportion of firms recorded in Compustat that do not have any R&D stock in 2017, as well as the joint distribution of capital and R&D stocks over those firms in that year. These moments are also affected by the choices of all other parameters as well, including those governing entry and exit, such as the fixed continuation cost ξ.

The output productivity shock is assumed to have a normal distribution with persistence ρ_{ε_o} and a standard deviation of disturbances of σ_{ε_o}, while the research productivity shock is assumed to follow a Pareto distribution with minimum value $\varepsilon_{r,low}$, maximum value $\varepsilon_{r,high}$, shape parameter Ψ, and a probability of keeping the current value of the shock φ. The parameters of these two distributions allow me to target the join distribution of firms over their stocks of physical capital and R&D, and my use of a Pareto distribution for the research productivity shock allows me to target the proportion of firms that have zero R&D stock.

My calibrated model does well matching most of the targeted aggregate moments, including those for the relative size of entrants. As shown in Figure 4, my model also reproduces the joint distribution of firms over R&D and capital discussed in Section 2.1.[32] This means that my model does well at capturing the heterogeneity of firms across these two variables that we see in the data, such that it is likely to be able to capture any heterogeneous response to changes in subsidies or other policies.

Figure 4: Comparison of model to data for joint distribution of Capital and R&D stocks



Note: Data in 2007 panel created using Compustat data. The capital stock is calculated by combining a firm's net property, plant, and equipment values with BEA industry-specific deflators to account for industry-specific rates of inflation. A firm's R&D stock is obtained using the perpetual inventory method on a firm's annual R&D expenditures along with an assumed annual depreciation rate of 15% and an initial R&D stock of 10^{-6} in the first year of a firm's appearance in the data. The zero row for the R&D stock shows the proportion of firms with zero R&D, with the R&D quintiles then reflecting a firm's R&D stock given that that stock is greater than zero. Model panel created from solved stationary outcome of my model using parameter values described in this Section.

The panel on the left-hand side shows this joint distribution over quintiles as found in the

[32] This is predominantly due to the calibration of the parameters governing the idiosyncratic exogenous output and research productivity shocks, as well as the fixed continuation cost and the scalar χ in the output productivity function.

data, and the panel on the right-hand side shows the joint distribution that arises in the steady-state of my model. In both my data and the model, of those firms that have some stock of R&D, those with a small amount of R&D are more likely to be firms towards the bottom of the capital distribution. For example, there are 4.9% of firms in the data, and 4.1% of firms in my model, in the bottom quintile of the capital distribution and in the first quintile of the non-zero R&D distribution (column 1 and the row labelled 1); and there are 0.5% of firms in the data, and 0.2% of firms in my model, that are at the top of both the capital distribution but in that same R&D "quintile" (column 5 and the row labelled 2). Hence, my model does well at matching this pattern of the proportion of firms declining from left to right for the rows labelled 1, 2, and 3.

Moreover, in both my model and the data, the top of the R&D distribution is populated more by firms that also have a lot of capital. For example, 6.1% of firms in the data, and 11% of firms in my model, are in the top quintiles of both R&D and capital, whereas a negligible proportion of firms are in the top quintile of R&D and the bottom quintile of the capital distribution. Hence, my model also matches the data in the row labelled 5. It also reproduces the relatively-flat pattern seen for firms in the data for those firms in the row labelled 4, although the data has a slight hump-shaped pattern moving left-to-right in this row, whereas my model has a slight U-shaped pattern.

However, my model does miss the pattern seen in the bottom row (i.e. those firms that have no stock of R&D). In particular, whereas the proportion of firms in this row generally increases from left to right in the data, it is somewhat hump-shaped in my model's stationary distribution, with too few firms having lots of physical capital and no R&D (the bottom right part of the graph) and slightly too many firms having little physical capital and no R&D (the bottom left part of the graph). This is driven by the fact that a firm's choice of R&D is affected by the firm's choice of capital as having higher capital means the marginal productivity of R&D stock is higher. This means that a firm with higher capital will have more of an incentive to invest in R&D because the marginal returns to doing so are higher in this model.

Nonetheless, overall, my overidentified model performs relatively well in matching the targeted moments.The specific parameter values used to obtain this calibration of my model are shown in Table 3. My model also performs well compared to untargeted moments of the data such as the ratio of total R&D stock to physical capital stock; and the paths of firm accumulation of physical capital and R&D stocks. Validation against these untargeted moments is discussed in more detail in Online Appendix D.

Table 3: Parameter values calibrated using targets from data

Parameter	Description	Value	Parameter	Description	Value
α	Cobb-Douglas Exponent on capital	0.29	φ	Probability of keeping current ε_r	0.996
γ	Cobb-Douglas Exponent on labour	0.61	M	Measure of potential entrants	0.15
S	Household disutility of working	2.2	ξ	Fixed continuation cost	0.025
θ	Collateral constraint	0.3	ξ_e	Fixed entry cost	0.05
χ	Scalar in $f(R, \bar{R}, \varepsilon_o)$	5.5	κ_K	Maximum proportion of K^U for potential entrants	0.05
ρ_{ε_o}	Persistence of output shock	0.60	κ_R	Maximum proportion of aggregate R for potential entrants	0.25
σ_{ε_o}	Standard deviation of output shock	0.05	$m_{e,low}$	Lowest value of m_e	0.025
$\varepsilon_{r,low}$	Lowest value of ε_r	5	$m_{e,high}$	Highest value of m_e	0.625
$\varepsilon_{r,high}$	Highest value of ε_r	20	Ξ	Pareto shape of m_e	-0.75
Ψ	Pareto shape of ε_r	-0.6	π_d	Exogenous probability of death	0.07

5. Results and Discussion for general R&D subsidies

In this Section I present the results obtained by varying the level of the general R&D subsidy in my model (i.e. varying the level of the R&D subsidy for all firms equally). I find that although high levels of the general R&D subsidy have a sizeable effect on the level of aggregate R&D in an economy, this does not translate into a large effect on other economic aggregates. For example, an increase in the general subsidy rate from 5% to 50% increases the aggregate stock of R&D by roughly 18%, but only increases output by 0.1%.

This is due to the negative spillovers that result from higher aggregate R&D and firms' heterogeneous response to changes in R&D subsidies. In particular, higher aggregate R&D reduces the effect of any individual firm's own stock of R&D on its productivity. However, this negative effect is higher for firms with lots of R&D (as it operates via the interactive term $R.\bar{R}$) this increase in aggregate R&D then "crowds out" the R&D stocks of firms that previously had high levels of this stock, causing them to reduce their

R&D stock levels. Conversely, a higher general R&D subsidy encourages firms who had previously had zero stocks of R&D to invest in R&D so as to obtain a positive stock of R&D, albeit only to a level where the negative spillover effect via the interaction term is not particularly high.

Without the negative spillover from higher aggregate R&D, output would increase by 3.2% when the general R&D subsidy is increased to 50%, indicating that the negative spillover serves to severely restrict the increase in output that results from higher general R&D subsidies.

Moreover, the effect of increasing subsidies on the growth of a cohort of entrants differs according to where that cohort is in their growth path. For example, higher subsidies increase the cohort's total output early on in their growth path, but this increase is then reversed after roughly 30 years.

Each of these results is discussed in more detail below. I also obtain similar results for alternative policies that try to incentivise firm R&D (such as targeted R&D subsidies or allowing firms to use some portion of their stock of R&D as collateral) and these are presented in Online Appendix E.

5.1. Aggregate effects of different general R&D subsidies

Table 4 shows how changing the R&D subsidy rate τ_r for all firms equally affects the stationary level of economic aggregates in my model. The second column of this table shows the levels of various aggregate variables in the calibrated base-case where the R&D subsidy rate is set at 5%, and the corporation tax is set at 35%. Columns 3-6 of the table show how these aggregates change relative to the base case when the subsidy rate is at the level shown in the first row, except for the penultimate and last rows of the table which contain, respectively, the actual proportion of firms with zero R&D stocks and the compensating variation required to return the representative household to the utility level it had in the baseline case (expressed as a percentage of the household's income) in each scenario.[33] [34] For example, the aggregate R&D stock in the base case scenario of

[33] The corporation tax rate is adjusted to ensure that the government surplus Λ from the base-case is constant in each scenario.

[34] Although a general R&D subsidy of 50% might seem high, OECD (2022) finds that some countries, such as Colombia and Slovakia, have general R&D subsidies as high as 70% and 55%, respectively.

a 5% R&D subsidy ($\tau_r = 0.05$) is 0.22, while it is 18.1% higher when the subsidy is 50% ($\tau_r = 0.5$).

Table 4: Effect of different levels of R&D subsidy on aggregate variables

Aggregates	$\tau_r = 0.05$	$\tau_r = 0.0$	$\tau_r = 0.1$	$\tau_r = 0.2$	$\tau_r = 0.5$
R&D stock	0.22	98.3	101.7	105.4	118.1
Mass of firms	1.31	99.9	100.1	100.2	100.7
Output	0.50	100.0	100.0	100.0	100.1
Capital	1.04	100.0	100.0	100.0	100.1
Consumption	0.44	100.01	99.99	99.96	99.84
Labour	0.31	99.97	100.03	100.08	100.28
Average productivity	1.01	100.00	100.00	99.98	99.93
% of firms with zero R&D stock	42.3%	43.1%	41.2%	39.6%	32.6%
Compensating Variation (% of income)	N/A	-0.02%	0.02%	0.076%	0.23%

Note: The results for the R&D subsidy rate $\tau_r = 0.05$ are the levels of each aggregate variable obtained in my calibrated model. The results for other values of τ_r are obtained by re-solving the stationary equilibrium for my model with that level of τ_r and allowing the level of the corporation tax τ_c to adjust so that the government surplus is kept the same as it was for the $\tau_r = 0.05$ scenario. The results for these other values of τ_r are presented relative to the scenario in which $\tau_r = 0.05$ except for the last two rows of the table, which are, respectively, the actual proportion of firms with zero R&D stock in each scenario and the percentage of income required to restore the representative household to its utility level in the baseline scenario (a positive compensating variation indicates that firms are worse off under that particular scenario, while a negative compensating variation indicates that firms are better off compared to the baseline).

It is clear from this table that increasing the level of the R&D subsidy has a sizeable effect on the aggregate level of R&D in the economy (the first row of the table) and the number of firms with zero R&D (the last row of the table). In particular, higher R&D subsidies reduce the number of firms with zero R&D stock and increase the aggregate level of R&D stock.[35]

Higher subsidies are also associated with a slightly higher number of firms in the steady-state: the second row of the table indicates that there are 0.7% more firms when the R&D subsidy is set at 50% compared to when it is 5%. However, the higher stocks of R&D,

[35] Hall and Van Reenen (2000) find that R&D expenditure has an elasticity of about 1-1.5 in response to changes in R&D subsidies. However, in my model this elasticity is roughly 0.7. As such, my model's responsiveness of R&D in the face of changes to its (relative) price is somewhat low. Nonetheless, this would not explain the explain the lack of change in other aggregates, because those are governed by the elasticity of those aggregates with respect to R&D, rather than the elasticity of R&D with respect to its price.

number of firms, and proportion of firms with positive R&D stocks do not translate into substantial changes in other economic aggregates such as output, consumption, or capital stock.

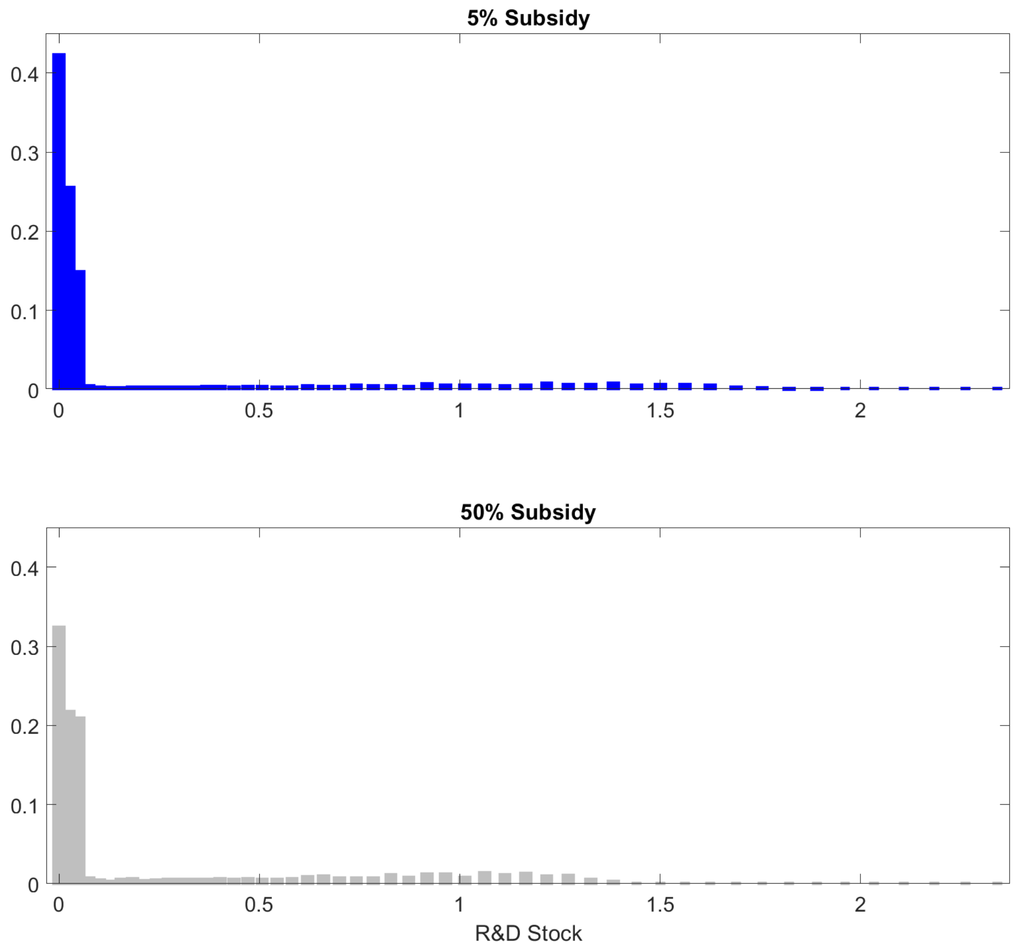
For example, even in the highest R&D subsidy level (the last column of the table), the 18.1% increase in the R&D stock, the 0.7% higher mass of firms, and the roughly ten percentage point increase in the number of firms with some R&D stock translate into only a 0.1% increase in output and a 0.16% decrease in consumption. The higher output is driven by the presence of a larger number of firms with higher aggregate stocks of R&D and physical capital, overcoming the slight reduction in the average firm's productivity associated with higher subsidies. The decrease in consumption despite the higher output stems from the presence of a larger mass of firms (and, in particular, continuing incumbent firms), meaning that more output is needed to fund investments in R&D and physical capital as well as the fixed continuation cost, such that less output is available for consumption.

Changes in aggregate labour reflect the change in R&D stock and physical capital stock, due to changes in those stocks affecting the marginal productivity of labour. Finally, the fact that hours worked by the representative household (i.e. aggregate labour) are increasing while consumption is decreasing in the level of the subsidy means that the compensating variation (i.e. the amount of money required to return the representative household to the level of utility it obtained in the baseline scenario) is increasing in the level of the general R&D subsidy (i.e. household utility decreases as the subsidy increases), with a positive compensating variation indicating that households are worse off under the alternative scenario compared to the baseline.

The distributional effects of higher general R&D subsidies are clear when examining the univariate distribution of firms over each of their stock of R&D and their stock of physical capital. Figure 5 shows the distribution of firms over R&D for the baseline 5% subsidy (the top panel) and the 50% subsidy (the bottom panel). It is immediately clear that higher levels of the R&D subsidy 1) increase the number of firms with non-zero R&D stocks, as the bar at the zero point in the distribution gets smaller; and 2) reduces the highest levels of R&D stocks chosen, as the highest level of R&D stock in the 5% subsidy scenario was above 1.5 but is barely above 1.2 when the subsidy is at 50%.

The mechanism underlying this is as follows: the increase in R&D subsidies means it is cheaper for firms to invest in R&D, so more firms find it viable to have non-zero

Figure 5: Effect of higher general R&D subsidies on the distribution of R&D stock



Note: The results for the R&D subsidy rate $\tau_r = 0.05$ are the levels of each aggregate variable obtained in my calibrated model. The results for $\tau_r = 0.5$ (the bottom panel) are obtained by re-solving the stationary equilibrium for my model with that level of τ_r and allowing the level of the corporation tax τ_c to adjust so that the government surplus is kept the same as it was for the $\tau_r = 0.05$ scenario. Each panel of the figure shows the steady-state proportion of firms at each level of the R&D stock.

R&D stocks, as shown by the decrease in the proportion of firms with zero stocks of R&D. However, this results in an increase in the aggregate level of R&D stocks $\bar{R}$, which reduces the impact of having a higher R&D stock on a firm's productivity via the negative spillovers that are represented by the negative estimated value of the ν_3 coefficient in $f(R, \varepsilon_o, \mu) = \chi(\nu_1 R + \nu_2 R^2 + \nu_3 R\bar{R}) + \varepsilon_o$. This means that the firms that had held large stocks of R&D when the subsidies were low now have a reduced incentive to hold high stocks of R&D when subsidies are higher, due to the effect of the negative spillover.

In other words, higher R&D subsidies have a heterogeneous effect on firms' stocks of

R&D, increasing them for some firms but reducing them for others. The fact that some firms reduce their R&D stocks in the face of higher subsidies is consistent with Berger (1993)'s finding that this happened after the introduction of the R&D Tax Credit in the USA.[36] The overall effect, when combined with the larger number of firms present at higher levels of the subsidy, is for the aggregate R&D stock to increase in the subsidy.

The heterogeneous effect on R&D stocks also feeds through into the distribution of firms over capital stocks. In particular, for those firms that previously had high R&D stocks, their lower level of R&D stock and the higher aggregate R&D stock means that the marginal productivity of capital is reduced, leading to them choosing slightly less capital in steady-state.

This is also reflected in the fact that the average size of firms is decreasing in the level of the R&D subsidy. In particular, firms are 0.5% smaller, on average, under a 50% R&D subsidy compared to the 5% level of the subsidy. However, the fact that there is a larger mass of firms as the subsidy increases means that the aggregate capital stock is slightly increasing in the level of the R&D subsidy, and this also drives aggregate output being increasing in the level of the subsidy.

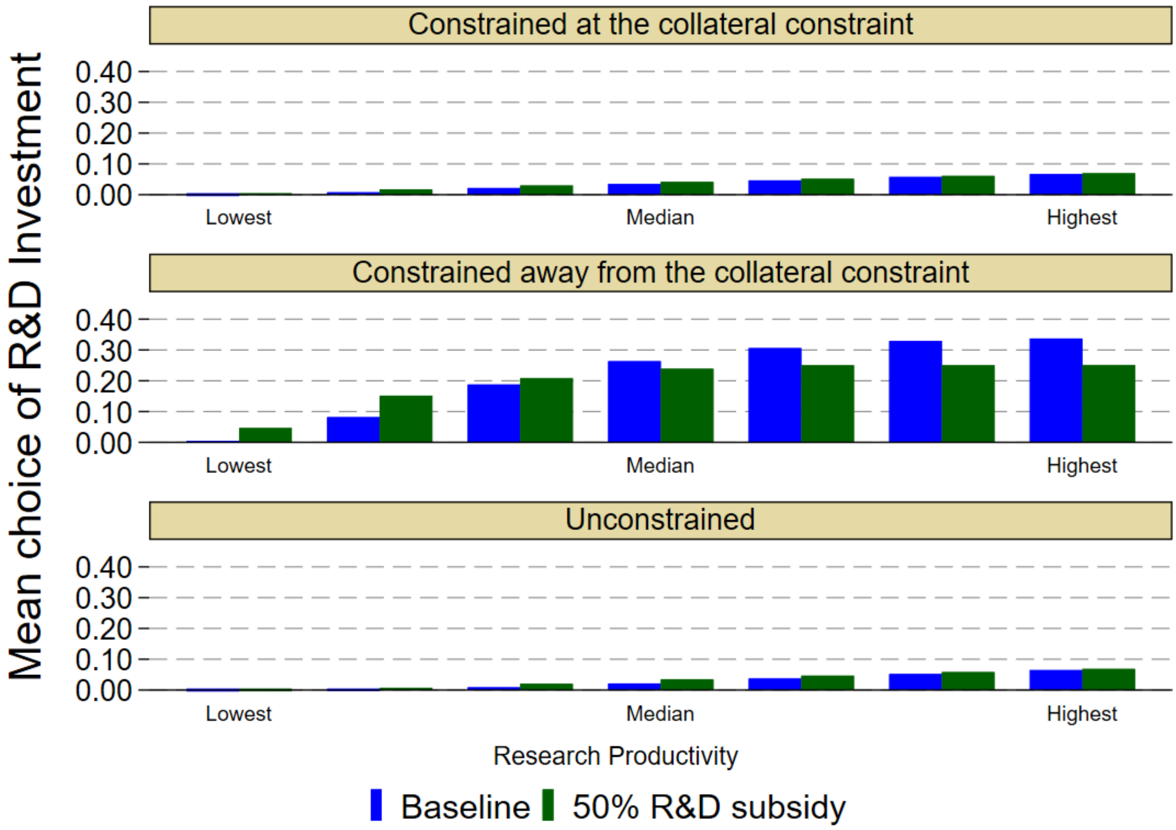
Hence, although the effect of large R&D subsidies is small, a policymaker still needs to decide whether it is better for potential innovation to have a larger number of smaller firms with R&D stocks (and so have high R&D subsidies) or have a smaller number of larger firms holding R&D stocks (and so have lower R&D subsidies). This reflects the trade-off between increased diversity of having many small firms and the increased appropriability of having a small number of large firms discussed in Cohen and Klepper (1992).

However, the fact that R&D subsidies tend to increase R&D investment by low research productivity firms and reduce those made by high productivity firms means that higher R&D subsidies might not be the best approach as it induces more relatively inefficient R&D spending. This can be seen from Figure 6, which shows the steady-state mean R&D investment for each level of research productivity, split according to whether a firm is 1) constrained with a debt level at the collateral constraint, in the top panel; 2) constrained

[36] The mechanism proposed by Berger is that a higher R&D subsidy increases the demand of inputs into R&D (such as researcher wages), thereby increasing the costs of conducting R&D, reducing the profitability of R&D for some firms.

but with debt that is below the collateral constraint level, in the middle panel; and 3) unconstrained firms, in the bottom panel.[37] The different coloured bars represent the R&D investment under the baseline scenario of a 5% R&D subsidy (the blue bars) and under the 50% R&D subsidy (the green bars).

Figure 6: Effect of higher general R&D subsidies on R&D investment by research productivity



Note: The results for the R&D subsidy rate $\tau_r = 0.05$ are the levels of each aggregate variable obtained in my calibrated model. The results for $\tau_r = 0.5$ (the bottom panel) are obtained by re-solving the stationary equilibrium for my model with that level of τ_r and allowing the level of the corporation tax τ_c to adjust so that the government surplus is kept the same as it was for the $\tau_r = 0.05$ scenario. Each panel shows the mean choice of R&D investment in the steady-state for firms with each idiosyncratic research productivity ε_r, split according to whether or not eh firms are 1) constrained with debt levels at the collateral constraint; 2) constrained but with debt below the collateral constraint; and 3) unconstrained.

The top panel of the graph shows that the mean R&D investment decisions by firms at the collateral constraint are higher under the R&D subsidy compared to the baseline case. Moreover, the proportion by which R&D investment is substantially higher for

[37] In both steady-states, roughly 87% of firms are constrained with a debt level at the collateral constraint, 4% are constrained but with debt not at the collateral constraint level, and the remaining 9% are unconstrained. Note that the latter firms are the only ones that pay non-zero dividends.

those firms with lower research productivity: under the 50% subsidy, R&D investment is almost 17 times higher compared to the baseline scenario for those firms with the lowest research productivity but only 1.04 times higher for those with the highest research productivity. This is also true for unconstrained firms as shown by the bottom panel: R&D investment is higher for all research productivities, but is nearly 70 times higher for firms with the lowest research productivity and only 1.06 times higher for firms with the highest research productivity.

Constrained firms that are not on the collateral constraint are growing quickly, so they invest more in R&D than do firms at the collateral constraint and unconstrained firms (the latter of which are investing just enough to cover the depreciation of their R&D stock). For these constrained firms, shown in the middle panel of the graph, the R&D subsidy increases the R&D investment for those firms with the lowest research productivity, but actually reduces it for those firms with the highest research productivity.

This means that firms with low research productivity increase their R&D investment more than do high research productivity firms for all types of constrained or unconstrained firms when R&D subsidies are increased. In other words, R&D subsidies do not appear to alleviate the effects of the collateral constraint on high research productivity firms to any substantial degree, but instead result in low research productivity firms substantially increasing their investments in R&D.

Moreover, the importance of the negative spillover of aggregate R&D is highlighted when re-running the comparison between a 5% general R&D subsidy and a 50% general R&D subsidy with the ν_3 coefficient in the productivity process (representing the spillovers from aggregate R&D) set to zero (rather than its estimated, negative, value). Such a comparison shows that output is increased by 3.2% under the higher general R&D subsidies without the negative spillover effect, compared to just 0.1% when the estimated negative spillover is present. In other words, the negative spillover from aggregate R&D serves to reduce substantially the effect of higher general R&D subsidies on output. R&D subsidies have small aggregate effects but large distribution ones, as seen by the response of heterogeneous firms in my model. The effect of different levels of the spillover from aggregate R&D is discussed in more detail in Online Appendix F.[38]

[38] It is possible that the use of Compustat overstates the extent of the negative spillovers that are present. In particular, as the vast majority of private R&D in the US is conducted by Compustat firms, the negative

Conversely, the change to the level of the corporation tax that ensures that the government's budget maintains the same surplus as in the baseline case of a 5% R&D subsidy has little effect on my results. Even when the R&D subsidy is increased to 50%, the corporation tax only increases by about six percentage points (i.e. it becomes roughly 41% rather than the baseline's 35%). Nonetheless, to check that these movements in the corporation tax are not driving my results (as changes to the corporation tax could affect firms' incentives to invest in either physical capital or R&D) I re-run each scenario keeping the corporation tax rate fixed.

These results are robust to this modification due to 1) only a small number (roughly 10%) of firms being unconstrained and, hence, paying dividends in the steady-state; and 2) firms taking more than 20 years to become unconstrained. The former means that very few firms could have their investment decisions affected by paying the adjusted corporation tax level in the current period and, even for these firms, the altered corporation tax does not affect their investment decisions substantially. The latter means that constrained firms very heavily discount (both due to the household rate of time preference and the exogenous probability of a firm exiting before it becomes unconstrained) the fact that they will have to pay a different corporation tax if they ever become unconstrained, such that their investment decisions are not meaningfully changed. Both of these factors mean that the choice of letting the corporation tax to adjust in order to maintain the government's budget surplus or keeping it fixed does not substantially affect my results.

5.2. *Effect on a cohort of entrants*

In order to examine the effect of changing the level of the general R&D subsidy on how firms grow over time, for each level of the R&D subsidy I simulate the growth of a cohort of entrants that are subject to a series of randomly drawn idiosyncratic shocks to output productivity ε_o and research productivity ε_r (the aggregate level of R&D stock that enters

spillovers resulting from product market rivalry and bidding up researcher wages (among others) could be larger than they are for non-Compustat firms as those non-Compustat firms are less likely to conduct any R&D and therefore not compete for researchers or introduce as many new products. Nonetheless, unless the negative spillovers for these non-Compustat firms are very small (or non-existent) and the positive spillovers are large, the spillovers that pertain overall are likely to be negative. And as shown in Online Appendix F, unless these overall negative spillovers are relatively small (less than one-fifth of the magnitude estimated here), then the boost to aggregate output from R&D subsidies is small.

$e^{f(R,\varepsilon_0,\mu)}$ is kept at the solved stationary equilibrium value for the relevant level of the subsidy). The result of this simulation is shown in Figure 7, in which the x-axis for each panel is the number of periods (years) of the simulation. The top left panel of this figure shows the number of firms present under each level of the R&D scenario relative to the base-case of a 5% R&D subsidy, with different coloured lines representing the different levels of the R&D subsidy. Although the same number of firms enter under each level of the subsidy, firms are more likely to survive (i.e. choose not to exit) as the level of the subsidy is increased.

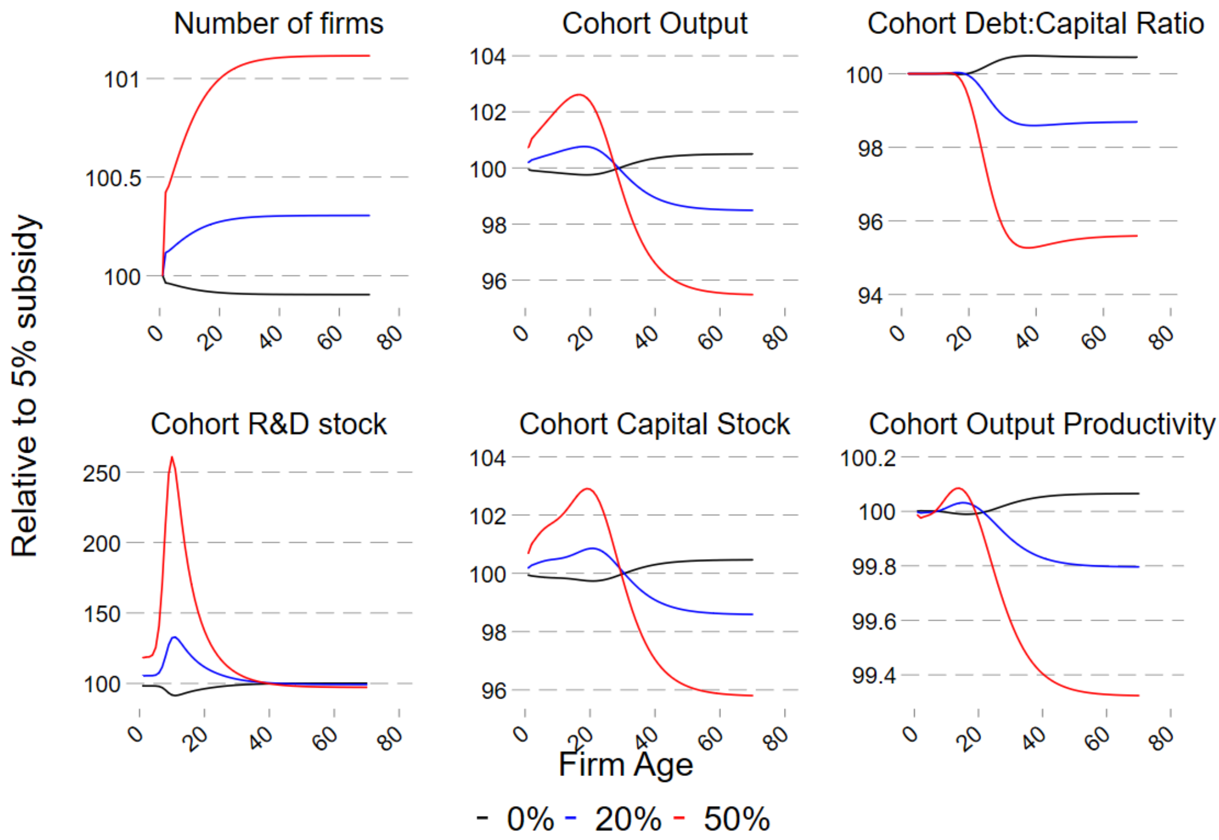
The remaining five panels show the stock of R&D, total output, physical capital stock, debt, and average productivity for the cohort of entrants under each level of the subsidy relative to the baseline case in which the subsidy is set to 5%. The bottom left panel, showing the stock of R&D held by the cohort, indicates that higher subsidies encourage entrants to choose much higher starting levels of R&D than under the base-case level of the subsidy. This difference in R&D stocks becomes even greater in the first few years after entry, before gradually declining such that the total R&D stock is roughly the same in all scenarios by period 40.

This higher initial R&D choice increases the marginal productivity of capital, so higher subsidies also result in entrants choosing higher capital stock. Similar to the path for the cohort's stock of R&D, the difference in the stock of physical capital also increases in initial years (again due to the effect R&D has on the marginal productivity of capital), before starting to decline relative to the base-case scenario after about 20 years. This decline is such that after 35 years, higher R&D subsidies are associated with lower stocks of physical capital. A similar pattern is found in the output produced by the cohort of entrants, and in the debt held by the cohort.[39] Finally, the average productivity of the cohort is roughly similar for the first few periods after entry, but then becomes decreasing in the R&D subsidy.

To disentangle the effect of the number of surviving firms from individual firm choices on these cohort totals, it is useful to look at the mean R&D stock, capital stock, output and debt across firms in each cohort relative to the base-case scenario. These are shown

[39] Even though debt is decreasing in the level of the subsidy for this cohort, aggregate debt for the entire steady-state mass of firms is unaffected by the level of the subsidy due to the presence of other cohorts of entrants at various stages in their lifecycles, and the fact that early on in their lifecycles, entrants' debt is increasing in the level of the subsidy.

Figure 7: Effect of different levels of R&D subsidies on aggregates from a cohort of entrants



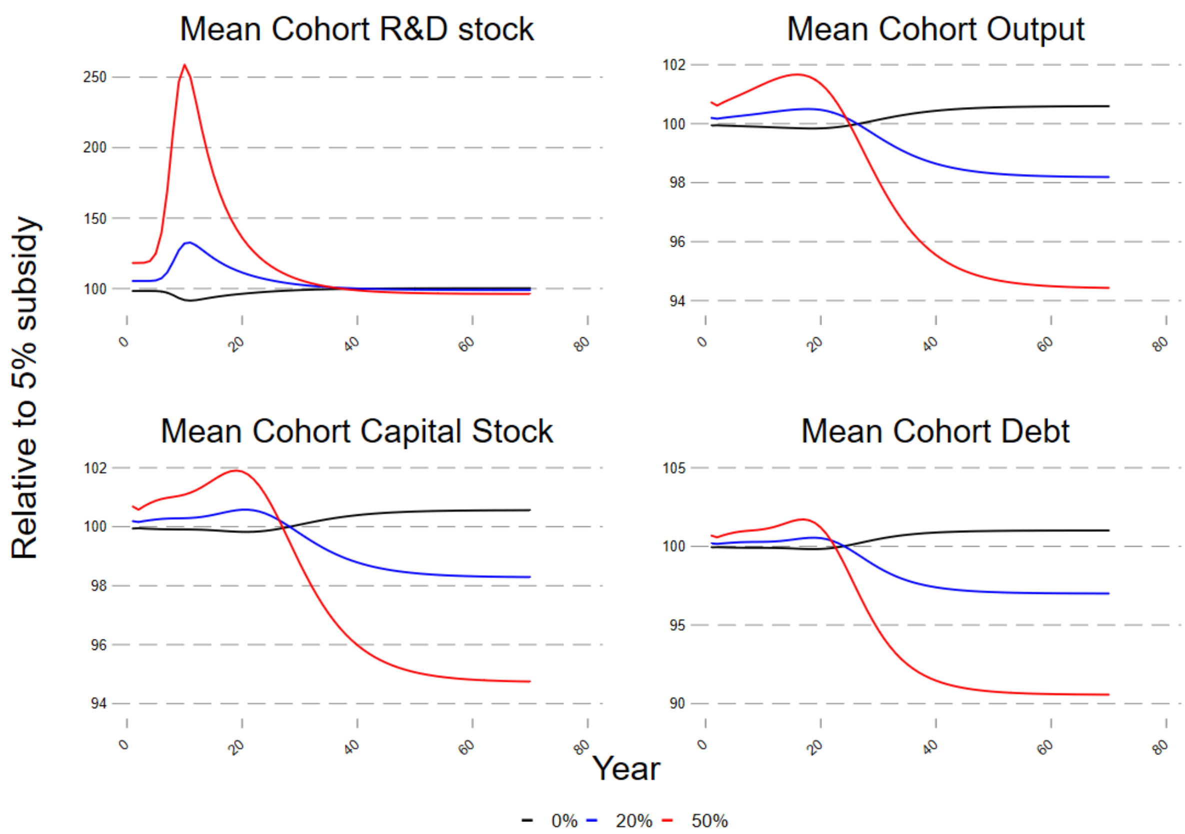
Note: The graphs show the time path of aggregate variables for a single cohort of entrants that are potential entrants in period 0 and choose whether or not to enter in period 1. The time paths are presented for different levels of the R&D subsidy τ_r relative to the base case of the R&D subsidy of $\tau_r = 5\%$.

in Figure 8. The top left panel shows that the mean stock of R&D follows the path of the aggregate stock for the cohort, with higher R&D subsidies having higher stocks of R&D in initial years, but that difference eroding over time.

The bottom left panel shows the mean stock of physical capital held by the cohort of entrants. Although the cohort's mean capital stock is higher initially under higher subsidies, that is reversed after roughly 25 years (as opposed to the 35 years it takes for this reversal to happen when looking at the totals for the cohort). A similar path also happens for the cohort's mean output and debt.

The mechanism underpinning these results is as follows. Higher R&D subsidies mean that investing in R&D is cheaper for entrants, so they start with a lot more of it (but the higher subsidies are not sufficient to induce more firms to enter). The higher starting

Figure 8: Effect of different levels of R&D subsidies on means of a cohort of entrants



Note: The graphs show the time path of the mean variables for a single cohort of entrants that are potential entrants in period 0 and choose whether or not to enter in period 1. The time paths are presented for different levels of the R&D subsidy τ_r relative to the base case of the R&D subsidy of $\tau_r = 5\%$.

R&D raises the marginal productivity of capital, such that entrants also start with higher capital. In the first few periods, the higher output resulting from these higher initial choices gives firms more cash-on-hand that they can use to invest further in R&D and physical capital, thereby enabling them to grow faster and potentially escape the collateral constraint earlier than otherwise.

Higher subsidies mean that firms choose to invest in R&D more than they do in physical capital (due to higher subsidies reducing the price of R&D investments relative to physical capital), so the average R&D stock continues to grow relative to the case with lower subsidies. The relative average physical capital stock is roughly constant for the initial years. This means that average output is slightly higher in those initial years as well, predominantly due to the slightly higher average physical capital stock. Moreover, the higher initial average output under higher subsidies means that more firms are able to afford the fixed continuation cost so more firms are able to survive (i.e. fewer firms choose to exit), with the result that higher subsidies result in a higher number of surviving firms. In the initial years, the combination of higher mean R&D, capital, and output with a higher number of surviving firms leads to a relatively sizeable effect on the cohort totals shown in Figure 7 previously.

After the first few years, however, the different level of the aggregate stock of R&D under different subsidy levels starts to affect firms' R&D decisions via the increased negative spillovers from higher aggregate R&D. Recall that a firm's productivity is given by $f(R, \varepsilon_o, \mu) = \chi(\nu_1 R + \nu_2 R^2 + \nu_3 R\bar{R}) + \varepsilon_o$, with the negative spillover coming via the value of ν_3 on the interaction term being less than zero. The fact that this negative spillover arises via an interaction effect means that the marginal effect of the aggregate level of R&D on an individual firm's productivity depends on the firm's own stock of R&D. In other words, the negative spillover has a small effect when a firm's own stock of R&D is small, but has more of an effect as an individual firm's stock of R&D increases. This means that the difference in an average firm's R&D relative to the base case starts to fall after ten years, once a firm's own stock of R&D becomes sufficiently large.

This effect increases as firms continue to grow, such that after 20 years the higher aggregate R&D stock (relative to the baseline scenario) obtained under higher R&D subsidies reduces the effect of a firm's own R&D on its productivity. The difference in mean R&D stock between higher subsidies and the base case continues to fall, but the combination of higher individual and aggregate stocks of R&D reduces the marginal productivity of physical capital (relative to the baseline scenario). Hence, average physical capital (and output) is now decreasing in the level of the subsidy. The reduction in output relative to the base-case does not result in a reduction in the number of surviving firms for higher levels of the subsidy because all surviving firms in the cohort have grown large enough after ten years that they can afford easily the fixed continuation cost and never would choose to exit.

The drives the hump-shape seen for the totals of these variables for the cohort. Specifically, when the cohort mean becomes decreasing in the level of the subsidy the total of that aggregate variable produced by the surviving firms for that cohort must start to decrease. It is only the fact that the number of surviving firms is larger for higher levels of the subsidy that the cohort totals remain increasing in the level of the subsidy for another 10 years after the cohort averages have become decreasing in the level of the subsidy. After 35 years, however, higher levels of the subsidy decrease the average cohort values sufficiently that even the larger number of surviving firms is not enough to have the cohort totals continue to be increasing in the subsidy.

These results indicate that higher levels of general R&D subsidies result in faster initial growth of entrants as well as more entrants surviving, but that those entrants then do not grow as large as entrants would under lower levels of the subsidy, due to the negative spillover having more of an effect as firms grow. This means that the evaluation of the effect of general R&D subsidies on the outcomes (such as survival, growth rates, profitability, and so on) of a firm or group of firms needs to take into account which part of their lifecycle the firm(s) are experiencing. For example, if a firm was at the 5-year point, then the apparent effect of a higher R&D subsidy would be different from the effect seen if the firm was at the 45 year point. In other words, the effect of R&D subsidies are likely to depend on a firm's age as much as they are on other characteristics.

6. The effect of subsidising entry

An alternative policy that could boost output by more than that obtained via subsidising R&D is to subsidise entry. This potentially could increase output due to the fact that some high productivity potential entrants do not have enough initial cash endowment to fund the fixed cost of entry and any starting investments in physical capital and R&D. Moreover, Luttmer (2011)'s finding that new entrants grow faster than incumbent firms suggests that if these high-productivity potential entrants were able to enter, they are likely to be able to rapidly increase their own production, thereby contributing substantially to aggregate output.

I find that entry subsidies are much more effective at increasing output than are R&D subsidies.[40] For example, a 60% entry subsidy increases output by 1.0%, ten times more than the most effective R&D subsidy. This is precisely because the entry subsidy does not

[40] As with general R&D subsidies in the previous Section, I allow the corporation tax to vary to ensure that the government surplus Λ from the base-case is constant in each scenario. Similar to the case with general R&D subsidies, the corporation tax only changes by a few percentage points at most. As a robustness check, I re-run each scenario keeping the corporation tax rate fixed and this does not substantially change my results due to the factors discussed previously.

explicitly target R&D and, therefore, does not substantially increase aggregate R&D. This means that it avoids the detrimental effect of negative spillovers from higher aggregate R&D. Instead, it results in the entry of more high productivity potential entrants who otherwise would not have chosen to do so, with these additional entrants surviving and growing quickly, thereby contributing to higher output.

Specifically, I examine the effect of providing entry subsidies that operate by the government paying some portion of the cost of entry that potential entrants must pay the period before they enter. This modifies Equation (7) representing the value function for a potential entrant such that they only pay $(1-\tau_e)\xi_e$ of the cost of entry rather than the full cost ξ_e if they choose to enter, where τ_e is the subsidy provided by the government. The government budget constraint is now $\int [\tau_c D]\, \mu(d[K \times R \times B \times \varepsilon_o \times \varepsilon_r) - \int \left[\tau_r(\mathbb{I}^O \frac{I_r}{\varepsilon_r} + \mathbb{I}^{\mathbb{N}} \frac{R^{N\prime}}{\varepsilon_r}) + \mathbb{I}^{\mathbb{N}}\tau_c D^N + \mathbb{I}^{\mathbb{N}}\tau_e\xi_e\right] \mu(d[K \times R \times B \times \varepsilon_o \times \varepsilon_r) = \Lambda$.

Table 5 presents the results of varying the entry subsidy. The second column of this table contains the results from the baseline model with a general R&D subsidy of 5% and no entry subsidy, while columns 3-5 contain the results of scenarios varying the level of the entry subsidy (the values of the entry subsidy shown are the minimum levels where the results change compared to the next highest level of the subsidy).

It is apparent that the entry subsidy has very different effects from the policies that target R&D directly. First, higher entry subsidies have very little effect on the aggregate R&D stock (the highest entry subsidy only increases the aggregate stock of R&D by 4%) while the number of firms with zero R&D stock increases slightly. This latter effect is driven by the fact that the idiosyncratic research productivity follows a Pareto distribution, so the entry subsidies encourage a relatively larger amount of low research productivity firms to enter compared to the baseline case, and these less productive firms have less of an incentive to invest in R&D due to it costing them (relatively) more to do so.

Second, the mass of firms is increased (by 5.5% under the highest entry subsidy compared to only 0.7% under the 50% general R&D subsidy), as one would expect since the entry subsidy enables more firms to enter. Most importantly, many of these firms that are now able to enter are high exogenous productivity firms that did not have a sufficiently high cash endowment to pay the entry cost without it being subsidised. The higher number of entrants means that the average size of firms decreases as the level of the entry subsidy increases; in particular, under the 60% entry subsidy, the average firm is roughly 4%

Table 5: Effect of subsidising entry on aggregate variables

Aggregates	Baseline	$\tau_e = 0.05$	$\tau_e = 0.3$	$\tau_e = 0.6$
R&D stock	0.22	100.5	102.1	104.0
Mass of firms	1.31	100.7	102.9	105.5
Output	0.50	100.1	100.5	101.0
Capital	1.04	100.1	100.4	100.7
Consumption	0.44	100.0	100.2	100.5
Labour	0.31	100.1	100.3	100.5
Average productivity	1.01	100.00	100.01	100.02
% of firms with zero R&D stock	42.3%	42.5%	42.6%	42.7%
Compensating Variation (% of income)	N/A	0.01%	-0.04%	-0.09%

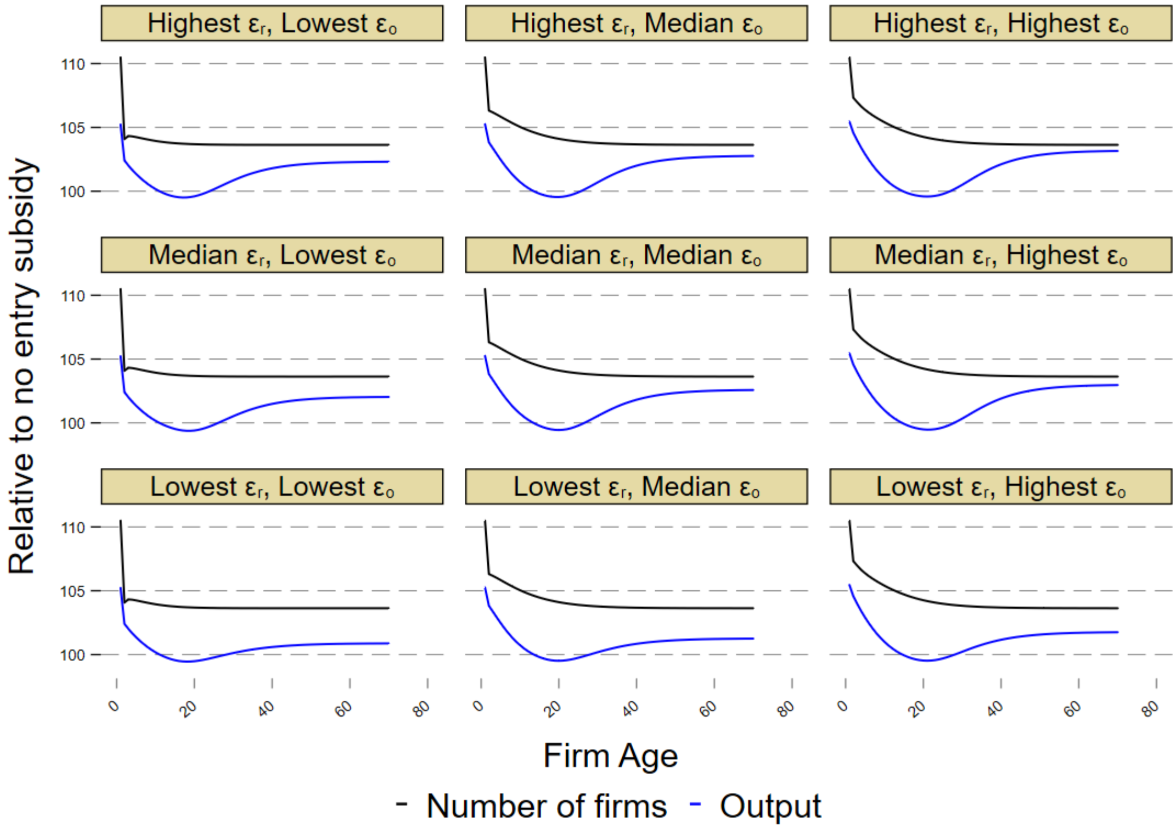
Note: The results for the baseline model are with an R&D subsidy rate $\tau_r = 0.05$ applying to all firms, with zero entry subsidy and are the levels of each aggregate variable obtained in my calibrated model. The results where there are non-zero entry subsidies (i.e. where $\tau_e > 0$) are obtained by re-solving the stationary equilibrium for my model with that level of τ_r and allowing the level of the corporation tax τ_c to adjust so that the government surplus is kept the same as it was for the baseline scenario. The results for these different entry subsidies are presented relative to the baseline scenario except for the last two rows of the table, which are, respectively, the actual proportion of firms with zero R&D stock in each scenario and the percentage of income required to restore the representative household to its utility level in the baseline scenario (a positive compensating variation indicates that firms are worse off under that particular scenario, while a negative compensating variation indicates that firms are better off compared to the baseline).

smaller than in the scenario without any entry subsidy due to the fact that there are more younger, smaller firms present.

Nonetheless, the entry subsidy means that more highly productive firms are now able to enter, produce, and survive. This results in a higher stock of physical capital and a higher aggregate level of labour alongside the higher mass of firms. Combined, this means that output is 1.0% higher under the highest level of entry subsidies, which is ten times the increase in output achieved by raising general R&D subsidies to 50%.

The higher number of entrants can be seen in Figure 9 which plots the cohort total number of firms (the black line) and the total output (the blue line) over time for a cohort of entrants under the 60% entry subsidy relative to the baseline case of no entry subsidy. Each panel shows these variables for the entrants in the cohort that hold each specific combination of idiosyncratic research ε_r and output ε_o productivities (where a productivity shock numbered 1 represents the lowest level of the shock, 5 represents the highest output productivity shock and 7 represents the highest research productivity shock).[41]

Figure 9: Effect of entry subsidies on number of firms and output from a cohort of entrants



Note: The graphs show the time path of aggregate variables for a single cohort of entrants that are potential entrants in period 0 and choose whether or not to enter in period 1. The time paths are presented for the 60% entry subsidy relative to the base case of the R&D subsidy of $\tau_r = 5\%$ and no entry subsidy. Each panel of the graph shows the cohort values for firms that have a specific combination of idiosyncratic output productivity ε_o and research productivity ε_r at each period. Note that firms can move between idiosyncratic productivities from period to period if they receive a shock to those productivities.

For the lowest output and research productivity (the top left panel), the entry subsidy results in 10% more entrants in period 1. However, as these firms are low productivity, many of them only produce for that single period before exiting, such that the number of these firms is only about 5% higher than the baseline case from period 2 on. This is because those firms that exited between period 1 and 2 were those at the very bottom of the distribution of the cash endowment, so could not afford a lot of physical capital or R&D even after the entry cost is subsidised. As such these firms find it profitable to enter and produce just for one period before exiting.

[41] Entrants in this exercise do transition between idiosyncratic productivities.

However, for those firms with the highest output and research productivity (the bottom right panel), the initial 10% increase in entrants as a result of the entry subsidy takes much longer to get to its ultimate level of roughly 4% higher than under the baseline scenario. This is because firms at this idiosyncratic productivity level are more able to pay the fixed continuation cost so fewer of them exit after the first period. Nonetheless, the number of firms at this level does gradually decrease relative to the baseline case due to some firms that transitioned to this productivity level only recently still not being sufficiently large to survive a negative shock. The larger number of firms at each productivity level and cohort age (plus the much higher number of firms in the initial period) drives the higher mass of firms under the entry subsidy.

The path of output for the cohort under the 60% entry subsidy is roughly similar for each combination of productivity levels: in the initial period, each productivity level's total output is roughly 5% higher than the baseline, before decreasing to slightly below the baseline output after roughly 15 years (the decrease is more gradual for higher productivity levels because those productivity levels do not have as many firms exit after the first period). Total cohort output then increases gradually for each productivity level until it ends up between 0.8% and 3% (depending on the productivity level) higher than the baseline case.

General equilibrium effects drive the dip in output of entrants between 15-20 years of age under the entry subsidy relative to the baseline scenario. In particular, higher consumption under the entry subsidy means that the wage is higher (as the wage is given by $w(\mu) = Sc(\mu)$ where S is the household's disutility of working). This reduces firms' optimum level of labour each period, also decreasing the marginal productivity of physical capital and R&D. Hence, new entrants grow more slowly under the entry subsidy such that even the presence of a larger mass of firms means that their total output relative to the baseline scenario decreases from years 2-20. However, once firms have surpassed 20 years of age, they are large enough that the fact they are more numerous under the 60% entry subsidy compared to the baseline scenario that their total output starts to increase relative to that baseline until reaching a plateau after roughly 50 years.

Indeed, higher consumption from higher entry subsidies means that the utility of the representative household is slightly increasing in the level of the entry subsidy (i.e. the compensating variation is decreasing), in contrast to the case with higher R&D subsidies.

This is despite the fact that higher entry subsidies increase aggregate labour by even more than do R&D subsidies, and is because entry subsidies allow more productive new entrants to come in rather than R&D subsidies mostly subsidising incumbents that do not grow as quickly.

This indicates that entry subsidies might be more effective at boosting output than are policies that focus specifically on R&D.

7. Conclusion

I have shown that firms can grow by increasing their stocks of R&D as an alternative to increasing their stock of physical capital, and that a firm's stock of R&D is important in determining its productivity. I estimate the impact of a firm's own stock of R&D and the aggregate stock of R&D on its productivity using a production function estimation procedure that allows a firm's productivity to depend on its own stock and the aggregate stock of R&D. I find that a firm's productivity is increasing, but with decreasing marginal returns, in its own stock of R&D such that a 1% increase in the average firm's own stock of R&D increases its productivity by 0.003%. Moreover, my results indicate the presence of negative spillovers from higher aggregate R&D as higher aggregate R&D reduces the effect of a firm's own stock of R&D on its productivity.

To investigate the effect of different potential government policies on long-run (steady-state) output and other economic aggregates, I developed a discrete time model in which firms choose their physical capital stock, R&D stocks, and debt for the next period subject to idiosyncratic shocks to their output and research productivity, and calibrated it to match aggregate moments for the US economy and the joint distribution of firms over stocks of R&D and physical capital. Although not targeted, my model also captures the different growth patterns of firms that are observed in the data.

I use my model to find that higher general R&D subsidies substantially increase the number of firms with non-zero R&D stocks and the aggregate stock of R&D in an economy, and slightly increase the total number of firms. However, the negative spillovers from higher aggregate R&D (caused by firms that previously did not invest in R&D choosing to do so, thereby increasing aggregate R&D) mean that increasing the level of the general R&D subsidy from 5% to 50% only leads to an increase in output of

0.1% . Absent these negative spillovers, output would increase 3.2% as a result of this change in the general R&D subsidy. The small output effects also result from alternative policies that aim to stimulate R&D (such as targeted R&D subsidies or allowing firms to use some proportion of their R&D stock as collateral when taking on debt), also driven by the negative spillover created by higher aggregate R&D.

The policy that has the largest impact on output is subsidising entry. A 60% entry subsidy boosts output by 1.0% due to a larger mass of firms as more potential entrants, many of which are highly productive, are now able to enter. In particular, highly productive potential entrants whose cash endowment is not sufficient to pay the fixed cost of entry without that cost being subsidised can now afford to pay the cost, enter, and start producing. These highly productive new entrants are then able to survive and keep contributing to output, such that the overall effect on output is much larger than policies that explicitly incentivise R&D (also due to the fact that entry subsidies do not increase aggregate R&D as much and therefore do not result in large negative spillovers).

I leave open for further work the issues of uncertainty and "jump-ahead" effects that might be associated with R&D investments. In particular, my model abstracts from explicitly modelling these aspects of R&D (although these factors are implicitly included in the empirically estimated coefficients determining R&D's effect on productivity) so further work in this area could explicitly model those elements and might find a larger effect of R&D subsidies than those presented here, particularly if any such model also incorporates diffusion of new technologies such that the effect of the estimated negative spillovers might be ameliorated. Additionally, distinguishing between certain types of R&D, such as basic research compared to applied research, and / or R&D aimed at developing new products rather than improving productivity (product vs process innovation) could affect the results of this analysis.

References

Acemoglu, Daron, Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William Kerr. 2018. "Innovation, Reallocation and Growth." *American Economic Review, forthcoming*.

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer. 2015. "Identification Properties of Recent Production Function Estimators." *Econometrica*, 83(6): 2411–2451.

Akcigit, Ufuk, Douglas Hanley, and Nicolas Serrano-Velarde. 2021. "Back to Basics: Basic Research Spillovers, Innovation Policy, and Growth." *Review of Economic Studies*, 88(1): 1–43.

Atkeson, Andrew, and Ariel Burstein. 2019. "Aggregate implications of innovation policy." *Journal of Political Economy*, 127(6): 2625–2683.

Berger, Patrick. 1993. "Explicit and Implicit Tax Effects of the R & D Tax Credit." *Journal of Accounting Research*, 2: 131–171.

Bloom, Nicholas, Charles I. Jones, John van Reenen, and Michael Webb. 2020. "Are ideas getting harder to find?" *American Economic Review*, 110(4): 1104–1144.

Bloom, Nicholas, John Van Reenen, and Heidi Williams. 2019. "A toolkit of policies to promote innovation." *Journal of Economic Literature*, 33(3): 163–184.

Bloom, Nicholas, Mark Schankerman, and John Van Reenen. 2013. "Identifying Technology Spillovers and Product Market Rivalry." *Econometrica*, 81(4): 1347–1393.

Cabral, Luís M.B., and José Mata. 2003. "On the Evolution of the Firm Size Distribution: Facts and Theory." *American Economic Review*, 93(4): 1075–1090.

Cohen, Wesley M., and Steven Klepper. 1992. "The tradeoff between firm size and diversity in the pursuit of technological progress." *Small Business Economics*, 4(1): 1–14.

Cooley, Thomas F., and Edward C. Prescott. 1995. *Economic Growth and Business Cycles.*

Davidson, Carl, and Paul Segerstrom. 1998. "R & D Subsidies and Economic Growth." *RAND Journal of Economics*, 29(3): 548–577.

De Loecker, Jan. 2013. "Detecting learning by exporting." *American Economic Journal: Microeconomics*, 5(3): 1–21.

Dieppe, Alistair, and Jan Mutl. 2013. "International R&D Spillovers: Technology Transfer vs. R&D Synergies."

Dinlersoz, Emin, Sebnem Kalemli-Ozcan, Henry Hyatt, and Veronika Penciakova. 2018. "Leverage over the Life Cycle and Implications for Firm Growth and Shock Responsiveness." *NBER Working Paper Series*, , (October).

Doraszelski, Ulrich, and Jordi Jaumandreu. 2013. "R and D and productivity: Estimating endogenous productivity." *Review of Economic Studies*, 80(4): 1338–1383.

Döttling, Robin, and Lev Ratnovski. 2020. "Monetary Policy and Intangible Investment." *IMF Working Papers*, 20(160).

Eisfeldt, Andrea L., and Adriano A. Rampini. 2006. "Capital reallocation and liquidity." *Journal of Monetary Economics*, 53(3): 369–399.

Ericson, Richard, and Ariel Pakes. 1995. "Markov-Perfect Industry Dynamics: A Framework for Empirical Work." *The Review of Economic Studies*, 62(1): 53.

Falato, Antonio, Dalida Kadyrzhanova, Jae W. Sim, and Roberto Steri. 2020. "Rising Intangible Capital, Shrinking Debt Capacity, and the US Corporate Savings Glut."

Galaasen, Sigurd Molster, and Alfonso Irarrazabal. 2021. "R&D Heterogeneity and the Impact of R&D Subsidies." *The Economic Journal*.

Gandhi, Amit, Salvador Navarro, and David A. Rivers. 2020. "On the identification of gross output production functions." *Journal of Political Economy*, 128(8): 2973–3016.

Gertler, Mark, and Simon Gilchrist. 1994. "Monetary policy, business cycles, and the behavior of small manufacturing firms." *Quarterly Journal of Economics*, 109(2): 309–340.

Hall, Bronwyn. 1990. "The Manufacturing Sector Master File: 1959-1987."

Hall, Bronwyn. 1993. "The Stock Market's Valuation of R&D Investment During the 1980's." *American Economic Review, Papers & Proceedings*, 83(2).

Hall, Bronwyn, and John Van Reenen. 2000. "How effective are fiscal incentives for R&D? a review of the evidence." *Research Policy*, 29(4-5): 449–469.

Haskel, Jonathan, and Stian Westlake. 2018. *Capitalism Without Capital.* Princeton University Press.

Hopenhayn, Hugo, and Richard Rogerson. 1993. "Job Turnover and Policy Evaluation: A General Equilibrium Analysis." *Journal of Political Economy*, 101(5): 915–938.

Howell, Sabrina T. 2017. "Financing innovation: Evidence from R&D grants." *American Economic Review*, 107(4): 1136–1164.

Jo, In Hwan. 2021. "Firm Size and Business Cycles with Credit Shocks."

Jovanovic, Boyan. 1982. "Selection and the Evolution of Industry." *Econometrica*, 50(3): 649–670.

Kehrig, Matthias, Weiting Miao, and Daniel Y Xu. 2022. "R D , Innovation and Productivity."

Khan, Aubhik, and Julia K. Thomas. 2013. "Credit shocks and aggregate fluctuations in an economy with production heterogeneity." *Journal of Political Economy*, 121(6): 1055–1107.

Kim, Minjung, and Moon Jung Choi. 2019. "R&D spillover effects on firms' export behavior: evidence from South Korea." *Applied Economics*, 51(28): 3066–3080.

Loumioti, Maria. 2012. "The Use of Intangible Assets as Loan Collateral."

Lucking, Brian, Nicholas Bloom, and John Van Reenen. 2019. "Have R&D Spillovers Declined in the 21st Century?" *Fiscal Studies*, 40(4): 561–590.

Luttmer, Erzo G.J. 2011. "On the Mechanics of Firm Growth." *Review of Economic Studies*, 78(3): 1042–1068.

Mann, William. 2018. "Creditor rights and innovation: Evidence from patent collateral." *Journal of Financial Economics*, 130(1): 25–47.

OECD. 2019. "R & D Tax Incentives : United States , 2019." 1–2.

OECD. 2022. "OECD R&D tax incentives database, 2021 edition." February.

Pakes, Ariel, and Richard Ericson. 1998. "Empirical Implications of Alternative Models of Firm Dynamics." *Journal of Economic Theory*, 79(1): 1–45.

Peters, Ryan H., and Lucian A. Taylor. 2017. "Intangible capital and the investment-q relation." *Journal of Financial Economics*, 123(2): 251–272.

Sterk, Vincent, Petr Sedlacek, and Benjamin W. Pugsley. 2020. "The Nature of Firm Growth." *American Economic Review*.

Syverson, Chad. 2011. "What determines productivity?" *Journal of Economic Literature*, 49(2): 326–365.