Financial Crises With Different Collateral Types

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Abstract

Firms borrow against earnings more than they do against their assets. How does this affect the aggregate response to financial crises? I take a dynamic stochastic general equilibrium model of heterogeneous firms that choose their capital and debt subject to a borrowing constraint and examine the recovery from a financial crisis when firms can use different types of collateral. I compare between two collateral types: assets, and earnings. I find that when firms borrow against earnings recessions are deeper, but recoveries are quicker compared to when firms borrow against assets. I also find that neither type of collateral can, buy itself, completely explain the recovery from the Great Recession. Instead, the path of investment after the 2007-2008 Financial crisis is better captured by firms borrowing against earnings than by firms borrowing against assets, but this is reversed when looking at the path of output. This suggests that a combination of collateral types is required to fully capture the recovery from the Great Recession.

Key words: Financial crisis, collateral constraints, financial frictions, heterogeneity, earnings as collateral

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1. Introduction

How does the type of collateral that firms borrow against matter for recoveries from recessions, particularly those caused by financial shocks? Recent evidence from the US indicates that the majority of firms use their earnings rather than their assets as collateral: Lian and Ma (2021) find that 80% of US firms borrow against their cashflow whereas only 20% put up specific assets as collateral and Drechsel (2023) indicates that the most popular type of debt firms take on is collateralised against a measure of their earnings.[1] Despite this, most business cycle models examining the effect of financial shocks rely on firms borrowing solely against assets.[2]

Therefore, I use a standard dynamic stochastic general equilibrium model of heterogeneous firms to examine the differences between recoveries from recessions when firms borrow against assets (in the form of capital) compared to when they borrow against their earnings. Firms invest in capital and take on debt subject to a borrowing constraint. In one specification, this borrowing constraint assumes that firms can borrow up to a certain fraction of their choice of capital, a la Kiyotaki and Moore (1997); in the other specification, the borrowing constraint assumes that firms can take on debt up to a multiple of their expected future earnings, as modelled by Drechsel (2023).

I find that the type of collateral that firms use to secure their debt causes substantial differences in the recoveries from negative TFP and financial shocks. In the case of a temporary decrease in aggregate TFP, the recovery is initially much more volatile when firms use expected earnings as collateral, but settles down and then has a quicker recovery (by 1-2 years) compared to when firms borrow against expected earnings.

Moreover, in the case of a financial shock, there are even greater differences between the two specifications: the troughs in output, consumption, investment, and capital are much deeper when firms borrow against expected earnings compared to when they use capital as collateral: each variable falls by roughly twice as much (relative to steady-state) in the former case. Once the financial shock is over, however, these variables return to their pre-shock level much more quickly when firms borrow against expected earnings compared to when they borrow against capital.

[1] Specifically, Drechsel (2023) finds that roughly 60% of loans are made against a firm's earnings before interest, tax, depreciation, and amortisation (EBITDA).

[2] See, for example, work by Jermann and Quadrini (2012), Khan and Thomas (2013), and Jo (2021).

These differences are driven by the effect of the borrowing constraint on firm's capital choices. When firms use their capital as collateral the effect of the collateral constraint on their choice of capital is just the multiple of capital that they can borrow. Hence, a reduction in this multiple has a simple, direct effect on a firm's choice of capital.

However, when firms borrow against expected earnings, the borrowing constraint has an effect on their capital choice via a combination of the multiple of those earnings that they can borrow and the expectation of what those earnings will be next period. As that expectation of earnings is affected by expectations of the aggregate state, consumption, and wages, there is (additional) feedback of those expectations into a firm's choice of capital.[3] This means that when firms expect a detrimental future state of the economy, they will reduce their capital by more when they borrow against earnings compared to when they borrow against capital; conversely, when they expect a beneficial future state of the economy, they increase their capital by more when they borrow against earnings.

Firms borrowing against expected earnings more accurately captures the path of investment after the Great Financial Crisis, but firms using capital as collateral does better at capturing the path of output during that recovery. In other words, neither collateral type is sufficient by itself to explain the recovery from the Great Recession, but instead some combination of the two is likely to be required in order to do so. This lends weight to Kariya (2022) and **?**'s empirical results that both loan-to-value (i.e. capital as collateral) and debt-to-income (i.e. earnings as collateral) borrowing constraints tightened at different times during the Financial Crisis and recovery from it.

I also find that there is substantial heterogeneity in firms' responses to a financial crisis. For example, in the face of a financial shock, larger firms (in terms of their capital stock) reduce the amount of capital that they hold, whereas smaller and medium-sized firms actually increase their stock of capital (so they can try to ameliorate the effect of the tightened borrowing constraint on the amount of debt they can hold). However, once the financial shock is over, smaller firms reduce their capital stock rapidly while larger firms gradually start to increase their capital holdings. It is only after roughly eight years that smaller firms then start increasing their capital again and returning to their steady-state level of capital. This holds regardless of the type of collateral that is used.

[3] Expectations of the aggregate state, consumption, and the wage also affect optimum capital choice via the marginal productivity of capital in output. This channel is present in both specifications of the borrowing constraint, but the additional expectations effect is only present in the expected earnnigs specification.

My results indicate that not only does a policymaker that wants to alleviate the effects of a financial shock need to be careful regarding which aspect of banking regulation (asset-based or earnings-based) they target for any reform, but they also need to take into account the fact that different firms respond very differently from each other and from the aggregate.

I contribute to two strands of the literature. My first contribution is to the literature on the relative importance of different types of borrowing constraints when examining business cycles. Seminal work such as Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999) focused on looking at the importance of financial frictions such as agency costs or borrowing constraints based on a firm's assets, while recent papers have given consideration to other types of collateral against which firms can borrow.

Perhaps closest to my paper is Drechsel (2023), who provides evidence that firms tend to borrow against earnings rather than collateral and uses this to develop a model in which firms are of two different types: ones that can borrow against only their earnings, and ones that can borrow only against their capital. Assuming that both types of firms are always subject to a binding collateral constraint, Dreschel finds that the type of collateral firms use matters for the aggregate responses to recoveries from recessions due to TFP or investment-cost shocks. In contrast, I allow for more heterogeneity across firms (by introducing idiosyncratic output productivity) and examine the economy's recovery from a financial recession.[4]

Nonetheless, similar to Dreschel, I find that the type of collateral against which firms borrow matters substantially for the recovery from recessions, although I find that earnings-based constraints are not necessarily better than capital-based ones at capturing the recoveries of different aggregate variables after a financial crisis. This also matches Lorenzoni (2008)'s finding in a three-period model with a representative firm that the type of collateral used to secure a loan can have different aggregate effects, and Caglio, Darst and Kalemli-Ozcan (2022)'s empirical results that the type of collateral firms use matter for their responses to monetary policy shocks.

The second contribution of my paper concerns the effect of financial crises and recoveries from them, with particular focus on their heterogeneous nature across firms. This follows

[4] Choi (2022) also introduces heterogeneity in the form of idiosyncratic firm output productivity, but focuses on the impact that has on recoveries from TFP and investment-cost recessions, as well as the impact of firm default in such recoveries.

on from Jermann and Quadrini (2012) who use a representative firm that is always at the (capital-based) borrowing constraint, and Khan and Thomas (2013) who introduce firm heterogeneity and find that negative financial shocks can lead to prolonged recoveries. Similarly, Jo (2021) examines the importance of the firm-size distribution in determining the response to financial shocks. My findings reinforce these papers by showing that heterogeneity across firms matters even when firms can borrow against earnings rather than against capital.

Likewise, I provide support for Diamond, Hu and Rajan (2020)'s empirical results concerning the impact differences in enforceability of the contracts governing a firm's debts can affect business cycles, and for Chaney, Sraer and Thesmar (2012)'s finding that firms exhibit heterogeneous responses to balance sheet shocks, by showing that differences in loan contracts and firm responses also matter for recoveries from financial crises. Finally, my examination of heterogeneous responses to credit shocks relates to Buera and Moll (2015)'s assessment of the heterogeneous responses to such shocks, with my results indicating that the the aggregate and firm-specific responses to those shocks differ when firms borrow against different types of collateral.

The remainder of this paper proceeds as follows: Section 2 sets out the model I use to investigate the effect of different types of collateral, while Section 3 provides an analysis of the effect of the different types of collateral on firms' optimum choices. Section 4 contains a description of the calibration of my model, and Section 5 presents the responses of aggregate variables to recessions caused by each of a negative TFP shock and a negative financial shock. Section 6 then details how the responses to the negative financial shock differ across different groups of firms, and Section 7 concludes.

2. Model

In this Section I set out the features of the discrete-time stochastic general equilibrium model and characterise agents' optimisation problems. There are two categories of agents: an infinitely-lived representative household and a mass of heterogeneous firms that produce the single consumption good. The representative household obtains utility from the consumption good C and leisure $1 - N$; it also owns the firms and so receives any profits from them.[5]

[5] Households also have access to a full set of state-contingent claims, but as the household is representative in my model, these are in zero net supply in equilibrium. As such, I do not explicitly model these

Firms are characterised by their exogenous idiosyncratic output productivity ε and their stock of physical capital k and debt b (which can be combined into their cash-on-hand at the start of the period m as per Jo (2021)). The distribution of firms over these variables is represented by μ. Each period, incumbent firms face a probability π_d of exogenous exit and produce output y using labour ℓ along with their physical capital stock. In addition, a firm's stock of physical capital depreciates by δ, and surviving firms choose their physical capital and debt for the next period. Their choice of debt is subject to a borrowing constraint governed by a multiple θ of collateral against which firms can borrow, and which differs across two specifications of the model (one based on physical capital and the other based on earnings) and is discussed in more detail below.

These firms are also subject to aggregate shocks in the form of aggregate output productivity z and to the borrowing constraint; for ease of notation, I represent these exogenous state variables as $S = (z, \theta)$. Finally, there is a mass of entrants that enter each period with zero debt and physical capital k^N (which is some fraction χ of the aggregate capital stock).[6]

I present the optimisation problems of each agent in turn.

2.1. *Households*

There is a continuum of infinitely-lived identical households that care about consumption C and leisure $1 - N$ (with N representing the household's labour supply). These households have a subjective rate of time preference β and earn wages $w(S, \mu)$ by supplying labour, own shares (a vector λ) in the firms, and can choose to borrow or save in one-period discount bonds ϕ.

Hence, households solve the following problem:

$$V^h(\lambda, \phi; S, \mu) = \max_{C,N,\phi',\lambda'} U(C, 1 - N) + \beta E_{\pi_S} V^h(\lambda', \phi'; S', \mu') \tag{1}$$

claims here for convenience.

[6] As the aggregate capital stock changes over time, this means that the capital stock with which entrants begin the period also changes over time.

$$
\begin{aligned}
C + q(S,\mu)\phi' + \int \rho_1(k',b',\varepsilon';S,\mu)\lambda'(d[k'\times b'\times\varepsilon')] \\
= w(S,\mu)N + \phi + \int \rho_0(k,b,\varepsilon;S,\mu)\lambda(d[k\times b\times\varepsilon) \\
\mu' = \Gamma(S,\mu)
\end{aligned}
\tag{2}
$$

where Equation (2) is the household budget constraint, and with the price of a one-period discount bond (i.e. the loan discount factor) represented by $q(S,\mu)$; the ex-dividend price of shares in a firm with capital stock k, debt b, and exogenous output productivity ε denoted by $\rho_1(k',b',\varepsilon';S,\mu)$; and $\rho_0(k,b,\varepsilon;S,\mu)$ indicating the share price including dividend.

The equilibrium wage w is determined by the marginal rates of substitution between consumption and leisure for the household, and the bond price (or loan discount factor) q by the inverse of the (expected) gross real interest rate. In other words, $w(S,\mu) = \frac{D_2U(C,1-N)}{D_1U(C,1-N)}$ and $q(S,\mu) = \beta\frac{E\pi_{S'}D_1U(C'_{S'},1-N'_{S'})}{D_1U(C,1-N)}$. The distribution of firms moves according to the law of motion Γ.

Following Khan and Thomas (2013), I can use the household's share optimisation problem to establish the firm's stochastic discount factor due to the fact that household owns all firms. Therefore, I can re-write the firm problem to incorporate that stochastic factor such that the firm problem scales any current period profit by $p(S,\mu) = D_1U(C,1-N)$. The specific for of the household utility function takes Rogerson-Hansen preferences such that $U(C,1-N) = \log C + \varphi(1-N)$, with φ representing the household's disutility of working.

2.2. *Firms*

Firms learn their persistent idiosyncratic output productivity at the start of the period, at the same time as they learn if they will be affected by the exogenous death shock or if they get a choice of continuing on to the next period.[7] After these shocks are observed each period, firms hire labour ℓ, and produce using their stock of physical capital k via a Cobb-Douglas production function with weights α on capital and γ on labour. A firm's

[7] The presence of an exogenous probability of death each period is necessary in my model to ensure that my model economy is not entirely populated by large firms. Without the exogenous death shock, all firms would survive until they reached their optimum size, such that the steady-state would consist only of large firms.

output productivity is determined by their exogenous output productivity shock, which is persistent according to a one-period Markov Chain. As there is complete reversibility of physical capital, as per Jo (2021), I can represent a firm at the start of a period by its cash-on-hand. In particular, a firm's cash-on-hand is given by $m = \max_\ell z\varepsilon k^\alpha \ell^\gamma - w(S,\mu)\ell + (1-\delta)k - b$, which allows me to reduce the state space used to describe firms from two variables (physical capital and debt) down to one (its cash-on-hand).

After production has occurred, those firms that did not receive the exogenous death shock choose their capital stock k' and debt b' (subject to a borrowing constraint). Firms that received the exogenous death shock pay off their current debts, sell off their undepreciated capital stock, and pay out whatever is left as dividends to the household. Each period there is also a mass of entrants that match the number of exiting firms in number. Firms that continue on to the next period save any profits they make and pay zero dividends to households.[8]

I use two different specifications of the model, that differ only in the form of their borrowing constraint. In particular, in one specification (the physical capital collateral specification), firms can borrow up to θ_k of their choice of physical capital choice for the next period. This is the form of the borrowing constraint used in Kiyotaki and Moore (1997) and similar to that used by Khan and Thomas (2013) and is presented in Section 2.2.1.

The borrowing constraint in the second specification of the model instead allows firms to borrow up to θ_e against their expected future earnings, reflecting arguments made by Drechsel (2023) and Lian and Ma (2021) that the majority of loan contracts are based on future earnings rather than physical capital and other assets to determine the amount that firms can borrow and is presented in Section 2.2.2.[9]

My use of distinct specifications of the model with different borrowing constraints in each (rather than one model that combines the two borrowing constraints) is supported by Ivashina, Laeven and Moral-Benito (2022)'s results that the vast majority of firms do not switch between different loan types (i.e. they do not switch between borrowing against

[8] An alternative to firms saving all of their profits and paying zero dividends would be the approach taken by Khan and Thomas (2013) and Jo (2021) which distinguish between constrained and unconstrained firms, with only the latter paying dividends. That approach would require the use of a "minimum savings policy" for the unconstrained firms in order to determine their choice of debt each period, which greatly complicates the computational requirements for the solution.

[9] I use expected future, rather than current, earnings to ensure that both specifications of the constraint reflect a lender's desire to be able to recoup any debt in the next period, and (expected) future earnings better reflects the ability of lenders to recoup those debts than do current earnings.

capital and borrowing against expected earnings), indicating that firms pick either one or the other rather than regularly switch between the two alternatives.

2.2.1. *Specification with physical capital as collateral*

Let V represent the value of an incumbent firm at the start of the period and V^1 be the value of a firm that does not receive the exogenous death shock. The incumbent firm's optimisation problem can be defined recursively as

$$\begin{aligned} V(m,\varepsilon;S,\mu) = \pi_d p(S,\mu)m \\ + (1-\pi_d)V^1(m,\varepsilon;S,\mu) \end{aligned} \tag{3}$$

$$V^1(m,\varepsilon;S,\mu) = \max_{k',b'} E_{\pi_S} E_{\pi_\varepsilon} V(m',\varepsilon';S',\mu') \tag{4}$$

subject to

$$\begin{aligned} 0 &= m - k' + q(S,\mu)b' \\ b' &\leq \theta_k k' \\ \mu' &= \Gamma(\mu) \\ m' &= m(K',B',\varepsilon';S',\mu') \\ &= \max_{\ell'} z\varepsilon k'^{\alpha} \ell'^{\gamma} - w(S,\mu')\ell' + (1-\delta)k' - b \end{aligned} \tag{5}$$

Equation 3 indicates that firms simply take a weighted average over its values of exiting or continuing before it knows whether or not it receives the exogenous death shock. If the firm survives, then it chooses its physical capital stock k' and debt b' to maximise its expected future value (with expectations over the aggregate state and its own idiosyncratic productivity) as per Equation 4. This optimisation is subject to the constraints that its dividends (i.e. within-period profit) must be zero and its choice of debt must be no more than $\theta_k k'$ (the physical capital as collateral form of the borrowing constraint). The firm's cash-on-hand depends on its output, labour expenses, undepreciated capital, and debt brought forward in the next period, while $\mu' = \Gamma(\mu)$ represents the law of motion of the distribution of firms.

2.2.2. *Specification with expected earnings constraint*

In the specification of the model where firms face a borrowing constraint based on their expected earnings, firms' optimisation problem is the same as that presented above except for the borrowing constraint. This borrowing constraint instead takes the form

$$b' \leq \theta_e E_{\pi_S} E_{\pi_\varepsilon} (\max_{\ell'} z\varepsilon k'^{\alpha} \ell'^{\gamma} - w(S, \mu')\ell') \tag{6}$$

In other words, firms can borrow up to θ_e of their expected earnings (output minus labour expenses), where those earnings depend on the firm's own physical capital, optimal labour choice, and exogenous idiosyncratic productivity as well as the aggregate state next period. This formulation of earnings captures a firm's Earnings before Interest, Depreciation, and Amortisation (EBITDA) as highlighted by Drechsel (2023). Note that expectations are taken over aggregate output productivity z and the level of the multiple of expected earnings against which firms can borrow θ_e (since that affects the wage that firms must pay), as well as the firm's own idiosyncratic productivity ε.[10]

2.3. *Equilibrium*

Let $\mathbb{I}^O$ represent those firms in the distribution that are continuing incumbent firms and $\mathbb{I}^{\mathbb{N}}$ denote entrants, a Recursive Competitive Equilibrium is a set of prices p, w, q, ρ_0, ρ_1, and functions for quantities k', l, b', C, N,ϕ', λ', k^N and values V^h, V, V^1 that solve the firm and household problems and clear markets as described below:

1. V and V^1solve Equations (3) and (4), and ℓ, k', b' are the associated policy functions for incumbent firms
2. V^h solves Equation (1) with C, N, ϕ, and λ being the household's associated policy functions
3. The distribution of firms over $k \times b \times \varepsilon$ is given by μ and evolves according to $\mu' = \Gamma(\mu)$
4. The labour market clears $N = \int \ell(k, \varepsilon; S, \mu)\mu(d[k \times b \times \varepsilon)$

10 As both the physical capital and earnings-based constraints are based on future values (k' in the case of the physical capital constraint and expected earnings in the case of the earnings constraint), I abstract from the distinction between current-valued and future-valued borrowing constraints discussed in Ottonello, Perez and Varraso (2022).

5. The goods market clears $C = \int \left(z\varepsilon k^{\alpha}\ell^{\gamma} + (1-\delta)k - \mathbb{I}^{O}[k'] - \mathbb{I}^{\mathbb{N}}[k^{N\prime}] \right) \mu(d[k \times b \times \varepsilon])$
6. Price q clears the bond market
7. Prices ρ_0 and ρ_1 clear the market for shares

3. Analysis of firm decision rules

In each period all incumbent firms optimally choose their labour demand depending on the wage rate, their capital stock, and their output productivity (determined by the level of aggregate TFP and each firm's idiosyncratic productivity shock). In other words, their optimal choice of labour is given by $\ell = \left(\frac{w(S,\mu)}{\gamma z \varepsilon}\right)^{\frac{1}{\gamma-1}} k^{\frac{-\alpha}{\gamma-1}}$.[11]

Firms' first-best (i.e. absent any constraints on borrowing or loss-making) choice of capital next period is obtained by substituting for ℓ in Equation (4) and then maximising it with respect to K' before applying the Benveniste-Scheinkman condition to replace $V(m', \varepsilon'; S', \mu')$ (and noting that the borrowing constraint does not bind for this first-best choice). This gives the analytical expression for the first-best choice of capital k'^{*}:

$$k'^{*} = \left(\frac{\frac{p(S,\mu)}{\beta} - (1-\delta_k) E_{\pi_S} p(S',\mu')}{E_{\pi_S} p(S',\mu') E_{\pi_\varepsilon} \left[\frac{-\alpha}{\gamma-1} z'\varepsilon' \left(\frac{w(S',\mu')}{\gamma z'\varepsilon'}\right)^{\frac{\gamma}{\gamma-1}} + \frac{\alpha}{\gamma-1} w(S',\mu') \left(\frac{w(S',\mu')}{\gamma z'\varepsilon'}\right)^{\frac{1}{\gamma-1}} \right]} \right)^{\frac{\gamma-1}{1-\alpha-\gamma}} \tag{7}$$

Firms pay zero dividends, so their choice of debt must ensure that $0 = m - k' + q(S,\mu)b'$. Moreover, a firm's choice of debt cannot exceed the collateral constraint. Firms that can afford the first-best level of physical capital k'^{*} without violating the borrowing constraint and without their dividends being negative choose that first-best level of capital for next period. Thus, their choice of debt becomes $b' = \frac{1}{q(S,\mu)}(k'^{*} - m)$.

[11] This relies on labour only entering the firm's problem via the production function (and labour expenses). If, in the earnings-based specification of the model, firms were to borrow against their current earnings rather than their expected earnings, then firms that were borrowing constrained would have an extra term in their optimum choice of labour as their labour choice would also directly affect how much they could borrow.

Firms that cannot afford the first-best choice of capital without paying negative dividends and / or exceeding the borrowing constraint instead choose the maximum amount of capital that they can afford without violating those two constraints. In the specification of the model with capital as collateral, this means that Equation (7) becomes:

$$k'^{*} = \left(\frac{\frac{p(S,\mu)}{\beta} + \frac{\theta_k \lambda}{\beta} - (1-\delta_k) E_{\pi_S} p(S',\mu')}{E_{\pi_S} p(S',\mu') E_{\pi_\varepsilon} \left[\frac{-\alpha}{\gamma-1} z'\varepsilon' \left(\frac{w(S',\mu')}{\gamma z'\varepsilon'} \right)^{\frac{\gamma}{\gamma-1}} + \frac{\alpha}{\gamma-1} w(S',\mu') \left(\frac{w(S',\mu')}{\gamma z'\varepsilon'} \right)^{\frac{1}{\gamma-1}} \right]} \right)^{\frac{\gamma-1}{1-\alpha-\gamma}} \tag{8}$$

where λ is the multiplier on the borrowing constraint. In other words, the second-best choice of capital for constrained firms that can borrow against their capital depends only on the current value of the borrowing parameter θ_k, the household rate of time preference β, and the value of the multiplier on the borrowing constraint. The fact that firms must pay zero dividends means that firms choose $k' = \frac{m}{1-q(S,\mu)\theta_k}$ and their choice of debt is thus $b' = \frac{1}{q(S,\mu)}(k'-m)$.

In the specification where firms borrow against expected earnings Equation (7) becomes:

$$k'^{*} = \left(\frac{\frac{p(S,\mu)}{\beta} - (1-\delta_k) E_{\pi_S} p(S',\mu')}{\frac{\lambda}{\beta}\theta_e \Omega + E_{\pi_S} p(S',\mu') E_{\pi_\varepsilon} \left[\frac{-\alpha}{\gamma-1} z'\varepsilon' \left(\frac{w(S',\mu')}{\gamma z'\varepsilon'} \right)^{\frac{\gamma}{\gamma-1}} + \frac{\alpha}{\gamma-1} w(S',\mu') \left(\frac{w(S',\mu')}{\gamma z'\varepsilon'} \right)^{\frac{1}{\gamma-1}} \right]} \right)^{\frac{\gamma-1}{1-\alpha-\gamma}} \tag{9}$$

where $\Omega = E_{\pi_S} E_{\pi_\varepsilon} z'\varepsilon' \left(\frac{w(S',\mu')}{\gamma z'\varepsilon'} \right)^{\frac{\gamma}{\gamma-1}} - w(S',\mu') \left(\frac{w(S',\mu')}{\gamma z'\varepsilon'} \right)^{\frac{1}{\gamma-1}}$, and λ is again the multiplier on the borrowing constraint. This means that when firms borrow against their expected future earnings, their choice of capital is modified not only by the current value of the borrowing parameter, θ_e, the household rate of time preference β, and the value of the multiplier on the borrowing constraint, but also the interaction between these terms and the expected future state of the economy and the firm. Specifically, aggregate output productivity, individual firm productivity, and the wage (which itself depends on the aggregate state of the economy and the distribution of firms) each affect a firm's expected future earnings, thereby affecting their choice of capital to take into the next period.

Nonetheless, in this case, these firms' choice of k' can be determined by the expression $k' = m + q(S,\mu)\theta_e E_{\pi_S} E_{\pi_\varepsilon} \max_{\ell'}[z\varepsilon k'^{\alpha} \ell'^{\gamma} - w(S,\mu')\ell']$ via the zero dividend condition. Note that k' appears on both sides of this last expression, such that there is no analytical expression for the choice of k' and it must be solved computationally.[12]

4. Quantitative strategy

In this Section, I set out how I calibrate the values of parameters in my model, in which one period represents one year, to match target moments.[13] The number of target moments (17) is almost double the number of calibrated parameters (10), such that my model is overidentified. I first describe the calibration of the model's non-stochastic steady-state before setting out how the parameters governing the exogenous aggregate shocks are determined.

4.1. Steady-state calibration

Two parameters are chosen outside the model. In particular, the value of β representing the representative household's rate of time preference is set to obtain a risk-free interest rate of roughly 4%, while the annual depreciation rate of physical capital is taken from Khan and Thomas (2013).

The targets and model moments for the parameters calibrated via the non-stochastic steady-state are presented in Table 1 and in Figure 1. The first column of the table contains the targeted moment while, although the parameters are all determined jointly, the second column of the table shows what parameters are most-closely associated with each target.[14] The third and fourth columns show, respectively, the source of the numerical target and the value of that target. Finally, the fifth and sixth columns show the actual moments from the specification of the model with physical capital as collateral constraint (the fifth column) and the specification with the earnings borrowing constraint (the sixth column).

[12] The solution algorithm can be sped up by solving the maximisation problem with respect to labour ℓ and then noting that k' can be taken outside the expectation. In this way, the expectation part of the expression only needs to be calculated once at each starting point rather than for each guess of k'.

[13] Details regarding the approach I take to solve my model are contained in Appendix A

[14] The parameters are obtained via the Neldor-Mead optimisation algorithm using the sum of squared percentage deviations as the objective criterion.

The exponents on capital α and labour γ in the Cobb-Douglas production function are chosen to target, respectively, the aggregate capital:output ratio and labour's share of output. Household's dis-utility of working φ is set to target the proportion of time that is spent working, while the exogenous death probability π_d is set to match the average entry rate over the period 1980-2007. The relative size of entrants, and the proportion of total employment accounted for by entrants are predominantly determined by the fraction χ of aggregate physical capital with which entrants begin.

All of the aforementioned parameters are kept the same across both specifications of the model. Only the value of the borrowing constraint (θ_k in the capital-as-collateral specification of the model and θ_c in the expected earnings specification of the model) changes across the two specifications, and in both cases it is chosen to target the aggregate debt-to-capital ratio that prevailed from 1954 to 2006.

My model is able to match the majority of these moments well and only slightly misses the relative size and employment contribution of entrants (both of these are relatively unimportant targets and are only included to ensure that entrants are not too large relative to incumbents). Note that the calibrated value of the multiple $\theta_e = 4.98$ of expected earnings up to which firms can borrow in the expected earnings specification is close to the 4.58 figures used by Drechsel (2023) (stemming from his finding that the mean debt:EBITDA ratio has that value) and also similar in magnitude to the value of 4.33 chosen by Choi (2022) to match a different set of targets.[15]

Finally, the parameters governing the distribution of the idiosyncratic output productivity shock are chosen to target the average firm-size distribution in terms of the proportion of firms in different employment-bins from 1980 to 2019. In particular, following Jo (2021) I use a Pareto distribution for this productivity term so there are four values that need to be calibrated (the upper and lower bounds of the distribution, the shape of its distribution, and the probability of a firm keeping its current productivity level). The blue bars in this graph show the proportion of firms in each employment-bin according to the average of the BEA data over the period 1980 to 2009, while the orange and grey bars show the proportion of firms in each employment bin in the steady-state of, respectively, the specification of the model with physical capital as collateral and the specification with an expected earnings borrowing constraint. I am able to capture the Pareto distribution of

[15] Specifically, Choi (2022) chooses the multiple of expected earnings up to which firms can borrow so that the real wage when firms borrow against expected earnings is the same as the real wage in the specification in which firms borrow against capital.

Table 1: Comparison of model specifications and targeted moments

Moment	Main parameter	Source	Data	Physical capital Model	Earnings Model
Capital: Output ratio	Cobb-Douglas Exponent α	Literature	2.5	2.47	2.142
Labour Share	Cobb-Douglas Exponent γ	Cooley & Prescott (1995)	60%	60%	60%
% of time worked	Household disutility of working φ	Literature	33%	34%	34%
Entry rate	Exogenous death probability π_d	BDS	10.6%	10.6%	10.6%
Relative average size of entrants vs incumbents	Entrant fraction of the aggregate capital stock χ	BDS	25.6%	15.9%	14.2%
Proportion of total employment by entrants	Entrant fraction of the aggregate capital stock χ	BDS	3.0%	1.9%	1.7%
Aggregate debt-to-Capital ratio (1954-2006)	Borrowing constraint θ_k or θ_c	FRB and NIPA	0.37	0.36	0.34

firms over these employment bins relatively well, with just a few too many firms that are small relative to others.

The parameters chosen via this calibration process are presented in Table 2, with only the multiples (θ_k for capital or θ_e for expected earnings) of collateral up which firms can borrow differing between the two specifications of the model.[16]

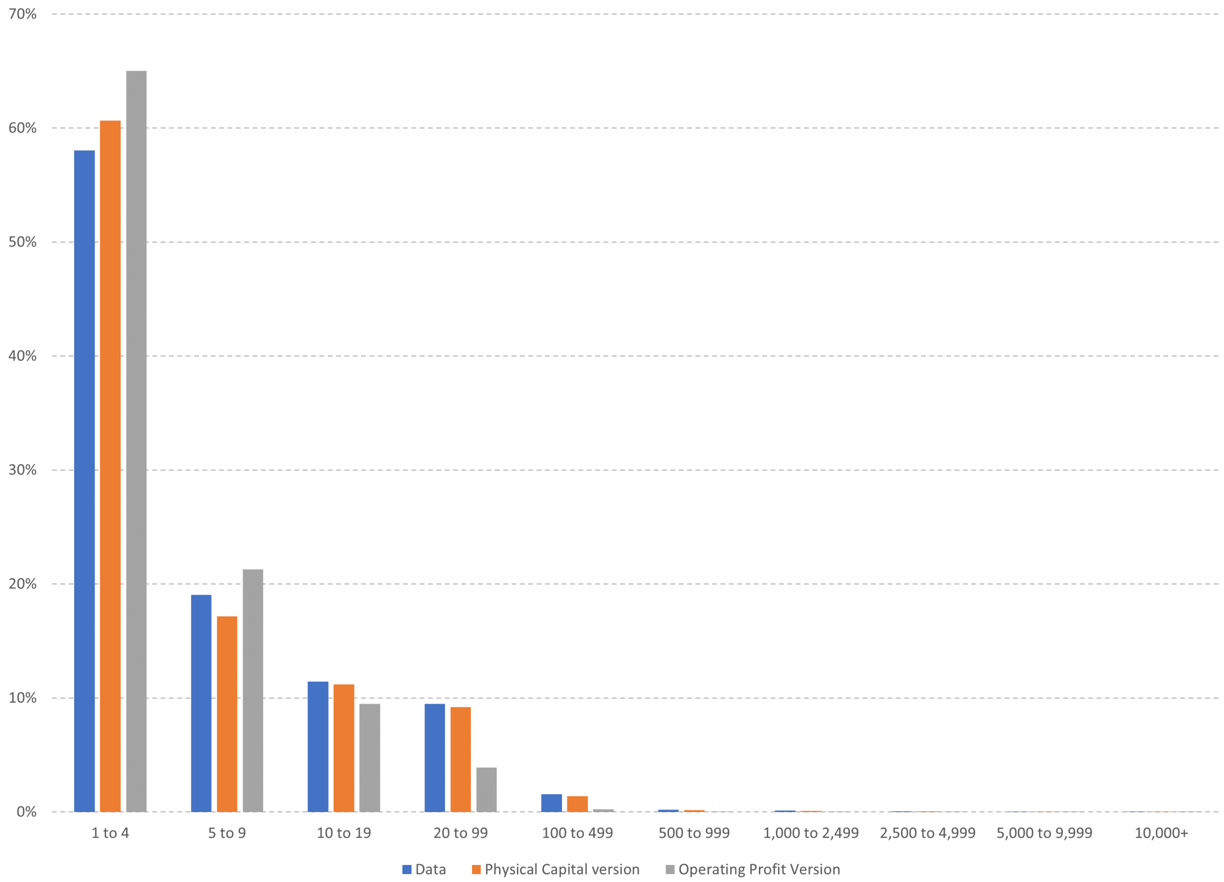
4.2. *Calibration of aggregate shocks*

Following Khan and Thomas (2013) I assume that the aggregate productivity term z follows a mean zero AR(1) process such that $\log z' = \rho_z \log z + \eta'_z$ where η'_z is normally distributed with a mean of zero and variance of $\sigma^2_{\eta'_z}$ and I set $\rho_z = 0.909$ and $\sigma_{\eta'_z} = 0.014$.[17]

[16] These parameter values mean that all firms in the physical capital specification of the model can repay their debts in the steady-state, while only 0.08% of firms are technically insolvent (i.e. have negative cash-on-hand) in the steady-state of the expected earnings specification of the model. This means that only a very small proportion of the exiting exogenous firms cannot repay their debt when their exit, with these unpaid debts ultimately covered by the representative household.

[17] Khan and Thomas (2013) estimates these figures via the Solow residuals on US data.

Figure 1: Comparison of firm-size distribution targets and calibrated model results



Note: The data (the blue bar) is the BEA average firm-size distribution over the period 1980 - 2009, while the orange and grey bars show the results obtained in the non-stochastic steady state of the specification of the model with, respectively, the physical caputal borrowing constraint and the earnings borrowing constraint.

This productivity process is discretised using the Rouwenhorst algorithm over three grid-points.

I also take the transition probabilities of the borrowing constraint parameter (θ_k or θ_e) from Khan and Thomas (2013). Specifically, I assume that the borrowing constraint parameter can take only two values (with its calibrated steady-state value representing "normal times" and a lower, calibrated value, representing a "financial crisis") and follows a Markov chain process represented by the matrix:

$$\Pi_\theta = \begin{bmatrix} p_0 & 1 - p_0 \\ 1 - p_l & p_l \end{bmatrix}$$

with $p_0 = 0.9765$ and $p_l = 0.3125$ set so as to achieve an average financial crisis duration

Table 2: Parameter values

Parameter	Description	Value	Parameter	Description	Value
α	Cobb-Douglas Exponent on capital	0.30	β	Household rate of time preference	0.96
γ	Cobb-Douglas Exponent on labour	0.60	δ	Capital depreciation rate	0.069
φ	Household disutility of working	2.11	ε_{low}	Lowest value of ε	0.72
π_d	Exogenous probability of death	0.106	ε_{high}	Highest value of ε	2.32
χ	Entrant starting capital as % of aggregate	15.4%	Ψ	Pareto shape of ε	6.68
θ_k	Borrowing constraint in physical capital specification	0.92	ξ	Probability of keeping current ε	0.70
θ_e	Borrowing constraint in earnings specification	4.98			

of roughly 3 years and ensure that the economy spends roughly 7% of the time with the borrowing constraint at its lower value.

The aforementioned parameter values governing transition probabilities are the same across both the physical capital and expected earnings specifications of the model. The parameter that differs is the lower value of the borrowing constraint term θ_k or θ_e. In both cases it is chosen so that the level of debt falls by roughly 26% when the economy is subject to just a financial crisis (i.e. aggregate output productivity is at its median level and the borrowing constraint parameter is at its lowest level).[18] This results in the low value of the borrowing constraint parameter in the collateral specification of the model being set to $\theta_{k,low} = 0.894$, while for the expected earnings specification of the model it is set to $\theta_{e,low} = 4.478$.

These calibrations of the different model specifications produce the business cycle moments in Table 3 for the specification with capital as collateral and Table 4 for the specification with the expected earnings borrowing constraint. In each table, the first row shows the mean of each series, while the second shows their coefficient of variation. The third row shows the volatility of each series relative to that of output.

Although most series' relative volatilities are similar to what one would expect (e.g. con-

[18] See Khan and Thomas (2013) for a discussion of the effect of the financial crisis on the effect of the 2007 US recession on the path of lending and firm debt over time.

Table 3: Simulated business cycle moments for physical capital borrowing constraint

	Y	C	K	I	L
Mean	0.49	0.42	1.08	0.07	0.34
Coefficient of Variation	5.7%	4.6%	6.9%	14.8%	1.9%
$\frac{\sigma_x}{\sigma_Y}$	1	0.69	2.67	0.39	0.22
Correlation with GDP	0.93	0.98	0.99	0.81	0.77
Auto-correlation	1	0.96	0.82	0.85	0.67

Note: These moments are obtained from a simulation of 5,000 periods of the specification in which capital is used as collateral, dropping the first 500 periods of that simulation. These moments have not been filtered or smoothed. The first two rows show, respectively, the mean and coefficient of variation of each series. The third row shows the standard deviation of each series relative to that of output, except for the first column, which shows the actual standard deviation of output. The fourth row shows each series' contemporaneous correlation with output, while the last row contains each series 1-period auto-correlation.

Table 4: Simulated business cycle moments for expected earnings borrowing constraint

	Y	C	K	I	L
Mean	0.53	0.45	1.19	0.08	0.34
Coefficient of Variation	5.9%	4.7%	7.1%	18.2%	2.5%
$\frac{\sigma_x}{\sigma_Y}$	1	0.67	2.70	0.48	0.27
Correlation with GDP	0.92	0.98	0.99	0.80	0.77
Auto-correlation	1	0.91	0.80	0.82	0.65

Note: These moments are obtained from a simulation of 5,000 periods of the specification in which expected earnings is used as colalteral, dropping the first 500 periods of that simulation. These moments have not been filtered or smoothed. The first two rows show, respectively, the mean and coefficient of variation of each series. The third row shows the standard deviation of each series relative to that of output, except for the first column, which shows the actual standard deviation of output. The fourth row shows each series' contemporaneous correlation with output, while the last row contains each series 1-period auto-correlation.

sumption is less volatile than output, investment has a larger volatility relative to its own mean than do output and capital), aggregate capital K is more volatile than output in both specifications. This is predominantly due to the capital stock with which new entrants start the period varying as the aggregate capital stock itself varies. This means that fluctuations in the aggregate capital stock caused by changes in output productivity or the borrowing constraint are exacerbated by this variation in new entrants' initial capital stock. All series also have the expected sign of their correlation with output and their own-auto-correlation, as shown in the fifth and sixth rows of each table.

Nonetheless, the business cycle moments are similar between the two specifications, albeit with the series in the expected earnings specification of the model being slightly more volatile and less correlated with output and themselves than they are in the capital as collateral specification.

5. Aggregate responses to TFP and financial shocks

In this Section I set out the aggregate results of two exercises and compare these results between the two specifications of the model. First, I investigate the recovery from a recession caused by a 1% drop in aggregate TFP that gradually returns to its steady-state level. Second, I examine the recovery from a recession caused by a three-period drop in the multiple of collateral that firms can borrow against. In both exercises, the economy is allowed to run for 75 periods at the median aggregate productivity level and the steady-state value of the colalteral multiple before the shock is applied.

I find that the response to both a TFP recession and a financial recession differ substantially according to whether firms can borrow against their capital stock or their expected earnings. In the case of a recession caused by a drop in aggregate TFP, the aggregate response when firms can borrow against expected earnings initially is much more volatile than when firms borrow against their capital stock. When that volatility difference has disappeared after a few periods, the economy recovers from its trough more quickly when firms borrow against expected earnings and catches back up to the recovery path of the economy in which firms borrow against capital. This is due to the expected lower TFP causing lower expected earnings and, therefore, a larger drop in investment and the aggregate capital stock when firms borrow against expected earnings. This larger reduction in capital leads to a bigger drop in consumption (and hence lower wage), allowing firms to hire more labour and catch up to the economy in which firms borrow against capital.

Similarly, this effect of expected earnings on a firm's choice of capital in the expected earnings as collateral specification of the model drives its different responses compared to the specification with capital as collateral. In particular, the financial shock has a larger reduction on firms' incentives to invest in capital when those firms borrow against expected earnings compared to when they borrow against capital, via the effect of the expectations and constraint multiplier as discussed in Section 3. This bigger drop in capital results in a bigger drop in output and consumption and, importantly, the wage when firms borrow against expected earnings. This subsequent drop in the wage (and expected wage) means that firms can hire more labour, thereby increasing their current output and expected earnings. This means that they can borrow more to invest in capital during the recovery, such that the recovery is quicker when firms borrow against expected earnings. Hence, despite the deeper troughs in the expected earnings specification of the model, most aggregates soon catch up with the levels (relative to steady state) of those variables in the capital as collateral specification of the model.

As such, reinforcing Drechsel (2023)'s findings, there are substantial differences in the recoveries of an economy from both a TFP recession or a financial recession depending on whether firms can borrow against their capital stock or against their expected earnings. However, I find that although firms borrowing against expected earnings more accurately captures the path of investment in response to the Great Recession, the specification in which firms borrow against capital does better at capturing the path of output during the recovery from the Great Recession. I discuss the results from each of these two exercises in more detail below.

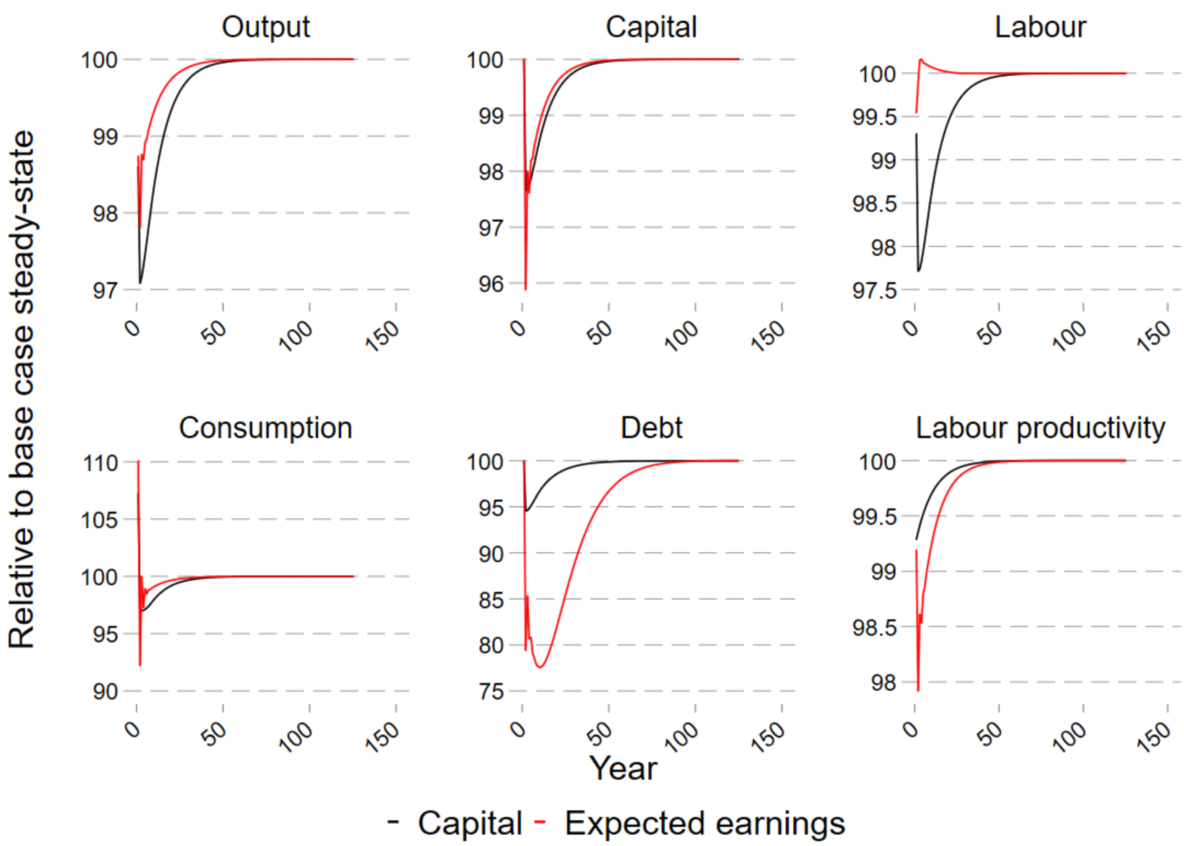
5.1. Response to a 1% drop in TFP

A comparison between the economy's recovery from a 1% drop in (and subsequent gradual return to the steady-state level of) aggregate TFP in the specification of the model with capital as collateral and the specification with expected earnings as collateral is shown in Figure 2. Each panel in this graph shows a different aggregate variable, with the black line representing the path of each variable in the capital-as-collateral specification and the red line showing the path for the specification with expected earnings as collateral.

In both specifications of the model, there is an instantaneous drop in labour hired, output, capital, and debt when the shock hits. When firms borrow against capital, the initial 1% drop in TFP reduces the marginal productivity of both capital and labour, causing firms to hire less labour and invest in less capital. The resulting lower output also means that consumption falls despite the reduction in investment. The reduction in capital further leads to a reduction in available collateral, so firms take on less debt. However, as TFP gradually returns back to its median value, the marginal productivities of both capital and labour increase, such that firms also slowly increase the amount their capital stock and labour hiring. Coupled with the increasing TFP itself, this results in output gradually returning to its steady-state level, as does consumption. Finally, the incremental return of capital to its steady-state value means that debt does likewise.

The initial recovery of each aggregate variable is substantially more volatile in the expected future earnings specification of the model than it is in the capital as collateral specification.[19] This difference in volatility across different borrowing constraints is similar to Lorenzoni (2008)'s finding that certain financial contracts can result in increased volatility of economic aggregates.

Figure 2: Simulated recovery of aggregate variables after a TFP recession



Note: The graphs show the simulated time path of aggregate variables in the model economies. Each economy is started at the steady-state distribution and runs for 75 periods with exogenous aggregate productivity at the median value of 1. In the 76th period aggregate TFP is subjected to a 1% reduction (i.e. is set to 0.99) before gradually recovering back to its median value of 1 at a rate determined by ρ_z. The borrowing constraint parameter θ_k or θ_e is kept at its steady-state level throughout. The black line shows the results for the capital as collateral specification of the model, while the red line shows the path of the economy in the specification of the model with the expected earnings borrowing constraint. All series are expressed in terms of their percentage deviation from steady-state.

This higher volatility is due to the fact that the shock to TFP has a direct effect on the borrowing constraint (via the expectation of earnings over aggregate TFP) in the expected

[19] As the same solution method is used for both specifications, this difference in volatility is unlikely to be due to the solution method itself.

earnings specification of the model, whereas there is only an indirect effect (via a reduction in funds available to invest in capital) in the capital as collateral specification of the model. The effect on expectations means that firms and households are likely to be less certain regarding the future state of the economy, resulting in a wider variance in aggregate outcomes for the initial period of the recovery. In particular, this increased uncertainty affects firms' capital choice via an additional interaction with the borrowing constraint when firms borrow against expected future earnings (as shown in Equation 9) that is not present when firms borrow against capital.

Moreover, once the economy has settled down (after the first five periods) the large drop in consumption at the start of the recession when firms borrow against expected future earnings means that there is a large decrease in the wage (that firms expect to be persistent over time). This means that firms choose to hire more labour such that labour hiring is actually above the steady-state level from the second period on. The increased labour hiring also means that output is much closer to its steady-state level when firms borrow against expected future earnings compared to when they borrow against capital. This higher output, along with the increased marginal productivity of capital through hiring more labour, results in firms investing in more capital during the recovery when they borrow against expected future earnings. Hence, despite the much larger initial drops in output, capital, and consumption when firms borrow against expected earnings, the recovery of these variables in the face of a TFP shock is actually quicker when firms borrow against expected earnings compared to when they borrow against capital.

The two variables where this is not the case are debt and labour productivity. For labour productivity, the fact that firms hire more labour when they borrow against expected earnings means that output per worker (i.e. labour productivity) will be lower due to decreasing marginal returns to labour. This difference between the expected earnings and capital specifications of the model only disappears once labour hired returns to its steady-state level in both specifications.

Debt falls much more when firms borrow against expected earnings because of the gradual recovery of TFP to its steady-state level and the prolonged effect that has on expected earnings. This means that debt also takes much longer to return to the steady-state level because firms can only borrow a certain multiple of their expected earnings, meaning that they can only gradually increase their debt even when output and labour have returned to the steady-state level. Hence, the large trough in debt means that there is a much more gradual and prolonged return of debt to the steady-state level when firms borrow against expected earnings.

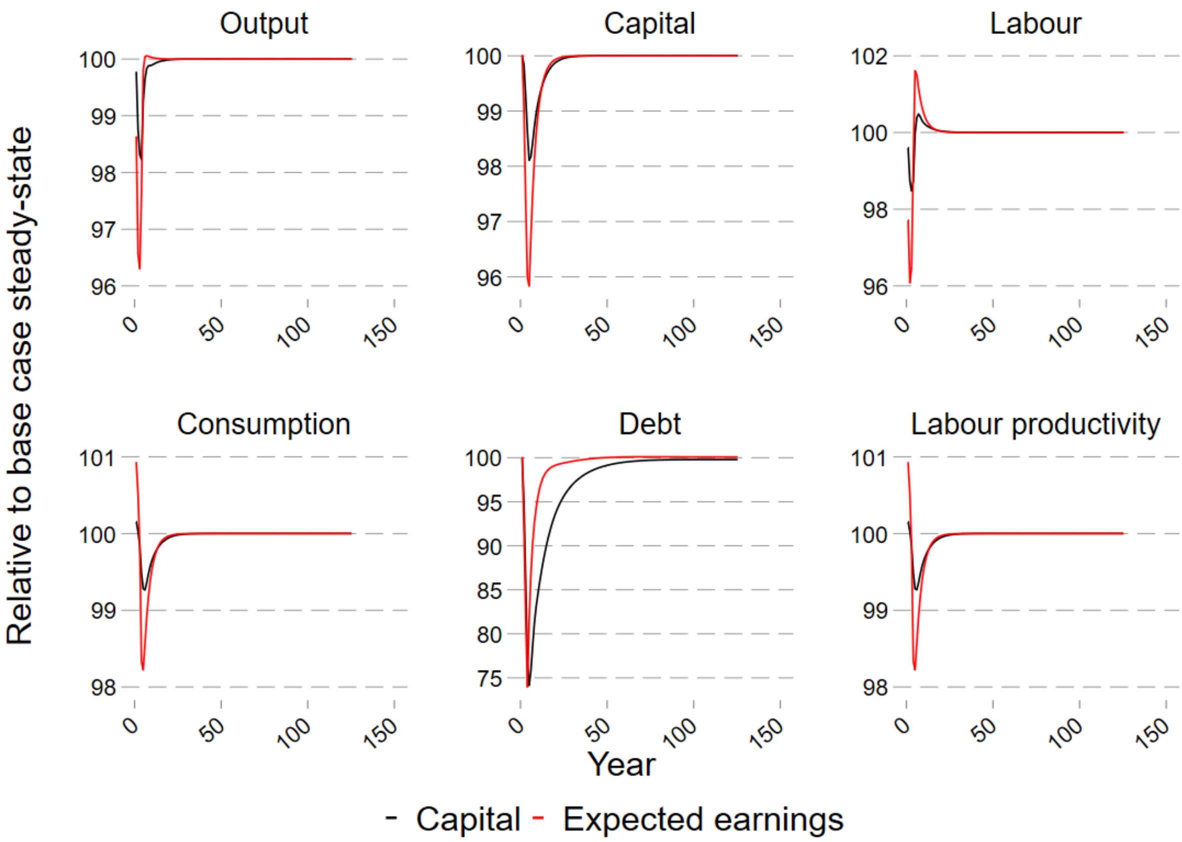
5.2. Response to a tightening of the borrowing constraint

A comparison between the responses in each of the two specifications to a three-period drop in the multiple of collateral up which firms can borrow against from its steady-state level to its calibrated "lower" level before returning to its steady-state level is shown in Figure 3 (as before, each panel in this graph shows a different aggregate variable, with the black line representing the path of each variable in the capital-as-collateral specification and the red line showing the path for the specification with expected earnings as collateral). In both specifications of the model, there is an immediate drop in labour hired, capital, output, and consumption, as well as these variables only reaching a trough a few years into the recession, in response to this financial shock.

When firms borrow against capital, the financial shock reduces their ability to borrow and finance investment in capital, such that the aggregate capital stock decreases by roughly 2% compared to the steady-state (and their preferred level of capital might also reduce as, via Equation 8, a constrained firm's choice of capital is affected by the term $\frac{\theta_k \lambda}{\beta}$ in the numerator, which is smaller when θ_k is smaller). This means that labour hired also decreases by about 1.5%, with the combined effect that output also decreases by roughly 1.6%. The initial sell off of capital stock means that consumption increases slightly relative to steady-state in the initial period of the shock, but soon the reduction in output means that consumption falls below the steady-state level as well, reaching a trough of about 0.8% below the steady-state level.

Once the credit shock is over, and firms can again borrow more against their stock of capital, investment increases such that firms' capital increases. Moreover, the lower level of consumption means that the wage is lower, leading to firms hiring more than the steady-state level of labour once the credit shock is over. This means that a firm's output increases substantially once the credit shock is over and then gradually returns to its steady-state level. The gradual return of capital to steady-state results in a gradual return of debt to its steady-state level as well, since firms can still only borrow up to a certain multiple of their capital stock. Finally, despite aggregate TFP remaining constant throughout the financial crisis, there is still an initial slight increase in labour productivity (since there is slightly lower labour hired in the first period of the shock), but labour productivity soon diminishes to about 0.75% less than the steady-state level due to the fall in the capital stock and the subsequent increase in labour hired.

Figure 3: Simulated recovery of aggregate variables after a credit shock



Note: The graphs show the simulated time path of aggregate variables in the model economies. Each economy is started at the steady-state distribution and runs for 75 periods with the borrowing constraint parameter θ_k or θ_e at its steady-state level. From the 76th - 78th period (inclusive) the borrowing constraint parameter θ_k or $\theta_e$is set to its (calibrated) low value to represent a tightening of the borrowing constraint and a financial crisis. The parameter is then returned to its steady-state level from the 79th period on. The exogenous aggregate productivity at the median value of 1 throughout. The black line shows the results for the capital as collateral specification of the model, while the red line shows the path of the economy in the specification of the model with the expected earnings borrowing constraint. All series are expressed in terms of their percentage deviation from steady-state.

It is immediately apparent that the recession is much deeper when firms instead borrow against expected earnings. In particular, the troughs of each of output, capital, consumption, and labour are much lower: output falls by about 3.5%; capital by 4%; consumption by 1.75%; and labour by 4%. These deeper troughs are driven by the effect of the expected earnings borrowing constraint on a firm's choice of capital. Recall from Equation 9 that the (constrained) firm's choice of capital is modified by the inclusion of the term $\frac{\lambda}{\beta}\theta_e[E_{\pi_S}E_{\pi_\varepsilon}z'\varepsilon'\left(\frac{w(S',\mu')}{\gamma z'\varepsilon'}\right)^{\frac{\gamma}{\gamma-1}} - w(S',\mu')\left(\frac{w(S',\mu')}{\gamma z'\varepsilon'}\right)^{\frac{1}{\gamma-1}}]$ in the denominator. The effect of the financial shock not only reduces θ_e directly in this term, but also affects firms' expectations of the wage $w(S',\mu')$.

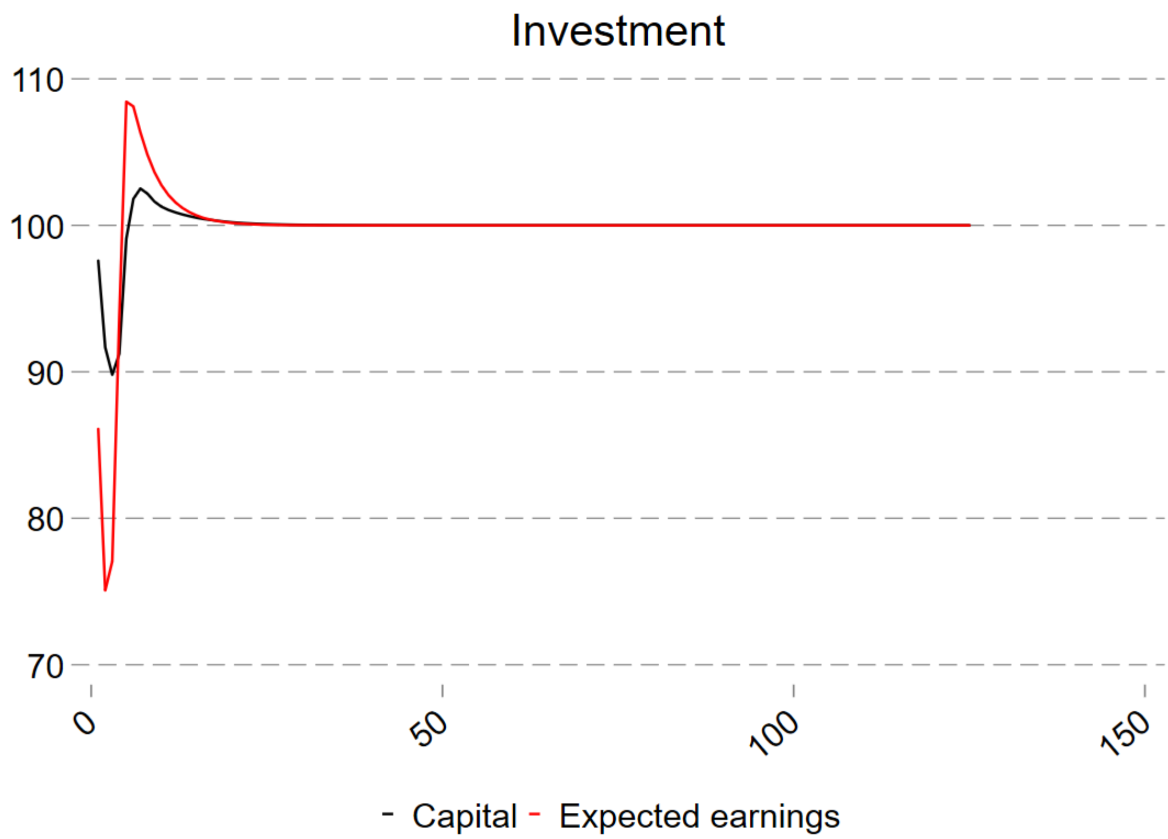
The direct effect of θ_e means that its lower value during the initial shock has a large impact on firms' capital choices (as it is also multiplied by expectations as well as $\frac{\lambda}{\beta}$, with only the latter appearing when firms borrow against capital). This results in a much deeper trough in aggregate capital as firms choose to invest less / sell off more of their capital. Hence, consumption in the initial period also increases by more compared to when firms borrow against capital.

The larger drop in the aggregate capital stock also results in a more pronounced reduction in output and subsequent decrease in consumption. However, once the shock ends, the recovery of output and capital is much quicker when firms borrow against expected earnings. This is because the large drop in consumption leads to a large drop in wages, which firms expect to continue (note that consumption in the expected earnings specification remains below consumption in the capital specification until around period fifteen). Hence, constrained firms increase their investment and capital choice more rapidly when they borrow against expected earnings due to the effect of (expected) wages on their optimum choice.

Moreover, the fact that consumption (and wages) are lower when firms borrow against expected earnings means that firms hire more labour in this specification: labour hired is nearly 2% above the steady-state level once the shock ends (compared to about 0.5% when firms borrow against capital). This means that output is actually slightly higher than the steady-state level only two periods after the shock ends (i.e. in period five). In addition, debt recovers very quickly because the higher output means that firms can borrow even more as their expected earnings are higher. Finally, the path of labour productivity reflects the path of labour hired in that it is initially higher due to the larger drop in labour, but then falls by more as labour hired increases more.

Overall, when firms borrow against expected earnings, a financial crisis results in a much deeper recession and a much quicker bounce-back from that trough, such that output actually returns to its steady-state level only five periods after the start of the credit shock (compared to roughly 20 periods when firms borrow against capital). The more gradual return of output to trend that results when firms borrow against capital more accurately captures the path of that variable after the Great Recession, but the deep trough in investment seen in the recovery from the Great Recession (and highlighted in Khan and Thomas (2013) as investment declining by as much as 25%) is more accurately captured by the specification in which firms borrow against expected earnings, as shown in Figure 4. These differences between the two specifications in capturing the recovery from the Great Recession suggest that neither type of collateral is sufficient on its own, but a combination of the two specifications (i.e. firms can choose to borrow against expected earnings or against capital) is likely to be necessary in order to capture both aspects of the recovery.

Figure 4: Simulated recovery of aggregate variables after a credit shock



Note: The graphs show the simulated time path of aggregate investment in the model economies. Each economy is started at the steady-state distribution and runs for 75 periods with the borrowing constraint parameter θ_k or θ_e at its steady-state level. From the 76th - 78th period (inclusive) the borrowing constraint parameter θ_k or $\theta_e$is set to its (calibrated) low value to represent a tightening of the borrowing constraint and a financial crisis. The parameter is then returned to its steady-state level from the 79th period on. The exogenous aggregate productivity at the median value of 1 throughout. The black line shows the results for the capital as collateral specification of the model, while the red line shows the path of the economy in the specification of the model with the expected earnings borrowing constraint. All series are expressed in terms of their percentage deviation from steady-state.

6. Different responses across the distribution of firms

It is also possible to examine the extent to which recoveries from shocks differ across different groups of firms. In particular, the presence of a distribution of firms over capital and debt means that firms can be grouped into quintiles according to where they are in the distribution of firms over those variables. I can then examine the path of total capital, output, and debt for each of these quintiles of firms as the economy recovers.

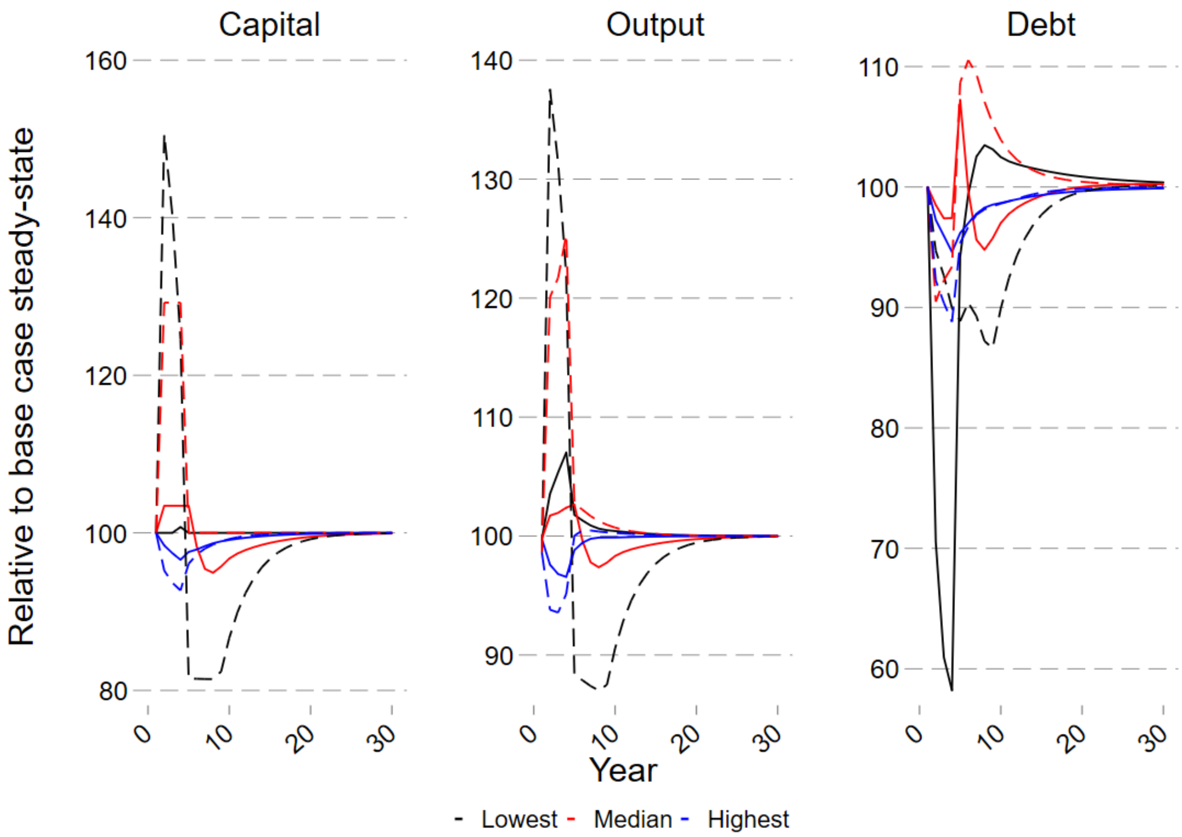
Focusing on the recovery from a financial shock as discussed in Section 5.2, Figure 5 shows the paths (relative to each group's steady-state level) of total capital, output, and debt for each of three aggregated groups of firms, split according to where those firms are in the distribution over capital. The black line shows the path for the lowest quintile in terms of capital (i.e. the smallest firms); the red line for the middle quintile; and the blue line for the top quintile. The results for the capital as collateral specification are shown by the solid lines while those for the specification with expected earnings as collateral are shown using dashed lines.

It is immediately apparent that, for all quintiles shown, the capital and output responses of firms in those quintiles are more extreme in the expected earnings specification of the model than they are in the capital as collateral specification: for each quintile, the dashed lines reach higher peaks / lower troughs than do the solid lines of the same colour. Moreover, there is a distinct difference in the response of capital and output the largest firms (in the highest quintile) compared to small and medium-sized firms: specifically, large firms reduce their capital and debt in the initial years of the shock whereas small and medium-sized firms actually increase their capital stocks (by a large amount in the expected earnings specification of the model and by a small amount in the capital as collateral specification). This increase in capital soon reduces after the shock to the borrowing constraint parameter disappears (i.e. from period four on) as firms' incentives to hold capital are now back to "normal".

However, for the smallest firms in the expected earnings specification of the model, capital actually falls below the steady-state level once the shock is over, and it falls back to the steady-state level immediately for medium-sized firms. This is a combination of a general equilibrium effect due to households, who own the firms, experiencing a large drop in consumption and wanting to ameliorate that by having the least productive, i.e. smallest, firms sell off their capital; and the fact that firms no longer need to keep as much capital to boost their expected earnings and maximum amount they can borrow. As a result, firms that over-accumulated capital sell it off as soon as the financial shock ends in the expected earnings specification of the model. There is a similar patter for the path of output for all three groups of firms.

This difference between groups of firms is driven by the fact that the largest firms have

Figure 5: Simulated recovery of aggregate variables after a financial crisis, split by quintile of capital



Note: The graphs show the simulated time path of aggregate variables in the model economies, with separate series for each of three different quintiles of the amount of capital owned by a firm. In this graph, the level of capital owned by the firm that denote each quintile changes as the distribution of firms changes over time, such that each quintile always contains 20% of firms. The black lines shows the path for all of those firms in the bottom quintile; the red lines for all firms in the middle quintile; and the blue lines for those firms in the highest quintile. The solid lines show these paths for the specification of the model with the capital-based borrowing constraint while the dashed lines shows the paths for the specification of the model with the borrowing constraint based on expected earnings. Each economy is started at the steady-state distribution and runs for 75 periods with the borrowing constraint parameter θ_k or θ_e at its steady-state level. From the 76th - 78th period (inclusive) the borrowing constraint parameter θ_k or $\theta_e$is set to its (calibrated) low value to represent a tightening of the borrowing constraint and a financial crisis. The parameter is then returned to its steady-state level from the 79th period on. The exogenous aggregate productivity at the median value of 1 throughout. All series are expressed in terms of their percentage deviation from steady-state.

no need nor incentive to grow substantially more; they are already at (or close to) their optimum size. As a result, when the borrowing constraint tightens, they have less of an incentive to hold capital as they cannot borrow as much (either against the capital itself or against the expected earnings that capital implies), so they reduce their capital stock. On the other hand, small and medium-sized firms are far from their optimum size and still have the desire to grow so they can reach that size. These smaller firms, therefore, still need and want to try to grow as quickly as they can, so they increase (or do not reduce) their stocks of capital to try to counter the fact that they can borrow only a smaller multiple of collateral during the shock. This feeds through into the respective paths of output for each group of firms.

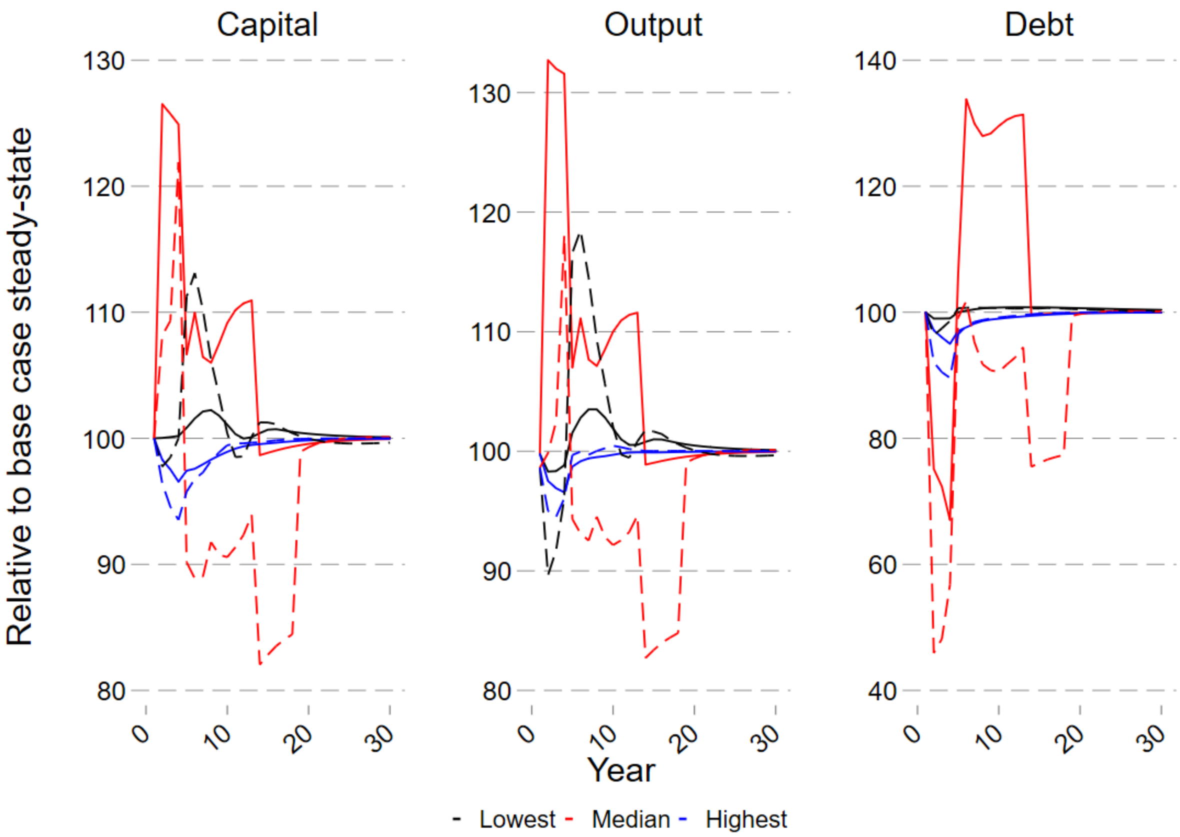
The path of debt is also affected by this mechanism, but the more powerful effect is from the tightening of the constraint itself. In particular, small firms are already borrowing as much as they possibly can, so the tightening of the borrowing constraint means the amount that they can borrow decreases dramatically. In the case of capital as collateral, once the shock to the borrowing constraint has passed, small firms then increase their debt relative to their steady-state level in order to try to catch up to where they would have been absent the shock; this process takes much longer for these small firms in the expected earnings specification of the model as the effects of the changes in those firms' capital stocks has a longer-lasting effect on the amount they can borrow (via the effect of those capital stocks on output as described previously).

Debt held by large firms also decreases, albeit for a different reason: they are far from the borrowing constraint and already close to their optimum size, so do not need to borrow as much in order to get to that optimum. There is a lot more volatility for medium-sized firms: first their debt falls as the tighter borrowing constraint reduces how much they can borrow, but debt increases as soon as the financial shock ends as these firms try to catch up to where they would have been, before debt gradually returning back to its steady-state level.

Nonetheless, in both specifications of the model, the largest responses relative to their steady-states comes from firms that are smaller in terms of their capital stocks.

Looking at the paths of these variables for firms grouped according to their debt levels (rather than their levels of capital stock) reveals a slightly different story. Figure 6 shows the paths of the same variables in response to the same financial shock as previously, except now the black line shows the path for the lowest quintile in terms of debt (i.e. those firms with the least amount of debt); the red line for the middle quintile; and the blue line for the top quintile (those firms with the most debt). This indicates that the largest response in terms of capital, output, and debt comes from those firms in the middle of the distribution. This is because those firms with the least debt (i.e. most savings) do not need to borrow substantial sums anyway and therefore are less affected by the tightening of the borrowing constraint, whereas firms with the most amount of debt are those that are most constrained by the financial shock and therefore keep borrowing as much as they can anyway to ensure they can keep growing as quickly as possible.

Figure 6: Simulated recovery of aggregate variables after a financial crisis, split by quintile of debt



Note: The graphs show the simulated time path of aggregate variables in the model economies, with separate series for each of three different quintiles of the amount of debt owned by a firm. In this graph, the level of capital owned by the firm that denote each quintile changes as the distribution of firms changes over time, such that each quintile always contains 20% of firms. The black lines shows the path for all of those firms in the bottom quintile; the red lines for all firms in the middle quintile; and the blue lines for those firms in the highest quintile. The solid lines show these paths for the specification of the model with the capital-based borrowing constraint while the dashed lines shows the paths for the specification of the model with the borrowing constraint based on expected earnings. Each economy is started at the steady-state distribution and runs for 75 periods with the borrowing constraint parameter θ_k or θ_e at its steady-state level. From the 76th - 78th period (inclusive) the borrowing constraint parameter θ_k or $\theta_e$is set to its (calibrated) low value to represent a tightening of the borrowing constraint and a financial crisis. The parameter is then returned to its steady-state level from the 79th period on. The exogenous aggregate productivity at the median value of 1 throughout. All series are expressed in terms of their percentage deviation from steady-state.

Moderately-indebted firms, on the other hand, are affected by both the tightening of the borrowing constraint directly as well as the changes to incentives that tightening implies. In particular, they try to hold more capital so that they can produce more and soften the reduction in the amount that they can borrow during the periods when the shock is present and then sell off that excess capital, reducing their output after the shock has disappeared.

When comparing between the two specifications of the model, the most-indebted firms (the blue lines) reduce their capital stock, debt, and output more when they borrow against their expected earnings. This reflects the presence of the additional term in firms' capital choices when they are at the expected-earnings based borrowing constraint (as shown in Section 3). In particular, the most indebted firms are the most constrained and the firms for whom the constraint binds most tightly, so their choice of capital is most affected by the presence of the additional term. This effect is least present in the firms with the least amount of debt (the black lines) as they are the least likely to be affected by the additional term in the choice of capital, so the gap between the two specifications of the model is narrowest for these firms.

These results imply that any policy that might try to alleviate the effects of a financial recession needs to take into account the fact that individual (groups) of firms are likely to respond very differently from the aggregate response; that there are also different (and even opposite) responses across different groups of firms; and that these responses differ according to whether firms face capital-based borrowing constraints or earnings-based ones.

7. Conclusion

I have shown that responses to aggregate TFP shocks and financial shocks differ substantially depending on whether firms face capital-based borrowing constraints or earnings-based ones. I have also shown that individual firms' responses differ according to how much capital or debt they hold at the onset of the shock and during the recovery from it. Using a discrete time dynamic stochastic general equilibrium model with heterogeneous firms that hold physical capital and debt subject to a borrowing constraint, I use two different specifications of the borrowing constraint: in one specification, firms can borrow up to a multiple of their choice of capital stock (a la Kiyotaki and Moore (1997)); while in the second specification, firms can borrow up to a multiple of their expected earnings (as put forward by Drechsel (2023)).

For each specification of the model, I simulate the responses to a 1% decrease in aggregate TFP that gradually returns to its steady-state level; and a three-period reduction in the multiple of the relevant collateral that firms can borrow against before that multiple returns to its steady-state level. I find that the aggregate response to is initially much more volatile when firms borrow against their expected earnings compared to when they borrow against capital, but after a few periods the recession is less harsh (i.e. output and consumption are higher on the recovery path) when firms borrow against expected earnings.

However, when considering a financial recession, the troughs in each of output, consumption, and capital are much deeper when firms borrow against expected earnings. This is due to the tighter borrowing constraint directly affecting firms' capital choices via its effect on expected earnings. The lower value of the borrowing constraint parameter therefore means firms' optimal capital choices are much lower initially compared to when firms borrow against capital. However, the (expected) lower wages also mean that once the financial shock is over, the recovery is much quicker when firms borrow against expected earnings as their capital choices take into account this expected lower wage and higher expected earnings.

I find that both specifications of the model can capture different aspects of the Great Recession: the specification in which firms borrow against capital does better in terms of matching the path of output during the recovery, whereas the specification in which firms borrow against expected earnings more accurately captures the path of investment.

Finally, I also show that responses differ across different firms. Firms with lots of capital reduce their capital stock and output in the face of a financial shock, whereas firms that have little or a moderate amount of capital actually increase both of these variables. Similarly, firms with a moderate amount of debt exhibit very volatile responses to a financial shock, whereas firms with either little or lots of debt have much calmer responses.

My findings suggest that a policymaker that wants to try to soften the effects of a recession needs to consider whether firms borrow against capital or against expected earnings, and the extent to which different firms respond differently to the shock in the first place. I leave open for further work issues such as allowing firms to choose between borrowing against capital or earnings and the effect of separate shocks to the multiples of which firms can borrow against when such a choice is available to them, akin to Greenwald (2018) and Greenwald (2019)'s assessment of the relative importance of the two types of borrowing constraints in the transmission of monetary policy.

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Appendix A. Solving the model

In this Appendix I set out the pseudo-algorithms I use to solve for the non-stochastic steady-state and the model with aggregate uncertainty.

Appendix A.1. Non-stochastic steady-state solution algorithm

In order to solve the model I discretise the output productivity shock ε using the Rouwenhorst algorithm with 15 gridpoints. I guess an aggregate level of physical capital $\bar{K}$ (which is necessary to determine the size at which new entrants start) and a consumption C, the latter implying a wage w through the household's labour-leisure condition. I then use I use the implied decision rules for k', and b' described above, as well as the distribution of new entrants alongside exogenous exits, to find the stationary distribution, which is then used to obtain values for implied $\bar{K}$ and C. The initial guesses of these variables are then updating using Broyden's algorithm with the updating-step obtained as per Santaeulalia-Llopis (2016).

Appendix A.2. Solution algorithm for aggregate uncertainty

The presence of aggregate uncertainty in the form of varying aggregate productivity z (which can take three values) and θ_k or θ_e (each of which can take two values) means that I use the Krusell-Smith solution method to solve my model, adapted for heterogeneous firms as per Khan and Thomas (2013). This means that I approximate the distribution of firms μ using the summary statistic of the aggregate level of capital $\bar{K}$ and use that in the forecasting rules that take the form

$$\log x = \beta_{0,i} + \beta_{1,i} \log \bar{K}_t + \beta_{2,i}\xi_{1,t} + \beta_{3,i}\xi_{2,t} + v_i$$

where x is the variable to be forecast, β_j are coefficients to be estimated, $\xi_{1,t}$ and $\xi_{2,t}$ are dummy variables taking the value 1 if there was a low value of θ_c or θ_e one period ago ($\xi_{1,t}$) or two periods ago ($\xi_{2,t}$), v is the error term, and i represents the level of aggregate productivity in the current period. Each forecast rule is obtained via regressions using the final 4,500 observations from a simulation of 5,000 periods (i.e. the first 500 periods of the simulation are dropped when estimating the forecast rules).

I require forecasting rules for two variables: the level of aggregate capital next period $\bar{K}'$ (to determine the expected level of the wage next period and, therefore, firms' optimum choices); and the household's marginal utility this period p (to determine the marginal utility value of firm profits today, as firms are owned by the household). The estimated coefficients, and some regression statistics for each of these forecast rules for the specification of the model with the capital borrowing constraint are shown in Table A.5. The first column indicates which variable is being forecast while the second and third columns show to which aggregate state in the current period the forecast rule applies. Columns 4-7 contain the estimated coefficient values, while column eight shows the standard error for the regression. The final column presents the R^2 of the regressions and shows that it is above 0.99 for all regressions.

Table A.5: Forecast rules for the capital as collateral specification

Forecast rule for	Aggregate productivity	Borrowing Constraint	β_0	β_1	β_2	β_3	SE	R^2
$\bar{K}'$	z_1	θ_k	0.02789	0.85129	-2.77E-03	5.16E-04	2.47E-03	0.99532
	z_1	$\theta_{k,low}$	0.03232	0.84509	-2.00E-03	2.73E-04	1.04E-03	0.99930
	z_2	θ_k	0.01681	0.84795	-1.80E-03	1.84E-04	9.87E-04	0.99933
	z_2	$\theta_{k,low}$	0.01350	0.84829	-2.97E-03	1.90E-04	2.25E-03	0.99671
	z_3	θ_k	-0.00209	0.85606	-2.53E-03	1.19E-04	2.44E-03	0.99695
	z_3	$\theta_{k,low}$	0.00127	0.85267	-2.10E-03	2.70E-04	9.67E-04	0.99935
p	z_1	θ_k	0.85415	-0.37821	-3.33E-04	-4.77E-05	1.72E-04	0.99990
	z_1	$\theta_{k,low}$	0.85507	-0.37620	-3.37E-04	-1.55E-05	8.12E-05	0.99998
	z_2	θ_k	0.89011	-0.37773	-2.87E-04	1.32E-05	9.41E-05	0.99997
	z_2	$\theta_{k,low}$	0.88897	-0.37879	-1.87E-04	-2.51E-05	2.43E-04	0.99983
	z_3	θ_k	0.92359	-0.37952	-2.31E-04	-3.20E-05	1.97E-04	0.99991
	z_3	$\theta_{k,low}$	0.92481	-0.37852	-2.75E-04	6.76E-06	6.39E-05	0.99999

Note: SE is the standard error in each regression

Similarly, Table A.6 shows the solved forecast rules for the specification with the expected earnings borrowing constraint. As before, the regression standard errors are very small and the R^2 is above or very close to 0.99 in all cases.

Table A.6: Forecast rules for the expected earnings specification

Forecast rule for	Aggregate productivity	Borrowing Constraint	β_0	β_1	β_2	β_3	SE	R^2
$\bar{K}'$	z_1	θ_e	0.05167	0.83977	-5.75E-04	3.36E-04	4.58E-03	0.98457
	z_1	$\theta_{e,low}$	0.05473	0.83820	-3.54E-04	1.64E-04	2.00E-03	0.99682
	z_2	θ_e	0.04795	0.83886	-3.49E-04	1.37E-04	1.88E-03	0.99695
	z_2	$\theta_{e,low}$	0.04568	0.83748	-7.58E-04	1.82E-04	4.55E-03	0.98529
	z_3	θ_e	0.03873	0.84077	-6.32E-04	1.95E-04	5.23E-03	0.98322
	z_3	$\theta_{e,low}$	0.04134	0.83985	-5.23E-04	8.93E-05	2.09E-03	0.99636
p	z_1	θ_e	0.81493	-0.35642	-1.40E-04	-3.47E-05	2.85E-04	0.99980
	z_1	$\theta_{e,low}$	0.81594	-0.35423	-7.40E-05	2.52E-05	2.26E-04	0.99986
	z_2	θ_e	0.85345	-0.35461	-8.83E-05	1.25E-05	2.09E-04	0.99987
	z_2	$\theta_{e,low}$	0.85227	-0.35678	-1.60E-04	-2.09E-05	2.80E-04	0.99981
	z_3	θ_e	0.89112	-0.35636	-1.52E-04	7.95E-06	3.93E-04	0.99968
	z_3	$\theta_{e,low}$	0.89249	-0.35480	-9.29E-05	-1.00E-06	1.66E-04	0.99992

Note: SE is the standard error in each regression