21EES101T-ELECTRICAL AND ELECTRONICS ENGINEERING

UNIT 1

Unit-1 -Electric Circuits

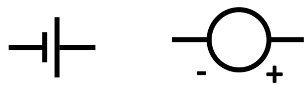
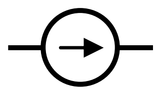
Introduction to basic terminologies in DC circuit, Kirchhoff's Current law, Kirchhoff's Voltage law, Mesh Current Analysis, Nodal Voltage Analysis, Thevenin's Theorem, Maximum power transfer Theorem, Superposition Theorem.

Basic terminologies of AC -RMS and Average value of half wave and Full wave alternating quantity, Fundamentals of single-phase AC circuits-Analysis of R-L, R-C, R-L-C series circuits-Fundamentals of three phase AC system, Three-Phase Winding Connections, Relationship of Line and Phase Voltages, and Currents in a Delta and Star-connected System

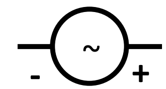
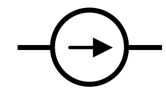
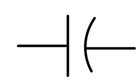
Practice on : Theorems, Halfwave, Full wave bridge rectifier circuits.

Electric circuits are broadly classified as Direct Current (D.C.) circuits and Alternating Current (A.C.) circuits. The following are the various elements that form electric circuits.

D.C. Circuits

Elements	Representation
Voltage source	
Current source	
Resistor	

A.C. Circuits

Elements	Representation
Voltage source	
Current source	
Resistor	
Inductor	
Capacitor	

Active and Passive two terminal elements

Active Components

An **active** component is an electronic component which supplies energy to a circuit. Active elements have the ability to electrically control electron flow (i.e. the flow of charge). All electronic circuits must contain at least one active component.

Examples

Voltage sources, Current sources, Generators , transistors, Diodes

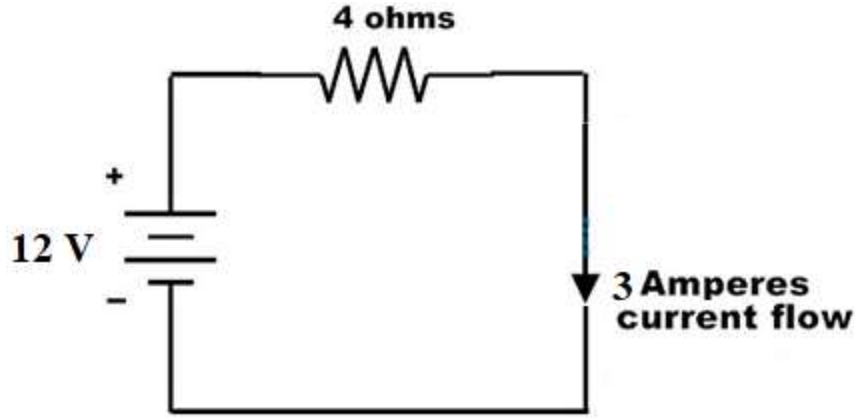
Passive Components

A **passive component** is an electronic component which can only receive energy, which it can either dissipate, absorb or store it in an electric field or a magnetic field

Examples

Resistors, Inductors, Capacitors, Transformers.

A simple DC circuit is given in below figure to get aware of DC circuit components and its parameters.

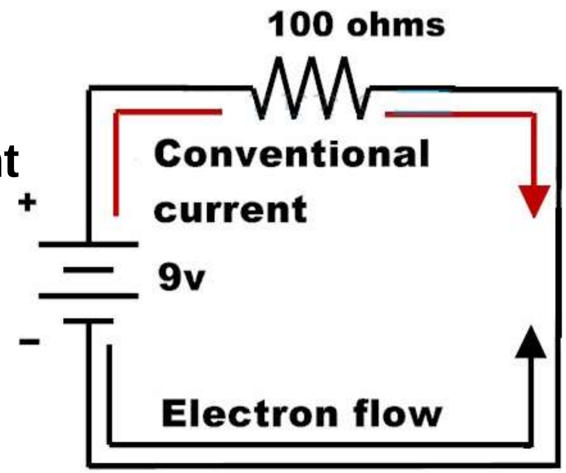


Electric Voltage: The potential difference between two points or voltage in an electric circuit is the amount of energy required to move a unit charge between two points. Unit: Volts

Electric Current

It is the flow of electrons or electric charge. Unit: Ampere

Difference Between Conventional and Electron Current Flow:



OHM'S LAW-STATEMENT

Statement The ratio of potential difference between any two points of a conductor to the current flowing between them is constant, provided the physical conditions (e.g. temperature, etc.) do not change.

i.e. V/I = constant (or) $V/I = R$

Ohm's law can be expressed in three forms:

i.e. $V/I = R; \quad V = IR; \quad I = V/R$

Resistance:

The resistance of a conducting material opposes the flow of electrons. It is measured in ohms (**Ω)**

Electric Power (P)

The power is termed as the work done in a given amount of time. Unit : Watts

$$\mathbf{P = VI \text{ or } I^2R \text{ or } V^2/R}$$

Electrical Energy

The rate at which electrical power consumed is generally referred as electrical energy. Unit: watt-seconds or watt-hr

$$\mathbf{E = P \times t}$$

Resistors connected in series

Two resistors are said to be connected in series when there is only one common point between them and no other element is connected in that common point. Resistors connected in series carry same current. Consider three resisters R_1, R_2 and R_3 connected in series as shown in Fig. 4. With the supply voltage of E, voltages across the three resistors are V_1, V_2 and V_3.

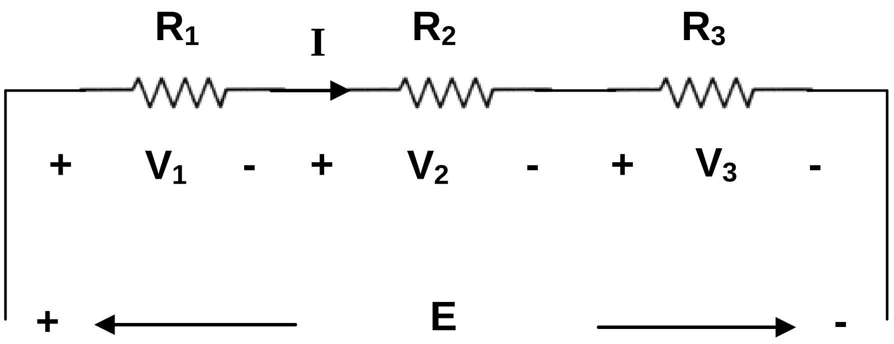


Fig. 4 Resistors connected in series

As per Ohm's law

$$\left.\begin{aligned} V_1 &= R_1 I \\ V_2 &= R_2 I \\ V_3 &= R_3 I \end{aligned}\right\} \qquad (14)$$

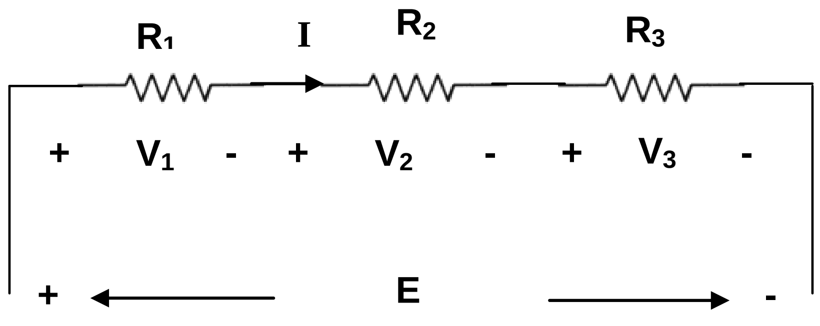


Fig. 4 Resistors connected in series

As per Ohm's law

$V_1 = R_1 I$

$V_2 = R_2 I$

$V_3 = R_3 I$

Applying KVL,

$$E = V_1 + V_2 + V_3 \tag{15}$$

$$= (R_1 + R_2 + R_3) I = R_{eq} I \tag{16}$$

Thus for the circuit shown in Fig. 4,

$$E = R_{eq} I \tag{17}$$

where E is the circuit voltage, I is the circuit current and R_{eq} is the equivalent resistance. Here

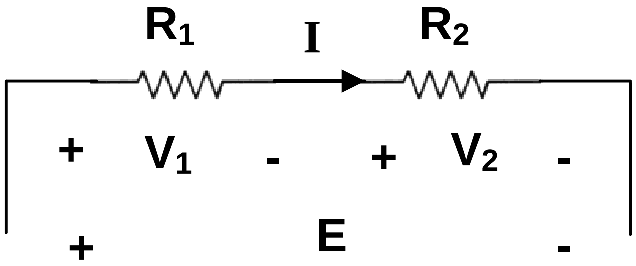
$$R_{eq} = R_1 + R_2 + R_3 \tag{18}$$

This is true when two are more resistors are connected in series. When n numbers of resistors are connected in series, the equivalent resistor is given by

$$R_{eq} = R_1 + R_2 + \ldots\ldots\ldots\ldots + R_n \tag{19}$$

Voltage division rule

Consider two resistors connected in series. Then



$V_1 = R_1 I$

$V_2 = R_2 I$

$E = (R_1 + R_2) I$ **and hence** $I = E / (R_1 + R_2)$

Total voltage of E is dropped in two resistors. Voltage across the resistors are given by

$$V_1 = \frac{R_1}{R_1 + R_2} E \quad \text{and} \tag{20}$$

$$V_2 = \frac{R_2}{R_1 + R_2} E \tag{21}$$

Resistors connected in parallel

Two resistors are said to be connected in parallel when both are connected across same pair of nodes. Voltages across resistors connected in parallel will be equal.

Consider two resistors R_1 and R_2 connected in parallel as shown in Fig. 5.

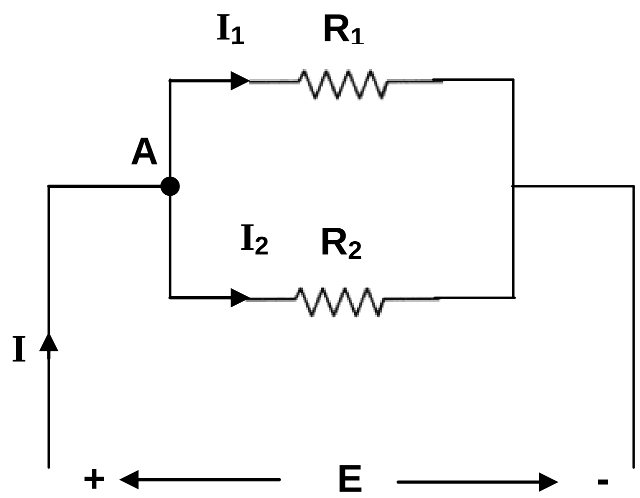
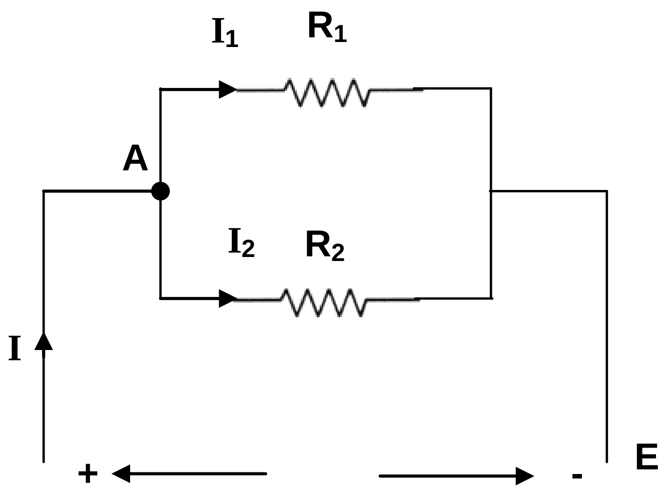


Fig. 5 Resistors connected in parallel

As per Ohm's law,

$$\left.\begin{aligned} I_1 &= \frac{E}{R_1} \\ I_2 &= \frac{E}{R_2} \end{aligned}\right\} \qquad (22)$$



As per Ohm's law

$$I_1 = \frac{E}{R_1}$$

$$I_2 = \frac{E}{R_2}$$

Applying KCL at node A

$$I = I_1 + I_2 = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_{eq}} \qquad (23)$$

Thus for the circuit shown in Fig. 5

$$I = \frac{E}{R_{eq}} \qquad (24)$$

where E is the circuit voltage, I is the circuit current and R_{eq} is the equivalent resistance. Here

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (25)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (25)$$

From the above $\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$

Thus $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$ (26)

When n numbers of resistors are connected in parallel, generalizing eq. (25), R_{eq} can be obtained from

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots \ldots\ldots + \frac{1}{R_n} \qquad (27)$$

Current division rule

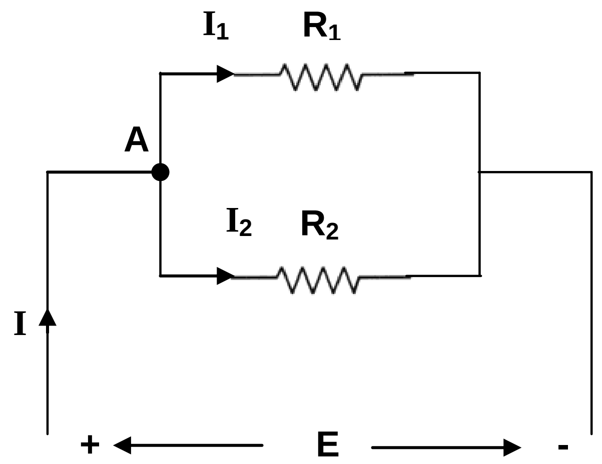


Fig. 5 Resistors connected in parallel

Referring to Fig. 5, it is noticed the total current gets divided as I_1 and I_2. The branch currents are obtained as follows.

From eq. (23)

$$E = \frac{R_1 R_2}{R_1 + R_2} I \qquad (29)$$

Substituting the above in eq. (22)

$$\left.\begin{aligned} I_1 &= \frac{R_2}{R_1 + R_2} I \\ I_2 &= \frac{R_1}{R_1 + R_2} I \end{aligned}\right\} \qquad (30)$$

KIRCHHOFF’s LAWS

There are two Kirchhoff’s laws. The first one is called Kirchhoff’s current law, KCL and the second one is Kirchhoff’s voltage law, KVL. Kirchhoff’s current law deals with the element currents meeting at a junction, which is a meeting point of two are more elements. Kirchhoff’s voltage law deals with element voltages in a closed loop also called as closed circuit.

Kirchhoff's current law

Kirchhoff's currents law states that the algebraic sum of element current meeting at a junction is zero.

Consider a junction P wherein four elements, carrying currents I_1, I_2, I_3 and I_4, are meeting as shown in Fig. 2.

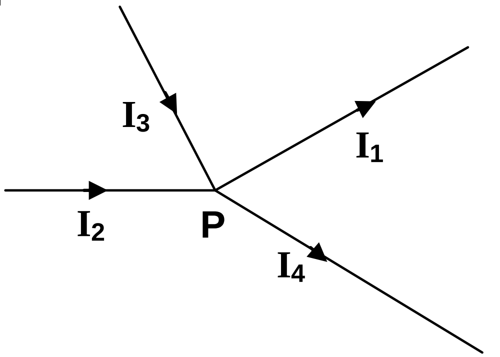


Fig. 2 Currents meeting at a junction

Note that currents I_1 and I_4 are flowing out from the junction while the currents I_2 and I_3 are flowing into the junction. According to KCL,

$$I_1 - I_2 - I_3 + I_4 = 0 \qquad (10)$$

The above equation can be rearranged as

$$I_1 + I_4 = I_2 + I_3 \qquad (11)$$

From equation (11), KCL can also stated as at a junction, the sum of element currents that flows out is equal to the sum of element currents that flows in.

Kirchhoff’s voltage law

Kirchhoff’s voltage law states that the algebraic sum of element voltages around a closed loop is zero.

Consider a closed loop in a circuit wherein four elements with voltages V_1, V_2, V_3 and V_4, are present as shown in Fig. 3.

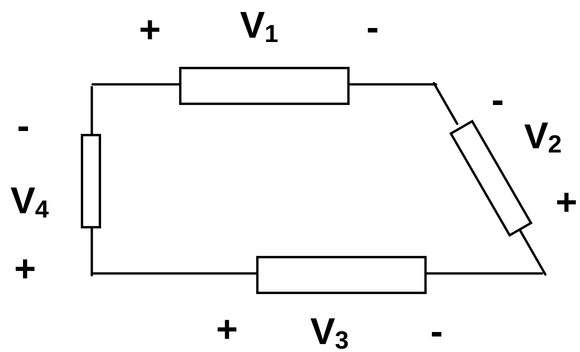


Fig. 3 Voltages in a closed loop

Assigning positive sign for voltage drop and negative sign for voltage rise, when the loop is traced in clockwise direction, according to KVL

$$V_1 - V_2 - V_3 + V_4 = 0 \quad (12)$$

The above equation can be rearranged as

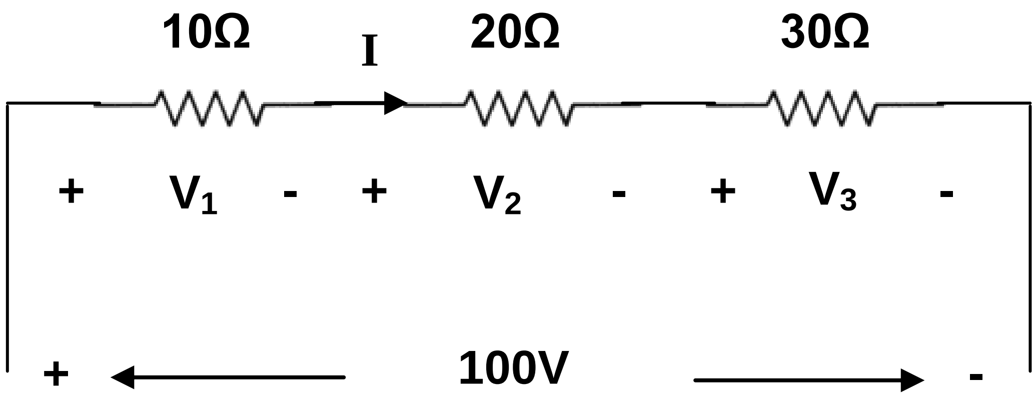
$$V_1 + V_4 = V_2 + V_3 \quad (13)$$

From equation (13), KVL can also stated as, in a closed loop, the sum of voltage drops is equal to the sum of voltage rises in that loop.

Example 1

Three resistors 10Ω, 20Ω and 30Ω are connected in series across 100 V supply. Find the voltage across each resistor.

Solution



Current I = 100 / (10 + 20 + 30) = 1.6667 A

Voltage across 10Ω = 10 x 1.6667 = 16.67 V

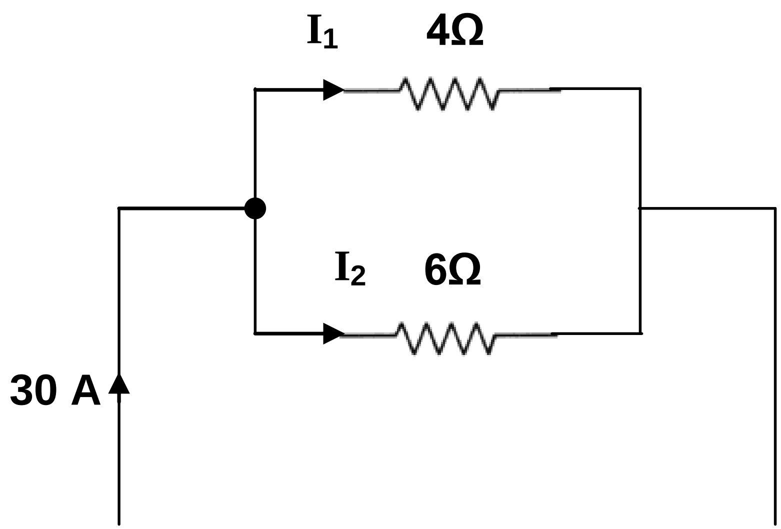
Voltage across 20Ω = 20 x 1.6667 = 33.33 V

Voltage across 30Ω = 30 x 1.6667 = 50 V

Example 2

Two resistors of 4Ω and 6Ω are connected in parallel. If the supply current is 30 A, find the current in each resistor.

Solution



Using the current division rule

$$\text{Current through } 4\Omega = \frac{6}{4+6} \times 30 = 18\,\text{A}$$

$$\text{Current through } 6\Omega = \frac{4}{4+6} \times 30 = 12\,\text{A}$$

Example 3

Four resistors of 2 ohms, 3 ohms, 4 ohms and 5 ohms respectively are connected in parallel. What voltage must be applied to the group in order that the total power of 100 W is absorbed?

Solution

Let R_T be the total equivalent resistor. Then

$$\frac{1}{R_T} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{60+40+30+24}{120} = \frac{154}{120}$$

Resistance $R_T = \frac{120}{154} = 0.7792\,\Omega$

Let E be the supply voltage. Then total current taken = E / 0.7792 A

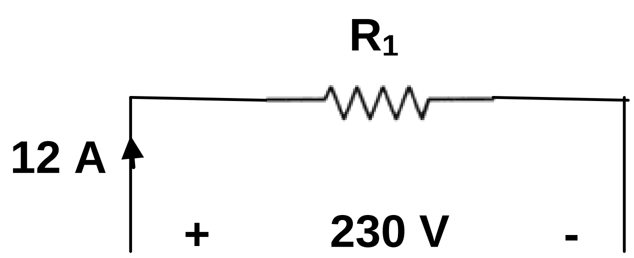
Thus $\left(\frac{E}{0.7792}\right)^2 \times 0.7792 = 100$ and hence $E^2 = 100 \times 0.7792 = 77.92$

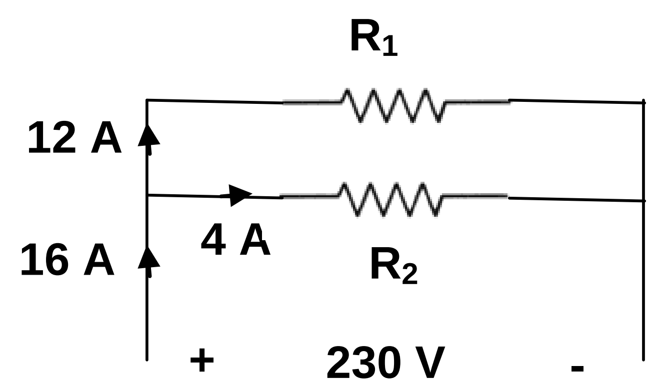
Required voltage = $\sqrt{77.92} = 8.8272$ V

Example 4

When a resistor is placed across a 230 V supply, the current is 12 A. What is the value of the resistor that must be placed in parallel, to increase the load to 16 A

Solution





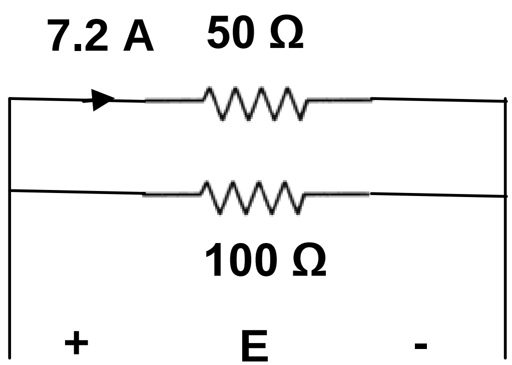
To make the load current 16 A, current through the second resistor = 16 –12 = 4 A

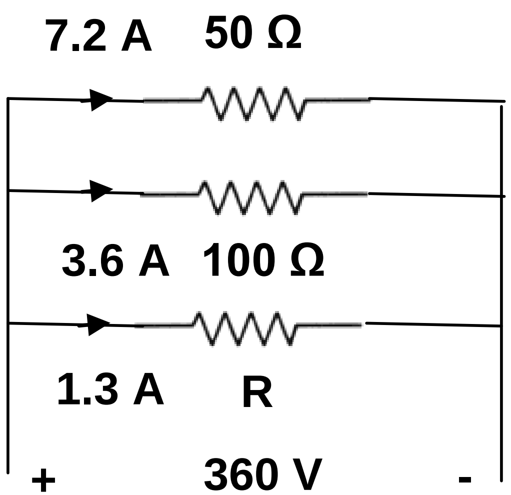
Value of second resistor R_2 = 230/4 = 57.5 Ω

Example 5

A 50 Ω resistor is in parallel with a 100 Ω resistor. The current in 50 Ω resistor is 7.2 A. What is the value of third resistor to be added in parallel to make the line current as 12.1A?

Solution





Supply voltage E = 50 x 7.2 = 360 V

Current through 100 Ω = 360/100 = 3.6 A

When the line current is 12.1 A, current through third resistor = 12.1 – (7.2 + 3.6)

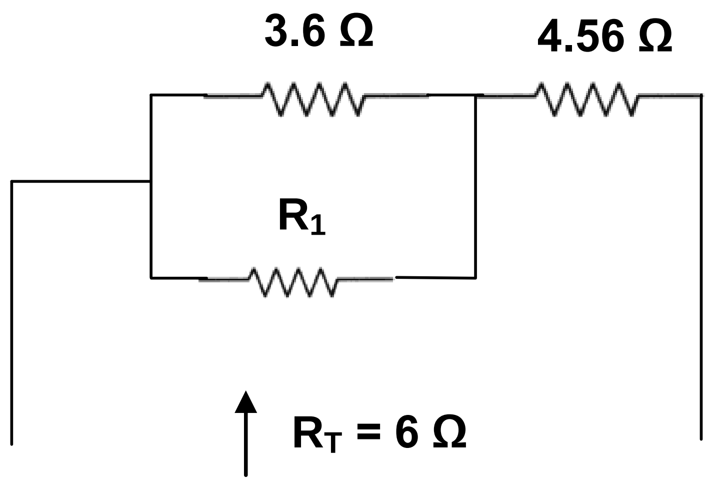
= 1.3 A

Value of third resistor = 360/1.3 = 276.9230 Ω

Example 6

A resistor of 3.6 ohms is connected in series with another of 4.56 ohms. What resistance must be placed across 3.6 ohms, so that the total resistance of the circuit shall be 6 ohms?

Solution



$3.6 \parallel R_1 = 6 - 4.56 = 1.44\ \Omega$

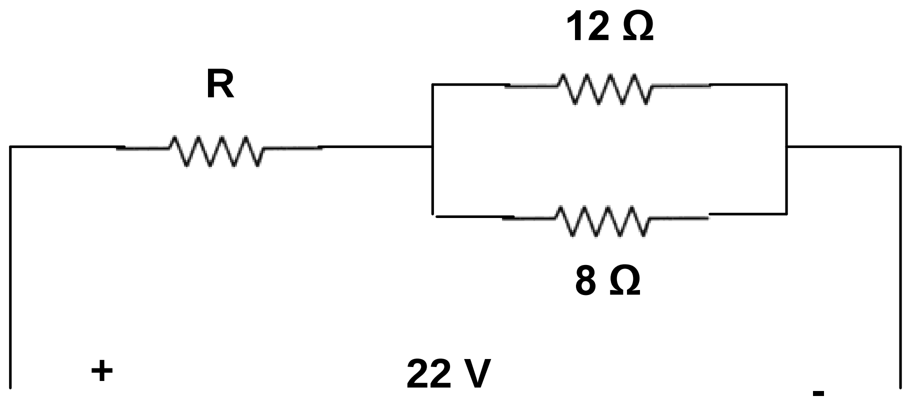
Thus $\dfrac{3.6 \times R_1}{3.6 + R_1} = 0.4$; Therefore $\dfrac{3.6 + R_1}{R_1} = \dfrac{1}{0.4} = 2.5$; $\dfrac{3.6}{R_1} = 1.5$

Required resistance $R_1 = 3.6/1.5 = 2.4\ \Omega$

Example 7

A resistance R is connected in series with a parallel circuit comprising two resistors 12 Ω and 8 Ω respectively. Total power dissipated in the circuit is 70 W when the applied voltage is 22 V. Calculate the value of the resistor R.

Solution



Total current taken = 70 / 22 = 3.1818 A

Equivalent of 12 Ω ║ 8 Ω = 96/20 = 4.8 Ω

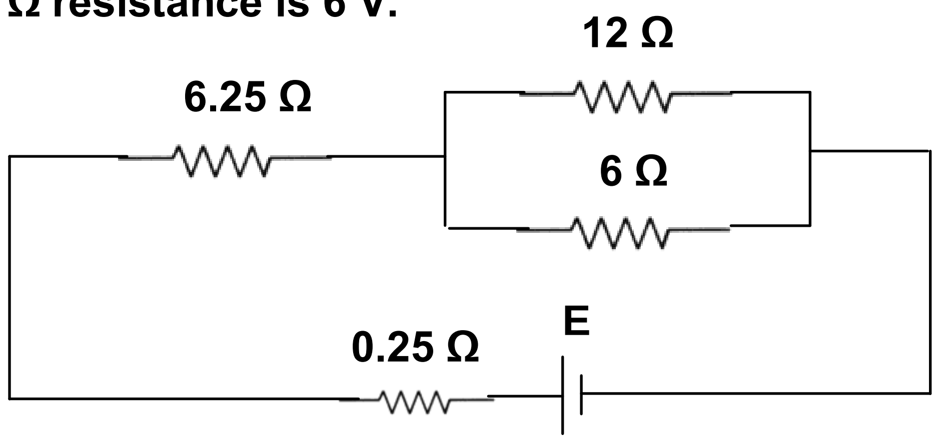
Voltage across parallel combination = 4.8 x 3.1818 = 15.2726 V

Voltage across resistor R = 22 – 15.2726 = 6.7274 V

Value of resistor R = 6.7274/3.1818 = 2.1143 Ω

Example 8

The resistors 12 Ω and 6 Ω are connected in parallel and this combination is connected in series with a 6.25 Ω resistance and a battery which has an internal resistance of 0.25 Ω. Determine the emf of the battery if the potential difference across 6 Ω resistance is 6 V.



Voltage across 6 Ω = 6 V

Solution

Current in 6 Ω = 6/6 = 1 A

Current in 12 Ω = 6/12 = 0.5 A

Therefore current in 25 Ω = 1.0 + 0.5 = 1.5 A

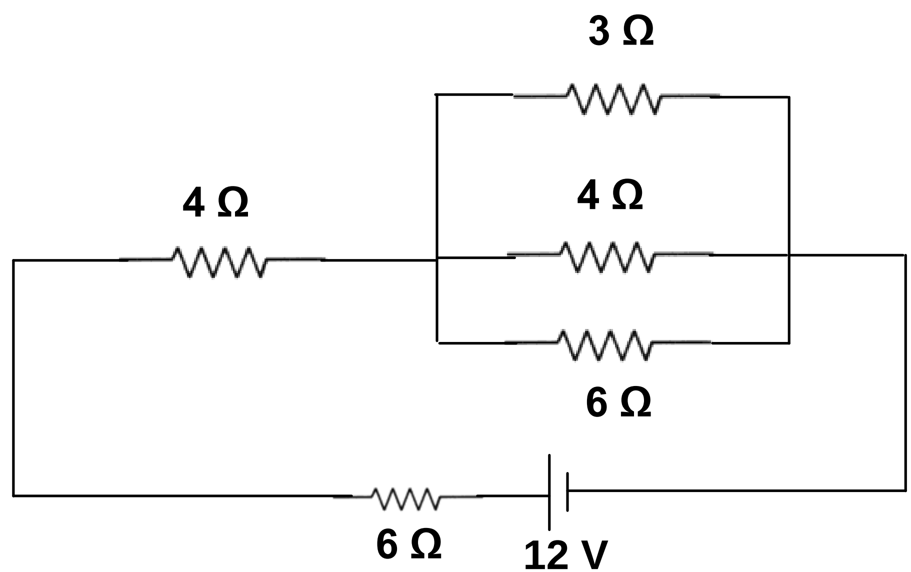
Using KVL   E = (0.25 x 1.5) + (6.25 x 1.5) + 6 = 15.75 V

Therefore battery emf E = 15.75 V

Example 9

A circuit consist of three resistors 3 Ω, 4 Ω and 6 Ω in parallel and a fourth resistor of 4 Ω in series. A battery of 12 V and an internal resistance of 6 Ω is connected across the circuit. Find the total current in the circuit and the terminal voltage across the battery.

Solution



$4\ \Omega \parallel 6\ \Omega = 24/10 = 2.4\ \Omega$
$1.4\ \Omega \parallel 3\ \Omega = 7.2/5.4 = 1.3333\ \Omega$

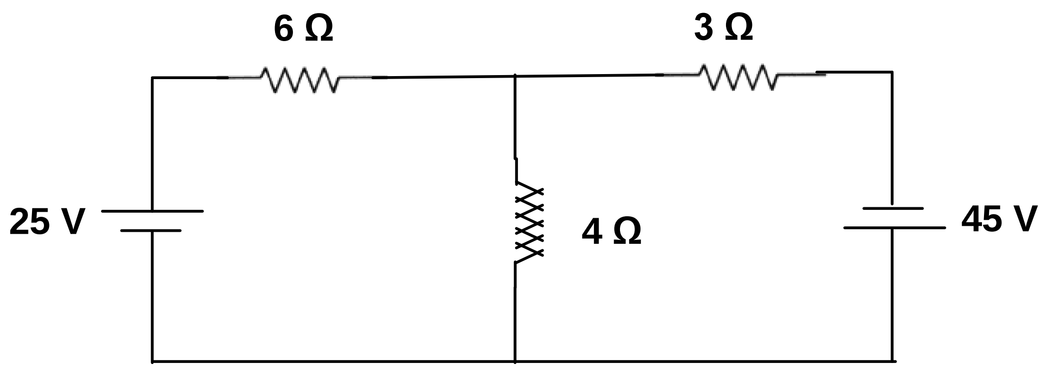
Total circuit resistance = 4 + 6 + 1.3333 = 11.3333 Ω

Circuit current = 12/11.3333 = 1.0588 A

Terminal voltage across the battery = 12 – (6 x 1.0588) = 5.6472 V

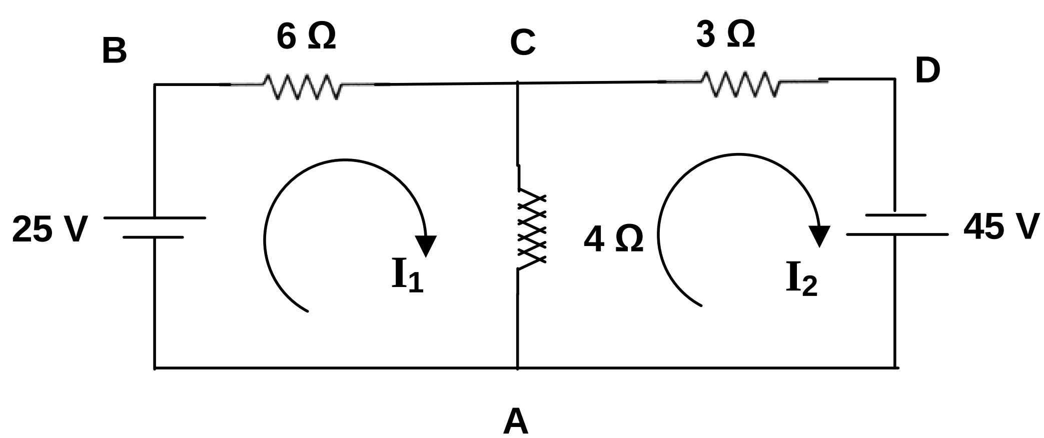
Example 11

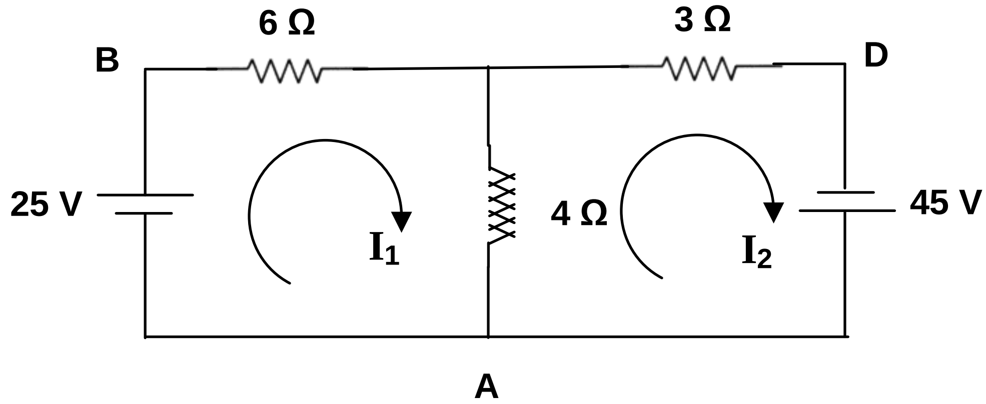
Using Kirchhoff's laws, find the current in various resistors in the circuit shown.



Solution

Let the loop current be I_1 and I_2





Considering the loop ABCA, KVL yields

$6\ I_1 + 4\ (I_1 - I_2) - 25 = 0$

For the loop CDAC, KVL yields

$3\ I_2 - 45 + 4\ (I_2 - I_1) = 0$

Thus $10\ I_1 - 4\ I_2 = 25$

$-4\ I_1 + 7\ I_2 = 45$

On solving the above $I_1 = 6.574$ A; $I_2 = 10.1852$ A

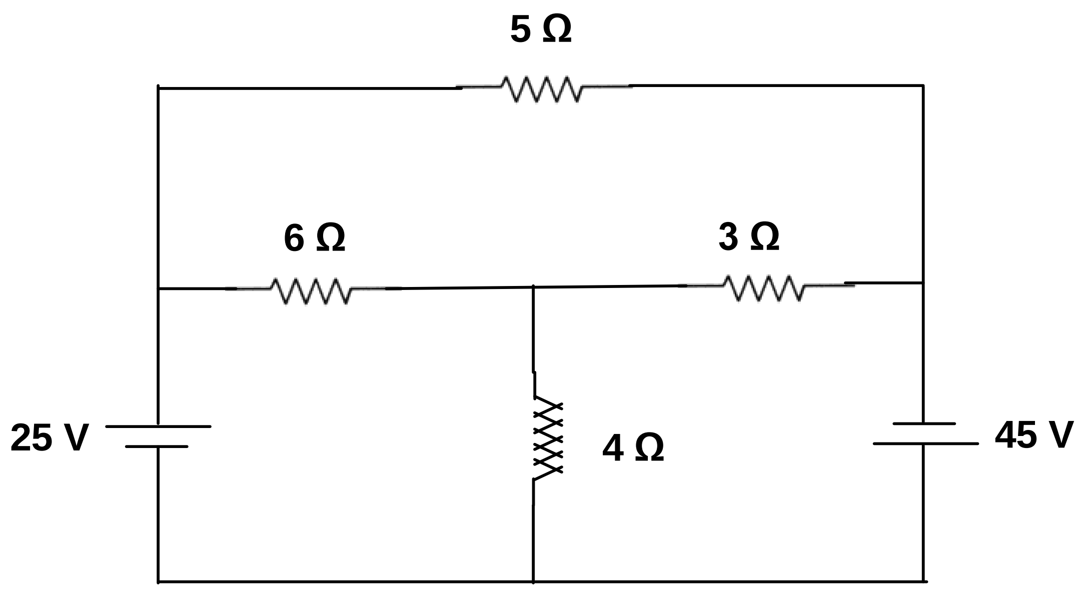
Current in 4Ω resistor = $I_1 - I_2 = 6.574 - 10.1852 = -3.6112$ A

Thus the current in 4Ω resistor is 3.6112 A from A to C

Current in 6 Ω resistor = 6.574 A; Current in 3 Ω resistor = 10.1852 A

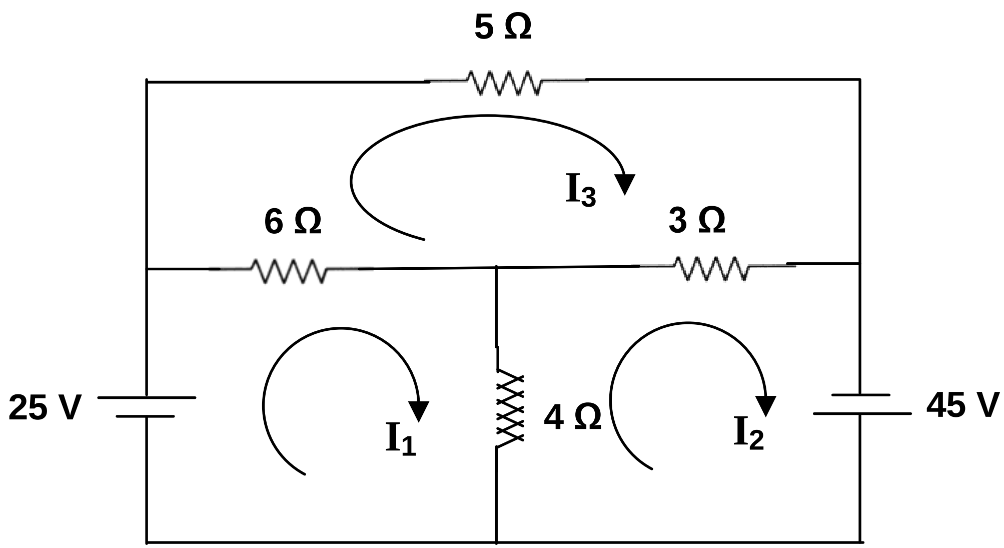
Example 12

Find the current in 5 Ω resistor in the circuit shown.



Solution

Let the loop current be I_1, I_2 and I_3.



Three loops equations are:

$$6\,(I_1 - I_3) + 4\,(I_1 - I_2) - 25 = 0$$

$$4\,(I_2 - I_1) + 3\,(I_2 - I_3) - 45 = 0$$

$$5\,I_3 + 3\,(I_3 - I_2) + 6\,(I_3 - I_1) = 0$$

On solving

Current in 5 Ω resistor, $I_3 = 14$ A

For MESH and NODAL Analysis

REFER THE PDF..\MESH AND NODAL ANALYSIS.PDF

CIRCUIT THEOREMS

THEVENIN'S THEOREM

In many practical applications, we may not be interested in getting the complete analysis of the circuit, namely finding the current through all the elements and voltages across all the elements. We may be interested to know the details of a portion of the circuit; as a special case **it may be a single element such as load impedance.** In such a situation it is very convenient to use Thevenin's theorem to get the solution.

Fig. illustrates the Thevenin's equivalent of sub-circuit A.

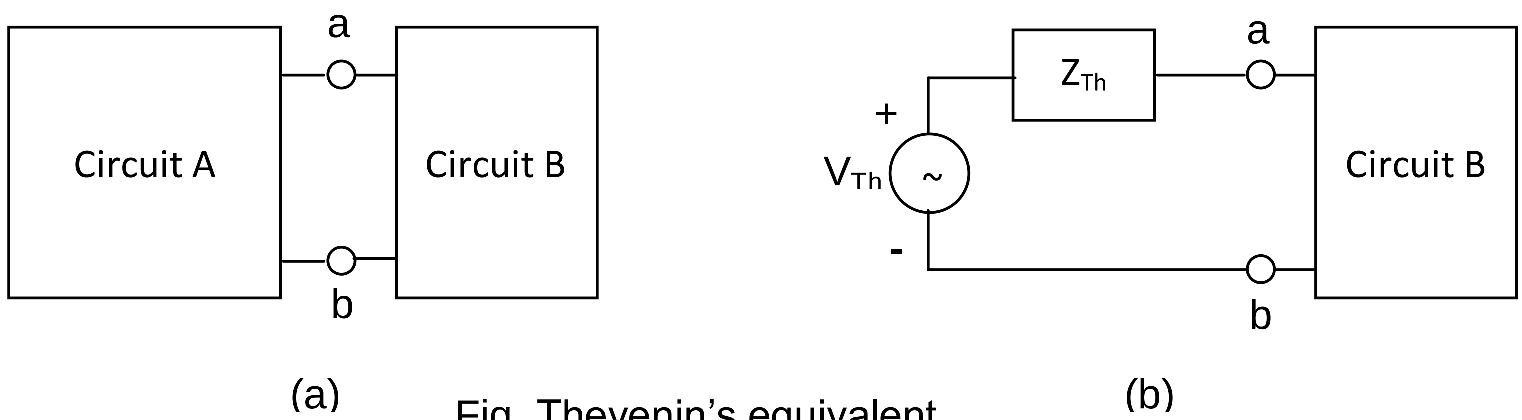


Fig. Thevenin's equivalent.

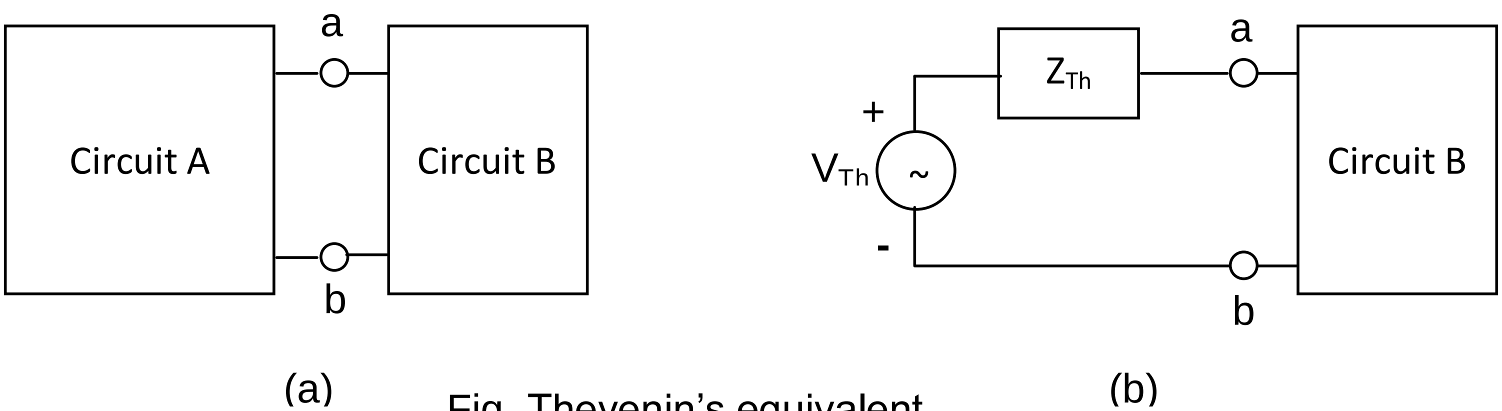


Fig. Thevenin's equivalent.

In Fig. (a) a circuit partitioned into two parts, namely circuit A and circuit B, is shown. They are connected by a single pair of terminals. In Fig.(b) circuit A is replaced by Thevenin's equivalent circuit, which consists of a voltage source V_{Th} in series with an impedance Z_{Th}.

To obtain the Thevenin's equivalent circuit, we need to find Thevenin's voltage V_{th} and Thevenin's impedance Z_{Th}. Unique procedure is available to find the Thevenin's voltage V_{Th}. When we need the Thevenin's voltage of circuit A, measure or calculate the **OPEN CIRCUIT VOLTAGE of circuit A.** This will be the Thevenin's voltage.

Thevenin's impedance can be calculated in three different ways **depending on the nature of voltage and current sources** in the circuit of our interest.

The circuit for which Thevenin's impedance is to be calculated consists of impedances and **one or more independent sources.** That is, **the circuit does not contain any dependent source**. To determine Thevenin's impedance, circuit shown in Fig. (b) is to be used.

Fig. Determining Thevenin's equivalents.

The circuit AA in Fig. (b) is obtained from circuit A by replacing all the independent voltage sources by short circuits and replacing all independent current sources by open circuits. Thus in circuit AA, all the independent sources are set to zero. Then, Thevenin's impedance is the equivalent circuit impedance of circuit AA which can be obtained using reduction techniques.

The methods of finding the Thevenin's impedance depend on the nature of the circuit for which the Thevenin's equivalent is sought for. These methods are summarized below:

Circuit with independent sources only - ANY ONE OF THE FOLLOWING

1. **Make independent sources zeros and use reduction techniques to find Z_{Th}.**

2. Short circuit terminals a and b and find the short circuit current I_{sc} flowing from *a* to *b*. Then $Z_{Th} = V_{Th} / I_{sc}$

3. Set all independent sources to zero. Apply 1 V across the open circuited terminals *a-b* and determine the source current I_s entering the circuit through *a.* Then $Z_{Th} = 1 / I_s$. Alternatively introduce a current source of 1 A from *b* to *a* and determine the voltage V_{ab}. Then, Thevenin's impedance $Z_{Th} = V_{ab}$.

Example 1

Find the Thevenin's voltage with respect to the load resistor R_L in circuit shown in Fig.

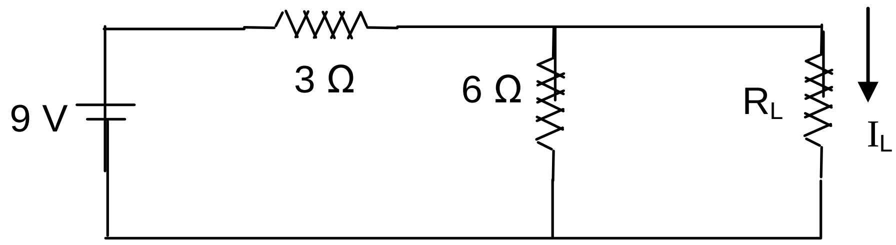
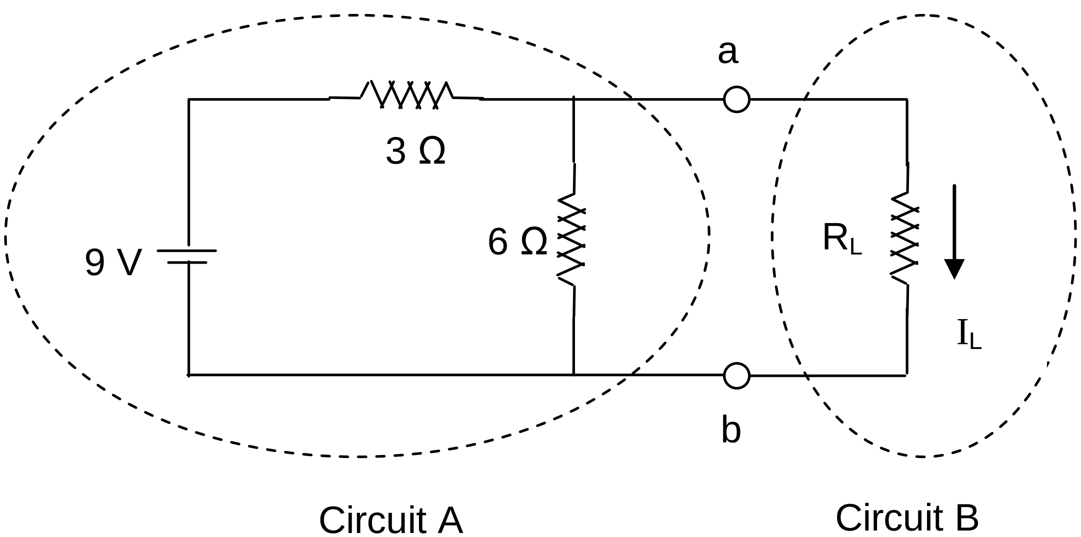
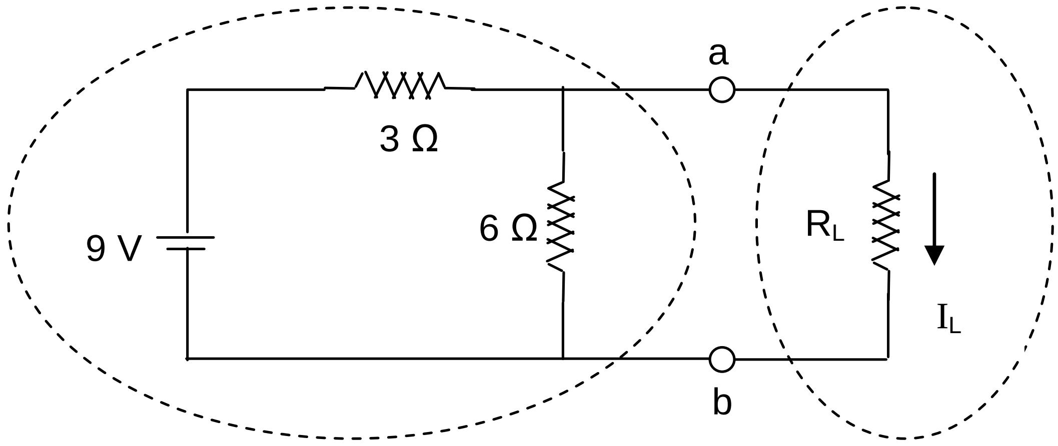


Fig. Circuit for Example1

Solution

The given circuit can be divided into two circuits as shown in Fig.





Thevenin's voltage of circuit A can be obtained from the circuit shown in Fig.

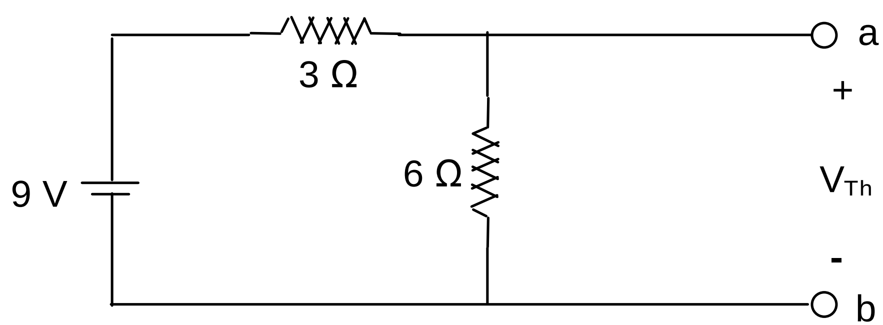


Fig. Circuit.

Using voltage division rule $V_{Th} = V_{6\Omega} = \frac{6}{9} \times 9 = 6\,V$

Example 2

Obtain the Thevenin's equivalent for the circuit shown in Fig.

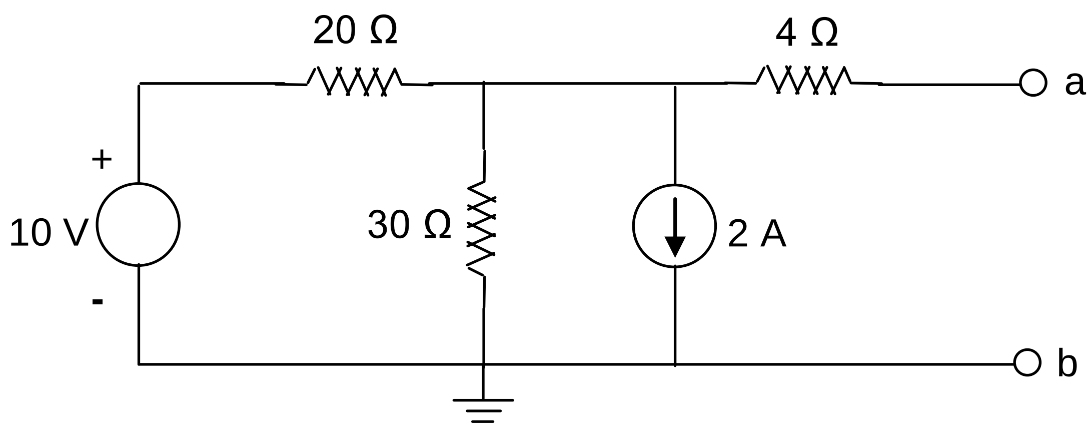


Fig. Circuit for Example 2.

Solution:

Open circuit voltage V_{ab} is the Thevenin's voltage V_{Th}.

To find Thevenin's voltage:

Note that there is no current flow in resistor of 4 Ω. Therefore, voltage V_{Th} is same as the voltage across 30 Ω resistor. Then, the node voltage equation is

$$\frac{V_{Th} - 10}{20} + \frac{V_{Th}}{30} + 2 = 0$$ On solving this, we get V_{Th} = - 18 V

To find Thevenin's impedance: Since the circuit has only independent sources, it falls under case 1

Reducing the sources to zero, the resulting circuit is shown in Fig.

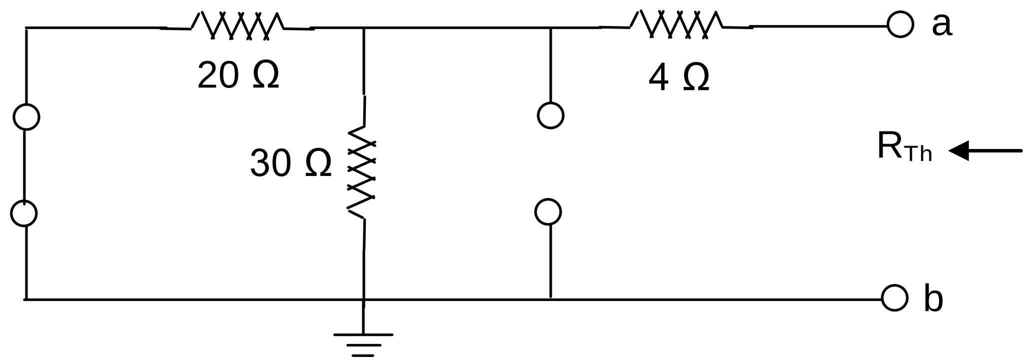


Fig. Circuit - Example 2.

Thus $R_{Th} = 4 + 20||30 = 16\ \Omega$ Thevenin's equivalent circuit is shown in Fig.

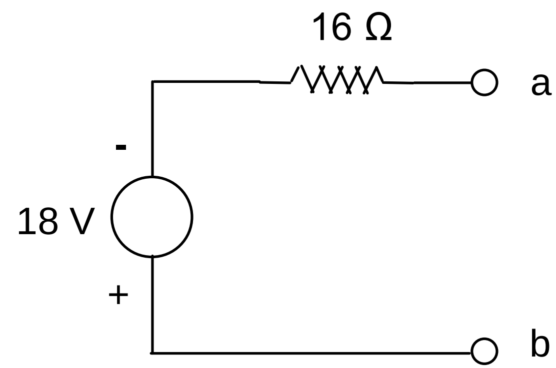
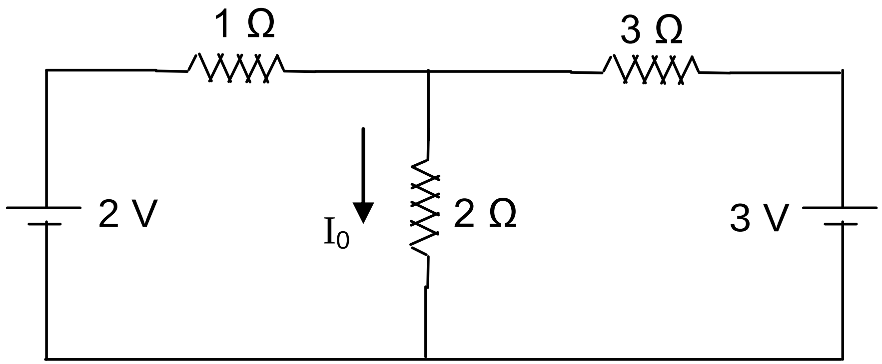


Fig. Thevenin's equivalent circuit – Example 2.

R_{Th} can be obtained by two other methods also

Example 3 Using Thevenin's equivalent circuit, calculate the current I_0 through the 2 Ω resistor in the circuit shown below.



Solution: Circuit by which V_{Th} and R_{Th} can be calculated are shown in Fig.

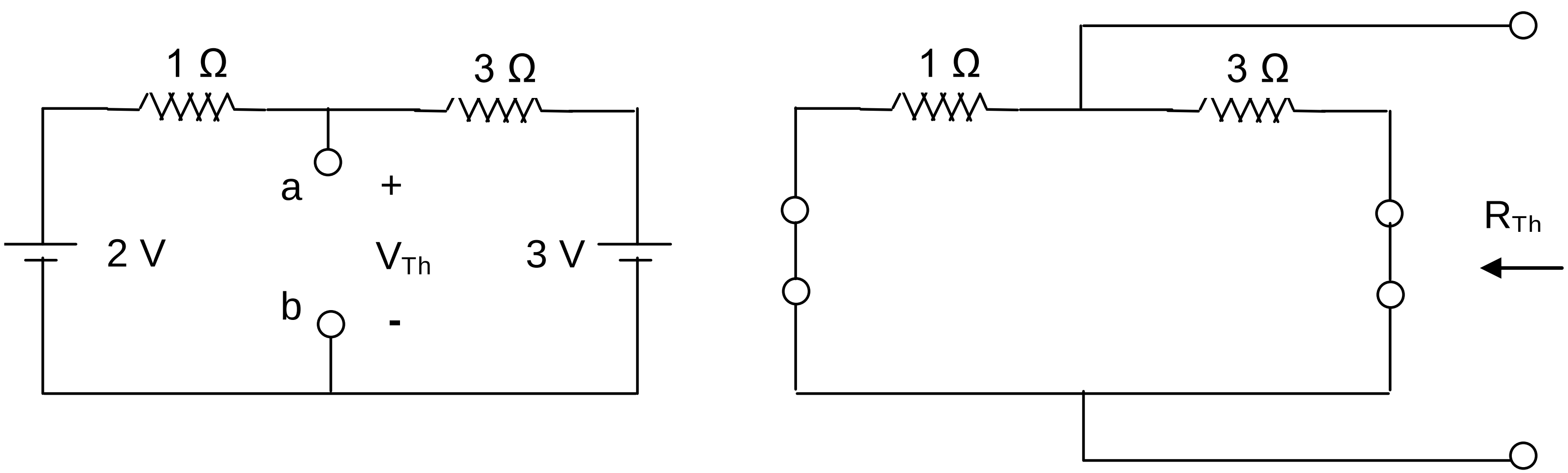
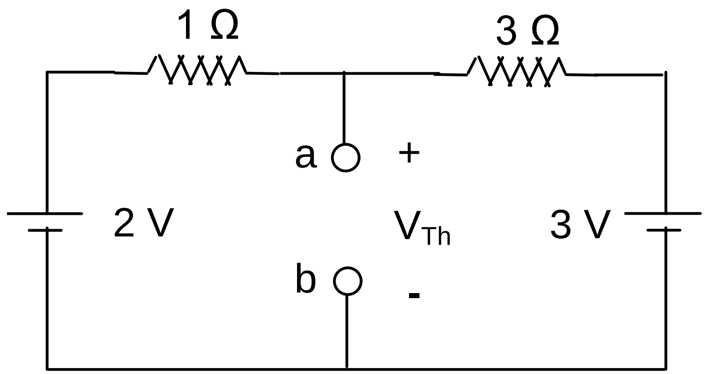
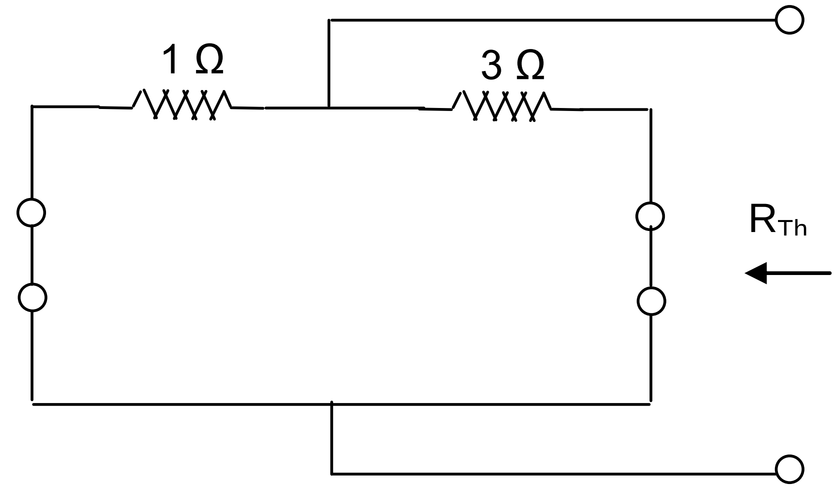


Fig. Circuits for V_{Th} and R_{Th} - Example 3.

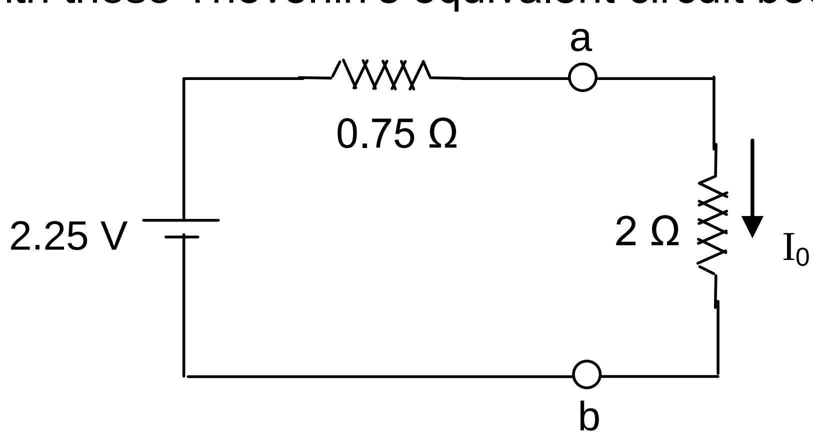




Knowing the anticlockwise current as 0.25 A

$- 2 - (1 \times 0.25) + V_{Th} = 0$. i.e. $V_{Th} = 2.25$ V; Also $R_{Th} = 1 || 3 = 0.75\ \Omega$

With these Thevenin's equivalent circuit becomes



Current $I_0 = 2.25 / 2.75 = 0.8182$ A

MAXIMUM POWER TRANSFER THEOREM

There are some applications wherein maximum power needs to be transferred to the load connected. Consider a linear ac circuit *A*, connected to a load of impedance Z_L as shown in Fig. (a). It is required to transfer maximum real power to the load. The circuit A can be replaced by its Thevenin's equivalent as shown in Fig. (b).

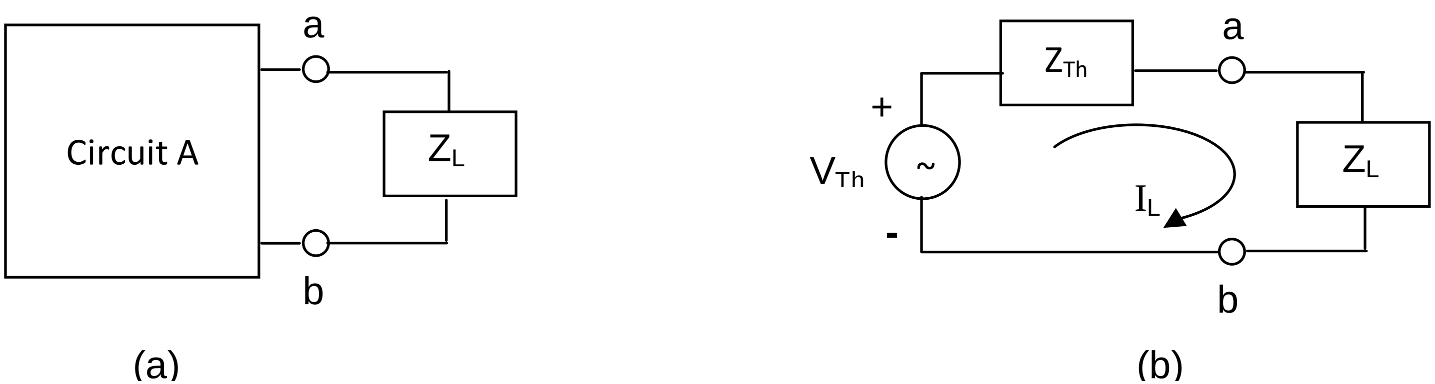


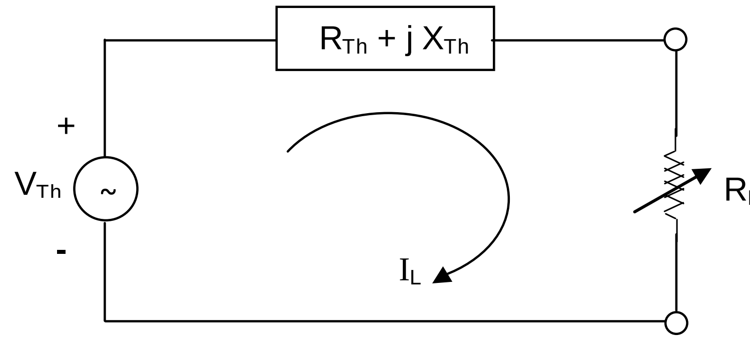
Fig. Maximum power transfer theorem - Illustration.

Let $Z_{Th} = (R_{Th} + j X_{Th})$ and $Z_L = (R_L + j X_L)$

The following maximum power transfer theorems determine the values of load impedance Z_L for which maximum real power is transferred to the load impedance.

Case 1:

Load is a variable resistance R_L



Load current $I_L = \dfrac{V_{Th}}{(R_{Th} + R_L) + jX_{Th}}$ This gives $|I_L| = \dfrac{|V_{Th}|}{\sqrt{(R_{Th} + R_L)^2 + X_{Th}^2}}$

Real power delivered to the load $P_L = |I_L|^2 R_L = \dfrac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + X_{Th}^2}$

This can be written as $P_L = \dfrac{|V_{Th}|^2}{\dfrac{R_{Th}^2}{R_L} + 2R_{Th} + R_L + \dfrac{X_{Th}^2}{R_L}}$

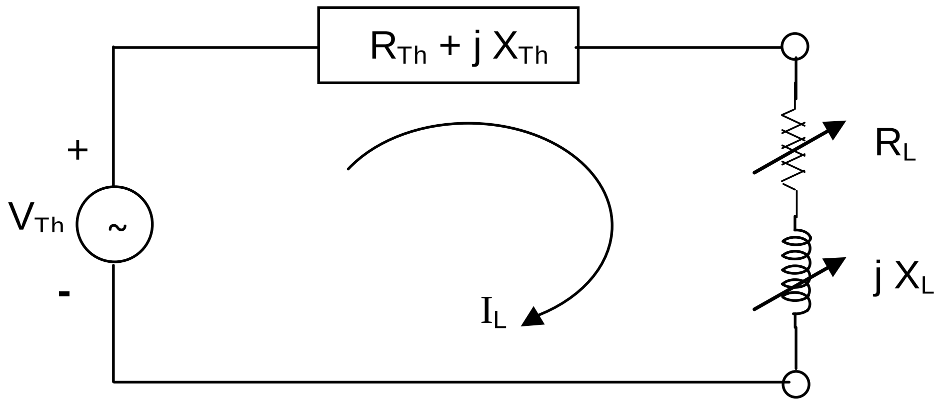
For power P_L to be maximum, $\dfrac{R_{Th}^2}{R_L} + 2R_{Th} + R_L + \dfrac{X_{Th}^2}{R_L}$ must be minimum. Thus power P_L will be maximum when

$$\frac{d}{dR_L}\left(\frac{R_{Th}^2}{R_L} + 2R_{Th} + R_L + \frac{X_{Th}^2}{R_L}\right) = 0 \quad \text{i.e. when} \quad -\frac{R_{Th}^2}{R_L^2} + 1 - \frac{X_{Th}^2}{R_L^2} = 0$$

i.e. when $R_L^2 = R_{Th}^2 + X_{Th}^2$ i.e. when $R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = |Z_{Th}|$

Using this value of R_L, the current I_L and hence maximum power can be computed.

Case 2 In the load impedance, R_L and X_L are varied independently as shown.



Load current $I_L = \dfrac{V_{Th}}{(R_{Th} + R_L) + j(X_{Th} + jX_L)}$ Thus $|I_L| = \dfrac{|V_{Th}|}{\sqrt{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}}$

Real power delivered to the load $P_L = |I_L|^2 R_L = \dfrac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$

If R_L in Eq. is held fixed, the value of P will be maximum when $(X_{Th} + X_L)^2$ is minimum.
This will occur when $X_{Th} + X_L = 0$ i.e. when

$X_L = - X_{Th}$

Keeping $X_L = - X_{Th}$ Eq. becomes

$$P_L = \frac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2} = \frac{|V_{Th}|^2}{\frac{R_{Th}^2}{R_L} + 2R_{Th} + R_L}$$

For P_L given by Eq. to become maximum, $\frac{R_{Th}^2}{R_L} + 2R_{Th} + R_L$ must be minimum. This will occur when $\frac{d}{dR_L}(\frac{R_{Th}^2}{R_L} + 2R_{Th} + R_L) = 0$ i.e. when $-\frac{R_{Th}^2}{R_L^2} + 1 = 0$ i.e. when

$R_L = R_{Th}$

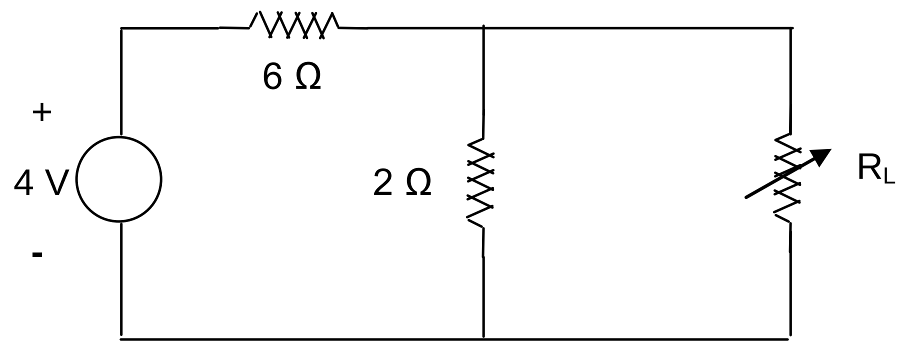
Combining Eqs. and, we can state that real power transferred to the load will be maximum when

$Z_L = R_{Th} - j X_{Th} = Z_{Th}^*$

Setting $R_L = R_{Th}$ and $X_L = - X_{Th}$ in Eq., maximum real power can be obtained as

$$P_{max} = \frac{|V_{Th}|^2}{4 R_{Th}}$$

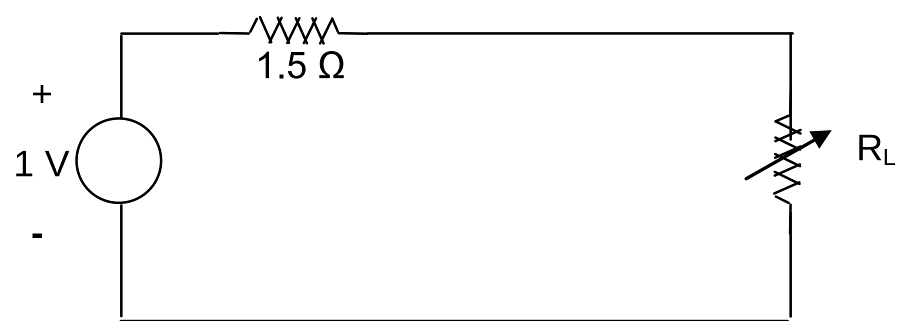
Example 1 Consider the circuit shown below. Determine the value of R_L when it is dissipating maximum power. Also find the value of maximum power dissipated.



Solution:

As a first step, Thevenin's equivalent across the load resistor is obtained.

$V_{Th} = \frac{2}{2+6} \times 4 = 1\ V$; $R_{Th} = 6||2 = 1.5\ \Omega$ Resulting circuit is shown.



For P_L to be maximum, $R_L = 1.5\ \Omega$; Then circuit current = 1/3 = 0.3333 A

Maximum power dissipated $P_{max} = 0.3333^2 \times 1.5 = 0.16667$ W

SUPERPOSITION THEOREM

The idea of superposition rests on the linearity property. Superposition theorem is applicable to linear circuits having two or more independent sources.

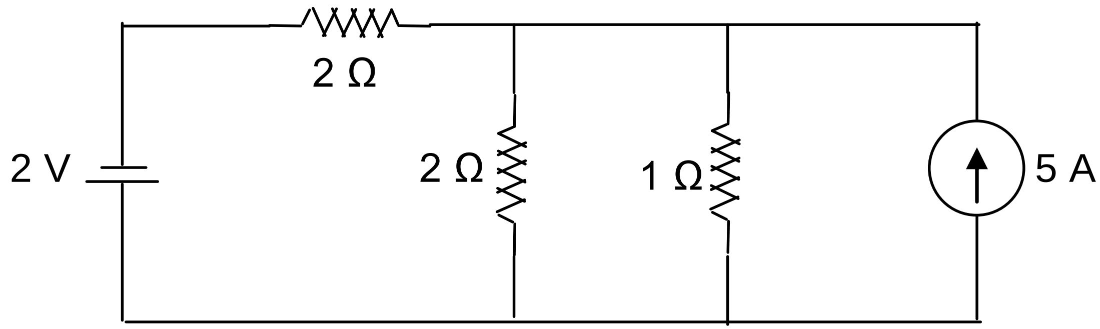
In a linear circuit having two or more independent sources, total response in an element (voltage across the element or current through the element) is equal to the algebraic sum of responses in that element due to each source applied separately while the other sources are reduced to zero.

To make a current source to zero, it must be open circuited. Similarly, **if any voltage source is to be made zero, it must be short circuited.** When this theorem is used in circuit with initial conditions, they are to be treated as sources. Further, dependent sources if any are left intact because they are controlled by circuit variables.

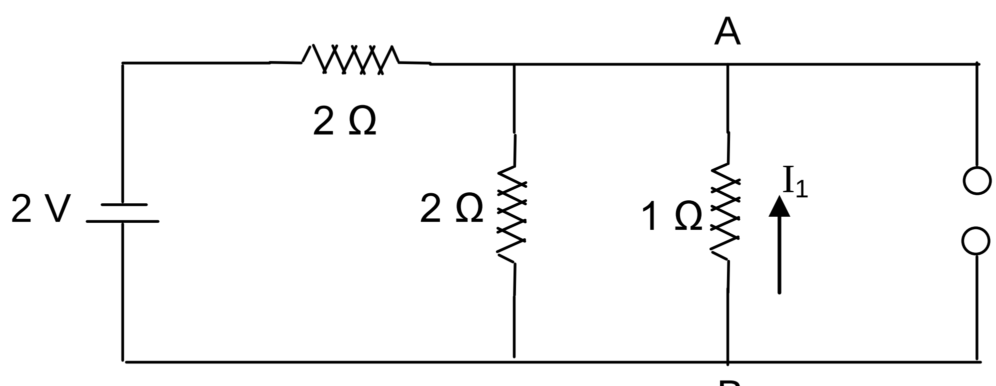
One disadvantage of analyzing a circuit using Superposition theorem is that it involves more calculations. If the circuit has three independent sources, we need to solve three simpler circuits each having only one independent source. However, when the circuit has only one independent source, several short-cut techniques can be readily applied to get the solution.

Major advantage of Superposition theorem is that it can be used to solve ac circuit having more than one source with **different frequencies.** In such case, solution in time frame is obtained corresponding to each source and added up to get the total solution.

Example 1 Calculate the current through the 1 Ω resistor in the circuit shown below.

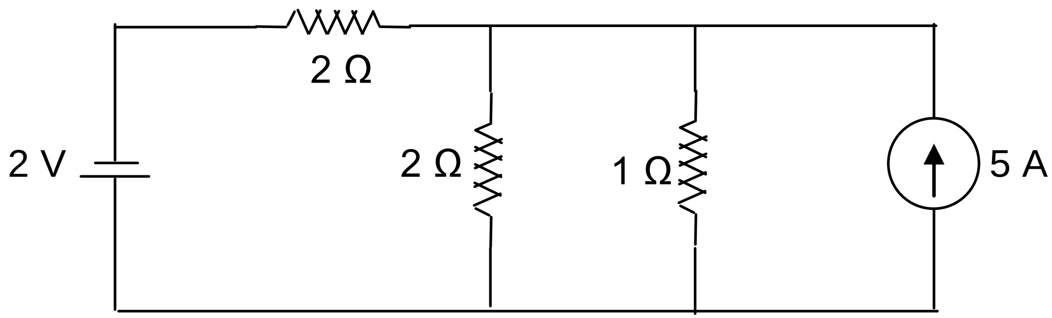


Solution: First calculate current I_1 due to voltage source alone. The current source is open circuited. The resulting circuit is shown below.



Total circuit resistance R_T = 2.6667 Ω. Circuit current $I_T = \dfrac{2}{2.6667} = 0.75$ A

$$\text{Current } I_1 = \frac{2}{3} \times 0.75 = 0.5 \text{ A from B to A}$$



Now calculate current I_2 due to current source alone. The voltage source is short circuited as shown in Fig.

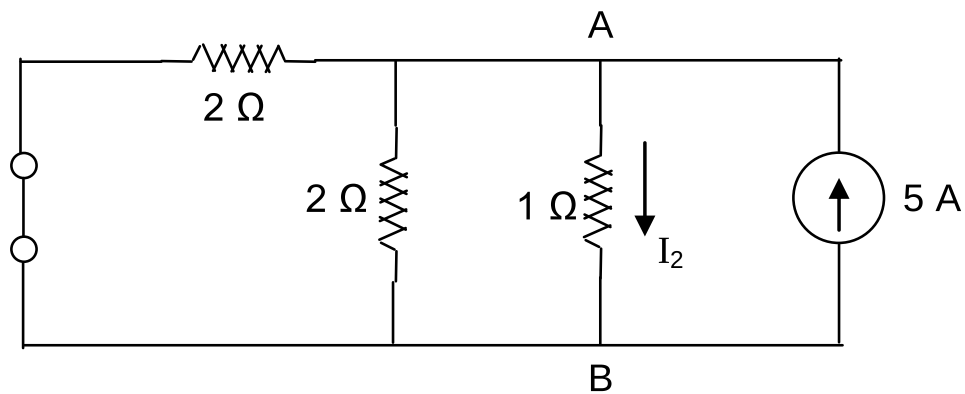


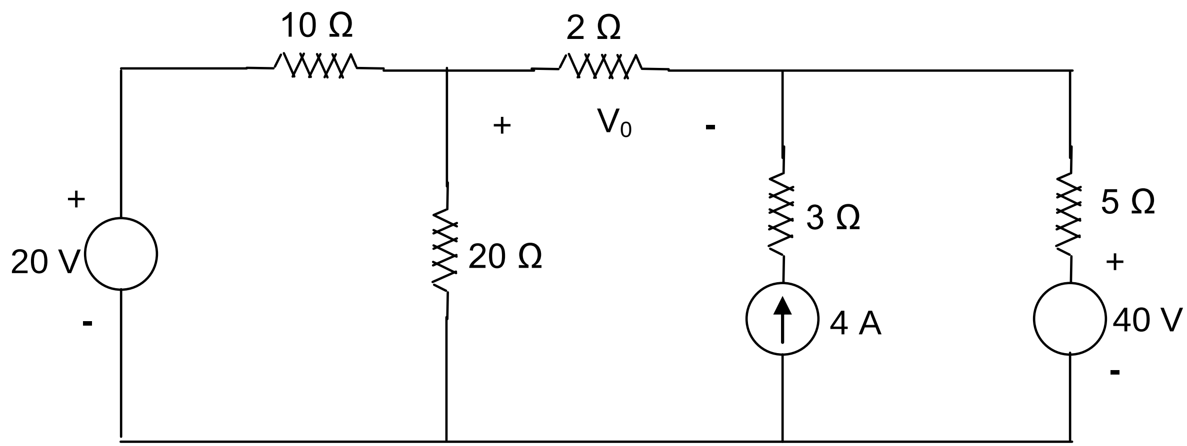
Fig. Circuit - Example 1

Noting that two 2 Ω resistors are in parallel, current I_2 = 2.5 A from A to B.

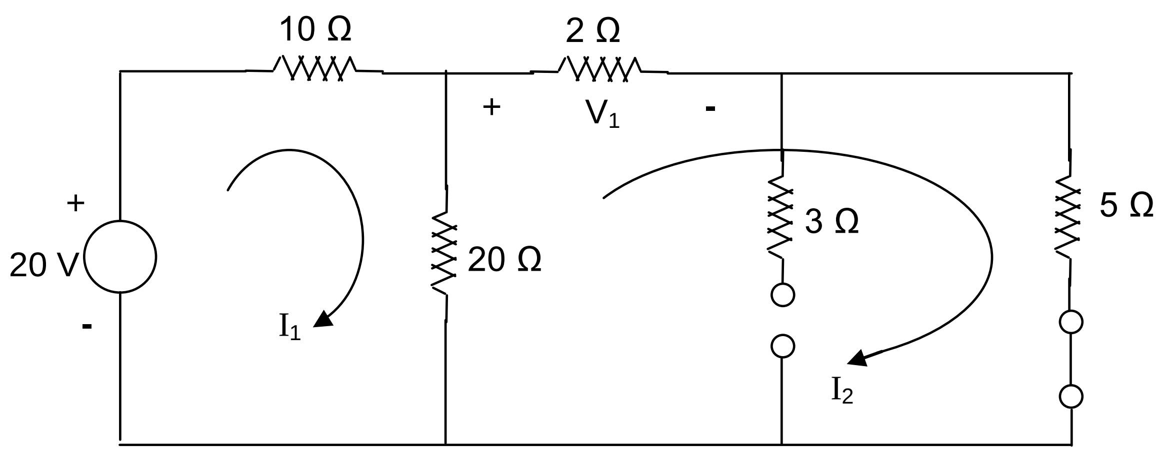
When both the sources are simultaneously present:

Current through 1 Ω resistor = 2.5 - 0.5 = 2 A from A to B.

Example 2 In the circuit shown, find the voltage drop, V_0 across the 2 Ω resistor using Superposition theorem.



Solution: 20 V source alone present: The circuit will be as shown below.

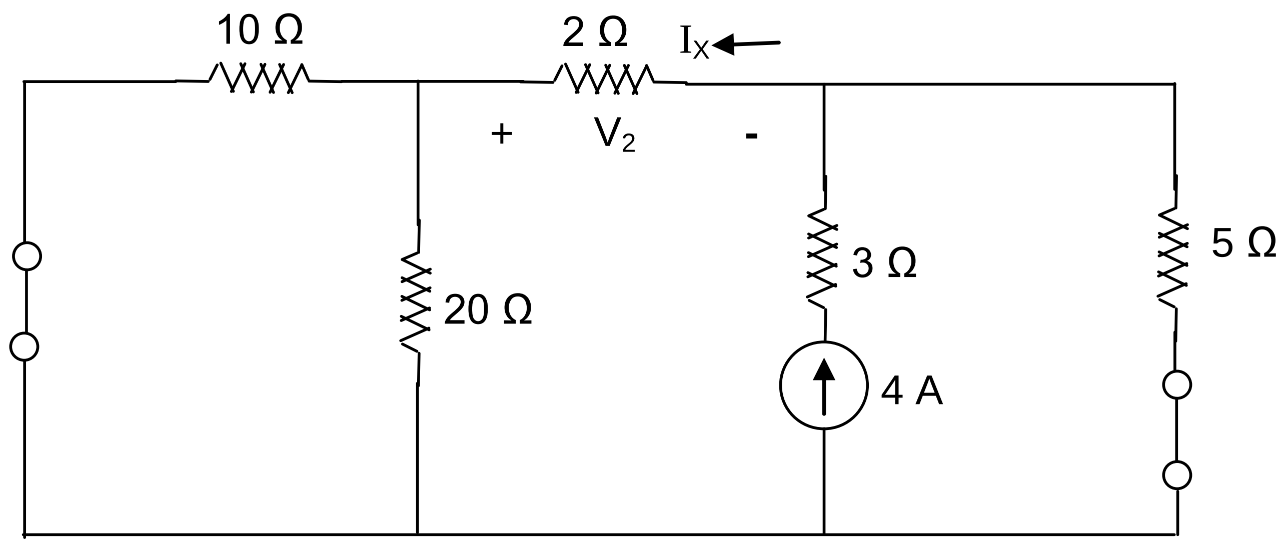


Mesh current equations : $\begin{bmatrix} 30 & -20 \\ -20 & 27 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$ On solving, $I_2 = 0.9756$ A

Thus voltage $V_1 = 2 \times 0.9756 = 1.9512$ V

<u>4 A source alone present:</u>

The circuit will be as shown below.



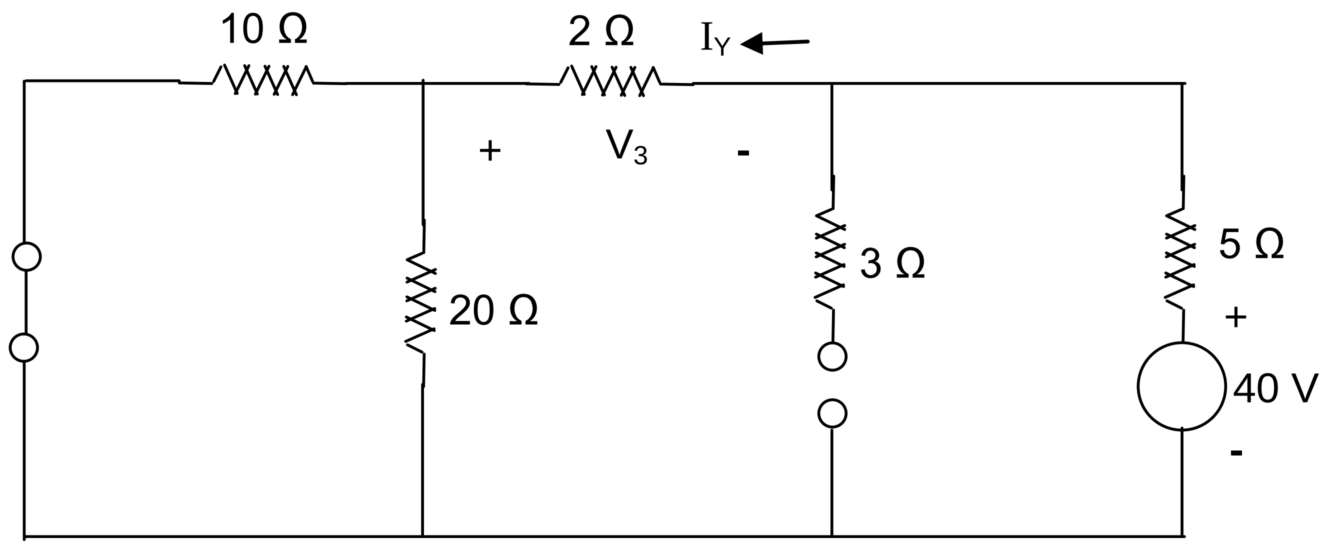
$$2 + 10 \,||\, 20 = 8.6667\ \Omega$$

Therefore current $I_X = \dfrac{5}{13.6667} \times 4 = 1.4634$ A

Thus voltage $V_2 = -\,2 \times 1.4634 = -\,2.9268$ V

40 V source alone present:

Resulting circuit is shown below.



Circuit resistance $R_T = 5 + 2 + (10||20) = 13.6667\ \Omega$

Current $I_Y = 40 / 13.6667 = 2.9268$ A; Thus voltage $V_3 = -2 \times 2.9268 = -5.8537$ V

When all the three sources are simultaneously present,

voltage across 2 Ω, i.e. $V_0 = V_1 + V_2 + V_3 = 1.9512 - 2.9268 - 5.8537 = -6.8293$ V

FUNDAMENTALS OF AC

GENERATION OF ALTERNATIVE EMF

Sinusoidal emf or Sinusoidal voltage

Consider a coil of n turns placed in a magnetic field of maximum value ϕ_m Webers [see Fig. 4.1 (a)]. The coil is initially along the reference axis. In this position, the field is perpendicular to the plane of the coil.

Let the coil be rotated in the anticlockwise direction with an angular velocity of ω rad/sec.

When the coil is along the reference axis at $\omega t = 0$, it is called as zero e.m.f. position. This is because the movement of the coil at this instant $\omega t = 0$ is along the field.

Let at any instant t sec. the coil takes a position as shown in Fig. 4.1(b).

At this instant, the coil makes an angle $\theta = \omega t$ with the reference axis.

At this position, the normal component of the mangnetic flux with respect to the plane of the coil is equal to

$$\phi_m \cos\theta \qquad (\therefore \quad \theta = \omega t)$$

The normal component $= \phi_m \cos \omega t$

Flux linkages (ψ) at this instant (y) is equal to $N\phi_m \cos \omega t$. According to Faraday's law.

The emf induced in the coil at the instant under consideration.

$$e = -\frac{d\psi}{dt} = \frac{-d}{dt}(N\phi_m \cos \omega t)$$

$$= -N\phi_m \omega(-\sin \omega t)$$

$$e = (N\phi_m \omega) \sin \omega t \tag{4.1}$$

With the above expression, we can calculate the emf induced at various instants.

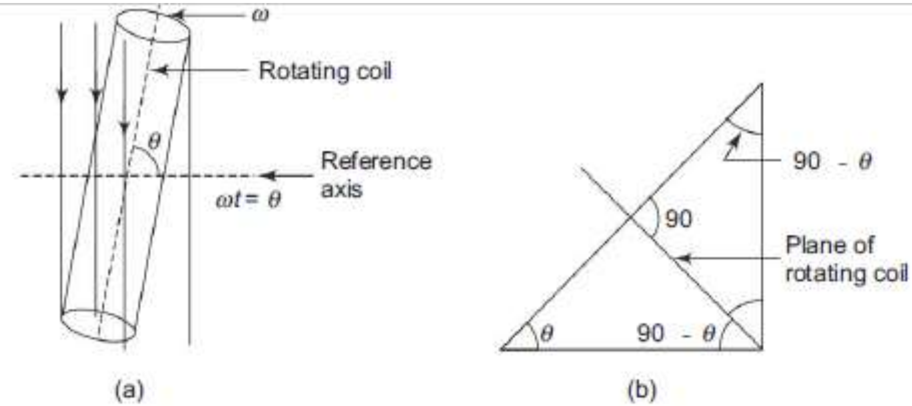


Fig. 4.1

When $\omega t = 0$ or $180°$

$$= (N\phi_m \omega) \sin 0 = (N\phi_m \omega) \sin 180°$$

$$e = 0$$

when $\omega t = 90°$ $e = (N\phi_m \omega) \sin 90°$

when $\omega t = 270°$ $e = N\phi_m \omega \sin (270°)$

$$e = -N\phi_m \omega$$

Let $N\phi_m \omega = E_m$ denote the maximum value of induced emf then from Eq. (4.1) we can write,

Instantaneous emf $\quad e = E_m \sin \omega t$ (Refer Fig. 4.2) $\quad$ (4.2)

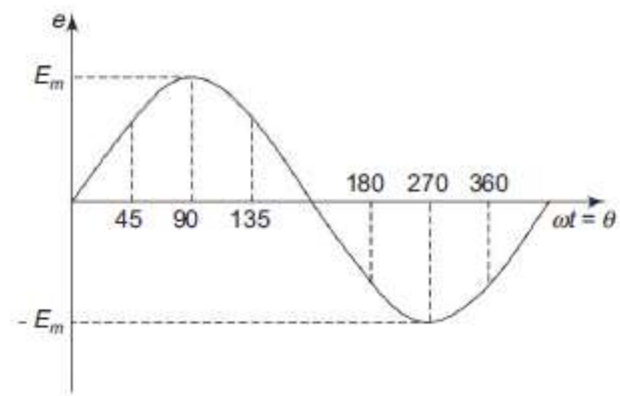


Fig. 4.2 Alternating emf wave for one complete cycle

4.2 TERMINOLOGY

1. Waveform A waveform is a graph in which the instantaneous value of any quantity is plotted against time. Examples of waveforms are shown in Fig. 4.3.

2. Alternating Waveform This is a wave which reverses its direction at regularly recurring intervals, e.g. Fig. 4.3(a).

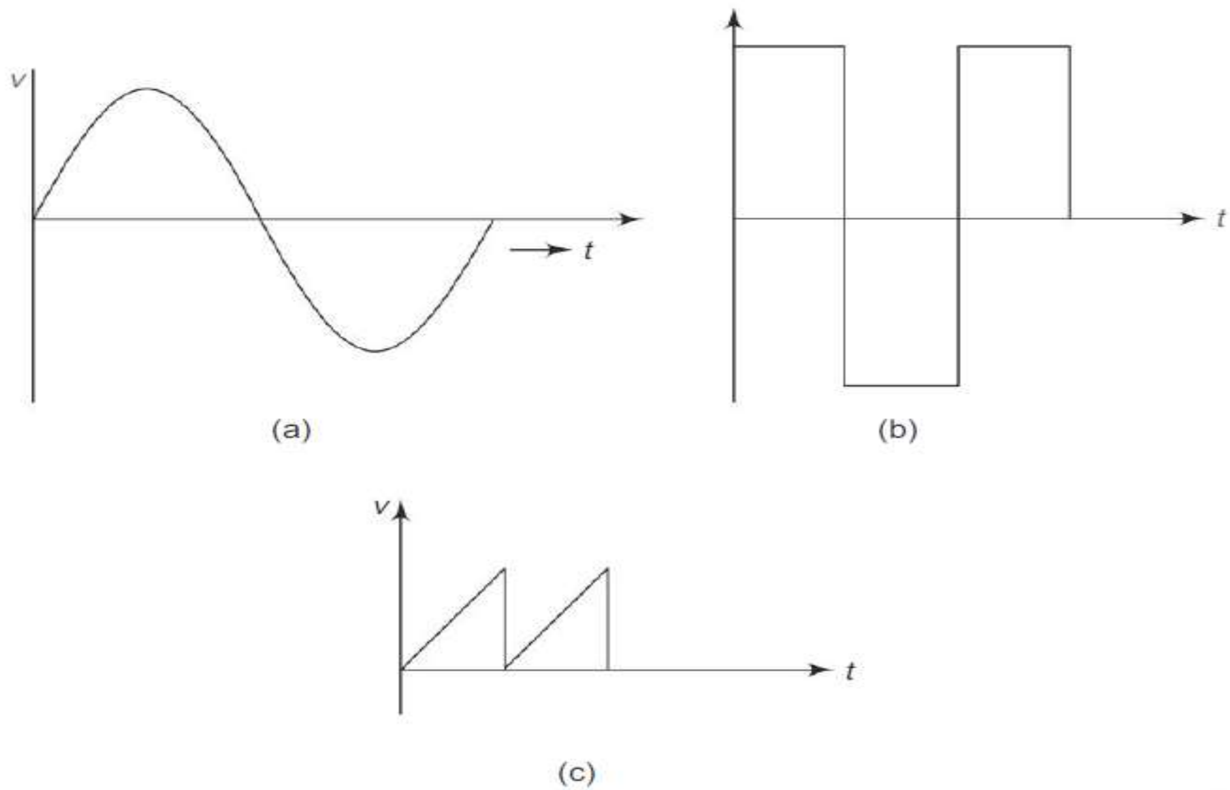


Fig. 4.3 (a) Sinusoidal waveform (b) Rectangular waveform (c) Sawtooth waveform

3. *Periodic Waveform* Periodic waveform is one which repeats itself after definite time intervals.

4. *Sinusoidal and Non-Sinusoidal Waveform*

Sinusoidal waveform It is an alternating waveform in which sine law is followed.

Non-sinusoidal waveform It is an alternating waveform in which sine law is not followed.

5. *Cycle* One complete set of positive and negative halves constitute a cycle.

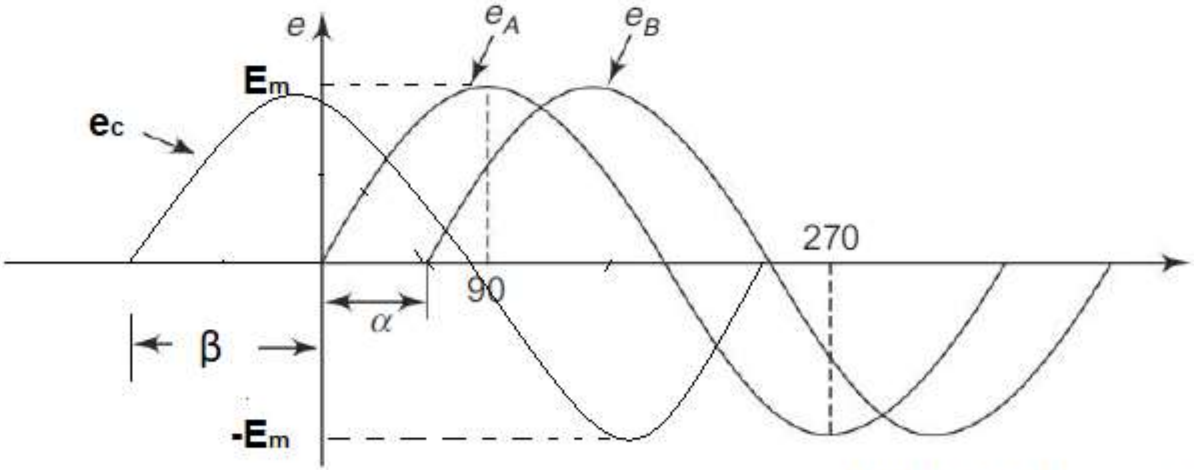
6. *Amplitude* The maximum positive or negative value of an alternating quantity is called the amplitude.

7. *Frequency* The number of cycles per second of an alternating quantity is known as frequency. Unit for frequency is expressed as c/s or Hertz (Hz).

8. *Period (T)* Time period of an alternating quantity is the time taken to complete one cycle. Time period is equal to the reciprocal of frequency. Time period is expressed in secs.

9. *Phase* The phase at any point on a given wave is the time that has elapsed since the quantity has last passed through zero point of reference and passed positively.

10. *Phase Difference* The term is used to compare the phase of two waveforms or alternating quantities.



$e_A = E_m \sin \omega t$ **Voltage A is reference**

$e_B = E_m \sin(\omega t - \alpha)$ **Voltage B lags voltage A by an angle** α

$e_c = E_m \sin(\omega t + \beta)$ **Voltage C leads voltage A by an angle** β

ROOT MEAN SQUARE (RMS) OR EFFECTIVE VALUE

Definition Effective or RMS value of an alternating current is deifned by that steady value of current (dc) which when flowing in a given circuit for a given time produces the same heat as would be produced by the alternating current flowing in the same circuit for the same time.

Method to Obtain the RMS Value for Sinusoidal Currents

Let the alternating current be represented by

$$i = I_m \sin \omega t$$

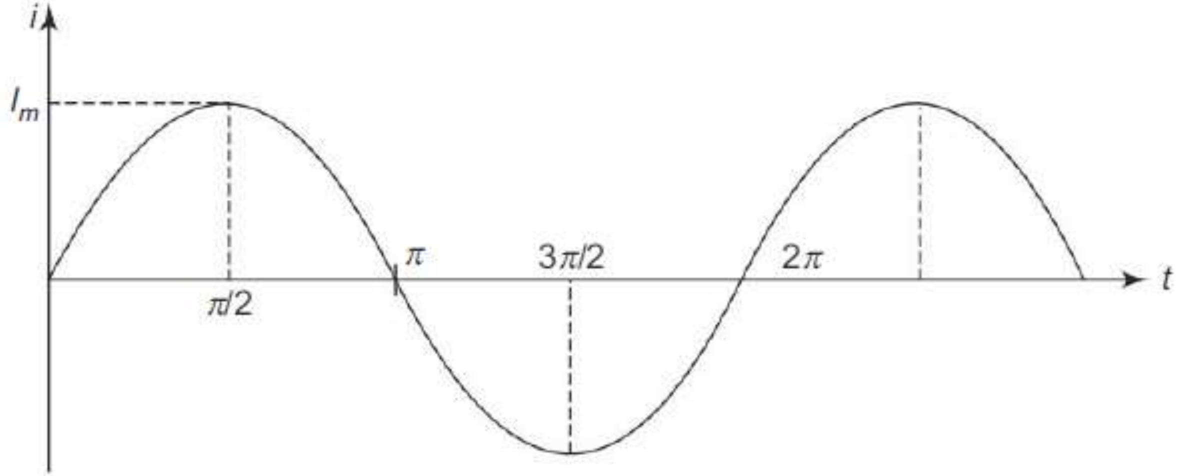
$$= I_m \sin \theta \, (\theta = \omega t)$$

$$i^2 = I_m^2 \sin^2 \theta$$

Mean square of $\quad \text{AC} = \int_0^{2x} \frac{I_m^2 \sin^2 \theta}{2\pi} d\theta$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta \, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \frac{I - \cos 2\theta}{2} \, d\theta$$



$$= \frac{I_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2\pi}\frac{2\pi}{2} = \frac{I_m^2}{2}$$

RMS value of the alternating sinusoidal current is

$$I = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

$$I_{\mathrm{RMS}} = 0.707\, I_m$$

Similarly, For a sinusoidal voltage

$$V_{\mathrm{RMS}} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

AVERAGE VALUE OF AC

Definition The average value of an ac is given by that steady current which transfers across a circuit the same charge as would be transferred by the ac across the same circuit in the same time.

Method to Obtain the Average Value for Sinusoidal Current

Let $$i = I_m \sin \theta$$

Since this is a symmetrical wave it has two equal half cycles namely positive and negative halves.

Considering one half cycle for this symmetrical wave the average value is obtained by

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i d\theta = \frac{1}{\pi} I_m \sin \theta \, d\theta$$

$$= \frac{I_m}{\pi}(-\cos \theta)_0^x$$

$$= \frac{I_m}{\pi}(1 + 1) = \frac{I_m}{\pi} \times 2$$

$$I_{av} = \frac{2 I_m}{\pi}$$

$$I_{av} = 0.637 \, I_m$$

where I_m is the maximum value of current.

For a sinusoidal voltage wave,

$$V_{av} = 0.637 \, V_m.$$

RMS and average value of half wave rectified sine wave

RMS value

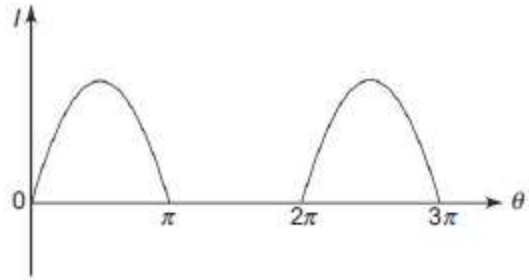


Fig. E.4.6

$$i = I_m \sin\theta \quad \text{for } 0 < \theta < \pi$$

$$= 0 \qquad \pi < \theta < 2\pi$$

$$\text{Mean square value} = \frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2\theta + \int_{\pi}^{2\pi} \theta d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos^2\theta}{2} d\theta = \frac{I_m^2}{4\pi}\int_0^{\pi} (1-\cos 2\theta)d\theta$$

$$= \frac{I_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$

$$= \frac{I_m^2}{4\pi}\left(\pi - \frac{\sin 2\pi}{2} - 0 - \frac{\sin 2\times 0}{2}\right)$$

$$= \frac{I_m^2}{4\pi} \times \pi = \frac{I_m^2}{4}$$

$$\text{RMS value} \sqrt{\frac{I^2 m}{4}} = \frac{Im}{2}$$

RMS and average value of half wave rectified sine wave

Average value

Average value Half-wave rectified wave is an unsymmetrical wave.

$$\text{Average value} = \frac{\text{Area under the curve for one complete cycle}}{\text{Period}}$$

$$\text{Area under one complete cycle} = \int_0^{\pi} I_m \sin\theta \, d\theta + \int_{\pi}^{2\pi} 0 \, d\theta$$

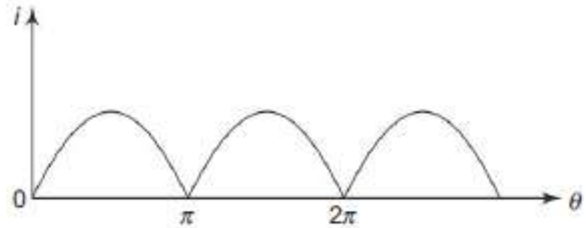
$$= I_m (-\cos\theta)_0^{\pi}$$

$$= I_m (-\cos\pi + \cos 0) = I_m (1 + 1) = 2I_m$$

$$\text{Average value} = \frac{2I_m}{2\pi} = I_m/\pi$$

RMS and average value of full wave rectified sine wave

RMS Value



$$\text{Mean square value} = \frac{\text{Area under one squared curve}}{\text{Period}}$$

$$\text{Mean square value} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta = \frac{I_m^2}{2\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left(\pi - \frac{\sin 2r}{2} - 0 + \frac{\sin 0}{2}\right)$$

$$= \frac{I_m^2}{2\pi} \times \pi = \frac{I_m^2}{2}$$

$$\boxed{\text{RMS value} = \frac{I_m}{\sqrt{2}}}$$

Average Value

$$\text{Average value} = \frac{\text{Area under the curve for one complete cycle}}{\text{Period}}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta \qquad (\because \text{the given wave is symmetrical})$$

$$= \frac{I_m}{\pi}(-\cos\theta)_0^{\pi} = \frac{I_m}{\pi}(1+1) = \boxed{\frac{2I_m}{\pi}}$$

ANALYSIS OF SINGLE PHASE AC CIRCUITS

1. Real power / True power / Average power/ Power

$P = V\,I\,\cos\Phi$ **or**

$P = V_{RMS}\, I_{RMS} \cos\Phi$

$P = |V|\,|I| \cos\Phi$

$P = I^2 R$

Unit: Watts

Where Φ Angle between voltage and current

2. Reactive Power

$Q = V\,I\,\sin\Phi$

$Q = V_{RMS}\, I_{RMS} \sin\Phi$

Unit: VAR

3. Apparent power

$S = VI$

$S = \sqrt{P^2 + Q^2}$

Unit : VA

4. Power Factor = $\cos\Phi$

$= \frac{R}{|Z|}$

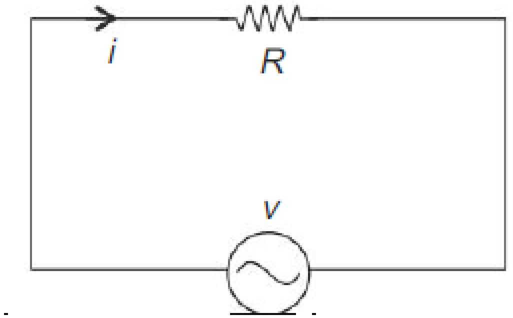
No Unit

Where Φ Angle between voltage and current

Where $|Z| = R^2 + (X_L - X_C)^2$

ANALYSIS OF AC CIRCUIT

The response of electric circuits to alternating current can be studied by passing an alternating current through the basic circuit elements resistor (R), inductor (L) and capacitor (C).



1 . Pure Resistive Circuit

Let the sinusoidal voltage applied across the resistance be

$$v = V_m \sin \omega t$$

The resulting current has an instantaneous value, i By Ohm's law,

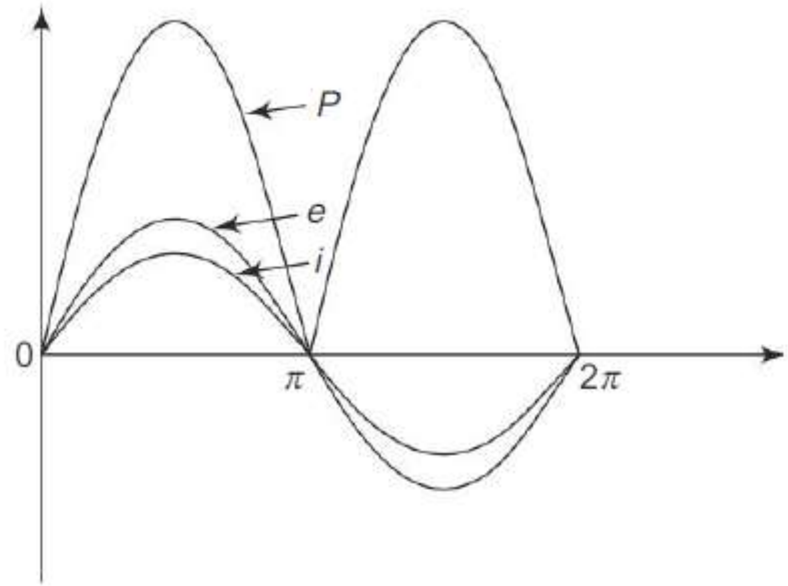
$$i = I_m \sin \omega t$$

$$\text{where } I_m = \frac{V_m}{R}$$

Phasor Representation

In a pure resistive circuit,there is no phase difference between the voltage applied and the resulting current, i.e the phase angle $\phi = 0$. .If the voltage is taken as the reference phasor, thephasor representation for voltage and current in a pure resistive circuit is given in Fig.

Waveform Representation

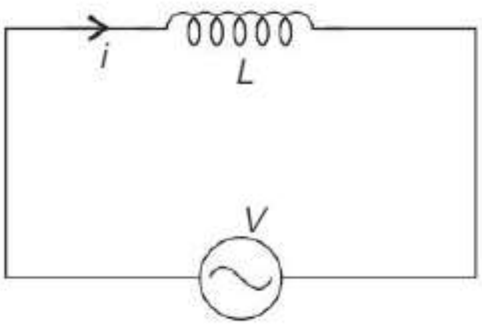


Power Factor It is the cosine of the phase angle between voltage and current

$\cos \phi = \cos 0 = 1$ (unity)

2. Pure Inductive Circuit

Consider the circuit of Fig. (4.16). In this circuit, an alternating voltage is applied across a pure inductor of self inductance L Henry.



Let the applied alternating voltage be

$$v = V_m \sin \omega t$$

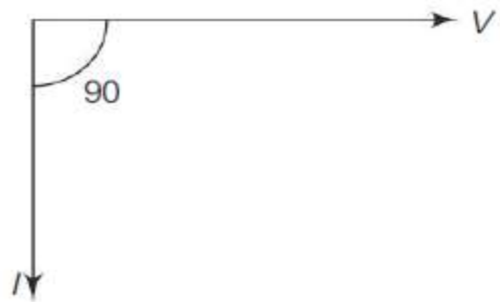
We know that the self induced emf always opposes the applied voltage.

$$v = L \frac{di}{dt}$$

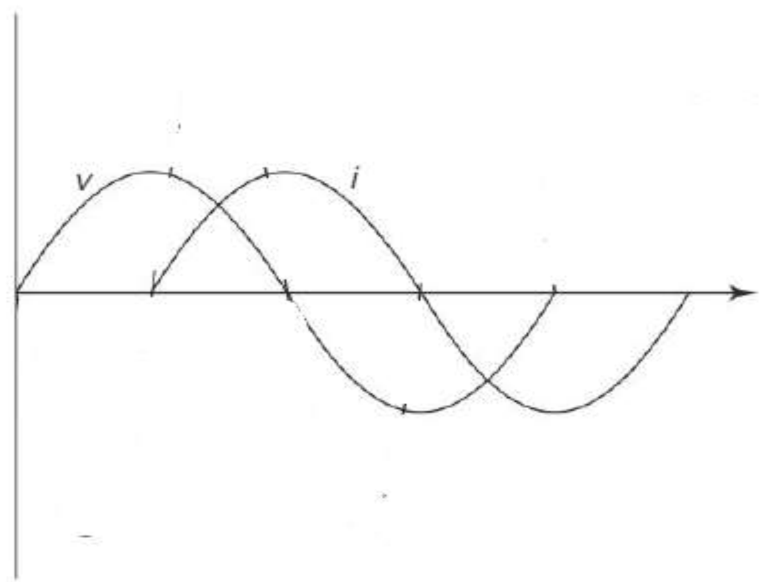
$$\therefore \quad i = \frac{1}{L}\int \cdot v\, dt = I_m \sin(\omega t - \pi/2)$$

we can say that the current through an inductor lags the applied voltage by 90°.

Phasor Representation Taking the voltage phasor as reference, the current phasor is shown to lag the voltage by 90° (Fig. 4.18).



Waveform Representation The current waveform is lagging behind the voltage waveforms by 90°.



Since $\phi = 90°$ Real power P=0

The pure inductor does not consume any real power

Power Factor In a pure inductor the phase angle between the current and the voltage phasors is 90°.

i.e. $\phi = 90°; \cos\theta = \cos 90° = 0$

Thus the power factor of a pure inductive circuit is zero lagging.

3 Pure Capacitive Circuit

Consider the circuit of Fig. 4.19 in which a capacitor of value C Farad is connected across an alternating voltage source.

Let the sinusoidal voltage applied across the capacitance be

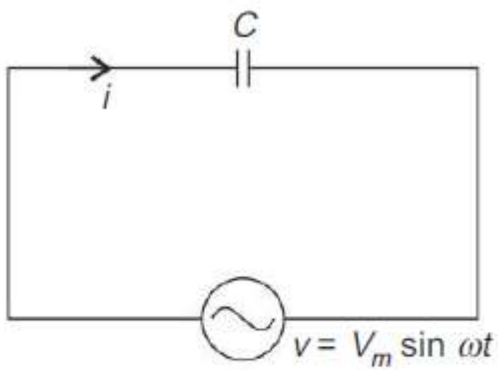
$$v = V_m \sin \omega t$$

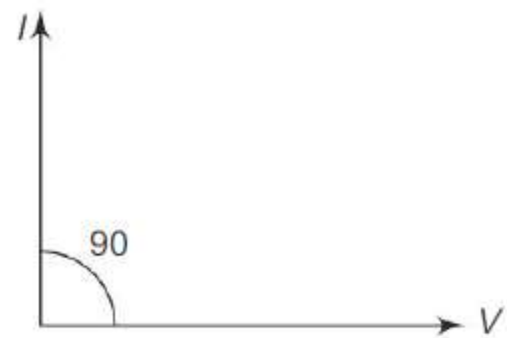
The characteristic equation of a capacitor is

$$V = \frac{1}{C}\int i\,dt$$

$$i = I_m \sin(\omega t + 90)$$

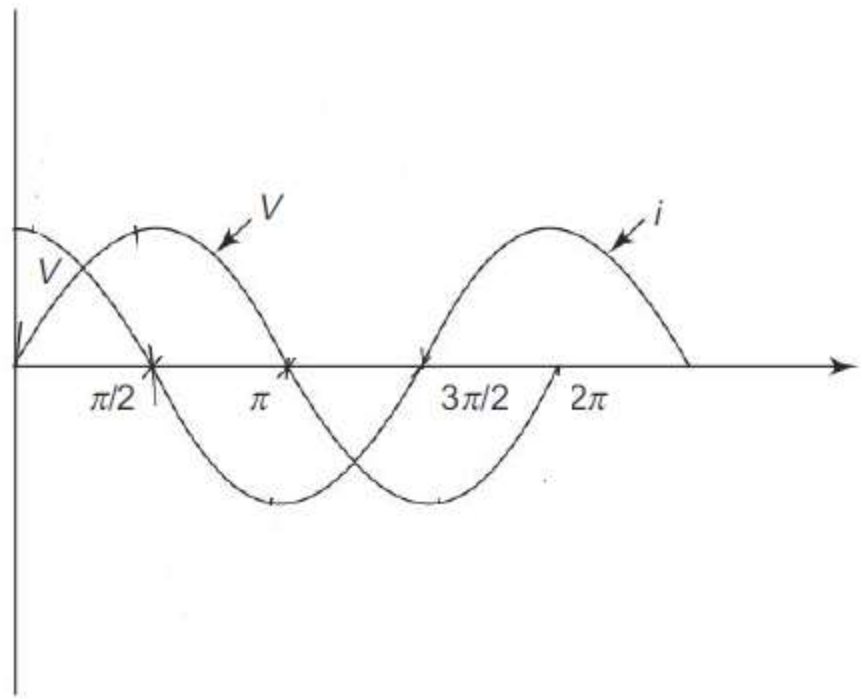
The current in a pure capacitor leads the applied voltage by an angle of 90°.



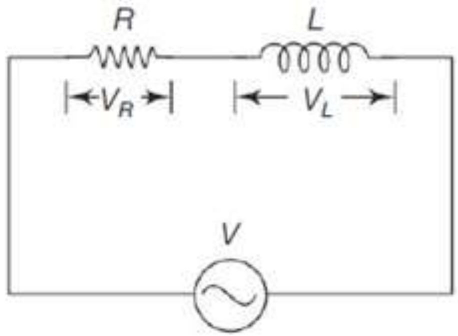


Phasor Representation In the phasor representation. voltage phasor is taken as the reference. The current phasor leads an angle of 90°

Waveform Representation The current waveform is ahead of the voltage waveform by an angle of 90°.



4 R–L SERIES CIRCUIT

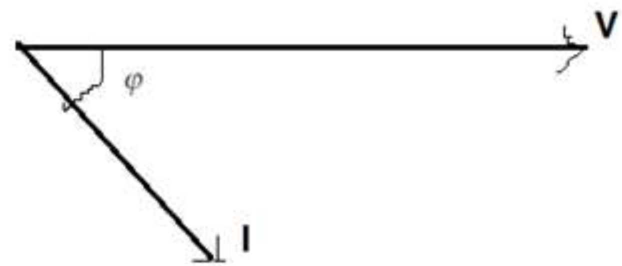


Let $v = V_m \sin \omega t$ be the applied voltage

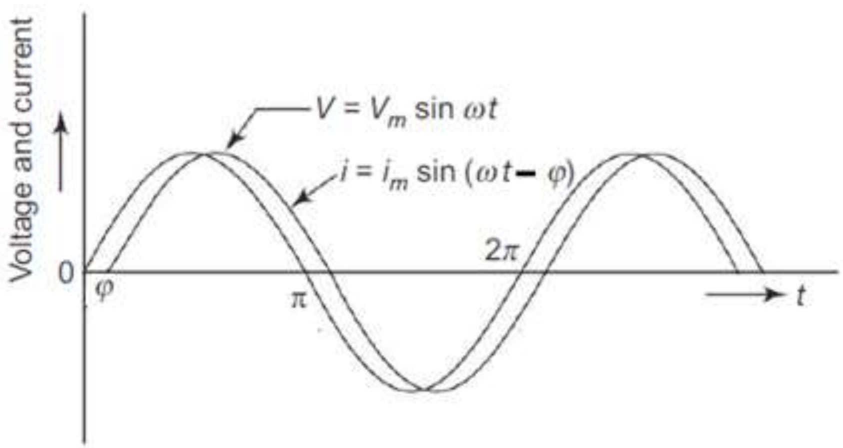
then the current equation is

$$i = I_m \sin(\omega t - \phi)$$

Phasor

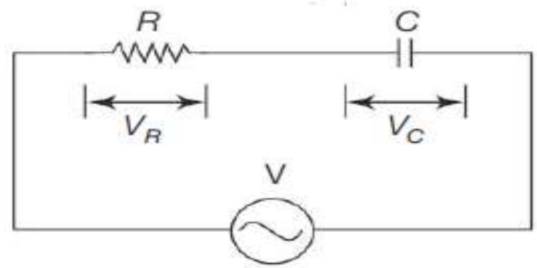


Waveform



The current I lag Voltage V by an angle φ

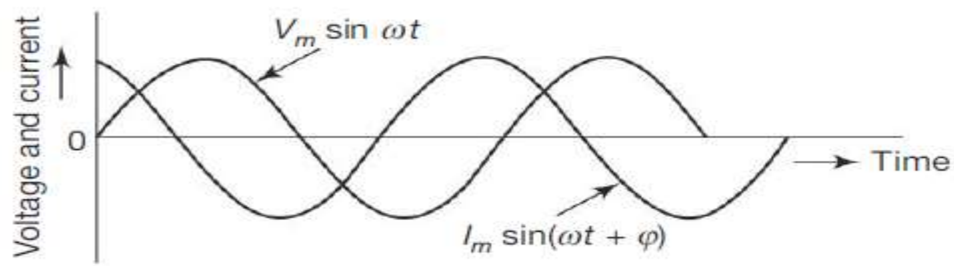
5 R-C SERIES CIRCUIT



Let $v = V_m \sin \omega t$ be the applied voltage

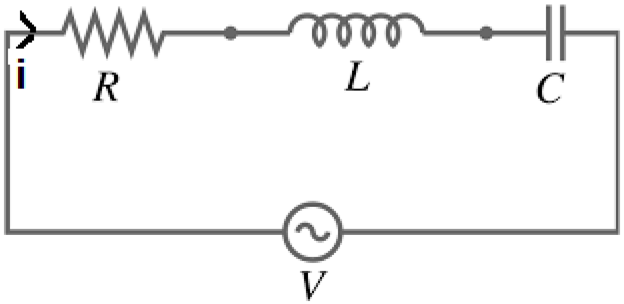
Then the current equation is $i = I_m \sin(\omega t + \phi)$

Waveform



Phasor

6. R-L-C SERIES CIRCUIT



Depends upon the value of X_L and X_C, the circuit behaves

If $X_L > X_C$, then the circuit behaves as RL circuit

If $X_L < X_C$, then the circuit behaves as RC circuit

Impedance $Z = R + j X_L - j X_C$

or

$Z = R + j(X_L - X_C)$

Unit: Ω

Where i or j = $\sqrt{-1}$

$X_L = 2\pi fL$ Unit: Ω

$X_C = \dfrac{1}{2\pi fC}$ Unit: Ω

f-Supply Frequency in Hz

PROBLEMS

1 A voltage 100 sin ωt is applied to a 10-ohm resistor. Find the instantaneous current, the current (rms) and the average power

Solution: V or $e = 100 \sin \omega t$

$R = 10$ ohms

$i = e/R = 100/10 \sin \omega t = 10 \sin \omega t$ A

$I_{rms} = I_m / \sqrt{2}$

$= 10 / \sqrt{2}$

$= 7.07$ A

$P = V\,I \cos \phi$

$= \frac{100}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos 0$

$= 500$ Watts

2. A voltage v = 340 sin 314t is applied to a circuit and the resulting current, i = 42.5 sin 314t. Identify and hence find the values of the component. Find the value of power consumed.

Solution: $v = 340 \sin 314t = \frac{340}{\sqrt{2}} \angle 0$ Volts $= 240 \angle 0$ Volts

$i = 42.5 \sin 314\,t = \frac{42.5}{\sqrt{2}} \angle 0$ Amps $= 30 \angle 0$ Amps

From the above voltage and current equations, we find that they are in phase with each other. Thatis angle between V and I is 0. Hence, the basic component connected in the circuit must be resistor.

Note

$V_{rms} = |V| = V$

$R = V / I$

$= 240.4 / 30$

$= 8\ \Omega$

$P = VI \cos \Phi$

$= 240.\ \ x\ 30\ x \cos 0$

$= 7200$ Watts

$P = I^2 R$

$= 30x30x8$

$= 7200$ Watts

Example 3

The voltage of $v = \sqrt{2}\ 20 \sin(50t - 25^0)$ V is applied across an inductor of 0.1 H. Find the steady state current through the inductor.

Solution

Phasor voltage $V = 20 \angle -25^0 - 90^0 = 20 \angle -115^0$ V

Impedance $Z = j\omega L = j\,50 \times 0.1 = 5 \angle 90^0\ \Omega$

Current $I = \dfrac{V}{Z} = \dfrac{20\angle -115^0}{5\angle 90^0} = 4\angle -205^0$ A

Converting this to the time domain

Current $i(t) = 5.6569 \cos(50t - 205^0)$ A

$= -\ 5.6569 \cos(50t - 25^0)$ A

Example 4

The voltage of v = $\sqrt{2}$ 12 cos (100 t - 25^0) V is applied across a capacitor of 50 μF. Find the steady state current through the capacitor.

Solution

Phasor voltage V = 12 $\angle$- 25^0 V

Impedance Z = - j $\dfrac{1}{\omega C} = -j\,\dfrac{1}{100 \text{ x } 50 \text{ x } 10^{-6}} = -j\,200\,\Omega$

Current I = $\dfrac{V}{Z} = \dfrac{12\angle -25^0}{200\angle -90^0}$ **A** $= 0.06\angle 65^0$ **A** $=$ $60\angle 65^0$ **mA**

Converting this to the time domain

Current i(t) = 84.8528 cos (100 t + 65^0) mA

Example 5

In a series circuit containing pure resistance and pure inductance, the current and voltage are: $i(t) = 5 \sin (314 + \frac{2\pi}{3})$ and $v(t) = 20 \sin (314 + \frac{5\pi}{6})$. (i) What is the impedance of the circuit? (ii) What are the values of resistance, inductance and power factor? (iii) What is the power drawn by the circuit?

Solution

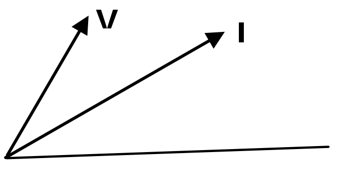
Current $I = \frac{5}{\sqrt{2}} \angle 120\text{-}90 = \frac{5}{\sqrt{2}} \angle 30^0$; Voltage $V = \frac{20}{\sqrt{2}} \angle 150\text{-}90 = \frac{20}{\sqrt{2}} \angle 60^0$

Impedance Z = $\frac{V}{I} = \frac{20\angle 60}{5\angle 30} = 4 \angle 30^0 \ \Omega = (3.4641 + j2)\Omega$

Resistance R = $3.4641\ \Omega$

$X_L = 2\ \Omega$; $314\,L = 2$; $L = \frac{2}{314}$ H ; Inductance L = 6.3694 mH

Angle between voltage and current = 30^0



p.f. = $\cos 30^0 = 0.866$ lagging

Power P = $|V||I| \cos\theta = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866 = 43.3$ W

Example 6

An inductive coil takes 10 A and dissipates 1000 W when connected to a supply of 250 V, 25 Hz. Calculate the impedance, resistance, reactance, inductance and the power factor.

Solution

$$P = |I|^2 R; \quad \text{Resistance } R = \frac{1000}{100} = 10\ \Omega$$

$$|Z| = \frac{250}{10} = 25\,\Omega; \quad \text{From impedance triangle } X = \sqrt{25^2 - 10^2} = 22.9128\,\Omega$$

Thus impedance $Z = (10 + j\,22.9128)\ \Omega = 25 \angle 66.42^0\ \Omega$

Resistance $R = 10\ \Omega$ **Reactance $X = 22.9128\ \Omega$**

Inductance $L = \dfrac{X}{2\pi f} = \dfrac{22.9128}{2\pi \times 25} = 0.1459\ \text{H}$

From impedance triangle, power factor $= \dfrac{R}{|Z|} = \dfrac{10}{25} = 0.4$ **lagging**

Example 7

A resistance is connected in series with a coil. With a supply of 250 V, 50 Hz, the circuit takes a current of 5 A. If the voltages across the resistance and the coil are 125 V and 200 V respectively, calculate (i) impedance, resistance and reactance of the coil (ii) power absorbed by the coil and the total power. Draw the phasor diagram.

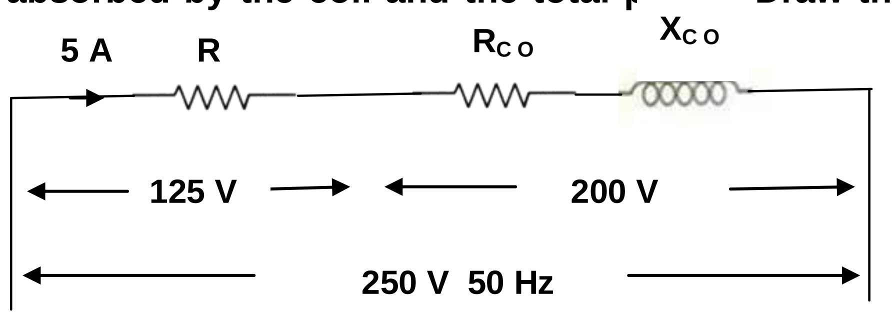


Fig. 21 Example 7

Resistance R = $\frac{125}{5} = 25\ \Omega$

$|Z_{CO}| = \frac{200}{5} = 40\ \Omega$; $|Z_T| = \frac{250}{5} = 50\ \Omega$

$|Z_{CO}| = \frac{200}{5} = 40\ \Omega$; Therefore $|R_{CO} + jX_{CO}| = 40$; $R^2_{CO} + X^2_{CO} = 1600$

$|Z_T| = \frac{250}{5} = 50\ \Omega$; $|(25 + R_{CO}) + jX_{CO}| = 50$

$625 + 50\ R_{CO} + R^2_{CO} + X^2_{CO} = 2500$ i.e. $50\ R_{CO} = 2500 - 625 - 1600 = 275$

Resistance of the coil $R_{CO} = 5.5\ \Omega$ Also $X^2_{CO} = 1600 - 5.5^2 = 1569.75$

<u>Reactance of the coil</u> X_C = 39.62 Ω

<u>Impedance of the coil</u> $Z_{CO} = (5.5 + j\,39.62) = 40\ \angle 82.1^0\ \Omega$

<u>Power absorbed by the coil</u> $P_{CO} = 5^2 \times 5.5 = 137.5$ W

<u>Total power</u> $P_T = (5^2 \times 25) + 137.5 = 762.5$ W

Total impedance $Z_T = (30.5 + j\,39.62) = 50\ \angle 52.41^0 \Omega$

$$\left|\mathbf{I}\right| R_{CO} = 5 \times 5.5 = 27.5\ \text{V}; \qquad \left|\mathbf{I}\right| X_{CO} = 5 \times 39.62 = 198.1\ \text{V}$$

Phasor diagram is shown in Fig. 22.

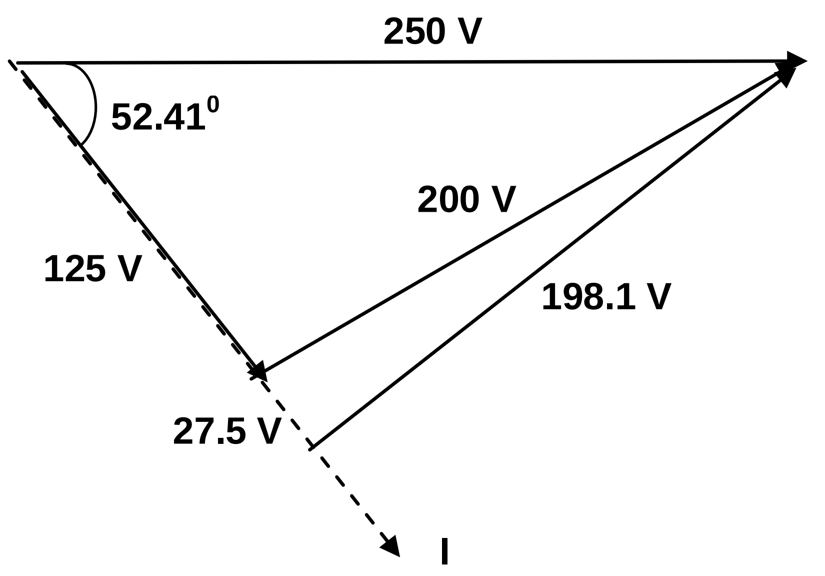


Fig. 22 Phasor diagram-Example 7

Example 10

A resistance of 100 ohm is connected in series with a 50 μF capacitor. When the supply voltage is 200 V, 50 Hz, find the (i) impedance, current and power factor (ii) the voltage across resistor and across capacitor. Draw the phasor diagram.

Solution

Resistor R = 100 Ω; Reactance of the capacitor $X_C = \dfrac{10^6}{2\pi \times 50 \times 50} = 63.662\ \Omega$

Impedance $Z = (100 - j\,63.662) = 118.5447\ \angle -32.48^0$

Taking the supply voltage as reference, $E = 200\ \angle 0^0$ **V**

Current $I = \dfrac{E}{Z} = \dfrac{200\angle 0^0}{118.5447\angle -32.48^0} = 1.6871\angle 32.48^0$ **A**

Power factor $= \cos 32.48^0 = 0.8436$ **leading**

Voltage across resistor $V_R = 100 \times 1.6871\ \angle 32.48^0 = 168.71\ \angle 32.48^0$ **V**

Voltage across capacitor $V_C = -\,j\,63.662 \times 1.6871\ \angle 32.48^0 = 107.4042\ \angle -57.52^0$ **V**

Phasor diagram is shown in Fig. 24.

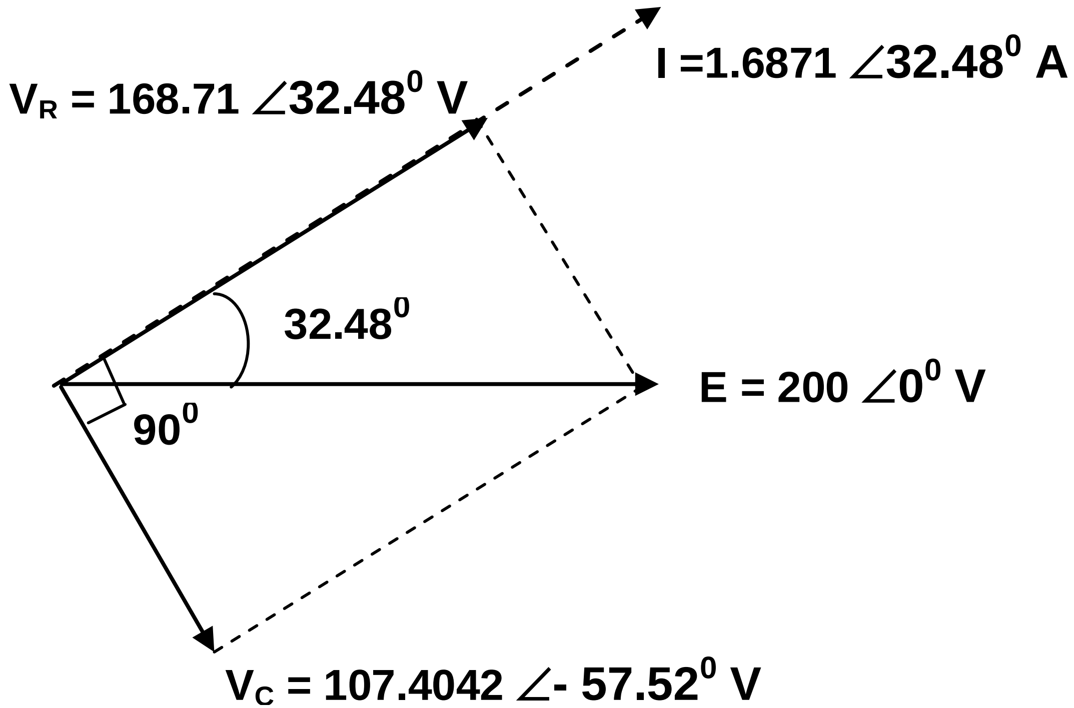


Fig. 24 Phasor diagram - Example 10

Example 11

In a circuit, the applied voltage of 150 V lags the current of 8 A by 40^0. (i) Find the power factor (ii) Is the circuit inductive or capacitive? (iii) Find the active and reactive power.

Solution

Power Factor = 0.766 leading

Circuit is capacitive.

Active Power P = 150 x 8 x 0.766 = 919.2 W

Reactive Power Q = 150 x 8 x 0.6428 = 771.36 VAR

Example 12

Find the circuit constants of a two elements series circuit which consumes 700 W with 0.707 leading power factor. The applied voltage is V = 141.4 sin 314 t volts.

Solution

$$|V| = \frac{141.4}{\sqrt{2}} = 99.9849 \text{ V} ; \quad \text{Since Power } P = |V||I| \cos\theta$$

$$|I| = \frac{700}{99.9849 \times 0.707} = 9.9025 \text{ A} \quad \text{and} \quad |Z| = \frac{99.9849}{9.9025} = 10.0969 \ \Omega$$

From the impedance triangle

Resistance R = $|Z| \cos\theta = 10.0969 \times 0.707 = 7.1385 \ \Omega$

Reactance X_C = $|Z| \sin\theta = 10.0969 \times 0.707 = 7.1385 \ \Omega$

Capacitance C = $\dfrac{1}{314 \times 7.1385} = 446.132 \ \mu F$

4. Find the circuit constants of a two element series circuit which consumes 700 W with 0.707 leading p.f. The applied voltage is $V = 141.4 \sin 314\,t$.

Solution

$$v = 141.4 \sin 314t$$

$$P = 700 \text{ W}, \quad \text{p.f.} = 0.707 \text{ leading}$$

Leading p.f. means $R–C$ circuit

Max. value of supply voltage = 141.4 V

R.M.S. value of supply voltage $= \dfrac{141.4}{\sqrt{2}} = 99.98$ V

$$\cos \phi = 0.\,707 \text{ leading; Power} = VI \cos \phi$$

$$700 = 99.98 \times I \times 0.707; \quad I = 9.9 \text{ A}$$

Impedance $\quad |Z| = \dfrac{V}{I} = \dfrac{99.98}{9.9} = 10.09$ ohms

$$\cos \phi = \frac{R}{|Z|} \Rightarrow R = |Z| \cos \phi$$

$$R = 10.09 \times 0.707 = 7.13\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{10.09^2 - 7.13^2} = 7.13 \text{ ohms}$$

$$\frac{1}{\omega C} = 7.13; \quad \frac{1}{314 \times C} = 7.13 \Rightarrow C = \frac{1}{314 \times 7.13}$$

$$C = 4.466 \times 10^{-4} \text{ F}$$

$$= 446.6 \times 10^{-6} \text{ F}$$

$$C = 446.6 \ \mu\text{F}$$

ANOTHER METHOD IN NEXT PAGE

ANOTHER METHOD

$$v = 141.4 \sin 314t$$

$$P = 700 \text{ W}, \quad \text{p.f.} = 0.707 \text{ leading}$$

Leading p.f. means R–C circuit

Max. value of supply voltage = 141.4 V

R.M.S. value of supply voltage $= \dfrac{141.4}{\sqrt{2}} = 99.98$ V ; $V = 99.98\angle 0^\circ$ V

$$\cos\phi = 0.707 \text{ leading; Power} = VI\cos\phi$$

$$700 = 99.98 \times I \times 0.707; \quad I = 9.9 \text{ A} ; \quad I = 9.9 \angle 45^\circ \text{ A}$$

Impedance $Z = V / I$ Since Current is leading, it is $+45^\circ$

$$= \frac{99.98\angle 0^\circ}{9.9\angle 45^\circ}$$

$$= 7.13 - j\,7.13 \text{ Ohms}$$

$$= R - j\,X_c$$

$$R = 7.13\ \Omega \qquad X_C = 7.13 \text{ ohms}$$

$$\frac{1}{\omega C} = 7.13; \quad \frac{1}{314 \times C} = 7.13 \Rightarrow C = \frac{1}{314 \times 7.13}$$

$$C = 4.466 \times 10^{-4} \text{ F}$$

$$= 446.6 \times 10^{-6} \text{ F}$$

$$C = 446.6\ \mu\text{F}$$

THREE PHASE AC CIRCUITS

Three Phase Connections There are two possible connections in 3-phase system. One is star (or wye) connection and the other is delta (or mesh) connection. Each type of connection is governed by characteristic equations relating the currents and the voltages. Phasor diagrams plays a vital role in this analysis.

Star Connection

Here three similar ends of the three phase coils are joined together to form a common point. Such a point is called the starpoint or the neutral point. The free ends of the three phase coils will be operating at specific potentials with respect to the potential at the star point.

It may also be noted that wires are drawn from the three free ends for connecting loads. We actually have here three phase four wire system (Fig. 5.32) and three phase three wire system (Fig. 5.33).

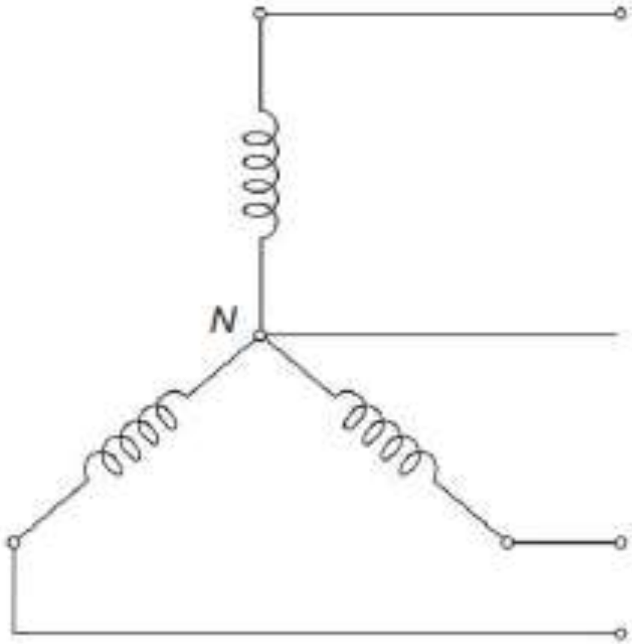


Fig. 5.32

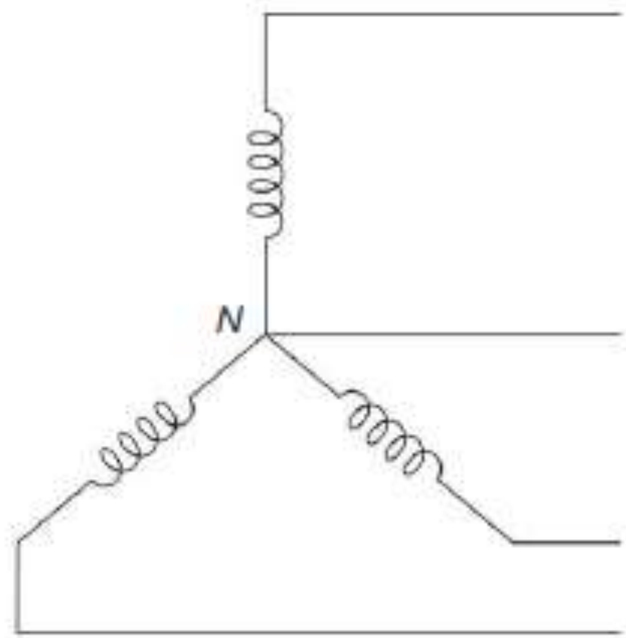


Fig. 5.33

Analysis Let us analyse the relationship between currents and relationship between voltages. We also arrive at the power equations.

Notations Defined

E_R, E_Y, E_B : Phase voltages of R, Y and B phases
I_R, I_Y, I_B : Phase currents
V_{RY}, V_{YB}, V_{BR} : Line voltages
I_{L1}, I_{L2}, I_{L3} : Line currents

In a balanced system,

$$E_R = E_Y = E_B = E_P \qquad V_{RY} = V_{YB} = V_{BR} = V_L$$
$$I_R = I_Y = I_B = I_P \qquad I_{L1} = I_{L2} = I_{L3} = I_L$$

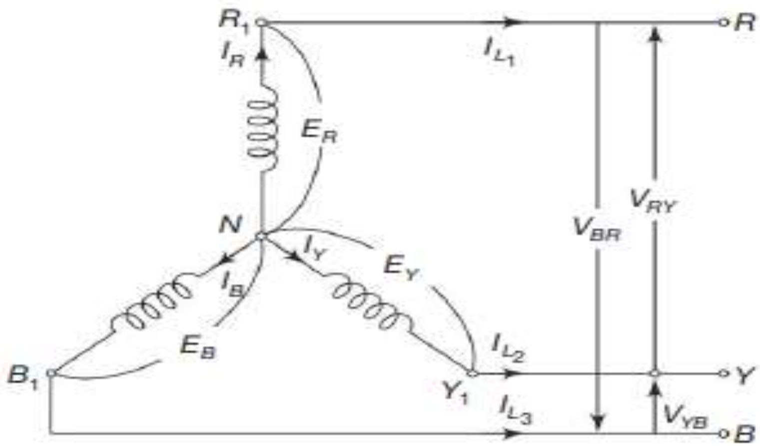


Fig. 5.34

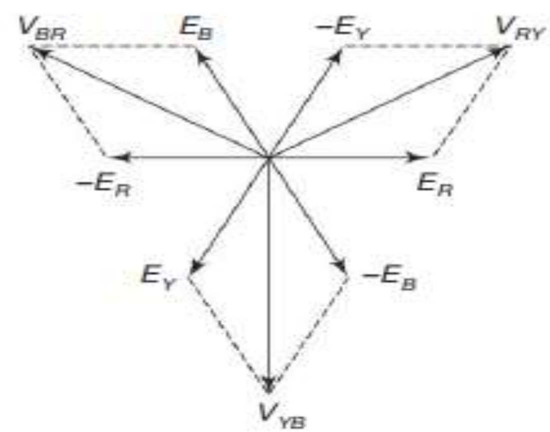


Fig. 5.35

Current Relationship Applying Kirchhoff's current law at nodes R_1, Y_1, B_1 we get $I_R = I_{L1}$; $I_Y = I_{L2}$; $I_B = I_{L3}$.

This means that in a balanced star connected system, phase current equals the line current

$$I_P = I_L.$$

Voltage Relationship Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R, V_{RY} and E_Y. We have

$$\overline{E}_R - \overline{E}_Y = \overline{V}_{RY}$$

Using law of parallelogram,

$$|\overline{V}_{RY}| = V_{RY} = \sqrt{E_R^2 + E_Y^2 + 2\,E_R\,E_Y \cos 60^\circ}$$

$$= \sqrt{E_p^2 + E_P^2 + 2\,E_P\,E_P\,(\%0)} = E_P\sqrt{3}$$

Similarly,

$$\overline{E}_Y - \overline{E}_B = \overline{V}_{YB} \quad \text{and} \quad \overline{E}_B - \overline{E}_R = \overline{V}_{BR}$$

$\therefore$ $$\overline{V}_{YB} = E_P\sqrt{3} \text{ and } \quad V_{BR} = E_P\sqrt{3}$$

Thus, $$V_L = \sqrt{3}\,E_P$$

Line voltage = $\sqrt{3}$ phase voltage

Power Relationship Let $\cos\phi$ be the power factor of the system.

Power consumed in one phase = $E_P\,I_P \cos\phi$

Power consumed in three phases = $3E_P\,I_P \cos\phi$

$$= 3\,\frac{V_L}{\sqrt{3}}\,I_L \cos\phi$$

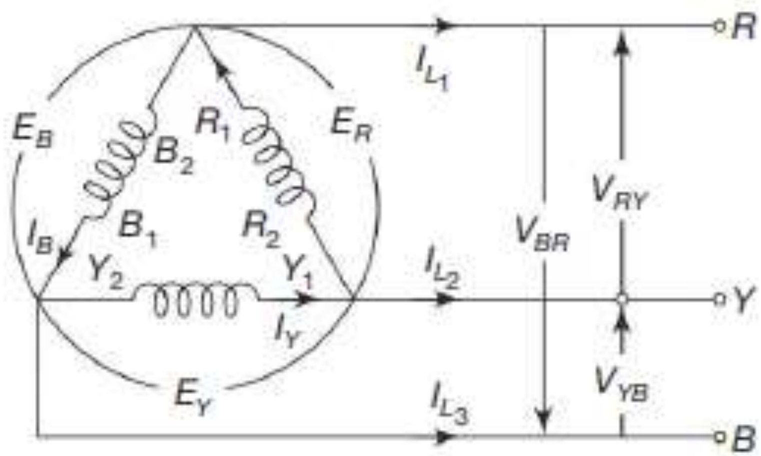
$$= \sqrt{3}\,V_L\,I_L \cos\phi \text{ watts}$$

Delta Connection

Here the dissimilar ends of the three phase coils are connected together to form a mesh. Wires are drawn from each junction for connecting load. We can connect only three phase loads as there is no fourth wire available.

Let us now analyse the above connection.

The system is a balanced one. Hence the currents and the voltages will be balanced. Notations used in the star connection are used here.



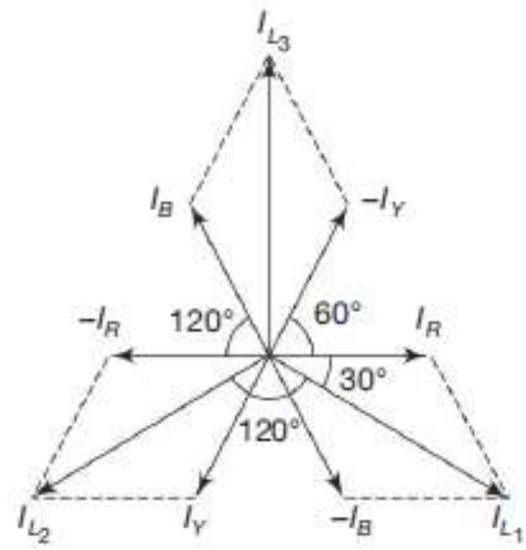


Fig. 5.37

$E_R = E_Y = E_B = E_P$, phase voltage

$I_R = I_Y = I_B = I_P$, phase voltage

$V_{RY} = V_{YB} = V_{BR} = V_L$, line voltage

$I_{L1} = I_{L2} = I_{L3} = I_L$, line voltage

Voltage Relationship Applying Kirchhoff's voltage law to the loop consisting of E_R and V_{RY}, we have $E_R = V_{RY}$.

Similarly, $E_Y = V_{YB}$ and $E_B = V_{BR}$,

Thus $E_P = V_L$.

Phase Voltage = Line Voltage

Current Relationship Applying Kirchhoff's current law at the junction of R_1 and B_2, we have $\bar{I}_R - \bar{I}_B = \bar{I}_L 1$.

Referring to the phasor diagram and applying the law of parallelogram, we have

$$I_L I = \sqrt{I_R^2 + I_B^2 + 2I_R I_B \cos 60^\circ}$$

$$= \sqrt{I_P^2 I_P^2 + 2I_P I_P (\%0)}$$

$$= I_P\sqrt{3}$$

Similarly, we have

$$\bar{I}_Y - \bar{I}_R = \bar{I}_L 2 \quad \text{and} \quad \bar{I}_B - \bar{I}_Y = \bar{I}_L 3$$

Hence, $\quad I_L 2 = I_P\sqrt{3} \;$ and $\; I_L^3 = I_P\sqrt{3}$

Thus, line current $= \sqrt{3}$ phase current

$$I_L = \sqrt{3}\, I_P$$

Power Relationship Let $\cos\phi$ be the power factor of the system.

Power per phase $= E_P\, I_P \cos\phi$

Total power for all the three phases $= 3\, E_P\, I_P \cos\phi$

$$= 3\, V_L \frac{I_L}{\sqrt{3}} \cos\phi$$

$$= \sqrt{3}\, V_L\, I_L \cos\phi \text{ watts}$$

Sl. No.	Star (Y) Connected System	Delta (Δ) Connected System
1.	In star connected system there is common point known as neutral 'n' or star point. It can be earthed.	There is no neutral point in delta connected system
2.	In star connected system we get 3-phase, three wire system and also 3-phase, 4 wire system is taken out.	Only 3-phase, 3 wire system is possible in delta connected system
3.	Line voltage $V_L = \sqrt{3}\, V_{ph}$ or, $V_{ph} = \frac{1}{\sqrt{3}} V_L$	Line voltage = Phase voltage $V_L = V_{ph}$
4.	Line current = Phase current $I_L = I_{ph}$	Line current $I_L = \sqrt{3}\, I_{ph}$ $I_{ph} = \frac{1}{\sqrt{3}} I_L$

1. For a three phase star connected system with a line voltage of 400 V, calculate the value of phase voltage.

A. 400 V

B. 692.8 V

C. 331.33 V

D. 230.94 V

ANSWER: D

2. Phase voltages of the windings of a 3-wire star-connected machine are 2 kV. Line voltages of the machine is

a. 1732.05 V
b. 1154.70 V
c. 2309.4 V
d. 3464.10 V

ANSWER: (d)

3. For a three phase delta connected system with a line voltage of 400 V and line current is 100 A, calculate the value of phase voltage and phase current.

A. 400 V, 173.2 A
B. 400 V, 57.7 A
C. 230.9 V, 100 A
D. 230.9 V, 57.7 A

ANSWER:B

In a series RC circuit, 12 V is measured across the resistor and 15 V is measured across the capacitor. The source voltage is

(A) 3 V
(B) 27 V
(C) 19.2 V
(D) 12 V

Krichoff's voltage law is based on

(A) Law of conservation of energy
(B) Law of conservation of charge
(C) Faraday's law of electromagnetic induction
(D) Fleming's right hand rule

Superposition theorem is applied to

(A) Only linear circuit
(B) Only non linear circuit
(C) Either on linear or non linear circuit
(D) Only on DC circuit

Three equal resistances of 3 Ω are connected in star. What is the resistance in one of the arms in an equivalent delta circuit?

(A) 10 Ω
(B) 27Ω
(C) 9 Ω
(D) 3Ω

In a certain series RC circuit, the true power is 2W, and the reactive power is 3.5 VAR. What is the apparent power?

(A) 3.5 VA
(B) 2.03 VA
(C) 4.03 VA
(D) 3 VA

A power factor of '0' indicates

(A) Purely resistive element
(B) Purely inductive element
(C) Combination of both (A) and (B)
(D) Purely capacitive element and resistive element

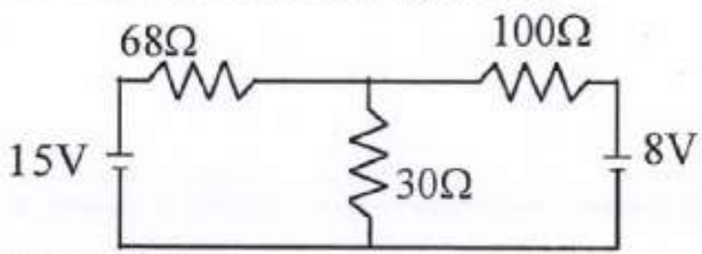
A 10Ω resistor is connected in parallel with a 15Ω resistor and the combination in series with a 12Ω resistor. The equivalent resistance of the circuit is

(A) 37Ω
(B) 18Ω
(C) 27Ω
(D) 4Ω

A current is said to be direct current when its

(A) Magnitude remains constant with time
(B) Magnitude changes with time
(C) Direction changes with time
(D) Magnitude and direction changes with time

What is the current through 30Ω?



(A) 3.19 A
(B) 319 mA
(C) 1.73 mA
(D) 173 mA

When an additional resistor is connected across an existing parallel circuit, the total resistance

(A) Remains the same
(B) Decreases by the value of the added resistor
(C) Increases by the value of the added resistor
(D) Decreases

When a fourth resistor is connected in series with three resistor, the total resistance

(A) Increase by one-fourth
(B) Increases
(C) Decreases
(D) Remains the same

A circuit consists of three resistors in parallel, when one resistor is removed the circuit current,

(A) Decreases
(B) Increases by one third
(C) Decreases by one-third
(D) Decrease by the amount of current through the removed resistor.

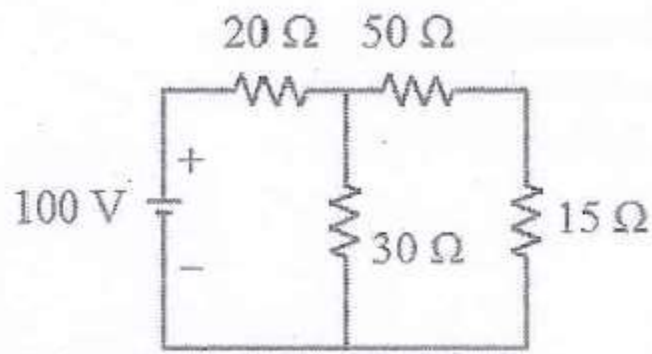
An algebraic sum of all the element voltages in a mesh is equal to

(A) The total of the voltage drops
(B) The source voltage
(C) Zero
(D) The total of the source voltage and the voltage drops.

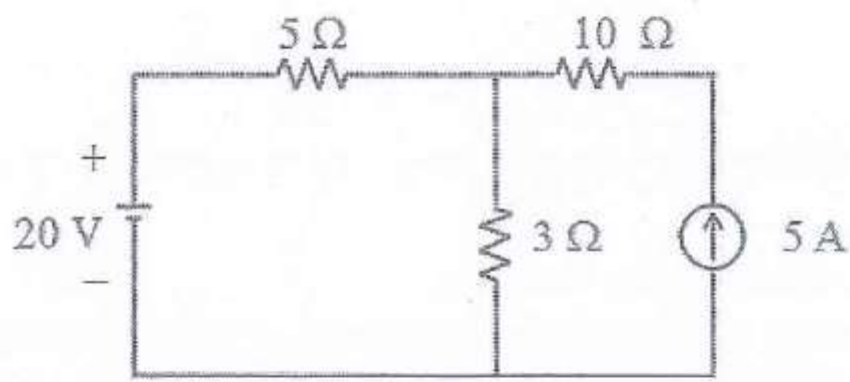
A 24V DC supply is applied across a voltage divider consisting of two 68kΩ resistors. The unknown output voltage is

(A) 12V
(B) 24V
(C) 0V
(D) 6V

Using KVL, determine total current drawn from the source and also current across 15 Ω resistor in the following circuit.



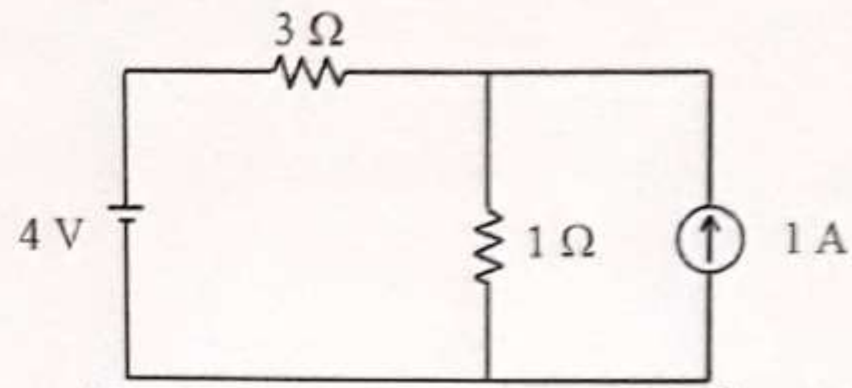
Identify the current through 3Ω resistor in the circuit shown below, using superposition theorem.



A resistance of 100 Ω is connected in series with a 50μF capacitor to a supply at 200 V, 50 Hz. Determine the

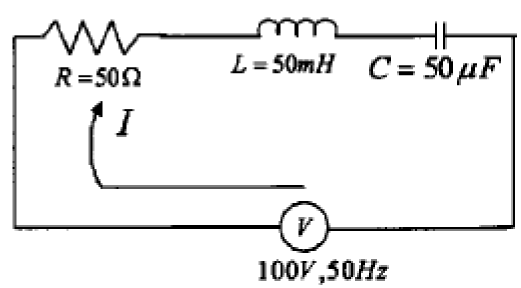
(i) Current
(ii) Power factor
(iii) Voltage across resistor and capacitor

For the circuit shown, find the voltage across the 1 ohm resistors.

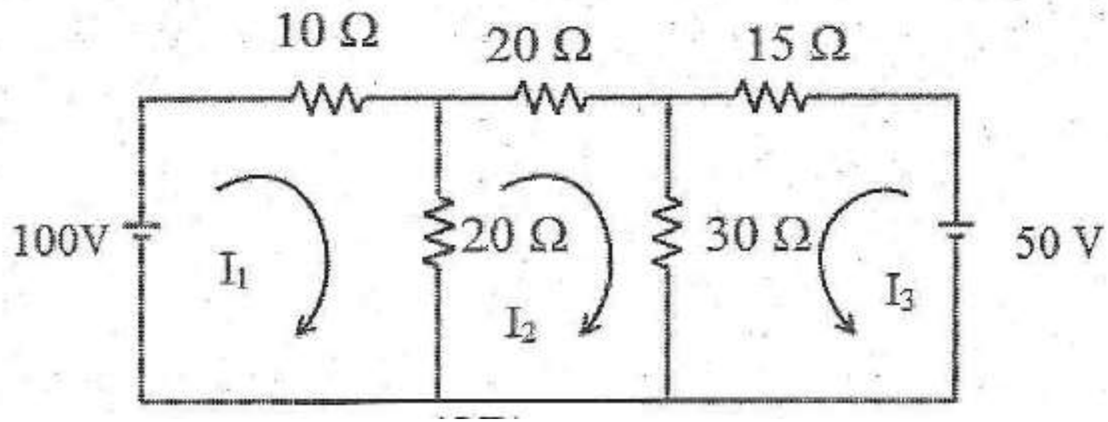


Two coils when connected in series have a resistance of 18Ω and when connected in parallel have a resistance of 4Ω. Find the resistance of each coil.

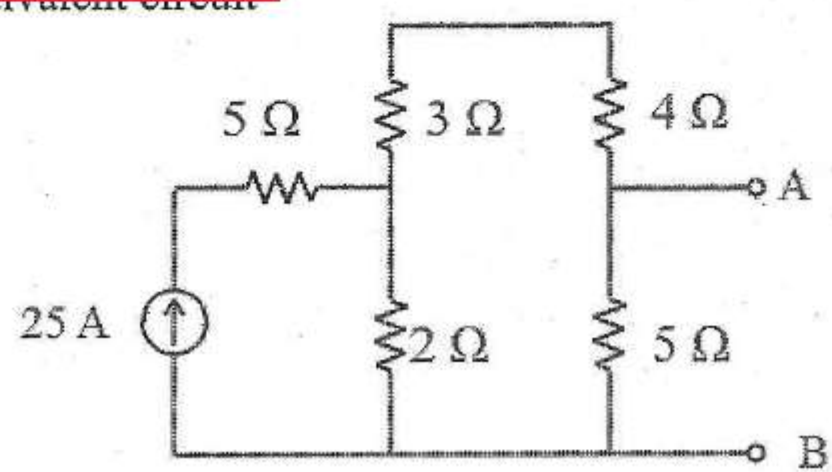
Find the current I in the circuit shown.



Using mesh analysis, determine mesh current in each loop given in the below circuit.



Simplify the given circuit into
(i) Thevenin's equivalent circuit and
~~(ii) Norton's equivalent circuit~~



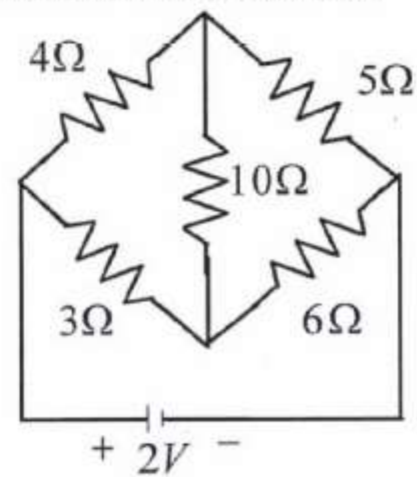
Analyze that the RMS and Average values of sinusoidal alternating current are $\frac{I_m}{\sqrt{2}}$ and $\frac{2I_m}{\pi}$.

Derive the average value, RMS value, ~~peak factor and form factor~~ for a full-wave rectified sinusoidal waveform. (8 Marks)

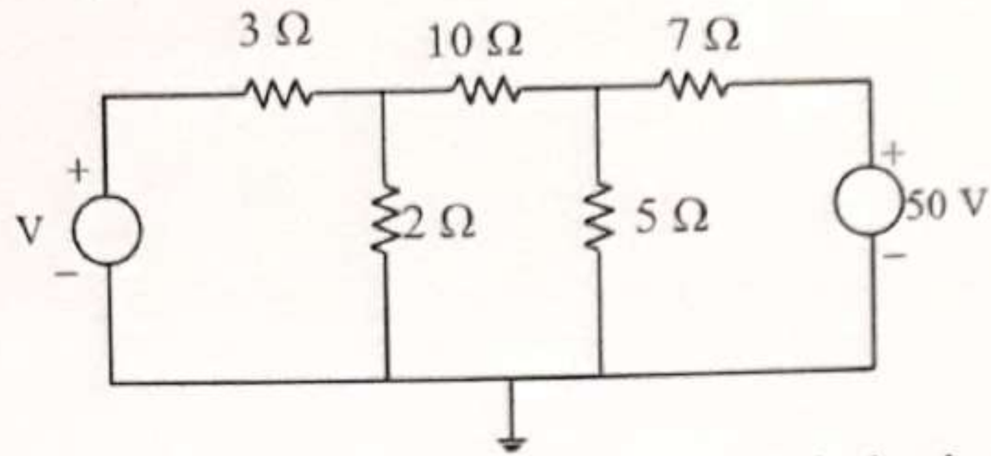
Derive the average value, RMS value, ~~form factor and peak factor~~ for the half-wave and full-wave rectified sine wave.

Derive the average and RMS value of the full wave rectified sine wave voltage.

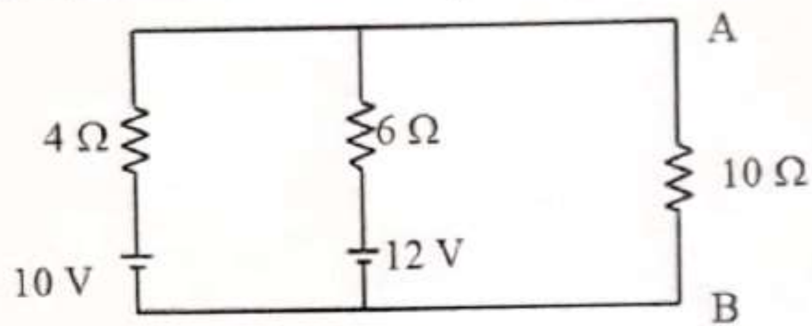
Determine the current through all the branches in circuit



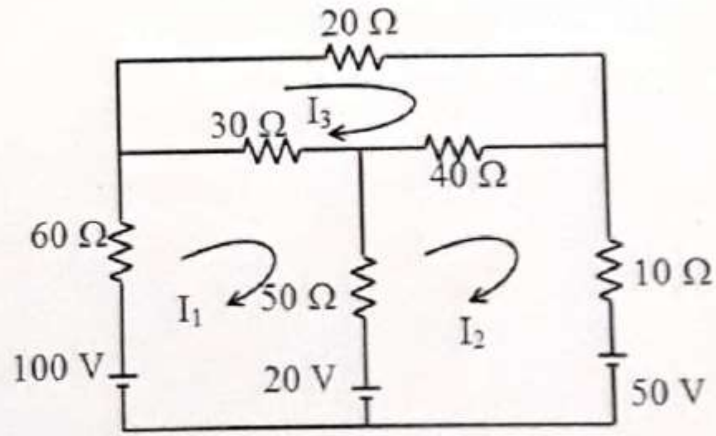
a.i. Find the voltage 'V' in the circuit shown below which makes the current in the 10 Ω resistor zero by using nodal analysis.



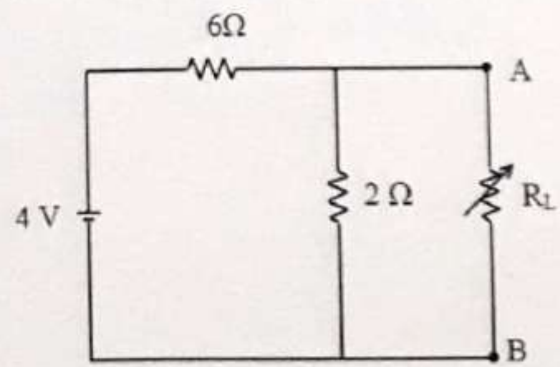
ii. Using Thevenin's theorem, find the current through 10Ω resistor in the circuit shown below.



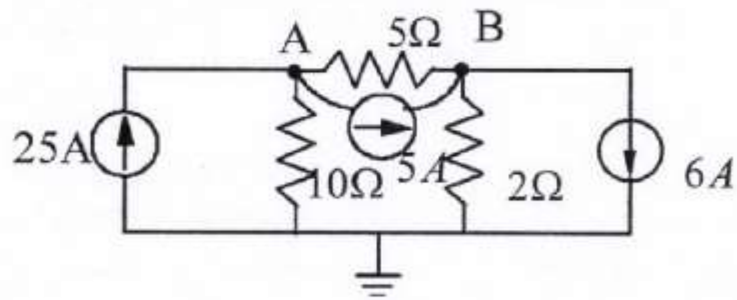
b.i. Find the current that flows through the 50 Ω resistor for the circuit shown below using mesh analysis.



ii. Determine the value of load resistance R_L when it is dissipating maximum power. Also find the maximum power dissipated in the load resistance for the circuit given below.



Compute the voltages at node A and B for the circuit shown below. (8 Marks)



Find the impedance, current and power factor of the following series circuit and draw the corresponding phasor diagrams

(i) R and L

(ii) R and C.

In each case the applied voltage is 200 volts and the frequency is 50Hz. Further $R = 10\Omega,\ L = 50mH\ \ and\, C = 100\mu F$

In an ac circuit, resistor R and inductor L are connected in series. Voltage and current equations are given as $e(t) = 200\sin 314t$ and $i(t) = 20\sin(314t - 30°)$ calculate (i) Rms value of the voltage and current. (ii) Frequency (iii) Power factor (iv) Power (v)Values of R and L.

A coil of resistance 10Ω and inductance 0.1H is connected in series with a 150μF capacitor across 200V, 50Hz supply. Calculate

(i) Inductive reactance, capacitive reactance, impedance, current and power factor

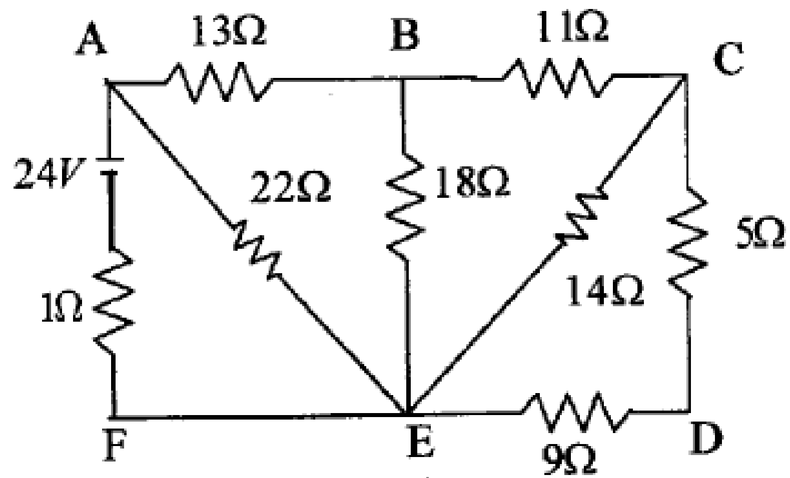
(ii) Voltage across the coil and capacitor

An electrical network is arranged as shown below. Find

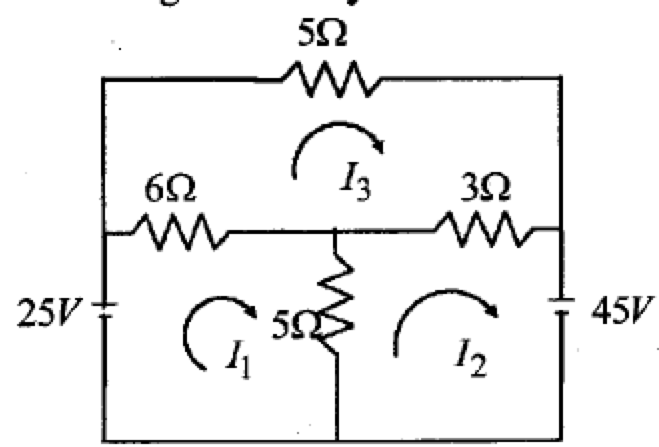
(i) Current in branch AF

(ii) Power absorbed in branch BE

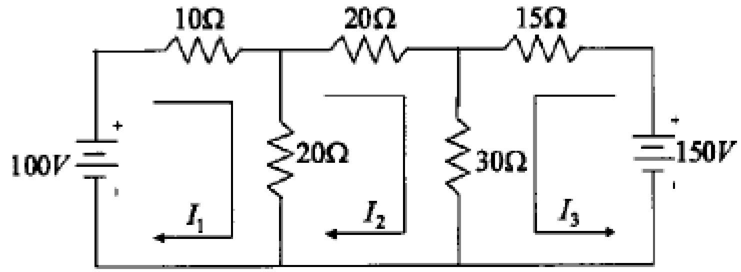
(iii) Potential difference across the branch CD



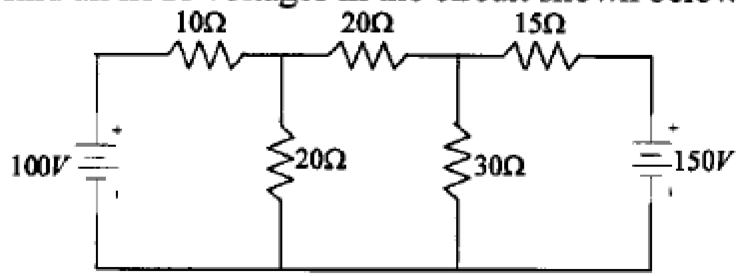
Find the current in 5Ω resistor using Mesh analysis in the circuit shown below.



Using mesh analysis, find mesh currents in the circuit shown below.



Using nodal analysis, find all node voltages in the circuit shown below.



For the circuit shown below, find currents I_{AB}, I_{AC}, I_{CD} and I_{EF} using mesh current method.